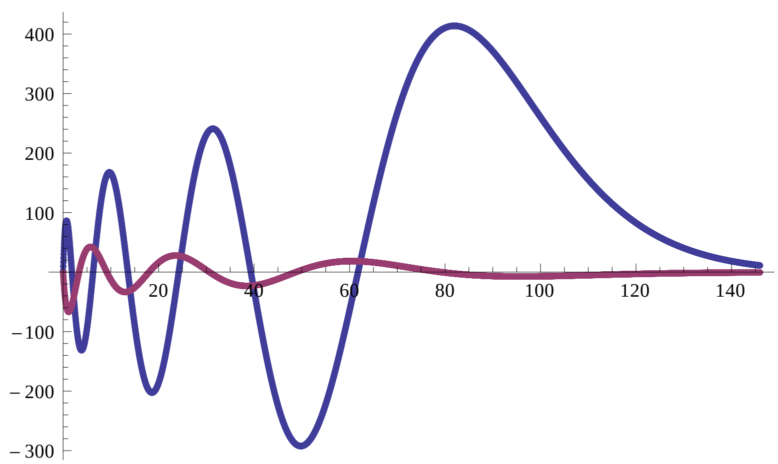
```
Exit[]

a = 7297352537.6 * 10^-12; M = 510998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k^2 - (Z * a)^2]))^2]);
Table[N[Energie[i]], {i, 10}]

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

'

f[u_, r_] := Simplify[{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 5000;
h = 20000 / n;
Enn = 0.27766906844567757` / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do[
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[U, {r, u}], {n}]; x =.;
ListPlot[Table[{#[[1]] / 137, 137^(i - 2) * #[[2, i]]} & /@ U[[1 ;; n]], {i, 2}] // N,
 PlotRange → All]
```



```
lambda[n_] := Sqrt[1 - (1 - En[n] / M) ^ 2];
gamma := Sqrt[k ^ 2 - Z ^ 2 * a ^ 2];
f[r_, n_, k_] := -r ^ gamma * Exp[-lambda[n] * r] * (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] +
     (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
g[r_, n_, k_] := Sqrt[2 * M / En[n] - 1] * r ^ gamma * Exp[-lambda[n] * r] *
   (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] -
     (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
```

```
g[1, 1, -1] / f[1, 1, -1]
```

```
-274.068
```

Randbedingungen

r << 1

```
Exit[]
```

```
a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

```
{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}
```

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - En) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

```
u := x ^ (s + n) * {a[n], b[n]}
```

```
g1 = Collect[Simplify[(f[u, x] - D[u, x]) / x ^ (s - 1)], x]; g1 // MatrixForm
```

$$\begin{pmatrix} -(-2+\mathrm{En})\, x^{1+n}\, b[n] - x^n\, ((k+n+s)\, a[n] - a\, Z\, b[n]) \\ \mathrm{En}\, x^{1+n}\, a[n] + x^n\, (-a\, Z\, a[n] + (k-n-s)\, b[n]) \end{pmatrix}$$

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\begin{pmatrix} -(k+s)\,a[0]+a\,Z\,b[0] & -a\,Z\,a[0]+(k-s)\,b[0] \\ -(1+k+s)\,a[1]-(-2+En)\,b[0]+a\,Z\,b[1] & En\,a[0]-a\,Z\,a[1]+(-1+k-s)\,b[1] \\ -(2+k+s)\,a[2]-(-2+En)\,b[1]+a\,Z\,b[2] & En\,a[1]-a\,Z\,a[2]+(-2+k-s)\,b[2] \\ -(3+k+s)\,a[3]-(-2+En)\,b[2]+a\,Z\,b[3] & En\,a[2]-a\,Z\,a[3]+(-3+k-s)\,b[3] \\ -(4+k+s)\,a[4]-(-2+En)\,b[3]+a\,Z\,b[4] & En\,a[3]-a\,Z\,a[4]+(-4+k-s)\,b[4] \\ -(5+k+s)\,a[5]-(-2+En)\,b[4]+a\,Z\,b[5] & En\,a[4]-a\,Z\,a[5]+(-5+k-s)\,b[5] \\ -(6+k+s)\,a[6]-(-2+En)\,b[5]+a\,Z\,b[6] & En\,a[5]-a\,Z\,a[6]+(-6+k-s)\,b[6] \\ -(7+k+s)\,a[7]-(-2+En)\,b[6]+a\,Z\,b[7] & En\,a[6]-a\,Z\,a[7]+(-7+k-s)\,b[7] \\ -(8+k+s)\,a[8]-(-2+En)\,b[7]+a\,Z\,b[8] & En\,a[7]-a\,Z\,a[8]+(-8+k-s)\,b[8] \\ -(9+k+s)\,a[9]-(-2+En)\,b[8]+a\,Z\,b[9] & En\,a[8]-a\,Z\,a[9]+(-9+k-s)\,b[9] \\ -(10+k+s)\,a[10]-(-2+En)\,b[9]+a\,Z\,b[10] & En\,a[9]-a\,Z\,a[10]+(-10+k-s)\,b[10] \end{pmatrix}$$

