```
<< NC`;
<< NCAlgebra`;
```

You are using the version of NCAlgebra which is found in:

d:\Users\Johannes\Codes and Libraries\NC.

You can now use "<< NCAlgebra`" to load NCAlgebra or "<< NCGB`" to load NCGB.

You have already loaded NCAlgebra.m

```
SetCommutative[h, b]; SNC[A]; SNC[B];
```

(*this is needed, because of bug in Series*)series[f_, {x_, x0_, n_}] :=

$$\mathrm{Sum}\left[\frac{(x - x0)^k}{k!}\ \mathrm{Simplify}[\mathrm{NCExpand}[D[f, \{x, k\}]\ /.\ x \to x0]], \{k, 0, n\}\right]$$

```
Pow[A_, n_] := Nest[(A ** # &), A, n - 1];
```

(*Calculates $\frac{1}{1+h\ A}$ as NC Series*)

$$\mathrm{Inv}[A\_, h\_, n\_] := 1 + \mathrm{Sum}\left[(-h)^k\ \mathrm{Pow}[A, k], \{k, 1, n\}\right]$$

$$\exp[A\_, h\_, n\_] := 1 + \mathrm{Sum}\left[h^k\ \mathrm{Pow}[A\ , k] / k!, \{k, 1, n\}\right]$$

```
b[A_ + B_, t_] := b[A, t] + b[B, t]
```

$$\mathrm{CN}[A\_, h\_, t\_] := \mathrm{Inv}[A, h/2, 5] ** \left((1 - h\ A/2) ** \# - \frac{1}{2}\ h\ (b[A, t] + b[A, h + t])\right) \&$$

- **Due to the non-commutative nature of A and B and the inhomogenity is the following approximation only first order!**

```
series[CN[A, h, t][CN[ B, h, t][u0]] - CN[A + B, h, t][u0], {h , 0, 2}]
```

$$\frac{1}{2}\ h^2\ (u0\ A ** B + A ** b[B, t] - u0\ B ** A - B ** b[A, t])$$

```
series[
 CN[A, h / 2, t + h / 2][CN[ B, h, t][CN[A, h / 2, t][u0]]] - CN[A + B, h, t][u0], {h, 0, 3}]
```

$$\frac{1}{16}\ h^3\ \Big(-A ** b^{(0,1)}[A, t] - 2\ B ** b^{(0,1)}[A, t] + u0\ A ** A ** A +$$
$$2\ u0\ A ** A ** B + A ** A ** b[A, t] + 2\ A ** A ** b[B, t] + 2\ u0\ B ** A ** A +$$
$$4\ u0\ B ** A ** B + 2\ B ** A ** b[A, t] + 4\ B ** A ** b[B, t] + b^{(0,2)}[A, t]\Big)$$

$$\mathrm{CN3}[A\_, \Delta\_] := \mathrm{Inv}[A, h/4, 5] ** \left((1 - h/4\ A) ** \# - \frac{h}{2}\ b[A, t + \Delta\ h]\right) \&;$$

```
b[A_ + B_ + C_, t_] := b[A, t] + b[B, t] + b[C, t]
```

```
series[CN3[A, 1][CN3[ B, 1][CN[C, h, t][CN3[ B, 0][CN3[A, 0][u0]]]]] -
  CN[A + B + C, h, t][u0], {h, 0, 3}]
```

$$\frac{1}{16} h^3 \left(4\, A\, C^2\, u0 + 4\, B\, C^2\, u0 + 4\, C\, u0\, A ** A + 4\, C\, u0\, A ** B + 2\, C\, A ** b[A, t] + 4\, C\, A ** b[B, t] + 4\, C\, A ** b[C, t] - 2\, A ** b^{(0,1)}[A, t] + 4\, C\, u0\, B ** A + 4\, C\, u0\, B ** B + 2\, C\, B ** b[B, t] + 4\, C\, B ** b[C, t] - 4\, B ** b^{(0,1)}[A, t] - 2\, B ** b^{(0,1)}[B, t] + u0\, A ** A ** A + 2\, u0\, A ** A ** B + A ** A ** b[A, t] + 2\, A ** A ** b[B, t] + 2\, A ** A ** b[C, t] + 2\, u0\, B ** A ** A + 4\, u0\, B ** A ** B + 2\, B ** A ** b[A, t] + 4\, B ** A ** b[B, t] + 4\, B ** A ** b[C, t] + u0\, B ** B ** B + B ** B ** b[B, t] + 2\, B ** B ** b[C, t] - 4\, C\, b^{(0,1)}[A, t] - 4\, C\, b^{(0,1)}[B, t]\right)$$

- **Due to the non-commutative nature of A and B and the inhomogenity is the following approximation only first order!**

```
series[exp[A + B, h, 7] b[0] + b[h] -
  exp[A, h / 2, 7] ** exp[B, h, 7] b[0] - exp[A, h / 2, 7] b[h], {h , 0, 3}]
```

$$\frac{1}{2} h^2 \left(\frac{1}{2}\, b[0]\, A ** A + b[0]\, B ** A - A\, b'[0]\right) + \frac{1}{24} h^3 \left(3\, b[0]\, A ** A ** A + b[0]\, A ** A ** B + 4\, b[0]\, A ** B ** A - 2\, b[0]\, A ** B ** B + 4\, b[0]\, B ** A ** A + 4\, b[0]\, B ** A ** B + 4\, b[0]\, B ** B ** A - 3\, A ** A\, b'[0] - 6\, A\, b''[0]\right)$$