```
Exit[];
```

```
$Assumptions = r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
```

$$r > 0 \&\& m \in \text{Integers} \&\& n \in \text{Integers} \&\& s > 0 \&\& k \in \text{Integers} \&\& k > 0$$

```
m = 5;
```

```
f[r_, En_] := {{i En - 1/(2 r) - i r^p, (1-2 m)/(2 r)}, {(1-2 m)/(2 r), -i En - 1/(2 r) + i r^p}}; f[r, En] // MatrixForm
```

$$\begin{pmatrix} i\,\text{En} - \frac{1}{2r} - i\,r^2 & \frac{1-2m}{2r} \\ \frac{1-2m}{2r} & -i\,\text{En} - \frac{1}{2r} + i\,r^2 \end{pmatrix}$$

```
En =.; n =.; p = 2; s = -1 / 2;
```

```
fE = D[f[r, En], En]
```

$$\{\{0, i\}, \{i, 0\}\}$$

```
u = {a[n] * x^(n), b[n] * x^(n)} * x^s
```

$$\left\{x^{-\frac{1}{2}+n}\,a[n],\ x^{-\frac{1}{2}+n}\,b[n]\right\}$$

```
r[x_] := x;
```

```
g1 = Collect[Expand[Simplify[Expand[(D[u, x] - r'[x] * f[r[x], En].u) * x^(-s + 1)]]],
   {x^n, a[n], b[n]}];
g1 // MatrixForm
```

$$\begin{pmatrix} x^n \left(\left(n - i\,\text{En}\,x + i\,x^3\right) a[n] + \left(-\frac{1}{2} + m\right) b[n]\right) \\ x^n \left(\left(-\frac{1}{2} + m\right) a[n] + \left(n + i\,\text{En}\,x - i\,x^3\right) b[n]\right) \end{pmatrix}$$

```
V = {{1, 2 m - 1}, {2 m - 1, 1}}; eV = Transpose[Eigenvectors[V]]
```

```
Simplify[eV.V.Inverse[eV]] // MatrixForm
```

$$\begin{pmatrix} 2 - 2\,m & 0 \\ 0 & 2\,m \end{pmatrix}$$

```
Eitg
```

```
s = -1 + m;
```

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

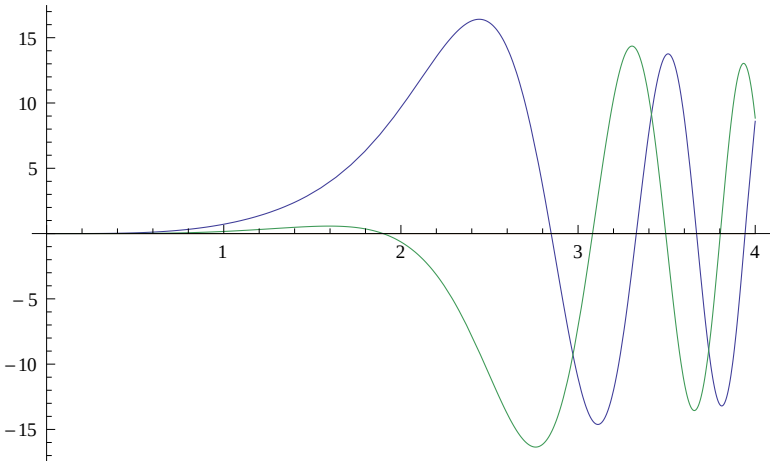
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
a[0] = 1;  b[0] = i En a[0] / 2 / m; a[1] = i En b[0] / 2;
b[1] = -(i En^3 / (4 m)) / 2 / (1 + m) ; a[2] = i En b[1] / 4;
b[n_] := Simplify[(-i a[n - 2] + i En a[n]) / (2 (n + m) )];
a[n_] := Simplify[-i (b[n - 3] - En b[n - 1]) / 2 / n]
b[4]
```

$$\frac{i\ En^4\ \left(-22\,464 + En^5\right)}{185\,794\,560}$$

