```
Exit[]
```

```
n = 4;
```

```
EW[1] = 0; EW[2] = 1; Moments = Table[W^n → EW[n], {n, n, 1, -1}]
```

$$\{W^4 \to EW[4],\ W^3 \to EW[3],\ W^2 \to 1,\ W \to 0\}$$

```
ExpValue[a_] := Simplify[a - a + Expand[Normal[a]] /. Moments]
```

```
Cov[a_, b_] := Simplify[ExpValue[a b] - ExpValue[a] ExpValue[b]]
```

```
Var[a_] := Cov[a, a]
```

```
dX = μ dt^2 + σ W dt;
```

```
dS = S (Series[Exp[dX], {dt, 0, n}] - 1);
```

```
dV = Series[V[t + dt^2, S + dS], {dt, 0, n}] - V[t, S];
```

```
dP[Δ_] := dV + Δ dS - (V[t, S] + Δ S) (Exp[dt^2 r] - 1)
```

```
VarHedgingError[Δ_] := Var[dP[Δ]]
```

```
Cov[dS, dS]
```

```
0
```

■ Hedging Ratios:

```
(* Variance minimizing *)
Δ0 = -Simplify[Cov[dS, dV] / Var[dS]]
```

```
Power::infy : Infinite expression 1/0 encountered. >>
```

```
ComplexInfinity
```

```
(* Wilmott's *)
ΔW = Δ0 /. EW[3] → 0
```

$$-V^{(0,1)}[t, S] + \left(-\frac{1}{4} S \left(4\mu + \sigma^2 (-1 + 3\, EW[4])\right) V^{(0,2)}[t, S] - \frac{1}{6} S^2 \sigma^2 EW[4]\, V^{(0,3)}[t, S] - V^{(1,1)}[t, S]\right) dt^2 + O[dt]^4$$

```
(* Black Scholes *)
ΔBS = ΔW /. dt → 0
```

$$-V^{(0,1)}[t, S]$$

■ Hedging results:

VarHedgingError[Δ0]

$$-\frac{1}{4}\left(S^4\,\sigma^4\left(1+\text{EW}[3]^2-\text{EW}[4]\right)V^{(0,2)}[t,S]^2\right)dt^4-\frac{1}{12}\left(S^4\,\sigma^5\,\text{EW}[3]\,V^{(0,2)}[t,S]\left(\left(3-3\,\text{EW}[3]^2+9\,\text{EW}[4]\right)V^{(0,2)}[t,S]+2\,S\,(1+\text{EW}[4])\,V^{(0,3)}[t,S]\right)\right)dt^5-\frac{1}{144}\left(S^4\,\sigma^4\left(3\left(48\,\mu\left(1+\text{EW}[3]^2-\text{EW}[4]\right)+\sigma^2\left(3+12\,\text{EW}[3]^4+\text{EW}[3]^2\,(11-43\,\text{EW}[4])-4\,\text{EW}[4]+27\,\text{EW}[4]^2\right)\right)V^{(0,2)}[t,S]^2+4\,S^2\,\sigma^2\left(\text{EW}[3]^2+\text{EW}[4]^2\right)V^{(0,3)}[t,S]^2+6\,V^{(0,2)}[t,S]\left(2\,S\left(6\,\mu\left(1+\text{EW}[3]^2-\text{EW}[4]\right)+\sigma^2\left(\text{EW}[3]^2\,(1-2\,\text{EW}[4])+\text{EW}[4]\,(2+3\,\text{EW}[4])\right)\right)V^{(0,3)}[t,S]+S^2\,\sigma^2\,\text{EW}[4]\,V^{(0,4)}[t,S]+12\left(1+\text{EW}[3]^2-\text{EW}[4]\right)V^{(1,2)}[t,S]\right)\right)\right)dt^6+O[dt]^7$$

ExpValue[dP[Δ0]]

$$\left(-r\,V[t,S]+r\,S\,V^{(0,1)}[t,S]+\frac{1}{2}\,S^2\,\sigma^2\,V^{(0,2)}[t,S]+V^{(1,0)}[t,S]\right)dt^2+\frac{1}{12}\left(6\,r\,S^2\,\sigma\,\text{EW}[3]\,V^{(0,2)}[t,S]-6\,S^2\,\mu\,\sigma\,\text{EW}[3]\,V^{(0,2)}[t,S]+3\,S^2\,\sigma^3\,\text{EW}[3]\,V^{(0,2)}[t,S]+2\,S^3\,\sigma^3\,\text{EW}[3]\,V^{(0,3)}[t,S]\right)dt^3+\frac{1}{24}\left(-12\,r^2\,V[t,S]+12\,r^2\,S\,V^{(0,1)}[t,S]+24\,r\,S^2\,\mu\,V^{(0,2)}[t,S]-12\,S^2\,\mu^2\,V^{(0,2)}[t,S]-6\,r\,S^2\,\sigma^2\,V^{(0,2)}[t,S]+30\,S^2\,\mu\,\sigma^2\,V^{(0,2)}[t,S]+3\,S^2\,\sigma^4\,V^{(0,2)}[t,S]-12\,r\,S^2\,\sigma^2\,\text{EW}[3]^2\,V^{(0,2)}[t,S]+12\,S^2\,\mu\,\sigma^2\,\text{EW}[3]^2\,V^{(0,2)}[t,S]+4\,S^2\,\sigma^4\,\text{EW}[3]^2\,V^{(0,2)}[t,S]+18\,r\,S^2\,\sigma^2\,\text{EW}[4]\,V^{(0,2)}[t,S]-18\,S^2\,\mu\,\sigma^2\,\text{EW}[4]\,V^{(0,2)}[t,S]-2\,S^2\,\sigma^4\,\text{EW}[4]\,V^{(0,2)}[t,S]+12\,S^3\,\mu\,\sigma^2\,V^{(0,3)}[t,S]+4\,r\,S^3\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]-4\,S^3\,\mu\,\sigma^2\,\text{EW}[4]\,V^{(0,3)}[t,S]+4\,S^3\,\sigma^4\,\text{EW}[4]\,V^{(0,3)}[t,S]+S^4\,\sigma^4\,\text{EW}[4]\,V^{(0,4)}[t,S]+24\,r\,S\,V^{(1,1)}[t,S]+12\,S^2\,\sigma^2\,V^{(1,2)}[t,S]+12\,V^{(2,0)}[t,S]\right)dt^4+O[dt]^5$$

