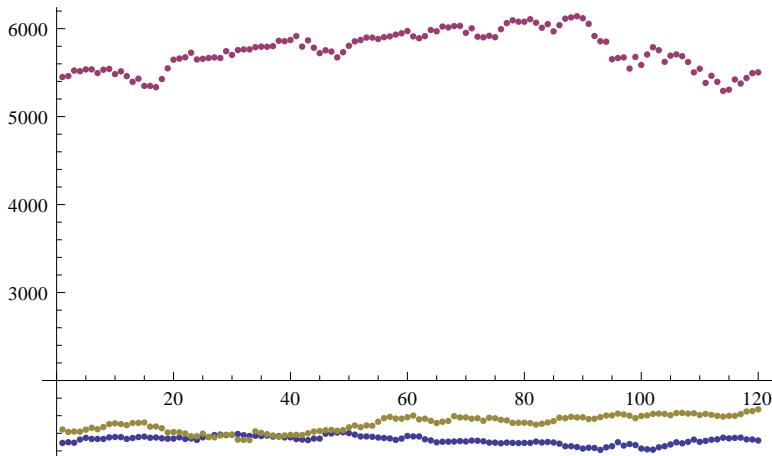
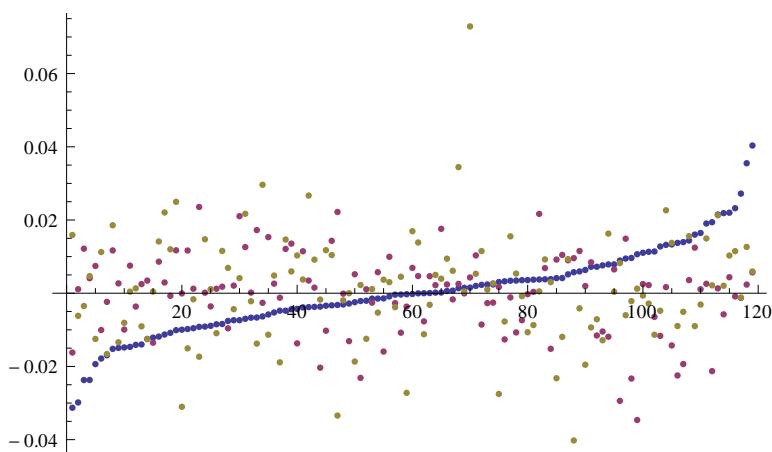


```
{hedge, dax, d2} = Transpose[Import["c:\\kurse.dat", "Table"]];
ListPlot[{hedge, dax, d2}]
```



```
hedge = Log[hedge]; dax = Log[dax]; d2 = Log[d2];
hedge = Differences[hedge];
dax = Differences[dax]; d2 = Differences[d2];
w = Transpose[{hedge, dax, d2}];
w = Sort[w, #1[[1]] < #2[[1]] &];
hedge = Transpose[w][[1]];
dax = Transpose[w][[2]];
d2 = Transpose[w][[3]];
ListPlot[Transpose[w][[1 ;; 3]], PlotRange -> All]
```



```
min0 = Min[Transpose[w][[1]]];
wn = Length[hedge];
nn = wn;
max0 = Max[Transpose[w][[1]]];
min1 = Min[Transpose[w][[2]]];
max1 = Max[Transpose[w][[2]]];
```

```

U = {} ; sdax = Sort[dax] ; AppendTo[U, {max0, max1, 1}] ;

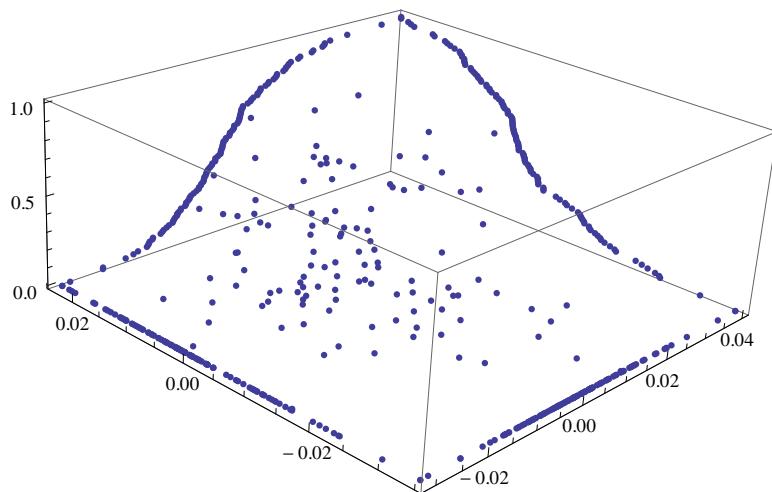
For[i = 1, i ≤ nn, i++,
  AppendTo[U, {hedge[[i]], max1, (i - 1) / nn}] ;
  AppendTo[U, {max0, sdax[[i]], (i - 1) / nn}] ;
  AppendTo[U, {hedge[[i]], min1, 0}] ;
  AppendTo[U, {min0, sdax[[i]], 0}] ;
]

F = {} ; For[i = 1, i ≤ wN, i++,
  AppendTo[F, {w[[i, 1]], w[[i, 2]],
    Length[Select[w, #[[1]] <= w[[i, 1]] && #[[2]] <= w[[i, 2]] &]] / wN}] ;
]

W = Join[F, U] ;

ListPointPlot3D[W]

```

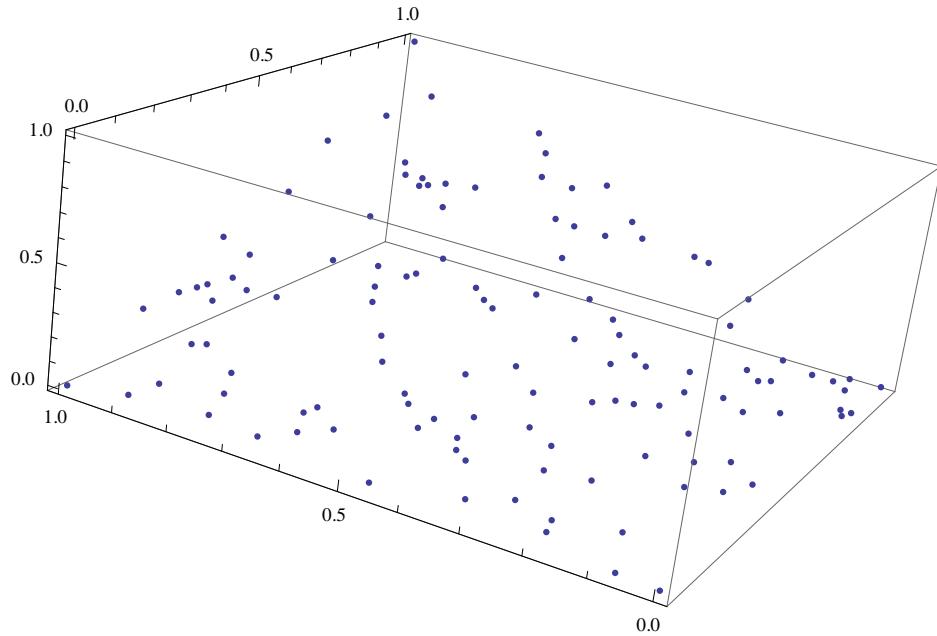


```

hedgeI = Table[{hedge[[i]], (i - 1) / (nn - 1)}, {i, nn}] ;
daxI = Table[{sdax[[i]], (i - 1) / (nn - 1)}, {i, nn}] ; W = F ;
Co = Table[{Select[hedgeI, #[[1]] == W[[i, 1]] &][[1, 2]],
  Select[daxI, #[[1]] == W[[i, 2]] &][[1, 2]], W[[i, 3]]}, {i, Length[W]}] ;
AppendTo[Co, {1, 0, 0}] ; AppendTo[Co, {0, 1, 0}] ; AppendTo[Co, {0, 0, 0}] ;
AppendTo[Co, {1, 1, 1}] ;

