```
Exit[];
```

```
$Assumptions = r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
```

$$r > 0 \&\& m \in \text{Integers} \&\& n \in \text{Integers} \&\& s > 0 \&\& k \in \text{Integers} \&\& k > 0$$

```
m = 5;
```

```
f[r_, En_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}} -
  0 * IdentityMatrix[2] * I * r ^ p; f[r, En] // MatrixForm
```

$$\begin{pmatrix} \frac{4}{r} & i\left(\text{En} - r^4\right) \\ i\left(\text{En} - r^4\right) & -\frac{5}{r} \end{pmatrix}$$

```
En =.; n =.; p = 4
```

$$4$$

```
fE = D[f[r, En], En]
```

$$\{\{0, i\}, \{i, 0\}\}$$

```
u = {a[n] * x ^ (2 * n), b[n] * x ^ (2 * n + 1)} * x ^ s
```

$$\left\{x^{2n+s}\, a[n],\ x^{1+2n+s}\, b[n]\right\}$$

```
r[x_] := x;
```

```
g1 = Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x], En].u) * x ^ (-s + 1)]]],
   {x ^ n, a[n], b[n]}];
g1 // MatrixForm
```

$$\begin{pmatrix} x^{2n}\left((-4 + 2n + s)\, a[n] + \left(-i\,\text{En}\, x^2 + i\, x^6\right) b[n]\right) \\ x^{2n}\left(\left(-i\,\text{En}\, x + i\, x^5\right) a[n] + (6x + 2n\, x + s\, x)\, b[n]\right) \end{pmatrix}$$

```
s = -1 + m;
```

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
a[0] = 1; b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
```

```
b[1] = -(i En^3 / (4 m)) / 2 / (1 + m) ; a[2] = i En b[1] / 4;
```

```
b[n_] := Simplify[(-i a[n - 2] + i En a[n]) / (2 (n + m) )];
a[n_] := Simplify[-i (b[n - 3] - En b[n - 1]) / 2 / n]
```

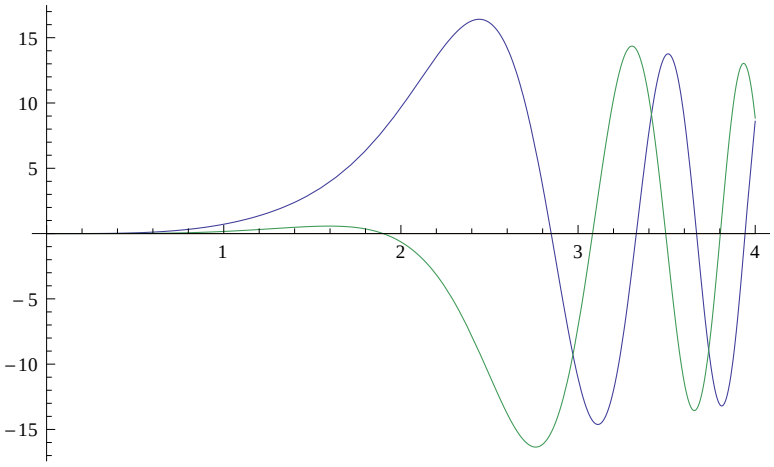
```
b[4]
```

$$\frac{i\ En^4\ \left(-22\,464+En^5\right)}{185\,794\,560}$$

```
Un[En_, m_, nN_, x_] := Module[{n, U},
  U = {1}; AppendTo[U, -En^2/(4 m)]; AppendTo[U, En (4 + En^3 + 8 m)/(32 m (1 + m))]; G = {i En / 2 / m};

  For[n = 3, n < nN, n++,
   AppendTo[U,
    -((-1 + m + n) U[[-2 + n]] + En ((3 - 2 m - 2 n) U[[-1 + n]] + En (-2 + m + n) U[[n]])) /
      (4 n (-2 + m + n) (-1 + m + n))];
  ];
  ({1, i En / 2 / m * x} +
      Sum[{U[[n + 1]] * x^(2 * n), I * (En * U[[n + 1]] - U[[n]]) / 2 / (n + m) * x^(2 * n + 1)},
       {n, 1, nN - 1}]) * x^(-1 + m) // N]
```

```
G = {Re[#], Im[#]} &[Un[3, 5, 150, x]]; Plot[G, {x, 0, 4}, PlotRange → All]
```




```
U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
   n++;
   U = G; G = -Un[En, m, n + 1, X];

