```
Exit[]
```

```
f[x_] := a x^3 + b x^2 + c x + d
```

```
erg = Solve[{f[0] == bounds[1], f[nPoints + 1] == bounds[2],
   f'[nPoints + 1] / f'[0] == args[3],}, {a, b, c, d}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{\left\{b \to -\frac{a\,(1+\text{nPoints})\,(2+\text{args}[3])}{1+\text{args}[3]} - \frac{(-1+\text{args}[3])\,(\text{bounds}[1]-\text{bounds}[2])}{(1+\text{nPoints})^2\,(1+\text{args}[3])},\right.\right.$$

$$\left.\left. c \to -\frac{a\left(-1-2\,\text{nPoints}-\text{nPoints}^2\right)}{1+\text{args}[3]} - \frac{2\,(\text{bounds}[1]-\text{bounds}[2])}{(1+\text{nPoints})\,(1+\text{args}[3])},\ d \to \text{bounds}[1]\right\}\right\}$$

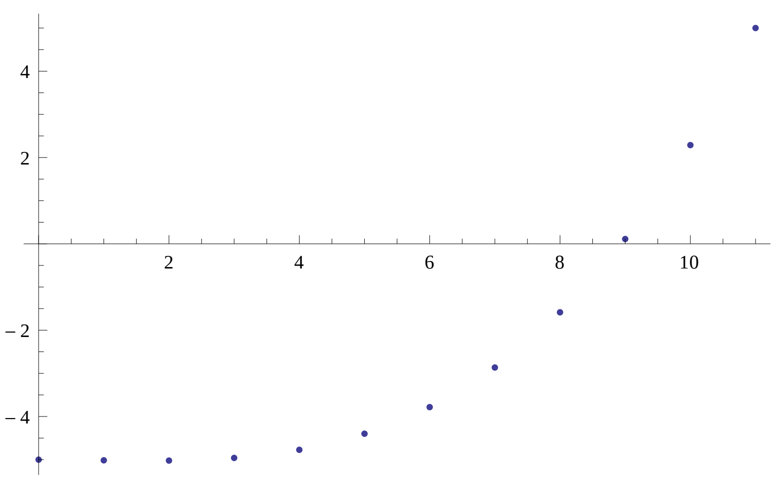
```
f[nPoints + 1] / f[0]
```

$$\frac{d + c\,(1+\text{nPoints}) + b\,(1+\text{nPoints})^2 + a\,(1+\text{nPoints})^3}{d}$$

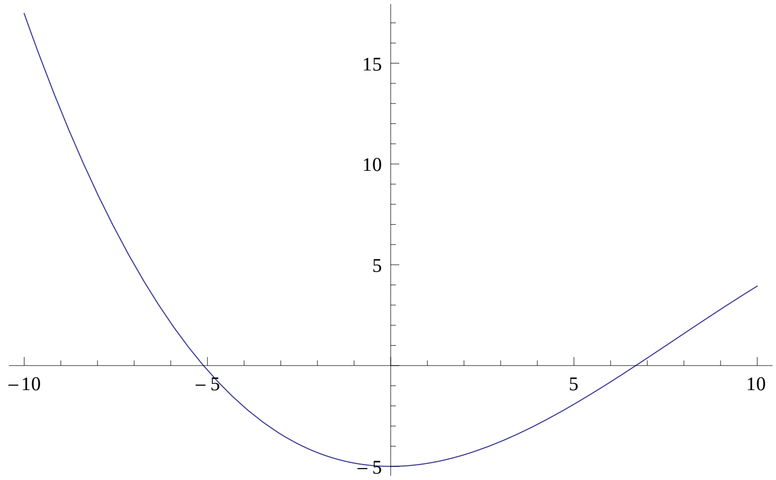
```
FortranForm[erg]
```

