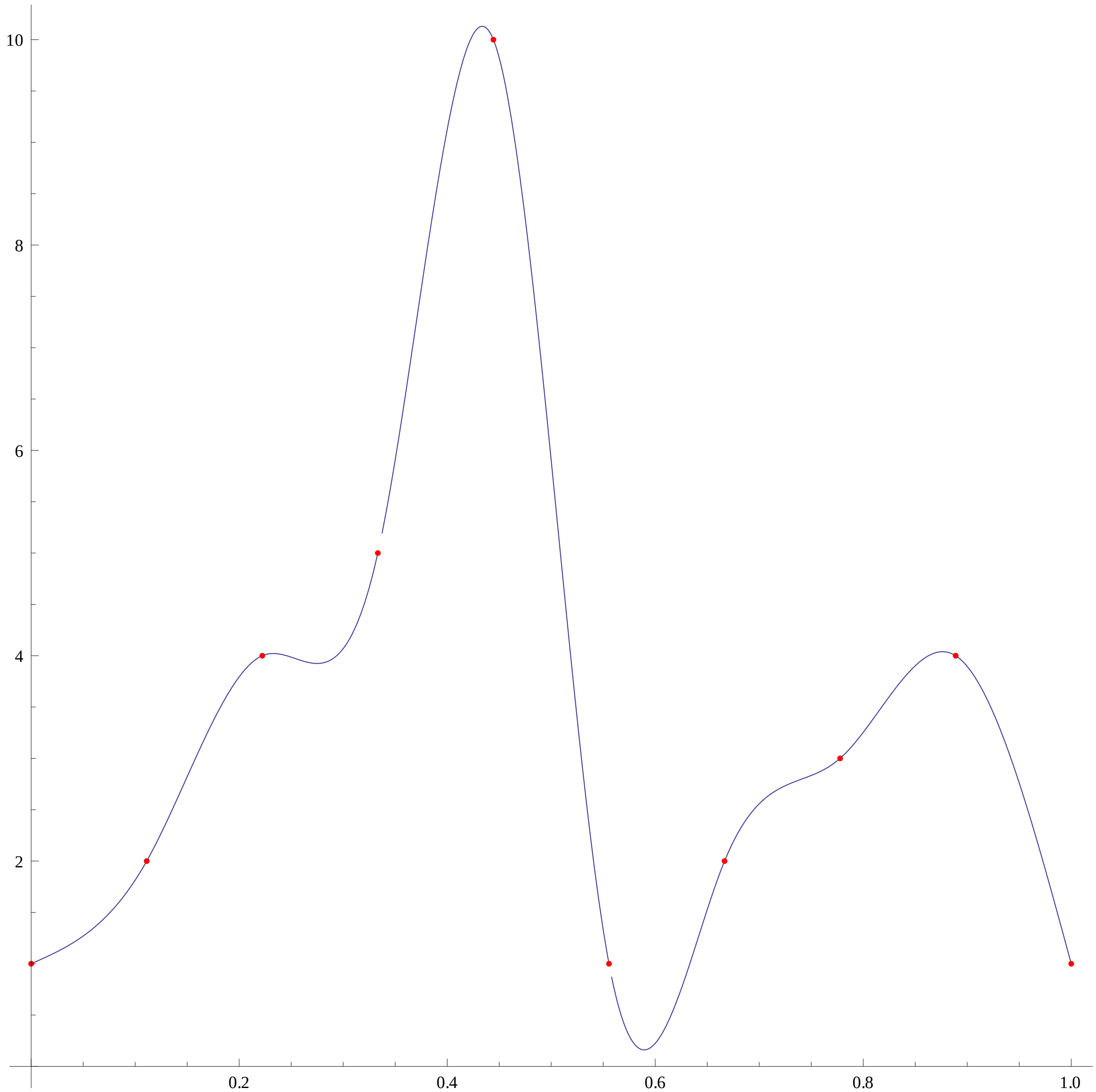
```
Exit[];

c = {1 - 3 n^2 x^2 + 2 n^3 x^3, 3 n^2 x^2 - 2 n^3 x^3, x - 2 n x^2 + n^2 x^3, -n x^2 + n^2 x^3}

{1 - 3 n^2 x^2 + 2 n^3 x^3, 3 n^2 x^2 - 2 n^3 x^3, x - 2 n x^2 + n^2 x^3, -n x^2 + n^2 x^3}

b = 1 / n;

Y[i_, h_] := {y[i], y[i + 1], m[i], m[i + 1]}.c /. x → h;
{Y[i, 0], Y[i, 1 / n], D[Y[i, x], x] /. x → 0, D[Y[i, x], x] /. x → 1 / n}

{y[i], y[1 + i], m[i], m[1 + i]}

Simplify[(D[Y[1, x], {x, 2}] / 4 / n /. x → 0) == 0]

2 m[1] + m[2] + 3 n y[1] == 3 n y[2]

Simplify[(D[Y[n, x], {x, 2}] / 4 / n /. x → b) == 0]

m[n] + 2 m[1 + n] + 3 n y[n] == 3 n y[1 + n]

Simplify[(D[Y[i, x], {x, 2}] / 4 / n /. x → b) == (D[Y[i + 1, x], {x, 2}] / 4 / n /. x → 0)]

m[i] + 4 m[1 + i] + m[2 + i] + 3 n y[i] == 3 n y[2 + i]

M[n_] := SparseArray[{{1, 1} → -2, (n + 1) {1, 1} → 2,
