stochastics and probability

Lecture 3

Dr. Johannes Pahlke

accept-reject sampling

accept-reject sampling

want:

accept-reject sampling

want:

X with PDF $f_X(x)$

accept-reject sampling

want:

X with PDF $f_X(x)$

use:

accept-reject sampling

want:

X with PDF $f_X(x)$

use:

$f_X(x) \leq \alpha$

accept-reject sampling

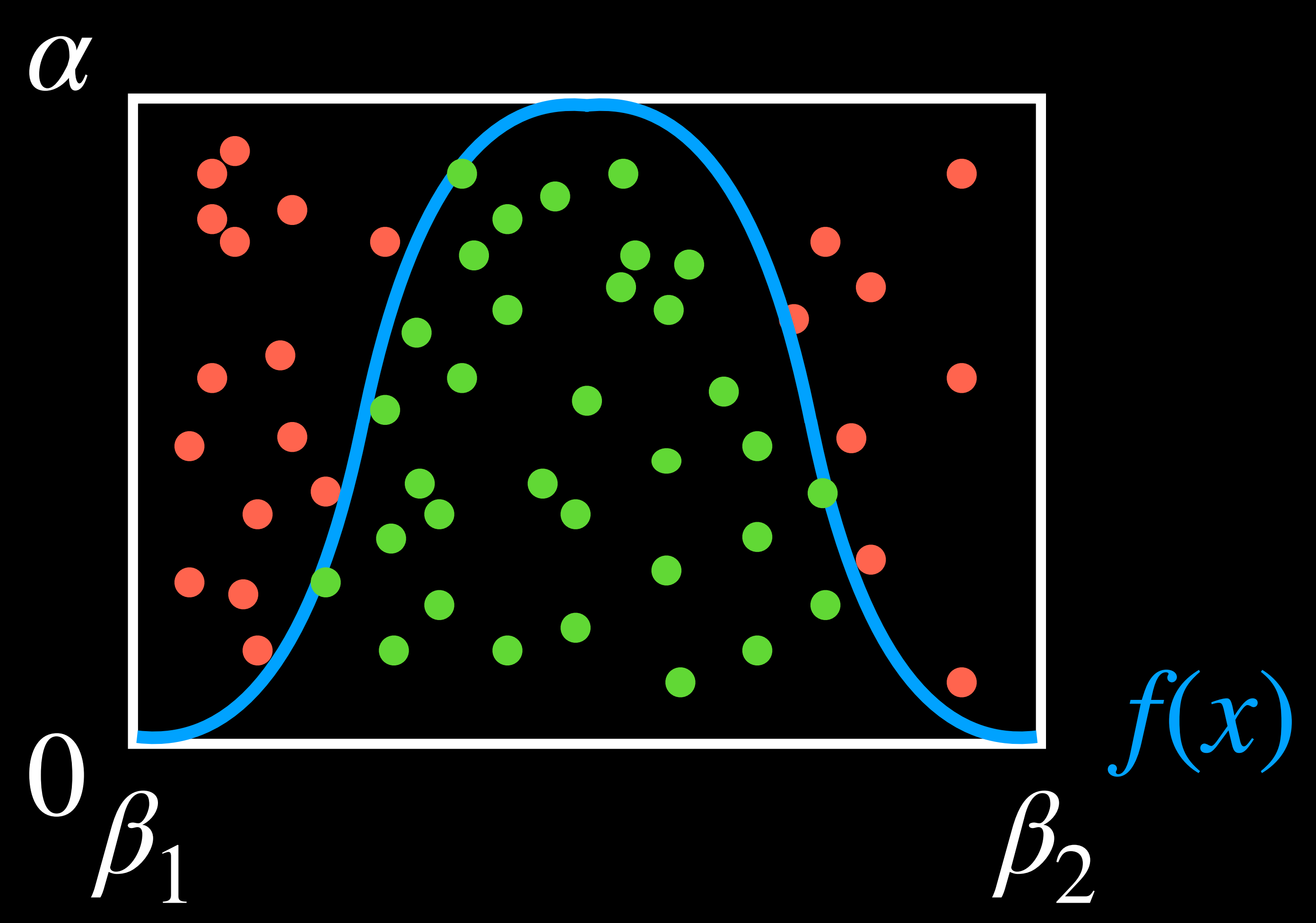
want:

X with PDF $f_X(x)$

use:

$f_X(x) \leq \alpha \qquad Range(X) \subseteq [\beta_1, \beta_2]$

accept-reject sampling



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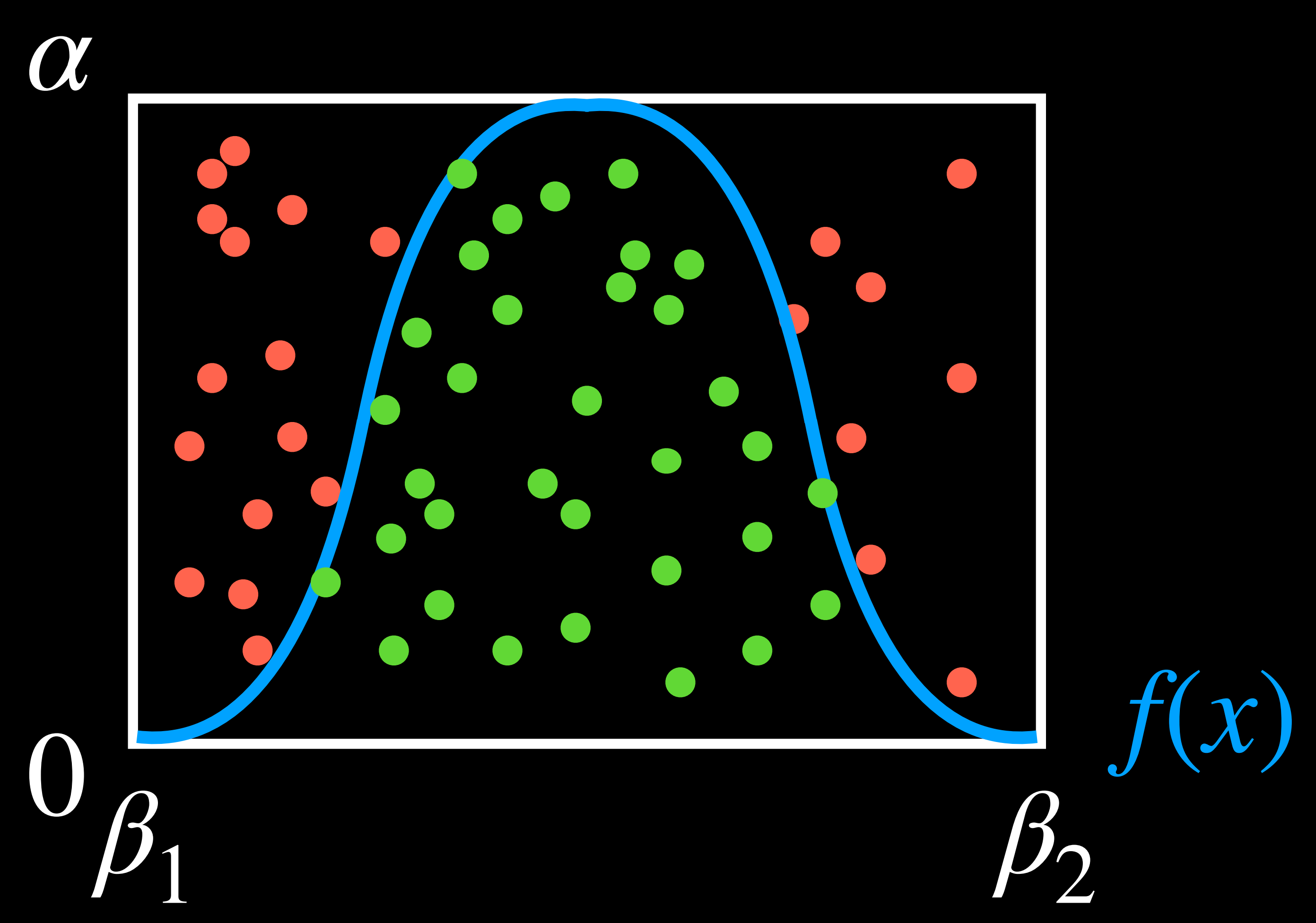

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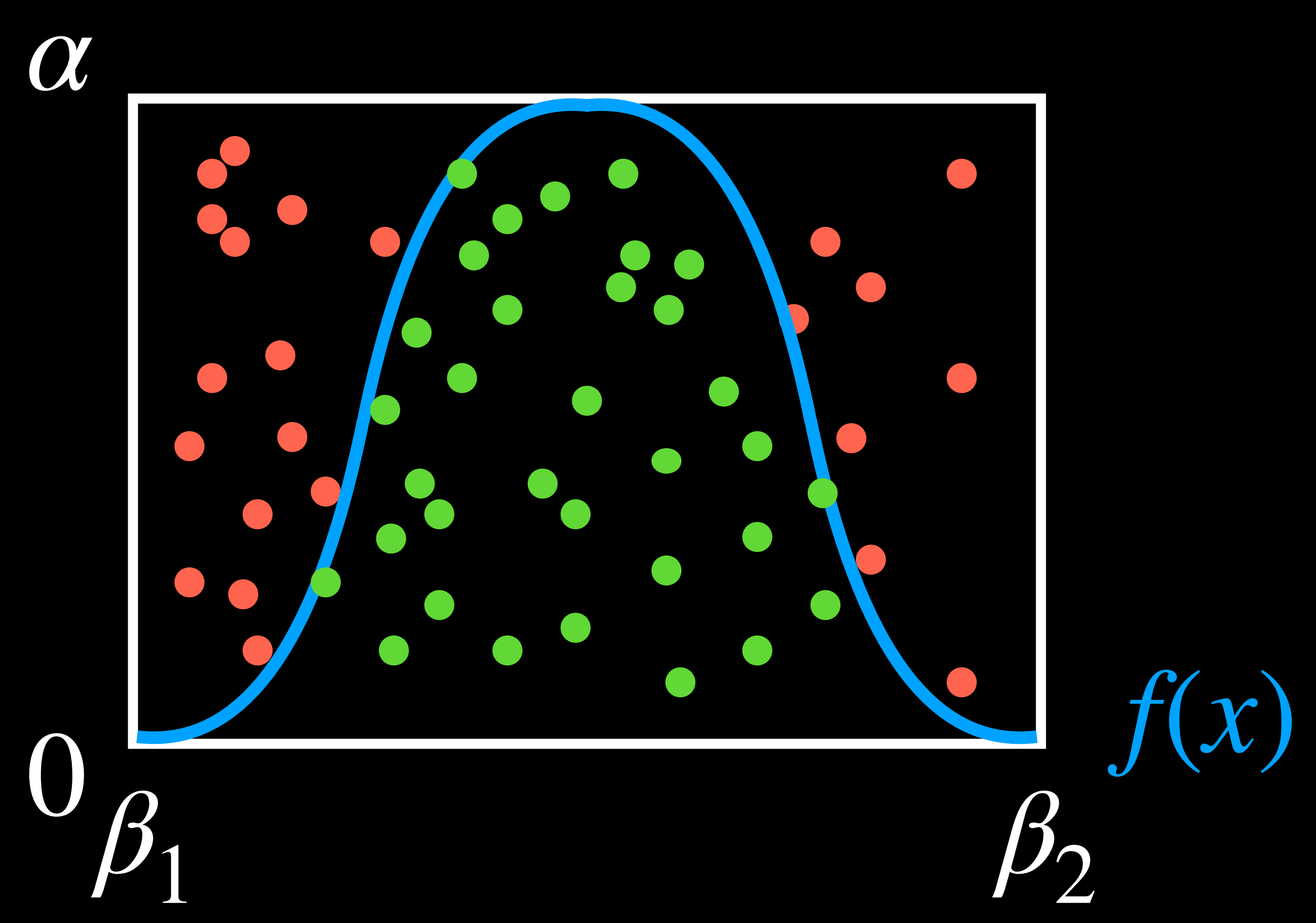

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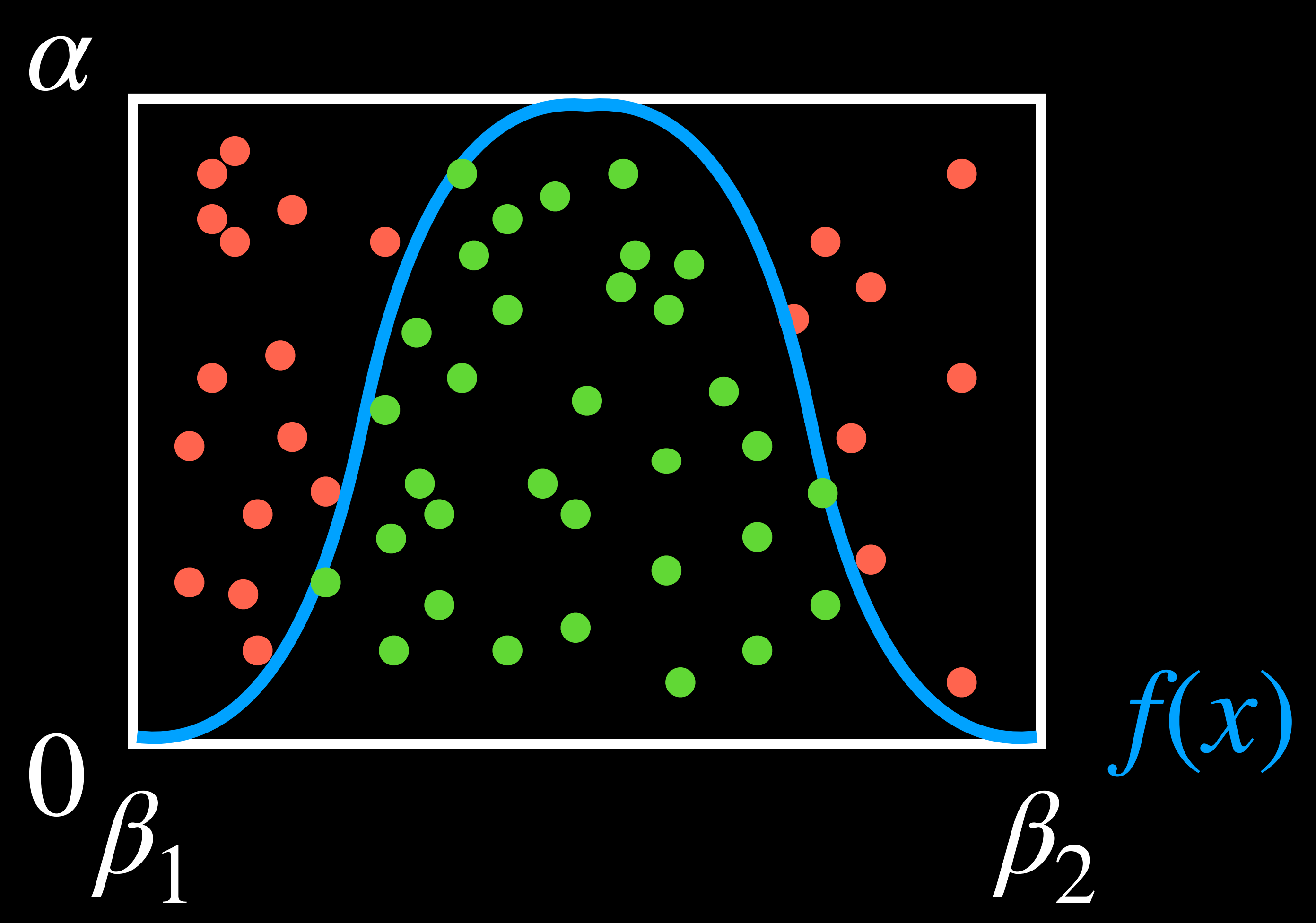
1.) draw $U_1 \sim \mathcal{U}(\beta_1, \beta_2), \quad U_2 \sim \mathcal{U}(0,\alpha)$

accept-reject sampling



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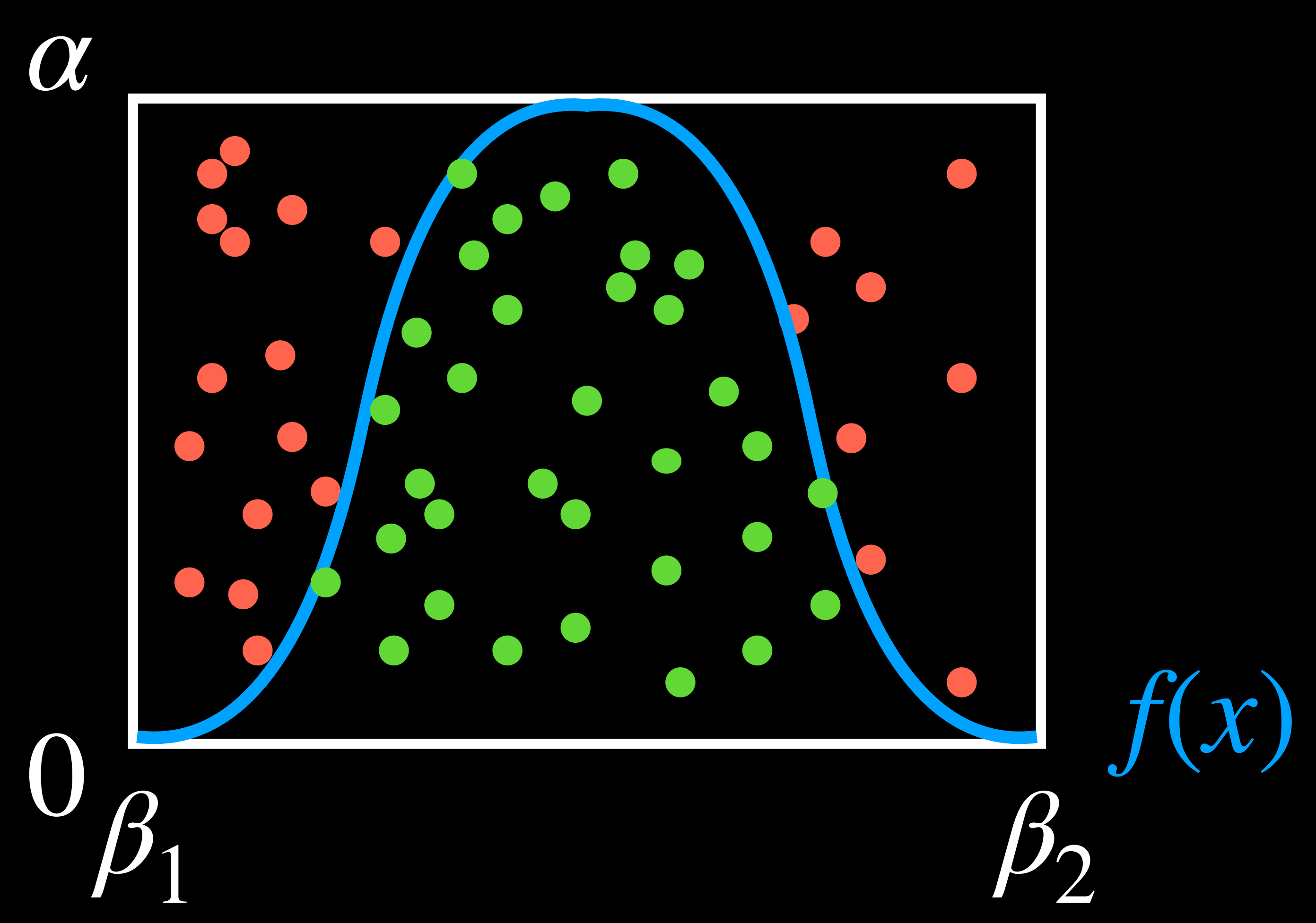
accept-reject sampling



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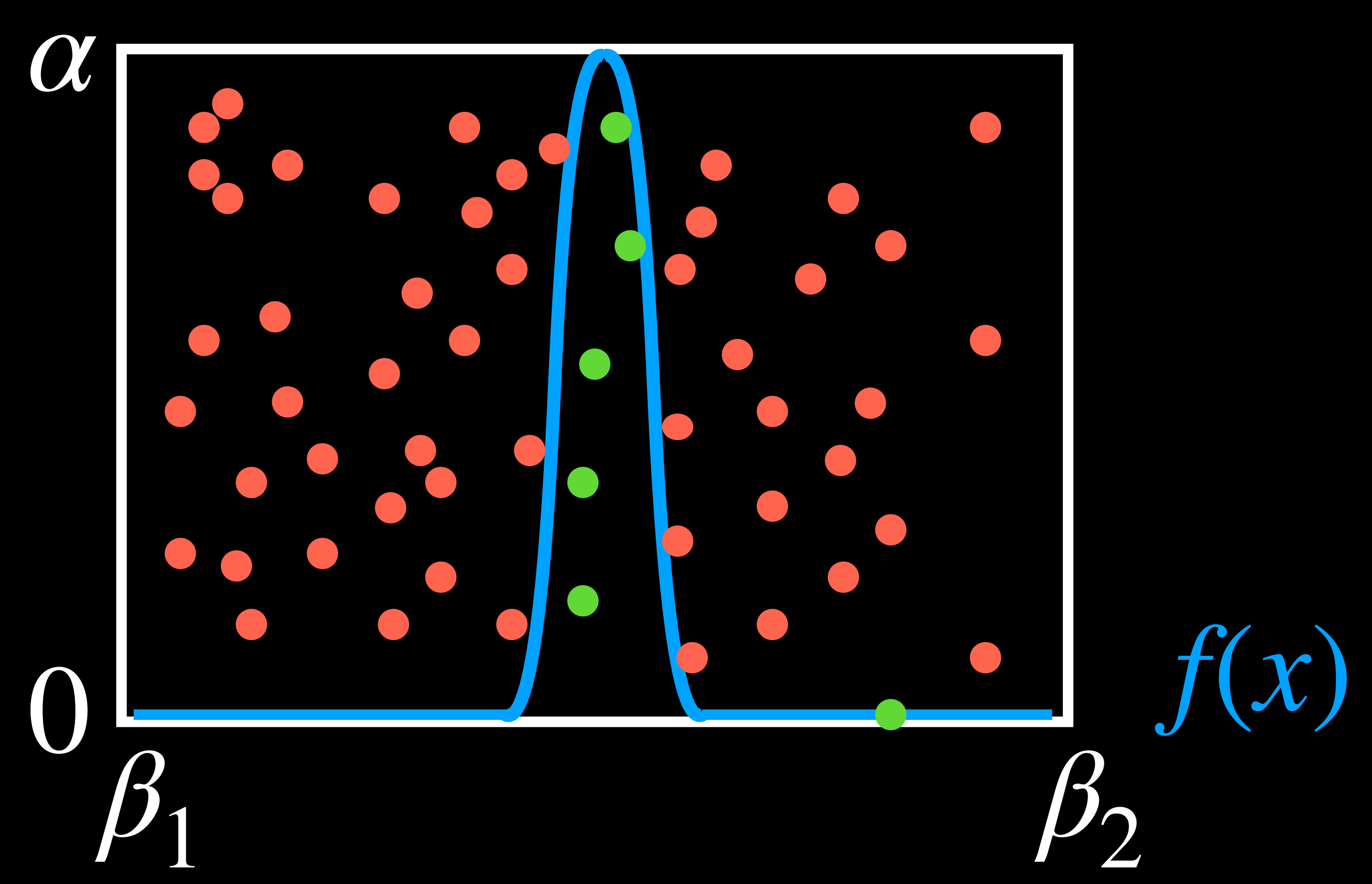

