```
σ = 2 / 10; ρ = 1 / 10; r = 3 / 100;
Σ[i_] := σ; Π[i_, j_] := { 1  i == j
                         { ρ  True ;
n = 3;
σI = Sqrt[ Sum[Σ[i] Σ[j] Π[i, j], {i, n}, {j, n}]] / n // N
qI = 1 / n Sum[1 / 2 Σ[i]^2, {i, n}] - 1 / 2 σI^2 // N
I0 = Product[100, {i, n}]^(1 / n);
```

```
0.126491
```

```
0.012
```

```
Timing[
 InputForm[
  FinancialDerivative[{"American", "Put"}, {"StrikePrice" → 100, "Expiration" → 1 / 4},
   {"InterestRate" → r, "Volatility" → σI, "CurrentPrice" → I0, "Dividend" → qI},
   "GridSize" → {2^16, 7000}]]
]
```

```
{71.261, 2.3271362219357212}
```

```
(*no strang 200*)
100 (2.32625975468735 / 2.325976 - 1)
```

```
0.0121994
```

```
(*strang 110*)
100 (2.32577169859819 / 2.325976 - 1)
```

```
-0.00878347
```

```
(*no strang 366*)
100 (2.32614310359909 / 2.325976 - 1)
```

```
0.00718424
```

```
(*strang 200*)
100 (2.32587716544954 / 2.325976 - 1)
```

```
-0.00424916
```

```
InputForm[
 FinancialDerivative[{"American", "Put"}, {"StrikePrice" → 100, "Expiration" → 1 / 4},
  {"InterestRate" → r, "Volatility" → σI, "CurrentPrice" → I0, "Dividend" → qI},
  "GridSize" → {2^16, 2500}]]
```

```
3.004451843936393
```

```
FinancialDerivative[{"American", "Put"},
 {"StrikePrice" → 100.00, "Expiration" → 0.25},
 {"InterestRate" → r - qI, "Volatility" → σI, "CurrentPrice" → I0, "Dividend" → 0},
 "GridSize" → {2^10, 10000}]
```

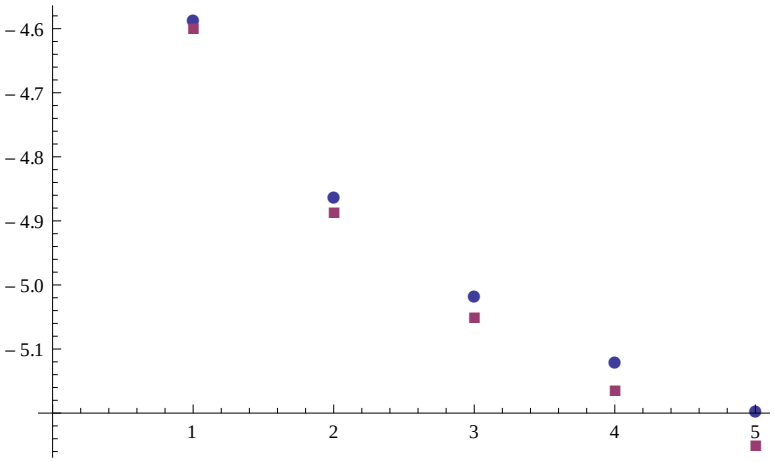
```
2.91137
```

```
l = Transpose[Table[FinancialDerivative[
      {"American", "Put"}, {"StrikePrice" → 100.00, "Expiration" → 0.25},
      {"InterestRate" → r, "Volatility" → σI, "CurrentPrice" → I0, "Dividend" → qI},
      "GridSize" → {2^i, j}], {i, 15, 16}, {j, 1000, 5000, 1000}]];
```

```
Log[Abs[1 - l / 2.90980]] / Log[10] // MatrixForm
```

