Optimal Severity of Stress-Test Scenarios*

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Abstract

Bank supervisors, such as the Federal Reserves, conduct regular stress tests to ensure stable lending. This constitutes a de facto constraint on balance sheets: equity must be sufficient to maintain current lending also in the future, even after absorbing severe loan losses. We study the effects of such forward looking constraints in a representative bank model. More severe stress tests scenarios lead to lower dividends, higher equity levels, and universally lower, albeit less volatile, lending. Subsequently, we calibrate our model to U.S. banks to compute the optimal, state-dependent severity of stress tests. Finally, we compare stress tests with several policy alternatives, such as the Covid-19 dividend ban, the counter-cyclical capital buffer (CCyB), and the dividend prudential target (DPT): while the first two perform well as complementary policies, a DPT is not welfare-improving for a supervisor seeking stable lending levels.

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1. Introduction

The financial crisis of 2008-09 has highlighted how crucial bank health is for economic stability and growth. To promote a safe and sound financial system going forward, supervisory authorities around the world have since introduced a wide range of new regulatory measures. As part of this policy package, the Federal Reserve (Fed), the European Central Bank (ECB), and many other authorities have begun to subject banks to regular, usually annual stress tests. The objective of stress tests is to ensure that banks are sufficiently capitalized to maintain their current lending activities even under severely adverse macroeconomic conditions in the future.[1] Banks found to be insufficiently capitalized in a hypothetical downturn are consequently restricted in their dividend payments: depending on the severity of violation, an increasing amount of net-income must be retained to boost equity levels.[2]

This regulatory pressure on dividend payments clashes with the banks' apparent objective to generate stable dividends that compensate shareholders for their investments (Koussis and Makrominas, 2019; Larkin et al., 2017).[3] These dividends are paid from both accumulated equity and returns on assets that are financed via equity capital and debt. To keep dividends smooth across the business cycle, banks deplete capital reserves when facing negative earning shocks (see Figure 1). Unregulated, simultaneously maintaining stable dividend levels and minimum capital ratio requirements may lead to asset shrinkage during crisis periods. Thus, intuitively, supervisory restrictions on dividend payments via stress tests seem warranted to maintain equity capital and thereby to ensure lending to viable firms.

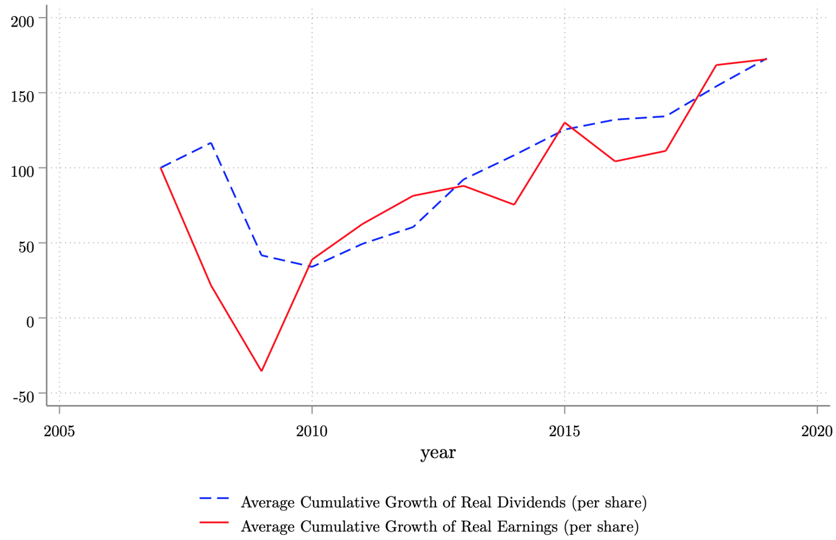
This argument, however, ignores how banks might change their behavior in anticipation of stress-test constrained dividend payments. To the banks' risk-averse shareholders, a safe payment today is worth more than an expected equal amount tomorrow that is subject to uncertainty. To pass the stress tests, banks therefore may avoid cutting dividends and instead reduce lending levels. Hence, one must account for the bank's margin of adjustment when evaluating the efficiency of stress tests. Thus far, the existing stress test literature provides little insights on *ex ante* dividend and lending choices by stress-tested banks, as it focuses mainly on the announcement effect of bank stress test results and the subsequent immediate stock-price responses (Beck et al., 2020; Goldstein and Leitner, 2018; Sahin et al.,

[1] Thus, stress tests extend the existing macro-prudential framework by going beyond point-in-time-estimates.

[2] A detailed description of the U.S. regulatory framework can be found in Appendix A. Similar restrictions exist in the European Union and China (Svoronos and Vrbaski, 2020).

[3] There is no shortage of potential explanations for banks' dividend smoothing policies, ranging from investor interests to managerial pay-out schemes directly linked to dividend stability (Lambrecht and Myers, 2012; Wu, 2018). In this paper we do not take a stand on the cause of this behaviour but rather take it as a given bank objective.

Figure 1: Cumulative Growth of BHC Earnings and Dividends (2007=100)



Note: Sample includes all banks that were registered as Bank Holding Companies (BHCs) in 2007 and were at any subsequent point subject to the stress tests of the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.

2020).

Research Agenda In this paper, we therefore study the effect of a forward-looking stress-test constraint on banks' dividend policies, equity levels, and lending activities. To answer this, we build a partial equilibrium framework that characterizes these three bank choices, given different return realizations and varying tightness of the stress-test constraint. We model the stress-test as a forward-looking constraint on the bank's balance sheet choices similar to a minimum equity ratio, thus mirroring the frequency in which stress tests are performed by supervisors (and abstracting from strategic balance sheet adjustments). We then derive the optimal tightness of the stress-test constraint for a supervisor who seeks to maximize lending levels while avoiding lending volatility. For this derivation we partially rely on a calibration of our model for a quantitatively meaningful discussion. Finally, we investigate how stress tests perform relative to other policies, such as the Covid-19 dividend ban, the countercyclical capital buffer, and the dividend prudential target by Muñoz (2020) (banks must pay a punishment fee when dividends deviate from a regulatory target).

Theoretical Framework To illustrate the effects of a stress-test constraint on bank balance sheet choices, we propose a three-period, partial equilibrium framework. The model is populated by a supervisor with mean-variance welfare over bank lending and a representative investor with mean-variance preferences over dividends received from investments in said bank loans.[4] The objective of the supervisor is, thus, in conflict with objective of the investor: while the investor prefers high and stable dividends, the supervisor prefers high and stable lending. The environment is characterized by a single source of uncertainty: loan returns evolve over all three periods following an AR(1) process. The parameter space additionally contains an initial bank equity endowment, an interest rate on bank deposits, and an exogenously given minimum equity-to-loan ratio requirement.

In period 0, an initial loan return state realizes and the representative investor is endowed with the equity holding in the bank. Observing both, the supervisor decides on the tightness of the forward looking stress-test constraint, our key novelty in this paper, with the objective to maximize welfare. The stress-test constraint will apply in period 1 and requires that the bank's retained equity is sufficient to absorb (simulated) severely adverse losses from the chosen lending levels without violating the minimum equity-to-loan ratio. In period 1, the bank observes the initial equity and an evolved loan return state. With the objective to maximize the shareholder's total expected dividends, the bank first decides how much equity to retain versus to pay out as period 1 dividends. The retained equity and additional external debt are used to invest in risky loans. Here, the degree of debt financing of loans is constrained by both the stress-test and the minimum equity-to-loan ratio constraint. In period 2, a further evolved loan return rate realizes and, together with last period's equity, lending, and debt choices, determines period 2 dividends. After paying out such to the investor, the bank ceases to exist.

Bank Choices First, we show that any meaningful stress test scenario results in a de facto increased minimum equity-to-asset ratio requirement. Hence, the forward looking stress-test constraint always binds before the minimum equity-to-loan ratio constraint. Moreover, the bank always lends as much as the stress-test constraint allows, given the level of optimal equity. The optimal equity follows a step function in return states: in bad return states no equity is retained as loans are very risky and investments not profitable; in medium states a portion of equity is retained for risky investments and a portion is paid out as dividends; only in high return states all inherited equity is retained to be fully invested in loans. Performing comparative statics over the stress-test constraint tightness (the severity of the adverse scenario) highlights the core supervisory trade-off: an increase in tightness leads to higher retained equity in (almost) all states of the world, but always reduces lending levels. At the same time, however, a tighter stress-test constraint leads to less volatile lending.

Optimal Tightness The underlying stochastic process together with the kinks in optimal lending and equity do not allow for a fully analytical expression of the opti-

[4] Assuming mean-variance preferences introduces the above described bank preference for smooth dividends. Lambrecht and Myers (2012) provide a micro-foundation for such an objective function.

mal stress-test tightness. To nevertheless provide a quantitative estimate, we calibrate the model parameters using the balance sheet data of U.S. bank holding companies that are subject to the stress tests implemented by the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.[5] We then numerically derive the ex-ante optimal tightness of the stress-test constraint that maximizes the supervisor's mean-variance preferences over expected lending. We find that the optimal tightness typically leads to additional capital buffers of $1\% - 9\%$, depending on the different initial return states and welfare weights: a supervisor more (less) concerned about the level than the volatility of lending imposes a looser (tighter) stress-test scenario; a supervisor in a higher (lower) initial return state imposes a looser (stricter) stress-test scenario. This numerical result closely matches the Federal Reserves' recently announced stress-test buffers for 2021 which are reported to lie between 2.5% to 7.5% (Federal Reserve Board, 2021), indicating that we are able to capture well the magnitude of bank balance sheet choices under stress tests.

We then perform two sensitivity analyses to investigate the robustness of our model assumptions. First, we show that it would be optimal for large U.S. banks to violate stress tests in high return states as long as there is no kind of cost associated with this violation. Furthermore, we show that a supervisor would have to put extraordinarily large weight on the investor's welfare for it to make a quantitatively meaningful difference for the optimal stress-test design.

Policy Extensions Utilizing the calibrated model, we first study bank choices when compliance with the stress-test constraint is voluntary and show that voluntary violation would often be optimal for stress-tested U.S. banks. We further use the model to evaluate several other policies in their ability to maintaining stable lending levels, acting both as complements and substitutes to stress tests. First, we investigate how a blanket dividend ban, as many supervisory agencies introduced at the beginning of the Covid-19 pandemic, impacts the lending of stress-tested banks. Here, we find that a ban successfully increases lending, but banks refrain from using as much debt financing as the stress-test constraint allows. Subsequently, we show that relaxing a counter-cyclical capital buffer (CCyB) increases lending in bad states. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects.[6] We conclude by comparing the performance of the dividend prudential target (DPT) of Muñoz (2020) with that of stress tests. Here, we find that a DPT is a useful policy instrument to maximise lending levels. However, if a supervisor also cares about the volatility of lending, even an optimal DPT

[5] See Appendix A for a detailed description of the regulatory environment.

[6] Here, we are thus able to provide an explanation for the current policy puzzle of unused CCyB buffers during the Covid-19 crisis (FSB, 2021).

policy leads to substantially lower welfare than the stress-test constraint.

Literature Our paper primarily contributes to the stress test literature. Thus far, the bulk of papers in this literature is empirical and studies the information revealing mechanism of stress-tests and their immediate impact on stock prices (Bird et al., 2020; Morgan et al., 2014; Petrella and Resti, 2013; Quijano, 2014). Even though the outcomes of stress tests are to a large extent predictable (Ahnert et al., 2020), a range of studies has shown that the release of stress test results nonetheless provides valuable information. Among others, Flannery et al. (2017) and more recently Fernandes et al. (2020) identify positive abnormal equity returns and negative responses of CDS spreads in response to stress-test disclosure. However, this effect is heterogeneous across the business cycle (Sahin et al., 2020), across banks' risk-exposure (Flannery et al., 2017), and between those banks passing and those failing the stress test (Sahin et al., 2020). As a result, the optimal disclosure policy of stress test results is not trivial: it depends non-linearly on a bank's capital gap (Goldstein and Leitner, 2018) and it is subject to a time inconsistency problem (Parlasca, 2021).