```
        List(Rule(a,-((-(((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*(args(2) - args(4)))
     -         (-2*args(1) + 2*args(3))*((1 + nPoints)*args(2) + bounds(1) - bounds(2)))/
     -       (((1 + nPoints)**3 - 3*(1 + nPoints)*args(1)**2)*(-2*args(1) + 2*args(3)) -
     -         ((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*(-3*args(1)**2 + 3*args(3)**2))
     -  Rule(b,-(-args(2) + args(4))/(2.*(args(1) - args(3))) +
     -    ((-3*args(1)**2 + 3*args(3)**2)*(-(((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*
     -          (-2*args(1) + 2*args(3))*((1 + nPoints)*args(2) + bounds(1) - bounds(2))))/
     -     ((-2*args(1) + 2*args(3))*(((1 + nPoints)**3 - 3*(1 + nPoints)*args(1)**2)*(-2*
     -          ((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*(-3*args(1)**2 + 3*args(3)**2)
     -  Rule(c,-((args(2)*args(3) - args(1)*args(4))/(args(1) - args(3))) -
     -    (3*args(1)*args(3)*(-(((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*(args(2) - ar
     -          (-2*args(1) + 2*args(3))*((1 + nPoints)*args(2) + bounds(1) - bounds(2))))/
     -     (((1 + nPoints)**3 - 3*(1 + nPoints)*args(1)**2)*(-2*args(1) + 2*args(3)) -
     -        ((1 + nPoints)**2 - 2*(1 + nPoints)*args(1))*(-3*args(1)**2 + 3*args(3)**2)))
```

```
nPoints = 10; args[1] = 0; args[2] = 0; args[3] = 11; args[4] = 3; bounds[1] = -5;
bounds[2] = 5; ListPlot[Table[{x, f[x] /. erg}, {x, 0, nPoints + 1}], PlotRange → All]
```



```
Plot[f[x] /. erg, {x, -10, 10}]
```




```
Table[f[x] /. erg, {x, 0, nPoints + 1}] // N
```

{−5., −5.01503, −5.02104, −4.95943, −4.7716,
−4.39895, −3.78287, −2.86476, −1.58603, 0.111946, 2.28775, 5.}

```
Differences[%]
```

{−0.0150263, −0.00601052, 0.0616078, 0.187829,
0.372652, 0.616078, 0.918107, 1.27874, 1.69797, 2.17581, 2.71225}

```
Exit[]

f[x_] := a x ^ 2 + b x + c

nPoints = 10; args[1] = -5; args[2] = 0; args[3] = 0.01; bounds[1] = -5; bounds[2] = 5;

erg1 = Solve[{f '[args[1]] == args[2], f '[1] == args[3]}, {a, b}][[1]]
```

{a → 0.000833333, b → 0.00833333}

```
ii = Integrate[1 / f[x] /. erg1, {x, bounds[1], bounds[2]}]
```

$$\text{ConditionalExpression}\left[\frac{2.\ \text{ArcTan}\left[\frac{0.0166667}{\sqrt{-0.0000694444+0.00333333\ c}}\right]}{\sqrt{-0.0000694444+0.00333333\ c}},\right.$$
$$\left(\text{Re}\left[\sqrt{0.0000694444-0.00333333\ c}\right]\geq 0.0166667\ ||\ c>0.0208333\ ||\right.$$
$$\left.\left(\sqrt{1.-48.\ c}\neq 0\ \&\&\ \sqrt{0.0000694444-0.00333333\ c}\notin \text{Reals}\right)\right)\ \&\&$$
$$\left.\sqrt{0.0000694444-0.00333333\ c}\neq 0\right]$$

```
erg2 = Simplify[NSolve[ii == nPoints + 1, c, Reals]][[1]]
```

NSolve::ratnz :
NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

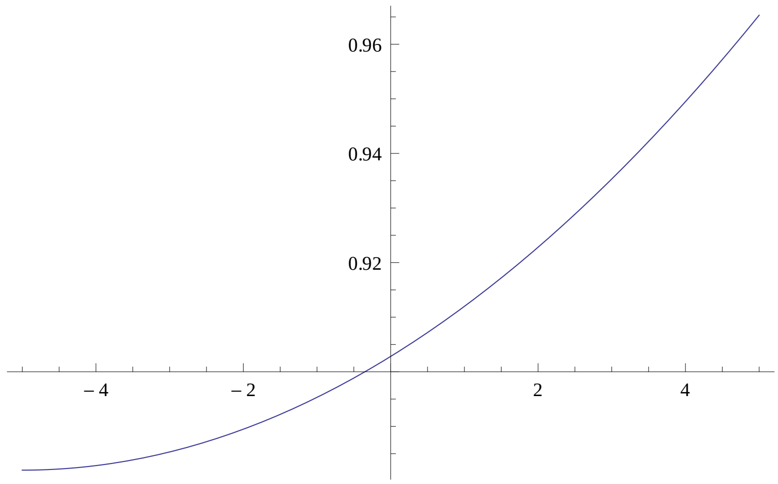
{c → 0.902814}

