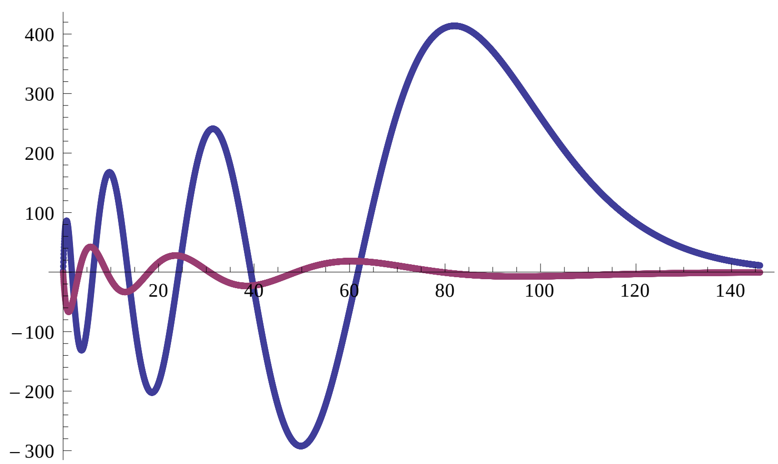
```
Exit[]

a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

'

f[u_, r_] := Simplify[{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 5000;
h = 20 000 / n;
Enn = 0.27766906844567757` / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do[
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[U, {r, u}], {n}]; x =.;
ListPlot[Table[{#[[1]] / 137, 137 ^ (i - 2) * #[[2, i]]} & /@ U[[1 ;; n]], {i, 2}] // N,
 PlotRange → All]
```



```
lambda[n_] := Sqrt[1 - (1 - En[n] / M) ^ 2];
gamma := Sqrt[k ^ 2 - Z ^ 2 * a ^ 2];
f[r_, n_, k_] := -r ^ gamma * Exp[-lambda[n] * r] * (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] +
     (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
g[r_, n_, k_] := Sqrt[2 * M / En[n] - 1] * r ^ gamma * Exp[-lambda[n] * r] *
   (((n - 1 + gamma) / (1 - En[n] / M) - k) *
      Hypergeometric1F1[-(n - 1), 2 * gamma + 1, 2 * lambda[n] * r] -
     (n - 1) * Hypergeometric1F1[1 - (n - 1), 2 * gamma + 1, 2 * lambda[n] * r]);
```

```
g[1, 1, -1] / f[1, 1, -1]
```

-274.068

Randbedingungen

r << 1

```
Exit[]
```

```
a = 7297352537.6 * 10 ^ -12; M = 510998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

```
f[u_, r_] := Simplify[{(Z * a / r + (2 - En)) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

```
u = x ^ (s + n) * {(b[n] - a[n]) * L / En, b[n] + a[n]} Exp[-1 / x * L];
```

```
r[x_] := 1 / x;
```

```
g1 =
 Collect[Simplify[(f[u, r[x]] * D[r[x], x] - D[u, x]) / x ^ (s - 2) / Exp[-1 / x * L] * En],
  x]; g1 // MatrixForm
```

$$\begin{pmatrix} x^n \left(-4\,En\,b[n] + 2\,En^2\,b[n]\right) + x^{1+n} \left(-\sqrt{-(-2+En)\,En}\;k\,a[n] + \sqrt{-(-2+En)\,En}\;n\,a[n] - a\,Z\,a \right. \\ -\frac{En\,x^n\,\left(4\,En\,b[n] - 2\,En^2\,b[n]\right)}{\sqrt{-(-2+En)\,En}} - \frac{En\,x^{1+n}\left(\sqrt{-(-2+En)\,En}\;k\,a[n] + \sqrt{-(-2+En)\,En}\;n\,a[n] + a\,Z\,a[n] + \sqrt{-(-2+En)\,En}\;k\,b[\right.}{\sqrt{-(-2+En)\,En}} \end{pmatrix}$$

```
s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En];
```

```
g2 = Table[Simplify[Sum[D[g1, {x, n2}] / n2!, {n, 0, 10}] /. x → 0], {n2, 0, 10}];
g2 // MatrixForm
```