```

```
ListPointPlot3D [Co]
```

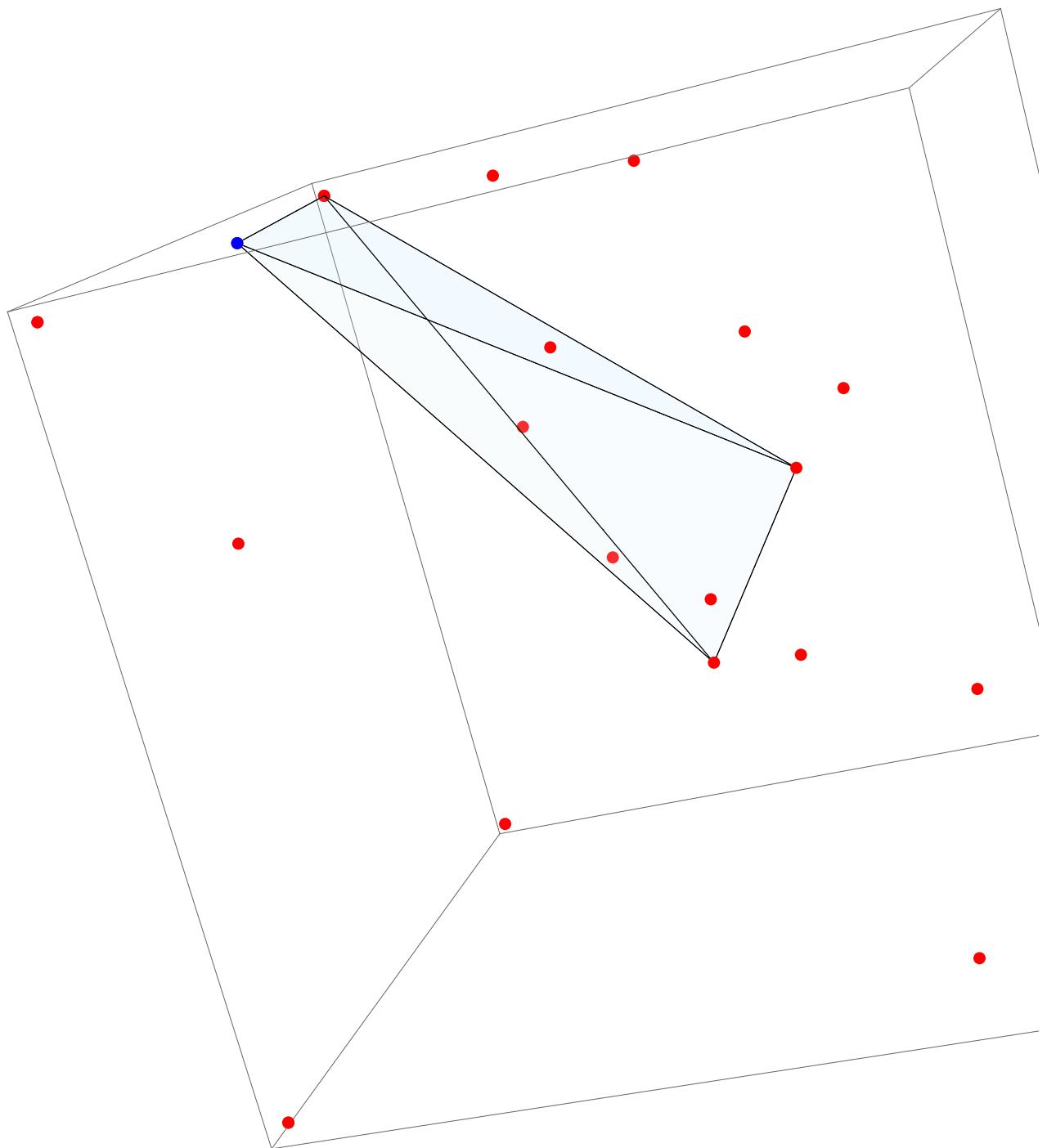


```

h = Import["C:\\\\delaunay.txt", "Table"]
g = Import["c:\\\\copulasample.txt", "Table"];
cc = Drop[Import["c:\\\\empcopula.dat", "Table"], 1];
c = Transpose[{#[[1]], #[[2]], #[[4]]} &[Transpose[cc]]];
cc = Transpose[Transpose[cc][[1 ;; 3]]];
hh = Table[Subsets[h[[i]], {3}], {i, Length[h]}][[3]];
Graphics3D[{{PointSize[Large], Red, Table[Point[c[[i]]], {i, Length[c]}]},
{PointSize[Large], Blue, Point[c[[8]]]},
{Opacity[0.1], Table[Polygon[c[[# + 1]] & /@ hh[[j]]], {j, Length[hh]}]}}]

```

```
{ {0, 7, 8, 12}, {0, 7, 8, 13}, {0, 7, 12, 22}, {0, 7, 13, 22}, {0, 8, 10, 13}, {0, 8, 10, 14},  
{0, 8, 12, 14}, {0, 10, 13, 21}, {0, 10, 14, 21}, {0, 12, 14, 16}, {0, 12, 16, 22},  
{0, 13, 17, 21}, {0, 13, 17, 22}, {0, 14, 16, 21}, {0, 16, 17, 21}, {0, 16, 17, 22},  
{0, 16, 21, 22}, {0, 17, 21, 22}, {1, 3, 4, 5}, {1, 3, 4, 6}, {1, 3, 5, 7}, {1, 3, 6, 8},  
{1, 3, 7, 8}, {1, 4, 5, 22}, {1, 4, 6, 18}, {1, 4, 18, 22}, {1, 5, 7, 22}, {1, 6, 8, 9},  
{1, 6, 9, 15}, {1, 6, 15, 18}, {1, 7, 8, 13}, {1, 7, 9, 13}, {1, 8, 9, 13}, {2, 3, 4, 6},  
{2, 3, 4, 11}, {2, 3, 6, 8}, {2, 3, 8, 11}, {2, 4, 6, 11}, {2, 6, 8, 9}, {2, 6, 9, 15},  
{2, 6, 11, 19}, {2, 6, 15, 19}, {2, 8, 9, 13}, {2, 8, 10, 13}, {2, 8, 10, 14},  
{2, 8, 11, 14}, {2, 9, 13, 19}, {2, 9, 15, 19}, {2, 10, 13, 19}, {2, 10, 14, 19},  
{2, 11, 14, 19}, {3, 4, 5, 11}, {3, 5, 7, 12}, {3, 5, 8, 12}, {3, 5, 8, 14}, {3, 5, 11, 14},  
{3, 7, 8, 12}, {3, 8, 11, 14}, {4, 5, 11, 18}, {4, 5, 18, 22}, {4, 6, 11, 18},  
{5, 7, 12, 22}, {5, 8, 12, 14}, {5, 11, 14, 20}, {5, 11, 18, 20}, {5, 12, 14, 16},  
{5, 12, 16, 22}, {5, 14, 16, 20}, {6, 11, 15, 18}, {6, 11, 15, 19}, {7, 9, 13, 17},  
{7, 13, 17, 22}, {9, 10, 13, 15}, {9, 10, 13, 19}, {9, 10, 15, 19}, {9, 13, 15, 17},  
{10, 13, 15, 17}, {10, 13, 17, 21}, {10, 14, 16, 20}, {10, 14, 16, 21}, {10, 14, 19, 20},  
{10, 16, 19, 20}, {10, 16, 19, 21}, {10, 16, 20, 21}, {10, 19, 20, 21}, {11, 14, 19, 20} }
```



