```
Exit[];
```

```
$Assumptions = r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
```

$r > 0 \&\& m \in \text{Integers} \&\& n \in \text{Integers} \&\& s > 0 \&\& k \in \text{Integers} \&\& k > 0$

```
m = 5;
```

2-d Dirac

```
f[r_, En_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}};
f[r, En] // MatrixForm
```

$$\begin{pmatrix} \frac{-1+m}{r} & i\left(\text{En}-r^2\right) \\ i\left(\text{En}-r^2\right) & -\frac{m}{r} \end{pmatrix}$$

Diagonaldarstellung für r gegen Infinity

```
V = {{1, 1}, {-1, 1}}; Simplify[V.f[r, En].Inverse[V]] // MatrixForm
```

$$\begin{pmatrix} i\,\text{En}-\frac{1}{2\,r}-i\,r^2 & \frac{1-2\,m}{2\,r} \\ \frac{1-2\,m}{2\,r} & -i\,\text{En}-\frac{1}{2\,r}+i\,r^2 \end{pmatrix}$$

```
Inverse[V].{0, C}
```

$$\left\{-\frac{C}{2}, \frac{C}{2}\right\}$$

```
En =.; n =.; p = 2
```

2

```
fE = D[f[r, En], En]
```

$\{\{0, i\}, \{i, 0\}\}$

```
u = {a[n], b[n]} * Exp[I x ^ 3 / 3] * x ^ n
```

$$\left\{e^{\frac{i\,x^3}{3}}\,x^n\,a[n],\ e^{\frac{i\,x^3}{3}}\,x^n\,b[n]\right\}$$

```
r[x_] := x;
```

```
g1 = Collect[
   Expand[Simplify[Expand[-(D[u, x] - r'[x] * f[r[x], En].u) * x / Exp[I x^3 / 3]]]],
   {x^n, a[n], b[n], F[x], G[x]}];
g1
```

$$\left\{x^n \left(\left(-n - i\, x^3\right) a[n] + \left(i\, En\, x - i\, x^3\right) b[n]\right),\ x^n \left(\left(i\, En\, x - i\, x^3\right) a[n] + \left(1 - 2\, m - n - i\, x^3\right) b[n]\right)\right\}$$

```
s = -m;
```

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 15}] /. x → 0], {n2, 0, 15}];
g2 // MatrixForm
```

$$\begin{pmatrix}
0 & (1-2\,m)\,b[0] \\
-a[1]+i\,En\,b[0] & i\,En\,a[0]-2\,m\,b[1] \\
-2\,a[2]+i\,En\,b[1] & i\,En\,a[1]-(1+2\,m)\,b[2] \\
-i\,(a[0]-3\,i\,a[3]+b[0]-En\,b[2]) & -i\,(a[0]-En\,a[2]+b[0]-2\,i\,b[3]-2\,i\,m\,b[3]) \\
-i\,(a[1]-4\,i\,a[4]+b[1]-En\,b[3]) & -i\,(a[1]-En\,a[3]+b[1]-3\,i\,b[4]-2\,i\,m\,b[4]) \\
-i\,(a[2]-5\,i\,a[5]+b[2]-En\,b[4]) & -i\,(a[2]-En\,a[4]+b[2]-4\,i\,b[5]-2\,i\,m\,b[5]) \\
-i\,(a[3]-6\,i\,a[6]+b[3]-En\,b[5]) & -i\,(a[3]-En\,a[5]+b[3]-5\,i\,b[6]-2\,i\,m\,b[6]) \\
-i\,(a[4]-7\,i\,a[7]+b[4]-En\,b[6]) & -i\,(a[4]-En\,a[6]+b[4]-6\,i\,b[7]-2\,i\,m\,b[7]) \\
-i\,(a[5]-8\,i\,a[8]+b[5]-En\,b[7]) & -i\,(a[5]-En\,a[7]+b[5]-7\,i\,b[8]-2\,i\,m\,b[8]) \\
-i\,(a[6]-9\,i\,a[9]+b[6]-En\,b[8]) & -i\,(a[6]-En\,a[8]+b[6]-8\,i\,b[9]-2\,i\,m\,b[9]) \\
-i\,(a[7]-10\,i\,a[10]+b[7]-En\,b[9]) & -i\,(a[7]-En\,a[9]+b[7]-9\,i\,b[10]-2\,i\,m\,b[10]) \\
-i\,(a[8]-11\,i\,a[11]+b[8]-En\,b[10]) & -i\,(a[8]-En\,a[10]+b[8]-10\,i\,b[11]-2\,i\,m\,b[11]) \\
-i\,(a[9]-12\,i\,a[12]+b[9]-En\,b[11]) & -i\,(a[9]-En\,a[11]+b[9]-11\,i\,b[12]-2\,i\,m\,b[12]) \\
-i\,(a[10]-13\,i\,a[13]+b[10]-En\,b[12]) & -i\,(a[10]-En\,a[12]+b[10]-12\,i\,b[13]-2\,i\,m\,b[13] \\
-i\,(a[11]-14\,i\,a[14]+b[11]-En\,b[13]) & -i\,(a[11]-En\,a[13]+b[11]-13\,i\,b[14]-2\,i\,m\,b[14 \\
-i\,(a[12]-15\,i\,a[15]+b[12]-En\,b[14]) & -i\,(a[12]-En\,a[14]+b[12]-14\,i\,b[15]-2\,i\,m\,b[15
\end{pmatrix}$$