ExpValue[dP[Δ0] − ((−r V[t, S] + r S V^(0,1)[t, S] + 1/2 S^2 σ^2 V^(0,2)[t, S] + V^(1,0)[t, S]) dt^2 + 1/12 (6 r S^2 σ EW[3] V^(0,2)[t, S] − 6 S^2 μ σ EW[3] V^(0,2)[t, S] + 3 S^2 σ^3 EW[3] V^(0,2)[t, S] + 2 S^3 σ^3 EW[3] V^(0,3)[t, S]) dt^3)] + O[dt]^n

$$O[dt]^4$$

VarHedgingError[ΔW]

$$\frac{1}{4} S^4 \sigma^4 (-1 + EW[4]) V^{(0,2)}[t, S]^2 dt^4 -$$
$$\frac{1}{12} \left(S^4 \sigma^5 EW[3] V^{(0,2)}[t, S] \left((3 + 9 EW[4]) V^{(0,2)}[t, S] + 2 S (1 + EW[4]) V^{(0,3)}[t, S]\right)\right) dt^5 -$$
$$\frac{1}{144} \left(S^4 \sigma^4 \left(3 \left(-48 \mu (-1 + EW[4]) + \sigma^2 \left(3 + 12 EW[3]^2 - 4 EW[4] + 27 EW[4]^2\right)\right) V^{(0,2)}[t, S]^2 + \right.\right.$$
$$4 S^2 \sigma^2 \left(EW[3]^2 + EW[4]^2\right) V^{(0,3)}[t, S]^2 + 6 V^{(0,2)}[t, S]$$
$$\left(2 S \left(-6 \mu (-1 + EW[4]) + \sigma^2 \left(2 EW[3]^2 + EW[4] (2 + 3 EW[4])\right)\right) V^{(0,3)}[t, S] + \right.$$
$$\left.\left.\left. S^2 \sigma^2 EW[4] V^{(0,4)}[t, S] - 12 (-1 + EW[4]) V^{(1,2)}[t, S]\right)\right)\right) dt^6 + O[dt]^7$$

ExpValue[dP[ΔW]]

$$\left(-r V[t, S] + r S V^{(0,1)}[t, S] + \frac{1}{2} S^2 \sigma^2 V^{(0,2)}[t, S] + V^{(1,0)}[t, S]\right) dt^2 +$$
$$\frac{1}{6} S^2 \sigma^3 EW[3] \left(3 V^{(0,2)}[t, S] + S V^{(0,3)}[t, S]\right) dt^3 +$$
$$\frac{1}{24} \left(-12 r^2 V[t, S] + 12 r^2 S V^{(0,1)}[t, S] + 24 r S^2 \mu V^{(0,2)}[t, S] - \right.$$
$$12 S^2 \mu^2 V^{(0,2)}[t, S] - 6 r S^2 \sigma^2 V^{(0,2)}[t, S] + 30 S^2 \mu \sigma^2 V^{(0,2)}[t, S] +$$
$$3 S^2 \sigma^4 V^{(0,2)}[t, S] + 18 r S^2 \sigma^2 EW[4] V^{(0,2)}[t, S] - 18 S^2 \mu \sigma^2 EW[4] V^{(0,2)}[t, S] -$$
$$2 S^2 \sigma^4 EW[4] V^{(0,2)}[t, S] + 12 S^3 \mu \sigma^2 V^{(0,3)}[t, S] + 4 r S^3 \sigma^2 EW[4] V^{(0,3)}[t, S] -$$
$$4 S^3 \mu \sigma^2 EW[4] V^{(0,3)}[t, S] + 4 S^3 \sigma^4 EW[4] V^{(0,3)}[t, S] + S^4 \sigma^4 EW[4] V^{(0,4)}[t, S] +$$
$$\left. 24 r S V^{(1,1)}[t, S] + 12 S^2 \sigma^2 V^{(1,2)}[t, S] + 12 V^{(2,0)}[t, S]\right) dt^4 + O[dt]^5$$

ExpValue[dP[ΔW] − ((−r V[t, S] + r S V^(0,1)[t, S] + 1/2 S^2 σ^2 V^(0,2)[t, S] + V^(1,0)[t, S]) dt^2 + O[dt]^n)]

$$\frac{1}{6} S^2 \sigma^3 EW[3] \left(3 V^{(0,2)}[t, S] + S V^{(0,3)}[t, S]\right) dt^3 + O[dt]^4$$