```
Exit[];

$Assumptions =  r > 0 && Element[m, Integers] &&
  Element[n, Integers] && s > 0 && Element[k, Integers] && k > 0
```

r > 0 && m ∈ Integers && n ∈ Integers && s > 0 && k ∈ Integers && k > 0

2-d Dirac

```
f1[r_, En_] := {{(m - 1) / r, I * (En - r ^ p)}, {I * (En - r ^ p), -m / r}};
f1[r, En] // MatrixForm
```

$$\begin{pmatrix} \frac{-1+m}{r} & i\,(En - r^p) \\ i\,(En - r^p) & -\frac{m}{r} \end{pmatrix}$$

Diagonaldarstellung für r gegen Infinity

```
V = {{1, 1}, {-1, 1}}; Simplify[V.f1[r, En].Inverse[V]] // MatrixForm
```

$$\begin{pmatrix} i\,En - \frac{1}{2r} - i\,r^p & \frac{1-2m}{2r} \\ \frac{1-2m}{2r} & -i\,En - \frac{1}{2r} + i\,r^p \end{pmatrix}$$

```
Inverse[V].{0, C}
```

$$\left\{-\frac{C}{2}, \frac{C}{2}\right\}$$

Potenzreihenansatz mit richtigem Rangverhalten

m<=0

```
s = -m; p = 2;
```

```
f1[r, En] // MatrixForm
```

$$\begin{pmatrix} \frac{-1+m}{r} & i\left(En - r^2\right) \\ i\left(En - r^2\right) & -\frac{m}{r} \end{pmatrix}$$

```
u = {F[x], G[x]} * x ^ (s)
```

$$\{x^{-m} F[x], x^{-m} G[x]\}$$

```
r[x_] := x;
```

```
g1 = Collect[Expand[Simplify[Expand[-(D[u, x] - r '[x] * f1[r[x], En].u) / x ^ (s - 0)]]],
   {x ^ n, a[n], b[n], F[x], G[x]}];
g1
```

$$\left\{\left(-\frac{1}{x} + \frac{2\,m}{x}\right) F[x] + \left(i\,En - i\,x^2\right) G[x] - F'[x],\ \left(i\,En - i\,x^2\right) F[x] - G'[x]\right\}$$

```
f[r_, En_] := {{(-1/x + 2 m/x), (i En - i x^p)}, {(i En - i x^p), 0}}; f[r, En] // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{x} + \frac{2\,m}{x} & i\,En - i\,x^2 \\ i\,En - i\,x^2 & 0 \end{pmatrix}$$

```
u = {a[n], b[n]} * x ^ n * Exp[I * x ^ (p + 1) / (p + 1)]
```

$$\left\{e^{\frac{i x^3}{3}} x^n a[n], e^{\frac{i x^3}{3}} x^n b[n]\right\}$$

```
g1 = Collect[Expand[Simplify[
     Expand[-(D[u, x] - r '[x] * f[r[x], En].u) * x / Exp[I * x ^ (p + 1) / (p + 1)]]]],
   {x ^ n, a[n], b[n], F[x], G[x]}];
g1
```

$$\left\{x^n \left(\left(-1 + 2\,m - n - i\,x^3\right) a[n] + \left(i\,En\,x - i\,x^3\right) b[n]\right),\ x^n \left(\left(i\,En\,x - i\,x^3\right) a[n] + \left(-n - i\,x^3\right) b[n]\right)\right\}$$

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
a[0] = 0; b[0] = 1;  b[1] = 0 ; a[2] = 0 ; a[1] = -i En / 2 / (-1 + m); b[2] = i En a[1] / 2;
```