**f[B[[1]]] / ccc // N**

0.723532

**s = h[[3]]**

{0, 7, 12, 22}

```

B = {{1, 2, 3, 4}};

For[i = 1, i ≤ 3, i++, AppendTo[B, RotateLeft[B[[Length[B]]]]]

B

{{1, 2, 3, 4}, {2, 3, 4, 1}, {3, 4, 1, 2}, {4, 1, 2, 3}};

RotateLeft[%]

{3, 4, 1, 2}

S[[#]] & /@ %

{7, 12, 22, 0}

f[u_] := Abs[2 * ArcTan[t[{#[[1]], #[[2]], #[[3]]} - {1, 1, 1} #[[4]] &[
c[[#+1]] & /@ (S[[#]] & /@ u)]]] / Pi // N

ccc = f[B[[1]]] + f[B[[2]]] + f[B[[3]]] + f[B[[4]]]

0.524036

4 / 3 // N

1.33333

t =.

t[R_] :=
Det[R] / (Norm[R[[1]]] Norm[R[[2]]] Norm[R[[3]]] + (R[[1]].R[[2]]) * Norm[R[[3]]] +
(R[[2]].R[[3]]) * Norm[R[[1]]] + (R[[3]].R[[1]]) * Norm[R[[2]]])

c[[1]]

{10, 7, 8}

ce = c[[3;;7]];

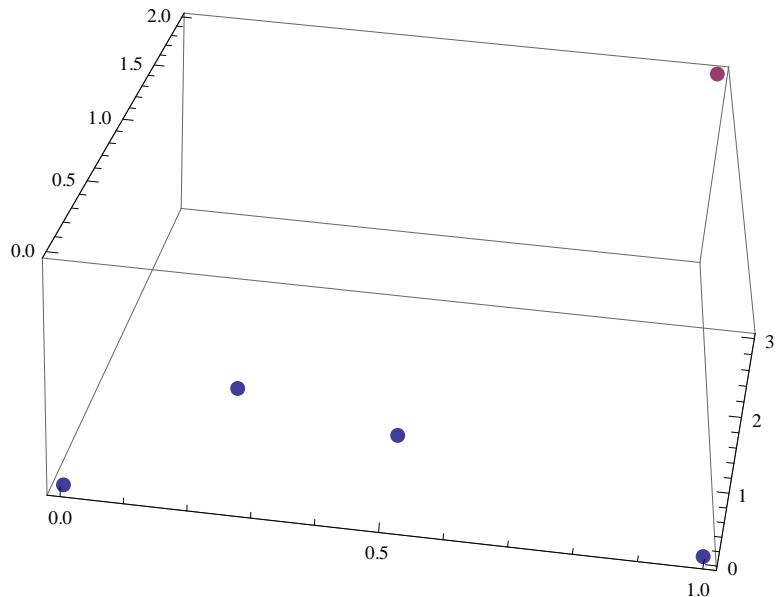
ce = {{0.23564142713114022`, 0.19380998628864599`, 0.594495511815069`},
{0.6525572153791819`, 0.24694329925782044`, 0.7672917678839029`},
{0.9163123444125463`, 0.026621532304647255`, 0.3715576391209683`},
{0.2668950744537262`, 0.7975001767040044`, 0.7934023887167383`},
{0.7641225411392012`, 0.2966207983489944`, 0.1456840182999719`}};

{{0.235641, 0.19381, 0.594496},
{0.652557, 0.246943, 0.767292}, {0.916312, 0.0266215, 0.371558},
{0.266895, 0.7975, 0.793402}, {0.764123, 0.296621, 0.145684}};

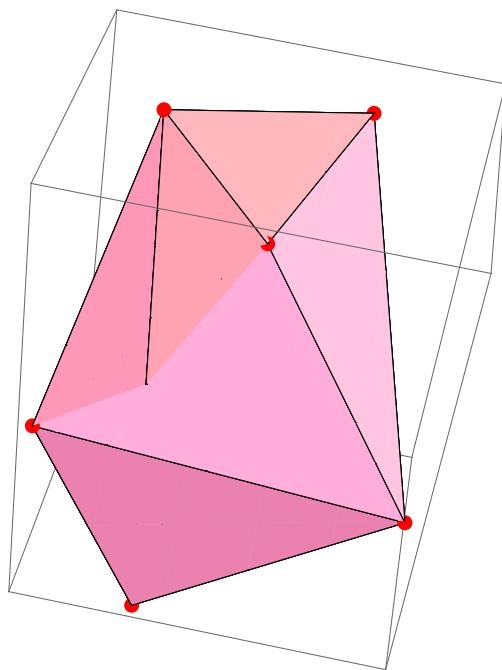
{{0.23564142713114022`, 0.19380998628864599`, 0.594495511815069`},
{0.6525572153791819`, 0.24694329925782044`, 0.7672917678839029`},
{0.3163123444125463`, 0.026621532304647255`, 0.3715576391209683`},
{0.2668950744537262`, 0.7975001767040044`, 0.7934023887167383`},
{0.7641225411392012`, 0.8966207983489944`, 0.1456840182999719`}};

```

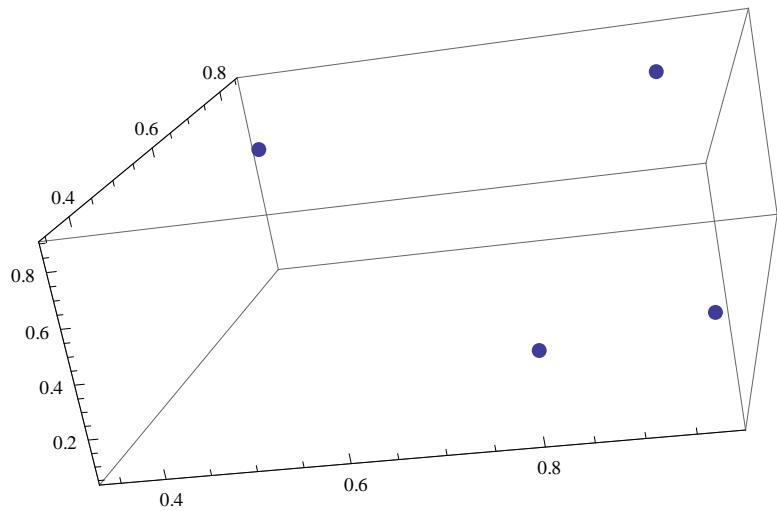
```
ListPointPlot3D[{ce, {a}}, PlotStyle -> PointSize[Large]]
```



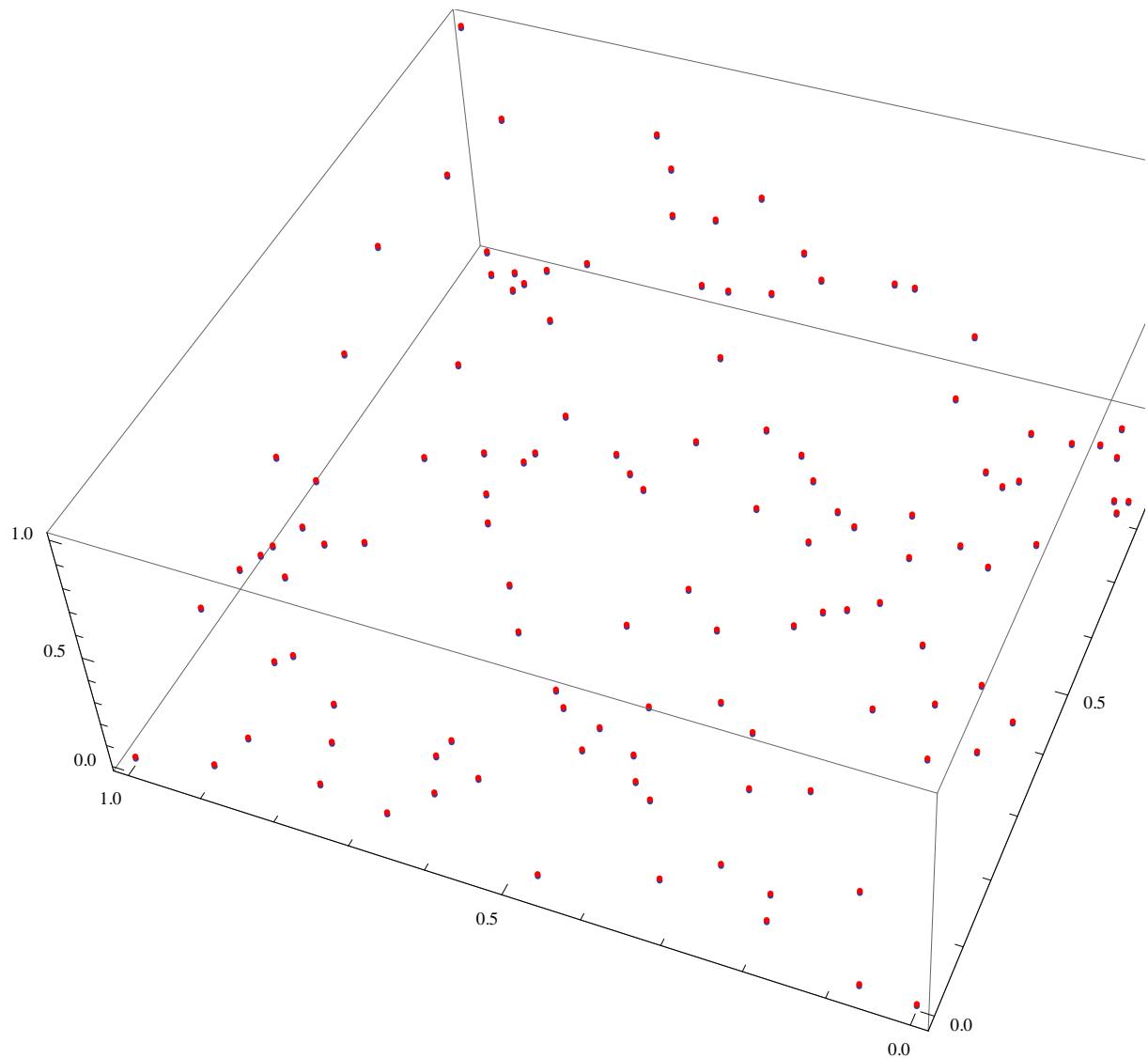
```
a = {1, 2, 3};  
Det [  
Export["c:\\empCopula.dat", RandomReal[1, {7, 4}]]  
c:\\empCopula.dat
```