```
Det[{{-s - k, a Z}, {-a Z, -s + k}}]
```

```
s =.
```

```
Simplify[
 Eigenvalues[Inverse[{{a / (En - 2), (n + s + k) / (2 - En)}, {(n + s - k) / En, a / En}}]]]
```

$$\left\{-\frac{a-a\,En+\sqrt{a^2+(-2+En)\,En\,\left(k^2-(n+s)^2\right)}}{a^2-k^2+(n+s)^2},\ \frac{-a+a\,En+\sqrt{a^2+(-2+En)\,En\,\left(k^2-(n+s)^2\right)}}{a^2-k^2+(n+s)^2}\right\}$$

```
Expand[(n + s) ^ 2 - s ^ 2]
```

$n^2+2\,n\,s$

```
(a^2 - k^2 + (n + s)^2) * Simplify[
   Inverse[{{a / (En - 2), (n + s + k) / (2 - En)}, {(n + s - k) / En, a / En}}]] // MatrixForm
```

$$\begin{pmatrix} a\,(-2+En) & En\,(k+n+s) \\ (-2+En)\,(k-n-s) & a\,En \end{pmatrix}$$

```
s = Sqrt[k ^ 2 - (Z * a) ^ 2];
```

```
En =.;
Simplify[Inverse[{{Z * a / En, (n + s - k) / En}, {(n + s + k) / (2 - En), Z * a / (En - 2)}}]]
```

$$S[n\_, En\_] := \left\{\left\{\frac{a\, En\, Z}{n^2 + 2\, n \sqrt{k^2 - a^2 Z^2}}, -\frac{(-2 + En)\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{En\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}, \frac{a\,(-2 + En)\, Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

$$DS[n\_, En\_] := \left\{\left\{\frac{a\, Z}{n^2 + 2\, n \sqrt{k^2 - a^2 Z^2}}, -\frac{\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}, \frac{a\, Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

```
S[m, En] // MatrixForm
DS[m, En] // MatrixForm
```

$$\left\{\left\{\frac{a\, En\, Z}{-k^2 + n^2 + 2\, n\, s + s^2 + a^2 Z^2}, \frac{(-2 + En)\,(k - n - s)}{-k^2 + n^2 + 2\, n\, s + s^2 + a^2 Z^2}\right\}, \left\{\frac{En\,(k + n + s)}{-k^2 + n^2 + 2\, n\, s + s^2 + a^2 Z^2}, \frac{a\,(-2 + En)\, Z}{-k^2 + n^2 + 2\, n\, s + s^2 + a^2 Z^2}\right\}\right\}$$

$$\begin{pmatrix} \frac{a\, En\, Z}{m^2 + 2\, m \sqrt{k^2 - a^2 Z^2}} & -\frac{(-2 + En)\left(-k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{En\left(k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} & \frac{a\,(-2 + En)\, Z}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

$$\begin{pmatrix} \frac{a\, Z}{m^2 + 2\, m \sqrt{k^2 - a^2 Z^2}} & -\frac{-k + m + \sqrt{k^2 - a^2 Z^2}}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{k + m + \sqrt{k^2 - a^2 Z^2}}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} & \frac{a\, Z}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

```
S[n, En].u
```

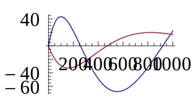
$S[n, En].\{x^{n+s}\, a[n], x^{n+s}\, b[n]\}$

```
UN[R_, N_, En_] := Module[{u = {1, (k + s) / Z / a}, U = {1, (k + s) / Z / a} * R ^ s},
  For[n = 1, n < N, n++,
   u = S[n, En].u;
   U += u * R ^ (s + n);
  ];

