```
Exit[]

a = 0.0072973525376; M = 510998.910;

En[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[En[i]], {i, 4}]

M = {{-k / r, En + m + Z * a / r}, {-(En - m + Z * a / r), k / r}}; M // MatrixForm
```

$$\begin{pmatrix} -\frac{k}{r} & \text{En} + m + \frac{a\,Z}{r} \\ -\text{En} + m - \frac{a\,Z}{r} & \frac{k}{r} \end{pmatrix}$$

```
Expand[(M.M + D[M, r])] // MatrixForm
```

$$\begin{pmatrix} \text{True} + \text{True}^2 + \left(f[-2\,h + x] == f[x] - 2\,h\,f'[x] + 2\,h^2\,f''[x] - \frac{4}{3}\,h^3\,f^{(3)}[x] + \frac{2}{3}\,h^4\,f^{(4)}[x]\right)^2 + \left(f[- \right. \\ \text{True} + \text{True}^2 + \left(f[-2\,h + x] == f[x] - 2\,h\,f'[x] + 2\,h^2\,f''[x] - \frac{4}{3}\,h^3\,f^{(3)}[x] + \frac{2}{3}\,h^4\,f^{(4)}[x]\right)^2 + \left(f[- \right. \\ \text{True}^2 + \left(f[-2\,h + x] == f[x] - 2\,h\,f'[x] + 2\,h^2\,f''[x] - \frac{4}{3}\,h^3\,f^{(3)}[x] + \frac{2}{3}\,h^4\,f^{(4)}[x]\right)^2 + \left(f[-h + x] \right. \\ \text{True} + \text{True}^2 + \left(f[-2\,h + x] == f[x] - 2\,h\,f'[x] + 2\,h^2\,f''[x] - \frac{4}{3}\,h^3\,f^{(3)}[x] + \frac{2}{3}\,h^4\,f^{(4)}[x]\right)^2 + \left(f[- \right. \\ \text{True} + \text{True}^2 + \left(f[-2\,h + x] == f[x] - 2\,h\,f'[x] + 2\,h^2\,f''[x] - \frac{4}{3}\,h^3\,f^{(3)}[x] + \frac{2}{3}\,h^4\,f^{(4)}[x]\right)^2 + \left(f[- \right. \end{pmatrix}$$

```
B = r ^ 2 * D[M, r]
```

$\{\{k, -a\,Z\}, \{a\,Z, -k\}\}$

```
S = Eigensystem[B, -2][[2]];
v = Transpose[{S[[2]], S[[1]]}] * a * Z; v // MatrixForm
```

$$\begin{pmatrix} k + \sqrt{k^2 - a^2\,Z^2} & k - \sqrt{k^2 - a^2\,Z^2} \\ a\,Z & a\,Z \end{pmatrix}$$

```
A = Transpose[{{Z * a, (k - Sqrt[k^2 - a^2 Z^2])}, {(k - Sqrt[k^2 - a^2 Z^2]), Z * a}}]; A // MatrixForm
```

$$\begin{pmatrix} a\,Z & k - \sqrt{k^2 - a^2\,Z^2} \\ k - \sqrt{k^2 - a^2\,Z^2} & a\,Z \end{pmatrix}$$

```
Simplify[v.{u1, u2}]
```

$\left\{k\,(u1 + u2) + (u1 - u2)\,\sqrt{k^2 - a^2\,Z^2},\ a\,(u1 + u2)\,Z\right\}$

```
Simplify[Inverse[v].B.v]
```

$\left\{\left\{\sqrt{k^2 - a^2\,Z^2},\ 0\right\},\ \left\{0,\ -\sqrt{k^2 - a^2\,Z^2}\right\}\right\}$

```
Expand[(M.M)[[1, 1]]]
```

$$-\text{En}^2 + m^2 + \frac{k^2}{r^2} - \frac{2\,a\,\text{En}\,Z}{r} - \frac{a^2\,Z^2}{r^2}$$

```
Exit[]
```

```
$Assumptions = k^2 > Z^2 * a^2 && Z > 0 && a > 0
```

$k^2 > a^2 Z^2$ && $Z > 0$ && $a > 0$

```
M = {{-(m + 1) / r - b * r, En - V}, {-En + V, m / r + b * r}};
```