```
Exit[];
Assumptions = r > 0 \& Element[m, Integers] \& Element[m, Integers] & Element[m, Integers] 
       Element [n, Integers] && s > 0 && Element [k, Integers] && k > 0
r > 0 \&\& m \in Integers \&\& n \in Integers \&\& s > 0 \&\& k \in Integers \&\& k > 0
f[r_] := \{ (m-1)/r, I * (En-r^p) \}, \{ I * (En-r^p), -m/r \} \}
       0 * IdentityMatrix[2] * I * r ^ p; f[r] // MatrixForm
En = E0 + I * Ga; En = .
u = \{a[n] * x^{(2*n)}, b[n] * x^{(2*n+1)}\} * x^s
\{x^{2\,n+s}\,a[n],\,x^{1+2\,n+s}\,b[n]\}
p = 2;
r[x_] := x;
g1 = Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x]].u) * x^(-s+1)]]],
           {x ^n, a[n], b[n]}];
g1 // MatrixForm
 s = -1 + m;
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x \rightarrow 0], {n2, 0, 10}];
g2 // MatrixForm
   0 0
    0 0
    0 0
    0 0
    0 0
    0 0
    0 0
    0 0
    0 0
a[0] = 1; b[0] = i En a[0] / 2/m; a[1] = i En b[0] / 2;
a[0] = 1; b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
b[n_] := I * (En * a[n] - a[n-1]) / 2 / (n+m);
a[n_] := Simplify [I * (En b[n-1] - b[n-2]) / 2 / n]
```

```
a[1]
 En^2
m = 2;
En = .
Un[En_{n, m_{n}}, nN_{n, x_{n}}] := Module [\{n, U\},
  U = {1}; AppendTo \left[ U, -\frac{En^2}{4 m} \right]; AppendTo \left[ U, \frac{En \left( 4 + En^3 + 8 m \right)}{32 m \left( 1 + m \right)} \right]; G = {i En / 2/m};
   For [n = 3, n < nN, n++,
    AppendTo[U,
        -((-1+m+n) U[[-2+n]] + En ((3-2m-2n) U[[-1+n]] + En (-2+m+n) U[[n]])) /
           (4 n (-2+m+n) (-1+m+n));
   ];
   ({1, i En / 2/m * x}) +
         Sum \left[ \left\{ U \left[ \left[ n+1 \right] \right] * x ^ (2*n) , I * \left( En * U \left[ \left[ n+1 \right] \right] - U \left[ \left[ n \right] \right] \right) / 2 / (n+m) * x ^ (2*n+1) \right\},
           \{n, 1, nN - 1\} \} \times x^{(-1+m)} / N
G = \{Re[\#], Im[\#]\} \& [Un[3, 2, 150, x]]; Plot[G, \{x, 0, 5\}, PlotRange \rightarrow \{-0.5, 0.5\}]
 0.4
 0.2
-0.2
-0.4
```

## RungeKutta von Links

Exit[]

s

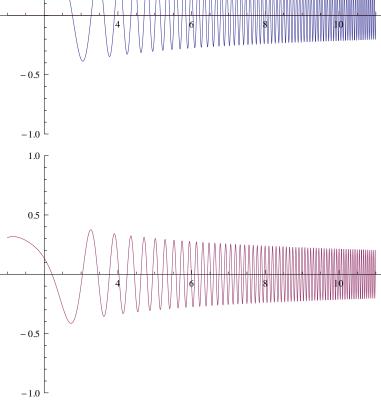
```
U[En_, m_, g_, X_] := Module[{n = 10, U, G},
    U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
    n++;
    U = G; G = -Un[En, m, n + 1, X];

];
Print[n];
{Un[En, m, n, X], n}]
p = 2; En =.

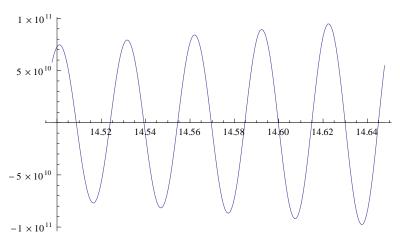
f[r_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}}; f[rr] // MatrixForm
    \[ \begin{align*} \frac{-1 + m}{rr} & i \left(En - rr^2) \\ i \left(En - rr^2) & -\frac{m}{rr} \\ i \left(En - rr^2) & -\frac{m}{rr} \\ \frac{-1 + m}{rr} & -\frac{m}{rr} & -\frac{m}{rr} \\ \frac{-1 + m}{rr} & -\frac{m}{rr} & -\frac{m}{rr} \\ \frac{-1 + m}{rr} & -\frac{m}{rr} \\ \frac{-1 + m}{rr} & -\frac{m}{rr} & -\frac{m}{rr} & -\frac{m}{rr} & -\fra
```