  U]
```

```
U[r_, g_, En_] :=
 Module[{u = {1, (k + s) / Z / a}, U = {0, 0}, DU = {0, 0}, du = {0, 0}, n = 0},
  Label[begin];
  U += u * r ^ (s + n);
  DU += du * r ^ (s + n);
  n++;
  du = DS[n, En].u + S[n, En].du;
  u = S[n, En].u;
  If[(#[[1]] > g  || #[[2]] > g) &[Abs[u * R ^ (s + n) / U /. r → R]], Goto[begin]];
  {n, U, DU}]
```

```
R = 1000; g = 0.01; rU = U[r, g, 1 / M];
Plot[{rU[[2, 1]], rU[[2, 2]] * 137}, {r, 0, R}, PlotRange → All]
```




```
EN[iEn_, g2_] := Module[{rU, fU, n = 0, i, En = iEn, ll},
  Label[begin];
  fU = U[r, g, En];

  rU = fU /. r → R;

  If[rU[[2, 1]] * rU[[2, 2]] > 0,

   En -= (rU[[2, 1]] + rU[[2, 2]]) / (rU[[3, 1]] + rU[[3, 2]]);

   n++
    Goto[begin];
  ];

  {n, En * M, Abs[rU[[2, 1]] - rU[[2, 2]]]}
 ]
```

```
R = 3000; g = 0.001; EN[13 / M, 0.1]
```

{8, 13.6059, 0.000441718}

```
- {0, 13.605873075061169
```

```
R = 2000; g = 0.001;
```

```
plot[{rU[[2, 1]], rU[[3, 1]]}, 100, R]
```

```
En = 4 / M; rU[[2, 1]]
```

```
129.072
```

```
n = 0; x
```

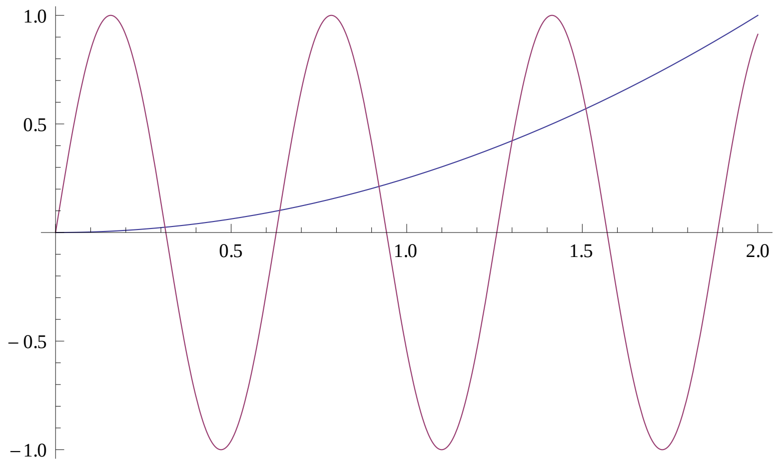
```
10
```

```
n = 0; While[x = n; x < 10, n++; Print[n]]
```

```
1
2
3
4
5
6
7
8
9
10
```

```
plot[liste_, R_] := Module[{nN = 100, table, max, st = {Red, Green, Blue}},
  liste / (Max[Abs[#]] & /@ (Table[# /. r → i * R / nN, {i, 0, nN}] & /@ liste))
 ]
ll = plot[{r ^ 2, Sin[10 * r]}, 2]
Plot[ll, {r, 0, 2}]
```

$\left\{\frac{r^2}{4}, -\text{Csc}[11]\ \text{Sin}[10\ r]\right\}$



r gegen Inifinity

```
Exit[]
```

```
a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

```
$Assumptions = 1 > En > 0;
```

```
s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En]; L =.; s =.
```

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - En) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

```
u =.
```

```
U = {(hb[r] - ha[r]) * L / En, hb[r] + ha[r]} * Exp[-r * L];
```

```
#[[2]] & /@ Simplify[Solve[Simplify[(f[U, r] - D[U, r]) / Exp[-r * L] * En * r] == 0,
    {ha'[r], hb'[r]}][[1]]] /. ha[r] → u[[1]] /. hb[r] → u[[2]] /. r → rr
```

