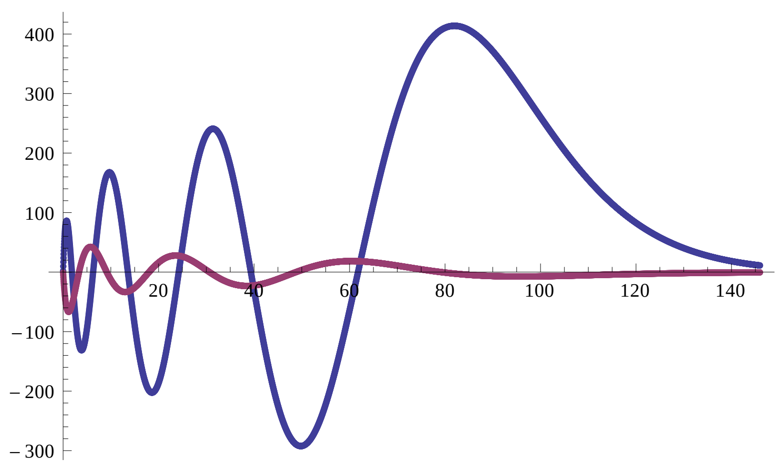
```
Exit[]

a = 7297352537.6 * 10 ^ -12; M = 510998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

'

f[u_, r_] := Simplify[{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 5000;
h = 20000 / n;
Enn = 0.27766906844567757` / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do[
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[U, {r, u}], {n}]; x =.;
ListPlot[Table[{#[[1]] / 137, 137 ^ (i - 2) * #[[2, i]]} & /@ U[[1 ;; n]], {i, 2}] // N,
 PlotRange → All]
```



```
lambda[n_] := Sqrt[1 - (1 - En[n] / M) ^ 2];
gamma := Sqrt[k ^ 2 - Z ^ 2 * a ^ 2];
f[r_, n_, k_] := -r ^ gamma * Exp[-lambda[n] * r] * (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] +
    (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
g[r_, n_, k_] := Sqrt[2 * M / En[n] - 1] * r ^ gamma * Exp[-lambda[n] * r] *
  (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] -
    (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
```

```
g[1, 1, -1] / f[1, 1, -1]
```

-274.068

Randbedingungen

r < < 1

```
Exit[]
```

```
a = 7297352537.6 * 10 ^ -12; M = 510998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

```
f[u_, r_] := Simplify[{(Z * a / r - En) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

$L = \sqrt{2\,\mathrm{En} - \mathrm{En}^2}$;

```
u := x ^ (s + n) * {a[n], b[n]} * Exp[-x * L];
```

```
g1 = Collect[Simplify[(f[u, x] - D[u, x]) / x ^ (s - 1) / Exp[-x * L]], x]; g1 // MatrixForm
```

$$\begin{pmatrix} x^{1+n} \left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\; a[n] + 2\, b[n] - \mathrm{En}\; b[n]\right) + x^n \left(-k\, a[n] - n\, a[n] - s\, a[n] + a\, Z\, b[n]\right) \\ x^{1+n} \left(\mathrm{En}\; a[n] + \sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\; b[n]\right) + x^n \left(-a\, Z\, a[n] + k\, b[n] - n\, b[n] - s\, b[n]\right) \end{pmatrix}$$