16

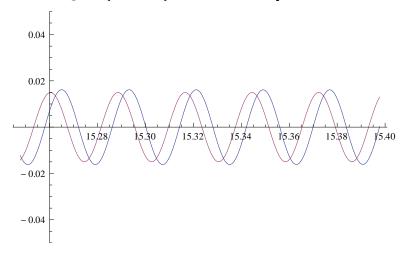
```
n = 5000; S = 1; h = 10 / n; ra = 1; En = 3; m = 2; r = 1; k = U[En, m, 10 ^-10, r][[1]];
kK = \{\{r, k\}\};
Do [
  k0 = h * f[r].k; k1 = h * f[r + h / 2].(k + k0 / 2);
  k2 = h * f[r + h / 2].(k + k1 / 2); k3 = h * f[r + h].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo [kK, \{r, k\}], \{n\}];
ListPlot[Join[{ \#[[1]], Re[\#[[2,1]]]} & /@ kK[[S;; n]] // N},
  { \#[[1]], Im [\#[[2,1]]] \& /@ kK [[S;;n]] // N}], }
 PlotRange → {-ra, ra}, Joined → True]
ListPlot[Join[{ \{\#[[1]], Re[\#[[2, 2]]\}\} \& /@ kK[[S;; n]] // N},
  \{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@kK[[S;; n]] // N \} \},
 PlotRange → {-ra, ra}, Joined → True]
m =.; r =.;
    1.0
    0.5
```



$$\begin{split} & \text{S = 18\,000; n = 200; ListPlot[Join[{ \{\#[[1]], Re[\#[[2, 2]]]\} \& /@ kK[[S ;; S + n]] // N}, \\ & \{\#[[1]], 0 \star Im[\#[[2, 2]]]\} \& /@ kK[[S ;; S + n]] // N}], \\ & \text{PlotRange} \to All, Joined} \to True] \end{aligned}$$



 $S = 19\,000; n = 200; ListPlot[Join[{ {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S;; S+n]]}, \\ { {#[[1]], -0.1 * Sin[(-#^3/3+10 * #)] & [#[[1]]] * 0.15} & /@ kK[[S;; S+n]]}], \\ PlotRange <math>\rightarrow$  {-ra, ra}, Joined  $\rightarrow$  True]



$$V = \{\{1, -1\}, \{1, 1\}\}$$

$$\{\{1, -1\}, \{1, 1\}\}.\{F[x], g[x]\}$$

Inverse[V].{C1, C2}

$$\left\{ \frac{\text{C1}}{2} + \frac{\text{C2}}{2} , -\frac{\text{C1}}{2} + \frac{\text{C2}}{2} \right\}$$

