

```

Exit[];

$Assumptions = r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
r > 0 && m ∈ Integers && n ∈ Integers && s > 0 && k ∈ Integers && k > 0

m = 2;

f[r_, En_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}} -
  0 * IdentityMatrix[2] * I * r ^ p; f[r, En] // MatrixForm

$$\begin{pmatrix} \frac{1}{r} & i (En - r^2) \\ i (En - r^2) & -\frac{2}{r} \end{pmatrix}$$


En = .

fE = D[f[r, En], En]

{{0, i}, {i, 0}}

u = {a[n] * x ^ (2 * n), b[n] * x ^ (2 * n + 1)} * x ^ s
{x ^ (2 * n + s) a[n], x ^ (1 + 2 * n + s) b[n]}

p = 2;

r[x_] := x;

g1 = Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x]].u) * x ^ (-s + 1)]]],
  {x ^ n, a[n], b[n]}];
g1 // MatrixForm

$$\begin{pmatrix} x^{2n} \left( (1 - m + 2n + s) a[n] + (-i En x^2 + i x^4) b[n] \right) \\ x^{2n} \left( (-i En x + i x^3) a[n] + (x + m x + 2n x + s x) b[n] \right) \end{pmatrix}$$


s = -1 + m;

g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm

$$\begin{pmatrix} 0 & 0 \\ 0 & -i En a[0] + 2 m b[0] \\ 2 a[1] - i En b[0] & 0 \\ 0 & i a[0] - i En a[1] + 2 (1 + m) b[1] \\ 4 a[2] + i (b[0] - En b[1]) & 0 \\ 0 & i a[1] - i En a[2] + 2 (2 + m) b[2] \\ 6 a[3] + i (b[1] - En b[2]) & 0 \\ 0 & i a[2] - i En a[3] + 2 (3 + m) b[3] \\ 8 a[4] + i (b[2] - En b[3]) & 0 \\ 0 & i a[3] - i En a[4] + 2 (4 + m) b[4] \\ 10 a[5] + i (b[3] - En b[4]) & 0 \end{pmatrix}$$


a[0] = 1; b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;

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a[0] = 1; b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
b[n_] := I * (En * a[n] - a[n-1]) / 2 / (n+m);
a[n_] := Simplify[I * (En b[n-1] - b[n-2]) / 2 / n]

a[1]

$$-\frac{En^2}{8}$$


m = 2;

En = .

Un[En_, m_, nN_, x_] := Module[{n, U},
  U = {1}; AppendTo[U,  $-\frac{En^2}{4 m}$ ]; AppendTo[U,  $\frac{En (4 + En^3 + 8 m)}{32 m (1 + m)}$ ]; G = {i En / 2 / m};

  For[n = 3, n < nN, n++,
    AppendTo[U,
      -((-1 + m + n) U[[-2 + n]] + En ((3 - 2 m - 2 n) U[[-1 + n]] + En (-2 + m + n) U[[n]])) /
      (4 n (-2 + m + n) (-1 + m + n))];
  ];
  ({1, i En / 2 / m * x} +