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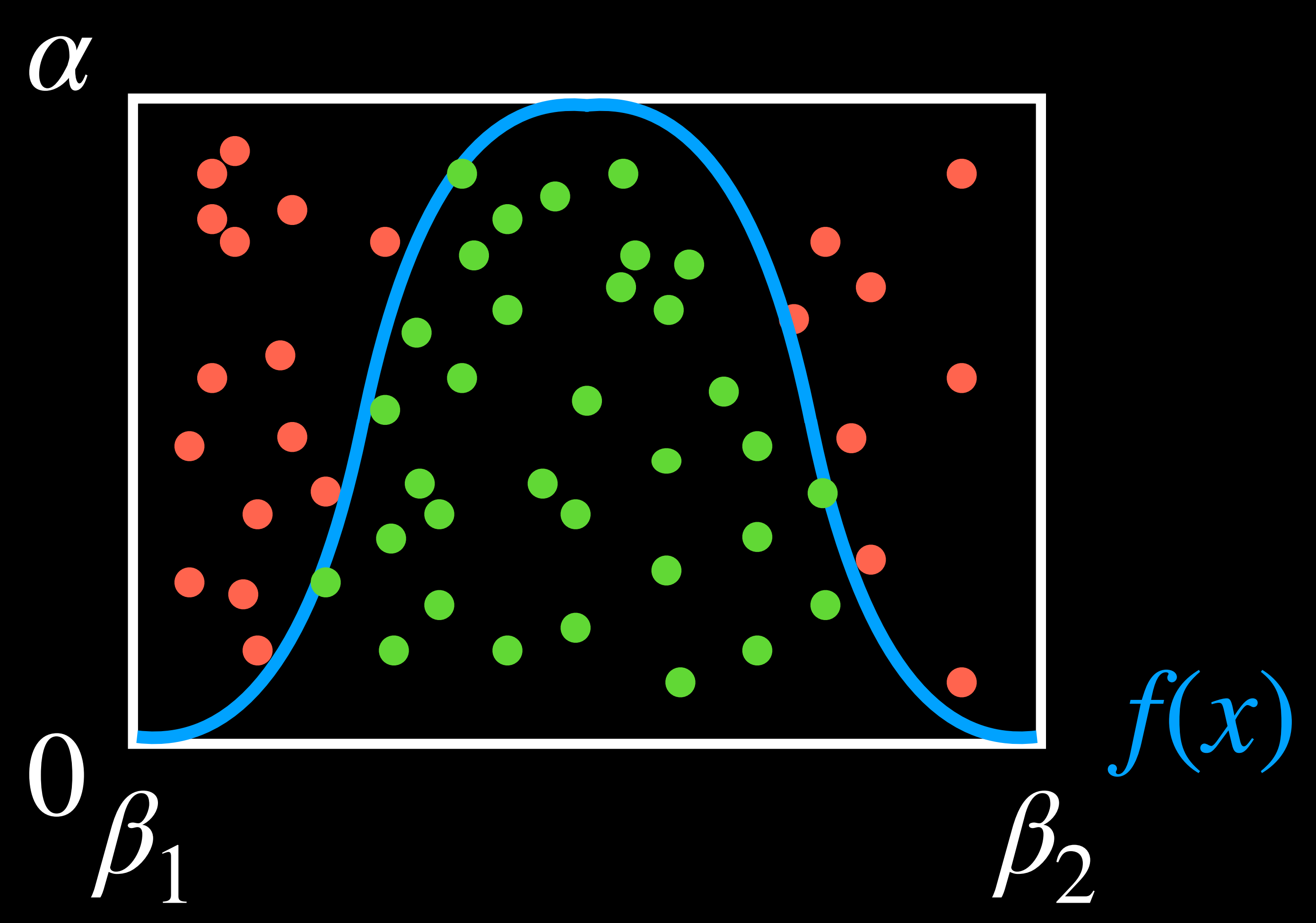

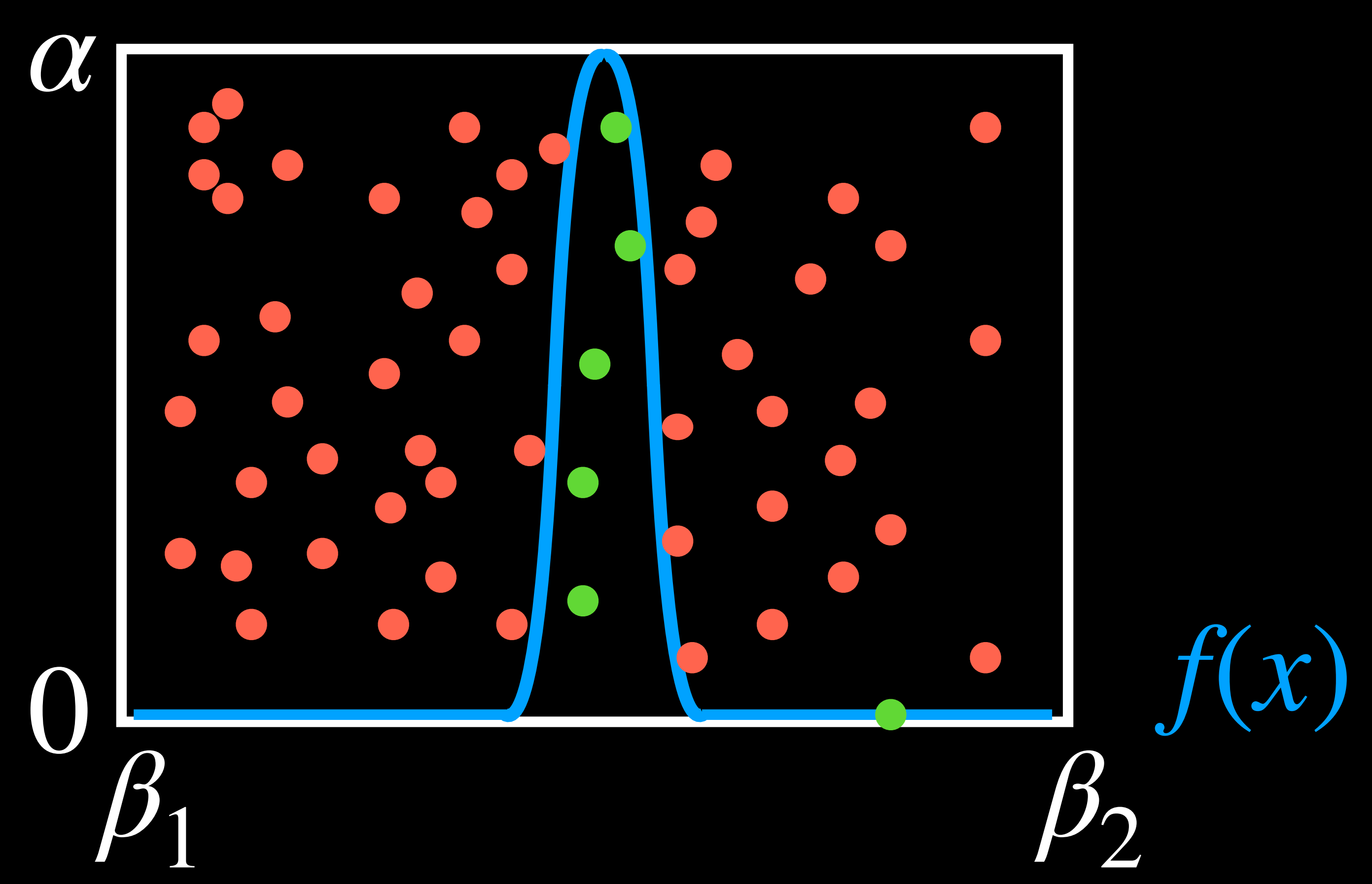

inefficient:

accept-reject sampling

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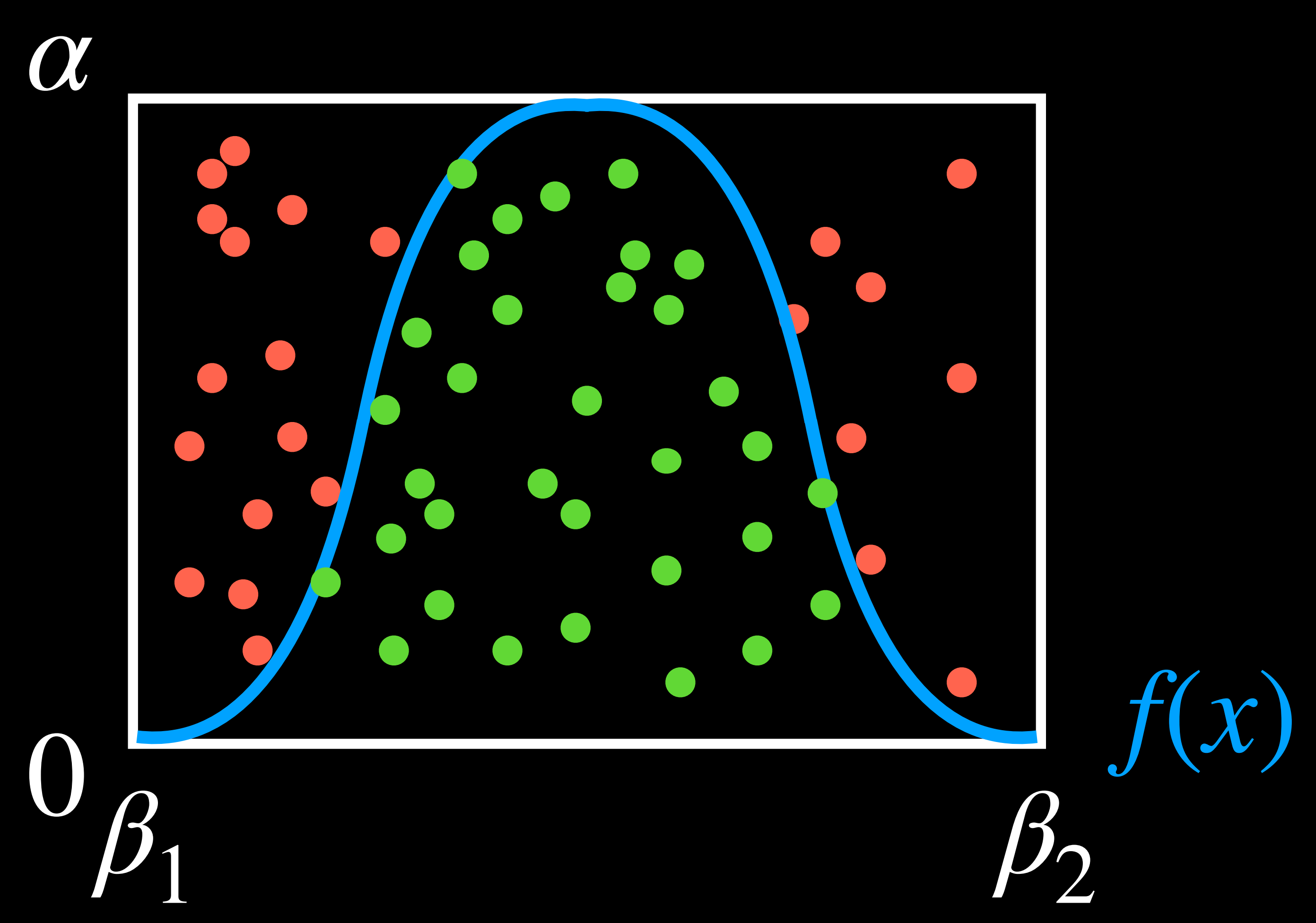

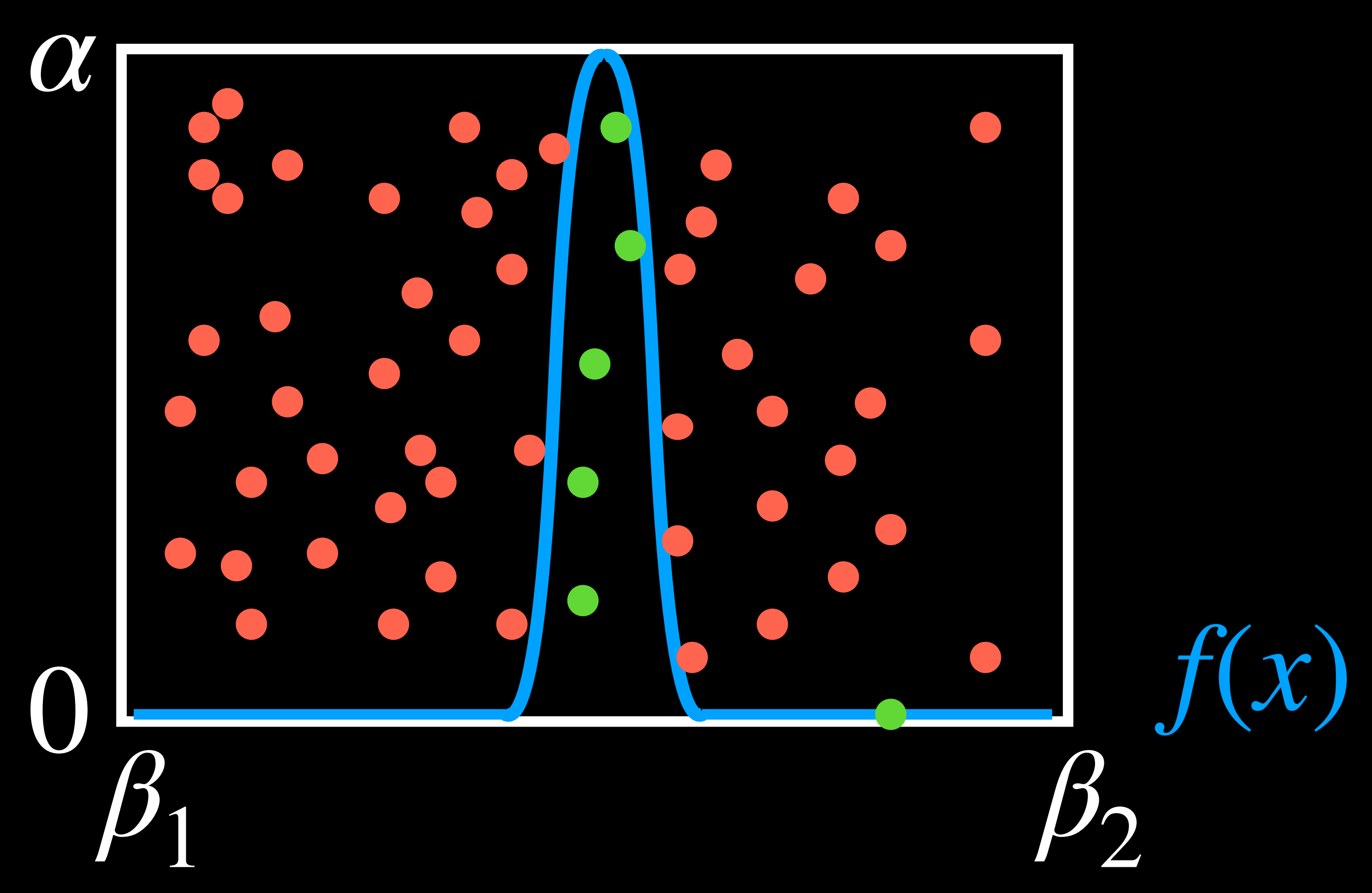

inefficient:

- peaky densities

accept-reject sampling

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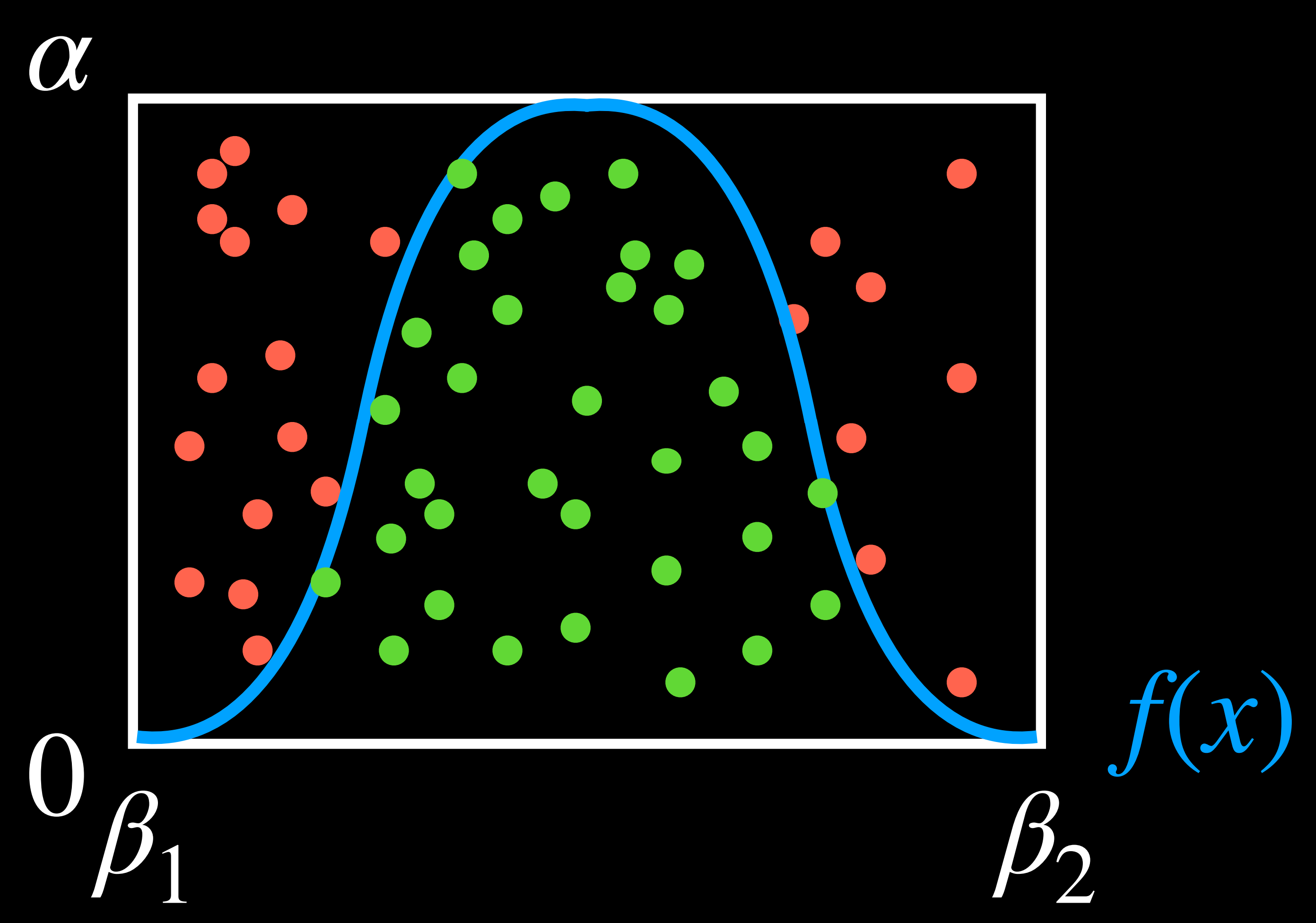

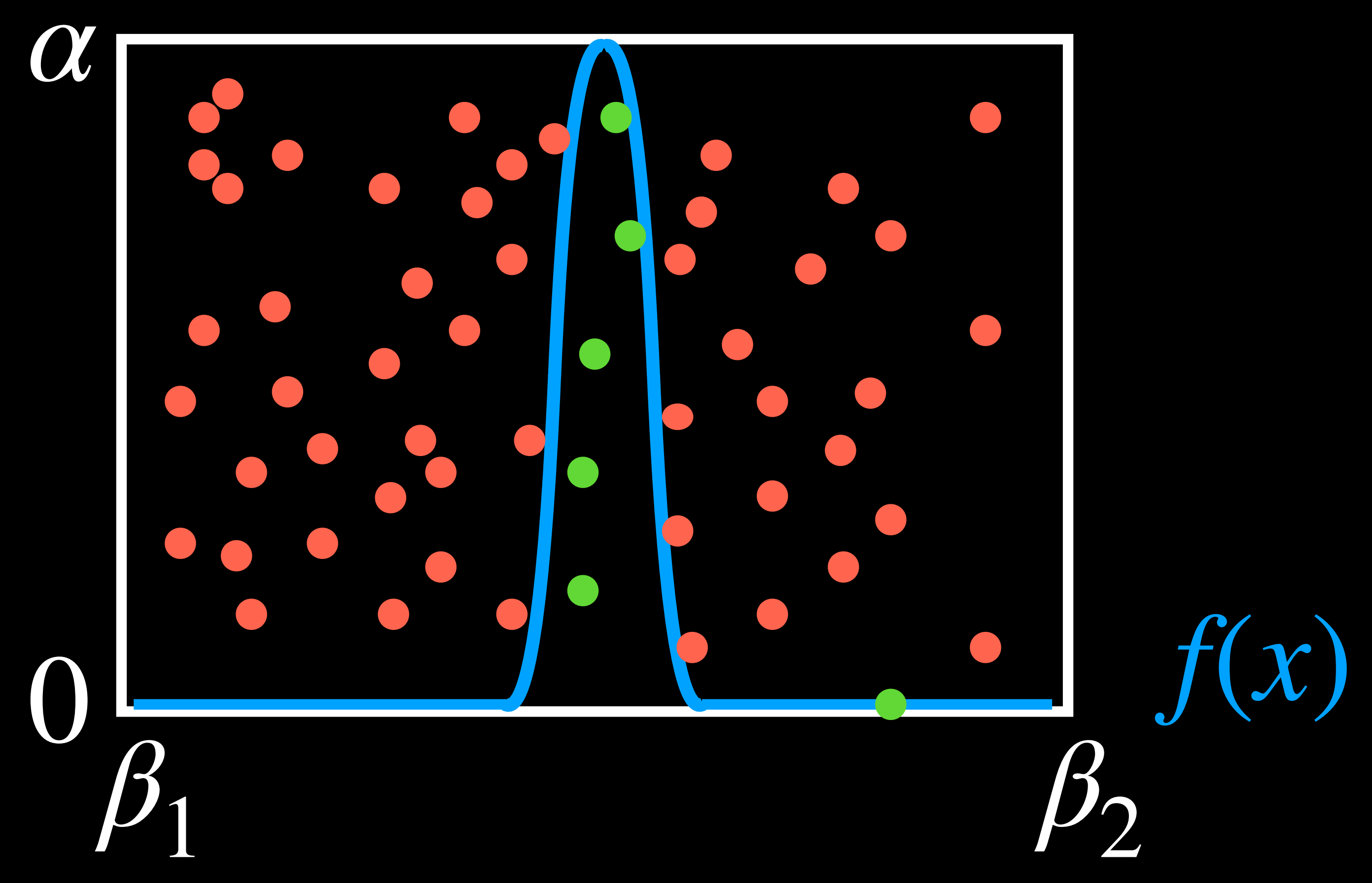

inefficient:

- peaky densities
- high dimensions

accept-reject sampling

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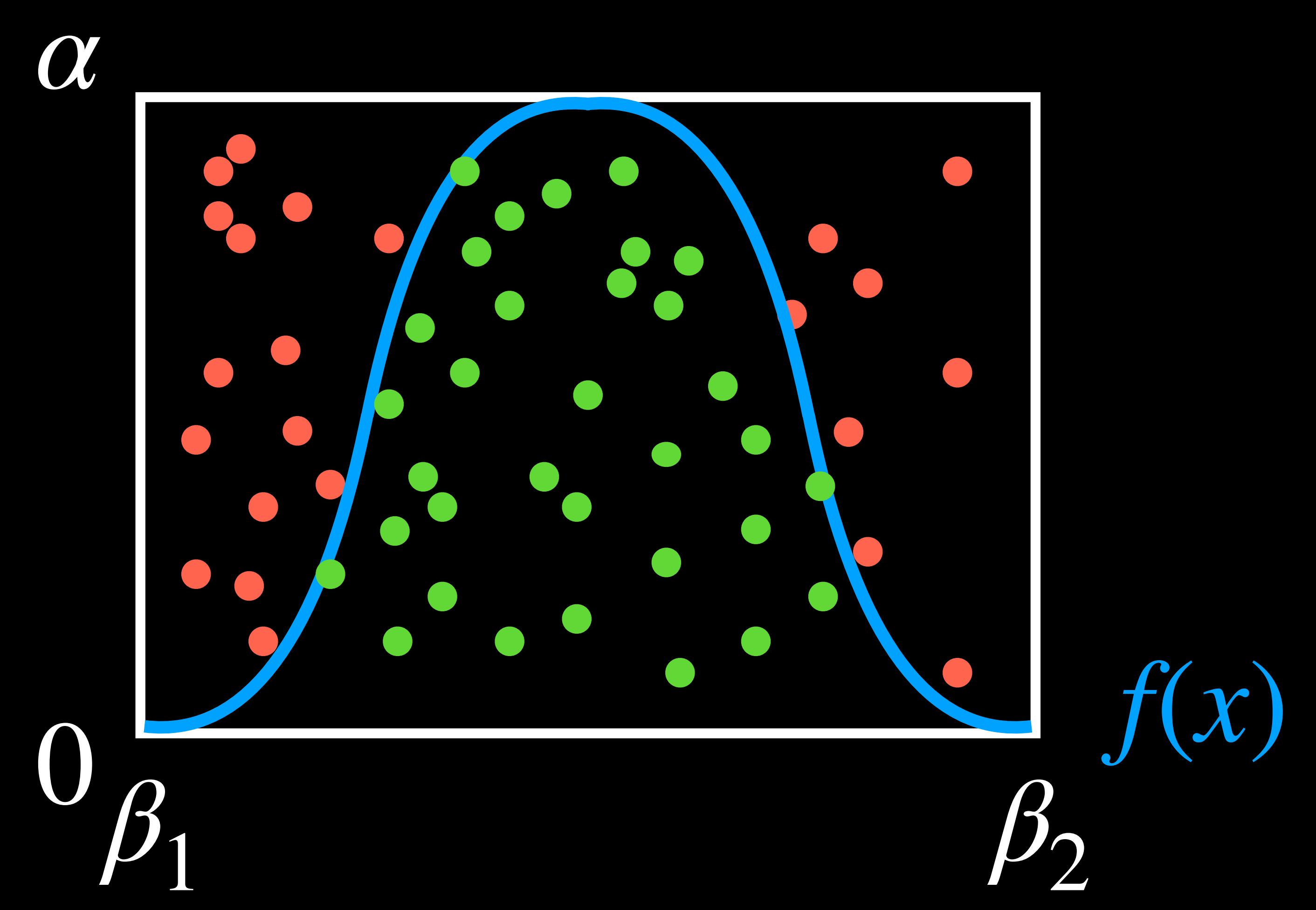

