```
DerivativeOrder = 3;

StencilPoints = GenStencilPoints[n, StencilType];
sys = {StencilPoints, DerivativeOrder, n};

{d, b} = GenDandB[sys]
```

$$\left\{\left\{\left\{\frac{5}{32}, -\frac{5}{32}, 0, 0, \frac{5}{32}, -\frac{5}{32}\right\}, \left\{0, 0, \frac{5}{32}, -\frac{5}{32}, \frac{5}{32}, -\frac{5}{32}\right\}, \left\{\frac{25}{2048}, \frac{25}{2048}, 0, 0, \frac{25}{2048}, \frac{25}{2048}\right\},\right.\right.$$

$$\left\{0, 0, 0, 0, \frac{25}{1024}, \frac{25}{1024}\right\}, \left\{0, 0, \frac{25}{2048}, \frac{25}{2048}, \frac{25}{2048}, \frac{25}{2048}\right\},$$

$$\left\{\frac{125}{196\,608}, -\frac{125}{196\,608}, 0, 0, \frac{125}{196\,608}, -\frac{125}{196\,608}\right\}, \left\{0, 0, 0, 0, \frac{125}{65\,536}, -\frac{125}{65\,536}\right\},$$

$$\left\{0, 0, 0, 0, \frac{125}{65\,536}, -\frac{125}{65\,536}\right\}, \left\{0, 0, \frac{125}{196\,608}, -\frac{125}{196\,608}, \frac{125}{196\,608}, -\frac{125}{196\,608}\right\}\right\},$$

$$\left\{0.03\ \text{S\$143967}[1], 0.03\ \text{S\$143967}[2], 0.02\ \text{S\$143967}[1]^2,\right.$$

$$\left.\left.0.02\ \text{S\$143967}[1]\ \text{S\$143967}[2], 0.02\ \text{S\$143967}[2]^2, 0, 0, 0, 0\right\}\right\}$$

```
c = RotationMatrix[α];

b2 = Join[c.{0.03` x, 0.03` y}, {#[[1, 1]], #[[1, 2]] * 2, #[[2, 2]]} &[
    c.{{0.020000000000000004` x^2, 0.020000000000000004` x y / 2},
      {0.020000000000000004` x y / 2, 0.020000000000000004` y^2}}.Transpose[c]], b[[6 ;;]]]
```

$$\left\{0.03\ x\ \text{Cos}[\alpha] - 0.03\ y\ \text{Sin}[\alpha], 0.03\ y\ \text{Cos}[\alpha] + 0.03\ x\ \text{Sin}[\alpha],\right.$$

$$\text{Cos}[\alpha]\ \left(0.02\ x^2\ \text{Cos}[\alpha] - 0.01\ x\ y\ \text{Sin}[\alpha]\right) - \text{Sin}[\alpha]\ \left(0.01\ x\ y\ \text{Cos}[\alpha] - 0.02\ y^2\ \text{Sin}[\alpha]\right),$$

$$2\ \left(\text{Sin}[\alpha]\ \left(0.02\ x^2\ \text{Cos}[\alpha] - 0.01\ x\ y\ \text{Sin}[\alpha]\right) + \text{Cos}[\alpha]\ \left(0.01\ x\ y\ \text{Cos}[\alpha] - 0.02\ y^2\ \text{Sin}[\alpha]\right)\right),$$

$$\text{Sin}[\alpha]\ \left(0.01\ x\ y\ \text{Cos}[\alpha] + 0.02\ x^2\ \text{Sin}[\alpha]\right) +$$

$$\left.\text{Cos}[\alpha]\ \left(0.02\ y^2\ \text{Cos}[\alpha] + 0.01\ x\ y\ \text{Sin}[\alpha]\right), 0, 0, 0, 0\right\}$$

```
weights = Table[w[i], {i, Length[StencilPoints]}]
```

