```
Exit[];
```

```
$Assumptions = r > 0 && Element[m, Integers] && Element[n, Integers] && s > 0
```

$$r > 0 \&\& m \in \text{Integers} \&\& n \in \text{Integers} \&\& s > 0$$

```
f[r_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}} -
  0 * IdentityMatrix[2] * I * r ^ p; f[r] // MatrixForm
```

$$\begin{pmatrix} \frac{-1+m}{r} & i\,(En - r^p) \\ i\,(En - r^p) & -\frac{m}{r} \end{pmatrix}$$

```
En = E0 + I * Ga; En =.
```

```
VV[x_] := Exp[I * Integrate[r ^ p - En, {r, 0, x}]]
```

```
u = {F[x], G[x]} * x ^ s
```

$$\{x^{-1+m} F[x],\ x^{-1+m} G[x]\}$$

```
p = 2;
```

```
r[x_] := x;
```

```
g1 = Collect[
  Expand[Simplify[Expand[(-D[u, x] + r'[x] * f[r[x]].u) * x ^ -s]]], {x ^ n, a[n], b[n]}]
```

$$\left\{\frac{2\,F[x]}{x} - \frac{2\,m\,F[x]}{x} + \frac{i\,G[x]}{x^4} - \frac{i\,En\,G[x]}{x^2} - F'[x],\ \frac{i\,F[x]}{x^4} - \frac{i\,En\,F[x]}{x^2} + \frac{G[x]}{x} - G'[x]\right\}$$

```
f2[r_] := {{1, I En - I x ^ 2}, {I En - I x ^ 2, (1 - 2 m) / x }}; f2[r] // MatrixForm
```

$$\begin{pmatrix} 1 & i\,En - i\,x^2 \\ i\,En - i\,x^2 & \frac{1-2\,m}{x} \end{pmatrix}$$

```
u = {a[n] * x ^ (n), b[n] * x ^ (n)} * x ^ S
```

$$\{x^{n+S}\,a[n],\ x^{n+S}\,b[n]\}$$

```
r[x_] := 1 / x;
```

```
s = m - 1;
```

```
g2 =
 Collect[Expand[Simplify[Expand[(-D[u, x] + r'[x] * f2[r[x]].u) * x ^ (3 - S) * {1, -1}]]],
  {x ^ n, a[n], b[n]}]
```

$$\left\{x^n \left(\left(-x - n\,x^2 - S\,x^2\right) a[n] + \left(-i\,En\,x + i\,x^3\right) b[n]\right),\right.$$
$$\left. x^n \left(\left(i\,En\,x - i\,x^3\right) a[n] + \left(1 - 2\,m + n\,x^2 + S\,x^2\right) b[n]\right)\right\}$$

```
g3 = Table[Simplify[Sum[D[g2, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g3 // MatrixForm
```

$$\begin{pmatrix} 0 & (1-2m)\,b[0] \\ -a[0]-i\,En\,b[0] & i\,En\,a[0]+b[1]-2m\,b[1] \\ -S\,a[0]-a[1]-i\,En\,b[1] & i\,En\,a[1]+S\,b[0]+b[2]-2m\,b[2] \\ -a[1]-S\,a[1]-a[2]+i\,b[0]-i\,En\,b[2] & -i\,a[0]+i\,En\,a[2]+b[1]+S\,b[1]+b[3]-2m\,b[3] \\ -2\,a[2]-S\,a[2]-a[3]+i\,b[1]-i\,En\,b[3] & -i\,a[1]+i\,En\,a[3]+2\,b[2]+S\,b[2]+b[4]-2m\,b[\\ -3\,a[3]-S\,a[3]-a[4]+i\,b[2]-i\,En\,b[4] & -i\,a[2]+i\,En\,a[4]+3\,b[3]+S\,b[3]+b[5]-2m\,b[5 \\ -4\,a[4]-S\,a[4]-a[5]+i\,b[3]-i\,En\,b[5] & -i\,a[3]+i\,En\,a[5]+4\,b[4]+S\,b[4]+b[6]-2m\,b[6 \\ -5\,a[5]-S\,a[5]-a[6]+i\,b[4]-i\,En\,b[6] & -i\,a[4]+i\,En\,a[6]+5\,b[5]+S\,b[5]+b[7]-2m\,b[7 \\ -6\,a[6]-S\,a[6]-a[7]+i\,b[5]-i\,En\,b[7] & -i\,a[5]+i\,En\,a[7]+6\,b[6]+S\,b[6]+b[8]-2m\,b[8 \\ -7\,a[7]-S\,a[7]-a[8]+i\,b[6]-i\,En\,b[8] & -i\,a[6]+i\,En\,a[8]+7\,b[7]+S\,b[7]+b[9]-2m\,b[9 \\ -8\,a[8]-S\,a[8]-a[9]+i\,b[7]-i\,En\,b[9] & -i\,a[7]+i\,En\,a[9]+8\,b[8]+S\,b[8]+b[10]-2m\,b[\end{pmatrix}$$

```
a[0] = 1;  b[0] = 0; a[1] = -i En a[0]; b[1] = En/(2 m);
a[2] = -i En a[1] / 2 - En b[1] / 2; b[2] = -1 / (1 + 2 m) (-En a[1] + i En b[1]);
```

```
Simplify[I * (i a[n] - i En a[n + 2] - (n + 3) a[n + 3] + b[n] - En b[n + 2]) +
   a[n] - En a[n + 2] - i b[n] + i En b[n + 2] + (n + 2) b[n + 3] + 2 m b[n + 3]]
```