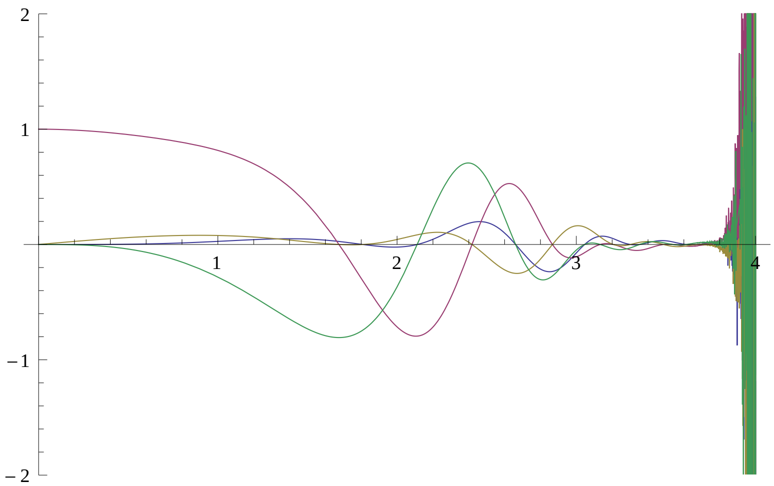
```
a[1] =.; a[2] =.; b[1] =.; b[0] =.; a[0] =.; b[2] =.; a[n_] =.; b[n_] =.
```

```
a[n_] := I / (n + 1 - 2 m) * (En b[n - 1] - a[n - 3] - b[n - 3]);
b[n_] := 1/n ⅈ (-a[n - 3] + En a[n - 1] - b[n - 3])
Un[Ene_, mm_, nN_, x_] := Module[{n, U1, U2, U3, U4, Erg},
  U1 = {a[0], b[0]}; U2 = {a[1], b[1]} /. m → mm /. En → Ene;
  U3 = {a[2], b[2]} /. m → mm /. En → Ene;
  Erg = U1 + U2 * x + U3 * x ^ 2;
  For[n = 3, n ≤ nN, n++,
   U4 = Simplify[{I / (n + 1 - 2 mm) * (Ene U3[[2]] - U1[[1]] - U1[[2]]),
       1/n ⅈ (Ene U3[[1]] - U1[[1]] - U1[[2]])}] // N;
   U1 = U2; U2 = U3; U3 = U4;
   Erg += U3 * x ^ n;

  ];

  Erg]
```

zur Probe

```
Uno[Ene_, m_, nN_, x_] := Module[{n, U, Erg = 0},
  U = {{a[0], b[0]}};
  For[n = 1, n ≤ nN + 1, n++,
   AppendTo[U, {a[n], b[n]}]
  ];
  For[n = 0, n ≤ nN, n++,
   Erg += U[[n + 1]] * x ^ n;
  ];
  Erg
 ]

Simplify[Un[En, m, 10, x] - Uno[En, m, 10, x]]

{0, 0}
```

```
G = {Re[#], Im[#]} &[Un[3, -10, 350, x]]; Plot[G, {x, 0, 4}, PlotRange → {-2, 2}]
```




```
U[En_, m_, g_, X_] := Module[{n = 10, U, G},
  U = Un[En, m, n, X]; G = -Un[En, m, n + 1, X];
  While[Sqrt[Abs[Conjugate[U - G].(U - G)]] > g,
   n++;
   U = G; G = -Un[En, m, n + 1, X];

  ];
  {Un[En, m, n, X], n}]
```

```
U[9, 5, 0.0001, 1]
```

```
{{-0.0126898, 0. + 0.00407705 i}, 14}
```