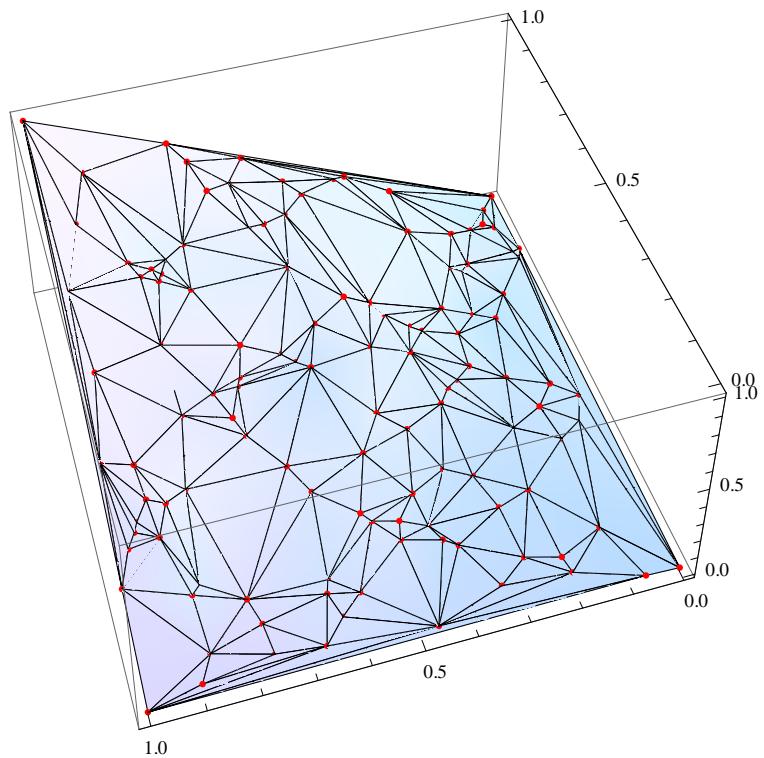
```
ListPointPlot3D[cc, PlotStyle -> PointSize[Large]]
```



```
Show[ListPointPlot3D[{#[[1]], #[[2]], #[[3]] + 0.01} & /@ c, PlotStyle -> Red],  
ListPointPlot3D[Co]]
```



```
Show[ListPlot3D[c, Mesh -> All], ListPointPlot3D[c, PlotStyle -> Red], AspectRatio -> 1]
```



**Y = .**

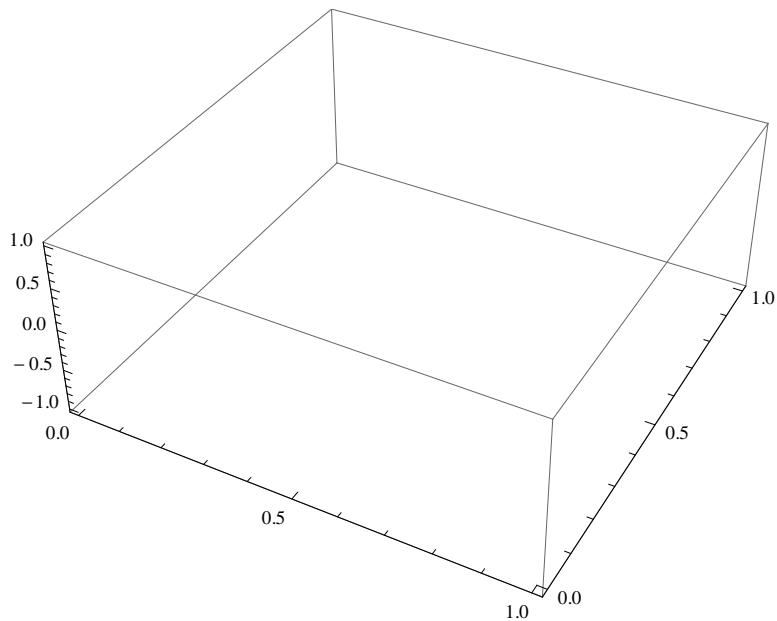
```
Plot3D[Interpolation[c][x, y], {x, 0, 1}, {y, 0, 1}]
```

Interpolation::indim: The coordinates do not lie on a structured tensor product grid.

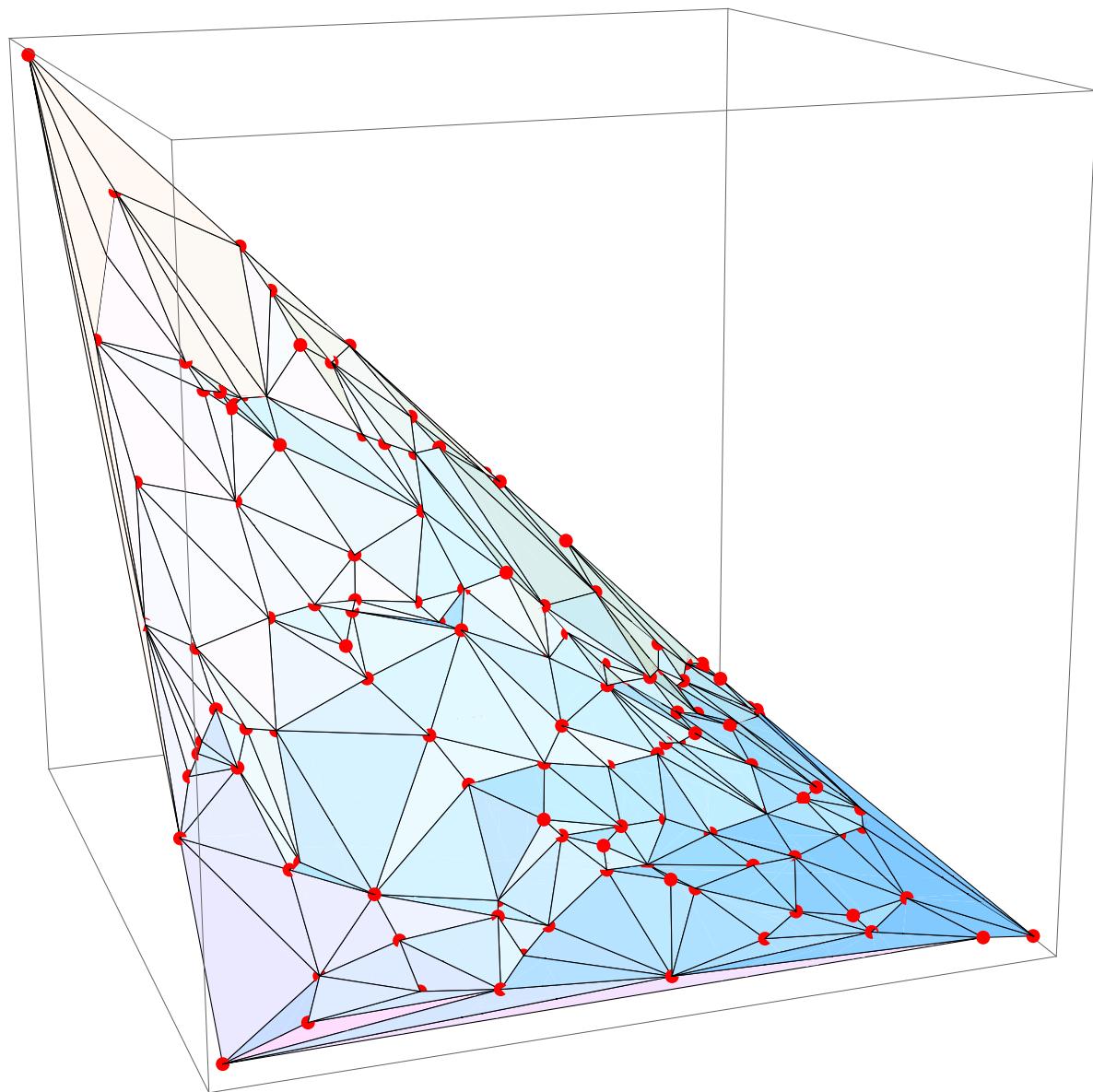
Interpolation::indim: The coordinates do not lie on a structured tensor product grid.

Interpolation::indim: The coordinates do not lie on a structured tensor product grid.

