```
$Assumptions = μ > 0 && σ > 0 && a ∈ Reals &&
  1 > k1 ≥ 0 && k0 ≥ 0 && S0 > 0 && K > 0 && r ≥ 0 && b ∈ Reals && rf ≥ 0
```

$\mu > 0 \&\& \sigma > 0 \&\& a \in \text{Reals} \&\& 1 > k1 \geq 0 \&\& k0 \geq 0 \&\& S0 > 0 \&\& K > 0 \&\& r \geq 0 \&\& b \in \text{Reals} \&\& rf \geq 0$

```
u[x_] := Abs[x]^2
u1 = Abs @* InverseFunction[u]
p[n_, f_] := Module[{e = n[f[x], x ≈ NormalDistribution[]]},
  Simplify[e - rf u1[n[u[(f[x] - e)]
      , x ≈ NormalDistribution[]]]]]
S[a_] := a Exp[μ - σ^2/2 + σ #] &
```

InverseFunction::ifun :
Inverse functions are being used. Values may be lost for multivalued inverses. >>

$$\text{Abs} \circ * \left(-\sqrt{\#1}\ \&\right)$$

```
p[Expectation, S[a]]
```

$$e^{\mu}\left(a - \sqrt{-1 + e^{\sigma^2}}\ rf\ \text{Abs}[a]\right)$$

```
(* a = Sig[phi] *)
g[a_, k1_] := Simplify[ p[Expectation, S[Exp[-r] * (a - k1)]] - a]
g[a, k1]
```

$$-a + e^{-r+\mu}\left(a - k1 - \sqrt{-1 + e^{\sigma^2}}\ rf\ \text{Abs}[a - k1]\right)$$

```
Simplify[g[1, k1] + k1 ≤ 0]
Simplify[g[-1, k1] + k1 ≤ 0]
```

$$e^{\mu} \leq e^{r} + e^{\mu}\sqrt{-1 + e^{\sigma^2}}\ rf$$

$$e^{r} \leq e^{\mu}\left(1 + \sqrt{-1 + e^{\sigma^2}}\ rf\right)$$

```
Simplify[g[1, k1] - k1 ≤ 0]
Simplify[g[-1, k1] - k1 ≤ 0]
```

$$e^{\mu}\ (-1 + k1)\left(-1 + \sqrt{-1 + e^{\sigma^2}}\ rf\right) \leq e^{r}\ (1 + k1)$$

$$k1 + e^{-r+\mu}\ (1 + k1)\left(1 + \sqrt{-1 + e^{\sigma^2}}\ rf\right) \geq 1$$

■ Market price of risk

```
asd = Solve[ℯ^(-h^2 σ rMinusMuOverSigma) (-1 + k1) (-1 + Sqrt[-1 + ℯ^(h^2 σ^2)] rf) == 1 + k1,
    rMinusMuOverSigma][[1, 1, 2]]
```

$$\text{ConditionalExpression}\left[\frac{2\,i\,\pi\,C[1]+\text{Log}\left[\frac{-1+k1+\sqrt{-1+e^{h^2\sigma^2}}\;rf-\sqrt{-1+e^{h^2\sigma^2}}\;k1\,rf}{-1-k1}\right]}{h^2\,\sigma},\ C[1]\in\text{Integers}\right]$$

```
Simplify[Series[asd /. C[1] → 0, {h, 0, 3}]]
```

$$\frac{\text{Log}\left[\frac{1-k1}{1+k1}\right]}{\sigma\,h^2}-\frac{rf}{h}-\frac{rf^2\,\sigma}{2}-\frac{1}{12}\left(rf\left(3+4\,rf^2\right)\sigma^2\right)h-\frac{1}{4}\left(rf^2\left(1+rf^2\right)\sigma^3\right)h^2-\frac{1}{480}\left(rf\left(25+120\,rf^2+96\,rf^4\right)\sigma^4\right)h^3+O[h]^4$$