$$\{w[1], w[2], w[3], w[4], w[5], w[6]\}$$

```
op = (d.weights - b2).(d.weights - b2)
```

$$\left(-0.03\ x\ \text{Cos}[\alpha] + 0.03\ y\ \text{Sin}[\alpha] + \frac{5\ w[1]}{32} - \frac{5\ w[2]}{32} + \frac{5\ w[5]}{32} - \frac{5\ w[6]}{32}\right)^2 +$$

$$\left(-0.03\ y\ \text{Cos}[\alpha] - 0.03\ x\ \text{Sin}[\alpha] + \frac{5\ w[3]}{32} - \frac{5\ w[4]}{32} + \frac{5\ w[5]}{32} - \frac{5\ w[6]}{32}\right)^2 +$$

$$2\left(\frac{125\ w[5]}{65536} - \frac{125\ w[6]}{65536}\right)^2 + \left(\frac{125\ w[1]}{196608} - \frac{125\ w[2]}{196608} + \frac{125\ w[5]}{196608} - \frac{125\ w[6]}{196608}\right)^2 +$$

$$\left(\frac{125\ w[3]}{196608} - \frac{125\ w[4]}{196608} + \frac{125\ w[5]}{196608} - \frac{125\ w[6]}{196608}\right)^2 +$$

$$\left(-\text{Cos}[\alpha]\left(0.02\ x^2\ \text{Cos}[\alpha] - 0.01\ x\ y\ \text{Sin}[\alpha]\right) + \text{Sin}[\alpha]\left(0.01\ x\ y\ \text{Cos}[\alpha] - 0.02\ y^2\ \text{Sin}[\alpha]\right) + \right.$$

$$\left.\frac{25\ w[1]}{2048} + \frac{25\ w[2]}{2048} + \frac{25\ w[5]}{2048} + \frac{25\ w[6]}{2048}\right)^2 + \left(-\text{Sin}[\alpha]\left(0.01\ x\ y\ \text{Cos}[\alpha] + 0.02\ x^2\ \text{Sin}[\alpha]\right) - \right.$$

$$\left.\text{Cos}[\alpha]\left(0.02\ y^2\ \text{Cos}[\alpha] + 0.01\ x\ y\ \text{Sin}[\alpha]\right) + \frac{25\ w[3]}{2048} + \frac{25\ w[4]}{2048} + \frac{25\ w[5]}{2048} + \frac{25\ w[6]}{2048}\right)^2 +$$

$$\left(-2\left(\text{Sin}[\alpha]\left(0.02\ x^2\ \text{Cos}[\alpha] - 0.01\ x\ y\ \text{Sin}[\alpha]\right) + \right.\right.$$

$$\left.\left.\text{Cos}[\alpha]\left(0.01\ x\ y\ \text{Cos}[\alpha] - 0.02\ y^2\ \text{Sin}[\alpha]\right)\right) + \frac{25\ w[5]}{1024} + \frac{25\ w[6]}{1024}\right)^2$$

```
NMinimize[Prepend[((# >= 0 &) /@ weights),
  Simplify[Expand[(d.weights - b2).(d.weights - b2) /. x → 50 /. y → 150 /.
     α → ArcCos[β]]]], Append[weights, β]]
```

$\{76\,965.5,\ \{w[1] \to 2566.15,\ w[2] \to 2583.53,\ w[3] \to 82.932,$
$\ w[4] \to 77.7553,\ w[5] \to 3662.66,\ w[6] \to 3674.92,\ \beta \to -0.536739\}\}$

```
{Sqrt[#[[1]]], weights /. #[[2]]} & [FindMinimum[Prepend[((# >= 0 &) /@ weights),
   Expand[(d.weights - b2).(d.weights - b2) /. x → 150 /. y → 50 /. α → 0]], weights]]
```