Part::partd : Part specification u⟦1⟧ is longer than depth of object. ≫

Part::partd : Part specification u⟦2⟧ is longer than depth of object. ≫

$$\Big\{\frac{1}{2\,\text{En}\,L\,\text{rr}}\Big(\big(\text{En}^3\,\text{rr}+\text{En}\,L^2\,\text{rr}+a\,L^2\,Z-\text{En}^2\,(2\,\text{rr}+a\,Z)\big)\,u[\![1]\!]+\big(\text{En}^3\,\text{rr}+\text{En}\,L\,(2\,k+L\,\text{rr})-a\,L^2\,Z-\text{En}^2\,(2\,\text{rr}+a\,Z)\big)\,u[\![2]\!]\Big),$$
$$\frac{1}{2\,\text{En}\,L\,\text{rr}}\Big(\big(-\text{En}^3\,\text{rr}+\text{En}\,L\,(2\,k-L\,\text{rr})+a\,L^2\,Z+\text{En}^2\,(2\,\text{rr}+a\,Z)\big)\,u[\![1]\!]+\big(-\text{En}^3\,\text{rr}+3\,\text{En}\,L^2\,\text{rr}-a\,L^2\,Z+\text{En}^2\,(2\,\text{rr}+a\,Z)\big)\,u[\![2]\!]\Big)\Big\}$$

$$\mathbf{F[u\_,\ rr\_] := Simplify}\Big[\Big\{\frac{-a\,(-1+\text{En})\,Z\,u[\![1]\!]+\left(\sqrt{-(-2+\text{En})\,\text{En}}\;k-a\,Z\right)u[\![2]\!]}{\sqrt{-(-2+\text{En})\,\text{En}}\;\text{rr}},$$
$$\left(\left(\sqrt{-(-2+\text{En})\,\text{En}}\;k+a\,Z\right)u[\![1]\!]+\left(4\,\text{En}\,\text{rr}-2\,\text{En}^2\,\text{rr}-a\,Z+a\,\text{En}\,Z\right)u[\![2]\!]\right)\Big/\left(\sqrt{-(-2+\text{En})\,\text{En}}\;\text{rr}\right)\Big\}\Big]$$

```
r[x_] := 1 / x; u := {a[n], b[n]} * x ^ (n + s)
```

```
g1 = Collect[Simplify[(F[u, r[x]] * D[r[x], x] - D[u, x]) * x ^ (2 - s)], {a[n], b[n]}]; g1
```

$$\Big\{-\left(x^{1+n}\left(a\,(-1+\text{En})\sqrt{-(-2+\text{En})\,\text{En}}\;Z+L\left(\sqrt{-(-2+\text{En})\,\text{En}}\;n+a\,Z-a\,\text{En}\,Z\right)\right)a[n]\right)\Big/\left(\sqrt{-(-2+\text{En})\,\text{En}}\;L\right)-\frac{x^{1+n}\left(\sqrt{-(-2+\text{En})\,\text{En}}\;k-a\,Z\right)b[n]}{\sqrt{-(-2+\text{En})\,\text{En}}},$$
$$-\frac{x^{1+n}\left(\sqrt{-(-2+\text{En})\,\text{En}}\;k+a\,Z\right)a[n]}{\sqrt{-(-2+\text{En})\,\text{En}}}-\frac{1}{\sqrt{-(-2+\text{En})\,\text{En}}\;L}x^n\Big(-2\,\text{En}^2\,L+x\left(\sqrt{-(-2+\text{En})\,\text{En}}\;L\,n-a\sqrt{-(-2+\text{En})\,\text{En}}\;Z-a\,L\,Z\right)+\text{En}\left(4\,L+a\sqrt{-(-2+\text{En})\,\text{En}}\;x\,Z+a\,L\,x\,Z\right)\Big)\,b[n]\Big\}$$

```
g2 := {x^n (-n) a[n] + x^n (-k + a Z / L) b[n],
  -x^(1+n) ( k + a Z / L) a[n] + x^n (-n x - 2 s x + -2 * L) b[n]}; g2 // MatrixForm
```

$$\begin{pmatrix} -n\,x^n\,a[n]+x^n\left(-k+\frac{a\,Z}{L}\right)b[n] \\ -x^{1+n}\left(k+\frac{a\,Z}{L}\right)a[n]+x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n] \end{pmatrix}$$

```
a[n_] := (Z * a / L - k) / n * b[n]
```

```
g2[[2]]
```

$$x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n]-\frac{x^{1+n}\left(-k+\frac{a\,Z}{L}\right)\left(k+\frac{a\,Z}{L}\right)b[n]}{n}$$

```
M1 = {{L, 2 - En}, {En, L}}; Eigenvalues[M1]; B = Transpose[Eigenvectors[M1]];
```

```
$Assumptions = 1 > En > 0
```