```
v = {bounds[1]}; For[i = 1, i ≤ nPoints + 1, i++,
 AppendTo[v, v[[i]] + f[v[[i]]] /. erg1 /. erg2];
]
```

```
Integrate[1 / f[x] /. erg1 /. erg2, {x, bounds[1], bounds[2]}]
```

```
11.
```

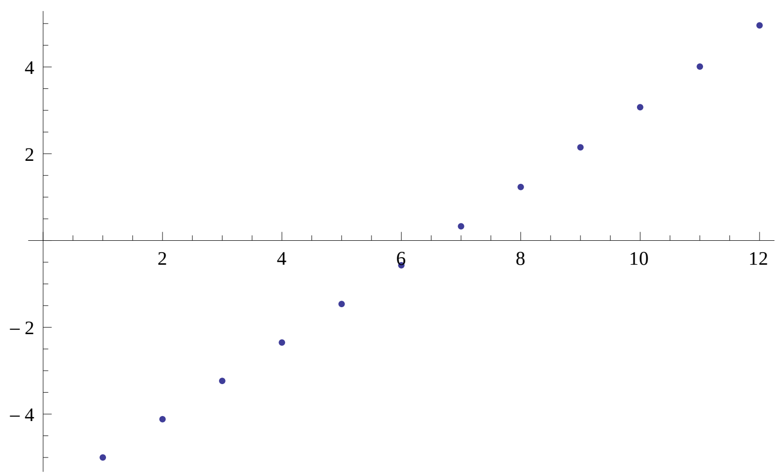
```
Plot[f[v] /. erg1 /. erg2, {v, bounds[1], bounds[2]}]
```



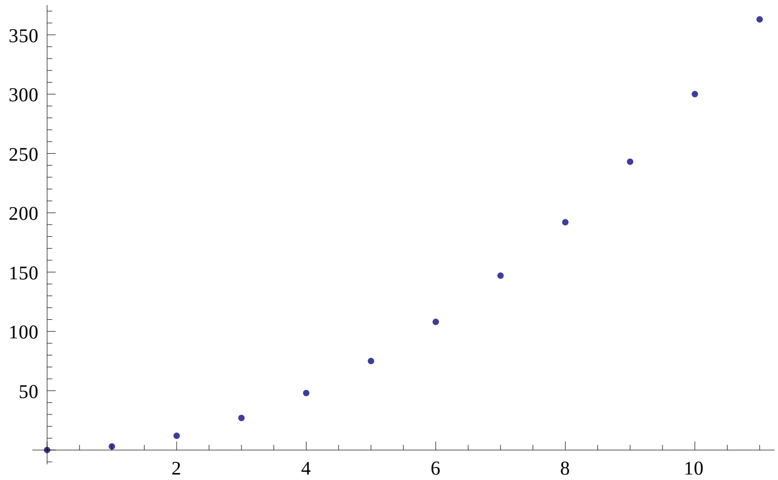

```
v
```

```
{−5, −4.11802, −3.23539, −2.35082, −1.46299,
 −0.570581, 0.327749, 1.23338, 2.14774, 3.0723, 4.00858, 4.95819}
```

```
ListPlot[v]
```



```
ListPlot[Table[{x, f[x] /. erg1 /. erg2}, {x, 0, nPoints + 1}], PlotRange → All]
```




```
Integrate[1 / f[x], {x, -5, 5}] /. er
```

$Aborted

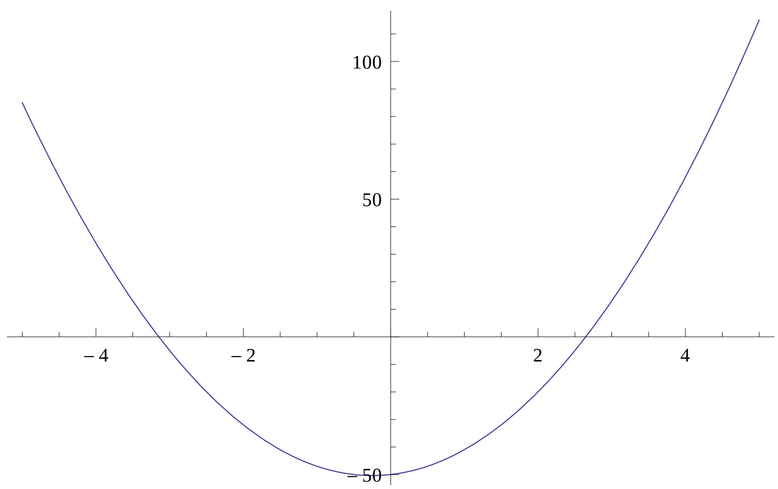
```
Integrate[1 / f[x], {x, -5, 5}]
```

$Aborted

