

```
Exit[]
```

```
$Assumptions = b > 0 && mpr < 0 &&  $\mu > 0$  &&  $\sigma > 0$  && a  $\in$  Reals &&
```

```
1 > k1  $\geq$  0 && k0  $\geq$  0 && S0 > 0 && K > 0 && r  $\geq$  0 && b  $\in$  Reals && rf  $\geq$  0 &&  $\gamma > 0$ ;
```

```
xx[W_, t_] := Exp[W + (mpr - 1/2) t^2] - 1;
```

```
 $\gamma = .2$ ; mpr = -0.2;
```

```
g[a_, t_, b_] := NIntegrate[Exp[-a xx[w t, t] - w^2/2], {w, -b, b}];
```

```
f[a_, w_, t_] := Exp[-a xx[w t, t] - w^2/2] xx[w t, t]
```

```
i[a_, w_, t_] :=
```

$$e^{-a \left( -1 + e^{\left( -\frac{1}{2} + \text{mpr} \right) t^2 + t w} \right) + \left( -\frac{1}{2} + \text{mpr} \right) t^2 + t w - \frac{w^2}{2}} \left( 2 \left( -\frac{1}{2} + \text{mpr} \right) t + w \right) -$$

$$a e^{-a \left( -1 + e^{\left( -\frac{1}{2} + \text{mpr} \right) t^2 + t w} \right) + \left( -\frac{1}{2} + \text{mpr} \right) t^2 + t w - \frac{w^2}{2}} \left( -1 + e^{\left( -\frac{1}{2} + \text{mpr} \right) t^2 + t w} \right) \left( 2 \left( -\frac{1}{2} + \text{mpr} \right) t + w \right)$$

```
i2[a_, w_, t_] := e^{a - a e^{t(-t+w)} - 3 t^2 + t w - \frac{w^2}{2}}
```

$$\left( a^2 e^{2 t w} (-2 t + w)^2 + (1 + a) e^{2 t^2} (-2 + 4 t^2 - 4 t w + w^2) - a e^{t(t+w)} (-2 + 4(3+a) t^2 - 4(3+a) t w + (3+a) w^2) \right);$$

```
j[a_, t_, b_] := NIntegrate[i[a, w, t], {w, -b, b}];
```

```
j2[a_, t_, b_] := NIntegrate[i2[a, w, t], {w, -b, b}];
```

```
gs[a_, t_, b_] := NIntegrate[f[a, w, t], {w, -b, b}];
```

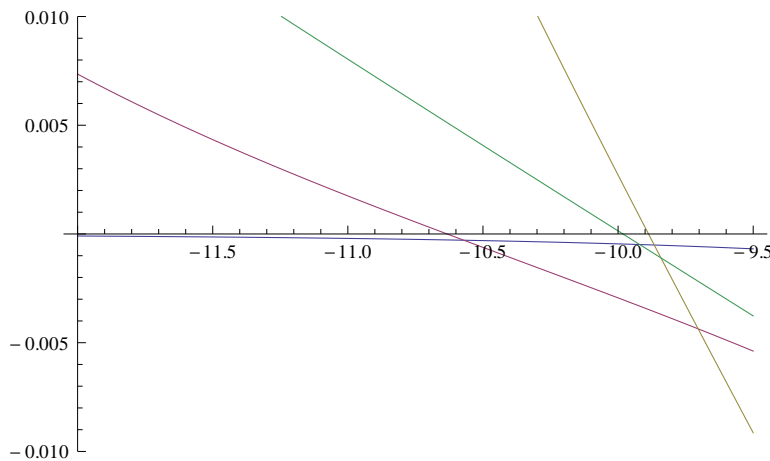
```
h[a_, w_, t_] := Exp[-a xx[w, t]] xx[w, t] + Exp[-a xx[-w, t]] xx[-w, t]
```

```
h2[w_, t_] := Exp[-w^2/2/t^2]/t
```

```
gs2[a_, t_, b_] := NIntegrate[h2[w, t] h[a, w, t], {w, 0, t b}]
```

```
as[t_, b_] := Quiet[FindRoot[gs[a, t, b] == 0, {a, -1, 0}][[1, 2]]]
```

```
b = 5; t = .3; Plot[{gs[a, t 1.5, b], gs[a, t 1, b], gs[a, t .5, b], gs[a, t .2, b]},  
{a, .95 mpr, 1.2 mpr}, PlotRange -> {- .01, .01}]
```



```
as[.1, b]
```

```
-0.994986
```

```
gs2[mpr, t, b]
```

```
0.00216389
```

```
mpr =.; a =.; t =.; Simplify[(i[a, w, t] + i[a, -w - c, t]) / e(-1/2 + mpr) t2]
```