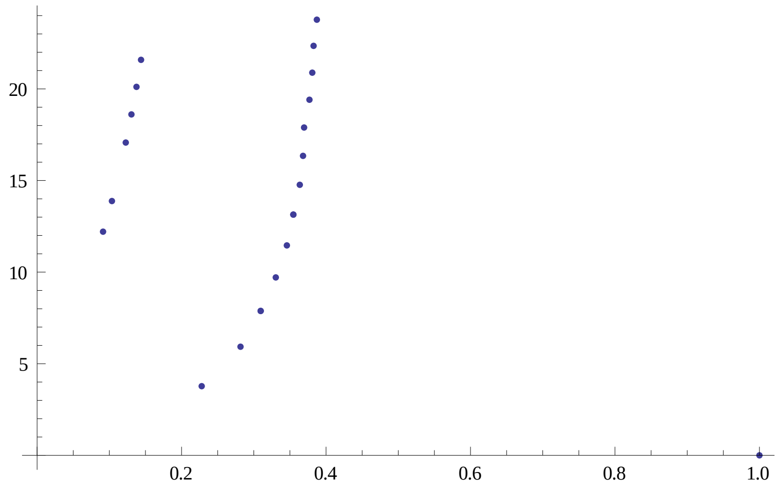
```
Ener[Ene_] :=
 Module[{U1, U2, U1S, U2S, VV = {{0, 1}, {-1, 0}}, En, Enn, NN, Erg, kE, k, n, m, r, h},
  En = Ene;
  Label[begin];
  n = 5000;
  m = 5;
  r = 7.2 // N; h = -7.0 / n;
  k = {1, -1};
  kE = {0, 0};
  Do[
   k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
   k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
   k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   k0 = h * (fE.k + f[r, En].kE); k1 = h * (fE.k + f[r + h / 2, En].(kE + k0 / 2));
   k2 = h * (fE.k + f[r + h / 2, En].(kE + k1 / 2)); k3 = h * (fE.k + f[r + h, En].(kE + k2));
   kE += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3);

   r += h;
   , {n}];

  NN = U[En, m, 0.0001, r][[2]];

  {U1, U2} = Un[En, m, NN, r];
  {U1S, U2S} = D[Un[Enn, m, NN, r], Enn] /. Enn → En;

  Erg = k[[1]] * U2 - U1 * k[[2]];

  If[Abs[Erg / U2 / k[[2]]] > 0.02,
   En -= Erg / (U2S k⟦1⟧ - U1S k⟦2⟧ + U2 kE⟦1⟧ - U1 kE⟦2⟧);
   Print[{En, Erg / U2 / k[[2]]}]; Goto[begin];
   ];
  {En, Erg / U2 / k[[2]]}
 ]

For[i = 0, i < 10, i += 0.1, Sepp = Ener[i];
 Print[{i, Sepp}]; AppendTo[Energie, {i, Sepp}];]

Ener[22]
```

```
{21.8924+0.168499 ⅈ, -0.262337+1.54006 ⅈ}
{21.7276+0.288878 ⅈ, 0.0977574+1.99468 ⅈ}
{21.5078+0.261371 ⅈ, 1.15369+1.40345 ⅈ}
{21.6825+0.0732421 ⅈ, 0.410727+0.725268 ⅈ}
{21.6223+0.137127 ⅈ, -1.84205+0.583011 ⅈ}
{21.5895+0.1468 ⅈ, -0.181458-0.525624 ⅈ}
{21.5862+0.143899 ⅈ, 0.0316819-0.0363388 ⅈ}
    {21.5862+0.143899 ⅈ, 0.000384958+0.000557679 ⅈ}
```

```
Energie = {3.77486283903418`+0.22786873407418717` ⅈ,
  5.928479968617718`+0.2815986526347655` ⅈ,
  7.8813588488087065`+0.30953294412328675` ⅈ,
  7.881329304880588`+0.3095336916562825` ⅈ,
  9.71036454189739`+0.3304415424573178` ⅈ,
  11.458077781781169`+0.3457848857997222` ⅈ,
  13.139764183242892`+0.3546744769440479` ⅈ,
  13.139114444715846`+0.3548172513559829` ⅈ,
  14.76566063867228`+0.3636637795053202` ⅈ,
  16.34527139193304`+0.3681521067888729` ⅈ,
  17.892138695416435`+0.3696213478944415` ⅈ,
  19.40831608562361`+0.37704222997528414` ⅈ,
  20.886015743223094`+0.38097023642873473` ⅈ,
  22.34958939297449`+0.38276309915136164` ⅈ,
  23.777494591007947`+0.3872991704766671` ⅈ, 12.207301904640477`+0.09130244038237889` ⅈ,
  13.879319291317909`+0.10359131716054014` ⅈ, 17.071331521063737`+0.12274708920757649` ⅈ,
  18.607651751873114`+0.1305970820854307` ⅈ, 20.11139546248976`+0.1375815129955693` ⅈ,
  21.586240037178456`+0.14389862151162078` ⅈ}; Energie // MatrixForm
```

