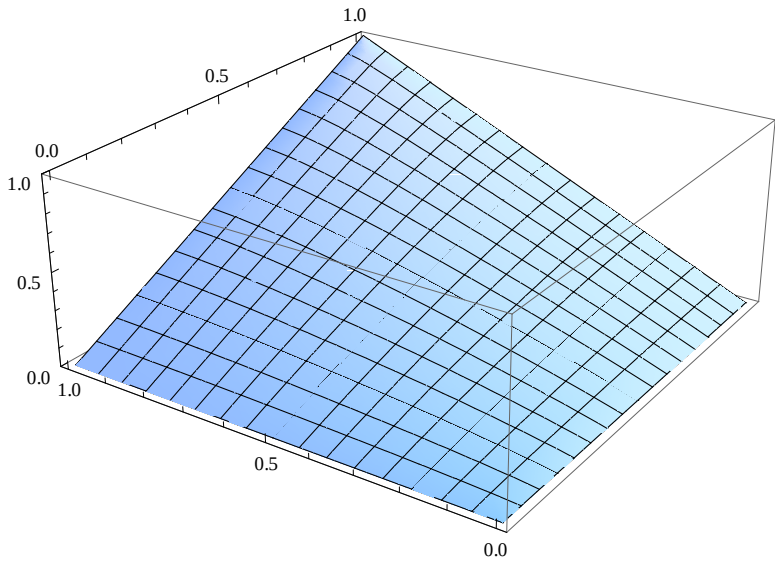
Maximum copula

```
A = 1.1; Plot3D[c[x, y, 1, A], {x, 0, 1}, {y, 0, 1}, PlotRange → {0, 1}]
```

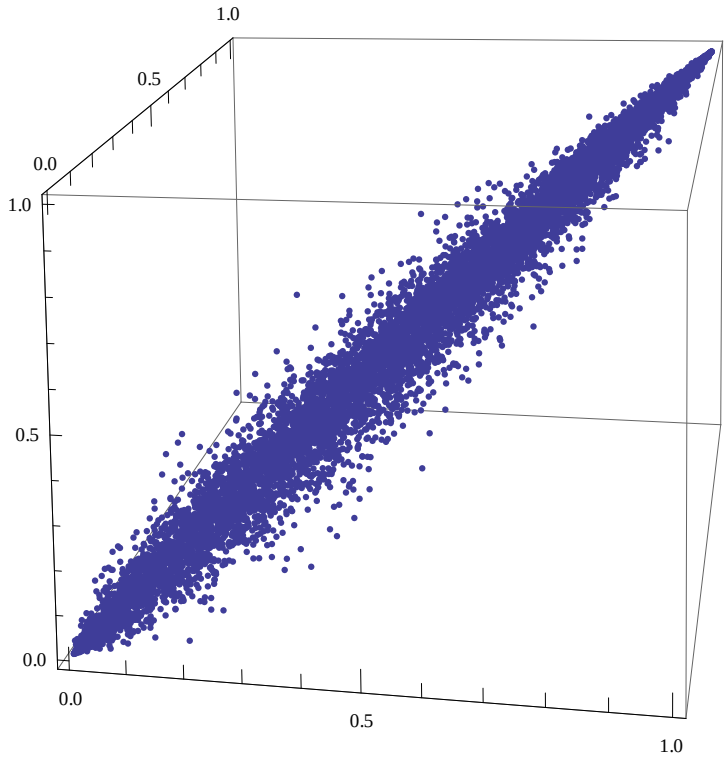



```
SeedRandom[];
```

```
Y = .
```

```
U = {}; A = 10;
Timing[For[i = 0, i < 10 000, i++,
  x = RandomReal[];
  y = f1[x, RandomReal[], A];
  z = f2[x, y, RandomReal[], A];
  AppendTo[U, {x, y, z}]]]
ListPointPlot3D[U, AspectRatio → 1]
```

```
{6.9, Null}
```




```
U[[4]]
```