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\begin{pmatrix}
-k\,a[0]-s\,a[0]+a\,Z\,b[0] & -a\,Z\,a[0]+(k-s)\,b[0] \\
\sqrt{-(-2+En)\,En}\;a[0]-(1+k+s)\,a[1]+2\,b[0]-En\,b[0]+a\,Z\,b[1] & En\,a[0]-a\,Z\,a[1]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[1]-(2+k+s)\,a[2]+2\,b[1]-En\,b[1]+a\,Z\,b[2] & En\,a[1]-a\,Z\,a[2]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[2]-(3+k+s)\,a[3]+2\,b[2]-En\,b[2]+a\,Z\,b[3] & En\,a[2]-a\,Z\,a[3]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[3]-(4+k+s)\,a[4]+2\,b[3]-En\,b[3]+a\,Z\,b[4] & En\,a[3]-a\,Z\,a[4]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[4]-(5+k+s)\,a[5]+2\,b[4]-En\,b[4]+a\,Z\,b[5] & En\,a[4]-a\,Z\,a[5]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[5]-(6+k+s)\,a[6]+2\,b[5]-En\,b[5]+a\,Z\,b[6] & En\,a[5]-a\,Z\,a[6]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[6]-(7+k+s)\,a[7]+2\,b[6]-En\,b[6]+a\,Z\,b[7] & En\,a[6]-a\,Z\,a[7]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[7]-(8+k+s)\,a[8]+2\,b[7]-En\,b[7]+a\,Z\,b[8] & En\,a[7]-a\,Z\,a[8]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[8]-(9+k+s)\,a[9]+2\,b[8]-En\,b[8]+a\,Z\,b[9] & En\,a[8]-a\,Z\,a[9]+\sqrt{-} \\
\sqrt{-(-2+En)\,En}\;a[9]-(10+k+s)\,a[10]+2\,b[9]-En\,b[9]+a\,Z\,b[10] & En\,a[9]-a\,Z\,a[10]+\sqrt{\cdot}
\end{pmatrix}$$

```
Det[{{-s - k, a Z}, {-a Z, -s + k}}]
```

$$-k^2+s^2+a^2\,Z^2$$

```
g4 = Simplify[Inverse[{{a, -(n + s + k)}, {-(n + s - k), -a}}].{{En - 2, -L}, {-L, -En}}]
```

$$\left\{\left\{\frac{a\,(-2+En)+L\,(k+n+s)}{a^2-k^2+(n+s)^2},\ \frac{-a\,L+En\,(k+n+s)}{a^2-k^2+(n+s)^2}\right\},\right.$$
$$\left.\left\{\frac{(-2+En)\,k+a\,L-(-2+En)\,(n+s)}{a^2-k^2+(n+s)^2},\ \frac{a\,En+L\,(-k+n+s)}{a^2-k^2+(n+s)^2}\right\}\right\}$$

$$\mathbf{L=\sqrt{2\,En-En^2}\ ;\ L=.}$$

```
Simplify[Eigenvectors[g4]]
```

$$\left\{\left\{\left(-a+k\,L-\frac{1}{2}\sqrt{\left(4\,(a\,(-1+En)+L\,(n+s))^2-4\left(-2\,En+En^2+L^2\right)\left(a^2-k^2+(n+s)^2\right)\right)}\right)\Big/\right.\right.$$
$$\left.((-2+En)\,k+a\,L-(-2+En)\,(n+s)),\ 1\right\},$$
$$\left\{\left(-a+k\,L+\frac{1}{2}\sqrt{\left(4\,(a\,(-1+En)+L\,(n+s))^2-4\left(-2\,En+En^2+L^2\right)\left(a^2-k^2+(n+s)^2\right)\right)}\right)\Big/\right.$$
$$\left.\left.((-2+En)\,k+a\,L-(-2+En)\,(n+s)),\ 1\right\}\right\}$$

$$\mathbf{g5=D\left[\left(-a+k\,L+\frac{1}{2}\sqrt{\left(4\,(a\,(-1+En)+L\,(n+s))^2-4\left(-2\,En+En^2+L^2\right)\left(a^2-k^2+(n+s)^2\right)\right)}\right)\Big/\right.}$$
$$\mathbf{\left.((-2+En)\,k+a\,L-(-2+En)\,(n+s)),\ n\right];}$$

Solve[g5 == 0, L]

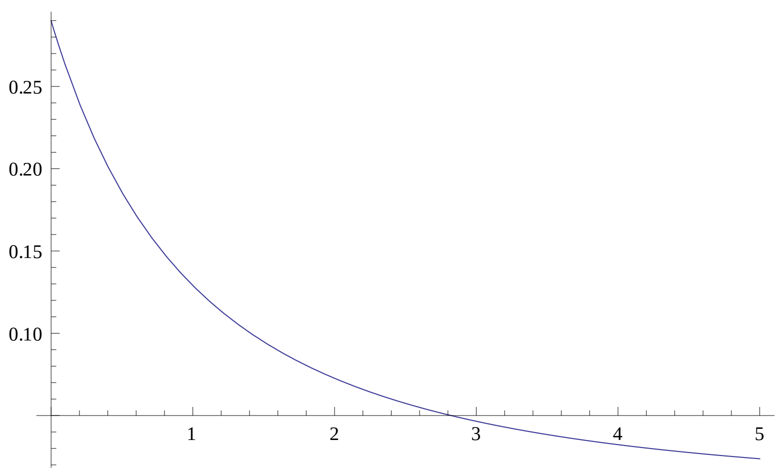
Solve::verif : Potential solution {L → ComplexInfinity} (possibly discarded by verifier) should be checked by hand. May require use of limits. ≫