Pay later

```
$Assumptions = $Assumptions && k1 < Exp[-r]
```

$$\mu>0\ \&\&\ \sigma>0\ \&\&\ a\in\text{Reals}\ \&\&\ 1>k1\geq 0\ \&\&\ k0\geq 0\ \&\&\ S0>0\ \&\&\ K>0\ \&\&\ r\geq 0\ \&\&\ b\in\text{Reals}\ \&\&\ rf\geq 0\ \&\&\ k1<e^{-r}$$

```
g[a_, k1_] := Simplify[ p[Expectation, S[(Exp[-r] * a - k1)]] - a]
g[a, k1]
```

$$-a+e^{-r+\mu}\left(a-e^{r}\,k1-\sqrt{-1+e^{\sigma^2}}\;rf\;\text{Abs}[a-e^{r}\,k1]\right)$$

```
Simplify[g[1, k1] + k1 ≤ 0]
Simplify[g[-1, k1] + k1 ≤ 0]
```

$$k1+e^{-r+\mu}\left(-1+e^{r}\,k1\right)\left(-1+\sqrt{-1+e^{\sigma^2}}\;rf\right)\leq 1$$

$$1+k1\leq e^{-r+\mu}\left(1+e^{r}\,k1\right)\left(1+\sqrt{-1+e^{\sigma^2}}\;rf\right)$$

```
Simplify[g[1, k1] - k1 ≤ 0]
Simplify[g[-1, k1] - k1 ≤ 0]
```

$$e^{-r+\mu}\left(-1+e^{r}\,k1\right)\left(-1+\sqrt{-1+e^{\sigma^2}}\;rf\right)\leq 1+k1$$

$$k1+e^{-r+\mu}\left(1+e^{r}\,k1\right)\left(1+\sqrt{-1+e^{\sigma^2}}\;rf\right)\geq 1$$

■ Market price of risk

```
asd = Solve[k1 + e^(-h^2 σ rMinusMuOverSigma) (-1 + e^(h^2 r) k1) (-1 + Sqrt[-1 + e^(h^2 σ^2)] rf) == 1,
    rMinusMuOverSigma][[1, 1, 2]]
```

$$\text{ConditionalExpression}\left[\frac{1}{h^2\,\sigma}\left(2\,i\,\pi\,C[1]+\text{Log}\left[\frac{-1+e^{h^2\,r}\,k1+\sqrt{-1+e^{h^2\,\sigma^2}}\,rf-e^{h^2\,r}\sqrt{-1+e^{h^2\,\sigma^2}}\,k1\,rf}{-1+k1}\right]\right),\ C[1]\in\text{Integers}\right]$$

```
Simplify[Series[asd /. C[1] → 0, {h, 0, 3}]]
```

$$-\frac{rf}{h}+\left(\frac{k1\,r}{(-1+k1)\,\sigma}-\frac{rf^2\,\sigma}{2}\right)-\frac{1}{12}\left(rf\left(3+4\,rf^2\right)\sigma^2\right)h-\left(\left(rf^2\left(1+rf^2\right)\sigma^4+k1^2\,rf^2\left(1+rf^2\right)\sigma^4+2\,k1\left(r^2-rf^2\left(1+rf^2\right)\sigma^4\right)\right)h^2\right)\Big/\left(4\left((-1+k1)^2\,\sigma\right)\right)-\frac{1}{480}\left(rf\left(25+120\,rf^2+96\,rf^4\right)\sigma^4\right)h^3+O[h]^4$$

```
asd = Solve[1 + k1 == e^(-h^2 rMinusMuOverSigma σ) (1 + e^(h^2 r) k1) (1 + Sqrt[-1 + e^(h^2 σ^2)] rf),
    rMinusMuOverSigma][[1, 1, 2]]
```

$$\text{ConditionalExpression}\left[\frac{1}{h^2\,\sigma}\left(2\,i\,\pi\,C[1]+\text{Log}\left[\frac{1}{1+k1}\left(1+e^{h^2\,r}\,k1+\sqrt{-1+e^{h^2\,\sigma^2}}\,rf+e^{h^2\,r}\sqrt{-1+e^{h^2\,\sigma^2}}\,k1\,rf\right)\right]\right),\ C[1]\in\text{Integers}\right]$$

```
Simplify[Series[asd /. C[1] → 0, {h, 0, 3}]]
```