A small but growing empirical literature furthermore studies the change in lending levels following stress test announcements, thus going beyond the immediate disclosure effects of stress tests. Using U.S. loan-level data, Acharya et al. (2018), Cortés et al. (2020), and Doerr (2021) document that stress-tested banks reduce credit supply, especially to risky borrowers. However, it remains unclear whether this results in an aggregate decrease of credit supply or whether the decrease of stress-tested banks is offset by unaffected banks. Cappelletti et al. (2019) argue that the 2016 stress-testing exercise in the euro area similarly has led banks to increase their capital ratios by reducing their lending and risk-taking. Finally, Cornett et al. (2020) find that the banks subject to stress tests lower dividends significantly compared to non-tested banks. However, this behavior reverses completely afterwards, suggesting that stress-tested banks may be managing financial performance. Our paper provides the theoretical counterpart to these empirical analyses by rationalizing these findings in a partial equilibrium framework.

To the best of our knowledge, we are the first to explicitly model the forward looking stress-test constraint and thus theoretically study its impact on banks' joined decision over lending, equity, and dividend payments. The closest to our paper are Shapiro and Zeng (2019), who study how banks optimally risk-adjust their portfolio in response to stress tests, holding dividends, equity, and debt levels fixed. We complement their work by endogenising the banks' balance sheet choices while abstracting from portfolio risk-adjustments. For this purpose, we extend the banking model by Gollier et al. (1997), borrowing several elements from the dynamic banking literature. For our objective function, we rely on Lambrecht and Myers (2012), who provide a micro-foundation for the dividend smoothing behavior of banks. Further, we extend the uncertainty of the asset to span all three periods, by utilizing the AR(1) process describing loan returns in Bolton et al. (2020). To maintain tractability in the face of an evolving return state, we abstract from the possibility of bank default as originally studied in Gollier et al. (1997). The result is an easily extendable model that not only highlights the effect of stress tests, but allows us to study a range of complementary and substitute policy measures.

Overview The remainder of the paper is organized as follows. In Section 2, we describe the baseline model environment and state the bank's optimal dividend, equity, and lending choices. In Section 3, we calibrate the model to obtain a numerical value, consequently quantify the marginal responses of equity and lending to changes in the stress-test tightness, and finally numerically establish the optimal stress-test tightness. Section 4.1 addresses the possibility for banks to voluntary violate stress tests and consider the behavior of a supervisor who also takes into account the welfare of the investor. In Section 5, we discuss several policy extensions, such as the Covid-19 dividend ban, the CCyB, and the dividend prudential target. Section 6 concludes and puts the theoretical and calibration exercise in perspective. The appendix contains a detailed description of the regulatory framework and all proofs.

2. Theoretical Analysis

The following section contains the representative bank problem and is structured in the three following sub-sections: Section 2.1 describes the baseline partial equilibrium framework inspired by the dynamic banking models of Bolton et al. (2020) and Lambrecht and Myers (2012), but modified to a three-period environment to allow for a tractable introduction of bank stress tests;[7] Section 2.2 subsequently derives the lending and equity choices by a stress-tested bank and, relying on this, Section 2.3 performs comparative statistics to study the response of equity and lending to the introduction of a stress test.

[7] We rely on the serially auto-correlated loan returns from Bolton et al. (2020), but abstract from bank default and investments in risk-free bonds for tractability, as these play a subordinate role in a three-period model, where the choice is only between consuming today versus tomorrow. Similarly to Lambrecht and Myers (2012), we further assume that deposit rates are fixed and we rely on their Proposition 1 that provides a micro-foundation for the bank objective function proposed here. Here, we take advantage of the fact that normally distributed future loan returns simplify their exponential utility function to mean-variance utility. We additionally include a supervisor constraining bank choices via stress tests.

2.1. Three-period Model

The model is populated by a representative risk-averse investor owning a bank, or a representative bank for short, and a welfare-maximizing supervisor. Both agents live for three periods, denoted with $t = \{0, 1, 2\}$ respectively, and share a common discount factor β.[8] Each period t is characterized by the stochastic return on loans $r_{l,t}$ which follows an AR(1) process (more below). In period $t = 0$, an initial bank equity endowment $E_0 > 0$ and initial return state $r_{l,0}$ realize. Observing these, the supervisor decides on the optimal stress-test tightness τ. In period $t = 1$, the representative bank observes an evolved loan return $r_{l,1}$ and E_0, and decides how much of the inherited equity to pay out as dividends versus to retain for loan investments. Here, the additional deposit financing of loans is constrained by both the stress test and a minimum equity-to-asset ratio requirement. In period $t = 2$, a further evolved loan return state $r_{l,2}$, together with inherited loan, deposit, and equity levels, determines the final dividend payment by the bank to the investor.

$t = 0$

- Equity E_0 and return state $r_{l,0}$ realize
- Supervisor sets τ

$t = 1$

- Return state $r_{l,1}$ realizes
- Bank decides on dividends d_1, retained equity E_1, loans L_1 and deposits D_1

$t = 2$

- Return state $r_{l,2}$ realizes
- Dividends d_2 are determined given $r_{l,2}$, E_1, L_1 and D_1

The Investor There exists a representative investor who is hand-to-mouth and subject to mean-variance utility $u(\cdot)$ from received time t dividends d_t.[9][10] We denote the resulting aversion to risk with γ, such that:

$$u(d_t) = \mathbb{E}[d_t] - \frac{\gamma}{2}\mathbb{VAR}[d_t] \tag{1}$$

The Bank's Balance Sheet The investor dividends are financed through an initial equity endowment E_0 in a representative bank. At time $t = 1$, the bank observes E_0, a loan return state $r_{l,1}$, and the two regulatory constraints (more below). Given these states,

[8] We make this assumption for simplicity but it does not affect the model outcomes. As will be discussed in more detail in Section 3.3, the supervisor has preferences only about the expected level and variance of lending in period $t = 1$. Therefore, there is no intertemporal trade-off for the supervisor that is influenced be the discount factor.

[9] This assumption is micro-founded by Lambrecht and Myers (2012), who show that payout smoothing naturally arises when insiders are risk averse and/or subject to habit formation. Here, we rely on their result from Proposition 1 and directly model an objective function over dividends rather than over managerial rents subject to investor participation constraints.

[10] Because the investor always maximizes expected utility given normally distributed returns, we directly maximize mean-variance utility, whose solutions are exactly equal those given exponential utility and Taylor-approximate those of all other concave (risk-averse) utility functions (Levy and Markowitz, 1979; Markowitz, 2014).

the bank first decides how much initial dividends d_1 to pay versus how much equity E_1 to retain.

$$d_1 = E_0 - E_1 \tag{2}$$

Subsequently, the bank additionally sources costly deposits D_1, at the exogenous interest rate r_d, to finance investments in the risky loans L_1:

$$L_1 = E_1 + D_1. \tag{3}$$

In period $t = 2$, a new loan return $r_{l,2}$ realizes, where we assume that the loan returns follow an AR(1) process:

$$r_{l,t} = \mu_l + \rho_l r_{l,t-1} + \sigma_l \epsilon_t \qquad \text{where} \quad \epsilon_t \sim \mathcal{N}(0,1), \quad \mu_l > r_d, \quad \rho_l \in (0,1). \tag{4}$$

Then the combined choices of equity E_1, deposits D_1, and lending L_2 determine dividends d_2. Accounting for the underlying AR(1) process and the loan return state $r_{l,1}$, this implies:

$$d_2 = r_{l,2} L_1 - r_d D_1 + E_1 \qquad \text{where} \quad d_2 \sim \mathcal{N}\left(\left(\mu_l + \rho_l r_{l,1}\right) L_1 - r_d D_1 + E_1,\ L_1^2 \sigma_l^2\right). \tag{5}$$

The Supervisory Constraints The choices of E_1, D_1, and L_1 are restricted by two supervisory constraints: a minimum equity-to-asset ratio constraint and a stress-test constraint. The first defines a minimum equity-to-asset ratio χ that effectively restricts the bank's debt financing of loans. Here, we assume that the minimum ratio χ is given exogenously.[11] For the choices E_1 and L_1 this implies:

$$\frac{E_1}{L_1} \geq \chi. \tag{6}$$

The stress-test constraint is forward looking instead, and requires that the bank's available equity at time $t = 2$ cannot drop below χ even under a severely adverse loan return state realization $r_{l,2}$. Here, the expected available equity is the sum of the retained equity E_1 and next period profits $\Pi_2(\tau)$ simulated for stress-test scenario τ:

$$\Pi_2(\tau) = (\overline{\mu}_l - \tau \sigma_l) L_1 - r_d D_1 \qquad \text{where} \quad \overline{\mu}_l = \frac{\mu_l}{1 - \rho_l}. \tag{7}$$

[11] This follows the narrative that global minimum capital standards, such at the Basel III requirements, are not quickly and easily adjustable by a national authority without severe costs. Furthermore, it allows us to focus on the effect of the forward looking constraint.

Here, $\overline{\mu}_l$ denotes the unconditional mean of the AR(1) process and τ defines the number of standard deviations below $\overline{\mu}_l$ that describe the adverse scenario of $r_{l,2}$. As τ defines the severity of the adverse scenario, we will refer to it as stress-test constraint tightness throughout the paper. For now, tightness $\tau \geq 0$ is taken as given and can be interpreted as a model parameter. In Section 3, we relax the latter assumption and explicitly determine the optimal τ numerically. With the definition of $\Pi_2(\tau)$ in mind, the stress-test constraint thus takes the following shape:

$$\frac{E_1 + \Pi_2(\tau)}{L_1} \geq \chi. \tag{8}$$

The Bank's Optimization Problem The above described constraints complete the model environment and we now turn to the bank optimization problem in period $t = 1$. For this, we denote the investor's total utility from d_1 and d_2 with $U(d_1, d_2)$. The bank's optimization problem is thus:

$$U(d_1, d_2) = \max_{E_1,\, L_1} \quad d_1 + \beta \left[\mathbb{E}[d_2] - \frac{\gamma}{2} \mathbb{VAR}(d_2) \right], \tag{9}$$

s.t.

$$d_1 = E_0 - E_1, \tag{10}$$

$$L_1 = E_1 + D_1, \tag{11}$$

$$d_2 = r_{l,2} L_1 - r_d D_1 + E_1 \qquad \sim \mathcal{N}\Big((\mu_l + \rho_l r_{l,1}) L_1 - r_d D_1 + E_1,\; \sigma_l^2 L_1^2 \Big), \tag{12}$$

$$E_1 \geq \chi L_1, \tag{13}$$

$$E_1 + \Pi_2(\tau) \geq \chi L_1 \qquad \text{where} \quad \Pi_2(\tau) = (\overline{\mu}_l - \tau \sigma_l) L_1 - r_d D_1, \tag{14}$$

$$L_1 \geq 0, \tag{15}$$

$$E_1 \in [0, E_0]. \tag{16}$$

Here, Equations (10) - (12) are the bank's balance sheet constraints, Inequalities (13) and (14) denote the two supervisory constraints on equity, and Constraints (15) and (16) are the feasibility constraints on lending and equity.[12]

Parameter Restrictions For the AR(1) process on loan returns, we assume that $\mu_l > 0$, $\rho_l \in (0, 1)$ and $\sigma_l > 0$. For the supervisory constraints, we assume $\chi \in (0, 1)$ and $\tau \geq 0$. For the risk-aversion we assume that $\gamma > 0$. For the initial equity endowment, we assume that $E_0 >> 0$, reflecting that we are dealing with large banks. Finally, for the

[12] Constraint (15) implies that the bank cannot short-sell loans. In (16), the lower bound implies that the bank cannot debt-finance dividends and the upper bound rules out additional equity injections.

deposit rate, we assume that $r_d < \mu_l$ and $1 + r_d < 1/\beta$, jointly ensuring that debt financing of loans is desirable.[13]

2.2. The Bank's Optimal Choices

We now turn to solving the bank optimization, starting with simplifying the two supervisory constraints: the minimum equity-to-asset ratio (13) and the stress-test constraint (14). First, we use the budget constraint in (11) and the definition of $\Pi_2(\tau)$ to rearrange the stress-test constraint:

$$E_1 + (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d(L_1 - E_1) \geq \chi L_1, \tag{17}$$

$$E_1 \geq \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} L_1. \tag{18}$$

Comparing this to the minimum equity-to-asset ratio constraint in (13), it is easy to see that, for sufficiently large τ, the stress-test constraint always binds first:

$$\frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} \geq \chi, \tag{19}$$

$$\tau \geq \frac{\overline{\mu} - r_d(1 + \chi)}{\sigma_l} = \tilde{\tau}. \tag{20}$$

And for τ below $\tilde{\tau}$, the minimum equity-to-asset ratio constraint binds first. In either case, the second constraint is binding exclusively in states where the first one is binding too.