    Sum[{U[[n+1]] * x^(2 * n), I * (En * U[[n+1]] - U[[n]]) / 2 / (n+m) * x^(2 * n+1)},
      {n, 1, nN - 1}]) * x^(-1 + m) // N]

U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n+1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
    n++;
    U = G; G = -Un[En, m, n+1, X];
  ];
  {Un[En, m, n, X], n}]

```

```

Ener[Ene_] :=
Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 3500;
  m = 2;
  r = 17 // N; h = -16.9 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
    k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
    k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
    k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

    k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
    k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
    kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

    r += h;

    , {n}];

  NN = U[En, m, 10^-10, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn -> En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.06,
    En -= Erg / (U2S k[[1]] - U1S k[[2]] + U2 kE[[1]] - U1 kE[[2]]);
    (*Print[{En, Erg/U2/k[[2]]}];*) Goto[begin];
  ];
  {En, Erg / U2 / k[[2]]}
]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
  Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[3.8]
{3.61902+0.265337 i, -0.00109633-0.000496109 i}

Energie // MatrixForm

$$\begin{pmatrix} 0 & \{-0.0348248+0.273429 i, 0.0824271+0.00377436 i\} \\ 0.1 & \{-0.0348248+0.273429 i, 0.0214202-0.0233512 i\} \\ 0.2 & \{0.161054+0.274461 i, -0.00191844+0.059646 i\} \\ 0.3 & \{0.161054+0.274461 i, 0.0430159+0.0247969 i\} \\ 0.4 & \{0.357691+0.275498 i, -0.0239546+0.0157214 i\} \\ 0.5 & \{0.357691+0.275498 i, 0.0119417+0.0477695 i\} \\ 0.6 & \{0.555088+0.276533 i, -0.0148071+0.000774966 i\} \end{pmatrix}$$


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```

0.7 {0.555088+0.276533 i, -0.00890224+0.0489729 i}
0.8 {0.753241+0.277561 i, -0.00846342-0.00200356 i}
0.9 {0.753241+0.277561 i, -0.0231779+0.047652 i}
1. {0.952144+0.278576 i, 0.0783007+0.0337232 i}
1.1 {0.952144+0.278576 i, -0.0355808+0.0442028 i}
