stochastics and probability

Lecture 10

Dr. Johannes Pahlke

optimization

optimization

Goal:

optimization

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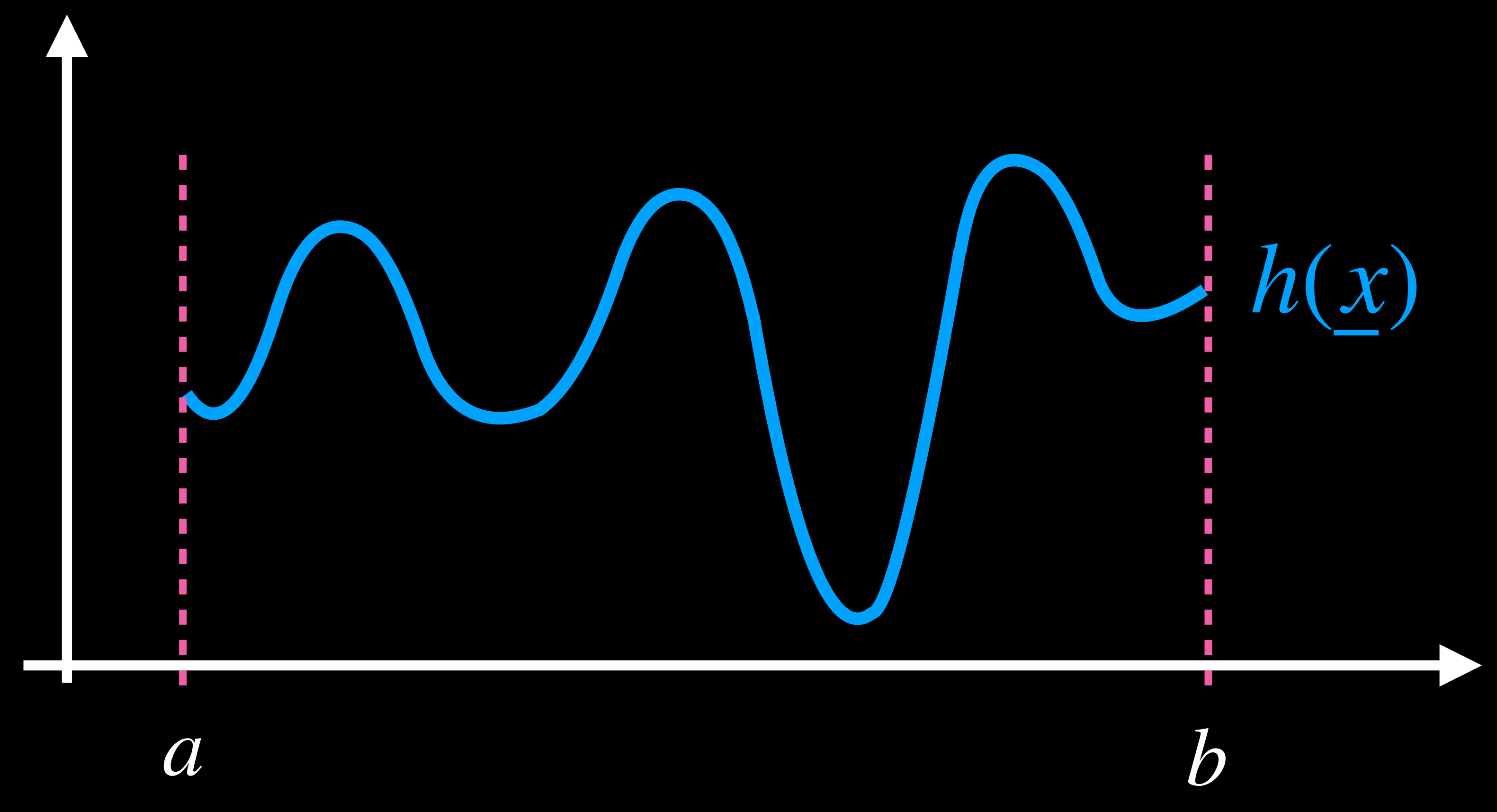

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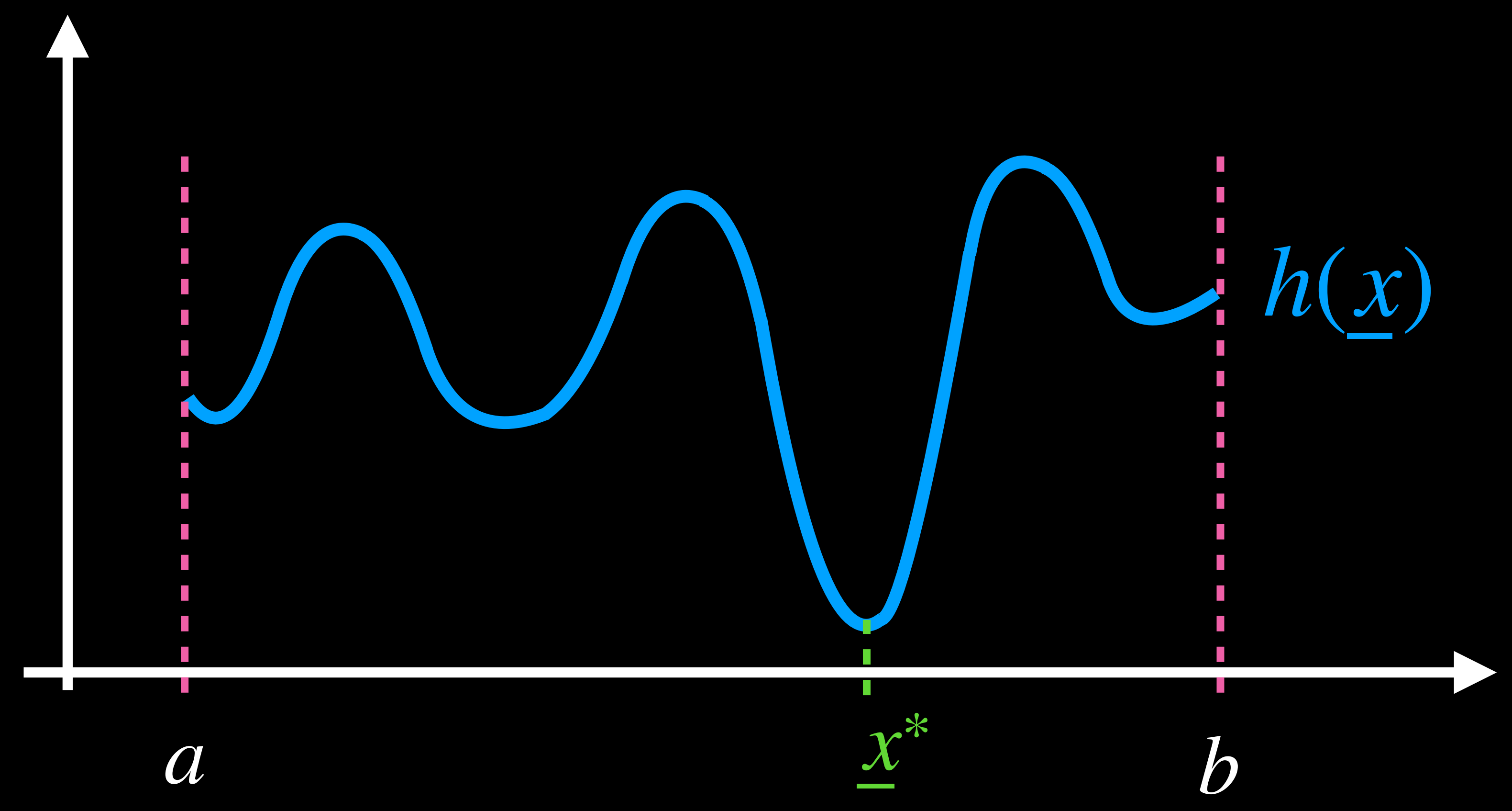

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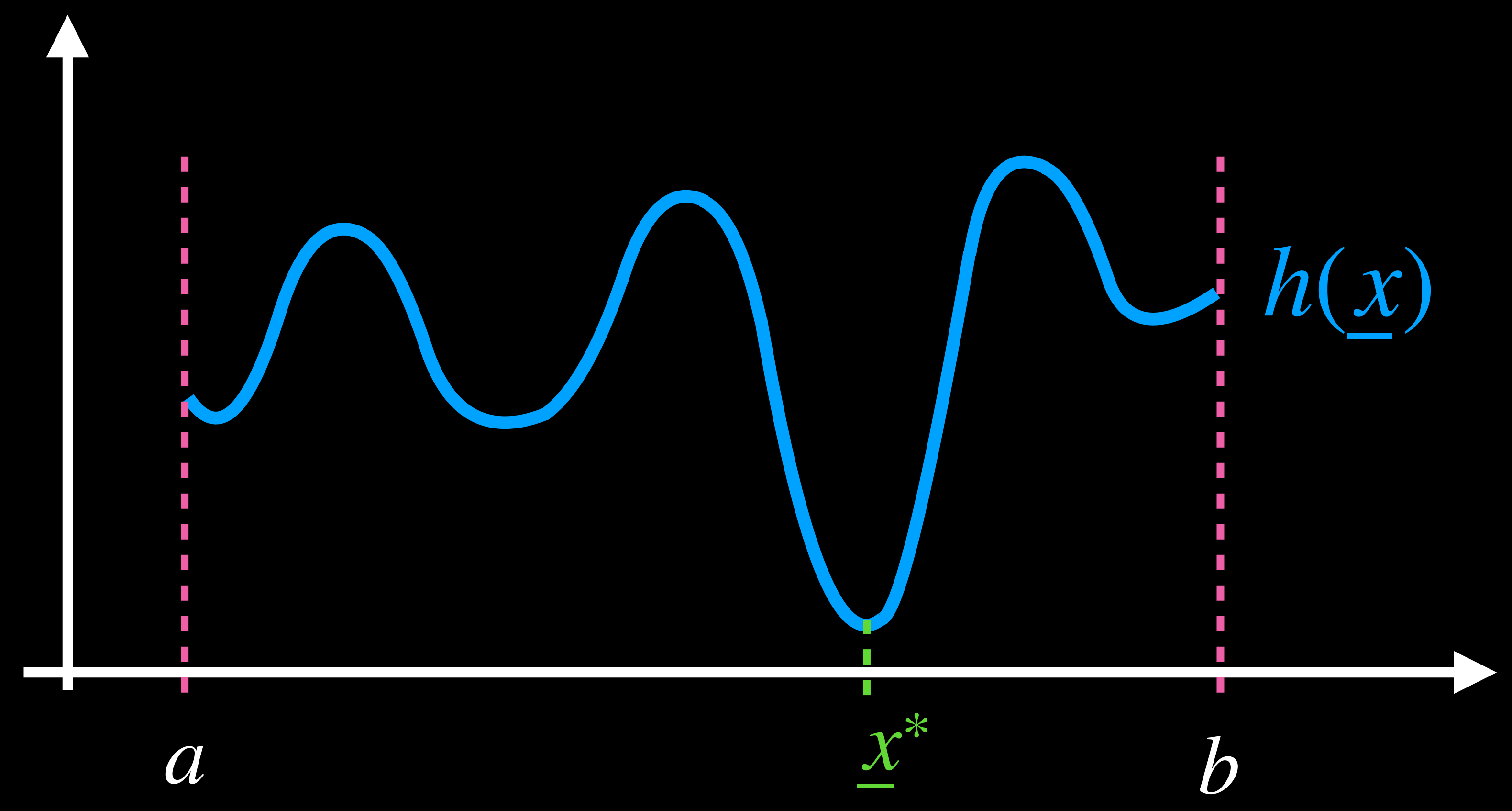
Goal:

$\underline{x}^* :=$



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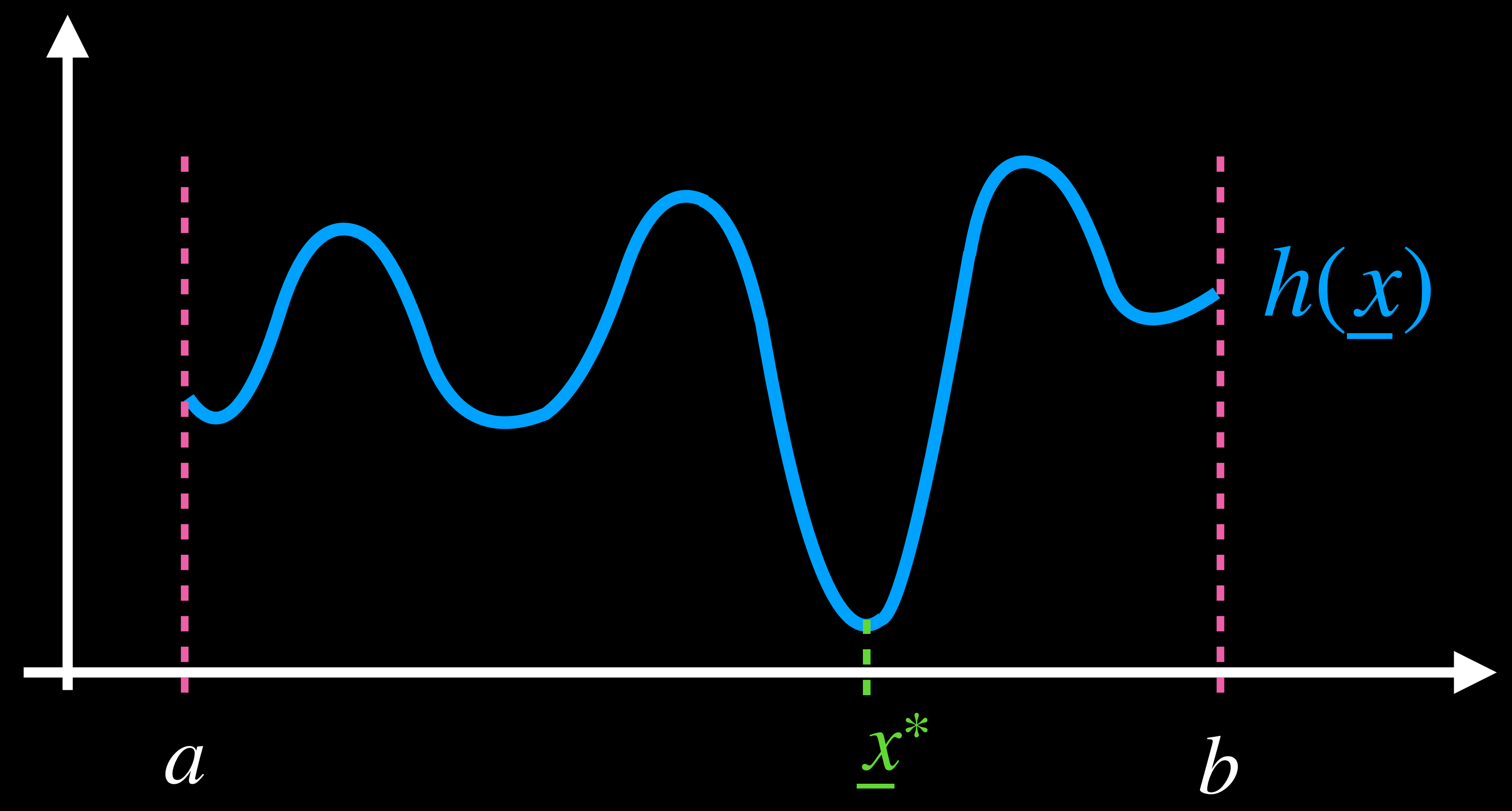
$$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\left(h(\underline{x})\right)$$



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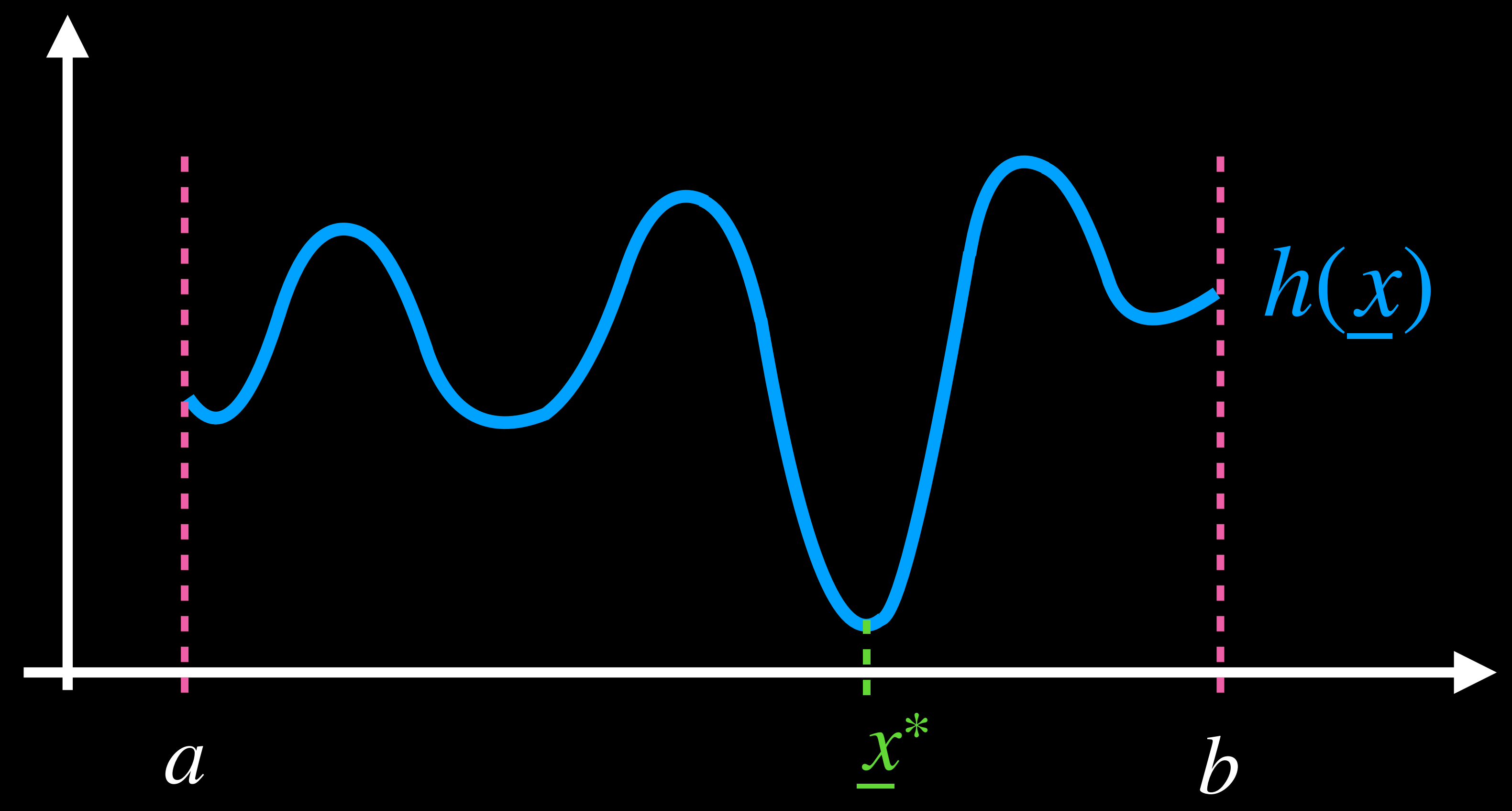

$\mathbb{S} := [a, b]$

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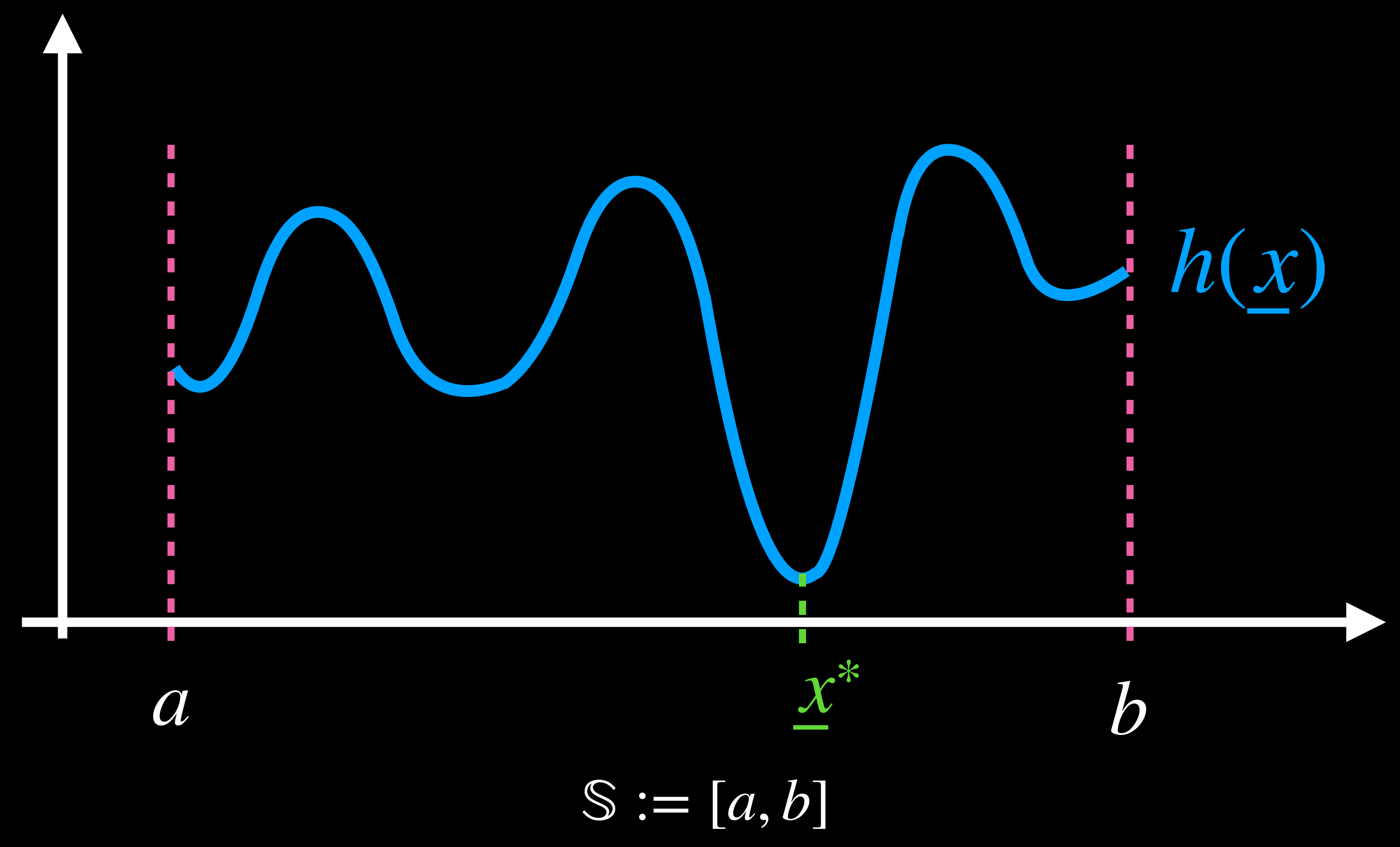
$$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\left(h(\underline{x})\right) \qquad \left(\longleftrightarrow \qquad h(\underline{x}^*) = \min_{\underline{x}\in\mathbb{S}}\left(h(\underline{x})\right)\right)$$



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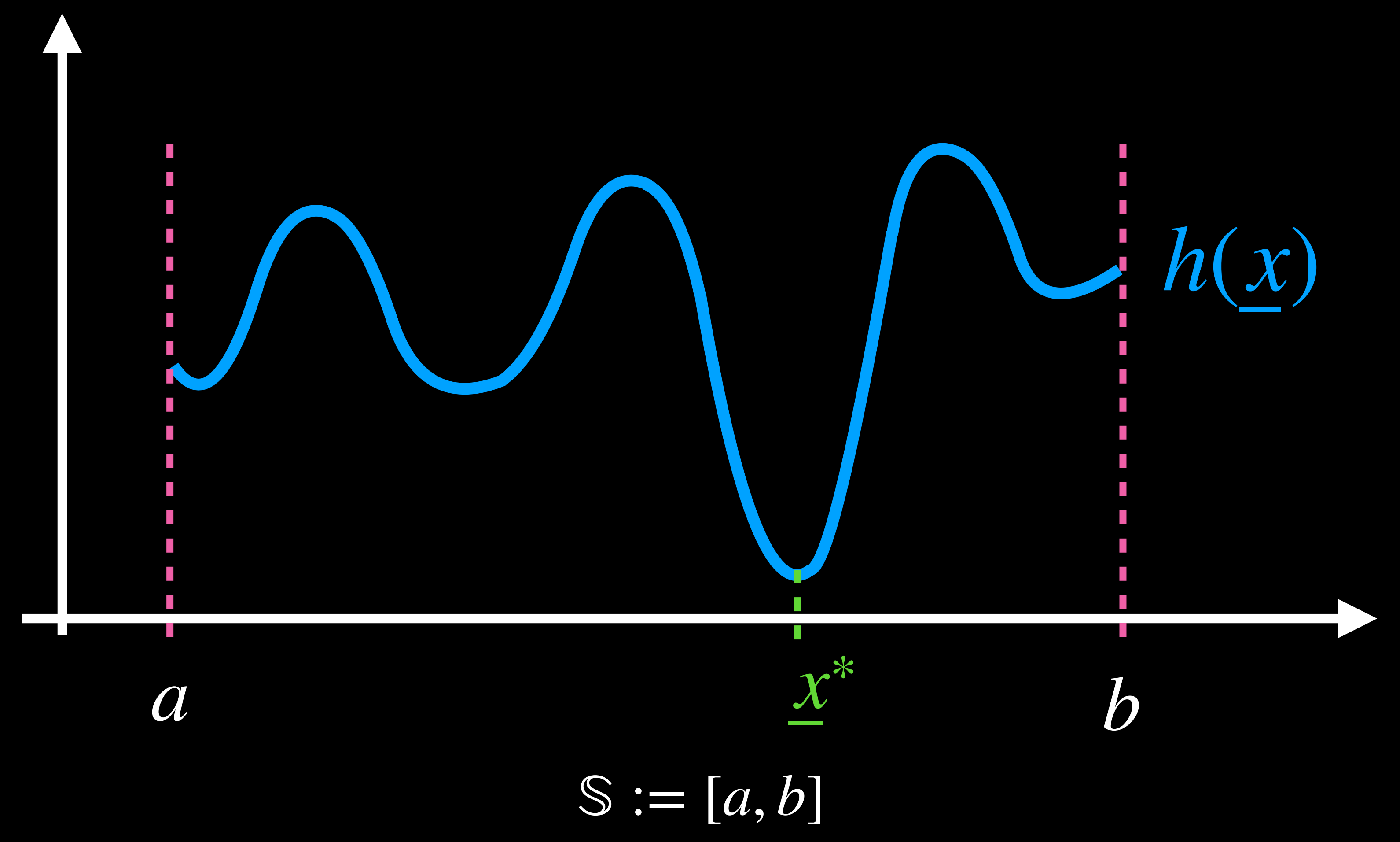

Challenges:

optimization

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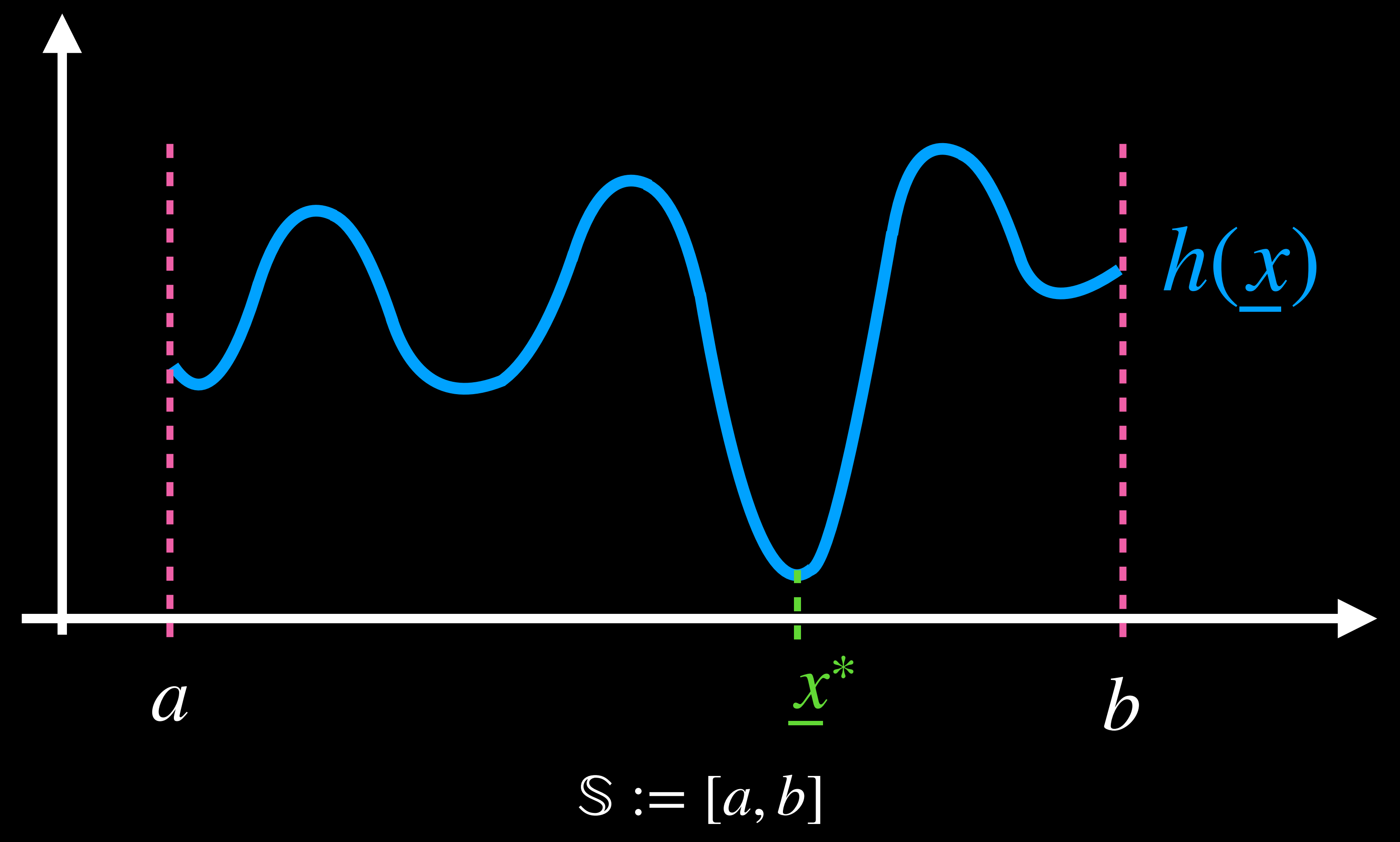
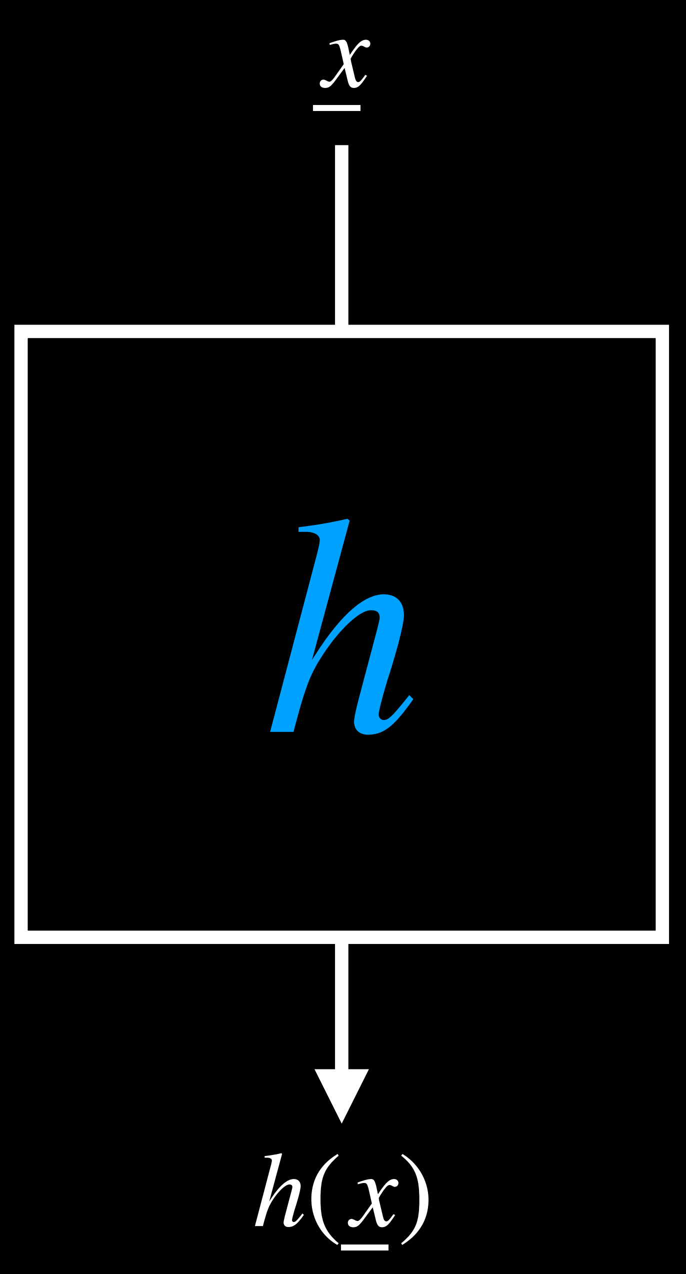
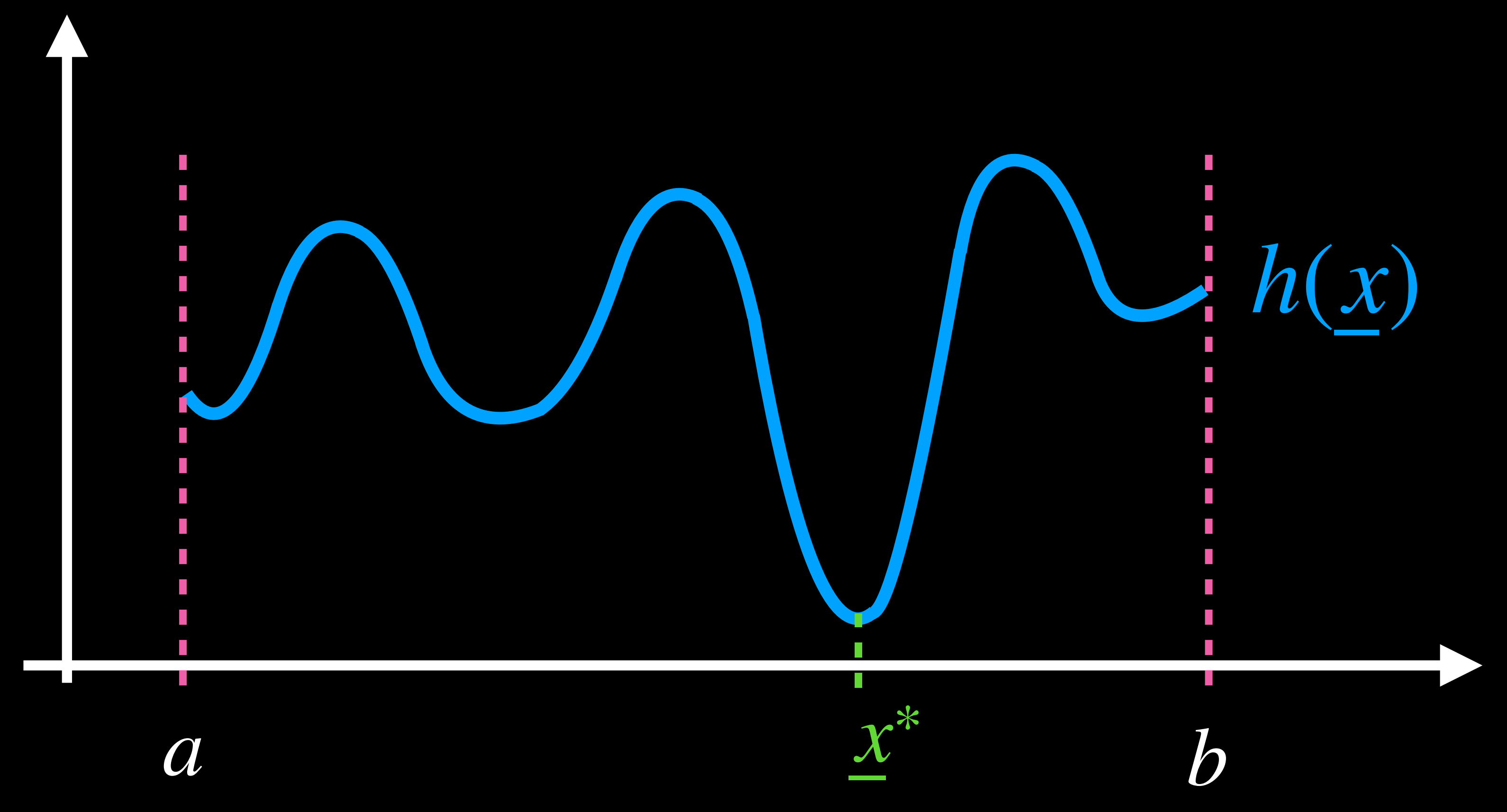


Challenges:

black box



optimization



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Challenges:

black box

$\underline{x}$

$h(\underline{x})$

high dimentional

$h(x_1, x_2, x_3, \ldots, x_{100})$

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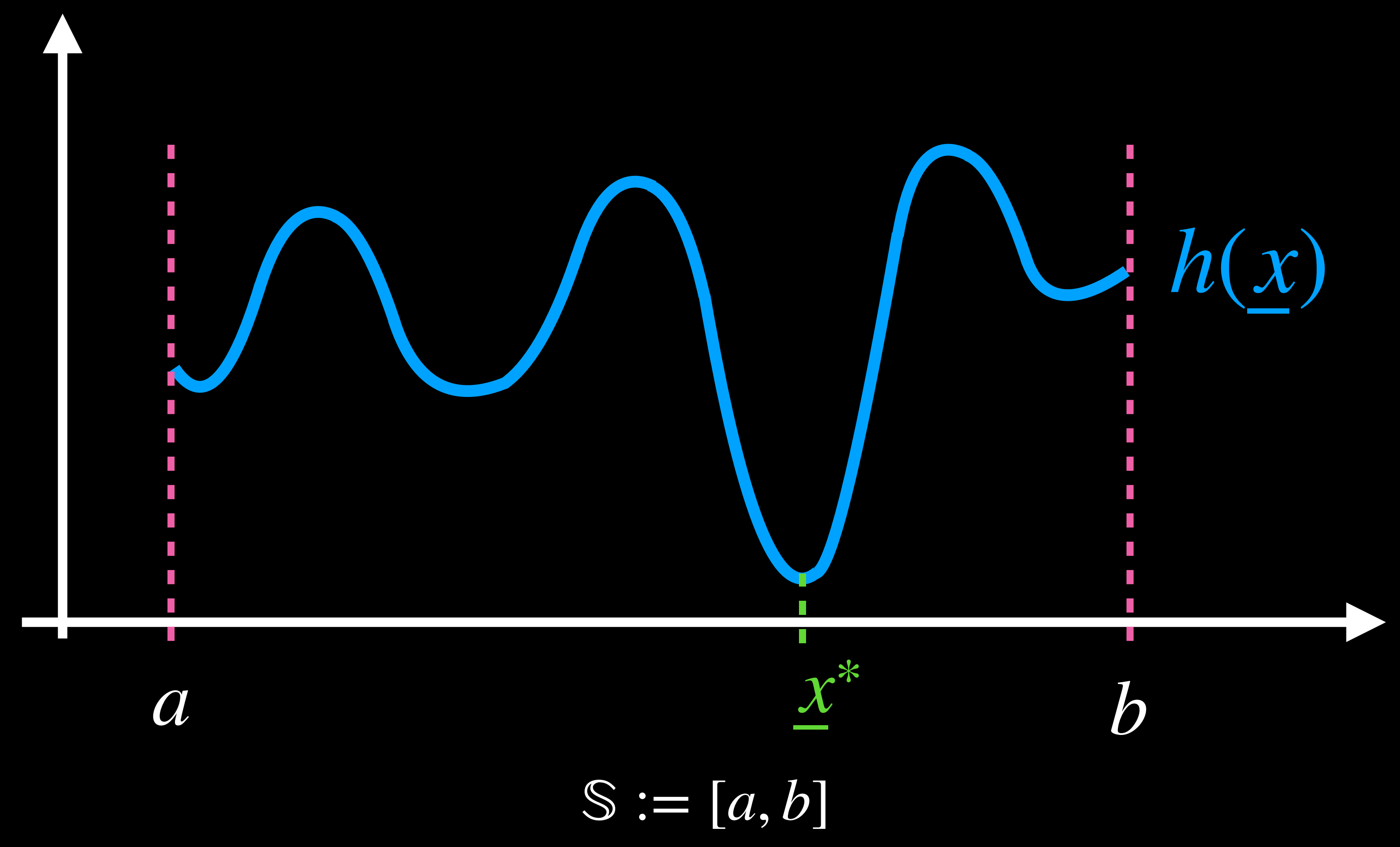

Challenges:

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function evaluation are expensive

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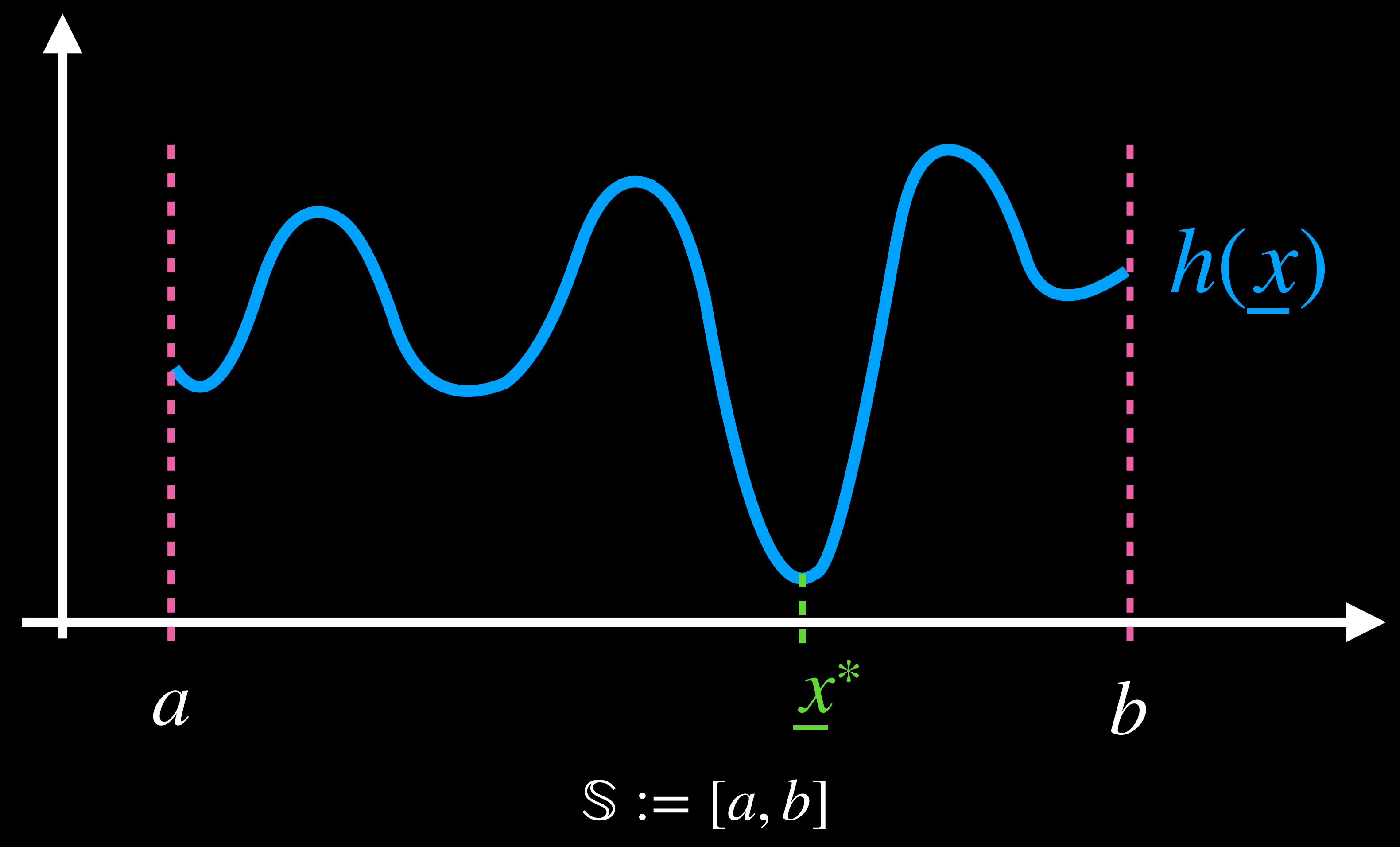

Challenges:

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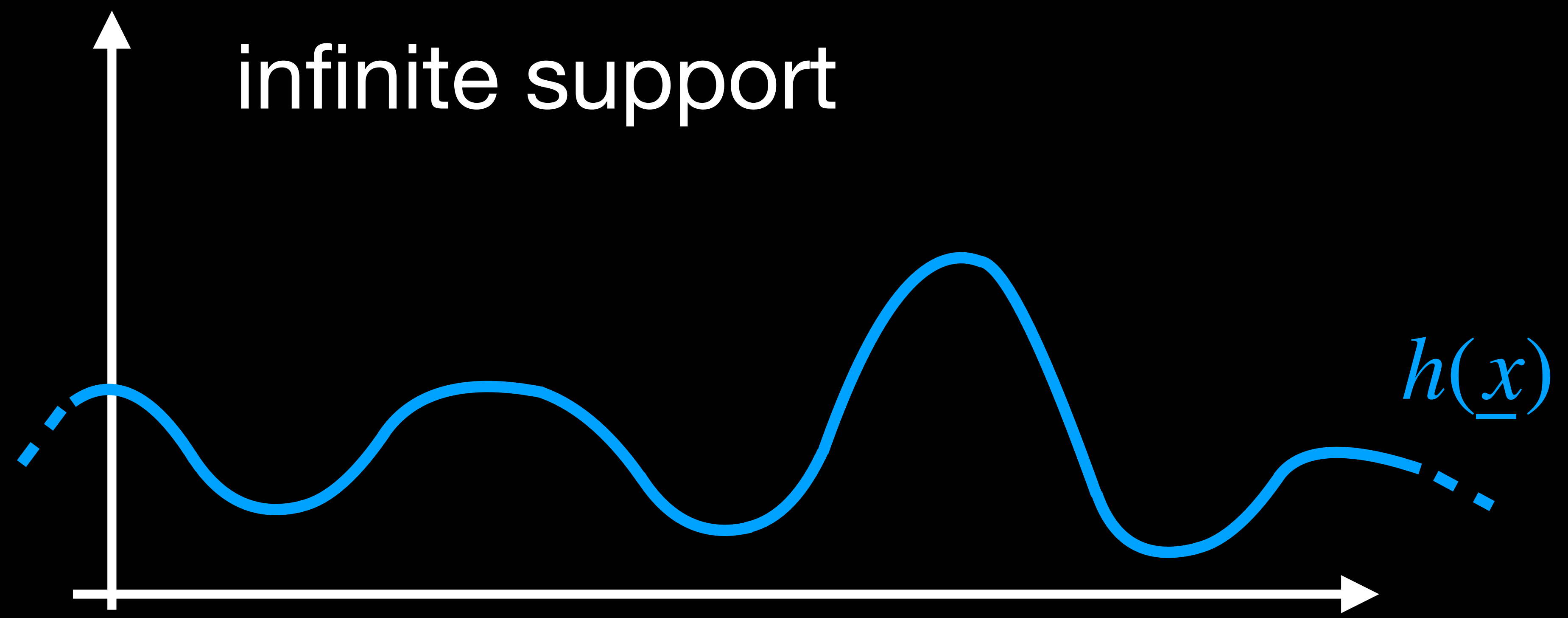
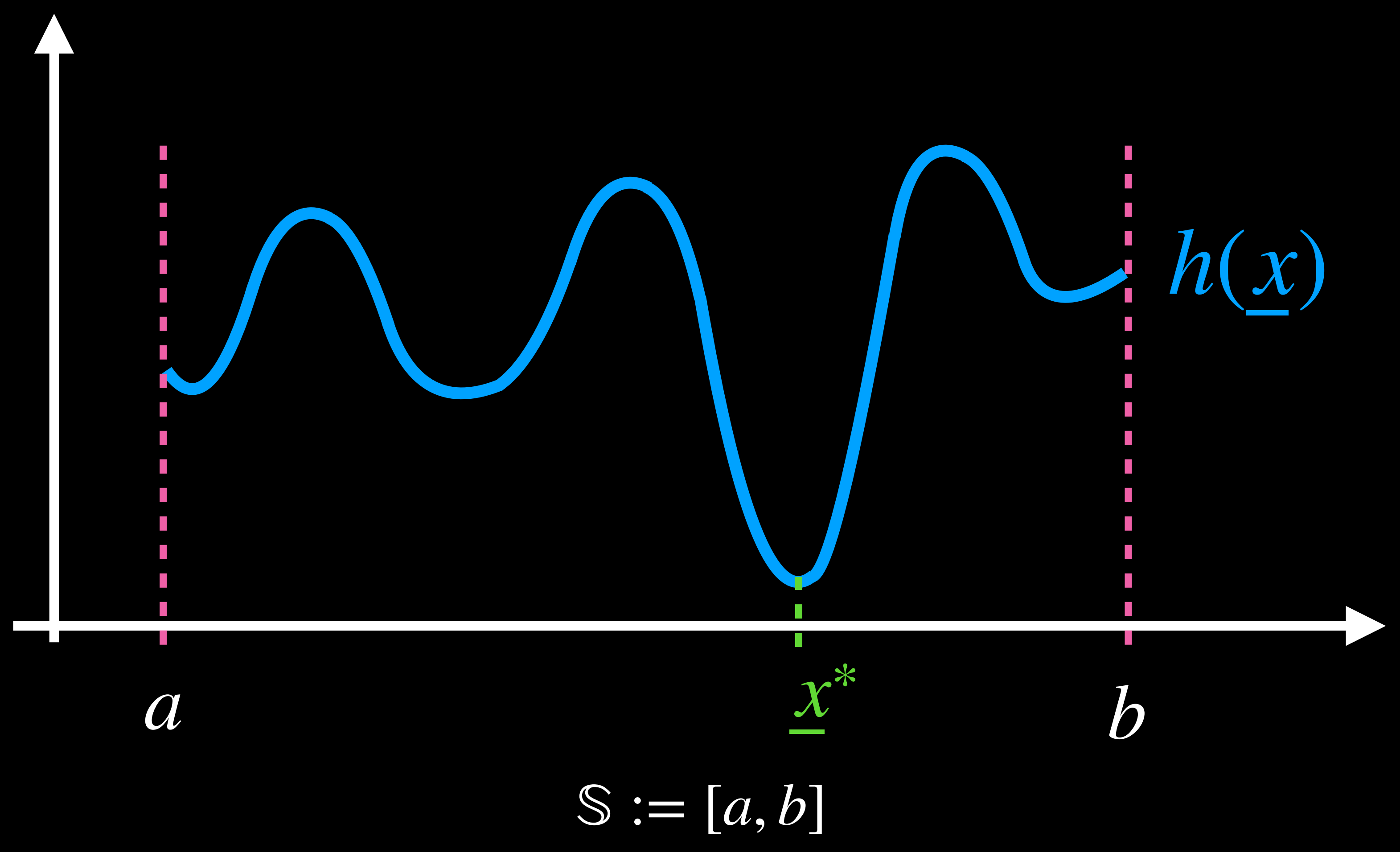
function evaluation are expensive

infinite support



optimization



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Challenges:

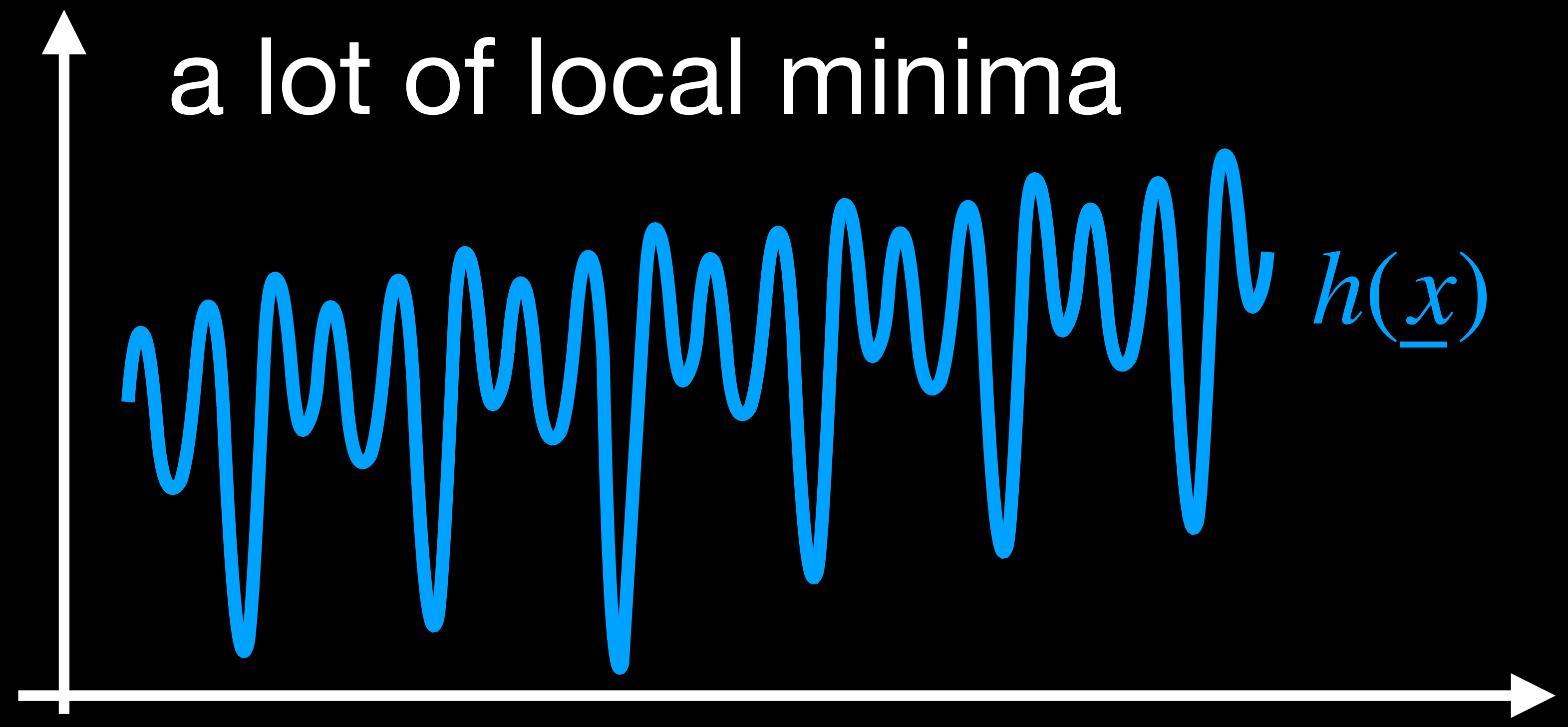
black box

high dimentional

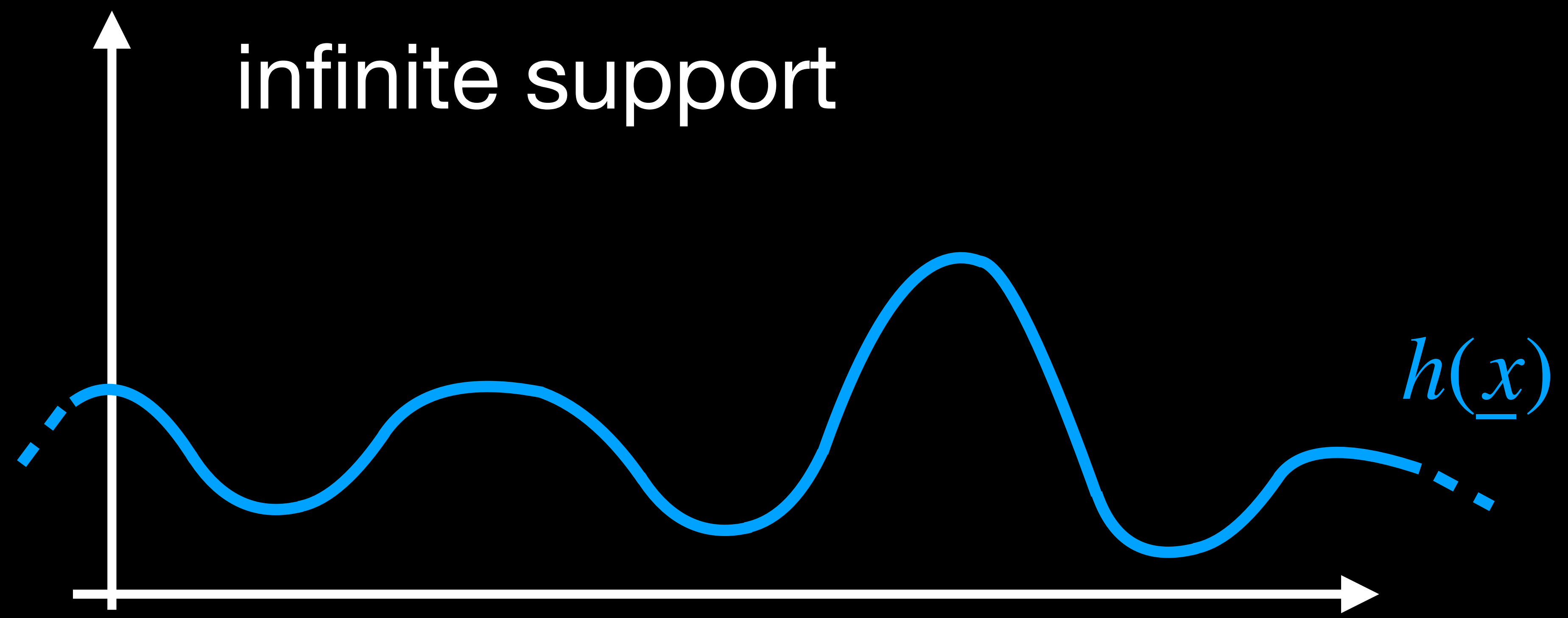
$h(x_1, x_2, x_3, \ldots, x_{100})$

function evaluation are expensive

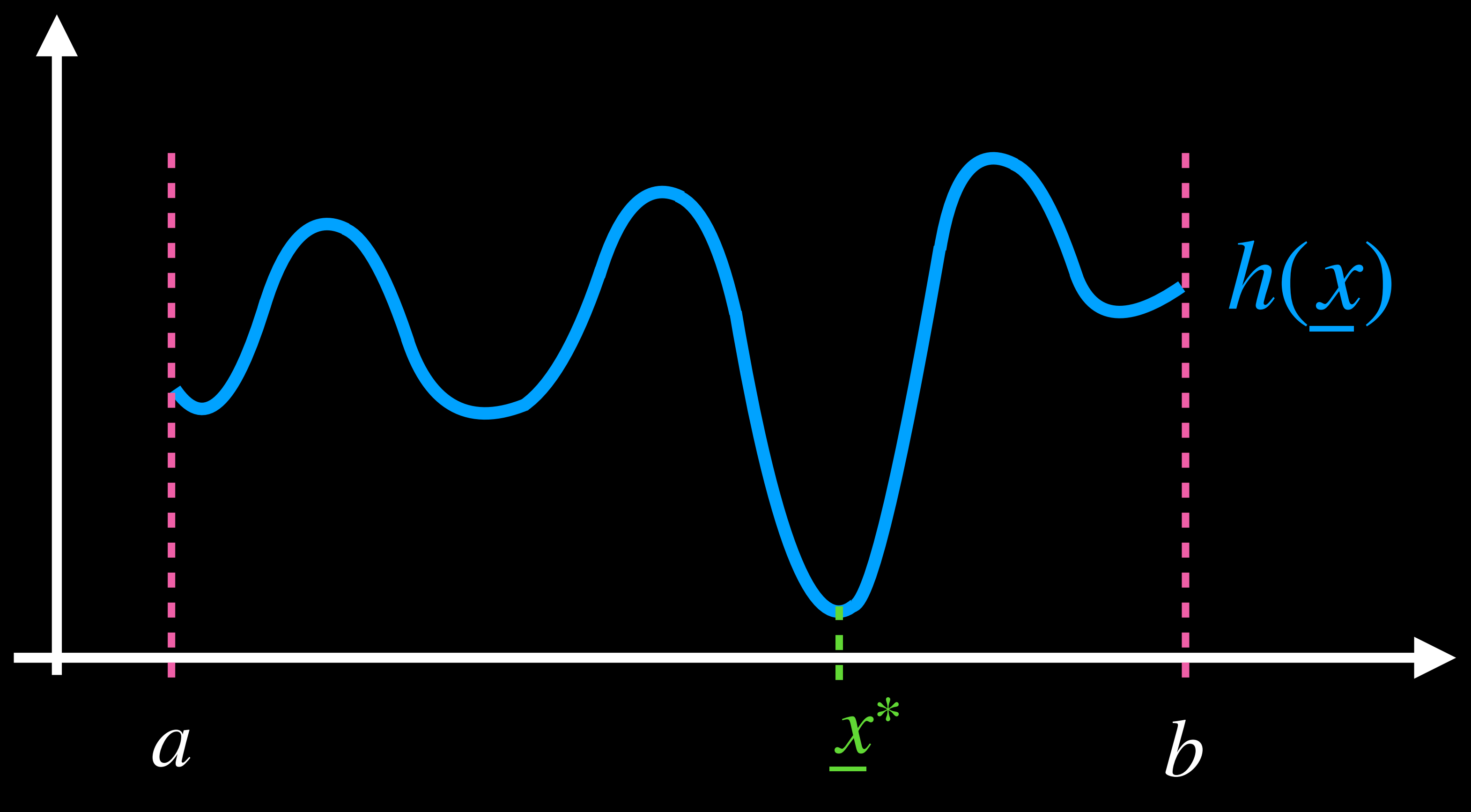
a lot of local minima



infinite support



optimization



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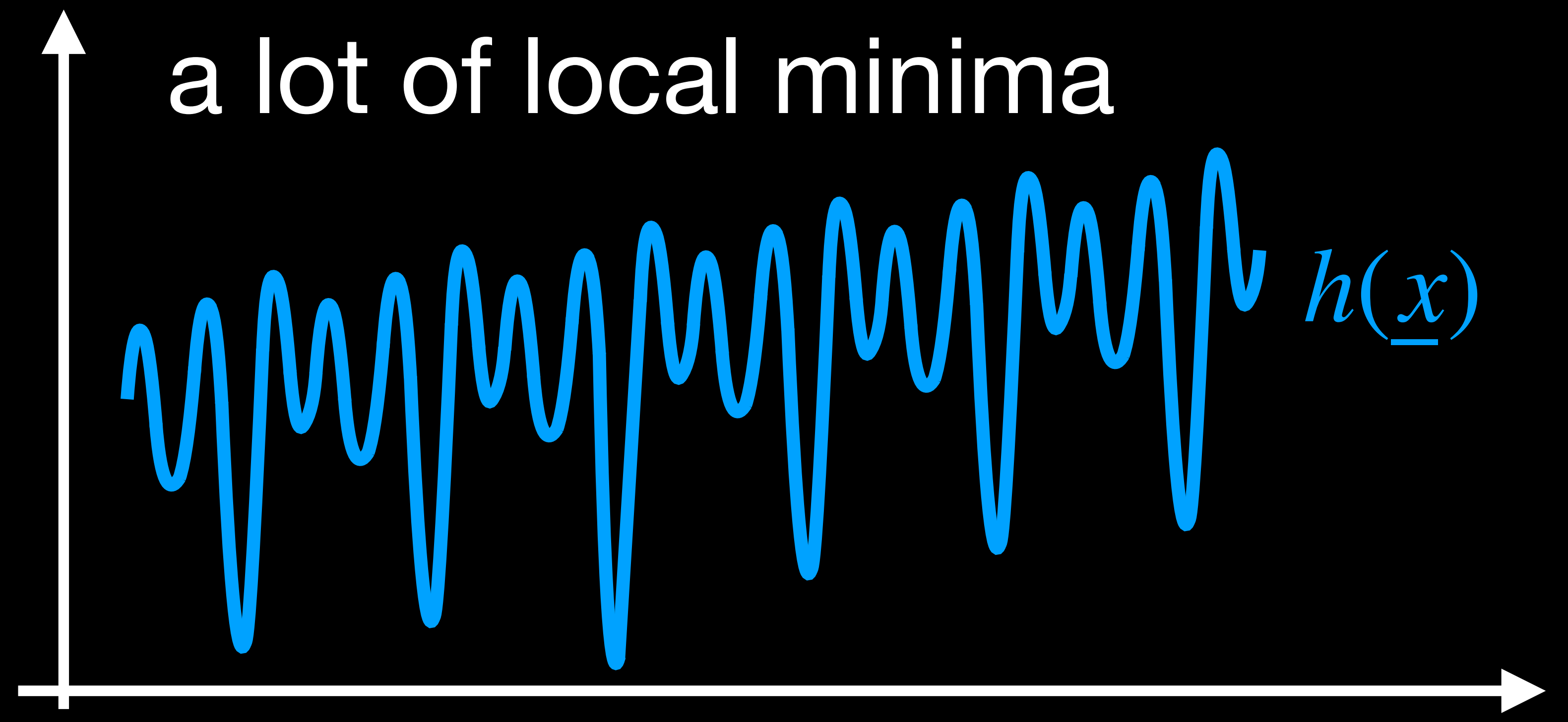

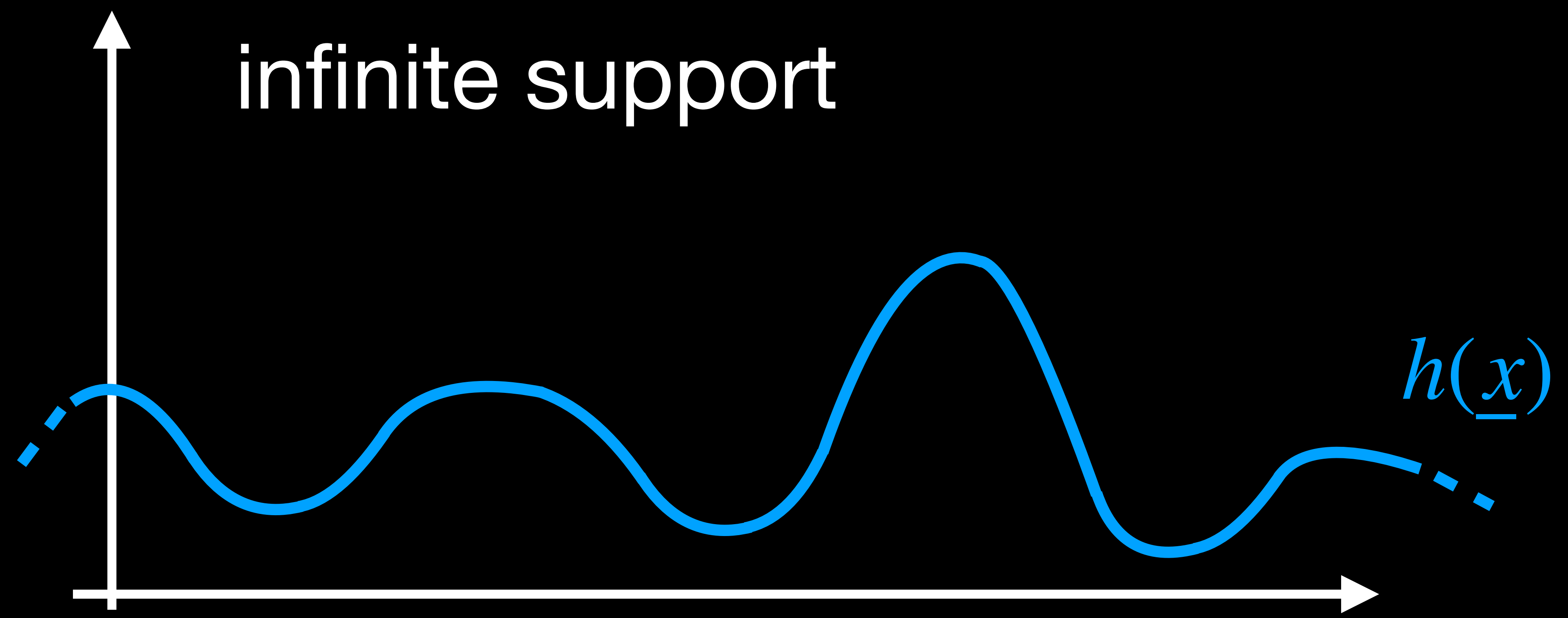

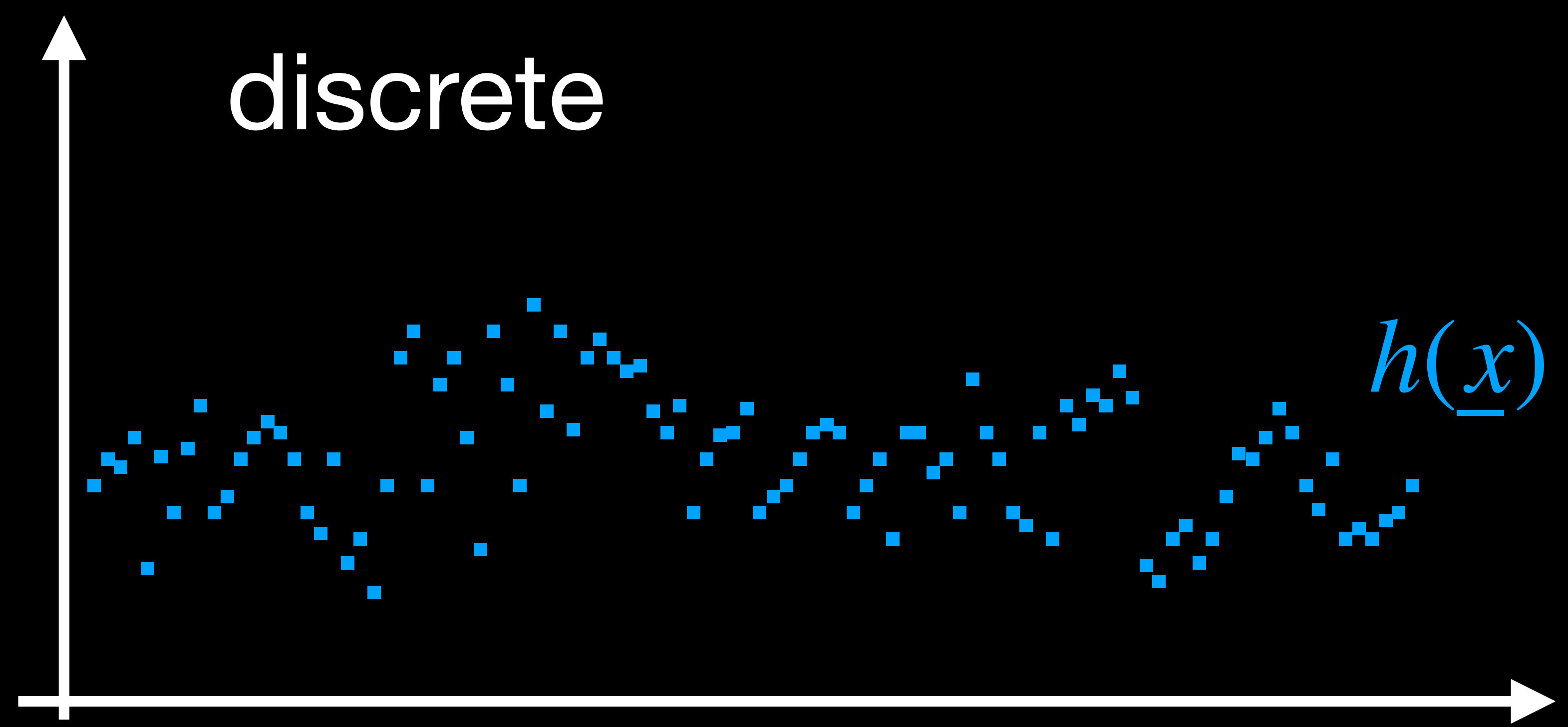

algorithm 1: stochastic exploration

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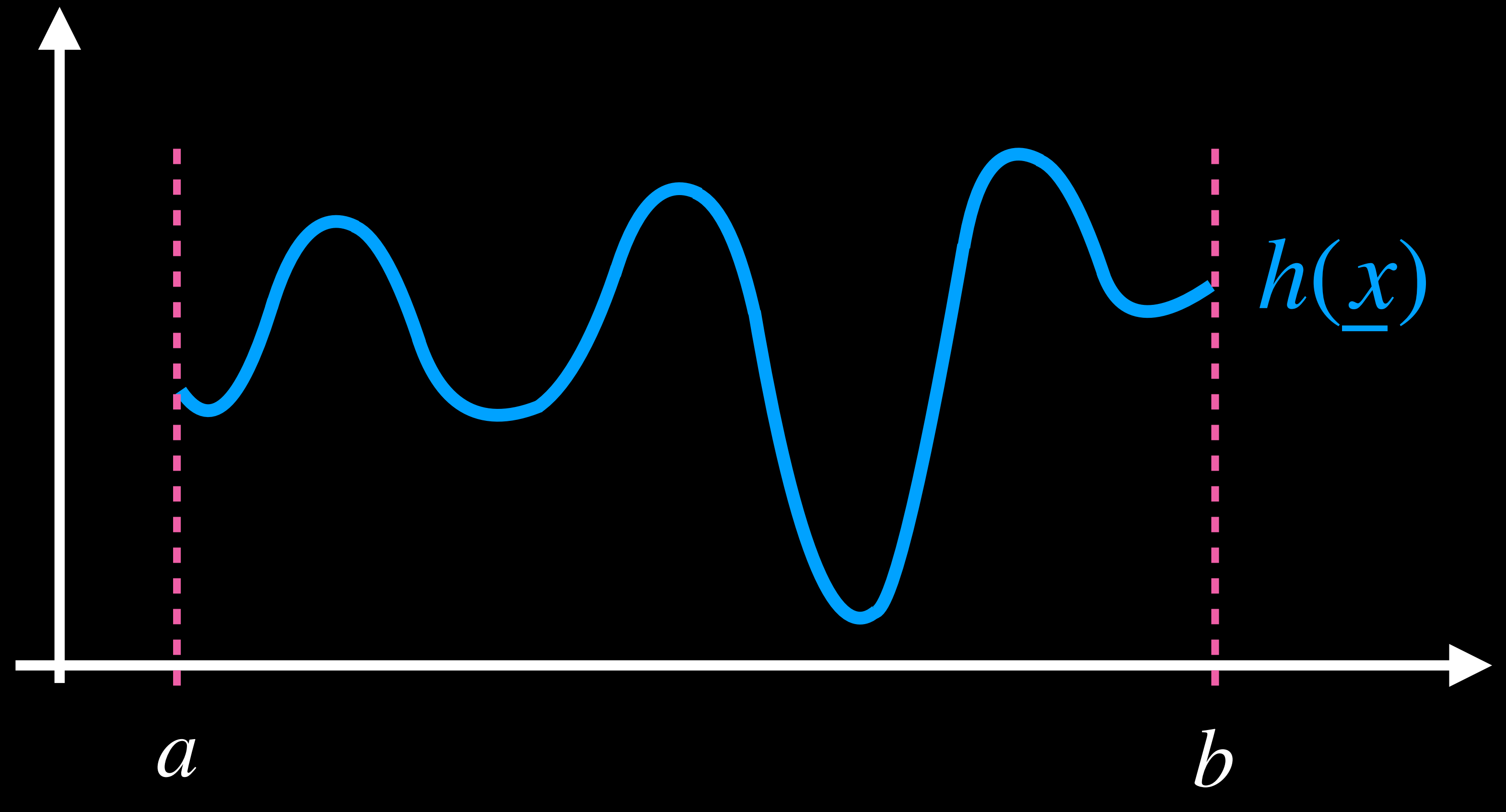

