

\$Assumptions = P ∈ Reals && -1 ≤ q ≤ 1 && t > 0

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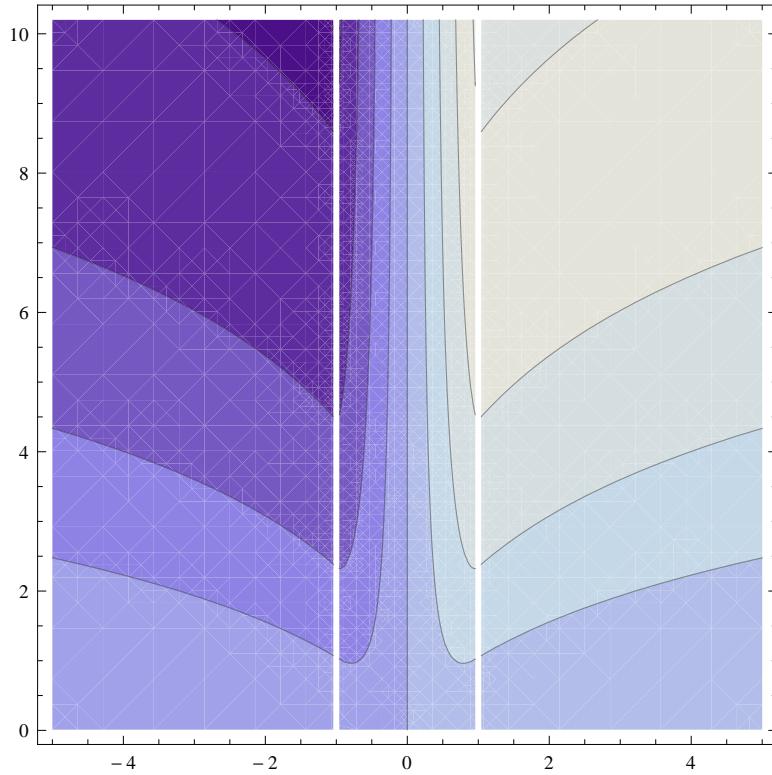
ExpectedValue[Max[0, 1 (Exp[#1 - 1 / 2 t] - 1) - P] - Max[0, -(Exp[#1 - 1 / 2 t] - 1) - P] &, NormalDistribution[0, Sqrt[t]]]

$$\begin{cases} \frac{1}{2} \left(-P + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & P \geq 1 \text{ && } t > 0 \\ \frac{1}{2} \left(-\operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] - \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & -1 < P < 1 \text{ && } t > 0 \\ \frac{1}{2} \left(-2 - P + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] - \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + 2 \operatorname{Erfc}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & \text{True} \end{cases}$$

g[t_, p_] :=

$$\begin{cases} \frac{1}{2} \left(-P + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & P \geq 1 \text{ && } t > 0 \\ \frac{1}{2} \left(-\operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] - \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & -1 < P < 1 \text{ && } t > 0 \\ \frac{1}{2} \left(-2 - P + \operatorname{Erf}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] - \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + P \operatorname{Erf}\left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] + 2 \operatorname{Erfc}\left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}}\right] \right) & \text{True} \end{cases}$$