Expand [V.f[r].Inverse[V]] // MatrixForm

# Finden und Einsetzen des Randverhaltens

```
Exit[];
  $Assumptions = r > 0 && Element[m, Integers] &&
                         Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
  r > 0 \&\& m \in Integers \&\& n \in Integers \&\& s > 0 \&\& k \in Integers \&\& k > 0
 f[r_] := \{ (m-1)/r, I * (En-r^p) \}, \{ I * (En-r^p), -m/r \} \}
                       0 * IdentityMatrix[2] * I * r ^ p; f[r] // MatrixForm
        \left( \begin{array}{ccc} \frac{-1+m}{r} & & \text{i.} & (\texttt{En-r}^p) \\ \\ \text{i.} & (\texttt{En-r}^p) & -\frac{m}{r} \end{array} \right) 
  En = E0 + I * Ga; En = .
  u = \{ Exp[I * (F1[x])] + Exp[I * F2[x]], Exp[I * (G1[x])] + Exp[I * G2[x]] \} * x ^s
  \left\{ \left( e^{i \operatorname{Fl}[x]} + e^{i \operatorname{F2}[x]} \right) x^{-1+m}, \left( e^{i \operatorname{Gl}[x]} + e^{i \operatorname{G2}[x]} \right) x^{-1+m} \right\}
 s = m - 1
  -1 + m
  p = 2;
  r[x_] := x;
    g1 = -Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x]].u) * F1[x] / u / I]]],
                                   {x ^n, a[n], b[n]}]
\Big\{ \frac{5 \, \, e^{\, i \, \, \text{G1} \, [\, x \,]} \, \, \, \text{F1} \, [\, x \,]}{e^{\, i \, \, \text{F1} \, [\, x \,]} + e^{\, i \, \, \text{F2} \, [\, x \,]}} \, + \frac{5 \, \, e^{\, i \, \, \text{G2} \, [\, x \,]} \, \, \text{F1} \, [\, x \,]}{e^{\, i \, \, \text{F1} \, [\, x \,]} + e^{\, i \, \, \text{F2} \, [\, x \,]}} \, \, - \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x \,]} \, + \frac{1}{2} \, e^{\, i \, \, \text{F1} \, [\, x
                   x^2 \left( \frac{e^{i \; \text{G1[x]} \; \text{F1[x]}}}{e^{i \; \text{F1[x]}} + e^{i \; \text{F2[x]}}} + \frac{e^{i \; \text{G2[x]} \; \text{F1[x]}}}{e^{i \; \text{F1[x]}} + e^{i \; \text{F2[x]}}} \right) - \frac{e^{i \; \text{F1[x]} \; \text{F1[x]} \; \text{F1[x]} \; \text{F1[x]} \; \text{F1[x]}}}{e^{i \; \text{F1[x]}} + e^{i \; \text{F2[x]}}} - \frac{e^{i \; \text{F2[x]} \; \text{F1[x]} \; \text{F1[x]} \; \text{F2[x]}}}{e^{i \; \text{F1[x]}} + e^{i \; \text{F2[x]}}} \, ,
              \frac{5 \, e^{\,\mathrm{i} \, \mathrm{F1}\,[\,\mathrm{x}\,]} \, \, \mathrm{F1}\,[\,\mathrm{x}\,]}{e^{\,\mathrm{i} \, \mathrm{G1}\,[\,\mathrm{x}\,]} + e^{\,\mathrm{i} \, \mathrm{G2}\,[\,\mathrm{x}\,]}} + \frac{5 \, e^{\,\mathrm{i} \, \mathrm{F2}\,[\,\mathrm{x}\,]} \, \, \mathrm{F1}\,[\,\mathrm{x}\,]}{e^{\,\mathrm{i} \, \mathrm{G1}\,[\,\mathrm{x}\,]} + e^{\,\mathrm{i} \, \mathrm{G2}\,[\,\mathrm{x}\,]}} - \, \mathrm{x}^{\,\mathrm{2}} \left( \frac{e^{\,\mathrm{i} \, \mathrm{F1}\,[\,\mathrm{x}\,]} \, \, \, \mathrm{F1}\,[\,\mathrm{x}\,]}{e^{\,\mathrm{i} \, \mathrm{G1}\,[\,\mathrm{x}\,]} + e^{\,\mathrm{i} \, \mathrm{G2}\,[\,\mathrm{x}\,]}} + \frac{e^{\,\mathrm{i} \, \mathrm{F2}\,[\,\mathrm{x}\,]} \, \, \, \mathrm{F1}\,[\,\mathrm{x}\,]}{e^{\,\mathrm{i} \, \mathrm{G1}\,[\,\mathrm{x}\,]} + e^{\,\mathrm{i} \, \mathrm{G2}\,[\,\mathrm{x}\,]}} \right) - \frac{1}{x}
                         \left(\frac{\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,+\,\,\frac{\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}\,\,m\,\,F1\,\,[\,x\,]}{\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,+\,\mathbb{e}^{\,i\,\,G2\,\,[\,x\,]}}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,\mathbb{e}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i\,\,G1\,\,[\,x\,]}\,\,m\,\,\mathcal{I}^{\,i
                          \frac{e^{\mathrm{i}\;\mathrm{G1}\,[\,\mathrm{x}\,]}\;\,\mathrm{F1}\,[\,\mathrm{x}\,]\;\;\mathrm{G1'}\,[\,\mathrm{x}\,]}{e^{\mathrm{i}\;\mathrm{G1}\,[\,\mathrm{x}\,]}+e^{\mathrm{i}\;\mathrm{G2}\,[\,\mathrm{x}\,]}}-\frac{e^{\mathrm{i}\;\mathrm{G2}\,[\,\mathrm{x}\,]}\;\,\mathrm{F1}\,[\,\mathrm{x}\,]\;\;\mathrm{G2'}\,[\,\mathrm{x}\,]}{e^{\mathrm{i}\;\mathrm{G1}\,[\,\mathrm{x}\,]}+e^{\mathrm{i}\;\mathrm{G2}\,[\,\mathrm{x}\,]}}\,\Big\}
```

s = -1 + m;