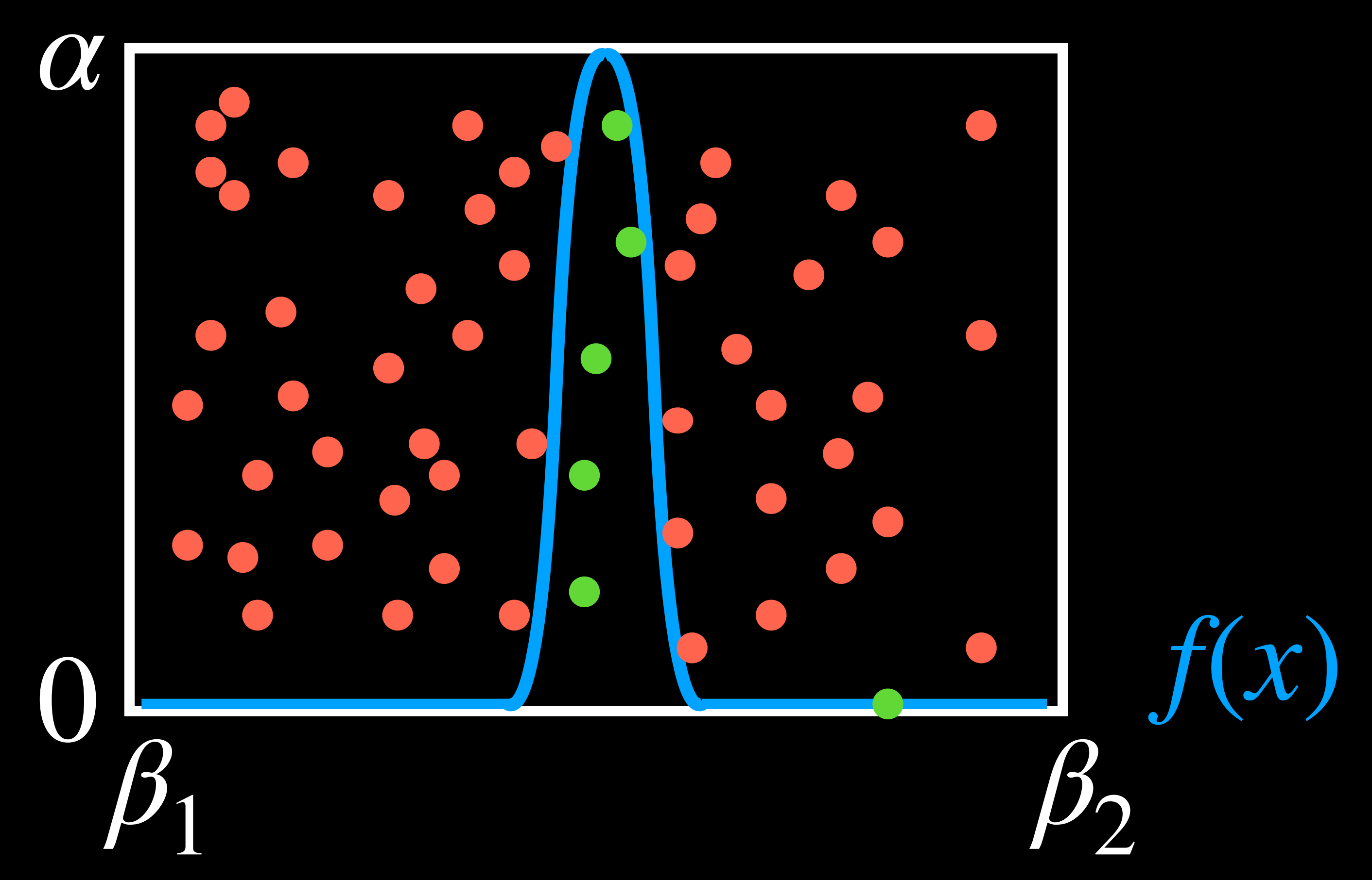

inefficient:

- peaky densities
- high dimensions
- long tails/infinite Range

accept-reject sampling

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X with PDF $f_X(x)$

use:

accept-reject sampling

want:

X with PDF $f_X(x)$

use:

$G \sim \mathscr{G}$ with PDF $g_G(x)$

accept-reject sampling

want:

X with PDF $f_X(x)$

use:

$G \sim \mathcal{G}$ with PDF $g_G(x)$ and $f_X(x) \leq c\, g_G(x)$

accept-reject sampling

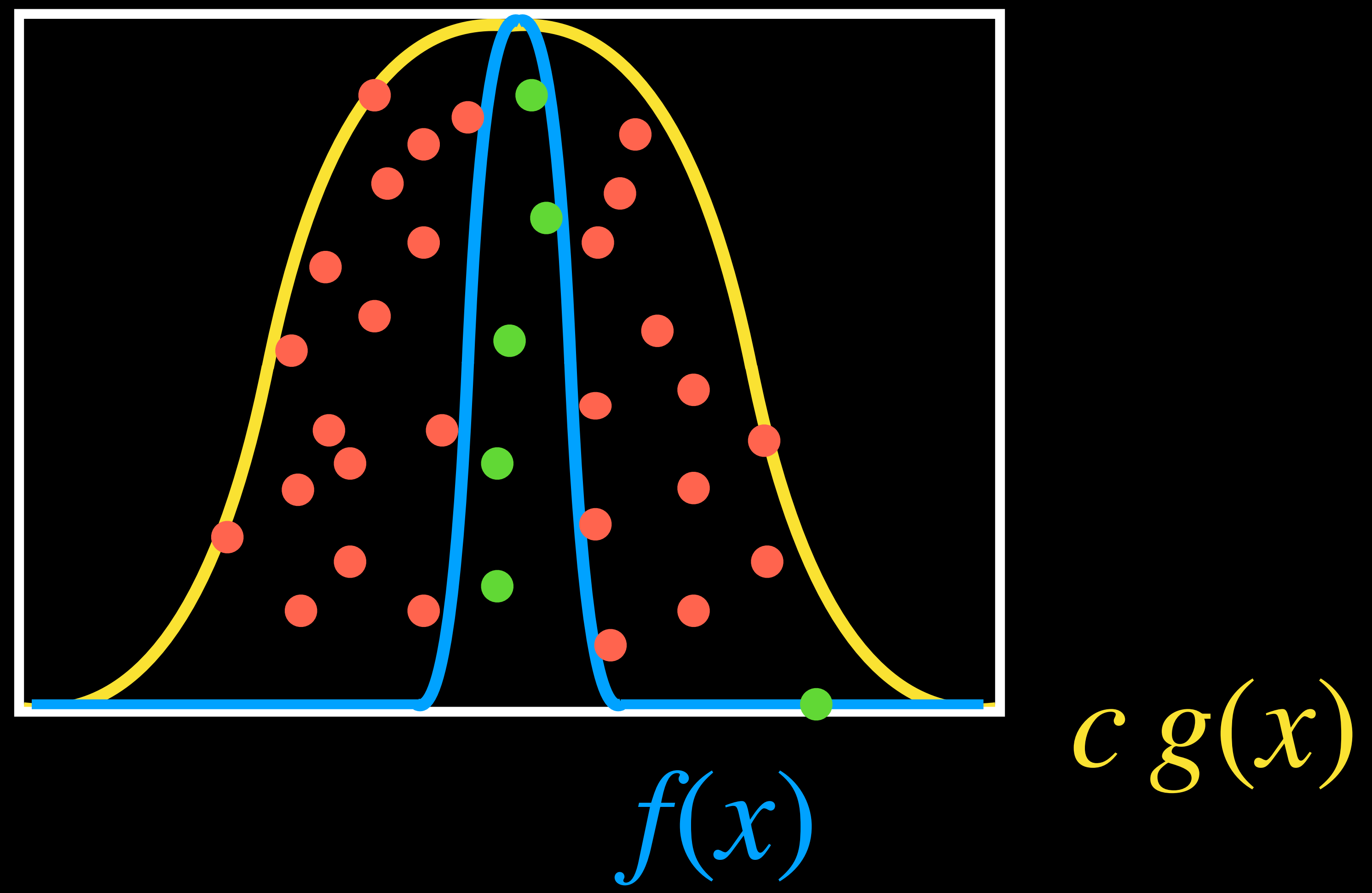
want:

X with PDF $f_X(x)$

use:

$G \sim \mathscr{G}$ with PDF $g_G(x)$ and $f_X(x) \leq c\, g_G(x)$ for all x

accept-reject sampling



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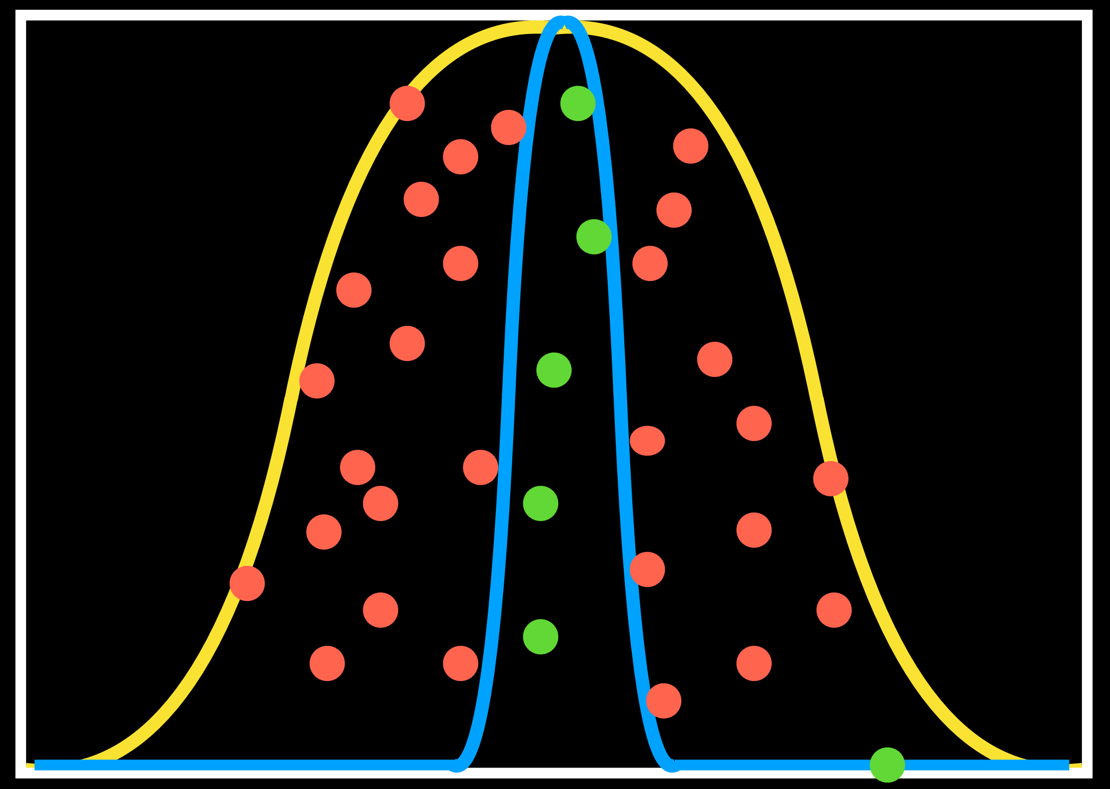

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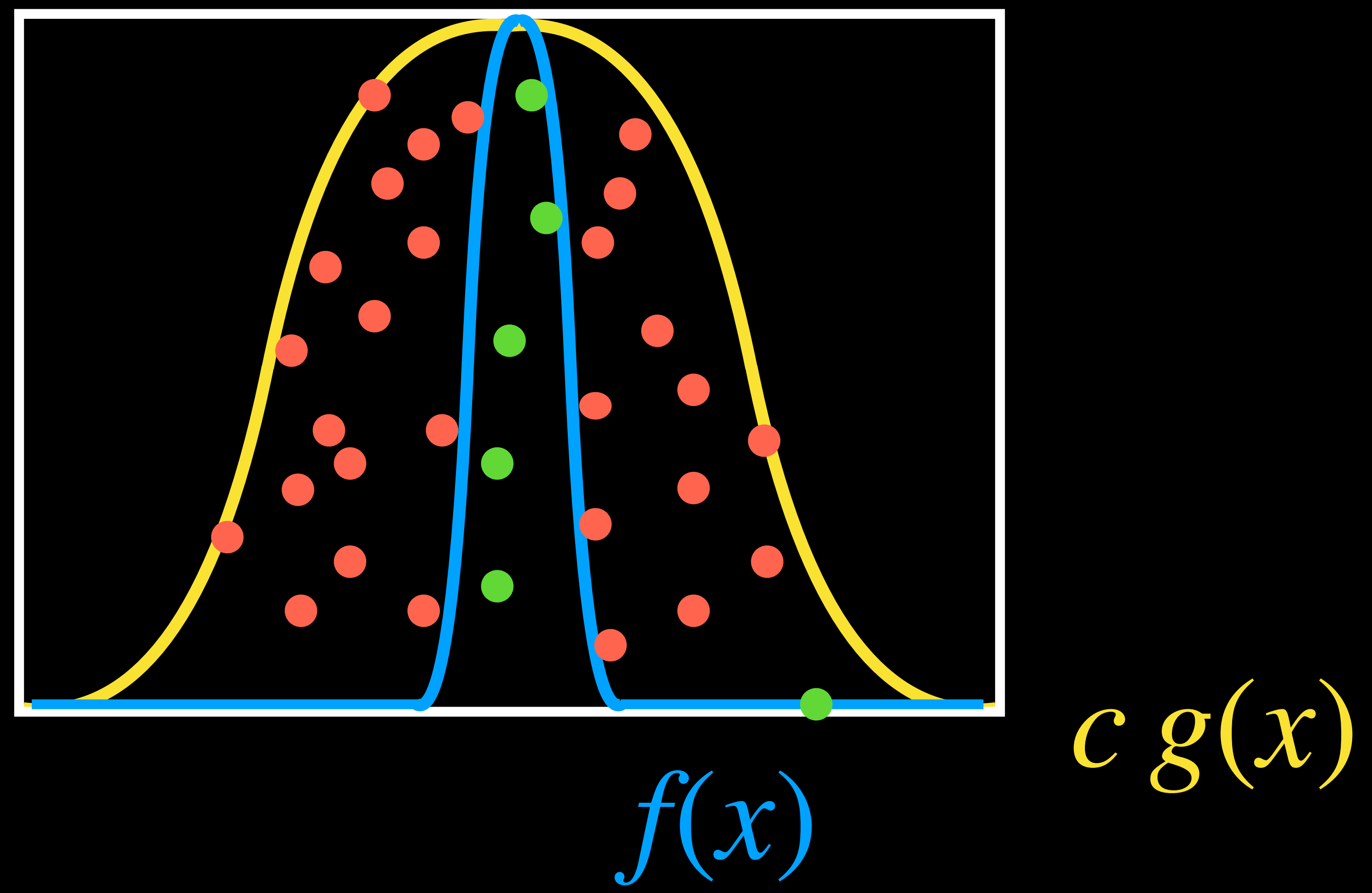

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algorithm:

1.) draw G from $\mathscr{G}$ and U from $\mathscr{U}(0,1)$

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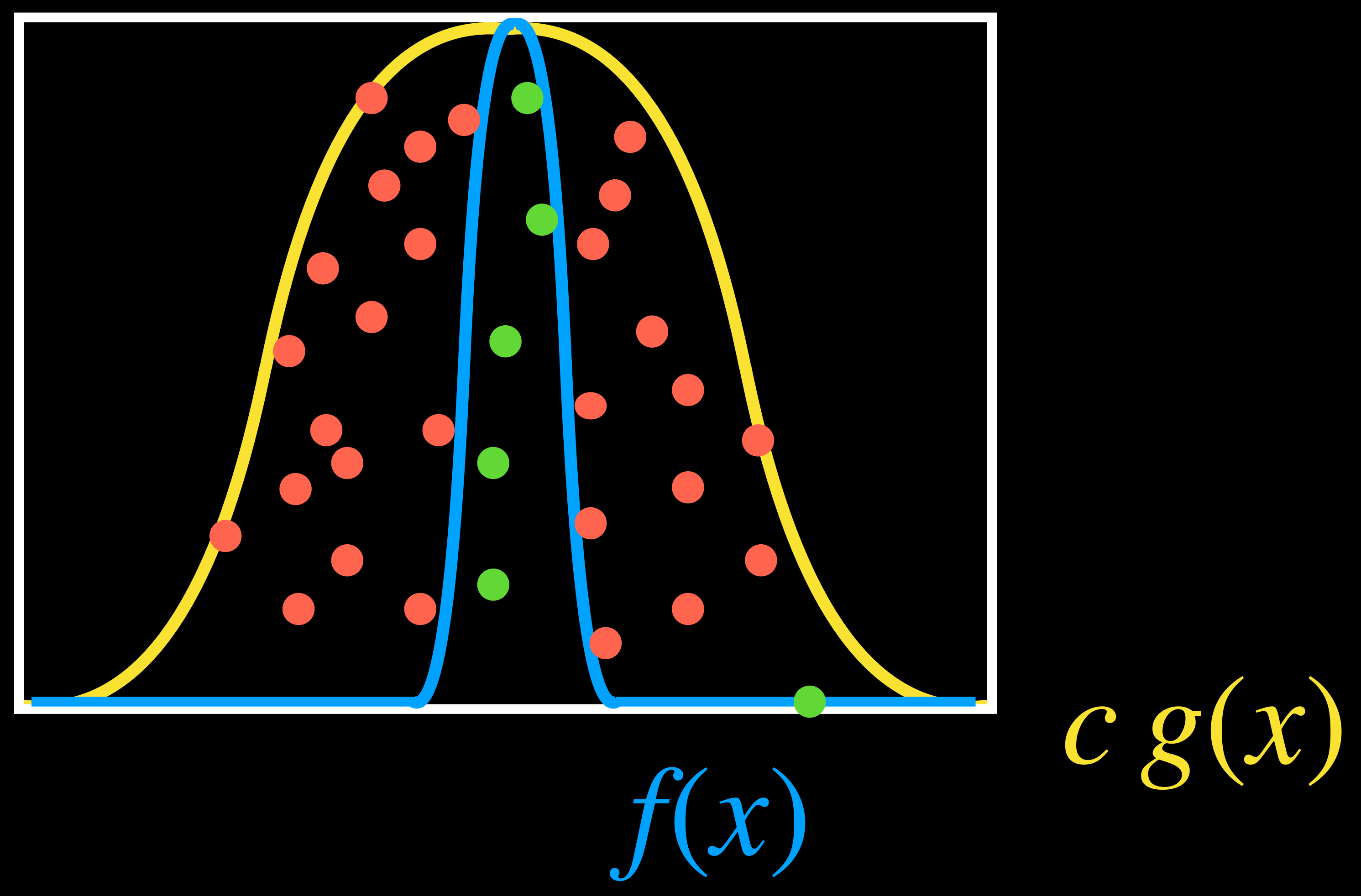

want:

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use:

$G \sim \mathscr{G}$ with PDF $g_G(x)$ and $f_X(x) \le c\, g_G(x)$ for all x

algorithm:

1.) draw G from $\mathscr{G}$ and U from $\mathscr{U}(0,1)$

2.) if $U \le \dfrac{f_X(G)}{c\, g_G(G)}$, then $X = G$, else go to 1.)

discrete-time stochastic processes

discrete-time stochastic processes

$n \in \mathbb{N}$

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

$X_0, X_1, X_2, \ldots$

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

$X_0, X_1, X_2, \dots$ "sequence of random variables"

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

$X_0, X_1, X_2, \ldots$ "sequence of random variables"

$\neq x_0, x_1, x_2, \ldots$

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$\{X_n : n \in \mathbb{N}\}$

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

$X_0, X_1, X_2, \ldots$ "sequence of random variables"

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$\{X_n : n \in \mathbb{N}\}$ discrete-time stochastic process

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x_n

discrete-time stochastic processes

$n \in \mathbb{N}$	discrete Time
$X_0, X_1, X_2, \ldots$	"sequence of random variables"
$\neq x_0, x_1, x_2, \ldots$	"sequence of random numbers"
$\{X_n : n \in \mathbb{N}\}$	discrete-time stochastic process
x_n	value of random variable X_n

discrete-time stochastic processes

$n \in \mathbb{N}$ discrete Time

$X_0, X_1, X_2, \ldots$ "sequence of random variables"

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$\{X_n : n \in \mathbb{N}\}$ discrete-time stochastic process

x_n value of random variable X_n

$X_n : \Omega \to \mathbb{S}$

discrete-time stochastic processes

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$X_n : \Omega \to \mathbb{S}$

$x_n \in \mathbb{S}$

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$\{X_n : n \in \mathbb{N}\}$ discrete-time stochastic process

x_n value of random variable X_n

$X_n : \Omega \to \mathbb{S}$ state space $\mathbb{S}$

$x_n \in \mathbb{S}$ (discrete or continuous)

discrete-time stochastic processes

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Examples:

discrete-time stochastic processes

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(discrete or continuous)

$x_n \in \mathbb{S}$

Examples:

population

discrete-time stochastic processes

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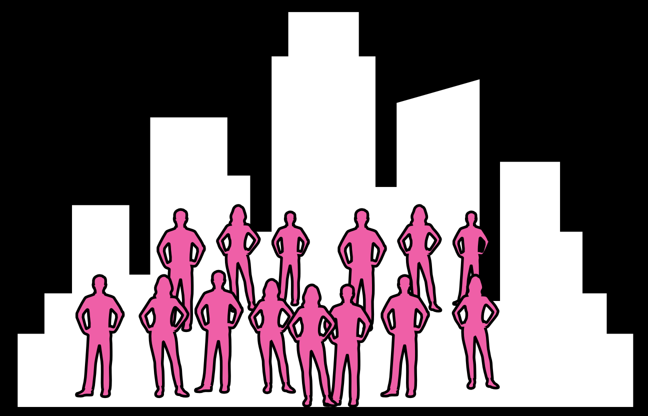
Examples:

population



2022

discrete-time stochastic processes

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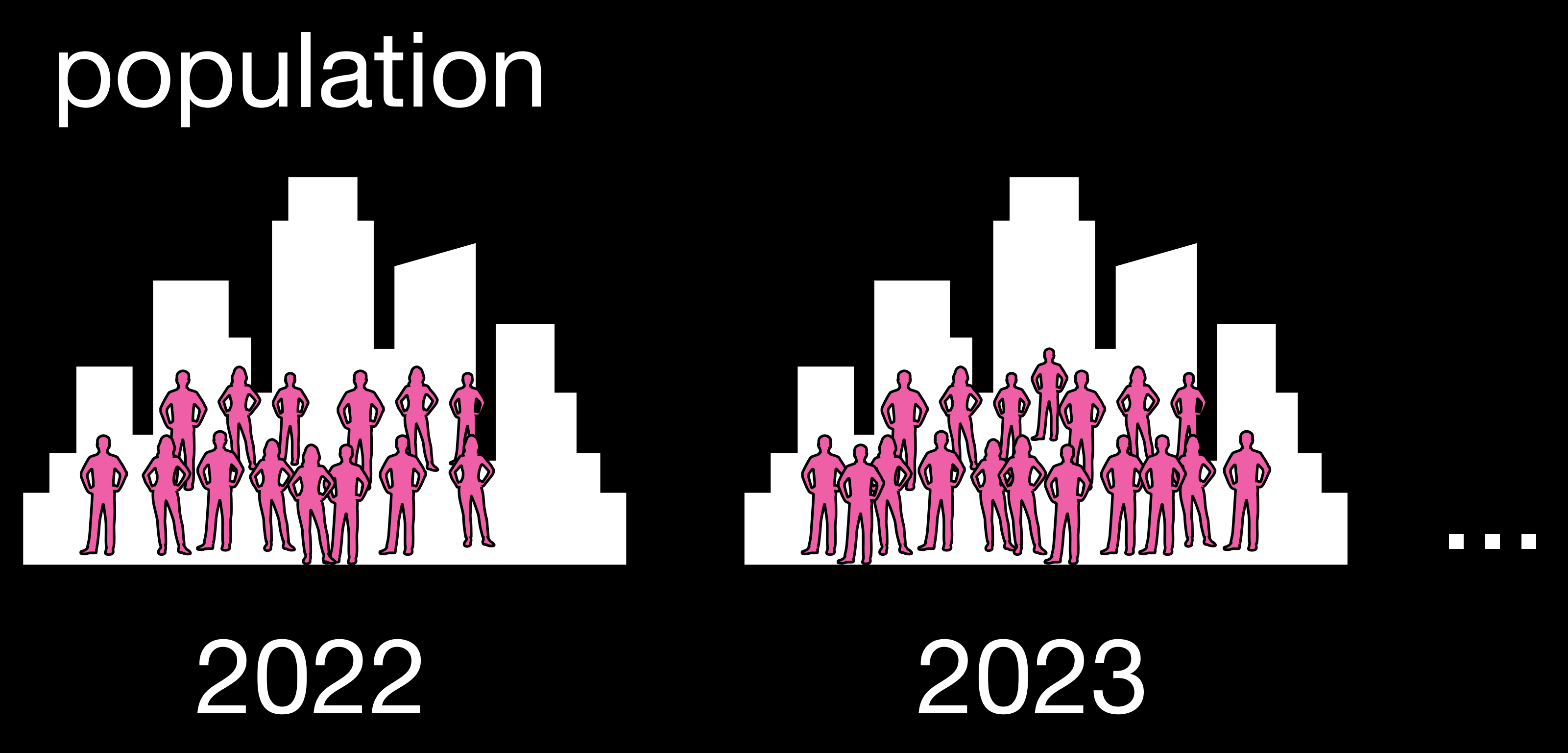
Examples:



