```
Exit[]

BS[s_, SK_, r_, T_] := SK CDF[NormalDistribution[], d[SK, s, r, T]] -
  Exp[-r T] CDF[NormalDistribution[], d[SK, s, r, T] - s Sqrt[T]];
d[sk_, s_, r_, T_] := (Log[sk] + (r + s ^ 2 / 2) T) / s / Sqrt[T]
Preis[Startkapital_, Gewinnschwelle_, Sigma_, Laufzeit_, r_] :=
  Gewinnschwelle * BS[Sigma, Startkapital / Gewinnschwelle, r, Laufzeit];
D[Preis[p, G, s, T, r], p];
Delta[p_, G_, s_, T_, r_] :=
```

$$G\left(\frac{e^{-\frac{\left(\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2s^2T}}}{G\sqrt{2\pi}\,s\sqrt{T}}-\frac{e^{-rT-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}}{p\sqrt{2\pi}\,s\sqrt{T}}+\frac{1+\mathrm{Erf}\left[\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{\sqrt{2}\,s\sqrt{T}}\right]}{2G}\right)$$

```
D[Delta[p, G, s, T, r] q p / S, p]

Gam[p_, G_, s_, T_, r_] :=
```

$$G\left(\frac{e^{-\frac{\left(\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2s^2T}}}{G\sqrt{2\pi}\,s\sqrt{T}}-\frac{e^{-rT-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}}{p\sqrt{2\pi}\,s\sqrt{T}}+\frac{1+\mathrm{Erf}\left[\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{\sqrt{2}\,s\sqrt{T}}\right]}{2G}\right)+$$

$$G\,p\left(\frac{e^{-rT-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}}{p^2\sqrt{2\pi}\,s\sqrt{T}}+\frac{e^{-\frac{\left(\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2s^2T}}}{G\,p\sqrt{2\pi}\,s\sqrt{T}}-\frac{e^{-\frac{\left(\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)^2}{2s^2T}}\left(\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]\right)}{G\,p\sqrt{2\pi}\,s^3\,T^{3/2}}+\left(e^{-rT-\frac{1}{2}\left(-s\sqrt{T}+\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)^2}\left(-s\sqrt{T}+\frac{\left(r+\frac{s^2}{2}\right)T+\mathrm{Log}\left[\frac{p}{G}\right]}{s\sqrt{T}}\right)\right)\Big/\left(p^2\sqrt{2\pi}\,s^2\,T\right)\right)$$

Processes (real-world, non-risk-neutral)

```
$Assumptions = dt ^ 2 == 0 && dt * dW == 0 && dW ^ 2 == dt && S > 0 && M > 0 && s > 0;

dS = α S dt + σ S dW; (*Aktie*)
dP = r (P - q * P) dt + q P / S dS; (*Kundenportfolio mit Zins r*)
 dX = Δ dS + r (X - Δ S) dt;(*Heding portfolio*)
Δ = q D[V[P, t], P];(*Heding rule*)

dLogP = Simplify[dP / P - 1 / 2 dP ^ 2 / P ^ 2](*LogKundenportfolio mit Zins r*)
Simplify[dX]
```

$$dW\ q\ \sigma + dt\left(r - q\ r + q\ \alpha - \frac{q^2\ \sigma^2}{2}\right)$$

