```
R[n_] := 2 Sqrt[2]/9801 Sum[(4 k)! (1103 + 26390 k)/((k!)^4 396^(4 k)), {k, 0, n}]

N[1 / R[5] - π, 100]
```

$$4.741011768567914974136850634834727161360394467082098721200536639730466696354233748949017936147287482 \times 10^{-48}$$

```
Table[R[n], {n, 10}]
```

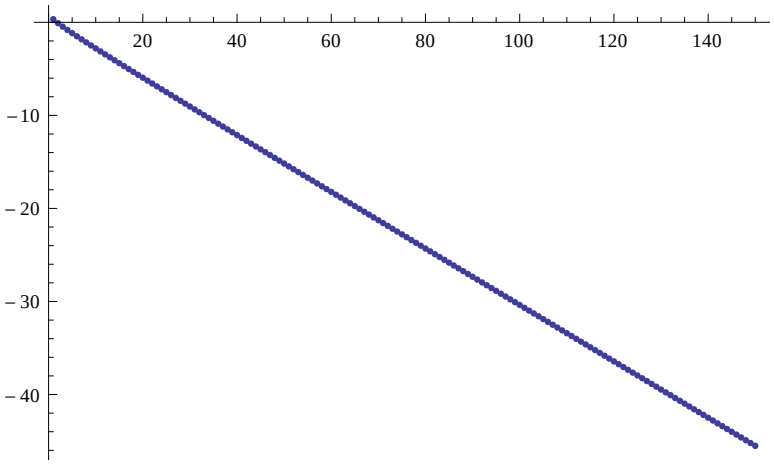
$$\left\{\frac{1\,130\,173\,253\,125}{2\,510\,613\,731\,736\sqrt{2}},\ \frac{1\,029\,347\,477\,390\,786\,609\,545}{2\,286\,635\,172\,367\,940\,241\,408\sqrt{2}},\right.$$
$$\frac{7\,766\,473\,062\,254\,307\,011\,793\,347\,201\,855}{17\,252\,765\,328\,978\,109\,815\,564\,789\,153\,792\sqrt{2}},$$
$$\frac{509\,299\,577\,881\,529\,611\,662\,930\,757\,403\,081\,523\,769\,055}{1\,131\,379\,202\,490\,552\,979\,877\,435\,552\,947\,122\,965\,839\,872\sqrt{2}},$$
$$\frac{57\,982\,950\,211\,280\,781\,944\,919\,792\,648\,021\,104\,999\,982\,386\,829\,481}{128\,805\,730\,098\,892\,711\,723\,125\,911\,845\,114\,081\,418\,091\,536\,842\,752\sqrt{2}},$$
$$3\,499\,871\,759\,747\,710\,499\,842\,768\,988\,784\,507\,373\,816\,789\,022\,688\,631\,739\,047\,925 \Big/ \left(7\,774\,760\,263\,562\,699\,859\,971\,501\,015\,139\,525\,269\,727\,219\,309\,055\,349\,184\,528\,384\sqrt{2}\right),$$
$$398\,454\,856\,050\,409\,400\,033\,667\,498\,427\,037\,929\,849\,361\,304\,439\,288\,784\,703\,764\,447\,270\,125 \Big/ \left(885\,144\,140\,786\,355\,895\,741\,177\,195\,716\,026\,970\,950\,416\,670\,420\,565\,960\,985\,448\,225\,439\,744\sqrt{2}\right),$$
$$6\,334\,387\,787\,708\,107\,824\,222\,495\,376\,281\,706\,107\,615\,730\,472\,323\,276\,284\,056\,009\,760\,393\,364\,218\,543\,125 \Big/ \left(14\,071\,471\,712\,843\,535\,798\,792\,494\,970\,078\,253\,119\,671\,801\,362\,717\,159\,118\,900\,747\,103\,370\,578\,550\,063 \times 104\sqrt{2}\right),$$
$$14\,194\,592\,594\,146\,827\,909\,170\,805\,406\,080\,156\,403\,980\,453\,284\,185\,387\,917\,579\,020\,073\,045\,561\,359\,013 \times 099\,552\,859\,053\,125 \Big/ \left(31\,532\,456\,625\,322\,022\,370\,765\,818\,276\,612\,583\,919\,584\,083\,811\,310\,999\,597\,255\,056\,965\,804\,073\,403\,194 \times 963\,043\,194\,765\,312\sqrt{2}\right),$$
$$116\,354\,295\,547\,844\,200\,479\,625\,540\,962\,705\,305\,445\,031\,010\,498\,388\,307\,062\,857\,519\,290\,687\,784\,871\,920 \times 308\,555\,177\,681\,218\,916\,232\,885 \Big/ \left(258\,474\,257\,235\,476\,477\,051\,634\,224\,477\,005\,861\,793\,643\,791\,092\,488\,013\,501\,737\,085\,215\,352\,314\,477 \times 706\,263\,478\,530\,979\,938\,932\,097\,024\sqrt{2}\right)\left.\right\}$$

```
Table[N[i/(2 i + 1), 50], {i, 100}]

P[n_, n_] := 1;

P[n_, i_] := 2 + i/(2 i + 1) P[n, i + 1]
```

```
ListPlot[Table[Log[10, Abs[N[P[n, 1] - π, 500]]], {n, 1, 150}]]
```




```
m[k_] := {{k, 4 * k + 2}, {0, 2 * k + 1}}

product[n_, n_] := IdentityMatrix[2];
product[n_, i_] := m[i].product[n, i + 1]
product[n_] := product[n, 1]

x[0] = 0; x[i_] := (1 + 2 i) (2 (i - 1)! + x[i - 1])

product2[n_] := {{n!, x[n]}, {0, Product[1 + 2 i, {i, n}]}}

Table[product2[i] - product[i + 1], {i, 10}]
```

```
{{{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}},
 {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}}
```

```
2 Product[1 + 2 i, {i, n}]
```