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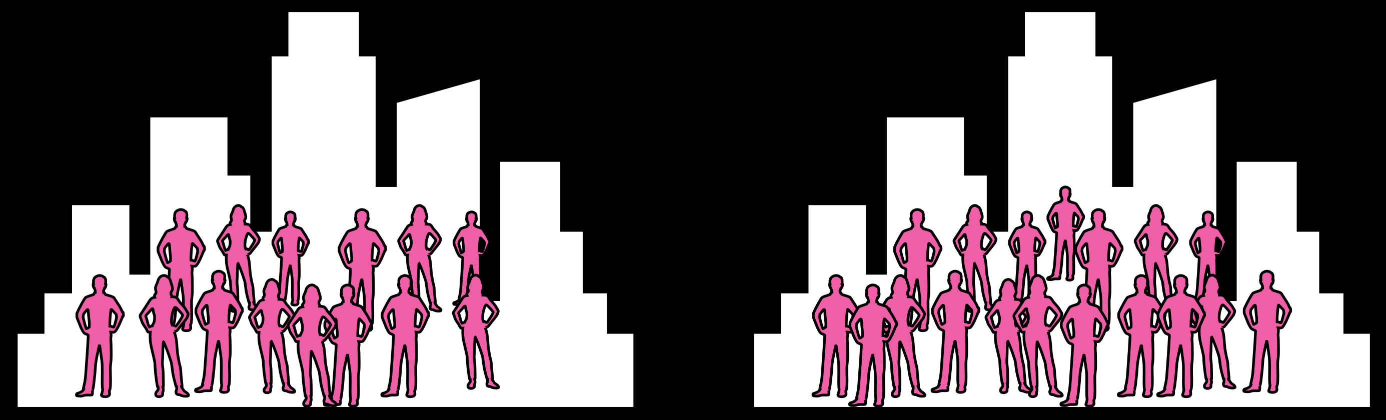

2022 2023 ...

rolling dice

discrete-time stochastic processes

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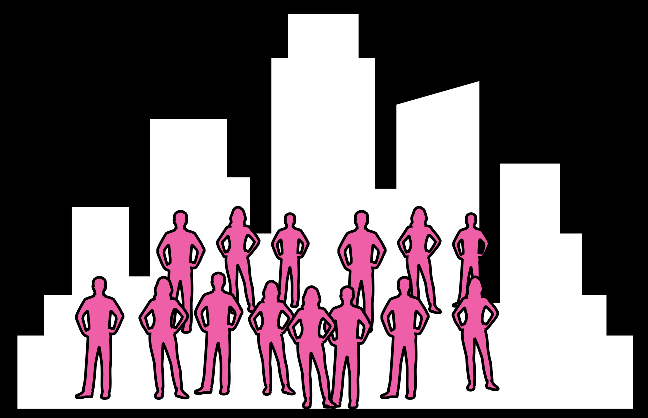
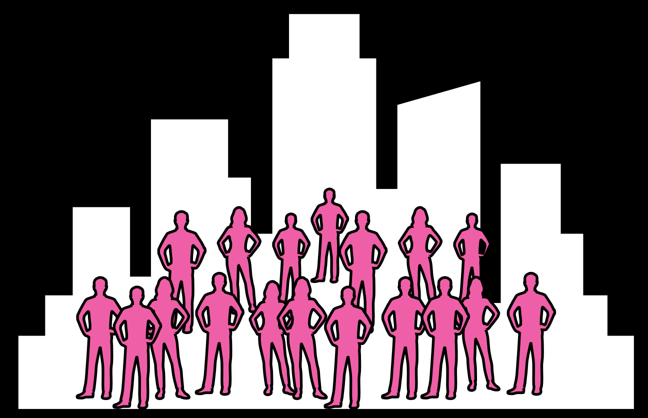


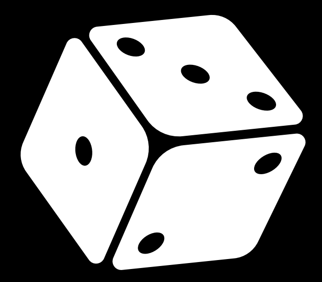
2022



2023

...

rolling dice



1st

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$\{X_n : n \in \mathbb{N}\}$ discrete-time stochastic process

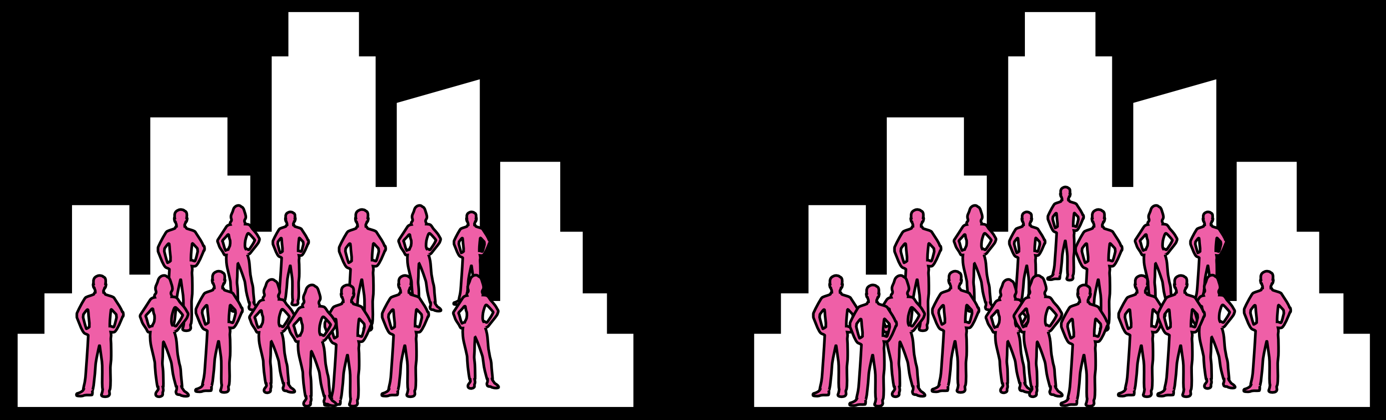
x_n value of random variable X_n

$X_n : \Omega \to \mathbb{S}$ state space $\mathbb{S}$

$x_n \in \mathbb{S}$ (discrete or continuous)

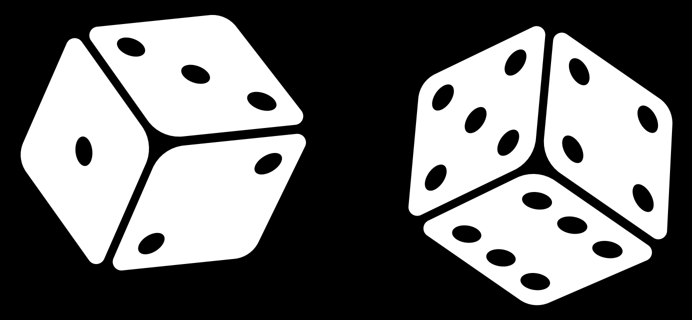
Examples:

population



2022 2023 ...

rolling dice



1st 2nd

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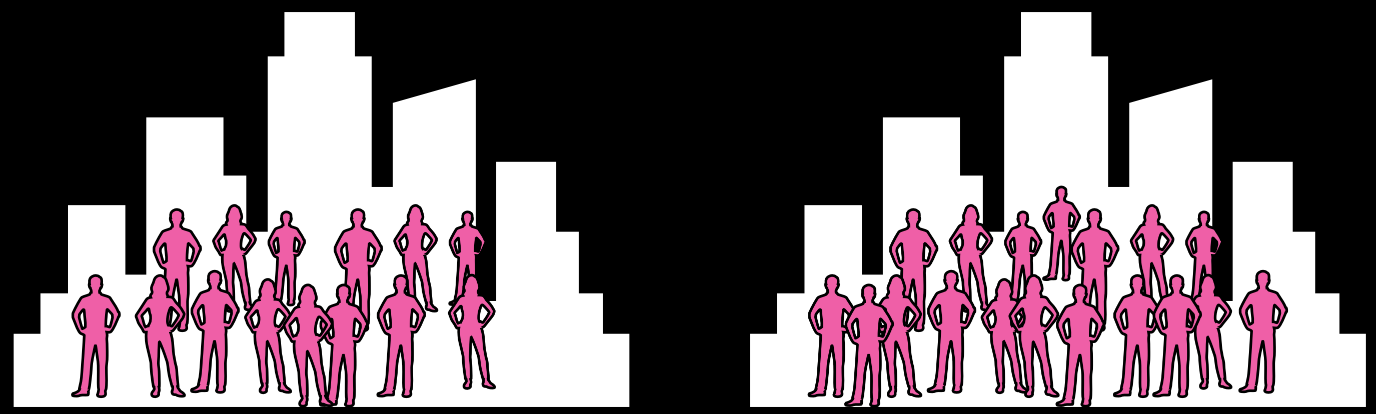
population



2022 2023 ...

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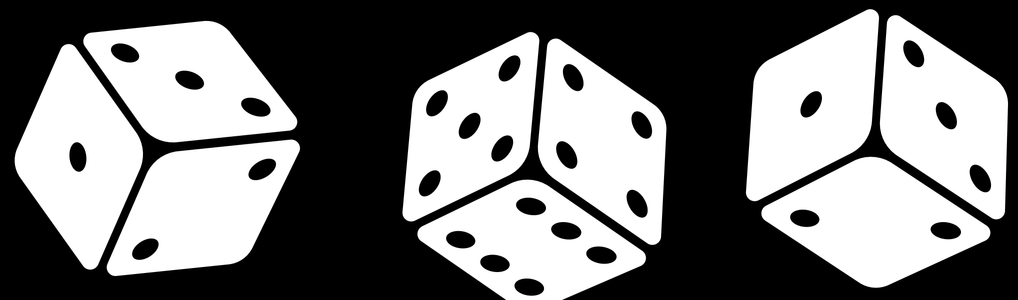

1st 2nd 3rd ...

independent identical distributed (i.i.d.) process

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$$\{X_n : n \in \mathbb{N}\}$$

independent identical distributed (i.i.d.) process

$\{X_n : n \in \mathbb{N}\}$ stochastic process

independent identical distributed (i.i.d.) process

$\{X_n : n \in \mathbb{N}\}$ stochastic process

$$F_{X_0}(x) = F_{X_1}(x) = F_{X_2}(x) = \dots$$

independent identical distributed (i.i.d.) process

$$\{X_n : n \in \mathbb{N}\}$$ stochastic process

$$F_{X_0}(x) = F_{X_1}(x) = F_{X_2}(x) = \dots$$ identical distributed

independent identical distributed (i.i.d.) process

$$\{X_n : n \in \mathbb{N}\}$$ stochastic process

$$F_{X_0}(x) = F_{X_1}(x) = F_{X_2}(x) = \dots$$ identical distributed

$$F_{X_0,\dots,X_n}(x_0,\dots,x_n) = F_{X_0}(x_0) \cdot \dots \cdot F_{X_n}(x_n)$$

independent identical distributed (i.i.d.) process

$\{X_n : n \in \mathbb{N}\}$ stochastic process

$F_{X_0}(x) = F_{X_1}(x) = F_{X_2}(x) = \ldots$ identical distributed

$F_{X_0,\ldots,X_n}(x_0, \ldots, x_n) = F_{X_0}(x_0) \cdot \ldots \cdot F_{X_n}(x_n)$ independent

independent identical distributed (i.i.d.) process

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exercise:

independent identical distributed (i.i.d.) process

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exercise:

Which system can be captured by an i.i.d. process?

independent identical distributed (i.i.d.) process

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$F_{X_0}(x) = F_{X_1}(x) = F_{X_2}(x) = \ldots$ identical distributed

$F_{X_0,\ldots,X_n}(x_0, \ldots, x_n) = F_{X_0}(x_0) \cdot \ldots \cdot F_{X_n}(x_n)$ independent

exercise:

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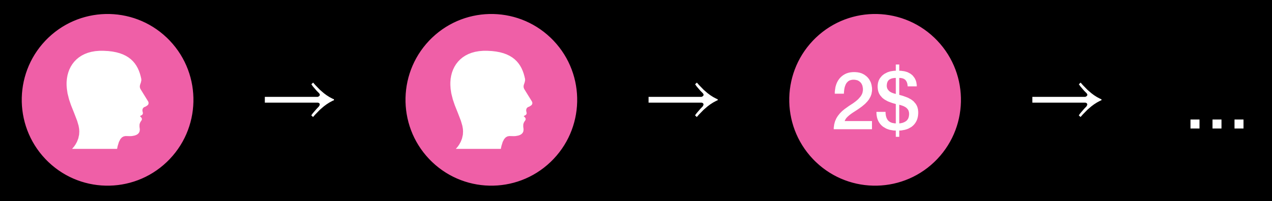
coin-flip



independent identical distributed (i.i.d.) process

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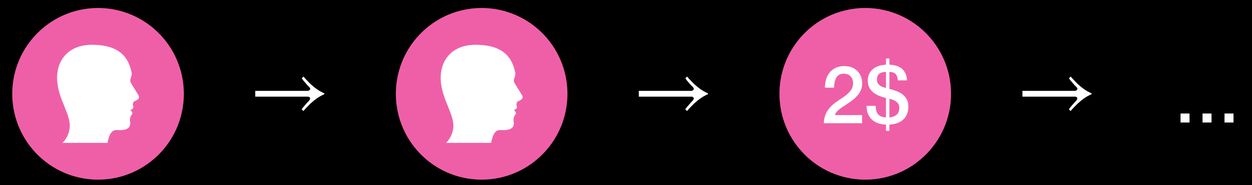
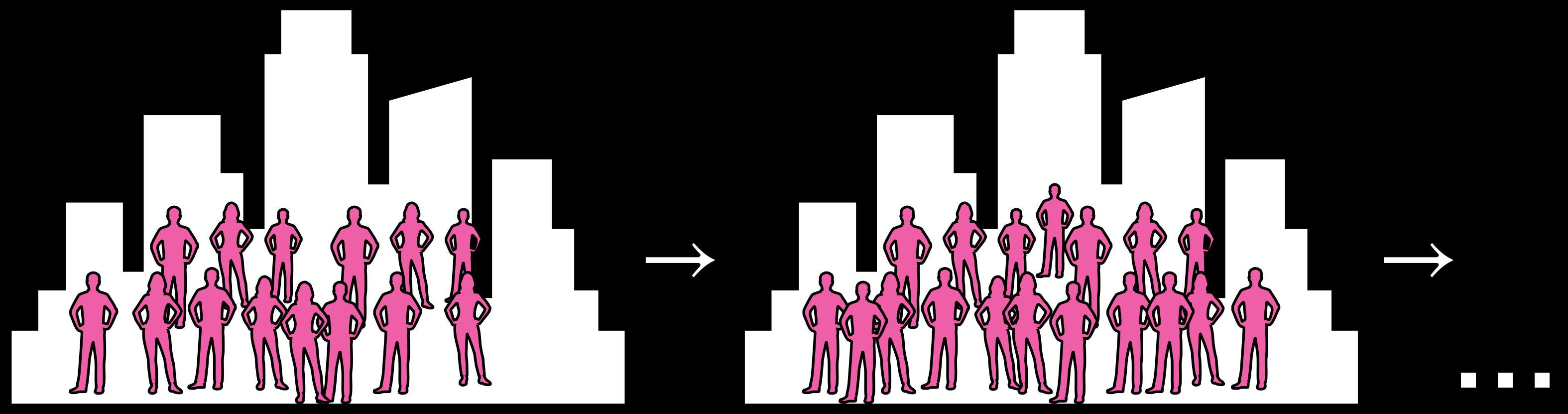
exercise:

Which system can be captured by an i.i.d. process?

coin-flip



population development



2022 2023

independent identical distributed (i.i.d.) process

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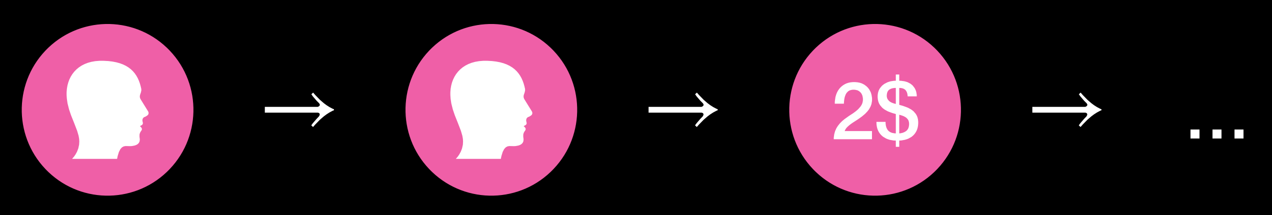

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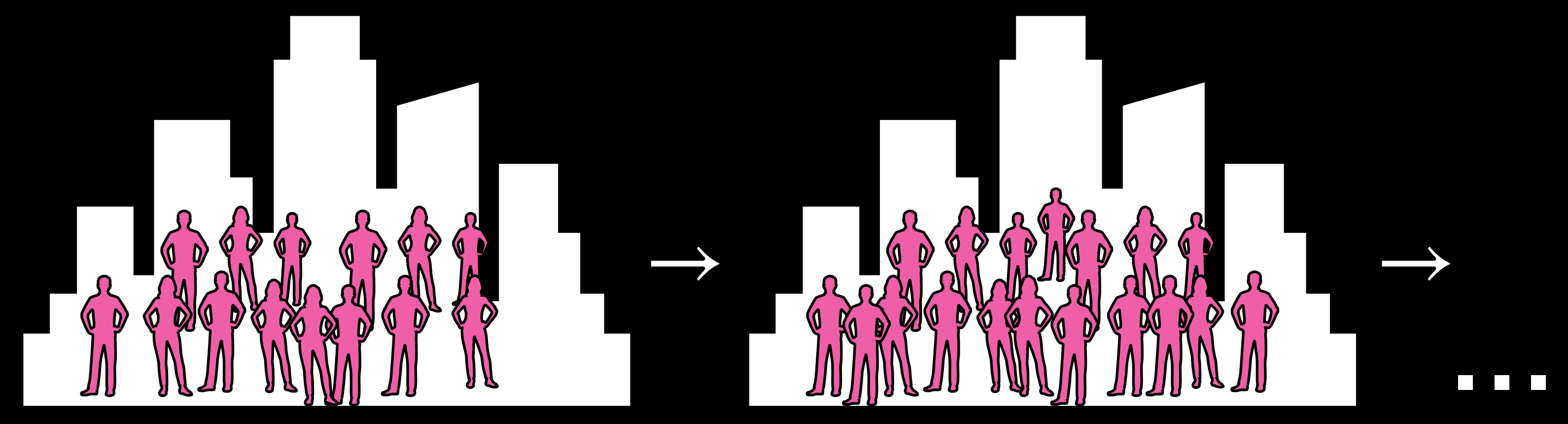
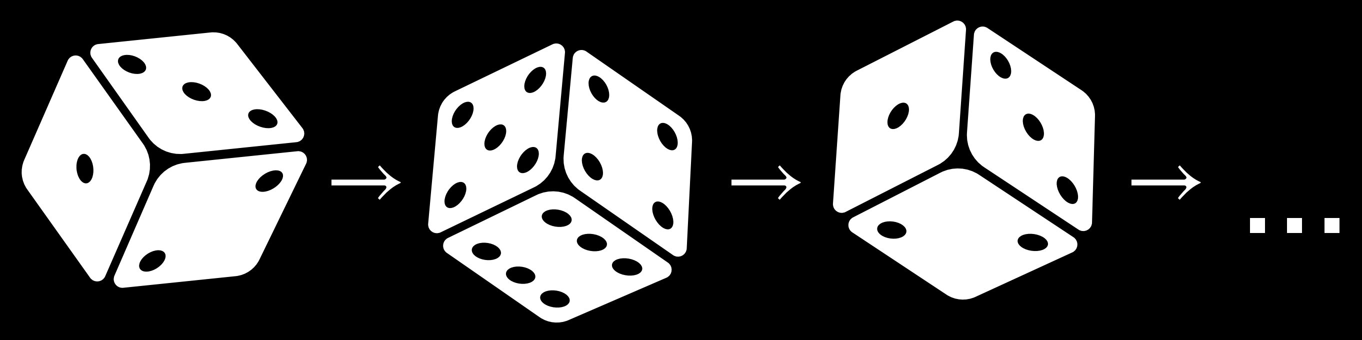


2022 2023

rolling dice



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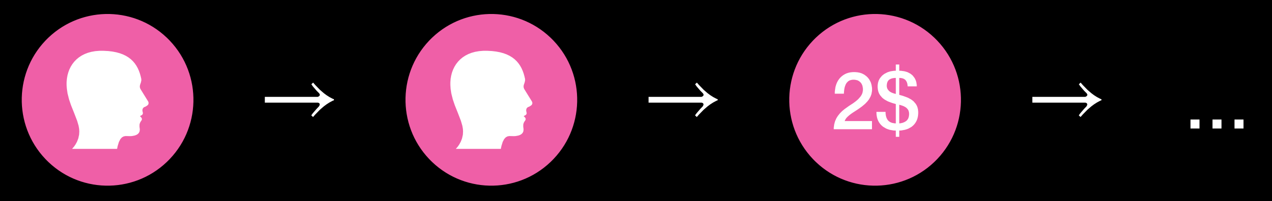

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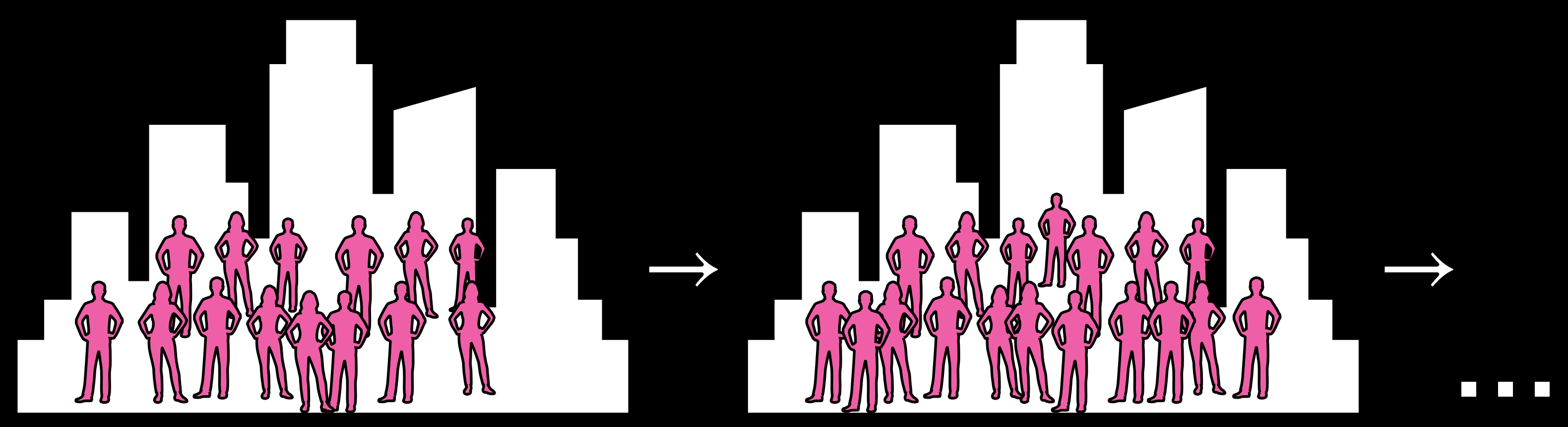

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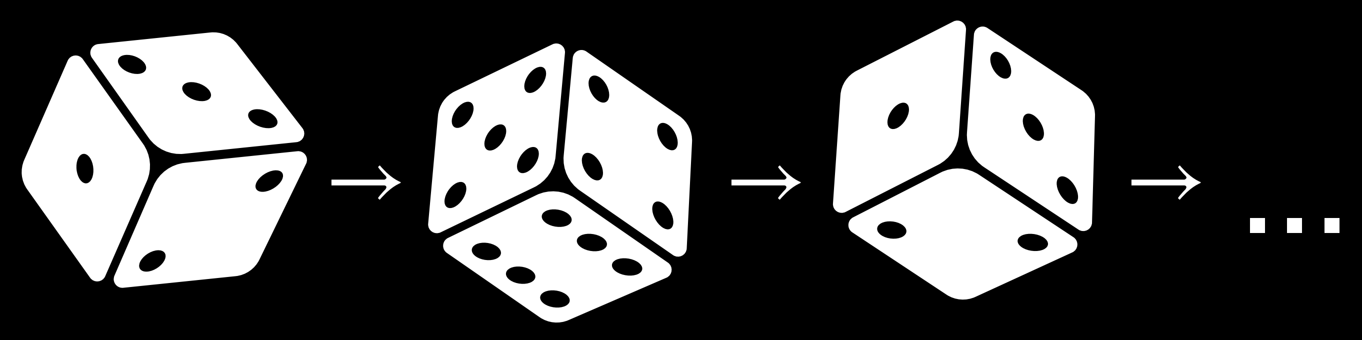