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ContourPlot[g[t, P], {P, -5, 5}, {t, 0.01, 10.2}]
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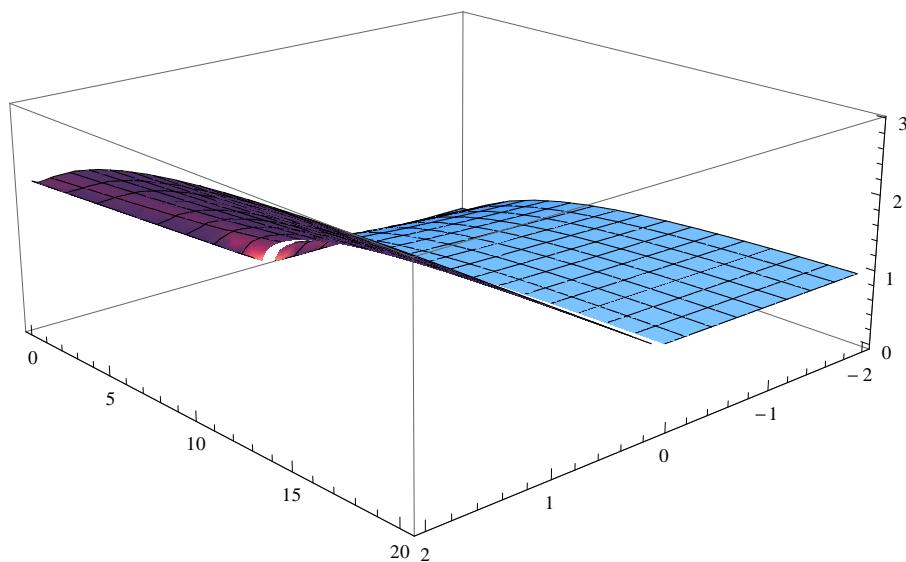
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ExpectedValue[Max[0, Sign[-P] (Exp[#1 - 1/2 t] - 1) + P] &, NormalDistribution[0, Sqrt[t]]]
```

$$\begin{cases} \frac{1}{2} \left(P + \operatorname{Erf} \left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] + \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] - P \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] \right) & P < 0 \& t > 0 \\ \frac{1}{2} \left(P + \operatorname{Erf} \left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] + \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] + P \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] \right) & P > 0 \& t > 0 \\ 0 & \text{True} \end{cases}$$

```
f[t_, P_] :=
```

$$\begin{cases} \frac{1}{2} \left(P + \operatorname{Erf} \left[\frac{t-2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] + \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] - P \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1-P]}{2 \sqrt{2} \sqrt{t}} \right] \right) & P < 0 \& t > 0 \\ \frac{1}{2} \left(P + \operatorname{Erf} \left[\frac{t-2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] + \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] + P \operatorname{Erf} \left[\frac{t+2 \operatorname{Log}[1+P]}{2 \sqrt{2} \sqrt{t}} \right] \right) & P > 0 \& t > 0 \\ 0 & \text{True} \end{cases}$$

```
Plot3D[f[t, p], {p, -2, 2}, {t, 0.01, 20.2}]
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```
ExpectedValue[Max[0, (Exp[#1 - 1/2 t] - 1) &, NormalDistribution[0, Sqrt[t]]]]
```

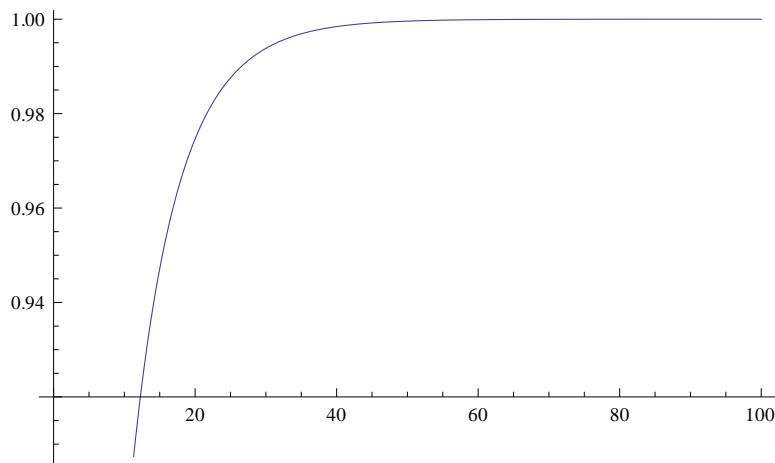
$$\text{Erf}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right]$$

$$\text{Erf}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right] / . t \rightarrow 1000000000000 // N$$

General::unfl : Underflow occurred in computation. >>

1.

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Plot[Erf[(Sqrt[t])/(2 Sqrt[2])], {t, 0, 100}]
```



```
ExpectedValue[Abs[Exp[#1 - 1/2 t] - 1] &, NormalDistribution[0, Sqrt[t]]]
```

$$2 \operatorname{Erf}\left[\frac{\sqrt{t}}{2 \sqrt{2}}\right]$$