$$\left(\begin{array}{l}
2\,(-2+En)\,En\,b[0] \\
\sqrt{-(-2+En)\,En}\;k\,(-a[0]+b[0])+a\,Z\,(-a[0]+b[0]-2\,En\,b[0])+2\,(-2+En)\,En\,b[1] \\
\left(\sqrt{-(-2+En)\,En}-\sqrt{-(-2+En)\,En}\;k-a\,Z\right)a[1]+\left(-\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z \right. \\
-\left(-2\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[2]+\left(-2\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-3\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[3]+\left(-3\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-4\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[4]+\left(-4\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-5\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[5]+\left(-5\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-6\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[6]+\left(-6\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-7\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[7]+\left(-7\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-8\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[8]+\left(-8\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right. \\
-\left(-9\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[9]+\left(-9\sqrt{-(-2+En)\,En}+\sqrt{-(-2+En)\,En} \right.
\end{array}\right.$$

```
Det[{{-s - k, a Z}, {-a Z, -s + k}}]
```

$$-k^2+s^2+a^2\,Z^2$$

```
Inverse[{{1, -1}, {1, 1}}]
```

$$\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\left\{-\frac{1}{2},\frac{1}{2}\right\}\right\}$$

```
Simplify[Eigenvalues[Inverse[{{-(n + s + k), a}, {a, n + s - k}}].{{-L, En}, {En, L}}]]
```

$$\left\{\left(a\ En + L\ n + L\ s - \sqrt{\left(-a^2 + k^2\right) L^2 + 2\ a\ En\ L\ (n + s) + En^2 \left(k^2 - (n + s)^2\right)}\right) \Big/ \left(a^2 - k^2 + (n + s)^2\right), \left(a\ En + L\ n + L\ s + \sqrt{\left(-a^2 + k^2\right) L^2 + 2\ a\ En\ L\ (n + s) + En^2 \left(k^2 - (n + s)^2\right)}\right) \Big/ \left(a^2 - k^2 + (n + s)^2\right)\right\}$$

```
Limit[n (a En + L n + L s - Sqrt[(-a^2 + k^2) L^2 + 2 a En L (n + s) + En^2 (k^2 - (n + s)^2)]) /
    (a^2 - k^2 + (n + s)^2), {n -> Infinity}]
```

$$\left\{-\sqrt{-En^2} + L\right\}$$

```
Solve[(a En + L n + L s - Sqrt[(-a^2 + k^2) L^2 + 2 a En L (n + s) + En^2 (k^2 - (n + s)^2)]) /
    (a^2 - k^2 + (n + s)^2) == 0, En]
```

$$\{\{En \to -i\ L\}, \{En \to i\ L\}\}$$

```
Expand[(n + s)^2 - s^2]
```

$$n^2 + 2\ n\ s$$

```
(a^2 - k^2 + (n + s)^2) * Simplify[
    Inverse[{{a / (En - 2), (n + s + k) / (2 - En)}, {(n + s - k) / En, a / En}}]] // MatrixForm
```

$$\begin{pmatrix} a\ (-2 + En) & En\ (k + n + s) \\ (-2 + En)\ (k - n - s) & a\ En \end{pmatrix}$$

```
s = Sqrt[k^2 - (Z * a)^2];
```

```
En =.;
Simplify[Inverse[{{Z * a / En, (n + s - k) / En}, {(n + s + k) / (2 - En), Z * a / (En - 2)}}]]
```

$$S[n\_, En\_] := \left\{\left\{\frac{a\,En\,Z}{n^2 + 2\,n\sqrt{k^2 - a^2 Z^2}}, -\frac{(-2 + En)\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{En\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}, \frac{a\,(-2 + En)\,Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

$$DS[n\_, En\_] := \left\{\left\{\frac{a\,Z}{n^2 + 2\,n\sqrt{k^2 - a^2 Z^2}}, -\frac{\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}, \left\{\frac{\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}, \frac{a\,Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\};$$

```
S[m, En] // MatrixForm
DS[m, En] // MatrixForm
```

$$\left\{\left\{\frac{a\,En\,Z}{-k^2 + n^2 + 2\,n\,s + s^2 + a^2 Z^2}, \frac{(-2 + En)\,(k - n - s)}{-k^2 + n^2 + 2\,n\,s + s^2 + a^2 Z^2}\right\}, \left\{\frac{En\,(k + n + s)}{-k^2 + n^2 + 2\,n\,s + s^2 + a^2 Z^2}, \frac{a\,(-2 + En)\,Z}{-k^2 + n^2 + 2\,n\,s + s^2 + a^2 Z^2}\right\}\right\}$$

$$\begin{pmatrix} \frac{a\,En\,Z}{m^2 + 2\,m\sqrt{k^2 - a^2 Z^2}} & -\frac{(-2 + En)\left(-k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{En\left(k + m + \sqrt{k^2 - a^2 Z^2}\right)}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} & \frac{a\,(-2 + En)\,Z}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

$$\begin{pmatrix} \frac{a\,Z}{m^2 + 2\,m\sqrt{k^2 - a^2 Z^2}} & -\frac{-k + m + \sqrt{k^2 - a^2 Z^2}}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \\ \frac{k + m + \sqrt{k^2 - a^2 Z^2}}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} & \frac{a\,Z}{m\left(m + 2\sqrt{k^2 - a^2 Z^2}\right)} \end{pmatrix}$$