  ];
  {Un[En, m, n, X], n}]
```

```
U[9, 5, 0.0001, 1]
```

```
{{-0.0126898, 0.+0.00407705 i}, 14}
```

```
Ener[Ene_] :=
 Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 5000;
  m = 5;
  r = 7.2 // N; h = -7.0 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
   k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
   k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
   k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
   k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
   kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   r += h;
   , {n}];

  NN = U[En, m, 0.0001, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn → En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.001,
   En -= Erg / (U2S k⟦1⟧ - U1S k⟦2⟧ + U2 kE⟦1⟧ - U1 kE⟦2⟧);
   Print[{En, Erg / U2 / k[[2]]}]; Goto[begin];
    ];
  {En, Erg / U2 / k[[2]]}
 ]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
 Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[24.236924123512843` + 0.7062238874780654` i]
```

{24.2369+ 0.706395 i, 0.00716219+ 0.0160044 i}

{24.2369+ 0.70624 i, −0.00746584 − 0.014522 i}

{24.2369+ 0.70638 i, 0.00722303+ 0.0122471 i}

{24.2369+ 0.706253 i, −0.00732641 − 0.0109237 i}

{24.2369+ 0.706367 i, 0.00693458+ 0.00920459 i}

{24.2369+ 0.706265 i, −0.00687771 − 0.00806601 i}

{24.2369+ 0.706356 i, 0.00642544+ 0.00677406 i}

{24.2369+ 0.706275 i, −0.00625369 − 0.00582549 i}

{24.2369+ 0.706347 i, 0.00579245+ 0.00485995 i}

{24.2369+ 0.706283 i, −0.00554783 − 0.00409206 i}

{24.2369+ 0.70634 i, 0.00510631+ 0.00337537 i}

{24.2369+ 0.70629 i, −0.00482361 − 0.00277015 i}

{24.2369+ 0.706333 i, 0.00441676+ 0.00224297 i}

{24.2369+ 0.706296 i, −0.00412247 − 0.00177825 i}

{24.2369+ 0.706328 i, 0.00375707+ 0.00139525 i}

{24.2369+ 0.7063 i, −0.00346992 − 0.00104786 i}

{24.2369+ 0.706324 i, 0.00314793+ 0.000774337 i}

{24.2369+ 0.706304 i, −0.00288004 − 0.000522185 i}

{24.2369+ 0.706321 i, 0.00260061+ 0.000331457 i}

{24.2369+ 0.706307 i, −0.00235896 − 0.00015467 i}

{24.2369+ 0.706319 i, 0.00211965+ 0.000026144 i}

{24.2369+ 0.706309 i, −0.00190737 + 0.0000924182 i}

{24.2369+ 0.706317 i, 0.00170489− 0.000174675 i}

{24.2369+ 0.70631 i, −0.00152244 + 0.000249309 i}

{24.2369+ 0.706316 i, 0.00135309− 0.000297629 i}

{24.2369+ 0.706311 i, −0.00119918 + 0.000339948 i}

{24.2369+ 0.706315 i, 0.00105913− 0.000363853 i}

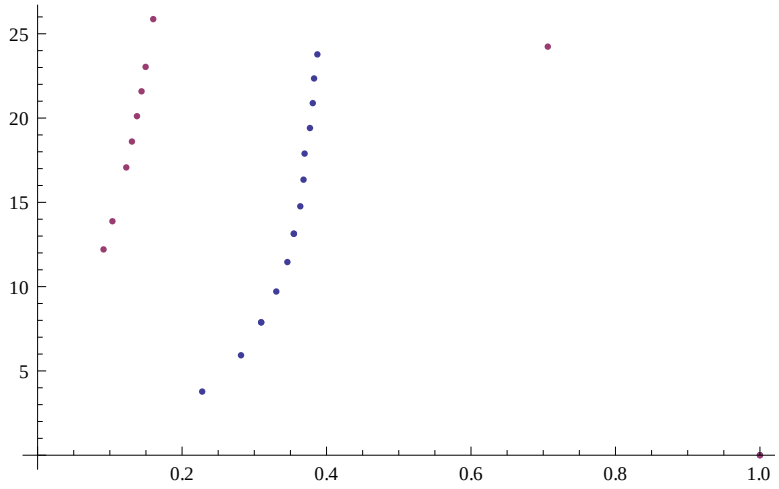
{24.2369+ 0.706312 i, −0.000931431 + 0.000383066 i}

{24.2369+ 0.706312 i, 0.00081691− 0.000389864 i}

