```
<< NC`;
<< NCAlgebra`;
```

You are using the version of NCAlgebra which is found in:

  d:\Users\Johannes\Codes and Libraries\NC.

You can now use "<< NCAlgebra`" to load NCAlgebra or "<< NCGB`" to load NCGB.

You have already loaded NCAlgebra.m

```
SetCommutative[h, b]; SNC[A]; SNC[B];

(*this is needed, because of bug in Series*)series[f_, {x_, x0_, n_}] :=
 Sum[(x - x0)^k / k! Simplify[NCExpand[D[f, {x, k}] /. x -> x0]], {k, 0, n}]

Pow[A_, n_] := Nest[(A ** # &), A, n - 1];

(*Calculates 1/(1+h A) as NC Series*)
Inv[A_, h_, n_] := 1 + Sum[(-h)^k Pow[A, k], {k, 1, n}]

exp[A_, h_, n_] := 1 + Sum[h^k Pow[A , k] / k!, {k, 1, n}]

CN[A_, h_, t1_, t2_] :=
 Inv[Through[A[t2], Plus], h / 2, 5] ** ((1 - h Through[A[t1], Plus] / 2) ** #) &
CN[A_, h_, t_] := CN[A, h, t, t + h];
```

Straight-forward CN splitting with intermediate A[t] values

```
series[
 CN[A, h / 2, t + h / 2][CN[ B, h, t][CN[A, h / 2, t][u0]]] - CN[A + B, h, t][u0], {h, 0, 3}]
```

$-\dfrac{1}{16} h^3$ u0 (A[t] ** A′[t] + 2 B[t] ** A′[t] +

2 A′[t] ** A[t] + 2 A′[t] ** B[t] + 4 B′[t] ** A[t] − A[t] ** A[t] ** A[t] −
2 A[t] ** A[t] ** B[t] − 2 B[t] ** A[t] ** A[t] − 4 B[t] ** A[t] ** B[t] − A″[t])

Alternative CN splitting without intermediate A[t] values

```
series[CN[A, h / 2, t, t + h][CN[ B, h, t][CN[A, h / 2, t, t + h][u0]]] -
  CN[A + B, h, t][u0], {h, 0, 3}]
```

$-\dfrac{1}{16} h^3$ u0 (2 A′[t] ** A[t] + 4 A′[t] ** B[t] + 4 B′[t] ** A[t] − A[t] ** A[t] ** A[t] −

2 A[t] ** A[t] ** B[t] − 2 B[t] ** A[t] ** A[t] − 4 B[t] ** A[t] ** B[t])

This only works due to an internal compenstion, of the follwing second order term:

```
series[CN[A, h / 2, t, t + h][u0] - CN[A, h / 2, t][u0], {h, 0, 2}]
```

$-\dfrac{1}{8} h^2$ u0 A′[t]

```
series[CN[A, h / 2, t, t + h][u0] - CN[A, h / 2, t + h / 2][u0], {h, 0, 2}]
```

$$\frac{1}{8} h^2 u0 A'[t]$$