

```

H[u_] := Simplify[x^2 * u - D[u, {x, 2}]];

ClearAll[e, g, f, h, p];
h := Exp[-2 * x^2]; f := h; n = 8; e = Table[0, {j, n+1}, {k, n+1}];

Print[Timing[
  i = 1;
  g[1] = f / Sqrt[Integrate[f^2, {x, -Infinity, Infinity}]];
  p = H[g[1]];
  e[[1, 1]] = Integrate[p * g[1], {x, -Infinity, Infinity}];
  e[[2, 1]] = Sqrt[Integrate[p^2, {x, -Infinity, Infinity}]] - e[[1, 1]]^2;
  g[2] = Simplify[(p - g[1] * e[[1, 1]]) / e[[2, 1]]];

  While[i < n, i++;
    p = Simplify[H[g[2]] / h] * h;
    e[[i, i]] =
      Integrate[Simplify[Collect[p * g[2] / h^2, x]] * h^2, {x, -Infinity, Infinity}];
    e[[i+1, i]] = Sqrt[Integrate[Simplify[Collect[p^2 / h^2, x]] * h^2,
      {x, -Infinity, Infinity}]] - e[[i, i]]^2 - e[[i, i-1]]^2;
    g[3] = Simplify[Collect[(p - e[[i, i]] * g[2] - e[[i, i-1]] * g[1]) / e[[i+1, i]] / h,
      x]] * h;
    g[1] = g[2]; g[2] = g[3];];

  e[[n+1, n+1]] =
    Integrate[Collect[H[g[3]] * g[3] / h^2, x] * h^2, {x, -Infinity, Infinity}];
  ]][[1]] Sekunden, N[e + Transpose[e] - DiagonalMatrix[Diagonal[e]] // MatrixForm],
  N[Eigenvalues[e + Transpose[e] - DiagonalMatrix[Diagonal[e]]] // MatrixForm, 50]]

```

13.88 Sekunden

2.125	2.65165	0.	0.	0.	0.	0.	0.	0.
2.65165	10.625	6.49519	0.	0.	0.	0.	0.	0.
0.	6.49519	19.125	10.2698	0.	0.	0.	0.	0.
0.	0.	10.2698	27.625	14.0312	0.	0.	0.	0.
0.	0.	0.	14.0312	36.125	17.7878	0.	0.	0.
0.	0.	0.	0.	17.7878	44.625	21.5421	0.	0.
0.	0.	0.	0.	0.	21.5421	53.125	25.2951	0.
0.	0.	0.	0.	0.	0.	25.2951	61.625	29.0474
0.	0.	0.	0.	0.	0.	0.	29.0474	70.125

102.92070561117968089448447430589402983368293574151
73.799103158735436931492382625375309546215052385423
53.155321262048790150977992145456420601503331577544
37.497960616644247580431574633370195629615356050247
25.475465217987905401620151125071629139222492579720
16.396829931794530712628280635620650344141908347845
9.8139961026166781338614096776662951492497242897140
5.0650965212261087793744230680352128302138574606633
1.0005215777666214151293117835102569261553415673304

**e + Transpose[e] - DiagonalMatrix[Diagonal[e]] // MatrixForm**

$$\begin{pmatrix} \frac{7}{4} & \sqrt{\frac{\frac{103}{10}}{2}} & 0 & 0 & 0 \\ \sqrt{\frac{\frac{103}{10}}{2}} & \frac{2653}{412} & 3\sqrt{\frac{291881}{5}} & 0 & 0 \\ 0 & 3\sqrt{\frac{291881}{5}} & \frac{1249472081}{120254972} & 3\sqrt{\frac{2241037129605}{2}} & 0 \\ 0 & 0 & 3\sqrt{\frac{2241037129605}{2}} & \frac{24138533844651055}{1693504617284756} & 3\sqrt{\frac{24166847972273674161}{2}} \\ 0 & 0 & 0 & 3\sqrt{\frac{24166847972273674161}{2}} & \frac{8897597912992941698788595}{480390869151008991060756} \end{pmatrix}$$