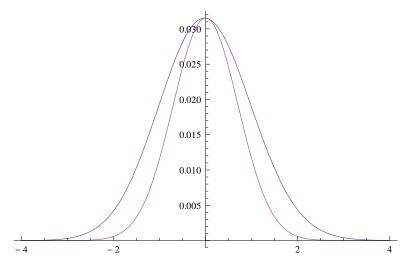
$P[k_{-}, n_{-}, p_{-}] := n! / (n-k)! / k! * p^k * (1-p)^(n-k)$

 $Plot[{R[z, 1000, 0.2], Exp[-z^2] * R[0, 1000, 0.2]}, {z, -4, 4}, PlotRange <math>\rightarrow All]$

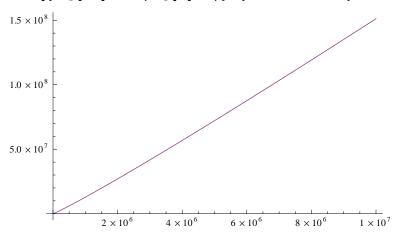


 $R[z_{n}, n_{p}] := P[z * Sqrt[p * (1 - p) * n] + n * p, n, p];$

Log[R[z,n,p]]

$$\text{Log}\, \Big[\, \frac{\left(1-p\right)^{\,n-n\,\,p-\sqrt{\,n\,\,\left(1-p\right)\,\,p\,\,\,\,z\,\,}}\,p^{\,n\,\,p+\sqrt{\,n\,\,\left(1-p\right)\,\,p\,\,\,\,z\,\,}}\,n\,\,!}{\left(n-n\,\,p-\sqrt{n\,\,\left(1-p\right)\,\,p\,\,\,}\,\,z\right)\,\,!\,\,\left(n\,\,p+\sqrt{n\,\,\left(1-p\right)\,\,p\,\,\,}\,z\right)\,!}\,\,\Big]$$

 $Plot[\{Log[n!], n * (Log[n]-1)\}, \{n, 1, 10000000\}, PlotRange \rightarrow All]$



 $f[n_] := n * (Log[n] - 1)$

$$\begin{split} & \text{Log} \left[\left(\mathbf{1} - \mathbf{p} \right)^{n-n} \, \mathbf{p} - \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \, \mathbf{p}^{n} \, \mathbf{p} + \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \, \right] + \\ & \mathbf{f} \left[\mathbf{n} \right] - \mathbf{f} \left[\mathbf{n} - \mathbf{n} \, \mathbf{p} - \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right] - \mathbf{f} \left[\mathbf{n} \, \mathbf{p} + \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right] \\ & \mathbf{n} \, \left(-1 + \text{Log} \left[\mathbf{n} \right] \right) + \text{Log} \left[\left(\mathbf{1} - \mathbf{p} \right)^{n-n} \, \mathbf{p} - \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \, \mathbf{p}^{n} \, \mathbf{p} + \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right] \\ & \left(\mathbf{n} - \mathbf{n} \, \mathbf{p} - \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right) \, \left(-1 + \text{Log} \left[\mathbf{n} - \mathbf{n} \, \mathbf{p} - \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right] \right) - \\ & \left(\mathbf{n} \, \mathbf{p} + \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right) \, \left(-1 + \text{Log} \left[\mathbf{n} \, \mathbf{p} + \sqrt{n \, \left(\mathbf{1} - \mathbf{p} \right) \, \mathbf{p}} \, \mathbf{z} \right] \right) \end{split}$$

Limit[%, n → Infinity]

\$Aborted

Exp[Log[Pi]]

π