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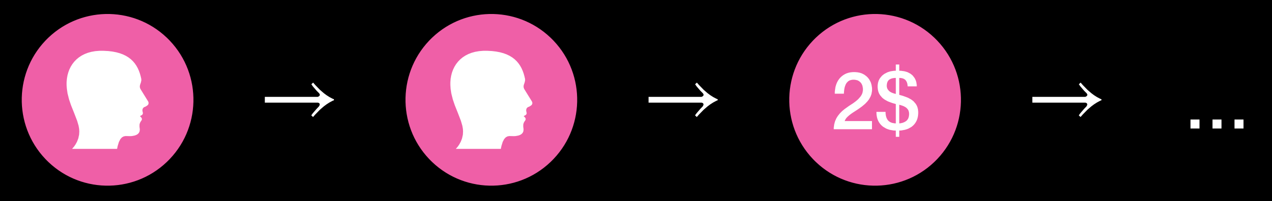

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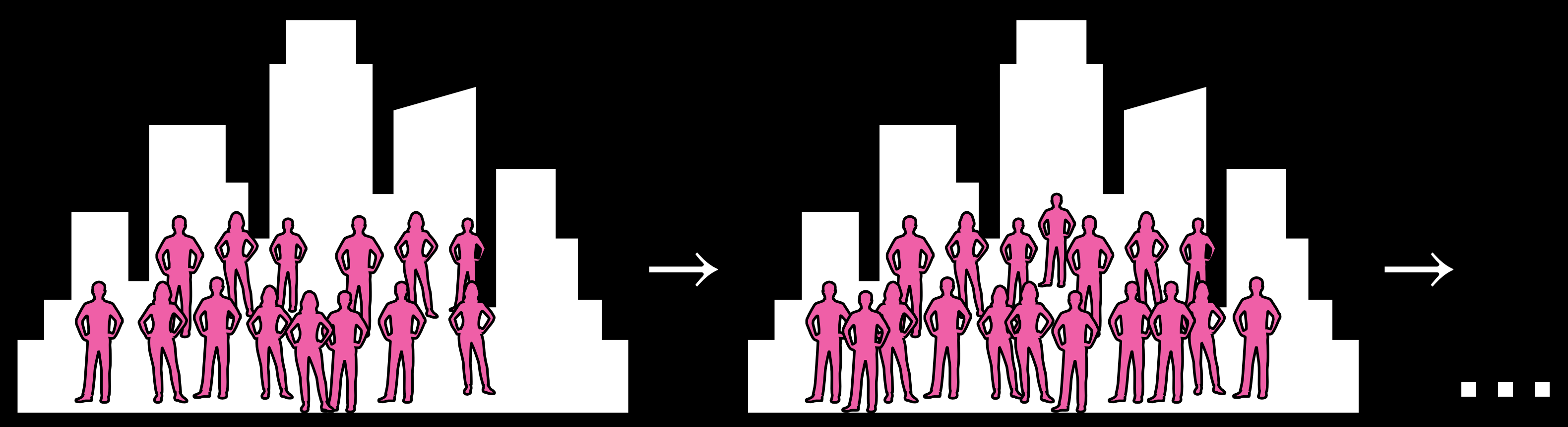

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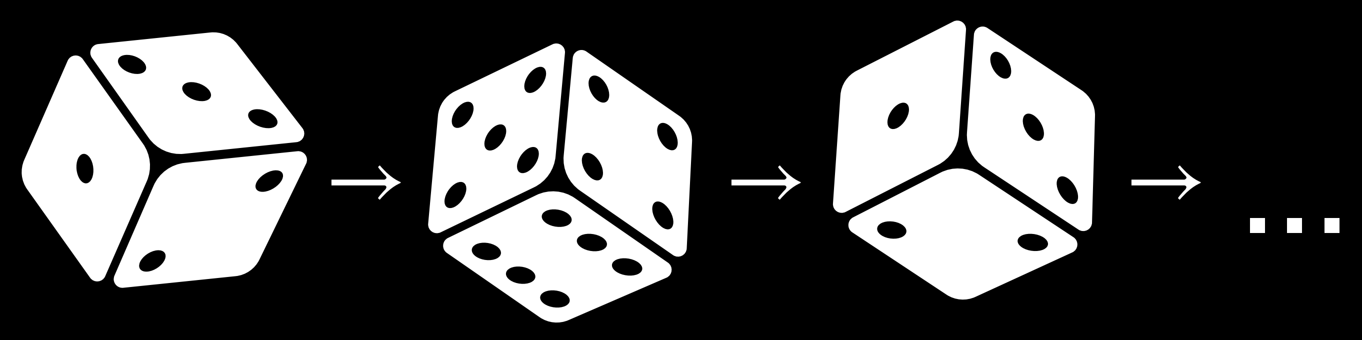

Markov chains

Markov chains

$$\{X_n : n \in \mathbb{N}\}$$

Markov chains

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$$P(X_{n+1} = x_j \,|\, X_n = x_i, X_{n-1} = x_{i-1}, \ldots, X_0 = x_0)$$

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Markov property

(X_{n+1} only depends on X_n)

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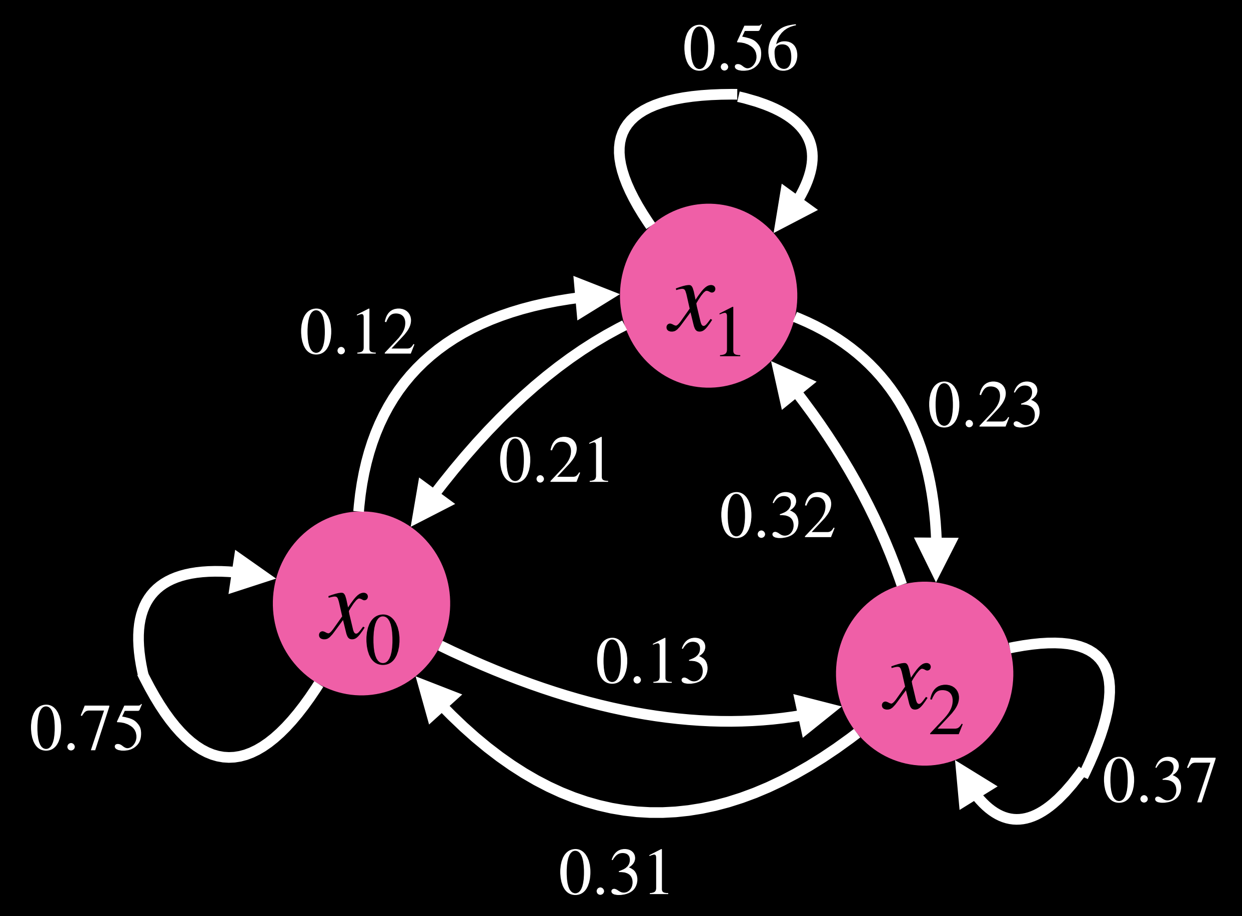

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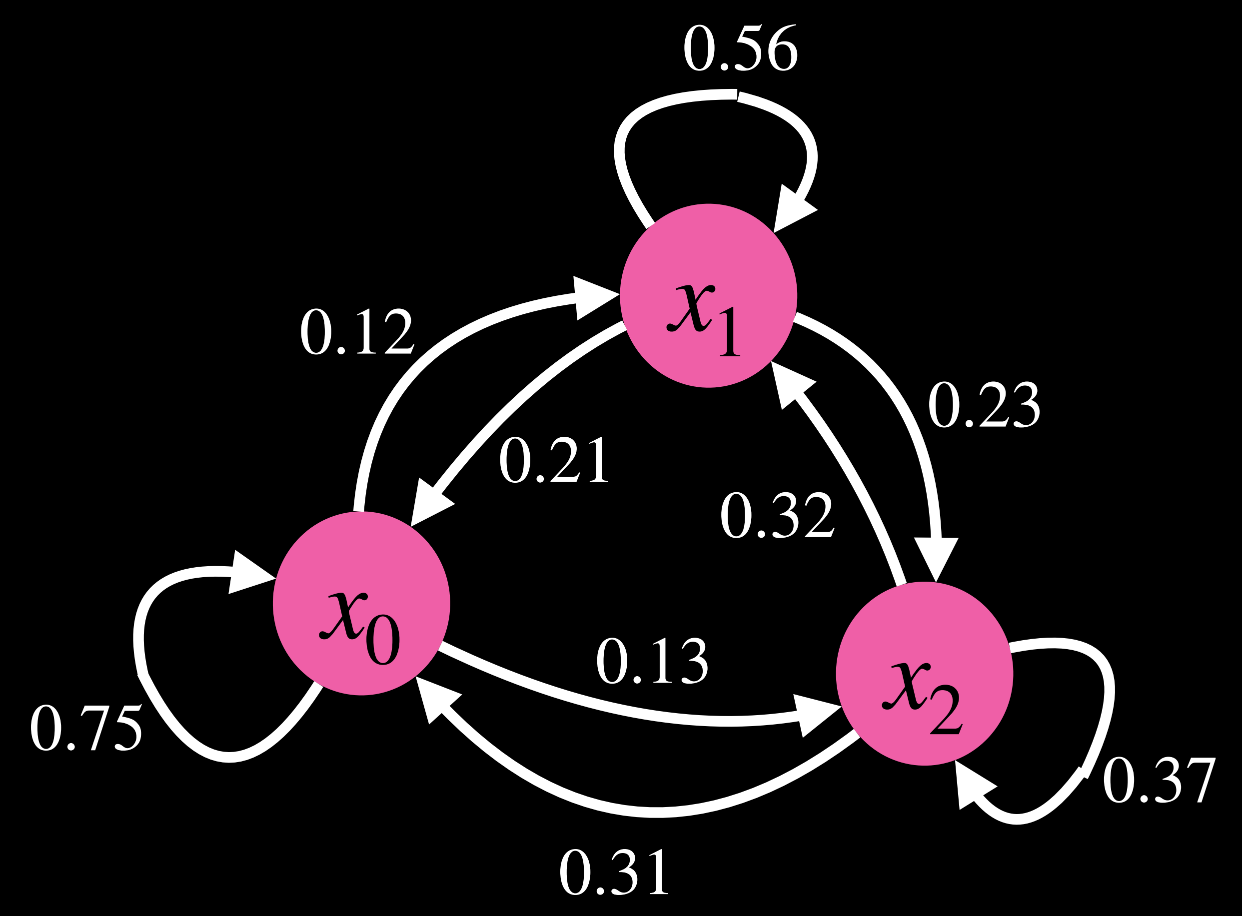

$$\underline{\underline{P}} = \begin{pmatrix} 0.75 & 0.12 & 0.13 \\ 0.21 & 0.56 & 0.23 \\ 0.31 & 0.32 & 0.37 \end{pmatrix}$$

Markov chains as recursions

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properties of Markov chains

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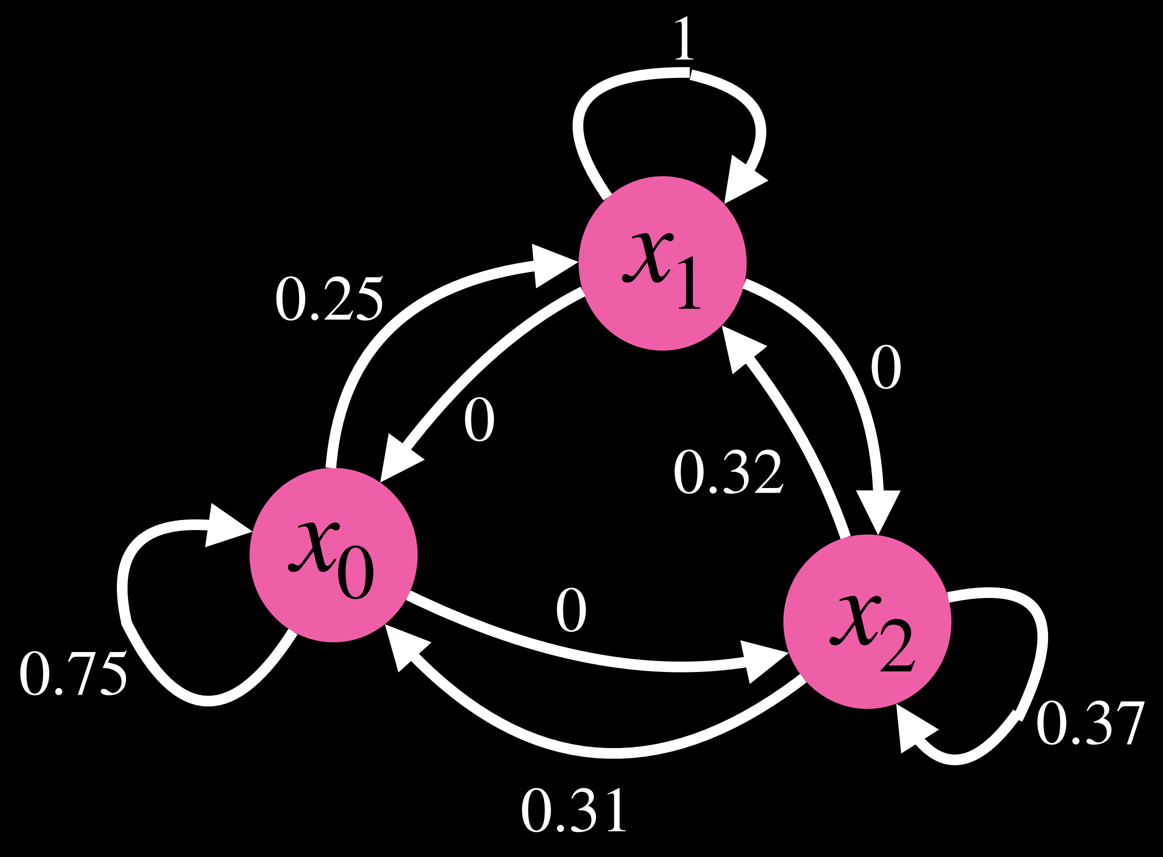

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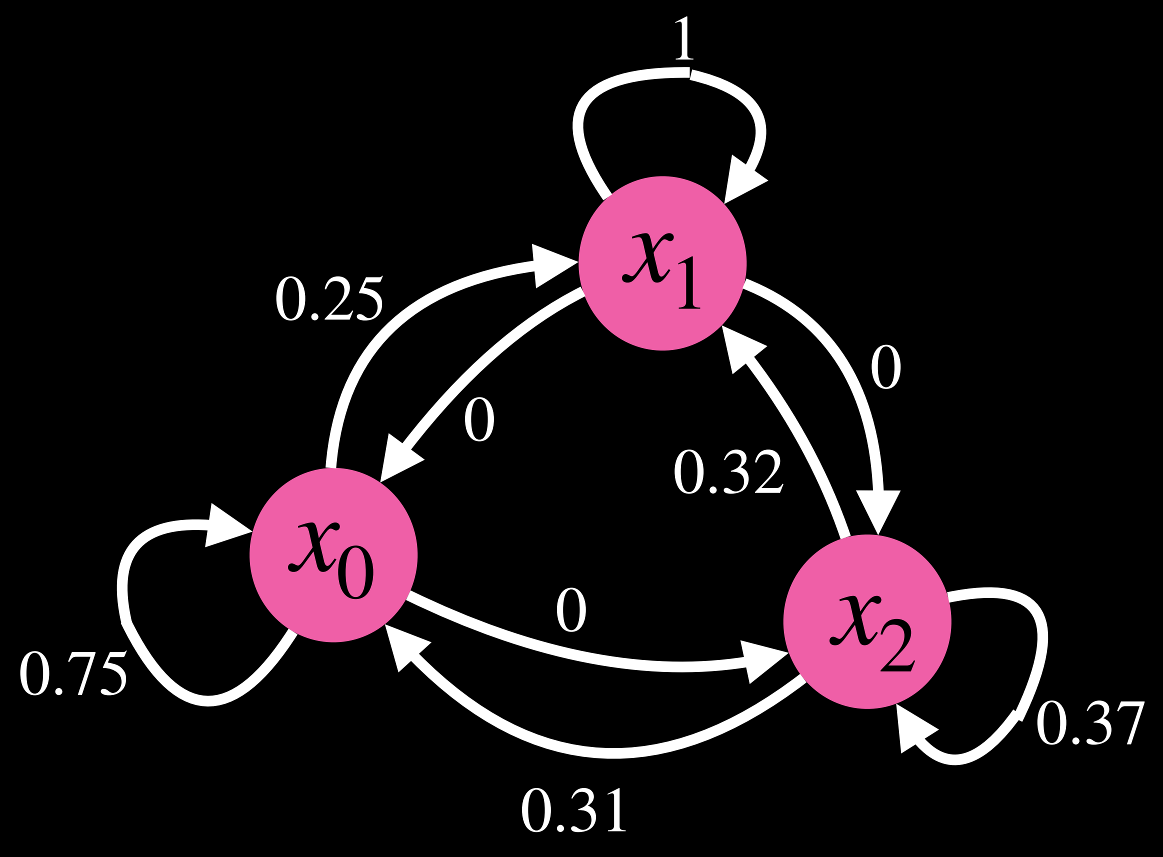

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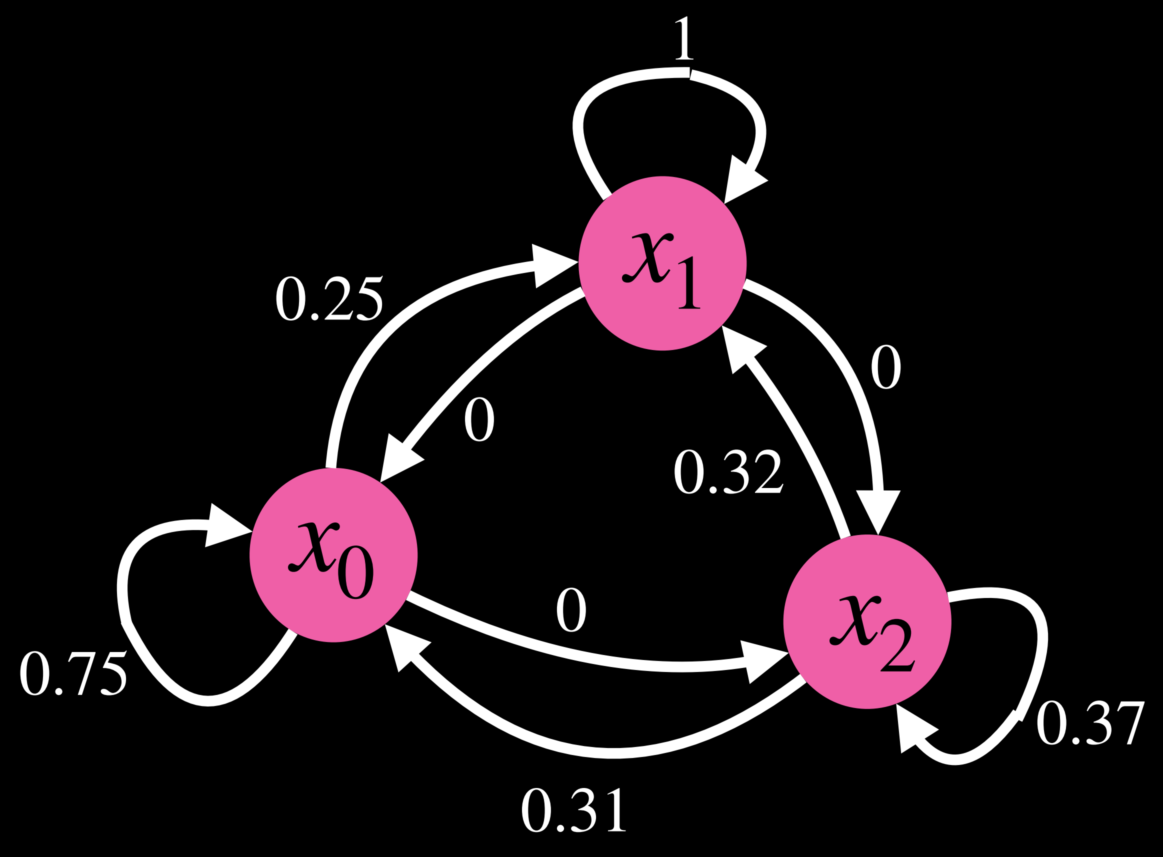

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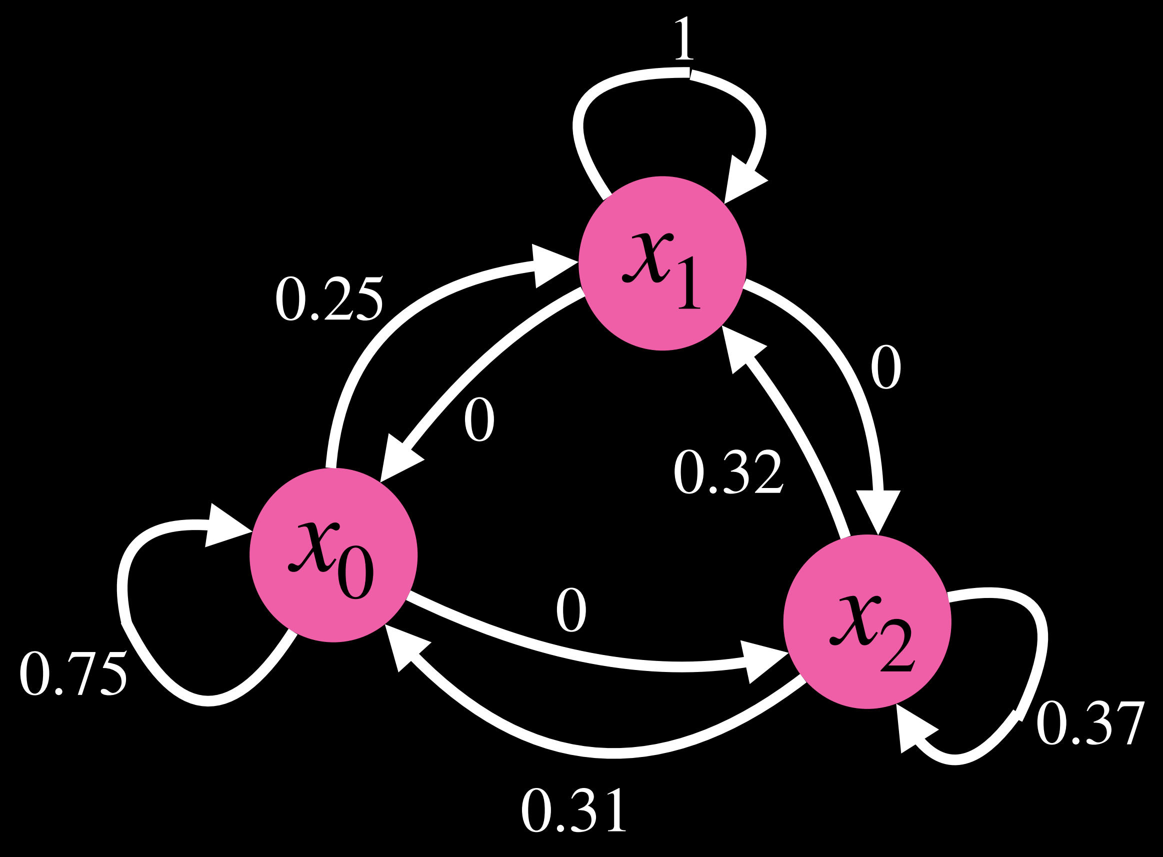

