

Series[(1 + h A1) (1 + (A1 + 2 A2) h), {h, 0, 1}]

$$1 + 2 (A1 + A2) h + O[h]^2$$

Series $\left[\frac{1 + h / 4 A1}{1 - h / 4 A1}, \{h, 0, 2\}\right]$

$$1 + \frac{A1 h}{2} + \frac{A1^2 h^2}{8} + O[h]^3$$

(* Crank Nicolson with Strang Symm*)

cn = Series $\left[\frac{1}{1 - h / 4 A1} \left((1 + h / 4 A1) \frac{1}{1 - h / 2 A2} \right. \right.$
 $\left. \left. \left((1 + h / 2 A2) \frac{1}{1 - h / 4 A1} ((1 + h / 4 A1) f3 + h / 2 b1) + h b2 \right) + h / 2 b1 \right), \{h, 0, 3\}\right]$

$$f3 + (b1 + b2 + A1 f3 + A2 f3) h + \frac{1}{2} (A1 b1 + A2 b1 + A1 b2 + A2 b2 + A1^2 f3 + 2 A1 A2 f3 + A2^2 f3) h^2 +$$

$$\frac{1}{16} (3 A1^2 b1 + 6 A1 A2 b1 + 4 A2^2 b1 + 2 A1^2 b2 + 4 A1 A2 b2 +$$

$$4 A2^2 b2 + 3 A1^3 f3 + 8 A1^2 A2 f3 + 8 A1 A2^2 f3 + 4 A2^3 f3) h^3 + O[h]^4$$

ex = Series[**Exp**[h (A1 + A2)] (f3 + (b1 + b2) / (A1 + A2)) - (b1 + b2) / (A1 + A2), {h, 0, 3}]

$$f3 + (b1 + b2 + A1 f3 + A2 f3) h + \frac{1}{2} (A1 + A2) (b1 + b2 + A1 f3 + A2 f3) h^2 +$$

$$\frac{1}{6} (A1 + A2)^2 (b1 + b2 + A1 f3 + A2 f3) h^3 + O[h]^4$$

Simplify[ex - cn]

$$\frac{1}{48} (-A1^2 (b1 - 2 b2) - 2 A1 A2 (b1 - 2 b2) - A1^3 f3 - 4 A2^2 (b1 + b2 + A2 f3)) h^3 + O[h]^4$$