## RungeKutta von Rechts

```
Exit[]

U[En_, m_, g_, X_] := Module[{n = 10, U, G},
    U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];

While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
    n++;
    U = G; G = -Un[En, m, n + 1, X];

];

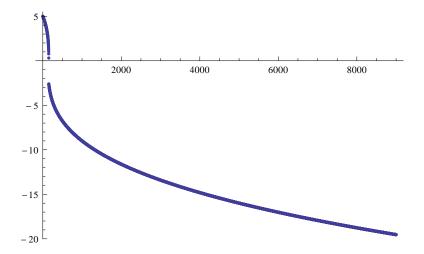
Print[n];
{Un[En, m, n, X], n}]

p = 2; En =.

f[r_] := {{(m - 1) / r, I * (En - r^p)}, {I * (En - r^p), -m/r}}; f[rr] // MatrixForm

\[ \begin{align*} \frac{-1+m}{rr} & i (3-rr^2) \\ i (3-rr^2) & -\frac{m}{rr} \end{align*} \]
\[ i (3-rr^2) & -\frac{m}{rr} \end{align*} \]
```

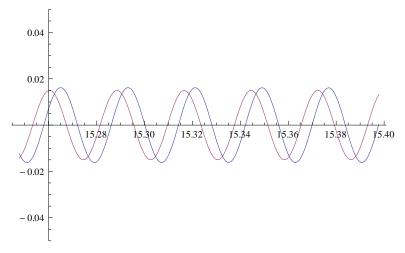
```
r = 5 // N; k = {r}; Do[
r -= 0.28 / r^2; AppendTo[k, r]
, {9000}]; r = .; ListPlot[k]
```



```
n = 3000; S = 1; ra = 0.05; En = 3; m = 2; r = 10 // N; h = 9 / n; k = \{1, -1\};
kK = \{\{r, k\}\};
Do [
  k0 = h * f[r].k; k1 = h * f[r + h / 2].(k + k0 / 2);
  k2 = h * f[r + h / 2].(k + k1 / 2); k3 = h * f[r + h].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r -= h;
   AppendTo [kK, \{r, k\}], \{n\}];
ListPlot[Join[{ \{\#[[1]], Re[\#[[2, 1]]\}\} \& /@ kK[[S;; n]] // N},
   \label{eq:localization} \begin{center} \{\#[[1]], Im[\#[[2,1]]]\} \& /@ kK[[S ;; n]] // N\}], PlotRange $\to All, Joined $\to True]$ \end{center}
 \{ \; \{\#[[1]] \; , \; \text{Im} \; [\#[[2,\,2]]] \} \; \& \; /@ \; kK \; [[S \; ;; \; n]] \; // \; N \} ] \; , \; PlotRange \; \rightarrow \; All \; , \; Joined \; \rightarrow \; True ] 
r =.;
      1.0
     0.5
    -1.0
      1.0
      0.5
    -0.5
    -1.0
```

```
r = 1 // N
               k = U[En, 2, 10 -10, r][[2]]; Un[En, 2, k, 1]
                G = \{Re[\#], Im[\#]\} \& [Un[3, 2, k, x]]; Plot[G, \{x, 0, r\}, PlotRange \rightarrow \{-0.5, 0.5\}]
                1.
19
                \{-0.0989079, 0.+0.0687335 i\}
                   0.4
                   0.2
                                                     0.2
                                                                                  0.4
                                                                                                               0.6
                                                                                                                                             0.8
                                                                                                                                                                          1.0
                -0.2
                -0.4
                S = 1; n = 200; ListPlot[Join[{ \{ \#[[1]], Re[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] \} & /@ kK[[S ;; S + n]] // N}, Re[\#[[2, 2]]] 
                       \{ \; \{ \#[[1]] \; , \; \text{Im} \; [\#[[2,\; 2]]] \} \; \& \; /@ \; kK \; [[S \; ;; \; S + n]] \; // \; N \} ] \; , \; PlotRange \; \rightarrow \; All \; , \; Joined \; \rightarrow \; False ] 
                    1.0
                   0.5
                -0.5
```

```
S = 19\,000; n = 200; ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1]], Im[\#[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[{ \{ \#[[1], Im[[M[[2, 2]]] \} \& /@ kK[[S ;; S + n]] \}, ListPlot[Join[[M[[2, 2]]]] \}, ListPlot[Join[[M[[2, 2]]]]] \}, ListPlot[Join[[M[[2, 2]]]]] \}, ListPlot[Join[[M[[2, 2]]]]]) \}
                   \{ \#[[1]], -0.1 * Sin[(-#^3/3+10*#)] \& [\#[[1]]] * 0.15 \} \& /@ kK[[S;;S+n]] \} \}
         PlotRange → {-ra, ra}, Joined → True]
```



