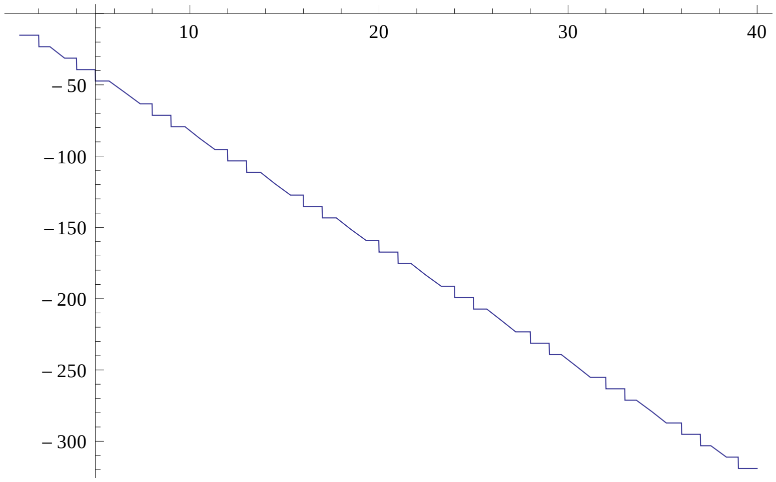
$$R[n\_] := \frac{2\,\mathrm{Sqrt}[2]}{9801}\ \mathrm{Sum}\left[\frac{(4\ k)!\ (1103 + 26390\ k)}{(k!)^{\wedge}4\ 396^{\wedge}(4\ k)},\ \{k, 0, n\}\right]$$

```
Plot[Log[10, Abs[N[1 / R[n] - π, 500]]], {n, 1, 40}]
```




```
N[1 / R[5] - π, 100]
```

4.741011768567914974136850634834727161360394467082098721200536639730466696354233748949017936147287482 $\times 10^{-48}$

Table[R[n], {n, 10}]

$$\left\{\frac{1\,130\,173\,253\,125}{2\,510\,613\,731\,736\,\sqrt{2}},\ \frac{1\,029\,347\,477\,390\,786\,609\,545}{2\,286\,635\,172\,367\,940\,241\,408\,\sqrt{2}},\right.$$

$$\frac{7\,766\,473\,062\,254\,307\,011\,793\,347\,201\,855}{17\,252\,765\,328\,978\,109\,815\,564\,789\,153\,792\,\sqrt{2}},$$

$$509\,299\,577\,881\,529\,611\,662\,930\,757\,403\,081\,523\,769\,055 \Big/ \left(1\,131\,379\,202\,490\,552\,979\,877\,435\,552\,947\,122\,965\,839\,872\,\sqrt{2}\right),$$

$$57\,982\,950\,211\,280\,781\,944\,919\,792\,648\,021\,104\,999\,982\,386\,829\,481 \Big/ \left(128\,805\,730\,098\,892\,711\,723\,125\,911\,845\,114\,081\,418\,091\,536\,842\,752\,\sqrt{2}\right),$$

$$3\,499\,871\,759\,747\,710\,499\,842\,768\,988\,784\,507\,373\,816\,789\,022\,688\,631\,739\,047\,925 \Big/ \left(7\,774\,760\,263\,562\,699\,859\,971\,501\,015\,139\,525\,269\,727\,219\,309\,055\,349\,184\,528\,384\,\sqrt{2}\right),$$

$$398\,454\,856\,050\,409\,400\,033\,667\,498\,427\,037\,929\,849\,361\,304\,439\,288\,784\,703\,764\,447\,270\,125 \Big/ \left(885\,144\,140\,786\,355\,895\,741\,177\,195\,716\,026\,970\,950\,416\,670\,420\,565\,960\,985\,448\,225\,439\,744\,\sqrt{2}\right),$$

$$6\,334\,387\,787\,708\,107\,824\,222\,495\,376\,281\,706\,107\,615\,730\,472\,323\,276\,284\,056\,009\,760\,393\,364\,218\,543\,125 \Big/ \left(14\,071\,471\,712\,843\,535\,798\,792\,494\,970\,078\,253\,119\,671\,801\,362\,717\,159\,118\,900\,747\,103\,370\,578\,550\,063\,104\,\sqrt{2}\right),$$