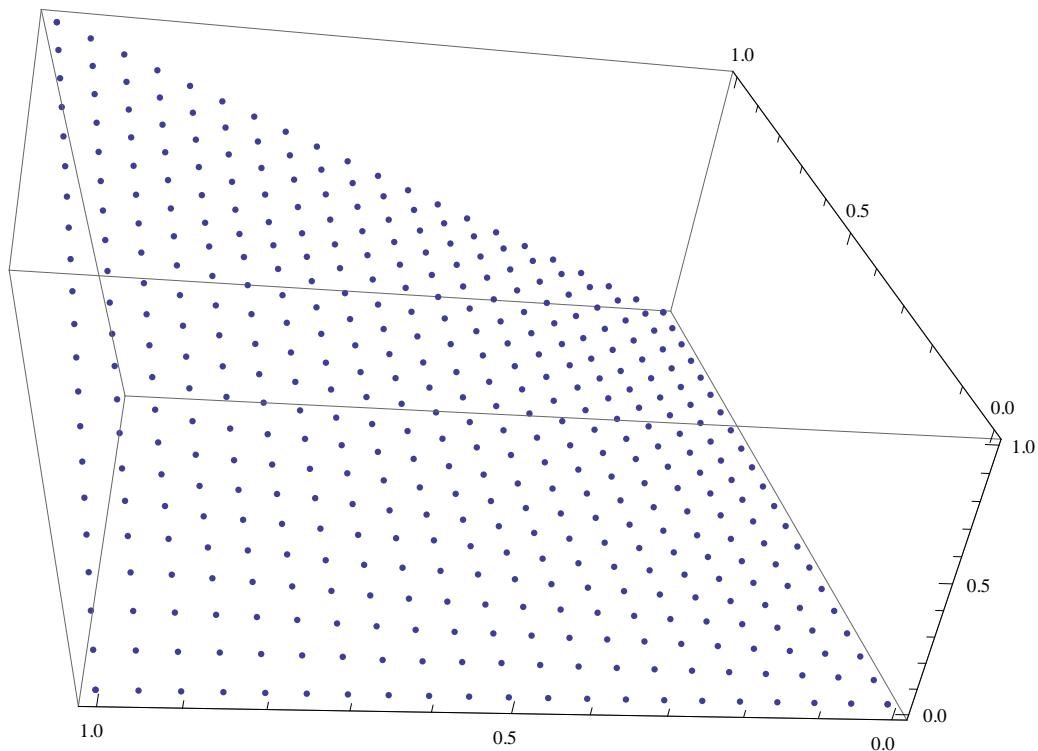
General::stop : Further output of Interpolation::indim will be suppressed during this calculation. >>



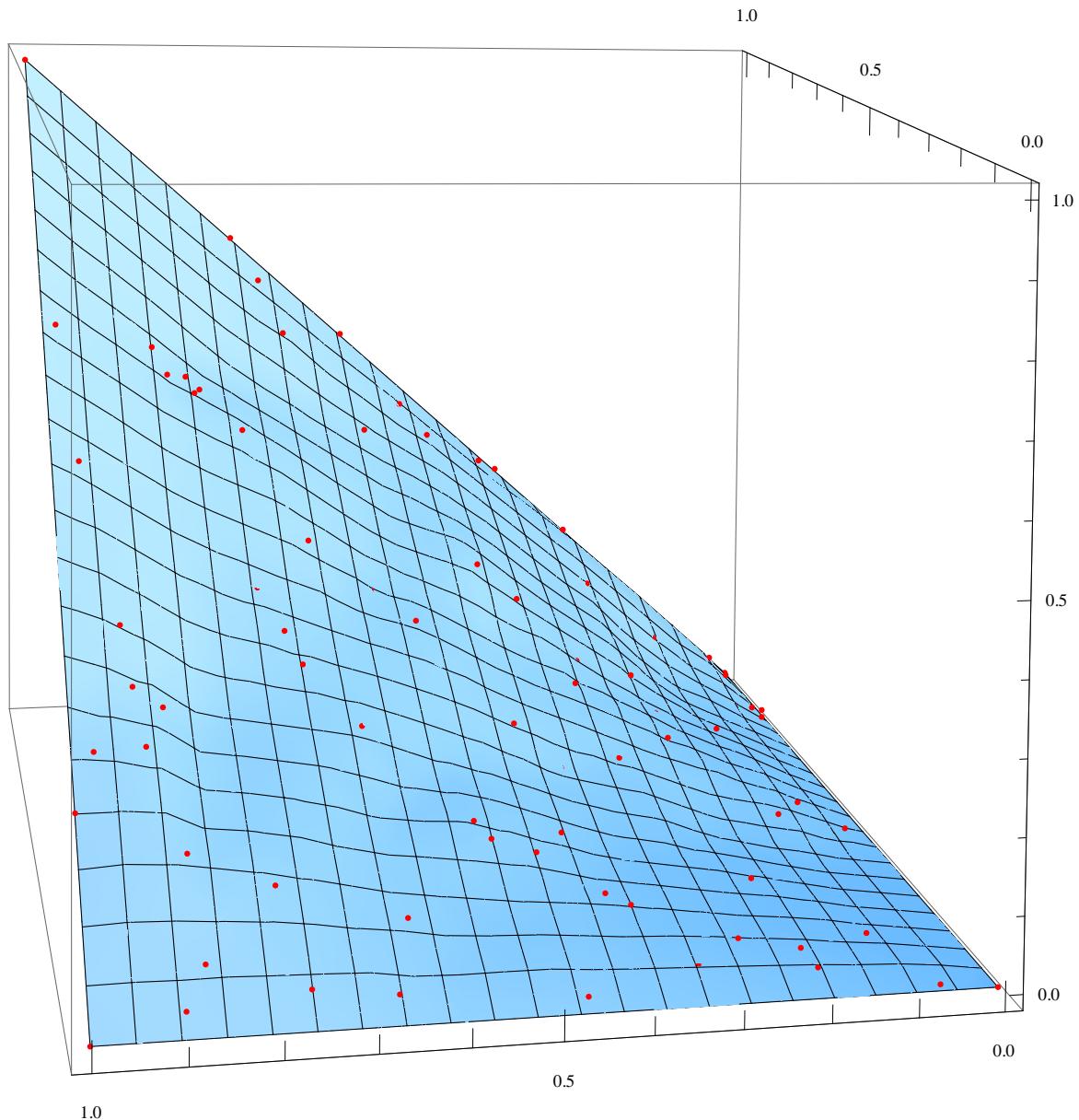
```
Graphics3D[{{PointSize[Large], Red, Table[Point[c[[i]]], {i, Length[c]}]},  
{Table[Polygon[c[[# + 1]] & /@ h[[j]]], {j, Length[h]}]}}]
```



```
ListPointPlot3D[{g}]
```



```
Show[ListPointPlot3D[c, PlotStyle -> Red],  
ListPlot3D[g, InterpolationOrder -> 1, Mesh -> {20, 20}], AspectRatio -> 1]
```



```

f[i_] := {C[i][0], C[i][1], C[i][2], C[i][0]^2 + C[i][1]^2 + C[i][2]^2};
Det[{f[h], f[i], f[j], f[k]}]

C[h][2] C[i][1] C[j][0]^2 C[k][0] - C[h][1] C[i][2] C[j][0]^2 C[k][0] -
C[h][2] C[i][0]^2 C[j][1] C[k][0] - C[h][2] C[i][1]^2 C[j][1] C[k][0] +
C[h][0]^2 C[i][2] C[j][1] C[k][0] + C[h][1]^2 C[i][2] C[j][1] C[k][0] +
C[h][2]^2 C[i][2] C[j][1] C[k][0] - C[h][2] C[i][2]^2 C[j][1] C[k][0] +
C[h][2] C[i][1] C[j][1]^2 C[k][0] - C[h][1] C[i][2] C[j][1]^2 C[k][0] +
C[h][1] C[i][0]^2 C[j][2] C[k][0] - C[h][0]^2 C[i][1] C[j][2] C[k][0] -
C[h][1]^2 C[i][1] C[j][2] C[k][0] - C[h][2]^2 C[i][1] C[j][2] C[k][0] +
C[h][1] C[i][1]^2 C[j][2] C[k][0] + C[h][1] C[i][2]^2 C[j][2] C[k][0] +
C[h][2] C[i][1] C[j][2]^2 C[k][0] - C[h][1] C[i][2] C[j][2]^2 C[k][0] -
C[h][2] C[i][1] C[j][0] C[k][0]^2 + C[h][1] C[i][2] C[j][0] C[k][0]^2 +
C[h][2] C[i][0] C[j][1] C[k][0]^2 - C[h][0] C[i][2] C[j][1] C[k][0]^2 -
C[h][1] C[i][0] C[j][2] C[k][0]^2 + C[h][0] C[i][1] C[j][2] C[k][0]^2 +
C[h][2] C[i][0]^2 C[j][0] C[k][1] + C[h][2] C[i][1]^2 C[j][0] C[k][1] -
C[h][0]^2 C[i][2] C[j][0] C[k][1] - C[h][1]^2 C[i][2] C[j][0] C[k][1] -
C[h][2]^2 C[i][2] C[j][0] C[k][1] + C[h][2] C[i][2]^2 C[j][0] C[k][1] -
C[h][2] C[i][0] C[j][0] C[k][1] + C[h][0] C[i][2] C[j][0] C[k][1]^2 -
C[h][2] C[i][0] C[j][1]^2 C[k][1] + C[h][0] C[i][2] C[j][1]^2 C[k][1] +
C[h][0]^2 C[i][0] C[j][2] C[k][1] + C[h][1]^2 C[i][0] C[j][2] C[k][1] +
C[h][2]^2 C[i][0] C[j][2] C[k][1] - C[h][0] C[i][0]^2 C[j][2] C[k][1] -
C[h][0] C[i][1]^2 C[j][2] C[k][1] - C[h][0] C[i][2]^2 C[j][2] C[k][1] -
C[h][2] C[i][0] C[j][2]^2 C[k][1] + C[h][0] C[i][2] C[j][2]^2 C[k][1] -
C[h][2] C[i][1] C[j][0] C[k][1]^2 + C[h][1] C[i][2] C[j][0] C[k][1]^2 +
C[h][2] C[i][0] C[j][1] C[k][1]^2 - C[h][0] C[i][2] C[j][1] C[k][1]^2 -
C[h][1] C[i][0] C[j][2] C[k][1]^2 + C[h][0] C[i][1] C[j][2] C[k][1]^2 -
C[h][1] C[i][0]^2 C[j][0] C[k][2] + C[h][0]^2 C[i][1] C[j][0] C[k][2] +
C[h][1]^2 C[i][1] C[j][0] C[k][2] + C[h][2]^2 C[i][1] C[j][0] C[k][2] -
C[h][1] C[i][1]^2 C[j][0] C[k][2] - C[h][1] C[i][2]^2 C[j][0] C[k][2] +
C[h][1] C[i][0] C[j][0]^2 C[k][2] - C[h][0] C[i][1] C[j][0]^2 C[k][2] -
C[h][0]^2 C[i][0] C[j][1] C[k][2] - C[h][1]^2 C[i][0] C[j][1] C[k][2] -
C[h][2]^2 C[i][0] C[j][1] C[k][2] + C[h][0] C[i][0]^2 C[j][1] C[k][2] +
C[h][0] C[i][1]^2 C[j][1] C[k][2] + C[h][0] C[i][2]^2 C[j][1] C[k][2] +
C[h][1] C[i][0] C[j][1]^2 C[k][2] - C[h][0] C[i][1] C[j][1]^2 C[k][2] +
C[h][1] C[i][0] C[j][2]^2 C[k][2] - C[h][0] C[i][1] C[j][2]^2 C[k][2] -
C[h][2] C[i][1] C[j][0] C[k][2]^2 + C[h][1] C[i][2] C[j][0] C[k][2]^2 +
C[h][2] C[i][0] C[j][1] C[k][2]^2 - C[h][0] C[i][2] C[j][1] C[k][2]^2 -
C[h][1] C[i][0] C[j][2] C[k][2]^2 + C[h][0] C[i][1] C[j][2] C[k][2]^2