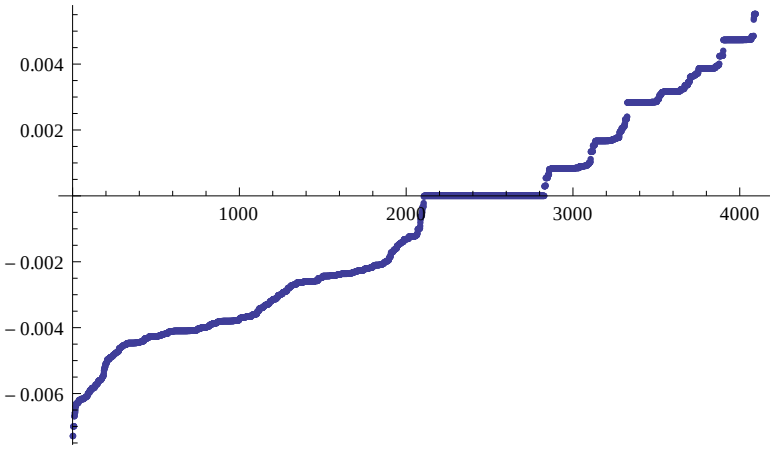
```
{0.170666, 0.0953471, 0.0502946}
```

```
M = {}; m = {}; l = Length[U]; h = 1 / 15;
For[i = 0, i ≤ 1, i += h,
 For[j = 0, j ≤ 1, j += h,
  For[k = 0, k ≤ 1, k += h,
   AppendTo[m,
    Length[Select[U, #[[1]] ≤ i && #[[2]] ≤ j && #[[3]] ≤ k &]] / l - c[i, j, k, A] // N]
  ]
 ]]; m = Sort[m];
```

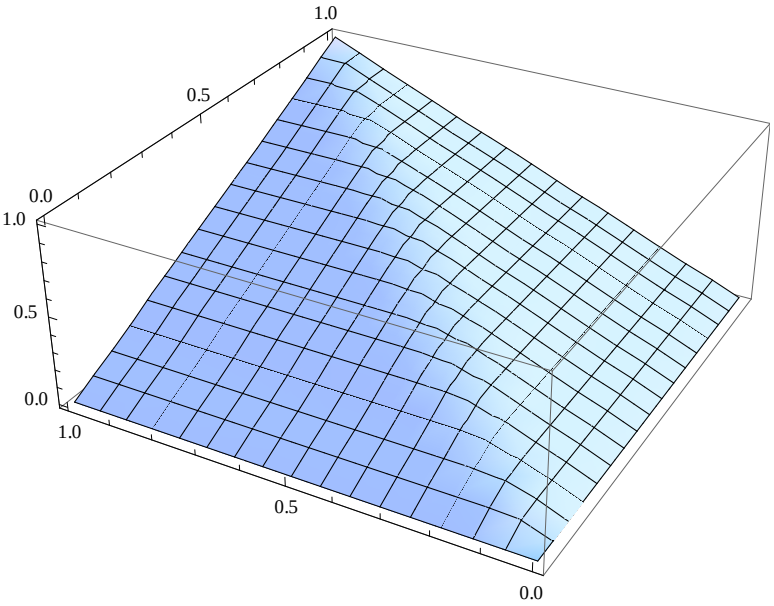
```
m[[3]]
```

```
0.
```

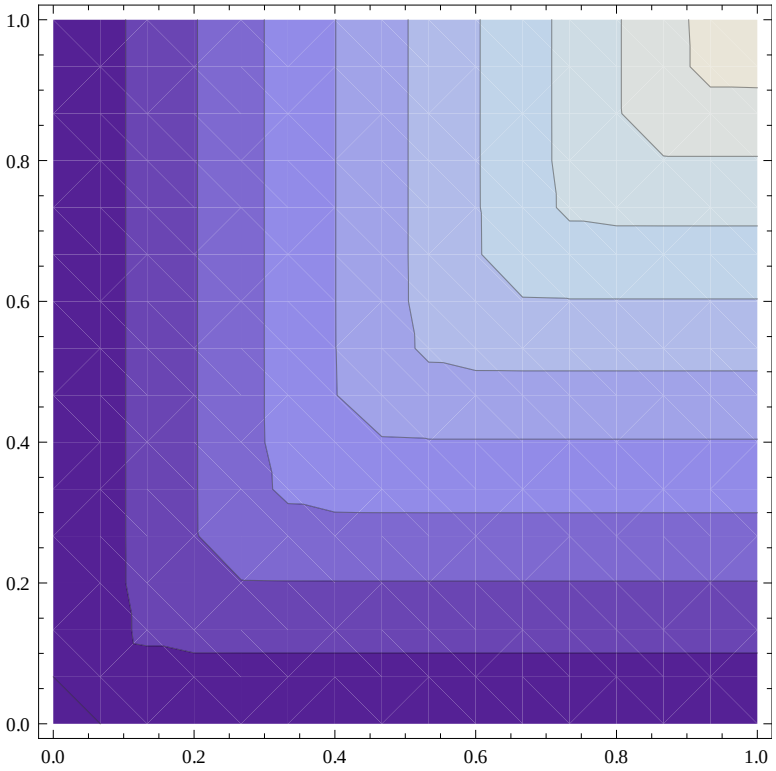
ListPlot[m]