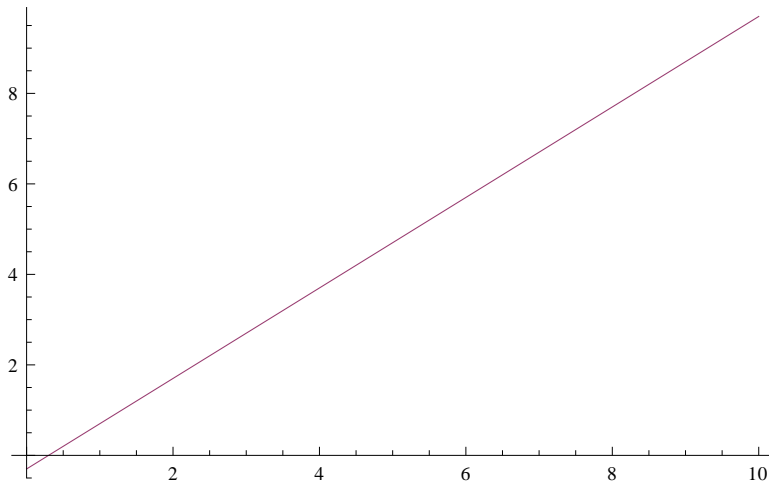
$$e^{a-a} e^{t \left( -c + \left( -\frac{1}{2} + \text{mpr} \right) t - w \right) - \frac{1}{2} (c+w) (c+2t+w) (-c + (-1+2\text{mpr}) t - w) -}$$

$$a e^{a-a} e^{t \left( -c + \left( -\frac{1}{2} + \text{mpr} \right) t - w \right) - \frac{1}{2} (c+w) (c+2t+w) \left( -1 + e^{t \left( -c + \left( -\frac{1}{2} + \text{mpr} \right) t - w \right)} \right) (-c + (-1+2\text{mpr}) t - w) +}$$

$$e^{a-a} e^{t \left( \left( -\frac{1}{2} + \text{mpr} \right) t + w \right) + t w - \frac{w^2}{2}} ((-1+2\text{mpr}) t + w) -$$

$$a e^{a-a} e^{t \left( \left( -\frac{1}{2} + \text{mpr} \right) t + w \right) + t w - \frac{w^2}{2}} \left( -1 + e^{t \left( \left( -\frac{1}{2} + \text{mpr} \right) t + w \right)} \right) ((-1+2\text{mpr}) t + w)$$

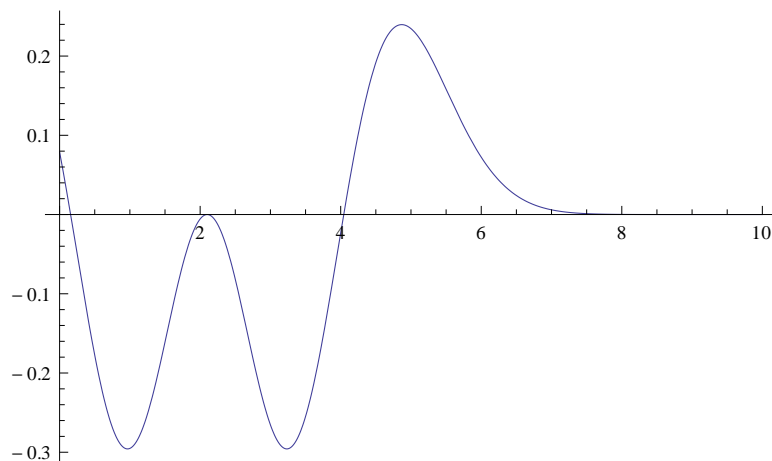
```
Plot[{-c + (-1 + 2 mpr) t - w, (-1 + 2 mpr) t + w}, {w, 0, 2 b}]
```



```
t = 0.1; mpr = -10; a = mpr;  $\left( -\frac{1}{2} + \text{mpr} \right) t$  4
```

```
Plot[{i[a, w, t] + i[a, -w + 4.2, t]}, {w, 0, 2 b}, PlotRange -> All]
```

```
-4.2
```



```
a = -.19901; NIntegrate[
  Exp[-w^2/2/t^2]/t (Exp[-a xx[w, t]] xx[w, t] + Exp[-a xx[-w, t]] xx[-w, t])
, {w, 0, t b}]
```

```
-1.70561 × 10-8
```

```
mpr =.;
```