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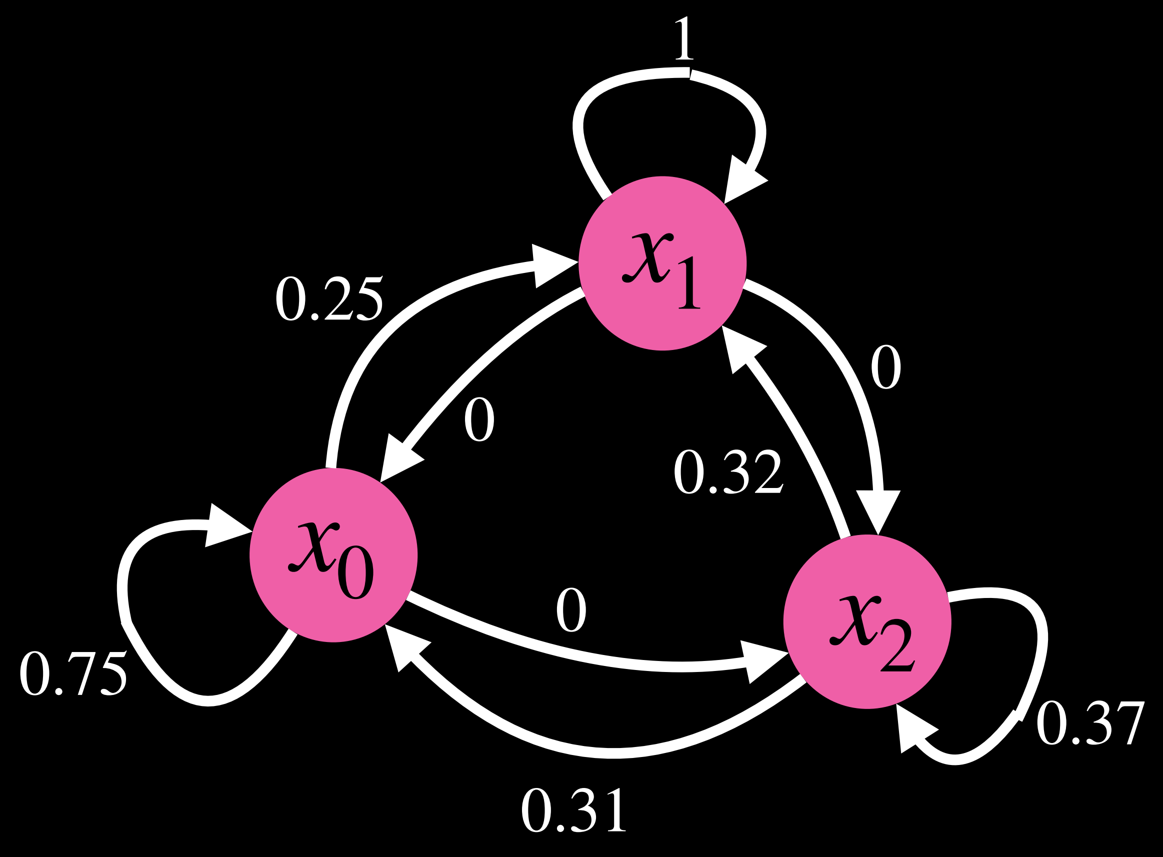

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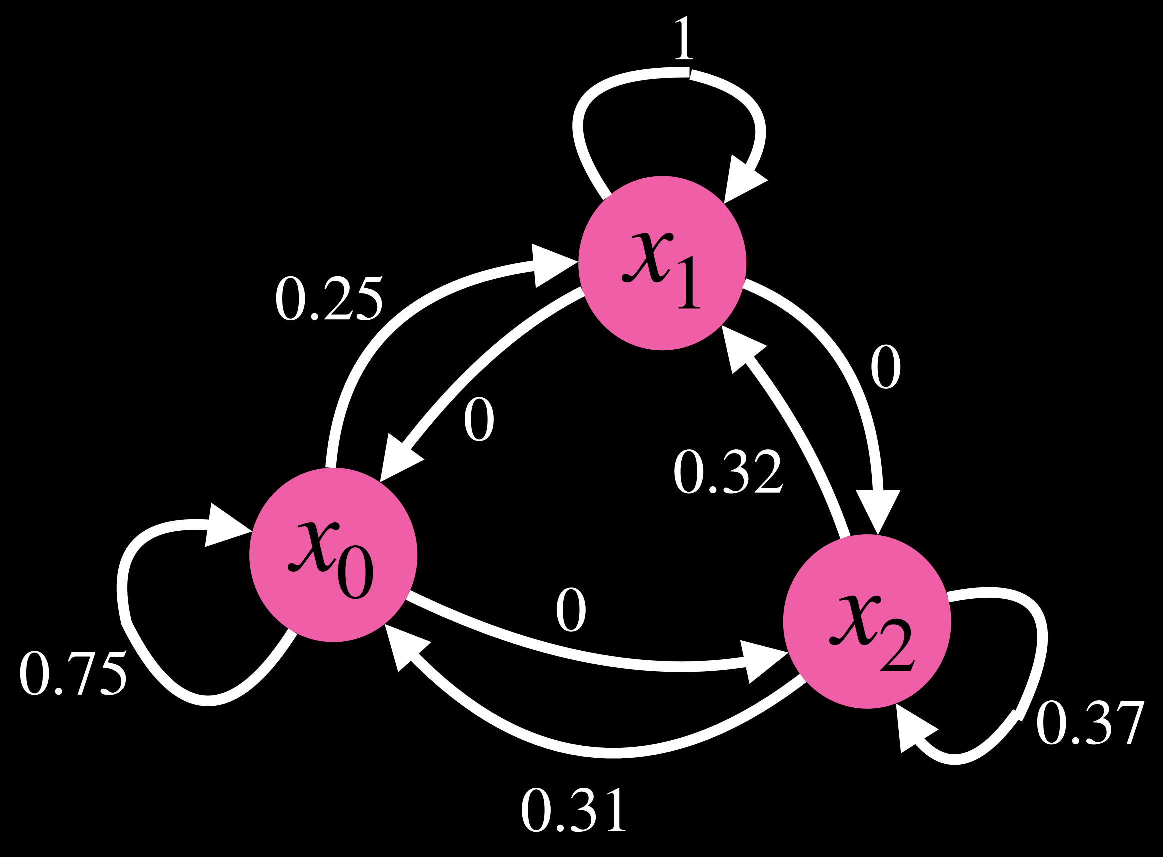

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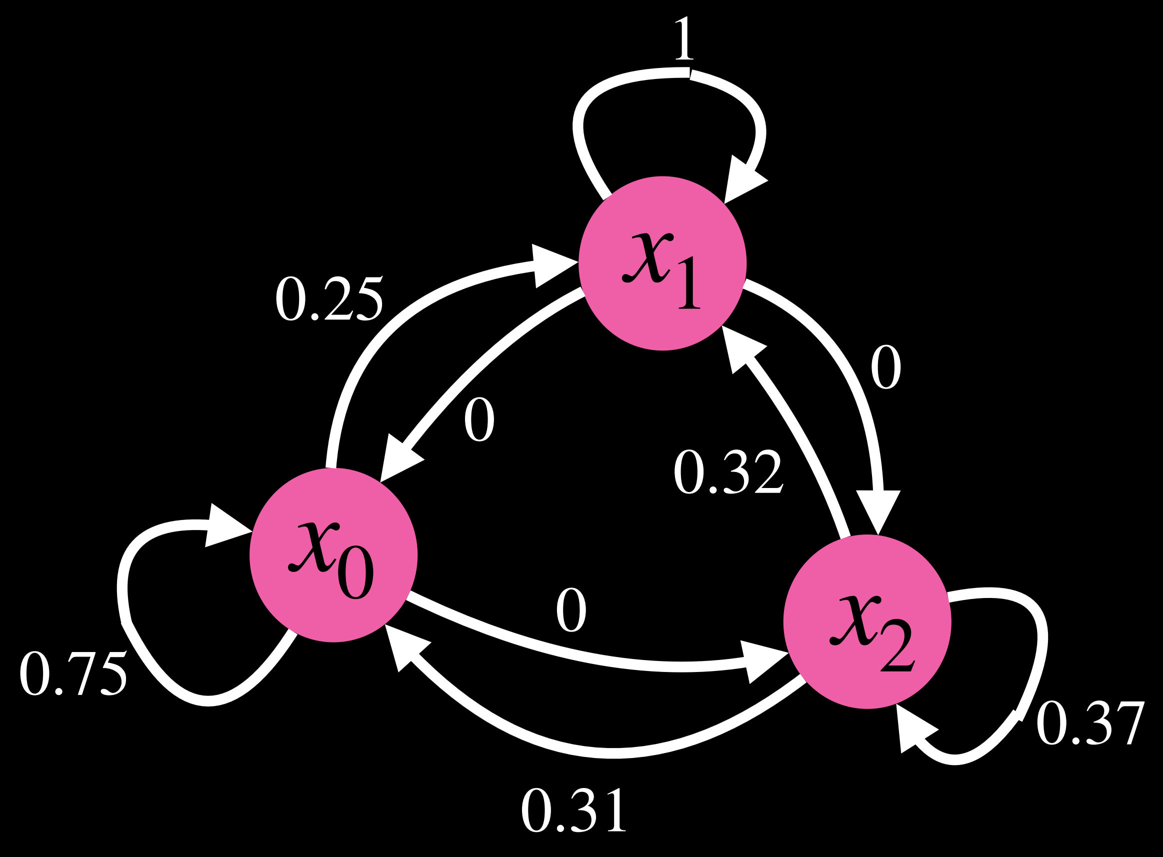

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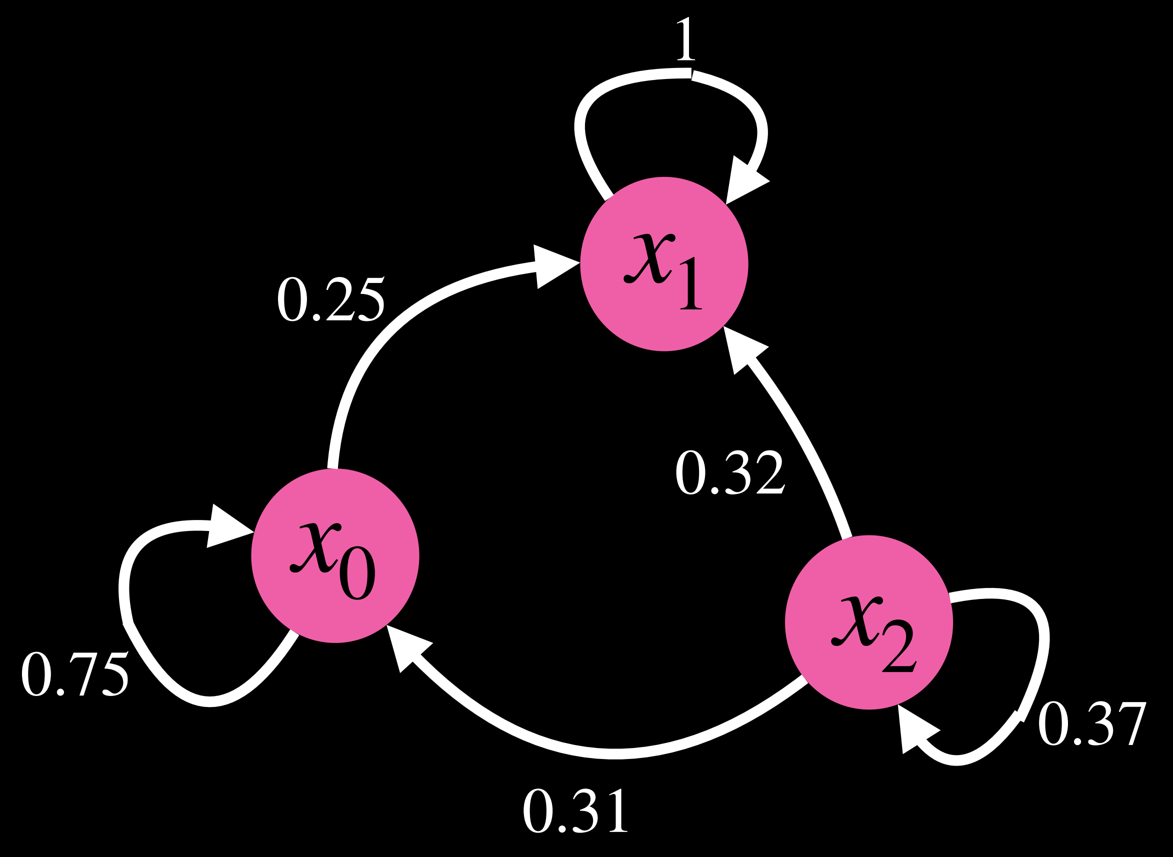

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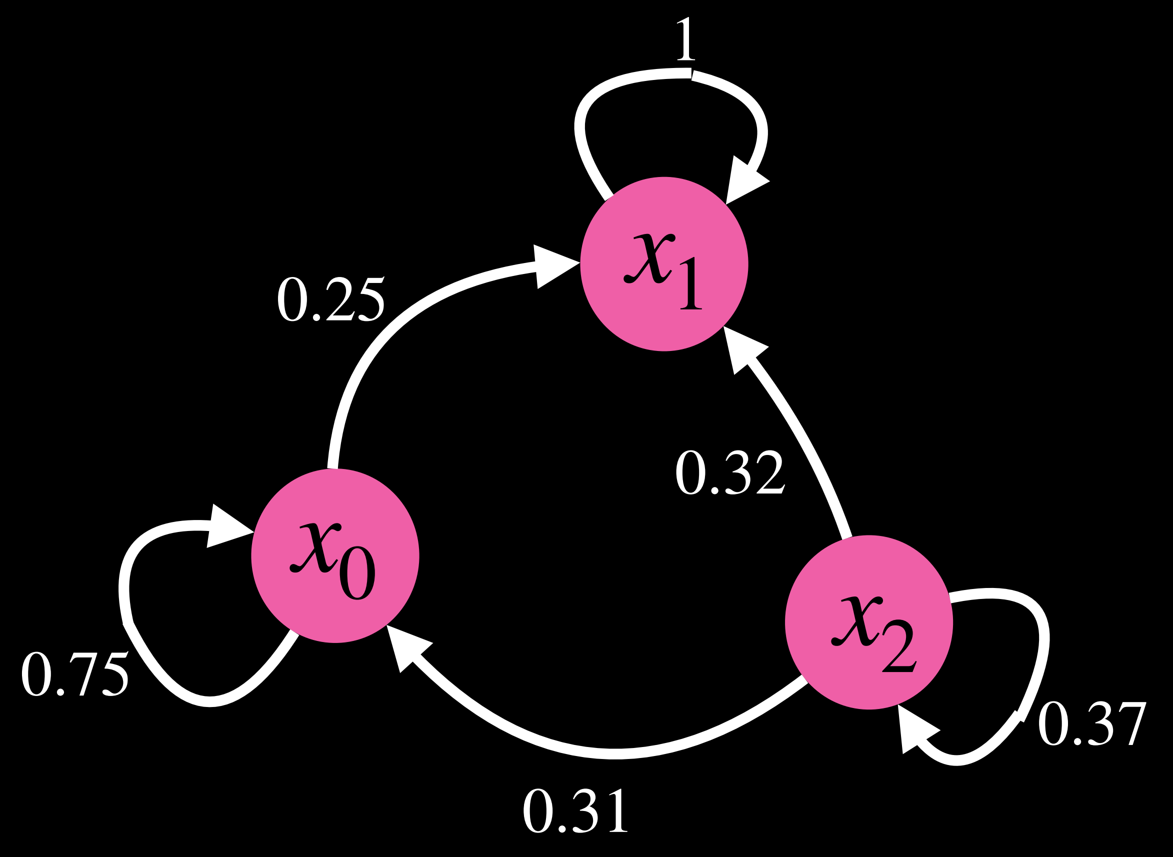

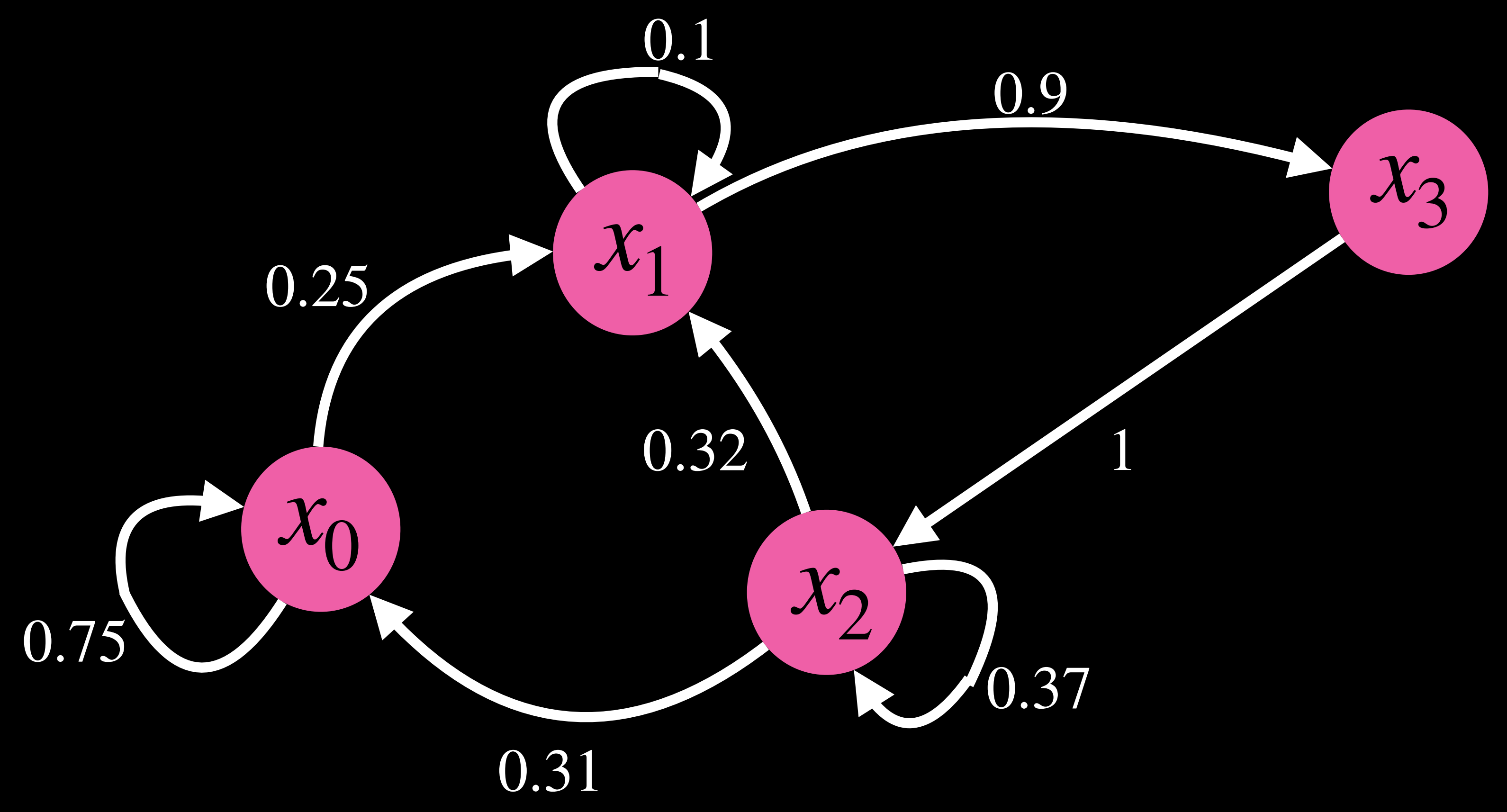

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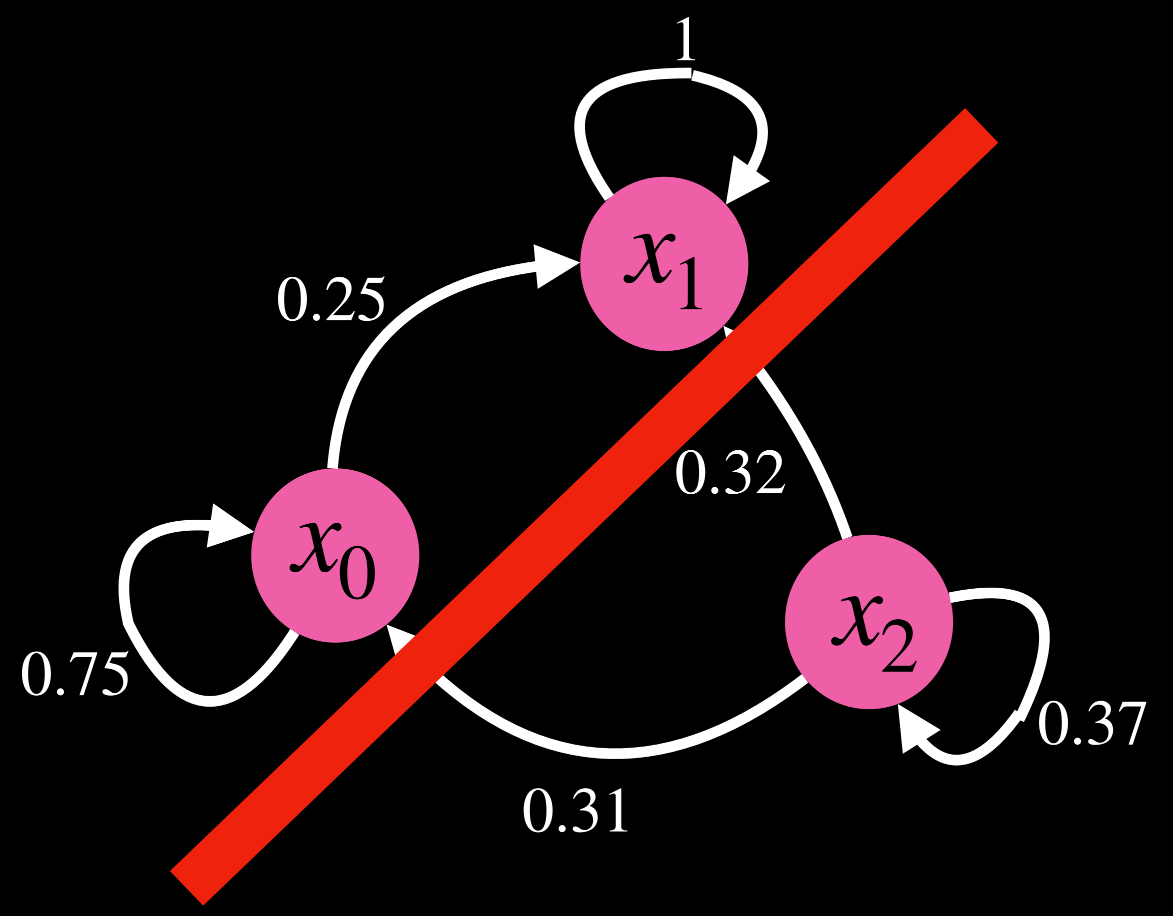

