

```

Exit[];

$Assumptions = r > 0 && Element[m, Integers] && Element[n, Integers] && s > 0
r > 0 && m ∈ Integers && n ∈ Integers && s > 0

f[r_] := {{(m - 1) / r, I * (En - r^p)}, {I * (En - r^p), -m / r}} -
0 * IdentityMatrix[2] * I * r^p; f[r] // MatrixForm

$$\begin{pmatrix} \frac{-1+m}{r} & i (En - r^p) \\ i (En - r^p) & -\frac{m}{r} \end{pmatrix}$$


En = E0 + I * Ga; En =.

VW[x_] := Exp[I * Integrate[r^p - En, {r, 0, x}]]

u = {F[x], I * G[x]} / VW[x]

$$\left\{ e^{-i \left( -En x + \frac{x^3}{3} \right)} F[x], i e^{-i \left( -En x + \frac{x^3}{3} \right)} G[x] \right\}$$


p = 2;

r[x_] := x;

g1 = Collect[Expand[Simplify[Expand[(-D[u, x] + r'[x] * f[r[x]].u) * VW[x] * {1, -I}]]],
{x^n, a[n], b[n]}]

$$\left\{ -i En F[x] - \frac{F[x]}{x} + \frac{m F[x]}{x} + i x^2 F[x] - En G[x] + x^2 G[x] - F'[x], \right.$$


$$\left. En F[x] - x^2 F[x] - i En G[x] - \frac{m G[x]}{x} + i x^2 G[x] - G'[x] \right\}$$


f2[r_] := {{-En I - \frac{1}{x} + \frac{m}{x} + i x^2, -En + x^2}, {+En - x^2, -I En - \frac{m}{x} + i x^2}};
f2[r] // MatrixForm

$$\begin{pmatrix} -i En - \frac{1}{x} + \frac{m}{x} + i x^2 & -En + x^2 \\ En - x^2 & -i En - \frac{m}{x} + i x^2 \end{pmatrix}$$


u = {a[n] * x^(n), b[n] * x^(n)} * x^s
{x^(n+s) a[n], x^(n+s) b[n]}

r[x_] := x;

s = -1 + m;

g2 =
Collect[Expand[Simplify[Expand[(-D[u, x] + r'[x] * f2[r[x]].u) * x^(1-s) * {1, -1}]]],
{x^n, a[n], b[n]}]

$$\left\{ x^n \left( (-n - i En x + i x^3) a[n] + (-En x + x^3) b[n] \right), \right.$$


$$\left. x^n \left( (-En x + x^3) a[n] + (-1 + 2 m + n + i En x - i x^3) b[n] \right) \right\}$$


```

```
g3 = Table[Simplify[Sum[D[g2, {x, n2}] / n2!, {n, 0, 10}] /. x -> 0], {n2, 0, 10}];
g3 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$a[0] = 1; \quad b[0] = 0; \quad a[1] = -i \, \text{En} \, a[0]; \quad b[1] = \frac{\text{En}}{2 \, m};$$

$$a[2] = -i \, \text{En} \, a[1] / 2 - \text{En} \, b[1] / 2; \quad b[2] = -1 / (1 + 2 \, m) \, (-\text{En} \, a[1] + i \, \text{En} \, b[1]);$$

$$\text{Simplify}[I * (i \, a[n] - i \, \text{En} \, a[n+2] - (n+3) \, a[n+3] + b[n] - \text{En} \, b[n+2]) + \\ a[n] - \text{En} \, a[n+2] - i \, b[n] + i \, \text{En} \, b[n+2] + (n+2) \, b[n+3] + 2 \, m \, b[n+3]]$$

$$-i \, (3+n) \, a[3+n] + (2+2 \, m+n) \, b[3+n]$$

$$b[n\_]:=i \, (n) \, a[n] / (-1+2 \, m+n)$$

$$\text{Collect}[i \, a[n] - i \, \text{En} \, a[n+2] - (n+3) \, a[n+3] + b[n] - \text{En} \, b[n+2], \\ \{a[n], a[n+3], a[n+2]\}] /. n \rightarrow n-3$$

$$\left(i + \frac{i \, (-3+n)}{-4+2 \, m+n}\right) a[-3+n] + \left(-i \, \text{En} - \frac{i \, \text{En} \, (-1+n)}{-2+2 \, m+n}\right) a[-1+n] - n \, a[n]$$

$$a[n\_]:=1/n * \left(\left(i + \frac{i \, (-3+n)}{-4+2 \, m+n}\right) a[-3+n] + \left(-i \, \text{En} - \frac{i \, \text{En} \, (-1+n)}{-2+2 \, m+n}\right) a[-1+n]\right)$$

$$\text{Simplify}\left[1/n * \left(\left(i + \frac{i \, (-3+n)}{-4+2 \, m+n}\right)\right)\right]$$

$$a[n-1]$$

\$RecursionLimit::reclim: Recursion depth of 256 exceeded. >>

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General::stop: Further output of \$RecursionLimit::reclim will be suppressed during this calculation. >>

\$Aborted

n=.