$$14\,194\,592\,594\,146\,827\,909\,170\,805\,406\,080\,156\,403\,980\,453\,284\,185\,387\,917\,579\,020\,073\,045\,561\,359\,013\,099\,552\,859\,053\,125 \Big/ \left(31\,532\,456\,625\,322\,022\,370\,765\,818\,276\,612\,583\,919\,584\,083\,811\,310\,999\,597\,255\,056\,965\,804\,073\,403\,194\,963\,043\,194\,765\,312\,\sqrt{2}\right),$$

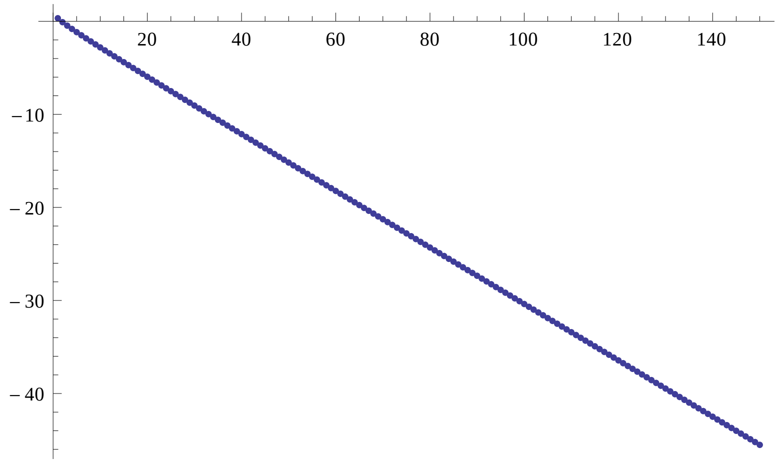
$$\left.116\,354\,295\,547\,844\,200\,479\,625\,540\,962\,705\,305\,445\,031\,010\,498\,388\,307\,062\,857\,519\,290\,687\,784\,871\,920\,308\,555\,177\,681\,218\,916\,232\,885 \Big/ \left(258\,474\,257\,235\,476\,477\,051\,634\,224\,477\,005\,861\,793\,643\,791\,092\,488\,013\,501\,737\,085\,215\,352\,314\,477\,706\,263\,478\,530\,979\,938\,932\,097\,024\,\sqrt{2}\right)\right\}$$

$$\mathbf{Table}\left[\mathbf{N}\left[\frac{\mathbf{i}}{\mathbf{2\,i+1}},\ \mathbf{50}\right],\ \{\mathbf{i},\ \mathbf{100}\}\right]$$

P[n_, n_] := 1;

$$\mathbf{P[n\_,\ i\_] := 2 + \frac{i}{2\,i+1}\ P[n,\ i+1]}$$

```
ListPlot[Table[Log[10, Abs[N[P[n, 1] - π, 500]]], {n, 1, 150}]]
```




```
m[k_] := {{k, 4 * k + 2}, {0, 2 * k + 1}}

product[n_, n_] := IdentityMatrix[2];
product[n_, i_] := m[i].product[n, i + 1]
product[n_] := product[n, 1]

x[0] = 0; x[i_] := (1 + 2 i) (2 (i - 1) ! + x[i - 1])

product2[n_] := {{n !, x[n]}, {0, Product[1 + 2 i, {i, n}]}}

Table[product2[i] - product[i + 1], {i, 10}]

{{{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}},
 {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}}

2 Product[1 + 2 i, {i, n}]
```

$$\frac{2^{1-n}\,(1+2\,n)\,!}{n\,!}$$

```
reduce[m_, x_] := #[[1]] / #[[2]] &[m.{x, 1}]

P2[n_, x_] := reduce[product2[n], x]

err[n_, x_] := Log[10, Abs[N[P2[n, x] - π, 500]]]
```

```
ListPlot[Table[err[n, #], {n, 1, 250}] & /@ {4, 6}]
```

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
7210733839176380643520042592933480014509748286712455265520571332741829025736722 72⋱
22400467362865517999180750505546389284420845568 / 2295247867649841671113802985006
87872079204589138925981326030390833146152064944380459789310141526372662940 06⋱
8439977718408373625 − π. ≫