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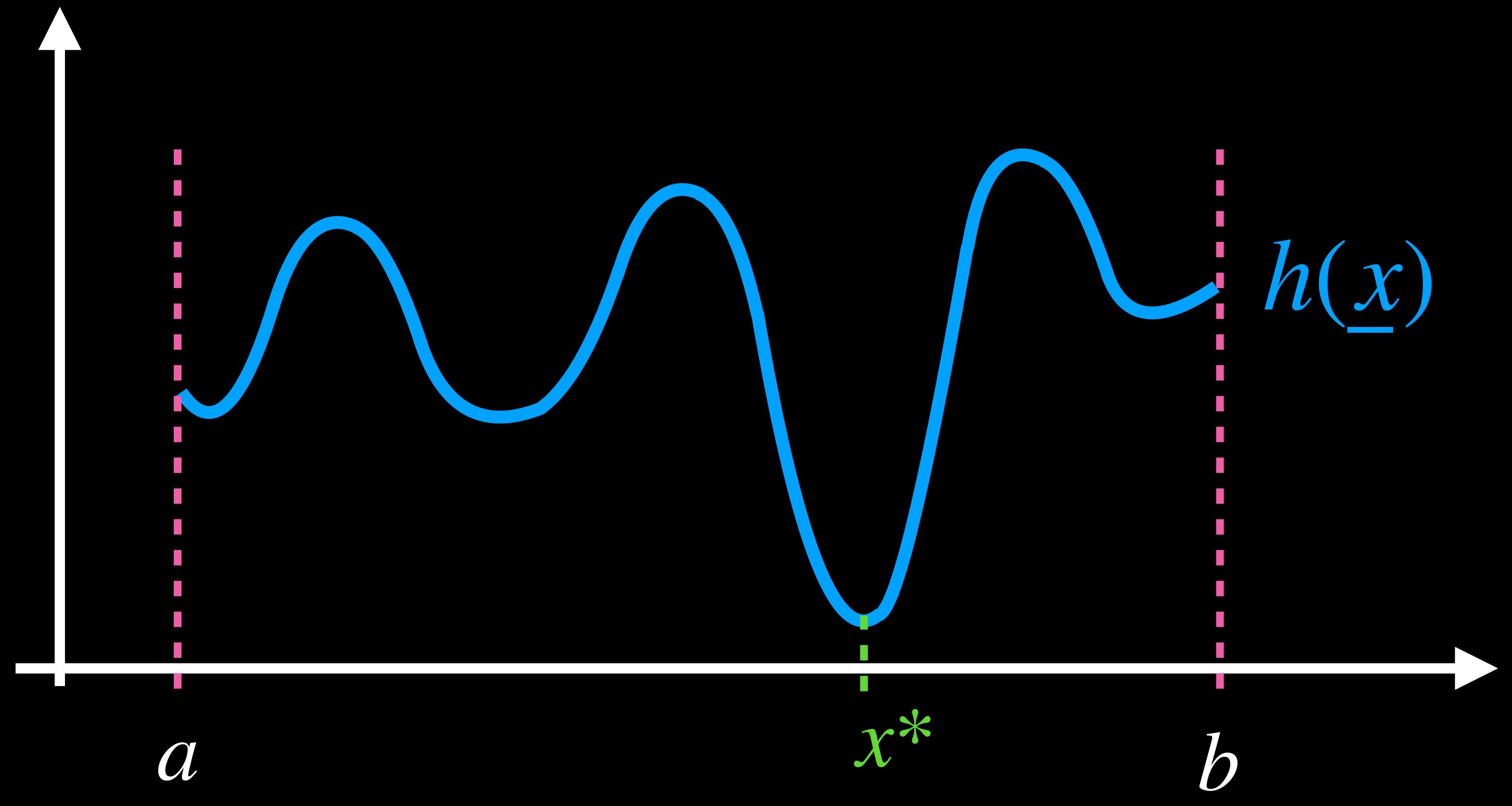

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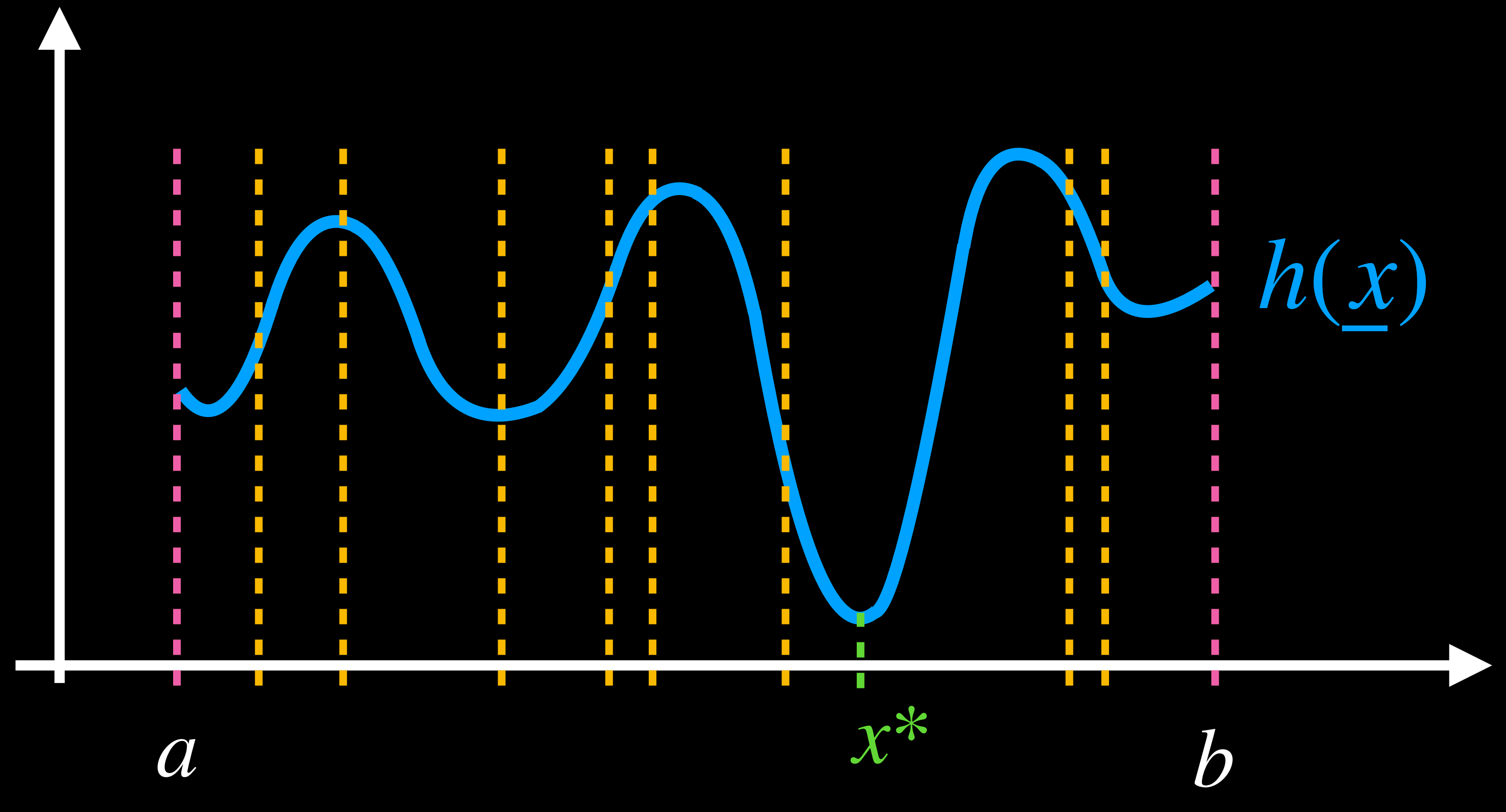

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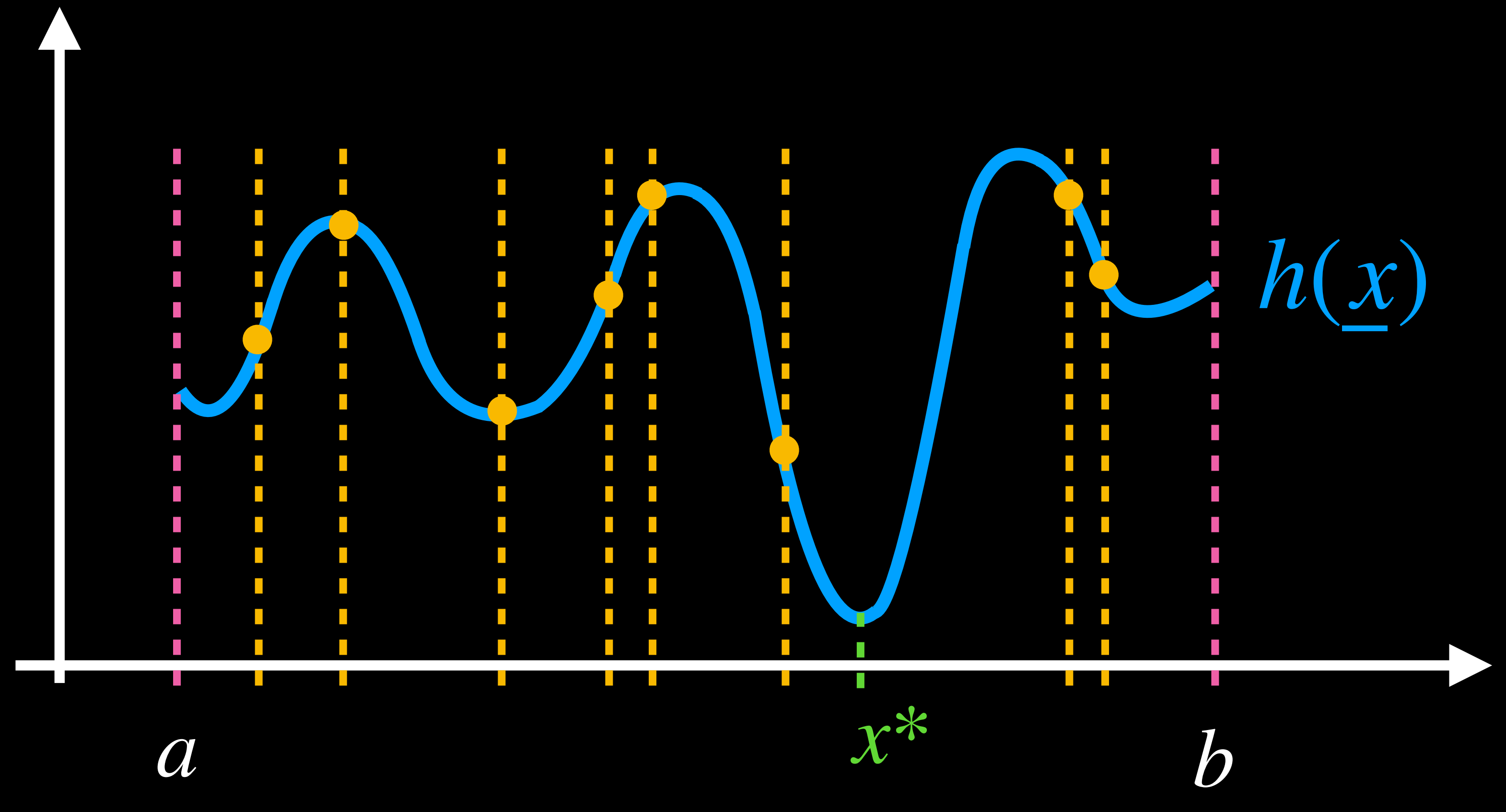

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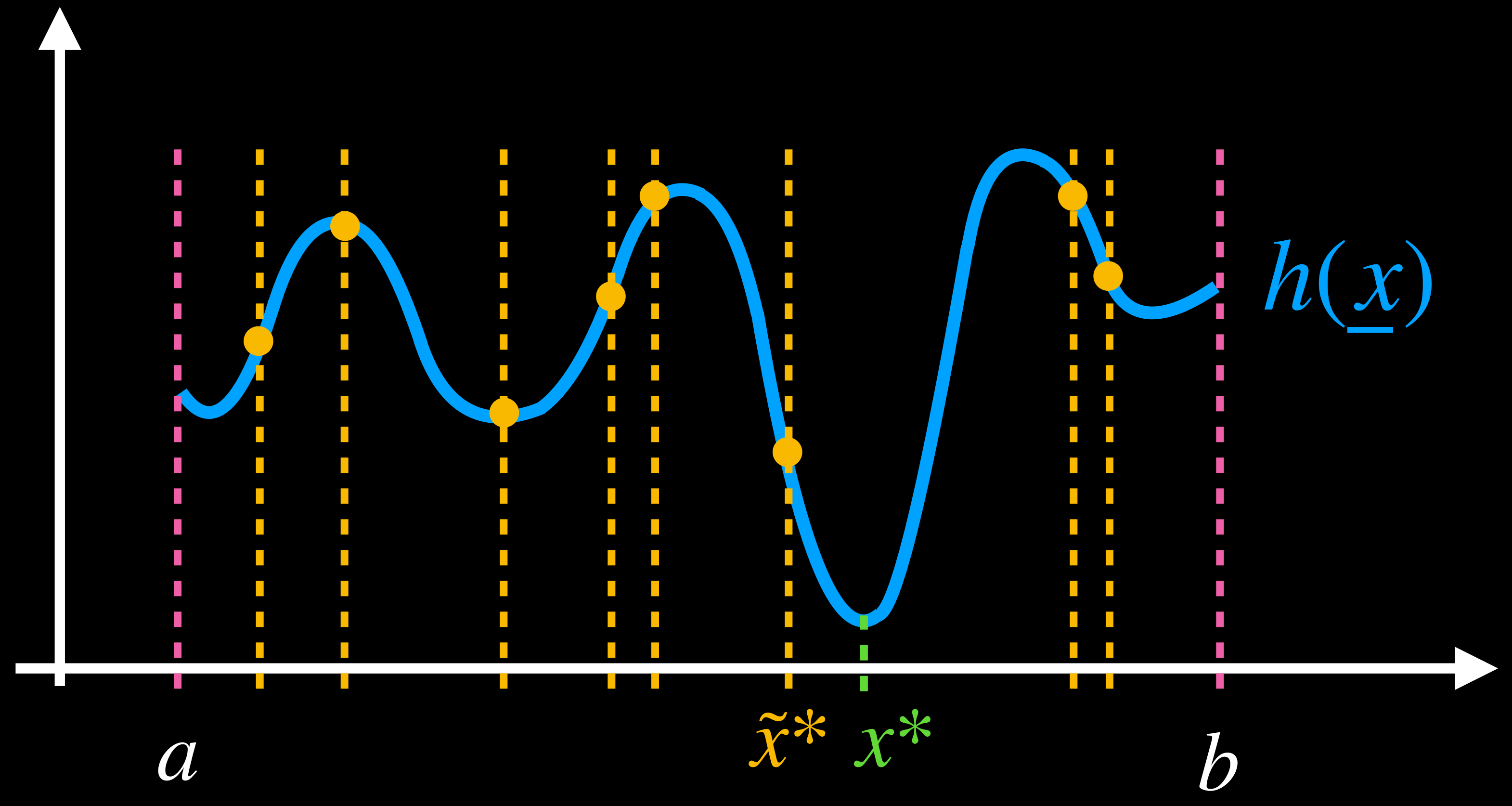

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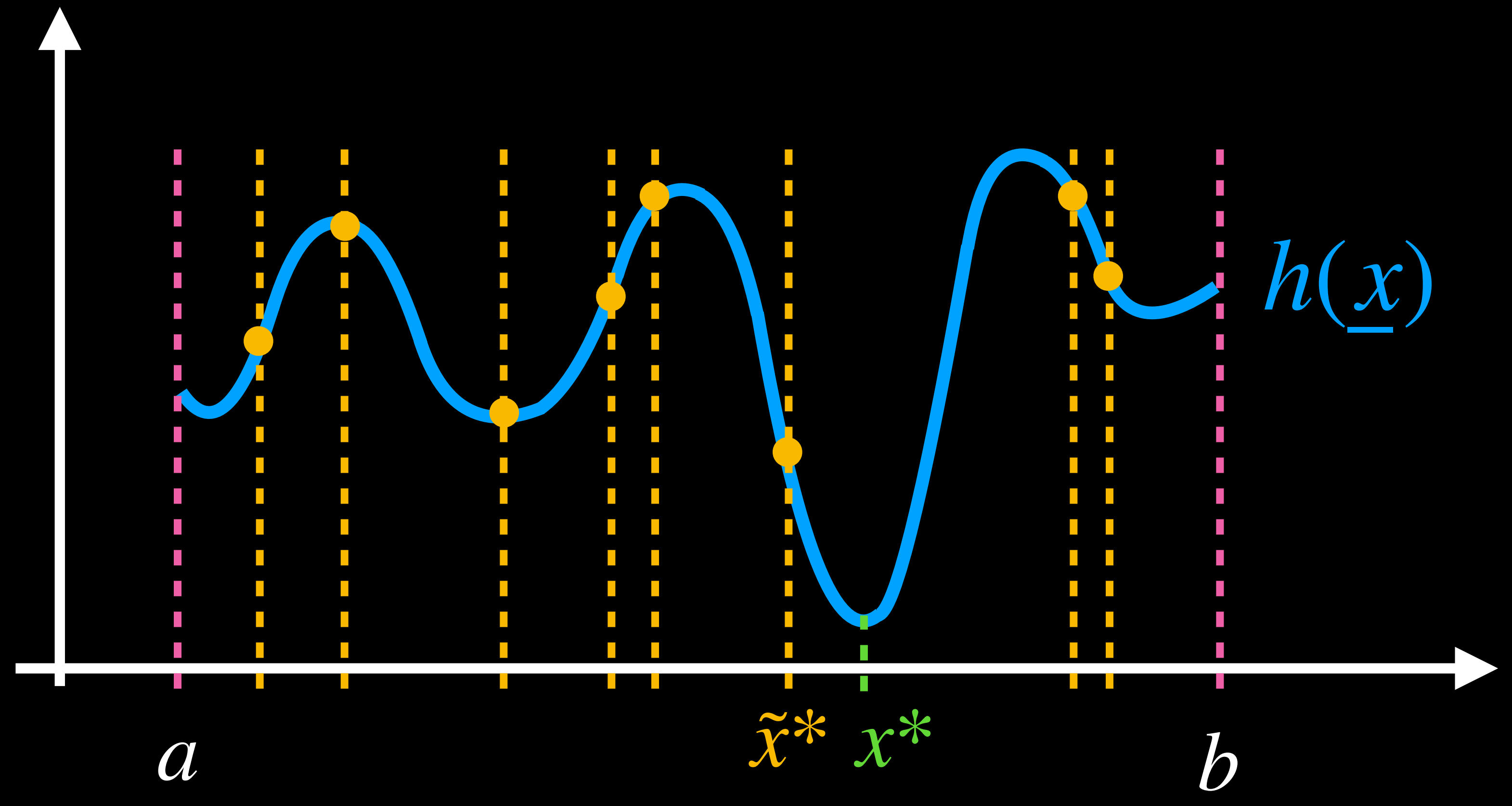

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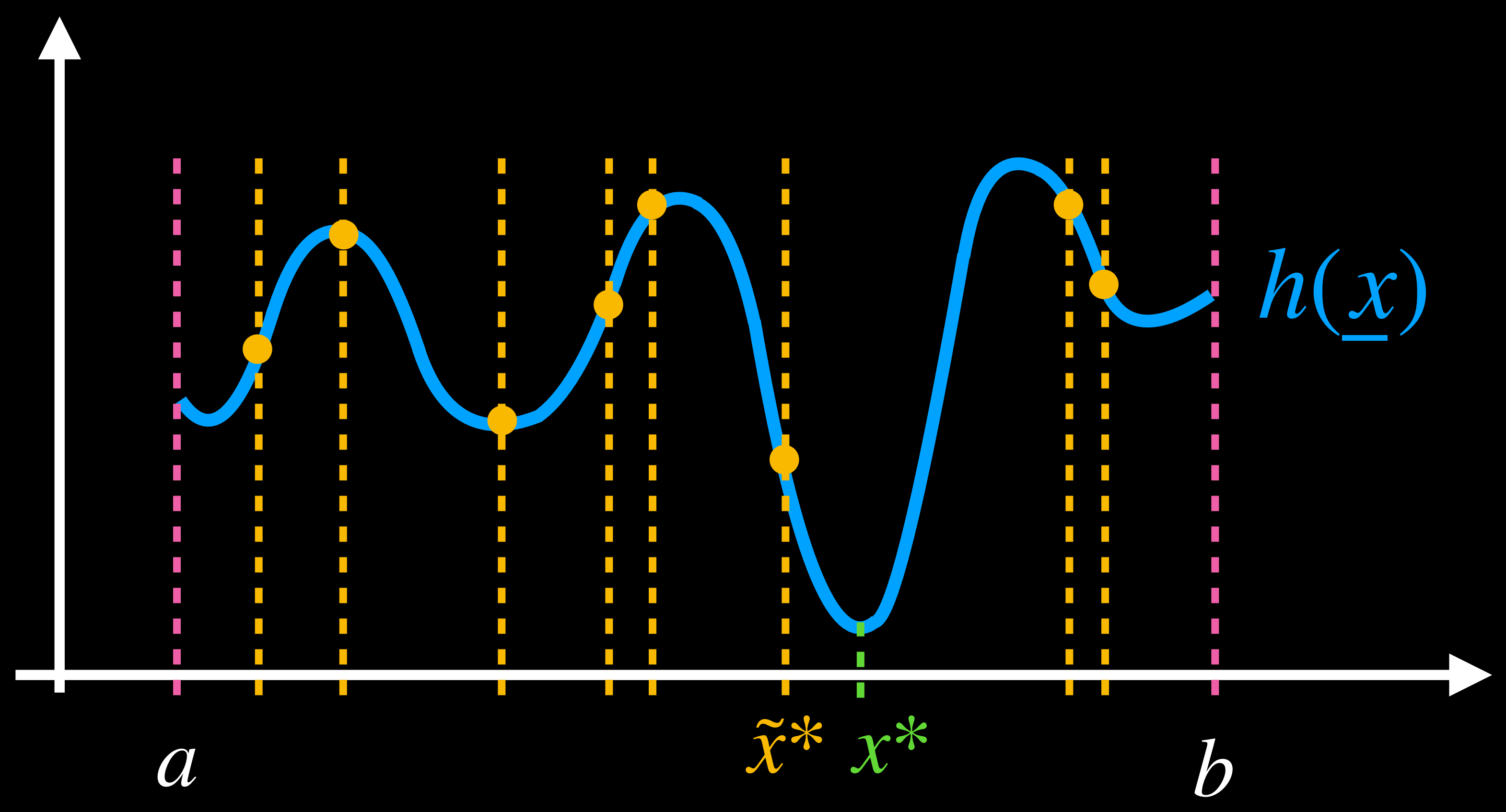

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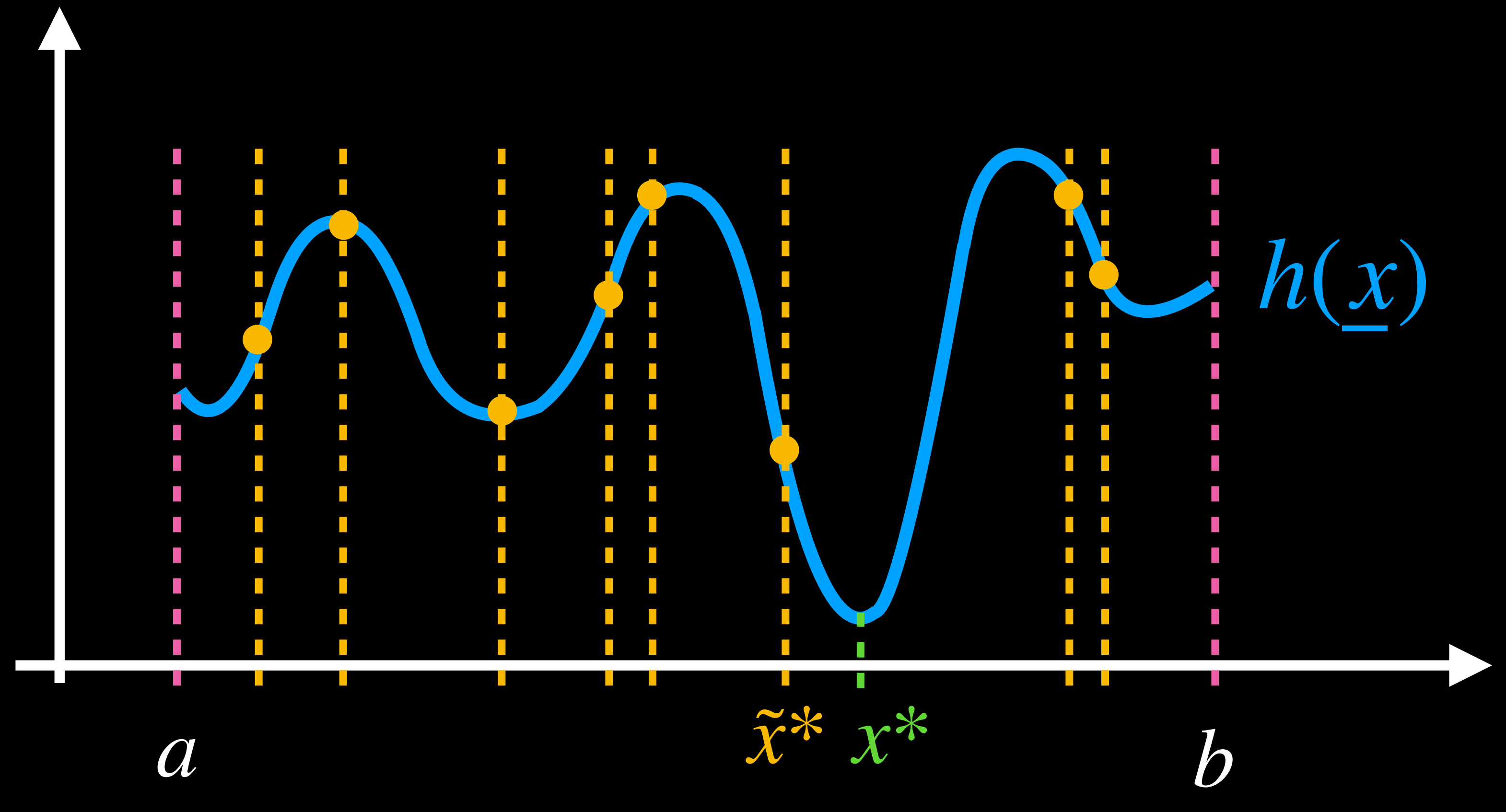

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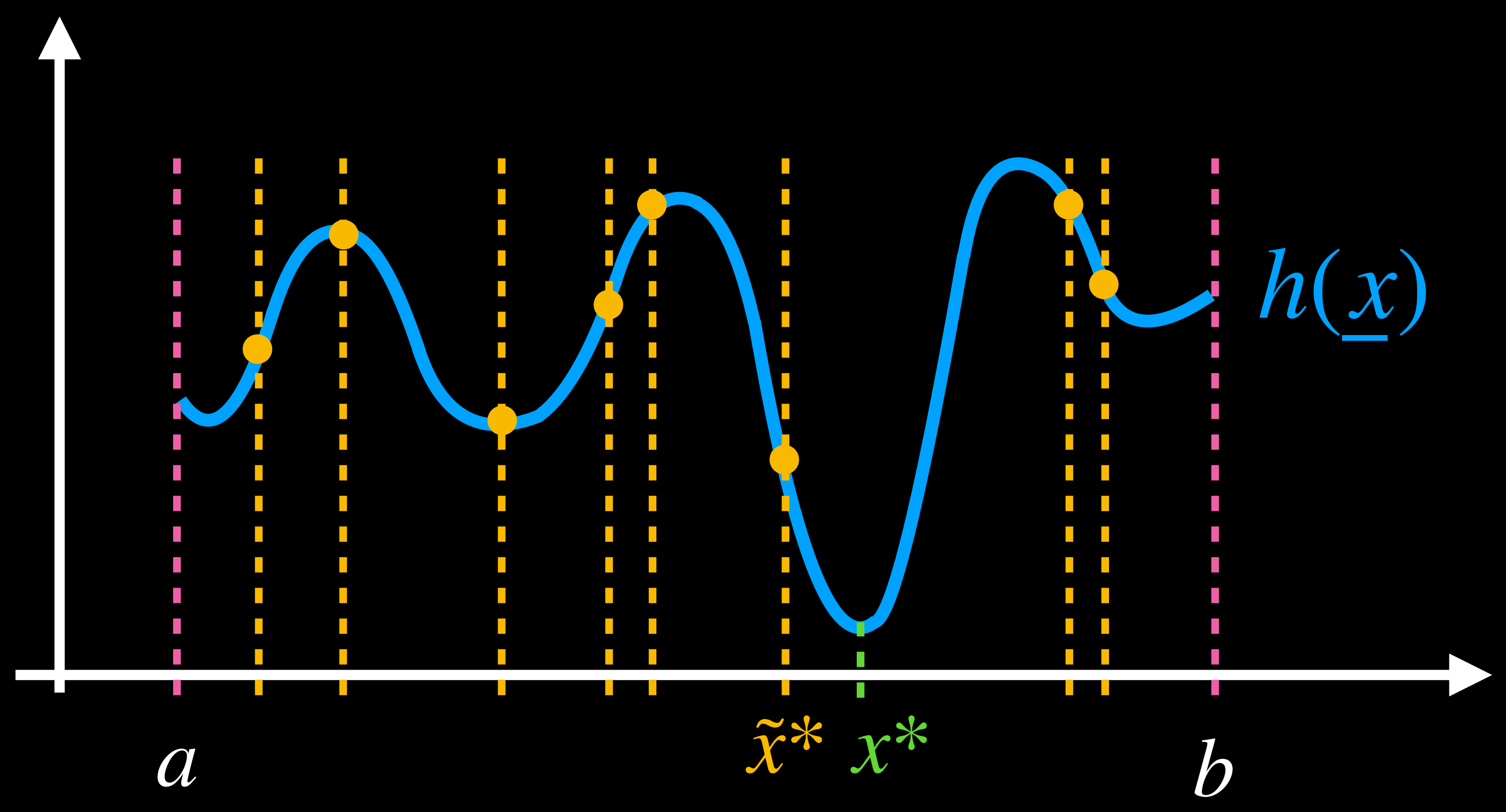

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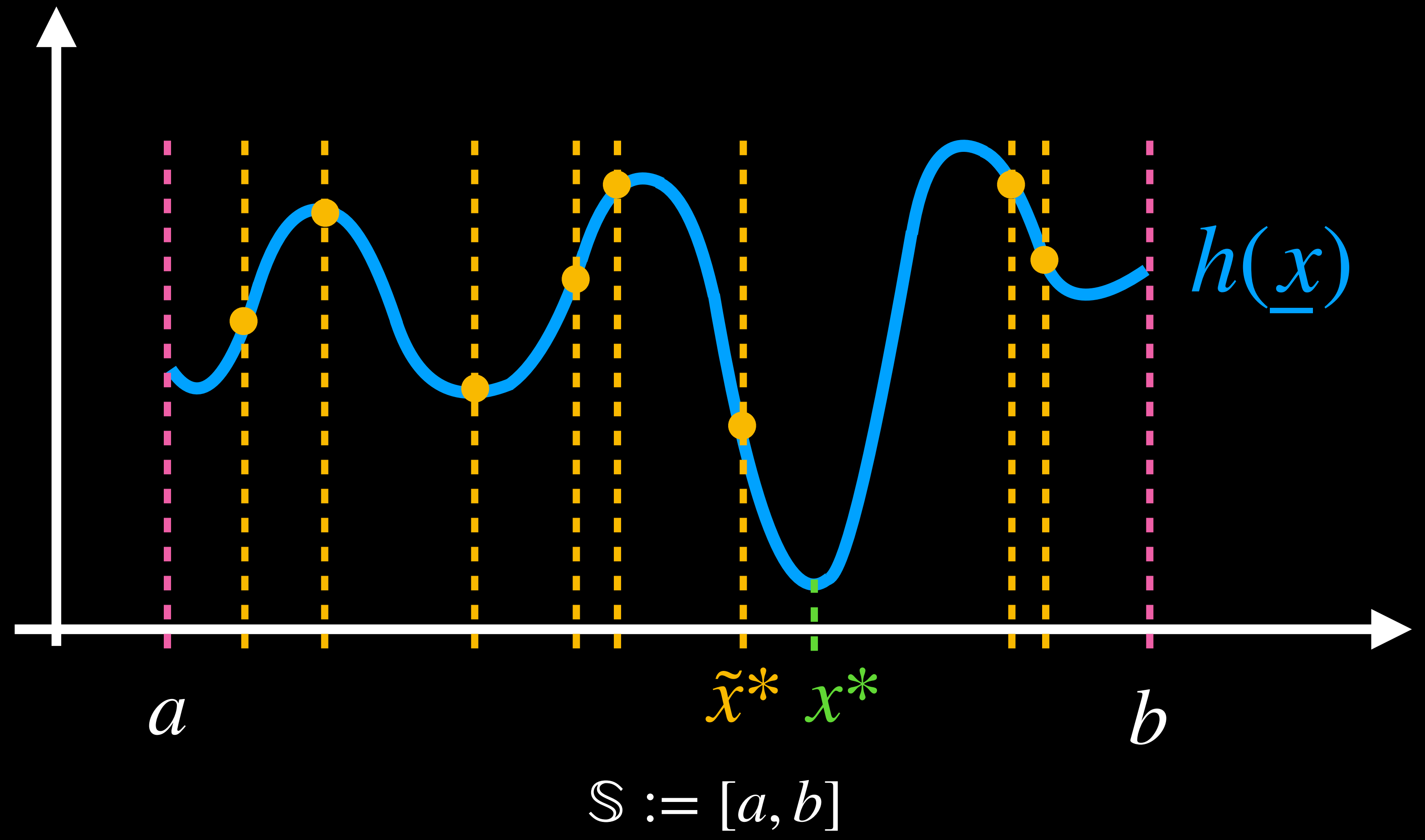

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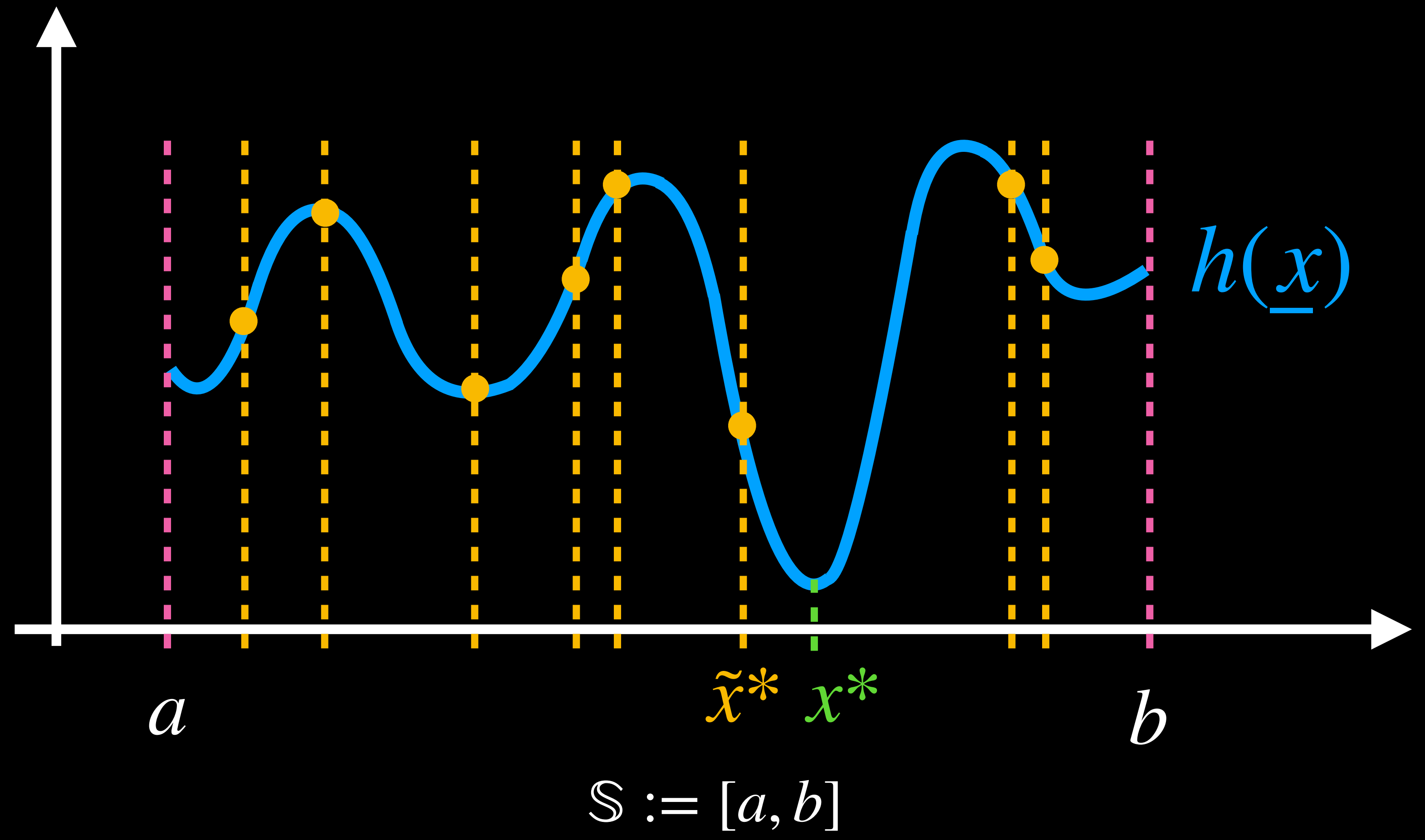

global convergence:

algorithm 1: stochastic exploration

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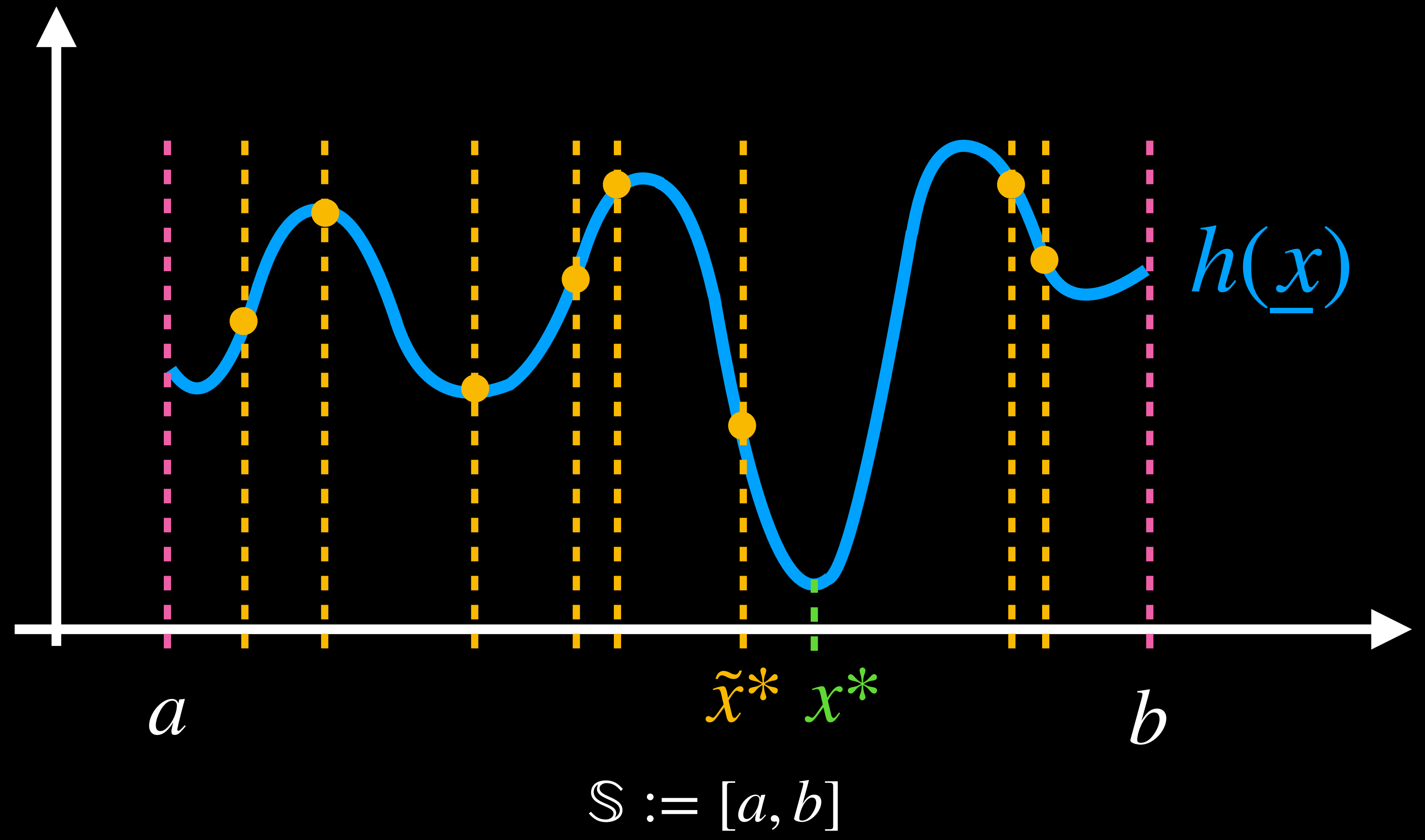

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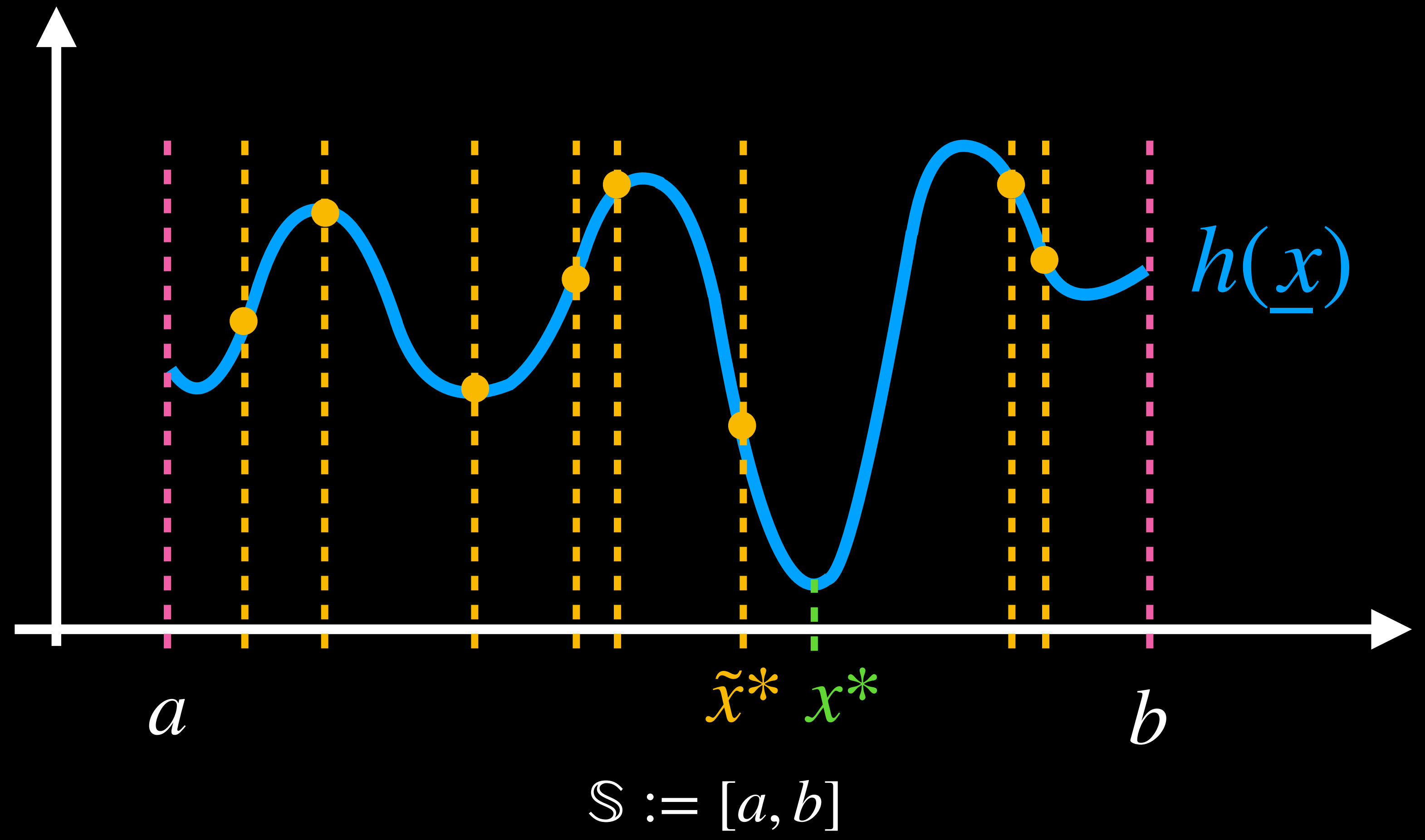

global convergence:

$$\underline{x}^* = \lim_{n\to\infty} \tilde{\underline{X}}^*_n$$

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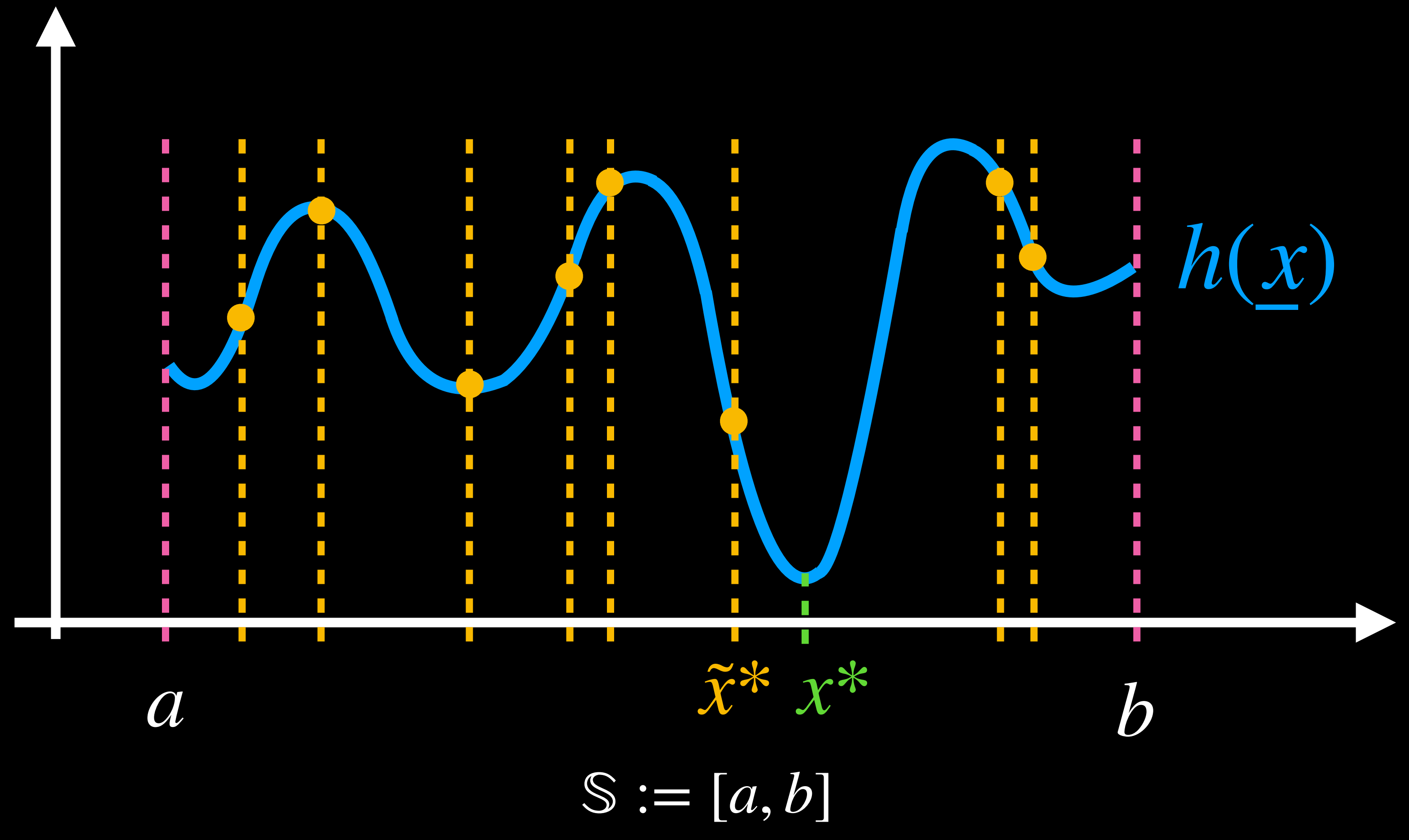

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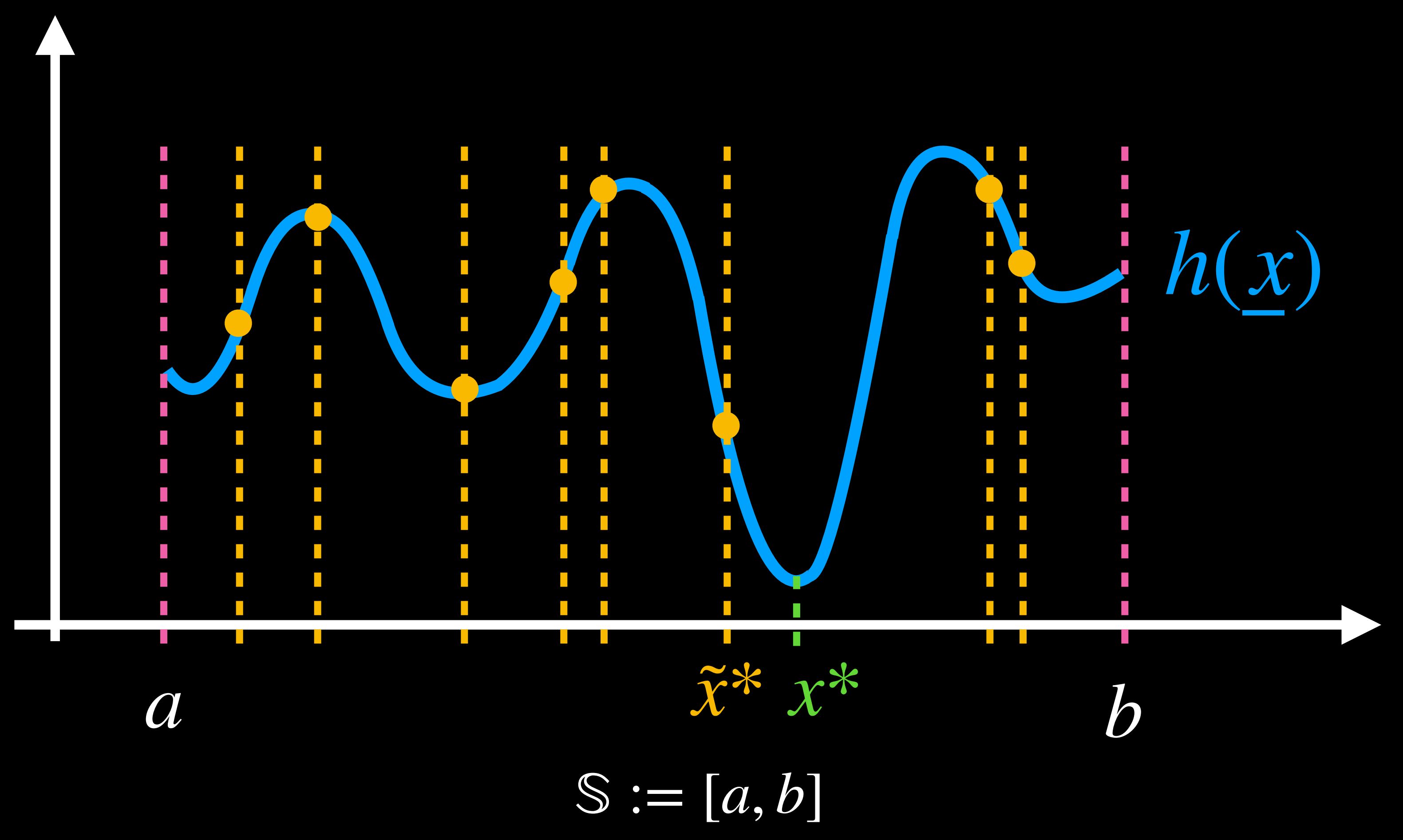
1d:

algorithm 1: stochastic exploration



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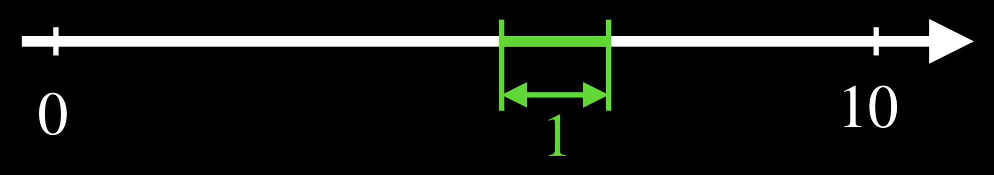

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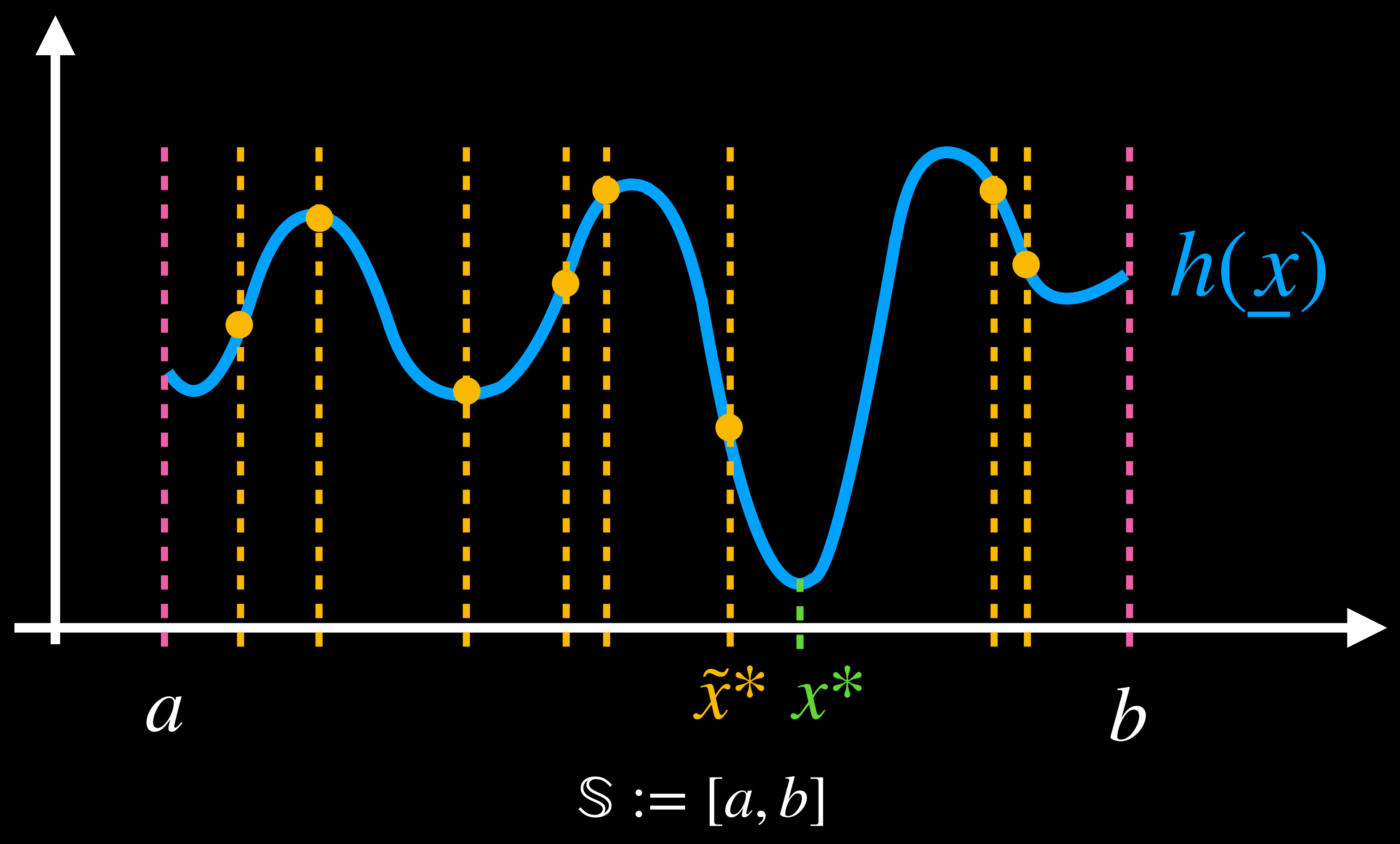

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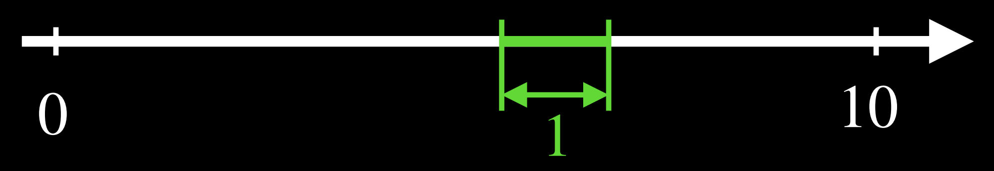

$P(X \in \text{—}) =$

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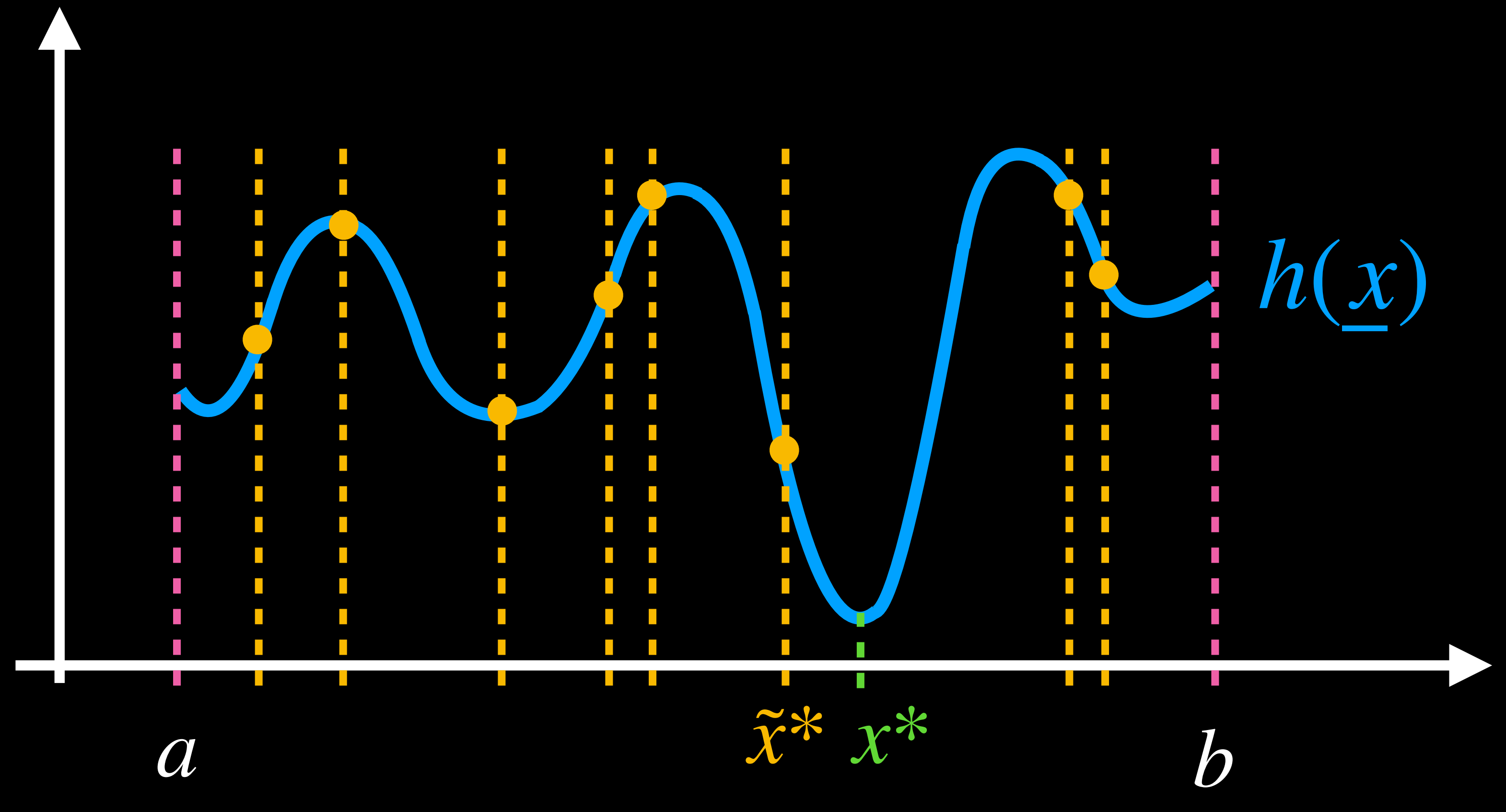

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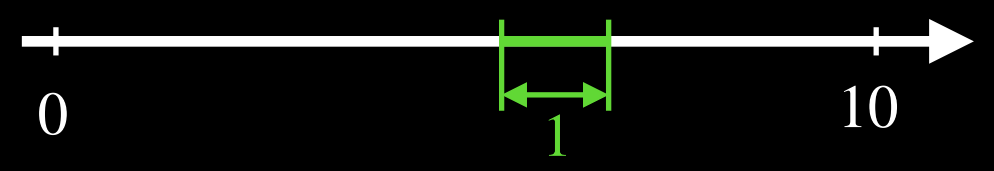
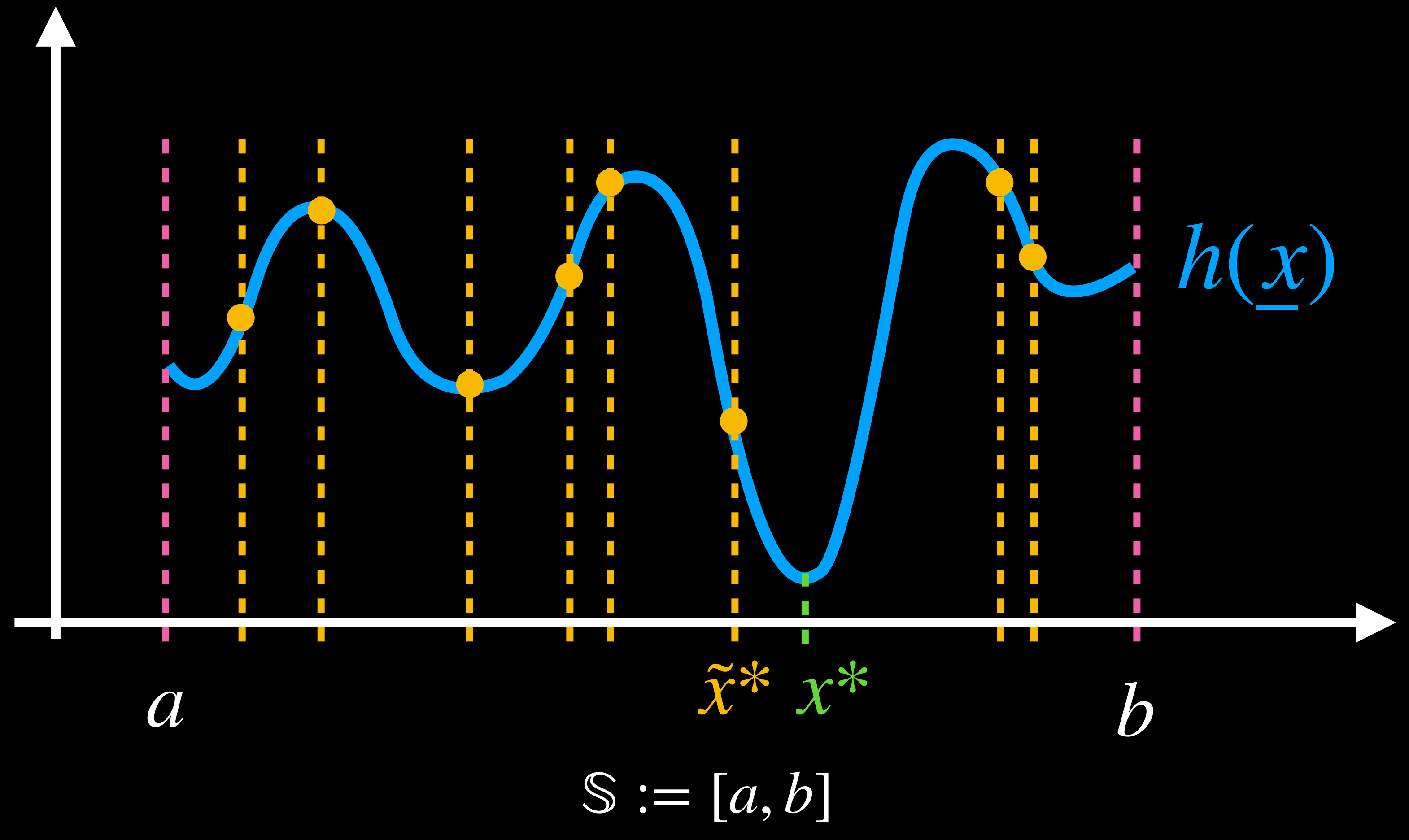
1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

algorithm 1: stochastic exploration



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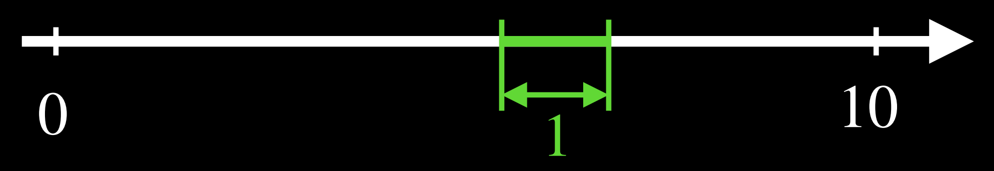

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2d:

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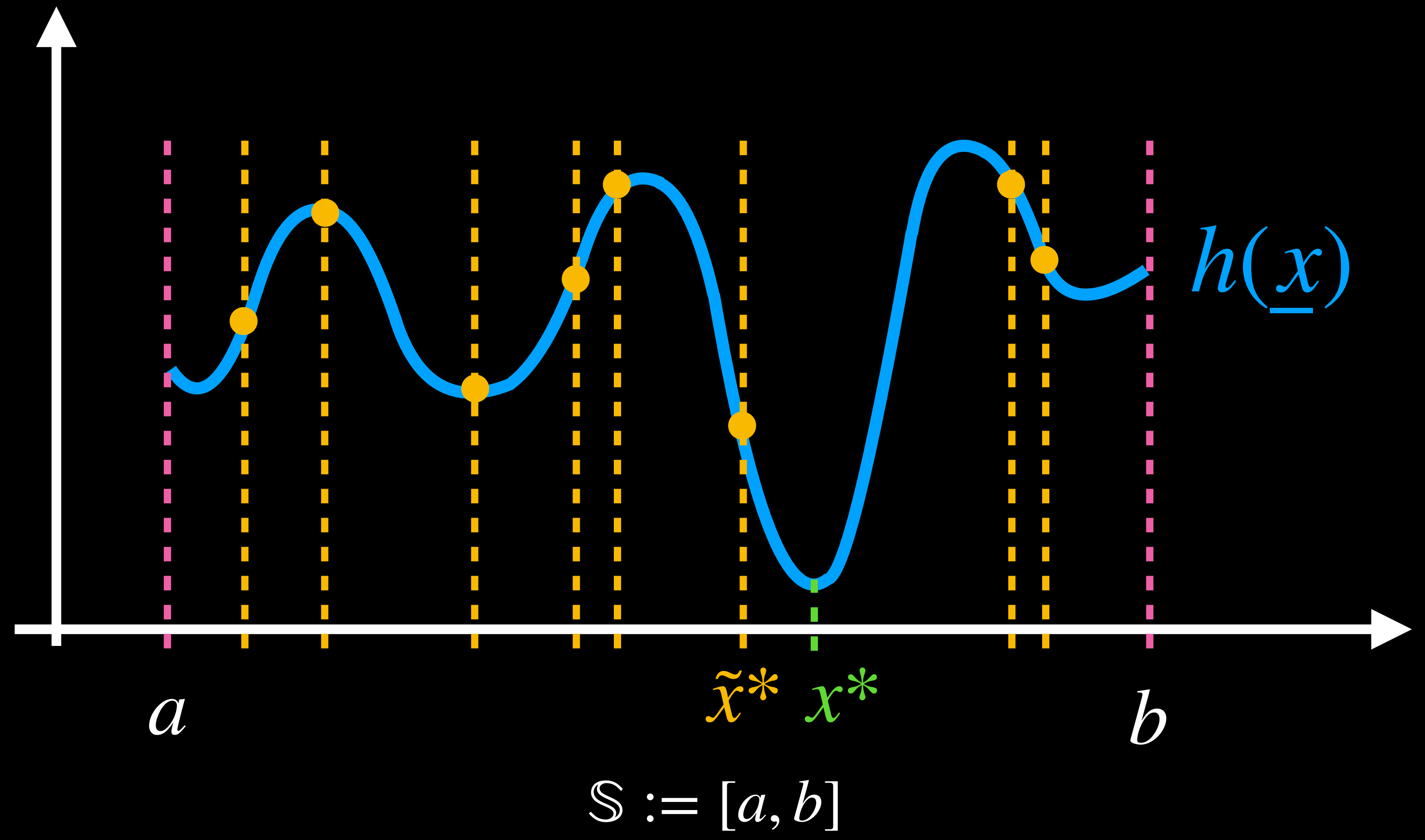

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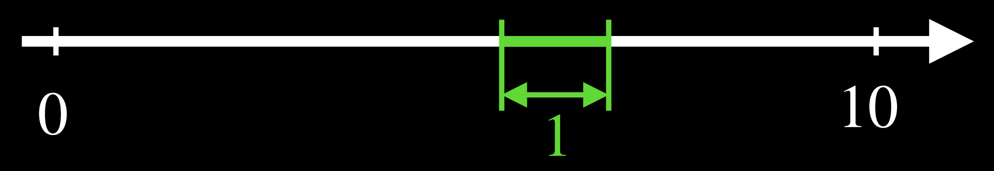

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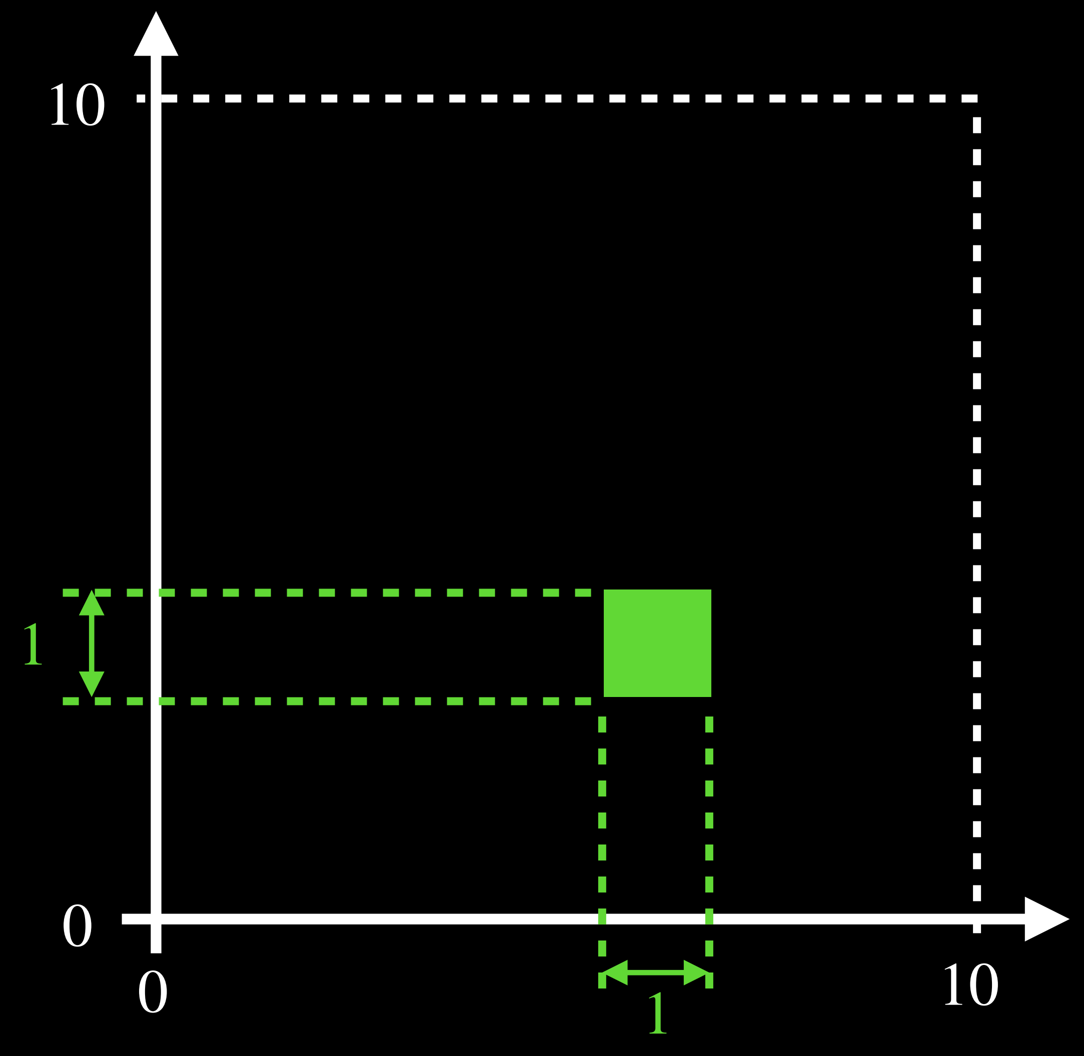
2d:

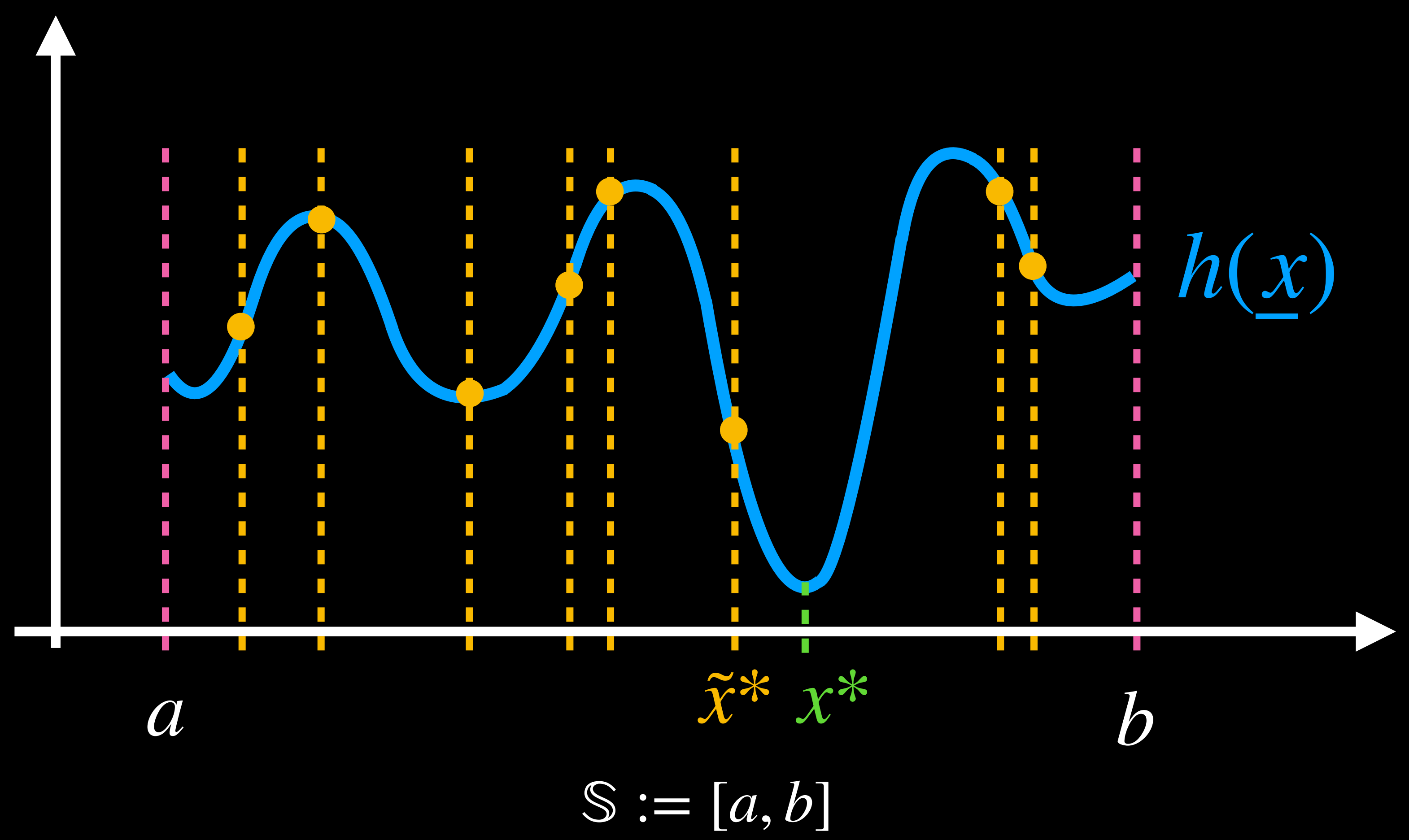


algorithm 1: stochastic exploration

$$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \left(h(\underline{x})\right) \approx \arg\min \left(h(\underline{X}_1), \ldots, h(\underline{X}_n)\right) =: \underline{\tilde{X}}_n^*$$

$$\underline{X}_t : \Omega \to \mathbb{S} \subseteq \mathbb{R}^d$$

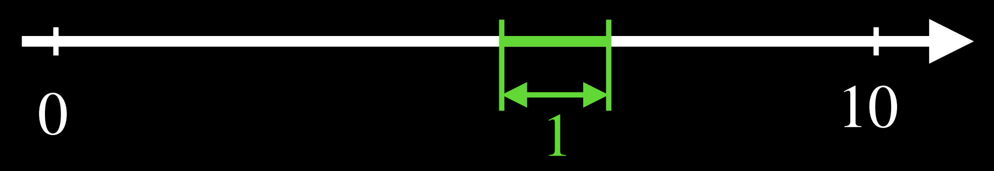
$$\underline{X}_t \sim \mathcal{U}(\mathbb{S})$$



global convergence:

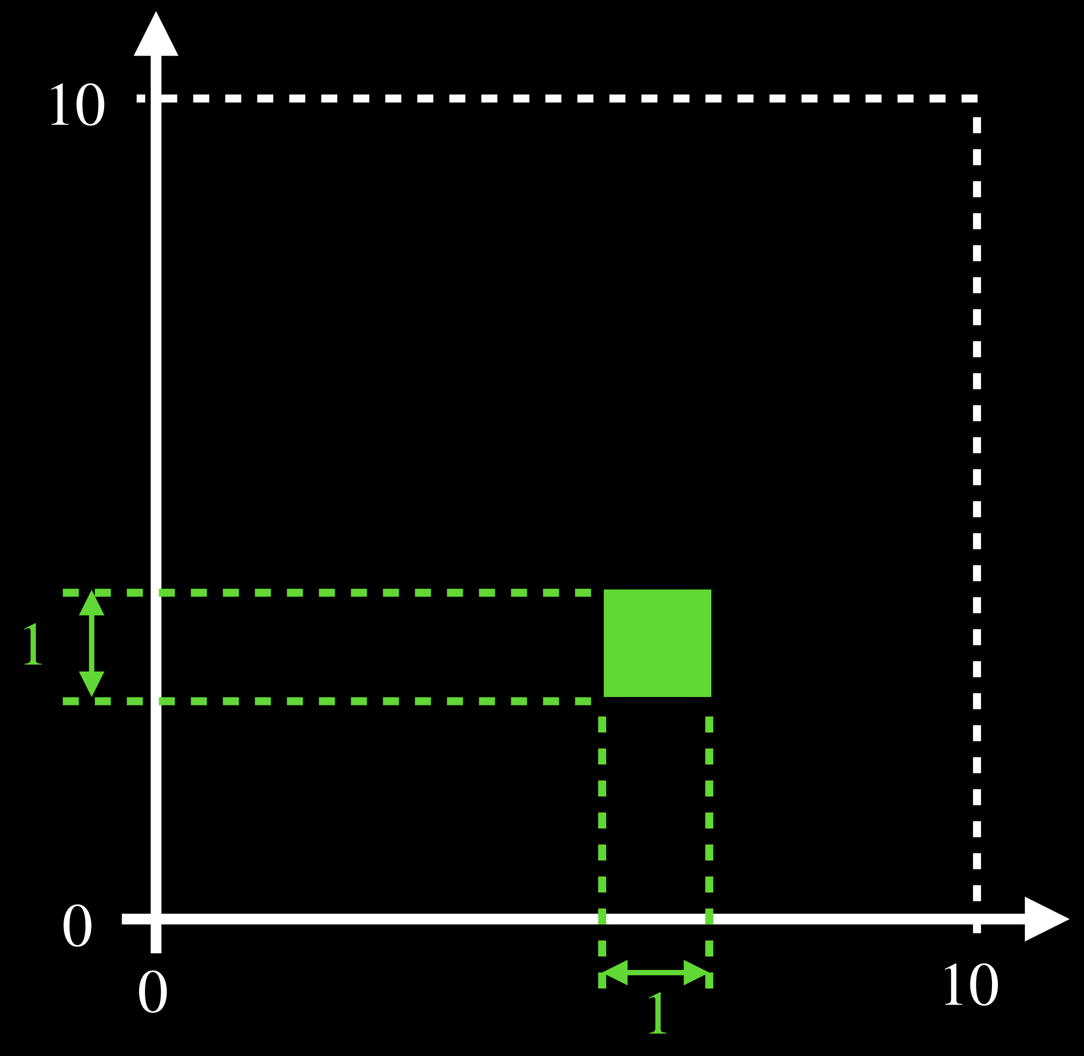
$$\underline{x}^* = \lim_{n\to\infty} \underline{\tilde{X}}_n^*$$

1d:



$P(X \in$ — $) = \dfrac{1}{10}$

2d:



$P(X \in$ ■ $) =$

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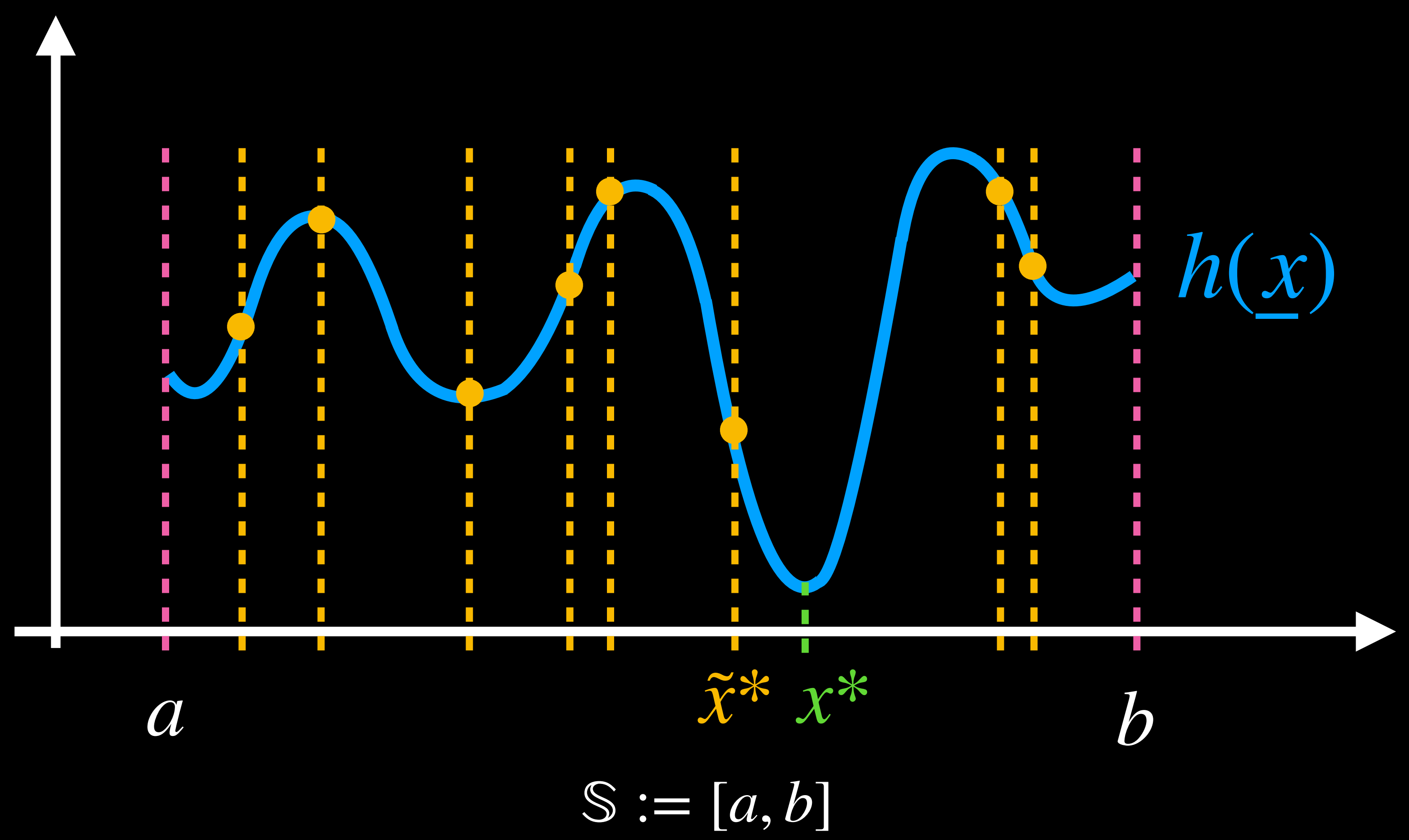

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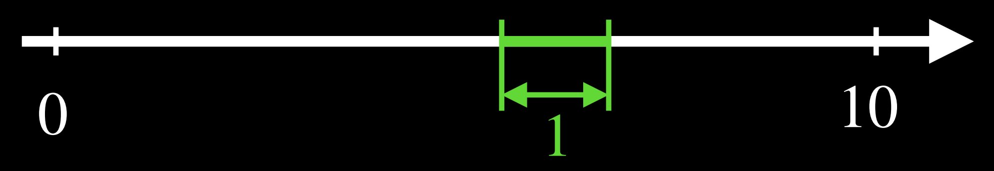
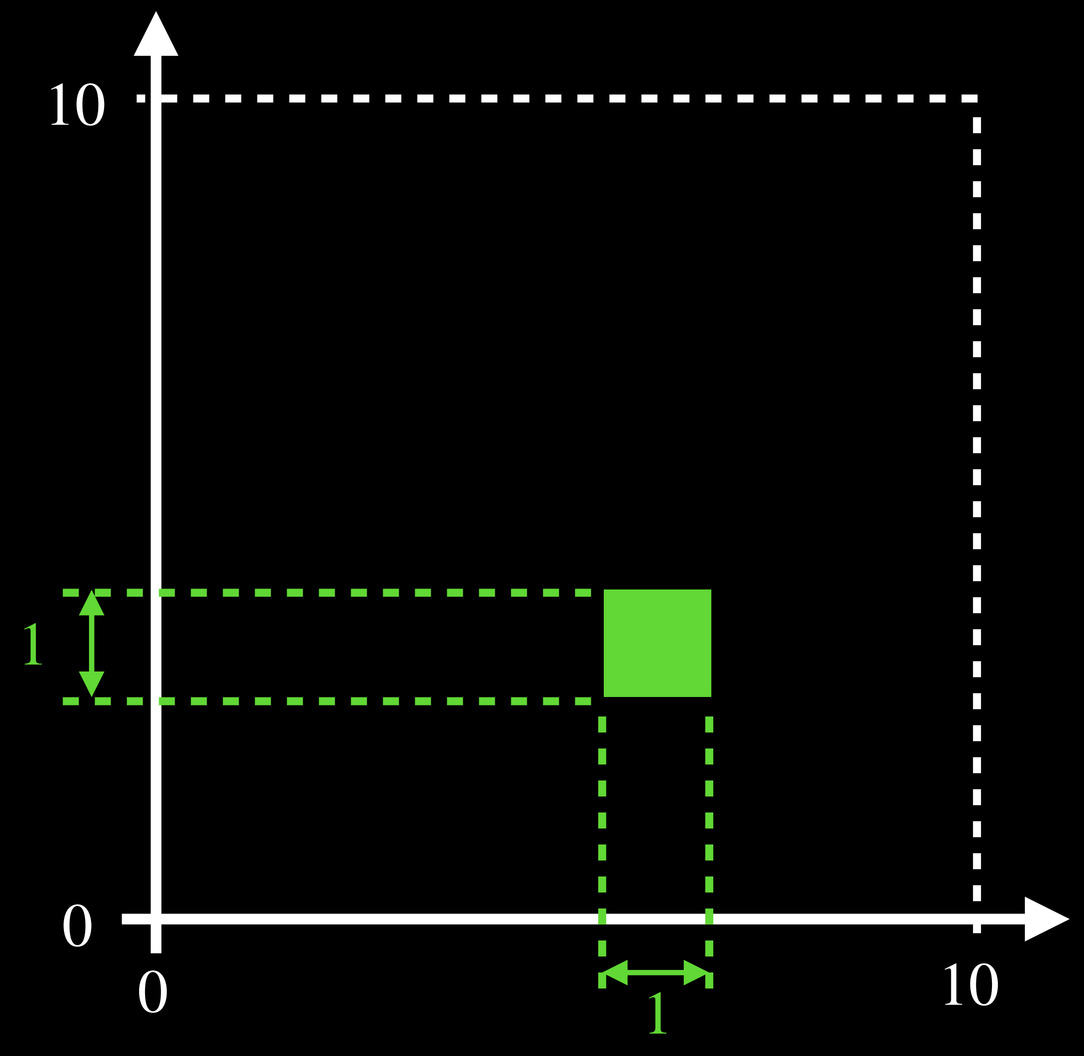


$$P(X \in \text{—}) = \frac{1}{10}$$

2d:



$$P(X \in \blacksquare) = \frac{1}{10^2}$$

algorithm 1: stochastic exploration

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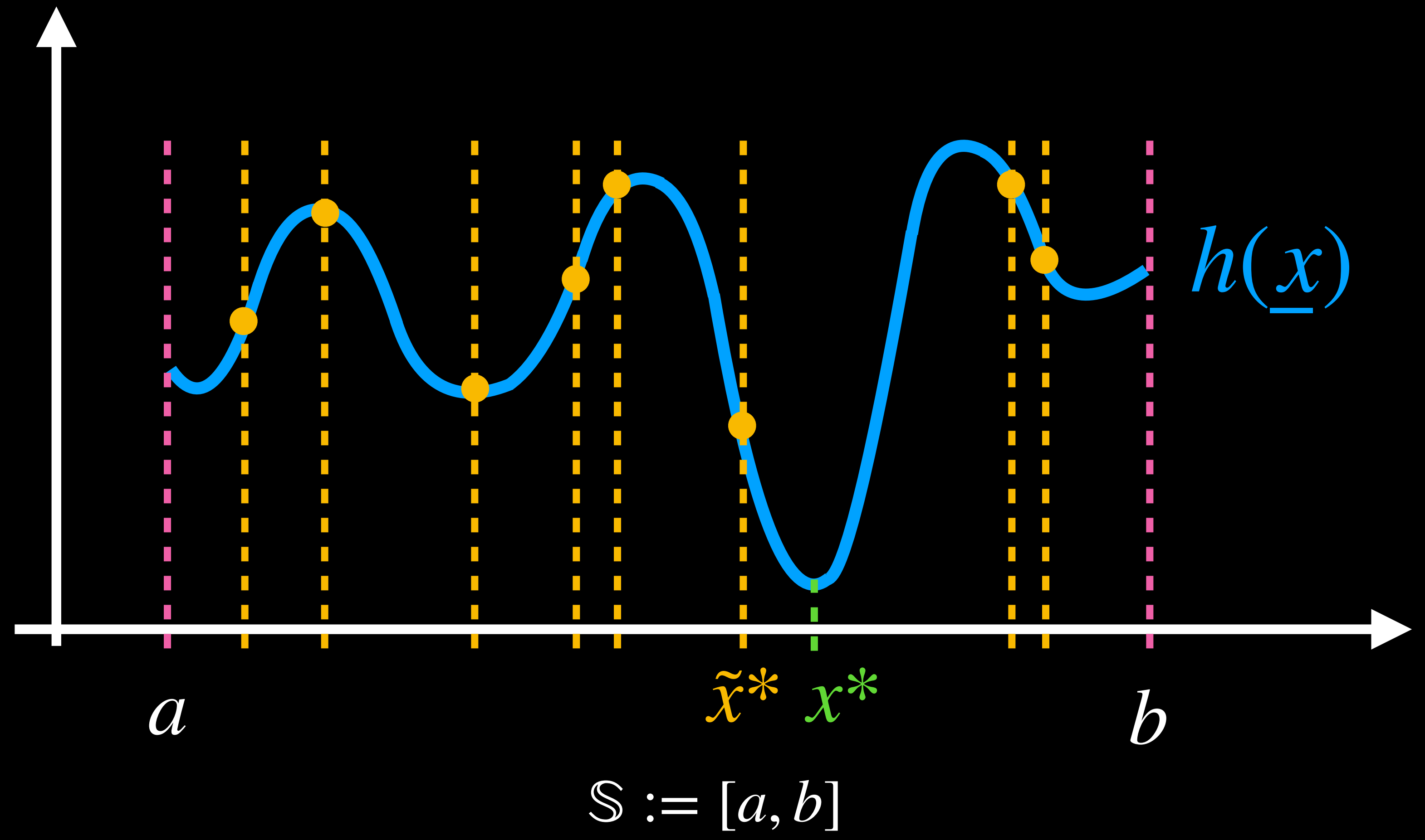

global convergence:

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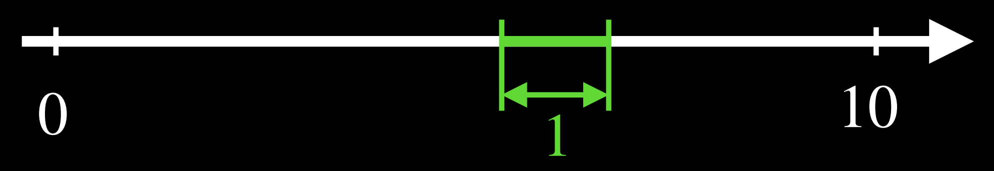
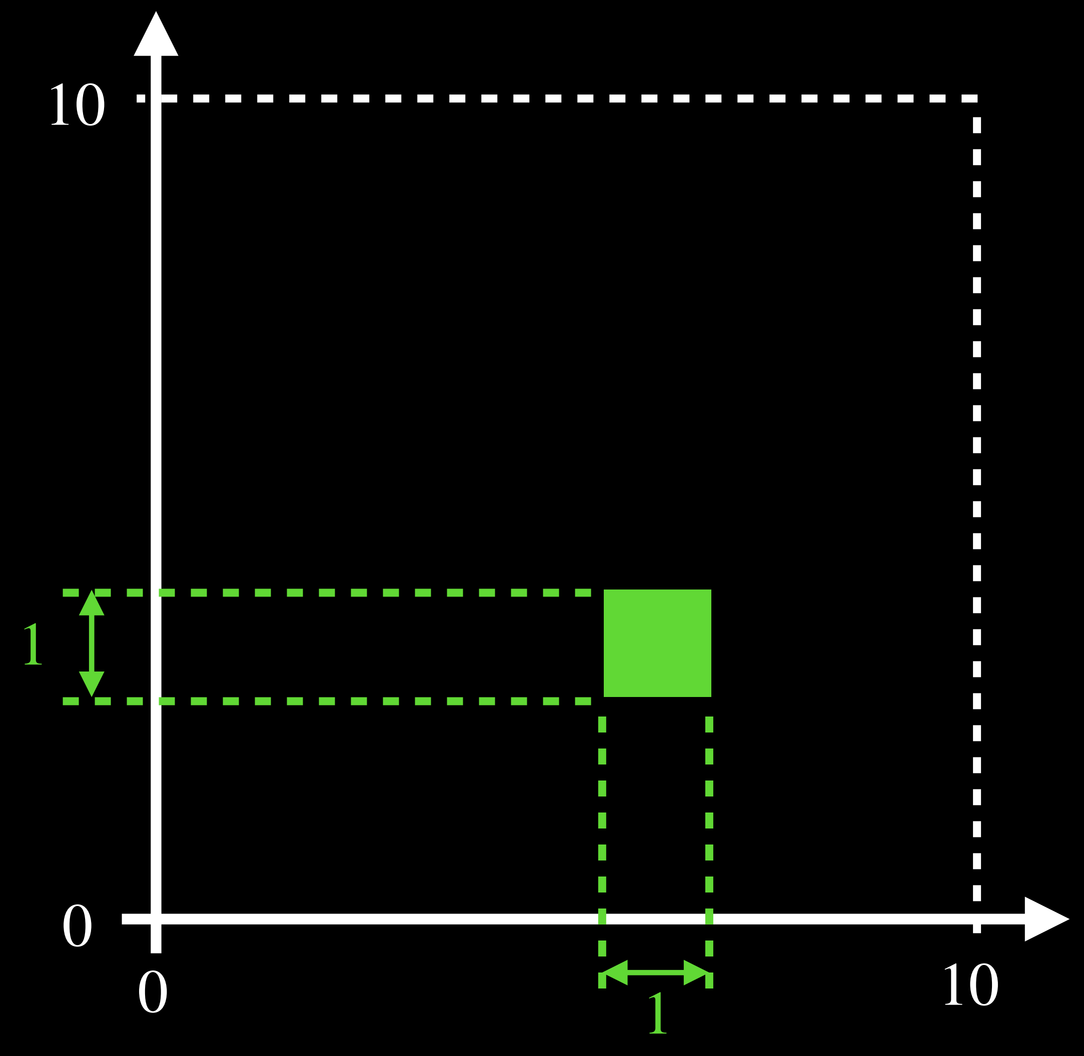
1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

2d:



$$P(X \in \blacksquare) = \frac{1}{10^2}$$

$$error \propto n^{-\frac{1}{d}}$$

algorithm 1: stochastic exploration

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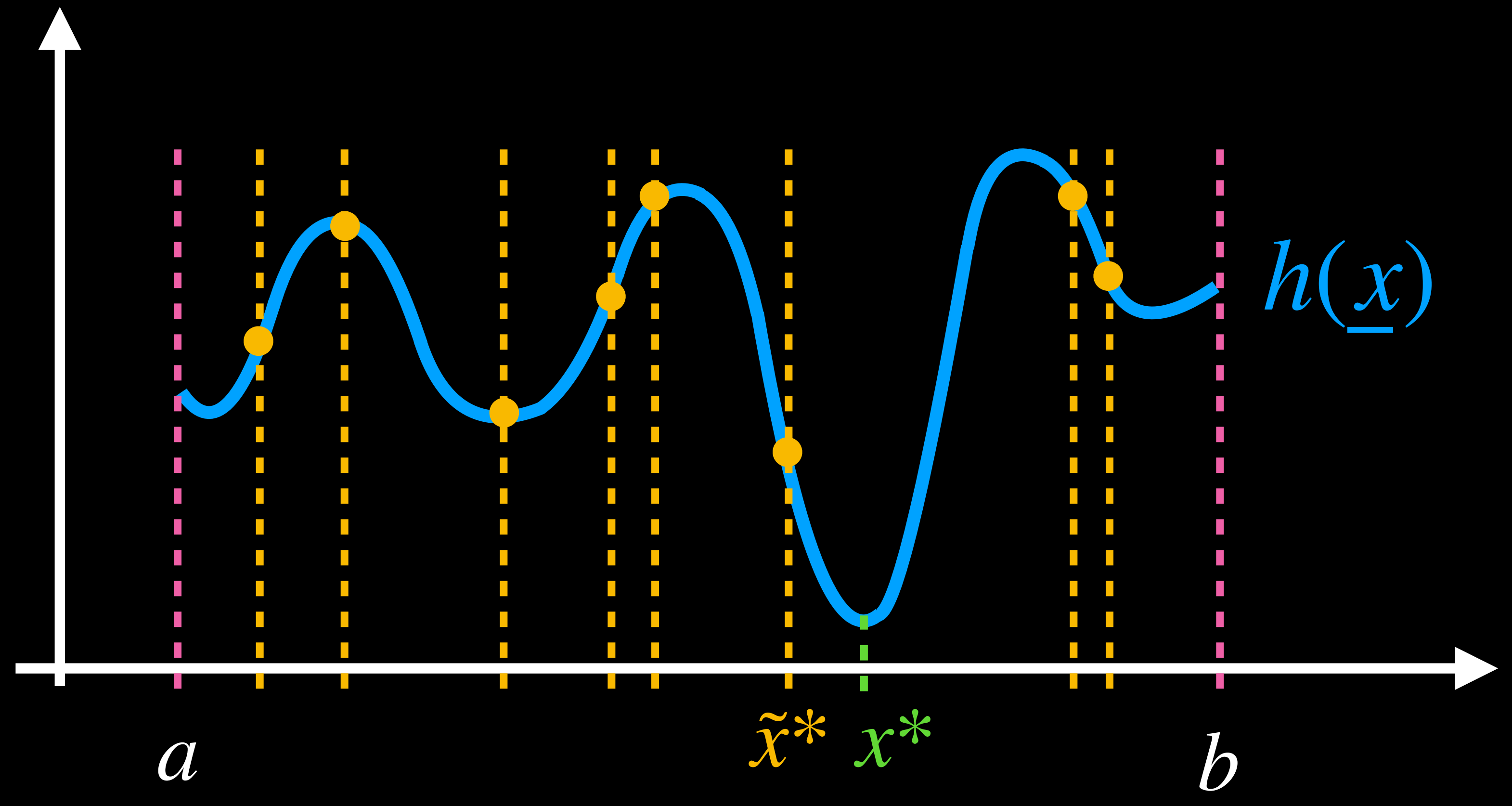

$\mathbb{S} := [a, b]$

global convergence:

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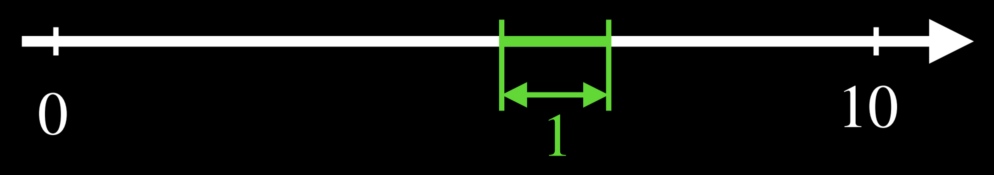
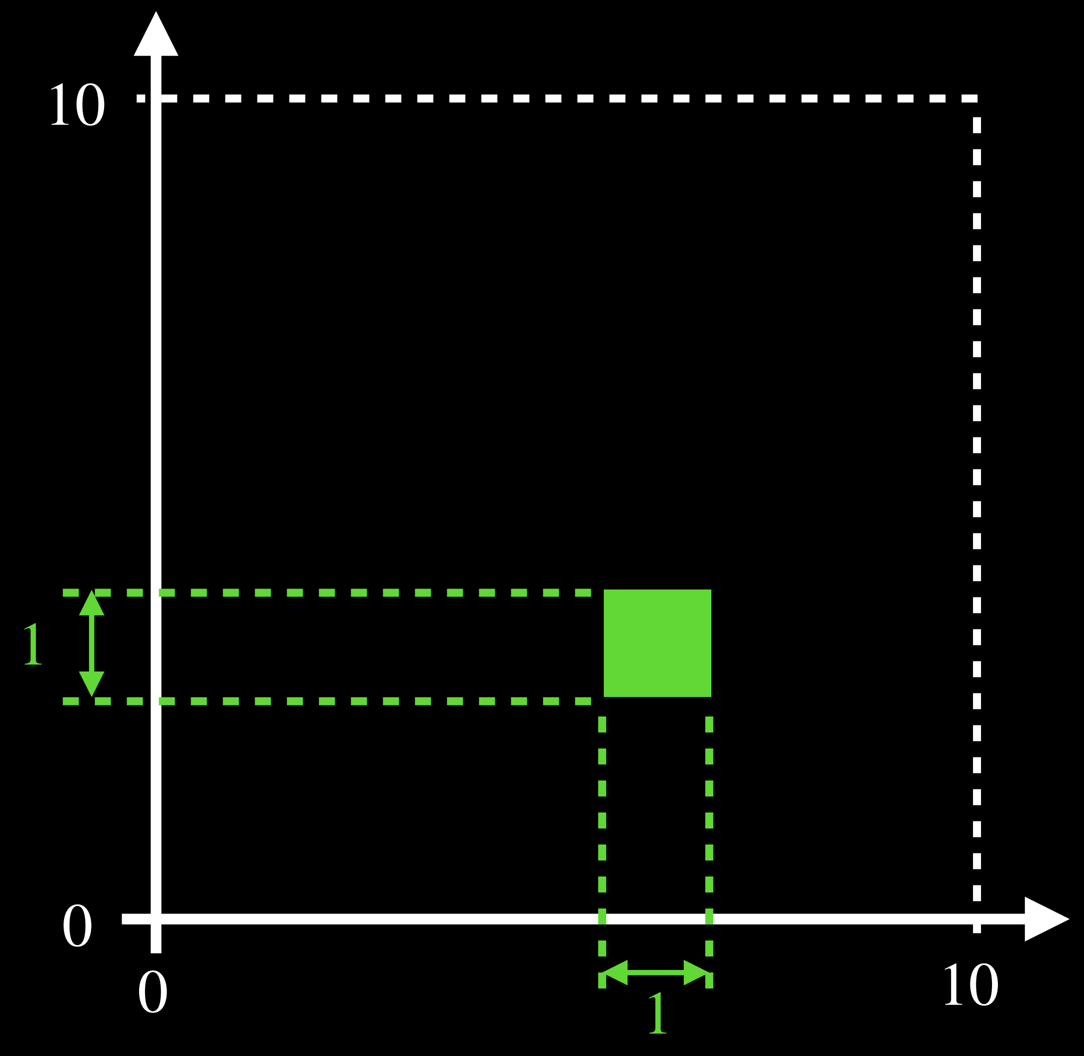
1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

properties:

2d:



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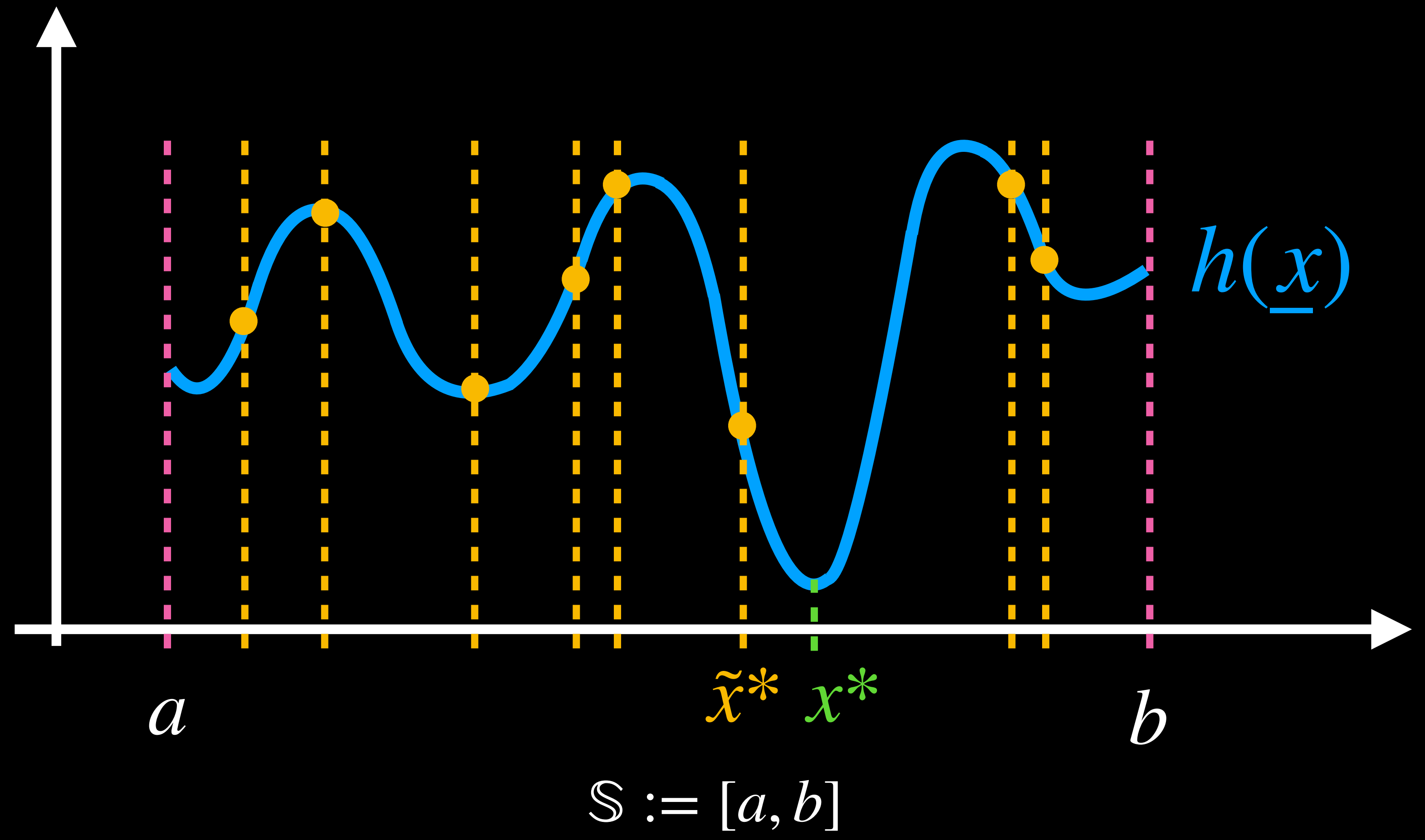
algorithm 1: stochastic exploration



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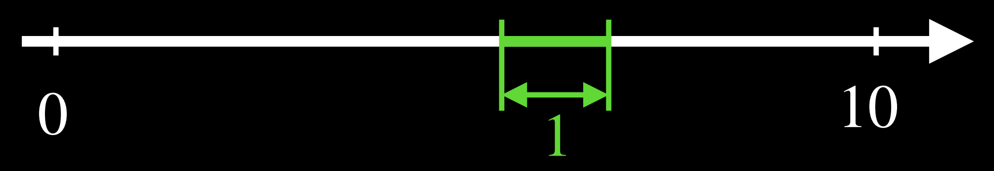
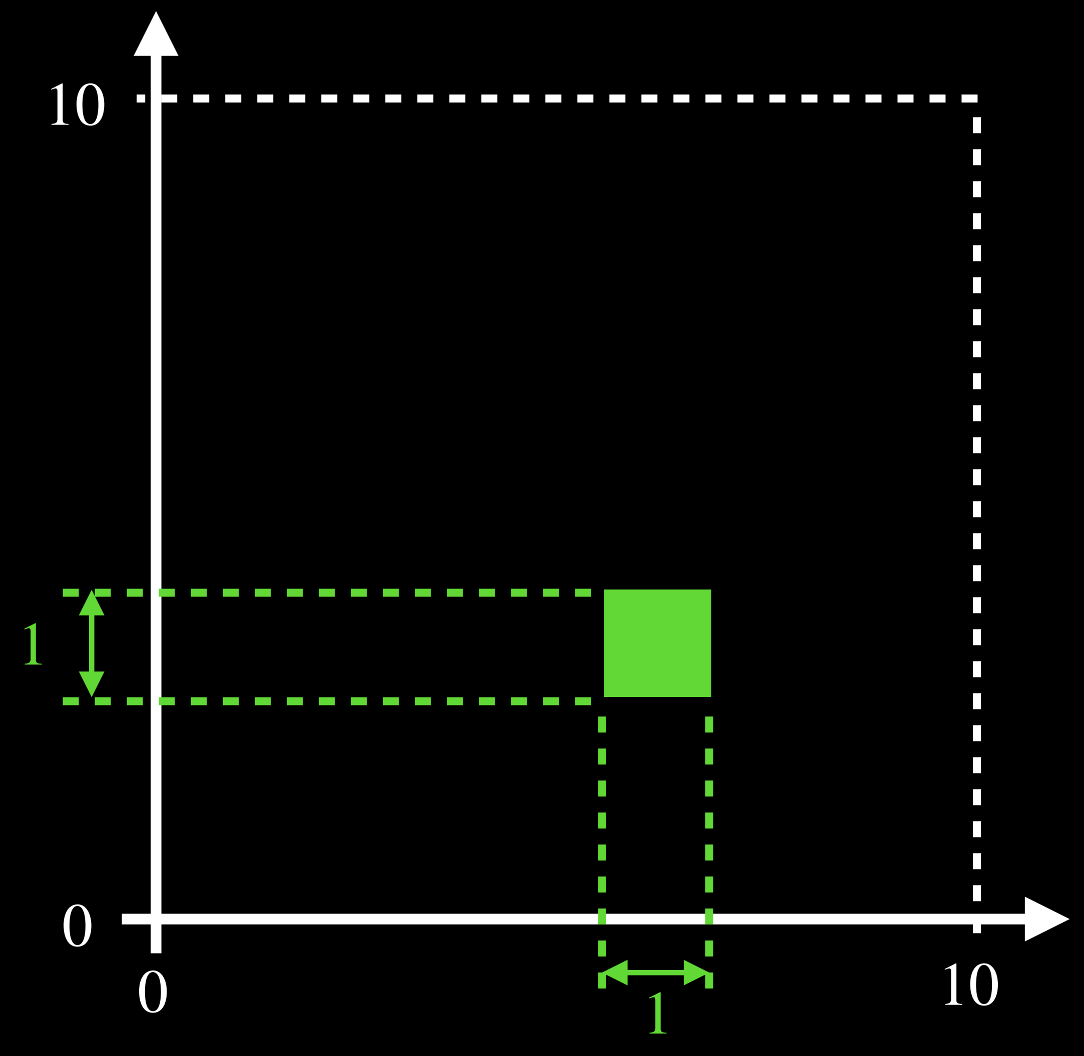
1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

properties:
- very simple

2d:



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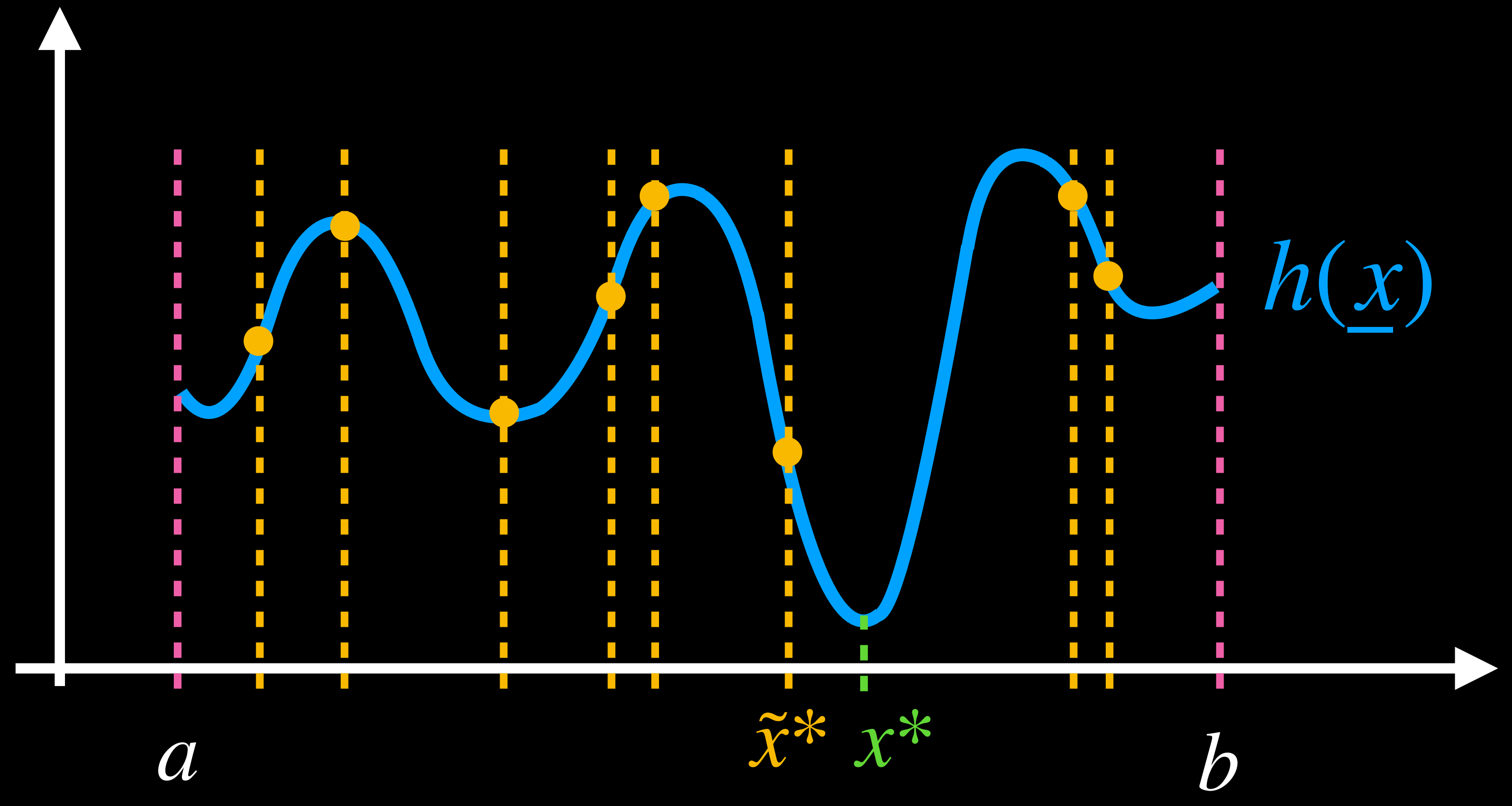

$\mathbb{S} := [a, b]$

global convergence:

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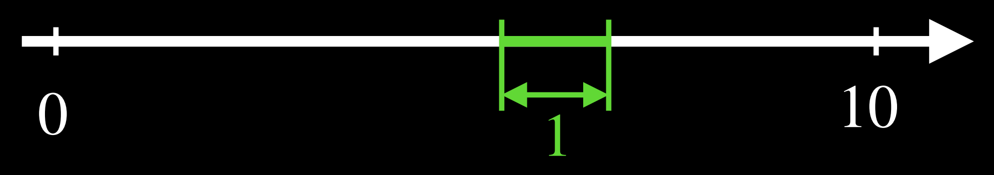
properties:

- very simple
- converges to $\underline{x}^*$

1d:



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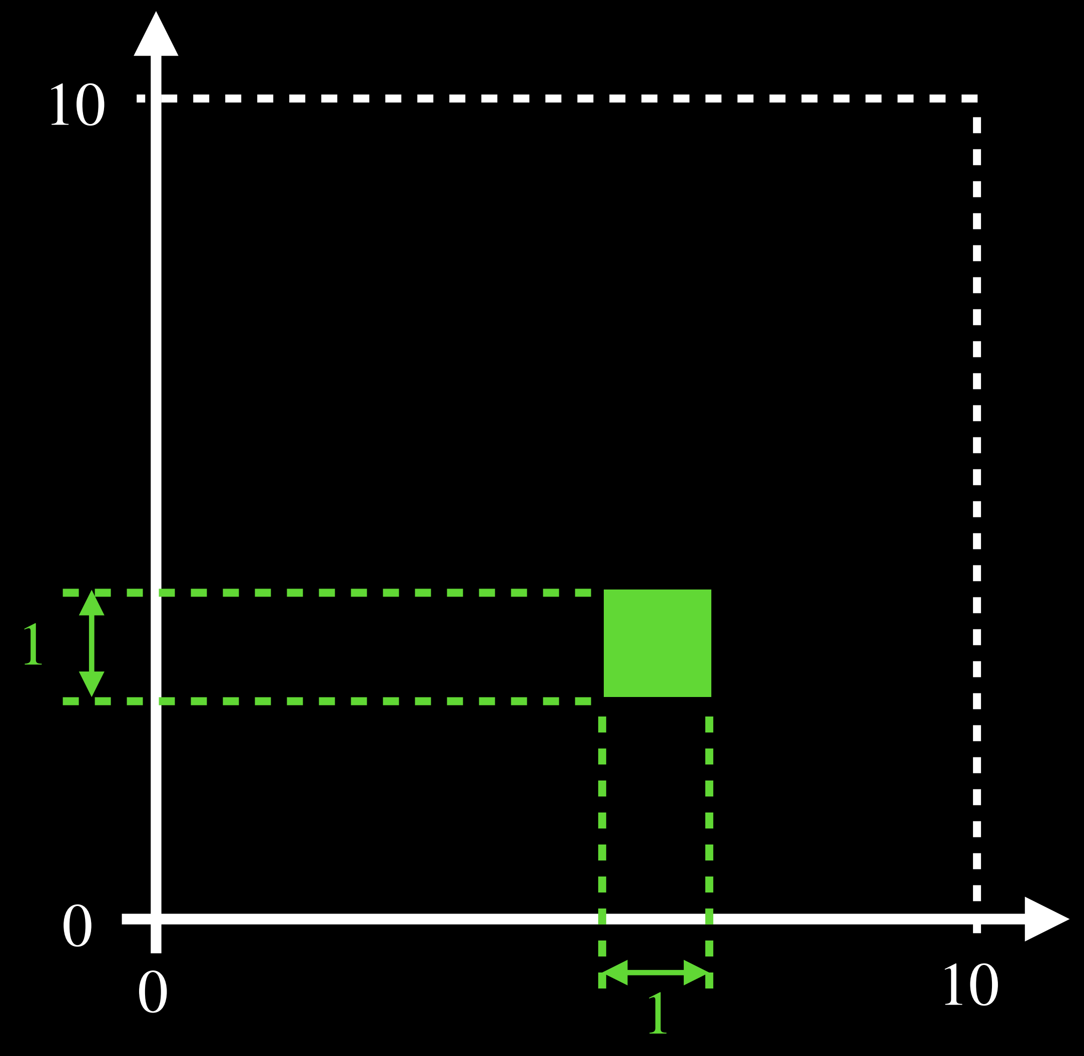
2d:



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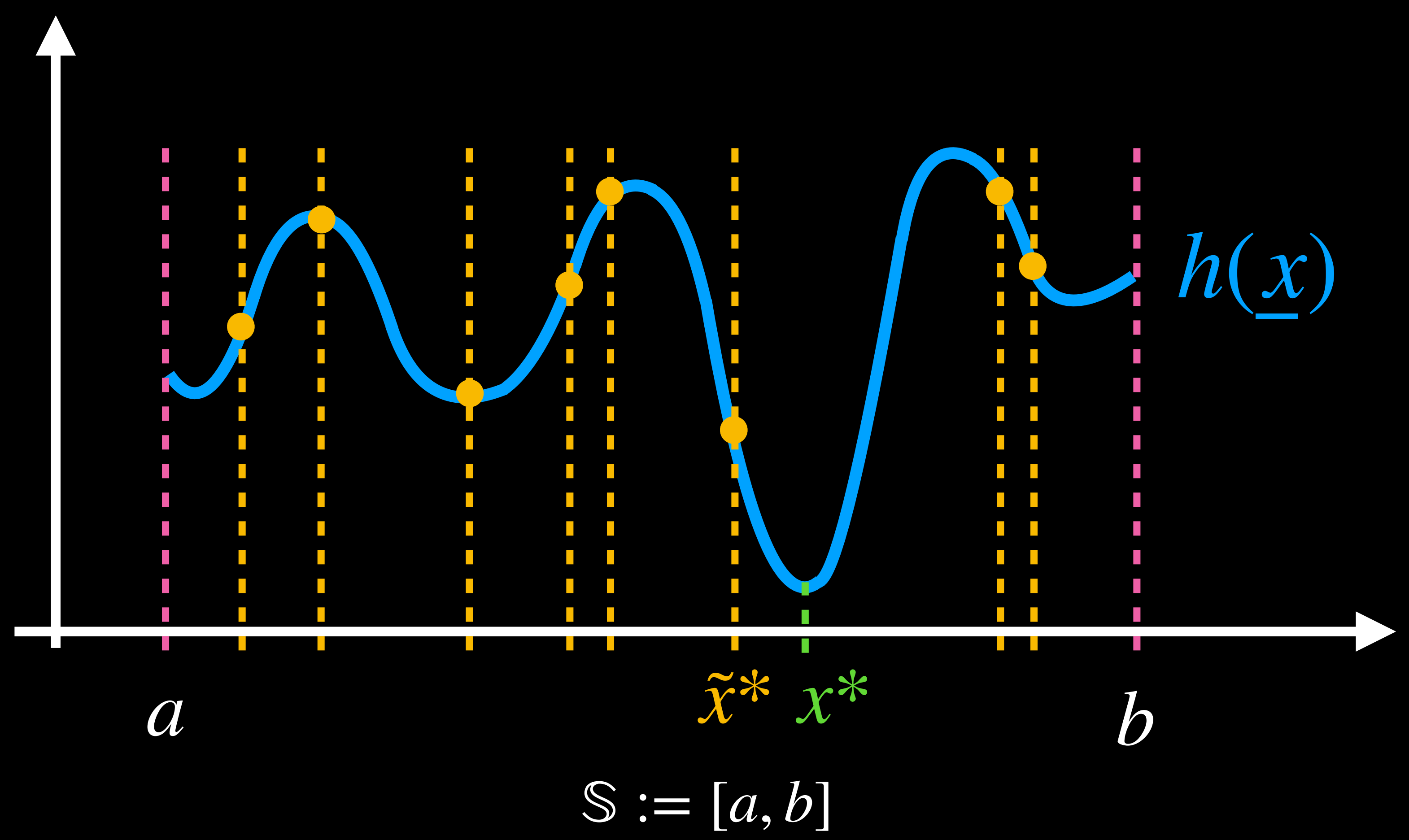
algorithm 1: stochastic exploration



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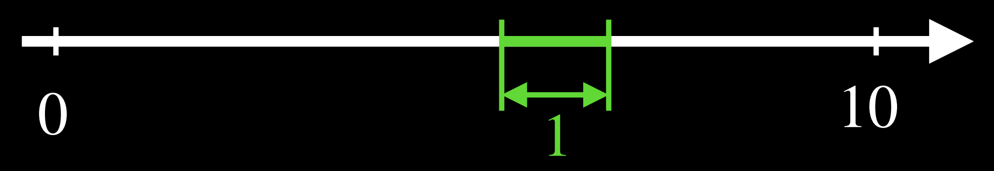
global convergence:

$$\underline{x}^* = \lim_{n\to\infty} \tilde{\underline{X}}^*_n$$

properties:
- very simple
- converges to $\underline{x}^*$

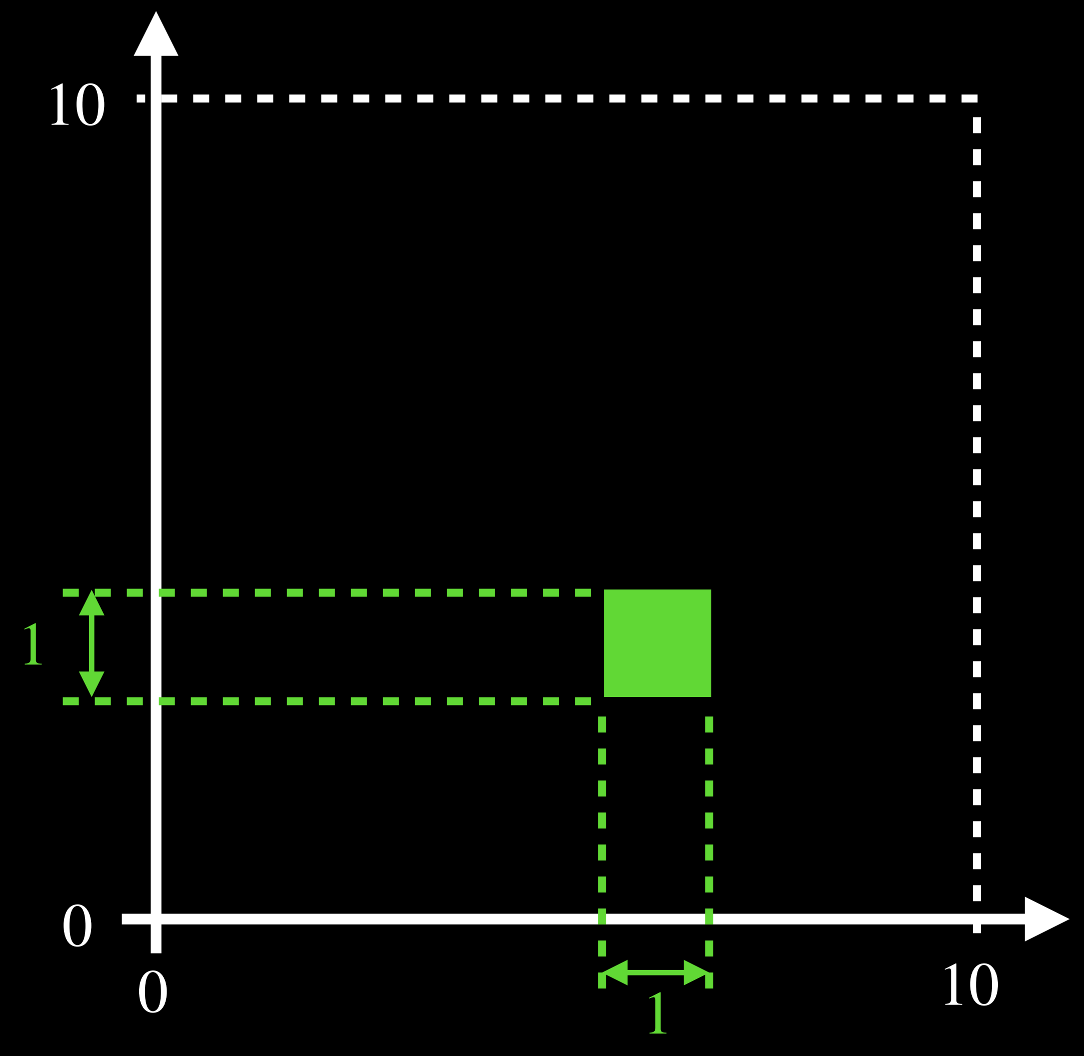
useful if:

1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

2d:



$$P(X \in \blacksquare) = \frac{1}{10^2}$$

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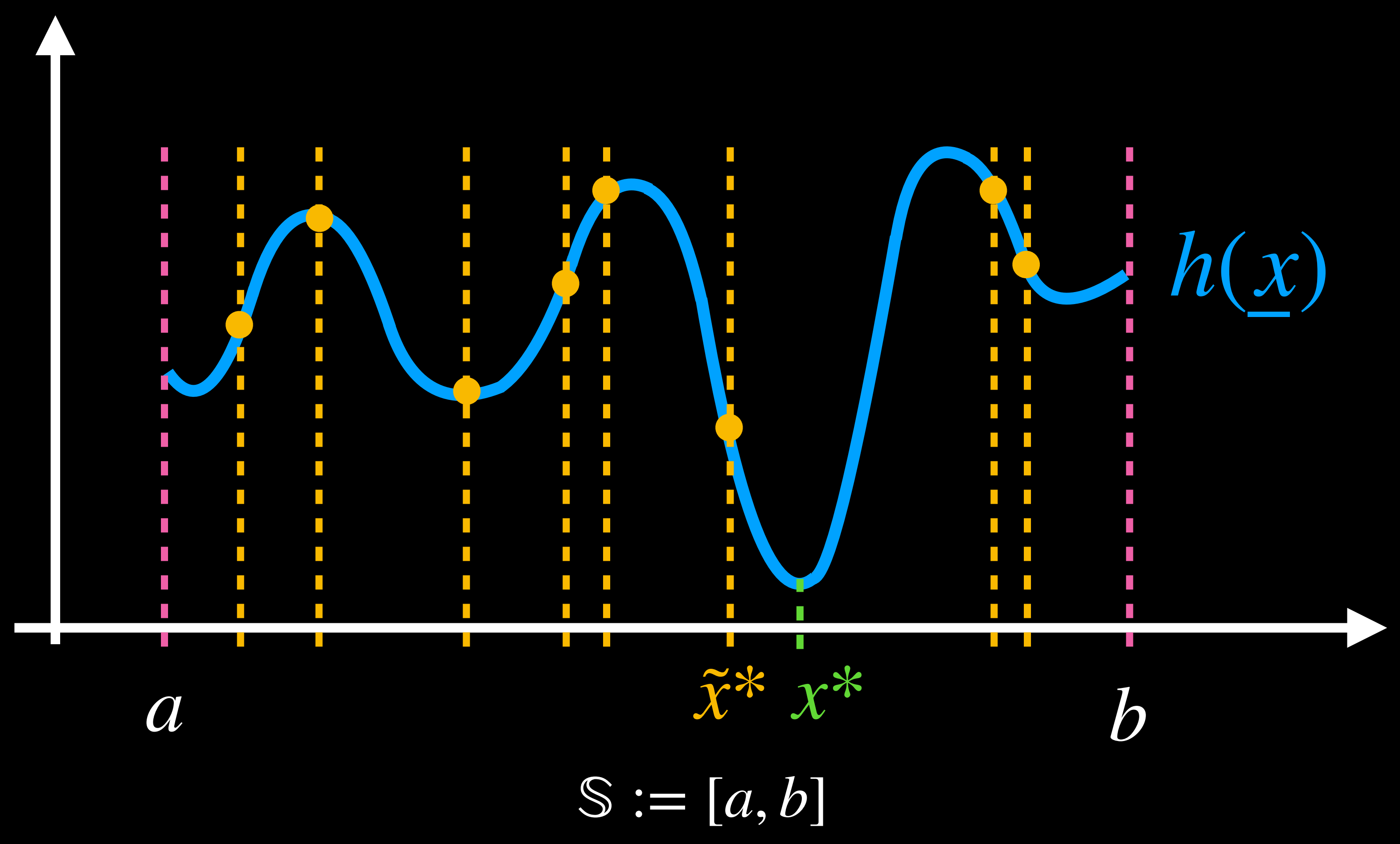
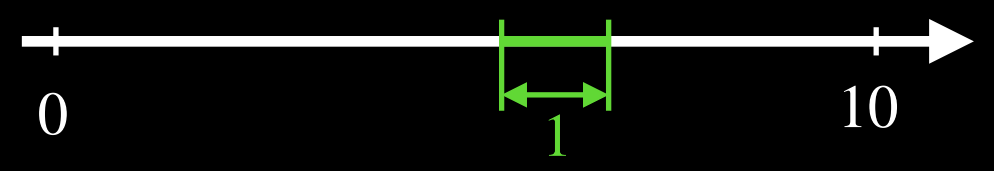


global convergence:

$$\underline{x}^* = \lim_{n\to\infty} \tilde{\underline{X}}^*_n$$

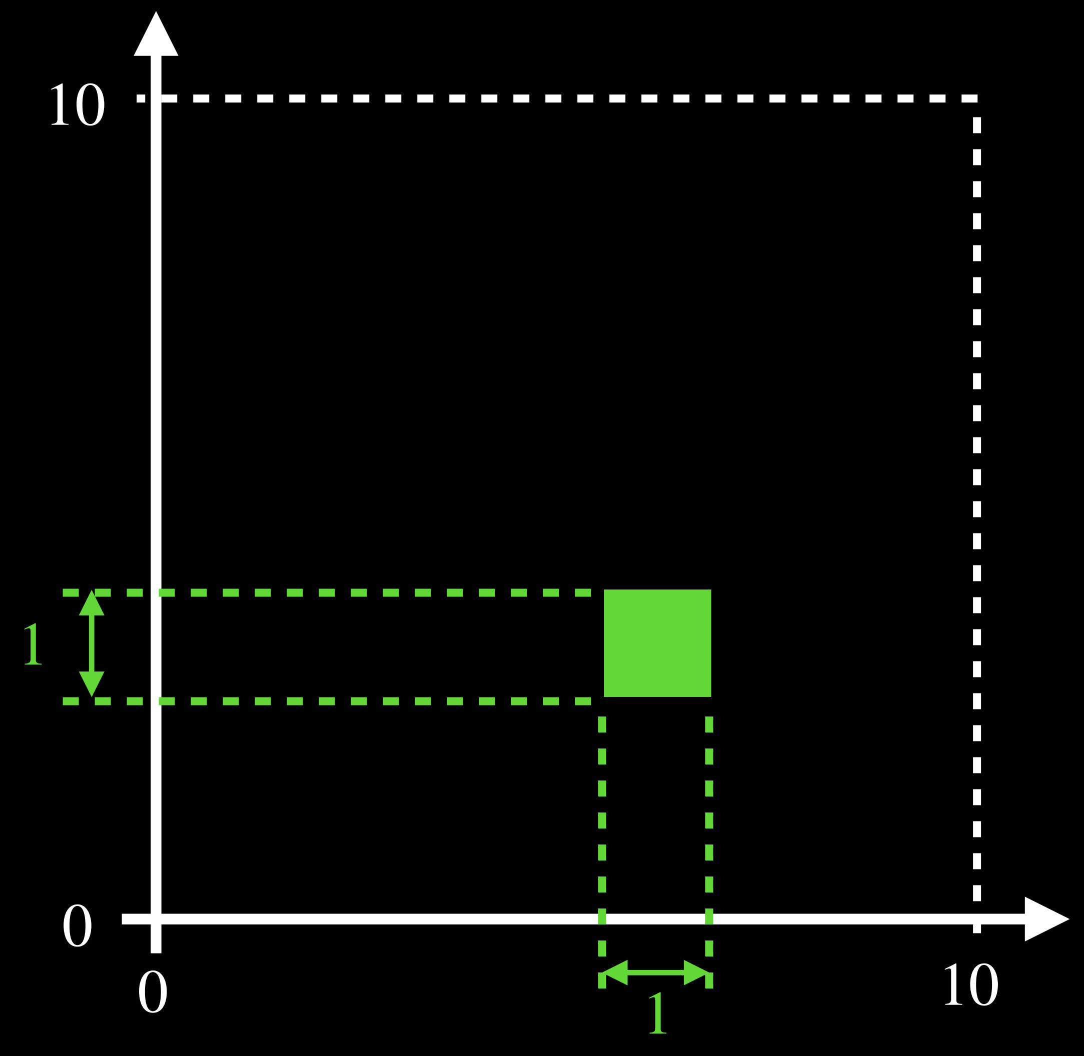
1d:



$$P(X \in \text{—}) = \frac{1}{10}$$

properties:
- very simple
- converges to $\underline{x}^*$

2d:



$$P(X \in \blacksquare) = \frac{1}{10^2}$$

useful if:
- $\mathbb{S}$ is low-dimensional and bounded

$$error \propto n^{-\frac{1}{d}}$$

algorithm 1: stochastic exploration

$$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\left(h(\underline{x})\right) \approx \arg\min\left(h(\underline{X}_1),\ldots,h(\underline{X}_n)\right) =: \underline{\tilde{X}}^*_n$$

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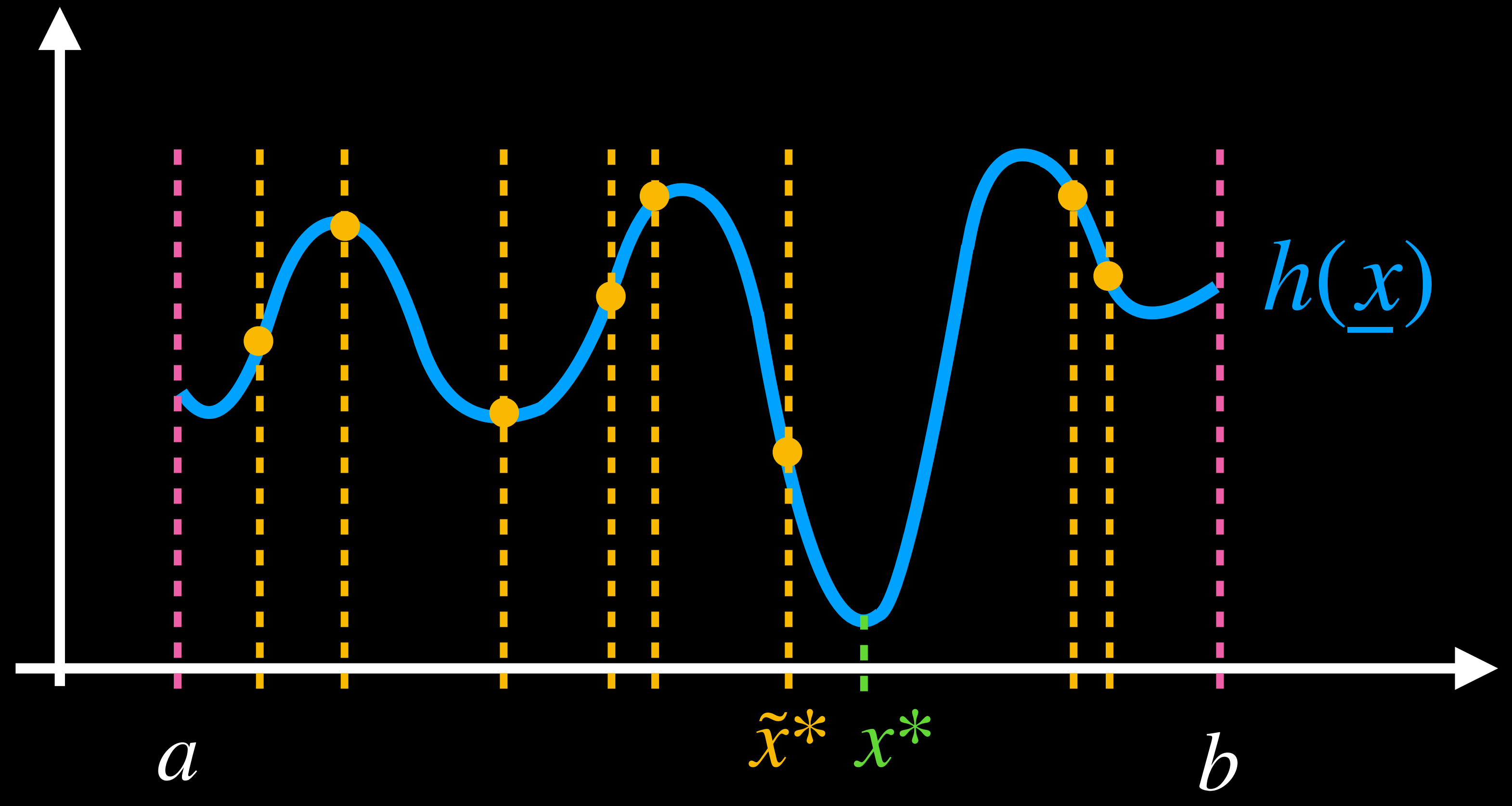

global convergence:

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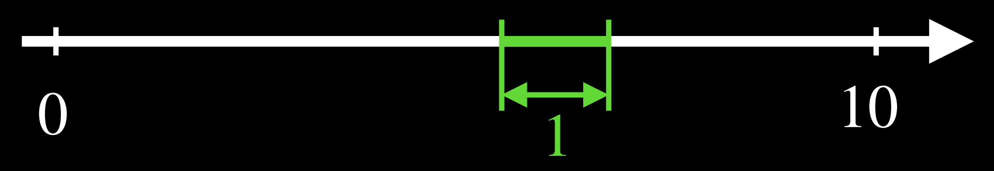
1d:

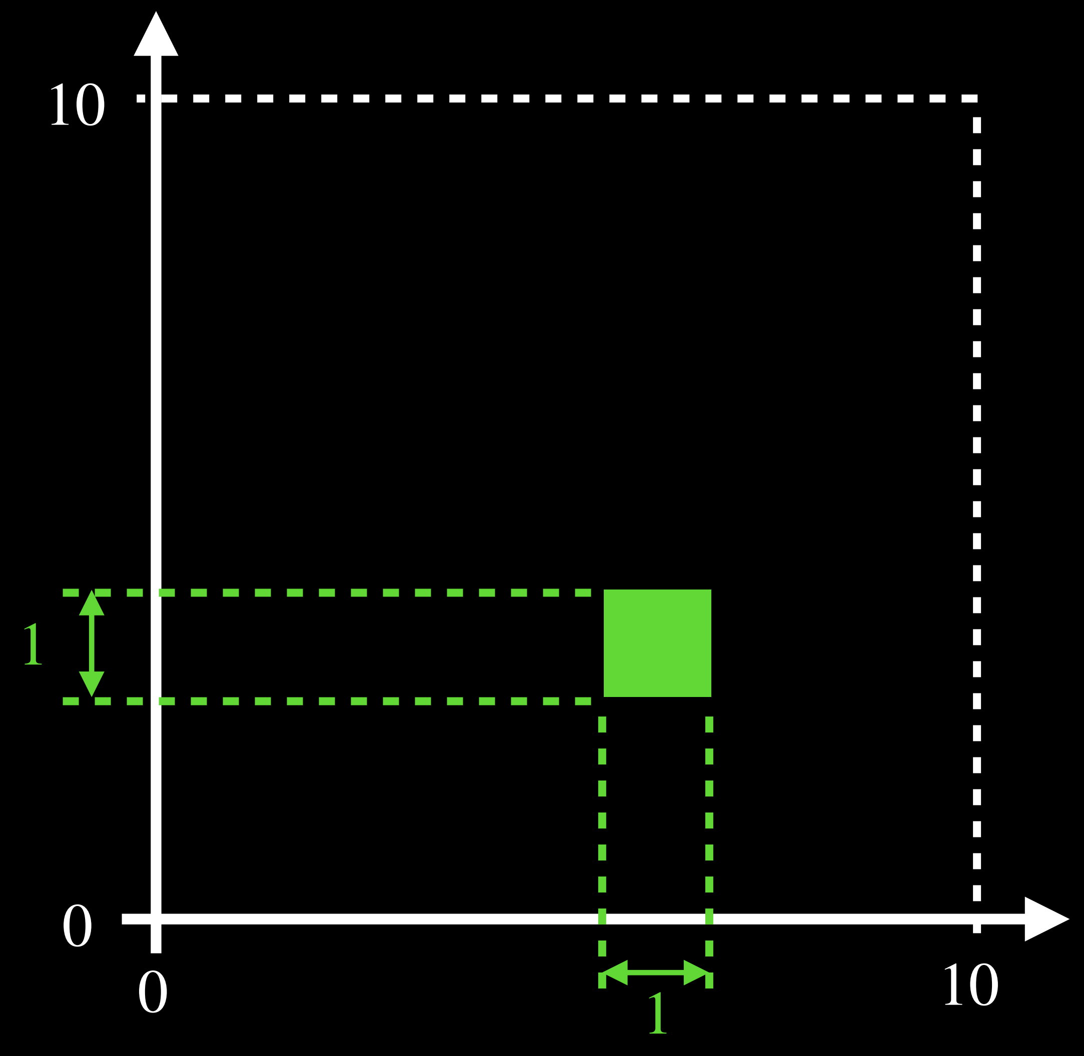


$P(X \in \text{—}) = \frac{1}{10}$

properties:

- very simple
- converges to $\underline{x}^*$

2d:



$P(X \in \blacksquare) = \frac{1}{10^2}$

useful if:

- $\mathbb{S}$ is low-dimensional and bounded
- (h is cheap to evaluate for higher dimensions)

$$error \propto n^{-\frac{1}{d}}$$

algorithm 2: stochastic descent

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$$\underline{x}^* := \arg\min_{\underline{x} \in \mathbb{S}} \left(h(\underline{x}) \right)$$

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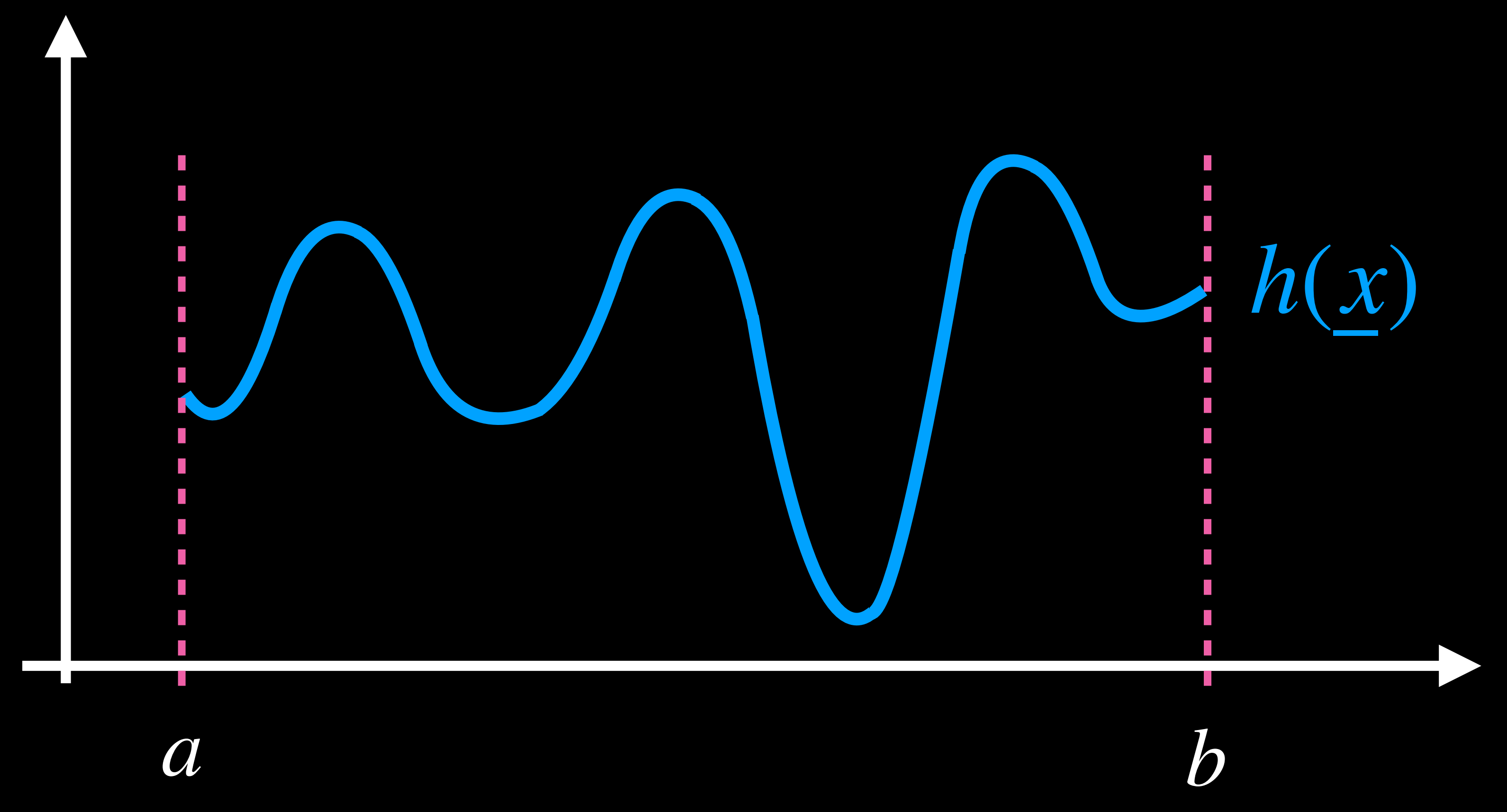