    {n + 1, n} → -2, {1, 2} → 2, {i_, j_} /; (i == j + 1 && i < n + 1 && i > 1) → -1,
    {i_, j_} /; (i == j - 1 && i < n + 1 && i > 1) → 1}, (n + 1) {1, 1}];
M[5] // MatrixForm
```

$$\begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix}$$

los:

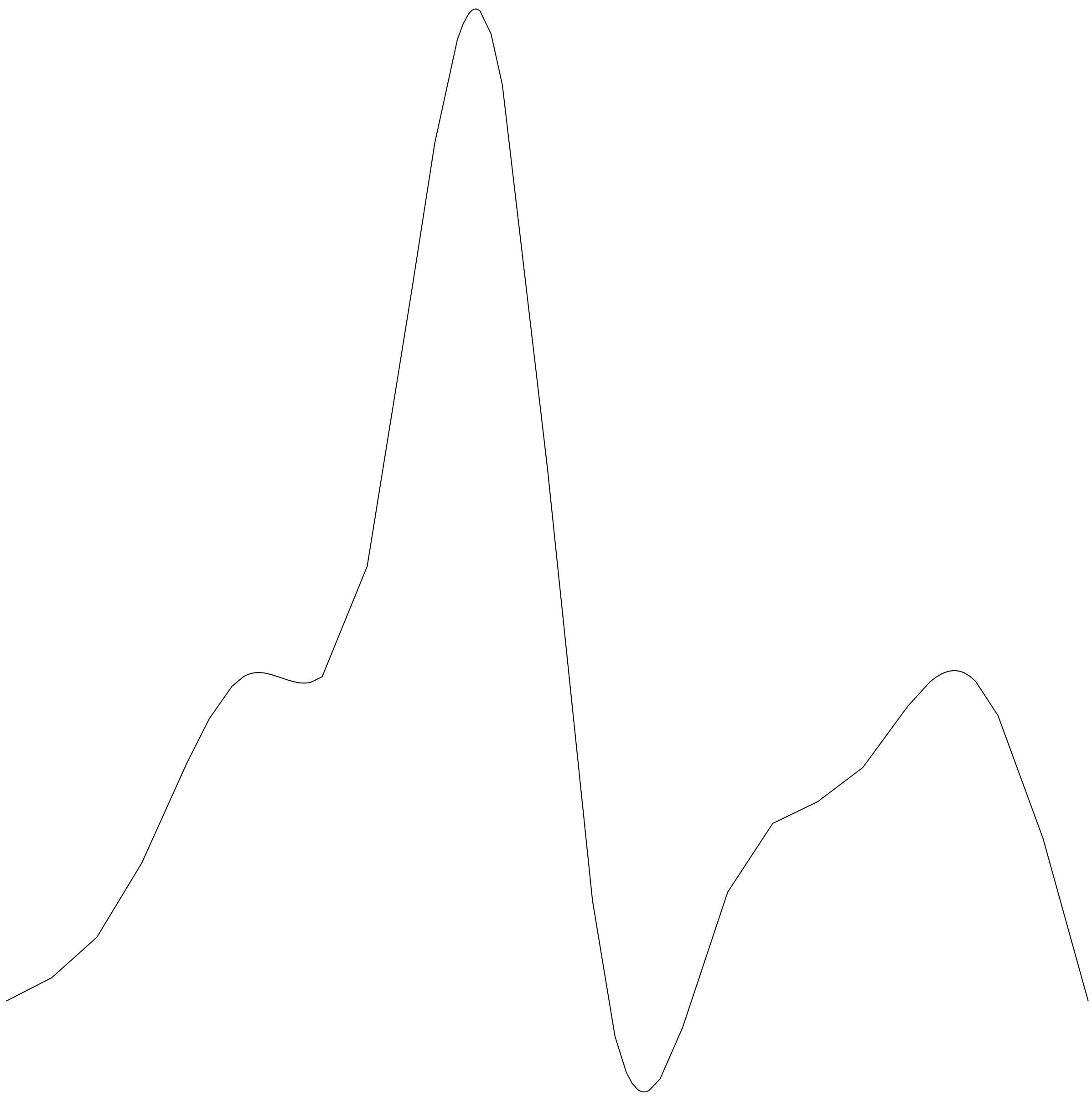
```
y = {1, 2, 4, 5, 10, 1, 2, 3, 4, 1}; m = M[Length[y] - 1].y; n = Length[y] - 1;

Y[x0_] := Module[{i, x = x0, y = y, m = m, c = c, n = n},
  i = Ceiling[x * n];
  x -= (i - 1) / n;
  {y[[i]], y[[i + 1]], m[[i]], m[[i + 1]]}.
   {1 - 3 n^2 x^2 + 2 n^3 x^3, 3 n^2 x^2 - 2 n^3 x^3, x - 2 n x^2 + n^2 x^3, -n x^2 + n^2 x^3}
 ]
```

```
Show[Plot[Y[x], {x, 0, 1}, AspectRatio → 1, PlotPoints → 250],
 ListPlot[Table[{i / (Length[y] - 1), y[[i + 1]]}, {i, 0, Length[y] - 1}], PlotStyle → Red]]
```




```
<< Splines`
```

```