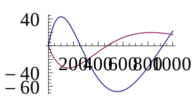
```
S[n, En].u
```

$S[n, En].\{x^{n+s}\, a[n],\, x^{n+s}\, b[n]\}$

```
UN[R_, N_, En_] := Module[{u = {1, (k + s) / Z / a}, U = {1, (k + s) / Z / a} * R ^s},
  For[n = 1, n < N, n++,
   u = S[n, En].u;
   U += u * R ^ (s + n);
  ];

  U]
```

```
U[r_, g_, En_] :=
 Module[{u = {1, (k + s) / Z / a}, U = {0, 0}, DU = {0, 0}, du = {0, 0}, n = 0},
  Label[begin];
  U += u * r ^ (s + n);
  DU += du * r ^ (s + n);
  n++;
  du = DS[n, En].u + S[n, En].du;
  u = S[n, En].u;
  If[(#[[1]] > g  || #[[2]] > g) &[Abs[u * R ^ (s + n) / U /. r → R]], Goto[begin]];
  {n, U, DU}]
```

```
R = 1000; g = 0.01; rU = U[r, g, 1 / M];
Plot[{rU[[2, 1]], rU[[2, 2]] * 137}, {r, 0, R}, PlotRange → All]
```




```
EN[iEn_, g2_] := Module[{rU, fU, n = 0, i, En = iEn, ll},
  Label[begin];
  fU = U[r, g, En];

  rU = fU /. r → R;

  If[rU[[2, 1]] * rU[[2, 2]] > 0,

   En -= (rU[[2, 1]] + rU[[2, 2]]) / (rU[[3, 1]] + rU[[3, 2]]);

   n++
    Goto[begin];
  ];

  {n, En * M, Abs[rU[[2, 1]] - rU[[2, 2]]]}
 ]
```

```
R = 3000; g = 0.001; EN[13 / M, 0.1]
```

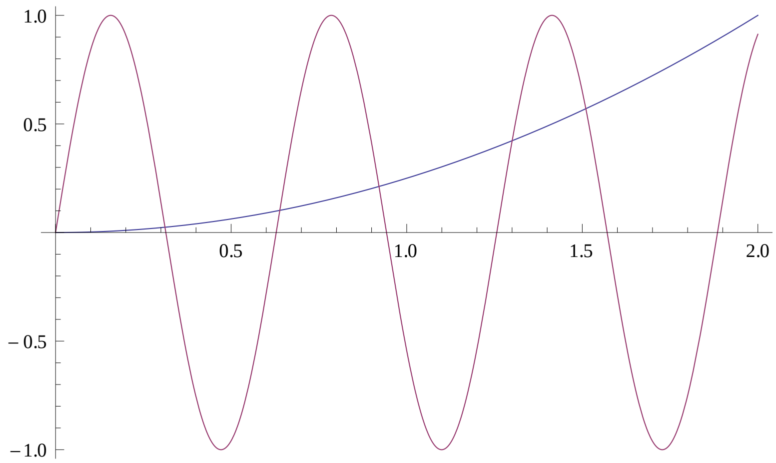
{8, 13.6059, 0.000441718}

```
- {0, 13.605873075061169
```

```
R = 2000; g = 0.001;

plot[{rU[[2, 1]], rU[[3, 1]]}, 100, R]

En = 4 / M; rU[[2, 1]]

129.072

n = 0; x

10

n = 0; While[x = n; x < 10, n++; Print[n]]

1

2

3

4

5

6

7

8

9

10
```

```
plot[liste_, R_] := Module[{nN = 100, table, max, st = {Red, Green, Blue}},
  liste / (Max[Abs[#]] & /@ (Table[# /. r → i * R / nN, {i, 0, nN}] & /@ liste))
 ]
ll = plot[{r ^ 2, Sin[10 * r]}, 2]
Plot[ll, {r, 0, 2}]
```

$$\left\{\frac{r^2}{4}, -\text{Csc}[11]\ \text{Sin}[10\ r]\right\}$$



r gegen Inifinity

```
Exit[]
```

```
a = 7 297 352 537.6 * 10 ^ -12; M = 510 998.910; k = -1; Z = 1;
```

```
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k ^ 2 - (Z * a) ^ 2])) ^ 2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365,
 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