1.2 {1.15178+0.279569 i, 0.0505876+0.0270738 i}
1.3 {1.15178+0.279569 i, -0.045905+0.0363295 i}
1.4 {1.35215+0.280534 i, 0.03465+0.019346 i}
1.5 {1.35215+0.280534 i, -0.0511494+0.0227949 i}
1.6 {1.55321+0.281461 i, 0.0245336+0.0125636 i}
1.7 {1.55321+0.281461 i, -0.0479442+0.00618525 i}
1.8 {1.75494+0.282337 i, 0.0174311+0.00717699 i}
1.9 {1.75494+0.282337 i, -0.0358656-0.00734238 i}
2. {1.9573+0.283143 i, 0.0119518+0.00328444 i}
2.1 {1.9573+0.283143 i, -0.0202808-0.0123854 i}
2.2 {2.16024+0.283843 i, 0.00759541+0.000846598 i}
2.3 {2.16024+0.283843 i, -0.00901651-0.00938155 i}
2.4 {2.36368+0.284381 i, -0.0649912+0.00531311 i}
2.5 {2.36368+0.284381 i, -0.00765636-0.00670322 i}
2.6 {2.56752+0.284638 i, -0.0300368+0.00916306 i}
2.7 {2.56752+0.284638 i, 0.017086+0.00489736 i}
2.8 {2.7716+0.284365 i, -0.0102766+0.00631036 i}
2.9 {2.7716+0.284365 i, 0.00255367-0.007645 i}
3. {2.97573+0.282951 i, 0.0324564-0.0390568 i}
3.1 {2.97573+0.282951 i, -0.0129679+0.0493867 i}
3.2 {3.18002+0.278733 i, 0.00561139-0.0124434 i}
3.3 {3.38843+0.267181 i, -0.00706132-0.00222546 i}
3.4 {3.38843+0.267181 i, 0.0716424+0.0146798 i}
3.5 {3.51788+0.111334 i, 0.00243169+0.000403973 i}
3.6 {3.51788+0.111334 i, -0.0000168755+0.000409094 i}
3.7 {3.51788+0.111334 i, 0.0000574665-0.000997058 i}
3.8 {3.61902+0.265337 i, 0.0159846+0.00559245 i}
3.9 {3.83002+0.279236 i, -0.00334078-0.00754872 i}
4. {3.83002+0.279236 i, -0.0031197-0.0264905 i}
4.1 {4.03602+0.284821 i, 0.0202834+0.030361 i}
4.2 {4.03602+0.284821 i, 0.00672479-0.00922651 i}
4.3 {4.24235+0.286678 i, 0.011193+0.00918883 i}
4.4 {4.03602+0.284821 i, 0.00496042+0.0116323 i}
4.5 {4.44907+0.286035 i, -0.0699411-0.0157006 i}
4.6 {4.65592+0.281786 i, -0.0128516+0.0066142 i}
4.7 {4.65592+0.281786 i, -0.0326016+0.0063652 i}
4.8 {4.8653+0.268593 i, -0.00101728+0.00822936 i}
4.9 {4.8653+0.268593 i, 0.0328955+0.00350871 i}
5. {5.02602+0.142922 i, 0.00271581+0.0632135 i}
5.1 {5.02603+0.142926 i, -0.000932623-0.000541224 i}
5.2 {5.32464+0.279517 i, -0.00715488+0.00675674 i}
5.3 {5.10897+0.258743 i, 0.0175854-0.0479305 i}
5.4 {5.32464+0.279518 i, 0.000118089+0.0186131 i}