$$V = \{\{1, -1\}, \{1, 1\}\}$$

$$\{\{1, -1\}, \{1, 1\}\}.\{F[x], g[x]\}$$

Inverse[V].{C1, C2}

$$\left\{ \frac{\text{C1}}{2} + \frac{\text{C2}}{2}, -\frac{\text{C1}}{2} + \frac{\text{C2}}{2} \right\}$$

Expand [V.f[r].Inverse[V]] // MatrixForm

### Nullstellensuche

```
n = 3000; S = 1; ra = 0.05; En = 5; m = 2; r = 10 // N; h = 9 / n; k = \{1, -1\};
Do [
  k0 = h * f[r].k; k1 = h * f[r + h / 2].(k + k0 / 2);
  k2 = h * f[r + h / 2].(k + k1 / 2); k3 = h * f[r + h].(k + k2);
  k += 1/6 * (k0 + 2 * k1 + 2 * k2 + k3); r -= h;
  , {n}];
```

 $Erg = k - Un[En, m, U[En, m, 10^-10, 1][[2]], 1]$ 

 $\{0.273782-0.220723 i, -0.422103+0.0503209 i\}$ 

D[f[R], En]

19

General::ivar: 5 is not a valid variable. >>

$$\partial_{5}\left\{\left\{\frac{1}{R}, i\left(5-R^{2}\right)\right\}, \left\{i\left(5-R^{2}\right), -\frac{2}{R}\right\}\right\}$$

```
r = 1 \text{ // N}
k = U[En, 2, 10^{-10}, r][[2]]; Un[En, 2, k, 1]
G = \{Re[\#], Im[\#]\} \& [Un[3, 2, k, x]]; Plot[G, \{x, 0, r\}, PlotRange \rightarrow \{-0.5, 0.5\}]
r = .
1.

\{-0.0989079, 0.+0.0687335 i\}
0.4
0.2
0.4
0.2
0.4
0.6
0.8
1.0
```

# RungeKutta von Links mit Randverhalten

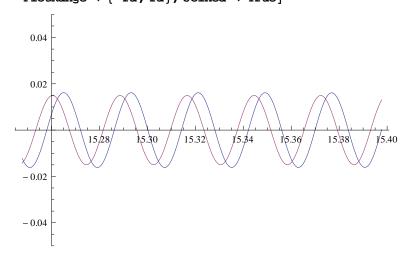
```
Exit[]
U [En_, m_, g_, X_] := Module[{n = 10, U, G},
    U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
While [Sqrt [Abs [Conjugate [U - G].(U - G)]] > g,
    n++;
    U = G; G = -Un[En, m, n + 1, X];
];
Print[n];
{Un[En, m, n, X], n}]
p = 2;
```

