From Bensoussan 1982

```
Exit[]
```

```
σ = {{x[1] Σ[x[2]] v, 0}, {Σ[x[2]], 0}};
```

```
H = -Sum[K[i, j] σ[[i, j]], {i, 2}, {j, 2}]
```

$$-K[2, 1]\ \Sigma[x[2]] - v\ K[1, 1]\ x[1]\ \Sigma[x[2]]$$

```
D[H, v]
```

$$-K[1, 1]\ x[1]\ \Sigma[x[2]]$$

```
(*  K[1,1]:  *)
Simplify[Sum[D[σ[[1, 1]], x[b]] p[b] - ψ[b, 1] G[b, 1], {b, 2}]]
```

$$v\ p[1]\ \Sigma[x[2]] - G[1, 1]\ \psi[1, 1] - G[2, 1]\ \psi[2, 1] + v\ p[2]\ x[1]\ \Sigma'[x[2]]$$

```
(*  K[1,2]:  *)
Simplify[Sum[D[σ[[1, 2]], x[b]] p[b] - ψ[b, 1] G[b, 2], {b, 2}]]
```

$$-G[1, 2]\ \psi[1, 1] - G[2, 2]\ \psi[2, 1]$$

```
(*  p:  *)
Simplify[Table[Sum[D[σ[[i, j]], x[a]] K[a, j], {a, 2}, {j, 2}], {i, 2}]]
```

$$\{v\ (K[1, 1]\ \Sigma[x[2]] + K[2, 1]\ x[1]\ \Sigma'[x[2]]),\ K[2, 1]\ \Sigma'[x[2]]\}$$

```
(*  ϕ[1,j]:  *)
Simplify[Table[Sum[D[σ[[1, i]], x[n]] ϕ[n, j] dW[i], {i, 2}, {n, 2}], {j, 2}]]
```

$$\{v\ dW[1]\ (\Sigma[x[2]]\ \phi[1, 1] + x[1]\ \phi[2, 1]\ \Sigma'[x[2]]),\ v\ dW[1]\ (\Sigma[x[2]]\ \phi[1, 2] + x[1]\ \phi[2, 2]\ \Sigma'[x[2]])\}$$

```
h // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & q1\ u\ \sigma^{(2,0)}[S, t] & q1\ \sigma^{(1,0)}[S, t] \\ 0 & q1\ \sigma^{(1,0)}[S, t] & 0 \end{pmatrix}$$

```
Simplify[Det[h]]
```

$$0$$

```
Simplify[Eigenvalues[h]]
```

$$\left\{0,\ \frac{1}{2}\left(q1\ u\ \sigma^{(2,0)}[S, t] - q1\ \sqrt{4\ \sigma^{(1,0)}[S, t]^2 + u^2\ \sigma^{(2,0)}[S, t]^2}\right),\ \frac{1}{2}\ q1\left(u\ \sigma^{(2,0)}[S, t] + \sqrt{4\ \sigma^{(1,0)}[S, t]^2 + u^2\ \sigma^{(2,0)}[S, t]^2}\right)\right\}$$

