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PrependTo[\$Path, "D:\\Users\\Johannes\\Promotion\\Mathematica\\Packages"]; << JoFin`

n = 1;

dS[i\_] := r S[i] dt + σ[i] S[i] dB[i];

dP = Expand[ $\sum_{i=1}^n q[i] dS[i] + \left(P - \sum_{i=1}^n q[i] S[i]\right) r dt$ ]

coef = CoefficientArrays[dP, Prepend[Table[dB[i], {i, n+1}], dt]][[2]];

S[n+1] := P;

mm = MMC[{coef[[1]]}, {coef[[2 ;; n+2]]}];

Print["Resulting system of SDEs:", MatrixForm /@ mm];

mmd = mm; MMDisc[mm] / {1, {{1, 1}, {S[1], 1}}, 1};

Print["Resulting system of SDEs:", MatrixForm /@ mmd];

dfk = Expand[FK[V, mmd, r]]

dt P r + dB[1] q[1] S[1] σ[1]

Resulting system of SDEs:  $\left\{ \begin{pmatrix} r S[1] \\ P r \end{pmatrix}, \begin{pmatrix} S[1] \sigma[1] & 0 \\ q[1] S[1] \sigma[1] & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

The resulting system of SDEs:  $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} S[1] \sigma[1] & 0 \\ q[1] S[1] \sigma[1] & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

Resulting system of SDEs:  $\left\{ \begin{pmatrix} r S[1] \\ P r \end{pmatrix}, \begin{pmatrix} S[1] \sigma[1] & 0 \\ q[1] S[1] \sigma[1] & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

$$\begin{aligned} & -r V[t, S[1], P] + P r V^{(0,0,1)}[t, S[1], P] + \frac{1}{2} q[1]^2 S[1]^2 \sigma[1]^2 V^{(0,0,2)}[t, S[1], P] + \\ & r S[1] V^{(0,1,0)}[t, S[1], P] + q[1] S[1]^2 \sigma[1]^2 V^{(0,1,1)}[t, S[1], P] + \\ & \frac{1}{2} S[1]^2 \sigma[1]^2 V^{(0,2,0)}[t, S[1], P] + V^{(1,0,0)}[t, S[1], P] \end{aligned}$$

V3 := #2 V[#1, #3 / #2] &;

dfk = Expand[DFK[V3, mmd] / S[1] /. P → P S[1]]

$$r V[t, P] + \frac{1}{2} P^2 \sigma[1]^2 V^{(0,2)}[t, P] -$$

$$P q[1] \sigma[1]^2 V^{(0,2)}[t, P] + \frac{1}{2} q[1]^2 \sigma[1]^2 V^{(0,2)}[t, P] + V^{(1,0)}[t, P]$$

MatrixForm /@ CoefficientArrays[dfk, Table[q[i], {i, n}], Symmetric → True]

$$\begin{aligned} & \left\{ -r V[t, S[1], P] + P r V^{(0,0,1)}[t, S[1], P] + r S[1] V^{(0,1,0)}[t, S[1], P] + \right. \\ & \quad \frac{1}{2} S[1]^2 \sigma[1]^2 V^{(0,2,0)}[t, S[1], P] + V^{(1,0,0)}[t, S[1], P], \\ & \quad \left. \left( S[1]^2 \sigma[1]^2 V^{(0,1,1)}[t, S[1], P] \right), \left( \frac{1}{2} S[1]^2 \sigma[1]^2 V^{(0,0,2)}[t, S[1], P] \right) \right\} \end{aligned}$$

## constant volatility

The option value will be independent of the asset prices, as can be verified by the absence of S terms in the following equation:

```
V2 := ToExpression[StringJoin["V[#1,#", ToString[n + 2], "&"]];
dfkC = Simplify[DFK[V2, mm]]
```

$$P r V^{(0,1)}[t, P] + \frac{1}{2} q[1]^2 \sigma[1]^2 V^{(0,2)}[t, P] + V^{(1,0)}[t, P]$$

```
coef = CoefficientArrays[dfkC, Table[q[i], {i, n}], Symmetric -> True]; MatrixForm /@ coef
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$$\{V^{(1,0)}[t, P], \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \sigma[1]^2 V^{(0,2)}[t, P] & \frac{1}{2} \rho[1, 2] \sigma[1] \sigma[2] V^{(0,2)}[t, P] & \frac{1}{2} \rho \\ \frac{1}{2} \rho[1, 2] \sigma[1] \sigma[2] V^{(0,2)}[t, P] & \frac{1}{2} \sigma[2]^2 V^{(0,2)}[t, P] & \frac{1}{2} \rho \\ \frac{1}{2} \rho[1, 3] \sigma[1] \sigma[3] V^{(0,2)}[t, P] & \frac{1}{2} \rho[2, 3] \sigma[2] \sigma[3] V^{(0,2)}[t, P] & \\ \frac{1}{2} \rho[1, 4] \sigma[1] \sigma[4] V^{(0,2)}[t, P] & \frac{1}{2} \rho[2, 4] \sigma[2] \sigma[4] V^{(0,2)}[t, P] & \frac{1}{2} \rho \end{pmatrix}$$