```
Exit[];
```

```
$Assumptions = r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
```

$$r > 0 \&\& m \in \text{Integers} \&\& n \in \text{Integers} \&\& s > 0 \&\& k \in \text{Integers} \&\& k > 0$$

```
m = 2;
```

```
f[r_, En_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}} -
  0 * IdentityMatrix[2] * I * r ^ p; f[r, En] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{r} & i\,(\text{En} - r^2) \\ i\,(\text{En} - r^2) & -\frac{2}{r} \end{pmatrix}$$

```
En =.; n =.
```

```
fE = D[f[r, En], En]
```

$$\{\{0, i\}, \{i, 0\}\}$$

```
u = {a[n] * x ^ (2 * n), b[n] * x ^ (2 * n + 1)} * x ^ s
```

$$\left\{x^{209}\, a[100],\ x^{210}\, b[100]\right\}$$

```
p = 2;
```

```
r[x_] := x;
```

```
g1 = Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x]].u) * x ^ (-s + 1)]]],
   {x ^ n, a[n], b[n]}];
g1 // MatrixForm
```

$$\begin{pmatrix} x^{2n}\left((1 - m + 2n + s)\, a[n] + \left(-i\,\text{En}\, x^2 + i\, x^4\right) b[n]\right) \\ x^{2n}\left(\left(-i\,\text{En}\, x + i\, x^3\right) a[n] + (x + m\, x + 2n\, x + s\, x)\, b[n]\right) \end{pmatrix}$$

```
s = -1 + m;
```

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 0 & -i\,\text{En}\, a[0] + 2\, m\, b[0] \\ 2\, a[1] - i\,\text{En}\, b[0] & 0 \\ 0 & i\, a[0] - i\,\text{En}\, a[1] + 2\,(1 + m)\, b[1] \\ 4\, a[2] + i\,(b[0] - \text{En}\, b[1]) & 0 \\ 0 & i\, a[1] - i\,\text{En}\, a[2] + 2\,(2 + m)\, b[2] \\ 6\, a[3] + i\,(b[1] - \text{En}\, b[2]) & 0 \\ 0 & i\, a[2] - i\,\text{En}\, a[3] + 2\,(3 + m)\, b[3] \\ 8\, a[4] + i\,(b[2] - \text{En}\, b[3]) & 0 \\ 0 & i\, a[3] - i\,\text{En}\, a[4] + 2\,(4 + m)\, b[4] \\ 10\, a[5] + i\,(b[3] - \text{En}\, b[4]) & 0 \end{pmatrix}$$

```
a[0] = 1;  b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
```

```
a[0] = 1;  b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
b[n_] := I * (En * a[n] - a[n - 1]) / 2 / (n + m);
a[n_] := Simplify[I * (En b[n - 1] - b[n - 2]) / 2 / n]
```

```
a[1]
```

$$-\frac{En^2}{8}$$

```
m = 2;
```

```
En =.
```

```
Un[En_, m_, nN_, x_] := Module[{n, U},
  U = {1}; AppendTo[U, -En^2/(4 m)]; AppendTo[U, En (4 + En^3 + 8 m)/(32 m (1 + m))]; G = {i En / 2 / m};

  For[n = 3, n < nN, n++,
   AppendTo[U,
    -((-1 + m + n) U[[-2 + n]] + En ((3 - 2 m - 2 n) U[[-1 + n]] + En (-2 + m + n) U[[n]])) /
      (4 n (-2 + m + n) (-1 + m + n))];
  ];
  ({1, i En / 2 / m * x} +
     Sum[{U[[n + 1]] * x^(2 * n), I * (En * U[[n + 1]] - U[[n]]) / 2 / (n + m) * x^(2 * n + 1)},
      {n, 1, nN - 1}]) * x^(-1 + m) // N]


U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
   n++;
   U = G; G = -Un[En, m, n + 1, X];

  ];
  {Un[En, m, n, X], n}]
```

```
Ener[Ene_] :=
 Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 3500;
  m = 2;
  r = 17 // N; h = -16.9 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
   k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
   k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
   k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
   k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
   kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   r += h;

   , {n}];

  NN = U[En, m, 10^-10, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn → En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.06,
   En -= Erg / (U2S k⟦1⟧ - U1S k⟦2⟧ + U2 kE⟦1⟧ - U1 kE⟦2⟧);
   (*Print[{En,Erg/U2/k[[2]]}];*)Goto[begin];
   ];
  {En, Erg / U2 / k[[2]]}
 ]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
 Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[3.8]
```

$$\{3.61902 + 0.265337\,i,\ -0.00109633 - 0.000496109\,i\}$$