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
75254436255824244038351020955364147737165431613882337260130728906217430893033 6070⋱
226503064287334052066836423873847875338 24 / 2395423103941674533153775064004788026
525584525566789209038074898758922795896585276025419014657326732479268464483 2⋱
3459625 − π. ≫

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
577177870763057602755988720795349227325356523926725023471167843785942732487010632⋱
49816673813936950729301912513476906411113815474176 / 183721422350391507008973210 4
03936998852644630719448178527968621200164379784836709257543158135742757344 48⋱
15528150621648643550653 75 − π. ≫

General::stop : Further output of N::meprec will be suppressed during this calculation. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

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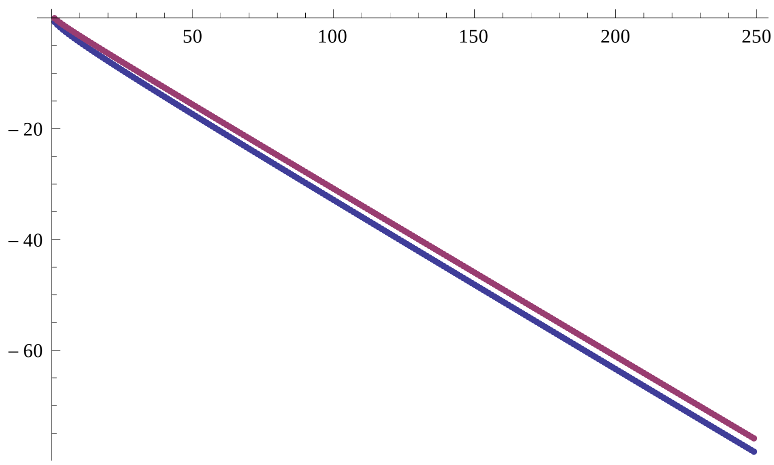
General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫




```
Simplify[{{A[1, 1], A[1, 2]}, {0, A[2, 2]}}.m[i]] // MatrixForm
```

$$\begin{pmatrix} i\,A[1,1] & (1+2i)\,(2\,A[1,1]+A[1,2]) \\ 0 & (1+2i)\,A[2,2] \end{pmatrix}$$

```
product[200]
```

```
product[200]
```

```
Exit[]
```

```
RSolve[x[i + 1] == (3 + 2 i) (2 i! + x[i]), x[i], i]
```

$$\left\{\left\{x[i] \to 2^{-1+i}\, C[1]\, \text{Pochhammer}\left[\frac{5}{2}, -1+i\right] + 3 \times 2^{-1+i} \sqrt{\pi} \left(\sqrt{\pi} - \frac{2^{-i}\, i!\, \text{Hypergeometric2F1}\left[1, 1+i, \frac{3}{2}+i, \frac{1}{2}\right]}{\left(\frac{1}{2}(1+2 i)\right)!}\right) \text{Pochhammer}\left[\frac{5}{2}, -1+i\right]\right\}\right\}$$

```
p = π/4
```

$$\frac{\pi}{4}$$

```
Solve[Sqrt[p (1 - p) / 10 ^ n] == 10 ^ -k, k]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{k \to -\frac{\text{Log}\left[\frac{1}{2}\sqrt{10^{-n}}\sqrt{\left(1-\frac{\pi}{4}\right)\pi}\right]}{\text{Log}[10]}\right\}\right\}$$

```
Plot[Log[10, Sqrt[(1 - p) p / 10 ^ n]], {n, 0, 1
```

```
Simplify[Log[10, Sqrt[p (1 - p) / 10 ^ n]]]
```

$$\frac{\text{Log}\left[\frac{1}{4}\sqrt{10^{-n}}\sqrt{(4-\pi)\pi}\right]}{\text{Log}[10]}$$

```
Log[10, p (1 - p)] / 2 // N
```

-0.386638

```
n = 10 ^ 13; n / Log[n] // N
```

3.34073×10^{11}

```
Plot[{n Log[n]^3, n^2}, {n, 0, 10000}]
```