$$\left\{\left\{L \to -\frac{\sqrt{2\,a^2\,En - a^2\,En^2 - 4\,En^2\,k^2 + 4\,En^3\,k^2 - En^4\,k^2}}{\sqrt{a^2 - 2\,En\,k^2 + En^2\,k^2}}\right\}, \left\{L \to \frac{\sqrt{2\,a^2\,En - a^2\,En^2 - 4\,En^2\,k^2 + 4\,En^3\,k^2 - En^4\,k^2}}{\sqrt{a^2 - 2\,En\,k^2 + En^2\,k^2}}\right\}\right\}$$

$$-\frac{\sqrt{2\,a^2\,En - a^2\,En^2 - 4\,En^2\,k^2 + 4\,En^3\,k^2 - En^4\,k^2}}{\sqrt{a^2 - 2\,En\,k^2 + En^2\,k^2}} + \sqrt{2\,En - En^2}$$

$$\sqrt{2\,En - En^2} - \frac{\sqrt{2\,a^2\,En - a^2\,En^2 - 4\,En^2\,k^2 + 4\,En^3\,k^2 - En^4\,k^2}}{\sqrt{a^2 - 2\,En\,k^2 + En^2\,k^2}}$$

$$\textbf{Plot}\left[g5\ /.\ L \to \frac{\sqrt{2\,a^2\,En - a^2\,En^2 - 4\,En^2\,k^2 + 4\,En^3\,k^2 - En^4\,k^2}}{\sqrt{a^2 - 2\,En\,k^2 + En^2\,k^2}}\ /.\ s \to \text{Sqrt}[k\,\hat{}\,2 - (Z * a)\,\hat{}\,2]\ /.\ Z \to 1\ /.\ En \to 0.5\ /.\ a \to 1/137\ /.\ k \to -1,\ \{n, 0, 5\}\right]$$



$$\textbf{Limit}\left[n\left(-a + a\,En + k\,L - \frac{1}{2}\sqrt{\left(4\,(a\,(-1 + En) + k\,L)^2 - 4\,\left(-2\,En + En^2 - L^2\right)\,\left(a^2 - k^2 + (n + s)^2\right)\right)}\right) \Big/ \left(a^2 - k^2 + (n + s)^2\right),\ \{n \to \text{Infinity}\}\right]$$

{0}

```
Expand[(n + s)^2 - s^2]
```

$n^2 + 2\,n\,s$

```
(a^2 - k^2 + (n + s)^2) * Simplify[
  Inverse[{{a / (En - 2), (n + s + k) / (2 - En)}, {(n + s - k) / En, a / En}}]] // MatrixForm
```

$$\begin{pmatrix} a\,(-2+En) & En\,(k+n+s) \\ (-2+En)\,(k-n-s) & a\,En \end{pmatrix}$$

```
s = Sqrt[k^2 - (Z * a)^2];
```

```
En =.;
Simplify[Inverse[{{Z * a / En, (n + s - k) / En}, {(n + s + k) / (2 - En), Z * a / (En - 2)}}]]
```

$$S[n\_, En\_] := \left\{\left\{\frac{a\, En\, Z}{n^2 + 2 n \sqrt{k^2 - a^2 Z^2}}, -\frac{(-2 + En) \left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{En \left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}, \frac{a (-2 + En) Z}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

$$DS[n\_, En\_] := \left\{\left\{\frac{a\, Z}{n^2 + 2 n \sqrt{k^2 - a^2 Z^2}}, -\frac{\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}, \frac{a\, Z}{n \left(n + 2 \sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

```
S[m, En] // MatrixForm
DS[m, En] // MatrixForm
```

