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PrependTo[$Path, "D:\\Users\\Johannes\\Promotion\\SVN Rep\\Mathematica\\Packages"];
<< JoFin`

n = 1;
dS[i_] := r S[i] dt + σ[i] S[i] dB[i];
Q[i_] := q[i] P / S[i];

dP = Expand[Sum[Q[i] dS[i] + (P - Sum[Q[i] S[i], {i, 1, n}]) r dt, {i, 1, n}]]
coef = CoefficientArrays[dP, Prepend[Table[dB[i], {i, n + 1}], dt]][[2]];
S[n + 1] := P;
mm = MMC[{coef[[1]]}, {coef[[2 ;; n + 2]]}];
Print["Resulting system of SDEs:", MatrixForm /@ mm];
mmlogd = MMdisc[mm];
dfk = Simplify[DFK[V, mmlogd]]

dt P r + P dB[1] q[1] σ[1]

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Resulting system of SDEs: $\left\{ \begin{pmatrix} r S[1] \\ P r \end{pmatrix}, \begin{pmatrix} S[1] \sigma[1] & 0 \\ P q[1] \sigma[1] & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

The resulting system of SDEs: $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} S[1] \sigma[1] & 0 \\ P q[1] \sigma[1] & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

$$\frac{1}{2} \sigma[1]^2 (P^2 q[1]^2 V^{(0,0,2)}[t, S[1], P] + S[1] (2 P q[1] V^{(0,1,1)}[t, S[1], P] + S[1] V^{(0,2,0)}[t, S[1], P])) + V^{(1,0,0)}[t, S[1], P]$$

MatrixForm /@ CoefficientArrays[dfk, Table[q[i], {i, n}], Symmetric → True]

$$\left\{ \frac{1}{2} S[1]^2 \sigma[1]^2 V^{(0,2,0)}[t, S[1], P] + V^{(1,0,0)}[t, S[1], P], \begin{pmatrix} P S[1] \sigma[1]^2 V^{(0,1,1)}[t, S[1], P] \\ \frac{1}{2} P^2 \sigma[1]^2 V^{(0,0,2)}[t, S[1], P] \end{pmatrix} \right\}$$

constant volatility

The option value will be independent of the asset prices, as can be verified by the absence of S terms in the following equation:

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V2 := ToExpression[StringJoin["V[#1,#", ToString[n + 2], "&"]]];
dfkC = Simplify[DFK[V2, mmlogd]]

1/2 P^2 q[1]^2 σ[1]^2 V^{(0,2)}[t, P] + V^{(1,0)}[t, P]

coef = CoefficientArrays[dfkC, Table[q[i], {i, n}], Symmetric → True]; MatrixForm /@ coef
{V^{(1,0)}[t, P], (0), (1/2 P^2 σ[1]^2 V^{(0,2)}[t, P])}

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autonomous volatility

$S[1] = P; S[2] = \sigma;$

$\text{coefsSDE} = \{\{0, a\}, \{\{P \sigma q, 0\}, \{0, b\}\}, \{\{1, \rho\}, \{\rho, 1\}\}\}; \text{MatrixForm} /@ \text{coefsSDE}$

$\text{dfkA} = \text{DFK}[V, \text{coefsSDE}]$

$$\left\{ \begin{pmatrix} 0 \\ a \end{pmatrix}, \begin{pmatrix} P q \sigma & 0 \\ 0 & b \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right\}$$

$a V^{(0,0,1)}[t, P, \sigma] +$

$$\frac{1}{2} \left(b^2 V^{(0,0,2)}[t, P, \sigma] + 2 b P q \rho \sigma V^{(0,1,1)}[t, P, \sigma] + P^2 q^2 \sigma^2 V^{(0,2,0)}[t, P, \sigma] \right) +$$

$V^{(1,0,0)}[t, P, \sigma]$

$\text{coef} = \text{CoefficientArrays}[\text{dfkA}, q, \text{Symmetric} \rightarrow \text{True}]; \text{MatrixForm} /@ \text{coef}$

$$\left\{ a V^{(0,0,1)}[t, P, \sigma] + \frac{1}{2} b^2 V^{(0,0,2)}[t, P, \sigma] + V^{(1,0,0)}[t, P, \sigma], \right. \\ \left. \left(b P q \sigma V^{(0,1,1)}[t, P, \sigma] \right), \left(\frac{1}{2} P^2 \sigma^2 V^{(0,2,0)}[t, P, \sigma] \right) \right\}$$

passport

$S[1] = P; S[2] = S$

$\text{coefsSDE} = \{\{0, 0\}, \{\{S \sigma q, 0\}, \{\sigma S, 0\}\}, \{\{1, 0\}, \{0, 0\}\}\}; \text{MatrixForm} /@ \text{coefsSDE}$

$\text{dfkA} = \text{DFK}[V, \text{coefsSDE}]$

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} q S \sigma & 0 \\ S \sigma & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$$

$$\frac{1}{2} \left(S^2 \sigma^2 V^{(0,0,2)}[t, P, \sigma] + 2 q S^2 \sigma^2 V^{(0,1,1)}[t, P, \sigma] + q^2 S^2 \sigma^2 V^{(0,2,0)}[t, P, \sigma] \right) +$$

$V^{(1,0,0)}[t, P, \sigma]$

$\text{coef} = \text{CoefficientArrays}[\text{dfkA}, q, \text{Symmetric} \rightarrow \text{True}]; \text{MatrixForm} /@ \text{coef}$

$$\left\{ \frac{1}{2} S^2 \sigma^2 V^{(0,0,2)}[t, P, \sigma] + V^{(1,0,0)}[t, P, \sigma], \right. \\ \left. \left(S^2 \sigma^2 V^{(0,1,1)}[t, P, \sigma] \right), \left(\frac{1}{2} S^2 \sigma^2 V^{(0,2,0)}[t, P, \sigma] \right) \right\}$$

$$\left\{ \frac{1}{2} S[1]^2 \sigma[1]^2 V^{(0,2,0)}[t, S[1], P] + V^{(1,0,0)}[t, S[1], P], \right.$$

$$\left. \left(P S[1] \sigma[1]^2 V^{(0,1,1)}[t, S[1], P] \right), \left(\frac{1}{2} P^2 \sigma[1]^2 V^{(0,0,2)}[t, S[1], P] \right) \right\}$$