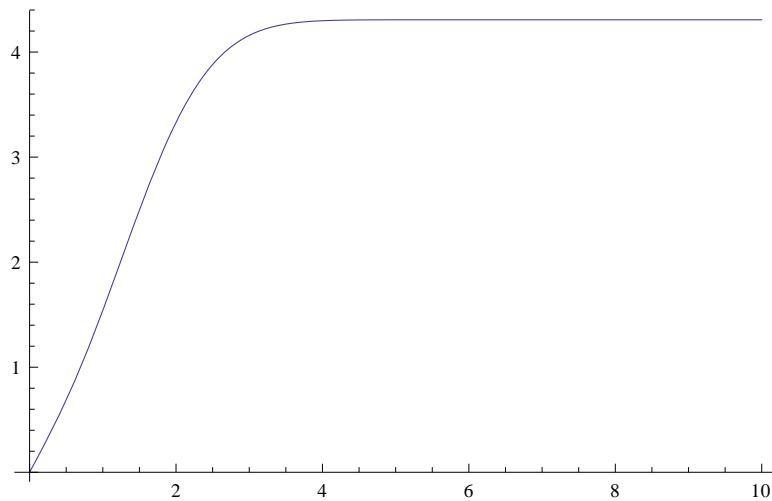
```
ds = Table[as[1/n, b/n], {n, 1, 60}]
```

```
{-0.0525742, -0.154073, -0.458295, -0.948096, -1.59442, -2.38981, -3.33203,
 -4.42028, -5.6542, -7.03363, -8.55848, -10.2287, -12.0442, -14.0051, -16.1112,
 -18.3626, -20.7594, -23.3014, -25.9886, -28.8212, -31.799, -34.922, -38.1904,
 -41.604, -45.1629, -48.867, -52.7164, -56.7111, -60.851, -65.1362, -69.5666,
 -74.1424, -78.8633, -83.7296, -88.7411, -93.8979, -99.1999, -104.647,
 -110.24, -115.978, -121.861, -127.889, -134.063, -140.381, -146.846, -153.455,
 -160.21, -167.109, -174.155, -181.345, -188.681, -196.162, -203.788,
 -211.559, -219.476, -227.538, -235.745, -244.098, -252.596, -261.239}
```

```
Integrate[xx[w, 1] Exp[-w^2/2], {w, -b, b}]
```

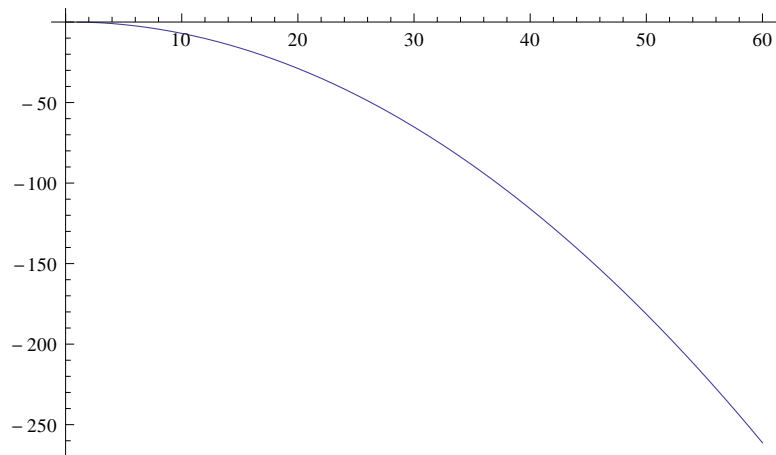
$$\sqrt{\frac{\pi}{2}} \left( -2 \operatorname{Erf}\left[\frac{b}{\sqrt{2}}\right] + e^{\operatorname{mpr}} \left( \operatorname{Erf}\left[\frac{-1+b}{\sqrt{2}}\right] + \operatorname{Erf}\left[\frac{1+b}{\sqrt{2}}\right] \right) \right)$$

```
mpr = 2/2; Plot[ $\sqrt{\frac{\pi}{2}} \left( -2 \operatorname{Erf}\left[\frac{b}{\sqrt{2}}\right] + e^{\operatorname{mpr}} \left( \operatorname{Erf}\left[\frac{-1+b}{\sqrt{2}}\right] + \operatorname{Erf}\left[\frac{1+b}{\sqrt{2}}\right] \right) \right)$ , {b, 0, 10}]
```



```
Exit[]
```

```
ListLinePlot[ds, PlotRange → All]
```



