Definitions

```
$Assumptions = {Bax ∈ Reals, Bay ∈ Reals, Baz ∈ Reals, Bsx ∈ Reals,
    Bsy ∈ Reals, Bsz ∈ Reals, ΓL ∈ Reals, ΓR ∈ Reals, ΓM ∈ Reals, ΓL ∈ Reals,
    Δ ∈ Reals, tc ∈ Reals, J ∈ Reals, ℏ ∈ Reals, tc > 0, J > 0, ℏ > 0, ΓL > 0,
    ΓR > 0, ΓM > 0, lu1 ∈ Reals, lu2 != 0, Bax != 0, Bay != 0, Baz != 0,
    Bsx != 0, Bsy != 0, Bsz != 0, ΓL != 0, ΓR != 0, ΓM != 0, ΓL != 0, Δ != 0};
```

LinearSystem =

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\Gamma M\ \hbar}{2}, 0, tc, -\frac{\Gamma R\ \hbar}{2}, 0\right\},\right.$$

$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Bsx, -Bsy, 0, -2\,tc, 0, -Bsz + 4\,J + 2\,\Delta,$
$\quad \Gamma R\ \hbar, 0, Bax, -Bay, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Bsy, Bsx,$
$\quad 0, 0, -2\,tc, -\Gamma R\ \hbar, -Bsz + 4\,J + 2\,\Delta, 0, Bay, Bax, 0, 0\}, \left\{0, 0, 0, 0, 0, 0, 0,\right.$

$$\left.0, 0, 0, 2\,Bsy, 2\,Bsx, 0, 0, 0, 0, 0, 2\,Bay, -2\,Bax, 0, 0, 0, 0, 0, \frac{\Gamma R\ \hbar}{2}, 0\right\},$$

$\{0, 0, 0, 0, 0, 0, 0, Bax, -Bay, 0, 0, 0, 0, 0, -Baz, 0, 0, 0, 0, -Bax, -Bay,$
$\quad 2\,tc, -2\,\Delta, -(\Gamma M + \Gamma R)\ \hbar, -2\,tc, 0\}, \{0, 0, 0, 0, 0, 0, 0, Bay, Bax, 0,$
$\quad 0, 0, 0, 0, -Baz, 0, 0, 0, Bay, -Bax, 0, (\Gamma M + \Gamma R)\ \hbar, -2\,\Delta, 0, 0\},$
$\{0, 0, 0, 0, 0, 0, 0, Bsx, -Bsy, 0, 0, 0, -2\,tc, 0, 2\,(2\,J + \Delta), \Gamma R\ \hbar, 0,$
$\quad 0, 0, Bsx, Bsy, 0, Baz, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, Bsy, Bsx, 0, 0,$
$\quad 0, 0, -2\,tc, -\Gamma R\ \hbar, 2\,(2\,J + \Delta), 0, 0, 0, -Bsy, Bsx, 0, 0, Baz, 0, 0\},$
$\{0, 0, 0, 0, 0, 0, 2\,tc, \Gamma R\ \hbar, -Bsz - 2\,(2\,J + \Delta), 0, 0, 0, 0, 0, Bsy, -Bsx,$
$\quad 0, 0, 0, 0, 0, 0, -Bay, Bax, 0, 0\}, \left\{0, 0, 0, 0, 0, 2\,Bay, 2\,Bax, 0, 0, 0,\right.$

$$\left.0, 0, 0, -2\,Baz, 0, 0, 0, 2\,Bay, -2\,Bax, 0, 0, 2\,\Gamma M\ \hbar, 0, 4\,tc, -\frac{\Gamma R\ \hbar}{2}, 0\right\},$$