***Lemma* 1.** *There exists a stress-test tightness threshold $\tilde{\tau}$, such that :*

(i) If $\tau < \tilde{\tau}$, the minimum equity-to-asset ratio constraint always binds first.
(ii) If $\tau \geq \tilde{\tau}$, the stress-test constraint always binds first.

The results from *Lemma 1* allow us to generalize the bank optimization problem to nest both supervisory constraints in a single equity constraint:

$$E_1 \geq \chi(\tau)L_1 \qquad \text{where} \qquad \chi(\tau) = \begin{cases} \chi & \tau < \tilde{\tau} \\ \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} & \tau \geq \tilde{\tau} \end{cases}. \tag{21}$$

Relying on this, we then derive the bank's optimal equity, dividend, and lending choices as a function of $\chi(\tau)$. The proof is described in detail in Appendix B, but follows a few very intuitive steps. First, it can be shown that, given the parameter assumptions, equity-

[13]The latter implies that shareholders are less patient than depositors and thus have a preference for debt-financing of loans. As Gollier et al. (1997) discuss, this is a necessary assumption for this type of banking models and thus commonly found. The alternatives with $1/\beta = 1 + r_d$ and $1/\beta < 1 + r_d$ would respectively imply that the Modigliani Miller theorem holds or that the bank exclusively equity-finances loans.

financing loans is never desirable. Thus, the revised minimum equity constraint is always binding at the optimum. Denote the optimal loan level with L_1^*. Then this implies:

$$L_1^* = \frac{E_1}{\chi(\tau)}. \tag{22}$$

This result can be substituted into the bank optimization problem to simplify it further. Temporarily ignoring the feasibility constraints on equity, equating the first-order-condition with respect to retained equity with zero, yields the following optimal equity level E_1^*:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{23}$$

However, E_1 is feasibility-constrained from below at zero and from above at E_0. Inserting these bounds in the above Equation (23) and rearranging allows us to derive two thresholds $\underline{r_l}$ and $\overline{r_l}$:

$$\underline{r_l} = \frac{1}{\rho_l}\left[r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{24}$$

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{25}$$

Here, threshold $\underline{r_l}$ denotes the return state $r_{l,1}$ below which no equity is retained and $d_1^* = E_0$. $\overline{r_l}$ denotes the return threshold above which equity is fully retained and $E_1^* = E_0$. With this, the optimal choices are fully characterized for a given $\chi(\tau)$, and summarized in *Proposition 1.*

***Proposition* 1.** *A given constraint tightness τ, equity endowment E_0, and return state $r_{l,1}$ imply the following optimal bank choices:*

(i) If $r_{l,1} \leq \underline{r_l}$ all initial equity is paid out, such that:

$$d_1^* = E_0, \tag{26}$$

$$E_1^* = L_1^* = d_2^* = 0. \tag{27}$$

(ii) If $r_{l,1} \in (\underline{r_l}, \overline{r_l})$, some equity is paid out and some retained, such that:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{28}$$

$$d_1^* = E_0 - E_1^*, \tag{29}$$

$$L_1^* = \frac{E_1^*}{\chi(\tau)}, \tag{30}$$

$$d_2^* = \frac{E_1^*}{\chi(\tau)}(r_{l,2} - r_d) + E_1^*(1 + r_d). \tag{31}$$

(iii) If $r_{l,1} \geq \overline{r_l}$, the initial equity is fully retained, such that:

$$E_1^* = E_0, \tag{32}$$

$$d_1^* = 0, \tag{33}$$

$$L_1^* = \frac{E_0}{\chi(\tau)}, \tag{34}$$

$$d_2^* = \frac{E_0}{\chi(\tau)}(r_{l,2} - r_d) + E_0(1 + r_d). \tag{35}$$

It is important to note that the kinks in the lending function are not just outliers of the return distribution but are quantitatively important: For an initial equity level equal to the optimal equity level at the unconditional mean of the return process (i.e. $E_0 = E^{ss}(\tau)$), the full-retainment return level is exactly equal to the unconditional mean of the return process. To see this, first define the steady state equity level for a given stress-test tightness τ;

$$E^{ss}(\tau) = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{36}$$

and substitute it into the full-retainment return level:

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]\right) + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{37}$$

which simplifies to

$$\overline{r_l} = \frac{1}{\rho_l}(\bar{\mu}_l - \mu_l) = \bar{\mu}_l. \tag{38}$$

Therefore, the bank will retain all its initial equity for all return states equal to or larger than the unconditional mean of the return process. The associated lending function will, thus, be also flat for all return states above the unconditional mean. This discontinuity prevents us from deriving a closed-form solution for the optimal stress-test tightness τ^* so that we rely on a numerically solution in Section 3.3 instead.

2.3. The Effect of Stress Tests

In this section, we analyze how E_1^* and L_1^* change when the supervisor decides to introduce a stress-test constraint by raising τ above $\tilde{\tau}$. For this purpose, we introduce two additional superscripts ^{e} and ^{s}, denoting the equilibrium outcomes under a binding minimum equity-to-asset ratio and a binding stress-test constraint, respectively.

First it can be shown that raising τ implies a higher $\chi(\tau) > \chi$, which consequently results in a higher no-retainment state $\underline{r_l}$. We thus have that:

$$\underline{r^s_{l,1}} > \underline{r^e_{l,1}}. \tag{39}$$

An introduction of a τ above $\tilde{\tau}$ also implies that the full-retainment state is reached earlier:

$$\overline{r^s_{l,1}} < \overline{r^e_{l,1}}. \tag{40}$$

This implies that at the low end of the return distribution, a stress-test constraint incentivizes banks to retain equity only in relatively better states. At the high end of the return state distribution, full retainment is reached already at relatively worse states. Complementing this, it can be shown that for all $r_{l,1}$ above $\underline{r_l}$ and below $\overline{r_l}$, the optimal retained equity E_1^* increases linearly in $r_{l,1}$ but with a steeper slope, the higher the τ:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma \sigma_l^2}\rho_l \qquad\qquad \frac{\partial^2 E_1}{\partial r_{l,1}\partial \tau} = \frac{\rho_l}{(1+r_d)\gamma\sigma_l} > 0. \tag{41}$$

Therefore, there exists a return state $\tilde{r} \in (\underline{r^e_{l,1}}, \overline{r^e_{l,1}})$, below (above) which a stress-test constrained bank retains less (more) equity than if it was constrained by the minimum-equity constraint only. Using Equation (23) we can characterize this threshold $\tilde{r}$ as:

$$\tilde{r}_l = \frac{1}{\rho_l}\left[r_d - \mu_l + \left(\chi\left(\tau\right) + \chi\right)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{42}$$

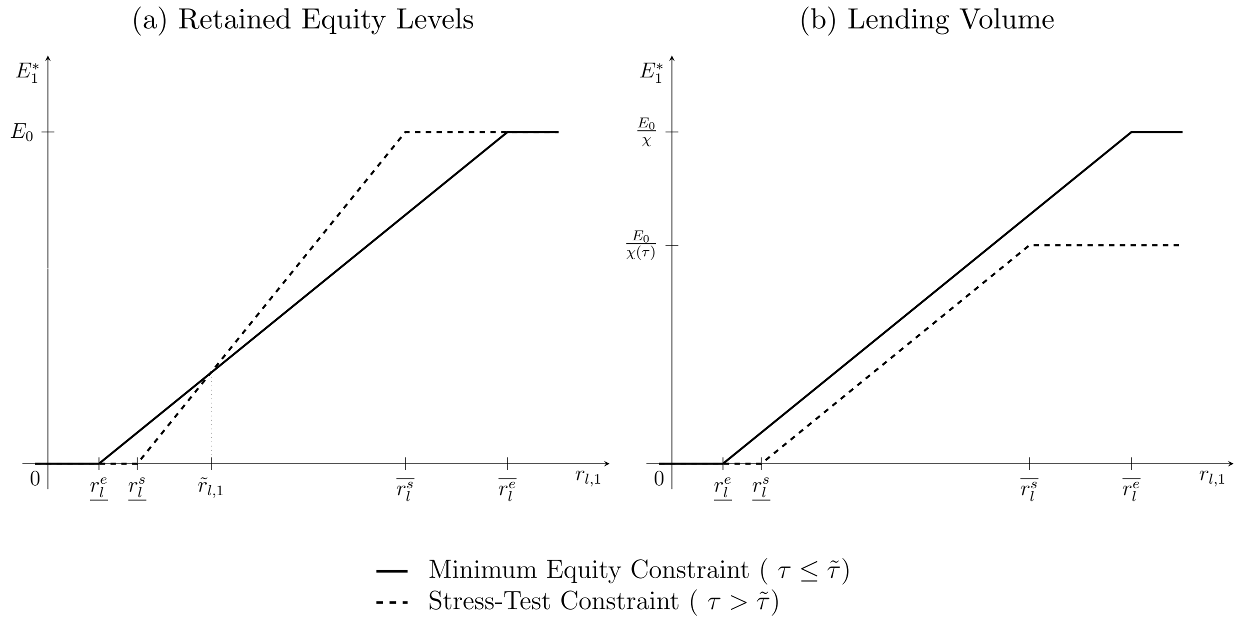
$$= \underline{r^s_{l,1}} + \frac{\chi}{\rho_l}\left(\frac{1}{\beta} - 1 - r_d\right). \tag{43}$$

Here, Equation (43) rearranges $\tilde{r}_l$ as a function of the no-retainment state, showing it to be only marginally higher. Thus, in most return states (and definitely the positive states) more equity is retained under stress tests. Figure 2a below illustrates this effect of a stress-test constraint on retained equity.

Corollary 1. *Raising τ above $\tilde{\tau}$ leads to more retained equity in almost all states of the world.*

Figure 2b complements the comparison, by illustrating the effect of the stress-test constraint on lending. Here, we can see that the higher retained equity levels between $\tilde{r}_{l,1}$ and $r^s_{l,1}$ never translate into higher lending volumes. The extra equity is lower than the equity

Figure 2: Minimum Equity-to-asset Ratio Versus Stress-test Constraint



level that would be required to maintain the same level of lending under the tighter equity ratio constraint which is implied by the stress-test constraint. Thus:

$$L_1^{*,s} < L_1^{*,e} \quad \forall r_{l,1} > \underline{r_{l,1}^e}. \tag{44}$$

Furthermore, the volatility of lending also decreases under the stress-test constraint, given that equity retainment starts only at a relatively better state but the full-retainment state is reached earlier.

Corollary 2. *Raising τ above $\tilde{\tau}$ implies strictly lower but less volatile lending.*

3. Calibration & Optimal Stress-test Tightness τ

We now turn to the supervisory choice of τ in period 0 and the resulting impact on lending and equity levels. Since this analysis requires a realistic model calibration, we use balance sheet data of stress-tested U.S. banks and discuss our model calibration in Section 3.1. We then use the calibrated model to quantify the marginal effects of adjusting the stress-test tightness τ on lending and equity in Section 3.2. In a final step, we compute the optimal choice of τ in Section 3.3.

3.1. Model Calibration

To provide a quantitative estimate of the optimal τ, we calibrate our model with two sets of parameters (see Table 1).