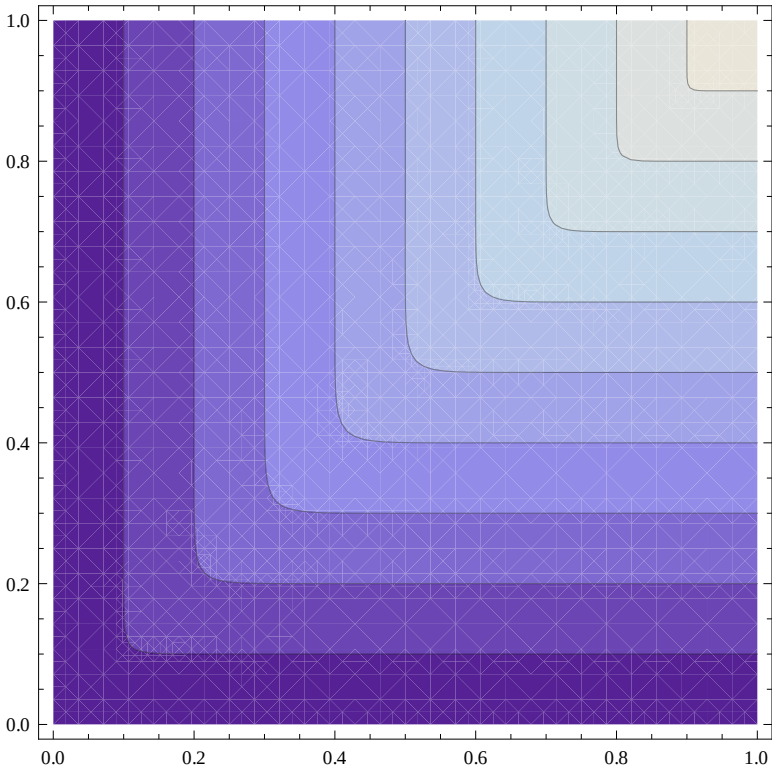
ListPlot3D[M]



ListContourPlot[M]



ContourPlot[c[x, y, 15], {x, 0, 1}, {y, 0, 1}]



```
f1 = Compile[{{x, _Real}, {Z2, _Real}, {a, _Real}},
  E^(-(-(-Log[x])^a + ((-1 + a) ProductLog[(-x Z2 (-Log[x])^-a Log[x] (-Log[x])^(-1/(-1 + a)))/(-1 + a)])^a)^(1/a)];

Exit[]

c[x_, y_, z_, a_] := Exp[-((-Log[x])^a + (-Log[y])^a + (-Log[z])^a)^(1/a)]

Simplify[D[c[x, y, 1, a], x]]
```

$$\frac{1}{x}\, e^{-\left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{\frac{1}{a}}}\, (-\text{Log}[x])^{-1+a}\, \left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{-1+\frac{1}{a}}$$

```
Solve[% == Z2, y]
```

InverseFunction::ifun :
Inverse functions are being used. Values may be lost for multivalued inverses. ≫

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{y \to e^{-\left(-(-\text{Log}[x])^a+\left((-1+a)\,\text{ProductLog}\left[\frac{(-x\ Z2\ (-\text{Log}[x])^{-a}\ \text{Log}[x])^{-\frac{1}{-1+a}}}{-1+a}\right]\right)^a\right)^{\frac{1}{a}}}\right\}\right\}$$

```
a =.

z =.

x =.; y =.

Simplify[D[D[c[x, y, z, a], x], y] / D[D[c[x, y, 1, a], x], y]
```

$$\left(e^{\left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{\frac{1}{a}}-\left((-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a\right)^{\frac{1}{a}}}\right.$$
$$\left(-1+a+\left((-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a\right)^{\frac{1}{a}}\right)$$
$$\left.\left(\frac{(-\text{Log}[x])^a+(-\text{Log}[y])^a}{(-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a}\right)^{2-\frac{1}{a}}\right) \Big/ \left(-1+a+\left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{\frac{1}{a}}\right)$$

```
Solve[% == Z3, z]

f2[x_, y_, Z3_, a_] := FindRoot[c2[x, y, z, a] == Z3, {z, 0.5}][[1, 2]]

Clear[f2]
```