$$dt\ r\ X + q\ S\ (dt\ (-r + \alpha) + dW\ \sigma)\ V^{(1,0)}[P, t]$$

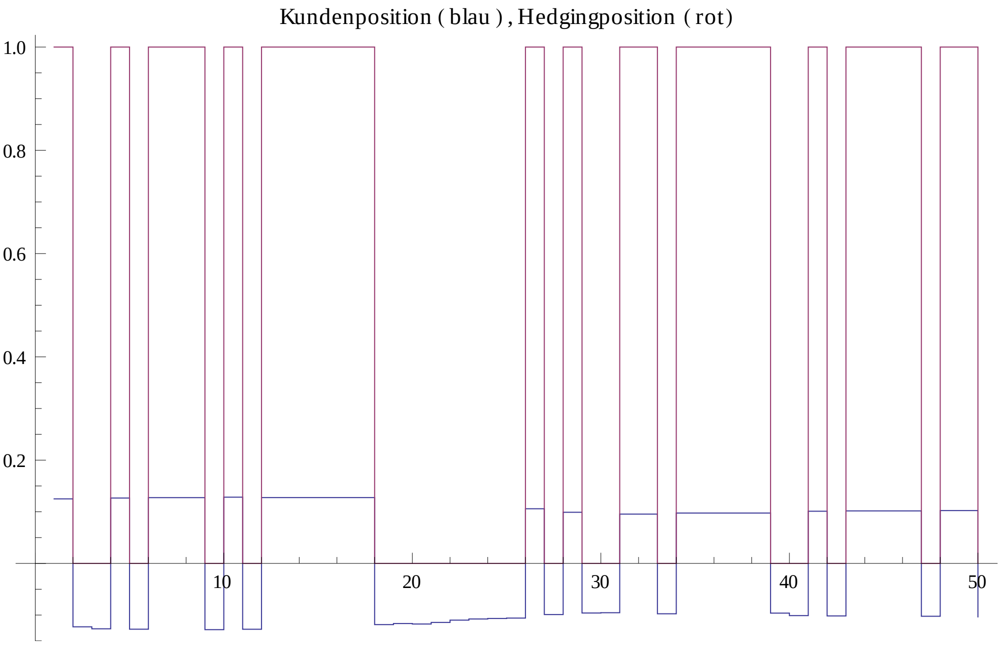
Hedging Simulation:

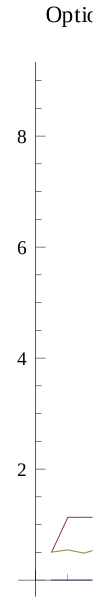
```
P0 = 10; M = 10; S0 = 80;
σ = 0.3;(*Volaitlität*)
r = 0.09;(*risk-free Zinssatz*)
T = 50 / 365;(*Laufzeit in jahren*)
α = 0.2;(*Stock drift*)
K = 1;(*Hedges täglich*)
nt = Ceiling[365 T ] K; dt = N[T / nt]
n = 1;(*MonteCarlo Durchläufe*)



(*Kunde2*) qk = RandomReal[{-1, 1}, {nt / K}];
(*schlimmster Kunde*) qk = Table[1, {nt / K}];
(*Kunde3*) qk = RandomInteger[{0, 1}, {nt / K}];
(*schlimmster Kunde*) qk = 2 * RandomInteger[{0, 1}, {nt / K}] - 1;

dW = RandomReal[NormalDistribution[], {nt n}] Sqrt[dt];
Timing[
 PE = 0; PV = 0;

 (*MonteCarlo Loop*)
 For[j = 0, j < n, j++,

  P = Log[P0]; W = 0; S = S0; s = {S0}; p = {P0 10 000};
  X = Preis[P0, M, σ, T, r]; x = {X}; h = {}; v = {}; Q = {};

  (*Time loop*)
  For[i = 1, i < nt + 1, i++,
```