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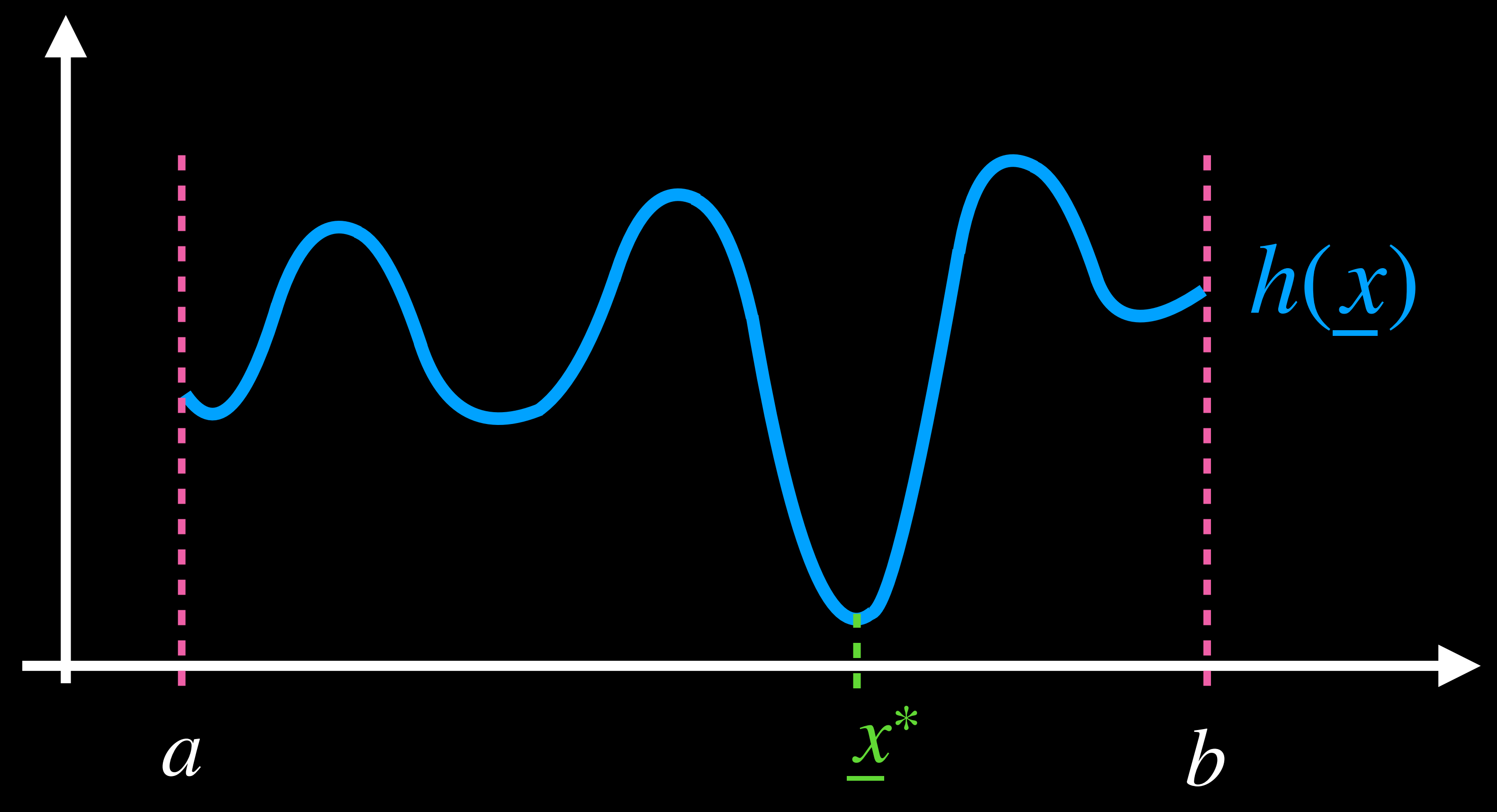

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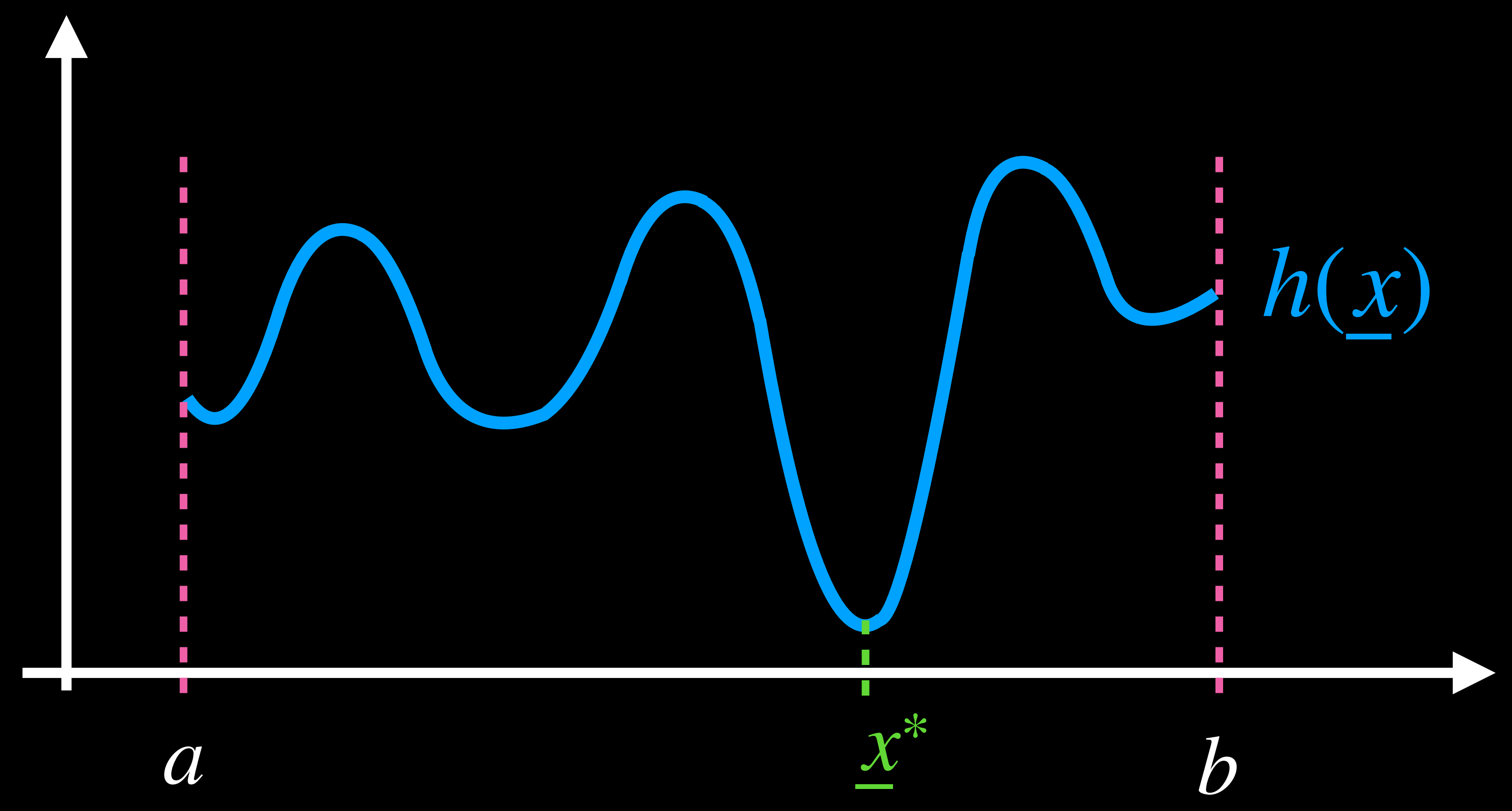
gradient descent:



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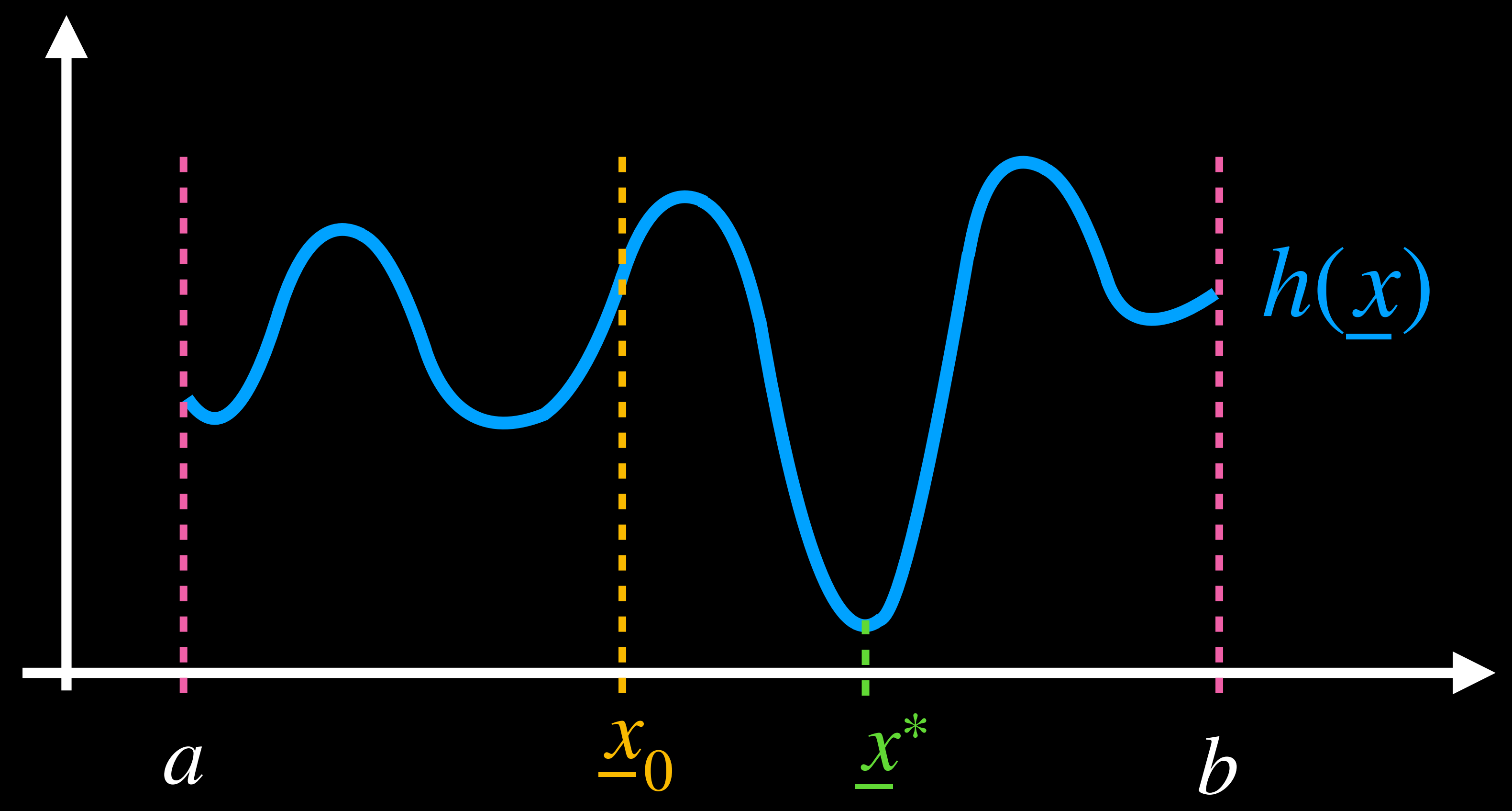
gradient descent:



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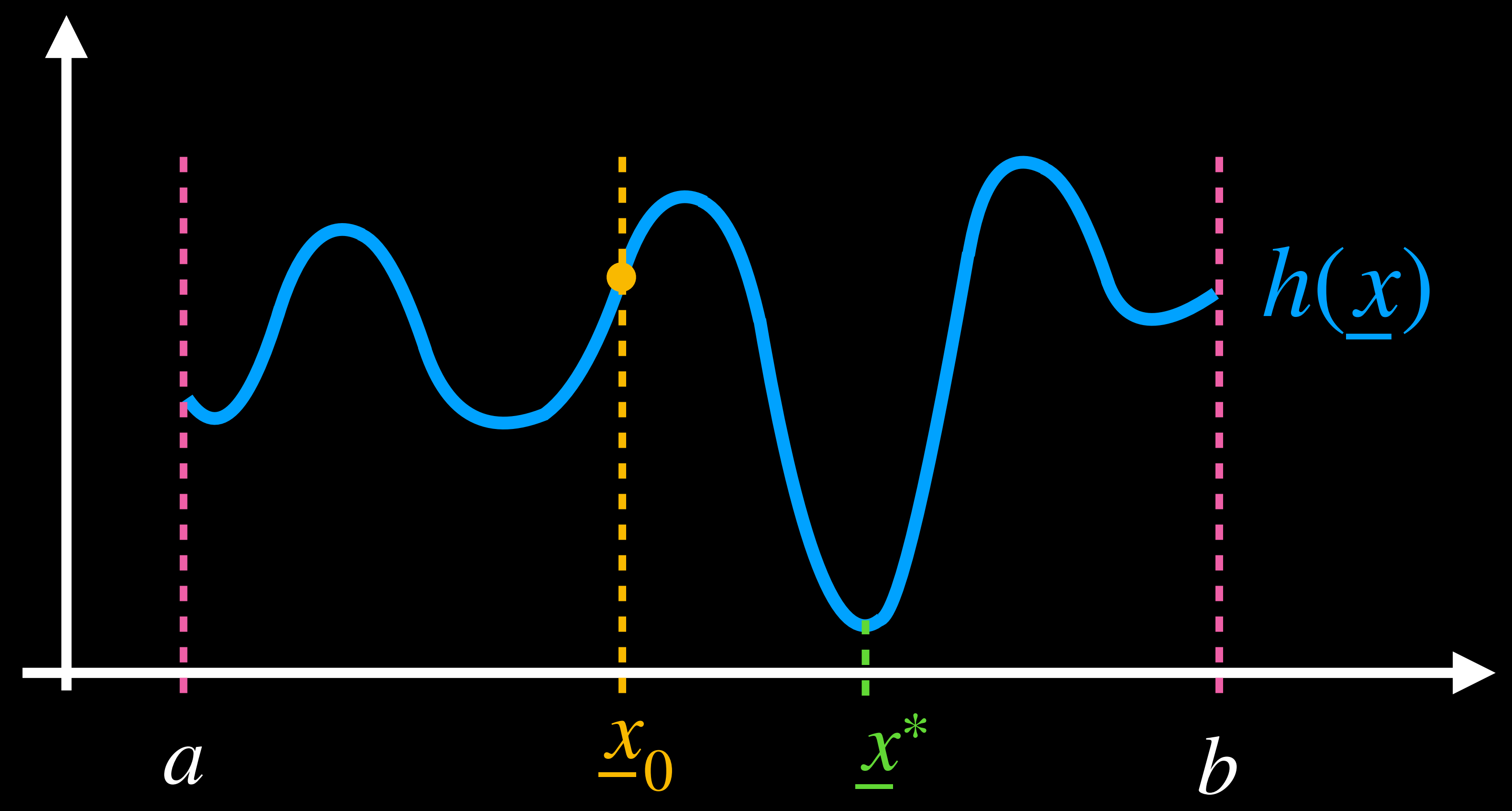

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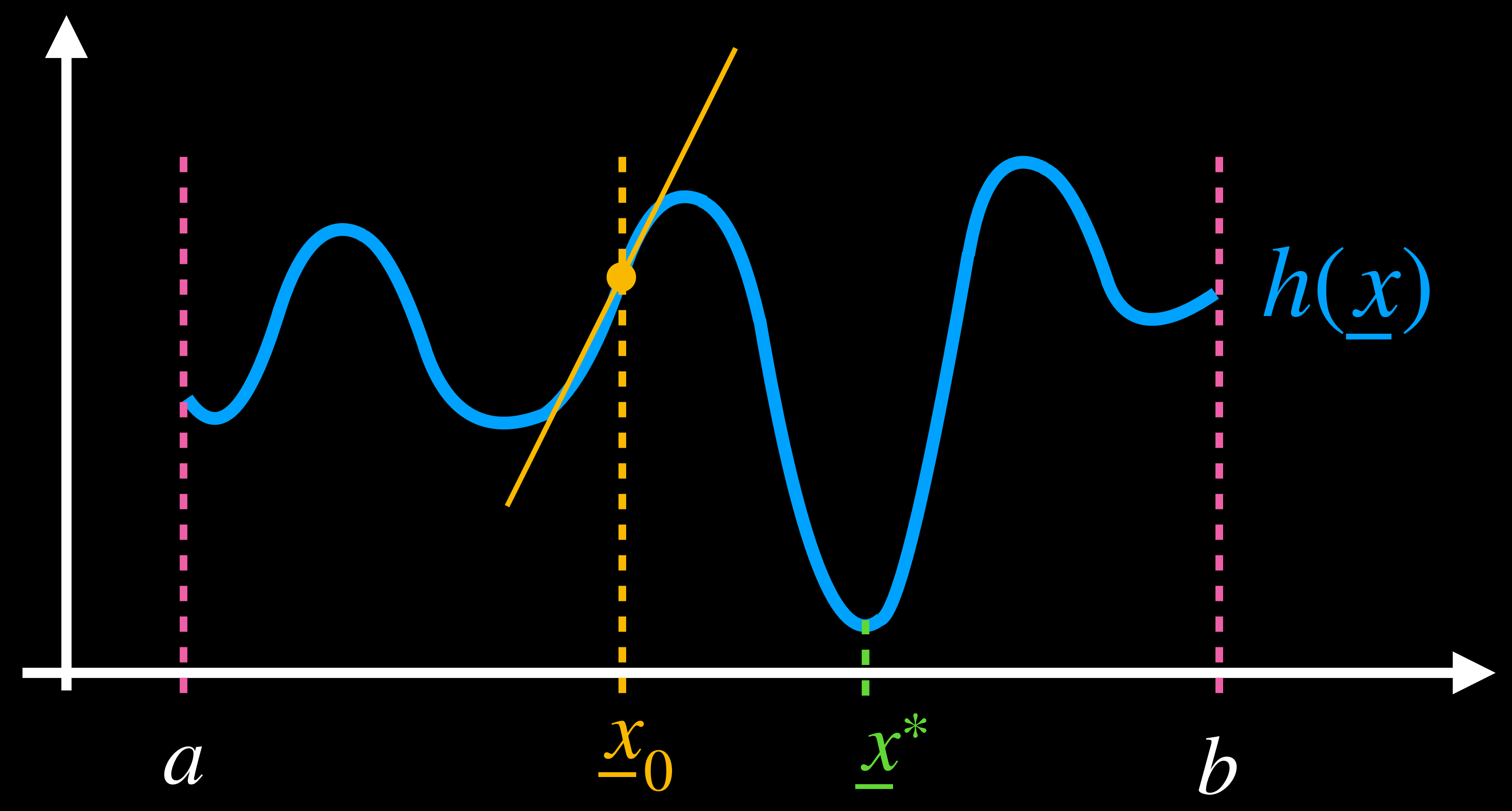

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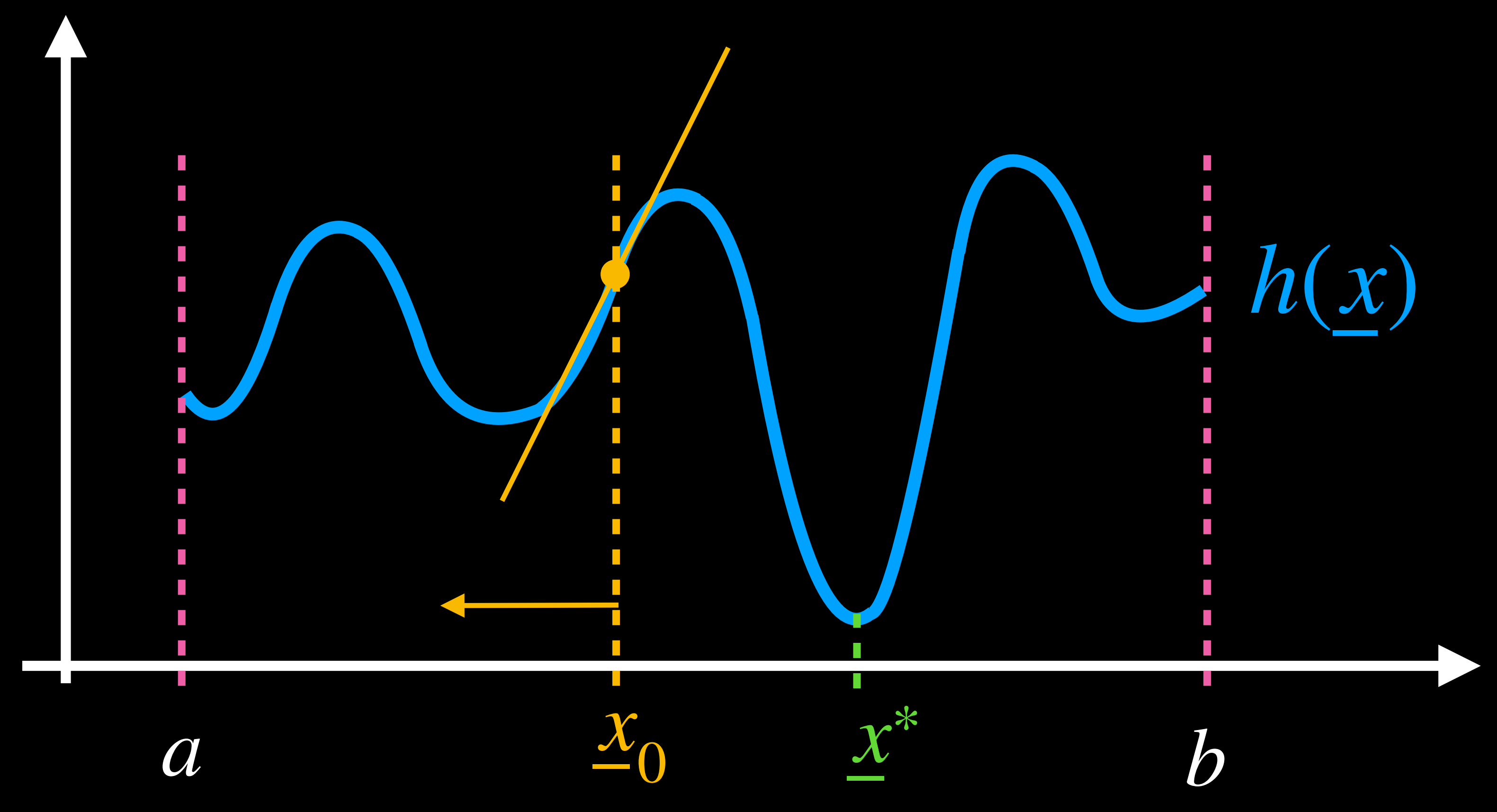

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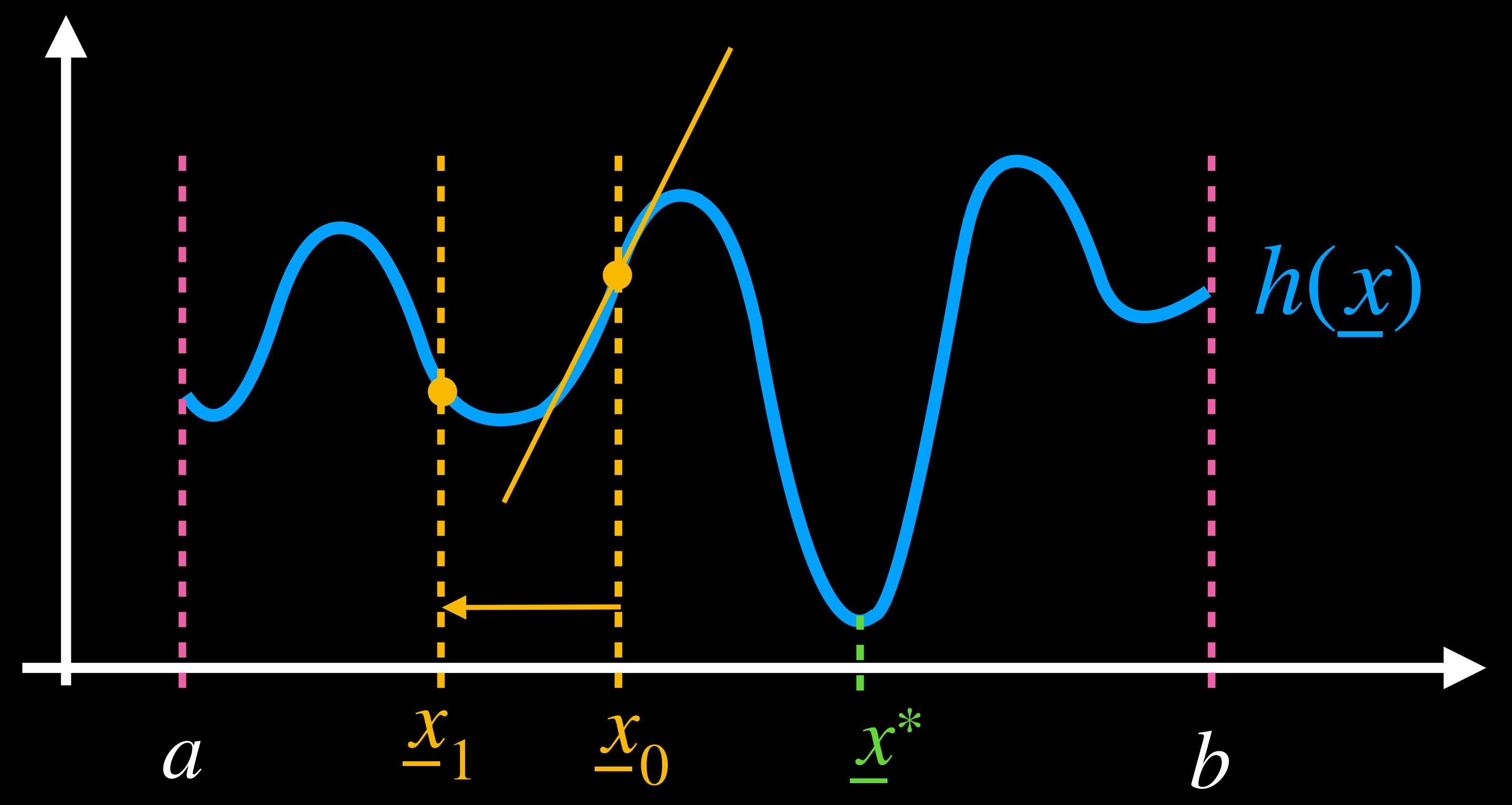

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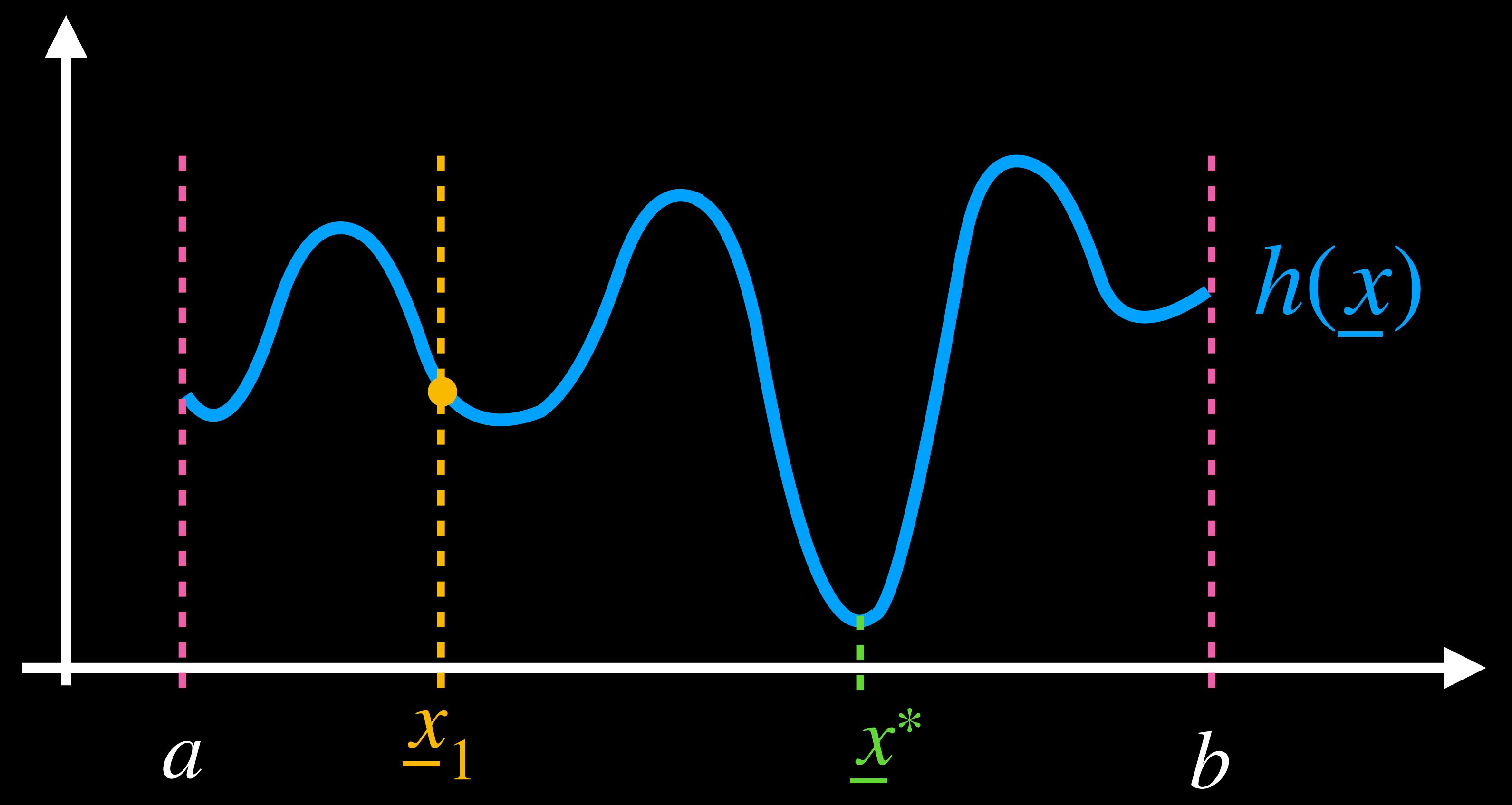

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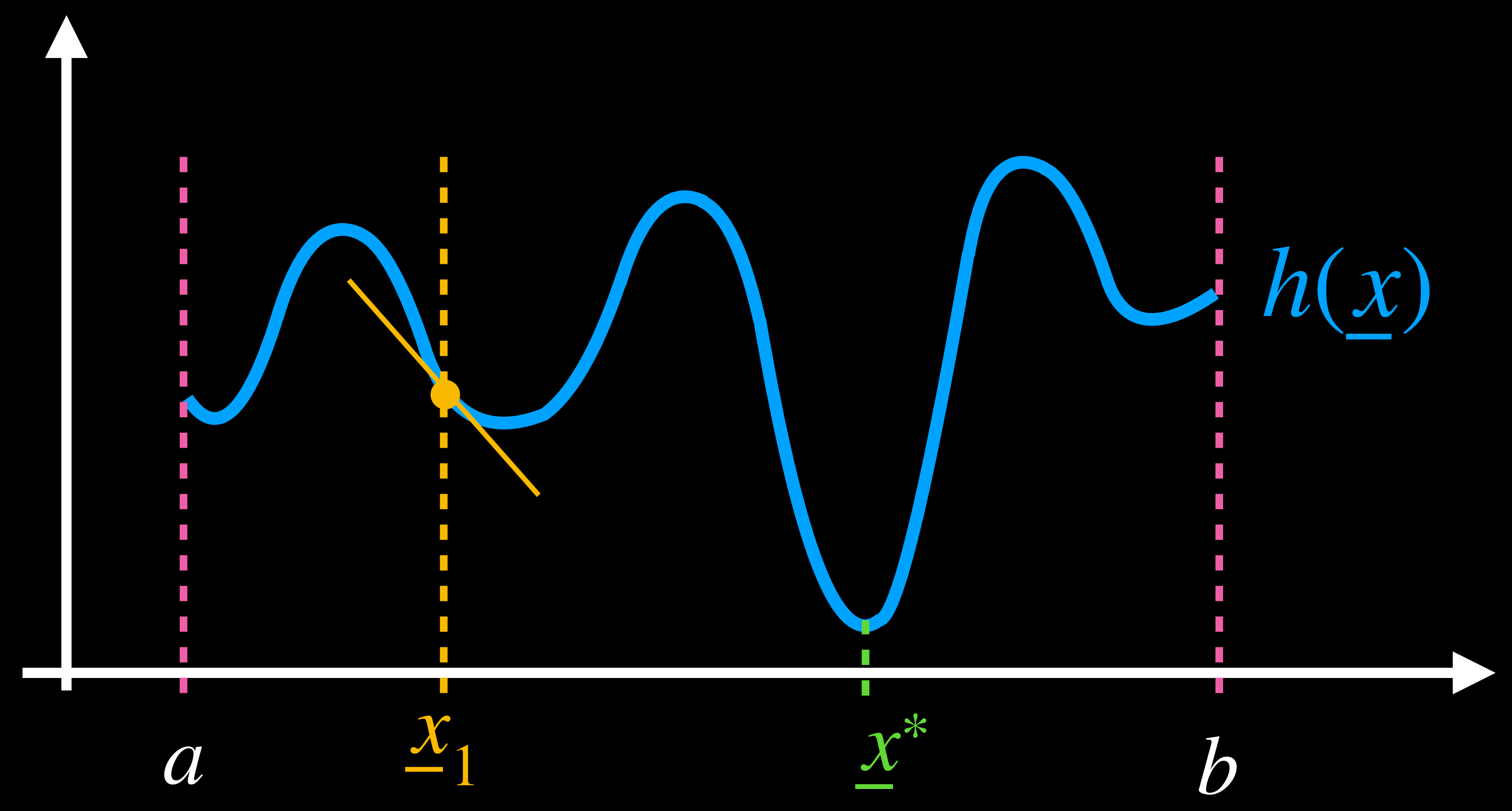

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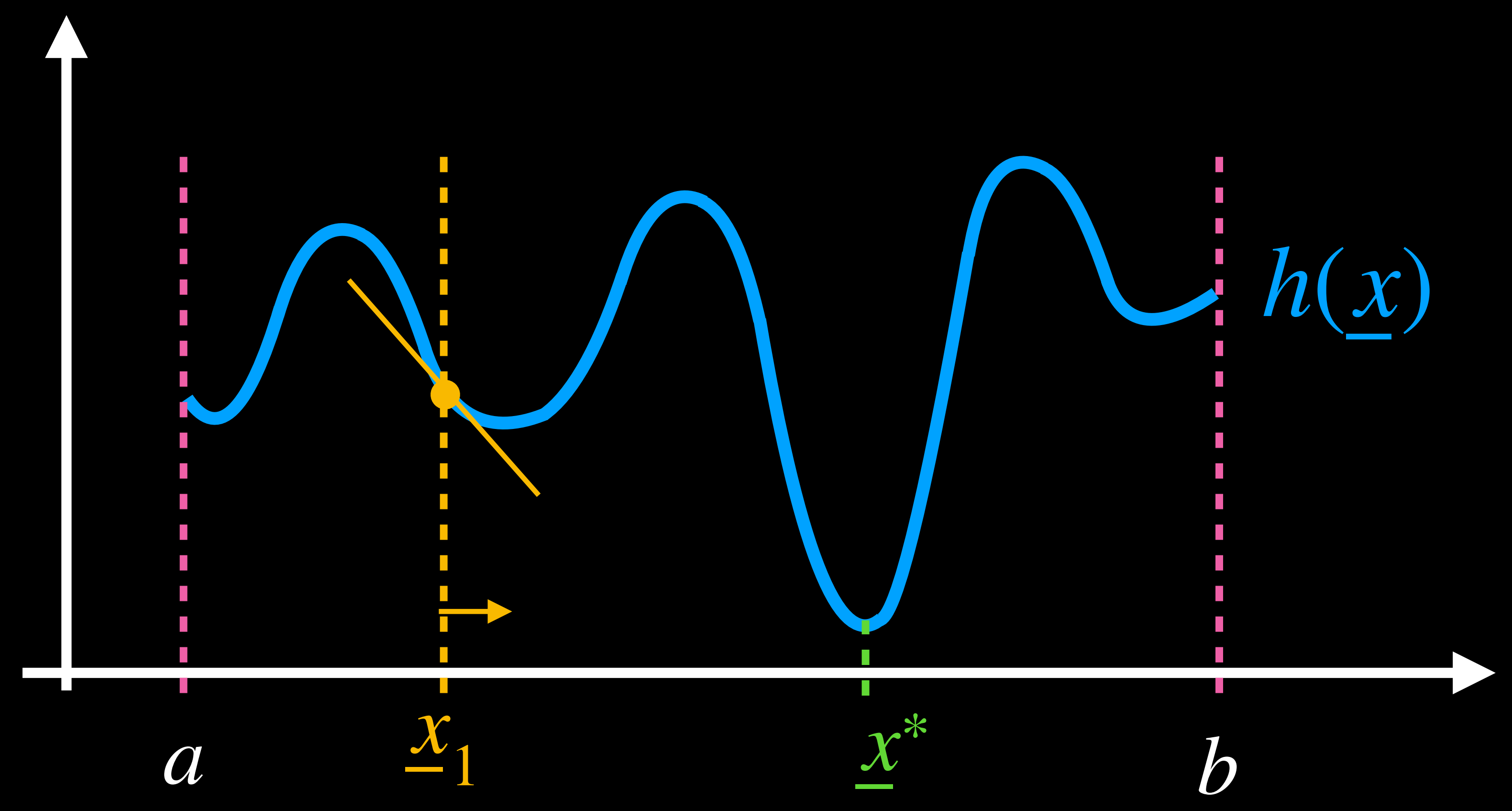

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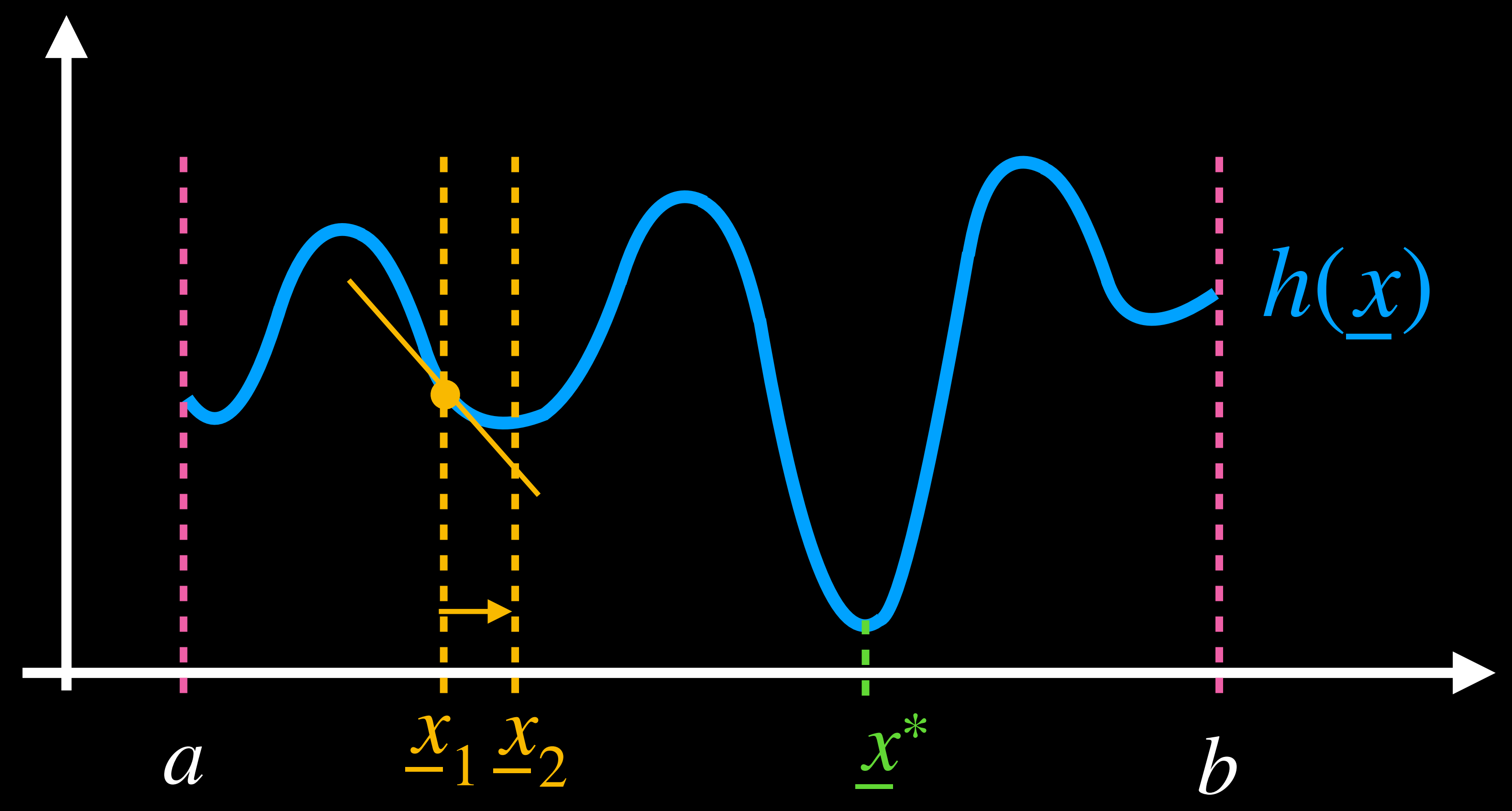

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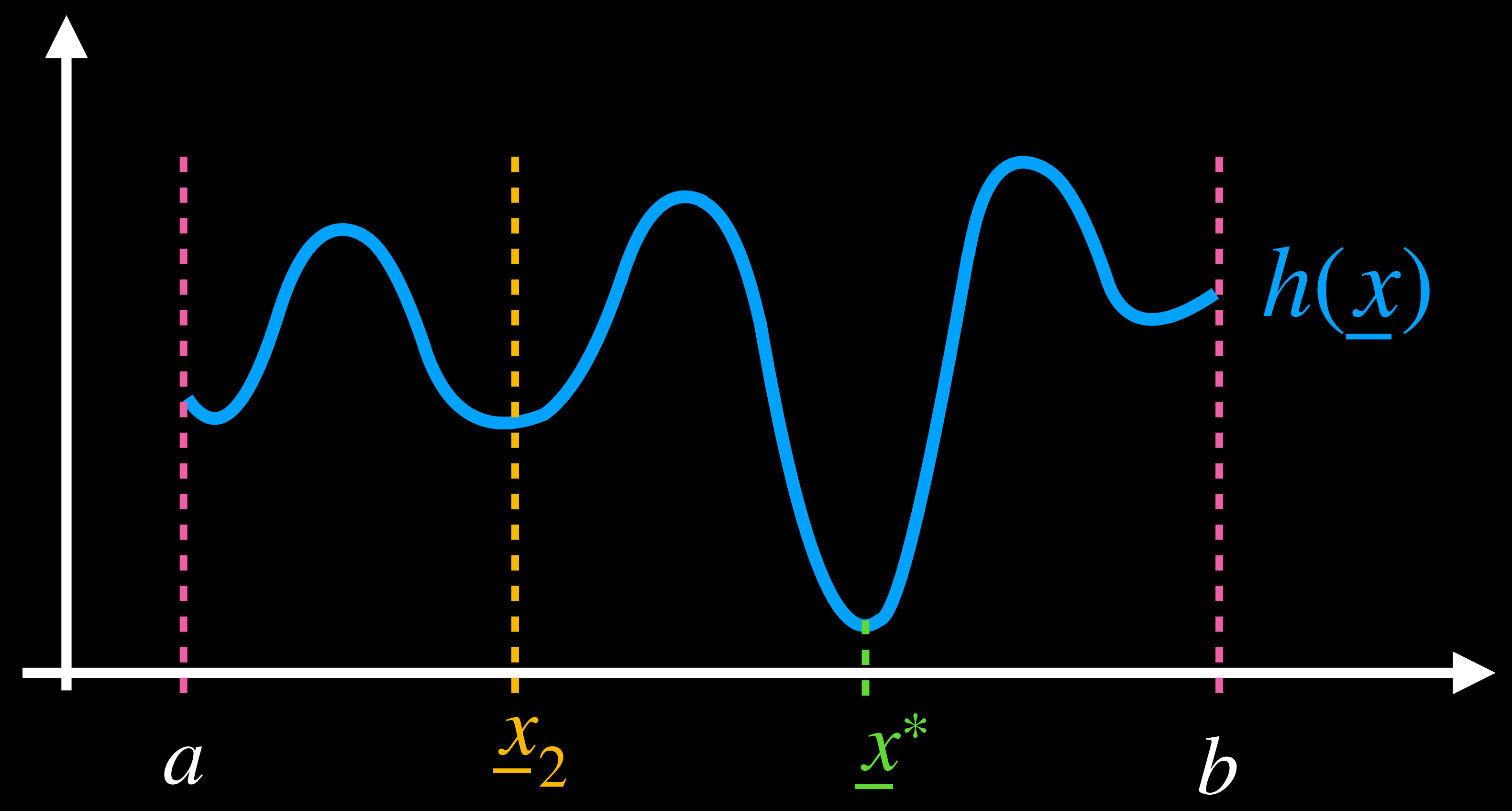

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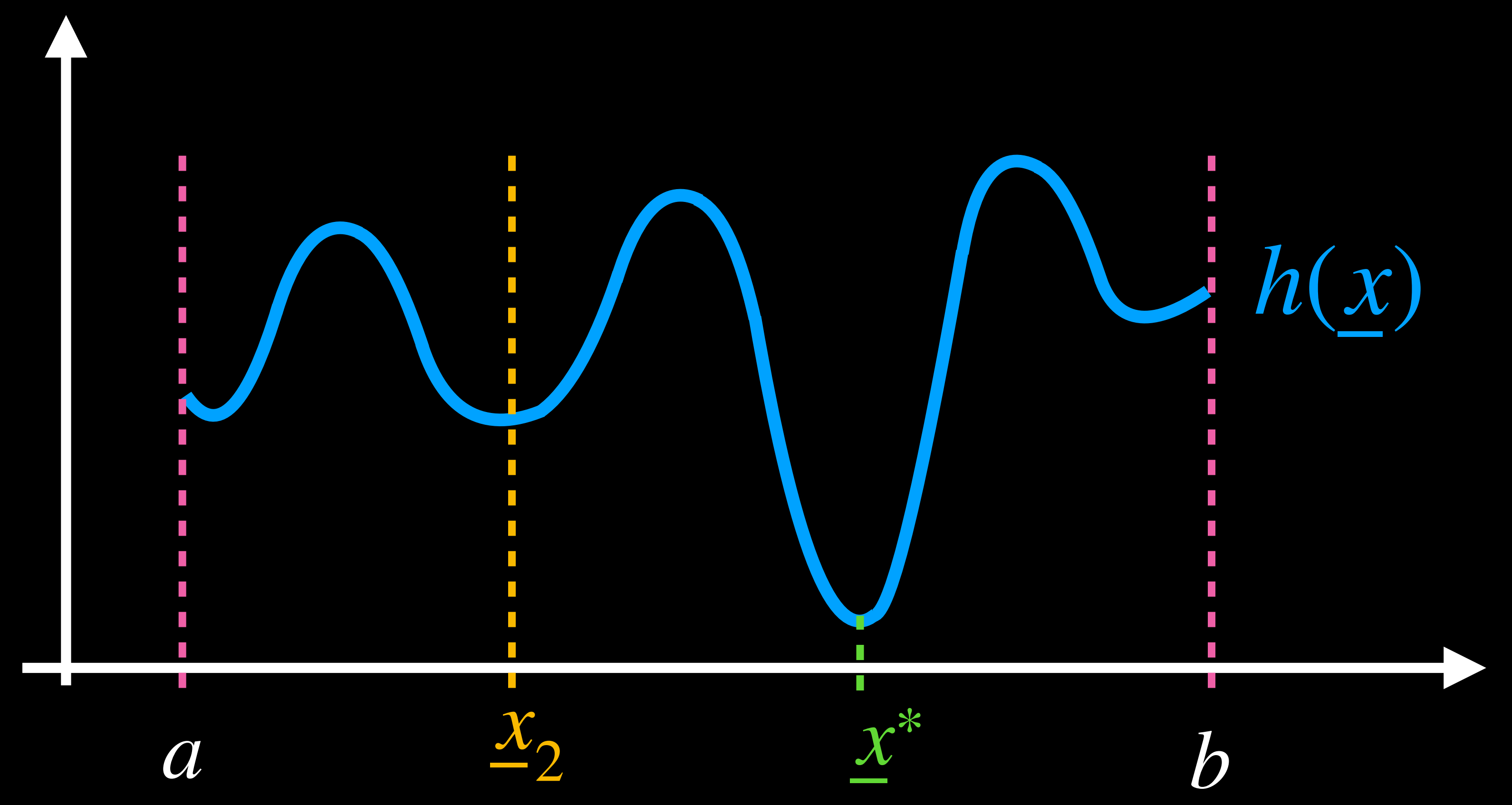
$$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\,\big(h(\underline{x})\big)$$

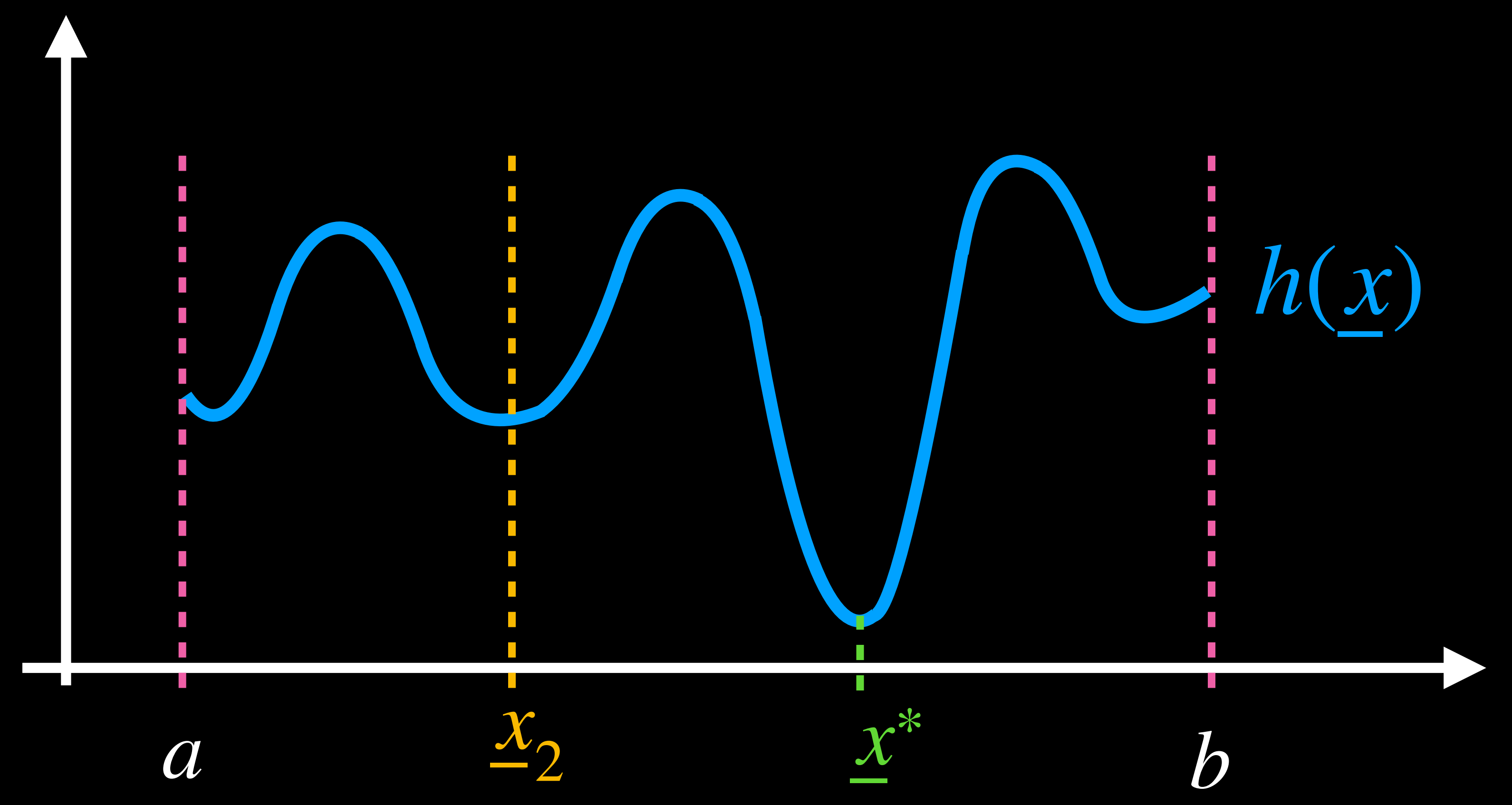
gradient descent: $\underline{x}_{t+1}$



algorithm 2: stochastic descent

$$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \big(h(\underline{x})\big)$$

gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \nabla h(\underline{x}_t)$

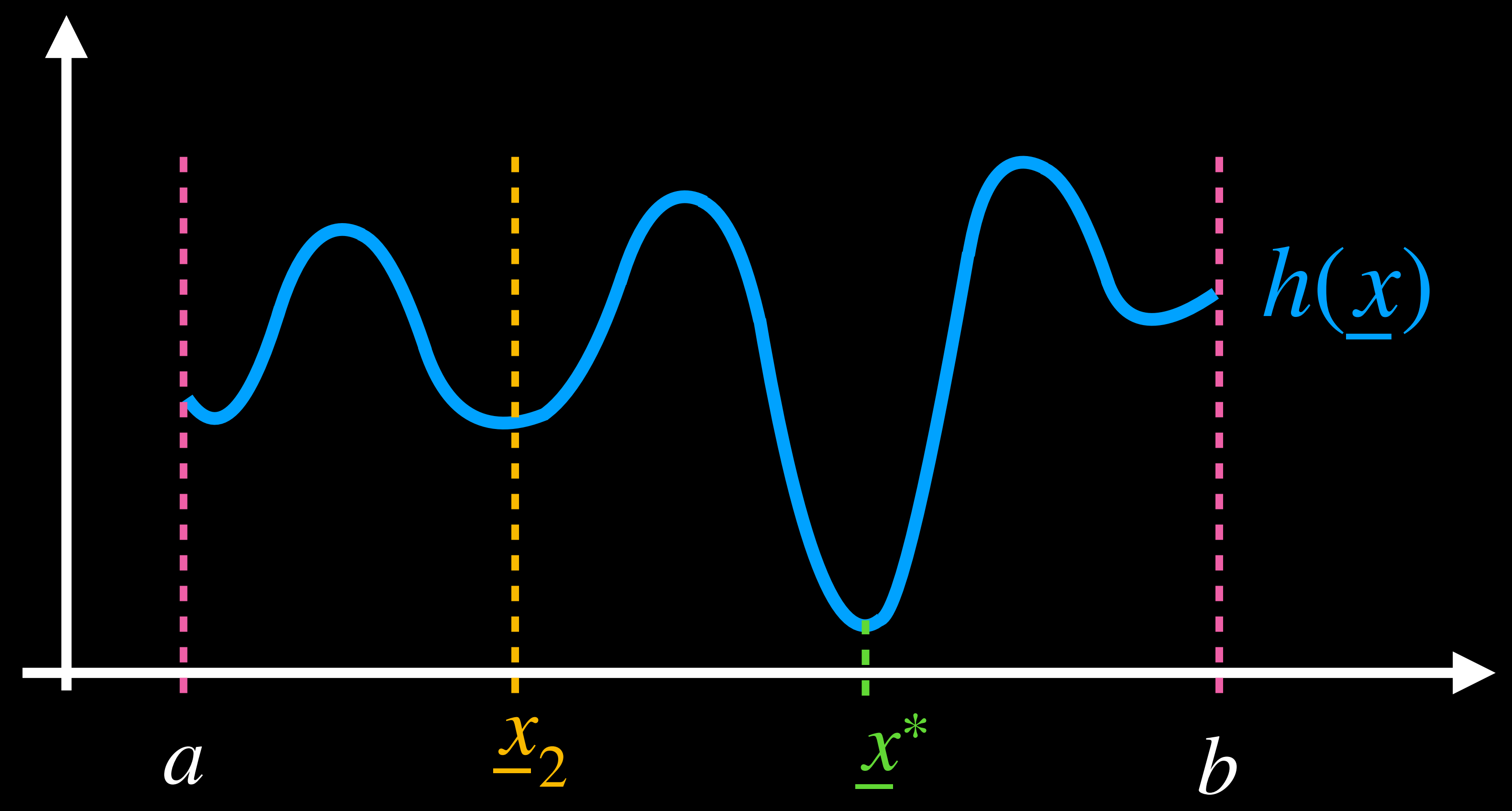


algorithm 2: stochastic descent

$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\left(h(\underline{x})\right)$

$t \in \{1,\ldots,n\}$ steps

gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \nabla h(\underline{x}_t)$



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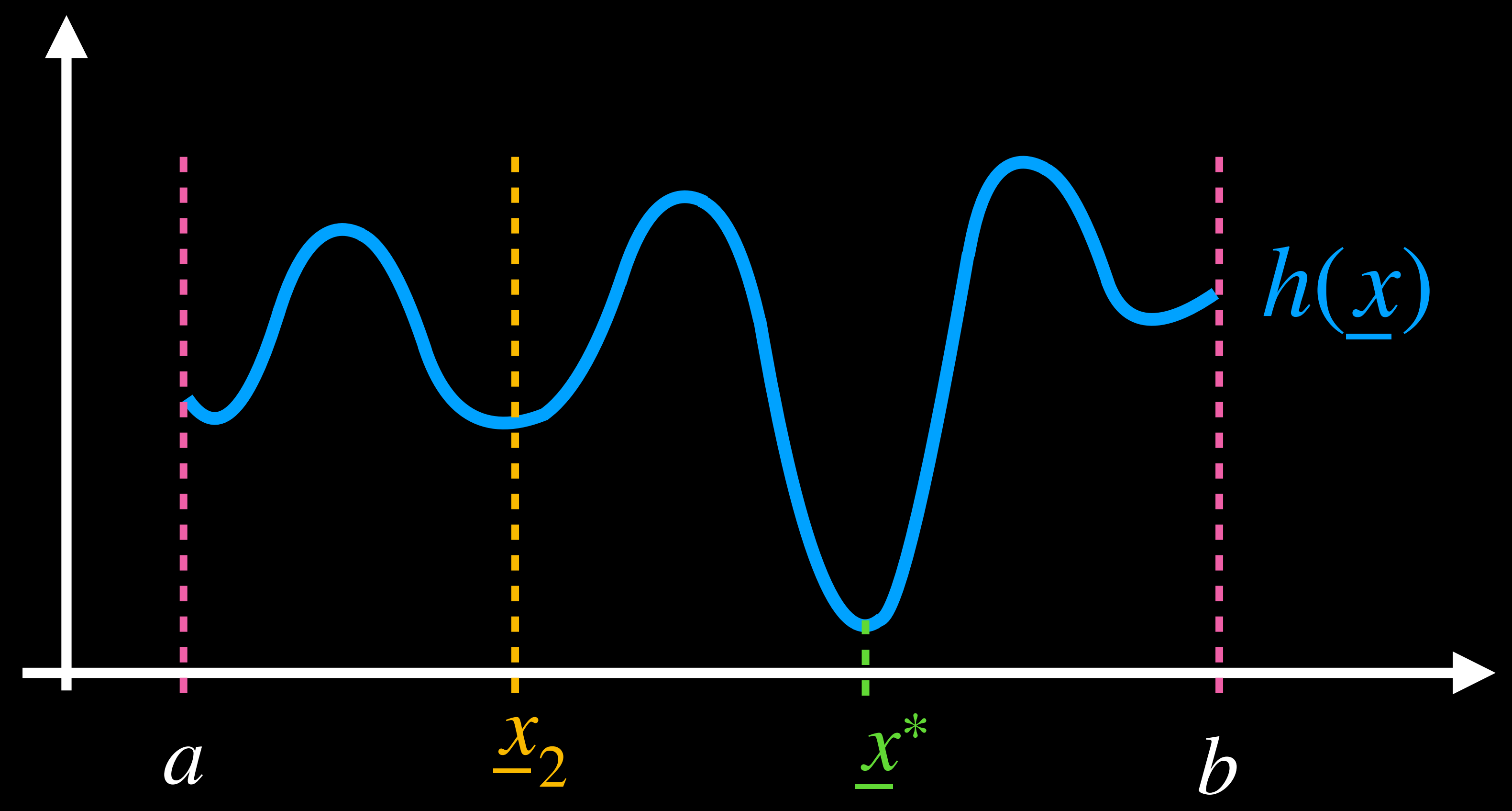
$t \in \{1,\ldots,n\}$ steps

$\alpha_t > 0$ step size

gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \nabla h(\underline{x}_t)$



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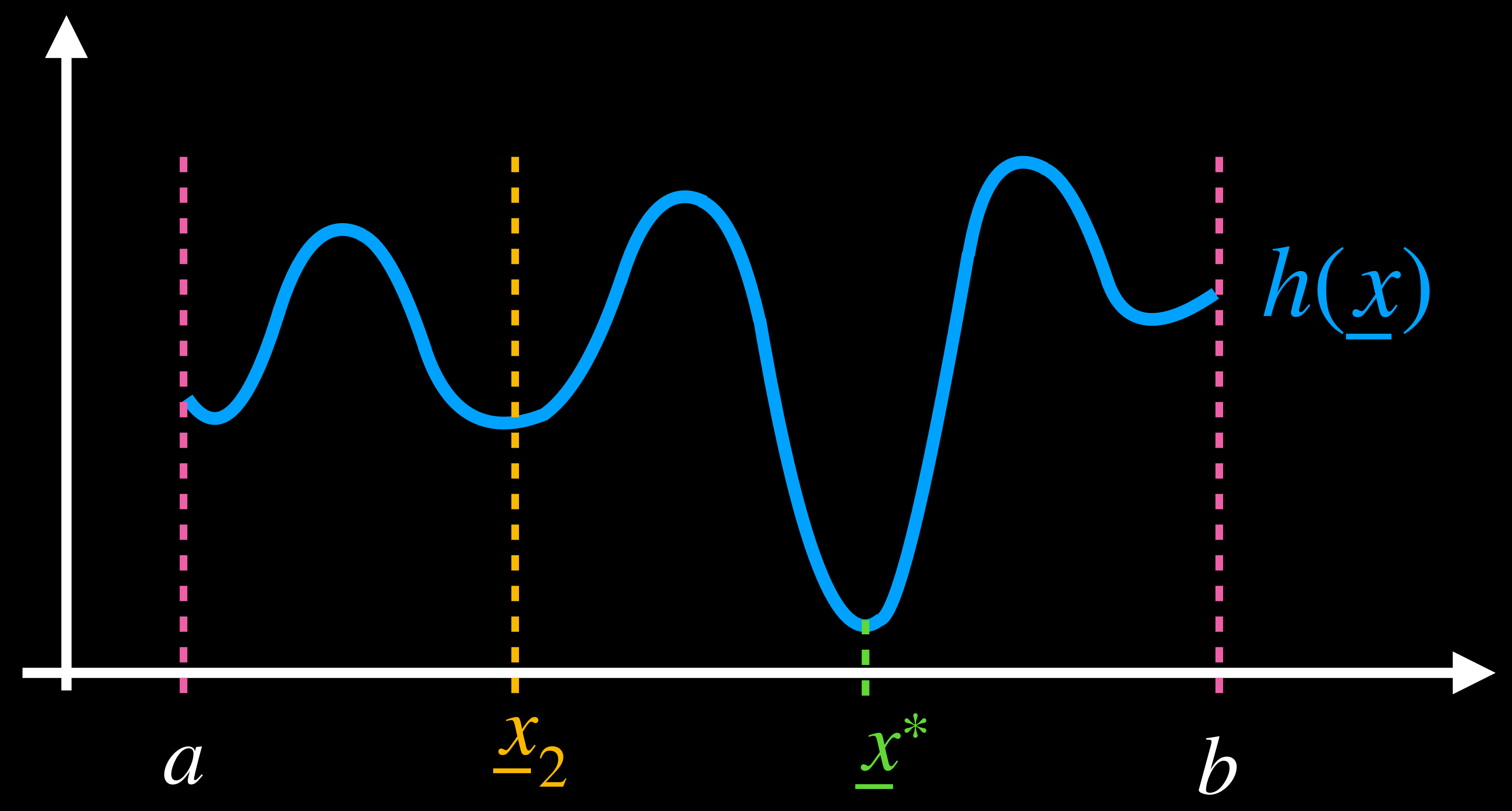
gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \nabla h(\underline{x}_t)$

$t \in \{1, \ldots, n\}$ steps

$\alpha_t > 0$ step size

$-\nabla h(\underline{x}_t)$ vector of steepest decrease



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$$\underline{x}^* := \underset{\underline{x}\in\mathbb{S}}{\arg\min}\left(h(\underline{x})\right)$$

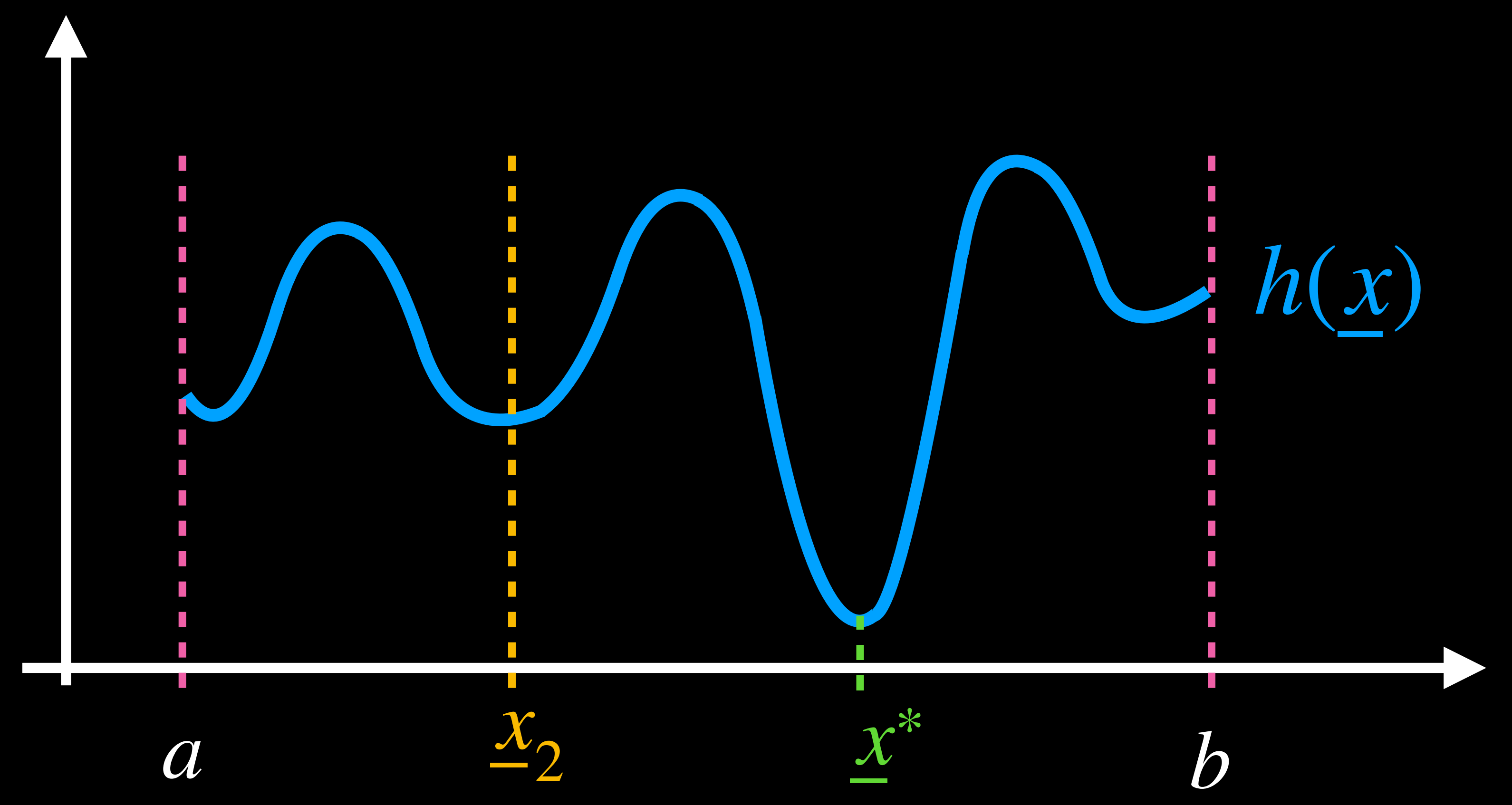
gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \nabla h(\underline{x}_t)$

$t \in \{1,\ldots,n\}$ steps

$\alpha_t > 0$ step size

$-\nabla h(\underline{x}_t)$ vector of steepest decrease

$\underline{x}_0$ starting point



algorithm 2: stochastic descent

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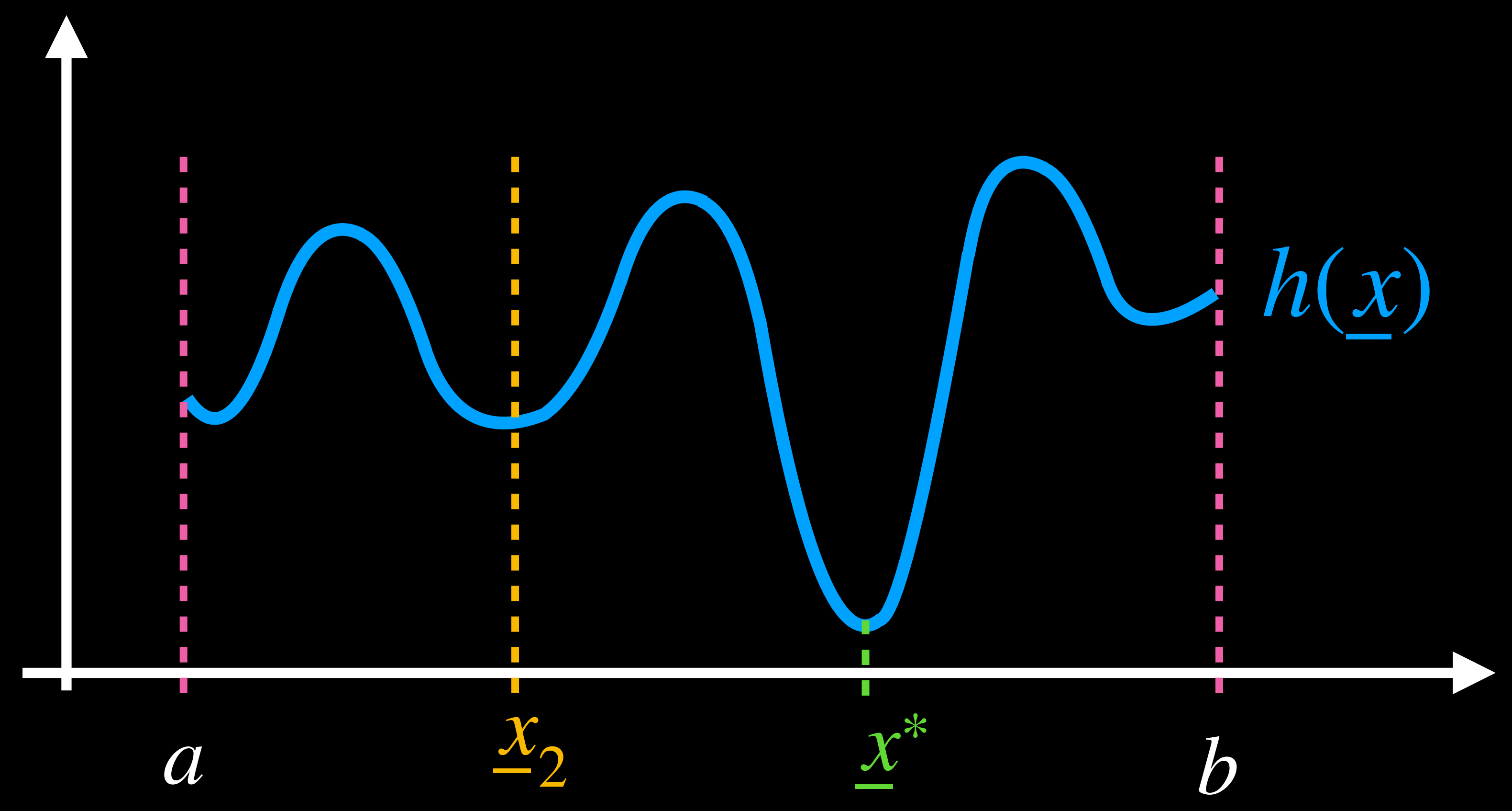