```

U[En_, m_, nN_] := Module[{U, te, n},

  U = {{1, 0}, {-i En,  $\frac{En}{2 m}$ }, {- $\frac{En^2}{2} - \frac{En^2}{4 m}$ , - $\frac{i En^2 + \frac{i En^2}{2 m}}{1 + 2 m}$ }}};

  For[n = 3, n < nN, n++,
    te = 1 / n *  $\left( \left( i + \frac{i (-3 + n)}{-4 + 2 m + n} \right) U[[-2 + n, 1]] + \left( -i En - \frac{i En (-1 + n)}{-2 + 2 m + n} \right) U[[n, 1]] \right)$ ;
    AppendTo[U, {te, i (n) te / (-1 + 2 m + n)}];
  ];
  {1, 1} * Exp[-I * Integrate[r ^ p - En, {r, 0, x}]];
  Expand[Sum[U[[n + 1]] * x ^ n, {n, 0, nN - 1}] * x ^ (-1 + m)]

]

```

```
Simplify[U[En, m, 15] - Table[{a[n], b[n]}, {n, 0, 14}]]
```

Thread::tdlen: Objects of unequal length in

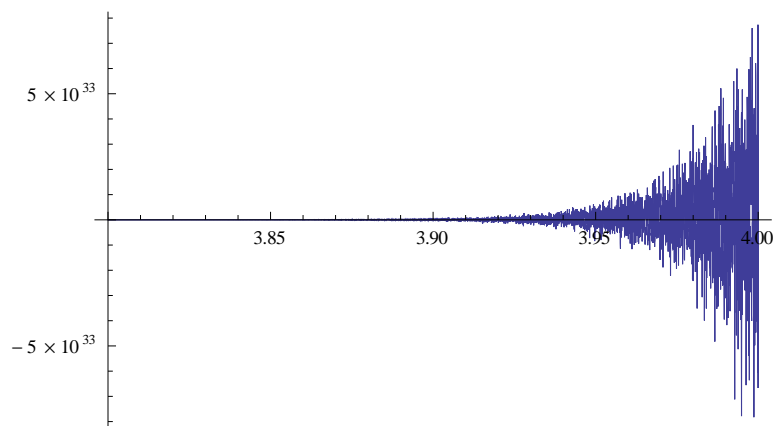
$$\left\{ x^{-1+m} - i En x^m - \frac{1}{2} En^2 x^{1+m} - \frac{En^2 x^{1 \ll 1 \gg \ll 1 \gg}}{4 m} + \ll 3 \gg + \frac{i \ll 1 \gg x^{\ll 1 \gg}}{3 (1 + \ll 1 \gg)} + \frac{i En^3 x^{2+m}}{6 m (1 + 2 m)} + \right. \\ \left. \frac{1}{3} En x^{3+m} + \ll 43458 \gg, \ll 1 \gg \right\} + \{ \ll 1 \gg \} \text{ cannot be combined. } \gg$$

\$Aborted

```
U[10, 2, 5] // N // MatrixForm
```

$$\begin{pmatrix} x - (0. + 10. i) x^2 - 62.5 x^3 + (0. + 292. i) x^4 + 1098.13 x^5 \\ 2.5 x^2 - (0. + 25. i) x^3 - 146. x^4 + (0. + 627.5 i) x^5 \end{pmatrix}$$

```
G = U[10 + I, 4, 600][[2]]; Plot[{Re[#], Im[#]} &[G], {x, 3.8, 4}, PlotRange -> All]
```



**G**

$$\begin{aligned}
& \left\{ \left\{ \operatorname{Re} \left[ e^{-i \left( -E n x + \frac{x^3}{3} \right)} \left( x - 10 i x^2 - \frac{125 x^3}{2} + 292 i x^4 + \frac{8785 x^5}{8} \right) \right] \right. \right. \\
& \quad \left. \operatorname{Re} \left[ e^{-i \left( -E n x + \frac{x^3}{3} \right)} \left( \frac{5 x^2}{2} - 25 i x^3 - 146 x^4 + \frac{1255 i x^5}{2} \right) \right] \right\} \right\}, \\
& \left\{ \operatorname{Im} \left[ e^{-i \left( -E n x + \frac{x^3}{3} \right)} \left( x - 10 i x^2 - \frac{125 x^3}{2} + 292 i x^4 + \frac{8785 x^5}{8} \right) \right] \right\}, \\
& \quad \operatorname{Im} \left[ e^{-i \left( -E n x + \frac{x^3}{3} \right)} \left( \frac{5 x^2}{2} - 25 i x^3 - 146 x^4 + \frac{1255 i x^5}{2} \right) \right] \} \}
\end{aligned}$$

**WV[x]**

$$e^{i \left( -E n x + \frac{x^3}{3} \right)}$$