$$\texttt{c2[x\_, y\_, z\_, a\_] :=}\ \left(e^{\left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{\frac{1}{a}}-\left((-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a\right)^{\frac{1}{a}}}\right.$$
$$\left(-1+a+\left((-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a\right)^{\frac{1}{a}}\right)$$
$$\left.\left(\frac{(-\text{Log}[x])^a+(-\text{Log}[y])^a}{(-\text{Log}[x])^a+(-\text{Log}[y])^a+(-\text{Log}[z])^a}\right)^{2-\frac{1}{a}}\right)\Big/\ \left(-1+a+\left((-\text{Log}[x])^a+(-\text{Log}[y])^a\right)^{\frac{1}{a}}\right)$$

```
f2[x_, y_, z3_, a_] :=
 FindRoot[c2[x, y, z, a] - z3, {z, 0.0000000000001, 1}, Method → "Brent"][[1, 2]]
```

```
Clear[z]
```

```
Exit[]
```

```
c2[xx, yy, z, 3]
```

$$1.38392\ e^{1.35983-\left(2.51453-\text{Log}[z]^3\right)^{1/3}}\left(\frac{1}{2.51453-\text{Log}[z]^3}\right)^{5/3}\left(2+\left(2.51453-\text{Log}[z]^3\right)^{1/3}\right)$$

```
FindRoot[c2[xx, yy, z, 3] == 0.1, {z, 0.5}]
```

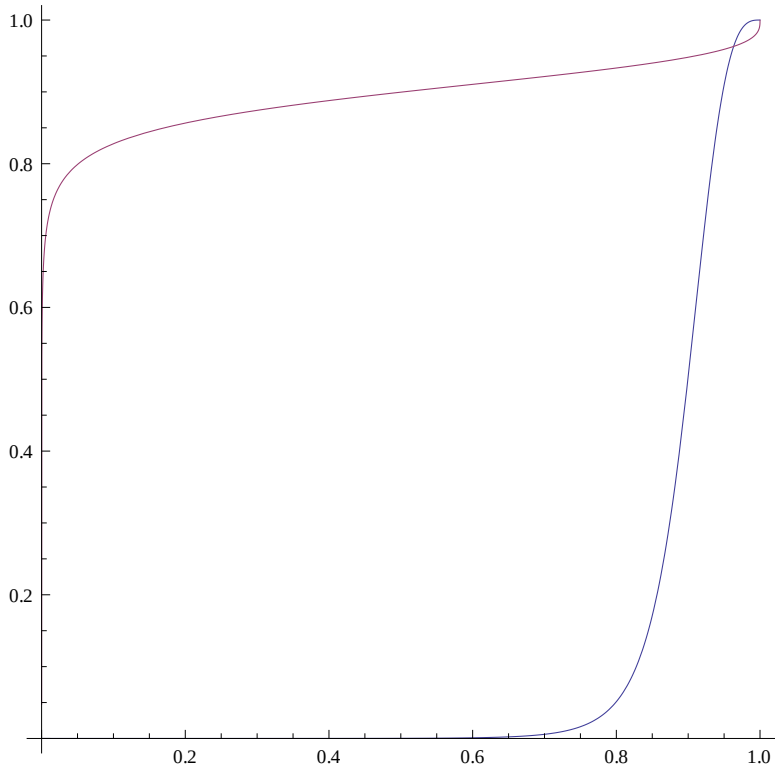
```
{z → 0.177796}
```

```
f2[.1, 0.3, 0.1, 3]
```