```
$Assumptions = 1 > En > 0;
```

```
s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En];
```

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - En) * u[[2]] - k / r * u[[1]],
     k / r * u[[2]] + (En - Z * a / r) * u[[1]]}];
```

```
u := {u1, u2}
```

```
U = {(hb[r] - ha[r]) * L / En, hb[r] + ha[r]} * Exp[-r * L];
```

```
#[[2]] & /@ Simplify[Solve[Simplify[(f[U, r] - D[U, r]) / Exp[-r * L] * En * r] == 0,
      {ha'[r], hb'[r]}][[1]]] /. ha[r] → u[[1]] /. hb[r] → u[[2]] /. r → rr
```

$$\left\{\frac{-a\,(-1+En)\,u1\,Z+u2\left(\sqrt{-(-2+En)\,En}\;k-a\,Z\right)}{\sqrt{-(-2+En)\,En}\;rr},\;\left(u1\left(\sqrt{-(-2+En)\,En}\;k+a\,Z\right)+u2\left(4\,En\,rr-2\,En^2\,rr-a\,Z+a\,En\,Z\right)\right)\Big/\left(\sqrt{-(-2+En)\,En}\;rr\right)\right\}$$

$$\text{F[u\_, rr\_] := Simplify}\left[\left\{\frac{-a\,(-1+En)\,Z\,u[\![1]\!]+\left(\sqrt{-(-2+En)\,En}\;k-a\,Z\right)u[\![2]\!]}{\sqrt{-(-2+En)\,En}\;rr},\;\left(\left(\sqrt{-(-2+En)\,En}\;k+a\,Z\right)u[\![1]\!]+\left(4\,En\,rr-2\,En^2\,rr-a\,Z+a\,En\,Z\right)u[\![2]\!]\right)\Big/\left(\sqrt{-(-2+En)\,En}\;rr\right)\right\}\right]$$

```
r[x_] := 1 / x; u := {a[n], b[n]} * x ^ (n + s)
```

```
g1 = Collect[Simplify[(F[u, r[x]] * D[r[x], x] - D[u, x]) * x ^ (2 - s)], {a[n], b[n]}]; g1
```

$$\left\{-n\,x^{1+n}\,a[n]+x^{1+n}\left(-k+\frac{a\,Z}{\sqrt{-(-2+En)\,En}}\right)b[n],\;-\frac{x^{1+n}\left(\sqrt{-(-2+En)\,En}\;k+a\,Z\right)a[n]}{\sqrt{-(-2+En)\,En}}-\left(x^n\left(4\,En-2\,En^2+\sqrt{-(-2+En)\,En}\;n\,x-2\,a\,x\,Z+2\,a\,En\,x\,Z\right)b[n]\right)\Big/\left(\sqrt{-(-2+En)\,En}\right)\right\}$$

$$\text{g2} := \left\{x^n\,(-n)\,a[n]+x^n\,(-k+a\,Z/L)\,b[n],\;-x^{1+n}\,(k+a\,Z/L)\,a[n]+x^n\,(-n\,x-2\,s\,x+-2*L)\,b[n]\right\};\ \text{g2 // MatrixForm}$$

$$\begin{pmatrix} -n\,x^n\,a[n]+x^n\left(-k+\frac{a\,Z}{L}\right)b[n] \\ -x^{1+n}\left(k+\frac{a\,Z}{L}\right)a[n]+x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n] \end{pmatrix}$$

```
a[n_] := (Z * a / L - k) / n * b[n]
```

```
g2[[2]]
```

$$x^n\,(-2\,L-n\,x-2\,s\,x)\,b[n]-\frac{x^{1+n}\left(-k+\frac{a\,Z}{L}\right)\left(k+\frac{a\,Z}{L}\right)b[n]}{n}$$

```
M1 = {{L, 2 - En}, {En, L}}; Eigenvalues[M1]; B = Transpose[Eigenvectors[M1]];
```

```
$Assumptions = 1 > En > 0
```