The first set of parameters (Panel A. of Table 1) consists of the discount factor, the risk aversion parameter, and the minimum equity-to-asset ratio. We pick a discount factor β equal to 0.99, which corresponds to an annualized real interest rate of 1%. We take the risk aversion parameter from Eisfeldt et al. (2020) and set it to 4.37. Furthermore, we take a minimum equity-to-asset ratio of 7% as given.

The second set of parameters (Panel B. of Table 1) describes the loan return process as well as the return on deposits. For these parameters we use balance sheet data of U.S. Bank Holding Companies with more than \$10bn in assets between 2009 - 2019 (i.e. banks subject to CCAR stress tests) to calibrate the parameters of the loan return process as well as the return on deposits.

To calibrate the return process, we follow De Nicolò et al. (2014) and estimate an AR(1) process on the mean excess return on assets. We use the excess return over the risk-free interest rate to make sure that return movements are not driven by movements in the risk-free interest. We compute the excess return on assets as the ratio of the interest and non-interest revenues to lagged assets (items *bhcp4000* and *bhck2170* respectively in the FR Y–9C reports) minus the 1-year Treasury rate. We then add this excess return to our implied (time-invariant) risk-free rate $1/\beta - 1$ to arrive at the mean of the return process. The calibrated return process has a mean of 1.02% with a standard deviation of 0.52% and a autocorrelation of $\rho_l = 0.62$, which implies an unconditional mean return of 2.66%.

To calibrate the deposit rate r_d, we again start by eliminating the movements of the risk-free rate and first estimate the deposit spread. We compute the deposit spread as the mean difference between the 1-year Treasury rate and the mean deposit rate, given by the ratio of interest paid on deposits (the sum of items *bhckhk03*, *bhckhk04*, *bhck6761*, and *bhck4172*) to lagged deposits (the sum of items *bhdm6631*, *bhdm6636*, *bhfn6631*, *bhfn6636*). We then subtract this deposit spread from our implied risk-free rate $1/\beta - 1$ to arrive at the deposit rate. Over our sample period, bank deposits yielded on average 0.39 percentage points less than the 1-year Treasury rate, yielding a return on deposits of 0.62% for our implied risk-free rate of 1%.

Alongside the calibrated parameters, Panel B. of Table 1 also reports the bootstrapped standard deviations (in parenthesis) of the estimated parameters. We use these estimates to conduct a robustness analysis for the supervisory choice of τ.

Table 1: Calibration

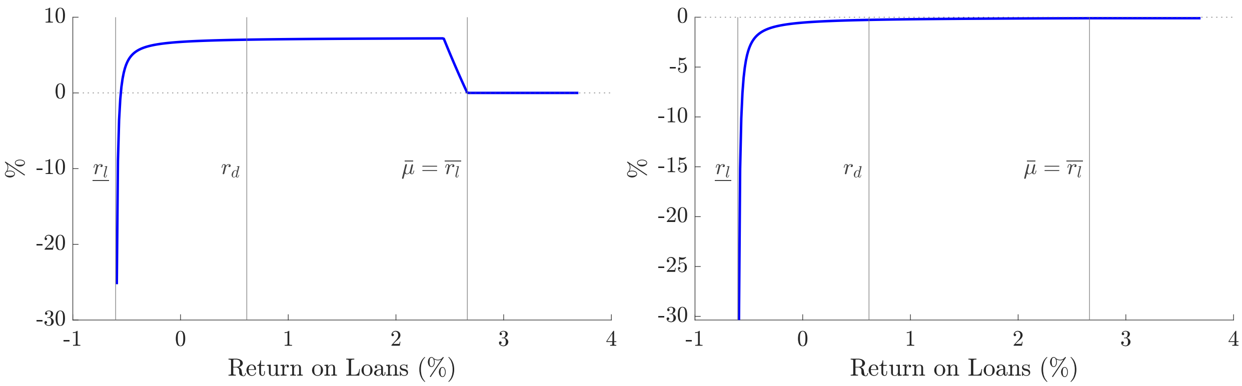
Description	Parameter	Value
A. Parameters assumed / obtained from literature		
Discount Factor	β	0.99
Risk Aversion	γ	4.370
Minimum Equity-to-asset Ratio	χ	0.07
B. Parameters estimated from data		
Mean Return of Risky Asset (%)	μ_l	1.02
		(0.03)
AR(1) of Risky Asset	ρ_l	0.62
		(0.04)
SD of Risky Asset (%)	σ_l	0.52
		(0.03)
Lending Spread (%)	$1/\beta - 1 - r_d$	0.39
		(0.01)
Return on Deposits (%)	r_d	0.62
		(0.01)

Note: Bootstrapped standard deviations reported in parenthesis.

3.2. *Effect of Stress Tests on Equity and Lending*

To illustrate the effect of stress tests, we use the calibrated model and plot the marginal responses of equity and loan levels (in %) to a one unit increase in the tightness of the stress-test constraint τ in Figure 3. It is clear that the effect of a higher stress-test constraint is highly non-linear in the state of the business cycle, i.e. the return state.

Figure 3: Marginal Response of Equity and Lending to a Unit Increase in τ



(a) Marginal Response of Equity (%)

(b) Marginal Response of Lending (%)

Following an increase of the stress-test tightness τ, equity (left panel) is lower for very bad states of the world due to an increased no-retainment threshold (see Equation 43). However, for most of the return realizations below the unconditional mean return $\bar{\mu}_l$, equity is higher following the increase of τ. For return realizations above $\bar{\mu}_l$ the increase of τ does not lead to higher equity retainment, since banks retain all of their equity either way.

Lending volumes (right panel) are affected by changes of both retained equity as well as the minimum equity constraint in response to an increase of the stress-test tightness τ. For all return states below the unconditional mean return $\bar{\mu}_l$, an increase in τ reduces lending because the the increase in the minimum equity constraint offsets the increase in retained equity. For all return states above the unconditional mean return $\bar{\mu}_l$, retained equity is unchanged but the increased minimum equity constraint leads to lower lending. However, this effect is marginal because a unit increase in τ increases the implied equity constrained only by $^{\sigma_l}/_{1+r_d}$.

This demonstrates that in all but very bad states of the world, the increase of τ can weakly enhance the safety of banks, but this unequivocally comes at the cost of lower lending levels, as the right panel shows. This reduction in lending, however, approaches zero as the return realisations increase.

3.3. The Supervisory Choice of τ

We now investigate how a supervisor optimally sets the severity of simulated losses used in the stress test (i.e. the number of standard deviationsτ below the mean return $\bar{\mu}$) with the objective to ensure stable lending levels.[14] Here, Corollary 2 highlights the supervisory trade-off between reduced but consequently less volatile lending. To capture this trade-off, we assign the welfare weight $\omega \geq 0$ to the expected variance of optimal bank lending L_1^* (we explore the implications of an alternative supervisor welfare function in Section 4.2). Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \omega \mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0], \tag{45}$$

s.t.

$$\chi(\tau) \in [\chi, 1), \tag{46}$$

where

$$r_{l,1} \leq \underline{r_l}: \quad L_1^* = 0, \tag{47}$$

$$r_{l,1} \in (\underline{r_l}, \overline{r_l}): \quad L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2}, \tag{48}$$

$$r_{l,1} \geq \overline{r_l}: \quad L_1^* = \frac{E_0}{\chi(\tau)}. \tag{49}$$

Equations (47) to (49) show that the supervisor anticipates a rectified normally distributed L_1^* with upper and lower bounds: (47) states that below $\underline{r_l}$, lending L_1^* is set to zero; (48) implies that between $\underline{r_l}$ and $\overline{r_l}$ lending is normally distributed with $\mathcal{N}\left(\mu_{L_1}, \sigma_{L_1}^2\right)$;[15] (49) states that above $\overline{r_l}$, lending is set to $E_0/\chi(\tau)$.

To identify the optimal stress-test tightness τ^*, we utilize our parameterization from Section 3.1 and computationally maximize the supervisor's welfare directly, subject to the respective constraints. As argued previously, the fact that loans follow a two-sided rectified distribution prevents us from deriving a closed-form expression for the optimal stress-test tightness so that we solve this problem numerically. Since the results depend to a large degree on the amount of initial equity E_0, we first define the steady state level E_1^{ss} in the absence of stress tests as

$$E_1^{ss} = \frac{\chi}{\gamma \sigma_l^2} \left[\bar{\mu}_l - r_d - \chi \left(\frac{1}{\beta} - 1 - r_d \right) \right], \tag{50}$$

and fix the initial equity endowment E_0 at this level to ensure comparable results.

To examine the supervisor's decision in more detail, we compute the optimal τ^* for different relative welfare weights ω. In particular, we compute the optimal stress-test tightness

[14] Note that this supervisory objective is taken directly from the Federal Reserve Board (2020c).

[15] Closed form expressions for μ_{L_1} and $\sigma_{L_1}^2$ can be found in Appendix C.

for a supervisor who does not care about lending volatility (i.e. $\omega = 0$), a supervisor who cares as much about lending volatility as about lending levels (i.e. $\omega = 1$), a supervisor who dislikes lending volatility as much as the investor (i.e. $\omega = \gamma/2$), and a supervisor who dislikes lending volatility twice as much as the investor (i.e. $\omega = \gamma$).

Table 2 below states the resulting, numerically derived optimal stress-test tightness τ^*, the implied minimum equity to asset ratio $\chi(\tau)^*$ (see Equation 21), and the associated supervisory welfare for the different welfare weights given an initial return realization of $r_{l,0} = \bar{\bar{\mu}} = 2.66\%$. Table 2 also reports the 95% confidence intervals of each optimal policy in square brackets. These confidence intervals are obtained by taking 10,000 draws from the distribution of parameters reported in Table 1 and computing the associated optimal supervisory policy. Based on the implied welfare for the respective τ^*, it is clear that the supervisory welfare function is decreasing in the weight given to the variance of loans. The supervisor therefore optimally sets $\tau^* = 4.05$ such that $\chi(\tau^*) = \chi$ when she does not derive any disutility from the variance of loans (i.e. when $\omega = 0$) to maximize the level of loans. However, as ω increases and she derives more disutility from the variance of loans, she optimally sets a higher τ^* to reduce that variance. In the other extreme case, i.e. when the supervisor dislikes volatility twice as much as the investor, she forces the bank to retain additional stress-test capital buffer of 4%. The estimates of these additional capital buffers generally are associated with confidence bands of up to 2%, indicating that it might be optimal for a very risk averse supervisor to require additional stress-test capital buffers of up to 6%. This matches well the Federal Reserve's publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021) and indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

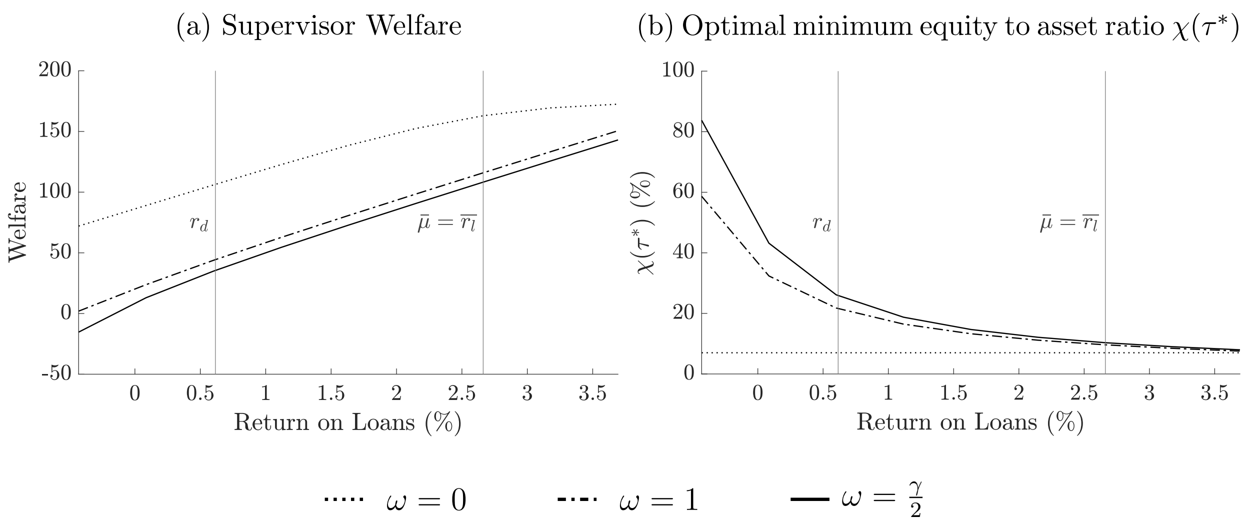
To further illustrate how these findings vary over different realizations of the initial return state r_0, Figure 4 displays the supervisor welfare (left panel) as well as the minimum equity to asset ratio $\chi(\tau^*)$ (right panel) implied by the optimal stress-test severity τ^* as a function of $r_{l,0}$ for different welfare weights ω. It is clear that the supervisory welfare function is increasing in the initial return realization $r_{l,0}$ and decreasing in the weight given to the variance of loans. However, the differences in welfare and the minimum equity to asset ratio decrease for higher realizations of $r_{l,0}$. Therfore, the supervisor would find it optimal to conduct significantly more severe stress tests during crises, imposing additional capital buffers of around 15% when loan returns fall below funding costs (i.e. for $r_{l,0}$ just below r_d).