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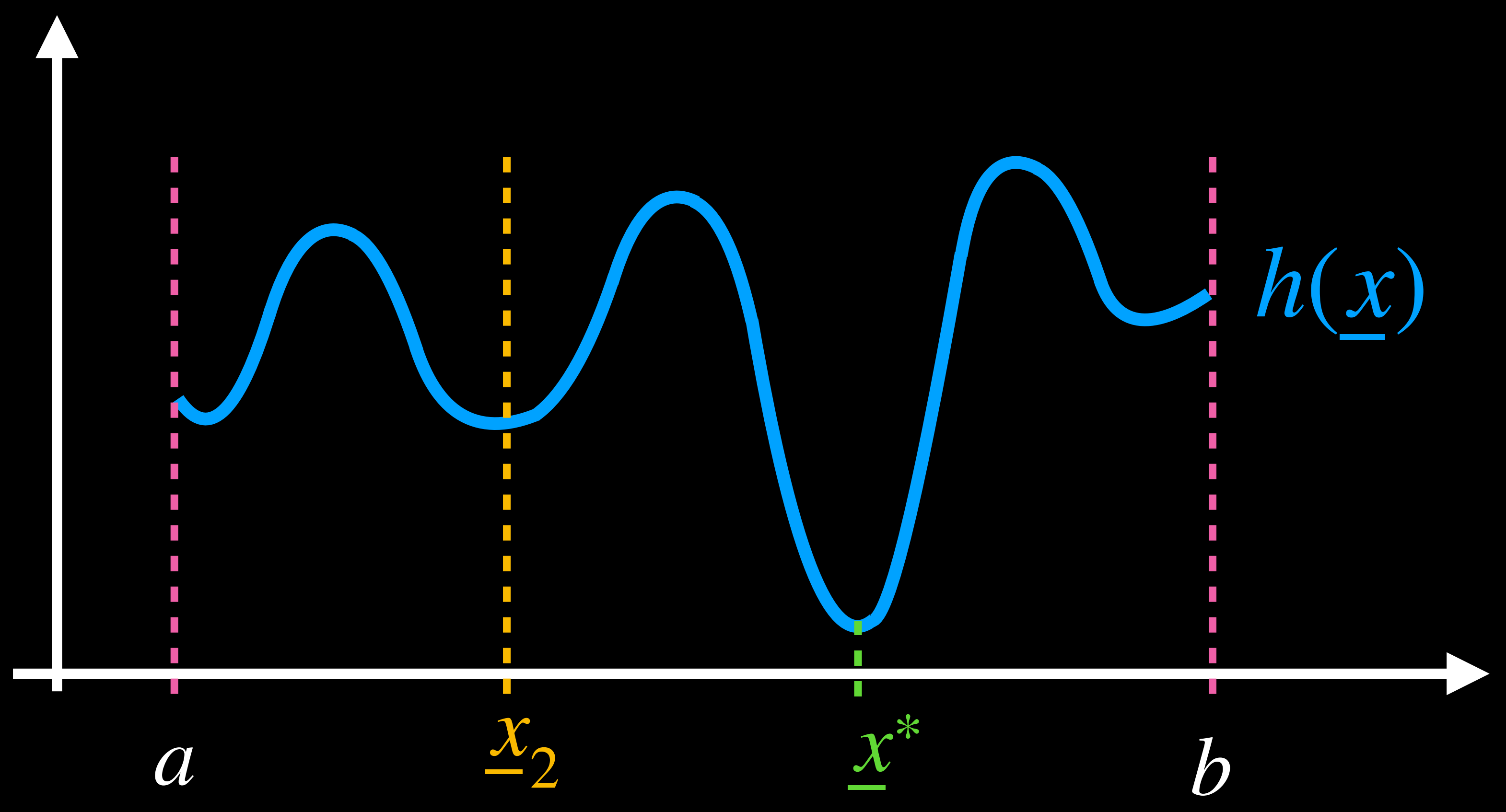

gradient:

algorithm 2: stochastic descent

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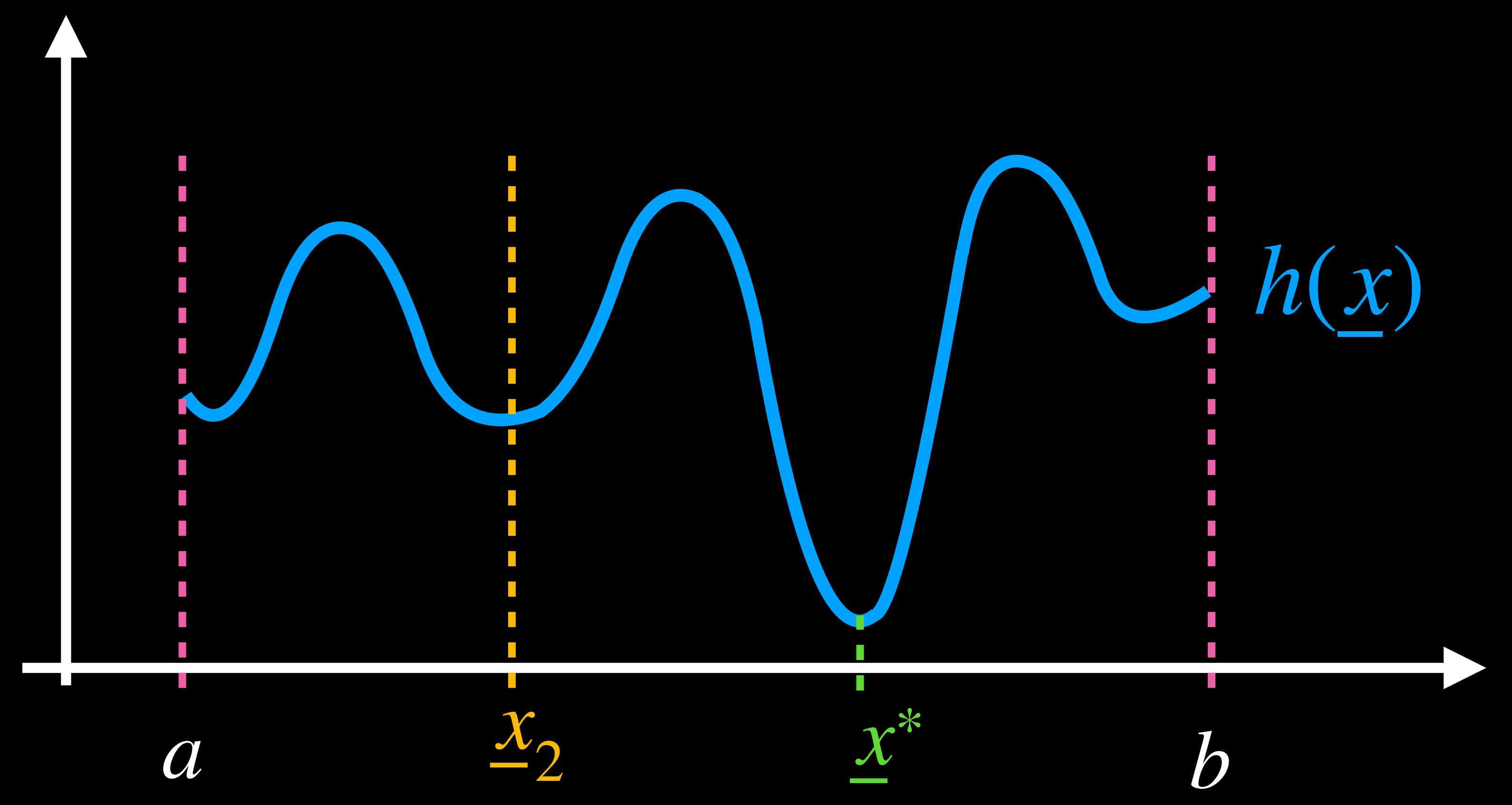

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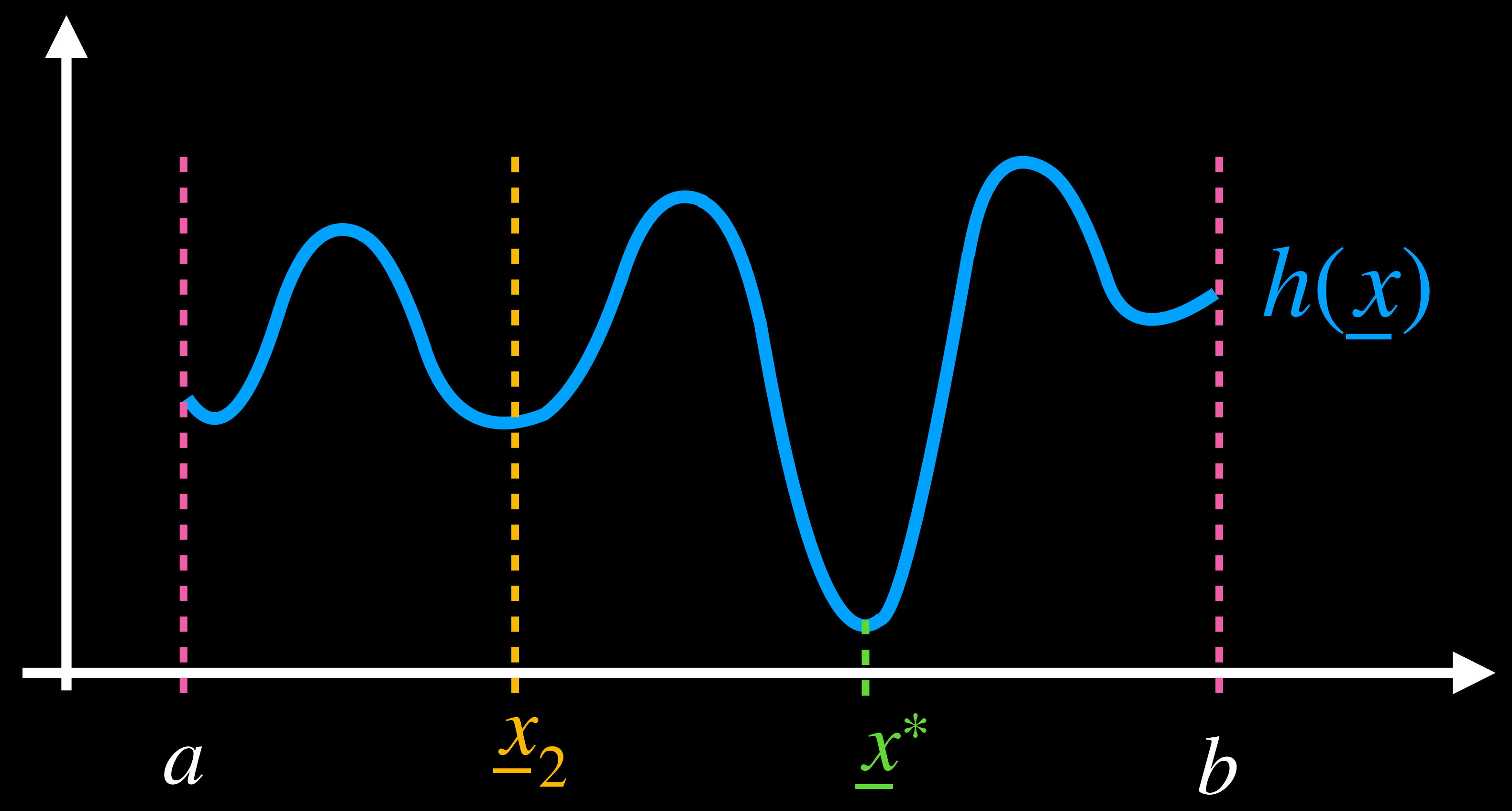

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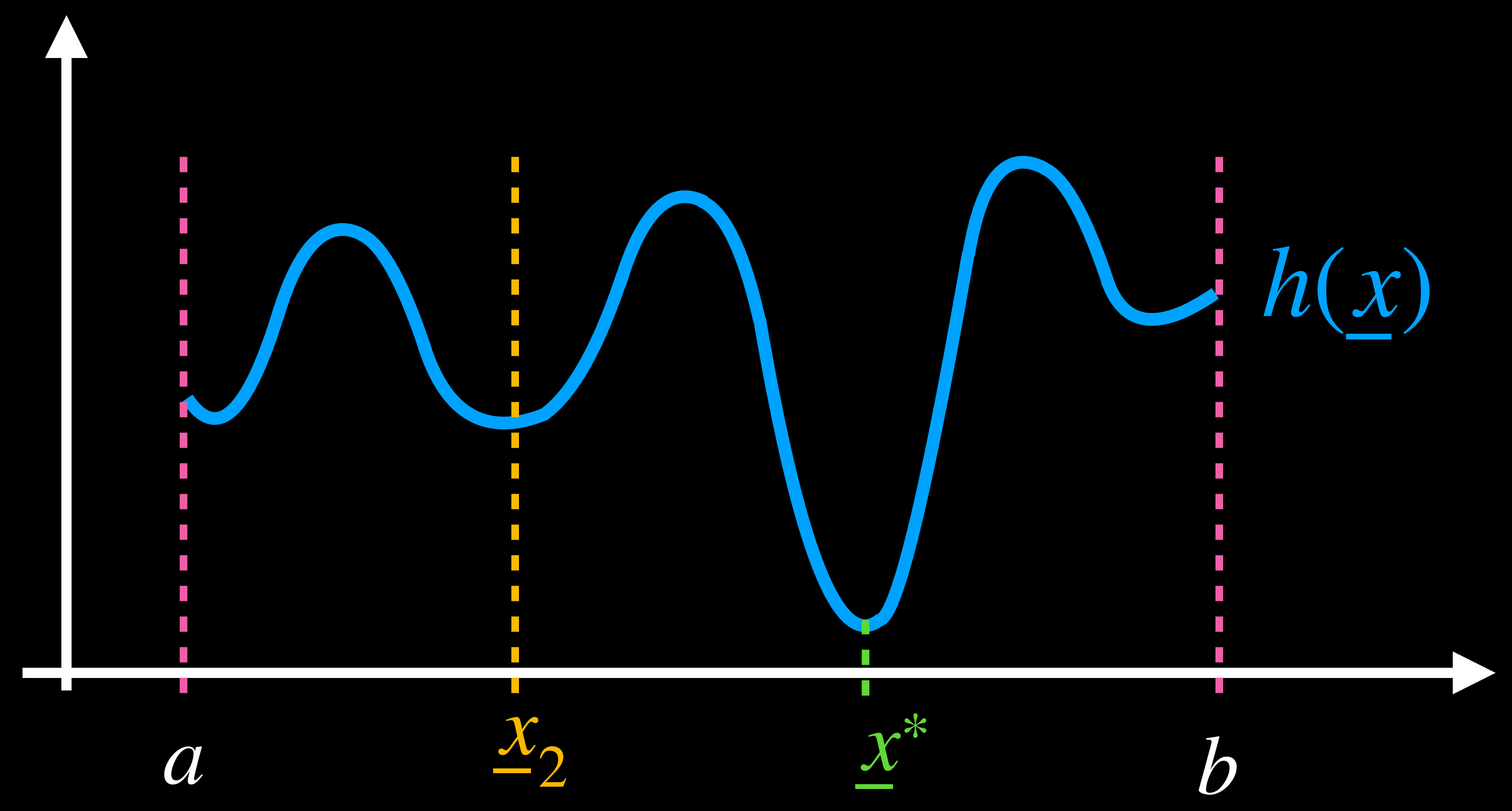

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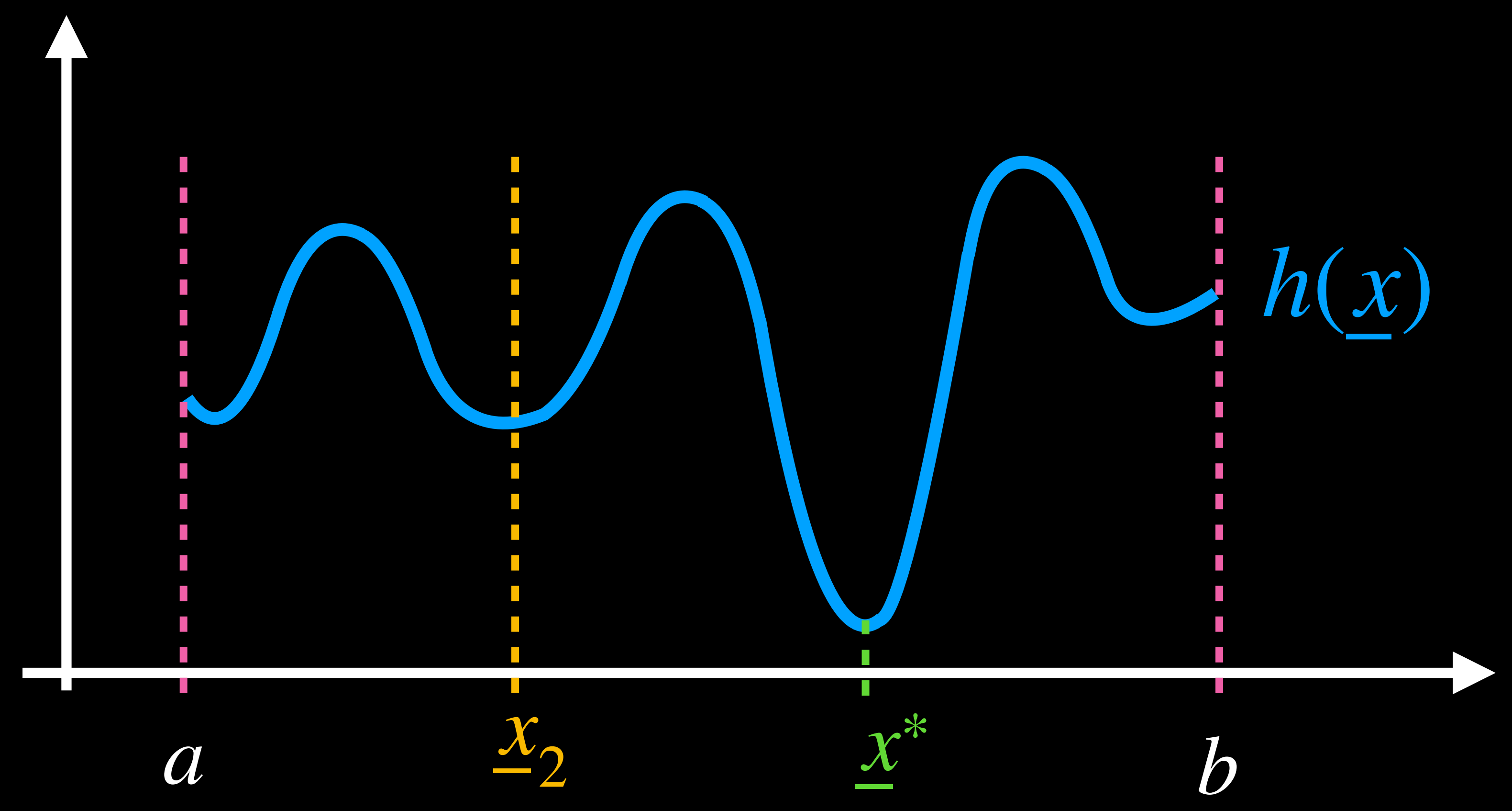
→ approximating $\nabla h(\underline{x})$ has $2d$ evaluations of h (expensive)

algorithm 2: stochastic descent



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stochastic descent:

algorithm 2: stochastic descent

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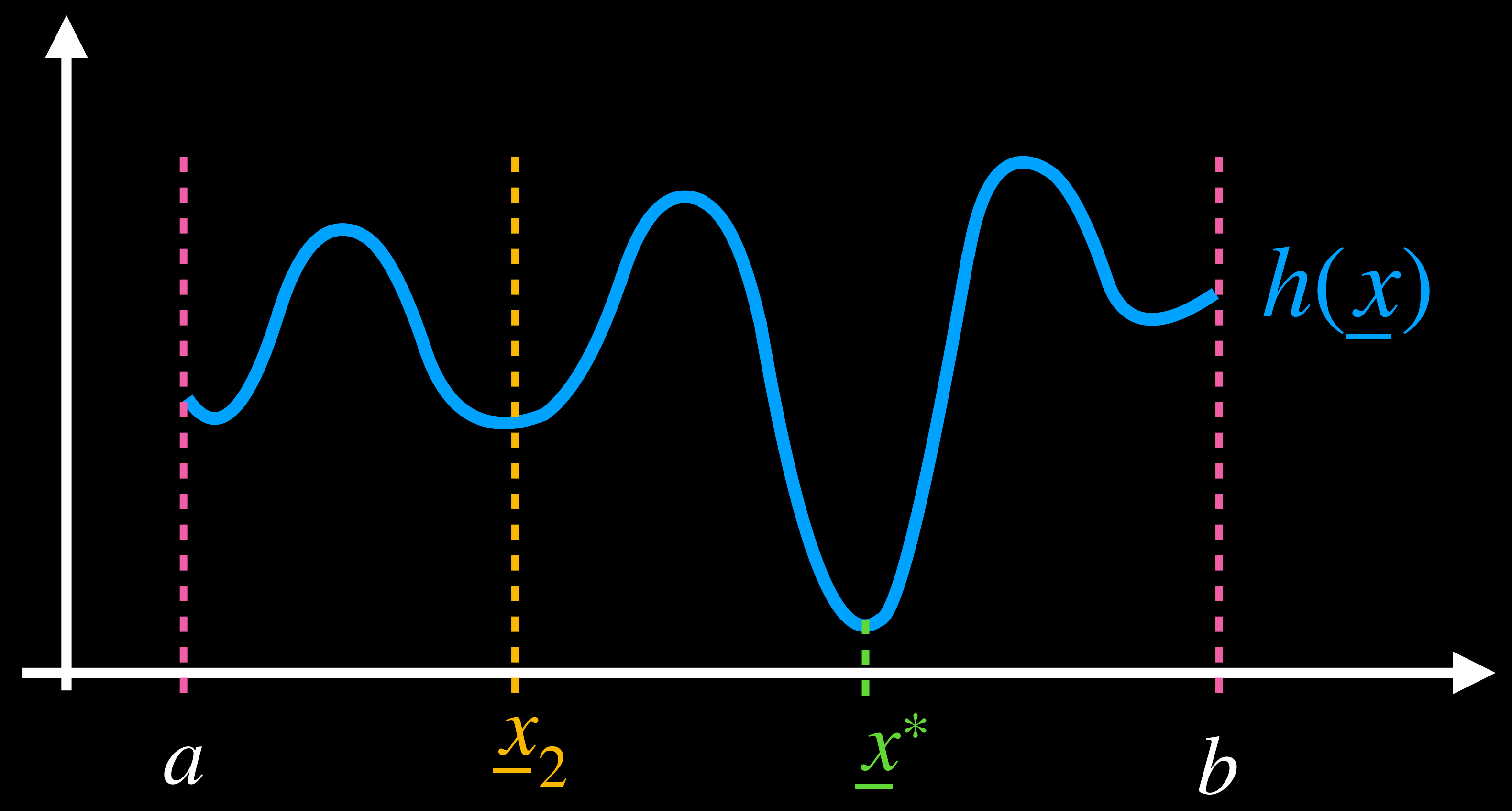

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stochastic descent:

$\underline{X}_{t+1}$

algorithm 2: stochastic descent

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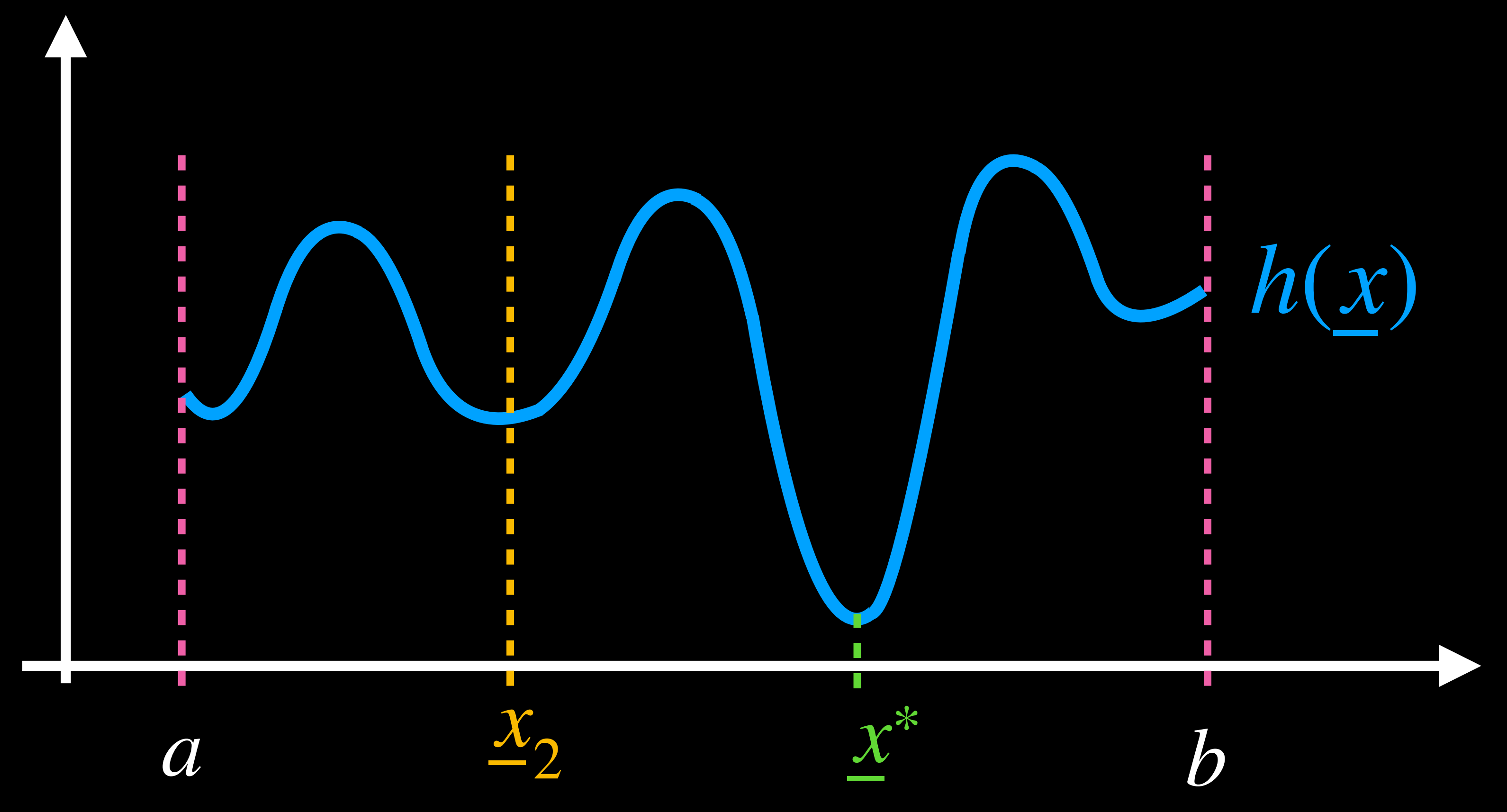

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→ approximating $\nabla h(\underline{x})$ has $2d$ evaluations of h (expensive)

stochastic descent:

$$\underline{X}_{t+1} = \underline{X}_t + \alpha_t \underline{U}_t$$

algorithm 2: stochastic descent

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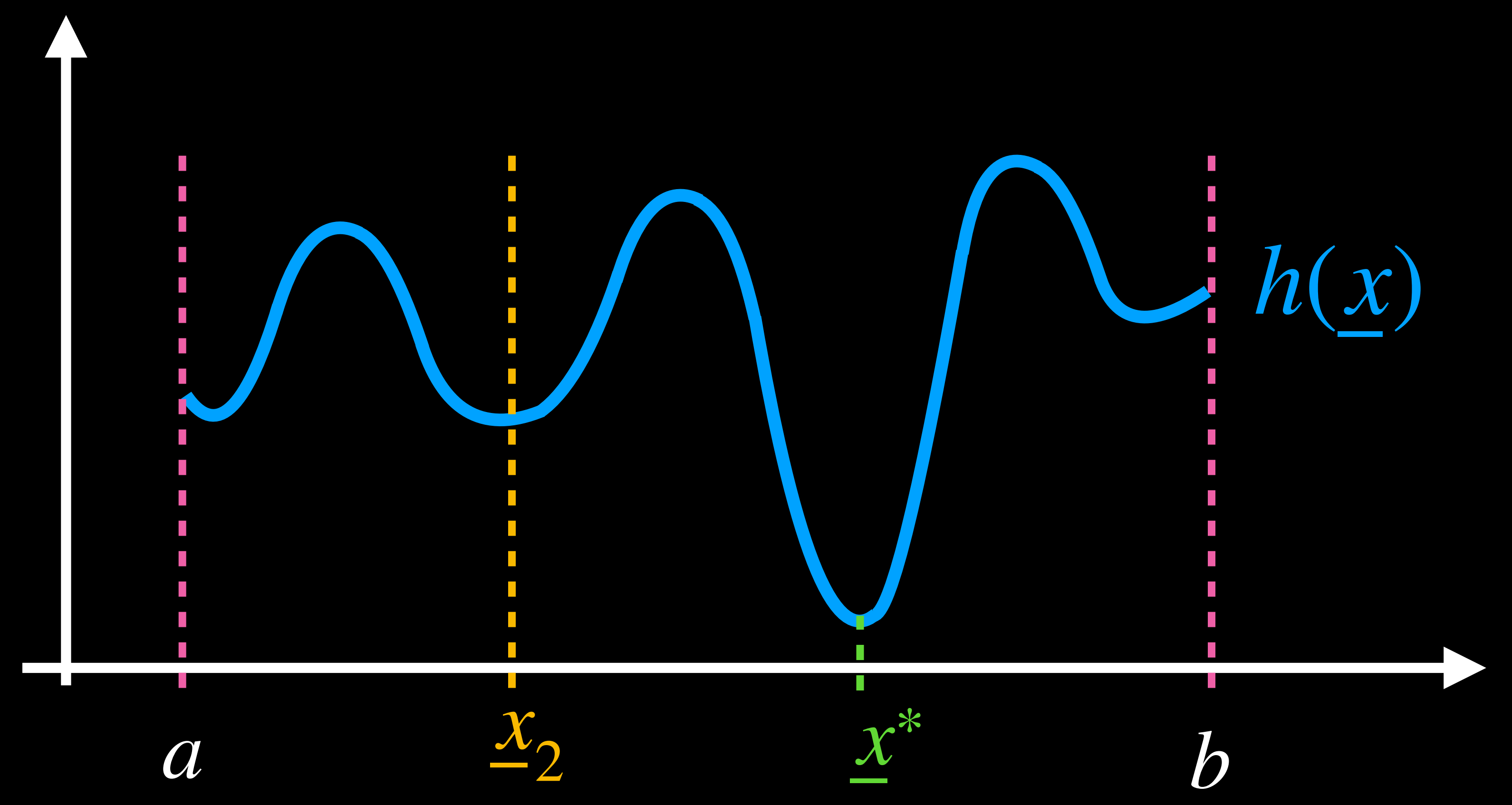

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stochastic descent:

$$\underline{X}_{t+1} = \underline{X}_t + \alpha_t \underline{U}_t \qquad \underline{U}_t \sim \mathcal{U}(\mathbb{S})$$

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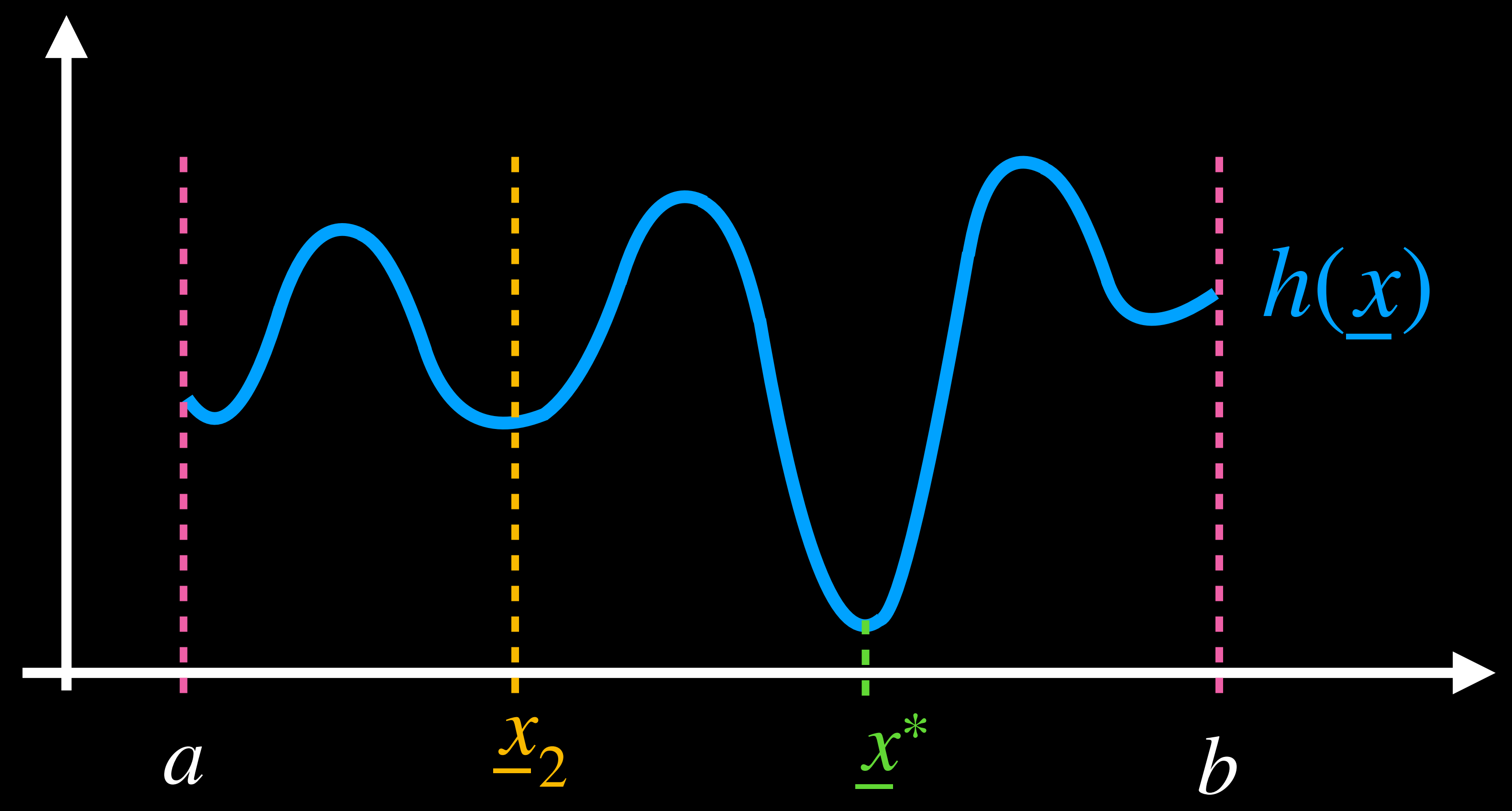

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stochastic descent:

$$\underline{X}_{t+1} = \underline{X}_t + \alpha_t \underline{U}_t$$

$\underline{U}_t \sim \mathcal{U}(\mathbb{S})$

$\mathbb{S}$ be the d-dimensional unit sphere

algorithm 2: stochastic descent

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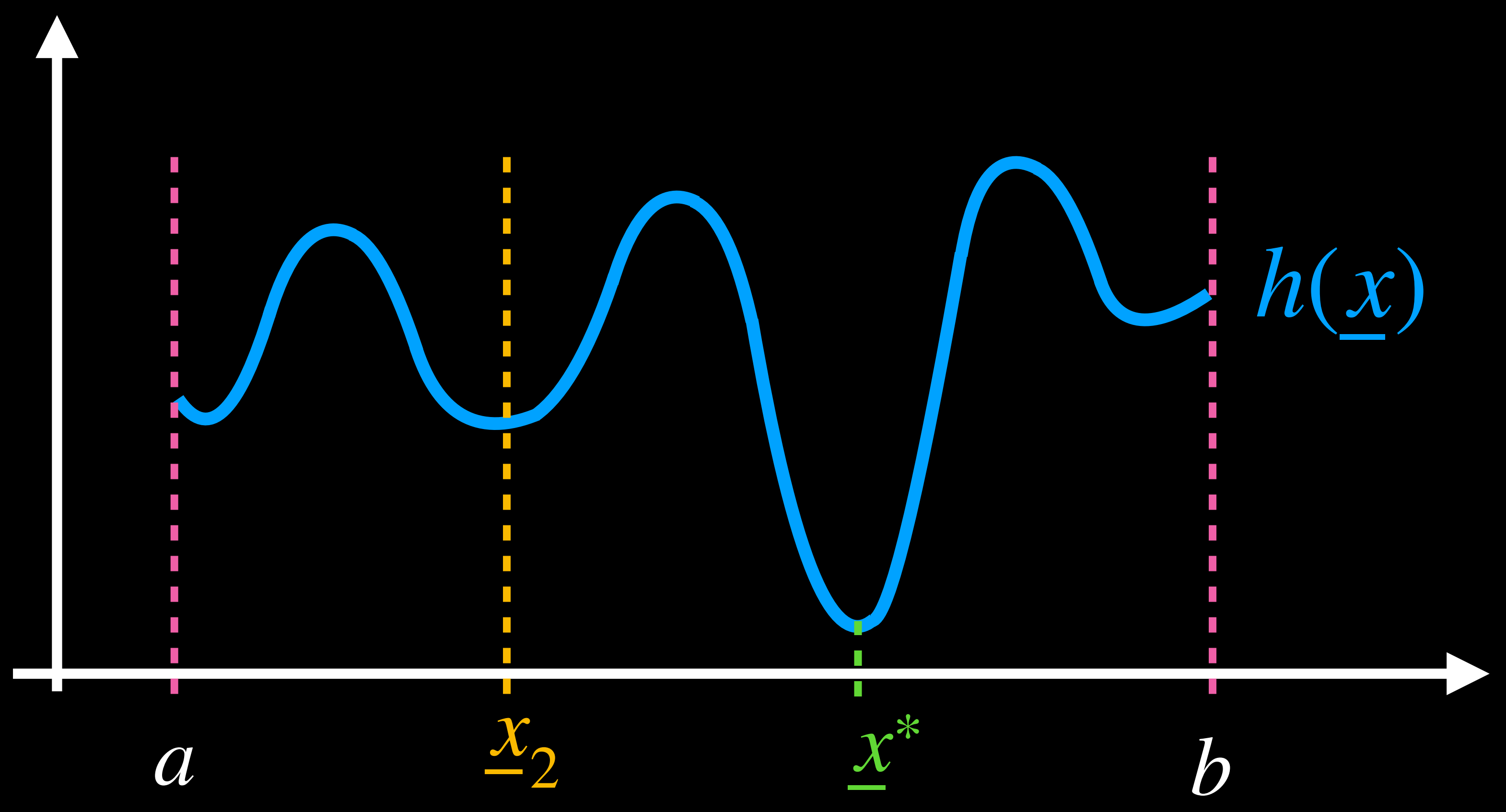

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$$\underline{X}_{t+1} = \underline{X}_t - \alpha_t \left(\nabla h(\underline{X}_t) \cdot \underline{U}_t \right) \underline{U}_t$$

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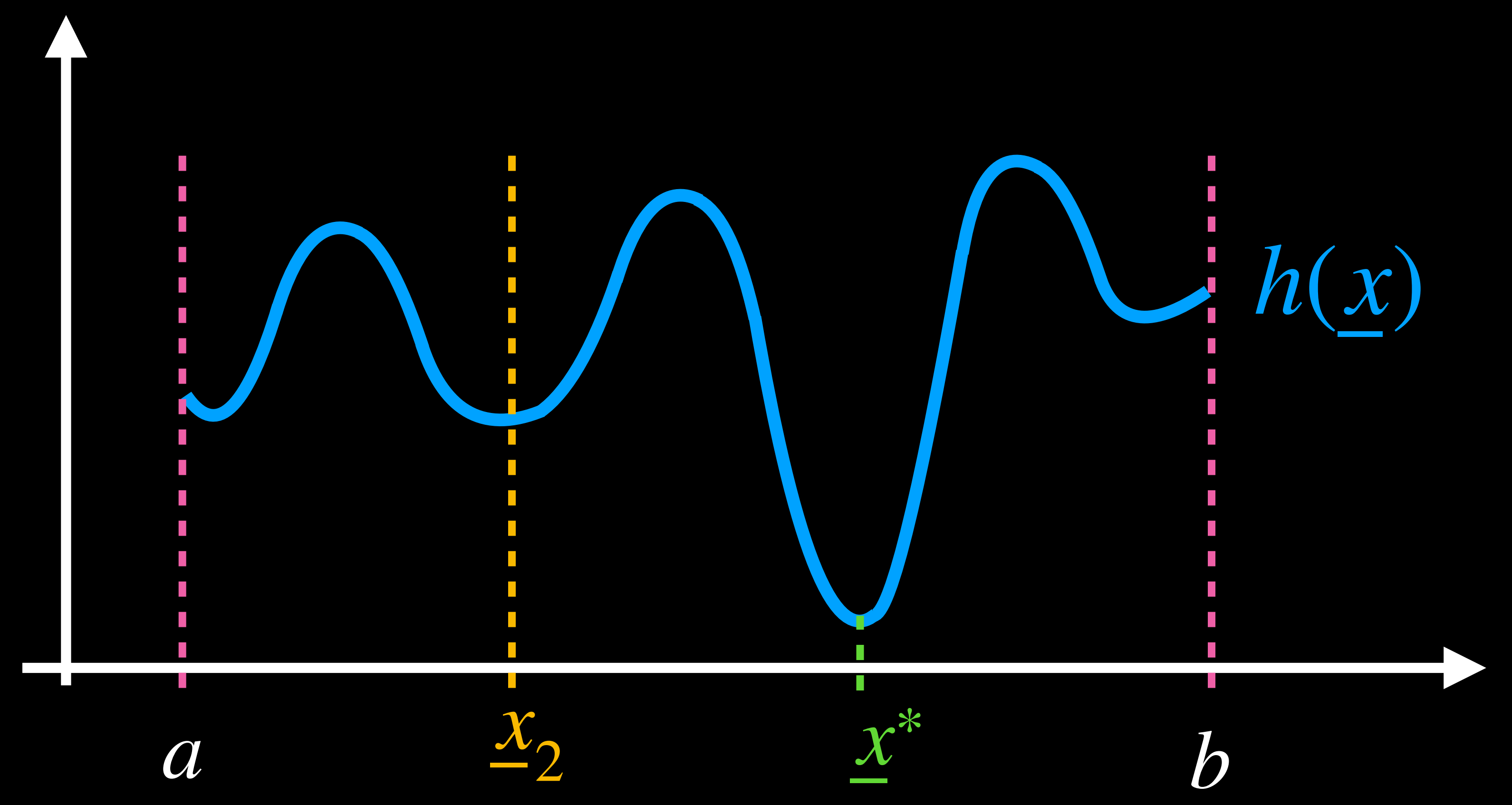

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dot product:

stochastic descent:

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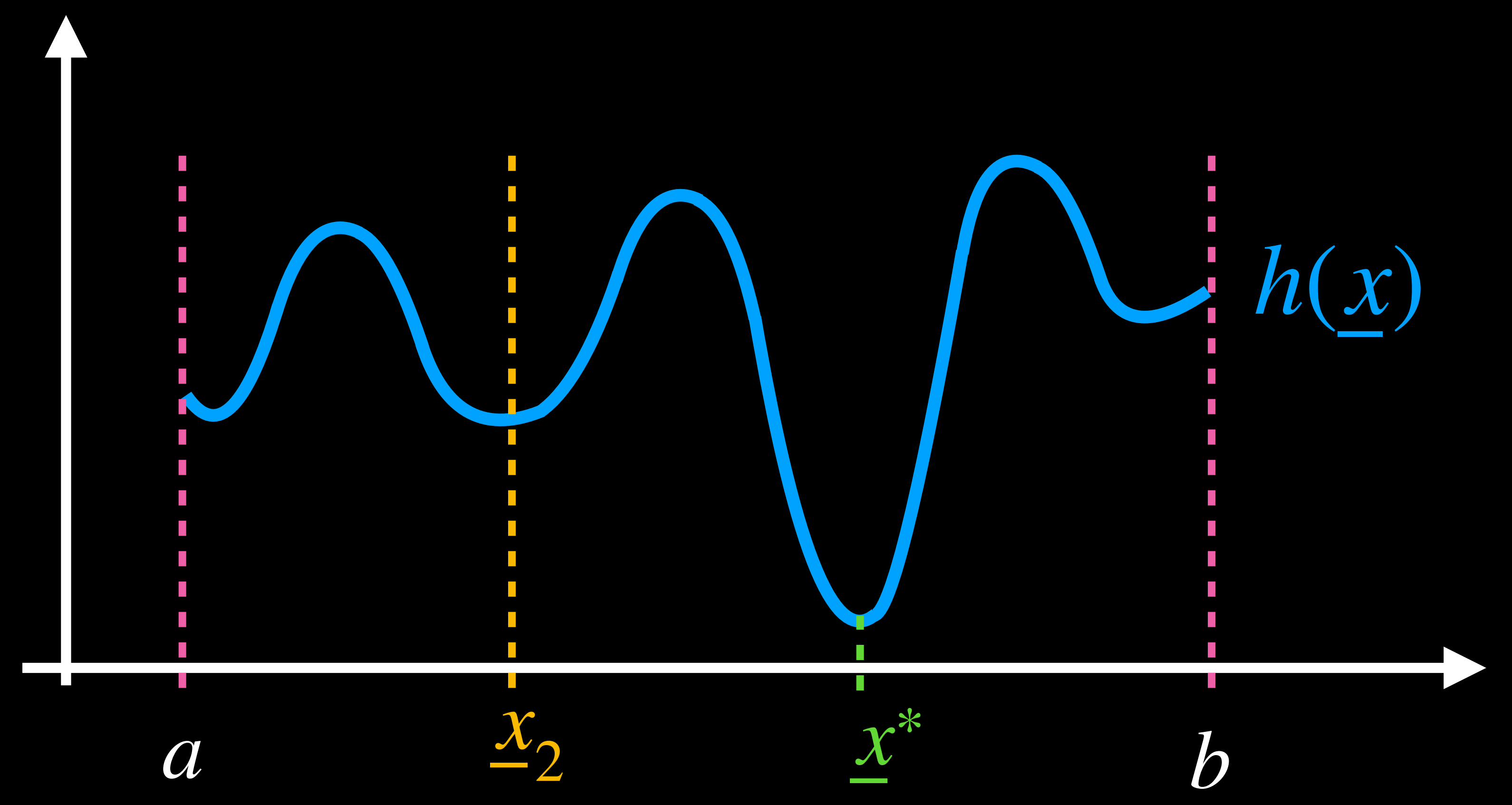

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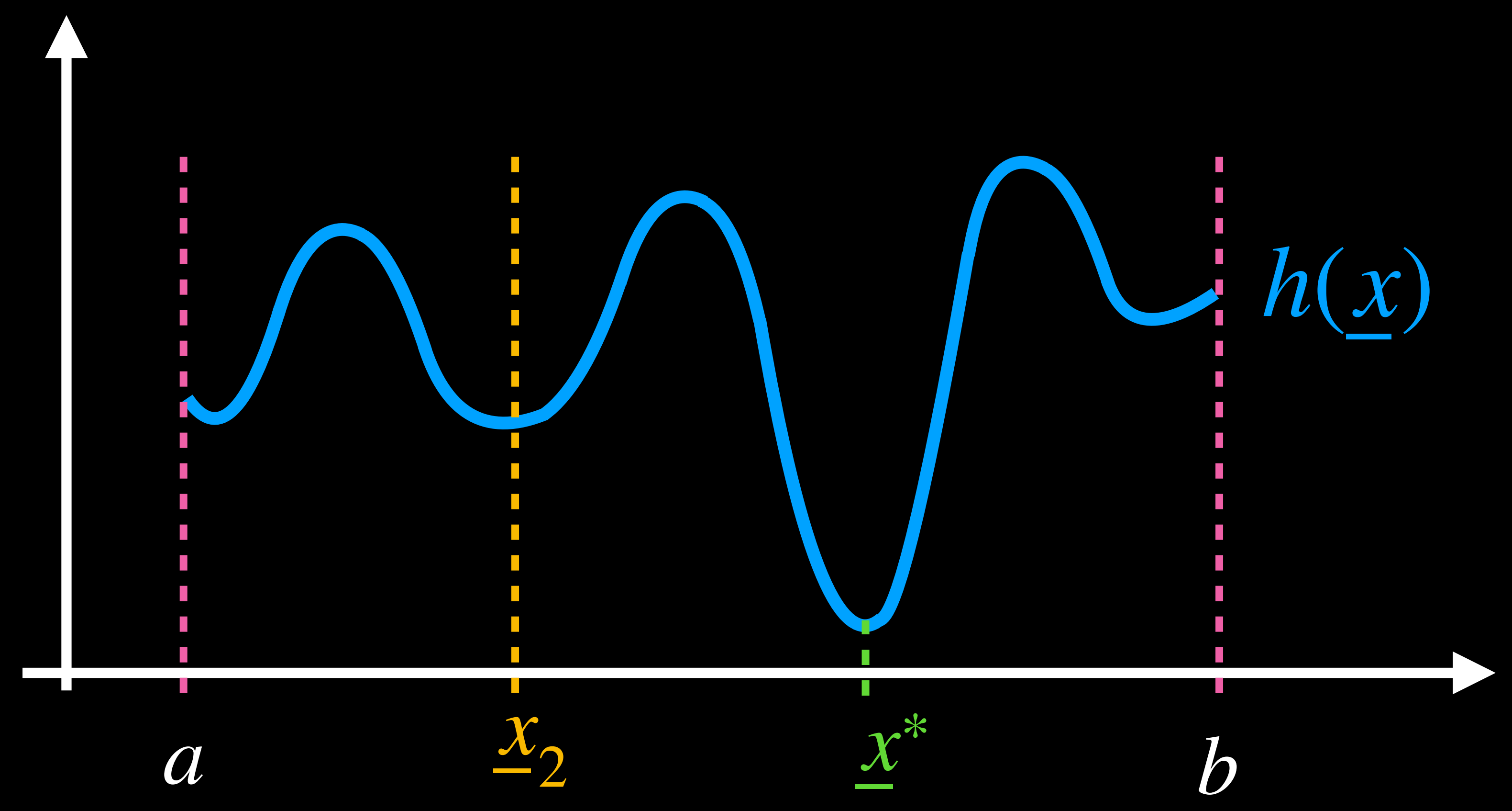

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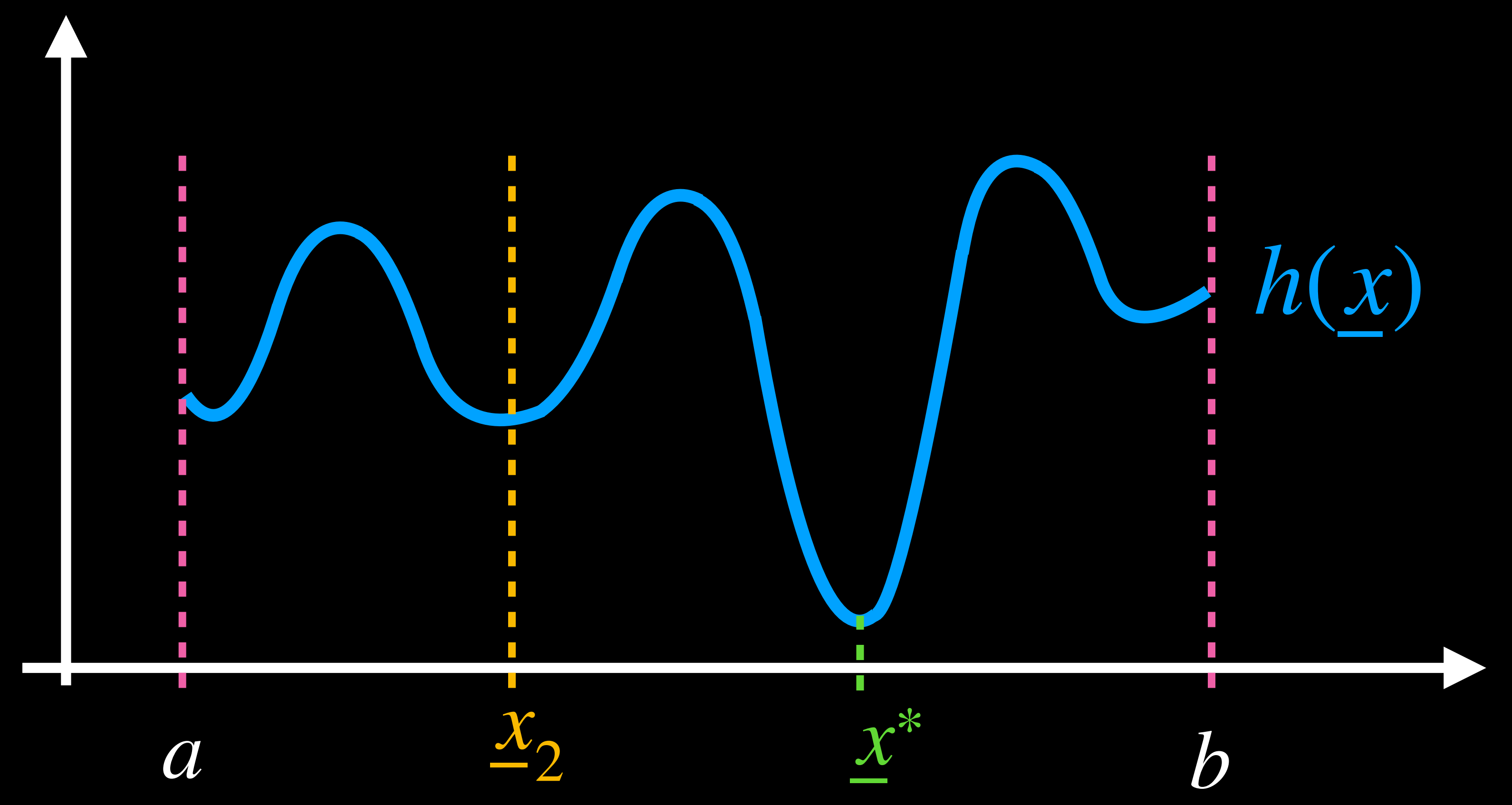

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directional derivative:

stochastic descent:

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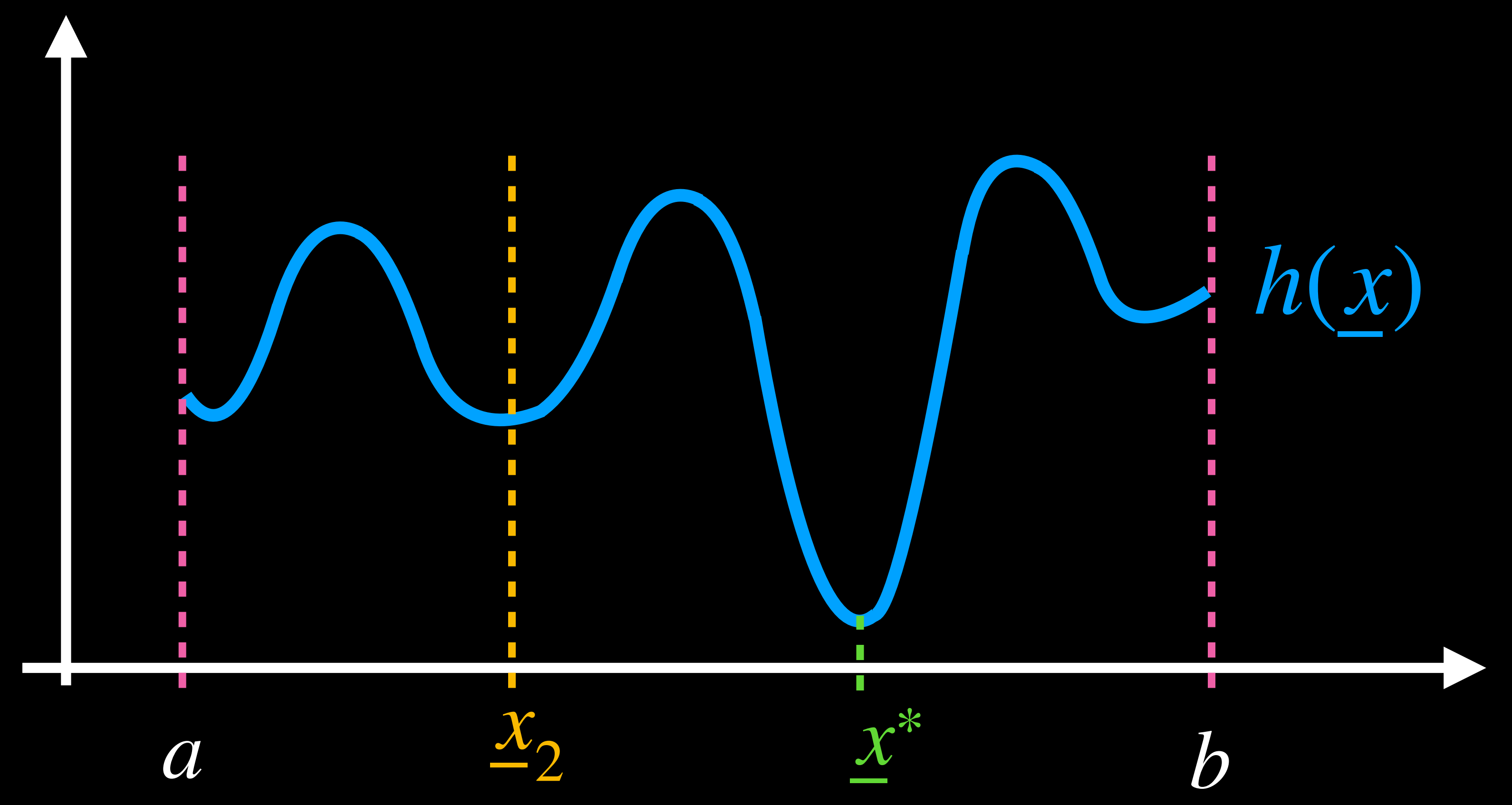

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directional derivative:

$$\nabla h(\underline{x}) \cdot \frac{\underline{y}}{|\underline{y}|} \approx$$

stochastic descent:

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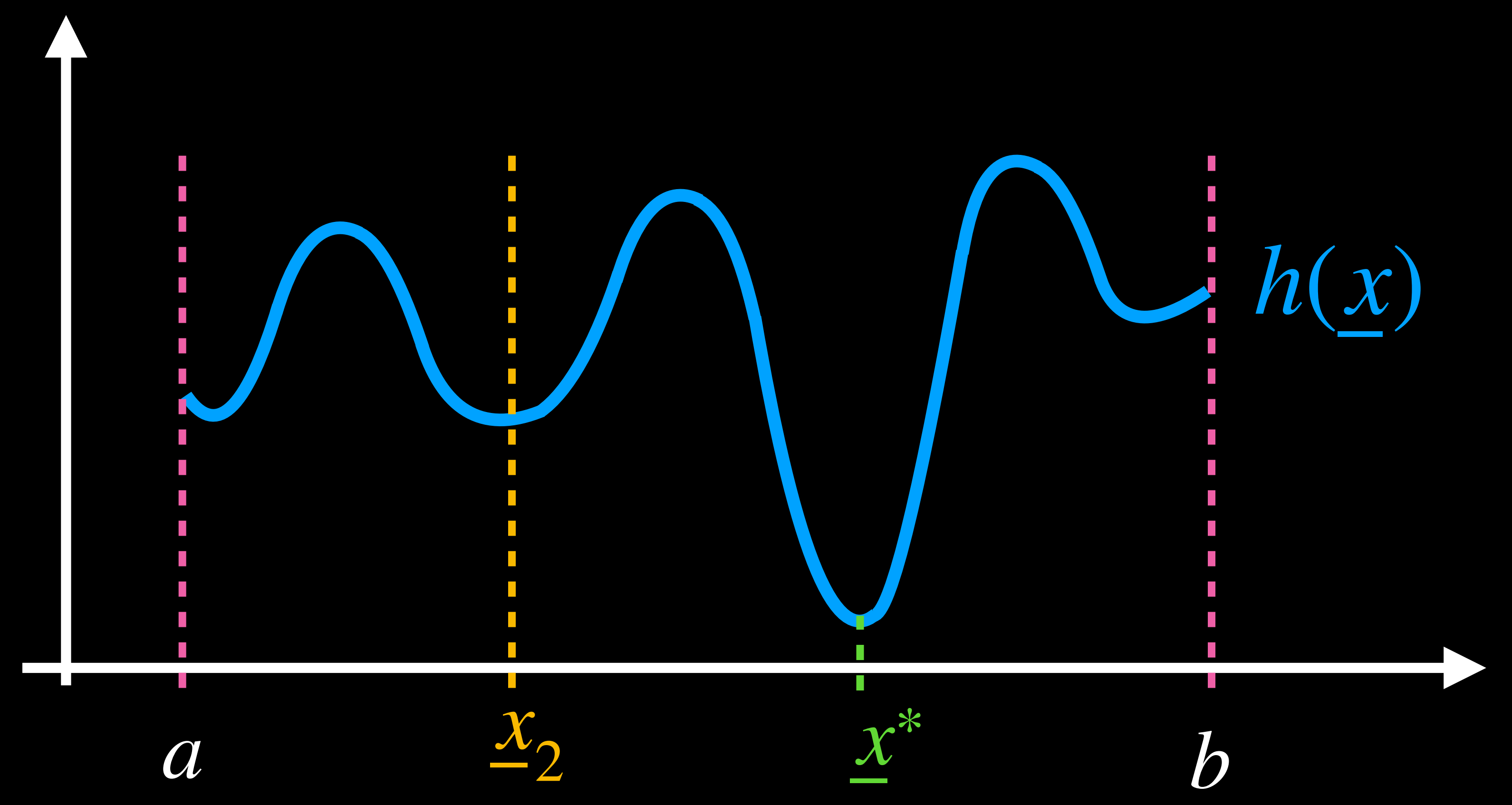

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directional derivative:

$$\nabla h(\underline{x}) \cdot \frac{\underline{y}}{|\underline{y}|} \approx \frac{\Delta h(\underline{x}, \underline{y})}{2|\underline{y}|},$$

stochastic descent:

$$\underline{X}_{t+1} = \underline{X}_t - \alpha_t \left(\nabla h(\underline{X}_t) \cdot \underline{U}_t \right) \underline{U}_t$$

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algorithm 2: stochastic descent

$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \left(h(\underline{x})\right)$

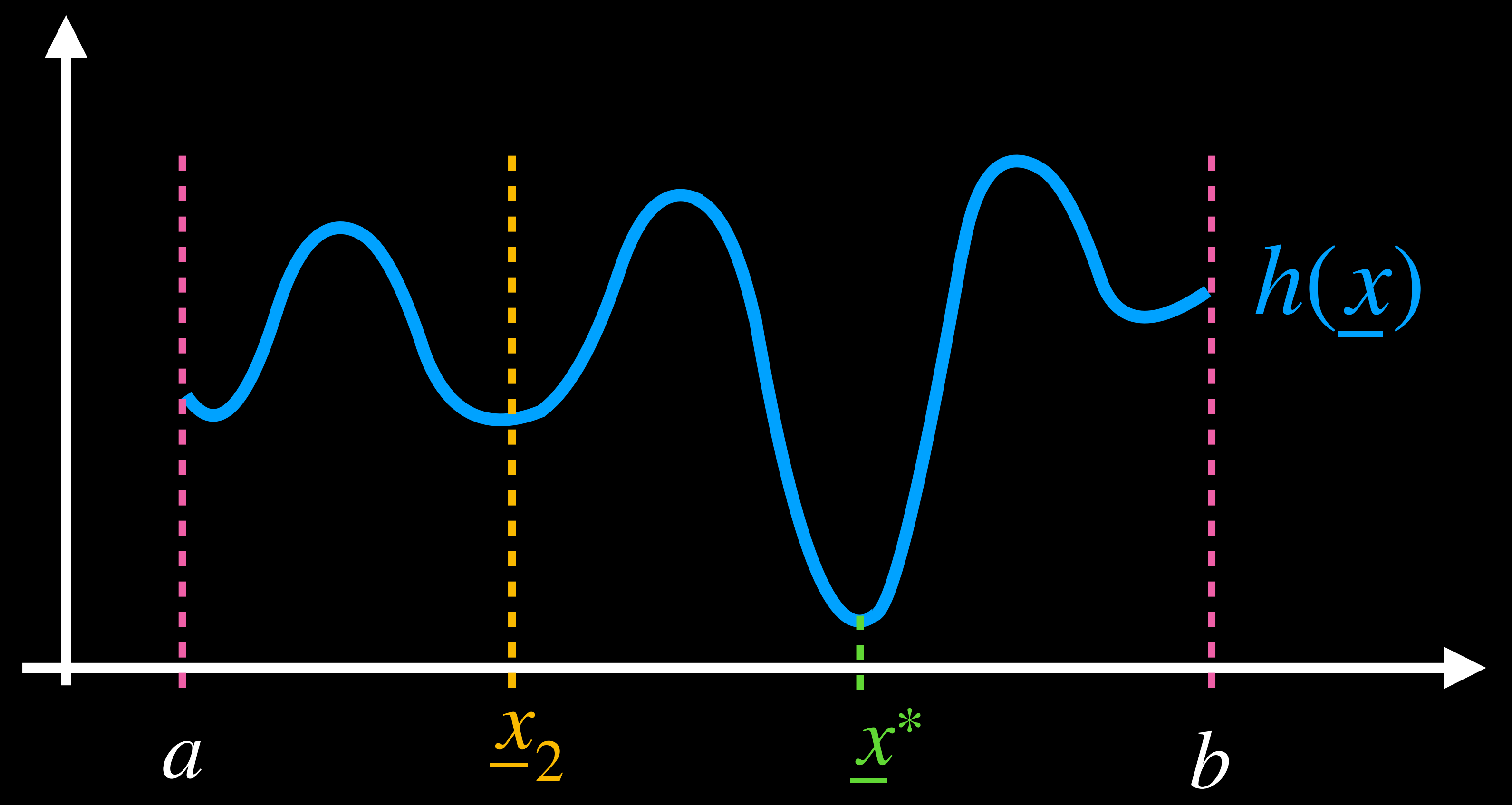
gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \boxed{\nabla h(\underline{x}_t)}$

$t \in \{1,\ldots,n\}$ steps

$\alpha_t > 0$ step size

$-\nabla h(\underline{x}_t)$ vector of steepest decrease

$\underline{x}_0$ starting point



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$\rightarrow$ approximating $\nabla h(\underline{x})$ has $2d$ evaluations of h (expensive)

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$$\underline{X}_{t+1} = \underline{X}_t - \alpha_t \left(\nabla h(\underline{X}_t) \cdot \underline{U}_t \right) \underline{U}_t$$

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directional derivative:

$$\nabla h(\underline{x}) \cdot \frac{\underline{y}}{|\underline{y}|} \approx \frac{\Delta h(\underline{x}, \underline{y})}{2|\underline{y}|}, \qquad \Delta h(\underline{x}, \underline{y}) := h(\underline{x} + \underline{y}) - h(\underline{x} - \underline{y})$$

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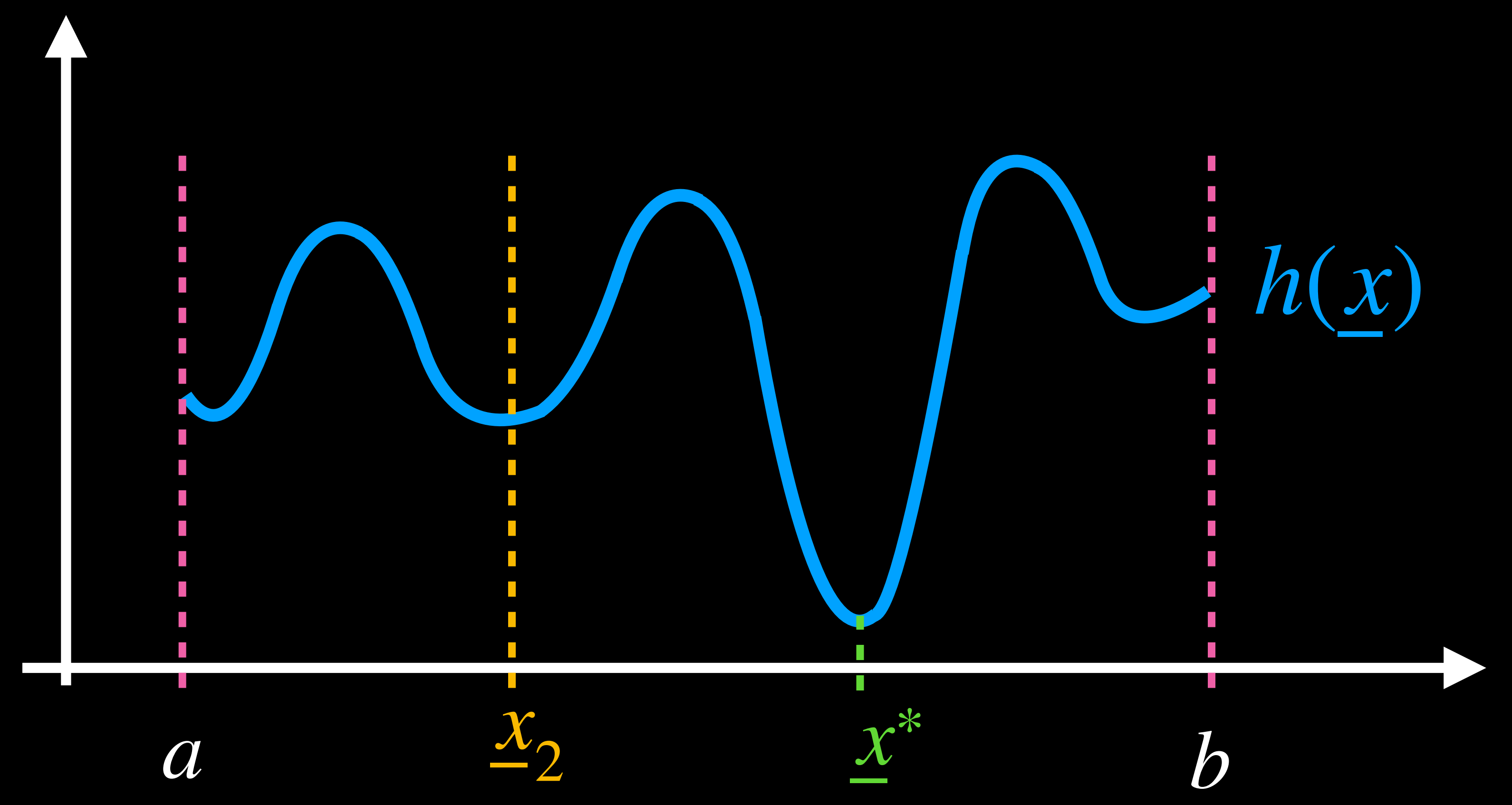

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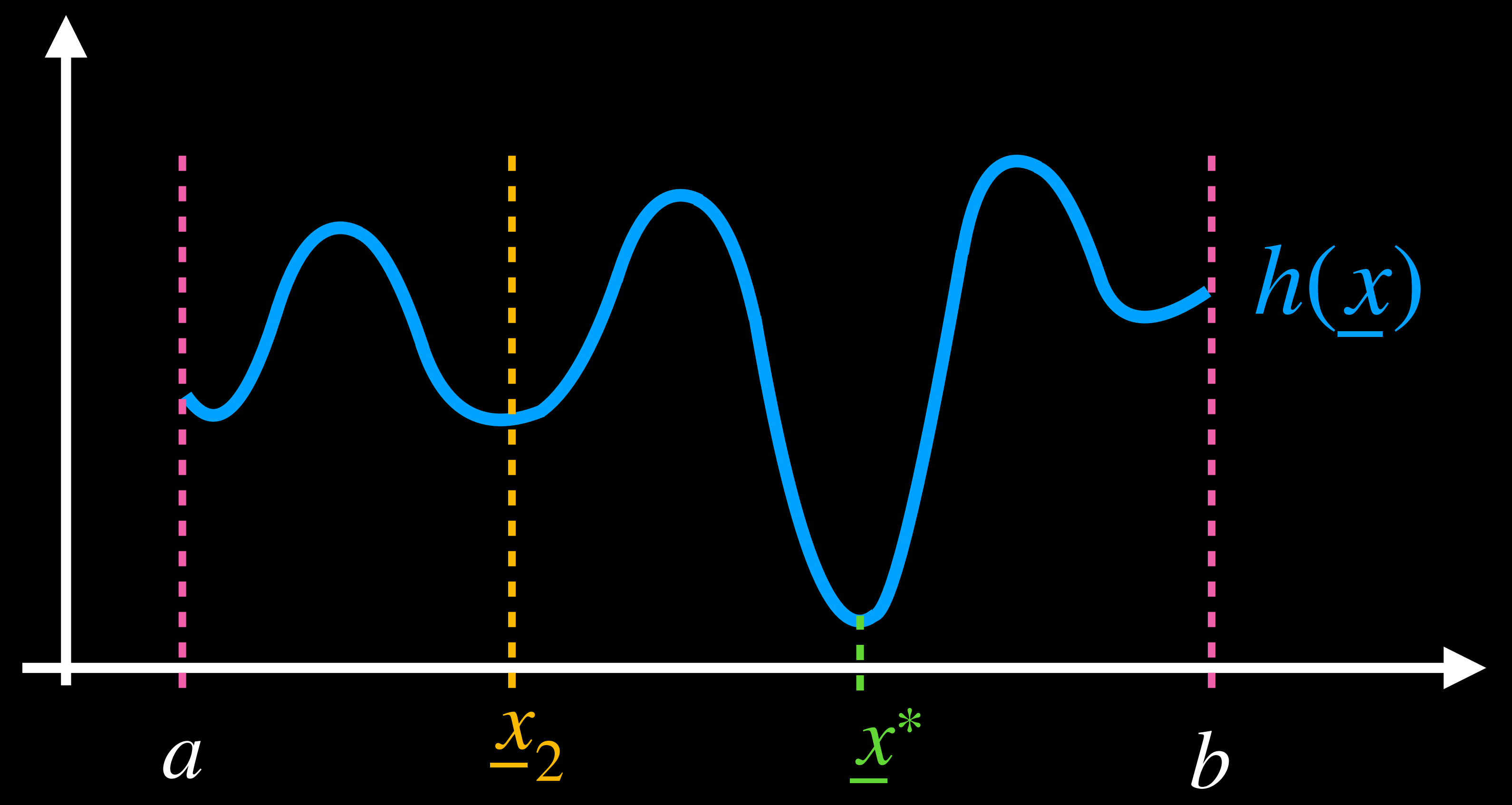

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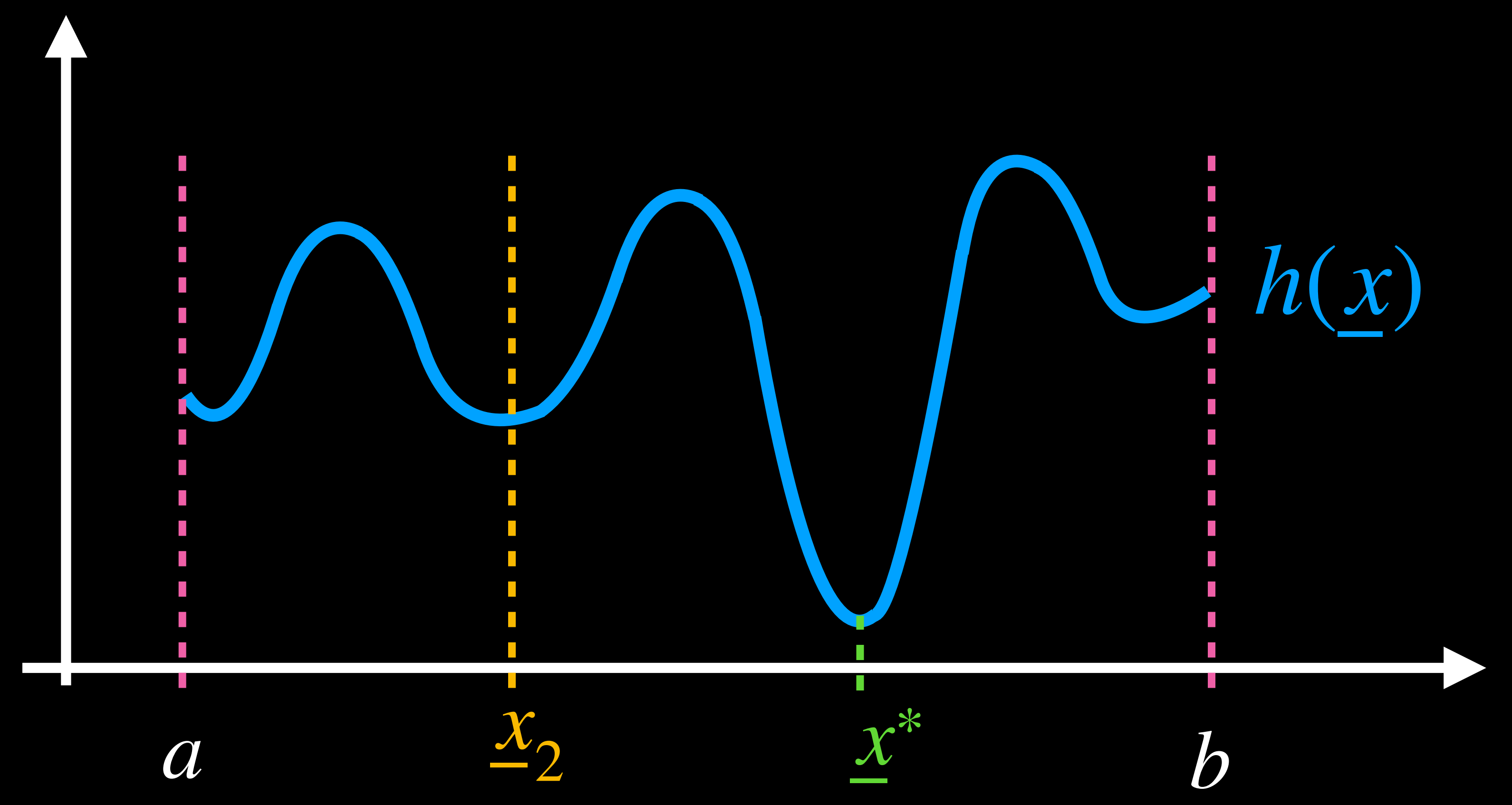