Graphics[Spline[Table[{i / (Length[y] - 1), y[[i + 1]]}, {i, 0, Length[y] - 1}], Cubic],
 AspectRatio → 1]
```



```
Y[x0_] := Module[{i = n, x = x0, p = P, m = m, kr = kr, n = n, b, y},
  If[x0 == 0, i = 1,
   While[x ≤ p[[i, 1]], i--];
  ];
  b = p[[i + 1, 1]] - p[[i, 1]];
  {p[[i, 2]], p[[i + 1, 2]], m[[i]], m[[i + 1]], kr[[i, 1]], kr[[i, 2]]}.
```

$$\left\{1-\frac{10 y^3}{b^3}+\frac{15 y^4}{b^4}-\frac{6 y^5}{b^5}, \frac{10 y^3}{b^3}-\frac{15 y^4}{b^4}+\frac{6 y^5}{b^5}, y-\frac{6 y^3}{b^2}+\frac{8 y^4}{b^3}-\frac{3 y^5}{b^4}, -\frac{4 y^3}{b^2}+\frac{7 y^4}{b^3}-\frac{3 y^5}{b^4}, \frac{y^2}{2}-\frac{3 y^3}{2 b}+\frac{3 y^4}{2 b^2}-\frac{y^5}{2 b^3}, \frac{y^3}{2 b}-\frac{y^4}{b^2}+\frac{y^5}{2 b^3}\right\} /.\ y \to (x - p[[i, 1]])$$

```
 ]
```

```
n = 20; nN = Length[XY];
P = Join[{1}, Table[i, {i, Ceiling[(nN - (Floor[nN / n] - 1) n) / 2], nN, n}], {nN}];
