```
$Assumptions = P ∈ Reals && -1 <= q <= 1 && t > 0
```

$$P \in \text{Reals} \;\&\&\; -1 \le q \le 1 \;\&\&\; t > 0$$

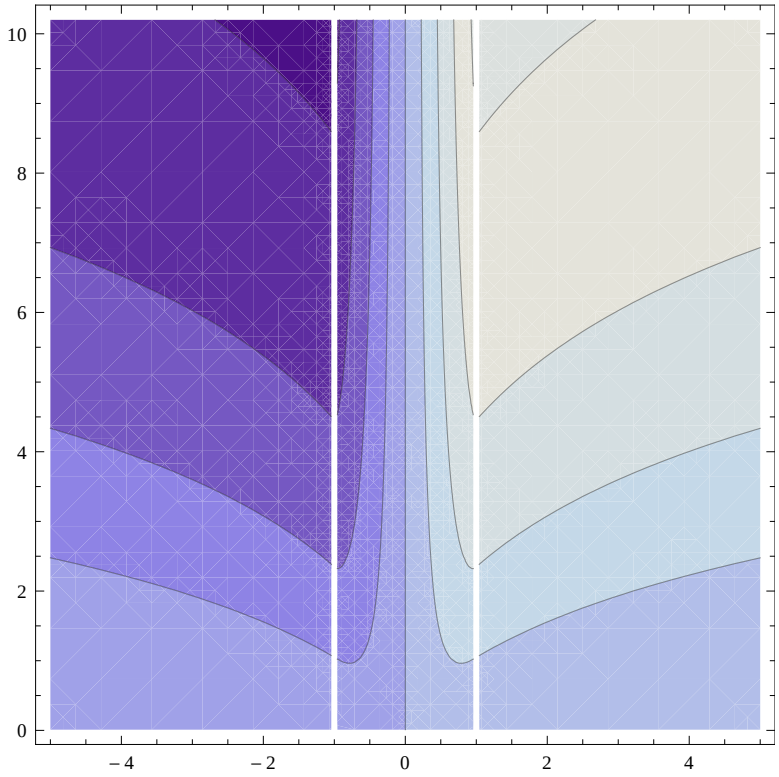
```
ExpectedValue[Max[0, 1 (Exp[#1 - 1 / 2 t] - 1) - P] - Max[0, -(Exp[#1 - 1 / 2 t] - 1) - P] &,
 NormalDistribution[0, Sqrt[t]]]
```

$$\begin{cases} \frac{1}{2}\left(-P+\text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P \ge 1 \;\&\&\; t > 0 \\ \frac{1}{2}\left(-\text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]-\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+\right. & -1 < P < 1 \;\&\&\; t > 0 \\ \quad \left.\text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & \\ \frac{1}{2}\left(-2-P+\text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]-\right. & \text{True} \\ \quad \left.\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+2\,\text{Erfc}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & \end{cases}$$

```
g[t_, P_] :=
```

$$\begin{cases} \frac{1}{2}\left(-P+\text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P \ge 1 \;\&\&\; t > 0 \\ \frac{1}{2}\left(-\text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]-\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+\right. & -1 < P < 1 \;\&\&\; t > 0 \\ \quad \left.\text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]+P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & \\ \frac{1}{2}\left(-2-P+\text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]-\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+\right. & \text{True} \\ \quad \left.P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]+2\,\text{Erfc}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & \end{cases}$$

```
ContourPlot[g[t, P], {P, -5, 5}, {t, 0.01, 10.2}]
```



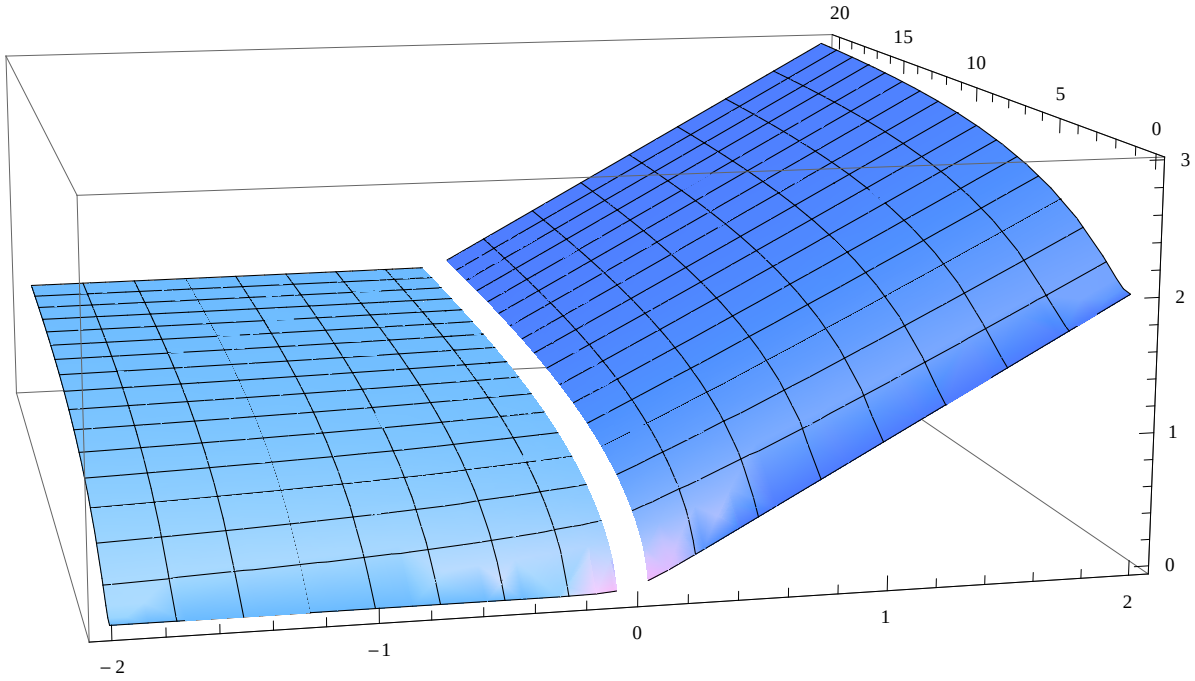

```
ExpectedValue[Max[0, Sign[-P] (Exp[#1 - 1 / 2 t] - 1) + P] &, NormalDistribution[0, Sqrt[t]]]
```