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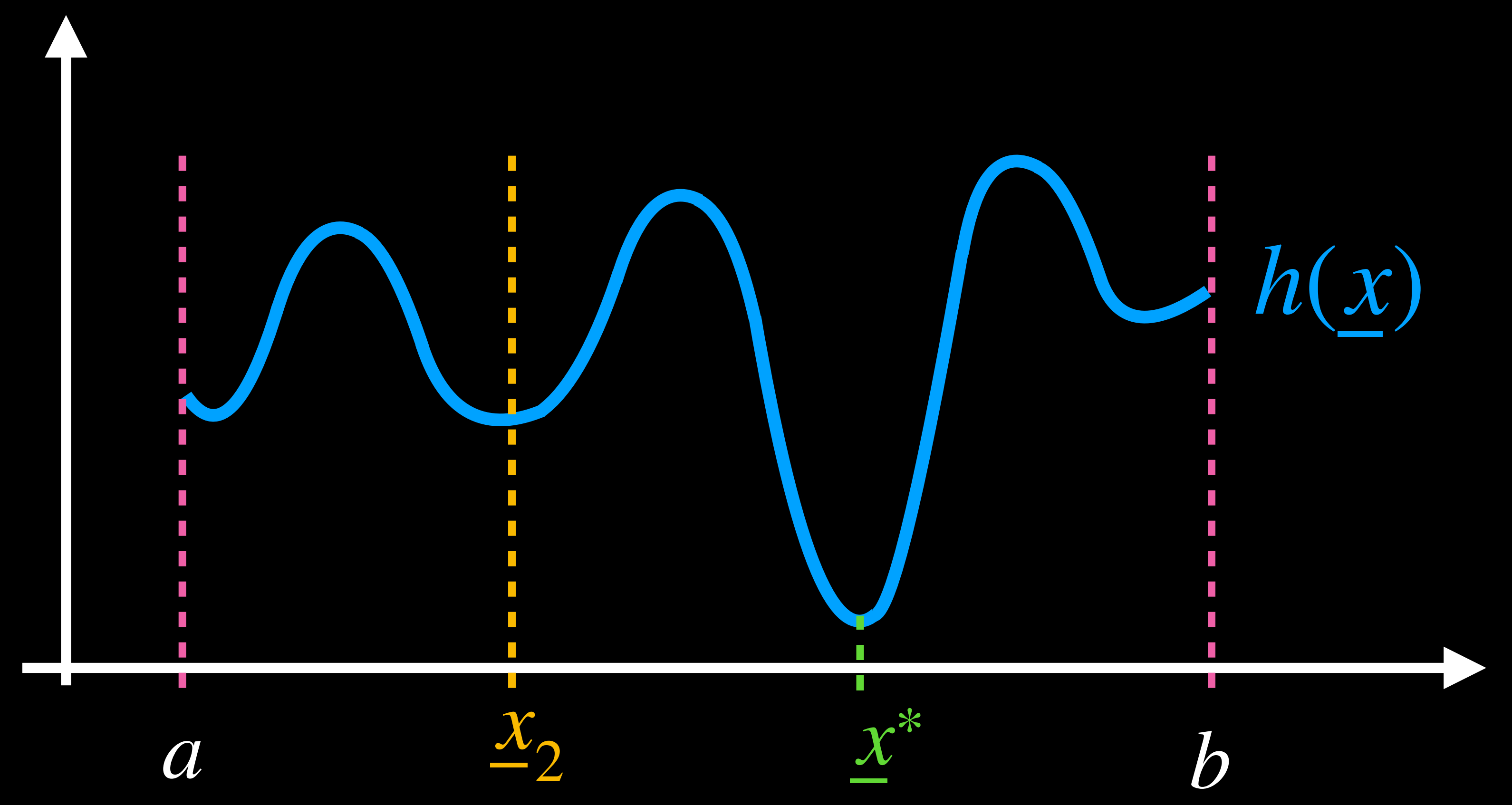

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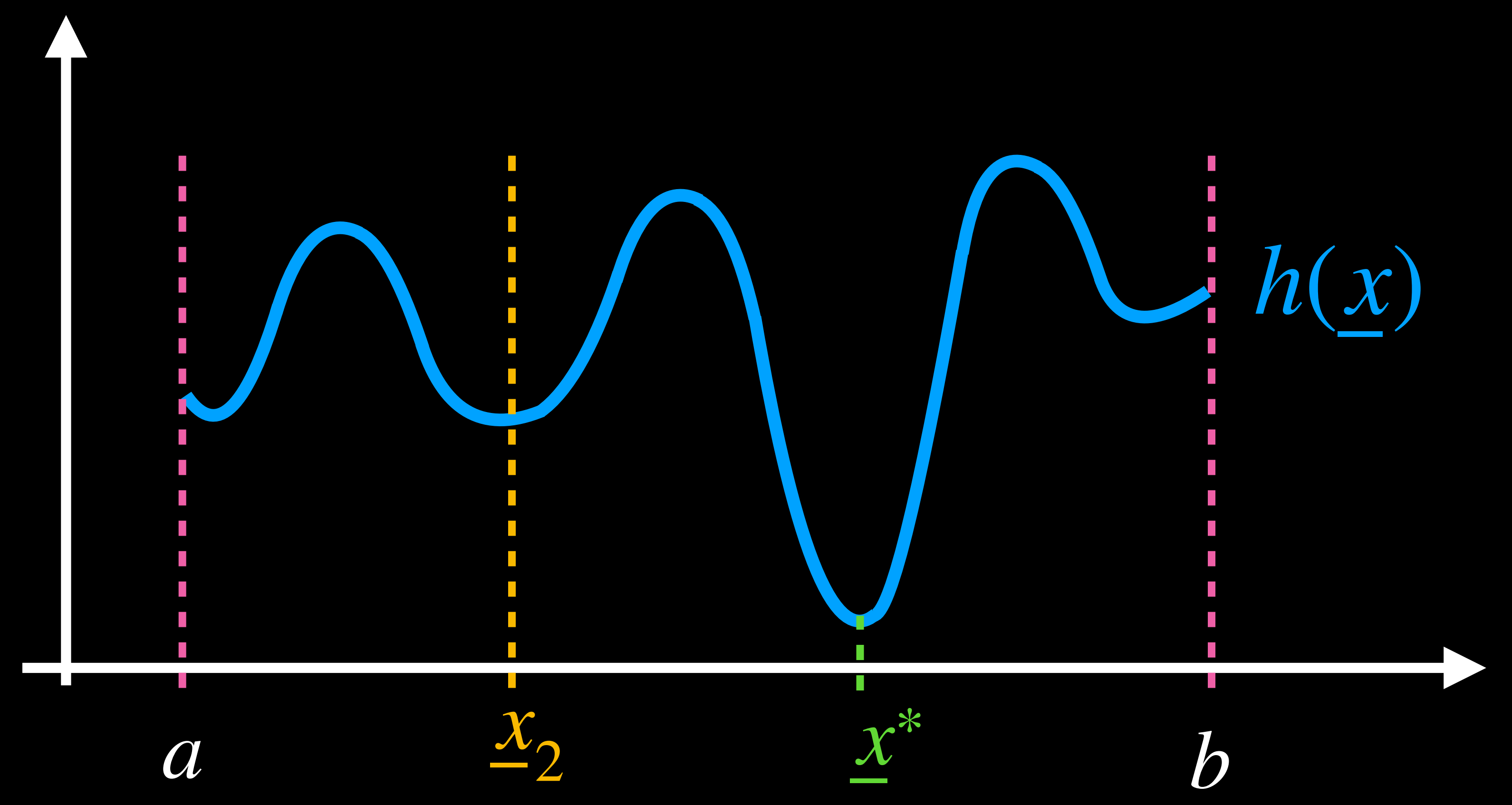

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properties:

- simple

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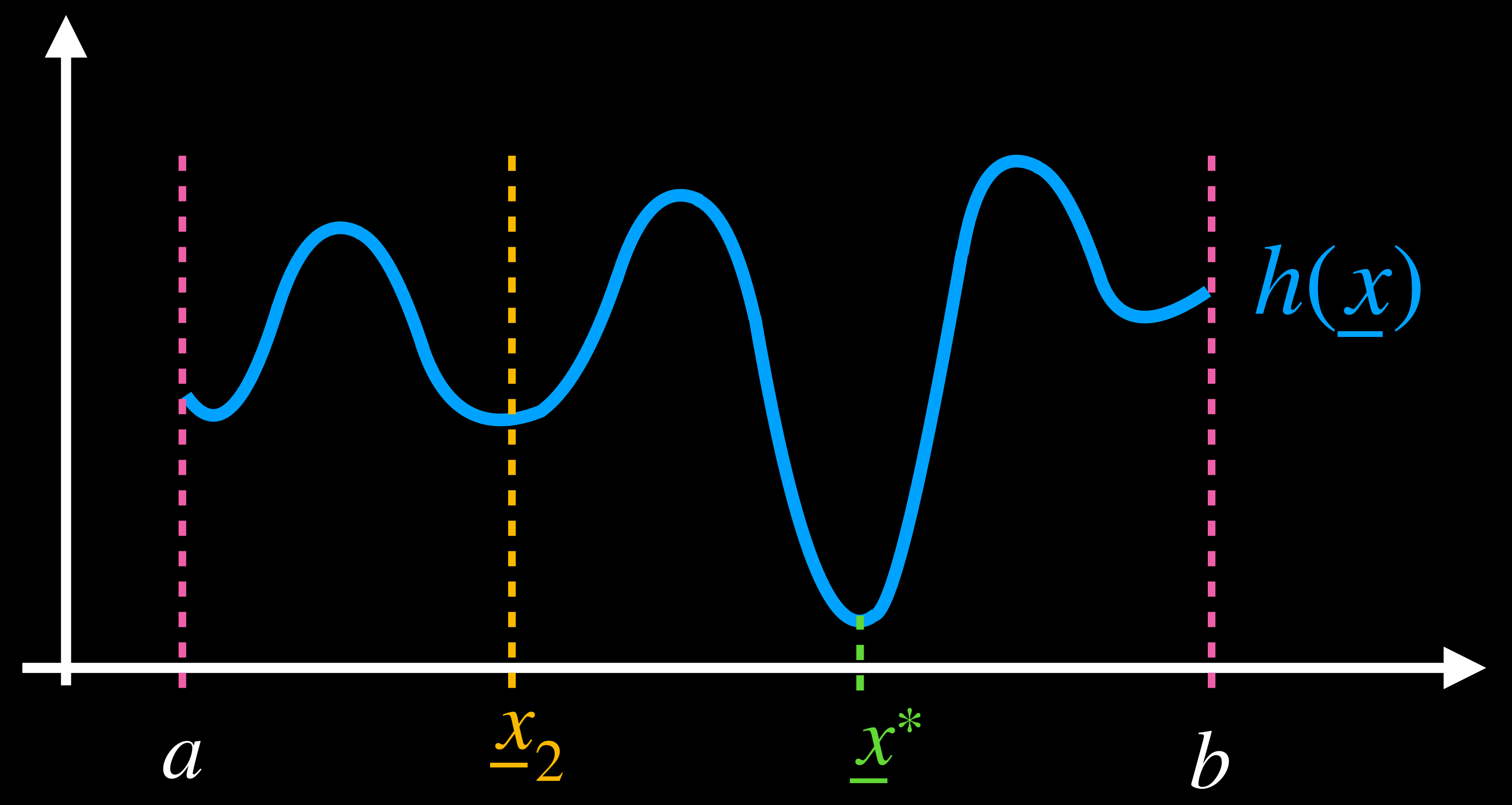

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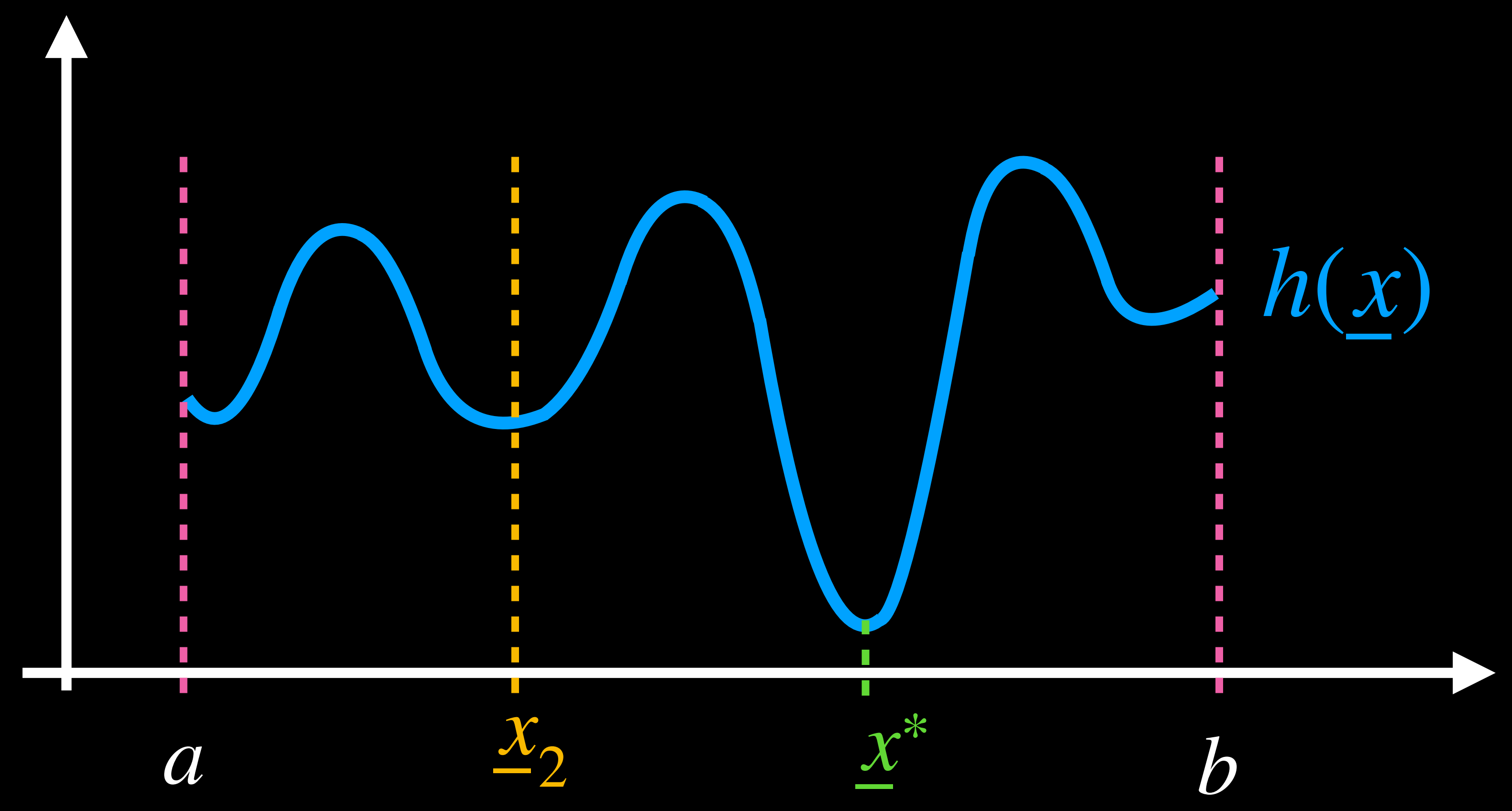

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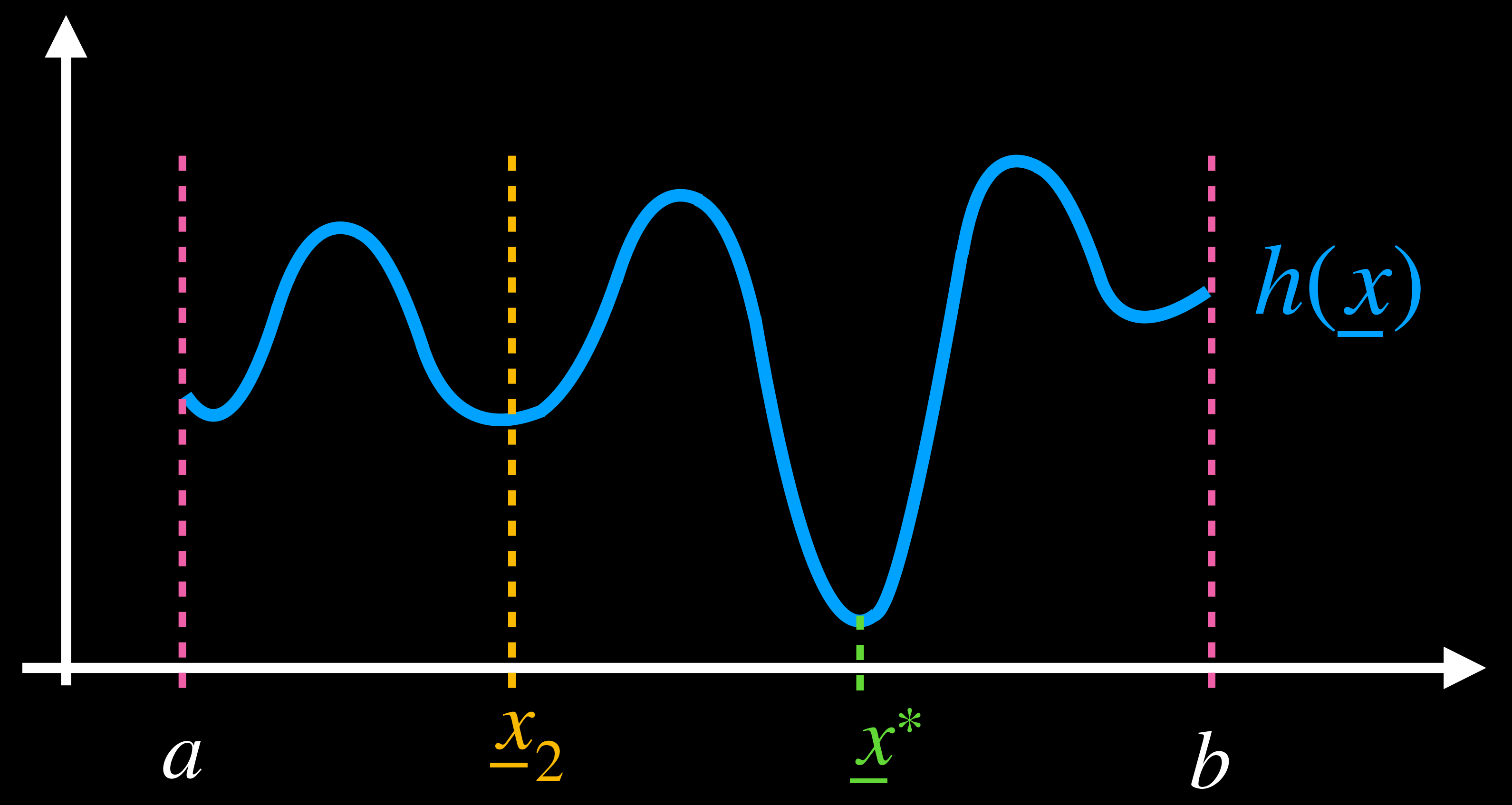

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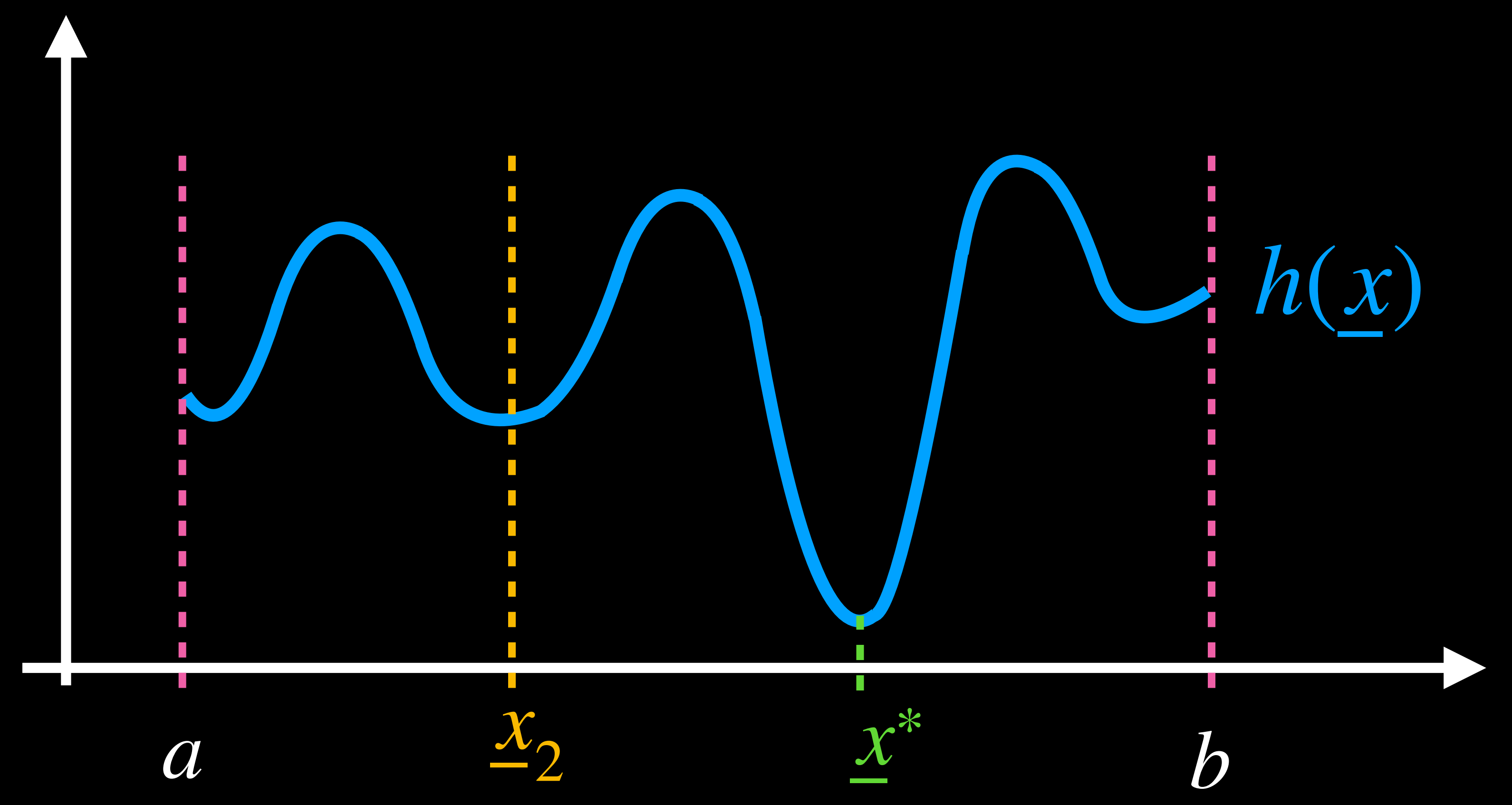

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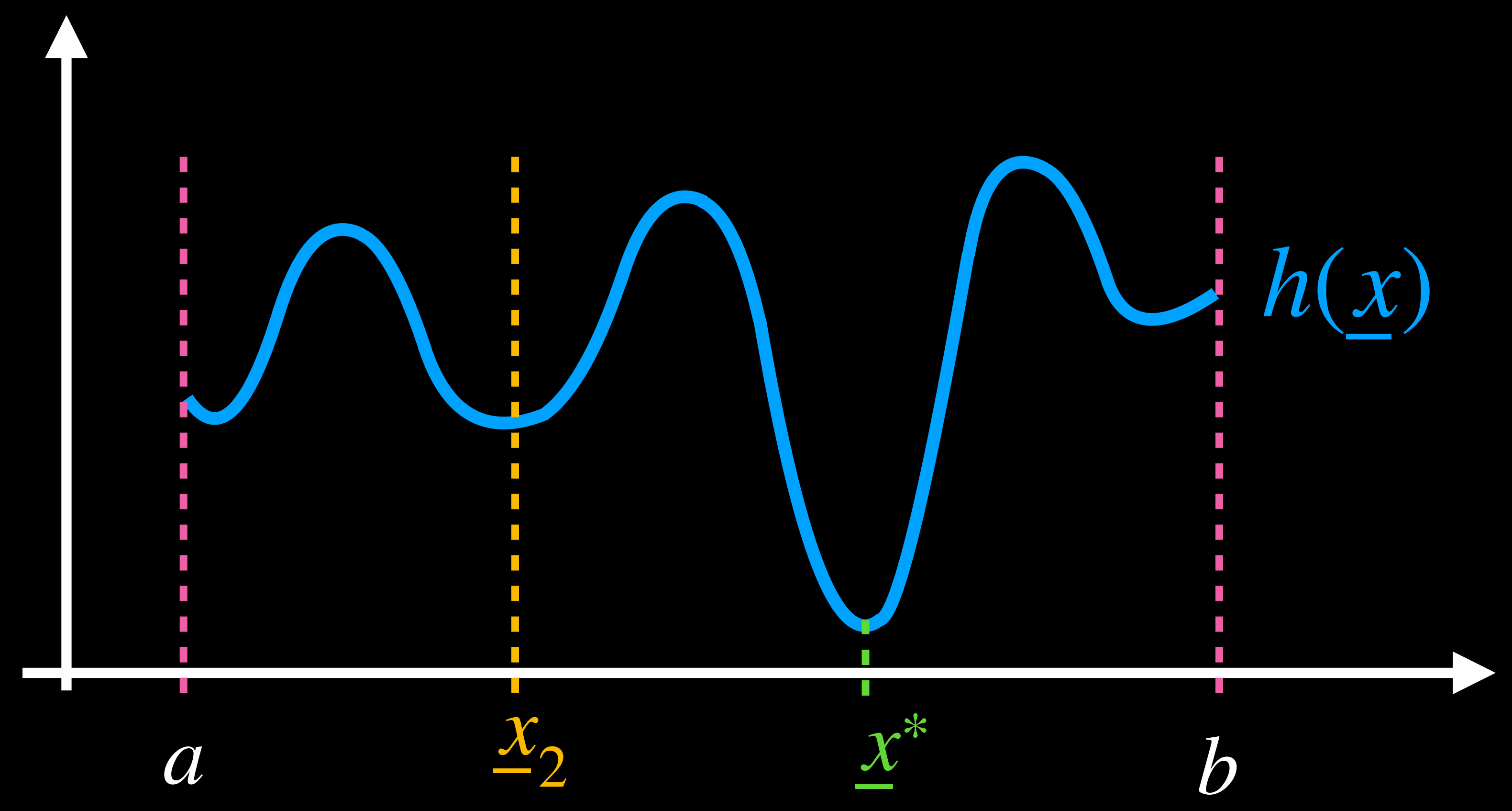

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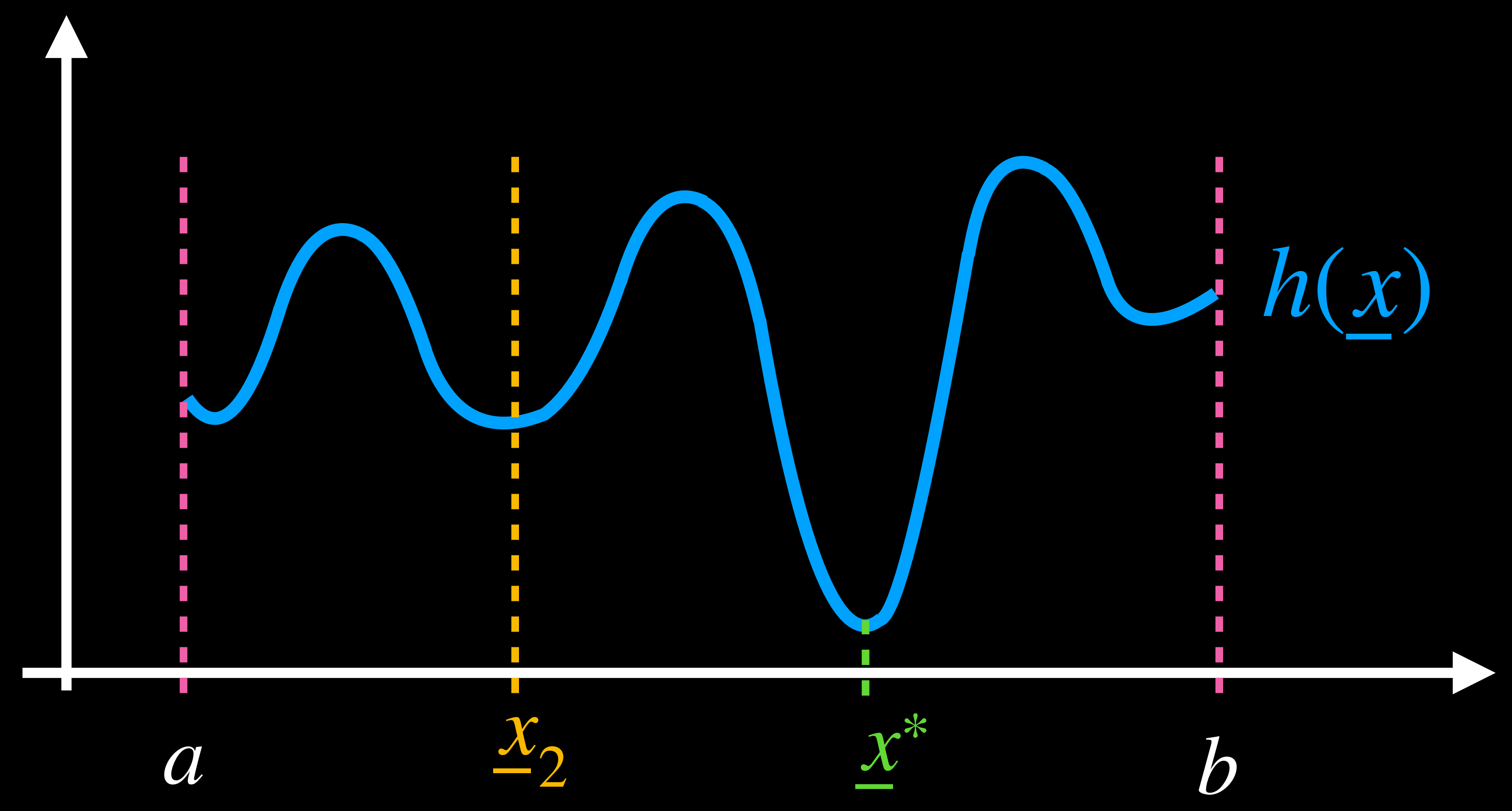
- need to choose $\alpha_t, \beta_t, \underline{X}_0$

algorithm 2: stochastic descent



$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \left(h(\underline{x})\right)$

$t \in \{1,\ldots,n\}$ steps
$\alpha_t > 0$ step size
$-\nabla h(\underline{x}_t)$ vector of steepest decrease
$\underline{x}_0$ starting point

gradient descent: $\underline{x}_{t+1} = \underline{x}_t - \alpha_t \boxed{\nabla h(\underline{x}_t)}$

gradient:

$$\nabla h(\underline{x}) := \begin{pmatrix} \frac{\partial}{\partial x_1} h(\underline{x}) \\ \vdots \\ \frac{\partial}{\partial x_d} h(\underline{x}) \end{pmatrix}, \qquad \frac{\partial}{\partial x_1} h(\underline{x}) \approx \left(h\begin{pmatrix} x_1 + \Delta x \\ x_2 \\ \vdots \\ x_d \end{pmatrix} - h\begin{pmatrix} x_1 - \Delta x \\ x_2 \\ \vdots \\ x_d \end{pmatrix}\right) \cdot \frac{1}{2\Delta x}$$

→ approximating $\nabla h(\underline{x})$ has $2d$ evaluations of h (expensive)

dot product: $\underline{x} \cdot \underline{y} = \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_d \end{pmatrix} = x_1 y_1 + \ldots + x_d y_d$

directional derivative:

$$\nabla h(\underline{x}) \cdot \frac{\underline{y}}{|\underline{y}|} \approx \frac{\Delta h(\underline{x}, \underline{y})}{2|\underline{y}|}, \qquad \Delta h(\underline{x}, \underline{y}) := h(\underline{x} + \underline{y}) - h(\underline{x} - \underline{y})$$

→ approximation has 2 evaluations of h (cheaper)

stochastic descent:

$$\underline{X}_{t+1} = \underline{X}_t - \frac{\alpha_t}{2\beta_t} \Delta h(\underline{X}_t, \beta_t \underline{U}_t)\, \underline{U}_t$$

$\underline{U}_t \sim \mathcal{U}(\mathbb{S})$
$\mathbb{S}$ be the d-dimensional unit sphere
β_t sampling radius

properties:

- simple
- no global convergence guarantee
- local convergence if $\lim_{n\to\infty} \alpha_n = 0$ and $\lim_{n\to\infty} \frac{\alpha_n}{\beta_n} = const.$
- converges fast ($\propto \frac{1}{n}$)
- needs uniform random numbers on the unit sphere
- need to choose $\alpha_t, \beta_t, \underline{X}_0$

useful if:

algorithm 2: stochastic descent

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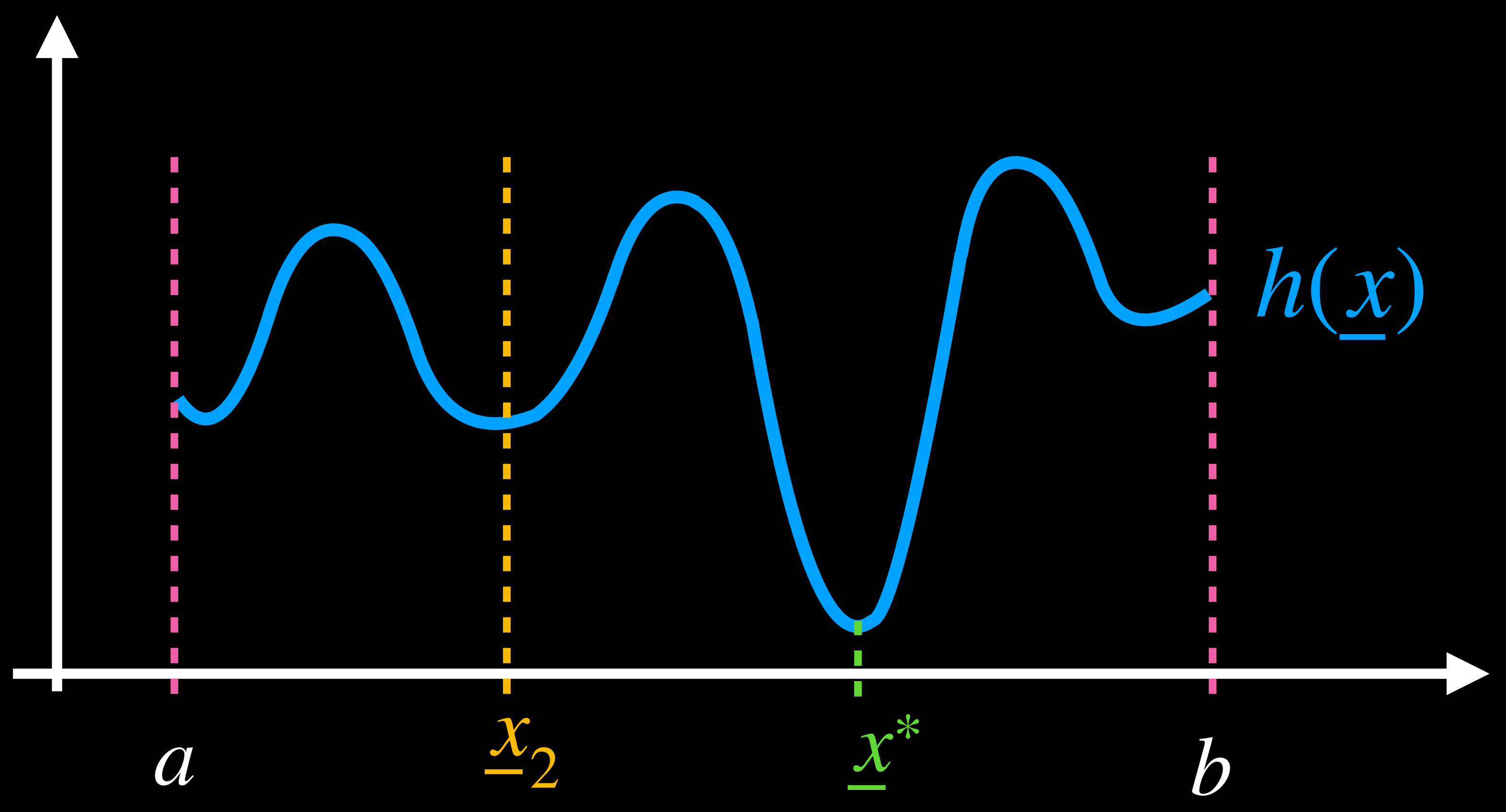

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useful if:

- $\mathbb{S}$ is high-dimensional and unbounded

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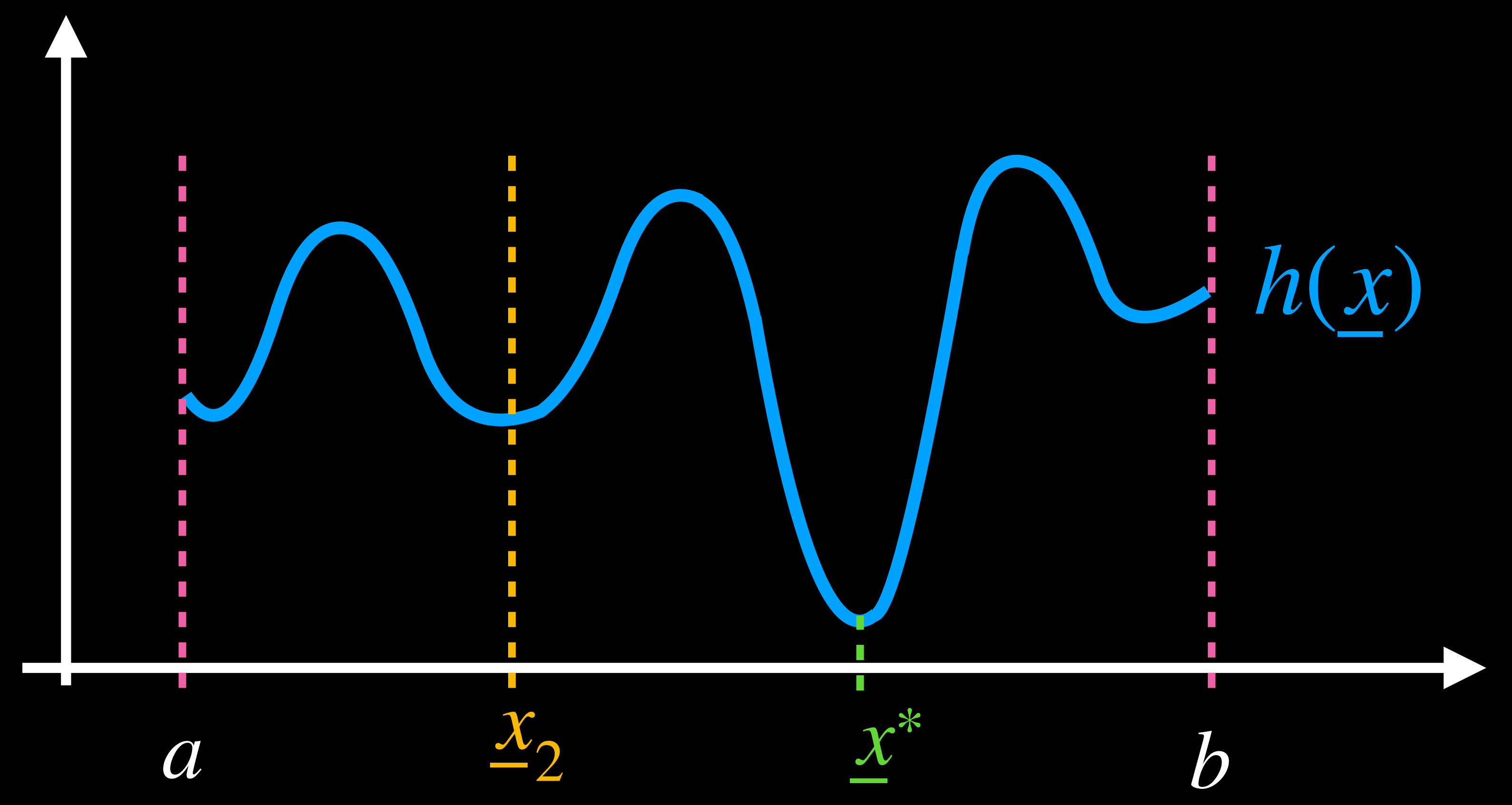

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- needs uniform random numbers on the unit sphere
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useful if:

- $\mathbb{S}$ is high-dimensional and unbounded
- h is convex or $\underline{X}_0$ is close to $\underline{x}^*$

algorithm 3: random pursuit

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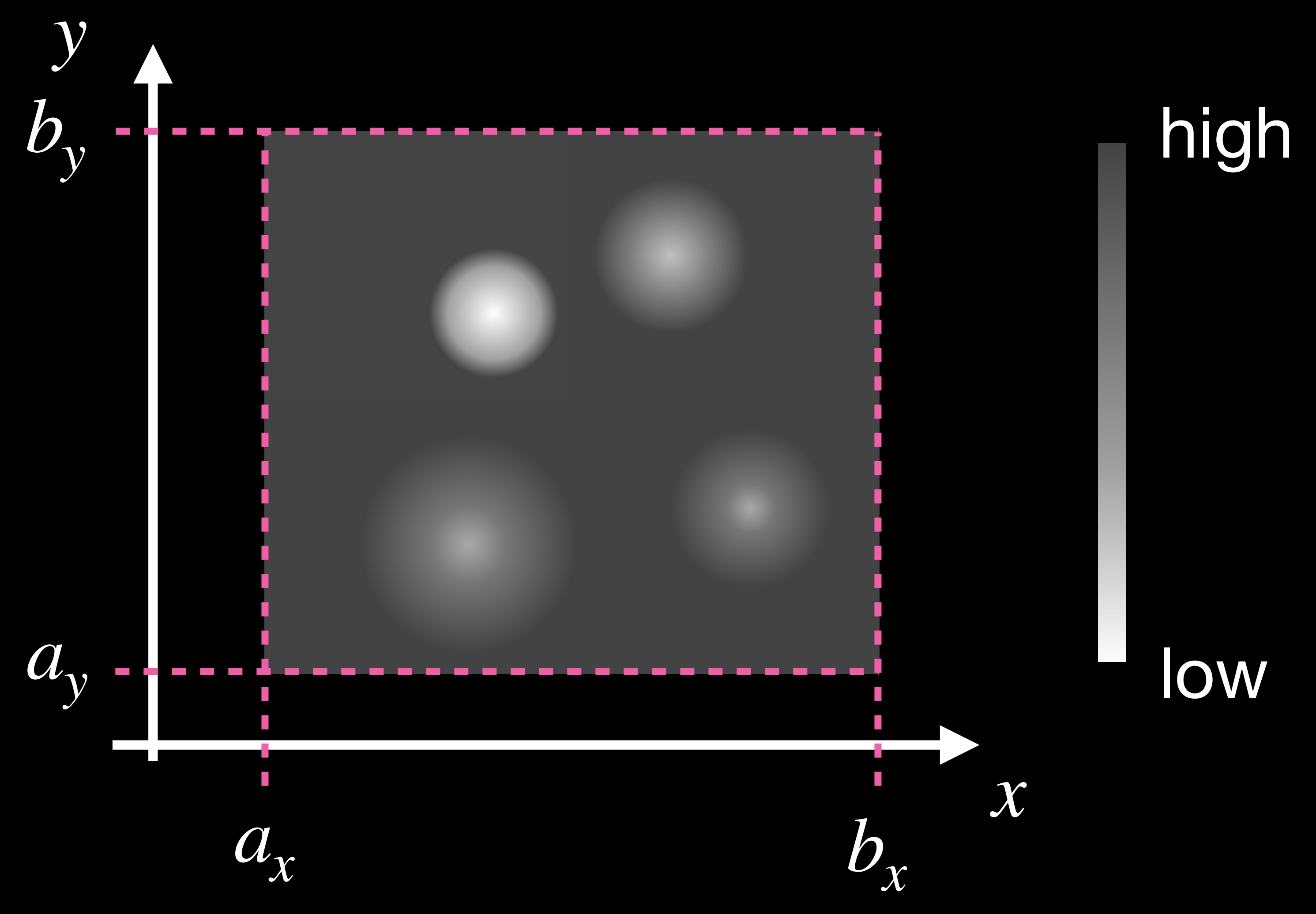

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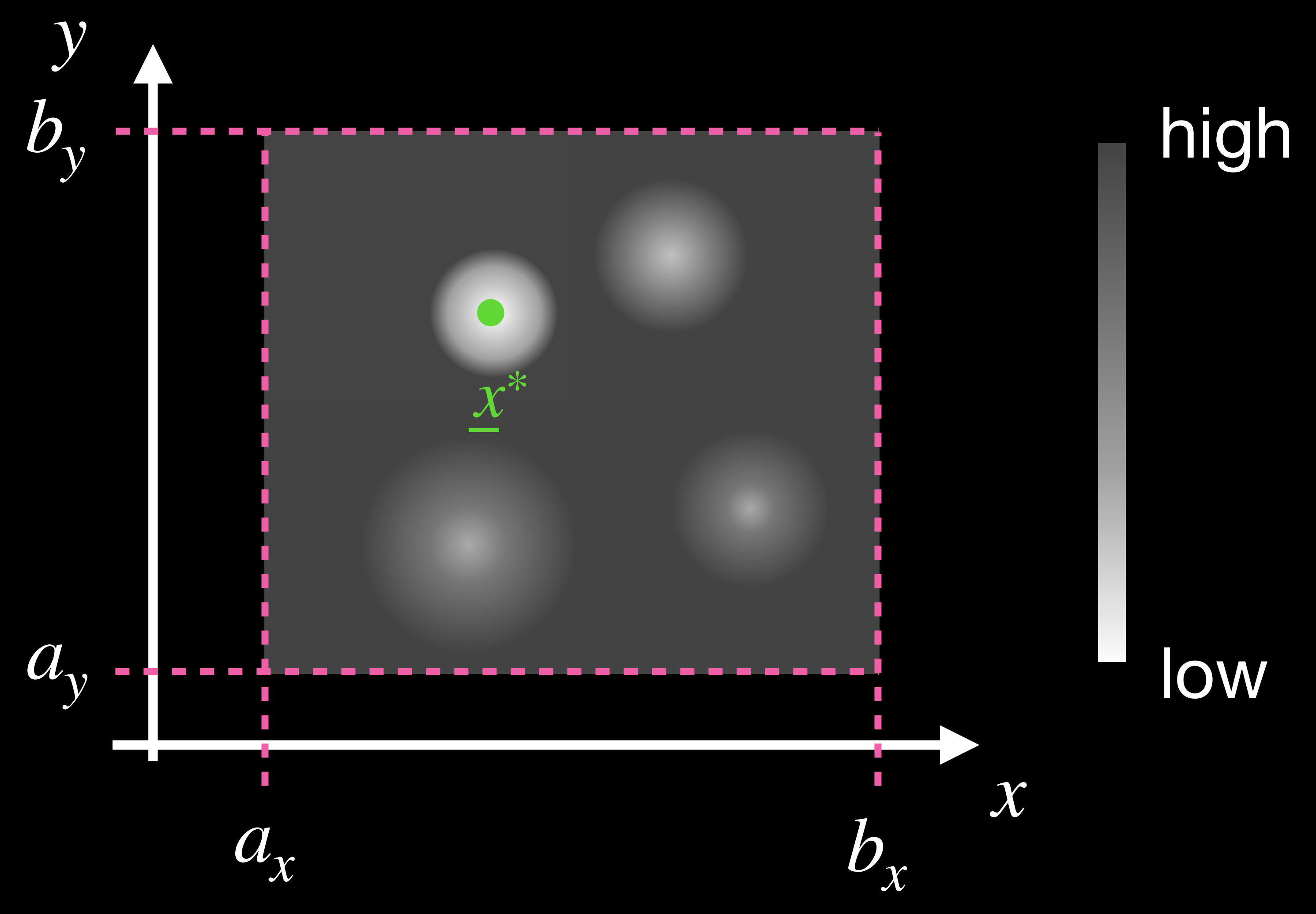

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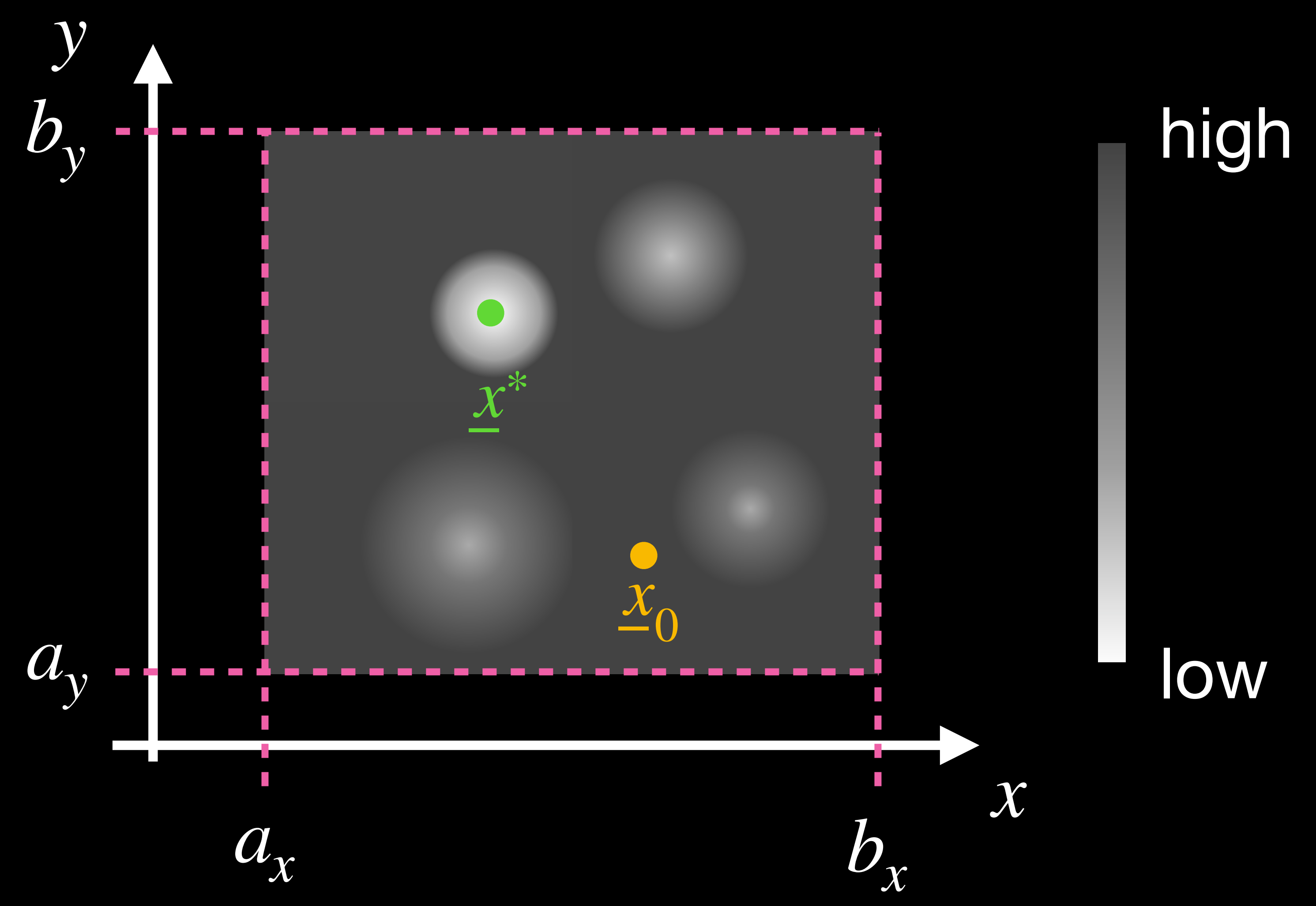

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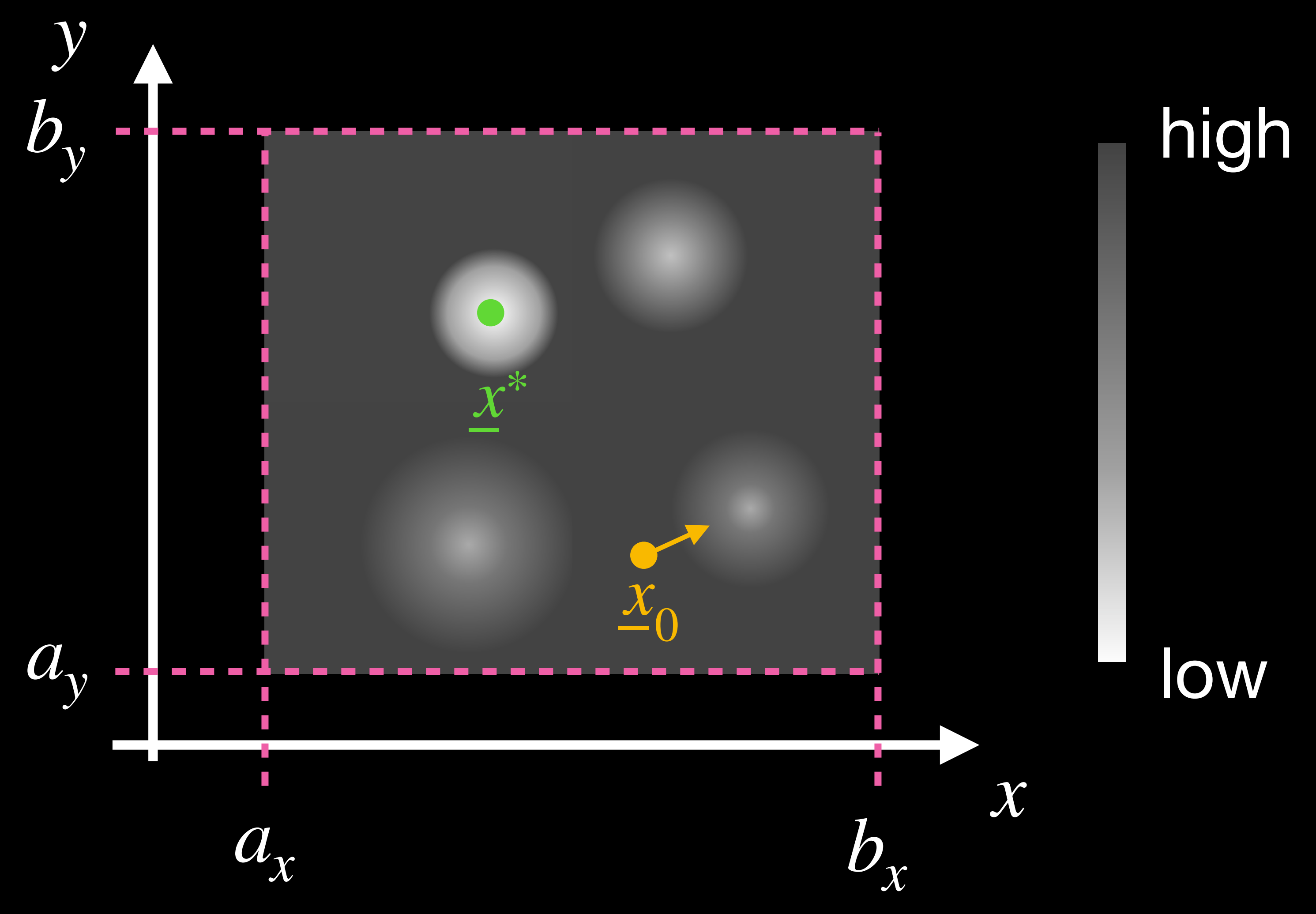

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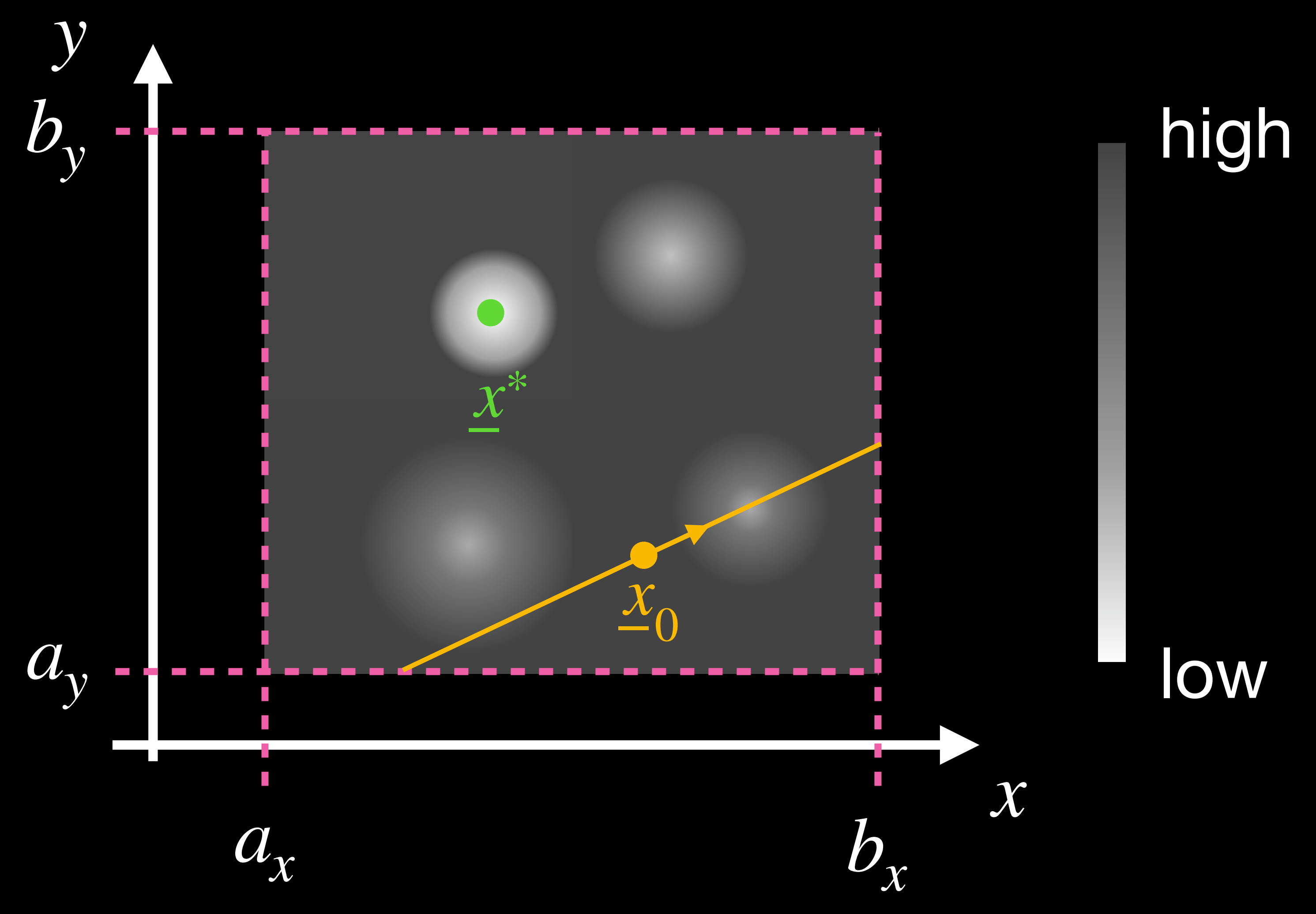

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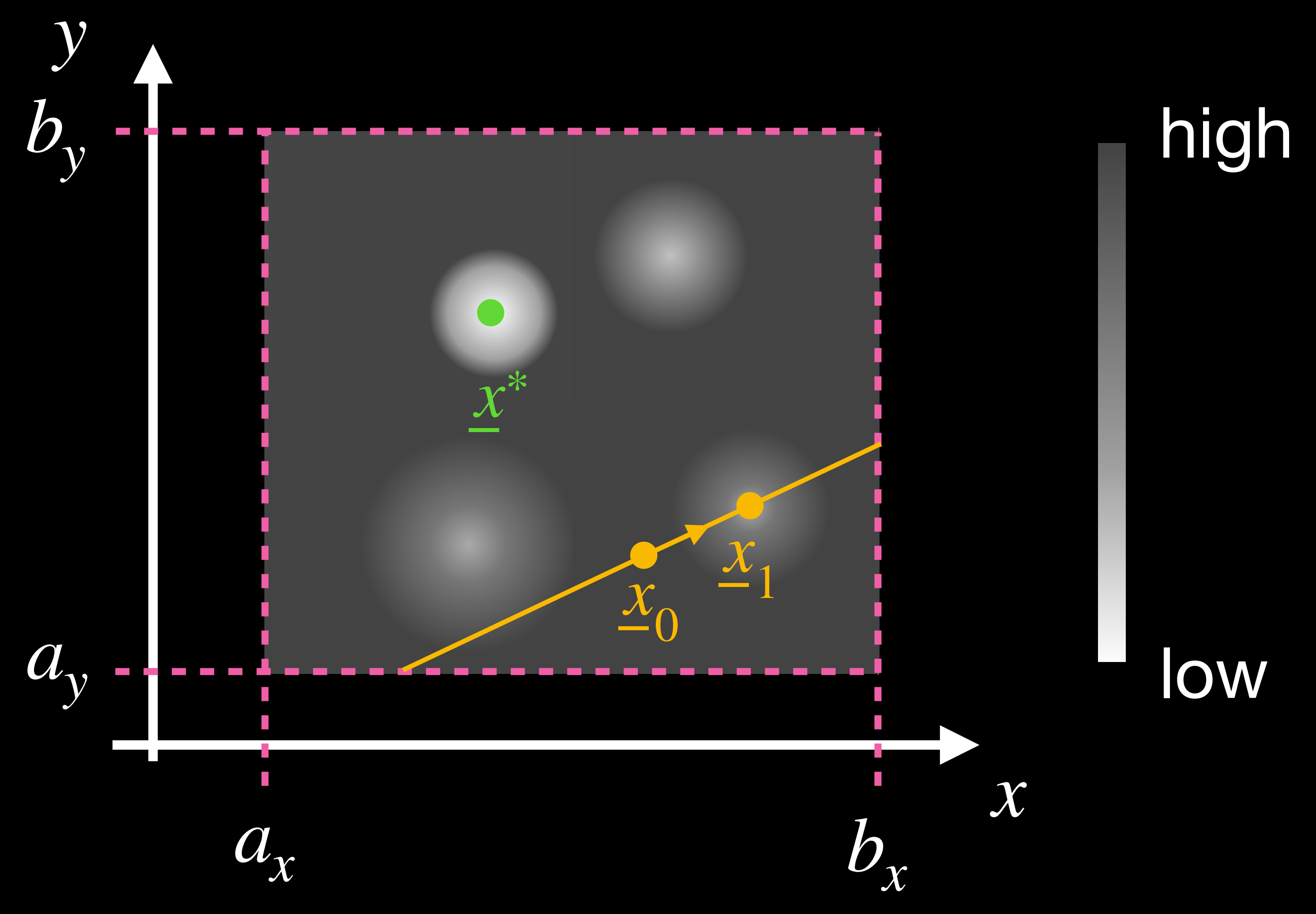

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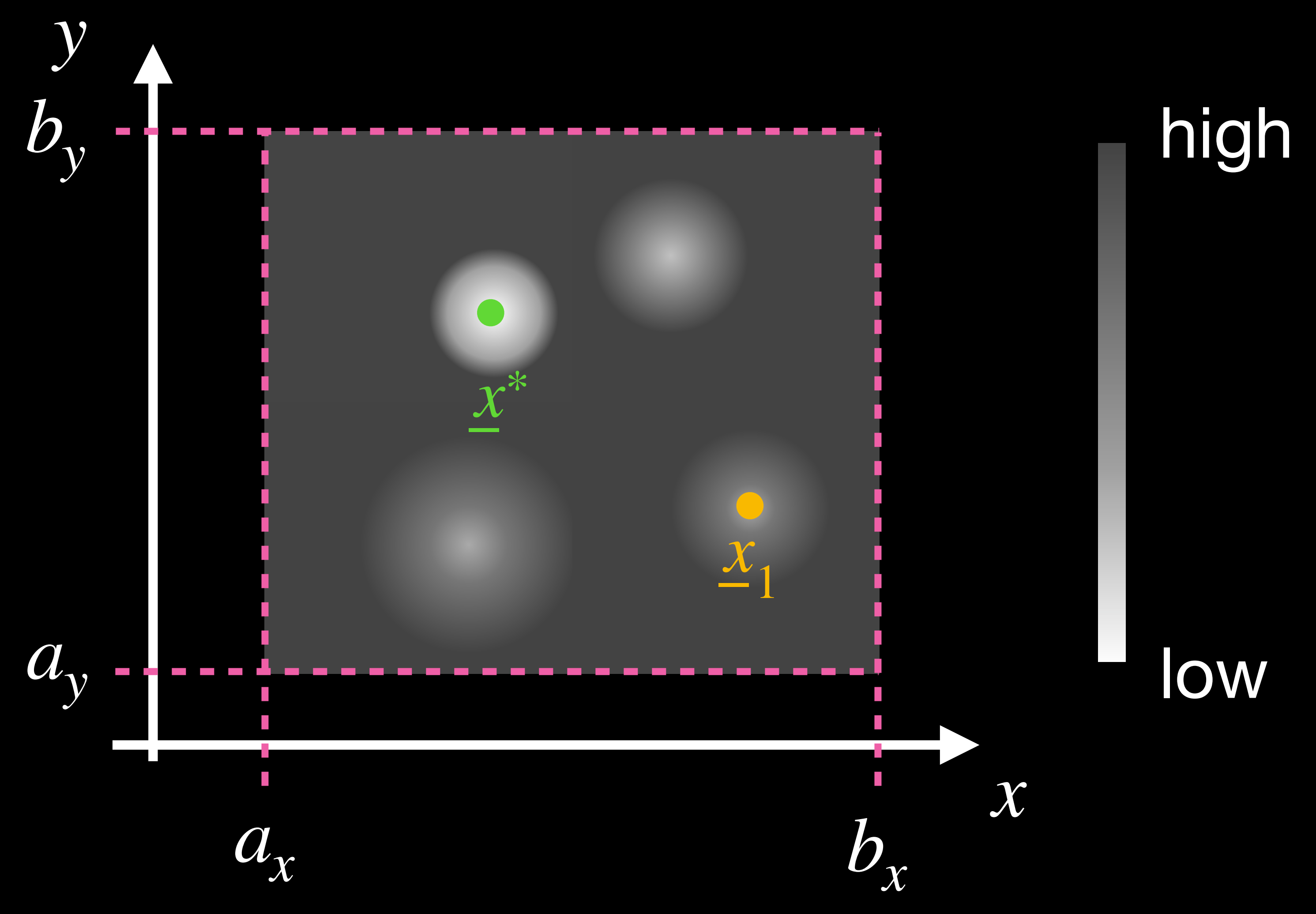

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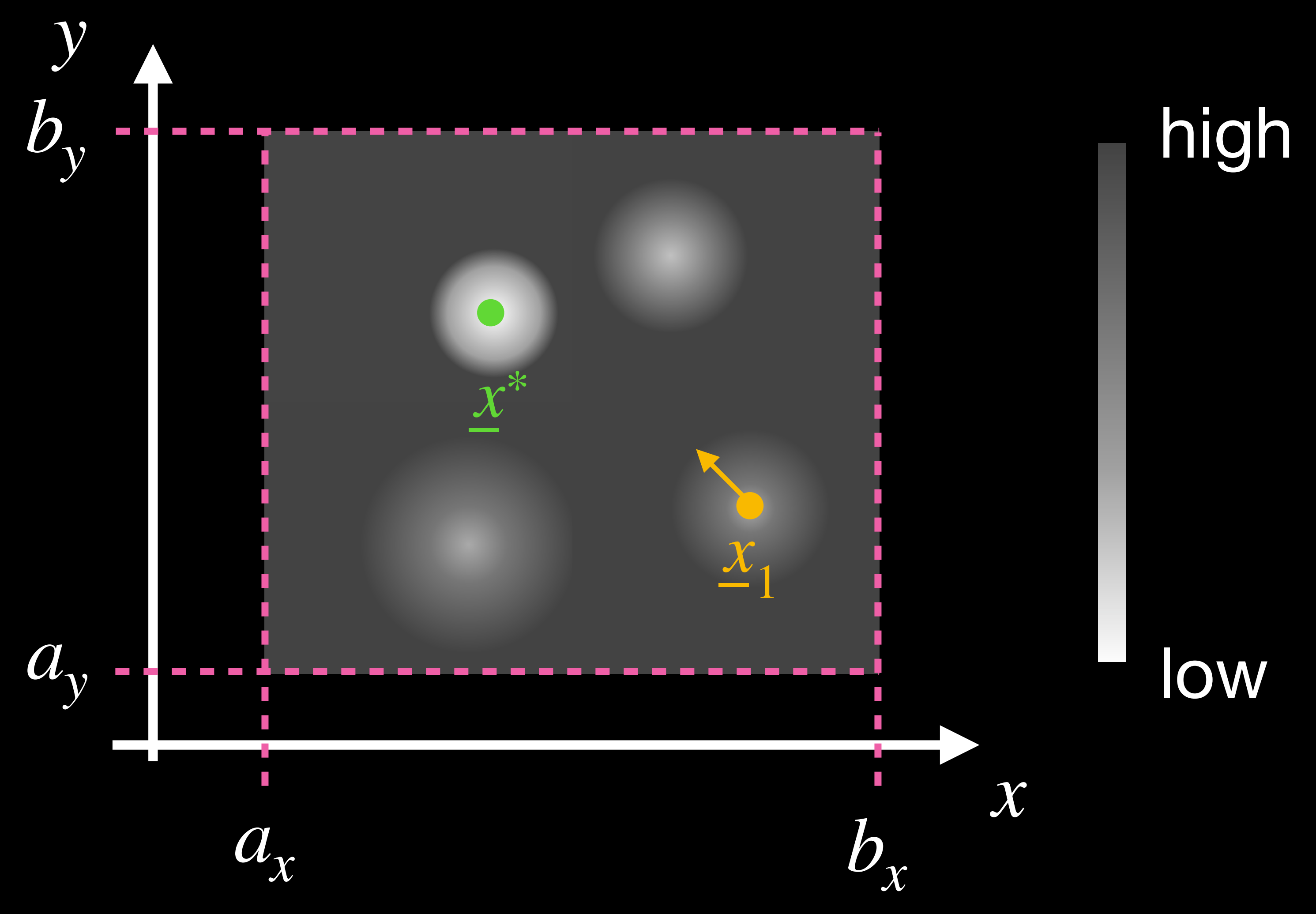

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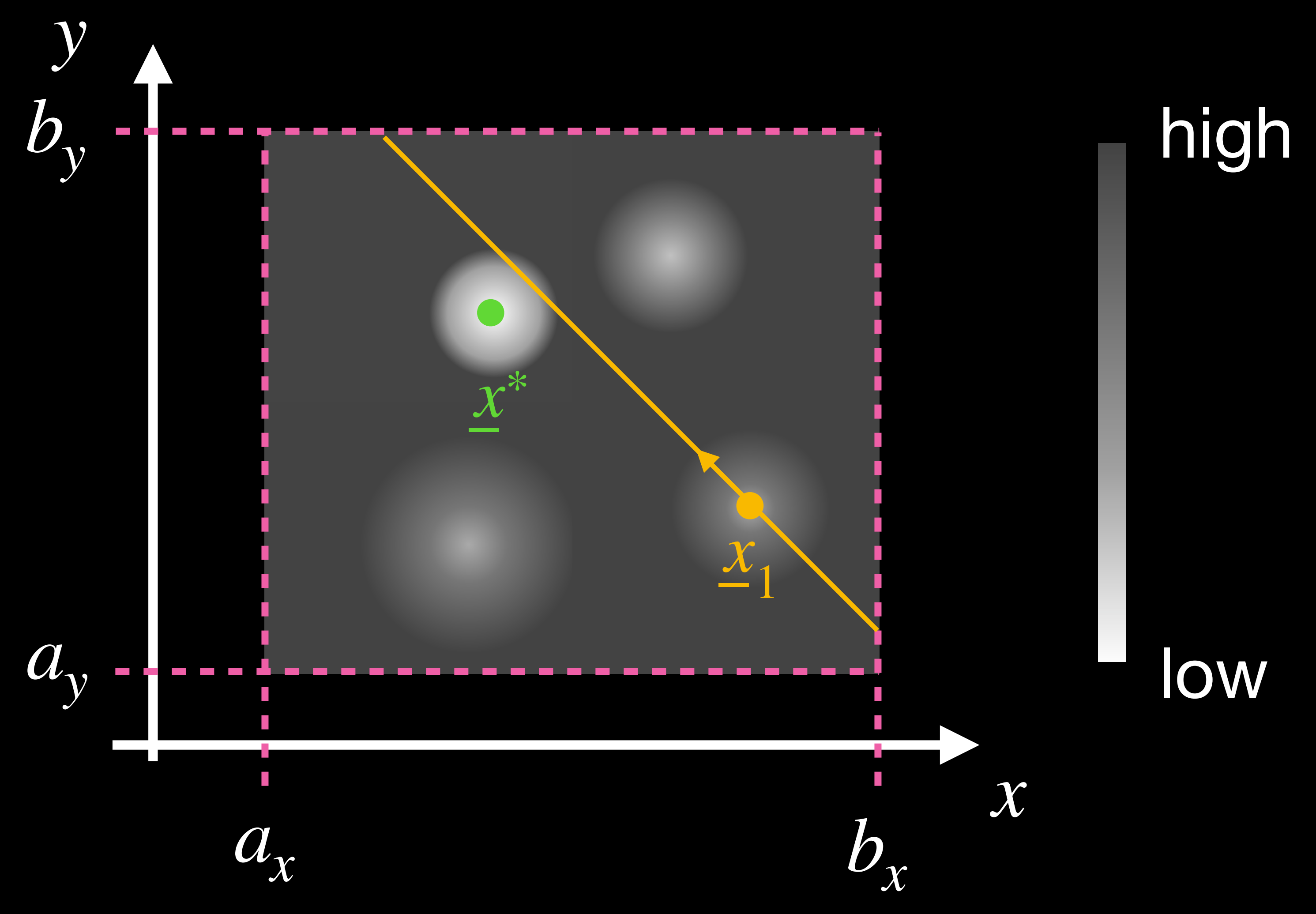

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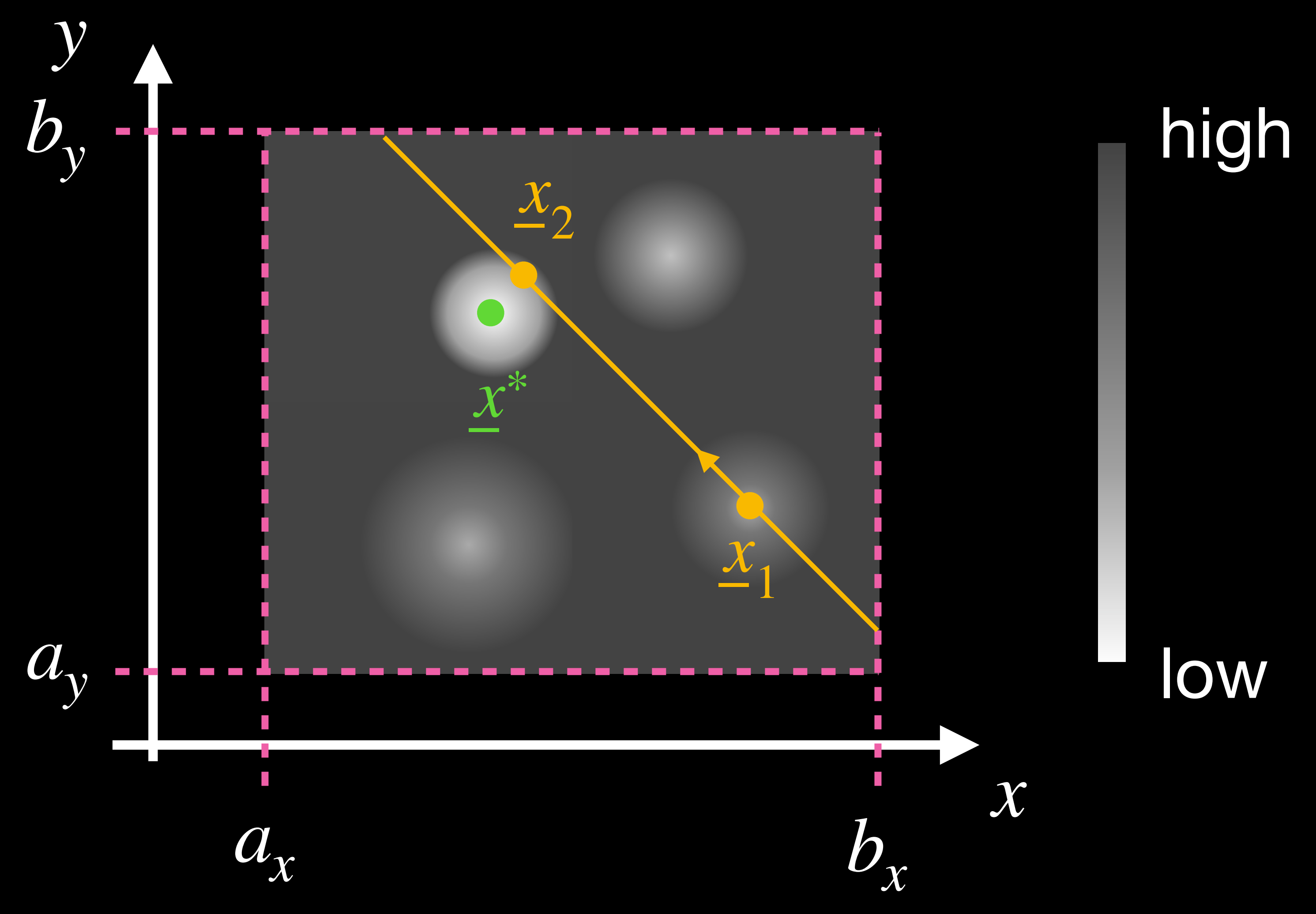