$$\left\{\left\{\frac{a\, En\, Z}{-k^2 + n^2 + 2 n s + s^2 + a^2 Z^2}, \frac{(-2 + En) (k - n - s)}{-k^2 + n^2 + 2 n s + s^2 + a^2 Z^2}\right\}, \left\{\frac{En (k + n + s)}{-k^2 + n^2 + 2 n s + s^2 + a^2 Z^2}, \frac{a (-2 + En) Z}{-k^2 + n^2 + 2 n s + s^2 + a^2 Z^2}\right\}\right\}$$

$$\begin{pmatrix} \frac{a\, En\, Z}{m^2 + 2 m \sqrt{k^2 - a^2 Z^2}} & -\frac{(-2 + En) \left(-k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{En \left(k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} & \frac{a (-2 + En) Z}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

$$\begin{pmatrix} \frac{a\, Z}{m^2 + 2 m \sqrt{k^2 - a^2 Z^2}} & -\frac{-k + m + \sqrt{k^2 - a^2 Z^2}}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{k + m + \sqrt{k^2 - a^2 Z^2}}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} & \frac{a\, Z}{m \left(m + 2 \sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

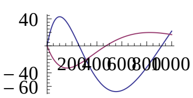
```
S[n, En].u
```

$$S[n, En].\{x^{n+s}\, a[n], x^{n+s}\, b[n]\}$$

```
UN[R_, N_, En_] := Module[{u = {1, (k + s) / Z / a}, U = {1, (k + s) / Z / a} * R ^ s},
  For[n = 1, n < N, n++,
   u = S[n, En].u;
   U += u * R ^ (s + n);
  ];

  U]
```

```
U[r_, g_, En_] :=
 Module[{u = {1, (k + s) / Z / a}, U = {0, 0}, DU = {0, 0}, du = {0, 0}, n = 0},
  Label[begin];
  U += u * r ^ (s + n);
  DU += du * r ^ (s + n);
  n++;
  du = DS[n, En].u + S[n, En].du;
  u = S[n, En].u;
  If[(#[[1]] > g  || #[[2]] > g) &[Abs[u * R ^ (s + n) / U /. r → R]], Goto[begin]];
  {n, U, DU}]
```

```
R = 1000; g = 0.01; rU = U[r, g, 1 / M];
Plot[{rU[[2, 1]], rU[[2, 2]] * 137}, {r, 0, R}, PlotRange → All]
```




```
EN[iEn_, g2_] := Module[{rU, fU, n = 0, i, En = iEn, ll},
  Label[begin];
  fU = U[r, g, En];

  rU = fU /. r → R;

  If[rU[[2, 1]] * rU[[2, 2]] > 0,

   En -= (rU[[2, 1]] + rU[[2, 2]]) / (rU[[3, 1]] + rU[[3, 2]]);

   n++
    Goto[begin];
  ];

  {n, En * M, Abs[rU[[2, 1]] - rU[[2, 2]]]}
 ]
```

```
R = 3000; g = 0.001; EN[13 / M, 0.1]
```

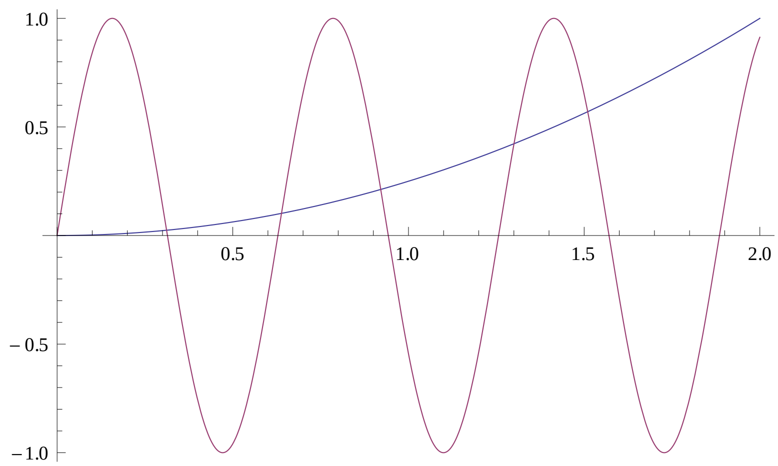
{8, 13.6059, 0.000441718}

```
- {0, 13.605873075061169
```

```
R = 2000; g = 0.001;

plot[{rU[[2, 1]], rU[[3, 1]]}, 100, R]

En = 4 / M; rU[[2, 1]]

129.072

n = 0; x

10

n = 0; While[x = n; x < 10, n++; Print[n]]

1
2
3
4
5
6
7
8
9
10
```

```
plot[liste_, R_] := Module[{nN = 100, table, max, st = {Red, Green, Blue}},
  liste / (Max[Abs[#]] & /@ (Table[# /. r → i * R / nN, {i, 0, nN}] & /@ liste))
 ]
ll = plot[{r ^ 2, Sin[10 * r]}, 2]
Plot[ll, {r, 0, 2}]
```

