```
$Assumptions = μ > 0 && σ > 0 && a ∈ Reals &&
  1 ≥ k1 ≥ 0 && k0 ≥ 0 && S0 > 0 && K > 0 && r ≥ 0 && b ∈ Reals && rf ≥ 0
```

$$\mu > 0 \;\&\&\; \sigma > 0 \;\&\&\; a \in \text{Reals} \;\&\&\; 1 \geq k1 \geq 0 \;\&\&\; k0 \geq 0 \;\&\&\; S0 > 0 \;\&\&\; K > 0 \;\&\&\; r \geq 0 \;\&\&\; b \in \text{Reals} \;\&\&\; rf \geq 0$$

```
u[x_] := Abs[x]^2
u1 = Abs @* InverseFunction[u]
p[n_, f_] := Module[{e = n[f[x], x ≈ NormalDistribution[]]},
  Simplify[Exp[-r] e + rf u1[n[u[Exp[-r] (f[x] - e)], x ≈ NormalDistribution[]]]]]
S[a_] := a Exp[μ - σ^2/2 + σ #] &
```

InverseFunction::ifun :
  Inverse functions are being used. Values may be lost for multivalued inverses. ≫

$$\text{Abs} \;@*\; \left(-\sqrt{\#1}\;\&\right)$$

```
p[Expectation, S[a]]
```

$$e^{-r+\mu}\left(a + \sqrt{-1 + e^{\sigma^2}}\; rf\; \text{Abs}[a]\right)$$

```
(* a = Sig[phi] *)
g[a_, k1_] := Simplify[ p[Expectation, S[(a + k1)]] - a]
g[a, k1]
```

$$-a + e^{-r+\mu}\left(a + k1 + \sqrt{-1 + e^{\sigma^2}}\; rf\; \text{Abs}[a + k1]\right)$$

```
g[1, k1]
```

$$-1 + e^{-r+\mu}\,(1 + k1)\left(1 + \sqrt{-1 + e^{\sigma^2}}\; rf\right)$$

```
Expand[Simplify[rf (1 + k1) Solve[-1 + e^(-r+μ) (1 + rf s) (1 + k1) + x == 0, s][[1, 1, 2]]]]
```

$$-1 + e^{r-\mu} - k1 - e^{r-\mu}\, x$$

```
(* arbitrage happens if equation ≠ 0 for x=k1. solve this for Sqrt[-1+e^(σ^2)] *)
l1 = (-1 + Exp[r - μ] (1 - k1)/(1 + k1))/rf; Simplify[-1 + e^(-r+μ) (1 + rf l1) (1 + k1) + k1]
```

$$0$$

```
g[-1, k1]
```

$$1 - e^{-r+\mu}\,(-1 + k1)\left(-1 + \sqrt{-1 + e^{\sigma^2}}\; rf\right)$$

```
Expand[Simplify[rf (1 - k1) Solve[1 - e^(-r+μ) (-1 + rf s) (-1 + k1) + x == 0, s][[1, 1, 2]]]]
```

$$1 - e^{r-\mu} - k1 - e^{r-\mu}\, x$$

(* arbitrage happens if equation ≠ 0 for x = k1. solve this for $\sqrt{-1+e^{\sigma^2}}$ *)

$$\text{l2} = \frac{1 - \text{Exp}[r - \mu]^{\frac{1+\text{k1}}{1-\text{k1}}}}{\text{rf}};\ \text{Simplify}[1 - e^{-r+\mu}\ (-1 + \text{rf l2})\ (-1 + \text{k1}) + \text{k1}]$$

0

g1[a_, b_, k1_] := Simplify[p[Expectation, S[(b + a + k1)]] – a]

g1[a, b, k1]

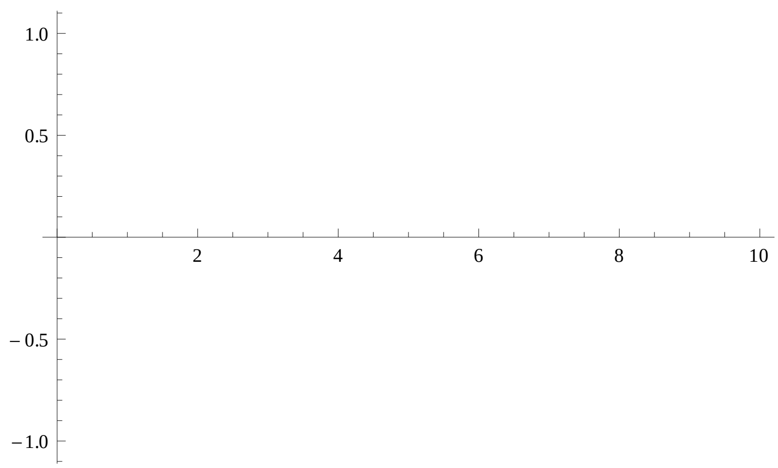
$$-a + e^{-r+\mu}\left(a + b + \text{k1} + \sqrt{-1 + e^{\sigma^2}}\ \text{rf Abs}[a + b + \text{k1}]\right)$$

g2[a_, k1_] := Simplify[p[Max[0, K – S[#]] +
S[#] &] – a]

g2[a, k1]