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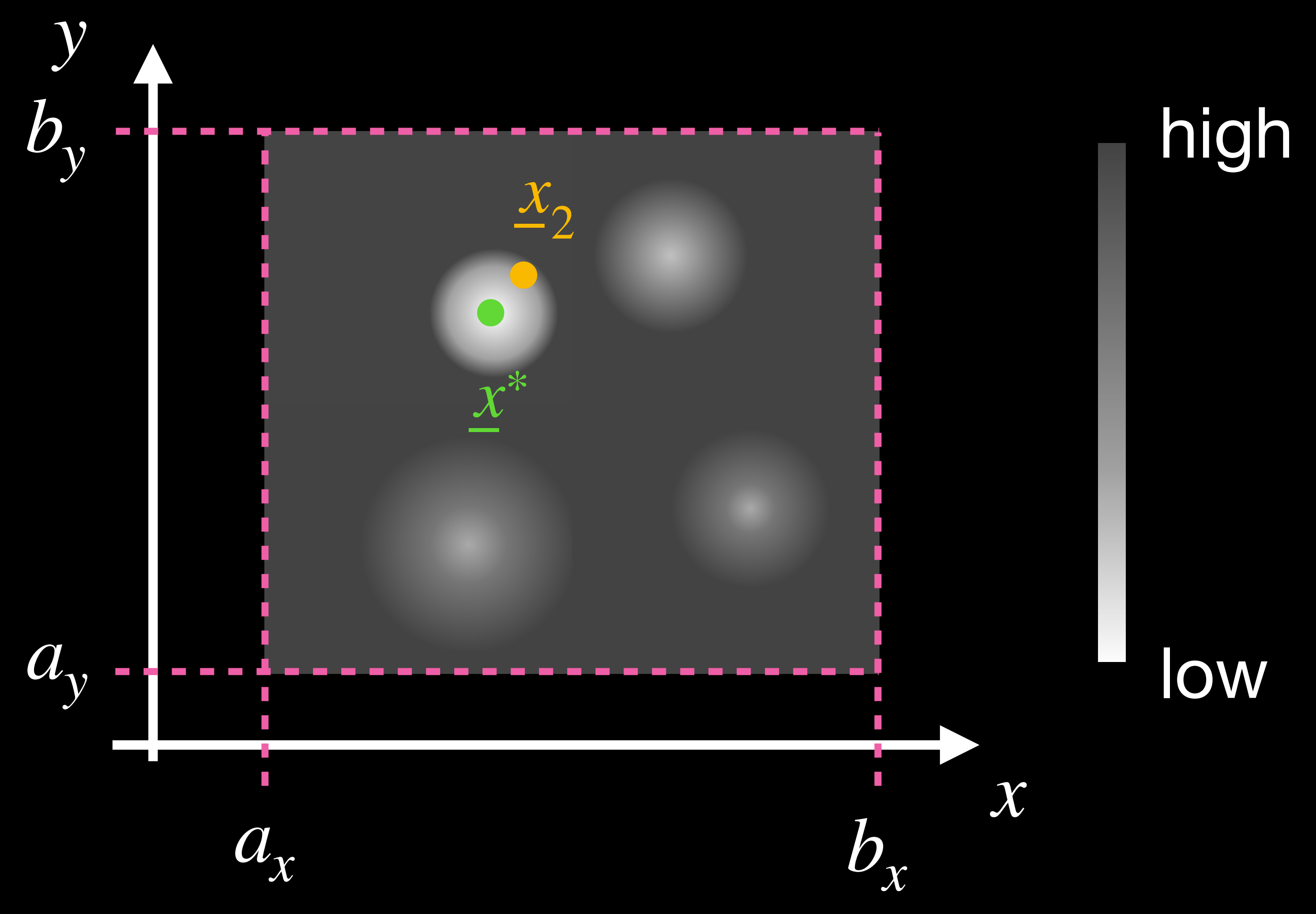

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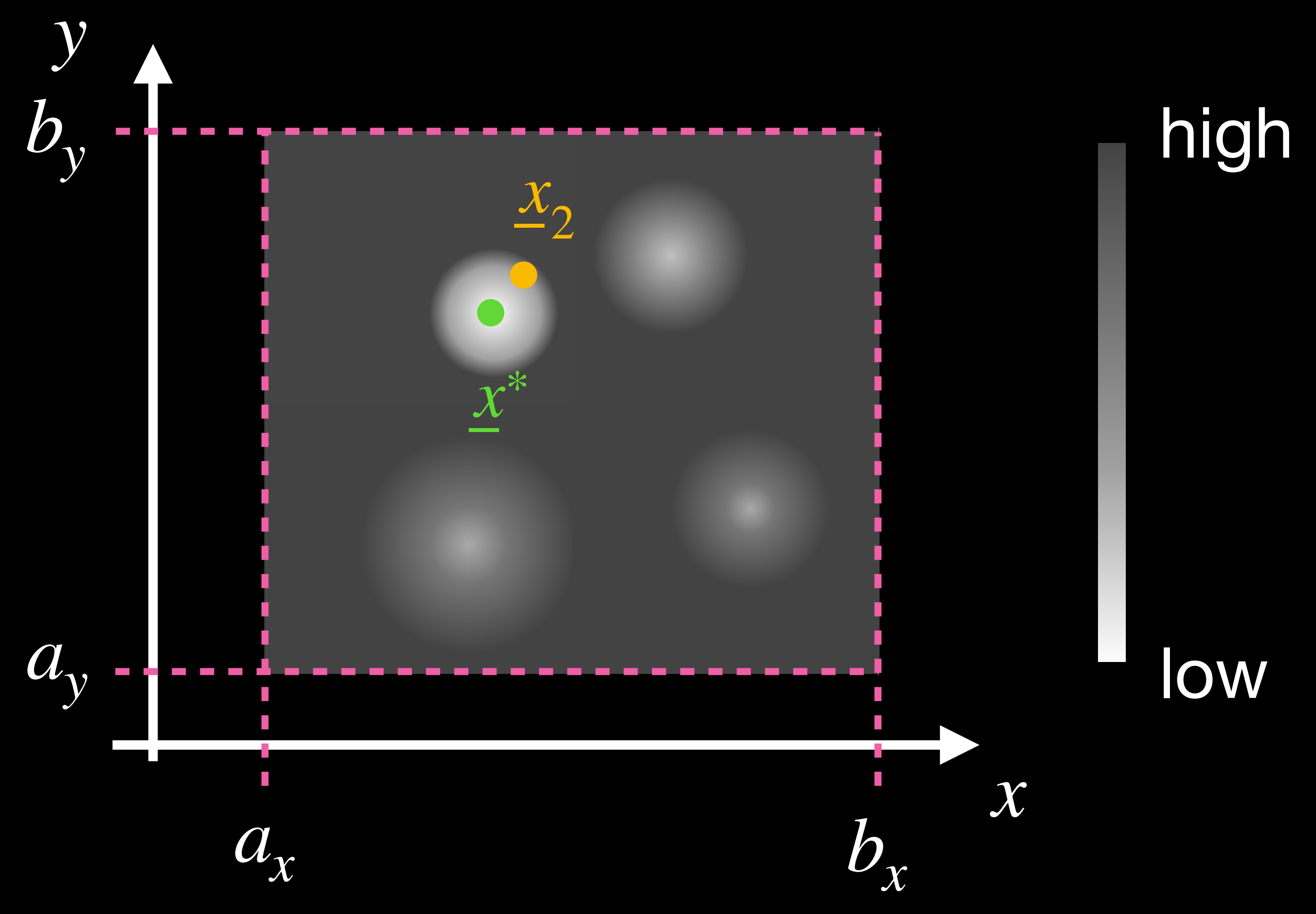
$\underline{X}_{t+1}$



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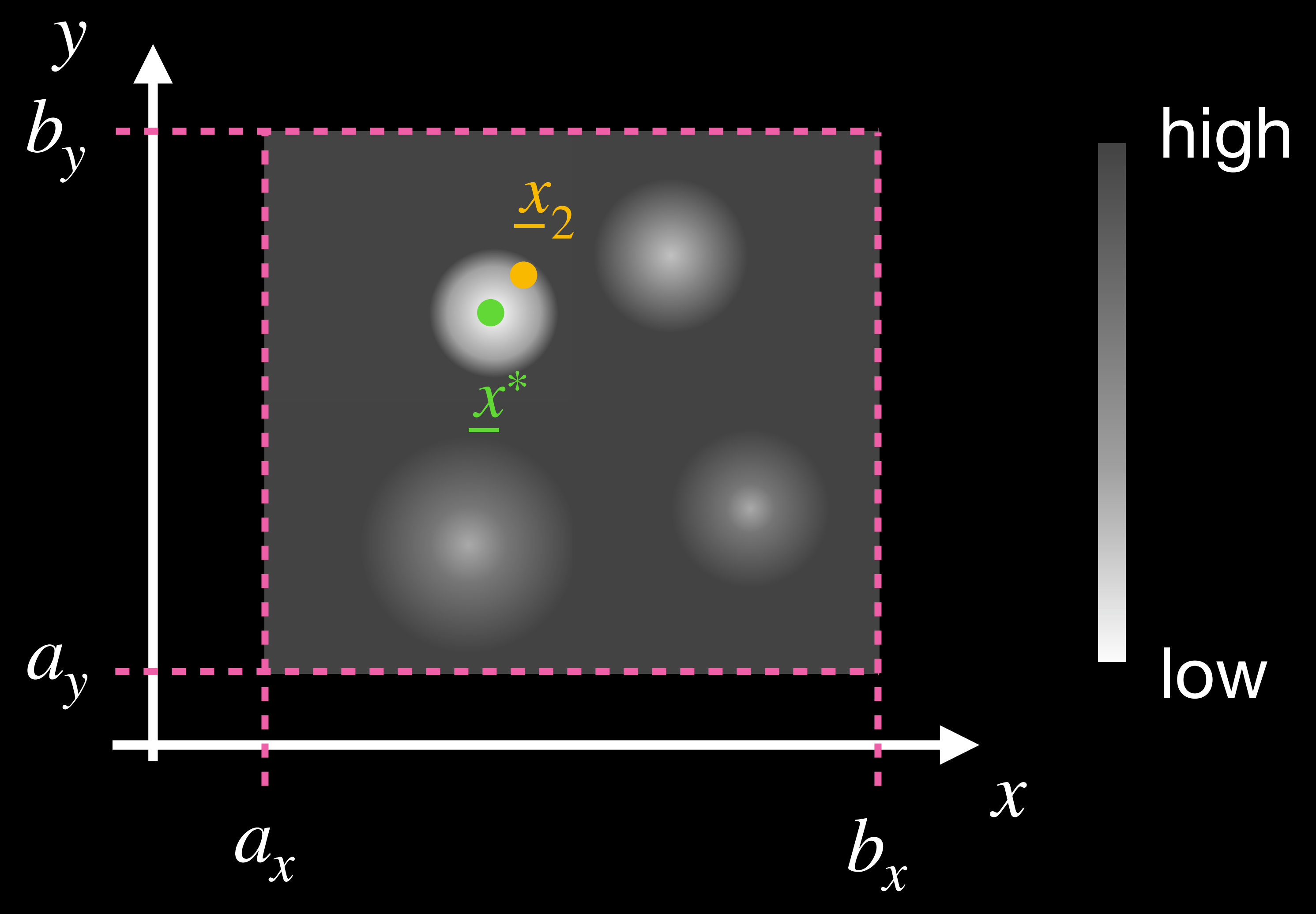

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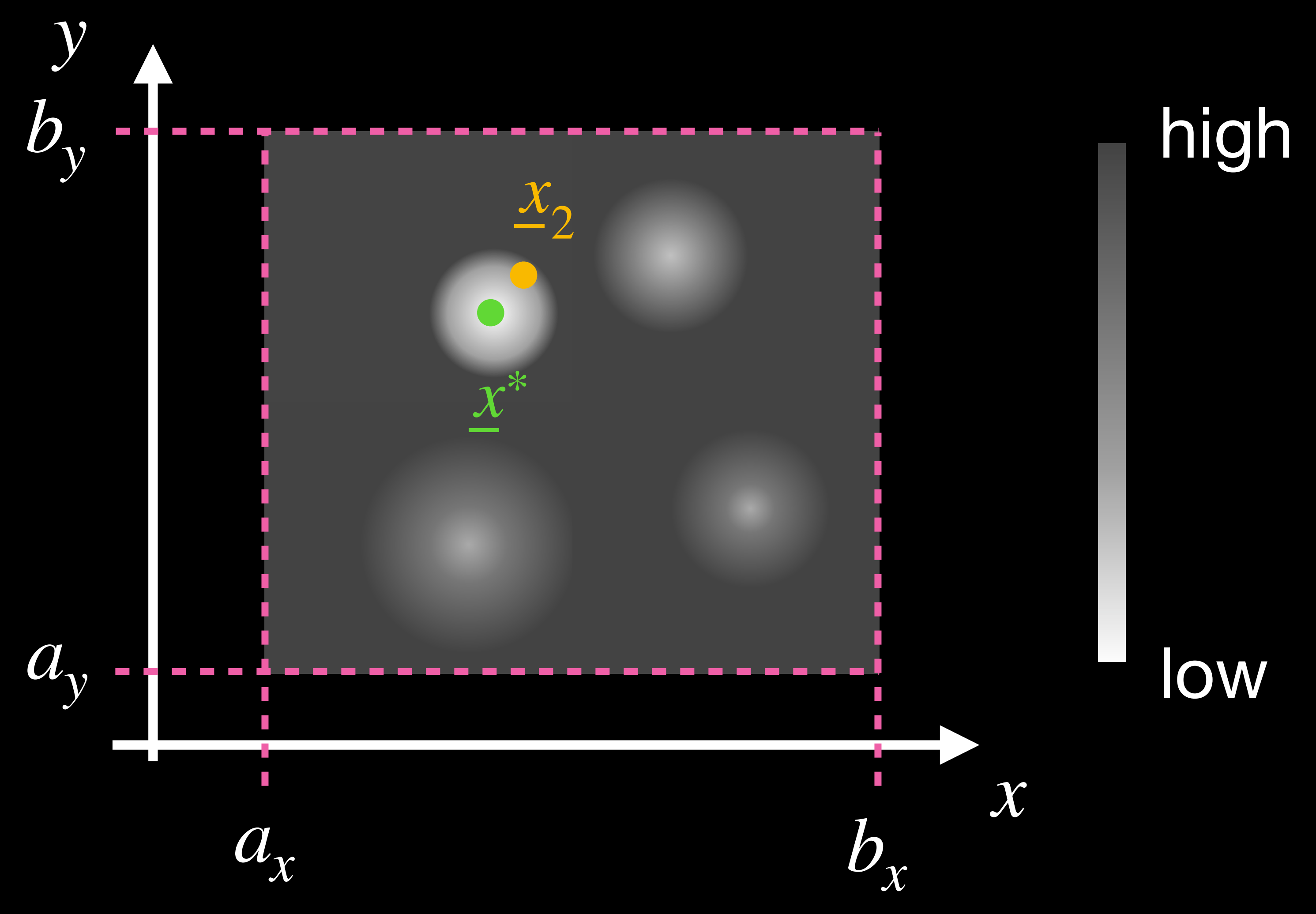

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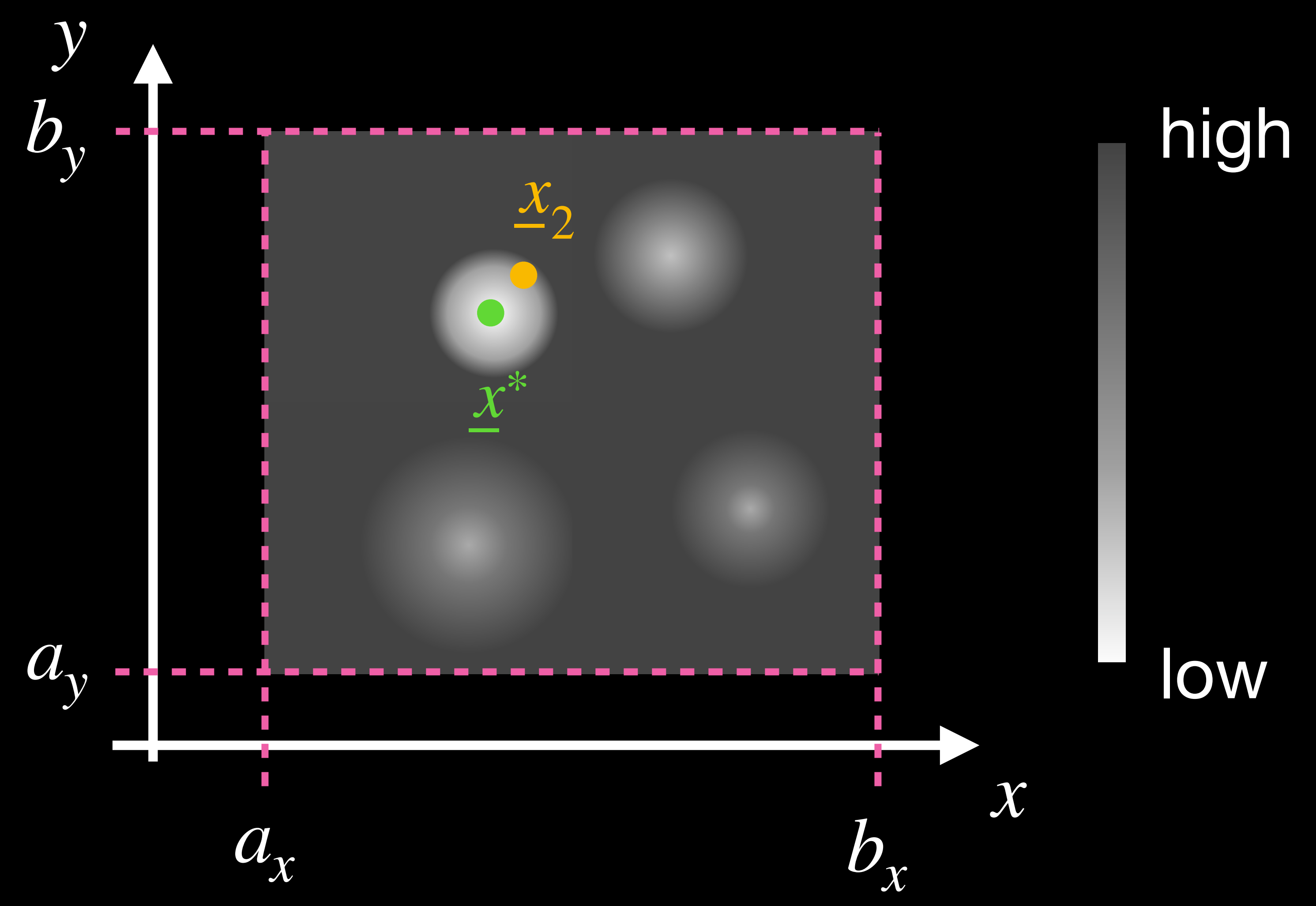

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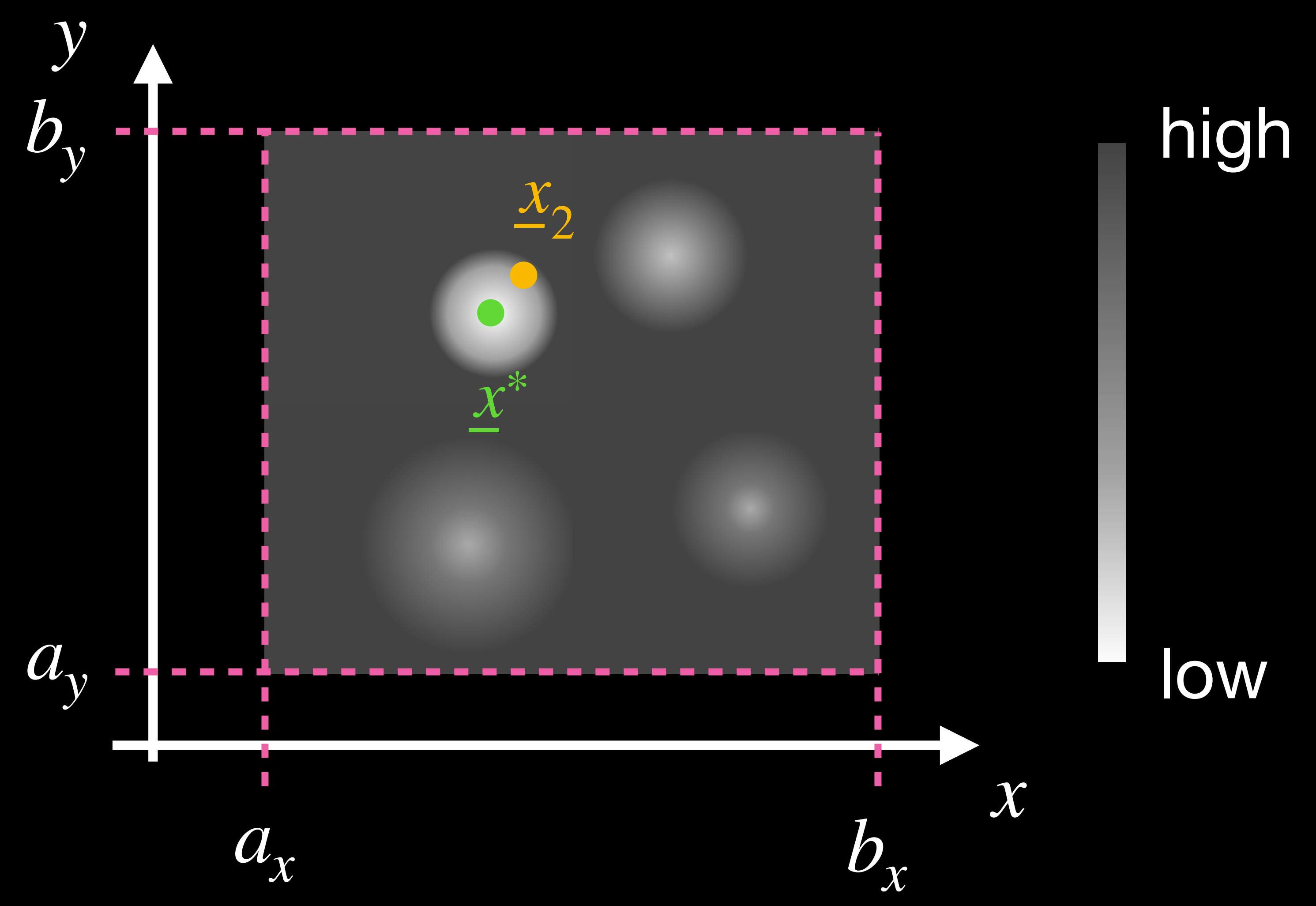

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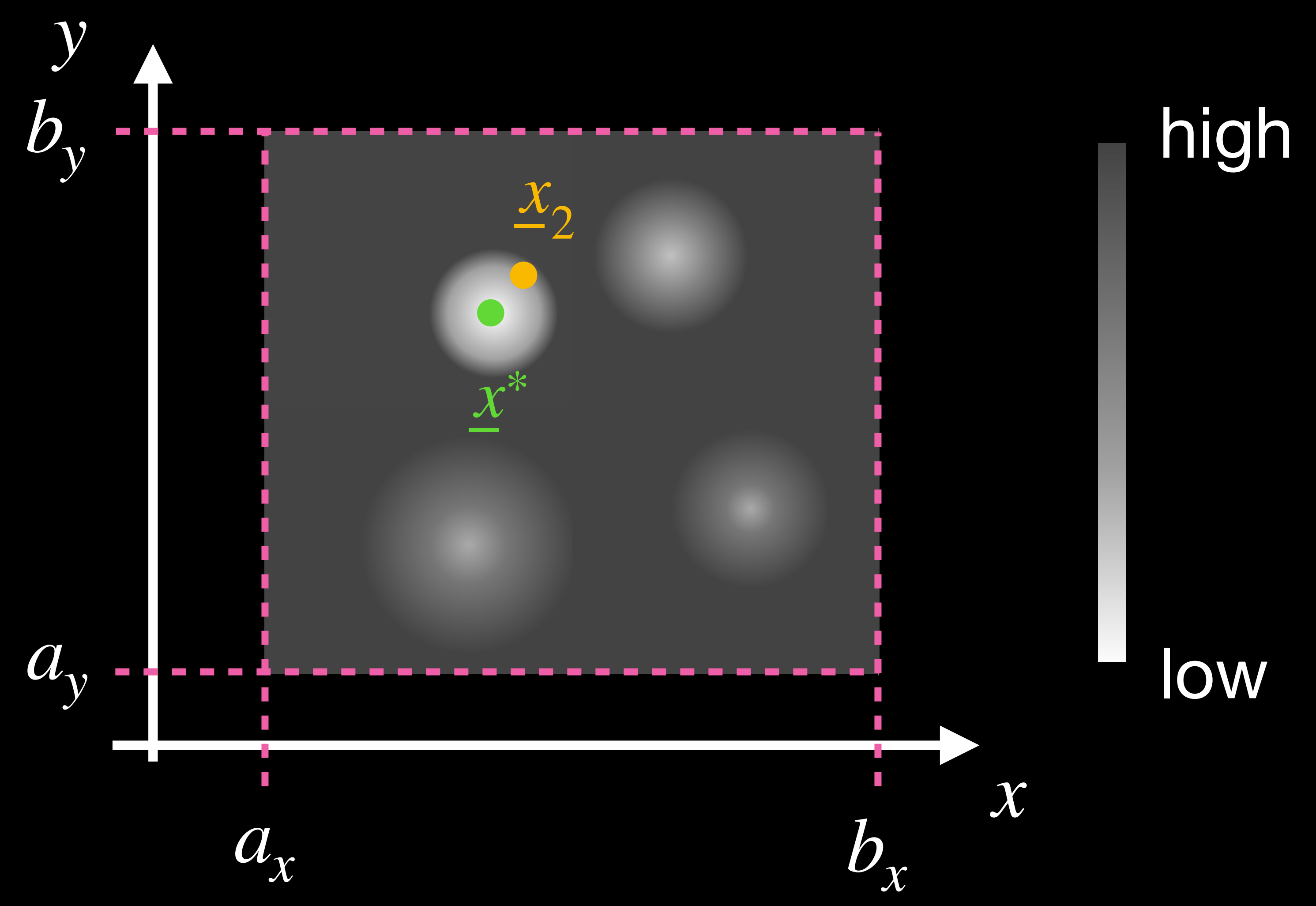

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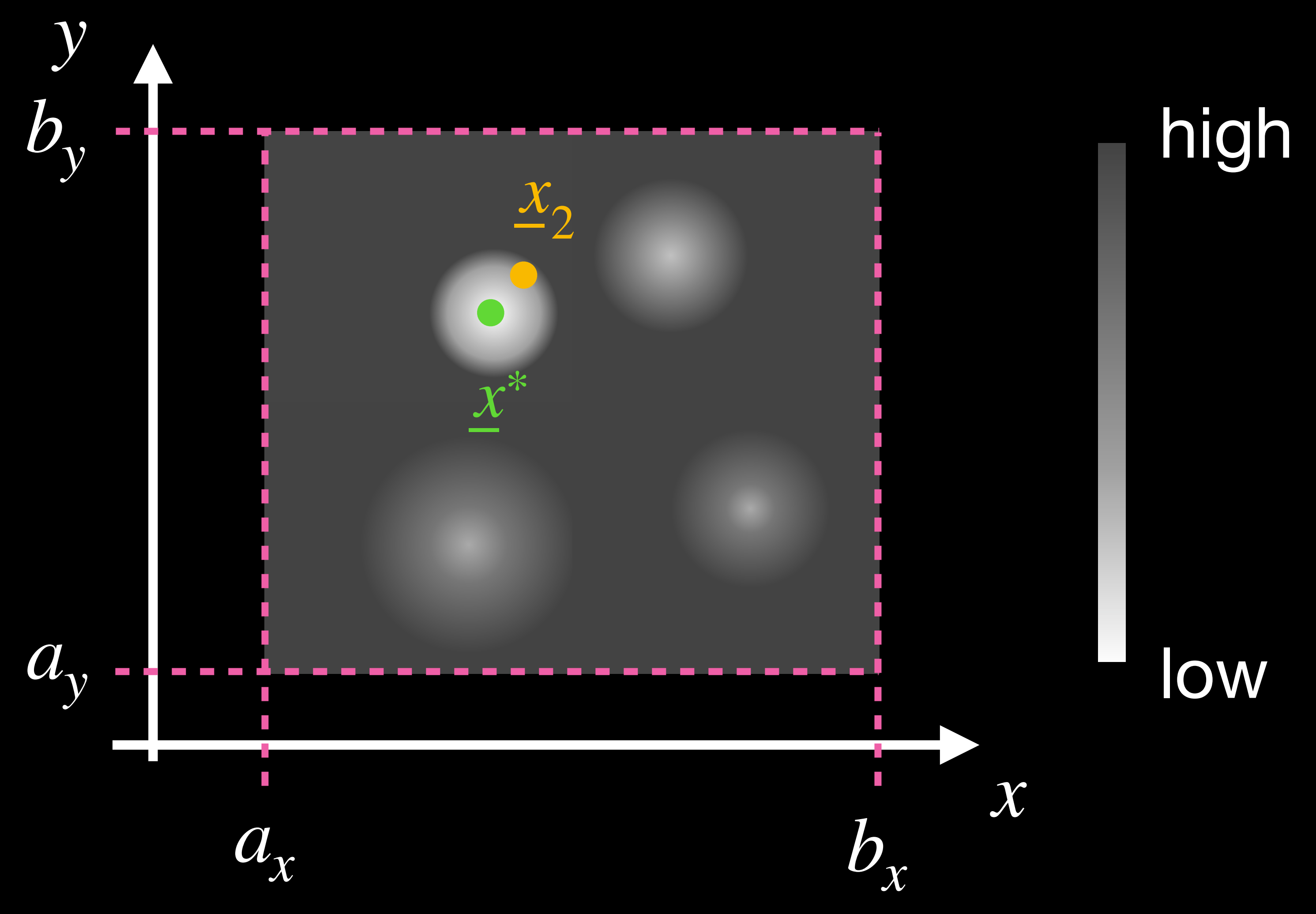

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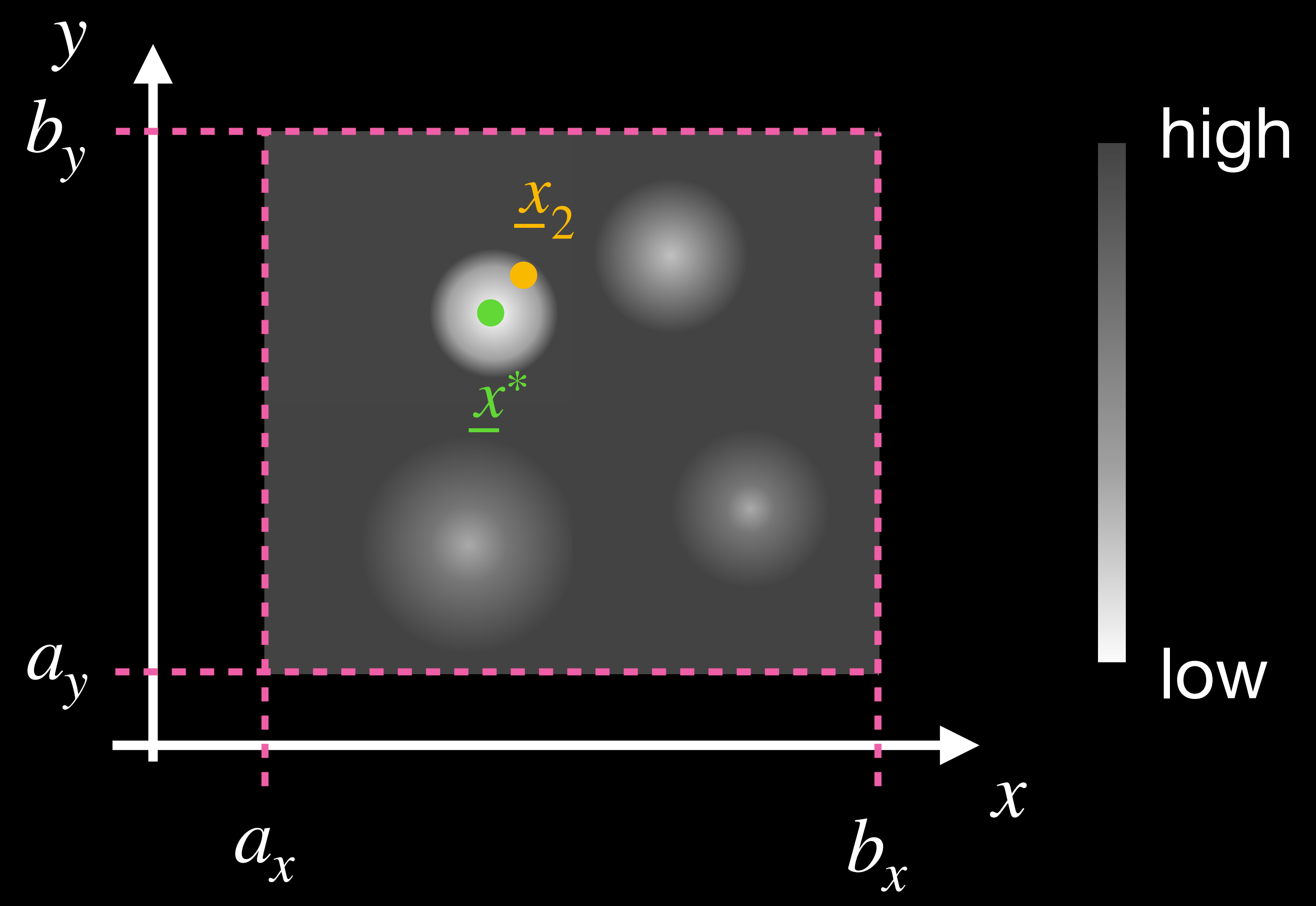

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$\rightarrow$ reduction to 1d-optimisation problem

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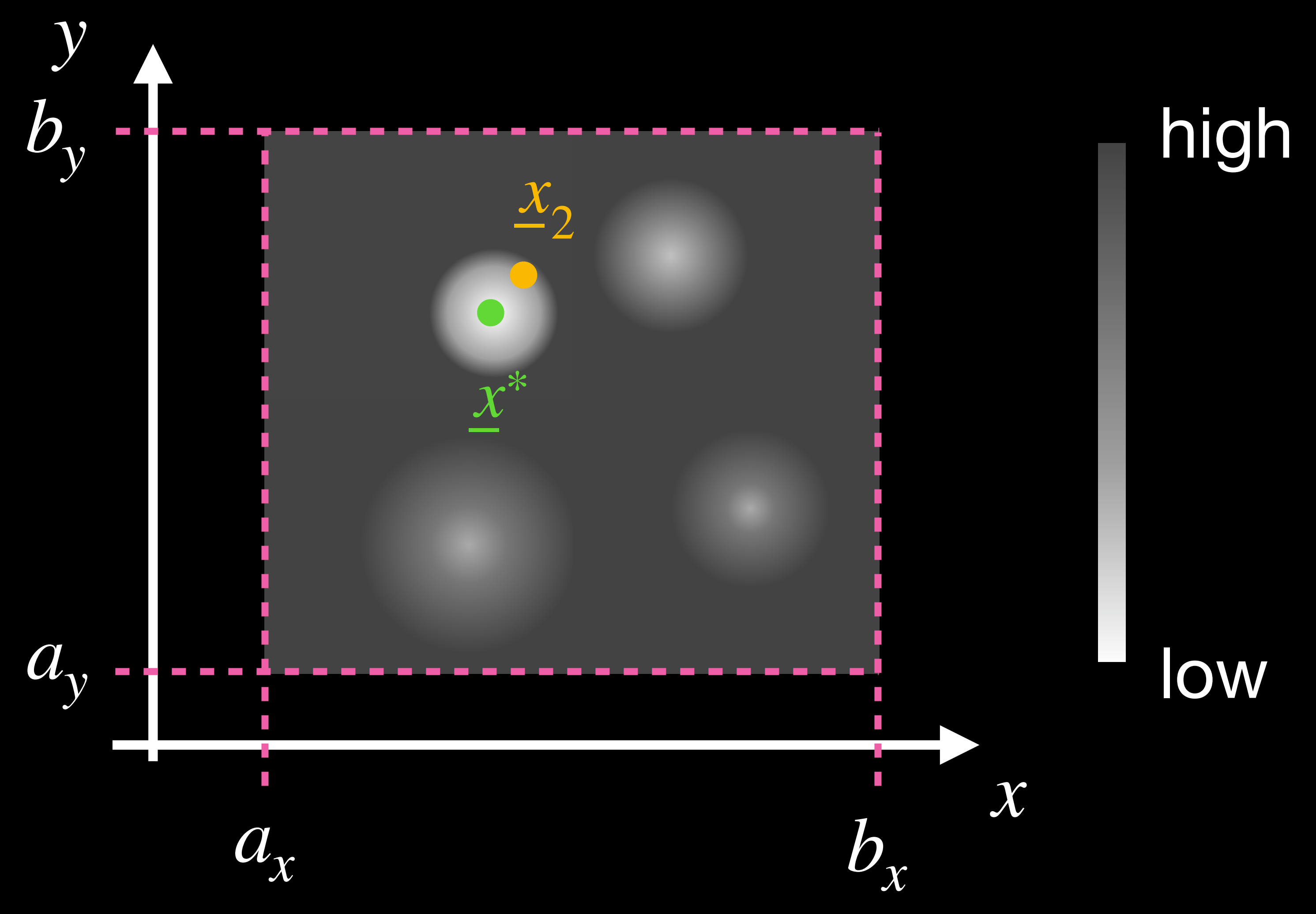

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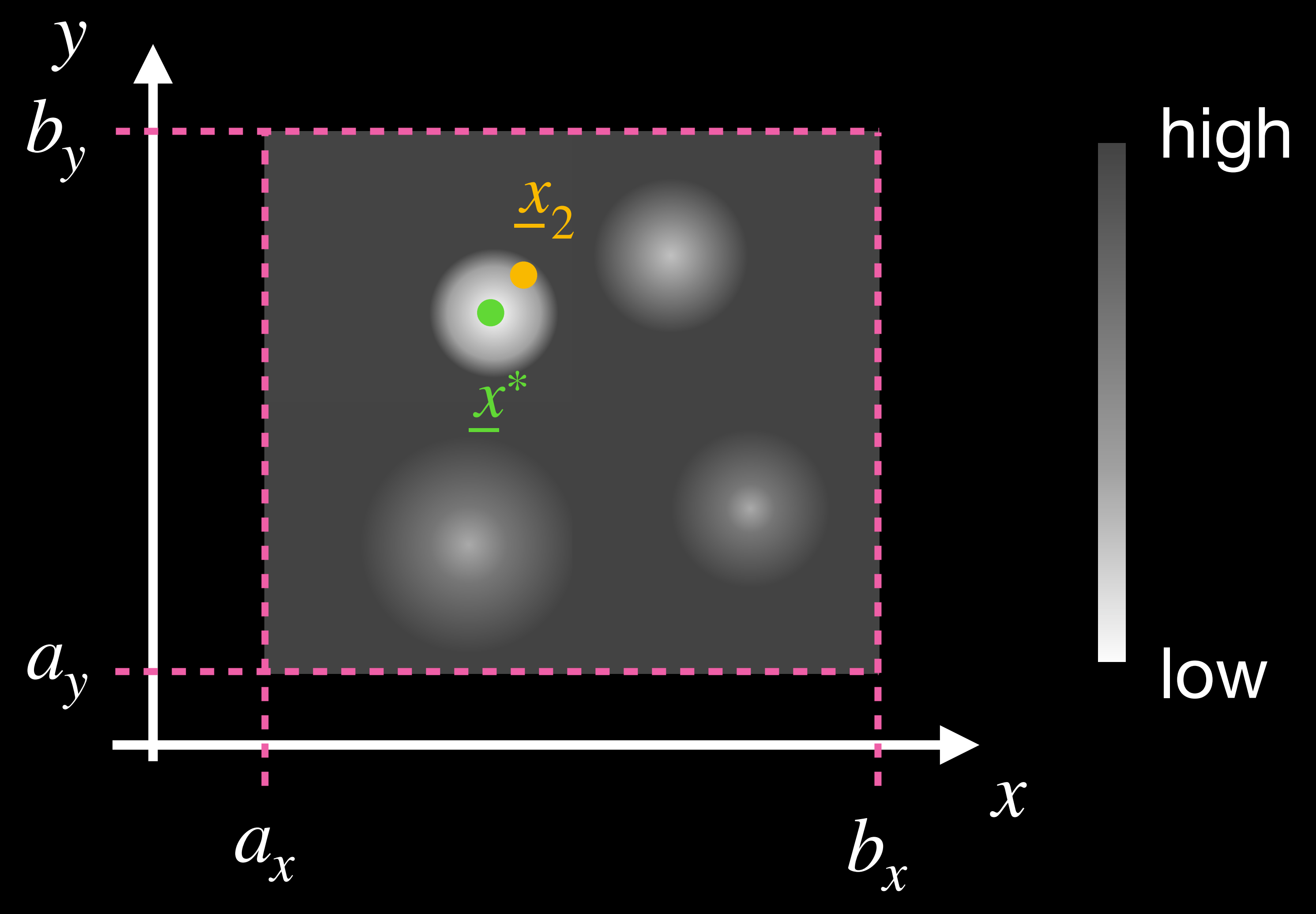

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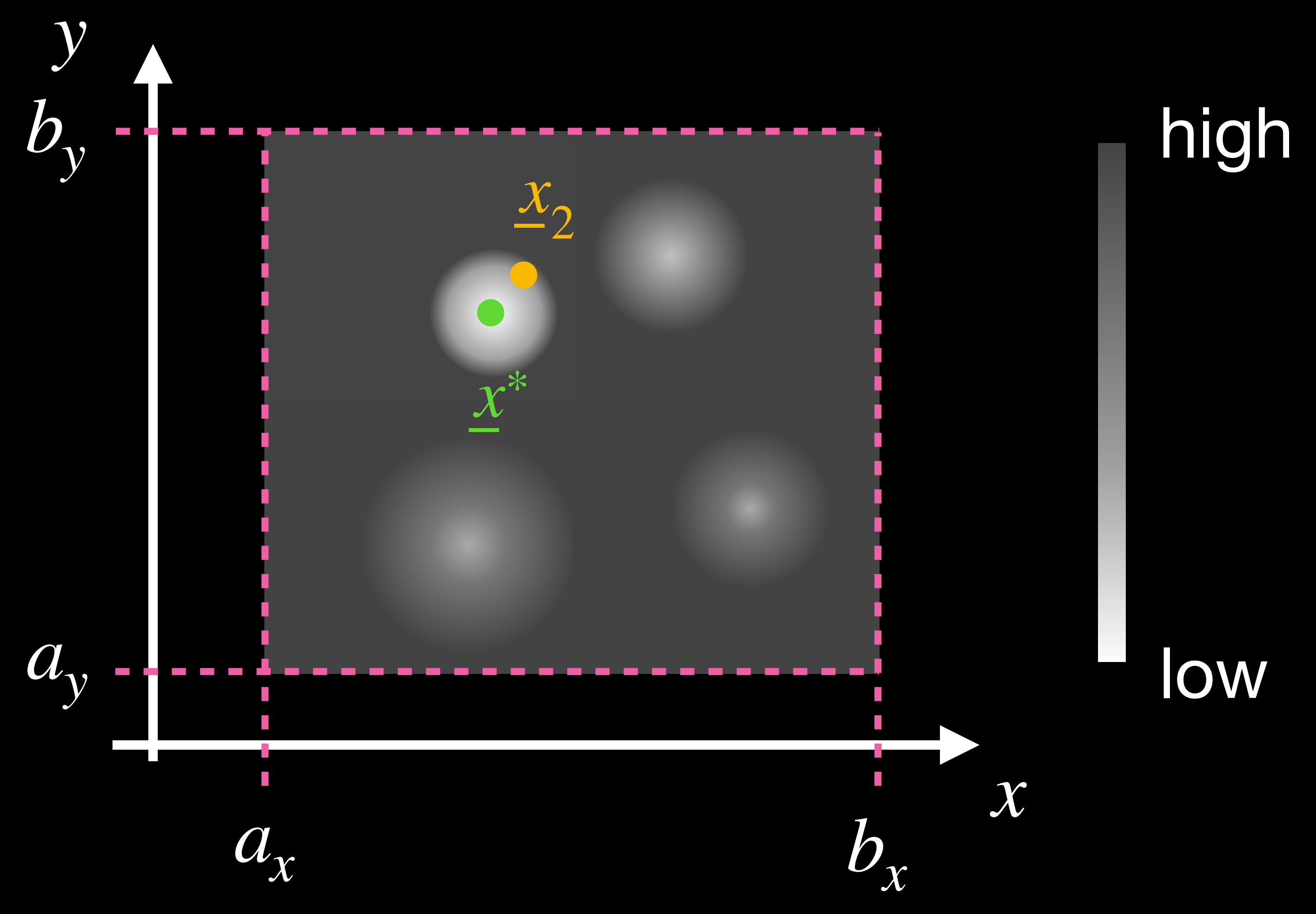

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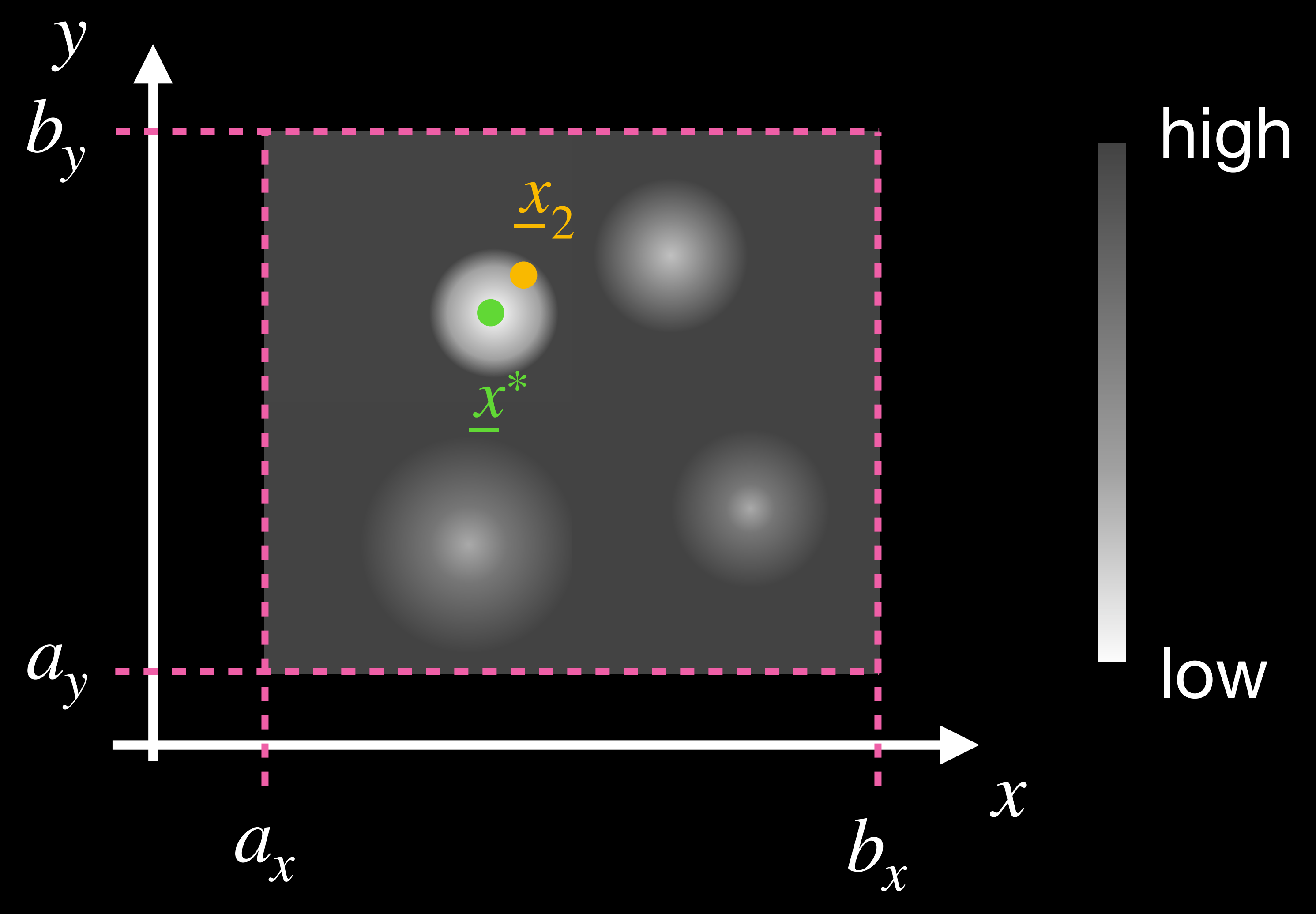

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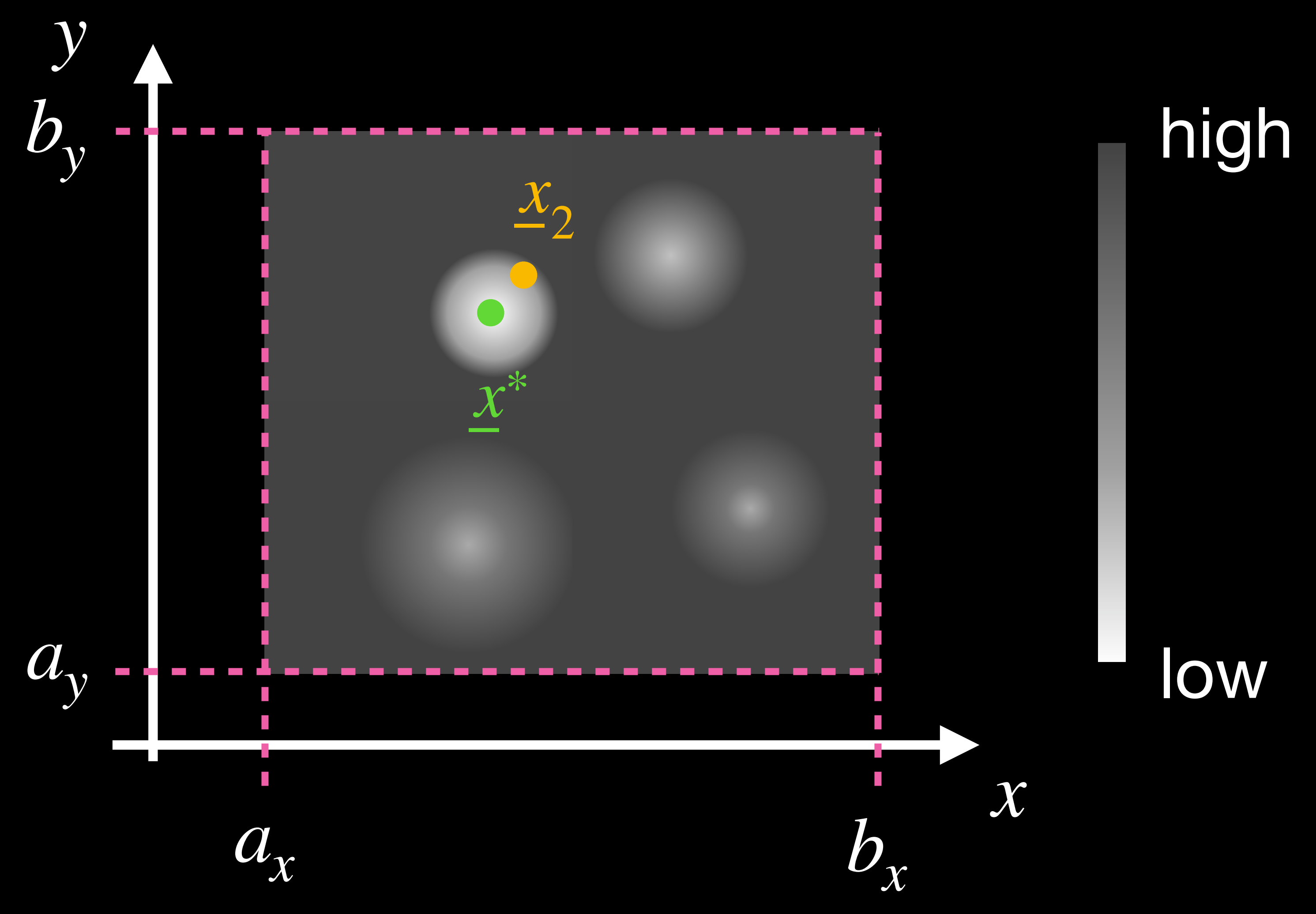

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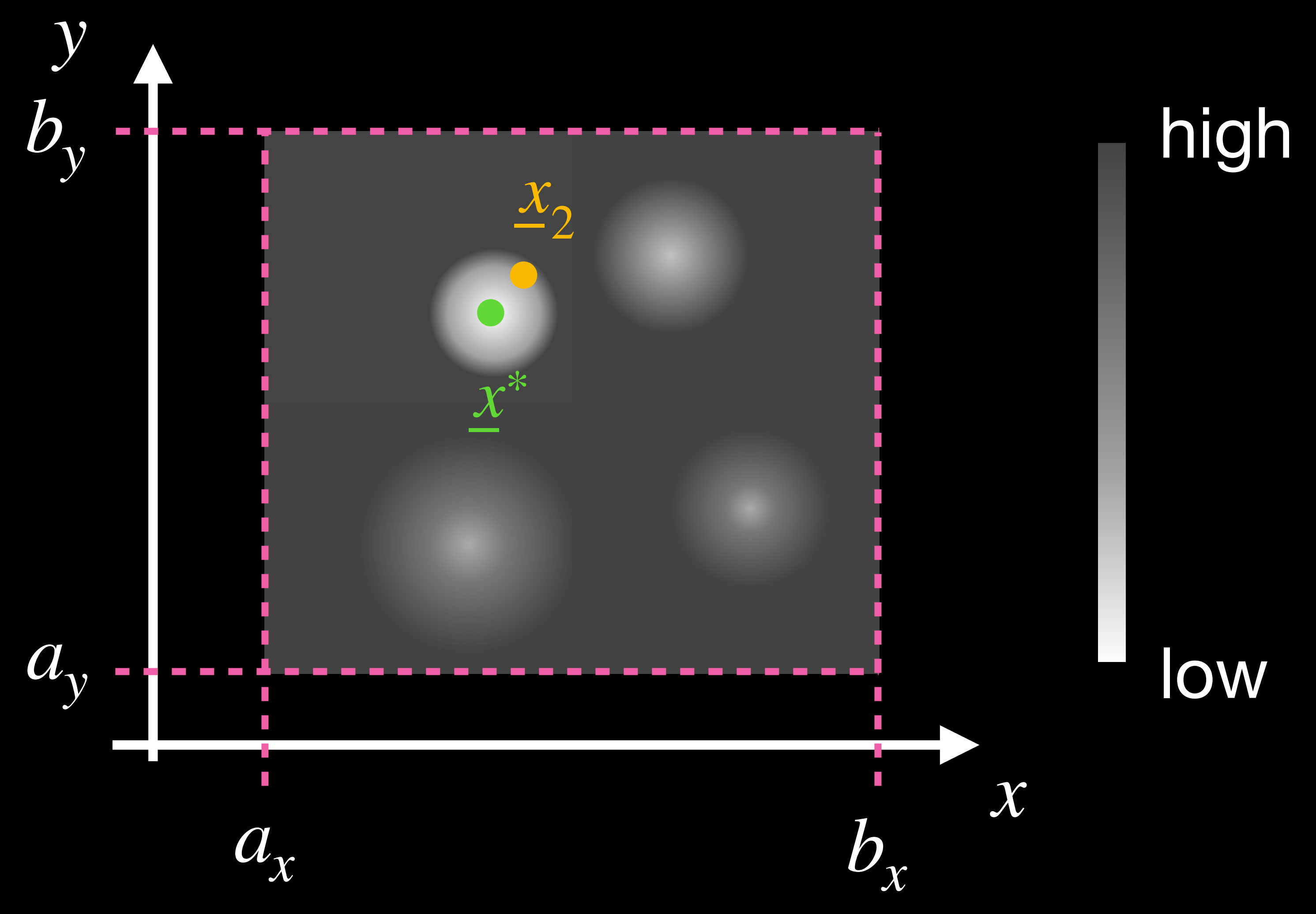

$$\underline{X}_{t+1} = \underline{X}_t + linesearch(\underline{X}_t, \underline{U}_t) \qquad \underline{U}_t \sim \mathcal{U}(\mathbb{S})$$

$\mathbb{S}$ be the d-dimensional unit sphere

$$linesearch_h(\underline{X}_t, \underline{U}_t) := \arg\min_{\beta} \Big(h(\underline{X}_t + \beta \cdot \underline{U}_t)\Big) \cdot \underline{U}_t$$

β is distance between X_t and X_{t+1}

(finds the vector to the point with minimal value on the line that goes through $\underline{X}_t$ in the direction $\underline{U}_t$)

$\rightarrow$ reduction to 1d-optimisation problem

properties:
- global convergence guarantee
- converges fast
- needs uniform random numbers (on the unit sphere)

useful if:
- $\mathbb{S}$ is high-dimensional and bounded

algorithm 4: simulated annealing

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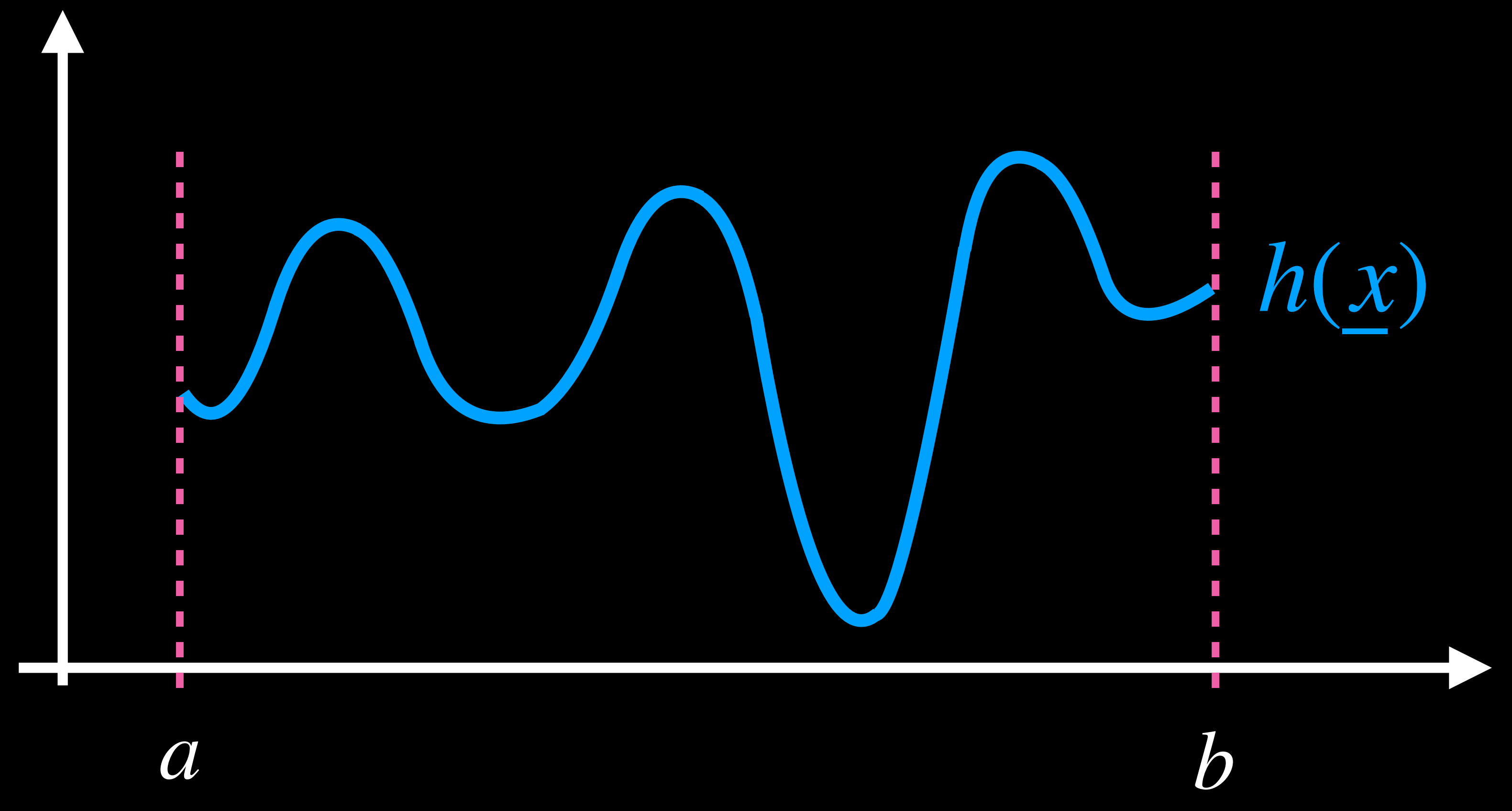

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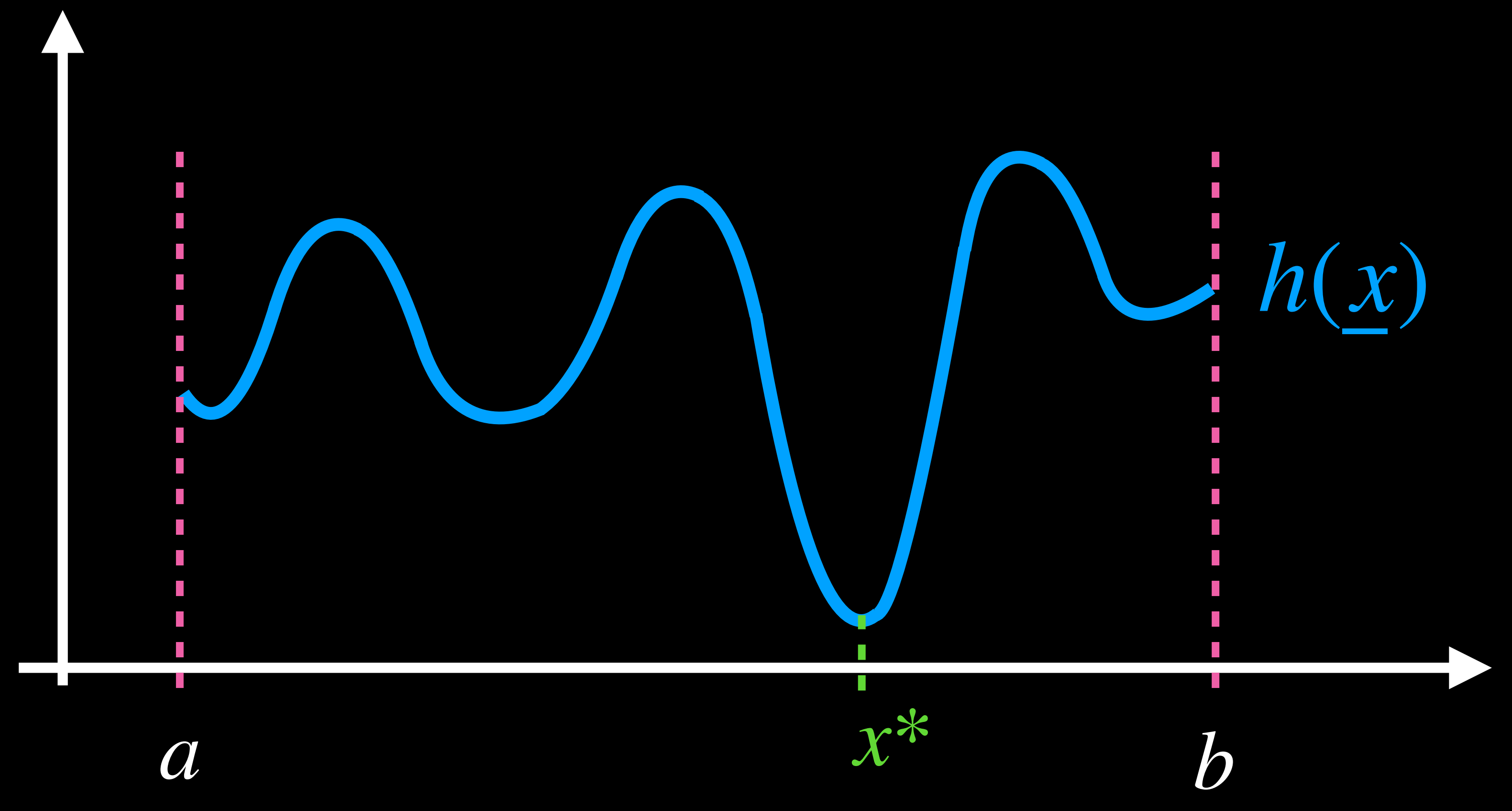

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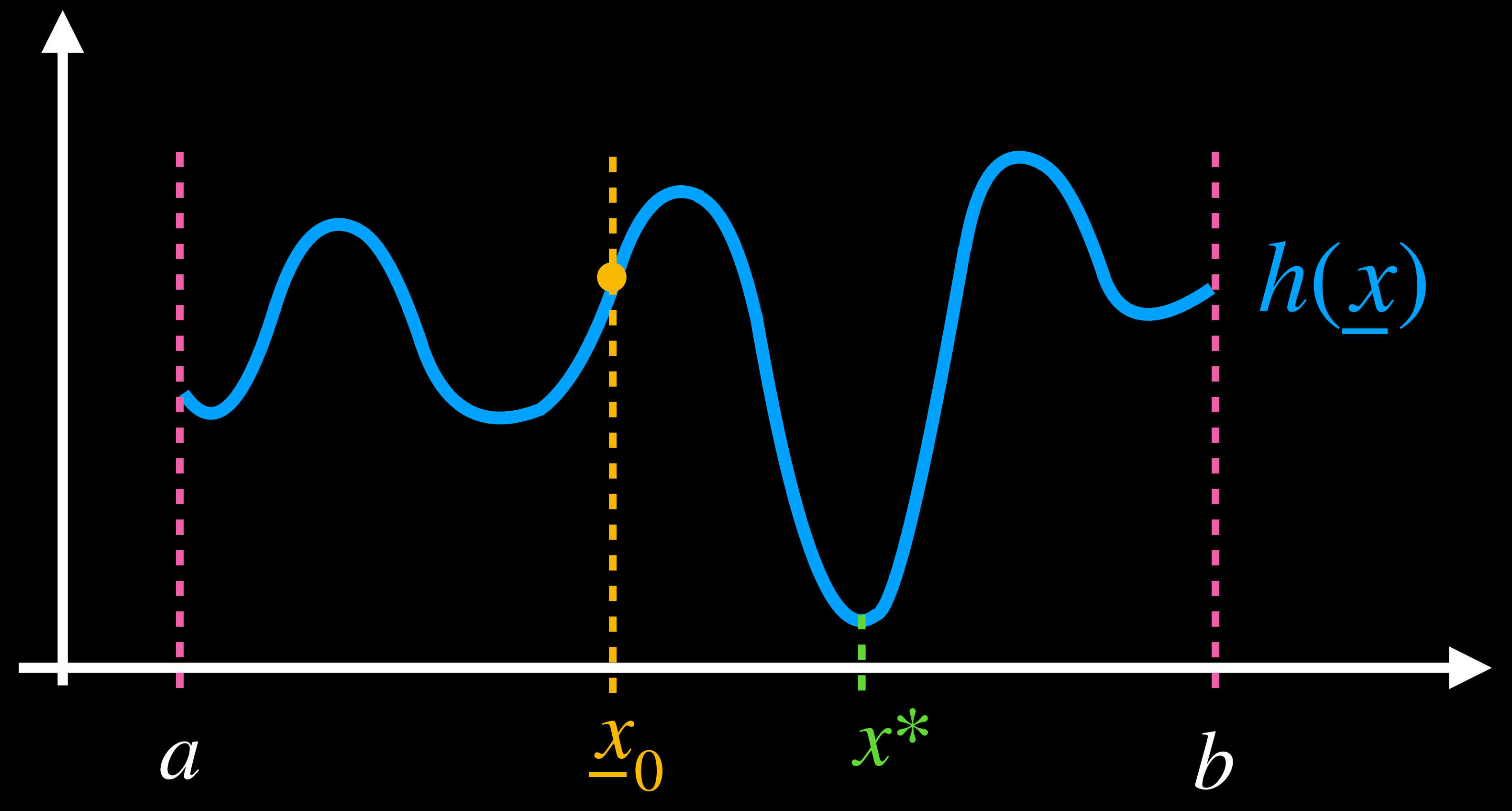

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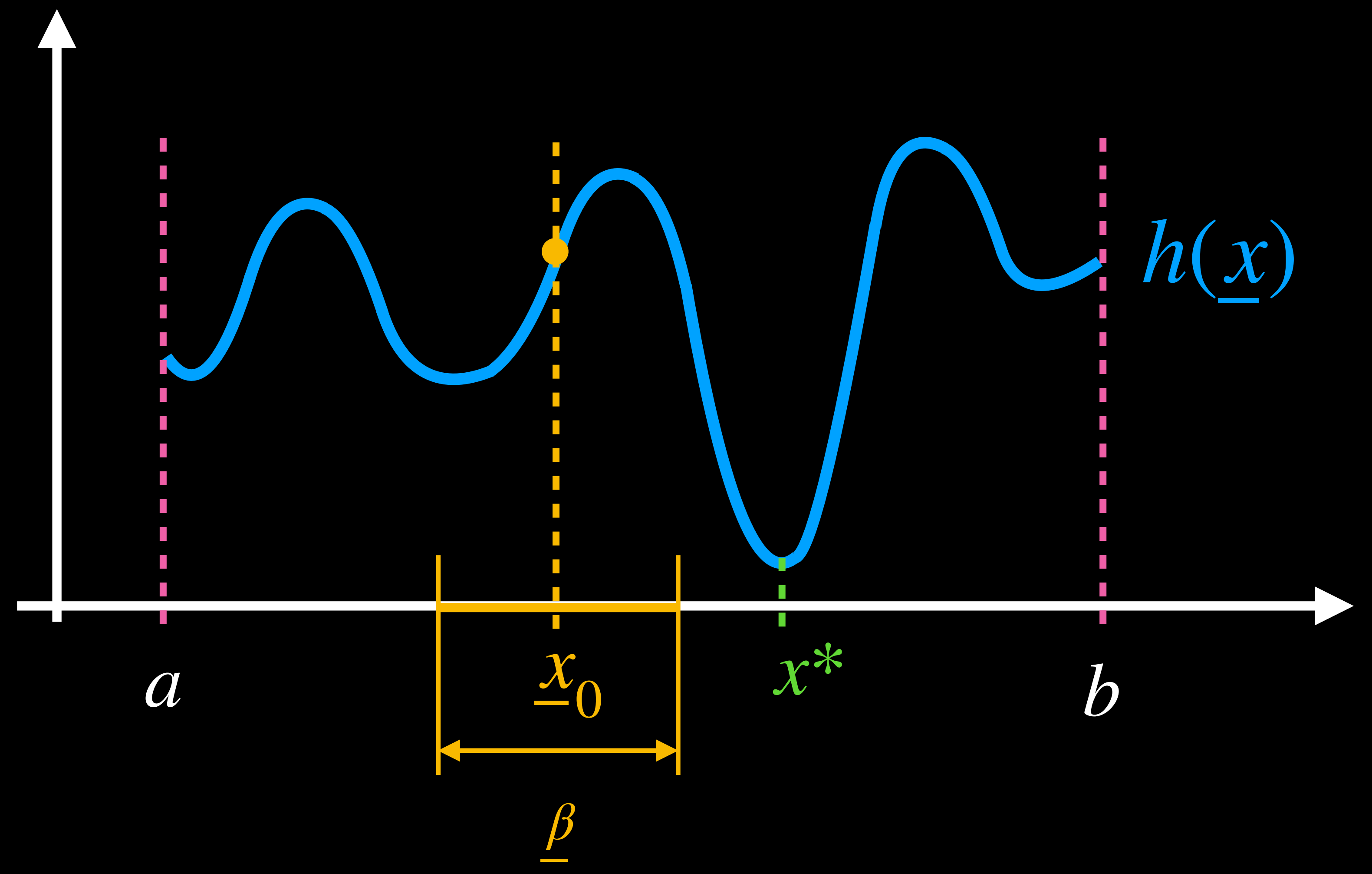

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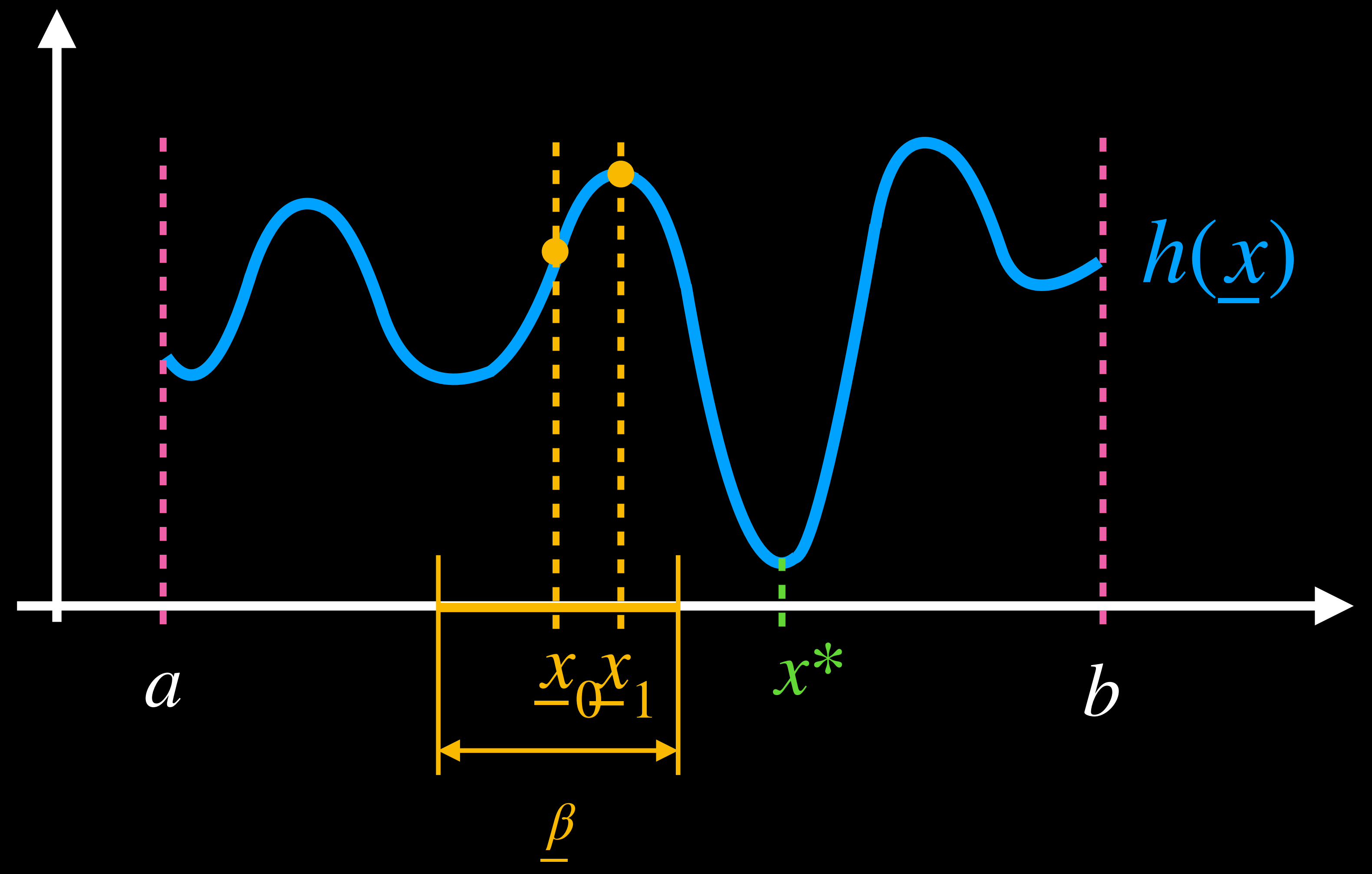