Table 2: Optimal Stress-test Tightness and Supervisor Welfare

Welfare Weight	Optimal Tightness τ^* (%)	$\chi(\tau^*)$	Welfare
$\omega = 0$	4.05	7.00	162.96
	[4.05 4.05]	[7.00 7.00]	[123.28 209.56]
$\omega = 1$	9.16	9.62	115.93
	[6.90 13.15]	[8.46 11.66]	[95.22 143.09]
$\omega = \gamma/2$	10.53	10.32	108.25
	[8.23 14.57]	[9.14 12.39]	[88.45 134.51]
$\omega = \gamma$	11.81	10.97	101.84
	[9.48 15.93]	[9.78 13.08]	[82.78 127.40]

Note: This table shows the results of computationally maximizing the supervisor's welfare, subject to the respective constraints (see Equation 45-49). We rely on the calibration from Section 3.1 to derive the optimal stress-test tightness τ, the implied minimum equity to asset ratio $\chi(\tau^*)$ (see Equation 21) and the associated supervisor welfare for different supervisory welfare weights ω. Values in square brackets indicate the 95% confidence intervals for each estimate constructed by taking 10,000 draws from the distribution of parameters reported in Table 1 and computing the associated optimal supervisory policy.

Figure 4: Welfare and Minimum Equity to Asset Ratio under Optimal Stress Tests



4. Sensitivity Analysis

In this section, we perform two sensitivity analyses to investigate the robustness of the model and findings. In Section 4.1, we study whether banks ever voluntarily fail stress tests. In Section 4.2, we derive the optimal τ^* in the case, where the supervisor not only cares ab out lending stability but also partially considers bank investor utility.

4.1. Voluntary Stress-test Violation

In our baseline model environment, banks can neither violate the minimum equity-to-asset ratio nor the stress-test constraint. The U.S. stress test framework, however, allows for voluntary violation of the stress-test constraint, albeit automatically triggering a (partial) ban on dividend payments (see Appendix A for details). This violation allows the bank to invest up to a binding minimum-equity-to-asset ratio constraint instead. In this section, we investigate when a bank might find it optimal to purposely violate the stress-test constraint. For simplicity, we assume that this immediately triggers a total ban on dividend payments in that period. Then, voluntary violation implies the following equalities:

$$d_1 = 0, \tag{51}$$

$$E_1 = E_0, \tag{52}$$

$$D_1 = L_1 - E_0. \tag{53}$$

Inserting these equalities in the original maximization problem results in the following revised bank objective:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2, \tag{54}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right]. \tag{55}$$

Because full retainment implies sub-optimally high equity levels, the bank no longer chooses to equity-finance as little as possible. The upper feasibility limit in (55) reflects this, where now χ applies instead of $\chi(\tau)$. Quite intuitively, the upper feasibility limit is binding in very high return states above a threshold $\overline{r_{l,1}^V}$, where the bank would like to invest more in loans than the minimum equity requirements allow. Hence:

$$L_1^{*V} = \frac{E_0}{\chi} \qquad \forall r_{l,1} \geq \overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right]. \tag{56}$$

On the contrary, the lower feasibility limit is binding in bad return states, where the bank would like to invest nothing but must at least invest E_0. This applies to all return states below threshold $\underline{r_{l,1}^V}$:

$$L_1^{*V} = 0 \qquad \forall r_{l,1} \leq \underline{r_l^V} = \frac{1}{\rho_l}\left[\sigma_l^2 E_0 + r_d - \mu_l\right]. \tag{57}$$

In between the two return thresholds, the bank equity-finances loans with a share strictly above χ but below one. The optimal loan level is determined by the first-order-condition of the objective function (54), when both feasibility constraint multipliers are zero. For $r_{l,1}$ above $\underline{r_{l,1}^V}$ and below $\overline{r_{l,1}^V}$, this implies an optimal lending:

$$L_1^{*V} = \frac{\mu_l - \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_{l,1}^V},\ \overline{r_{l,1}^V}\right). \tag{58}$$

To derive when voluntary violation is optimal, we must compare the total shareholder utility from voluntary violation, denoted with $U^V(d_1, d_2)$, to the one from the baseline analysis, denoted $U(d_1, d_1)$.

Total utility under voluntary violation:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0, \tag{59}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + (1 + r_d)\, E_0\right], \tag{60}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}, \tag{61}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0\,(1 + r_d)\right], \tag{62}$$

Total utility under compliance (baseline):,

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0, \tag{63}$$

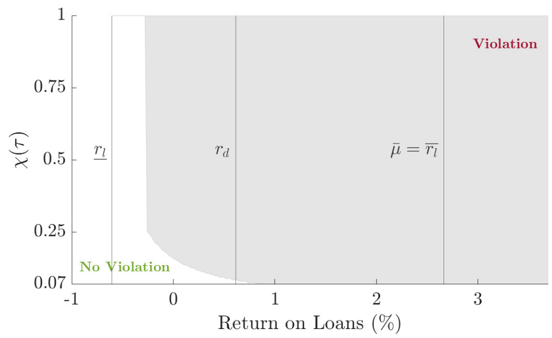
$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right], \tag{64}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2}, \tag{65}$$

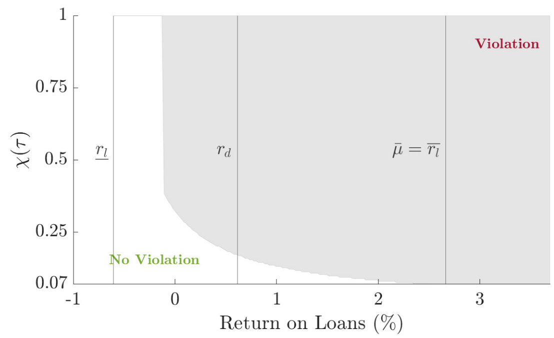
$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{2}\left(\frac{E_0}{\chi(\tau)}\right)^2 + E_0(1 + r_d)\right]. \tag{66}$$

To prove when $U^V(d_1, d_2)$ exceeds $U(d_1, d_1)$ is cumbersome, as the sizes of return thresholds $\underline{r_l^V}$ and $\overline{r_l^V}$ relative to $\underline{r_l}$ and $\overline{r_l}$ strongly depend on the initially inherited equity E_0 relative to other model parameters. Hence, a large number of different utility functions would have to be compared to cover all cases. Instead, we provide insights for a meaningful parameter space and numerically study the voluntary violation decision for large US banks, given our calibration. Figure 5 (below) illustrates when a bank violates the stress tests voluntarily for the above presented calibration and three levels of initial equity as a function of the steady state equity level E_1^{ss}.

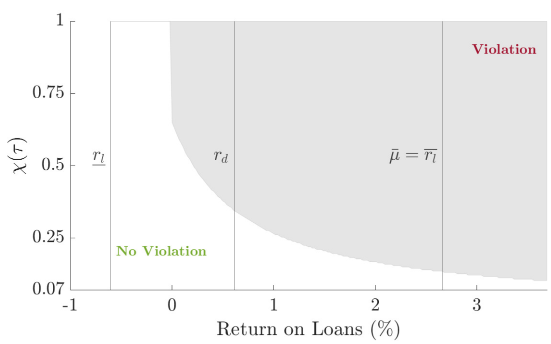
Figure 5: Optimal Choice of Stress-test Violation



(a) $E_0 = 0.5 \cdot E_1^{SS}$



(b) $E_0 = E_1^{SS}$



(c) $E_0 = 2 \cdot E_1^{SS}$

Each of the Panels 5a - 5c has the continuum of loan returns $r_{l,1}$ on the x-axis and the range of possible stress-test-implied minimum equity-to-loan ratio requirements on the y-axis. The gray shaded areas indicate when the bank finds it optimal to voluntarily violate the stress-test constraint. Here, we can see that this is generally the case for higher $\chi(\tau)$ and higher return states $r_{l,1}$. This should come as no surprise: the higher $\chi(\tau)$, the lower the total loans a stress-test compliant bank may issue and the more it can increase the loan capacity by voluntarily violating. Further, expanding loan capacity is more attractive in good states of the world, where risky loan investment is desirable. On the contrary, exposing (sub-optimally high) equity levels to risky loans in bad states by violating the stress-test constraint is not desirable. Therefore, the desirability of violation also decreases with the size of the initial equity endowment.

Remark 1. *For U.S. stress-tested banks, violation is optimal for higher tightness τ, higher loan return states $r_{l,1}$, and lower initial equity E_0.*

It should be noted, however, that the voluntary violation of the stress test constraint in our model does not incur any costs above and beyond the restriction on dividend payouts, such as financial market stigma or increased supervisory scrutiny. This explains why in reality, unlike our model would predict, large banks almost never violate the stress-test constraint.

4.2. Alternative Supervisor Welfare Function

In our baseline model environment we assumed that the supervisor only cares about the level (and potentially variance) of lending. We now allow the supervisor to also place weight on the investor's utility. Therefore, we let the supervisor set the optimal stress-severity with the goal to not only ensure high and stable lending levels, but also high and stable dividend levels in order to ensure that the bank is able to meet its obligation to its shareholder. To capture the trade-off between the supervisor's and the investor's preferences, we assign the welfare weight $\phi \geq 0$ to the time 0 expected utility of the bank's shareholder. We, furthermore, assume that both the supervisor and the bank shareholder assign the same welfare weight $\frac{\gamma}{2}$ to the expected variance of loans and dividends, respectively. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\begin{aligned} \max_{\tau} \quad & \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \frac{\gamma}{2}\mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0] \\ & + \phi\left(\mathbb{E}[d_1^* \mid r_{l,0}, E_0] + \beta\mathbb{E}[d_2^* \mid r_{l,0}, E_0] - \beta\frac{\gamma}{2}\mathbb{VAR}_0[d_2^* \mid r_{l,0}, E_0]\right), \end{aligned} \tag{67}$$

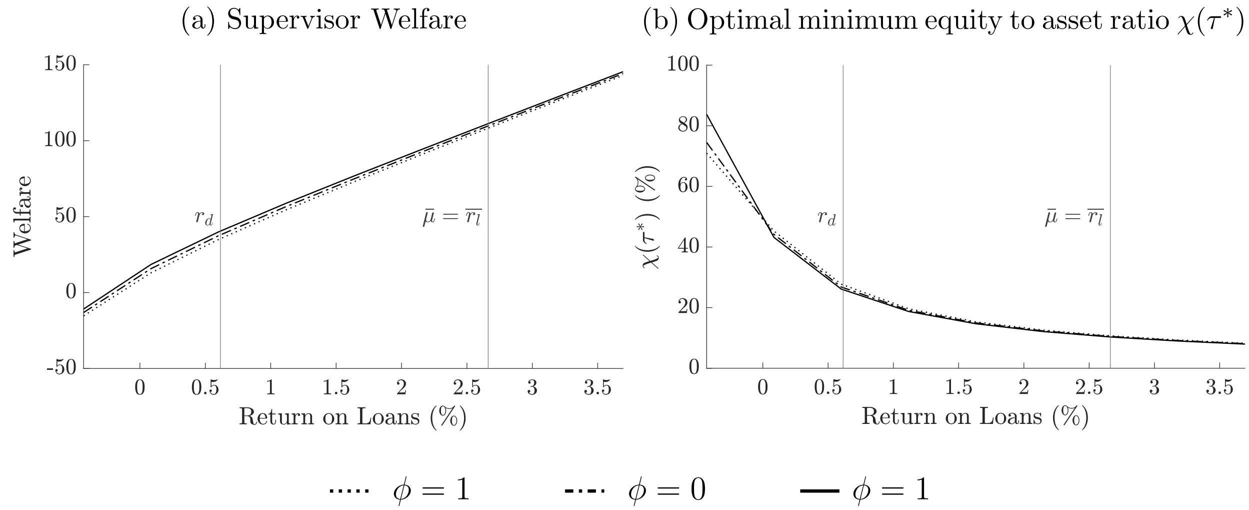
s.t.