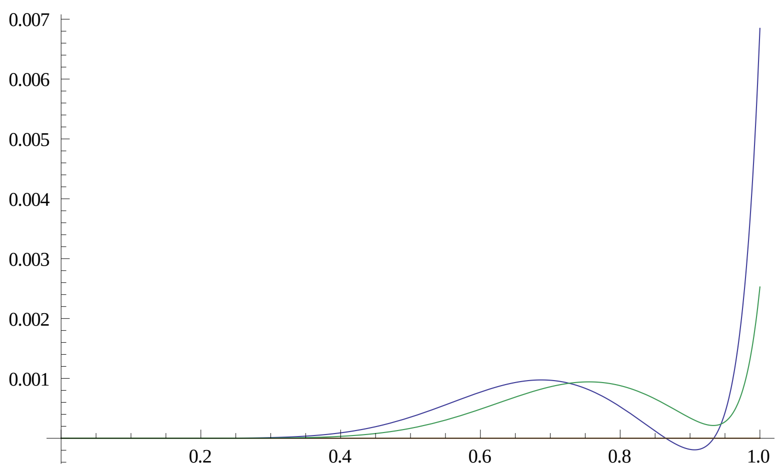
$$\begin{pmatrix} 3.77486+0.227869\,i \\ 5.92848+0.281599\,i \\ 7.88136+0.309533\,i \\ 7.88133+0.309534\,i \\ 9.71036+0.330442\,i \\ 11.4581+0.345785\,i \\ 13.1398+0.354674\,i \\ 13.1391+0.354817\,i \\ 14.7657+0.363664\,i \\ 16.3453+0.368152\,i \\ 17.8921+0.369621\,i \\ 19.4083+0.377042\,i \\ 20.886+0.38097\,i \\ 22.3496+0.382763\,i \\ 23.7775+0.387299\,i \\ 12.2073+0.0913024\,i \\ 13.8793+0.103591\,i \\ 17.0713+0.122747\,i \\ 18.6077+0.130597\,i \\ 20.1114+0.137582\,i \\ 21.5862+0.143899\,i \end{pmatrix}$$

```
ListPlot[Append[{Im[#], Re[#]} & /@ Energie, {1, 0}],
 AxesOrigin → {0, 0}, PlotRange → All]
```



:

```
G = {Re[#], Im[#]} &[Un[16, 10, 15, x]]; Plot[G, {x, 0, 1}, PlotRange → All]
```



```
n = 8000; S = 1; h = 6 / n; ra = 1; En = Energie[[17]]; m = 5; r = 1;
U[En, m, 10^-10, r][[2]]
k = U[En, m, 10^-10, r][[1]];
kK = {{r, k}};
Do[
  k0 = h * f[r, En].k; k1 = h * f[r + h / 2, En].(k + k0 / 2);
  k2 = h * f[r + h / 2, En].(k + k1 / 2); k3 = h * f[r + h, En].(k + k2);
  k += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
  AppendTo[kK, {r, k}], {n}];

ListPlot[Join[{ {#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; n]] // N},
  { {#[[1]], Im[#[[2, 1]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{ {#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; n]] // N},
  { {#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; n]] // N}], PlotRange → All, Joined → True]
En =.;
r =.;
```

