```
S[t_, x_] := Exp[σ Sqrt[t] x - σ^2/2 t]

$Assumptions = A > 0

A > 0

s = Expectation[Abs[Exp[x] - Exp[A]], x ≈ NormalDistribution[0, σ Sqrt[t]]]
```

$$\frac{1}{2}\left(e^{A}-e^{\frac{t\sigma^2}{2}}+e^{A}\,\mathrm{Erf}\left[\frac{A}{\sqrt{2}\sqrt{t}\,\sigma}\right]-e^{\frac{t\sigma^2}{2}}\,\mathrm{Erf}\left[\frac{A-t\,\sigma^2}{\sqrt{2}\sqrt{t}\,\sigma}\right]-e^{A}\,\mathrm{Erfc}\left[\frac{A}{\sqrt{2}\sqrt{t}\,\sigma}\right]+e^{\frac{t\sigma^2}{2}}\,\mathrm{Erfc}\left[\frac{A-t\,\sigma^2}{\sqrt{2}\sqrt{t}\,\sigma}\right]\right)$$

```
Simplify[Expand[s / Exp[A] /. A → σ^2 t/2]]
```

$$\frac{1}{2}\left(2\,\mathrm{Erf}\left[\frac{\sqrt{t}\,\sigma}{2\sqrt{2}}\right]+\mathrm{Erfc}\left[-\frac{\sqrt{t}\,\sigma}{2\sqrt{2}}\right]-\mathrm{Erfc}\left[\frac{\sqrt{t}\,\sigma}{2\sqrt{2}}\right]\right)$$

```
(*it holds*) Erf[t] = 1 - Erfc[t]; Erfc[A] == 2 - Erfc[-A];
```

$$\frac{1}{2}\left(2+2-4\,\mathrm{Erfc}\left[\frac{\sqrt{t}\,\sigma}{2\sqrt{2}}\right]\right)$$

```
CDF[NormalDistribution[0, 1]]
```

$$\frac{1}{2}\,\mathrm{Erfc}\left[\frac{0-\#1}{\sqrt{2}\;1}\right]\,\&$$

```
Expectation[(S[t, x] - 1)^2, x ≈ NormalDistribution[]]
```

$$-1+e^{t\,\sigma^2}$$

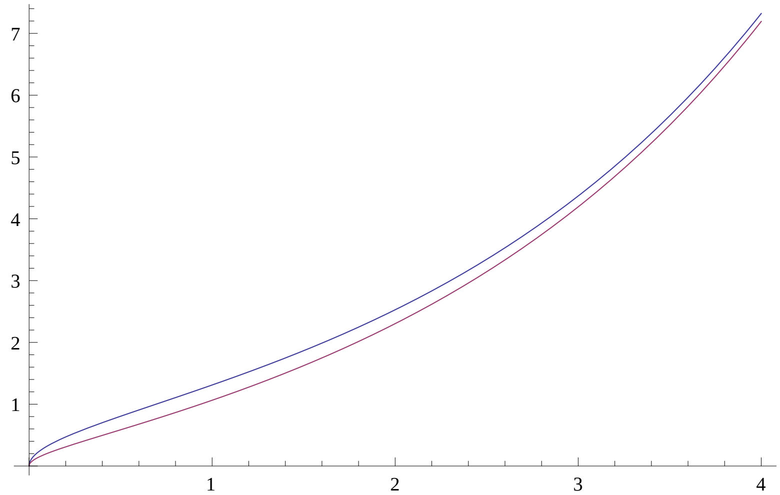
```
Expectation[Abs[x - 1], x ≈ NormalDistribution[0, Sqrt[t]]]
```

$$\mathrm{Expectation}\left[\mathrm{Abs}\left[x\,\sigma-\frac{t\,\sigma^2}{2}\right],\ x\approx\mathrm{NormalDistribution}\left[0,\sqrt{t}\right]\right]$$

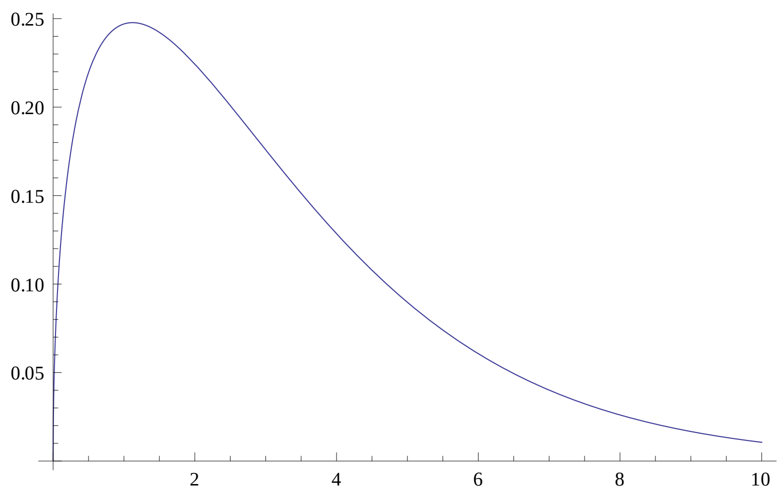
```
f[x_] := Sin[x] + 1
```

```
Plot[{Sqrt[Exp[t] - 1], Sqrt[Exp[t] - 1 - (2 (1 - Erfc[Sqrt[t]/(2 Sqrt[2])]))^2]}, {t, 0, 4}]
```

$$\texttt{Plot}\left[\left\{\texttt{Sqrt}[\texttt{Exp}[t]-1],\ \texttt{Sqrt}\left[\texttt{Exp}[t]-1-\left(2\left(1-\texttt{Erfc}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right]\right)\right)\texttt{\^{}}2\right]\right\},\ \{t,0,4\}\right]$$



$$\texttt{Plot}\left[\texttt{Sqrt}[\texttt{Exp}[t]-1]-\texttt{Sqrt}\left[\texttt{Exp}[t]-1-\left(2\left(1-\texttt{Erfc}\left[\frac{\sqrt{t}}{2\sqrt{2}}\right]\right)\right)\texttt{\^{}}2\right],\ \{t,0,10\}\right]$$



s =.

Eigenvalues[{{s ^ 2, -s}, {-s, 1}}]

$\{0, 1 + s^2\}$

Inverse[{{-a, 1}, {-b, a}}]

$$\left\{\left\{\frac{a}{-a^2+b}, -\frac{1}{-a^2+b}\right\}, \left\{\frac{b}{-a^2+b}, -\frac{a}{-a^2+b}\right\}\right\}$$

D[Sqrt[x], {x, 2}]

$$-\frac{1}{4\,x^{3/2}}$$