```
Energie = {{3.77486283903418` + 0.22786873407418717` i,
    5.928479968617718` + 0.2815986526347655` i,
    7.8813588488087065` + 0.30953294412328675` i,
    7.881329304880588` + 0.3095336916562825` i,
    9.71036454189739` + 0.3304415424573178` i,
    11.458077781781169` + 0.3457848857997222` i,
    13.139764183242892` + 0.3546744769440479` i,
    13.139114444715846` + 0.3548172513559829` i,
    14.76566063867228` + 0.3636637795053202` i,
    16.34527139193304` + 0.3681521067888729` i,
    17.892138695416435` + 0.3696213478944415` i,
    19.40831608562361` + 0.37704222997528414` i,
    20.886015743223094` + 0.38097023642873473` i,
    22.34958939297449` + 0.38276309915136164` i,
    23.777494591007947` + 0.3872991704766671` i},
   {12.207301904640477` + 0.09130244038237889` i, 13.879319291317909` +
     0.10359131716054014` i, 17.071331521063737` + 0.12274708920757649` i,
    18.607651751873114` + 0.1305970820854307` i, 20.11139546248976` + 0.1375815129955693` i,
    21.586240037178456` + 0.14389862151162078` i, 23.03567690195356` + 0.149533964468945` i,
    24.236898126799336` + 0.7063122785768635` i,
    25.86917074582206` + 0.1600892949325245` i}}; Energie // MatrixForm
```

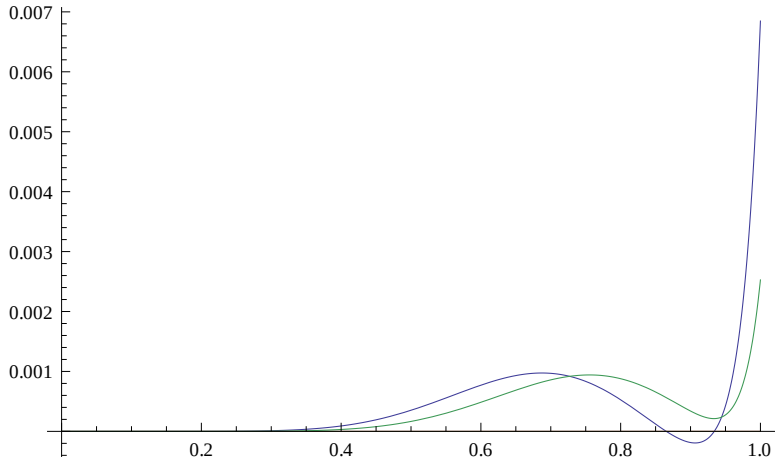
$$\begin{pmatrix} \{3.77486+0.227869\,i,\ 5.92848+0.281599\,i,\ 7.88136+0.309533\,i,\ 7.88133+0.309534\,i,\ 9.71036+0 \\ \{12.2073+0.0913024\,i,\ 13.8793+0.103591\,i,\ 17.0713+0.122747\,i,\ 18.6077+0.130597\,i,\ 20.1114+ \end{pmatrix}$$

```
ListPlot[Table[Append[{Im[#], Re[#]} & /@ Energie[[i]], {1, 0}], {i, 1, 2}],
 AxesOrigin → {0, 0}, PlotRange → All]
```



:

```
G = {Re[#], Im[#]} &[Un[16, 10, 15, x]]; Plot[G, {x, 0, 1}, PlotRange → All]
```