$\{0, 0, 0, 0, 0, 2\,tc, 0, -Bsz - 2\,(2\,J + \Delta), -\Gamma R\ \hbar, 0, 0, 0, 0, 0, -Bsx, -Bsy,$
$\quad 0, 0, 0, 0, 0, 0, Bax, Bay, 0, 0\}, \{0, 0, 0, Bax, -Bay, 0, 0, 0, 0, 0, -Baz,$
$\quad 0, Bsx, -Bsy, 0, 0, -Bax, -Bsz + 4\,J, \Gamma M\ \hbar, -2\,tc, 0, Bax, 0, 0, 0, 0\},$
$\{0, 0, 0, Bay, Bax, 0, 0, 0, 0, 0, 0, -Baz, -Bsy, -Bsx, 0, 0, Bay, \Gamma M\ \hbar,$
$\quad Bsz - 4\,J, 0, 2\,tc, -Bay, 0, 0, 0, 0\}, \{0, 0, 0, Bsx, -Bsy, 0, 0, 0,$
$\quad 0, -Bsx, Bsz, 0, -Bax, -Bay, 0, 0, Bsx, Baz, 0, 0, 0, 0, 0, 0, 0, 0\},$
$\{0, 0, 0, Bsy, Bsx, 0, 0, 0, 0, Bsy, 0, Bsz, Bay, -Bax, 0, 0, -Bsy, 0,$
$\quad -Baz, 0, 0, 0, 0, 0, 0, 0\}, \{0, Bax, -Bay, 0, 0, Bsx, -Bsy, 0, 0, -Baz,$
$\quad -Bax, Bay, 4\,J, \Gamma M\ \hbar, -2\,tc, 0, 0, Bsx, Bsy, 0, 0, Baz, 0, 0, 0, 0\},$
$\{0, Bay, Bax, 0, 0, -Bsy, -Bsx, 0, 0, 0, Bay, Bax, \Gamma M\ \hbar, -4\,J, 0, 2\,tc,$
$\quad 0, Bsy, -Bsx, 0, 0, 0, 0, 0, 0, 0\}, \{0, Bsx, Bsy, -2\,Bsz, 0, Bax, Bay,$
$\quad 0, 0, 0, -Bsx, -Bsy, 0, 0, 0, 0, 0, Bax, -Bay, 0, 0, 0, 0, 0, 0, 0\},$
$\{0, Bsy, -Bsx, 0, 2\,Bsz, Bay, -Bax, 0, 0, 0, -Bsy, Bsx, 0, 0, 0, 0, 0, Bay,$
$\quad Bax, 0, 0, 0, 0, 0, 0, 0\}, \left\{0, 2\,Bsy, 2\,Bsx, 0, 0, 0, 0, 0, 0, 0, -2\,Bsy, -2\,Bsx,\right.$

$$\left.0, -2\,Baz, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\Gamma R\ \hbar}{2}, 0\right\}, \left\{0, 2\,Bsy, 2\,Bsx, 0, 0,\right.$$

$$\left.-2\,Bay, -2\,Bax, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{\Gamma R\ \hbar}{2}, 0\right\},$$

$$\left\{\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, \frac{1}{2}, 0, 0, \frac{\Gamma L + \Gamma R}{2\,\Gamma L}, 0\right\},$$

```
{Bax, -Baz, 0, -Bax, Bay, Bsz + 4 J, ΓM ℏ, -2 tc, 0, 0, 0, 0, Bsx, Bsy,
 0, 0, 0, 0, 0, 0, 0, -Bax, 0, 0, 0, 0}, {Bay, 0, Baz, Bay, Bax, ΓM ℏ,
 -Bsz - 4 J, 0, 2 tc, 0, 0, 0, Bsy, -Bsx, 0, 0, 0, 0, 0, 0, 0, -Bay, 0, 0, 0, 0},
 {Bsx, -Bsz, 0, Bsx, -Bsy, Baz, 0, 0, 0, -Bsx, 0, 0, Bax, -Bay, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0}, {Bsy, 0, Bsz, -Bsy, -Bsx, 0, -Baz,
 0, 0, -Bsy, 0, 0, Bay, Bax, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}};
```

```
LinearSystem = Drop[LinearSystem, None, {26}];
A = Drop[LinearSystem, {21}];
b = ConstantArray[0, {26}]; b[[22]] = 1 / 2;
Dimensions[A]
```

{25, 25}

MatrixForm[A]