$$\begin{split} & f[u_-, \, x_-] := \\ & \left\{ e^{-i \, F[x] + i \, G[x]} \, En - e^{-i \, F[x] + i \, G[x]} \, x^2, \, e^{i \, F[x] - i \, G[x]} \, En - \frac{i}{x} + \frac{2 \, i \, m}{x} - e^{i \, F[x] - i \, G[x]} \, x^2 \right\} / \cdot \\ & F[x] \to u[[1]] / \cdot G[x] \to u[[2]]; \, f[\{F[x], G[x]\}, \, r] / / \, \text{MatrixForm} \\ & \left( e^{-i \, F[x] + i \, G[x]} \, En - e^{-i \, F[x] + i \, G[x]} \, r^2 \right) \\ & \left( e^{i \, F[x] - i \, G[x]} \, En - \frac{i}{r} + \frac{2 \, i \, m}{r} - e^{i \, F[x] - i \, G[x]} \, r^2 \right) \end{split}$$

```
n = 5000; S = 1; h = 10 / n; ra = 0.05; En = 5; m = 2;
       r = 0.1; k = Log[U[En, m, 10^-25, r][[1]]/r^s]/I
       kK = \{\{r, k\}\};
       Do [
        k0 = h * f[k, r]; k1 = h * f[(k+k0/2), r+h/2];
        k2 = h * f[(k + k1 / 2), r + h / 2]; k3 = h * f[(k + k2), r + h];
        k += 1/6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
        AppendTo [kK, \{r, k\}], \{n\}];
       ListPlot [Join [{ \{\#[[1]], Re[\#[[2,1]]\}\} \& /@kK[[S;;n]] // N},
         \{ \#[[1]], 0 * Im[\#[[2,1]]] \& /@ kK[[S ;; n]] // N \}], PlotRange <math>\rightarrow All, Joined \rightarrow True \}
       ListPlot [Join [{ \{\#[[1]], Re[\#[[2, 2]]\}\} \& /@kK[[S; n]] // N},
         \{ \#[[1]], 0 * Im[\#[[2, 2]]] \} \& /@ kK[[S;; n]] // N \} \}, PlotRange <math>\rightarrow All, Joined \rightarrow True \}
      m =.; r =.;
11
       \{0.0313618 i, 1.5708 + 2.10163 i\}
       General::ovfl: Overflow occurred in computation. >>
       General::unfl: Underflow occurred in computation. >>
       $Aborted
      Part::take: Cannot take positions 1 through
             5000 in \{\{0.1, \{0.0313618 \ i, 1.5708 + 2.10163 \ i\}\}, \ll 9 \gg, \ll 3501 \gg\}.
      Part::partd: Part specification (1;; 5000)[2, 2] is longer than depth of object. >>
      Part::pspec: Part specification \{1, \text{Re}[(1;;5000)][2,2]]\} is neither an integer nor a list of integers. \gg
      Part::partd: Part specification (1;; 5000)[2, 2] is longer than depth of object. >>
      Part::pspec: Part specification \{1, Re[(1; 5000)[2, 2]]\} is neither an integer nor a list of integers. \gg
      Part::take: Cannot take positions 1 through
             5000 in \{\{0.1, \{0.0313618 \ i, 1.5708 + 2.10163 \ i\}\}, \ll 9 \gg, \ll 3501 \gg\}.
      Part::partd: Part specification (1;; 5000)[2, 2] is longer than depth of object. >>
       General::stop: Further output of Part::partd will be suppressed during this calculation. >>
                                      1.0
                                     0.5
       -1.0
                       -0.5
                                                       0.5
                                                                       1.0
                                    -0.5
```

-1.0

```
3
Sqrt[5] // N
2.23607
S = 1; n = 10000;
 \{ \, \{ \#[[1]] \,,\, 0 \, * \, \text{Im} \, [\#[[2,\,1]]] \} \, \& \, / @ \, kK \, [[S \,\, ;; \, S \, + \, n]] \} ] \,, \, \, PlotRange \, \rightarrow \, All \,, \, \, Joined \, \rightarrow \, True \, ] 
1.0
0.8
0.6
0.4
0.2
S = 1; n = 5000; ListPlot[
 \label{eq:constraint} \mbox{Join[{ $\{\#[[1]], \mbox{Re}[\#[[2, 2]] - (\#[[1]] \mbox{$^3$/$3-En*$$$\#[[1]])]} \& /@ kK[[S ;; S+n]]}], $$
 PlotRange → All, Joined → True]
                                                                 12
-200
-400
-600
-\,800
En
5 + 0.2 i
```