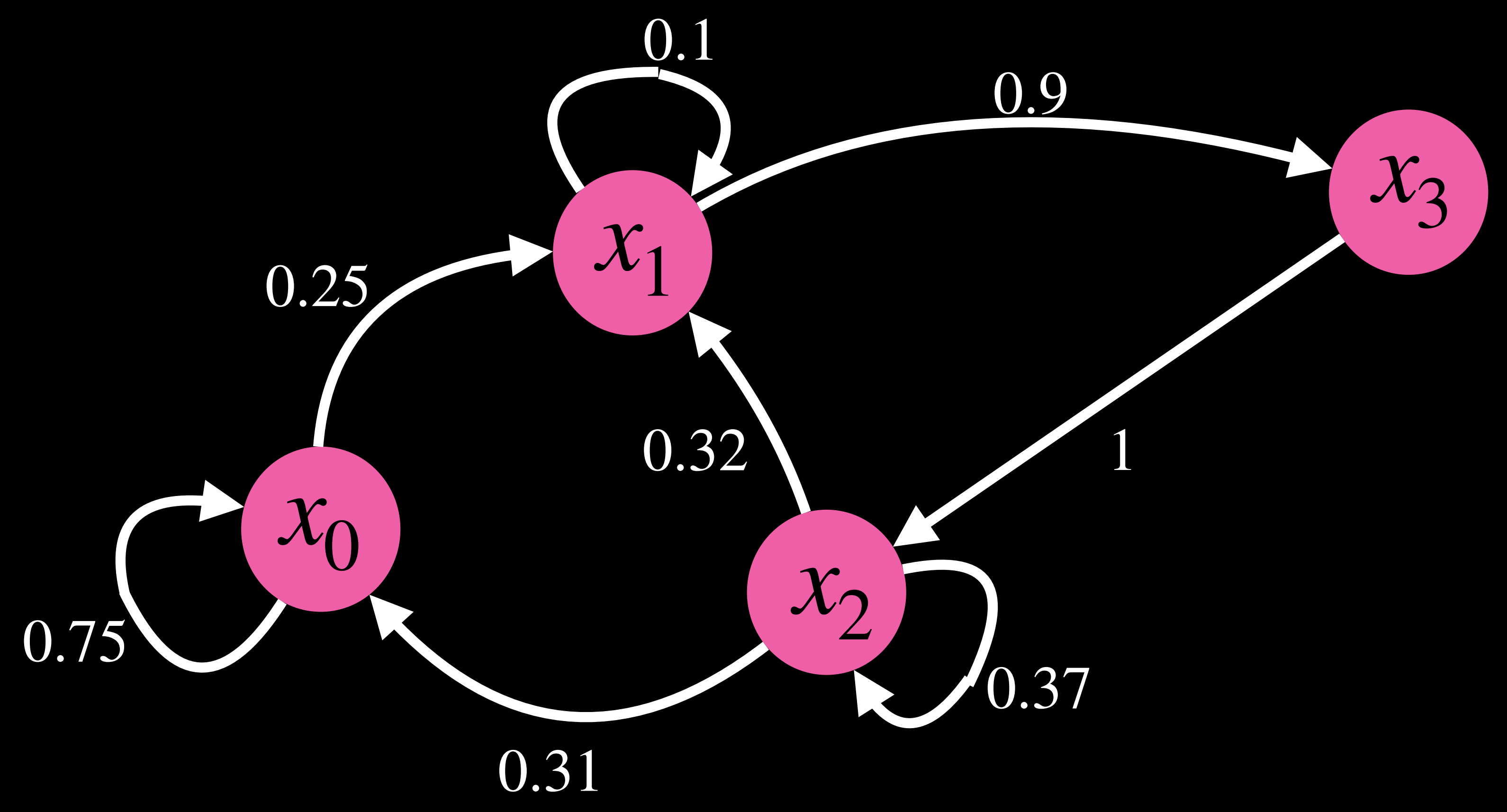

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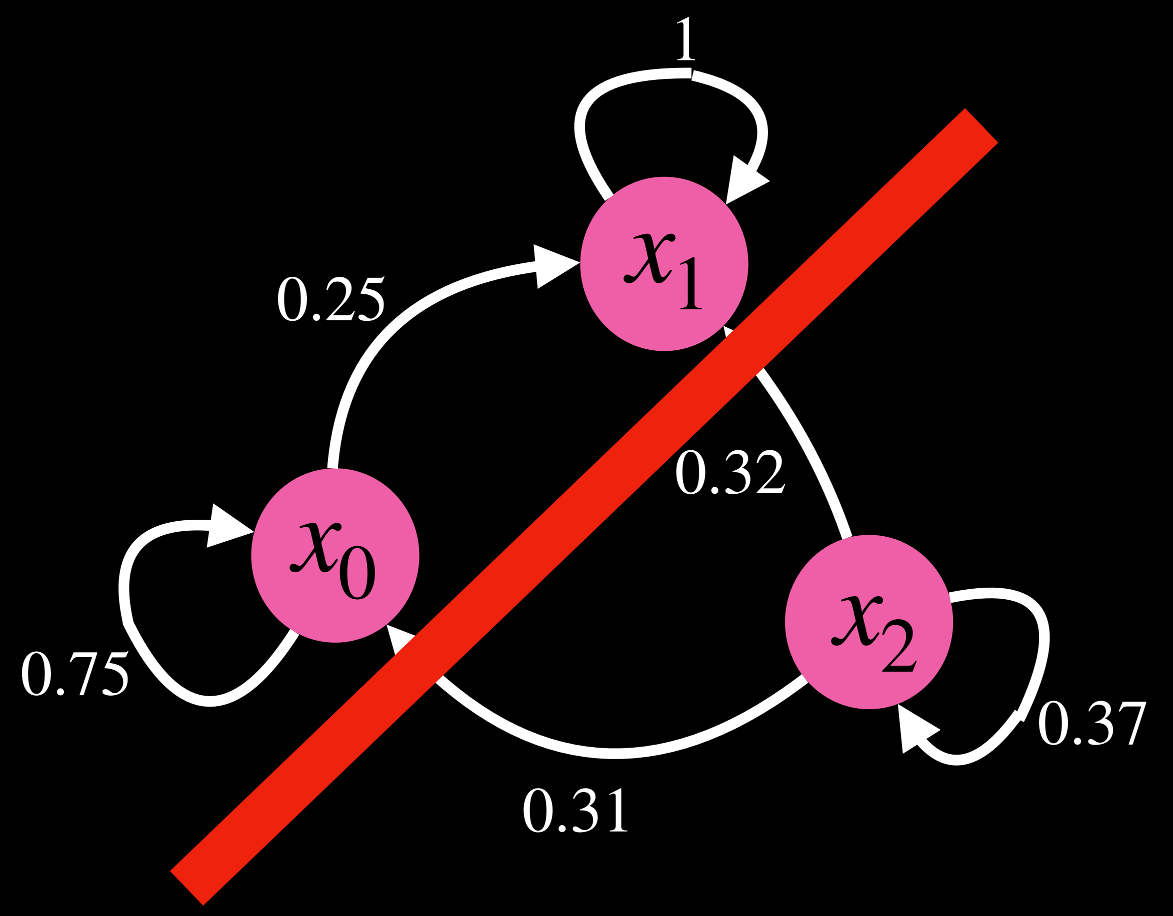

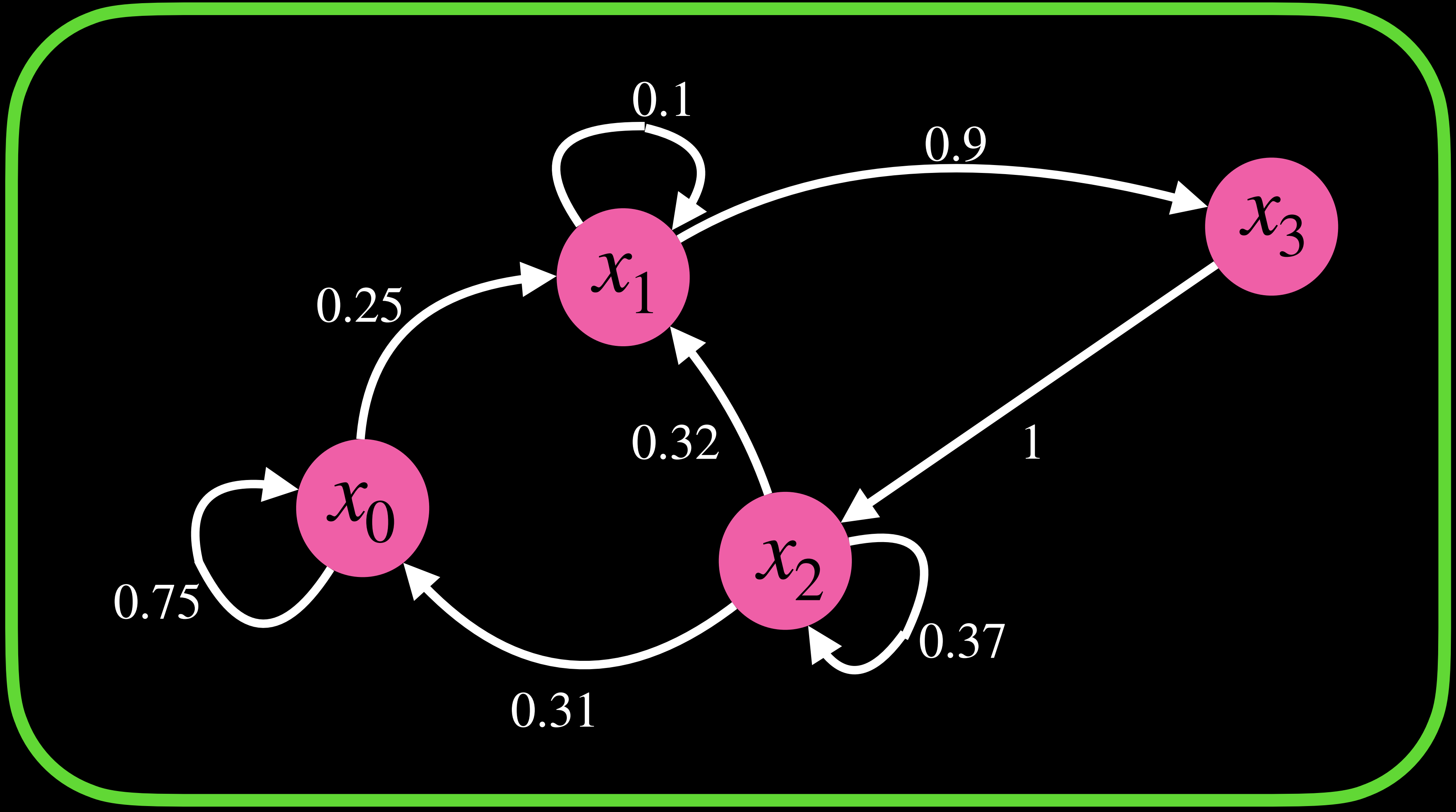

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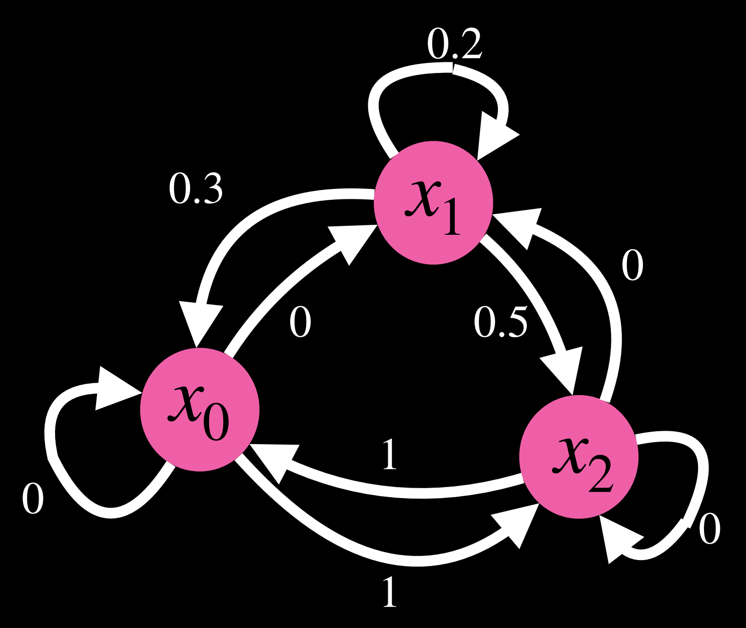

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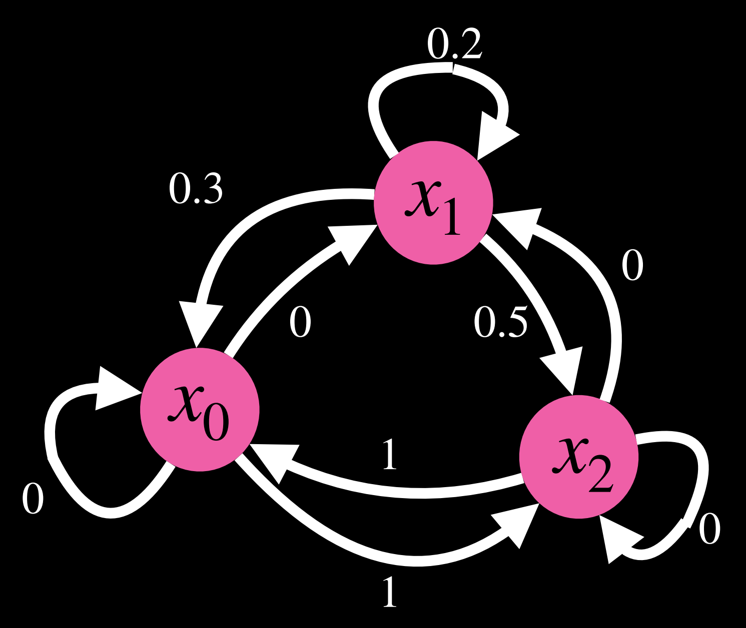

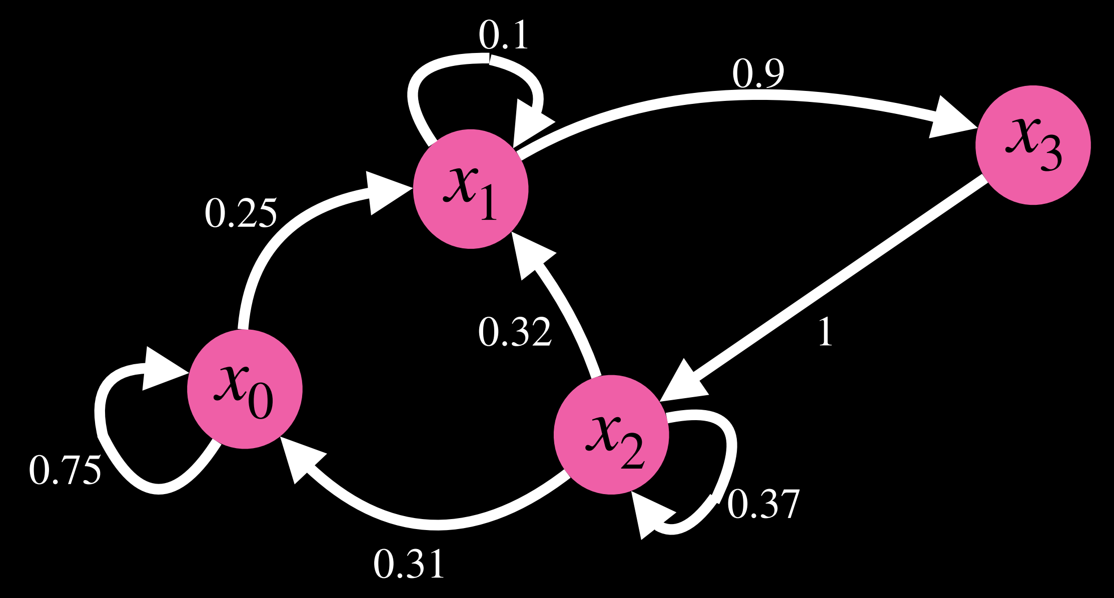

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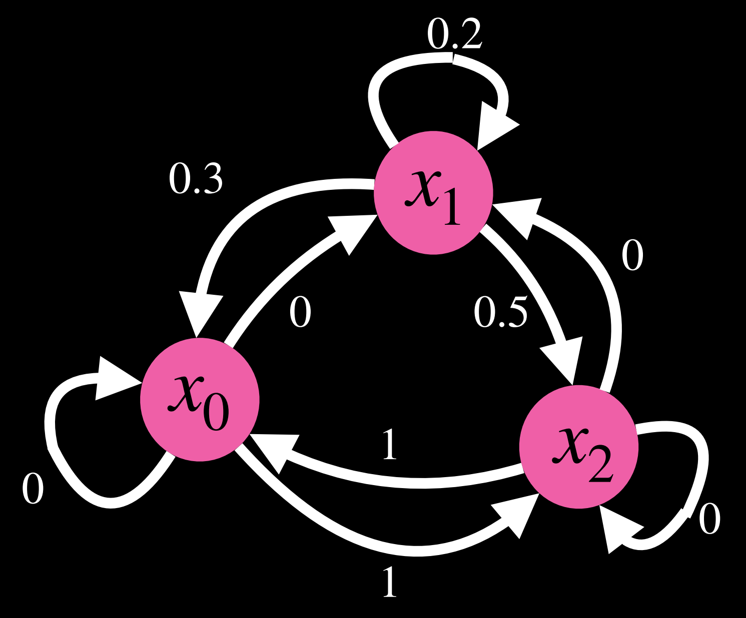

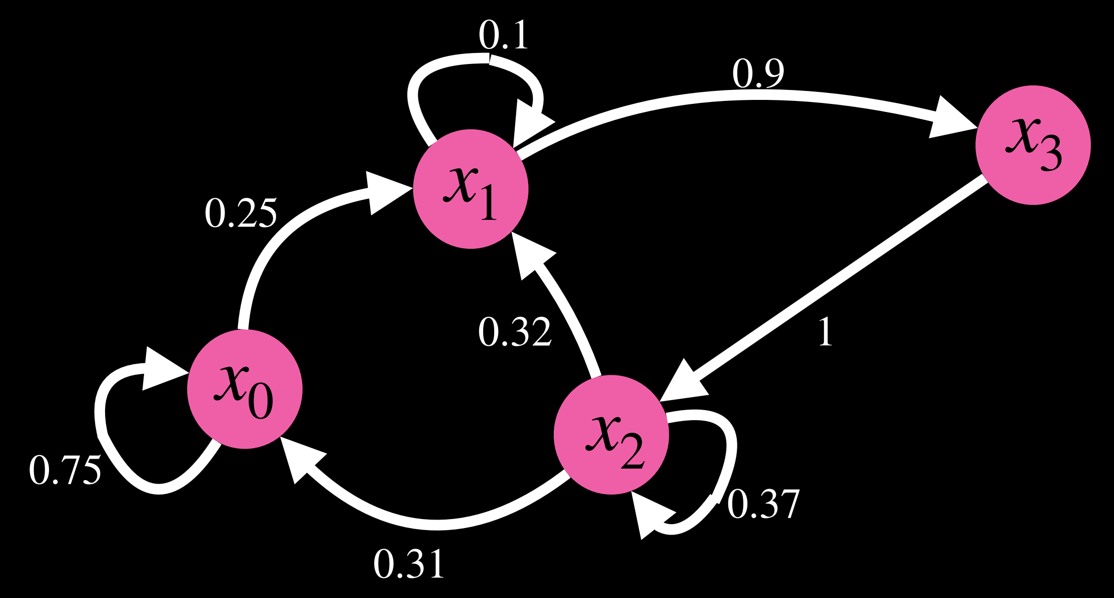

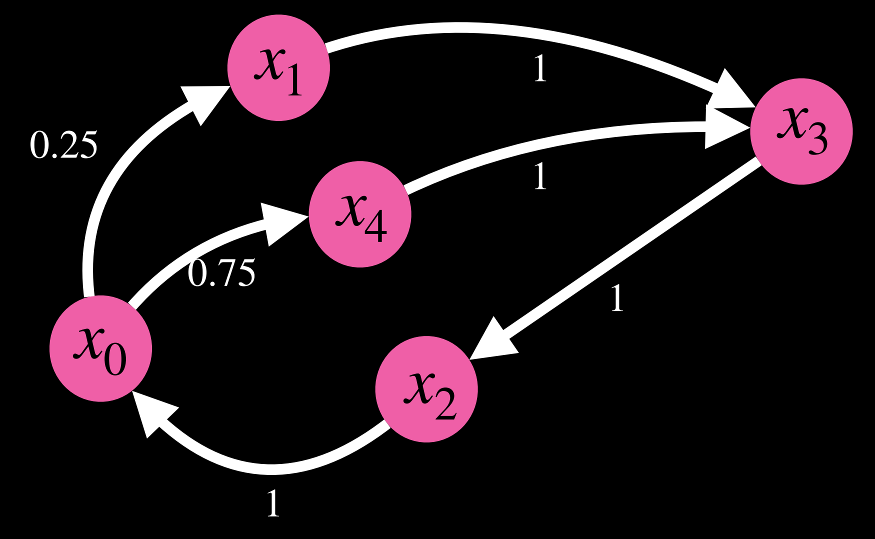

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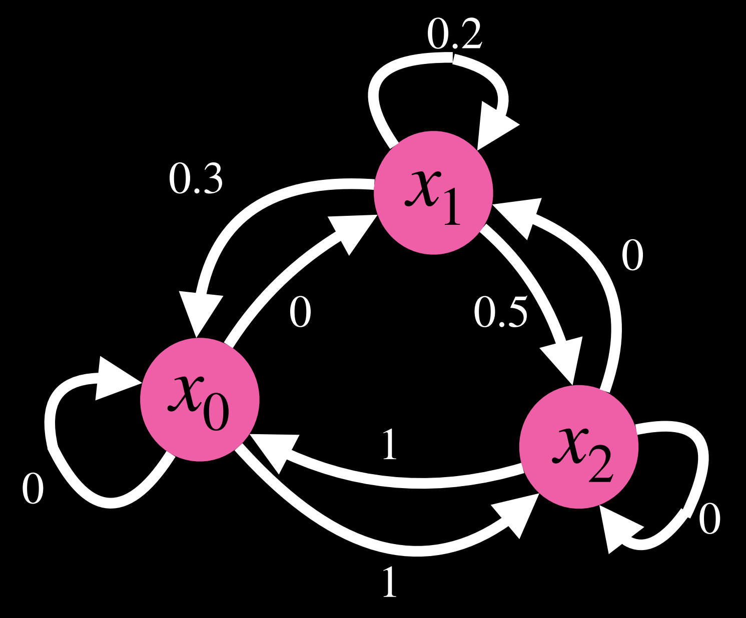

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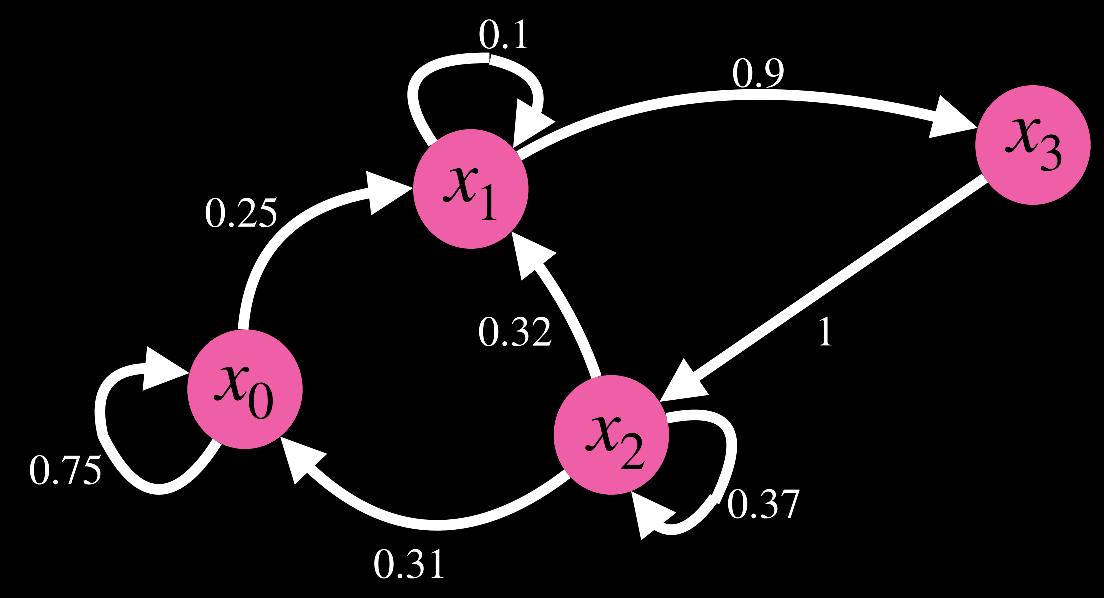

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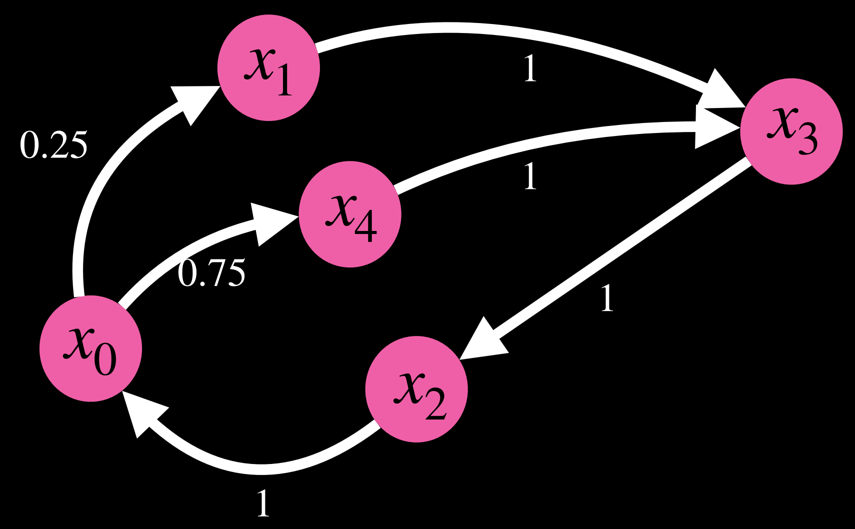

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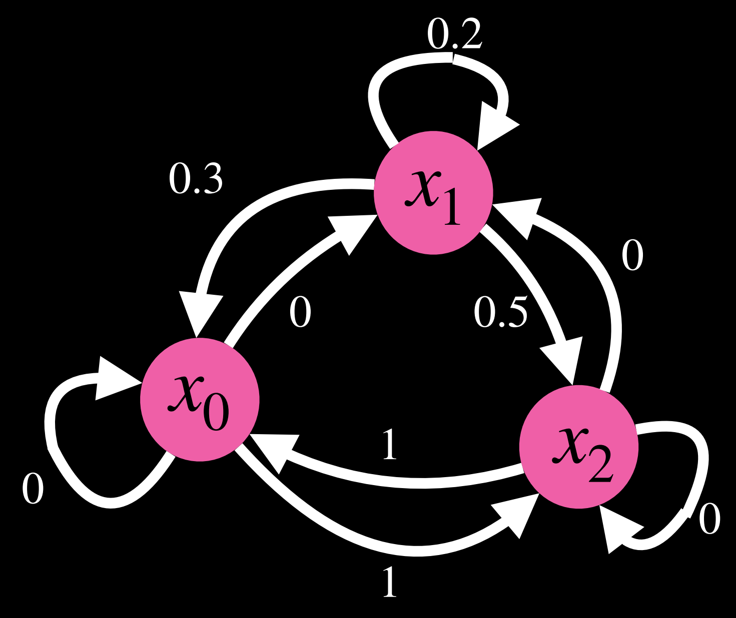