n = Length[P] - 1;
P = Table[{XY[[P[[i]], 1]], XY[[P[[i]], 2]]}, {i, 1, n + 1}]
tt = Table[ts[i], {i, 3 * n + 1}];
m = tt[[1 ;; n + 1]]; kr = Transpose[{tt[[n + 2 ;; 2 n + 1]], tt[[2 n + 2 ;; 3 n + 1]]}];
```

```
{{0, -0.011473}, {0.161017, -0.000802}, {0.330508, -0.000373},
 {0.5, 0.000026}, {0.669492, 0.00068}, {0.838983, 0.001158}, {1., 0.011826}}
```

```
d = (Y[#[[1]]] - #[[2]]) ^ 2 & /@ XY; d = Sum[d[[i]], {i, Length[d]}];
g = Solve[Table[D[d, ts[i]] == 0, {i, 3 * n + 3}], tt]
```

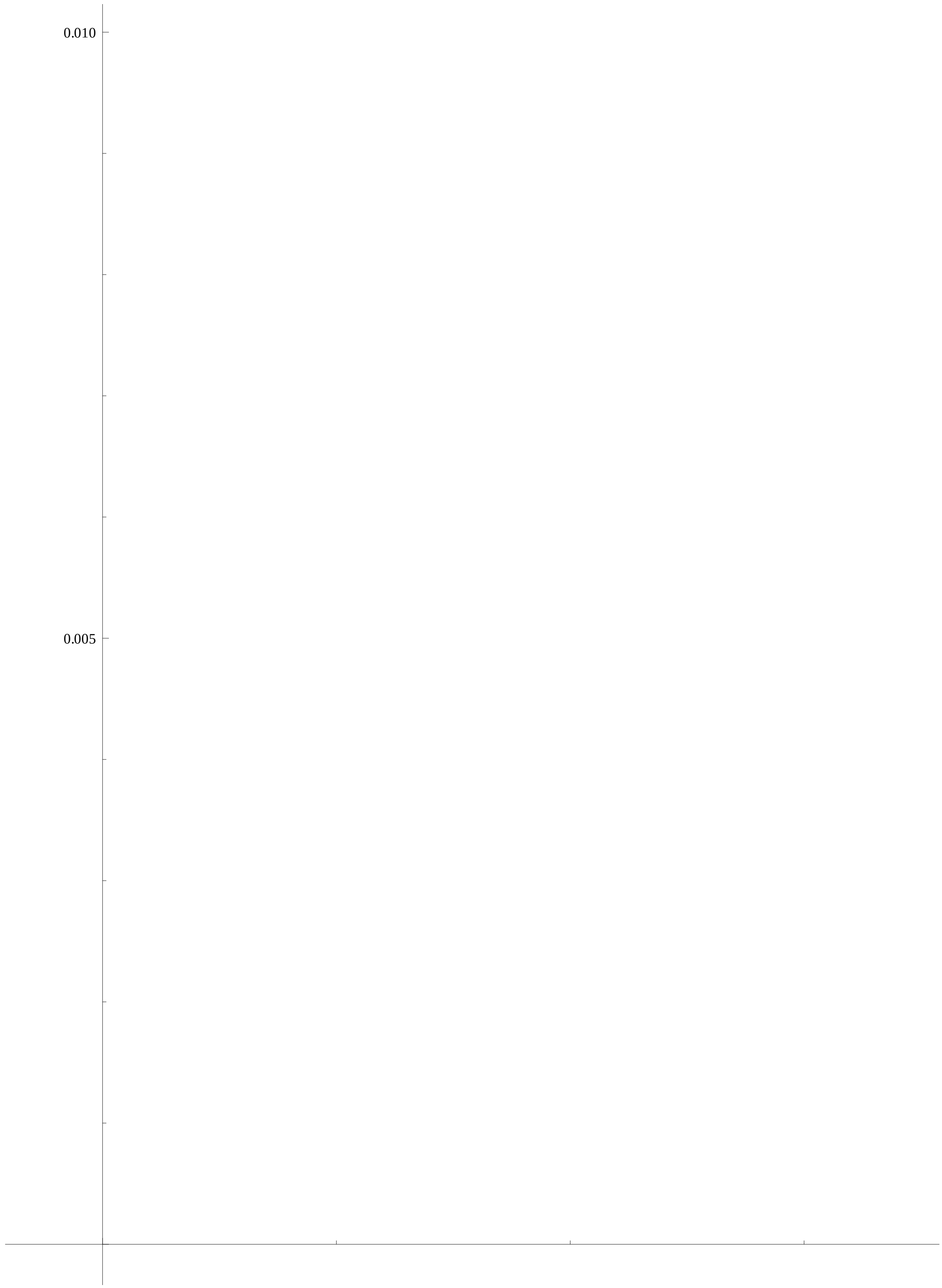
```
{{ts[1] → 0.732286, ts[2] → 0.0432745, ts[3] → 0.0155775,
  ts[4] → 0.00826988, ts[5] → 0.0258178, ts[6] → 0.0717768, ts[7] → 0.91313,
  ts[8] → -39.1332, ts[9] → -3.55332, ts[10] → -1.18621, ts[11] → -0.690975,
  ts[12] → -3.15685, ts[13] → -12.2057, ts[14] → 7.49265, ts[15] → 1.63073,