```
Un[En_, m_, nN_, x_] := Module[{n, U},
  U = {1}; AppendTo[U, -En^2 / (4 m)]; AppendTo[U, En (4 + En^3 + 8 m) / (32 m (1 + m))]; G = {i En / 2 / m};
  For[n = 3, n < nN, n++,
   AppendTo[U,
    -((-1 + m + n) U[[-2 + n]] + En ((3 - 2 m - 2 n) U[[-1 + n]] + En (-2 + m + n) U[[n]])) /
     (4 n (-2 + m + n) (-1 + m + n))];
  ];
  ({1, i En / 2 / m * x} +
     Sum[{U[[n + 1]] * x^(2 * n), I * (En * U[[n + 1]] - U[[n]]) / 2 / (n + m) * x^(2 * n + 1)},
      {n, 1, nN - 1}]) * x^(-1 + m) // N]
```

```
G = {Re[#], Im[#]} &[Un[3, 5, 150, x]]; Plot[G, {x, 0, 4}, PlotRange → All]
```




```
U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
   n++;
   U = G; G = -Un[En, m, n + 1, X];

  ];
  {Un[En, m, n, X], n}]
```

```
U[9, 5, 0.0001, 1]
```

```
{{-0.0126898, 0.+0.00407705 i}, 14}
```

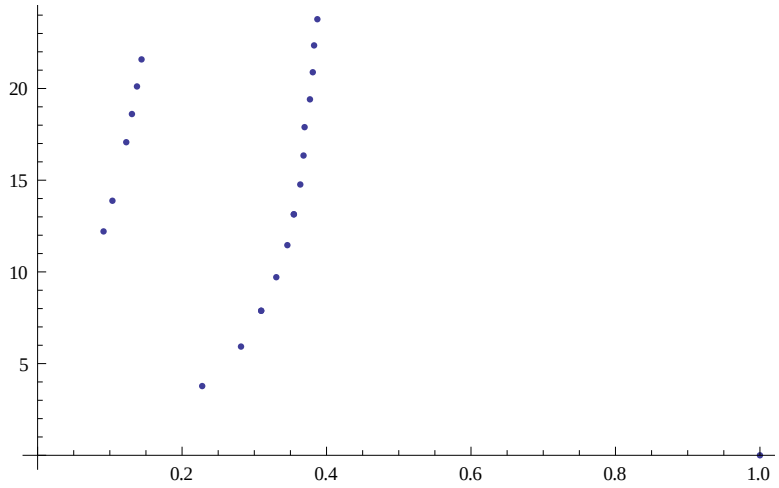
```
Ener[Ene_] :=
 Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 5000;
  m = 5;
  r = 7.2 // N; h = -7.0 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
   k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
   k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
   k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
   k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
   kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   r += h;
   , {n}];

  NN = U[En, m, 0.0001, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn → En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.02,
   En -= Erg / (U2S k⟦1⟧ - U1S k⟦2⟧ + U2 kE⟦1⟧ - U1 kE⟦2⟧);
   Print[{En, Erg / U2 / k[[2]]}]; Goto[begin];
    ];
  {En, Erg / U2 / k[[2]]}
 ]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
 Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[22]
```