# RungeKutta von Rechts-> Nullstellengesuche

```
Exit[]
p = 2;
f[r_, En_, m_] :=
    {{(m-1) / r, I * (En-r^p)}, {I * (En-r^p), -m/r}} - IdentityMatrix[2] * I * r^p;
```

```
n = 3000; En = 14 + I * 0.001; m = 2; r = 11.0; h = r / (n - 1); k = \{1, -1\};
      kK = \{\{r, k\}\};
      Do [
        k0 = h * f[r, En, m].k; k1 = h * f[r + h / 2, En, m].(k + k0 / 2);
        k2 = h * f[r + h / 2, En, m] . (k + k1 / 2); k3 = h * f[r + h, En, m] . (k + k2);
        k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r -= h;
        AppendTo [kK, \{r, k\}], \{n\}];
      Print[kK[[n, 2]]];
      ListPlot[Join[Table[ {#[[1]], Re[#[[2,i]]]} & /@ kK[[1;; n]] // N, {i, 2}],
        Table[{#[[1]], Im[#[[2,i]]]} & /@ kK[[1;; n]] // N, {i, 2}]],
       PlotRange \rightarrow \{-5, 5\}, Joined \rightarrow True]
\{2.19637 - 1.69195 i, 0.0535395 + 0.0697368 i\}
       2
      -2
      d[En_{m_{x}}, m_{x_{x}}, x_{x_{x}}, n_{x_{x}}] := Module[{k, k0, k1, k2, k3, k4},
        r = X; h = (X - x) / (n); k = \{1, -1\};
        Do [
         k0 = h * f[r, En, m].k; k1 = h * f[r + h / 2, En, m].(k + k0 / 2);
         k2 = h * f[r + h / 2, En, m].(k + k1 / 2); k3 = h * f[r + h, En, m].(k + k2);
         k += 1/6 * (k0 + 2 * k1 + 2 * k2 + k3); r -= h;, {n}];
        k4 = k;
        r = 0.001; h = (X - x - r) / (n); k = U[En, m, 0.001, r][[1]] * {1, -1} * e^{\frac{-3}{3}};
        Do [
         k0 = h * f[r, En, m].k; k1 = h * f[r + h / 2, En, m].(k + k0 / 2);
         k2 = h * f[r + h / 2, En, m] . (k + k1 / 2); k3 = h * f[r + h, En, m] . (k + k2);
         k += 1/6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;, {n}];
        k - k4
      d[7, 2, 2, 11.0, 3000]
      \{-21.3966 + 38.1887 i, 8.2075 + 3.56692 i\}
```

#### $AA = Table[Abs[d[i+I*j, 2, 2.0, 11.0, 3000][[1]]], {j, 8, 20, 2}, {i, 0, 5}]$

```
 \left\{ \left\{ 2.01539 \times 10^{72}, \ 1.70761 \times 10^{71}, \ 2.23889 \times 10^{69}, \ 3.61115 \times 10^{68}, \ 4.14497 \times 10^{66}, \ 5.36746 \times 10^{65} \right\}, \\ \left\{ 3.36981 \times 10^{79}, \ 8.41473 \times 10^{77}, \ 2.13402 \times 10^{76}, \ 4.93856 \times 10^{74}, \ 1.72678 \times 10^{73}, \ 4.46675 \times 10^{71} \right\}, \\ \left\{ 4.80477 \times 10^{86}, \ 1.27969 \times 10^{85}, \ 3.48353 \times 10^{83}, \ 9.67405 \times 10^{81}, \ 2.79312 \times 10^{80}, \ 8.22132 \times 10^{78} \right\}, \\ \left\{ 6.07865 \times 10^{93}, \ 1.78739 \times 10^{92}, \ 5.38671 \times 10^{90}, \ 1.66212 \times 10^{89}, \ 5.26181 \times 10^{87}, \ 1.70489 \times 10^{86} \right\}, \\ \left\{ 6.97571 \times 10^{100}, \ 2.27062 \times 10^{99}, \ 7.57069 \times 10^{97}, \ 2.58576 \times 10^{96}, \ 9.04737 \times 10^{94}, \\ 3.24291 \times 10^{93} \right\}, \\ \left\{ 7.27266 \times 10^{107}, \ 2.62014 \times 10^{106}, \ 9.66553 \times 10^{104}, \\ 3.65111 \times 10^{103}, \ 1.41238 \times 10^{102}, \ 5.59539 \times 10^{100} \right\}, \\ \left\{ 6.89962 \times 10^{114}, \\ 2.75055 \times 10^{113}, \ 1.12233 \times 10^{112}, \ 4.68769 \times 10^{110}, \ 2.00434 \times 10^{109}, \ 8.7738 \times 10^{107} \right\} \right\}
```

#### MatrixPlot[AA]