```
n = 8000; S = 1; h = 6 / n; ra = 1; En = Energie[[17]]; m = 5; r = 1;
U[En, m, 10^-10, r][[2]]
k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
En =.;
r =.;
```

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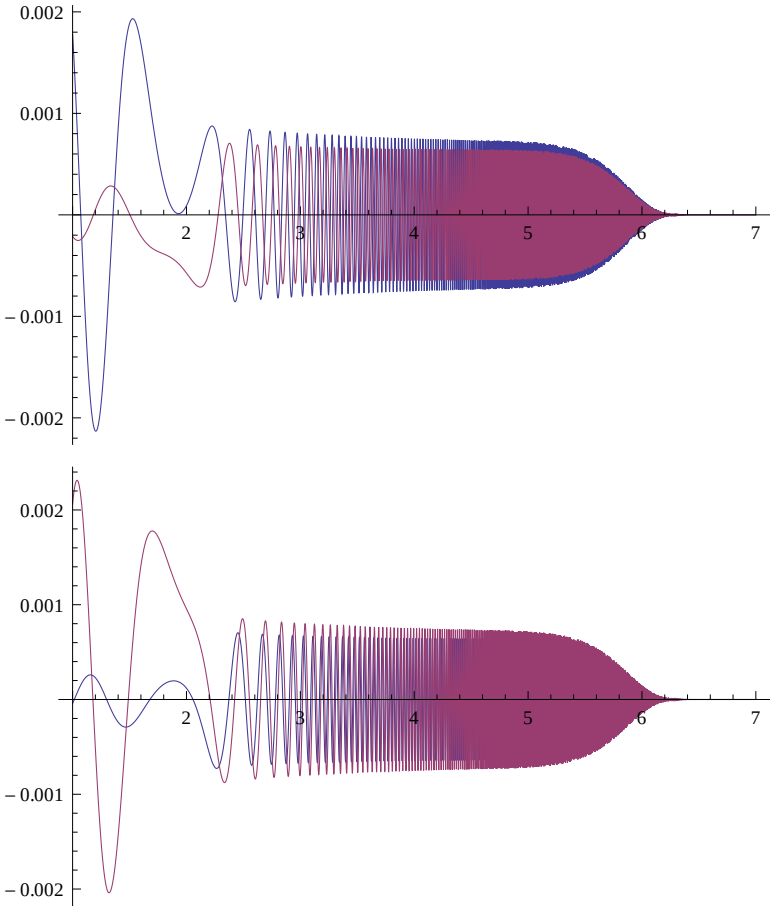
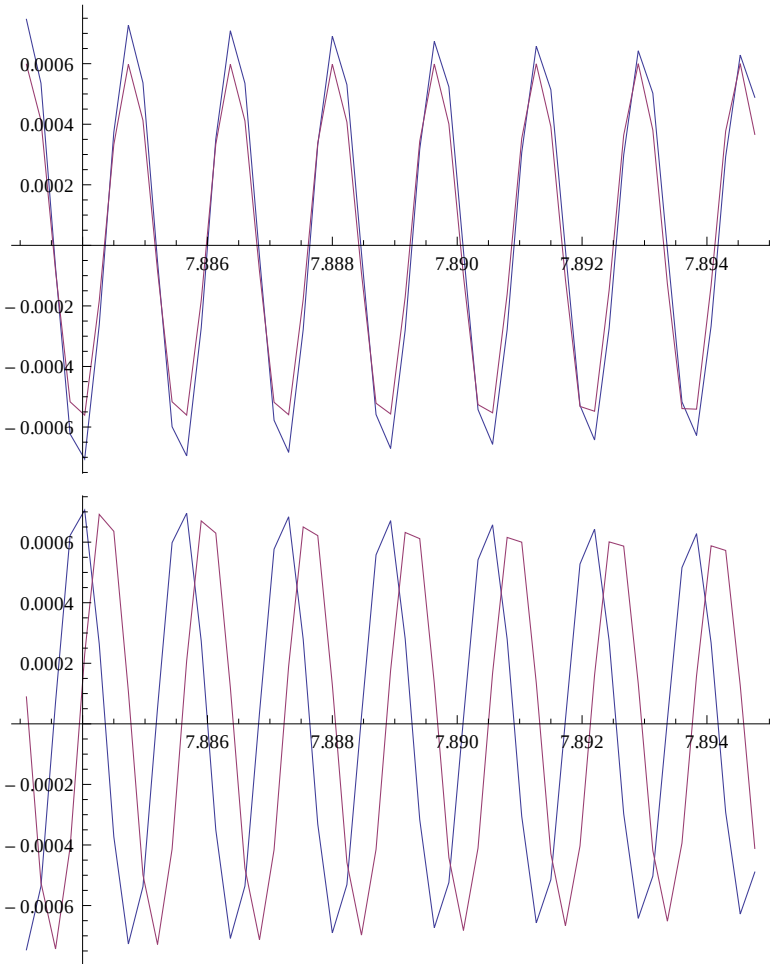



```
U[En, m, 10^-10, r][[1]]
```

```
S = 29500; n = 50; ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], -0.0006 * Sin[#[[1]]^5 / 5 + 1 - #[[1]] * Re[Energie[[11]]]]} & /@
      kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
```




```
Exp[I * Im[Energie[[1]]] * x]
```

$$e^{0.278733\, i\, x}$$