```

**CForm [Expand [%]]**

```
C(h)(2)*C(i)(1)*Power(C(j)(0),2)*C(k)(0) - C(h)(1)*C(i)(2)*Power(C(j)(0),2)*C(k)(0) - C(h)
C(h)(2)*Power(C(i)(1),2)*C(j)(1)*C(k)(0) + Power(C(h)(0),2)*C(i)(2)*C(j)(1)*C(k)(0) +
Power(C(h)(1),2)*C(i)(2)*C(j)(1)*C(k)(0) + Power(C(h)(2),2)*C(i)(2)*C(j)(1)*C(k)(0) -
C(h)(2)*Power(C(i)(2),2)*C(j)(1)*C(k)(0) + C(h)(2)*C(i)(1)*Power(C(j)(1),2)*C(k)(0) -
C(h)(1)*C(i)(2)*Power(C(j)(1),2)*C(k)(0) + C(h)(1)*Power(C(i)(0),2)*C(j)(2)*C(k)(0) -
Power(C(h)(0),2)*C(i)(1)*C(j)(2)*C(k)(0) - Power(C(h)(1),2)*C(i)(1)*C(j)(2)*C(k)(0) -
Power(C(h)(2),2)*C(i)(1)*C(j)(2)*C(k)(0) + C(h)(1)*Power(C(i)(1),2)*C(j)(2)*C(k)(0) +
C(h)(1)*Power(C(i)(2),2)*C(j)(2)*C(k)(0) + C(h)(2)*C(i)(1)*Power(C(j)(2),2)*C(k)(0) -
C(h)(1)*C(i)(2)*Power(C(j)(2),2)*C(k)(0) - C(h)(2)*C(i)(1)*C(j)(0)*Power(C(k)(0),2) +
C(h)(1)*C(i)(2)*C(j)(0)*Power(C(k)(0),2) + C(h)(2)*C(i)(0)*C(j)(1)*Power(C(k)(0),2) -
C(h)(0)*C(i)(2)*C(j)(1)*Power(C(k)(0),2) - C(h)(1)*C(i)(0)*C(j)(2)*Power(C(k)(0),2) +
C(h)(0)*C(i)(1)*C(j)(2)*Power(C(k)(0),2) + C(h)(2)*Power(C(i)(0),2)*C(j)(0)*C(k)(1) +
C(h)(2)*Power(C(i)(1),2)*C(j)(0)*C(k)(1) - Power(C(h)(0),2)*C(i)(2)*C(j)(0)*C(k)(1) -
Power(C(h)(1),2)*C(i)(2)*C(j)(0)*C(k)(1) - Power(C(h)(2),2)*C(i)(2)*C(j)(0)*C(k)(1) +
C(h)(2)*Power(C(i)(2),2)*C(j)(0)*C(k)(1) - C(h)(2)*C(i)(0)*Power(C(j)(0),2)*C(k)(1) +
C(h)(0)*C(i)(2)*Power(C(j)(0),2)*C(k)(1) - C(h)(2)*C(i)(0)*Power(C(j)(1),2)*C(k)(1) +
C(h)(0)*C(i)(2)*Power(C(j)(1),2)*C(k)(1) + Power(C(h)(0),2)*C(i)(0)*C(j)(2)*C(k)(1) +
Power(C(h)(1),2)*C(i)(0)*C(j)(2)*C(k)(1) + Power(C(h)(2),2)*C(i)(0)*C(j)(2)*C(k)(1) -
C(h)(0)*Power(C(i)(0),2)*C(j)(2)*C(k)(1) - C(h)(0)*Power(C(i)(1),2)*C(j)(2)*C(k)(1) -
C(h)(0)*Power(C(i)(2),2)*C(j)(2)*C(k)(1) - C(h)(2)*C(i)(0)*Power(C(j)(2),2)*C(k)(1) +
C(h)(0)*C(i)(2)*Power(C(j)(2),2)*C(k)(1) - C(h)(2)*C(i)(1)*C(j)(0)*Power(C(k)(1),2) +
C(h)(1)*C(i)(2)*C(j)(0)*Power(C(k)(1),2) + C(h)(2)*C(i)(0)*C(j)(1)*Power(C(k)(1),2) -
C(h)(0)*C(i)(2)*C(j)(1)*Power(C(k)(1),2) - C(h)(1)*C(i)(0)*C(j)(2)*Power(C(k)(1),2) +
C(h)(0)*C(i)(1)*C(j)(2)*Power(C(k)(1),2) - C(h)(1)*Power(C(i)(0),2)*C(j)(0)*C(k)(2) +
Power(C(h)(0),2)*C(i)(1)*C(j)(0)*C(k)(2) + Power(C(h)(1),2)*C(i)(1)*C(j)(0)*C(k)(2) +
Power(C(h)(2),2)*C(i)(1)*C(j)(0)*C(k)(2) - C(h)(1)*Power(C(i)(1),2)*C(j)(0)*C(k)(2) -
C(h)(1)*Power(C(i)(2),2)*C(j)(0)*C(k)(2) + C(h)(1)*C(i)(0)*Power(C(j)(0),2)*C(k)(2) -
C(h)(0)*C(i)(1)*Power(C(j)(0),2)*C(k)(2) - Power(C(h)(0),2)*C(i)(0)*C(j)(1)*C(k)(2) -
Power(C(h)(1),2)*C(i)(0)*C(j)(1)*C(k)(2) - Power(C(h)(2),2)*C(i)(0)*C(j)(1)*C(k)(2) +
C(h)(0)*Power(C(i)(0),2)*C(j)(1)*C(k)(2) + C(h)(0)*Power(C(i)(1),2)*C(j)(1)*C(k)(2) +
C(h)(0)*Power(C(i)(2),2)*C(j)(1)*C(k)(2) + C(h)(1)*C(i)(0)*Power(C(j)(1),2)*C(k)(2) -
C(h)(0)*C(i)(1)*Power(C(j)(1),2)*C(k)(2) + C(h)(1)*C(i)(0)*Power(C(j)(2),2)*C(k)(2) -
C(h)(0)*C(i)(1)*Power(C(j)(2),2)*C(k)(2) - C(h)(2)*C(i)(1)*C(j)(0)*Power(C(k)(2),2) +
C(h)(1)*C(i)(2)*C(j)(0)*Power(C(k)(2),2) + C(h)(2)*C(i)(0)*C(j)(1)*Power(C(k)(2),2) -
C(h)(0)*C(i)(2)*C(j)(1)*Power(C(k)(2),2) - C(h)(1)*C(i)(0)*C(j)(2)*Power(C(k)(2),2) + C
```