```
a[0] = 1; b[0] = 0;  b[1] = i En a[0] / 2 / m ; a[1] = 0 ; b[2] = 0; a[2] = -En^2 / 4 / m
```

$$-\frac{En^2}{4\,m}$$

```
a[1] =.; a[2] =.; b[1] =.; b[0] =.; a[0] =.; b[2] =.;
```

```
a[n_] := +3 i a[n + 3] - b[n + 0] + En b[n + 2]; a[n_] =.
```

```
b[n_] := a[n] * (-1)^n;
```

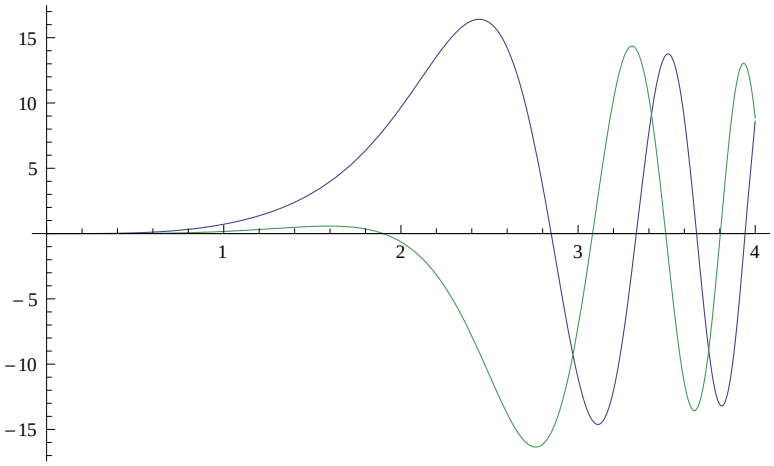
```
b[n_] := Simplify[(-i a[n - 2] + i En a[n]) / (2 (n + m))];
a[n_] := Simplify[-i (b[n - 3] - En b[n - 1]) / 2 / n]
```

```
b[4]
```

$$\frac{i\,En^4\left(-22\,464+En^5\right)}{185\,794\,560}$$

```
Un[En_, m_, nN_, x_] := Module[{n, U},
  U = {1}; AppendTo[U, -En^2/(4 m)]; AppendTo[U, En (4 + En^3 + 8 m)/(32 m (1 + m))]; G = {i En / 2 / m};

  For[n = 3, n < nN, n++,
   AppendTo[U,
    -((-1 + m + n) U[[-2 + n]] + En ((3 - 2 m - 2 n) U[[-1 + n]] + En (-2 + m + n) U[[n]])) /
      (4 n (-2 + m + n) (-1 + m + n))];
  ];
  ({1, i En / 2 / m * x} +
     Sum[{U[[n + 1]] * x^(2 * n), I * (En * U[[n + 1]] - U[[n]]) / 2 / (n + m) * x^(2 * n + 1)},
      {n, 1, nN - 1}]) * x^(-1 + m) // N]
```