$1 > En > 0$

=>

```
Exit[]
```

```
a[n_] := (Z * a / L - k) / n * b[n]
```

```
b[n_] := b0 * Product[((k^2 - a^2 * Z^2 / L^2) / i - (i + 2 * s)) / 2 / L, {i, 1, n}]
```

```
b[4]
```

$$\frac{1}{16\,L^4}\,b0\left(-1+k^2-2\,s-\frac{a^2\,Z^2}{L^2}\right)\left(-4-2\,s+\frac{1}{4}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)\left(-3-2\,s+\frac{1}{3}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)\left(-2-2\,s+\frac{1}{2}\left(k^2-\frac{a^2\,Z^2}{L^2}\right)\right)$$

```
((k^2 - a^2 * Z^2 / L^2) / i - (i + 2 * s)) / 2 / L
```

$$\frac{-i-2\,s+\frac{k^2-\frac{a^2\,Z^2}{L^2}}{i}}{2\,L}$$

```
Exit[];
```

```
En =.;
```

$$\text{Solve}\left[-i*\sqrt{(2-En)\,En}-2\,a\,(-1+En)+\frac{\sqrt{(2-En)\,En}-\frac{a^2}{\sqrt{(2-En)\,En}}}{i}==0,\,En\right]\text{ // MatrixForm}$$

$$\begin{pmatrix}
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4-\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2-2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4+\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2-2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4-\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2+2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}\\
En\to\frac{2-4\,i^2+8\,a^2\,i^2+2\,i^4+\sqrt{\left(-2+4\,i^2-8\,a^2\,i^2-2\,i^4\right)^2-4\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)\left(a^2+a^2\,i^2+2\sqrt{a^4\,i^2-a^6\,i^2}\right)}}{2\left(1-2\,i^2+4\,a^2\,i^2+i^4\right)}
\end{pmatrix}$$

'

$$\text{ET}[i\_,\,k\_,\,Z\_,\,a\_]:=\left(2\,i^4-4\,i^2\,k^2+2\,k^4+8\,a^2\,i^2\,Z^2-\sqrt{\left(\left(-2\,i^4+4\,i^2\,k^2-2\,k^4-8\,a^2\,i^2\,Z^2\right)^2-4\left(i^4-2\,i^2\,k^2+k^4+4\,a^2\,i^2\,Z^2\right)\left(a^2\,i^2\,Z^2+a^2\,k^2\,Z^2+2\sqrt{a^4\,i^2\,k^2\,Z^4-a^6\,i^2\,Z^6}\right)\right)}\right)\Big/\left(2\left(i^4-2\,i^2\,k^2+k^4+4\,a^2\,i^2\,Z^2\right)\right)$$

```
Table[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 10}]
```

$\{-5.67315\times10^{-11}, -1.9084\times10^{-7}, 5.30991\times10^{-11}, -1.3482\times10^{-10}, 3.24187\times10^{-11}, -5.86517\times10^{-11}, 5.5719\times10^{-11}, 2.10913\times10^{-11}, 1.87879\times10^{-11}, 1.01339\times10^{-10}, -1.46405\times10^{-12}\}$

```
Series[M * ET[n, -1, 1, a] - Energie[n + 1], {n, 0, 5}]
```

$-5.67315\times10^{-11} - 1.26477\times10^{-10}\, n^2 - 2.84217\times10^{-14}\, n^3 - 2.54019\times10^{-10}\, n^4 + 1.42109\times10^{-14}\, n^5 + O[n]^6$

```
M = 510998.910;
```

```
s = (En - 1) * Z * a / L; L := Sqrt[(2 - En) * En];
```

```
a = 7297352537.6 * 10^-12; M = 510998.910; Z = 1; k = -1;
Energie[n_] := M * (1 - 1 / Sqrt[1 + (Z * a / (n - Abs[k] + Sqrt[k^2 - (Z * a)^2]))^2]);
Table[N[Energie[i]], {i, 10}]
```

{13.6059, 3.40148, 1.51176, 0.850365, 0.544233, 0.377939, 0.277669, 0.21259, 0.167972, 0.136058}

Verhältnis bei r = 0

```
a = 7297352537.6 * 10^-12; M = 510998.910; k = -1; Z = 1;
s = Sqrt[k^2 - (Z * a)^2];
```

$$S[n\_] := \left\{\left\{\frac{a\,En\,Z}{n^2 + 2\,n\,\sqrt{k^2 - a^2 Z^2}},\ -\frac{(-2 + En)\left(-k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\},\ \left\{\frac{En\left(k + n + \sqrt{k^2 - a^2 Z^2}\right)}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)},\ \frac{a\,(-2 + En)\,Z}{n\left(n + 2\sqrt{k^2 - a^2 Z^2}\right)}\right\}\right\} \text{ /. En -> Enn;}$$

```
S[
 10]
```