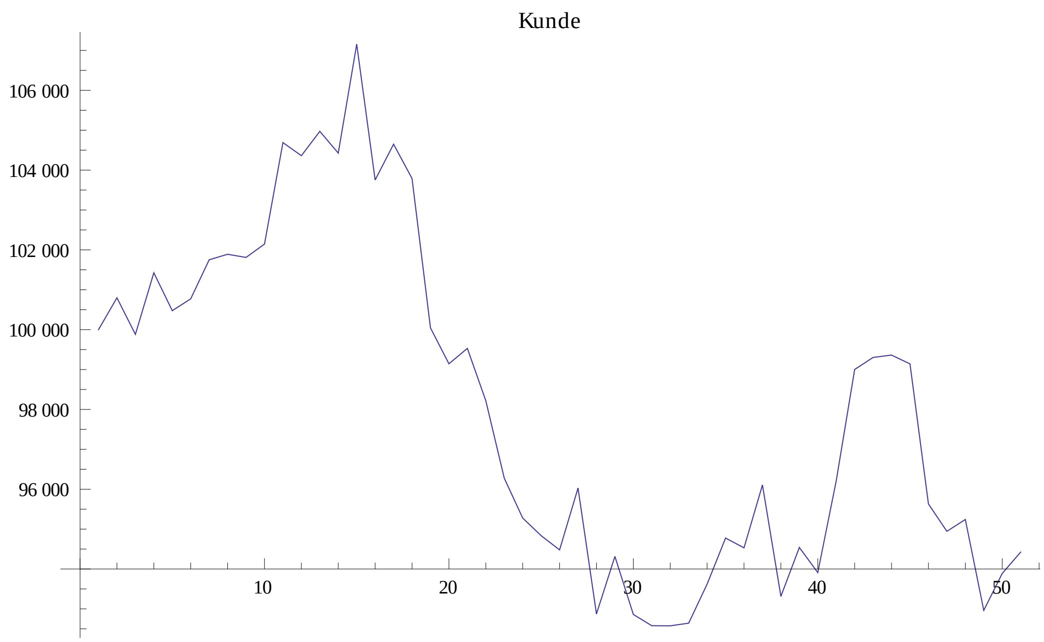
```
    AppendTo[v, Preis[Exp[P], M, σ, T - dt * (i - 1), r]];
    W += dW[[i]]; (*Brownian Motion*)
    dS = Exp[(α - σ ^ 2 / 2) i dt + σ W] S0 - S; (*Stock price Increment*)
    q = qk[[Ceiling[i / K]]] ;
    H = Ceiling[q Exp[P] / S Delta[Exp[P], M, σ, T - dt * (i - 1), r]];
    (*new Hedgingposition*)
    X += dt r X + H S (dt (-r + α) + dW[[i]] σ) ; (*new Hedgingportfolio*)
    P += dW[[i]] q σ + dt q (α - q σ^2/2); (*new Portfolio*)



    S += dS; (*new Stockprice*)

    AppendTo[Q, q Exp[P] / S];
    AppendTo[s, S];
    AppendTo[p, Exp[P] 10000];
    AppendTo[x, X];

    AppendTo[h, H];

   ]
   (*PE+=Max[Exp[P]*P0-M,0];PV+=Max[Exp[P]*P0-M,0]^2;{"Mean:",
    Exp[-r T]PE/n,"2 StD of Mean:",2 Sqrt[Exp[-2r T]/n/(n-1)(PV-PE^2/n)]}*)
  ];]
AppendTo[v, Max[p[[nt + 1]] / 10000 - M, 0]];
Ausz = Max[p[[nt + 1]] / 10000 - M, 0]; Print[""];
GraphicsGrid[{{ListLinePlot[{Q, h}, InterpolationOrder → 0,
     PlotLabel → Text["Kundenposition(blau),Hedgingposition(rot)"]],
    ListLinePlot[{Ausz & /@ x, x, v}, InterpolationOrder → 1, PlotRange → All,
     PlotLabel → Text["Auszahlung (const), Hedgingportfolio (rot)
Optionspreis: ", x[[1]], " / Auszahlung: ", Ausz, " / Hedgingergebnis: ", x[[nt + 1]], "
Kosten: ", x[[1]] + Ausz - x[[nt + 1]]]]},
   {ListLinePlot[{p}, InterpolationOrder → 1, PlotLabel -> Text["Kunde"]], ListLinePlot[
     {s}, InterpolationOrder → 1, PlotLabel -> Text["Aktie"]]}}, ImageSize → 1000]
```

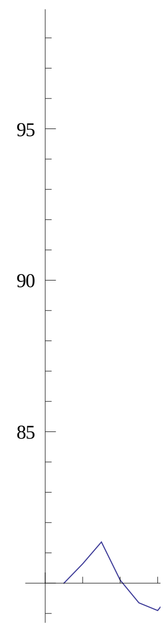
0.00273973

{0.016, Null}










```
Total[Abs[Differences[h]]] + h[[1]] + h[[nt]]
Length[h]
```

