```
VardP[Δ_] := Simplify[VardV + Δ^2 VardS + 2 Δ CovdSdV]

EdP[Δ_] := Simplify[dV + Δ dS - (V + Δ S) (Series[Exp[dt^2 r], {dt, 0, n}] - 1) /. Moments]

HedgeError[Δ_] :=
 Simplify[dV + Δ dS - (V + Δ S) (Series[Exp[dt^2 r], {dt, 0, n}] - 1) - EdP[Δ]]

dX = μ dt^2 + σ W dt;

n = 4;

EW[1] = 0; EW[2] = 1; Moments = Table[W^n → EW[n], {n, n, 1, -1}]
```

$$\{W^4 \to \text{EW}[4],\ W^3 \to \text{EW}[3],\ W^2 \to 1,\ W \to 0\}$$

```
dS = S (Series[Exp[dX], {dt, 0, n}] - 1);

EdS = Simplify[dS /. Moments];

EdS2 = Simplify[Series[Expand[Normal[dS^2]], {dt, 0, n + 1}] /. Moments];

VardS = Simplify[EdS2 - EdS^2];

dV = Series[V[t + dt^2, S + dS], {dt, 0, n}] - V[t, S];

EdV = Simplify[dV /. Moments];

EdV2 = Simplify[Series[Expand[Normal[dV^2]], {dt, 0, n + 1}] /. Moments];

VardV = Simplify[EdV2 - EdV^2];

CovdSdV = Simplify[(Series[Expand[Normal[dS dV]], {dt, 0, n + 1}] /. Moments) - EdS EdV];

Δ0 = -Simplify[CovdSdV / VardS]
```

$$-V^{(0,1)}[t, S] - \frac{1}{2}\left(S\,\sigma\,\text{EW}[3]\,V^{(0,2)}[t, S]\right) dt +$$
$$\left(-\frac{1}{4} S \left(4\,\mu + \sigma^2 \left(-1 - 2\,\text{EW}[3]^2 + 3\,\text{EW}[4]\right)\right) V^{(0,2)}[t, S] -\right.$$
$$\left.\frac{1}{6} S^2\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t, S] - V^{(1,1)}[t, S]\right) dt^2 -$$
$$\frac{1}{24}\left(S\,\sigma\,\text{EW}[3]\left(\left(24\,\mu + \sigma^2\left(7 + 12\,\text{EW}[3]^2 - 25\,\text{EW}[4]\right)\right) V^{(0,2)}[t, S] +\right.\right.$$
$$\left.\left.2\left(S\left(6\,\mu - \sigma^2\,(1 + 2\,\text{EW}[4])\right) V^{(0,3)}[t, S] + 6\,V^{(1,2)}[t, S]\right)\right)\right) dt^3 + O[dt]^4$$

```
ΔW = Δ0 /. EW[3] → 0
```

$$-V^{(0,1)}[t, S] +$$
$$\left(-\frac{1}{4} S \left(4\,\mu + \sigma^2\,(-1 + 3\,\text{EW}[4])\right) V^{(0,2)}[t, S] - \frac{1}{6} S^2\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t, S] - V^{(1,1)}[t, S]\right) dt^2 +$$
$$O[dt]^4$$

VardP[Δ0]

$$-\frac{1}{4}\left(S^4\,\sigma^4\left(1+\text{EW}[3]^2-\text{EW}[4]\right)V^{(0,2)}[t,S]^2\right)dt^4+$$
$$\frac{1}{12}S^2\,\sigma^3\,\text{EW}[3]\,V^{(0,2)}[t,S]\left(3\,S^2\,\sigma^2\left(1+\text{EW}[3]^2-3\,\text{EW}[4]\right)V^{(0,2)}[t,S]-\right.$$
$$\left.2\left(S^3\,\sigma^2\,(1+\text{EW}[4])\,V^{(0,3)}[t,S]-6\,V^{(1,0)}[t,S]\right)\right)dt^5+O[dt]^6$$

VardP[ΔW]

$$\frac{1}{4}S^4\,\sigma^4\,(-1+\text{EW}[4])\,V^{(0,2)}[t,S]^2\,dt^4+\frac{1}{12}S^2\,\sigma^3\,\text{EW}[3]\,V^{(0,2)}[t,S]$$
$$\left(3\,S^2\,\sigma^2\,(1-3\,\text{EW}[4])\,V^{(0,2)}[t,S]-2\left(S^3\,\sigma^2\,(1+\text{EW}[4])\,V^{(0,3)}[t,S]-6\,V^{(1,0)}[t,S]\right)\right)$$
$$dt^5+O[dt]^6$$

VardP[ΔW /. dt → 0]

$$\frac{1}{4}S^4\,\sigma^4\,(-1+\text{EW}[4])\,V^{(0,2)}[t,S]^2\,dt^4+\frac{1}{6}S^2\,\sigma^3\,\text{EW}[3]\,V^{(0,2)}[t,S]$$
$$\left(6\,S^2\,\mu\,V^{(0,2)}[t,S]-S^3\,\sigma^2\,V^{(0,3)}[t,S]+6\left(V^{(1,0)}[t,S]+S\,V^{(1,1)}[t,S]\right)\right)dt^5+O[dt]^6$$

