```
A = {Ax, Ay, Az, At}; d = {-dx, -dy, -dz, dt};

Aeu = {A[[1]], A[[2]], A[[3]], I A[[4]]}

deu = {-d[[1]], -d[[2]], -d[[3]], -I d[[4]]}
```

$$\{dx,\ dy,\ dz,\ -i\ dt\}$$

```
g = DiagonalMatrix[{-1, -1, -1, 1}];

F = Table[d[[a]] A[[b]] - d[[b]] A[[a]] - I G A[[a]] A[[b]], {b, 1, 4}, {a, 1, 4}]
```

$$\left\{\left\{-i\ Ax^2\ G,\ Ay\ dx - Ax\ dy - i\ Ax\ Ay\ G,\ Az\ dx - Ax\ dz - i\ Ax\ Az\ G,\ Ax\ dt + At\ dx - i\ At\ Ax\ G\right\},\right.$$
$$\left\{-Ay\ dx + Ax\ dy - i\ Ax\ Ay\ G,\ -i\ Ay^2\ G,\ Az\ dy - Ay\ dz - i\ Ay\ Az\ G,\ Ay\ dt + At\ dy - i\ At\ Ay\ G\right\},$$
$$\left\{-Az\ dx + Ax\ dz - i\ Ax\ Az\ G,\ -Az\ dy + Ay\ dz - i\ Ay\ Az\ G,\ -i\ Az^2\ G,\ Az\ dt + At\ dz - i\ At\ Az\ G\right\},$$
$$\left.\left\{-Ax\ dt - At\ dx - i\ At\ Ax\ G,\ -Ay\ dt - At\ dy - i\ At\ Ay\ G,\ -Az\ dt - At\ dz - i\ At\ Az\ G,\ -i\ At^2\ G\right\}\right\}$$

```
S1 = Expand[Tr[F.g.Transpose[F].g] / 2]
```

$$-Ax^2\ dt^2 - Ay^2\ dt^2 - Az^2\ dt^2 - 2\ At\ Ax\ dt\ dx - At^2\ dx^2 + Ay^2\ dx^2 + Az^2\ dx^2 -$$
$$2\ At\ Ay\ dt\ dy - 2\ Ax\ Ay\ dx\ dy - At^2\ dy^2 + Ax^2\ dy^2 + Az^2\ dy^2 - 2\ At\ Az\ dt\ dz -$$
$$2\ Ax\ Az\ dx\ dz - 2\ Ay\ Az\ dy\ dz - At^2\ dz^2 + Ax^2\ dz^2 + Ay^2\ dz^2 - \frac{At^4\ G^2}{2} + At^2\ Ax^2\ G^2 -$$
$$\frac{Ax^4\ G^2}{2} + At^2\ Ay^2\ G^2 - Ax^2\ Ay^2\ G^2 - \frac{Ay^4\ G^2}{2} + At^2\ Az^2\ G^2 - Ax^2\ Az^2\ G^2 - Ay^2\ Az^2\ G^2 - \frac{Az^4\ G^2}{2}$$

```
Feu = Table[
    deu[[a]] Aeu[[b]] - deu[[b]] Aeu[[a]] + I G Aeu[[a]] Aeu[[b]], {b, 1, 4}, {a, 1, 4}];

S2 = Expand[Tr[Feu.Transpose[Feu]] / 2]
```

$$-Ax^2\ dt^2 - Ay^2\ dt^2 - Az^2\ dt^2 - 2\ At\ Ax\ dt\ dx - At^2\ dx^2 + Ay^2\ dx^2 + Az^2\ dx^2 -$$
$$2\ At\ Ay\ dt\ dy - 2\ Ax\ Ay\ dx\ dy - At^2\ dy^2 + Ax^2\ dy^2 + Az^2\ dy^2 - 2\ At\ Az\ dt\ dz -$$
$$2\ Ax\ Az\ dx\ dz - 2\ Ay\ Az\ dy\ dz - At^2\ dz^2 + Ax^2\ dz^2 + Ay^2\ dz^2 - \frac{At^4\ G^2}{2} + At^2\ Ax^2\ G^2 -$$
$$\frac{Ax^4\ G^2}{2} + At^2\ Ay^2\ G^2 - Ax^2\ Ay^2\ G^2 - \frac{Ay^4\ G^2}{2} + At^2\ Az^2\ G^2 - Ax^2\ Az^2\ G^2 - Ay^2\ Az^2\ G^2 - \frac{Az^4\ G^2}{2}$$

```
Simplify[S1 - S2]
```

$$0$$

```
MatrixForm[F[[1]]
```