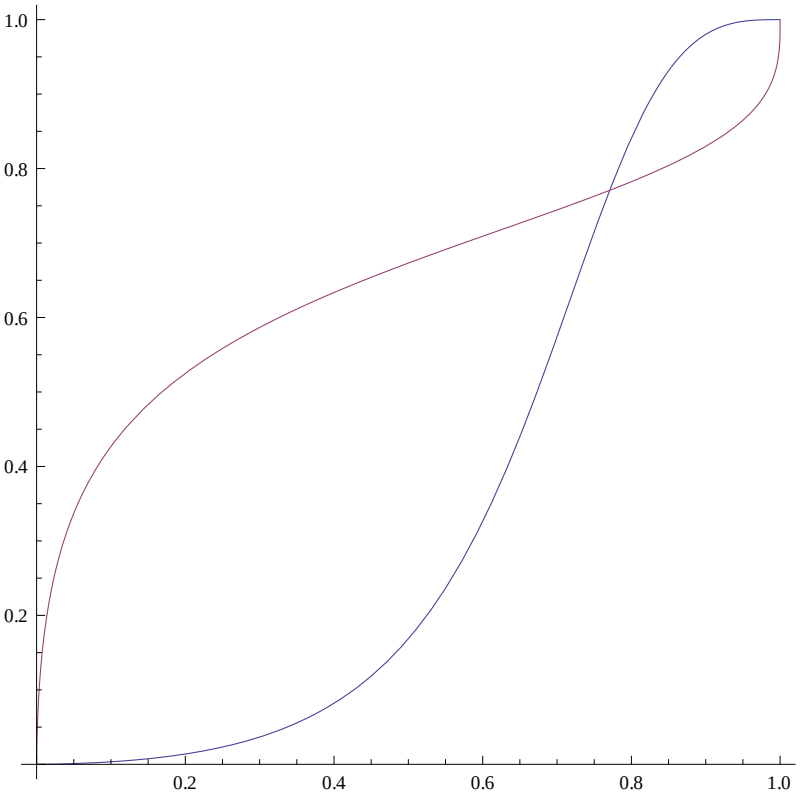
```
0.0600909
```

```
3
```

```
xx = 0.9; yy = 0.9; Plot[{c2[xx, yy, y, 3], f[xx, yy, y, 3]}, {y, 0, 1}, AspectRatio → 1]
```



```
xx = 0.7; A = 3; Plot[{D[c[x, y, A], x] /. x → xx, f[y, A, xx]},
 {y, 0, 1}, PlotRange → All, AspectRatio → 1]
```




```
D[c[x, y, A], x]
```

```
f[AA, BB, CC]
```

$$e^{-\left(-(-\text{Log}[CC])^{BB}+\left((-1+BB)\ \text{ProductLog}\left[\frac{(-AA\ CC\ (-\text{Log}[CC])^{-BB}\ \text{Log}[CC])^{-\frac{1}{-1+BB}}}{-1+BB}\right]\right)^{BB}\right)^{\frac{1}{BB}}}$$

```
Solve[D[c[x, y, A], x] == Z, y]
```

InverseFunction::ifun :
  Inverse functions are being used. Values may be lost for multivalued inverses. ≫

Solve::ifun : Inverse functions are being used by Solve, so some
    solutions may not be found; use Reduce for complete solution information. ≫

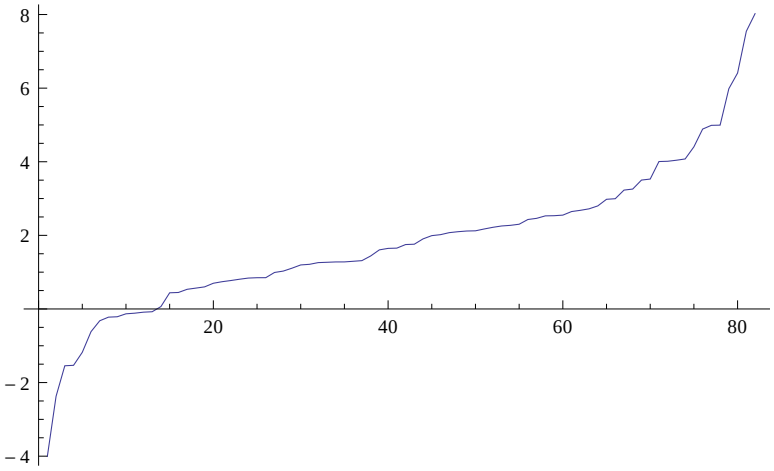
$$\left\{\left\{y \to e^{-\left(-(-\text{Log}[x])^{A}+\left((-1+A)\ \text{ProductLog}\left[\frac{(-x\ Z\ (-\text{Log}[x])^{-A}\ \text{Log}[x])^{-\frac{1}{-1+A}}}{-1+A}\right]\right)^{A}\right)^{\frac{1}{A}}}\right\}\right\}$$

```
$Assumptions =
 a > 1 && 0 < Z < 1 && 0 < x < 1 && 0 < y < 1 && 0 < z < 1 && 0 < Z1 < 1 && 0 < Z2 < 1 && A > 1
```