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Bax & -Bay & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Bay & Bax & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Bsx & -Bsy & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Bsy & Bsx & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\,tc & \Gamma R\,\hbar & -Bsz - 2\,(2\,J + \Delta) & 0 \\
0 & 0 & 0 & 0 & 0 & 2\,Bay & 2\,Bax & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\,tc & 0 & -Bsz - 2\,(2\,J + \Delta) & -\Gamma R\,\hbar & 0 \\
0 & 0 & 0 & Bax & -Bay & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Bay & Bax & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Bsx & -Bsy & 0 & 0 & 0 & 0 & -Bs \\
0 & 0 & 0 & Bsy & Bsx & 0 & 0 & 0 & 0 & Bsy \\
0 & Bax & -Bay & 0 & 0 & Bsx & -Bsy & 0 & 0 & -Ba \\
0 & Bay & Bax & 0 & 0 & -Bsy & -Bsx & 0 & 0 & 0 \\
0 & Bsx & Bsy & -2\,Bsz & 0 & Bax & Bay & 0 & 0 & 0 \\
0 & Bsy & -Bsx & 0 & 2\,Bsz & Bay & -Bax & 0 & 0 & 0 \\
0 & 2\,Bsy & 2\,Bsx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
Bax & -Baz & 0 & -Bax & Bay & Bsz + 4\,J & \Gamma M\,\hbar & -2\,tc & 0 & 0 \\
Bay & 0 & Baz & Bay & Bax & \Gamma M\,\hbar & -Bsz - 4\,J & 0 & 2\,tc & 0 \\
Bsx & -Bsz & 0 & Bsx & -Bsy & Baz & 0 & 0 & 0 & -Bs \\
Bsy & 0 & Bsz & -Bsy & -Bsx & 0 & -Baz & 0 & 0 & -Bs
\end{pmatrix}$$

```
n = 26; perm1 =
 SparseArray[{i_, i_} → 1, {n, n}][[Ordering[Length[Select[#, # =!= 0 &]] & /@ A]]];
```

```
Table[i, {i, 25}]
```

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}

```
n = 25; perm2 = SparseArray[{i_, i_} → 1, {n, n}][[{1, 2, 3, 4, 5, 6,
    7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}]];
```

```
Length[Select[#, # =!= 0 &]] & /@ (Transpose[perm1.A.perm2])
```

{5, 6, 8, 8, 5, 7, 7, 9, 9, 8, 8, 11, 9, 7, 13, 8, 8, 9, 7, 10, 10, 10, 7, 10, 7}

```
perm1.A.perm2 // MatrixForm
```

0	0	0	0	0	0	0	0	0	0	
0	0	2 Bsy	0	0	0	0	0	0	0	2
0	0	0	0	0	0	0	0	0	2 Bsx	
$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	0	0	0	
0	0	− 2 Bsy	0	0	0	0	0	0	2 Bsx	− 2
0	0	0	− Bsy	0	− Bsz + 4 J + 2 Δ	ΓR ℏ	0	0	0	
0	0	0	Bsx	0	− ΓR ℏ	− Bsz + 4 J + 2 Δ	0	0	0	
0	0	0	− Baz	0	Bay	− Bax	0	0	0	
0	0	0	− Bsx	0	0	0	0	0	0	
0	0	0	− Bsy	0	0	0	0	0	0	
0	0	0	ΓR ℏ	0	Bsx	Bsy	0	0	0	
0	0	0	2 (2 J + Δ)	0	− Bsy	Bsx	0	0	0	
0	0	0	0	0	0	0	0	0	0	
0	− Bsx	Bsz	0	Bsx	0	0	Bsx	− Bsy	0	
0	Bsy	0	0	− Bsy	0	0	Bsy	Bsx	0	B
Bsx	− Bsx	0	0	0	0	0	Bsx	− Bsy	0	
Bsy	− Bsy	0	0	0	0	0	− Bsy	− Bsx	Bsz	
0	0	0	0	0	− Bax	− Bay	0	0	0	
0	0	− Bsx	0	0	0	0	− 2 Bsz	0	Bsy	− I
0	0	− Bsy	0	0	0	0	0	2 Bsz	− Bsx	B
0	0	− Baz	0	− Bax	− 2 tc	0	Bax	− Bay	0	
0	0	0	0	Bay	0	2 tc	Bay	Bax	0	− I
Bax	0	0	0	0	0	0	− Bax	Bay	0	
Bay	0	0	0	0	0	0	Bay	Bax	Baz	
0	0	Bay	2 tc	0	0	0	0	0	Bax	B
0	− Baz	− Bax	0	0	0	0	0	0	− Bay	B