```
G = {Re[#], Im[#]} &[Un[3, 5, 150, x]]; Plot[G, {x, 0, 4}, PlotRange → All]
```




```
U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
   n++;
   U = G; G = -Un[En, m, n + 1, X];

  ];
  {Un[En, m, n, X], n}]
```

```
U[9, 5, 0.0001, 1]
```

```
{{-0.0126898, 0.+0.00407705 i}, 14}
```

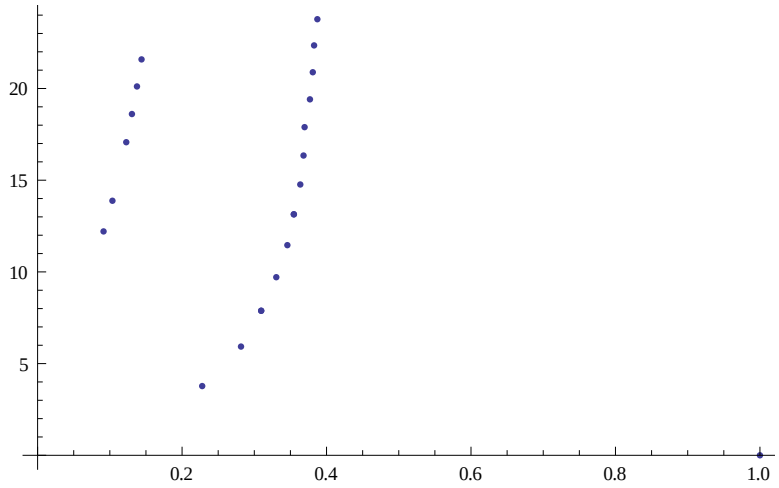
```
Ener[Ene_] :=
 Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 5000;
  m = 5;
  r = 7.2 // N; h = -7.0 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
   k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
   k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
   k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
   k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
   kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   r += h;
   , {n}];

  NN = U[En, m, 0.0001, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn → En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.02,
   En -= Erg / (U2S k⟦1⟧ - U1S k⟦2⟧ + U2 kE⟦1⟧ - U1 kE⟦2⟧);
   Print[{En, Erg / U2 / k[[2]]}]; Goto[begin];
    ];
  {En, Erg / U2 / k[[2]]}
 ]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
 Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[22]
```