5.5 {5.32464+0.279517 i, 0.00924017-0.00980417 i}
5.6 {5.53302+0.286516 i, -0.0380211-0.0675122 i}
5.7 {5.53302+0.286516 i, 0.0135533-0.015473 i}

```

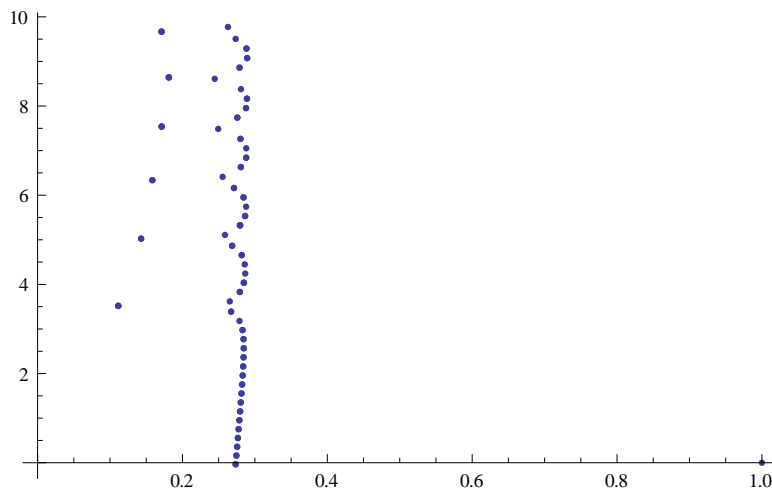
```

5.7 {5.32464+0.279518 i, 0.0105739+0.015473 i}
5.8 {5.74162+0.287838 i, -0.0199566-0.0134933 i}
5.9 {5.95051+0.284332 i, -0.0354819-0.0250697 i}
6. {5.95051+0.284332 i, -0.00913623-0.00047985 i}
6.1 {6.16131+0.271158 i, 0.0252866-0.0560522 i}
6.2 {6.16131+0.271152 i, 0.0101888+0.0123834 i}
6.3 {6.33701+0.158456 i, -0.00105735-0.000319741 i}
6.4 {6.33701+0.158456 i, -0.000671294+0.00110609 i}
6.5 {5.53302+0.286517 i, -0.0310481-0.0398119 i}
6.6 {6.41235+0.255483 i, 0.0235597-0.0317144 i}
6.7 {6.63113+0.280631 i, 0.00382499+0.00786223 i}
6.8 {6.63113+0.280631 i, -0.0077521+0.00818432 i}
6.9 {6.84154+0.287981 i, -0.022262-0.0132369 i}
7. {6.84154+0.287983 i, 0.00101079-0.0076893 i}
7.1 {7.05207+0.28802 i, -0.00646432-0.000525755 i}
7.2 {7.26328+0.28014 i, -0.00507427+0.00750115 i}
7.3 {7.26327+0.280144 i, 0.0673018+0.00510217 i}
7.4 {7.48771+0.249352 i, -0.0063826-0.00716244 i}
7.5 {7.53947+0.171252 i, 0.00125585+0.00522793 i}
7.6 {7.53948+0.171245 i, -0.0014772+0.00081522 i}
7.7 {7.53949+0.171252 i, 0.00201748-0.00137817 i}
7.8 {7.74042+0.275795 i, 0.0535+0.0351612 i}
7.9 {7.74041+0.27578 i, 0.0102428-0.0180593 i}
8. {7.9533+0.287735 i, -0.0215442+0.012056 i}
8.1 {7.95331+0.287745 i, 0.0599208-0.0229489 i}
8.2 {8.16549+0.289055 i, 0.0735446-0.0426555 i}
8.3 {8.16548+0.289041 i, 0.00452665+0.0332604 i}
8.4 {8.37849+0.280812 i, 0.0165815-0.0119099 i}
8.5 {8.6095+0.244542 i, -0.0122388-0.000407866 i}
8.6 {8.64233+0.181294 i, 0.00614591+0.00163093 i}
8.7 {8.64232+0.181244 i, -0.0052266+0.00204766 i}
8.8 {8.64234+0.181016 i, -0.0559045-0.00772336 i}
8.9 {8.86137+0.278848 i, 0.0378659+0.0469596 i}
9. {8.86138+0.27881 i, -0.00298264-0.0446443 i}
9.1 {9.07571+0.289363 i, 0.0192018-0.00629468 i}
9.2 {9.0757+0.289343 i, -0.0655082+0.0448379 i}
9.3 {9.28963+0.288376 i, 0.00576278-0.0502462 i}
9.4 {9.28962+0.288357 i, -0.0151671+0.0315395 i}