```
Energie // MatrixForm
```

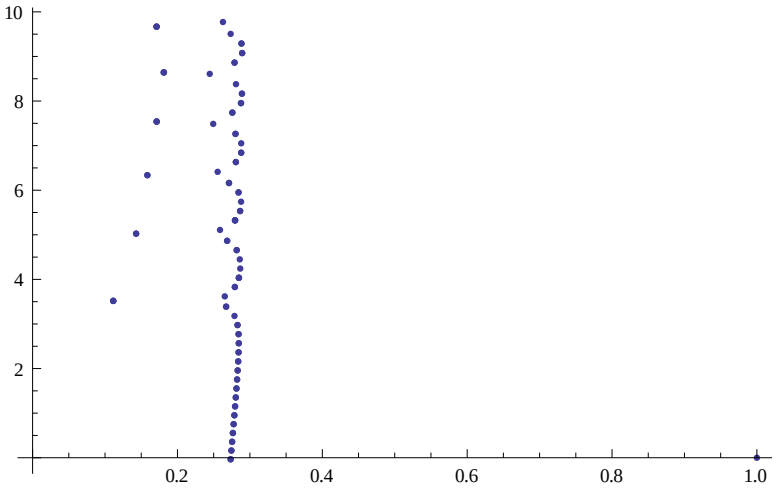
$$\begin{pmatrix}
0 & \{-0.0348248 + 0.273429\,i,\ 0.0824271 + 0.00377436\,i\} \\
0.1 & \{-0.0348248 + 0.273429\,i,\ 0.0214202 - 0.0233512\,i\} \\
0.2 & \{0.161054 + 0.274461\,i,\ -0.00191844 + 0.059646\,i\} \\
0.3 & \{0.161054 + 0.274461\,i,\ 0.0430159 + 0.0247969\,i\} \\
0.4 & \{0.357691 + 0.275498\,i,\ -0.0239546 + 0.0157214\,i\} \\
0.5 & \{0.357691 + 0.275498\,i,\ 0.0119417 + 0.0477695\,i\} \\
0.6 & \{0.555088 + 0.276533\,i,\ -0.0148071 + 0.000774966\,i\}
\end{pmatrix}$$