```
{21.8924+ 0.168499 i, −0.262337 + 1.54006 i}

{21.7276+ 0.288878 i, 0.0977574+ 1.99468 i}

{21.5078+ 0.261371 i, 1.15369+ 1.40345 i}

{21.6825+ 0.0732421 i, 0.410727+ 0.725268 i}

{21.6223+ 0.137127 i, −1.84205 + 0.583011 i}

{21.5895+ 0.1468 i, −0.181458 − 0.525624 i}

{21.5862+ 0.143899 i, 0.0316819− 0.0363388 i}

    {21.5862+ 0.143899 i, 0.000384958+ 0.000557679 i}
```

```
Energie = {3.77486283903418` + 0.22786873407418717` i,
  5.928479968617718` + 0.2815986526347655` i,
  7.8813588488087065` + 0.30953294412328675` i,
  7.881329304880588` + 0.3095336916562825` i,
  9.71036454189739` + 0.3304415424573178` i,
  11.458077781781169` + 0.3457848857997222` i,
  13.139764183242892` + 0.3546744769440479` i,
  13.139114444715846` + 0.3548172513559829` i,
  14.76566063867228` + 0.3636637795053202` i,
  16.34527139193304` + 0.3681521067888729` i,
  17.892138695416435` + 0.3696213478944415` i,
  19.40831608562361` + 0.37704222997528414` i,
  20.886015743223094` + 0.38097023642873473` i,
  22.34958939297449` + 0.38276309915136164` i,
  23.777494591007947` + 0.3872991704766671` i, 12.207301904640477` + 0.09130244038237889` i,
  13.879319291317909` + 0.10359131716054014` i, 17.071331521063737` + 0.12274708920757649` i,
  18.607651751873114` + 0.1305970820854307` i, 20.11139546248976` + 0.1375815129955693` i,
  21.586240037178456` + 0.14389862151162078` i}; Energie // MatrixForm
```

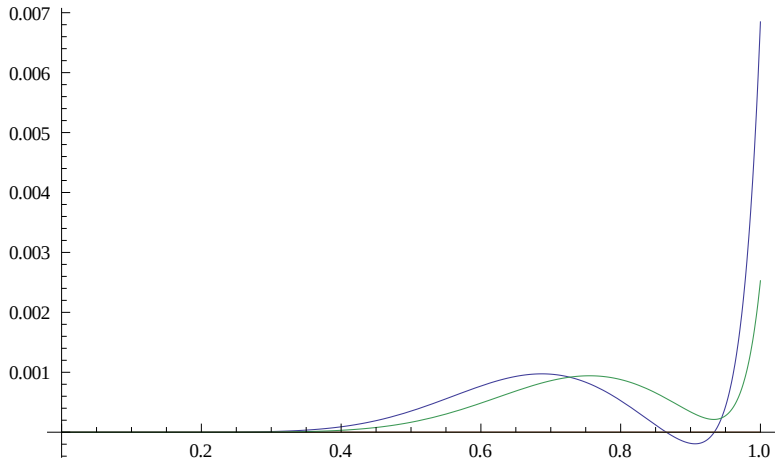
$$\begin{pmatrix} 3.77486+0.227869\,i \\ 5.92848+0.281599\,i \\ 7.88136+0.309533\,i \\ 7.88133+0.309534\,i \\ 9.71036+0.330442\,i \\ 11.4581+0.345785\,i \\ 13.1398+0.354674\,i \\ 13.1391+0.354817\,i \\ 14.7657+0.363664\,i \\ 16.3453+0.368152\,i \\ 17.8921+0.369621\,i \\ 19.4083+0.377042\,i \\ 20.886+0.38097\,i \\ 22.3496+0.382763\,i \\ 23.7775+0.387299\,i \\ 12.2073+0.0913024\,i \\ 13.8793+0.103591\,i \\ 17.0713+0.122747\,i \\ 18.6077+0.130597\,i \\ 20.1114+0.137582\,i \\ 21.5862+0.143899\,i \end{pmatrix}$$

```
ListPlot[Append[{Im[#], Re[#]} & /@ Energie, {1, 0}],
 AxesOrigin → {0, 0}, PlotRange → All]
```



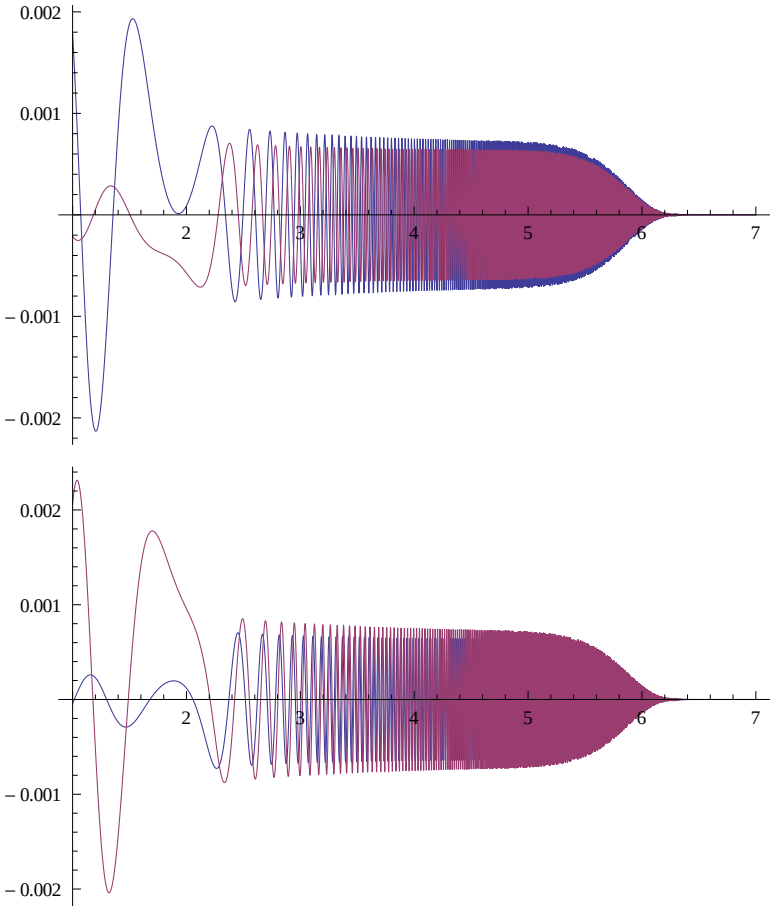