$$-i\,(3+n)\,a[3+n]+(2+2m+n)\,b[3+n]$$

```
b[n_] := i (n) a[n] / (-1 + 2 m + n)
```

```
Collect[i a[n] - i En a[n + 2] - (n + 3) a[n + 3] + b[n] - En b[n + 2],
   {a[n], a[n + 3], a[n + 2]}] /. n → n - 3
```

$$\left(i+\frac{i\,(-3+n)}{-4+2m+n}\right)a[-3+n]+\left(-i\,En-\frac{i\,En\,(-1+n)}{-2+2m+n}\right)a[-1+n]-n\,a[n]$$

```
a[n_] := 1 / n * (((i + i (-3 + n) / (-4 + 2 m + n)) a[-3 + n] + (-i En - i En (-1 + n) / (-2 + 2 m + n)) a[-1 + n]))
```

```
b[2]
```

$$-\frac{i\,En^2+\frac{i\,En^2}{2m}}{1+2m}$$

```
U[En_, m_, nN_] := Module[{U, te},
  U = {{1, 0}, {-i En, En/(2 m)}, {-En^2/2 - En^2/(4 m), -(i En^2 + i En^2/(2 m))/(1 + 2 m)}};
  For[n = 3, n < nN, n++,
   te = 1/n * ((i + (i (-3 + n))/(-4 + 2 m + n)) U[[-2 + n, 1]] + (-i En - (i En (-1 + n))/(-2 + 2 m + n)) U[[n, 1]]);
   AppendTo[U, {te, i (n) te/(-1 + 2 m + n)}];
  ];
  {1, 1} * Exp[-I * Integrate[r^p - En, {r, 0, x}]];
  Expand[Sum[U[[n + 1]] * x^n, {n, 0, nN - 1}] * x^(-1 + m)]
 ]
```

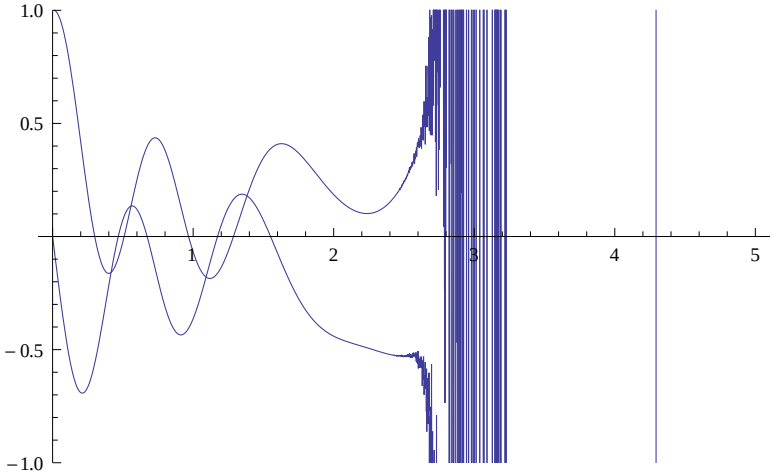
```
Simplify[U[En, m, 15] - Table[{a[n], b[n]}, {n, 0, 14}]]
```

$Aborted

```
U[10, 2, 5] // N // MatrixForm
```

$$\begin{pmatrix} x - (0.+10.\,i)\,x^2 - 62.5\,x^3 + (0.+292.\,i)\,x^4 + 1098.13\,x^5 \\ 2.5\,x^2 - (0.+25.\,i)\,x^3 - 146.\,x^4 + (0.+627.5\,i)\,x^5 \end{pmatrix}$$

```
G = U[5 + I * 0.25, 1, 250][[1]]; Plot[{Re[#], Im[#]} &[G], {x, 0, 5}, PlotRange → {-1, 1}]
```



G

$$\left\{\left\{\text{Re}\left[e^{-i\left(-\text{En}\,x+\frac{x^3}{3}\right)}\left(x-10\,i\,x^2-\frac{125\,x^3}{2}+292\,i\,x^4+\frac{8785\,x^5}{8}\right)\right],\right.\right.$$
$$\left.\left.\text{Re}\left[e^{-i\left(-\text{En}\,x+\frac{x^3}{3}\right)}\left(\frac{5\,x^2}{2}-25\,i\,x^3-146\,x^4+\frac{1255\,i\,x^5}{2}\right)\right]\right\},$$
$$\left\{\text{Im}\left[e^{-i\left(-\text{En}\,x+\frac{x^3}{3}\right)}\left(x-10\,i\,x^2-\frac{125\,x^3}{2}+292\,i\,x^4+\frac{8785\,x^5}{8}\right)\right],\right.$$
$$\left.\left.\text{Im}\left[e^{-i\left(-\text{En}\,x+\frac{x^3}{3}\right)}\left(\frac{5\,x^2}{2}-25\,i\,x^3-146\,x^4+\frac{1255\,i\,x^5}{2}\right)\right]\right\}\right\}$$

VV[x]

$$e^{i\left(-\text{En}\,x+\frac{x^3}{3}\right)}$$