**f[k]**

```
{C[k][0], C[k][1], C[k][2], C[i][0]*C[i][0] + C[i][1]*C[i][1] + C[i][2]*C[i][2], 1}
```

**Expand [Det [{f[h], f[i], f[j], f[k]}]]**

```
-C[h][2] C[i][1] C[j][0] + C[h][1] C[i][2] C[j][0] + C[h][2] C[i][0] C[j][1] -
C[h][0] C[i][2] C[j][1] - C[h][1] C[i][0] C[j][2] + C[h][0] C[i][1] C[j][2] +
C[h][2] C[i][1] C[k][0] - C[h][1] C[i][2] C[k][0] - C[h][2] C[j][1] C[k][0] +
C[i][2] C[j][1] C[k][0] + C[h][1] C[j][2] C[k][0] - C[i][1] C[j][2] C[k][0] -
C[h][2] C[i][0] C[k][1] + C[h][0] C[i][2] C[k][1] + C[h][2] C[j][0] C[k][1] -
C[i][2] C[j][0] C[k][1] - C[h][0] C[j][2] C[k][1] + C[i][0] C[j][2] C[k][1] +
C[h][1] C[i][0] C[k][2] - C[h][0] C[i][1] C[k][2] - C[h][1] C[j][0] C[k][2] +
C[i][1] C[j][0] C[k][2] + C[h][0] C[j][1] C[k][2] - C[i][0] C[j][1] C[k][2]
```

**f[i\_] := {C[i][0], C[i][1], C[i][2]}; CForm [Expand [Det [{f[h], f[i], f[j]}]]]**

```
-(C(h)(2)*C(i)(1)*C(j)(0)) + C(h)(1)*C(i)(2)*C(j)(0) + C(h)(2)*C(i)(0)*C(j)(1) - C(h)(0)*C(h)(1)*C(i)(0)*C(j)(2) + C(h)(0)*C(i)(1)*C(j)(2)
```

```

a = Expand [Det [{f[h], f[i], f[j]}]]

-C[h][2] C[i][1] C[j][0] + C[h][1] C[i][2] C[j][0] + C[h][2] C[i][0] C[j][1] -
C[h][0] C[i][2] C[j][1] - C[h][1] C[i][0] C[j][2] + C[h][0] C[i][1] C[j][2] +
C[h][2] C[i][1] C[k][0] - C[h][1] C[i][2] C[k][0] - C[h][2] C[j][1] C[k][0] +
C[i][2] C[j][1] C[k][0] + C[h][1] C[j][2] C[k][0] - C[i][1] C[j][2] C[k][0] -
C[h][2] C[i][0] C[k][1] + C[h][0] C[i][2] C[k][1] + C[h][2] C[j][0] C[k][1] -
C[i][2] C[j][0] C[k][1] - C[h][0] C[j][2] C[k][1] + C[i][0] C[j][2] C[k][1] +
C[h][1] C[i][0] C[k][2] - C[h][0] C[i][1] C[k][2] - C[h][1] C[j][0] C[k][2] +
C[i][1] C[j][0] C[k][2] + C[h][0] C[j][1] C[k][2] - C[i][0] C[j][1] C[k][2]

CForm[a]

-(C(h)(2)*C(i)(1)*C(j)(0)) + C(h)(1)*C(i)(2)*C(j)(0) + C(h)(2)*C(i)(0)*C(j)(1) - C(h)(0)*C
C(h)(1)*C(i)(0)*C(j)(2) + C(h)(0)*C(i)(1)*C(j)(2) + C(h)(2)*C(i)(1)*C(k)(0) - C(h)(1)*C
C(h)(2)*C(j)(1)*C(k)(0) + C(i)(2)*C(j)(1)*C(k)(0) + C(h)(1)*C(j)(2)*C(k)(0) - C(i)(1)*C
C(h)(2)*C(i)(0)*C(k)(1) + C(h)(0)*C(i)(2)*C(k)(1) + C(h)(2)*C(j)(0)*C(k)(1) - C(i)(2)*C
C(h)(0)*C(j)(2)*C(k)(1) + C(i)(0)*C(j)(2)*C(k)(1) + C(h)(1)*C(i)(0)*C(k)(2) - C(h)(0)*C
C(h)(1)*C(j)(0)*C(k)(2) + C(i)(1)*C(j)(0)*C(k)(2) + C(h)(0)*C(j)(1)*C(k)(2) - C(i)(0)*C

b = Expand [Det [{f[h], f[i], f[j], f[k]}]]

-C[h][2] C[i][1] C[j][0] + C[h][1] C[i][2] C[j][0] + C[h][2] C[i][0] C[j][1] -
C[h][0] C[i][2] C[j][1] - C[h][1] C[i][0] C[j][2] + C[h][0] C[i][1] C[j][2] +
C[h][2] C[i][1] C[k][0] - C[h][1] C[i][2] C[k][0] - C[h][2] C[j][1] C[k][0] +
C[i][2] C[j][1] C[k][0] + C[h][1] C[j][2] C[k][0] - C[i][1] C[j][2] C[k][0] -
C[h][2] C[i][0] C[k][1] + C[h][0] C[i][2] C[k][1] + C[h][2] C[j][0] C[k][1] -
C[i][2] C[j][0] C[k][1] - C[h][0] C[j][2] C[k][1] + C[i][0] C[j][2] C[k][1] +
C[h][1] C[i][0] C[k][2] - C[h][0] C[i][1] C[k][2] - C[h][1] C[j][0] C[k][2] +
C[i][1] C[j][0] C[k][2] + C[h][0] C[j][1] C[k][2] - C[i][0] C[j][1] C[k][2]

a - b

0

Needs["VectorAnalysis`"]

v[k_] := {C[k][0], C[k][1], C[k][2]};

n = CrossProduct[v[k] - v[i], v[j] - v[i]];
Solve[({A[0], A[1], z} - v[i]).n == 0, z][[1, 1, 2]]

(A[1] C[i][2] C[j][0] - A[0] C[i][2] C[j][1] - A[1] C[i][0] C[j][2] +
A[0] C[i][1] C[j][2] - A[1] C[i][2] C[k][0] + C[i][2] C[j][1] C[k][0] +
A[1] C[j][2] C[k][0] - C[i][1] C[j][2] C[k][0] + A[0] C[i][2] C[k][1] -
C[i][2] C[j][0] C[k][1] - A[0] C[j][2] C[k][1] + C[i][0] C[j][2] C[k][1] +
A[1] C[i][0] C[k][2] - A[0] C[i][1] C[k][2] - A[1] C[j][0] C[k][2] +
C[i][1] C[j][0] C[k][2] + A[0] C[j][1] C[k][2] - C[i][0] C[j][1] C[k][2]) /
(C[i][1] C[j][0] - C[i][0] C[j][1] - C[i][1] C[k][0] +
C[j][1] C[k][0] + C[i][0] C[k][1] - C[j][0] C[k][1])

