

g = .

g[a_ , b_] := Expand[Normal[Series[f[x, y], {x, 0, 2}, {y, 0, 2}]] /. x → a /. y → b

S = Flatten[Table[f[i h, j h] == g[i h, j h], {i, -1, 1}, {j, -1, 1}]]; S // MatrixForm

$$\begin{pmatrix} f[-h, -h] == f[0, 0] - h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] - h f^{(1,0)}[0, 0] + h^2 f^{(1,1)}[0, 0] - \frac{1}{2} h^3 f^{(1,2)}[0, 0] \\ f[-h, 0] == f[0, 0] - h f^{(1,0)}[0, 0] + \frac{1}{2} h^2 f^{(2,0)}[0, 0] \\ f[-h, h] == f[0, 0] + h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] - h f^{(1,0)}[0, 0] - h^2 f^{(1,1)}[0, 0] - \frac{1}{2} h^3 f^{(1,2)}[0, 0] \\ f[0, -h] == f[0, 0] - h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] \\ \text{True} \\ f[0, h] == f[0, 0] + h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] \\ f[h, -h] == f[0, 0] - h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] + h f^{(1,0)}[0, 0] - h^2 f^{(1,1)}[0, 0] + \frac{1}{2} h^3 f^{(1,2)}[0, 0] \\ f[h, 0] == f[0, 0] + h f^{(1,0)}[0, 0] + \frac{1}{2} h^2 f^{(2,0)}[0, 0] \\ f[h, h] == f[0, 0] + h f^{(0,1)}[0, 0] + \frac{1}{2} h^2 f^{(0,2)}[0, 0] + h f^{(1,0)}[0, 0] + h^2 f^{(1,1)}[0, 0] + \frac{1}{2} h^3 f^{(1,2)}[0, 0] \end{pmatrix}$$

Solve[S, G]

$$\begin{aligned} \{ \{ f^{(2,2)}[0, 0] \rightarrow -\frac{1}{h^4} (-4 f[0, 0] + 2 f[0, -h] + 2 f[0, h] + \\ 2 f[-h, 0] - f[-h, -h] - f[-h, h] + 2 f[h, 0] - f[h, -h] - f[h, h]), \\ f^{(2,1)}[0, 0] \rightarrow -\frac{1}{2 h^3} (-2 f[0, -h] + 2 f[0, h] + f[-h, -h] - f[-h, h] + f[h, -h] - f[h, h]), \\ f^{(1,1)}[0, 0] \rightarrow -\frac{-f[-h, -h] + f[-h, h] + f[h, -h] - f[h, h]}{4 h^2}, \\ f^{(1,2)}[0, 0] \rightarrow -\frac{1}{2 h^3} (-2 f[-h, 0] + f[-h, -h] + f[-h, h] + 2 f[h, 0] - f[h, -h] - f[h, h]), \\ f^{(2,0)}[0, 0] \rightarrow -\frac{2 f[0, 0] - f[-h, 0] - f[h, 0]}{h^2}, f^{(1,0)}[0, 0] \rightarrow -\frac{f[-h, 0] - f[h, 0]}{2 h}, \\ f^{(0,1)}[0, 0] \rightarrow -\frac{f[0, -h] - f[0, h]}{2 h}, f^{(0,2)}[0, 0] \rightarrow -\frac{2 f[0, 0] - f[0, -h] - f[0, h]}{h^2} \} \} \end{aligned}$$

G = Drop[

Flatten[Table[D[D[f[x, y], {x, i}], {y, j}], {i, 0, 2}, {j, 0, 2}]] /. x → 0 /. y → 0, 1]

$$\{ f^{(0,1)}[0, 0], f^{(0,2)}[0, 0], f^{(1,0)}[0, 0], \\ f^{(1,1)}[0, 0], f^{(1,2)}[0, 0], f^{(2,0)}[0, 0], f^{(2,1)}[0, 0], f^{(2,2)}[0, 0] \}$$