$$\frac{rf}{h}+\left(-\frac{rf^2\,\sigma}{2}+\frac{k1\,r}{\sigma+k1\,\sigma}\right)+\frac{1}{12}\,rf\left(3+4\,rf^2\right)\sigma^2\,h-\frac{1}{4\left((1+k1)^2\,\sigma\right)}\left(rf^2\left(1+rf^2\right)\sigma^4+k1^2\,rf^2\left(1+rf^2\right)\sigma^4+k1\left(-2\,r^2+2\,rf^2\left(1+rf^2\right)\sigma^4\right)\right)h^2+\frac{1}{480}\,rf\left(25+120\,rf^2+96\,rf^4\right)\sigma^4\,h^3+O[h]^4$$

```
asd = Solve[e^(-h^2 σ rMinusMuOverSigma) (-1 + e^(h^2 r) k1) (-1 + Sqrt[-1 + e^(h^2 σ^2)] rf) == 1 + k1,
    rMinusMuOverSigma][[1, 1, 2]]
```

$$\text{ConditionalExpression}\left[\frac{1}{h^2\,\sigma}\left(2\,i\,\pi\,C[1]+\text{Log}\left[\frac{1}{1+k1}\left(1-e^{h^2\,r}\,k1-\sqrt{-1+e^{h^2\,\sigma^2}}\,rf+e^{h^2\,r}\sqrt{-1+e^{h^2\,\sigma^2}}\,k1\,rf\right)\right]\right),\ C[1]\in\text{Integers}\right]$$

```
Simplify[Series[asd /. C[1] → 0, {h, 0, 3}]]
```

$$\frac{\text{Log}\left[\frac{1-k1}{1+k1}\right]}{\sigma\, h^2} - \frac{rf}{h} - \frac{2\, k1\, r + rf^2\, \sigma^2 - k1\, rf^2\, \sigma^2}{2\,\sigma - 2\, k1\,\sigma} - \frac{1}{12}\left(rf\left(3 + 4\, rf^2\right)\sigma^2\right) h - \left(\left(rf^2\left(1 + rf^2\right)\sigma^4 + k1^2\, rf^2\left(1 + rf^2\right)\sigma^4 + 2\, k1\left(r^2 - rf^2\left(1 + rf^2\right)\sigma^4\right)\right) h^2\right) \Big/ \left(4\left((-1 + k1)^2\,\sigma\right)\right) - \frac{1}{480}\left(rf\left(25 + 120\, rf^2 + 96\, rf^4\right)\sigma^4\right) h^3 + O[h]^4$$

```
asd = Solve[k1 + e^(-h^2 σ rMinusMuOverSigma) (1 + e^(h^2 r) k1) (1 + Sqrt[-1 + e^(h^2 σ^2)] rf) == 1,
    rMinusMuOverSigma][[1, 1, 2]]
```

$$\text{ConditionalExpression}\left[\frac{1}{h^2\,\sigma}\left(2\, i\,\pi\, C[1] + \text{Log}\left[\frac{-1 - e^{h^2 r}\, k1 - \sqrt{-1 + e^{h^2\sigma^2}}\, rf - e^{h^2 r}\sqrt{-1 + e^{h^2\sigma^2}}\, k1\, rf}{-1 + k1}\right]\right),\ C[1] \in \text{Integers}\right]$$

```
Simplify[Series[asd /. C[1] → 0, {h, 0, 3}]]
```

$$\frac{\text{Log}\left[\frac{1+k1}{1-k1}\right]}{\sigma\, h^2} + \frac{rf}{h} - \frac{-2\, k1\, r + rf^2\, \sigma^2 + k1\, rf^2\, \sigma^2}{2\,\sigma + 2\, k1\,\sigma} + \frac{1}{12}\, rf\left(3 + 4\, rf^2\right)\sigma^2\, h - \frac{1}{4\left((1 + k1)^2\,\sigma\right)}\left(rf^2\left(1 + rf^2\right)\sigma^4 + k1^2\, rf^2\left(1 + rf^2\right)\sigma^4 + k1\left(-2\, r^2 + 2\, rf^2\left(1 + rf^2\right)\sigma^4\right)\right) h^2 + \frac{1}{480}\, rf\left(25 + 120\, rf^2 + 96\, rf^4\right)\sigma^4\, h^3 + O[h]^4$$