$$\frac{2^{1-n}\ (1+2\,n)!}{n!}$$

```
reduce[m_, x_] := #[[1]] / #[[2]] &[m.{x, 1}]

P2[n_, x_] := reduce[product2[n], x]

err[n_, x_] := Log[10, Abs[N[P2[n, x] - π, 500]]]
```

```
ListPlot[Table[err[n, #], {n, 1, 250}] & /@ {4, 6}]
```

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
721073383917638064352004259293348001450974828671245526552057133274182902573672272`.
22400467362865517999180750505546389284420845568 / 22952478676498416711138029850064
87872079204589138925981326030390833146152064944380459789310141526372662940066`.
8439977718408373625 − π. ≫

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
752544362558242440383510209553641477371654316138823372601307289062174308930336070`.
22650306428733405206683642387384787533824 / 23954231039416745331537750640047880266
52558452556678920903807489875892279589658527602541901465732673247926846448326`.
3459625 − π. ≫

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating
577177870763057602755988720795349227325356523926725023471167843785942732487010632`.
49816673813936950729301912513476906411113815474176 / 18372142235039150700897321040
03936998852644630719448178527968621200164379784836709257543158135742757344486`.
1552815062164864355065375 − π. ≫

General::stop : Further output of N::meprec will be suppressed during this calculation. ≫

$RecursionLimit::reclim : Recursion depth of 256 exceeded. ≫

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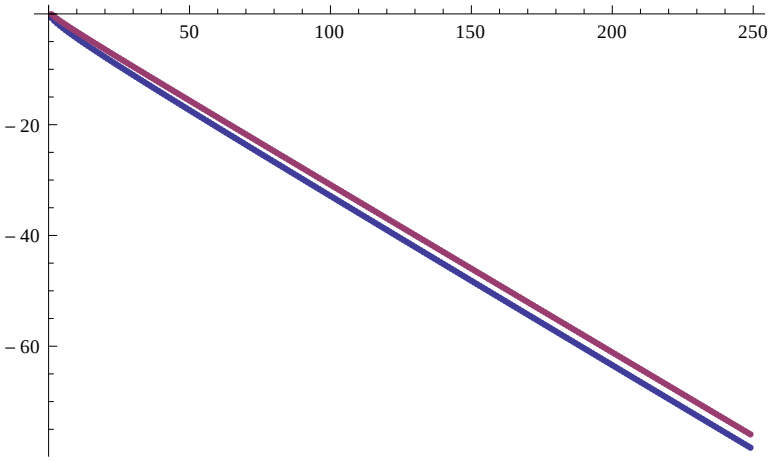
General::stop : Further output of $RecursionLimit::reclim will be suppressed during this calculation. ≫




```
Simplify[{{A[1, 1], A[1, 2]}, {0, A[2, 2]}}.m[i]] // MatrixForm
```

$$\begin{pmatrix} i\,A[1,1] & (1+2i)\,(2\,A[1,1]+A[1,2]) \\ 0 & (1+2i)\,A[2,2] \end{pmatrix}$$

```
product[200]
```

product[200]

```
Exit[]
```

```
RSolve[x[i + 1] == (3 + 2 i) (2 i! + x[i]), x[i], i]
```

$$\left\{\left\{x[i] \to 2^{-1+i}\, C[1]\ \text{Pochhammer}\left[\frac{5}{2}, -1+i\right] + 3 \times 2^{-1+i} \sqrt{\pi} \left(\sqrt{\pi} - \frac{2^{-i}\, i!\ \text{Hypergeometric2F1}\left[1, 1+i, \frac{3}{2}+i, \frac{1}{2}\right]}{\left(\frac{1}{2}\ (1+2\,i)\right)!}\right) \text{Pochhammer}\left[\frac{5}{2}, -1+i\right]\right\}\right\}$$