$$\chi(\tau) \in [\chi, 1). \tag{68}$$

As Section D.5 in the Appendix shows, the supervisor anticipates rectified normally distributed L_1^*, d_1^*, and d_2^*. To identify the optimal stress-test tightness τ^*, we utilize our parameterization from Section 3.1 and computationally maximize the supervisor's welfare directly, subject to the respective constraints. As argued previously, the fact that loans and dividends follow rectified distributions prevents us from deriving a closed-form expression for the optimal stress-test tightness so that we solve this problem numerically. Figure 6 plots the corresponding supervisor welfare function (left panel) and the minimum equity to asset ratio $\chi(\tau^*)$ (right panel) implied by the optimal stress-test severity τ^* as a function of the initial return realization $r_{l,0}$ for different welfare weights ϕ.

As in the original welfare function, the supervisor's welfare is increasing in the initial return realization $r_{l,0}$ (note that the solid line here corresponds to the solid line in Figure 4). Furthermore, the supervisor's welfare is also increasing in the weight given to the investors

Figure 6: Welfare and Minimum Equity to Asset Ratio under Optimal Stress Tests



utility. Interestingly, the optimal equity to asset ratio $\chi(\tau^*)$ is decreasing in the initial return realization $r_{l,0}$ but not in the welfare weight ϕ: For low levels of $r_{l,0}$ the supervisor optimally *decreases* the severity of the stress test the more weight he gives to the investor's preferences, whereas for high levels of $r_{l,0}$ the supervisor optimally *increases* the severity of the stress test the more weight he gives to the investor's preferences. However, in general the differences in the supervisor welfare and the minimum equity to asset ratio for different welfare weights ϕ are quantitatively small. This indicates that a supervisor would have to put extraordinarily large weight on the investor's welfare for it to make a quantitatively meaningful difference for the optimal stress-test design.

5. Stress Tests in the Wider Regulatory Environment

Bank stress tests are typically seen as a complementary measure to a rich set of additional prudential policies. To understand their relative effectiveness in stabilizing lending, we extend the model to include two currently utilized policy tools: the Covid-19 dividend ban and the counter-cyclical capital buffer (CCyB). Additionally, we provide a welfare comparison of stress tests to the dividend prudential target proposed by Muñoz (2020). In the latter, dividends are regulated directly, but less intensely than in an outright ban.

5.1. *Covid-19 Dividend Restrictions*

At the onset of the Covid-19 crisis, several jurisdictions introduced either an outright ban on dividend payments or a strong recommendation to stop payments temporarily (Beck et al., 2020). The goal was to boost equity and thereby counteract the procyclicality of lending. Here, we abstract from any moral suasion frictions between supervisors and banks, and analyze the effect of an outright dividend ban on bank lending levels.[16] A ban on dividends implies full equity retainment, such that:

$$d_1 = 0, \tag{69}$$

$$E_1 = E_0, \tag{70}$$

$$D_1 = L_1 - E_0. \tag{71}$$

Inserting these into the bank's original optimization problem results in a revised maximization similar to the one under voluntary violation:

$$U^B(d_1, d_2) = \max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2, \tag{72}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right]. \tag{73}$$

However, here the stress-test constraint still applies and determines the upper bound of loan investments in (73). Stress tests thus act as a feasibility constraint for the revised bank maximization problem. Again, the lower and upper feasibility bounds on L_1 imply two return thresholds denoted with $\underline{r_l^B}$ and $\overline{r_l^B}$ respectively:

$$\underline{r_l^B} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \qquad \overline{r_l^B} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + r_d - \mu_l\right]. \tag{74}$$

Unlike in the baseline model, however, the two thresholds determine the share of debt financing instead of the degree of equity retainment: for return states below $\underline{r_{l,1}^C}$, the bank fully equity-finances L_1 now equal to E_0. Intuitively, in these bad return states, the shareholder would prefer to liquidate the bank but this is prevented by the dividend ban. Thus the only remaining option is to invest the existing equity in loans.

$$L_1^{*B} = E_0 \qquad \forall\, r_{l,1} \le \underline{r_l^B}. \tag{75}$$

For intermediate return states, the bank sets an optimal loan level L_1^{*B} that requires a share of equity financing strictly below one but strictly above $\chi(\tau)$. Intuitively, in these return states the shareholder would actually prefer some dividends in period 1 but this is

[16] This is without loss of generality. As Beck et al. (2020) show, most European banks did indeed stop dividend payments following the ECB's recommendation.

prevented by the dividend ban. At the same time, the loans are still relatively risky, limiting the attractiveness of investing in them. Thus the bank utilizes all its equity, but does not lever up as much as it could. In this case, the level of lending is:

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_l^B}, \overline{r_l^B}\right). \tag{76}$$

For high return states above $\overline{r_l^B}$, the bank debt-finances as much as possible given E_0 and $\chi(\tau)$, where the stress-test constraint now becomes the upper feasibility limit:

$$L_1^{*B} = \frac{E_0}{\chi(\tau)} \qquad \forall r_{l,1} \geq \overline{r_l^B}. \tag{77}$$

Comparing the optimal lending of a bank with free reign over the dividend payments with the one subject to a ban, we show that for all return states below $\overline{r_l}$, the lending is higher under the latter. Only for return states above $\overline{r_l}$ is the feasibility constraint on total lending binding under both regimes and, thus, lending is identical.[17]

***Proposition* 2.** *A dividend ban leads to strictly higher lending during crises.*

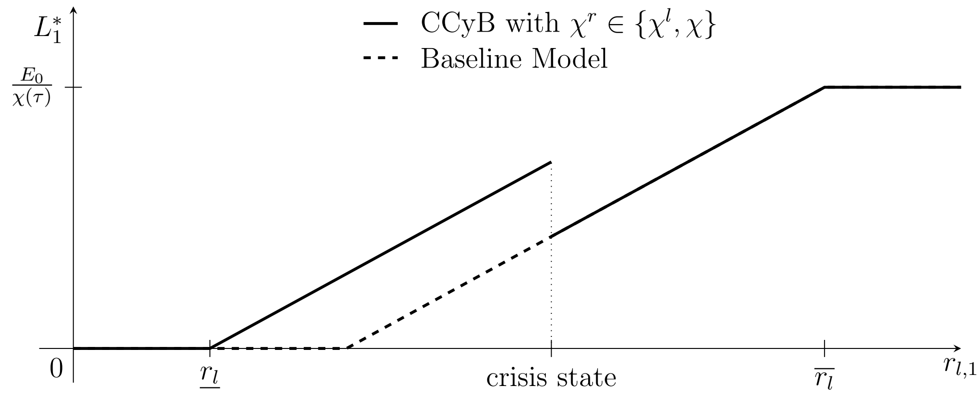
5.2. Counter-Cyclical Capital Buffer

A complementary policy tool to the dividend ban is the relaxation of the counter-cyclical capital buffer (CCyB) during times of crises. In the baseline model, we have assumed a constant χ that is state-independent. Instead, a CCyB implies a state-dependent χ^r that takes on a value $\chi^l < \chi$ for low return states. This relaxes the stress-test constraint in bad states via a reduction in $\chi(\tau)$. Relying on insights from Section 2.2, we know that this triggers an increase in lending and lowers the return thresholds below which no equity is retained. Figure 7 below illustrates this.

Relaxing a CCyB is often combined with other crisis measures, such as the Covid-19 dividend ban discussed above. Two natural question are thus, how both compare in their ability to increase lending during a crisis, but also how effective is a joint introduction of both. Here, it can be shown that lending under a dividend ban is strictly higher than under a relaxed CCyB. Intuitively, the main driver of lower loan levels in bad return states is equity withdrawal, which is not adequately addressed by relaxing the CCyB. Furthermore, the CCyB actually has no additional effect once a dividend ban is put in place. A bank subject to a ban already holds sub-optimally high equity and debt-finances less than allowed.

[17] Note here that for the formal proof, we account for the fact that the thresholds $\underline{r_l}$ may be above or below $\underline{r_l^B}$. However, $\overline{r_l^B}$ is always below $\overline{r_l}$.

Figure 7: The Impact of CCyBs on Lending



Therefore, a relaxed CCyB does not change the optimal loan levels when activated on top of a dividend ban during a crisis.

***Proposition* 3.** *Introduced individually, a relaxed CCyB increases lending during crises. However, the CCyB is less effective than a dividend ban and, if introduced additionally, has no further effect.*

We are thus able to provide an explanation for the recent policy puzzle regarding banks not using their CCyB buffers to finance lending during the Covid-19 crisis (FSB, 2021): additionally relaxing of CCyBs simply does not impact lending choices of already dividend restricted banks.

5.3. *Dividend Prudential Target*

Finally, we discuss a substitute regulatory approach to stress tests: the dividend prudential target (DPT). Initially suggested by Muñoz (2020), the DPT restricts dividends directly by encouraging retainment in bad states and pay-outs in good states. It, thereby, attempts to directly offset the banks' dividend smoothing behavior to avoid capital depletion in bad states and reduce the pro-cyclicality of lending. In a first step, a DPT defines an ideal dividend pay-out – usually the pay-out made by an unrestricted bank in steady-state. We follow this tradition and evaluate our baseline model at the unconditional mean $\bar{\mu}_l$ of the AR(1) process. The dividends in steady-state, denoted with d_1^{SS}, take on the following value:

$$d_1^{SS} = E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) - E_1^{SS}, \tag{78}$$

$$= \bar{\mu}_l\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right), \tag{79}$$

$$= \left[\frac{\bar{\mu}_l - r_d}{\chi} + r_d\right]\frac{\chi}{,}\gamma\sigma_l^2\left[\bar{\mu}_l - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{80}$$

Consequently, a state-dependent target dividend level d_1^T is defined that increases in the return state. The goal is to incentive more payouts in good and less payouts in bad states, thereby stabablizing both retained equity and lending.