EdP[Δ0]

$$\left(-r\,V+r\,S\,V^{(0,1)}[t,S]+\frac{1}{2}S^2\,\sigma^2\,V^{(0,2)}[t,S]+V^{(1,0)}[t,S]\right)dt^2+$$
$$\frac{1}{12}S^2\,\sigma\,\text{EW}[3]\left(\left(6\,r-3\left(2\,\mu+\sigma^2\right)\right)V^{(0,2)}[t,S]+2\,S\,\sigma^2\,V^{(0,3)}[t,S]\right)dt^3+$$
$$\frac{1}{24}\left(-12\,r^2\,V+12\,r^2\,S\,V^{(0,1)}[t,S]+S^2\left(-12\,\mu^2+\sigma^4\left(6+4\,\text{EW}[3]^2-9\,\text{EW}[4]\right)+\right.\right.$$
$$\left.\left.6\,\mu\,\sigma^2\left(1+2\,\text{EW}[3]^2-3\,\text{EW}[4]\right)+6\,r\left(4\,\mu+\sigma^2\left(-1-2\,\text{EW}[3]^2+3\,\text{EW}[4]\right)\right)\right)V^{(0,2)}[t,S]+$$
$$12\,S^3\,\mu\,\sigma^2\,V^{(0,3)}[t,S]+6\,S^3\,\sigma^4\,V^{(0,3)}[t,S]+4\,r\,S^3\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]-$$
$$4\,S^3\,\mu\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]-2\,S^3\,\sigma^4\,\text{EW}[4]\,V^{(0,3)}[t,S]+S^4\,\sigma^4\,\text{EW}[4]\,V^{(0,4)}[t,S]+$$
$$24\,r\,S\,V^{(1,1)}[t,S]+12\,S^2\,\sigma^2\,V^{(1,2)}[t,S]+12\,V^{(2,0)}[t,S]\right)dt^4+O[dt]^5$$

EdP[ΔW]

$$\left(-r\,V+r\,S\,V^{(0,1)}[t,S]+\frac{1}{2}S^2\,\sigma^2\,V^{(0,2)}[t,S]+V^{(1,0)}[t,S]\right)dt^2+$$
$$\frac{1}{6}S^3\,\sigma^3\,\text{EW}[3]\,V^{(0,3)}[t,S]\,dt^3+\frac{1}{24}\left(-12\,r^2\,V+12\,r^2\,S\,V^{(0,1)}[t,S]+\right.$$
$$\left.3\,S^2\left(-\left(2\,\mu+\sigma^2\right)\left(2\,\mu+\sigma^2\,(-2+3\,\text{EW}[4])\right)+r\left(8\,\mu+2\,\sigma^2\,(-1+3\,\text{EW}[4])\right)\right)V^{(0,2)}[t,S]+\right.$$
$$12\,S^3\,\mu\,\sigma^2\,V^{(0,3)}[t,S]+6\,S^3\,\sigma^4\,V^{(0,3)}[t,S]+4\,r\,S^3\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]-$$
$$4\,S^3\,\mu\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]-2\,S^3\,\sigma^4\,\text{EW}[4]\,V^{(0,3)}[t,S]+S^4\,\sigma^4\,\text{EW}[4]\,V^{(0,4)}[t,S]+$$
$$24\,r\,S\,V^{(1,1)}[t,S]+12\,S^2\,\sigma^2\,V^{(1,2)}[t,S]+12\,V^{(2,0)}[t,S]\right)dt^4+O[dt]^5$$

```
EdP0 =
 Simplify[dV + Δ0 dS - (V + Δ0 S) (Series[Exp[dt^2 r], {dt, 0, n}] - 1) /. Moments] / dt^2
```

$$\left(-r\,V + r\,S\,V^{(0,1)}[t,S] + \frac{1}{2}\,S^2\,\sigma^2\,V^{(0,2)}[t,S] + V^{(1,0)}[t,S]\right) +$$
$$\frac{1}{12}\,S^2\,\sigma\,EW[3]\,\left(\left(6\,r - 3\left(2\,\mu + \sigma^2\right)\right)\,V^{(0,2)}[t,S] + 2\,S\,\sigma^2\,V^{(0,3)}[t,S]\right)\,dt +$$
$$\frac{1}{24}\,\left(-12\,r^2\,V + 12\,r^2\,S\,V^{(0,1)}[t,S] + S^2\,\left(-12\,\mu^2 + \sigma^4\left(6 + 4\,EW[3]^2 - 9\,EW[4]\right) + \right.\right.$$
$$\left.6\,\mu\,\sigma^2\left(1 + 2\,EW[3]^2 - 3\,EW[4]\right) + 6\,r\,\left(4\,\mu + \sigma^2\left(-1 - 2\,EW[3]^2 + 3\,EW[4]\right)\right)\right)\,V^{(0,2)}[t,S] +$$
$$12\,S^3\,\mu\,\sigma^2\,V^{(0,3)}[t,S] + 6\,S^3\,\sigma^4\,V^{(0,3)}[t,S] + 4\,r\,S^3\,\sigma^2\,EW[4]\,V^{(0,3)}[t,S] -$$
$$4\,S^3\,\mu\,\sigma^2\,EW[4]\,V^{(0,3)}[t,S] - 2\,S^3\,\sigma^4\,EW[4]\,V^{(0,3)}[t,S] + S^4\,\sigma^4\,EW[4]\,V^{(0,4)}[t,S] +$$
$$\left.24\,r\,S\,V^{(1,1)}[t,S] + 12\,S^2\,\sigma^2\,V^{(1,2)}[t,S] + 12\,V^{(2,0)}[t,S]\right)\,dt^2 + O[dt]^3$$