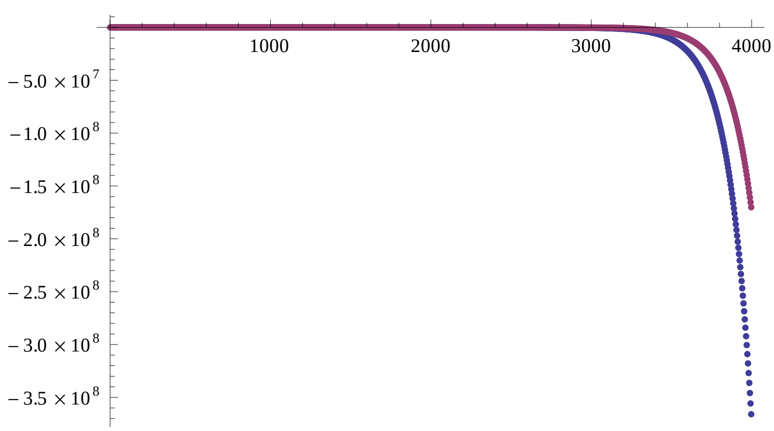
{{0.0000608115 Enn, -0.1 (-2 + Enn)}, {0.0833335 Enn, 0.0000608115 (-2 + Enn)}}

```
Enn =.; u = {1, (k + s) / Z / a}; U = u;
For [n = 1, n < 3, n++,
 u = S[n].u;
 U = Simplify[U + u];
]; n =.;
Simplify[U[[1]] / U[[2]]]
```

$$-\frac{91.3504\ (-3.66275 + \text{Enn})\ (1.62629 + \text{Enn})}{(-0.0109728 + \text{Enn})\ (181.383 + \text{Enn})}$$

Runge von links

```
f[u_, r_] := Simplify[{(Z * a / r + 2 - Enn) * u[[2]] - k / r * u[[1]],
    k / r * u[[2]] + (Enn - Z * a / r) * u[[1]]}];
k = -1; Z = 1; U =.
n = 1000;
h = 4000 / n;
Enn = 13.605 / M;
u = {(91.35044102604739` (-3.662751763692355` + Enn) (1.6262886176197724` + Enn)) /
    ((-0.0109728221664999` + Enn) (181.38339842774778` + Enn)), -1};
r = 1; U = {{r, u}};
Do[
 k0 = h * f[u, r]; k1 = h * f[u + k0 / 2, r + h / 2];
 k2 = h * f[u + k1 / 2, r + h / 2]; k3 = h * f[u + k2, r + h];
 u += 1 / 6 * (k0 + 2 * k1 + 2 * k2 + k3); r += h;
 AppendTo[U, {r, u}], {n}]; x =.;
ListPlot[
 Table[{#[[1]], 137 ^ (i - 2) * #[[2, i]]} & /@ U[[1 ;; n]], {i, 2}] // N, PlotRange → All]
```




```
Sum[A[n] * r ^ n / n!, {n, 0, 10}]
```

$$A[0] + r\,A[1] + \frac{1}{2}\,r^2\,A[2] + \frac{1}{6}\,r^3\,A[3] + \frac{1}{24}\,r^4\,A[4] + \frac{1}{120}\,r^5\,A[5] + \frac{1}{720}\,r^6\,A[6] + \frac{r^7\,A[7]}{5040} + \frac{r^8\,A[8]}{40\,320} + \frac{r^9\,A[9]}{362\,880} + \frac{r^{10}\,A[10]}{3\,628\,800}$$

D[%, {r, 4}]

$$A[4] + r\, A[5] + \frac{1}{2} r^2 A[6] + \frac{1}{6} r^3 A[7] + \frac{1}{24} r^4 A[8] + \frac{1}{120} r^5 A[9] + \frac{1}{720} r^6 A[10]$$

Sum[A[n + 4] * r ^ n / n !, {n, 0, 10}]

$$A[4] + r\, A[5] + \frac{1}{2} r^2 A[6] + \frac{1}{6} r^3 A[7] + \frac{1}{24} r^4 A[8] + \frac{1}{120} r^5 A[9] + \frac{1}{720} r^6 A[10] + \frac{r^7 A[11]}{5040} + \frac{r^8 A[12]}{40\,320} + \frac{r^9 A[13]}{362\,880} + \frac{r^{10} A[14]}{3\,628\,800}$$