$\left\{22.3607,\ \left\{15\,574.4,\ 15\,555.2,\ 2.44333 \times 10^{-8},\ 1.94271 \times 10^{-8},\ 2872.,\ 2862.4\right\}\right\}$

```
NMinimize[Prepend[((# >= 0 &) /@ weights),
  Simplify[Expand[(d.weights - b2).(d.weights - b2) /. x → 50 /. y → 50 /.
     α → ArcCos[-0.7614944037382076`]]]], weights]
```

$\{0.0000745046,\ \{w[1] \to 2888.5,\ w[2] \to 2902.03,$
$\ w[3] \to 873.025,\ w[4] \to 874.113,\ w[5] \to 163.581,\ w[6] \to 163.581\}\}$

```
NMinimize[Prepend[((# >= 0 &) /@ weights),
  Simplify[Expand[(d.weights - b2).(d.weights - b2) /. x → 50 /. y → 150 /.
     α → ArcCos[-0.7614944037382076`]]]], weights]
```

$\left\{1.80216,\ \left\{w[1] \to 3393.51,\ w[2] \to 3403.78, \right.\right.$
$\left.\left.\ w[3] \to 1.79256 \times 10^{-6},\ w[4] \to 2.64384 \times 10^{-6},\ w[5] \to 8557.4,\ w[6] \to 8573.11\right\}\right\}$

```
k = Eigenvectors[{{0.020000000000000004` x², 0.020000000000000004` x y / 2},
    {0.020000000000000004` x y / 2, 0.020000000000000004` y²}} /.
   x → 50 /. y → 150]; γ = -ArcCos[k[[1, 1]]];
```

```
NMinimize[Prepend[((# >= 0 &) /@ weights), Simplify[
    Expand[(d.weights - b2).(d.weights - b2) /. x → 50 /. y → 150 /. α → γ]]], weights]
```

$\{0.000373244, \{w[1] \to 19004.1, w[2] \to 18974.,$
$\quad w[3] \to 1488.77, w[4] \to 1493.08, w[5] \to 0.0179083, w[6] \to 0.01668\}\}$

```
NMinimize[Prepend[((# >= 0 &) /@ weights), Simplify[
    Expand[(d.weights - b2).(d.weights - b2) /. x → 50 /. y → 50 /. α → γ]]], weights]
```

$\{2191.78, \{w[1] \to 2413.13, w[2] \to 2401.97, w[3] \to 1684.58,$
$\quad w[4] \to 1692.32, w[5] \to 3.27779 \times 10^{-7}, w[6] \to 3.63408 \times 10^{-7}\}\}$

```
te = c.{{0.020000000000000004` x², 0.020000000000000004` x y / 2},
      {0.020000000000000004` x y / 2, 0.020000000000000004` y²}}.Transpose[c] /.
    x → 50 /. y → 150 /. α -> ArcCos[-0.7614944037382076`]
```

$\{\{292.087, 209.413\}, \{209.413, 207.913\}\}$

```
{{0.020000000000000004` x², 0.020000000000000004` x y / 2},
    {0.020000000000000004` x y / 2, 0.020000000000000004` y²}} /. x → 50 /.
  y → 150 /. α -> ArcCos[-0.7614944037382076`]
```

$\{\{50., 75.\}, \{75., 450.\}\}$

```
te = c.{{0.020000000000000004` x², 0.020000000000000004` x y / 2},
      {0.020000000000000004` x y / 2, 0.020000000000000004` y²}}.Transpose[c] /.
    x → 50 /. y → 50 /. α -> ArcCos[-0.7614944037382076`]
```

$\{\{74.6789, 3.99369\}, \{3.99369, 25.3211\}\}$

```
{{0.020000000000000004` x², 0.020000000000000004` x y / 2},
    {0.020000000000000004` x y / 2, 0.020000000000000004` y²}} /. x → 50 /.
  y → 50 /. α -> ArcCos[-0.7614944037382076`]
```

$\{\{50., 25.\}, \{25., 50.\}\}$