```
{21.8924+ 0.168499 i, −0.262337 + 1.54006 i}

{21.7276+ 0.288878 i, 0.0977574+ 1.99468 i}

{21.5078+ 0.261371 i, 1.15369+ 1.40345 i}

{21.6825+ 0.0732421 i, 0.410727+ 0.725268 i}

{21.6223+ 0.137127 i, −1.84205 + 0.583011 i}

{21.5895+ 0.1468 i, −0.181458 − 0.525624 i}

{21.5862+ 0.143899 i, 0.0316819− 0.0363388 i}

    {21.5862+ 0.143899 i, 0.000384958+ 0.000557679 i}
```

```
Energie = {3.77486283903418` + 0.22786873407418717` i,
  5.928479968617718` + 0.2815986526347655` i,
  7.8813588488087065` + 0.30953294412328675` i,
  7.881329304880588` + 0.3095336916562825` i,
  9.71036454189739` + 0.3304415424573178` i,
  11.458077781781169` + 0.3457848857997222` i,
  13.139764183242892` + 0.3546744769440479` i,
  13.139114444715846` + 0.3548172513559829` i,
  14.76566063867228` + 0.3636637795053202` i,
  16.34527139193304` + 0.3681521067888729` i,
  17.892138695416435` + 0.3696213478944415` i,
  19.40831608562361` + 0.37704222997528414` i,
  20.886015743223094` + 0.38097023642873473` i,
  22.34958939297449` + 0.38276309915136164` i,
  23.777494591007947` + 0.3872991704766671` i, 12.207301904640477` + 0.09130244038237889` i,
  13.879319291317909` + 0.10359131716054014` i, 17.071331521063737` + 0.12274708920757649` i,
  18.607651751873114` + 0.1305970820854307` i, 20.11139546248976` + 0.1375815129955693` i,
  21.586240037178456` + 0.14389862151162078` i}; Energie // MatrixForm
```

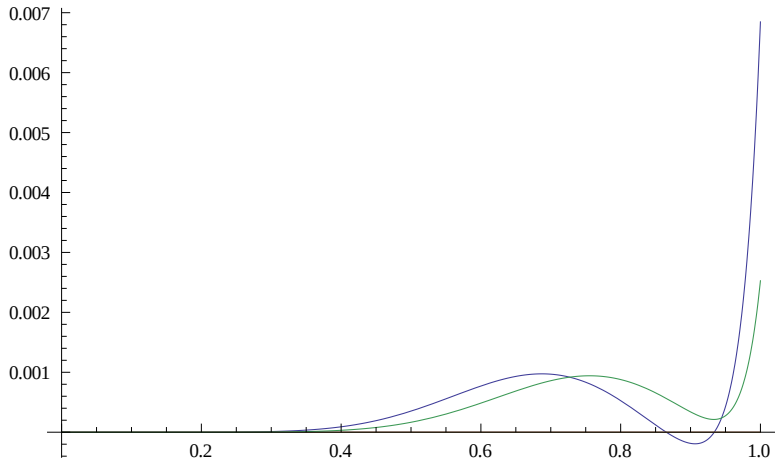
$$\begin{pmatrix}
3.77486+0.227869\,i\\
5.92848+0.281599\,i\\
7.88136+0.309533\,i\\
7.88133+0.309534\,i\\
9.71036+0.330442\,i\\
11.4581+0.345785\,i\\
13.1398+0.354674\,i\\
13.1391+0.354817\,i\\
14.7657+0.363664\,i\\
16.3453+0.368152\,i\\
17.8921+0.369621\,i\\
19.4083+0.377042\,i\\
20.886+0.38097\,i\\
22.3496+0.382763\,i\\
23.7775+0.387299\,i\\
12.2073+0.0913024\,i\\
13.8793+0.103591\,i\\
17.0713+0.122747\,i\\
18.6077+0.130597\,i\\
20.1114+0.137582\,i\\
21.5862+0.143899\,i
\end{pmatrix}$$

```
ListPlot[Append[{Im[#], Re[#]} & /@ Energie, {1, 0}],
 AxesOrigin → {0, 0}, PlotRange → All]
```




```
:
```

```
G = {Re[#], Im[#]} &[Un[16, 10, 15, x]]; Plot[G, {x, 0, 1}, PlotRange → All]
```