```
Simplify[Det[h]]
```

$$-q1^2\ u^2\ \sigma^{(1,0)}[S, t]^2$$

```
g[P_] := -HeavisideTheta[Exp[P] - m] * (Exp[P] - m)
```

```
D[-g[Log[p]], p]
```

$$(-m + p)\ \text{DiracDelta}[-m + p] + \text{HeavisideTheta}[-m + p]$$

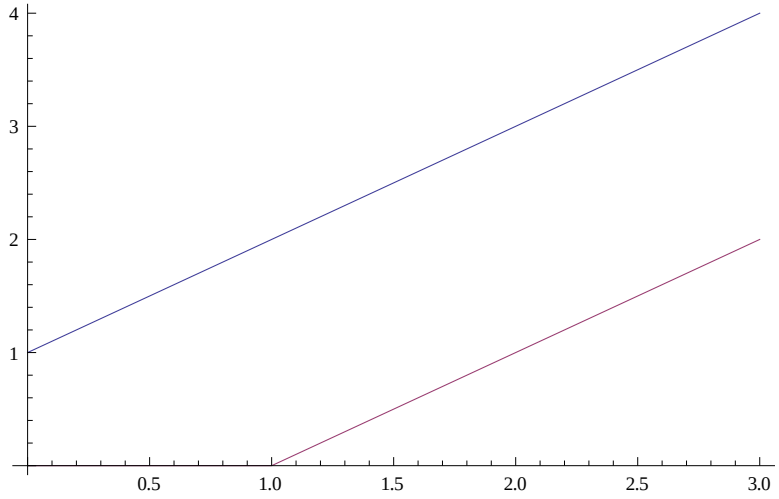
```
D[g[P], P]
```

$$-e^P\ (e^P - m)\ \text{DiracDelta}[e^P - m] - e^P\ \text{HeavisideTheta}[e^P - m]$$

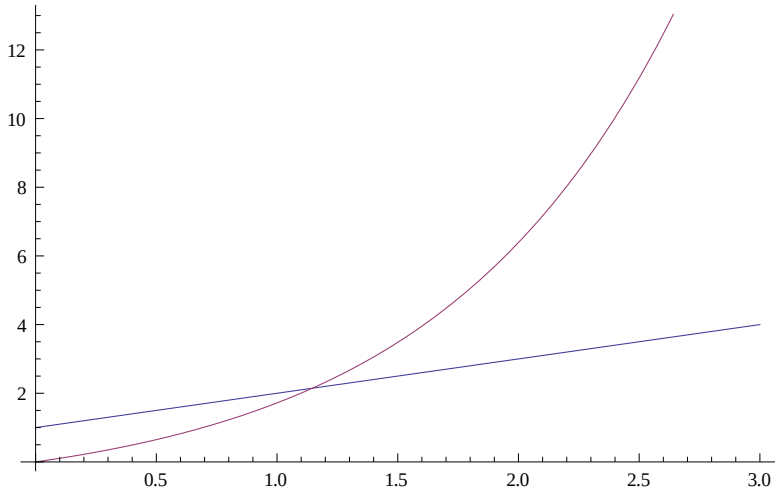
```
D[g[P], P] /. m → 1
```

$$-e^P\ (-1 + e^P)\ \text{DiracDelta}[-1 + e^P] - e^P\ \text{HeavisideTheta}[-1 + e^P]$$

```
Plot[{1 (P + 1), -g[Log[P]] /. m → 1}, {P, 0, 3}]
```




```
Plot[{1 (P + 1), -g[P] /. m → 1}, {P, 0, 3}]
```



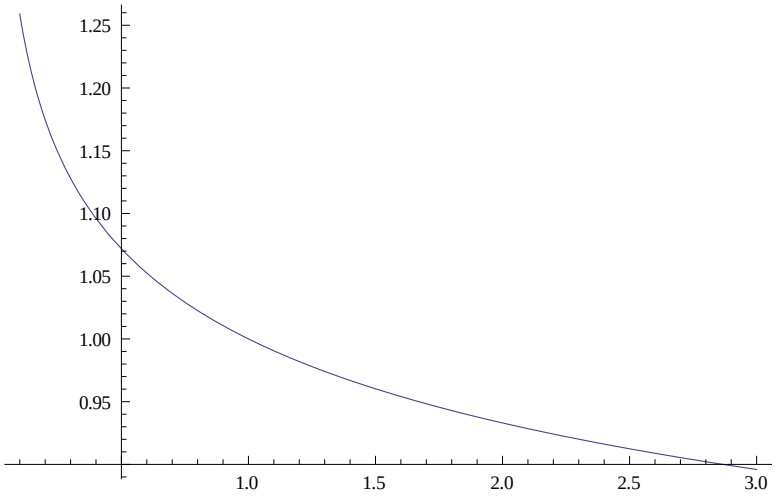

```
h = HessianH[ σ[S, t] u q1, {P, S}]
```

$$\{\{0, 0\}, \{0, q1\ u\ \sigma^{(2,0)}[S, t]\}\}$$

Eigenvalues[h]

$\{0, q1\ u\ \sigma^{(2,0)}[S, t]\}$

Plot[S^(0.9 - 1), {S, 0.1, 3}]



Simplify[D[S^(b - 1), S, S] / S^(-3+b)]

$(-2 + b)\ (-1 + b)$

Plot[(-2 + b) (-1 + b), {b, 0, 1}]