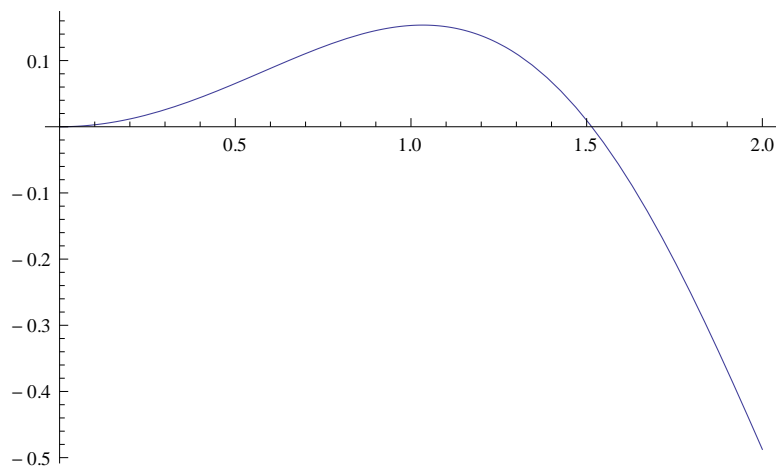
```
gs2[a_, t_] := NIntegrate[Exp[-a (Exp[-t w] - 1) - w^2 / 2] (1 - Exp[-t w]) +  
  Exp[-a (Exp[t w] - 1) - w^2 / 2] (1 - Exp[t w]), {w, 0, ∞}];
```

```
Integrate[Exp[t w - w^2 / 2], {w, -∞, ∞}]
```

$$e^{\frac{t^2}{2}} \sqrt{2\pi}$$

```
h[w_] := Exp[-a (Exp[w] - 1)] (Exp[w] - 1)
```

```
a = .7 / 2; Plot[h[x] + h[-x] /. x → w, {w, 0, 2}, PlotRange → All]
```



```
ie[s_, a_] := (a (s - 1) - 1) + (2 a - a^2 (s - 1)) s
```

```
a /. Solve[0 == ie[s, a], a]
```

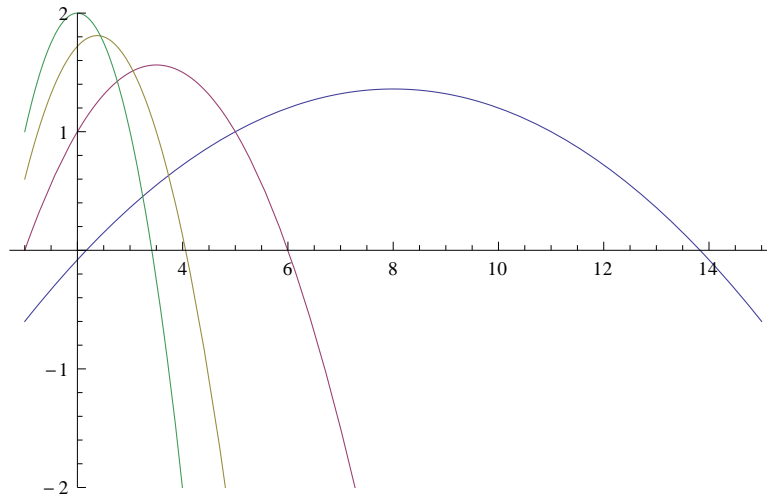
```
Limit[#, {s -> 1}] & /@ %
```

$$\left\{ \frac{-1 + 3s - \sqrt{1 - 2s + 5s^2}}{2(-s + s^2)}, \frac{-1 + 3s + \sqrt{1 - 2s + 5s^2}}{2(-s + s^2)} \right\}$$

$$\left\{ \left\{ \frac{1}{2} \right\}, \{\infty\} \right\}$$

```
asd = Simplify[Table[ie[s, a], {a, {.2, 1/2, .8, 1}}];
```

```
Plot[asd, {s, 1, 15}, PlotRange -> {-2, 2}]
```



```
u[s_] := -Exp[-a (s - 1)] (s - 1)
```

```
D[u[s], {s, 1}]
```

```
D[u[s], {s, 2}]
```

$$-e^{-a(-1+s)} + a e^{-a(-1+s)} (-1+s)$$

```
Simplify[D[Exp[-w^2/2/t^2]/t, t]]
```

$$\frac{e^{-\frac{w^2}{2t^2}} (-t^2 + w^2)}{t^4}$$