$\left\{\frac{r^2}{4}, -\mathrm{Csc}[11]\ \mathrm{Sin}[10\ r]\right\}$



r gegen Inifinity

```
Exit[]
```

```
a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

```
$Assumptions = 1 > En > 0;
```

```
s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En]; L =.; s =.
```

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - En) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

```
u =.
```

```
U = {(hb[r] - ha[r]) * L / En, hb[r] + ha[r]} * Exp[-r * L];
```

```
#[[2]] & /@ Simplify[Solve[Simplify[(f[U, r] - D[U, r]) / Exp[-r * L] * En * r] == 0,
      {ha'[r], hb'[r]}][[1]]] /. ha[r] → u[[1]] /. hb[r] → u[[2]] /. r → rr
```

Part::partd : Part specification u⟦1⟧ is longer than depth of object. ≫

Part::partd : Part specification u⟦2⟧ is longer than depth of object. ≫

$$\Big\{\frac{1}{2\,\mathrm{En}\,L\,rr}\Big(\big(\mathrm{En}^3\,rr+\mathrm{En}\,L^2\,rr+a\,L^2\,Z-\mathrm{En}^2\,(2\,rr+a\,Z)\big)\,u[\![1]\!]+\big(\mathrm{En}^3\,rr+\mathrm{En}\,L\,(2\,k+L\,rr)-a\,L^2\,Z-\mathrm{En}^2\,(2\,rr+a\,Z)\big)\,u[\![2]\!]\Big),$$
$$\frac{1}{2\,\mathrm{En}\,L\,rr}\Big(\big(-\mathrm{En}^3\,rr+\mathrm{En}\,L\,(2\,k-L\,rr)+a\,L^2\,Z+\mathrm{En}^2\,(2\,rr+a\,Z)\big)\,u[\![1]\!]+\big(-\mathrm{En}^3\,rr+3\,\mathrm{En}\,L^2\,rr-a\,L^2\,Z+\mathrm{En}^2\,(2\,rr+a\,Z)\big)\,u[\![2]\!]\Big)\Big\}$$

$$\mathbf{F[u\_,\,rr\_]:=Simplify}\Big[\Big\{\frac{-a\,(-1+\mathrm{En})\,Z\,u[\![1]\!]+\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;k-a\,Z\right)u[\![2]\!]}{\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;rr},$$
$$\left(\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;k+a\,Z\right)u[\![1]\!]+\left(4\,\mathrm{En}\,rr-2\,\mathrm{En}^2\,rr-a\,Z+a\,\mathrm{En}\,Z\right)u[\![2]\!]\right)\Big/\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;rr\right)\Big\}\Big]$$

```
r[x_] := 1 / x; u := {a[n], b[n]} * x ^ (n + s)
```

```
g1 = Collect[Simplify[(F[u, r[x]] * D[r[x], x] - D[u, x]) * x ^ (2 - s)], {a[n], b[n]}]; g1
```

$$\Big\{-\left(x^{1+n}\left(a\,(-1+\mathrm{En})\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;Z+L\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;n+a\,Z-a\,\mathrm{En}\,Z\right)\right)a[n]\right)\Big/\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;L\right)-\frac{x^{1+n}\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;k-a\,Z\right)b[n]}{\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}},$$
$$-\frac{x^{1+n}\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;k+a\,Z\right)a[n]}{\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}}-\frac{1}{\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;L}x^n\Big(-2\,\mathrm{En}^2\,L+x\left(\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;L\,n-a\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;Z-a\,L\,Z\right)+\mathrm{En}\left(4\,L+a\sqrt{-(-2+\mathrm{En})\,\mathrm{En}}\;x\,Z+a\,L\,x\,Z\right)\Big)b[n]\Big\}$$

$$\mathbf{g2:=}\Big\{x^n\,(-n)\,a[n]+x^n\,(-k+a\,Z/L)\,b[n],\; -x^{1+n}\,(k+a\,Z/L)\,a[n]+x^n\,(-n\,x-2\,s\,x+-2*L)\,b[n]\Big\};\ \mathbf{g2\ //\ MatrixForm}$$

$$\begin{pmatrix} -n\,x^n\,a[n]+x^n\left(-k+\frac{a\,Z}{L}\right)b[n] \\ -x^{1+n}\left(k+\frac{a\,Z}{L}\right)a[n]+x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n] \end{pmatrix}$$