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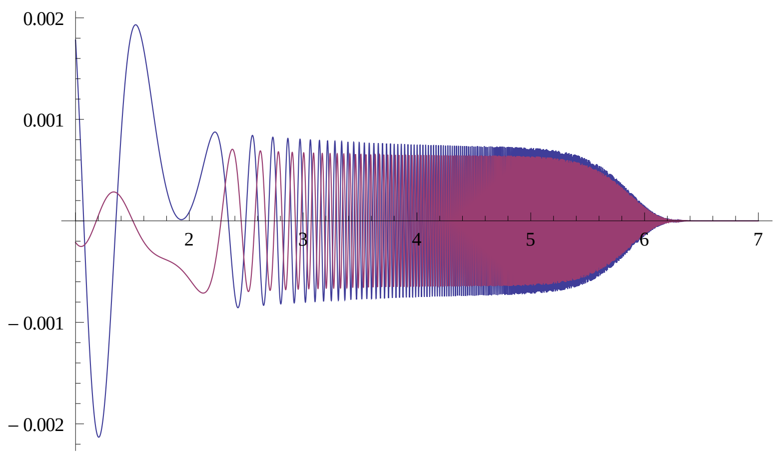
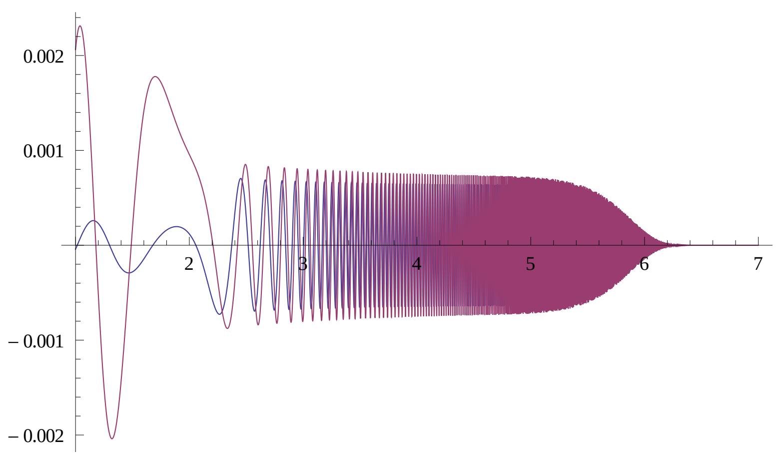
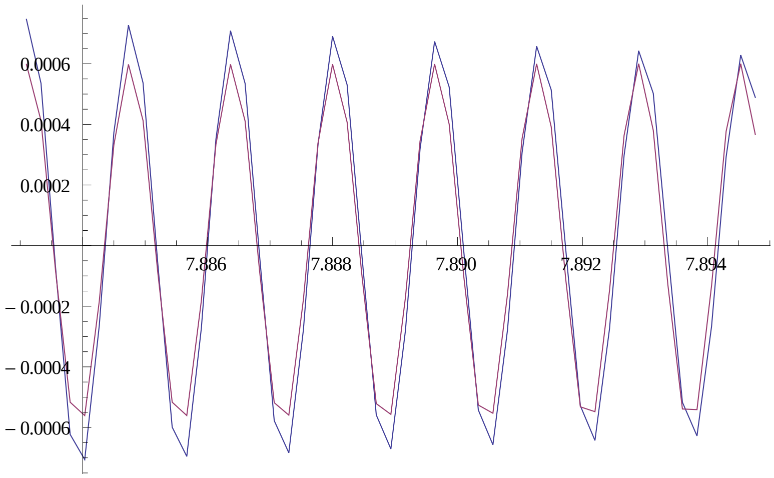
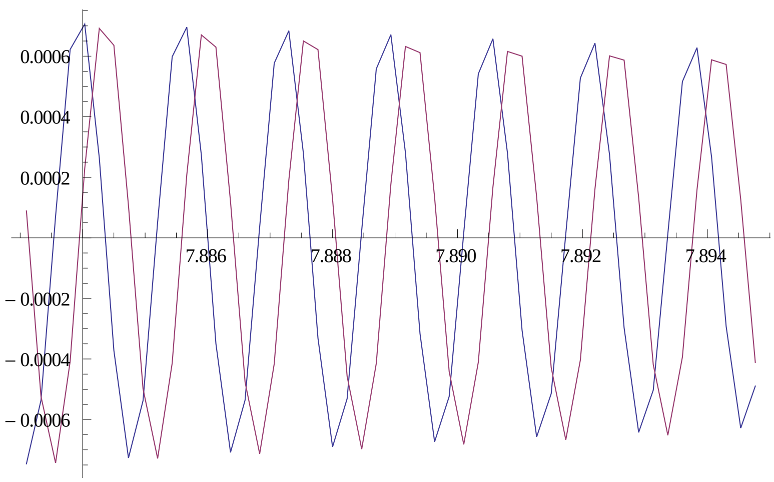





```
U[En, m, 10^-10, r][[1]]
```

```
S = 29500; n = 50; ListPlot[Join[{{#[[1]], Re[#[[2, 1]]]} & /@ kK[[S ;; S + n]] // N},
  {{#[[1]], -0.0006 * Sin[#[[1]] ^ 5 / 5 + 1 - #[[1]] * Re[Energie[[11]]]]} & /@
     kK[[S ;; S + n]] // N}], PlotRange → All, Joined → True]
ListPlot[Join[{{#[[1]], Re[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N},
  {{#[[1]], Im[#[[2, 2]]]} & /@ kK[[S ;; S + n]] // N}], PlotRange → All, Joined → True]
```






```
Exp[I * Im[Energie[[1]]] * x]
```

$e^{0.278733\, i\, x}$