9.5 {9.50588+0.273435 i, -0.000351129-0.0116833 i}
9.6 {9.66759+0.171037 i, -0.0194593+0.0666014 i}
9.7 {9.66791+0.171195 i, -0.00189307-0.00197333 i}
9.8 {9.66789+0.171216 i, 0.00293916+0.0000546853 i}
9.9 {9.77232+0.262805 i, -0.00545773+0.017099 i}

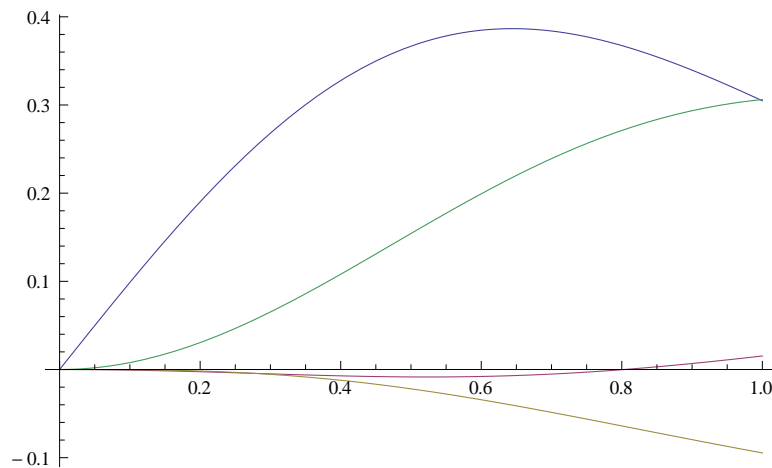
```

Energien = {Im[#[[2, 1]]], Re[#[[2, 1]]]} & /@ Energie;

```
ListPlot[Append[Energien, {1, 0}], AxesOrigin -> {0, 0}, PlotRange -> All]
```



```
G = {Re[#], Im[#]} &[Un[Energie[[1]], 2, 15, x]]; Plot[G, {x, 0, 1}, PlotRange -> All]
```



```
En = Energie[[1]]; n = 3500;
```

```
m = 2;
```

```
r = 17 // N; h = -16.9 / n;
```

```
k = {1, -1}; kK = {{r, k}};
```

```
Do[
```

```
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
```

```
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
```

```
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
```

```
  AppendTo[kK, {r, k}], {n}]; En =.
```

```
k / k[[1]] - Un[Energie[[1]], 2, 20, 0.1] / Un[Energie[[1]], 2, 20, 0.1][[1]]
```

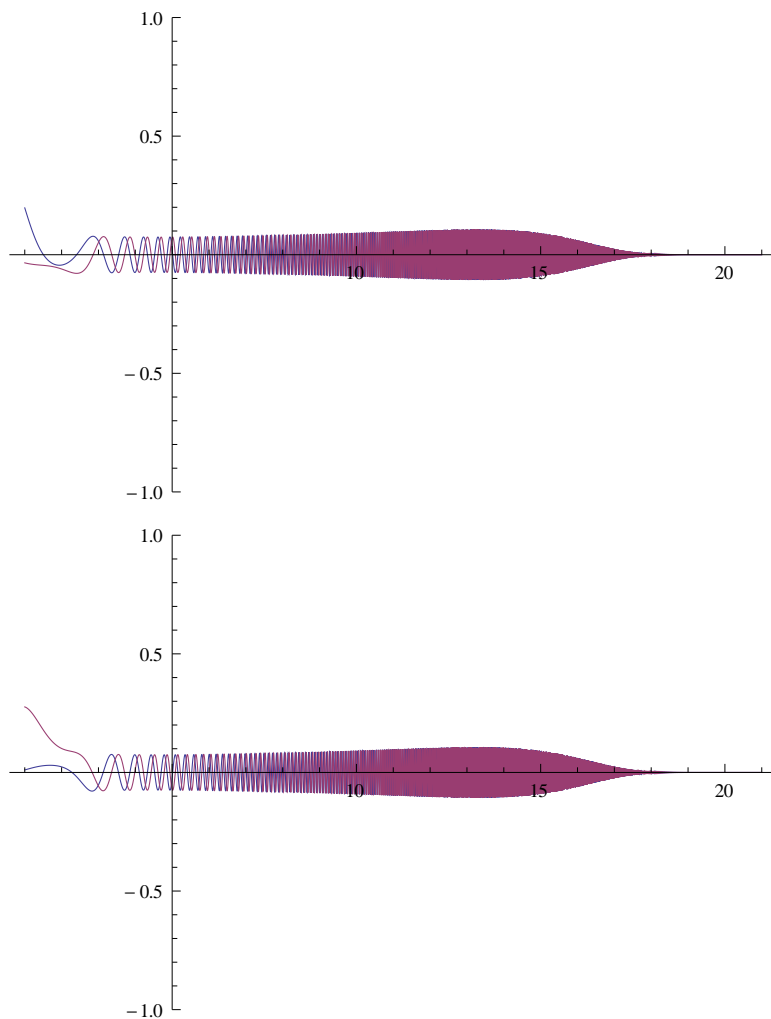
```
{0.+7.92188 × 10-19 i, 0.0000440082+0.0000213747 i}
```

```

n = 6000; S = 1; h = 20 / n; ra = 1; En = 3.517884643304668 + 0.11133398841863644 i;
m = 2; r = 1; k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N},
  {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N},
  PlotRange -> {-ra, ra}, Joined -> True]
ListPlot[Join[{#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N},
  {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N},
  PlotRange -> {-ra, ra}, Joined -> True]
En = .;
r = .;

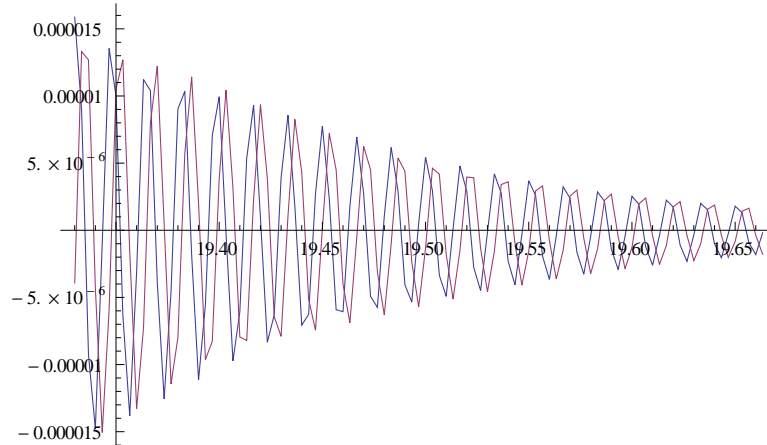
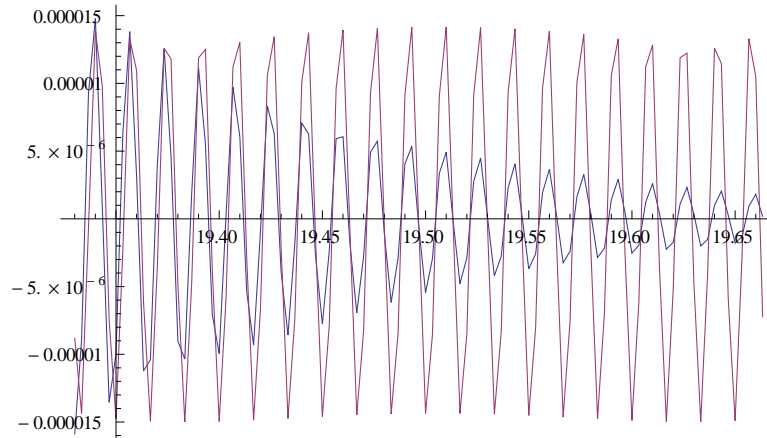
```



```

S = 5500; n = 100; ListPlot[Join[{#[[1]], Re#[[2, 1]]} & /@ kK[[S ;; S + n]] // N},
  {#[[1]], 0.000015 * Cos[#[[1]]^3 / 3 - #[[1]] * Re[3.517884613174765]]} & /@
    kK[[S ;; S + n]] // N], PlotRange -> All, Joined -> True]
ListPlot[Join[{#[[1]], Re#[[2, 2]]} & /@ kK[[S ;; S + n]] // N},
  {#[[1]], Im#[[2, 2]]} & /@ kK[[S ;; S + n]] // N], PlotRange -> All, Joined -> True]

```



Exp[I * Im[Energie[[1]]] * x]

$e^{0.278733 i x}$