```
0.7  {0.555088+0.276533 i, -0.00890224+0.0489729 i}
0.8  {0.753241+0.277561 i, -0.00846342-0.00200356 i}
0.9  {0.753241+0.277561 i, -0.0231779+0.047652 i}
1.   {0.952144+0.278576 i, 0.0783007+0.0337232 i}
1.1  {0.952144+0.278576 i, -0.0355808+0.0442028 i}
1.2  {1.15178+0.279569 i, 0.0505876+0.0270738 i}
1.3  {1.15178+0.279569 i, -0.045905+0.0363295 i}
1.4  {1.35215+0.280534 i, 0.03465+0.019346 i}
1.5  {1.35215+0.280534 i, -0.0511494+0.0227949 i}
1.6  {1.55321+0.281461 i, 0.0245336+0.0125636 i}
1.7  {1.55321+0.281461 i, -0.0479442+0.00618525 i}
1.8  {1.75494+0.282337 i, 0.0174311+0.00717699 i}
1.9  {1.75494+0.282337 i, -0.0358656-0.00734238 i}
2.   {1.9573+0.283143 i, 0.0119518+0.00328444 i}
2.1  {1.9573+0.283143 i, -0.0202808-0.0123854 i}
2.2  {2.16024+0.283843 i, 0.00759541+0.000846598 i}
2.3  {2.16024+0.283843 i, -0.00901651-0.00938155 i}
2.4  {2.36368+0.284381 i, -0.0649912+0.00531311 i}
2.5  {2.36368+0.284381 i, -0.00765636-0.00670322 i}
2.6  {2.56752+0.284638 i, -0.0300368+0.00916306 i}
2.7  {2.56752+0.284638 i, 0.017086+0.00489736 i}
2.8  {2.7716+0.284365 i, -0.0102766+0.00631036 i}
2.9  {2.7716+0.284365 i, 0.00255367-0.007645 i}
3.   {2.97573+0.282951 i, 0.0324564-0.0390568 i}
3.1  {2.97573+0.282951 i, -0.0129679+0.0493867 i}
3.2  {3.18002+0.278733 i, 0.00561139-0.0124434 i}
3.3  {3.38843+0.267181 i, -0.00706132-0.00222546 i}
3.4  {3.38843+0.267181 i, 0.0716424+0.0146798 i}
3.5  {3.51788+0.111334 i, 0.00243169+0.000403973 i}
3.6  {3.51788+0.111334 i, -0.0000168755+0.000409094 i}
3.7  {3.51788+0.111334 i, 0.0000574665-0.000997058 i}
3.8  {3.61902+0.265337 i, 0.0159846+0.00559245 i}
3.9  {3.83002+0.279236 i, -0.00334078-0.00754872 i}
4.   {3.83002+0.279236 i, -0.0031197-0.0264905 i}
4.1  {4.03602+0.284821 i, 0.0202834+0.030361 i}
4.2  {4.03602+0.284821 i, 0.00672479-0.00922651 i}
4.3  {4.24235+0.286678 i, 0.011193+0.00918883 i}
4.4  {4.03602+0.284821 i, 0.00496042+0.0116323 i}
4.5  {4.44907+0.286035 i, -0.0699411-0.0157006 i}
4.6  {4.65592+0.281786 i, -0.0128516+0.0066142 i}
4.7  {4.65592+0.281786 i, -0.0326016+0.0063652 i}
4.8  {4.8653+0.268593 i, -0.00101728+0.00822936 i}
4.9  {4.8653+0.268593 i, 0.0328955+0.00350871 i}
5.   {5.02602+0.142922 i, 0.00271581+0.0632135 i}
5.1  {5.02603+0.142926 i, -0.000932623-0.000541224 i}
5.2  {5.32464+0.279517 i, -0.00715488+0.00675674 i}
5.3  {5.10897+0.258743 i, 0.0175854-0.0479305 i}
5.4  {5.32464+0.279518 i, 0.000118089+0.0186131 i}
5.5  {5.32464+0.279517 i, 0.00924017-0.00980417 i}
5.6  {5.53302+0.286516 i, -0.0380211-0.0675122 i}
```

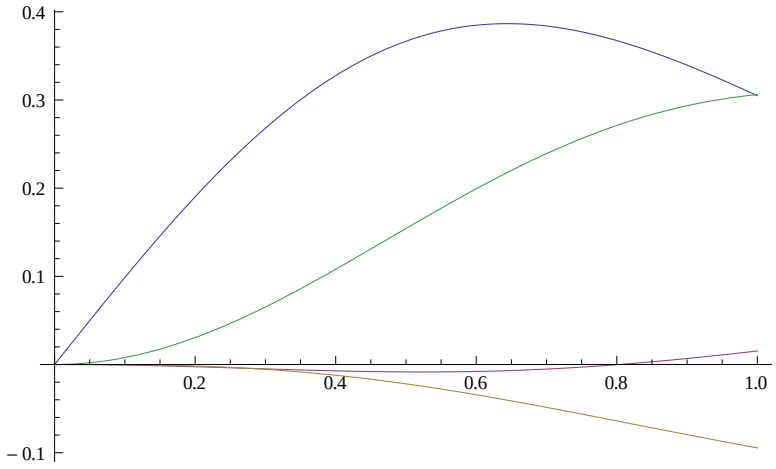
```
5.7  {5.32464+0.279518 i, 0.0105739+0.015473 i}
5.8  {5.74162+0.287838 i, -0.0199566-0.0134933 i}
5.9  {5.95051+0.284332 i, -0.0354819-0.0250697 i}
6.   {5.95051+0.284332 i, -0.00913623-0.00047985 i}
6.1  {6.16131+0.271158 i, 0.0252866-0.0560522 i}
6.2  {6.16131+0.271152 i, 0.0101888+0.0123834 i}
6.3  {6.33701+0.158456 i, -0.00105735-0.000319741 i}
6.4  {6.33701+0.158456 i, -0.000671294+0.00110609 i}
6.5  {5.53302+0.286517 i, -0.0310481-0.0398119 i}
6.6  {6.41235+0.255483 i, 0.0235597-0.0317144 i}
6.7  {6.63113+0.280631 i, 0.00382499+0.00786223 i}
6.8  {6.63113+0.280631 i, -0.0077521+0.00818432 i}
6.9  {6.84154+0.287981 i, -0.022262-0.0132369 i}
7.   {6.84154+0.287983 i, 0.00101079-0.0076893 i}
7.1  {7.05207+0.28802 i, -0.00646432-0.000525755 i}
7.2  {7.26328+0.28014 i, -0.00507427+0.00750115 i}
7.3  {7.26327+0.280144 i, 0.0673018+0.00510217 i}
7.4  {7.48771+0.249352 i, -0.0063826-0.00716244 i}
7.5  {7.53947+0.171252 i, 0.00125585+0.00522793 i}
7.6  {7.53948+0.171245 i, -0.0014772+0.00081522 i}
7.7  {7.53949+0.171252 i, 0.00201748-0.00137817 i}
7.8  {7.74042+0.275795 i, 0.0535+0.0351612 i}
7.9  {7.74041+0.27578 i, 0.0102428-0.0180593 i}
8.   {7.9533+0.287735 i, -0.0215442+0.012056 i}
8.1  {7.95331+0.287745 i, 0.0599208-0.0229489 i}
8.2  {8.16549+0.289055 i, 0.0735446-0.0426555 i}
8.3  {8.16548+0.289041 i, 0.00452665+0.0332604 i}
8.4  {8.37849+0.280812 i, 0.0165815-0.0119099 i}
8.5  {8.6095+0.244542 i, -0.0122388-0.000407866 i}
8.6  {8.64233+0.181294 i, 0.00614591+0.00163093 i}
8.7  {8.64232+0.181244 i, -0.0052266+0.00204766 i}
8.8  {8.64234+0.181016 i, -0.0559045-0.00772336 i}
8.9  {8.86137+0.278848 i, 0.0378659+0.0469596 i}
9.   {8.86138+0.27881 i, -0.00298264-0.0446443 i}
9.1  {9.07571+0.289363 i, 0.0192018-0.00629468 i}
9.2  {9.0757+0.289343 i, -0.0655082+0.0448379 i}
9.3  {9.28963+0.288376 i, 0.00576278-0.0502462 i}
9.4  {9.28962+0.288357 i, -0.0151671+0.0315395 i}
9.5  {9.50588+0.273435 i, -0.000351129-0.0116833 i}
9.6  {9.66759+0.171037 i, -0.0194593+0.0666014 i}
9.7  {9.66791+0.171195 i, -0.00189307-0.00197333 i}
9.8  {9.66789+0.171216 i, 0.00293916+0.0000546853 i}
9.9  {9.77232+0.262805 i, -0.00545773+0.017099 i}
```