```
:
```

```
G = {Re[#], Im[#]} &[Un[16, 10, 15, x]]; Plot[G, {x, 0, 1}, PlotRange → All]
```



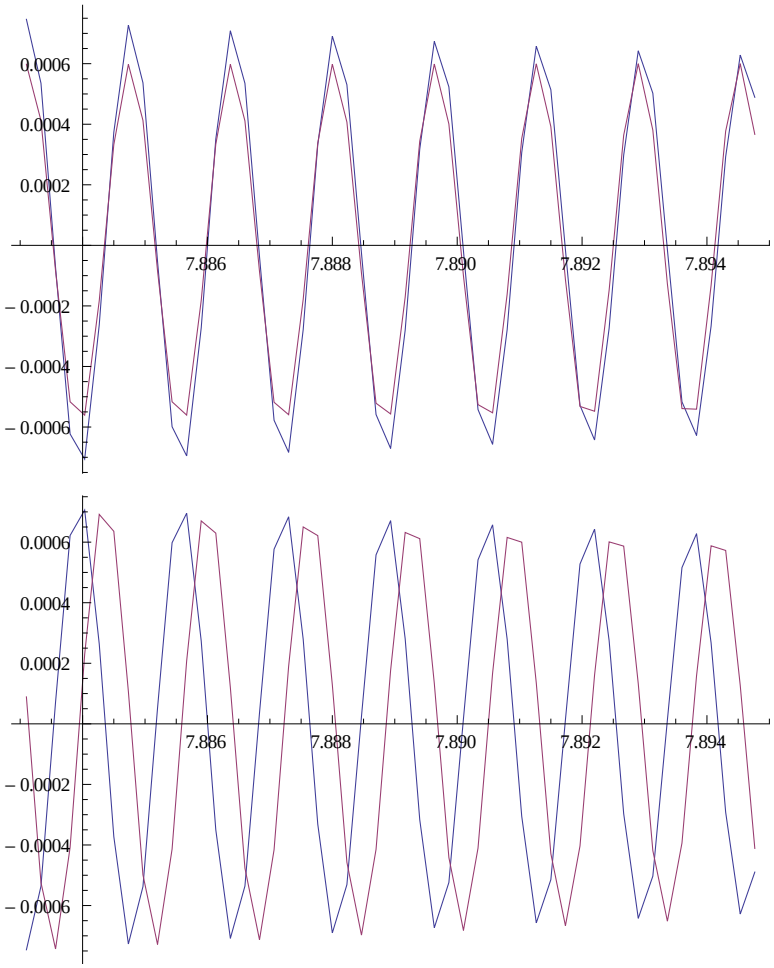
```
n = 8000; S = 1; h = 6 / n; ra = 1; En = Energie[[17]]; m = 5; r = 1;
U[En, m, 10^-10, r][[2]]
k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N },
  { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
En =.;
r =.;
```

27




```
U[En, m, 10^-10, r][[1]]
```

```
S = 29500; n = 50; ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], -0.0006 * Sin[#[[1]]^5 / 5 + 1 - #[[1]] * Re[Energie[[11]]]]} & /@
      kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N },
   { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N }], PlotRange → All, Joined → True]
```




```
Exp[I * Im[Energie[[1]]] * x]
```

$$e^{0.278733\, i\, x}$$