```
1 / f[x] /. er
```

$$\frac{1}{-\frac{4499}{90} + 3\,x + 6\,x^2}$$

```
Plot[f[x] /. er, {x, -5, 5}]
```



```
Integrate[1 / f[x], {x, , 5}]
```

Integrate::idiv : Integral of $\frac{1}{-\frac{4499}{90}+3\,x+6\,x^2}$ does not converge on $\{-5, 5\}$. ≫

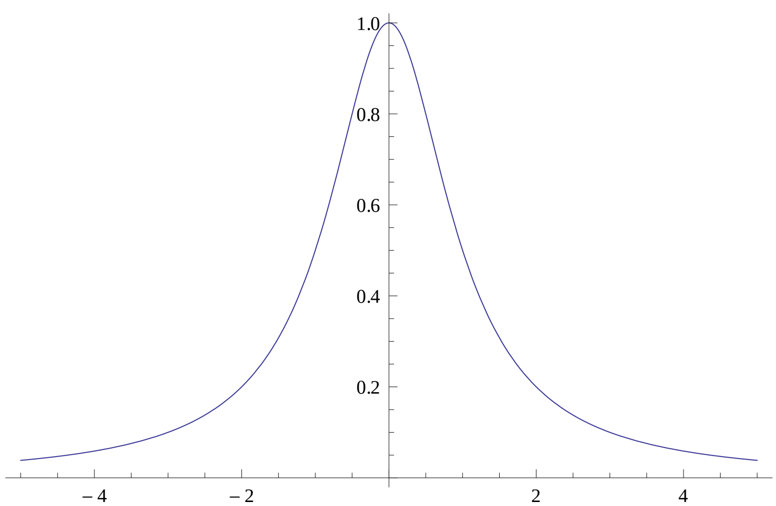
$$\int_{-5}^{5}\frac{1}{-\frac{4499}{90}+3\,x+6\,x^2}\,dx$$

```
Exit[];
```

```
Simplify[Integrate[1 / f[x], {x, bounds[1], bounds[2]}]]
```

$$\text{ConditionalExpression}\Big[-\frac{2\left(\text{ArcTan}\left[\frac{b+2\,a\,\text{bounds}[1]}{\sqrt{-b^2+4\,a\,c}}\right]-\text{ArcTan}\left[\frac{b+2\,a\,\text{bounds}[2]}{\sqrt{-b^2+4\,a\,c}}\right]\right)}{\sqrt{-b^2+4\,a\,c}},$$
$$(\text{Im}[\text{bounds}[2]]\,\text{Re}[\text{bounds}[1]]-\text{Im}[\text{bounds}[1]]\,\text{Re}[\text{bounds}[2]])^2\Big/(\text{Re}[\text{bounds}[1]]-\text{Re}[\text{bounds}[2]])^2\le 1\;\&\&\;\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\notin\text{Reals}\;||\;\text{Re}\left[\frac{b-\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\right]\ge 1\;||\;\text{Re}\left[\frac{b-\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\right]\le 0\right)\;\&\&$$
$$\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\notin\text{Reals}\;||\;\text{Re}\left[\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\right]\ge 1\;||\;\text{Re}\left[\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,\text{bounds}[1]}{2\,a\,\text{bounds}[1]-2\,a\,\text{bounds}[2]}\right]\le 0\right)\Big]$$

```
Plot[1 / (1 + x ^ 2), {x, -5, 5}]
```



```
$Assumptions = a ∈ Reals && b ∈ Reals
```

```
a Reals ∈ && b Reals ∈
```

```
Integrate[1 / (x ^ 2 + 1), {x, a, b}]
```

```
ConditionalExpression[ArcCot[a] - ArcCot[b], 0 < a < b]
```