$$S[N_{]} := k N \left[\frac{5}{2} + Log \left[\frac{2\sqrt{2} \pi^{3/2} (k m T)^{3/2} V}{h^3 N} \right] \right]$$

Solve
$$\left[\frac{2\sqrt{2} \pi^{3/2} (k m T)^{3/2} V}{h^3 N} = 1, N\right]$$

$$\left\{ \left\{ \mathbf{N} \; \rightarrow \; \frac{2\;\sqrt{2}\;\; \mathbf{k}\; \mathbf{m}\; \pi^{\,3/\,2}\; \mathbf{T}\; \sqrt{\mathbf{k}\; \mathbf{m}\; \mathbf{T}} }{\mathbf{h}^{\,3}} \; \right\} \right\}$$

Solve[1/D[S[N], En] = T, En]

$$\left\{\left\{\texttt{En}\,\rightarrow\,\frac{\texttt{3}\,\texttt{k}\,\texttt{N}\,\texttt{T}}{2}\right\}\right\}$$

$$\frac{p\ V}{T} = k\ N$$

Expand [S'[N]]

$$\text{k Log}\,\big[\,\frac{8\,\left(\frac{\text{En}\,\,\text{m}}{\text{N}}\right)^{\,3/\,\,2}\,\,\pi^{\,3/\,\,2}\,\,\text{V}}{3\,\sqrt{3}\,\,\,h^{\,3}\,\,\text{N}}\,\big]$$

$$-\frac{3 \text{ k T}}{2}$$

$$\text{Simplify}\left[k \text{ T Log}\left[\frac{2^{3/N} \left(\frac{En \text{ m}}{N}\right)^{\frac{3}{2N}} \left(\frac{\pi}{3}\right)^{\frac{3}{2N}} \text{ V}}{h^3 \text{ N}}\right]\right]$$

$$\text{k T Log}\left[\frac{8^{\frac{1}{N}} \left(\frac{\text{En m}}{N}\right)^{\frac{3}{2N}} \left(\frac{\pi}{3}\right)^{\frac{3}{2N}} V}{h^{3} N}\right]$$

Simplify
$$\left[k \text{ Log}\left[\frac{8 \left(\frac{En m}{N}\right)^{3/2} \pi^{3/2} V}{3 \sqrt{3} h^3 N}\right] - S[N]/N\right]$$

$$-\frac{5 \text{ k}}{2}$$

Exit[]

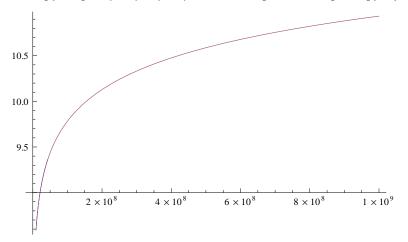
DSolve[
$$\{F'[N] == F[N] / N + k T / 2, F[N0] == F0\}, F[N], N$$
]

$$\left\{ \left\{ \text{F[N]} \rightarrow \frac{\text{2 F0 N} + \text{k N N0 T Log[N]} - \text{k N N0 T Log[N0]}}{\text{2 N0}} \right\} \right\}$$

$$\frac{\text{Simplify} \left[\frac{2 \text{ F0 N + k N N0 T Log[N] - k N N0 T Log[N0]}}{2 \text{ N0}} \right] }{2 \text{ N0}}$$

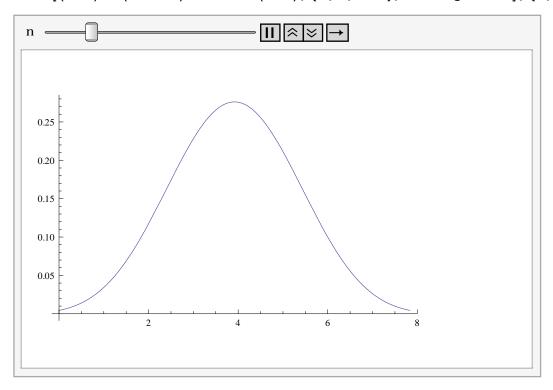
$$\frac{\text{N } (2 \text{ F0 + k N0 T Log[N] - k N0 T Log[N0]})}{2 \text{ N0}}$$

Plot[{Log[2^(2 N) / (2 N)! * N! * N!], 0.5 Log[Pi N]}, {N, 0, 10000000000}]



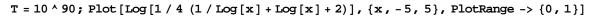
Animate[

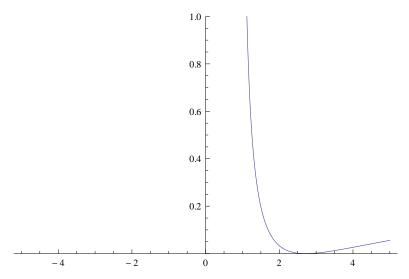
 $Plot[(n^2)!/(n^2-x)!/x!/2^(n^2), \{x, 0, n^2\}, PlotRange \rightarrow All], \{n, 1, 10\}]$



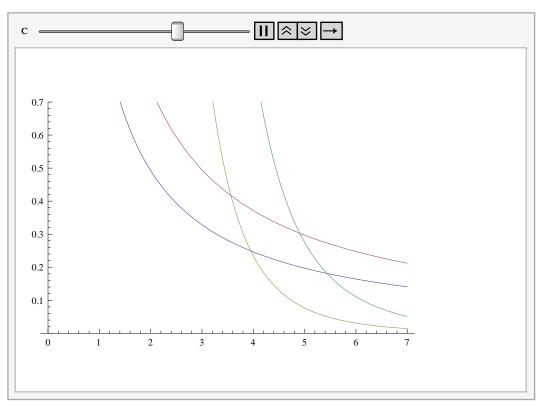
0.4!

0.887264





Animate[Plot[{ $c^2/3/4/V$, ($c^2/3+2$)/4/V, $c^5/V^5/2$, (c+1)^5/V^5/2}, {V, 0, 7}, PlotRange \rightarrow {0, 0.7}], {c, 0, 5}]



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