```
a[n_] := (Z * a / L - k) / n * b[n]
```

```
g2[[2]]
```

$$x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n]-\frac{x^{1+n}\left(-k+\frac{a\,Z}{L}\right)\left(k+\frac{a\,Z}{L}\right)b[n]}{n}$$

```
M1 = {{L, 2 - En}, {En, L}}; Eigenvalues[M1]; B = Transpose[Eigenvectors[M1]];
```

```
$Assumptions = 1 > En > 0
```

$1 > En > 0$

= >

```
Exit[]
a[n_] := (Z * a / L - k) / n * b[n]
b[n_] := b0 * Product[((k ^ 2 - a ^ 2 * Z ^ 2 / L ^ 2) / i - (i + 2 * s)) / 2 / L, {i, 1, n}]
b[4]
```

$$\frac{1}{16\,L^4}\,b0\left(-1+k^2-2\,s-\frac{a^2\,Z^2}{L^2}\right)\left(-4-2\,s+\frac{1}{4}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)\left(-3-2\,s+\frac{1}{3}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)\left(-2-2\,s+\frac{1}{2}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)$$

```
((k ^ 2 - a ^ 2 * Z ^ 2 / L ^ 2) / i - (i + 2 * s)) / 2 / L
```

$$\frac{-i-2\,s+\frac{k^2-\frac{a^2\,Z^2}{L^2}}{i}}{2\,L}$$

```
Exit[];
En =.;
```

$$\text{Solve}\left[-i*\sqrt{(2-En)\,En}-2\,a\,(-1+En)+\frac{\sqrt{(2-En)\,En}-\frac{a^2}{\sqrt{(2-En)\,En}}}{i}==0,\ En\right]\ //\ \text{MatrixForm}$$

$$\begin{pmatrix}
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4-\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2-2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4+\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2-2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4-\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2+2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4+\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2+2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}
\end{pmatrix}$$

'

$$\text{ET}[\text{i\_}, \text{k\_}, \text{Z\_}, \text{a\_}] := \left(2\, i^4 - 4\, i^2\, k^2 + 2\, k^4 + 8\, a^2\, i^2\, Z^2 - \sqrt{\left(\left(-2\, i^4 + 4\, i^2\, k^2 - 2\, k^4 - 8\, a^2\, i^2\, Z^2\right)^2 - 4\left(i^4 - 2\, i^2\, k^2 + k^4 + 4\, a^2\, i^2\, Z^2\right)\left(a^2\, i^2\, Z^2 + a^2\, k^2\, Z^2 + 2\sqrt{a^4\, i^2\, k^2\, Z^4 - a^6\, i^2\, Z^6}\right)\right)}\right) \Big/ \left(2\left(i^4 - 2\, i^2\, k^2 + k^4 + 4\, a^2\, i^2\, Z^2\right)\right)$$

Table[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 10}]

$\{-5.67315\times10^{-11}, -1.9084\times10^{-7}, 5.30991\times10^{-11}, -1.3482\times10^{-10}, 3.24187\times10^{-11}, -5.86517\times10^{-11}, 5.5719\times10^{-11}, 2.10913\times10^{-11}, 1.87879\times10^{-11}, 1.01339\times10^{-10}, -1.46405\times10^{-12}\}$

Series[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 5}]

$-5.67315\times10^{-11} - 1.26477\times10^{-10}\, n^2 - 2.84217\times10^{-14}\, n^3 - 2.54019\times10^{-10}\, n^4 + 1.42109\times10^{-14}\, n^5 + O[n]^6$

M = 510 998.910;

s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En];

a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]

{13.6059, 3.40148, 1.51176, 0.850365, 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

Verhältnis bei r=0

a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
s = Sqrt[k ^ 2 - (Z * a) ^ 2];

$$S[\text{n\_}] := \left\{\left\{\frac{a\, En\, Z}{n^2 + 2\, n\sqrt{k^2 - a^2 Z^2}},\ -\frac{(-2 + En)\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{En\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)},\ \frac{a\,(-2 + En)\, Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\} /.\ En \to Enn;$$

S[
10]

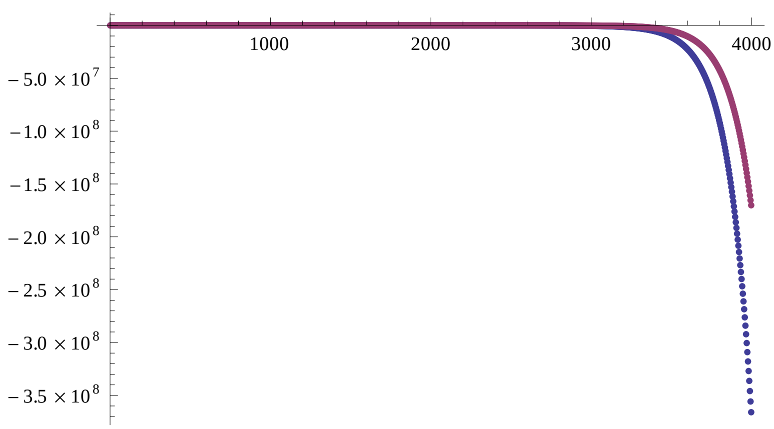
{{0.0000608115 Enn, -0.1 (-2 + Enn)}, {0.0833335 Enn, 0.0000608115 (-2 + Enn)}}