$1 > En > 0$

=>

```
Exit[]
a[n_] := (Z * a / L - k) / n * b[n]
b[n_] := b0 * Product[((k ^ 2 - a ^ 2 * Z ^ 2 / L ^ 2) / i - (i + 2 * s)) / 2 / L, {i, 1, n}]
b[4]
```

$$\frac{1}{16\, L^4}\, b0 \left(-1 + k^2 - 2\, s - \frac{a^2\, Z^2}{L^2}\right) \left(-4 - 2\, s + \frac{1}{4}\left(k^2 - \frac{a^2\, Z^2}{L^2}\right)\right) \left(-3 - 2\, s + \frac{1}{3}\left(k^2 - \frac{a^2\, Z^2}{L^2}\right)\right) \left(-2 - 2\, s + \frac{1}{2}\left(k^2 - \frac{a^2\, Z^2}{L^2}\right)\right)$$

```
((k ^ 2 - a ^ 2 * Z ^ 2 / L ^ 2) / i - (i + 2 * s)) / 2 / L
```

$$\frac{-i - 2\, s + \frac{k^2 - \frac{a^2\, Z^2}{L^2}}{i}}{2\, L}$$

```
Exit[];
En =.;
```

$$\text{Solve}\left[-i * \sqrt{(2 - En)\, En} - 2\, a\, (-1 + En) + \frac{\sqrt{(2 - En)\, En} - \frac{a^2}{\sqrt{(2 - En)\, En}}}{i} == 0,\ En\right] \text{// MatrixForm}$$

$$\begin{pmatrix}
En \to \frac{2 - 4\, i^2 + 8\, a^2\, i^2 + 2\, i^4 - \sqrt{\left(-2 + 4\, i^2 - 8\, a^2\, i^2 - 2\, i^4\right)^2 - 4\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)\left(a^2 + a^2\, i^2 - 2\sqrt{a^4\, i^2 - a^6\, i^2}\right)}}{2\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)} \\
En \to \frac{2 - 4\, i^2 + 8\, a^2\, i^2 + 2\, i^4 + \sqrt{\left(-2 + 4\, i^2 - 8\, a^2\, i^2 - 2\, i^4\right)^2 - 4\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)\left(a^2 + a^2\, i^2 - 2\sqrt{a^4\, i^2 - a^6\, i^2}\right)}}{2\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)} \\
En \to \frac{2 - 4\, i^2 + 8\, a^2\, i^2 + 2\, i^4 - \sqrt{\left(-2 + 4\, i^2 - 8\, a^2\, i^2 - 2\, i^4\right)^2 - 4\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)\left(a^2 + a^2\, i^2 + 2\sqrt{a^4\, i^2 - a^6\, i^2}\right)}}{2\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)} \\
En \to \frac{2 - 4\, i^2 + 8\, a^2\, i^2 + 2\, i^4 + \sqrt{\left(-2 + 4\, i^2 - 8\, a^2\, i^2 - 2\, i^4\right)^2 - 4\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)\left(a^2 + a^2\, i^2 + 2\sqrt{a^4\, i^2 - a^6\, i^2}\right)}}{2\left(1 - 2\, i^2 + 4\, a^2\, i^2 + i^4\right)}
\end{pmatrix}$$

'

$$\texttt{ET[i\_, k\_, Z\_, a\_]} := \left(2\, i^4 - 4\, i^2 k^2 + 2\, k^4 + 8\, a^2 i^2 Z^2 - \sqrt{\left(\left(-2\, i^4 + 4\, i^2 k^2 - 2\, k^4 - 8\, a^2 i^2 Z^2\right)^2 - 4\left(i^4 - 2\, i^2 k^2 + k^4 + 4\, a^2 i^2 Z^2\right)\left(a^2 i^2 Z^2 + a^2 k^2 Z^2 + 2\sqrt{a^4 i^2 k^2 Z^4 - a^6 i^2 Z^6}\right)\right)}\right) \Big/ \left(2\left(i^4 - 2\, i^2 k^2 + k^4 + 4\, a^2 i^2 Z^2\right)\right)$$

```
Table[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 10}]
```

$\{-5.67315\times10^{-11}, -1.9084\times10^{-7}, 5.30991\times10^{-11}, -1.3482\times10^{-10}, 3.24187\times10^{-11}, -5.86517\times10^{-11}, 5.5719\times10^{-11}, 2.10913\times10^{-11}, 1.87879\times10^{-11}, 1.01339\times10^{-10}, -1.46405\times10^{-12}\}$

```
Series[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 5}]
```