  ts[16] → 0.59971, ts[17] → 1.85439, ts[18] → 6.06894, ts[19] → 53.9935}}
```

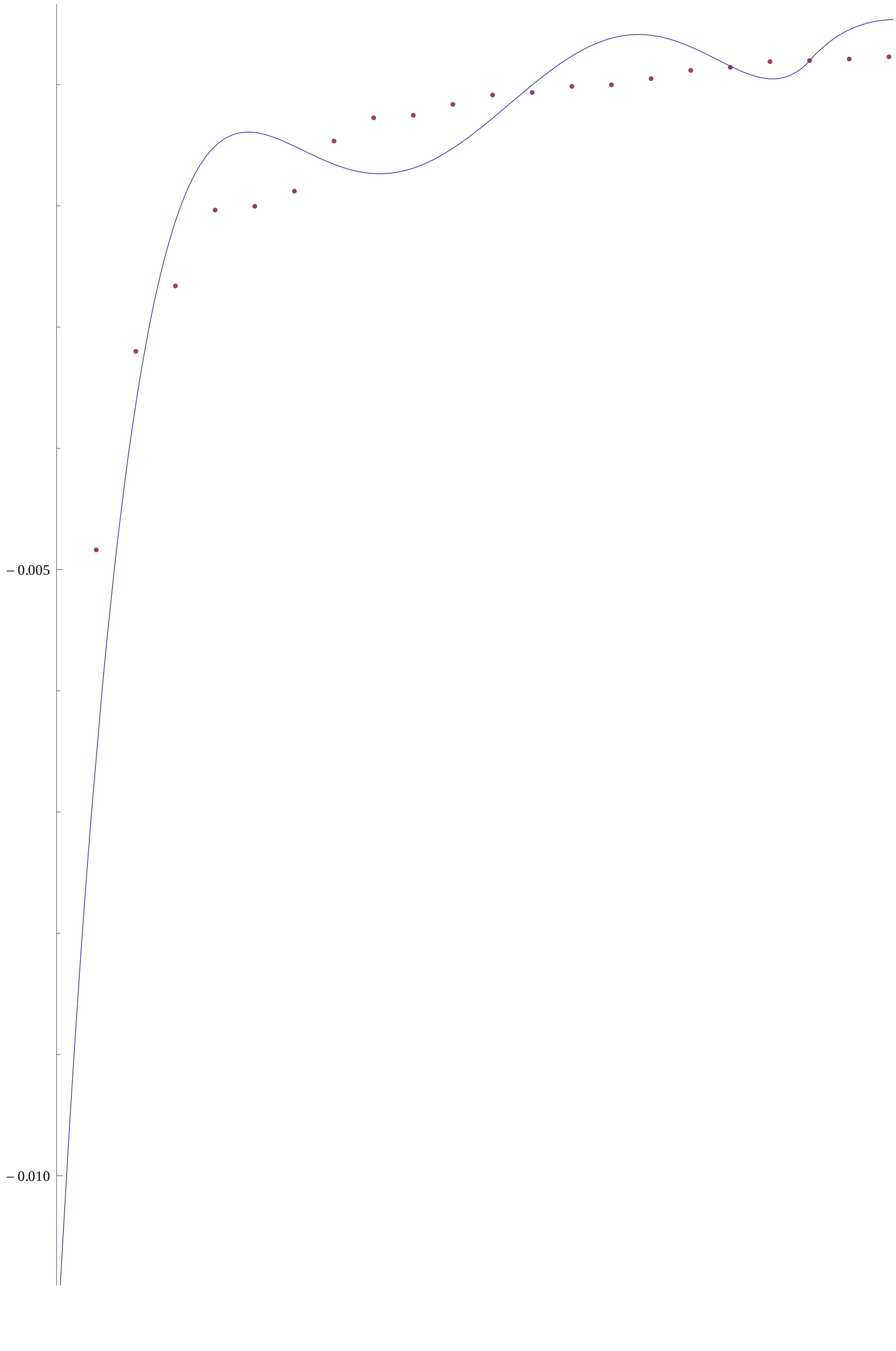
```
tt = Table[ts[i], {i, 3 * n + 1}] /. g[[1]];
```

```
d /. g[[1]]
```

```
0.0000251891
```

```
Show[ListPlot[{P, XY}, PlotRange → All],
 Plot[Y[x] /. g[[1]], {x, 0, 1}, PlotRange → All]]
```


0.010
0.005



– 0.005
– 0.010

```
Solve[Table[D[d, ys[i]] == 0, {i, nN}], Table[ys[i], {i, nN}]]
```

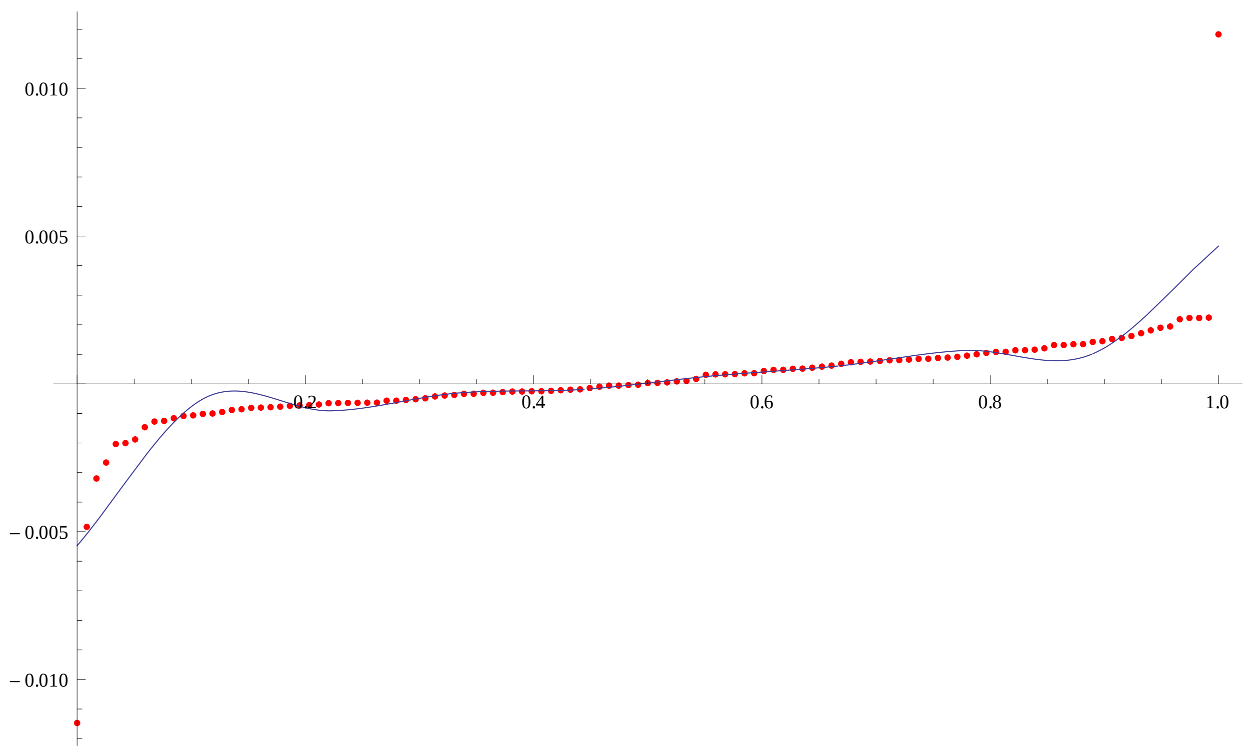
```
{{ys[1] → -0.00547464, ys[2] → -0.000492335, ys[3] → -0.000911088,
  ys[4] → -0.00031071, ys[5] → -0.000185091, ys[6] → 0.000265952,
  ys[7] → 0.00060474, ys[8] → 0.00113058, ys[9] → 0.00100276, ys[10] → 0.0046597}}
```

```
y = Table[ys[i], {i, nN}] /. %[[1]];
```

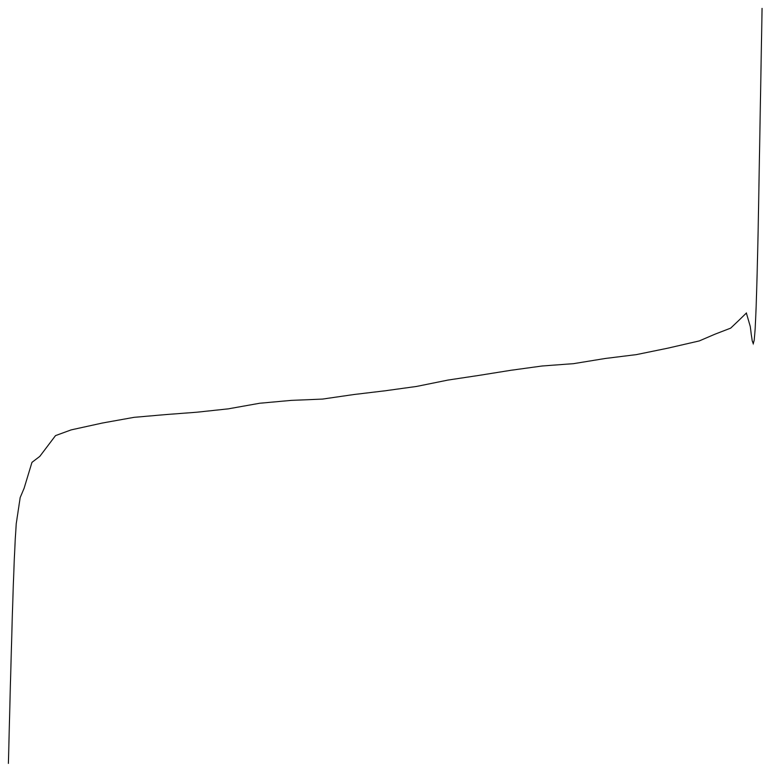
```
nN = 10; y = Table[ys[i], {i, nN}];
```

```
m = n / 2 M[n].y; n = Length[y] - 1;