$$\begin{pmatrix} -4.58559 & -4.59752 \\ -4.86154 & -4.8847 \\ -5.01534 & -5.04865 \\ -5.11871 & -5.16204 \\ -5.19533 & -5.24777 \end{pmatrix}$$

```
ListPlot[Log[Abs[1 - Transpose[l] / 2.90980]] / Log[10],
 PlotRange → All, PlotMarkers → Automatic]
```




```
l[[5, 2]]
```

```
2.90978
```

```
InputForm[l]
```

```
{11.73403343452288, 2.3549295950617433, 3.3524689555958487, 3.2045800655244165,
 3.25412776516467, 3.2458267025112217, 3.2604939057014337, 3.261751509328421,
 3.262145887167553}
```

```
Expand[(Sum[dS[j] D[ga, S[j]], {j, nn}] +
    1 / 2 Sum[dS[j] dS[k] D[ga, S[j], S[k]], {j, 5}, {k, nn}]) / ga]
```

$$-\frac{dS[1]^2}{9\,S[1]^2}+\frac{dS[1]}{3\,S[1]}-\frac{dS[2]^2}{9\,S[2]^2}+\frac{dS[2]}{3\,S[2]}+$$
$$\frac{dS[1]\,dS[2]}{9\,S[1]\,S[2]}-\frac{dS[3]^2}{9\,S[3]^2}+\frac{dS[3]}{3\,S[3]}+\frac{dS[1]\,dS[3]}{9\,S[1]\,S[3]}+\frac{dS[2]\,dS[3]}{9\,S[2]\,S[3]}$$

```
D[ga, S[1], S[1]]
```

$$-\frac{2\,S[2]^{1/3}\,S[3]^{1/3}}{9\,S[1]^{5/3}}$$

```
Exit[]
```

```
nn = 3;
ga := Product[S[i]^(1 / nn), {i, nn}];
Σ[i_] := σ[i]; Π[i_, j_] := Piecewise[{{1, i == j}, {ρ[i, j], i < j}}, ρ[j, i]]
repla = Prepend[Join[Table[dW[i] dt -> 0, {i, nn}],
    Flatten[Table[dW[i] dW[j] -> dt Π[i, j], {j, nn}, {i, j}]]], dt^2 → 0];
dS[i_] := S[i] (r dt + Σ[i] dW[i]);
σI = Sqrt[Sum[Σ[i] Σ[j] Π[i, j], {i, nn}, {j, nn}]] / nn;
qI = 3 / 2 Sum[Σ[i]^2, {i, nn}] / nn^2 - σI^2 / 2;

(* d(ga)/ga *)

Collect[Expand[(Sum[dS[j] D[ga, S[j]], {j, nn}] +
      1 / 2 Sum[dS[j] dS[k] D[ga, S[j], S[k]], {j, 5}, {k, nn}]) / ga] /. repla, dt]
```

$$\frac{1}{3}\,dW[1]\,\sigma[1]+\frac{1}{3}\,dW[2]\,\sigma[2]+\frac{1}{3}\,dW[3]\,\sigma[3]+dt\left(r-\frac{\sigma[1]^2}{9}+\frac{1}{9}\,\rho[1,2]\,\sigma[1]\,\sigma[2]-\frac{\sigma[2]^2}{9}+\frac{1}{9}\,\rho[1,3]\,\sigma[1]\,\sigma[3]+\frac{1}{9}\,\rho[2,3]\,\sigma[2]\,\sigma[3]-\frac{\sigma[3]^2}{9}\right)$$

```
(*test*)
Simplify[(Expand[(1/3 dW[1] σ[1] + 1/3 dW[2] σ[2] + 1/3 dW[3] σ[3])^2] /. repla) - σI^2 dt]
```

```
0
```

```
(* qI *)
qI = -(-σ[1]^2/9 + 1/9 ρ[1, 2] σ[1] σ[2] -
    σ[2]^2/9 + 1/9 ρ[1, 3] σ[1] σ[3] + 1/9 ρ[3] σ[2] σ[3] - σ[3]^2/9)
```

$$\frac{\sigma[1]^2}{9}-\frac{1}{9}\,\rho[1,2]\,\sigma[1]\,\sigma[2]+\frac{\sigma[2]^2}{9}-\frac{1}{9}\,\rho[1,3]\,\sigma[1]\,\sigma[3]-\frac{1}{9}\,\rho[2,3]\,\sigma[2]\,\sigma[3]+\frac{\sigma[3]^2}{9}$$