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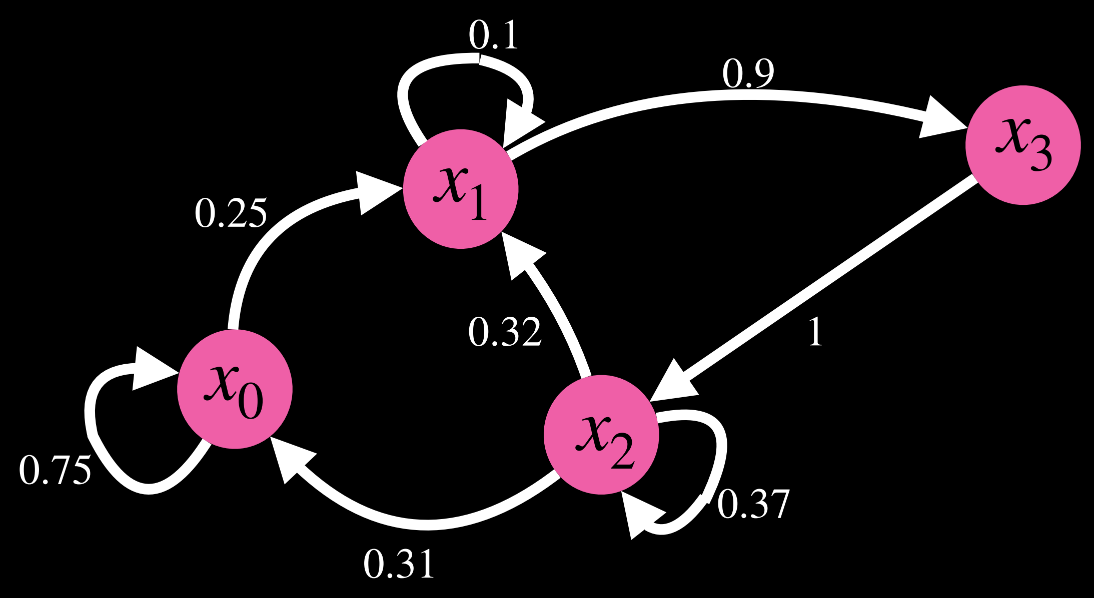

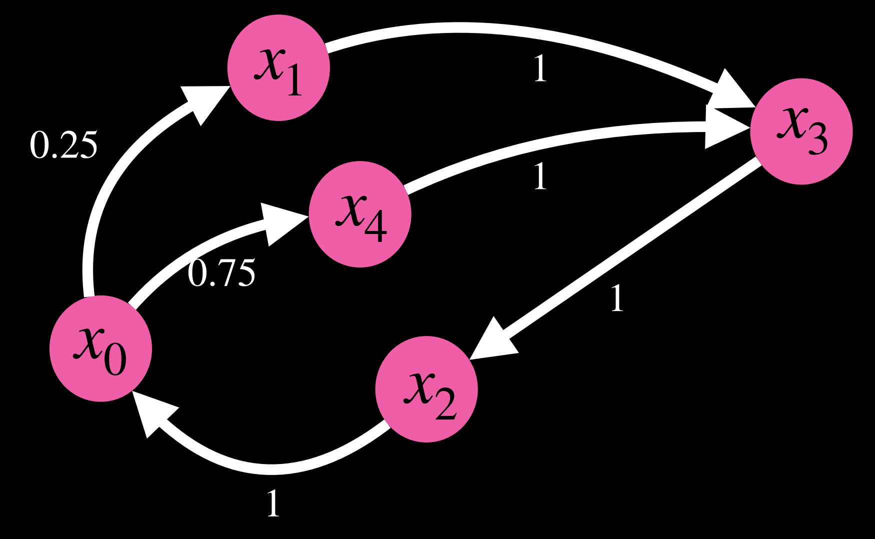

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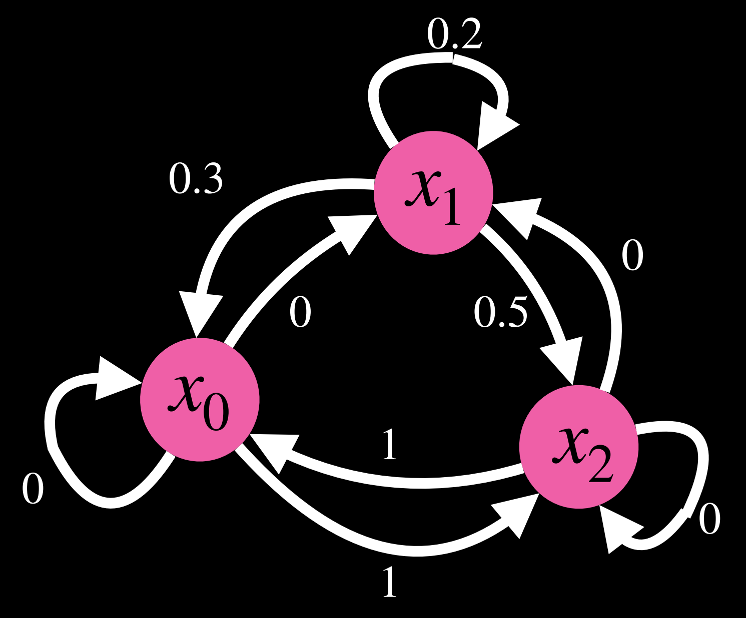

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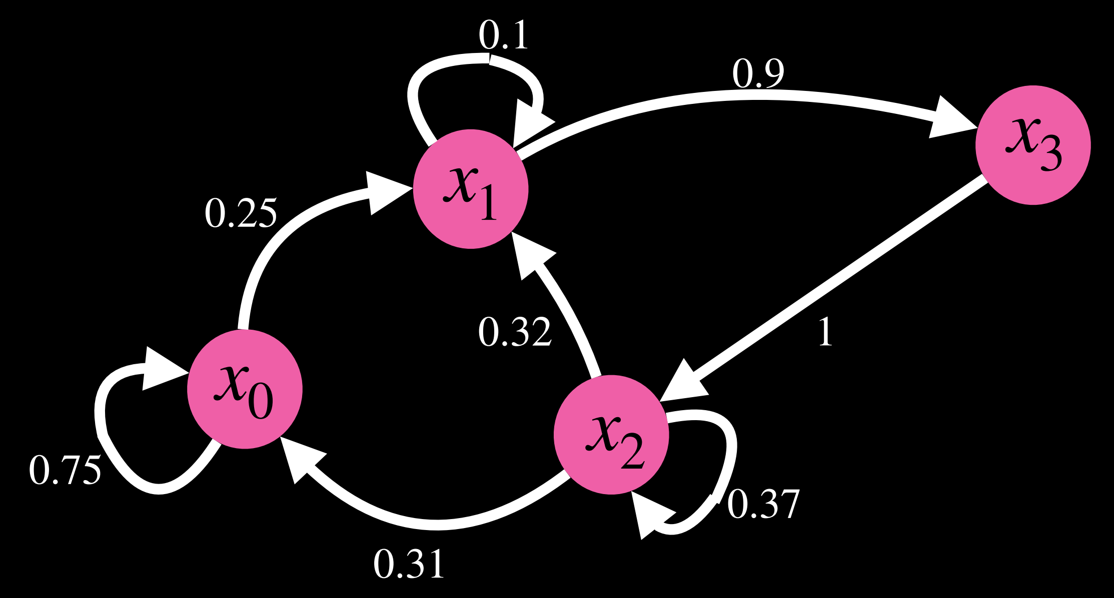

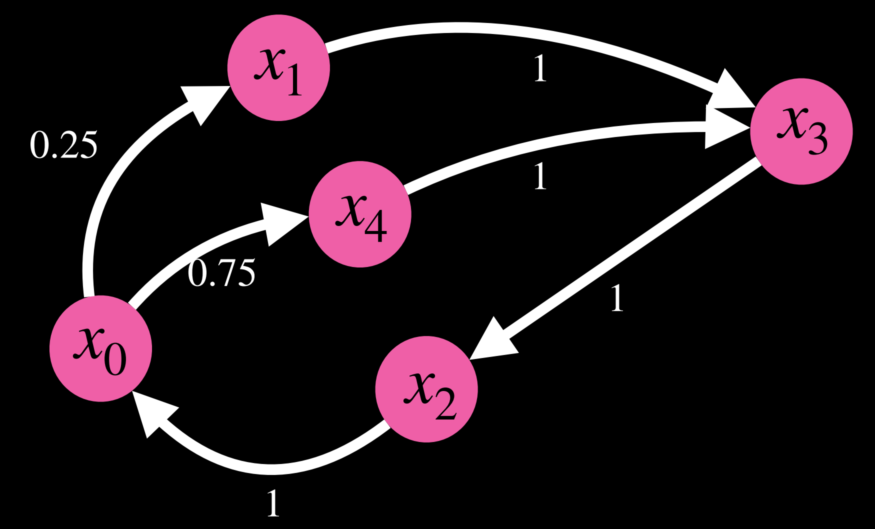

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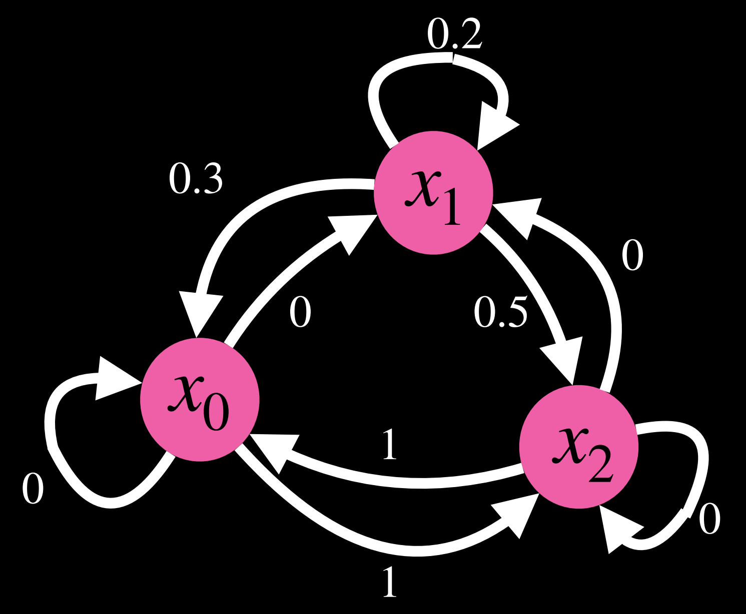

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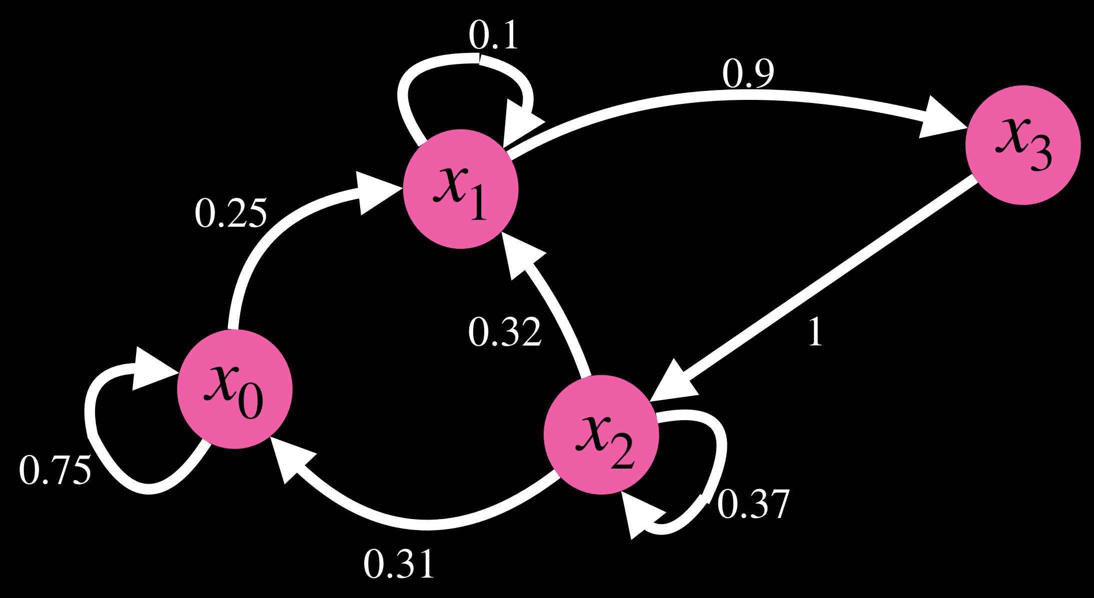

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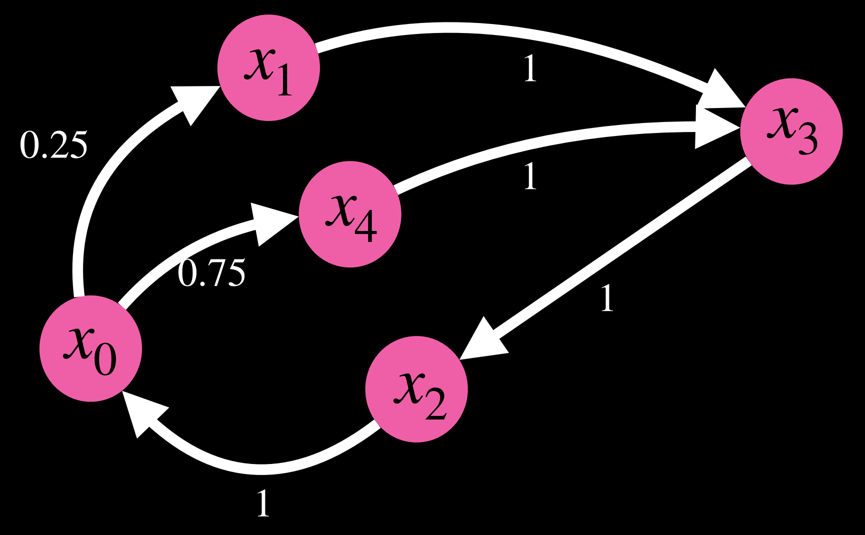

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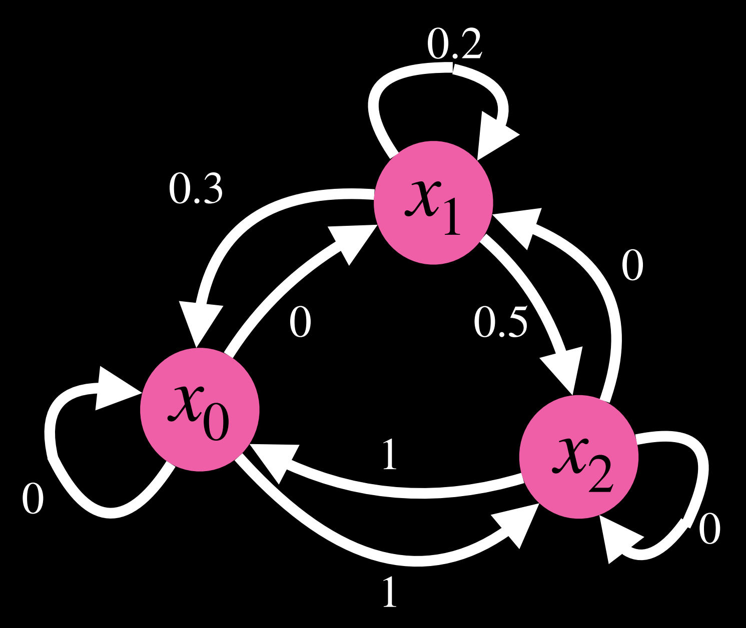

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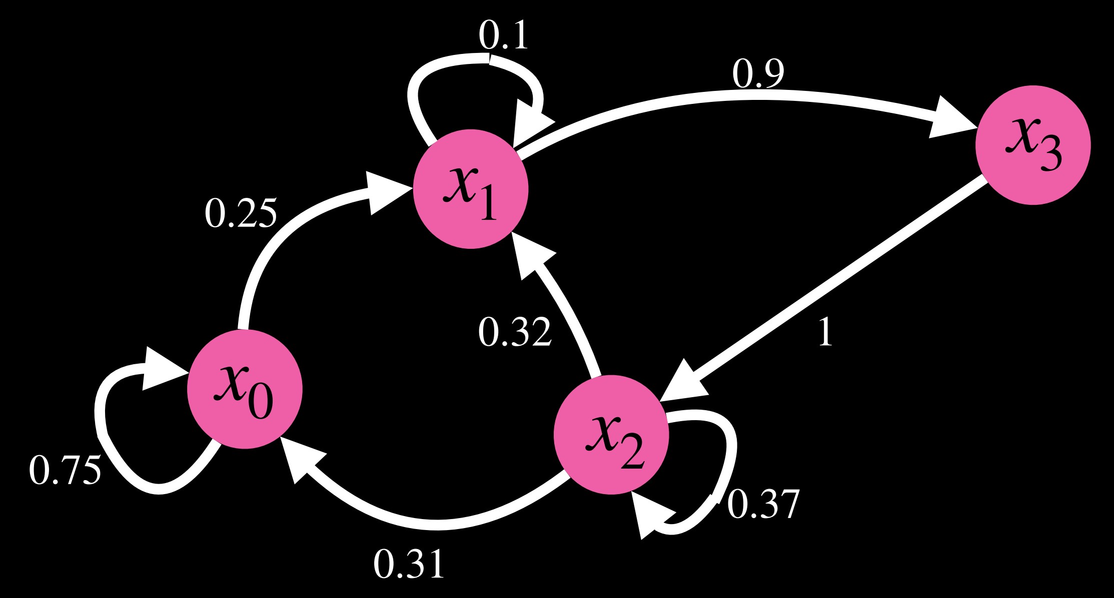

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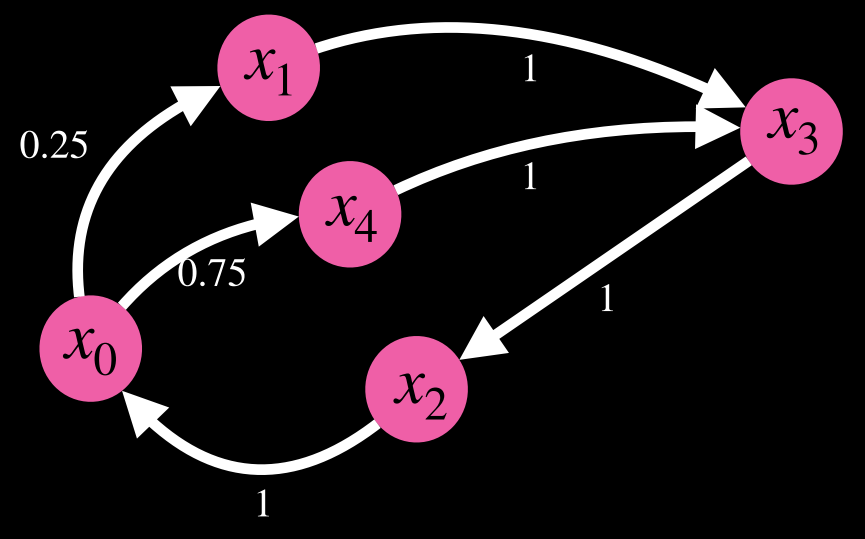

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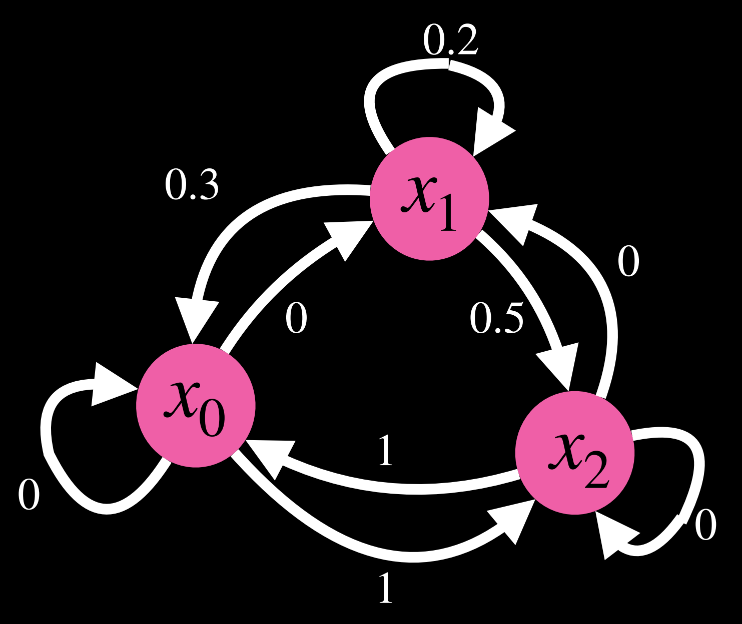

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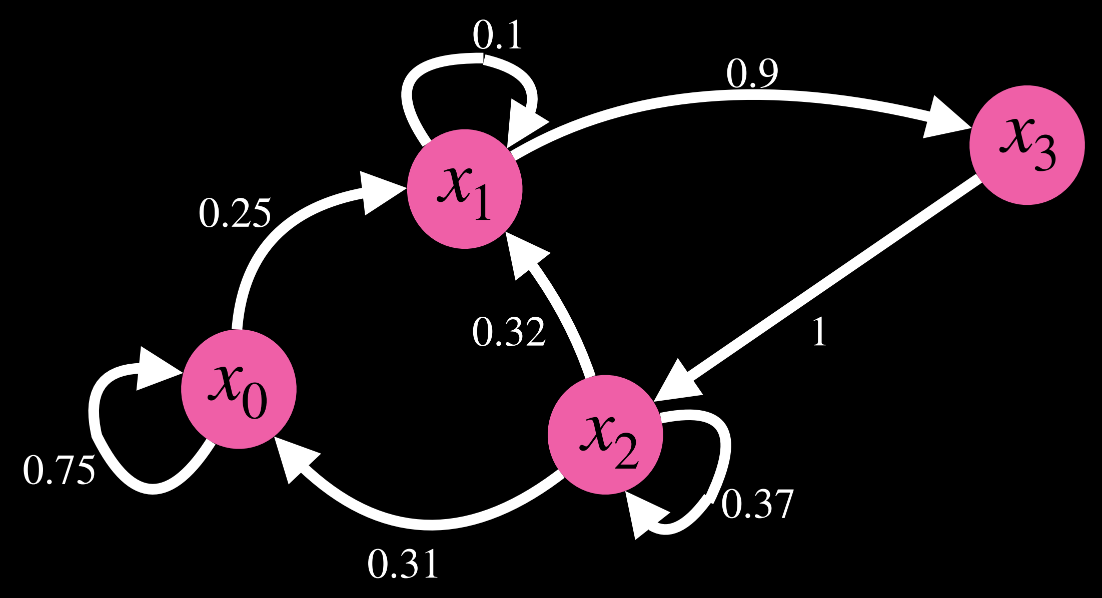

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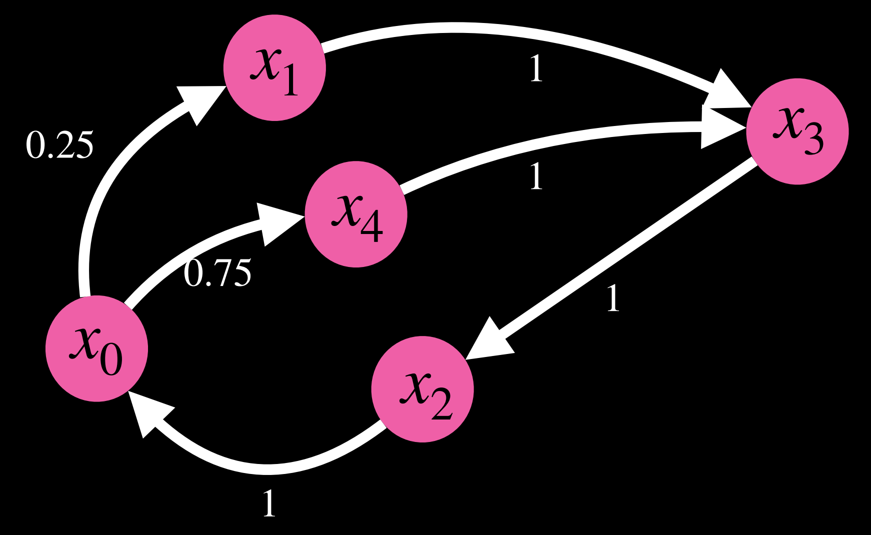

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(In praxis, statistical tests decide if the limiting distribution is reached.)

end