d = (Y[#[[1]]] - #[[2]])^2 & /@ XY; d = Sum[d[[i]], {i, Length[d]}]
Show[ListPlot[XY, PlotStyle → Red, PlotRange → All],
 Plot[Y[x], {x, 0, 1}, PlotRange → All]]
```

```
0.000118941
```



```
Graphics[Spline[XY, Cubic], AspectRatio → 1]
```



```
Y[0]
```

0.01

```
{#[[1]]} & /@ XY
```

{{0.}, {0.008475}, {0.016949}, {0.025424}, {0.033898}, {0.042373}, {0.050847},
{0.059322}, {0.067797}, {0.076271}, {0.084746}, {0.09322}, {0.101695}, {0.110169},
{0.118644}, {0.127119}, {0.135593}, {0.144068}, {0.152542}, {0.161017}, {0.169492},
{0.177966}, {0.186441}, {0.194915}, {0.20339}, {0.211864}, {0.220339}, {0.228814},
{0.237288}, {0.245763}, {0.254237}, {0.262712}, {0.271186}, {0.279661}, {0.288136},
{0.29661}, {0.305085}, {0.313559}, {0.322034}, {0.330508}, {0.338983}, {0.347458},
{0.355932}, {0.364407}, {0.372881}, {0.381356}, {0.389831}, {0.398305}, {0.40678},
{0.415254}, {0.423729}, {0.432203}, {0.440678}, {0.449153}, {0.457627}, {0.466102},
{0.474576}, {0.483051}, {0.491525}, {0.5}, {0.508475}, {0.516949}, {0.525424},
{0.533898}, {0.542373}, {0.550847}, {0.559322}, {0.567797}, {0.576271}, {0.584746},
{0.59322}, {0.601695}, {0.610169}, {0.618644}, {0.627119}, {0.635593}, {0.644068},
{0.652542}, {0.661017}, {0.669492}, {0.677966}, {0.686441}, {0.694915}, {0.70339},
{0.711864}, {0.720339}, {0.728814}, {0.737288}, {0.745763}, {0.754237}, {0.762712},
{0.771186}, {0.779661}, {0.788136}, {0.79661}, {0.805085}, {0.813559}, {0.822034},
{0.830508}, {0.838983}, {0.847458}, {0.855932}, {0.864407}, {0.872881}, {0.881356},
{0.889831}, {0.898305}, {0.90678}, {0.915254}, {0.923729}, {0.932203}, {0.940678},
{0.949153}, {0.957627}, {0.966102}, {0.974576}, {0.983051}, {0.991525}, {1.}}