# 9. Generating Functions

Generating functions are one of the most remarkable and useful tools in combinatorics, and perhaps all of mathematics. The idea is to encode an infinite sequence $\langle c_0, c_1, \ldots, c_n, \ldots \rangle$ as coefficients of a formal power series:

$$G(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n + \cdots$$

The word *formal* here is to emphasize the fact that we don't think of this as an expression in which we substitute for $x$. We are more interested in the sequence of coefficients. It doesn't matter, for example, whether the sum converges or not.

Let's see what this looks like for some sequences we can easily cook up.

$$\langle 1, 1, 1, 1, \ldots \rangle \quad \rightarrow \quad A(x) = 1 + x + x^2 + x^3 + \cdots \quad = \quad \frac{1}{1-x}$$

$$\langle 1, 2, 4, 8, \ldots \rangle \quad \rightarrow \quad B(x) = 1 + 2x + 4x^2 + 8x^3 + \cdots \quad = \quad \frac{1}{1-2x}$$