```
p[h_] := (Normal[Series[u[x], {x, y, 3}]] /. x → y + h) - u[h]
```

```
p[-h]
```

$$-u[-h]+u[y]-h\,u'[y]+\frac{1}{2}\,h^2\,u''[y]-\frac{1}{6}\,h^3\,u^{(3)}[y]$$

```
a = Expand[p[-h1] / h1^2 - p[h2] / h2^2]
```

$$-\frac{u[-h1]}{h1^2}+\frac{u[h2]}{h2^2}+\frac{u[y]}{h1^2}-\frac{u[y]}{h2^2}-\frac{u'[y]}{h1}-\frac{u'[y]}{h2}-\frac{1}{6}\,h1\,u^{(3)}[y]-\frac{1}{6}\,h2\,u^{(3)}[y]$$

```
b = Solve[a == 0, D[u[y], y]]
```

$$\left\{\left\{u'[y]\to\frac{1}{6\,h1\,h2\,(h1+h2)}\left(-6\,h2^2\,u[-h1]+6\,h1^2\,u[h2]-6\,h1^2\,u[y]+6\,h2^2\,u[y]-h1^3\,h2^2\,u^{(3)}[y]-h1^2\,h2^3\,u^{(3)}[y]\right)\right\}\right\}$$

```
Expand[b[[1, 1, 2]]]
```

$$-\frac{h2\,u[-h1]}{h1\,(h1+h2)}+\frac{h1\,u[h2]}{h2\,(h1+h2)}-\frac{h1\,u[y]}{h2\,(h1+h2)}+\frac{h2\,u[y]}{h1\,(h1+h2)}-\frac{h1^2\,h2\,u^{(3)}[y]}{6\,(h1+h2)}-\frac{h1\,h2^2\,u^{(3)}[y]}{6\,(h1+h2)}$$

```
Simplify[-h2 u[-h1] / (h1 (h1 + h2)) + h1 u[h2] / (h2 (h1 + h2)) - h1 u[y] / (h2 (h1 + h2)) + h2 u[y] / (h1 (h1 + h2))]
```

$$\frac{-h2^2\,u[-h1]+h1^2\,u[h2]+\left(-h1^2+h2^2\right)u[y]}{h1\,h2\,(h1+h2)}$$

```
a = -h2^2 / (h1 h2 (h1 + h2));
b = (h1^2 - h2^2) / (h1 h2 (h1 + h2));
c = h1^2 / (h1 h2 (h1 + h2));
```

```
1 / a
```

$$-\frac{h1\,(h1+h2)}{h2}$$

```
b / a
```

$$-\frac{h1^2-h2^2}{h2^2}$$

```
c / a
```

$$-\frac{h1^2}{h2^2}$$

```
Simplify[b / a + c / a]
```

$$1-\frac{2\,h1^2}{h2^2}$$