```
Enn =.; u = {1, (k + s) / Z / a}; U = u;
For [n = 1, n < 3, n++,
 u = S[n].u;
 U = Simplify [U + u];
]; n =.;
Simplify [U [[1]] / U [[2]]]
```

$$-\frac{91.3504\ (-3.66275 + \text{Enn})\ (1.62629 + \text{Enn})}{(-0.0109728 + \text{Enn})\ (181.383 + \text{Enn})}$$

Runge von links

```
f[u_, r_] := Simplify [{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 1000;
h = 4000 / n;
Enn = 13.605 / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do [
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo [U, {r, u}], {n}]; x =.;
ListPlot [
 Table [{#[[1]], 137 ^ (i - 2) * #[[2, i]]} & /@ U [[1 ;; n]], {i, 2}] // N, PlotRange → All]
```




```
Sum [A [n] * r ^ n / n !, {n, 0, 10}]
```

$$A[0] + r\,A[1] + \frac{1}{2}\,r^2\,A[2] + \frac{1}{6}\,r^3\,A[3] + \frac{1}{24}\,r^4\,A[4] + \frac{1}{120}\,r^5\,A[5] + \frac{1}{720}\,r^6\,A[6] + \frac{r^7\,A[7]}{5040} + \frac{r^8\,A[8]}{40\,320} + \frac{r^9\,A[9]}{362\,880} + \frac{r^{10}\,A[10]}{3\,628\,800}$$

```
D[%, {r, 4}]
```

$$A[4] + r\,A[5] + \frac{1}{2}\,r^2\,A[6] + \frac{1}{6}\,r^3\,A[7] + \frac{1}{24}\,r^4\,A[8] + \frac{1}{120}\,r^5\,A[9] + \frac{1}{720}\,r^6\,A[10]$$

```
Sum[A[n + 4] * r ^ n / n !, {n, 0, 10}]
```

$$A[4] + r\,A[5] + \frac{1}{2}\,r^2\,A[6] + \frac{1}{6}\,r^3\,A[7] + \frac{1}{24}\,r^4\,A[8] + \frac{1}{120}\,r^5\,A[9] + \frac{1}{720}\,r^6\,A[10] + \frac{r^7\,A[11]}{5040} + \frac{r^8\,A[12]}{40\,320} + \frac{r^9\,A[13]}{362\,880} + \frac{r^{10}\,A[14]}{3\,628\,800}$$