```

```

D[% , A[0]]

(-C[i][2] C[j][1] + C[i][1] C[j][2] + C[i][2] C[k][1] -
 C[j][2] C[k][1] - C[i][1] C[k][2] + C[j][1] C[k][2]) /
(C[i][1] C[j][0] - C[i][0] C[j][1] - C[i][1] C[k][0] +
 C[j][1] C[k][0] + C[i][0] C[k][1] - C[j][0] C[k][1])

CForm[(A[1] C[i][2] C[j][0] - A[0] C[i][2] C[j][1] - A[1] C[i][0] C[j][2] +
 A[0] C[i][1] C[j][2] - A[1] C[i][2] C[k][0] + C[i][2] C[j][1] C[k][0] +
 A[1] C[j][2] C[k][0] - C[i][1] C[j][2] C[k][0] + A[0] C[i][2] C[k][1] -
 C[i][2] C[j][0] C[k][1] - A[0] C[j][2] C[k][1] + C[i][0] C[j][2] C[k][1] +
 A[1] C[i][0] C[k][2] - A[0] C[i][1] C[k][2] - A[1] C[j][0] C[k][2] +
 C[i][1] C[j][0] C[k][2] + A[0] C[j][1] C[k][2] - C[i][0] C[j][1] C[k][2]) /
(C[i][1] C[j][0] - C[i][0] C[j][1] - C[i][1] C[k][0] +
 C[j][1] C[k][0] + C[i][0] C[k][1] - C[j][0] C[k][1])

(A(1)*C(i)(2)*C(j)(0) - A(0)*C(i)(2)*C(j)(1) - A(1)*C(i)(0)*C(j)(2) + A(0)*C(i)(1)*C(j)(2)
 C(i)(1)*C(j)(2)*C(k)(0) + A(0)*C(i)(2)*C(k)(1) - C(i)(2)*C(j)(0)*C(k)(1) - A(0)*C(j)(0)*C(i)(1)*C(k)(2) - A(1)*C(j)(0)*C(k)(2) + C(i)(1)*C(j)(0)*C(k)(2) + A(0)*C(j)(1)*C(i)(1)*C(k)(0) - C(i)(0)*C(j)(1) - C(i)(1)*C(k)(0) + C(j)(1)*C(k)(0) + C(i)(0)*C(k)(1)

Det[{{C[i][1], C[i][2], "C[i][1]*C[i][1]" + "C[i][2]*C[i][2]", 1},
 {C[j][1], C[j][2], "C[j][1]*C[j][1]" + "C[j][2]*C[j][2]", 1},
 {C[k][1], C[k][2], "C[k][1]*C[k][1]" + "C[k][2]*C[k][2]", 1},
 {C[p][1], C[p][2], "C[p][1]*C[p][1]" + "C[p][2]*C[p][2]", 1}}]

-C[k][1]*C[k][1] C[i][2] C[j][1] - C[k][2]*C[k][2] C[i][2] C[j][1] +
C[p][1]*C[p][1] C[i][2] C[j][1] + C[p][2]*C[p][2] C[i][2] C[j][1] +
C[k][1]*C[k][1] C[i][1] C[j][2] + C[k][2]*C[k][2] C[i][1] C[j][2] -
C[p][1]*C[p][1] C[i][1] C[j][2] - C[p][2]*C[p][2] C[i][1] C[j][2] +
C[j][1]*C[j][1] C[i][2] C[k][1] + C[j][2]*C[j][2] C[i][2] C[k][1] -
C[p][1]*C[p][1] C[i][2] C[k][1] - C[p][2]*C[p][2] C[i][2] C[k][1] -
C[i][1]*C[i][1] C[j][2] C[k][1] - C[i][2]*C[i][2] C[j][2] C[k][1] +
C[p][1]*C[p][1] C[j][2] C[k][1] + C[p][2]*C[p][2] C[j][2] C[k][1] -
C[j][1]*C[j][1] C[i][1] C[k][2] - C[j][2]*C[j][2] C[i][1] C[k][2] +
C[p][1]*C[p][1] C[i][1] C[k][2] + C[p][2]*C[p][2] C[i][1] C[k][2] +
C[i][1]*C[i][1] C[j][2] C[k][2] + C[i][2]*C[i][2] C[j][2] C[k][2] -
C[p][1]*C[p][1] C[j][1] C[k][2] - C[p][2]*C[p][2] C[j][1] C[k][2] -
C[j][1]*C[j][1] C[i][2] C[p][1] - C[j][2]*C[j][2] C[i][2] C[p][1] +
C[k][1]*C[k][1] C[i][2] C[p][1] + C[k][2]*C[k][2] C[i][2] C[p][1] +
C[i][1]*C[i][1] C[j][2] C[p][1] + C[i][2]*C[i][2] C[j][2] C[p][1] -
C[k][1]*C[k][1] C[j][2] C[p][1] - C[k][2]*C[k][2] C[j][2] C[p][1] -
C[i][1]*C[i][1] C[k][2] C[p][1] - C[i][2]*C[i][2] C[k][2] C[p][1] +
C[j][1]*C[j][1] C[k][2] C[p][1] + C[j][2]*C[j][2] C[k][2] C[p][1] +
C[j][1]*C[j][1] C[i][1] C[p][2] + C[j][2]*C[j][2] C[i][1] C[p][2] -
C[k][1]*C[k][1] C[i][1] C[p][2] - C[k][2]*C[k][2] C[i][1] C[p][2] -
C[i][1]*C[i][1] C[j][1] C[p][2] - C[i][2]*C[i][2] C[j][1] C[p][2] +
C[k][1]*C[k][1] C[j][1] C[p][2] + C[k][2]*C[k][2] C[j][1] C[p][2] +
C[i][1]*C[i][1] C[k][1] C[p][2] + C[i][2]*C[i][2] C[k][1] C[p][2] -
C[j][1]*C[j][1] C[k][1] C[p][2] - C[j][2]*C[j][2] C[k][1] C[p][2]

```

**CForm [%]**

```
C(i)(2)*Power(C(j)(1),2)*C(k)(1) - Power(C(i)(1),2)*C(j)(2)*C(k)(1) - Power(C(i)(2),2)*C(j)
C(i)(2)*C(j)(1)*Power(C(k)(1),2) + C(i)(1)*C(j)(2)*Power(C(k)(1),2) + Power(C(i)(1),2)*
C(i)(1)*Power(C(j)(1),2)*C(k)(2) - C(i)(1)*Power(C(j)(2),2)*C(k)(2) - C(i)(2)*C(j)(1)*P
C(i)(2)*Power(C(j)(1),2)*C(p)(1) + Power(C(i)(1),2)*C(j)(2)*C(p)(1) + Power(C(i)(2),2)*
C(i)(2)*Power(C(k)(1),2)*C(p)(1) - C(j)(2)*Power(C(k)(1),2)*C(p)(1) - Power(C(i)(1),2)*
Power(C(j)(1),2)*C(k)(2)*C(p)(1) + Power(C(j)(2),2)*C(k)(2)*C(p)(1) + C(i)(2)*Power(C(k
C(i)(2)*C(j)(1)*Power(C(p)(1),2) - C(i)(1)*C(j)(2)*Power(C(p)(1),2) - C(i)(2)*C(k)(1)*P
C(i)(1)*C(k)(2)*Power(C(p)(1),2) - C(j)(1)*C(k)(2)*Power(C(p)(1),2) - Power(C(i)(1),2)*
C(i)(1)*Power(C(j)(1),2)*C(p)(2) + C(i)(1)*Power(C(j)(2),2)*C(p)(2) + Power(C(i)(1),2)*
Power(C(j)(1),2)*C(k)(1)*C(p)(2) - Power(C(j)(2),2)*C(k)(1)*C(p)(2) - C(i)(1)*Power(C(k
C(i)(1)*Power(C(k)(2),2)*C(p)(2) + C(j)(1)*Power(C(k)(2),2)*C(p)(2) + C(i)(2)*C(j)(1)*P
C(i)(2)*C(k)(1)*Power(C(p)(2),2) + C(j)(2)*C(k)(1)*Power(C(p)(2),2) + C(i)(1)*C(k)(2)*P
```

**(n ^ 4 - (n ^ 3 - (n ^ 2 - n) / 2) / 2) / 2**

**(n ^ 3 - (n ^ 2 - n) / 2) / 2**

$$\frac{1}{2} \left( n^3 + \frac{1}{2} (n - n^2) \right)$$

**% /. n → 3**

**5**

**7 ^ 3**

**343**

**(n ^ 2 - n) / 2**

$$\frac{1}{2} (-n + n^2)$$

**(n ^ 3 - n ^ 2 - 2 \* (n + 1)) / 2**

$$\frac{1}{2} (-n^2 + n^3 - 2 (1 + n))$$