$$\begin{cases} \frac{1}{2}\left(P + \text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right] + \text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right] - P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P < 0 \&\& t > 0 \\ \frac{1}{2}\left(P + \text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right] + \text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right] + P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P > 0 \&\& t > 0 \\ 0 & \text{True} \end{cases}$$

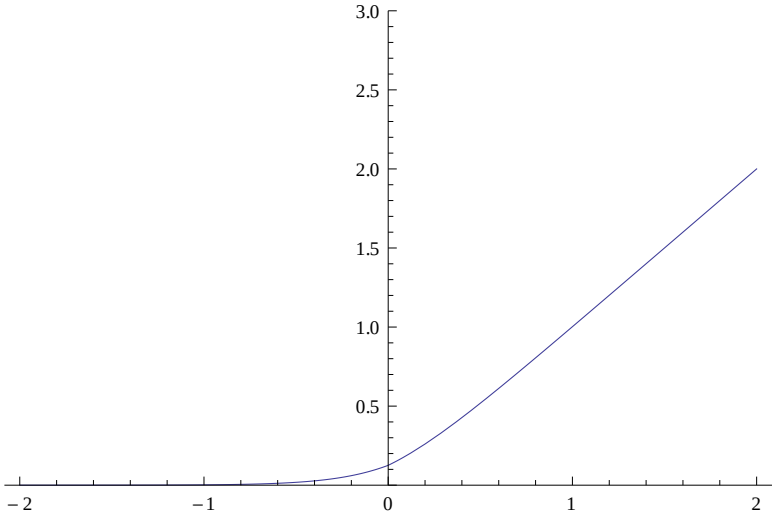
```
f[t_, P_] :=
```

$$\begin{cases} \frac{1}{2}\left(P + \text{Erf}\left[\frac{t-2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right] + \text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right] - P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1-P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P < 0 \&\& t > 0 \\ \frac{1}{2}\left(P + \text{Erf}\left[\frac{t-2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right] + \text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right] + P\,\text{Erf}\left[\frac{t+2\,\text{Log}[1+P]}{2\sqrt{2}\,\sqrt{t}}\right]\right) & P > 0 \&\& t > 0 \\ 0 & \text{True} \end{cases}$$

```
Plot3D[f[t, P], {P, -2, 2}, {t, 0.01, 20.2}]
```




```
Plot[f[0.1, P], {P, -2, 2}, PlotRange → {0, 3}]
```




```
ExpectedValue[Max[0, (Exp[#1 - 1 / 2 t] - 1)] &, NormalDistribution[0, Sqrt[t]]]
```

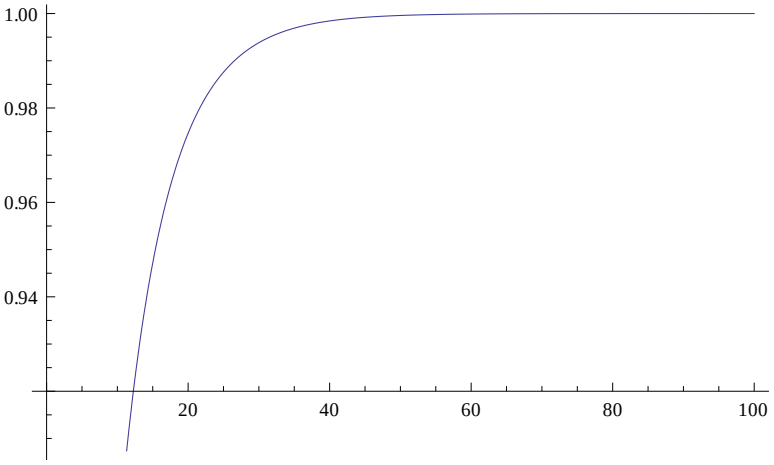
$$\text{Erf}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right]$$

$$\text{Erf}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right] /. t \to 1000000000000000 // N$$

General::unfl : Underflow occurred in computation. ≫

1.

```
Plot[Erf[Sqrt[t]/(2 Sqrt[2])], {t, 0, 100}]
```




```
ExpectedValue[Abs[Exp[#1 - 1 / 2 t] - 1] &, NormalDistribution[0, Sqrt[t]]]
```

$$2\,\text{Erf}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right]$$

```
Plot3D[{(Max[0, q (S - 1) + P] /. q → 1), (Max[0, q (S - 1) + P] /. q → -1)},
 {P, -2, 2}, {S, 0, 5}]
```