```
Energien = {Im[#[[2, 1]]], Re[#[[2, 1]]]} & /@ Energie;
```

```
ListPlot[Append[Energien, {1, 0}], AxesOrigin → {0, 0}, PlotRange → All]
```




```
G = {Re[#], Im[#]} &[Un[Energie[[1]], 2, 15, x]]; Plot[G, {x, 0, 1}, PlotRange → All]
```



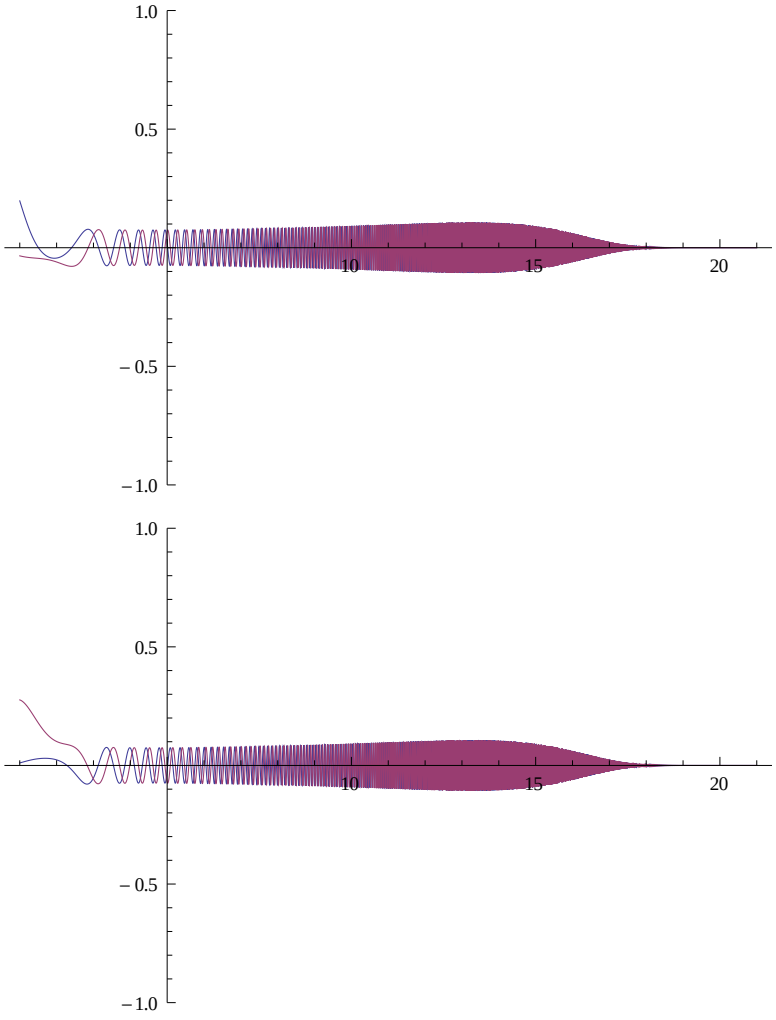