90.5624

50

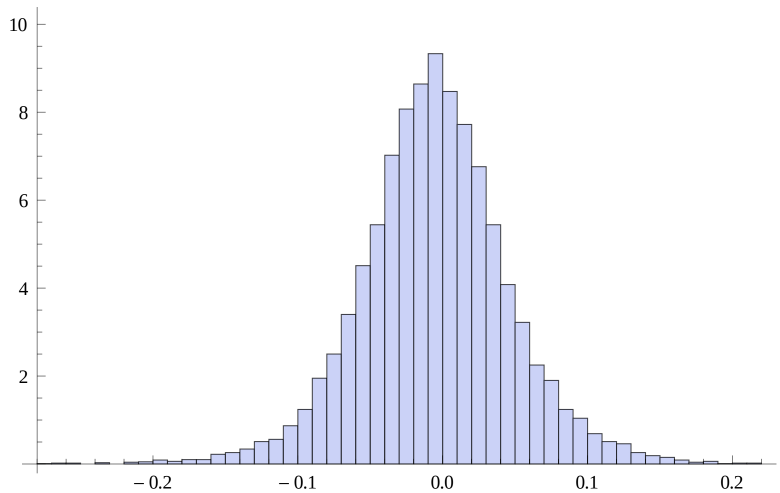
```
P0 = 10; M = 10; S0 = 6;
σ = 0.3;(*Volaitlität*)
r = 0.09;(*risk-free Zinssatz*)
T = 1 / 12;(*Laufzeit in jahren*)
α = 1.2;(*Stock drift*)
```

```
K = 1;(*Hedges täglich*)
nt = Ceiling[365 T ] K; dt = N[T / nt];
n = 10 000;(*MonteCarlo Durchläufe beeinflussen die Genauigkeit des ergebnisses*)

(*Kunde2*)qk = RandomReal[{-1, 1}, {nt / K}];
(*Kunde3*)qk = RandomInteger[{0, 1}, {nt / K}];
(*schlimmster Kunde*)qk = 2 * RandomInteger[{0, 1}, {nt / K}] - 1;
(*schlimmster Kunde*)qk = Table[1, {nt / K}];

dW = RandomReal[NormalDistribution[], {nt n}] Sqrt[dt];

Timing[
 PE = 0; PV = 0; PVV = 0; PVVV = 0; pe = {};

 (*MonteCarlo Loop*)
 For[j = 0, j < n, j++,

   P = Log[P0]; W = 0; S = S0; X = Preis[P0, M, σ, T, r];

   (*Time loop*)
   For[i = 1, i < nt + 1, i ++,

    W += dW[[i + j nt]]; (*Brownian Motion*)
    dS = Exp[(α - σ ^ 2 / 2) i dt + σ W] S0 - S; (*Stock price Increment*)
    q = qk[[Ceiling[i / K]]];
    H = q Exp[P] / S Delta[Exp[P], M, σ, T - dt * (i - 1), r]; (*new Hedgingposition*)
    X += dt r X + H S (dt (-r + α) + dW[[i + j nt]] σ) ; (*new Hedgingportfolio*)
    P += dW[[i + j nt]] q σ + dt q (α - q σ^2/2); (*new Portfolio*)

    S += dS;(*new Stockprice*)
   ];

   AppendTo[pe, X - Max[Exp[P] - M, 0]];
   PE += X - Max[Exp[P] - M, 0]; PV += (Max[Exp[P] - M, 0] - X) ^ 2;
   PVV += (-Max[Exp[P] - M, 0] + X) ^ 3; PVVV += (Max[Exp[P] - M, 0] - X) ^ 4]
  Print["Option Price:        ", Preis[P0, M, σ, T, r]];
 Print["Hedgegewinn:        ", Exp[-r T] PE / n,
  " (Error", Exp[-r T] Sqrt[1 / n / (n - 1) (PV - PE ^ 2 / n)], ")"];
 Print["2 StandardDeviations:  ", 2 Exp[-r T] Sqrt[1 / (n - 1) (PV - PE ^ 2 / n)],
  "  (Error ", 2 Exp[-r T]
   Sqrt[Sqrt[(PVVV - 4 PVV PE / n + 6 PV PE ^ 2 / n ^ 2 - 3 PE ^ 4 / n ^ 3) / (n - 1)] / n], " ⇔ ",
  100 Sqrt[Sqrt[(PVVV - 4 PVV PE / n + 6 PV PE ^ 2 / n ^ 2 - 3 PE ^ 4 / n ^ 3) / (n - 1)] / n] /
    Sqrt[1 / (n - 1) (PV - PE ^ 2 / n)], "%)"];
 Print["Shortfall(Verlust)wahrscheinlichkeit:        ",
  N[Length[Select[pe, # < 0 &]] / n 100], " %"];
 Print["(Error verringern durch höhreres n)"];
```

```
]
Histogram[pe, Automatic, "ProbabilityDensity", Epilog → First@Plot[
    PDF[NormalDistribution[Exp[-r T] PE / n, Exp[-r T] Sqrt[1 / (n - 1) (PV - PE ^ 2 / n)]],
     x], {x, -4, 4}, PlotStyle → Red], PlotRange → All]
```

```
Option Price:       0.382744
Hedgegewinn:        -0.00551279 (Error0.000509853)
2 StandardDeviations:  0.101971  (Error 0.00147121 ⇔ 1.44278%)
Shortfall(Verlust)wahrscheinlichkeit:       55.38 %
(Error verringern durch höhreres n)
```

{23.649, Null}



P2 – P

0.