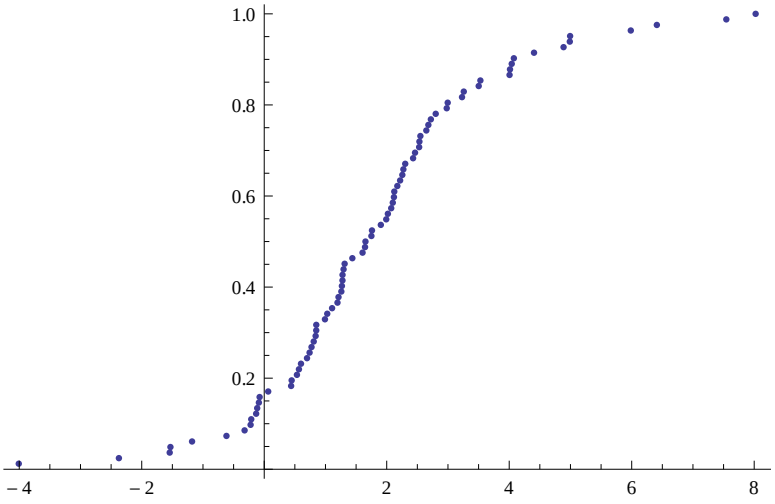
a > 1 && 0 < Z < 1 && 0 < z < 1 && 0 < Z1 < 1 && 0 < Z2 < 1

```
a =.
```

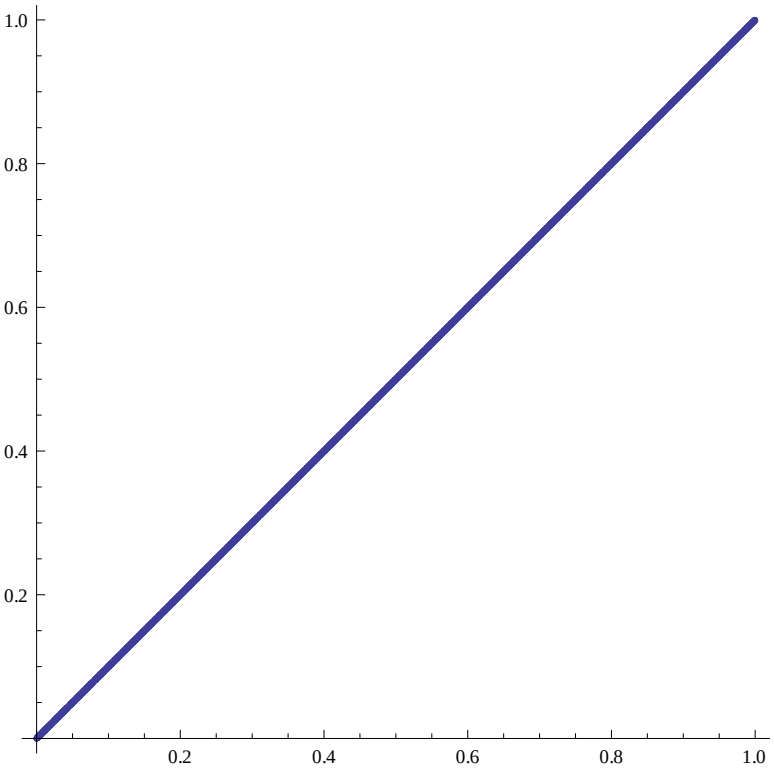
```
U = Sort[RandomReal[NormalDistribution[2, 2], 82]];
ListPlot[U, Joined -> True]
n = Length[U]
F = Table[{U[[i]], i / n}, {i, 1, n}];
ListPlot[F]
```



82



```
ListPlot[U, AspectRatio → 1]
```




```
F[a_, b_] := 10 000 - Sort[
    Tally[UnitStep[a - #1[[2]]] UnitStep[b - #1[[1]]] & /@ U], #1[[1]] < #2[[1]] &][[1, 2]]
```

```
Sort[Tally[UnitStep[20 / 20 - #1[[2]]] UnitStep[20 / 20 - #1[[1]]] & /@ U],
 #1[[1]] < #2[[1]] &]
```

```
{{1, 10 000}}
```

```
n = 40; G = {};
For[i = 0, i < n, i++,
 For[j = 0, j < n, j++,
  AppendTo[G, {i / n, j / n, F[i / n, j / n]}];
 ]]
```

```
G[[3]]
```

$$\left\{0, \frac{1}{10}, 0\right\}$$

```
ListPlot3D[G, PlotRange → All, BoxRatios → {1, 1, 1}]
```