```
n = 8000; S = 1; h = 6 / n; ra = 1; En = Energie[[17]]; m = 5; r = 1;
U[En, m, 10^-10, r][[2]]
k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
En =.;
r =.;
```

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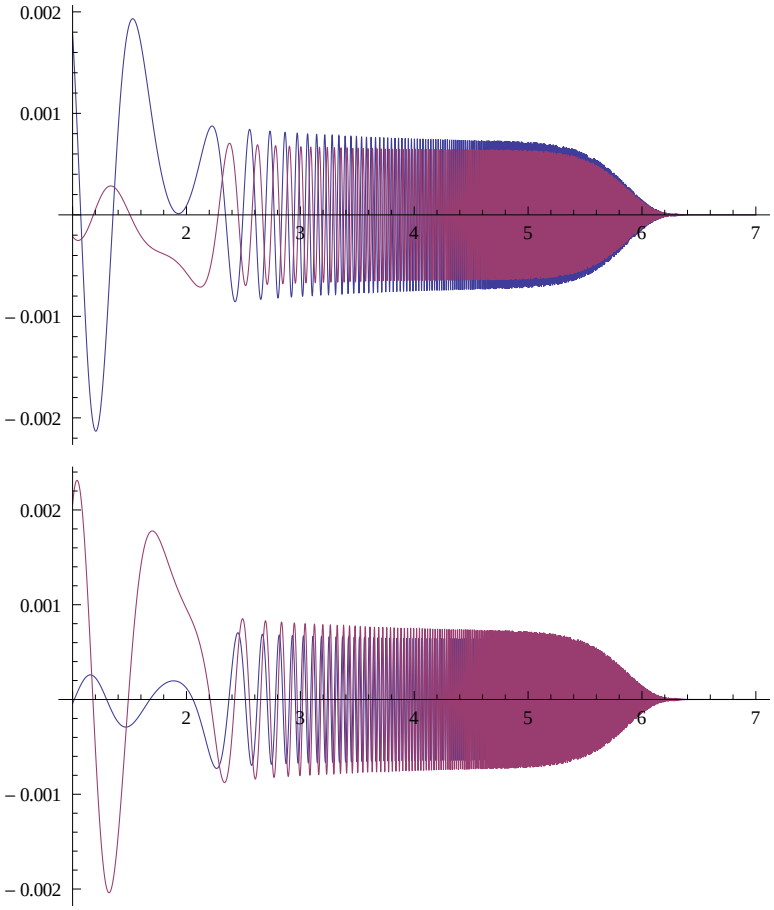
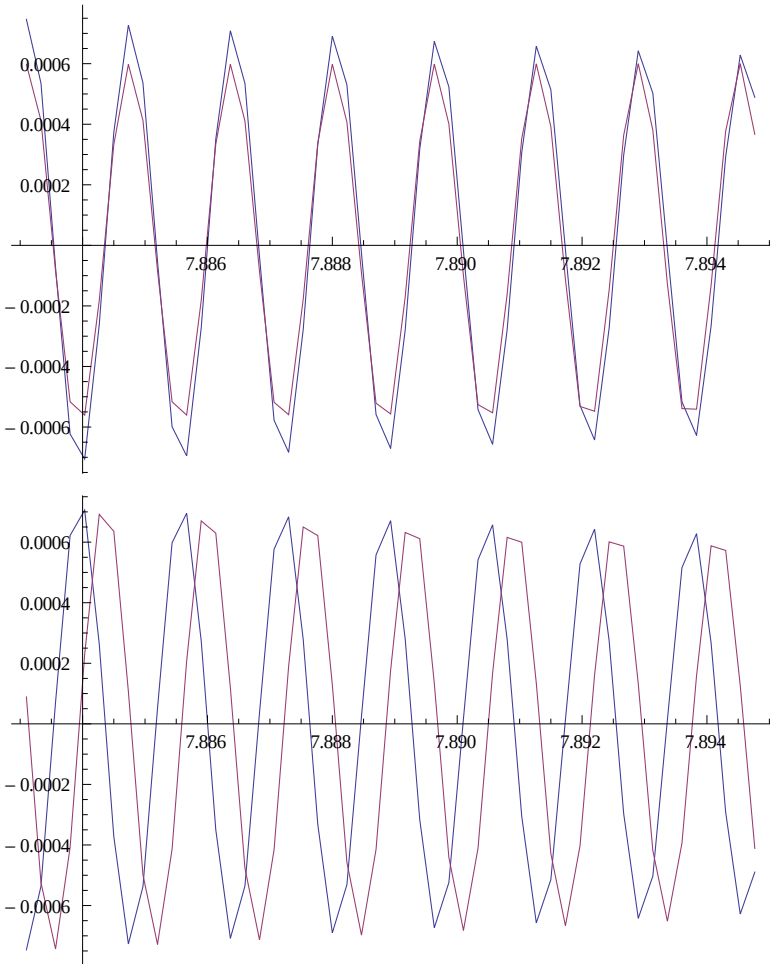



```
U[En, m, 10^-10, r][[1]]
```

```
S = 29500; n = 50; ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], -0.0006 * Sin[#[[1]]^5 / 5 + 1 - #[[1]] * Re[Energie[[11]]]]} & /@
      kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
```




```
Exp[I * Im[Energie[[1]]] * x]
```

$$e^{0.278733\, i\, x}$$