```
Exit[]
```

```
BS[s_, SK_, r_, T_] := SK CDF[NormalDistribution[], d[SK, s, r, T]] -
  Exp[-r T] CDF[NormalDistribution[], d[SK, s, r, T] - s Sqrt[T]];
d[sk_, s_, r_, T_] := (Log[sk] + (r + s ^ 2 / 2) T) / s / Sqrt[T]
```

```
Preis[Startkapital_, Gewinnschwelle_, Sigma_, Laufzeit_] :=
  Gewinnschwelle * BS[Sigma, Startkapital / Gewinnschwelle, 0.0, Laufzeit];
D[Preis[p, G, s, T], p]
```

$$G\left(\frac{e^{-\frac{\left(\left(0.+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2 s^2 T}}}{G\sqrt{2\pi}\, s\sqrt{T}}-\frac{e^{0.\,T-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(0.+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}}{p\sqrt{2\pi}\, s\sqrt{T}}+\frac{1+\mathrm{Erf}\left[\frac{\left(0.+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{\sqrt{2}\, s\sqrt{T}}\right]}{2G}\right)$$

```
Delta[p_, G_, s_, T_] :=
```

$$G\left(\frac{e^{-\frac{\left(\left(0.\grave{}+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2 s^2 T}}}{G\sqrt{2\pi}\, s\sqrt{T}}-\frac{e^{0.\grave{}\,T-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(0.\grave{}+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}}{p\sqrt{2\pi}\, s\sqrt{T}}+\frac{1+\mathrm{Erf}\left[\frac{\left(0.\grave{}+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{\sqrt{2}\, s\sqrt{T}}\right]}{2G}\right)$$

```
P = 80; G = 100; s = 0.2; T = 0.25; Kundenposition = 80;
Print["Preis: ", Preis[100, 100, 0.2, 0.25]];
Print["Hedging position: ", Kundenposition * /. p → P];
```

Preis: 3.98776

Hedging position: 1.16605

Processes (real-world, non-risk-neutral)

```
$Assumptions = dt ^ 2 == 0 && dt * dW == 0 && dW ^ 2 == dt && S > 0 && M > 0 && s > 0;
```

```
dS = α S dt + σ S dW; (*Aktie*)
dP = r (P - q * P) dt + q P / S dS; (*Kundenportfolio mit Zins r*)
 dX = Δ dS + r (X - Δ S) dt;(*Heding portfolio*)
Δ = q D[V[P, t], P];(*Heding rule*)
```

```
dLogP = Simplify[dP / P - 1 / 2 dP ^ 2 / P ^ 2](*LogKundenportfolio mit Zins r*)
Simplify[dX]
```

$$dW\, q\, \sigma + dt\left(r - q\, r + q\, \alpha - \frac{q^2\sigma^2}{2}\right)$$

$$dt\, r\, X + q\, S\, (dt\, (-r+\alpha) + dW\, \sigma)\, V^{(1,0)}[P, t]$$

Hedging Simulation:

```
P0 = 100; M = 100; S0 = 100;
σ = 0.3;(*Volaitlität*)
r = 0.04;(*risk-free Zinssatz*)
T = 1 / 365;(*Laufzeit in jahren*)
α = 0.1;(*Stock drift*)
K = 20;(*Hedges täglich*)
nt = Ceiling[365 T ] K; dt = N[T / nt]
n = 1;(*MonteCarlo Durchläufe*)

(*zufälliger Kunde*)qk = RandomReal[{-1, 1}, {nt / K}];

dW = RandomReal[NormalDistribution[], {nt n}] Sqrt[dt];
Timing[
 PE = 0; PV = 0;

 (*MonteCarlo Loop*)
 For[j = 0, j < n, j++,

  P = Log[P0]; W = 0; S = S0; s = {S0}; p = {P0}; X = Preis[P0, M, σ, T];

  (*Time loop*)
  For[i = 1, i < nt + 1, i++,

   W += dW[[i]]; (*Brownian Motion*)
   dS = Exp[(α - σ ^ 2 / 2) i dt + σ W] S0 - S; (*Stock price Increment*)
   q = qk[[Ceiling[i / K]]];
   X += dt r X + q S (dt (-r + α) + dW[[i]] σ) Delta[P, M, σ, T - dt * i];
   (*new Hedgingportfolio*)
   P += dW[[i]] q σ + dt (r + q (-r + α - q σ^2/2)); (*new Portfolio*)
   S += dS; (*new Stockprice*)

   AppendTo[s, S];
   AppendTo[p, Exp[P]];
  ]
  (*PE+=Max[Exp[P]*P0-M,0];PV+=Max[Exp[P]*P0-M,0]^2;{"Mean:",
   Exp[-r T]PE/n,"2 StD of Mean:",2 Sqrt[Exp[-2r T]/n/(n-1)(PV-PE^2/n)]}*)
 ];]
Print["Aktie(rot), Kunde (blau)"]; ListLinePlot[{p, s}, InterpolationOrder → 1]
Print["Kundenposition"]
ListLinePlot[Table[qk[[Ceiling[i / K]]], {i, 1, nt}], InterpolationOrder → 0]
Print["Auszahlung (const), Hedgingportfolio"]; ListLinePlot[
 {{0, Max[p[[nt]] - M, 0]}, {nt, Max[p[[nt]] - M, 0]}}, InterpolationOrder → 1]
```

0.000136986

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

∞::indet : Indeterminate expression $e^{\text{ComplexInfinity}}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

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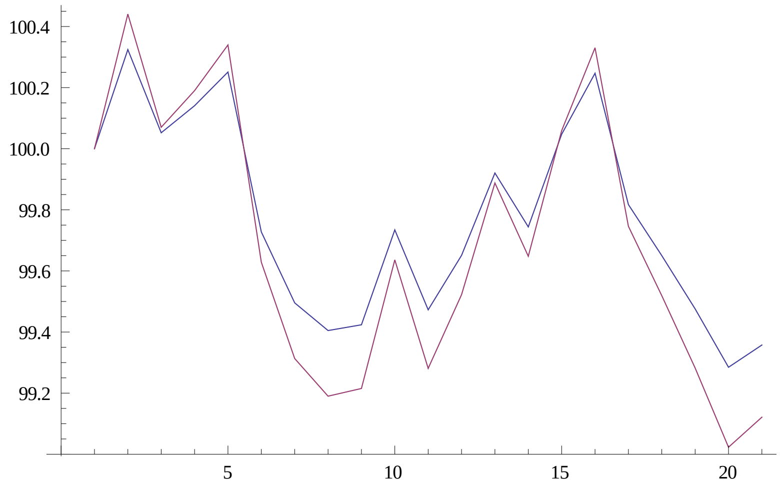
General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

∞::indet : Indeterminate expression $e^{\text{ComplexInfinity}}$ encountered. ≫

{0., Null}

Aktie(rot), Kunde (blau)



Kundenposition