```
G = SparseArray[{
    {1, 1, 1} → 1, {1, 2, 2} → 1, {1, 3, 3} → -1, {1, 4, 4} → -1,
    {2, 4, 1} → -1, {2, 3, 2} → -1, {2, 2, 3} → 1, {2, 1, 4} → 1, {3, 3, 2} → I,
    {3, 4, 1} → -I, {3, 1, 4} → -I, {3, 2, 3} → I, {4, 3, 1} → -1,
    {4, 1, 3} → 1, {4, 2, 4} → -1, {4, 4, 2} → 1}]; G[[3]] // MatrixForm
G5 = I G[[1]].G[[2]].G[[3]].G[[4]]; G[[1]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

```
e = Sqrt[p^2 + m^2];

w = {{1, 0, p / (e + m), 0}, {0, 1, 0, -p / (e + m)},
     {+p / (e + m), 0, -1, 0}, {0, -p / (e + m), 0, 1}} * Sqrt[(e + m)];

W = Join[Table[(G[[1]] * e - G[[4]] * p + IdentityMatrix[4] * m).
     (w[[i]] / Sqrt[m] /. p → 0 /. e → 0), {i, 2}],
   Table[(G[[1]] * e - G[[4]] * p - IdentityMatrix[4] * m).
     (w[[i]] / Sqrt[m] /. p → 0 /. e → 0), {i, 3, 4}]]
```

```
{{e + m, 0, p, 0}, {0, e + m, 0, -p}, {p, 0, e + m, 0}, {0, p, 0, -e - m}}
```

```
p =.; e =.

Simplify[I * G[[3]].G[[1]].W[[1]].G[[1]]] - W[[4]]
```

```
{0, 0, 0, 0}
```

```
$Assumptions = -s[1]^2 + s[2]^2 + s[3]^2 + s[4]^2 == 1
```

```
-s[1]^2 + s[2]^2 + s[3]^2 + s[4]^2 == 1
```

```
spin[a_] := (IdentityMatrix[4] + a G5.G[[4]]) / 2;
p = (IdentityMatrix[4] + G[[1]]) / 2; A = Table[a[i, j], {i, 1, 4}, {j, 1, 4}];

Simplify[spin[1].p.A.p.spin[-1]] // MatrixForm
Simplify[spin[-1].p.A.p.spin[1]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ a[2,1] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & a[1,2] & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Det[IdentityMatrix[4] + G5.G[[4]]] // MatrixForm
```

```
0
```

```
p.spin[1] - spin[1].p
```

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
p.A.p
```

```
{{a[1, 1], a[1, 2], 0, 0}, {a[2, 1], a[2, 2], 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
Simplify[p.spin[1].A.spin[1].p] // MatrixForm
Simplify[spin[-1].p.A.p.spin[-1]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a[2,2] & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a[1,1] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
spin[{s0, s1, s2, s3}].p
```

$$\left\{\left\{\frac{1}{4}+\frac{1-s0}{4}+\frac{1}{4}\,(1-s1+i\,s2)+\frac{1-s3}{2},\ \frac{1}{4}+\frac{1-s0}{4}+\frac{1}{2}\,(1-s1+i\,s2)+\frac{1-s3}{4},\right.\right.$$
$$\left.\frac{1}{4}+\frac{1}{4}\,(1-s1+i\,s2)+\frac{1-s3}{4},\ \frac{1-s0}{4}+\frac{1}{4}\,(1-s1+i\,s2)+\frac{1-s3}{4}\right\},$$
$$\left\{\frac{1}{4}+\frac{1-s0}{4}+\frac{1}{2}\,(1-s1-i\,s2)+\frac{1+s3}{4},\ \frac{1}{4}+\frac{1-s0}{4}+\frac{1}{4}\,(1-s1-i\,s2)+\frac{1+s3}{2},\right.$$
$$\left.\frac{1-s0}{4}+\frac{1}{4}\,(1-s1-i\,s2)+\frac{1+s3}{4},\ \frac{1}{4}+\frac{1}{4}\,(1-s1-i\,s2)+\frac{1+s3}{4}\right\},$$
$$\left\{\frac{1}{4}+\frac{1+s0}{2}+\frac{1}{4}\,(1+s1-i\,s2)+\frac{1+s3}{4},\ \frac{1}{2}+\frac{1+s0}{4}+\frac{1}{4}\,(1+s1-i\,s2)+\frac{1+s3}{4},\right.$$
$$\left.\frac{1}{4}+\frac{1+s0}{4}+\frac{1}{4}\,(1+s1-i\,s2),\ \frac{1}{4}+\frac{1+s0}{4}+\frac{1+s3}{4}\right\},\ \left\{\frac{1}{2}+\frac{1+s0}{4}+\frac{1}{4}\,(1+s1+i\,s2)+\frac{1-s3}{4},\right.$$
$$\left.\left.\frac{1}{4}+\frac{1+s0}{2}+\frac{1}{4}\,(1+s1+i\,s2)+\frac{1-s3}{4},\ \frac{1}{4}+\frac{1+s0}{4}+\frac{1-s3}{4},\ \frac{1}{4}+\frac{1+s0}{4}+\frac{1}{4}\,(1+s1+i\,s2)\right\}\right\}$$

```
spin[{s0, s1, s2, s3}].p.A.p.spin[-{s0, s1, s2, s3}]
```

A very large output was generated. Here is a sample of it:

```
{{<<1>>}, {<<1>>}, {<<1>>}, {<<1>>}}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
(1 + G5.Sum[G[[u]] s[u], {u, 1, 4}]).(1 + G[[1]]) / 4 // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4}\,(3 - s[1] - s[2] + i\,s[3] + 2\,(1 - s[4])) & \frac{1}{4}\,(3 - s[1] + 2\,(1 - s[2] + i\,s[3]) - s[4]) & \frac{1}{4}\,(3 - s[2] - \\ \frac{1}{4}\,(3 - s[1] + 2\,(1 - s[2] - i\,s[3]) + s[4]) & \frac{1}{4}\,(3 - s[1] - s[2] - i\,s[3] + 2\,(1 + s[4])) & \frac{1}{4}\,(3 - s[1] - \\ \frac{1}{4}\,(3 + 2\,(1 + s[1]) + s[2] - i\,s[3] + s[4]) & \frac{1}{4}\,(5 + s[1] + s[2] - i\,s[3] + s[4]) & \frac{1}{4}\,(3 + s[1] - \\ \frac{1}{4}\,(5 + s[1] + s[2] + i\,s[3] - s[4]) & \frac{1}{4}\,(3 + 2\,(1 + s[1]) + s[2] + i\,s[3] - s[4]) & \frac{1}{4}\,(3 + s[1] - \end{pmatrix}$$