Here, we opt for the simplest possible option by scaling d_1^{SS} with the factor $r_{l,1}/\bar{\mu}_l$. This choice ensures that the target pay-out increases in return states and is exactly equally to the steady-state level in steady state:

$$d_1^T = \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}. \tag{81}$$

Consequently, any (squared) deviations in dividend payouts d_1 from the target d_t^T are punished with a cost κ:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2, \tag{82}$$

$$\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2. \tag{83}$$

The cost κ is set by the supervisor at $t = 0$ and, similar to Jermann and Quadrini (2012), accounts for both fines to be paid and reputation costs from non-compliance. It is taken as given by the bank at $t = 1$ and enters the optimization problem in the following fashion:

$$\begin{aligned} U(d_1, d_2) = \quad & \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \\ & \qquad + \beta E_1(1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2\right], \end{aligned} \tag{84}$$

$$s.t.$$

$$\lambda_1: \qquad L_1 \in \left[E_1, \frac{E_1}{\chi}\right], \tag{85}$$

$$\lambda_2: \qquad E_1 \in \left[0, E_0\right]. \tag{86}$$

An important feature to note from condition (85) in the maximization problem is that the optimal choice of E_1 impacts directly the feasibility constraints of L_1. Thus, both when deriving the the optimal equity and lending choices under the DPT, we need to take this co-dependency into account. We, nevertheless, start by deriving the optimal equity level assuming away its impact on lending. After taking the FOC condition with respect to E_1, equating it to zero, and checking feasibility, we get the following constrained-optimal equity levels:

$$E_1^* = 0 \qquad \forall\, r_{l,1} \leq r_l^* = \frac{\bar{\mu}_l}{d_1^{SS}} \frac{1}{\kappa}\left(\beta\left(1 + r_d\right) - 1\right), \tag{87}$$

$$E_1^* = \frac{1}{\kappa}\left(\beta\left(1+r_d\right)-1\right)+E_0-\frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS} \quad \forall\, r_{l,1}\in(r_l^*,r_l^{**}], \tag{88}$$

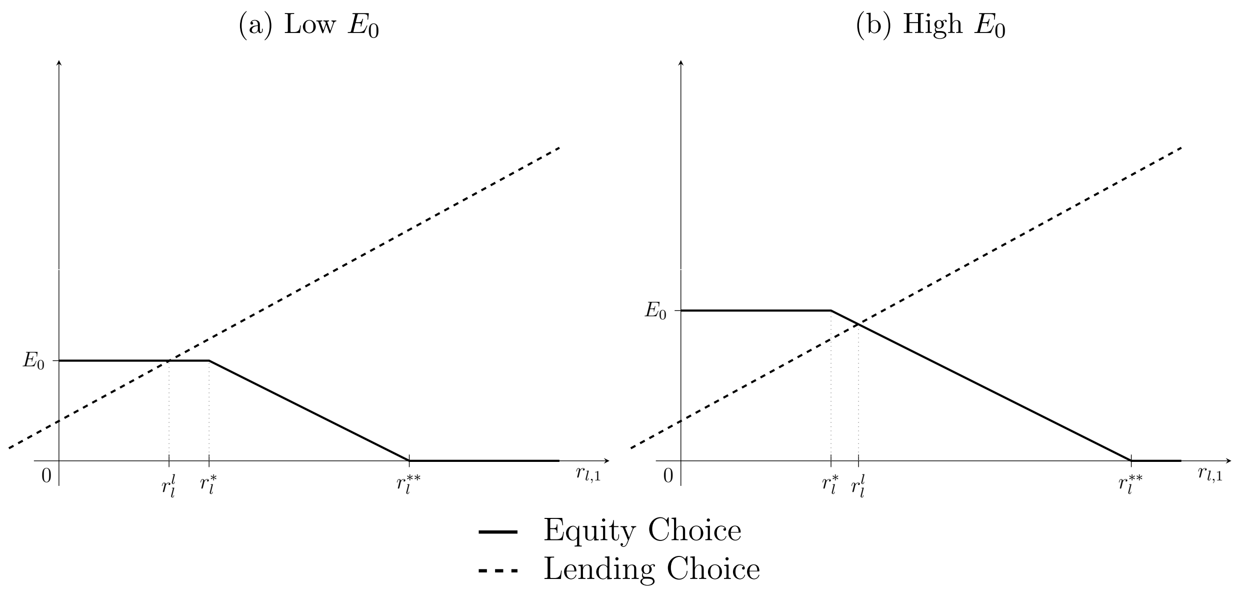
$$E_1^* = 0 \quad \forall\, r_{l,1} > r_l^{**} = \frac{\bar{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}\left(\beta\left(1+r_d\right)-1\right)+E_0\right]. \tag{89}$$

Here, we would immediately like to point out that equity now behaves quite differently than under stress tests: more equity is retained in bad states and less in good. This also impacts the optimal lending. Abstracting from feasibility constraints, taking the FOC with respect to L_1 and consequently equating it to zero yields the following optimal lending level:

$$L_1^* = \frac{\mu_l+\rho_l r_{l,1}-r_d}{\gamma\sigma_l^2}. \tag{90}$$

The Figure 8 illustrates both the optimal equity described in equations (87)-(89) and the unconstrained optimal lending in (90). Here, it is immediately visible that L_1^* in (90) is not feasible for low return states $r_{l,1}$, where the bank would ideally like to lend out less than the equity it would like to retain.

Figure 8: Optimal Equity for Unrestricted Lending under the DPT



Further, we can observe two cases: For low E_0 the feasibility constraint only binds for return states below the full-retainment state (Figure 9a); for high E_0, the feasibility constraint already binds above the full retainment state (Figure 9b). The threshold level on initial equity $\overline{E_0}$ distinguising the two cases is:

$$\overline{E_0} = \frac{\rho_l\bar{\mu}_l}{\gamma\sigma_l^2 d_1^{SS}}\frac{1}{\kappa}\left(\beta\left(1+r_d\right)-1\right)+\frac{\mu_l-r_d}{\gamma\sigma_l^2}. \tag{91}$$

We denote the return state below which the lower feasibility limit on L_1^* binds with r_l^l. For the case of low $E_0 \leq \overline{E_0}$ it can be shown, after some cumbersome re-arranging, that the bank is not willing to reduce the equity level in any return state below r_l^l to relax the lower limit on lending. Hence, the optimal equity choice is as defined as:

$$\text{If} \quad E_0 \leq \overline{E_0}:$$

$$r_l^l = \frac{\gamma\sigma_l^2 E_0 - \mu_l + r_d}{\rho_l}, \tag{92}$$

$$L_1^* = E_1^* = E_0 \quad \forall\, r_{l,1} \leq r_l^l. \tag{93}$$

.

For the case with high $E_0 \geq \overline{E_0}$, the bank does find it optimal to take the impact on lending into account, when deciding how much to retain in low return states. Here, we find that for all states below r_l^l, the bank solves a slightly revised optimization problem, where $E_1 = L_1$. Taking again FOCs with respect to E_1 and equating it to zero allows us to derive a slightly different optimal equity below r_l^l (see equation (96)). The bank can of course only retain additional equity as long as it is below E_0. Even for high E_0, the upper-feasibility constraint is eventually binding below return states r_l^{ll}:

$$\text{If} \quad E_0 \geq \overline{E_0}:$$

$$r_l^{ll} = \frac{\bar{\mu}_l}{\beta\rho_l\bar{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{94}$$

$$r_l^l = \frac{\bar{\mu}_l}{\rho_l\bar{\mu}_l + \gamma\sigma_l^2 d_1^{ss}}\left[\frac{\gamma\sigma_l^2}{\kappa}\big(\beta(1+r_d) - 1\big) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{95}$$

$$L_1^* = E_1^* = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \qquad \forall\, r_{l,1} \in (r_l^{ll}, r_l^l] \tag{96}$$

$$L_1^* = E_1^* = E_0 \qquad \forall\, r_{l,1} \leq r_l^{ll}. \tag{97}$$

Regardless whether E_0 is high or low, the banks preferred lending level will ultimately violate the minimum equity-to-asset ratio in high return states: while optimal equity decreases in returns, optimal lending increases. Lending exceed the feasible amount, given equity, in all return states above a threshold r_l^h. From there on-wards the bank takes into account that (costly) retainment allows for more (profitable) lending. Nevertheless, there exists a threshold r_l^{hh} above which the bank never retains any equity no matter how profitable lending would be:

$$r_l^h = \frac{\chi\bar{\mu}_l}{\chi\rho_l\bar{\mu}_l + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\chi\kappa}\big(\beta(1+r_d) - 1\big) + \frac{\gamma\sigma_l^2}{\chi}E_0 - \mu_l + r_d\right], \tag{98}$$

$$r_l^{hh} = \frac{\bar{\mu}_l\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1+r_d) + \beta\frac{\mu_l - r_d}{\chi}\right]. \tag{99}$$

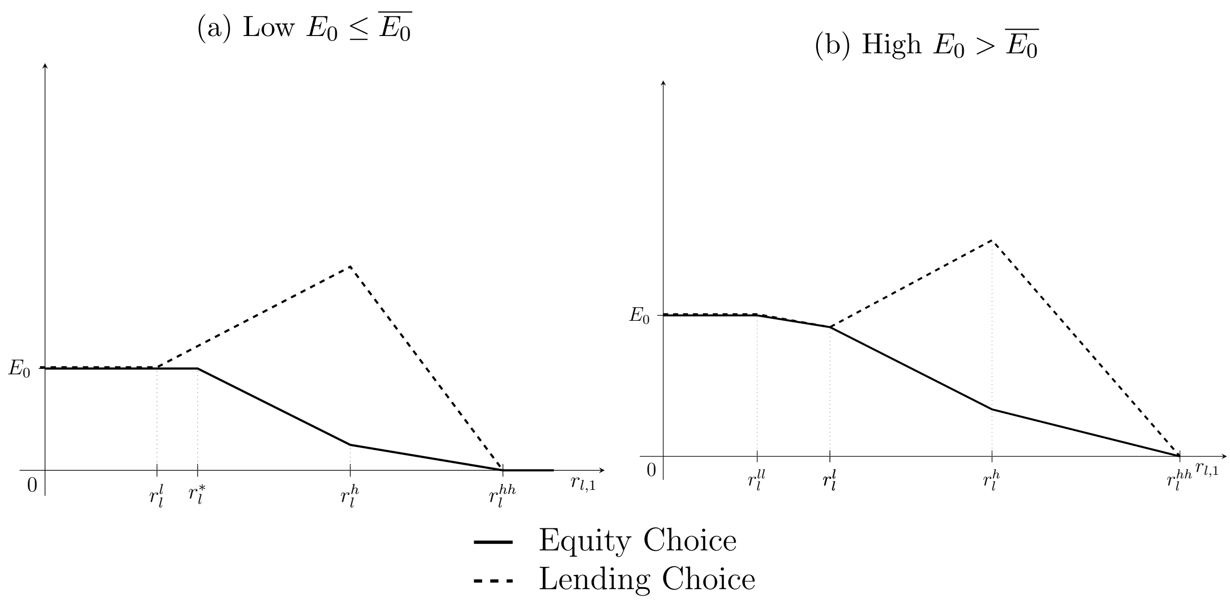
Inserting $L_1 = E_1/\chi$ and, again, taking FOCs yields the following optimal lending and equity in high return states.

$$E_1^* = \chi L_1^* = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu_l}}d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}\right] \quad \forall\, r_{l,1} \in (r_l^h, r_l^{hh}] \tag{100}$$

$$E_1^* = \chi L_1^* = 0 \quad \forall\, r_{l,1} > r_l^{hh} \tag{101}$$

Summarizing the just derived solutions is a bit cumbersome and, we believe, not very informative for the reader. Therefore, we rather display the functional forms of L_1^* and E_1^* for both the low and high initial equity case in Figure 9 below. For the full set of analytical expressions on the return thresholds, the reader is kindly asked to refer to the Appendix D.4.

Figure 9: Feasibility-Constrained Optimal Equity and Lending under the DPT



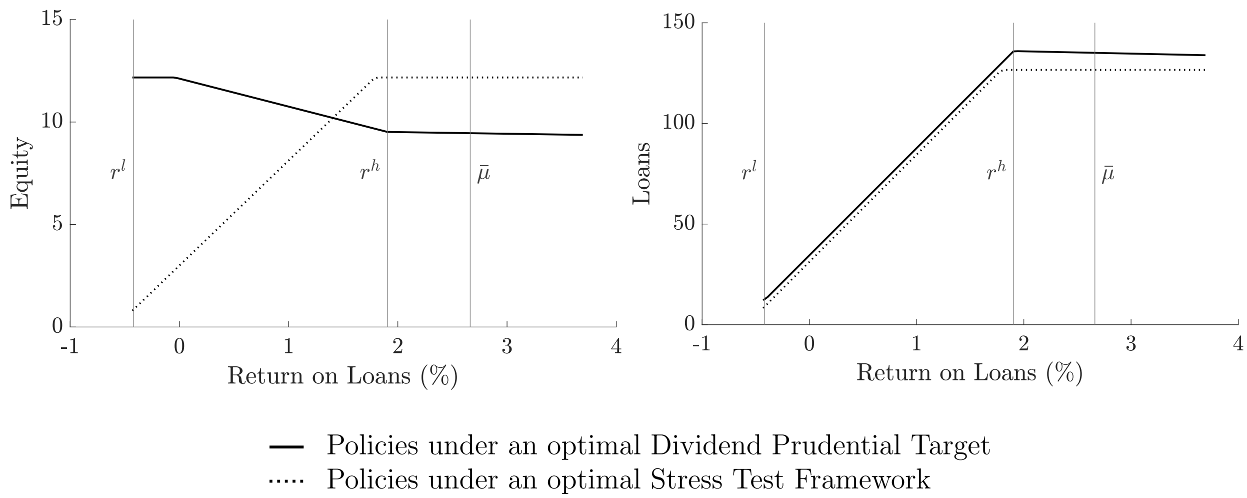
It can be seen in Figure 9 that a DPT results in a hump shaped policy function over the state-space for both equity and lending. The punishment parameter κ influences the mean and variance of both by affecting equity choices directly and, furthermore, affecting the threshold interest rate levels. Recall that the supervisory authority sets κ in period 0 with the objective to stabilize lending, putting welfare weight ω on the expected lending variance.