-a + p[Max[0, K - S[#1]] + S[#1] &]

$$\text{Plot}\left[\text{W}\left[\frac{10}{x}\right], \{k, 0, 10\}\right]$$



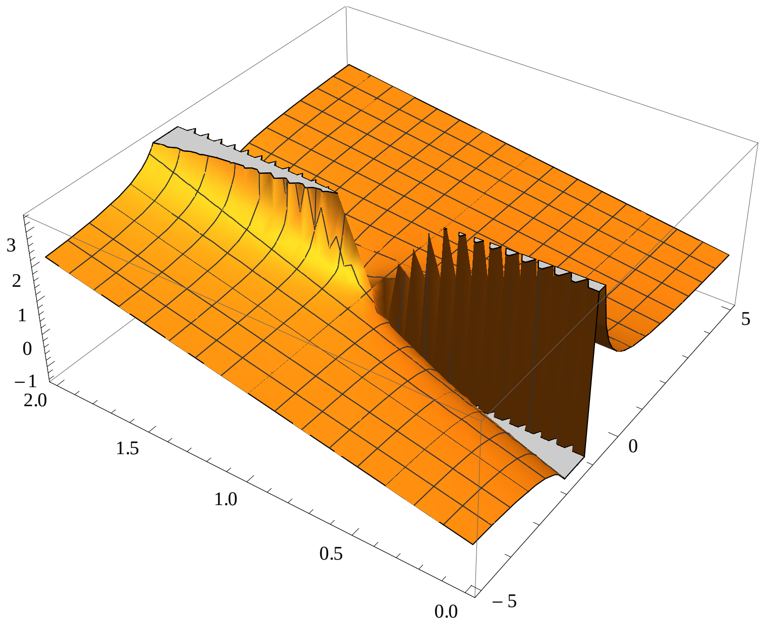
μ = 0.1; σ = 0.1;

μ =.; σ =.;

Expand[Normal[Series[U[x – e], {e, 0, 4}]]]

$$\text{U}[x] - e\ \text{U}'[x] + \frac{1}{2}\ e^2\ \text{U}''[x] - \frac{1}{6}\ e^3\ \text{U}^{(3)}[x] + \frac{1}{24}\ e^4\ \text{U}^{(4)}[x]$$

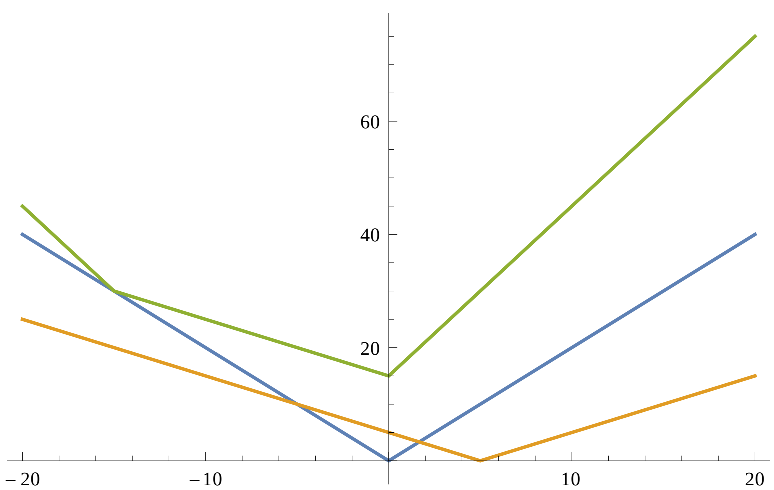
```
Plot3D[(1 + a b)/(1 + a), {a, -5, 5}, {b, 0, 2}]
```




```
a = 0.5
```

0.5

```
a = 2.; b = 5; Plot[{a Abs[x], Abs[b - x], a Abs[x] + Abs[15 + x]}, {x, -20, 20}]
```




```
Series[(1 + x)/(1 - x), {x, 0, 5}]
```

$1 + 2 x + 2 x^2 + 2 x^3 + 2 x^4 + 2 x^5 + O[x]^6$

```
Sqrt[Log[2]] // N
```

0.832555

Multiple Periods

```
s = 0.2; r = -0.2; Plot[{-s Sqrt[t] + 1 - Exp[r t]}, {t, 0, 3}]
```