

```
$Assumptions = dt ^ 2 == 0 && dt * dW == 0 && dW ^ 2 == dt && S > 0 && M > 0 && s > 0;
```

```
dS = r S dt + s S dW;
```

```
dP = r (P - q S) dt + q dS;
```

```
dDX = Δ dS - r Δ S dt;
```

```
dDV = Expand[Simplify[
  Normal[Series[V[a, b, c], {a, S, 2}, {b, P, 2}, {c, t, 1}]] - V[S, P, t]
  - r V[S, P, t] dt /. a → S + dS /. b → P + dP /. c → t + dt
]];
```

```
HR = Δ /. Solve[dDX == dDV /. dt → 0, Δ][[1, 1]]
```

```
FKE = Expand[dDV - dDX /. dW → 0 /. dt → 1]
```

```
q V(0,1,0)[S, P, t] + V(1,0,0)[S, P, t]
```

```
-r V[S, P, t] + V(0,0,1)[S, P, t] + P r V(0,1,0)[S, P, t] +  $\frac{1}{2}$  q2 s2 S2 V(0,2,0)[S, P, t] +
```

```
r S V(1,0,0)[S, P, t] + q s2 S2 V(1,1,0)[S, P, t] +  $\frac{1}{2}$  s2 S2 V(2,0,0)[S, P, t]
```

## Similarity reduction

```
Vr[S_, P_, t_] := S H[P/S, t]; (*P=e*S*)
```

```
FKE2 = Simplify[Simplify[(FKE /. V → Vr /. P → e * S) == 0][[1]] / 2]
```

```
 $\frac{1}{2}$  (2 H(0,1)[e, t] + (e - q)2 s2 H(2,0)[e, t])
```