$-5.67315\times10^{-11} - 1.26477\times10^{-10}\, n^2 - 2.84217\times10^{-14}\, n^3 - 2.54019\times10^{-10}\, n^4 + 1.42109\times10^{-14}\, n^5 + O[n]^6$

```
M = 510 998.910;

s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En];

a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365, 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

Verhältnis bei r = 0

```
a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
s = Sqrt[k ^ 2 - (Z * a) ^ 2];
```

$$\texttt{S[n\_]} := \left\{\left\{\frac{a\, En\, Z}{n^2 + 2\, n\sqrt{k^2 - a^2 Z^2}},\ -\frac{(-2 + En)\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\},\ \left\{\frac{En\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)},\ \frac{a\,(-2 + En)\, Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\} \texttt{ /. En -> Enn;}$$

```
S[
 10]
```

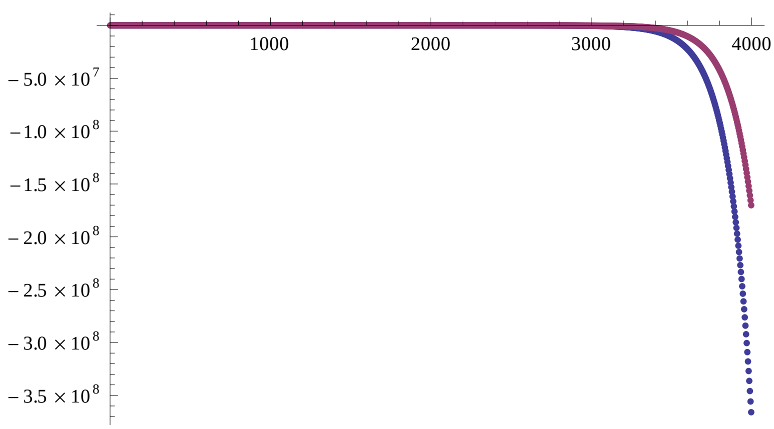
{{0.0000608115 Enn, −0.1 (−2 + Enn)}, {0.0833335 Enn, 0.0000608115 (−2 + Enn)}}

```
Enn =.; u = {1, (k + s) / Z / a}; U = u;
For[n = 1, n < 3, n++,
 u = S[n].u;
 U = Simplify[U + u];
]; n =.;
Simplify[U[[1]] / U[[2]]]
```

$$-\frac{91.3504\ (-3.66275 + \text{Enn})\ (1.62629 + \text{Enn})}{(-0.0109728 + \text{Enn})\ (181.383 + \text{Enn})}$$

Runge von links

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 1000;
h = 4000 / n;
Enn = 13.605 / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do[
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[U, {r, u}], {n}]; x =.;
ListPlot[
 Table[{#[[1]], 137 ^ (i - 2) * #[[2, i]]} & /@ U[[1 ;; n]], {i, 2}] // N, PlotRange → All]
```




```
Sum[A[n] * r ^ n / n!, {n, 0, 10}]
```

$$A[0] + r\,A[1] + \frac{1}{2}\,r^2\,A[2] + \frac{1}{6}\,r^3\,A[3] + \frac{1}{24}\,r^4\,A[4] + \frac{1}{120}\,r^5\,A[5] + \frac{1}{720}\,r^6\,A[6] + \frac{r^7\,A[7]}{5040} + \frac{r^8\,A[8]}{40\,320} + \frac{r^9\,A[9]}{362\,880} + \frac{r^{10}\,A[10]}{3\,628\,800}$$

```
D[%, {r, 4}]
```

$$A[4] + r\,A[5] + \frac{1}{2}\,r^2\,A[6] + \frac{1}{6}\,r^3\,A[7] + \frac{1}{24}\,r^4\,A[8] + \frac{1}{120}\,r^5\,A[9] + \frac{1}{720}\,r^6\,A[10]$$

```
Sum[A[n + 4] * r ^ n / n !, {n, 0, 10}]
```

$$A[4] + r\,A[5] + \frac{1}{2}\,r^2\,A[6] + \frac{1}{6}\,r^3\,A[7] + \frac{1}{24}\,r^4\,A[8] + \frac{1}{120}\,r^5\,A[9] + \frac{1}{720}\,r^6\,A[10] + \frac{r^7\,A[11]}{5040} + \frac{r^8\,A[12]}{40\,320} + \frac{r^9\,A[13]}{362\,880} + \frac{r^{10}\,A[14]}{3\,628\,800}$$