

Identity[Normal[Series[U[x - e], {x, e + 1, 4}]]]

$$U[1] + (-1 - e + x) U'[1] + \frac{1}{2} (-1 - e + x)^2 U''[1] + \frac{1}{6} (-1 - e + x)^3 U^{(3)}[1] + \frac{1}{24} (-1 - e + x)^4 U^{(4)}[1]$$

Series[x^a, {x, 1, 4}]

$$1 + a (x - 1) + \frac{1}{2} (-1 + a) a (x - 1)^2 + \frac{1}{6} (-2 + a) (-1 + a) a (x - 1)^3 + \frac{1}{24} (-3 + a) (-2 + a) (-1 + a) a (x - 1)^4 + O[x - 1]^5$$

Expand[Normal[Series[U[x - e], {e, 0, 4}]]]