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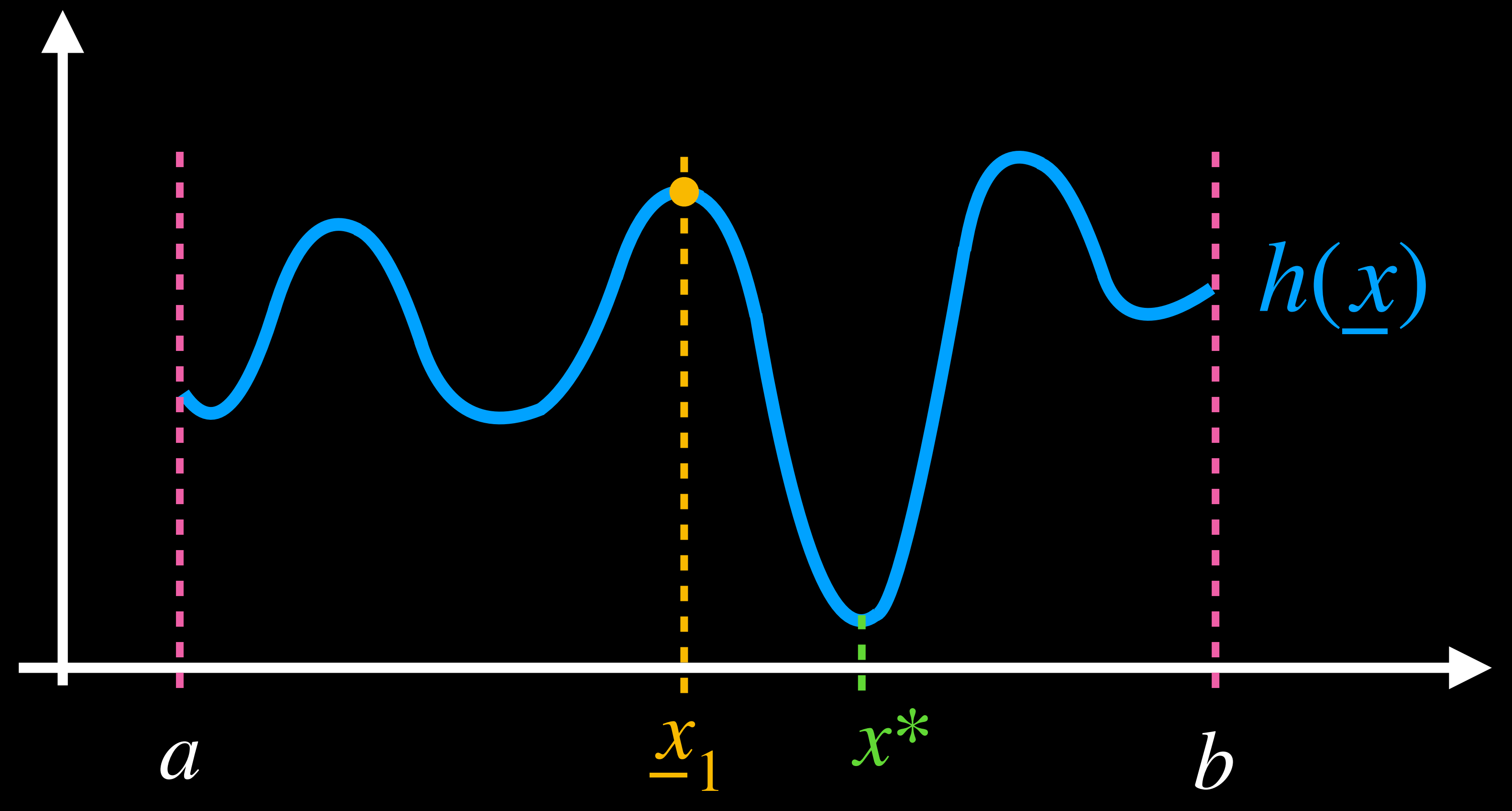

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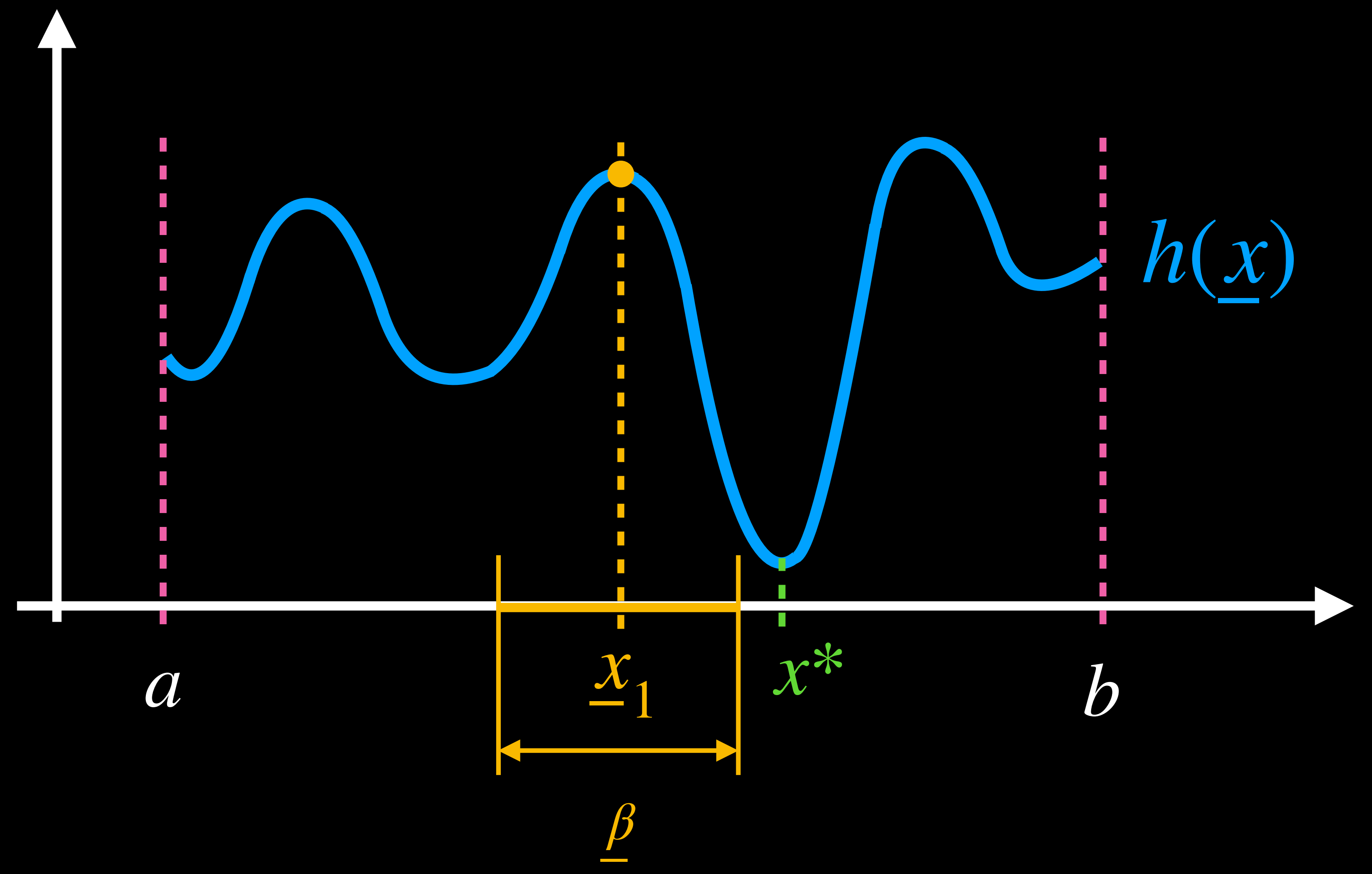

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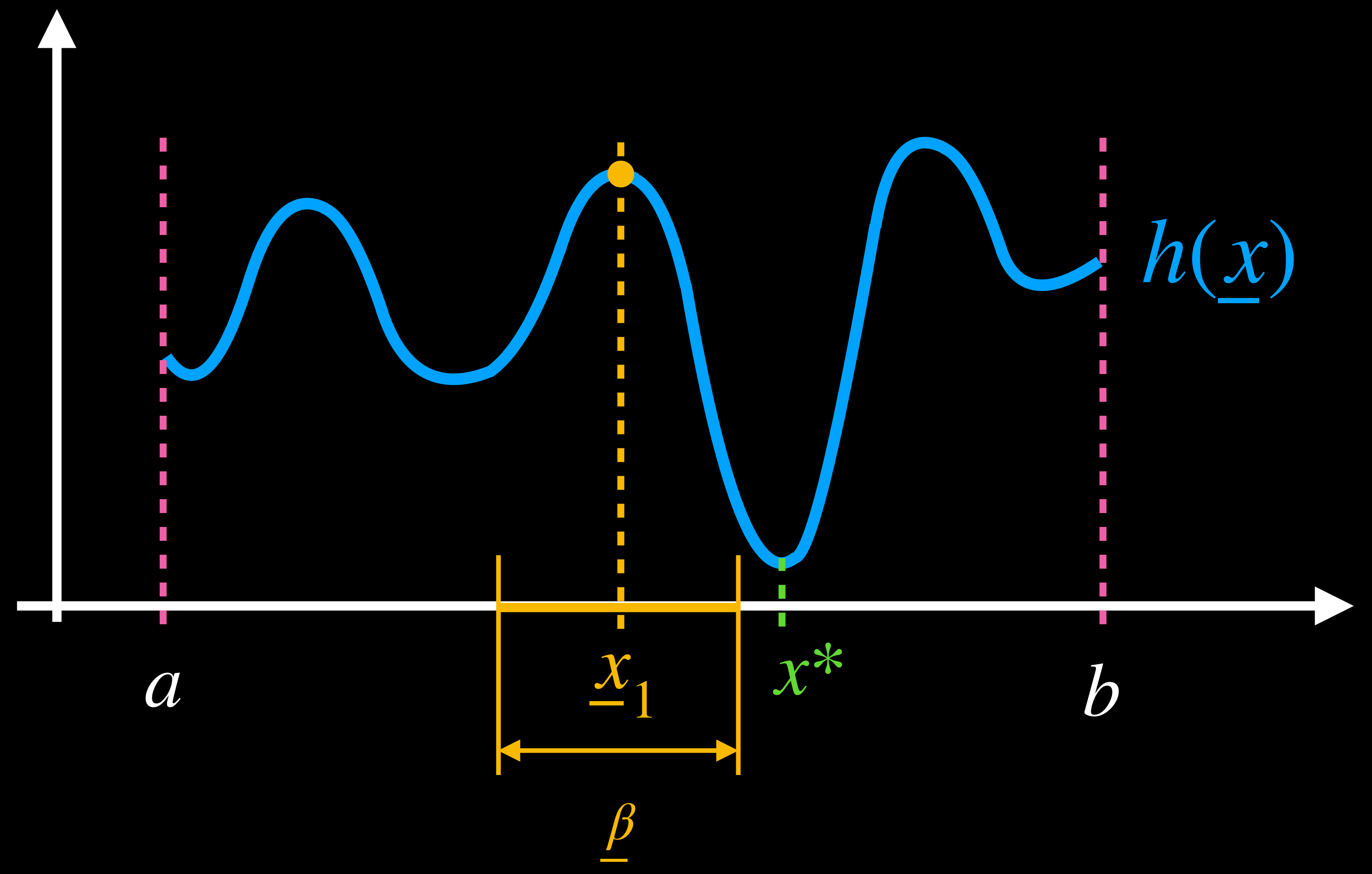

$\underline{X}_{t+1}$

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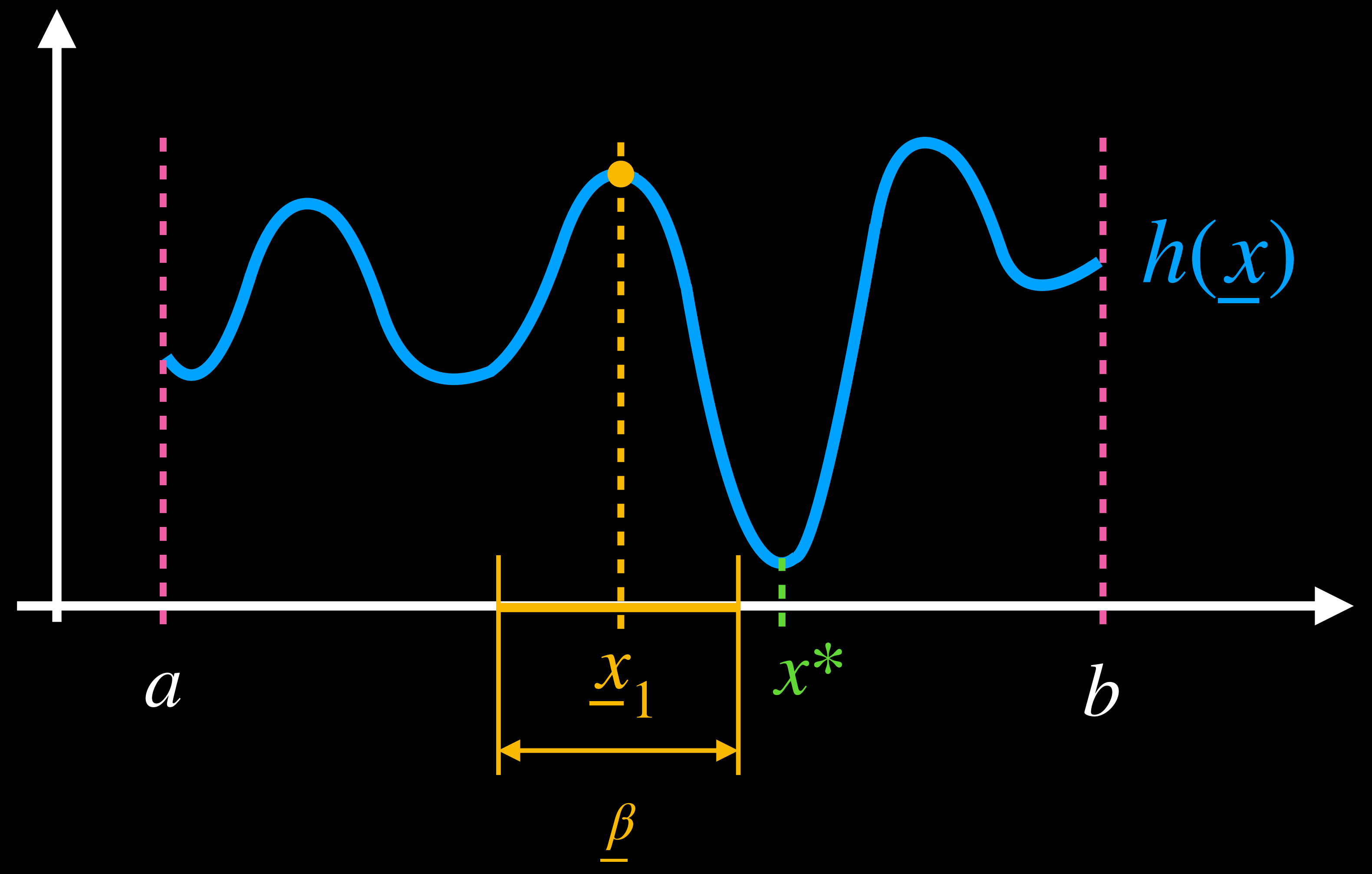

$\underline{X}_{t+1}$

$U_t \sim \mathcal{U}(0,1)$

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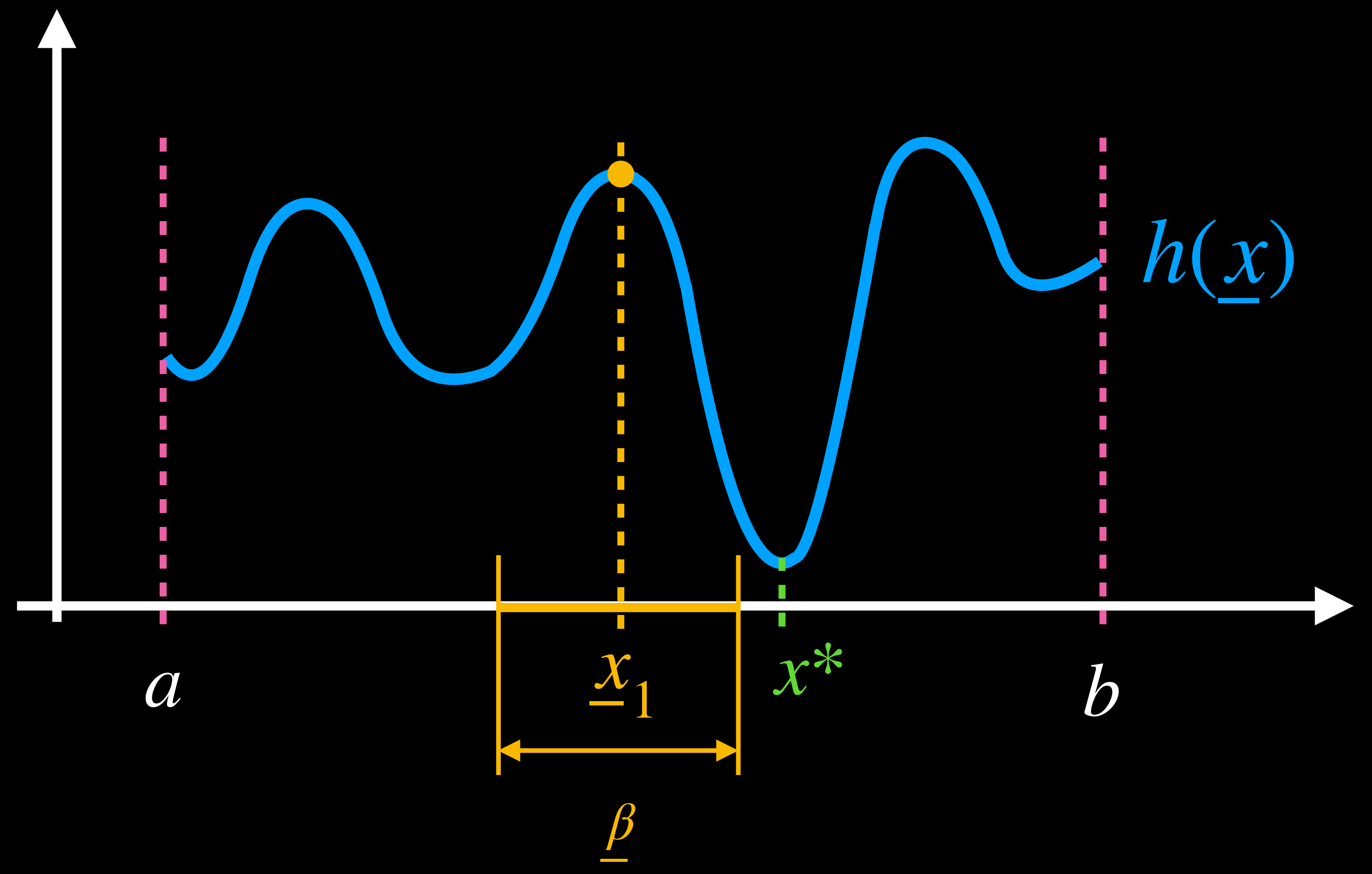

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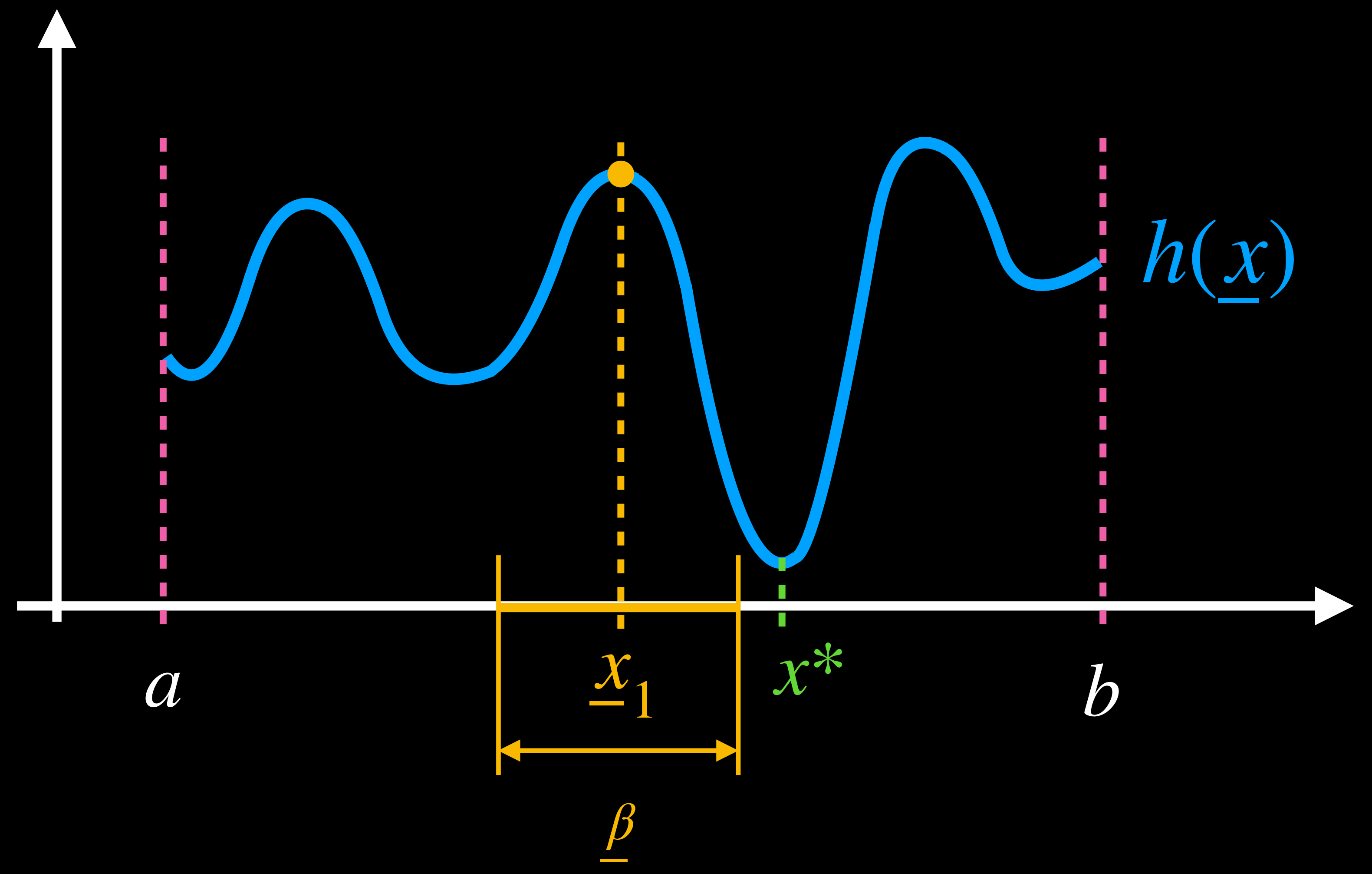

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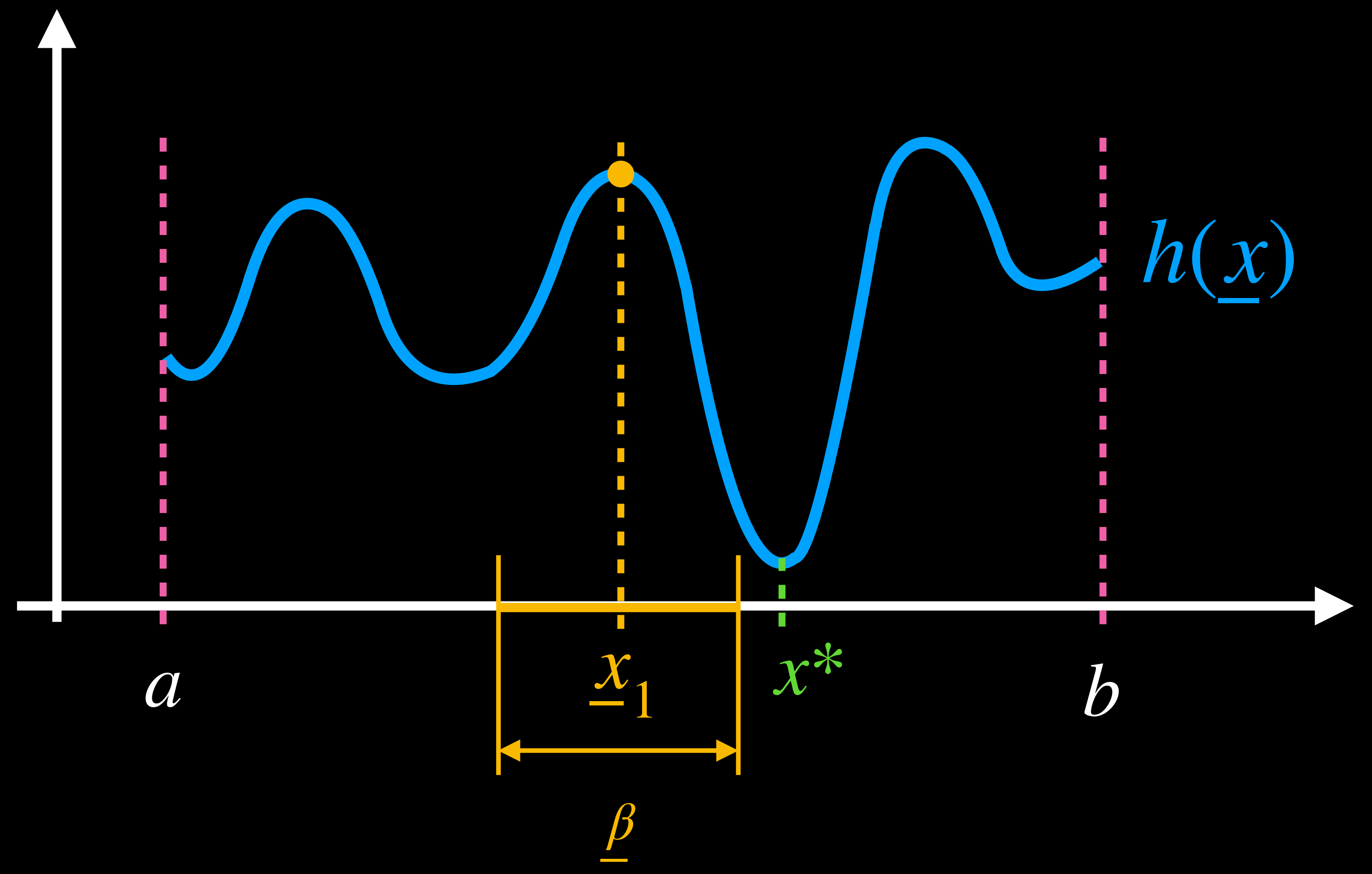

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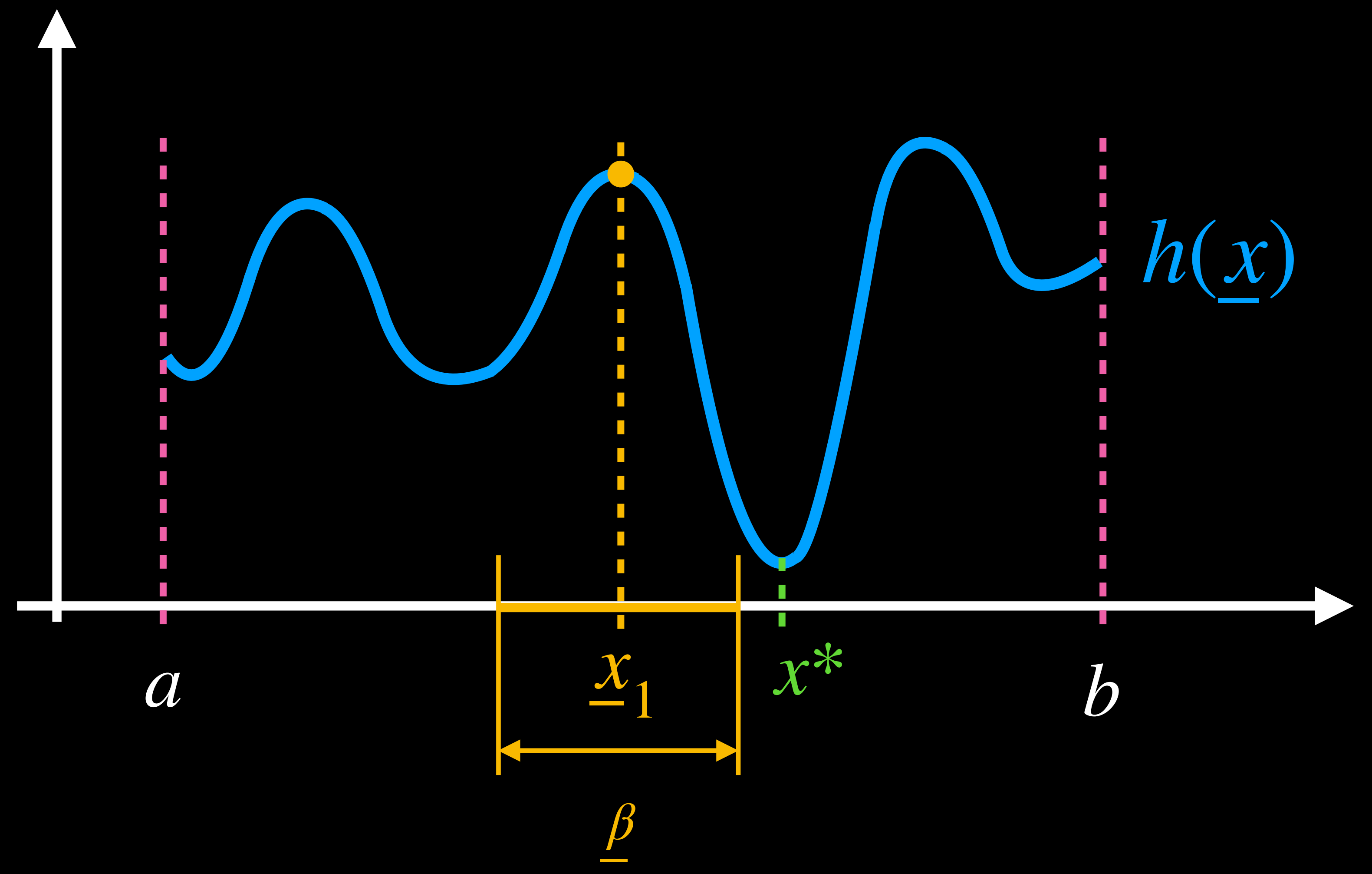

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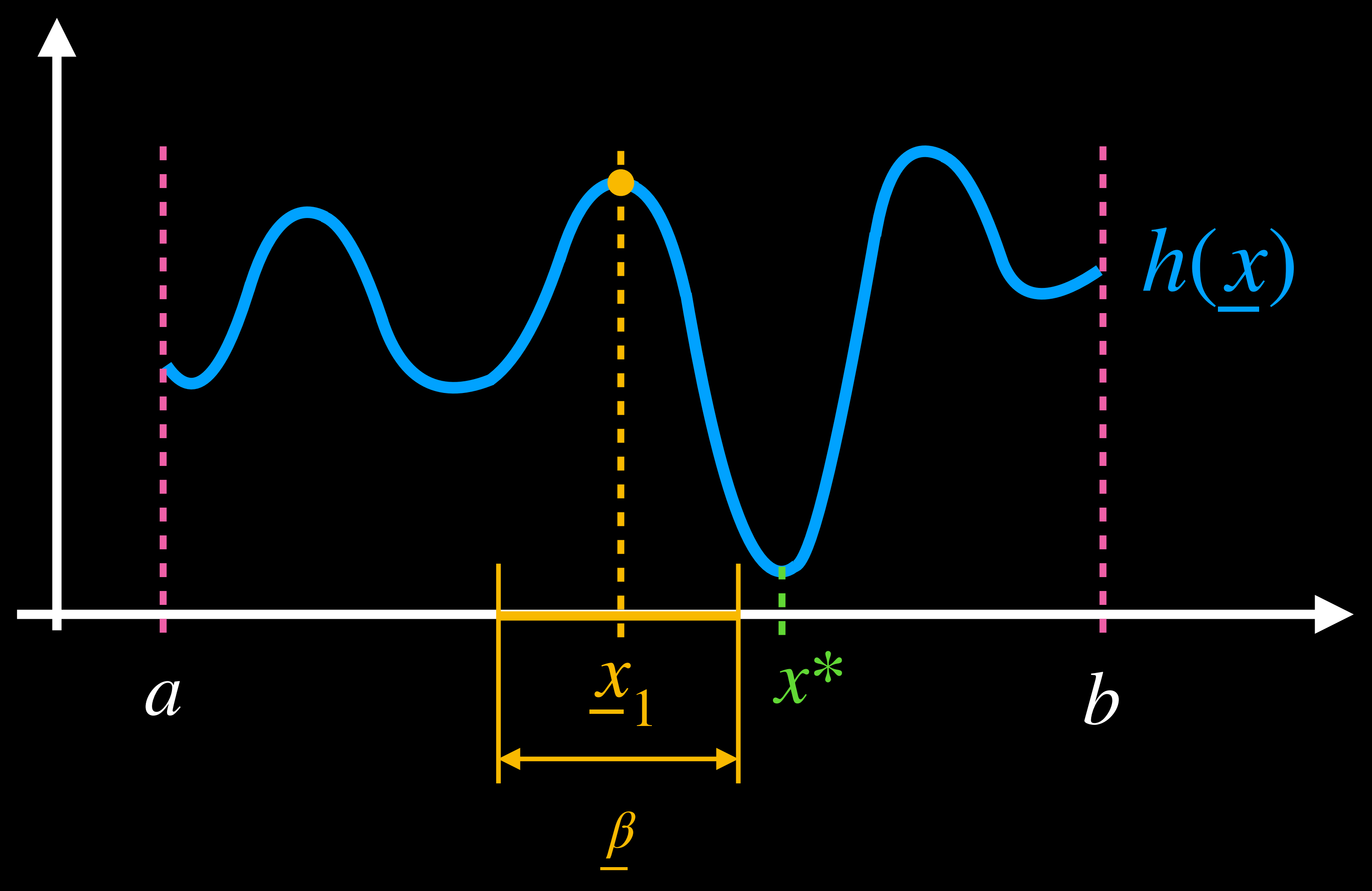

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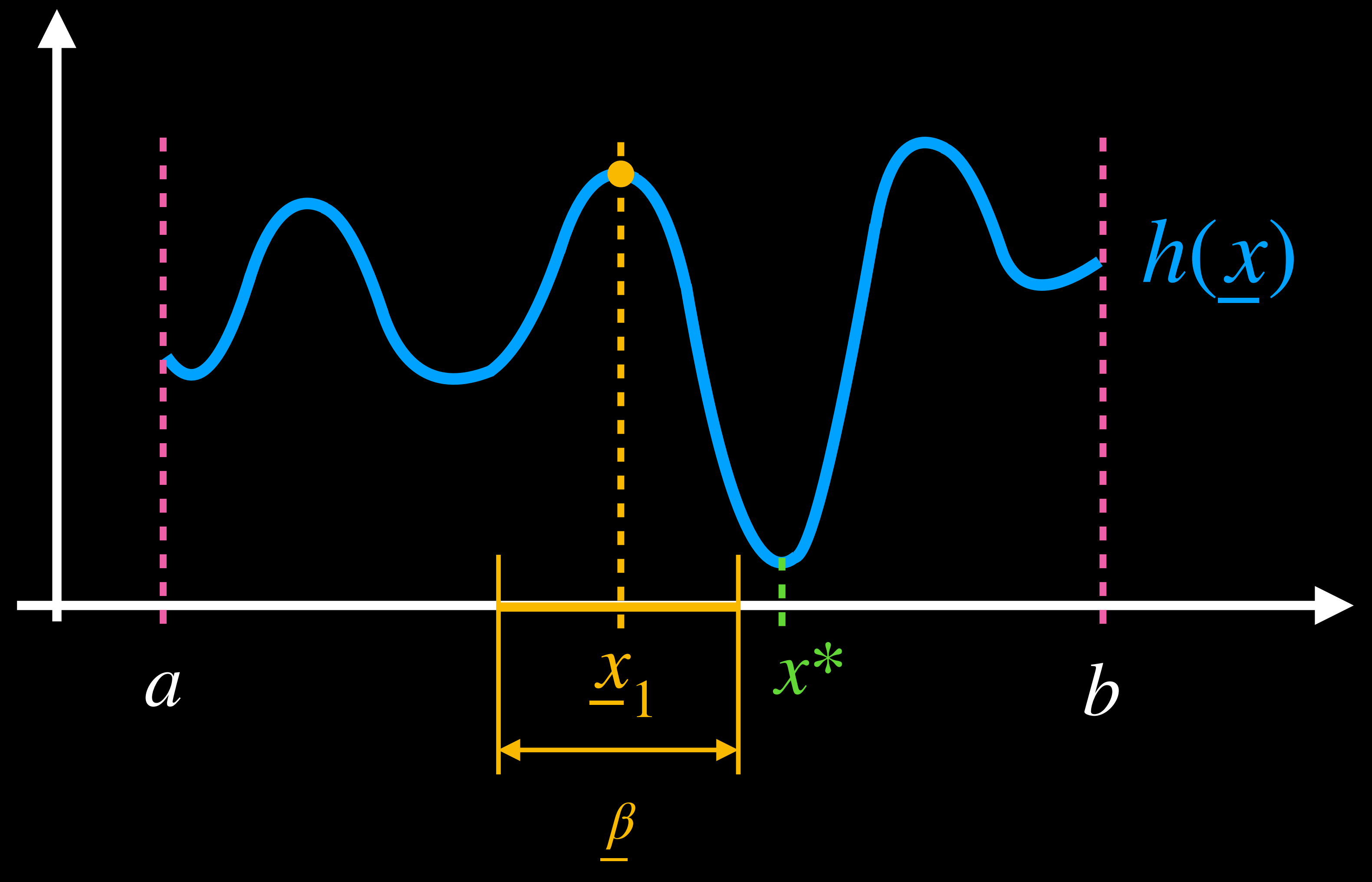

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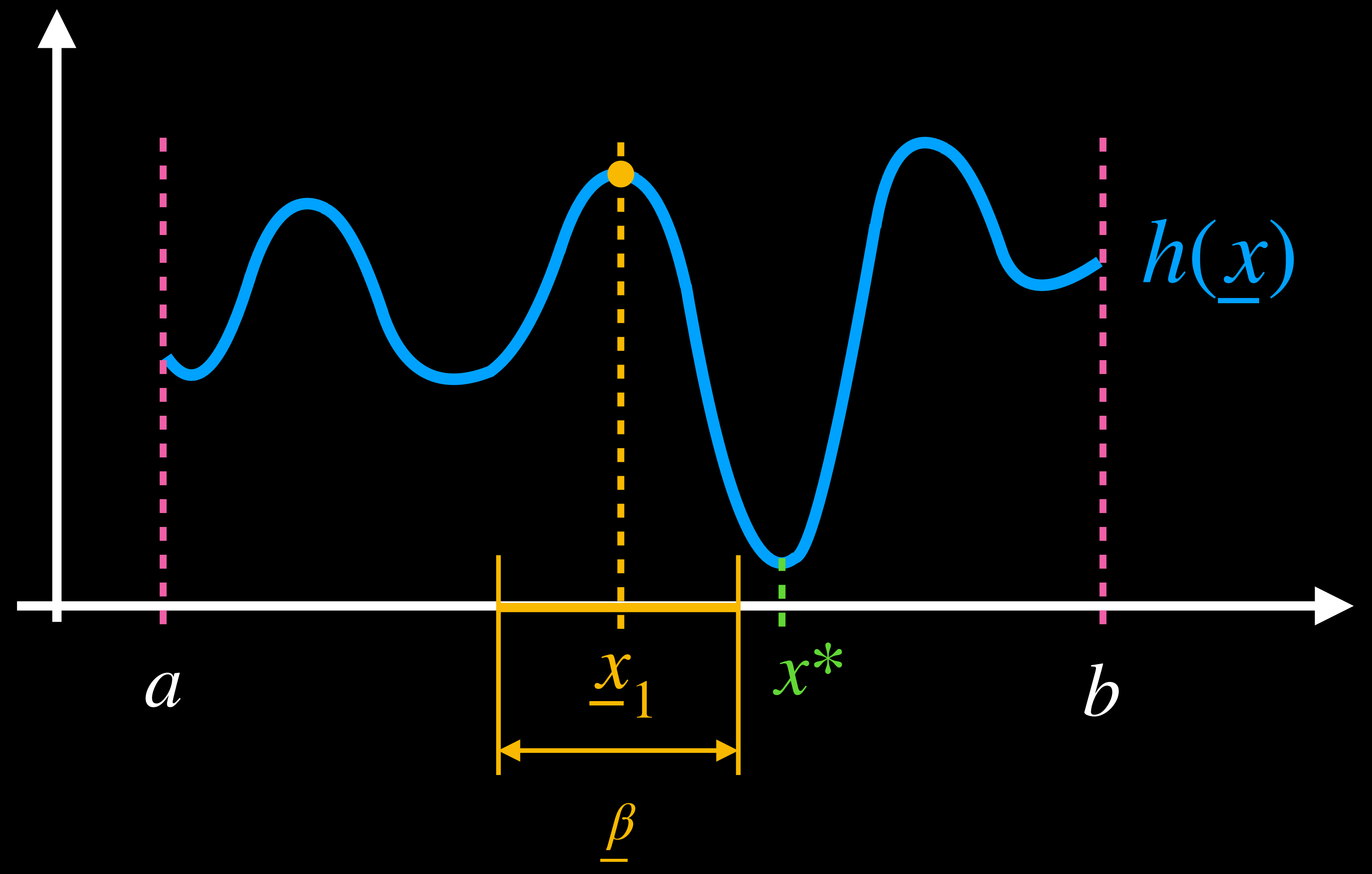
properties:

- simple

algorithm 4: simulated annealing



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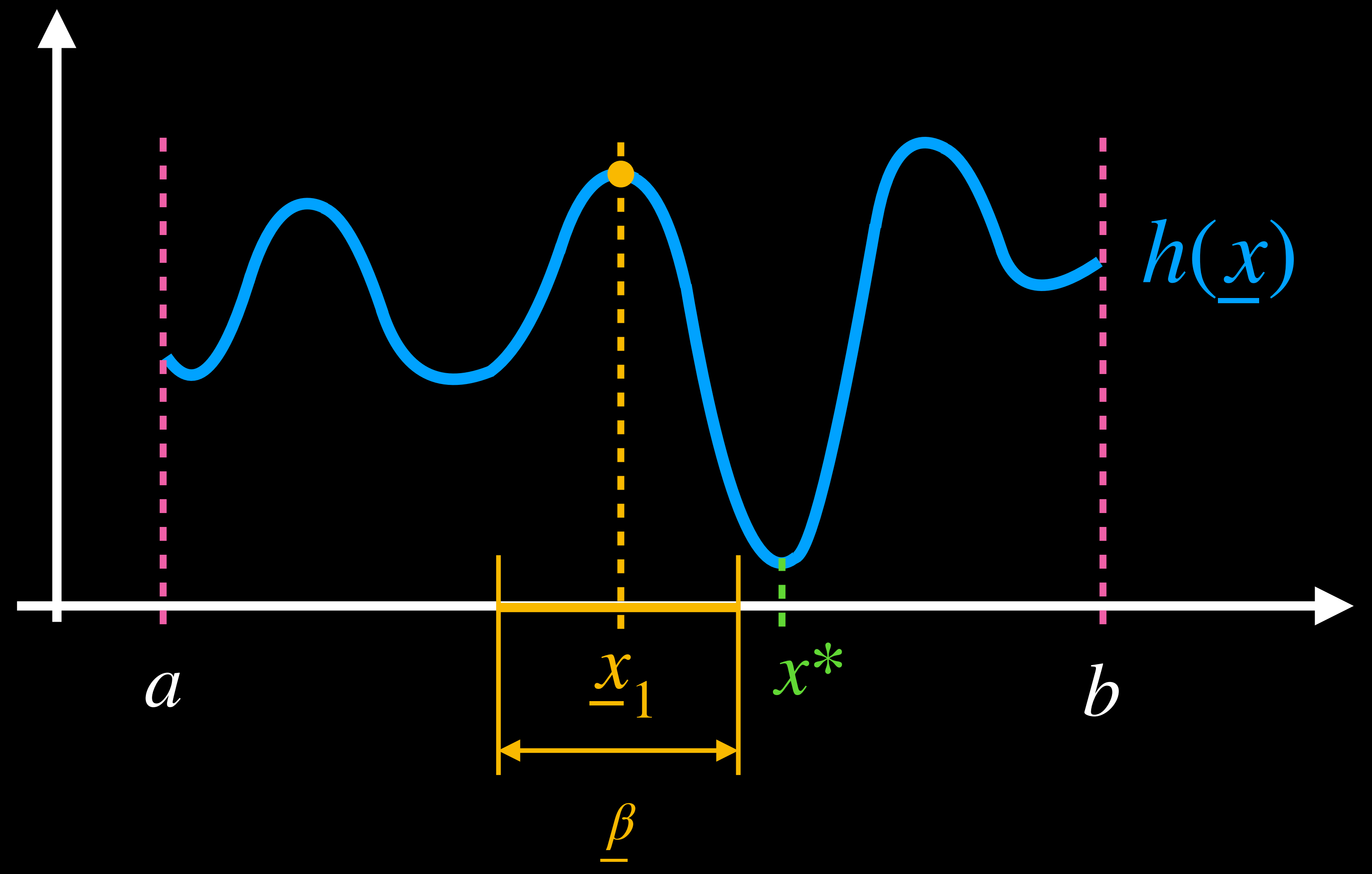

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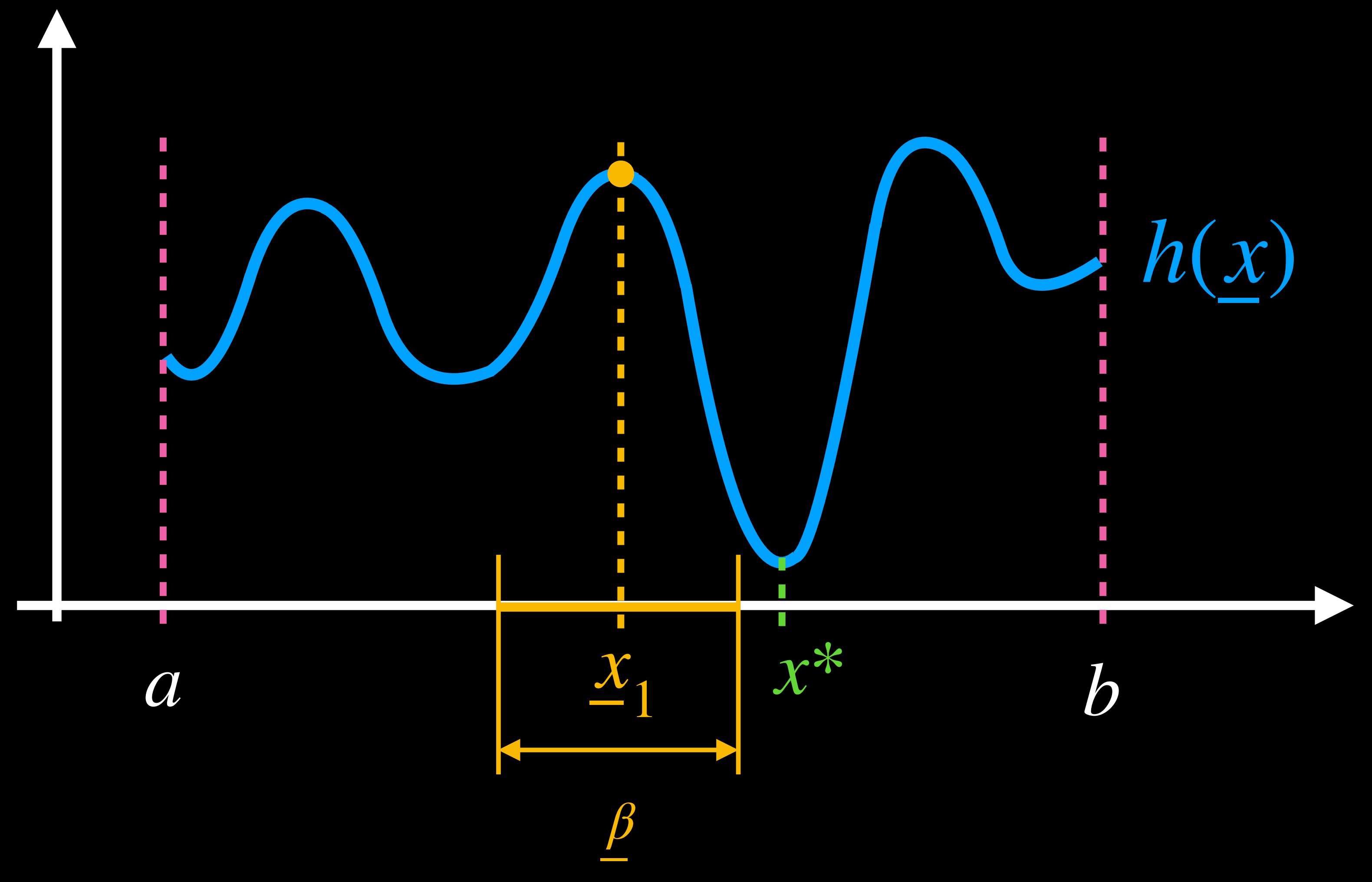

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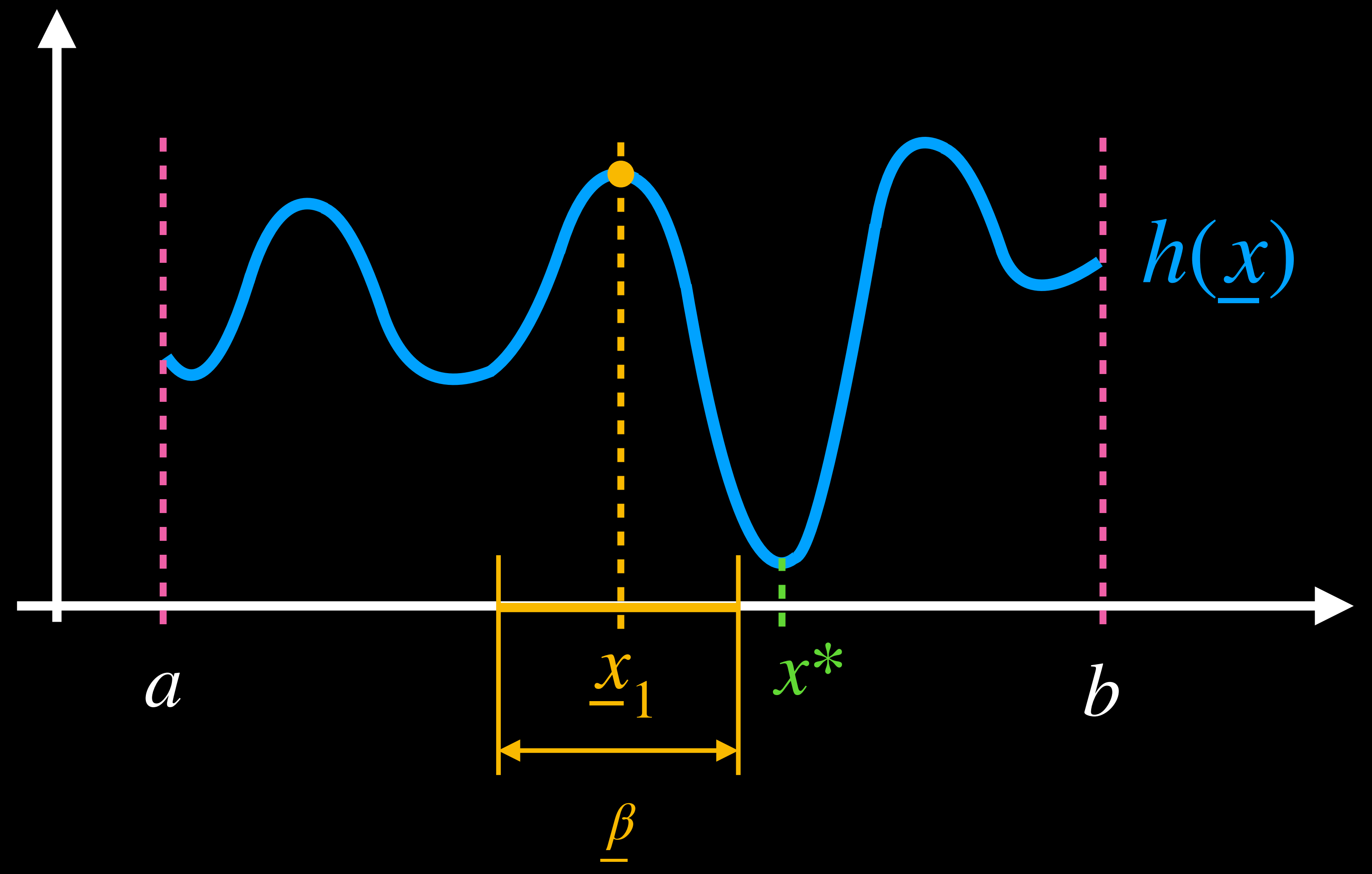

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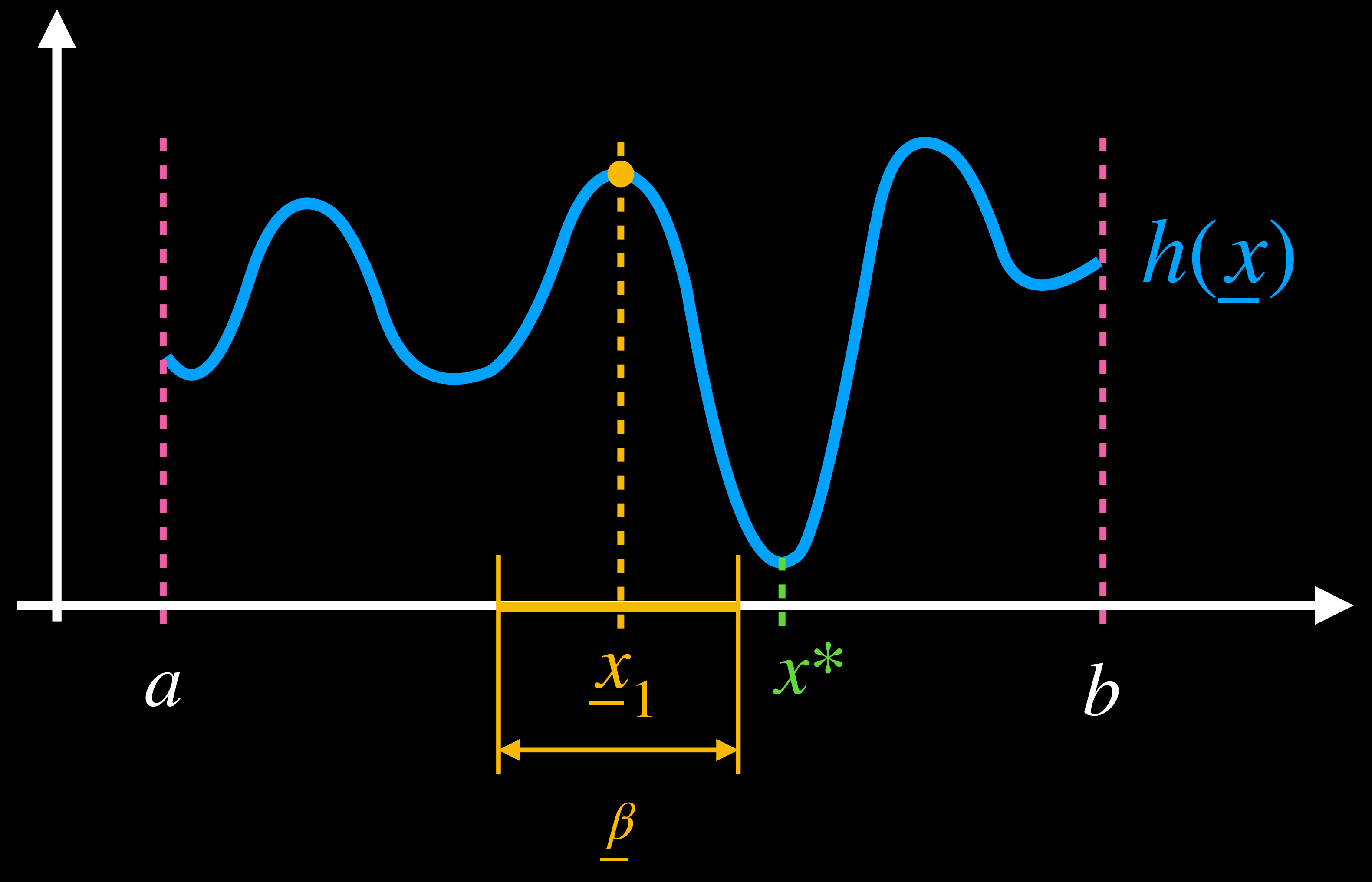

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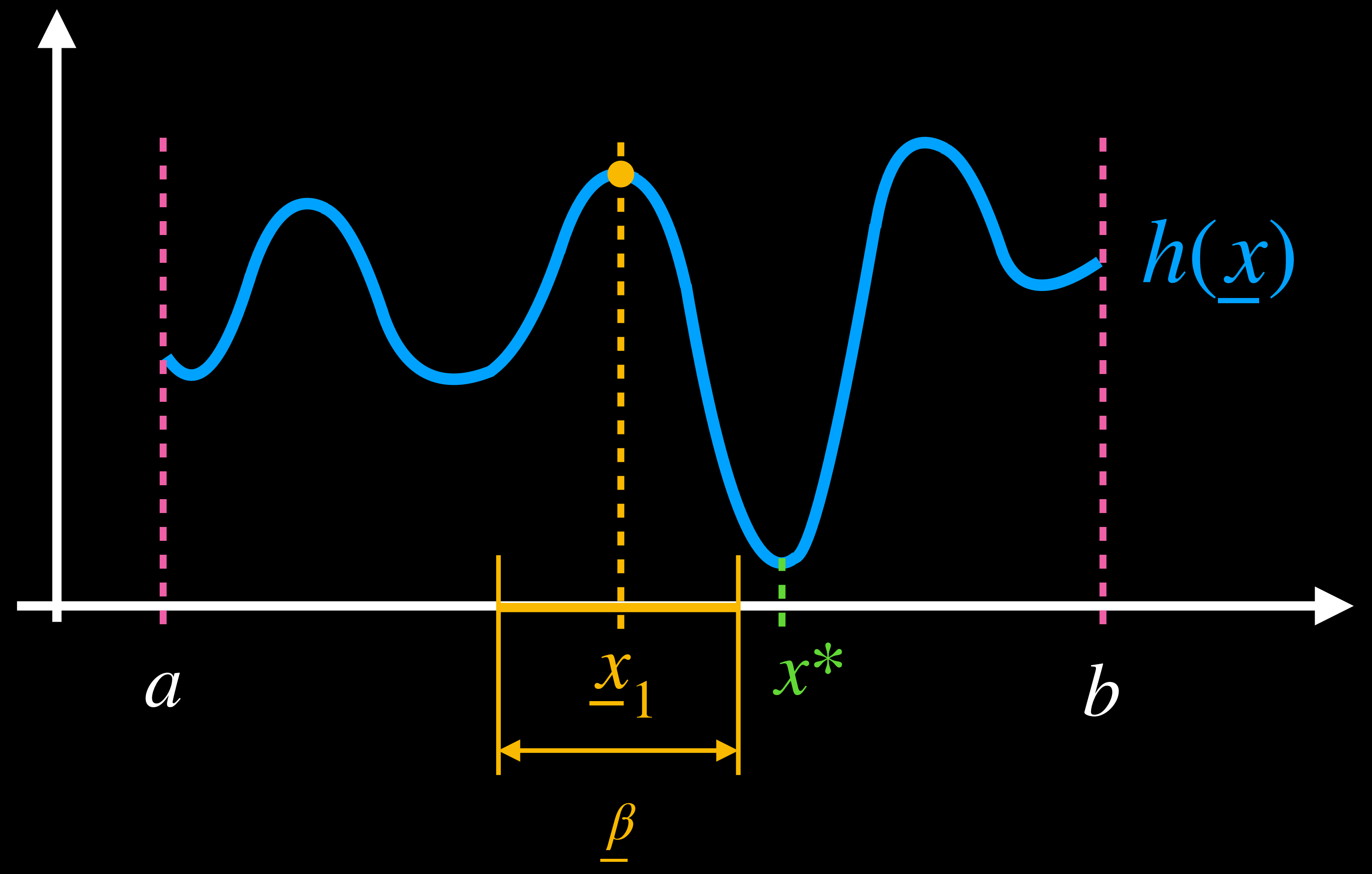

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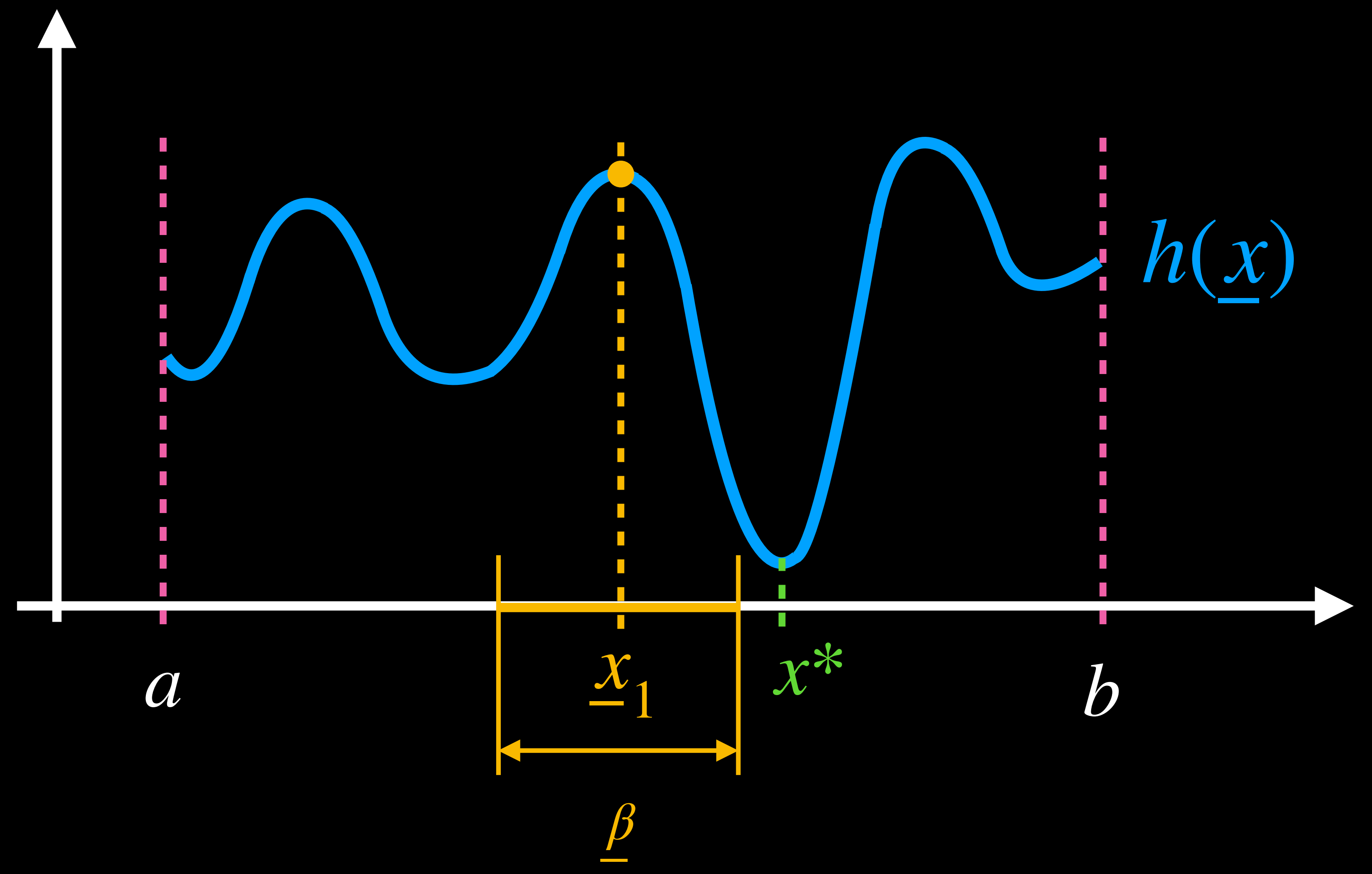

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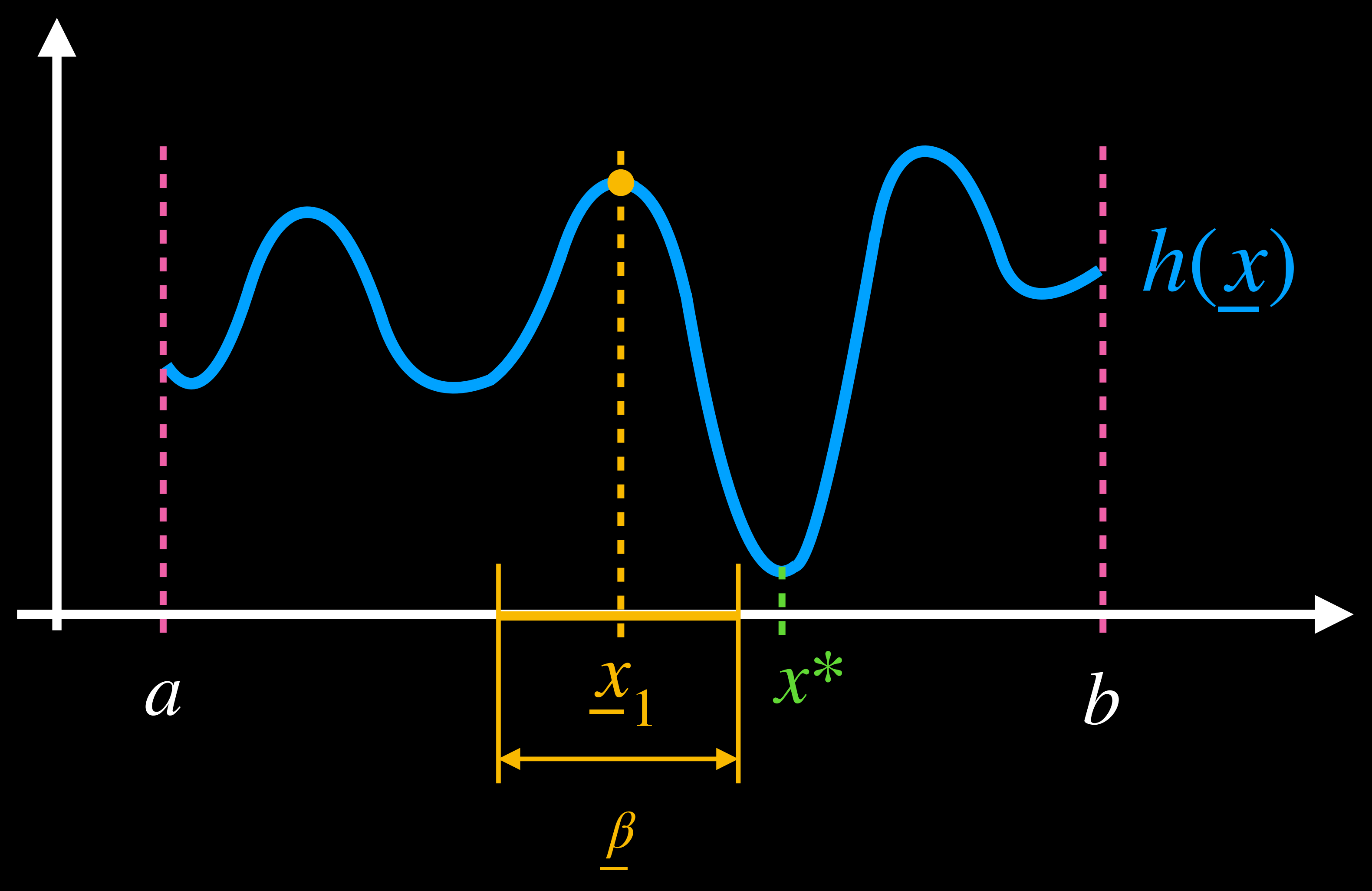

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properties:
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- convergence to a local minimum
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useful if:
- $\mathbb{S}$ is high-dimensional
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- h has a lot of local minima
- h is discrete

algorithm 5: evolutionary algorithms

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$\underline{Y}_t$

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(1+1)-ES: (ES = Evolution Strategy)

$$\underline{Y}_t = \underline{X}_t + \underline{N}_t$$

algorithm 5: evolutionary algorithms

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algorithm 5: evolutionary algorithms

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$\underline{Y}_t$ child

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$\underline{X}_t$ parent

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$\underline{Y}_t$ child
$\underline{X}_t$ parent
$\underline{N}_t$ mutation

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$\mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)$ multivariate Gaussian

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$\underline{\underline{\Sigma}}$ co-variance matrix

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algorithm 5: evolutionary algorithms

$$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \left(h(\underline{x})\right)$$

(1+1)-ES: (ES = Evolution Strategy)

$$\underline{Y}_t = \underline{X}_t + \underline{N}_t \qquad \underline{N}_t \sim \mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)$$

$$\underline{X}_{t+1} = \arg\min\left(h(\underline{Y}_t), h(\underline{X}_t)\right)$$

$\underline{Y}_t$ child

$\underline{X}_t$ parent

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(μ,λ)-ES & (μ+λ)-ES :

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recombination

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mutation

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(μ,λ)-ES & (μ+λ)-ES :

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$$\underline{Y}_{t,k} = \underline{Y}'_{t,k} + \underline{N}_{t,k} \qquad \underline{N}_{t,k} \sim \mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)$$ mutation

$$\left(\underline{X}_{t+1,1}, \ldots, \underline{X}_{t+1,\mu}\right) = selection_{(\mu,\lambda)\text{-ES}} \left(\underline{Y}_{t,1}, \ldots, \underline{Y}_{t,\lambda}\right)$$

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algorithm 5: evolutionary algorithms

$\underline{x}^* := \arg\min_{\underline{x}\in\mathbb{S}} \left(h(\underline{x})\right)$

$\underline{Y}_t$ child

$\underline{X}_t$ parent

$\underline{N}_t$ mutation

$\mathcal{N}\left(\underline{0},\underline{\underline{\Sigma}}\right)$ multivariate Gaussian

$\underline{\underline{\Sigma}}$ co-variance matrix

properties:

(1+1)-ES: (ES = Evolution Strategy)

$$\underline{Y}_t = \underline{X}_t + \underline{N}_t \qquad \underline{N}_t \sim \mathcal{N}\left(\underline{0},\underline{\underline{\Sigma}}\right)$$

$$\underline{X}_{t+1} = \arg\min\left(h(\underline{Y}_t), h(\underline{X}_t)\right)$$

(1+λ)-ES:

$$\underline{Y}_{t,k} = \underline{X}_t + \underline{N}_{t,k} \qquad \underline{N}_{t,k} \sim \mathcal{N}\left(\underline{0},\underline{\underline{\Sigma}}\right)$$

$$\underline{X}_{t+1} = \arg\min\left(h(\underline{Y}_{t,1}),\ldots,h(\underline{Y}_{t,\lambda}), h(\underline{X}_t)\right)$$

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recombination

$$\boxed{\underline{Y}_{t,k} = \underline{Y}'_{t,k} + \underline{N}_{t,k} \qquad \underline{N}_{t,k} \sim \mathcal{N}\left(\underline{0},\underline{\underline{\Sigma}}\right)}$$

mutation

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$$\left(\underline{X}_{t+1,1},\ldots,\underline{X}_{t+1,\mu}\right) = selection_{(\mu+\lambda)\text{-ES}}\left(\underline{Y}_{t,1},\ldots,\underline{Y}_{t,\lambda},\underline{X}_{t,1},\ldots,\underline{X}_{t,\mu}\right)$$

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- global convergence posible
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useful if:
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(μ,λ)-ES & (μ+λ)-ES :

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mutation

$$\left(\underline{X}_{t+1,1}, \ldots, \underline{X}_{t+1,\mu}\right) = \underset{(\mu,\lambda)\text{-ES}}{selection}\left(\underline{Y}_{t,1}, \ldots, \underline{Y}_{t,\lambda}\right)$$

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selection

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properties:
- global convergence posible
- (can go out of local minima)

useful if:
- $\mathbb{S}$ is high-dimensional
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(μ,λ)-ES & (μ+λ)-ES :

$$\boxed{\left(\underline{Y}'_{t,1}, \ldots, \underline{Y}'_{t,\lambda}\right) = recombination\left(\underline{X}_{t,1}, \ldots, \underline{X}_{t,\mu}\right)}$$

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(μ,λ)-ES & (μ+λ)-ES :

recombination

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improvement by
covariance matrix adaptation (CMA)

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improvement by
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$\underline{\underline{\Sigma}}_t$

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(μ,λ)-ES & (μ+λ)-ES :

recombination

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improvement by
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$$\underline{\underline{\Sigma}}_t$$

- scaling per dimension (different diagonal entries)

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- $\underline{X}_t$ parent
- $\underline{N}_t$ mutation
- $\mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)$ multivariate Gaussian
- $\underline{\underline{\Sigma}}$ co-variance matrix

(1+λ)-ES:

$$\underline{Y}_{t,k} = \underline{X}_t + \underline{N}_{t,k} \qquad \underline{N}_{t,k} \sim \mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)$$

$$\underline{X}_{t+1} = \arg\min\left(h(\underline{Y}_{t,1}), \ldots, h(\underline{Y}_{t,\lambda}), h(\underline{X}_t)\right)$$

(μ,λ)-ES & (μ+λ)-ES :

recombination

$$\boxed{\left(\underline{Y}'_{t,1}, \ldots, \underline{Y}'_{t,\lambda}\right) = recombination\left(\underline{X}_{t,1}, \ldots, \underline{X}_{t,\mu}\right)}$$

mutation

$$\boxed{\underline{Y}_{t,k} = \underline{Y}'_{t,k} + \underline{N}_{t,k} \qquad \underline{N}_{t,k} \sim \mathcal{N}\left(\underline{0}, \underline{\underline{\Sigma}}\right)}$$

selection

$$\left(\underline{X}_{t+1,1}, \ldots, \underline{X}_{t+1,\mu}\right) = selection_{(\mu,\lambda)\text{-ES}}\left(\underline{Y}_{t,1}, \ldots, \underline{Y}_{t,\lambda}\right)$$

$$\left(\underline{X}_{t+1,1}, \ldots, \underline{X}_{t+1,\mu}\right) = selection_{(\mu+\lambda)\text{-ES}}\left(\underline{Y}_{t,1}, \ldots, \underline{Y}_{t,\lambda}, \underline{X}_{t,1}, \ldots, \underline{X}_{t,\mu}\right)$$

properties:
- global convergence posible
- (can go out of local minima)

useful if:
- $\mathbb{S}$ is high-dimensional
- finding the global optimum is more important than finding the exact position of a (local/ global) optimum
- h has a lot of local minima
- h is discrete

improvement by
covariance matrix adaptation (CMA)

$\underline{\underline{\Sigma}}_t$

- scaling per dimension (different diagonal entries)
- correlations (off-diagonal entries)

end