```
En = Energie[[1]]; n = 3500;
m = 2;
r = 17 // N; h = -16.9 / n;
k = {1, -1}; kK = {{r, k}};
Do[
 k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
 k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
 k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[kK, {r, k}], {n}]; En =.
```

```
k / k[[1]] - Un[Energie[[1]], 2, 20, 0.1] / Un[Energie[[1]], 2, 20, 0.1][[1]]
```

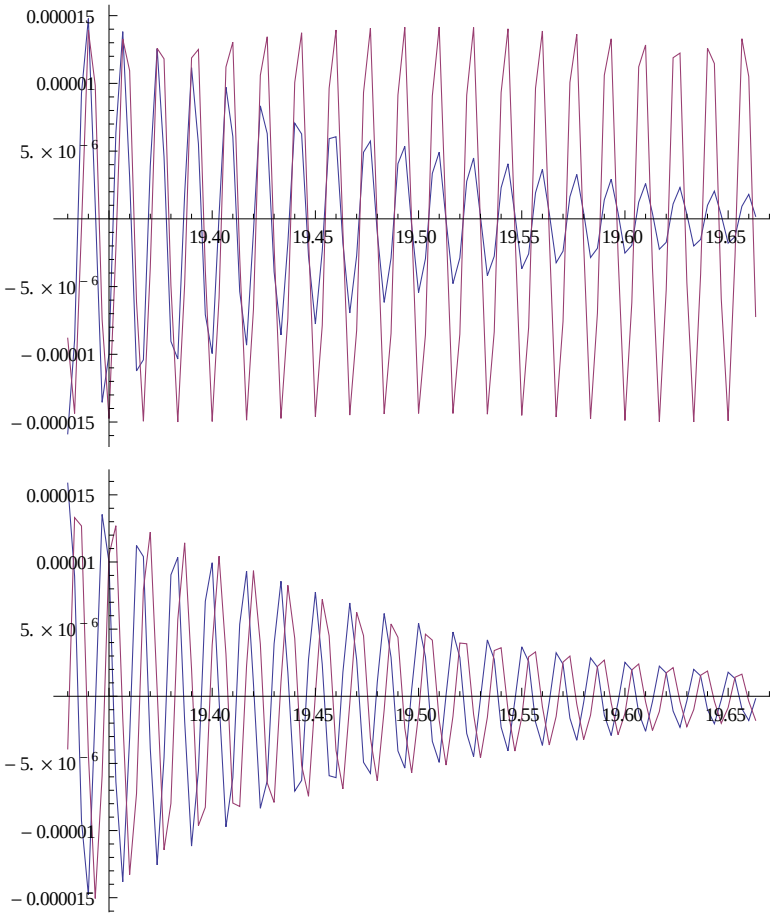
$$\left\{0. + 7.92188 \times 10^{-19}\, i,\ 0.0000440082 + 0.0000213747\, i\right\}$$

```
n = 6000; S = 1; h = 20 / n; ra = 1; En = 3.517884643304668 + 0.11133398841863644 i;
m = 2; r = 1; k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N},
  { {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N}],
 PlotRange → {-ra, ra}, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N},
  { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N}],
 PlotRange → {-ra, ra}, Joined → True]
En =.;
r =.;
```



```
S = 5500; n = 100; ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], 0.000015 * Cos[#[[1]]^3 / 3 - #[[1]] * Re[3.517884613174765]]} & /@
      kK[[S ;; S + n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N}], PlotRange → All, Joined → True]
```




```
Exp[I * Im[Energie[[1]]] * x]
```

$$e^{0.278733\, i\, x}$$