```
Exit[]
```

```
RandomMatrix[n_] := RandomReal[10 {-1, 1}, {n, n}];
Symmetrize[A_] := (A + Transpose[A]) / 2;
AntiSymmetrize[A_] := (A - Transpose[A]) / 2;
Positiviate[A_] := If[PositiveDefiniteMatrixQ[A],
  A, A - IdentityMatrix[Length[A]] Min[Eigenvalues[A]] 1.1];
ParamMatrix[c_, n_] := Table[c[i, j], {i, n}, {j, n}];
ParamAntiSymMatrix[c_, n_] :=
  2 AntiSymmetrize[LowerTriangularize[Table[c[i, j], {i, n}, {j, n}]]];
```

```
n = 2;
```

```
A = Symmetrize[RandomReal[10 {-1, 1}, {n, n}]];
B = Positiviate[Symmetrize[RandomMatrix[n]]];
```

```
c = Eigenvectors[{A, B}];
```

```
Round[c.A.Transpose[c], 10^-7] // MatrixForm // N
```

$$\begin{pmatrix} -9.52714 & 0. \\ 0. & 5.55156 \end{pmatrix}$$

```
Round[c.B.Transpose[c], 10^-7] // MatrixForm // N
```

$$\begin{pmatrix} 3.37544 & 0. \\ 0. & 11.6217 \end{pmatrix}$$

```
Eigenvalues[{{b[1, 1], b[1, 2]}, {b[1, 2], b [2, 2]}}]
```

$$\left\{\frac{1}{2}\left(b[1,1]+b[2,2]-\sqrt{\left(b[1,1]^2+4\,b[1,2]^2-2\,b[1,1]\,b[2,2]+b[2,2]^2\right)}\right),\right.$$
$$\left.\frac{1}{2}\left(b[1,1]+b[2,2]+\sqrt{\left(b[1,1]^2+4\,b[1,2]^2-2\,b[1,1]\,b[2,2]+b[2,2]^2\right)}\right)\right\}$$

```
B = RotationMatrix[α].DiagonalMatrix[{b1, b2}].RotationMatrix[-α]
```

$$\left\{\left\{b1\,\text{Cos}[\alpha]^2+b2\,\text{Sin}[\alpha]^2,\ b1\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]-b2\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]\right\},\right.$$
$$\left.\left\{b1\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]-b2\,\text{Cos}[\alpha]\,\text{Sin}[\alpha],\ b2\,\text{Cos}[\alpha]^2+b1\,\text{Sin}[\alpha]^2\right\}\right\}$$

```
Eigenvectors[{A, B}]
```

Eigenvectors::exnum : Eigenvectors has received a matrix with non-numerical or exact elements. ≫

$$\text{Eigenvectors}\left[\left\{\{\{0.875694,\ -7.05171\},\ \{-7.05171,\ -4.93852\}\},\right.\right.$$
$$\left.\left.\left\{\left\{b1\,\text{Cos}[\alpha]^2+b2\,\text{Sin}[\alpha]^2,\ b1\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]-b2\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]\right\},\right.\right.\right.$$
$$\left.\left.\left.\left\{b1\,\text{Cos}[\alpha]\,\text{Sin}[\alpha]-b2\,\text{Cos}[\alpha]\,\text{Sin}[\alpha],\ b2\,\text{Cos}[\alpha]^2+b1\,\text{Sin}[\alpha]^2\right\}\right\}\right\}\right]$$

```
A = ParamMatrix[a, n]
```

$$\{\{a[1,1],\ a[1,2]\},\ \{a[2,1],\ a[2,2]\}\}$$

```
A.ParamMatrix[b, n].Transpose[A] == Table[K
```

```
{{a[1, 1] (a[1, 1] b[1, 1] + a[1, 2] b[2, 1]) + a[1, 2] (a[1, 1] b[1, 2] + a[1, 2] b[2, 2]),
  a[2, 1] (a[1, 1] b[1, 1] + a[1, 2] b[2, 1]) + a[2, 2] (a[1, 1] b[1, 2] + a[1, 2] b[2, 2])},
 {a[1, 1] (a[2, 1] b[1, 1] + a[2, 2] b[2, 1]) + a[1, 2] (a[2, 1] b[1, 2] + a[2, 2] b[2, 2]),
  a[2, 1] (a[2, 1] b[1, 1] + a[2, 2] b[2, 1]) + a[2, 2] (a[2, 1] b[1, 2] + a[2, 2] b[2, 2])}}
```

```
n = 3;
```

```
mmm = Positiviate[Symmetrize[RandomMatrix[n]]];
```

```
Eigenvalues[mmm]
```

```
{23.2891, 10.9102, 1.28081}
```

```
mm = KroneckerProduct[Table[1, {i, n}, {j, n}] - IdentityMatrix[n], mmm]
```

```
{{0., 0., 0., 19.574, 3.57968, -7.38333, 19.574, 3.57968, -7.38333},
 {0., 0., 0., 3.57968, 10.5608, 1.60555, 3.57968, 10.5608, 1.60555},
 {0., 0., 0., -7.38333, 1.60555, 5.34533, -7.38333, 1.60555, 5.34533},
 {19.574, 3.57968, -7.38333, 0., 0., 0., 19.574, 3.57968, -7.38333},
 {3.57968, 10.5608, 1.60555, 0., 0., 0., 3.57968, 10.5608, 1.60555},
 {-7.38333, 1.60555, 5.34533, 0., 0., 0., -7.38333, 1.60555, 5.34533},
 {19.574, 3.57968, -7.38333, 19.574, 3.57968, -7.38333, 0., 0., 0.},
 {3.57968, 10.5608, 1.60555, 3.57968, 10.5608, 1.60555, 0., 0., 0.},
 {-7.38333, 1.60555, 5.34533, -7.38333, 1.60555, 5.34533, 0., 0., 0.}}
```

```
mm // MatrixForm
```

$$\begin{pmatrix} 0. & 0. & 0. & 19.574 & 3.57968 & -7.38333 & 19.574 & 3.57968 & -7.38333 \\ 0. & 0. & 0. & 3.57968 & 10.5608 & 1.60555 & 3.57968 & 10.5608 & 1.60555 \\ 0. & 0. & 0. & -7.38333 & 1.60555 & 5.34533 & -7.38333 & 1.60555 & 5.34533 \\ 19.574 & 3.57968 & -7.38333 & 0. & 0. & 0. & 19.574 & 3.57968 & -7.38333 \\ 3.57968 & 10.5608 & 1.60555 & 0. & 0. & 0. & 3.57968 & 10.5608 & 1.60555 \\ -7.38333 & 1.60555 & 5.34533 & 0. & 0. & 0. & -7.38333 & 1.60555 & 5.34533 \\ 19.574 & 3.57968 & -7.38333 & 19.574 & 3.57968 & -7.38333 & 0. & 0. & 0. \\ 3.57968 & 10.5608 & 1.60555 & 3.57968 & 10.5608 & 1.60555 & 0. & 0. & 0. \\ -7.38333 & 1.60555 & 5.34533 & -7.38333 & 1.60555 & 5.34533 & 0. & 0. & 0. \end{pmatrix}$$

```
Eigenvalues[mm]
```

```
{46.5782, -23.2891, -23.2891, 21.8203, -10.9102, -10.9102, 2.56162, -1.28081, -1.28081}
```

```
Eigenvalues
```