Unfortunately, a full closed-form characterisation of the mean and variance of lending is cumbersome and provides few general insights. We therefore again immediately rely on the calibrated model (assuming $E_0 = E^{ss}$) to derive the optimal κ and resulting supervisory welfare.[18] Again, we derive the optimal κ for a range of different initial return states $r_{l,1}$ and welfare weights ω (see Appendix D.4).

For intuition, we first plot the resulting policy functions for equity and loans under the optimal $\kappa^* = 0.07$ that maximizes the supervisor's welfare function (see Equation 45), assuming that the initial return realization is at the unconditional mean of the process (i.e. $r_{l,0} = \bar{\mu}$) and that the supervisor cares equally about the level and the variance of lending (i.e $\omega = 1$). The left panel of Figure 10 shows that, relative to the stress-test framework, retained equity under the DPT is higher and lower for bad and good states, respective. The DPT, thus, successfully addresses pro-cyclical retainment of equity and dividend smoothing.

Further, the right panel shows that, for low, intermediate and moderately high return states, the bank uses this equity to lever up more than under the stress-test framework. Only for very high return states, substantially above 4%, the dividend prudential target leads to lower loan levels than the stress-testing framework.

Figure 10: Optimal Policies under Stress Tests and a Dividend Prudential Target

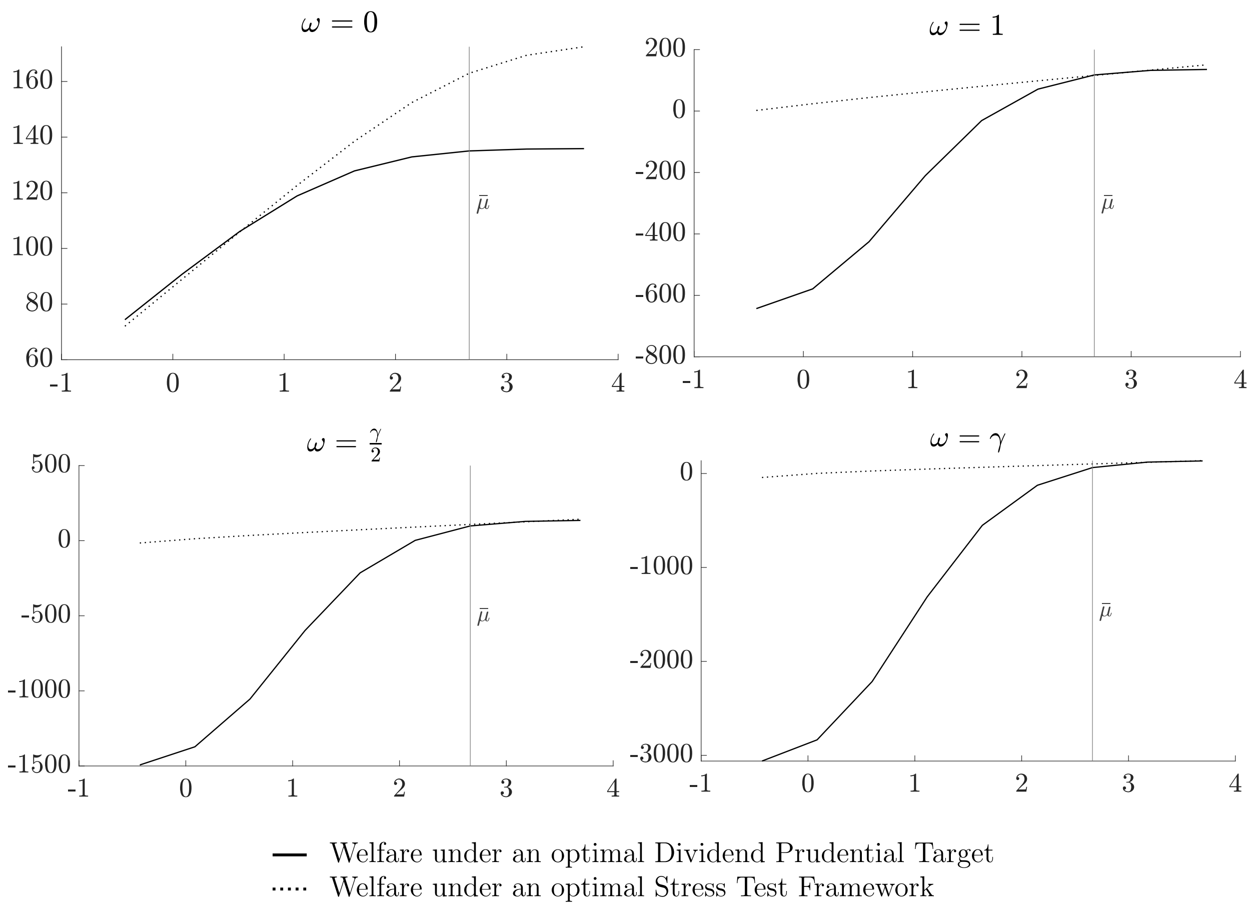


Unfortunately, the DPT's higher lending levels increase more in bad states than in good states. And, hence, while the DPT increases the mean of lending, it also increases the variance. This is reflected in the welfare comparison between the dividend prudential target and the stress-testing framework. Figure 11 plots the supervisor's welfare under each policy framework given the optimal severity of tress test scenarios τ^* and dividend deviation punishment parameter $\kappa*$, respectively. Th four panels of Figure 11 illustrate the associated

[18]When numerically maximizing the supervisor's welfare function we impose that the supervisor cannot set κ in such a way that $r_l^{hh} < r_l^l$. That way loans would be set to zero for basically all loan return states which would of course minimize the volatility of lending. This that loans cannot be zero resembles the standard Inada condition.

welfare for different realizations of the initial return on loans $r_{l,0}$ and different degrees of risk aversion ω.

Figure 11: Welfare under Optimal Stress Tests and a Dividend Prudential Target



As the comparison of supervisory welfare between stress tests and the DPT show, the former is in a better position to maximize the supervisor's welfare in almost all circumstances. Only when supervisor cares about the level of loans alone (i.e. $\omega = 0$) and $r_{l,0}$ is relatively low, a dividend prudential target can increase welfare by forcing banks to hold on to more equity, resulting in higher expected loan levels. In all other circumstances stress tests increase the supervisor's welfare. Therefore, stress tests are overall a much better tool to generate stable lending than a DPT.

6. Conclusion

Bank stress tests have become an increasingly important policy tool designed with the intent to ensure stable lending and, thereby, to foster financial stability. In this paper, we derive the optimal bank balance sheet choices anticipating subsequent stress testing. Here, we explicitly model the forward-looking constraint that stress tests place on the bank's degree of debt financing: equity capital levels should be sufficient to maintain current lending tomorrow, even after absorbing severe losses from said lending. We find that stress tests influence the banks' joint decision over (retained) equity, dividends, and lending. Here, we document the core supervisory trade-off: the more severe the assumed losses, the lower are both expected lending and lending volatility.

To quantitatively assess how such a trade-off plays out in practice, we calibrate our model to the U.S. banks subject to the CCAR stress tests. We derive the optimal stress-test tightness (severity of the adverse scenario) and the implied stress test capital buffer. We find that a supervisor who prefers to maximize lending levels while minimize lending volatility finds stress test capital buffers in the range of 2.67% to 4% to be optimal. This matches well the Federal Reserves' publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

Next, we turn to placing the stress test framework in the wider net of prudential policies. Here, we highlight in particular how stress tests may be complemented with a dividend ban and/or the relaxation of the CCyB in a crisis period. We find that separately introduced, both relax lending of stress-tested banks in bad states of the world. They can, thus, be utilized to dampen the stress-test induced decrease in lending during downturns. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects. We are thus able to rationalize why the relaxation of the CCyB during the onset of the Covid-19 pandemic had no measurable effect on lending by stress-tested banks subject to the dividend bans (FSB, 2021).

We conclude the paper by studying a hypothetical substitute policy: the dividend prudential target. Contrary to stress tests regulating equity levels, the dividend prudential target directly regulates bank dividend payments by introducing a quadratic cost function for deviations from a target. This discourages both equity extraction in bad states and excessive leveraging in good states. We find, however, that such a policy mainly increases the mean of lending but fails to reduce volatility. It is thus not welfare-improving for a supervisor seeking stable lending levels.

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Appendix A. Regulatory Framework

Following the financial crisis 08/09, the Federal Reserve Board (FED) was mandated to perform two complementary stress tests: the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act stress testing (DFAST). The CCAR is a forward-looking exercise and assesses bank holding companies' (BHC) capital adequacy accounting for individual dividend payment plans. Banks with assets of 10$bn and above are required to take part in the CCAR. The DFAST takes the last three quarters' dividend policy as given and mainly focuses on the sufficiency of loss-absorbing capital (Federal Reserve Board, 2020c). Banks with assets of 250$bn and above are required to take part in the DFAST. For the purpose of this study (apart from the calibration), we focus on the CCAR stress test framework, which is described in detail in the following paragraphs.

CCAR Stress Test As part of the CCAR stress test, the FED calculates the individual BHCs' capitalization under three scenarios: baseline, supervisory adverse, supervisory severely adverse. Here, they account for the BHCs' proposed future dividend payments and capital repurchase plans. Subsequently, the FED decides whether to approve a BHC's planned capital actions by compare the post-stress capital levels under the severely adverse scenarios to the minimum capital requirements plus surcharges (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Minimum Capital Requirements From 2009-2013, all stress-test eligible BHCs were subject to a minimum tier 1 common ratio of 5%. In 2014, all banks with at least $250 billion total assets or more than $10 billion foreign asset exposure were subject to a 4% minimum common equity tier 1 ratio (CET1) instead. The remaining banks continued to be subject to the 5% minimum tier 1 common ratio for one more year. From 2015 onward, all BHCs were subject to a 4.5% minimum common equity tier 1 ratio (Federal Reserve Board, 2015, 2016). This change in minimum capital measures was part of the phase-in of the Basel III framework, which also introduced additional capital surcharges.

Capital Surcharge BHCs identified as globally systemically important banks (G-SIB) are subject to additional minimum risk-adjusted capital measures of 1%-3.5%. From 2014 to 2016, the Basel Committee on Banking Supervisisons capital add-on is applied. Since 2017, the maximum of the surcharges calculated under the Basel capital framework and the Federal Reserve Board's assessment methodology titled "Method II" applies (Office of Financial Research, 2021). Additionally, a 2.5% conservation buffer was phased in from 2016-2019 (Federal Reserve Board, 2013, 2014). For our sample period, the banks are not subject to any countercyclical capital buffer (Federal Reserve Board, 2019a).

Table A.1: Maximum Dividend to Net-income Ratio Given CET1

CET1	Maximum Pay-out Ratio
< 5.125%	0%
5.125% – 5.75%	20%
5.75% – 6.375%	40%
6.375% – 7%	60%
> 7%	no limitations

Source: BIS (2019)

Supervisory Power over Dividend Payments Stress-test eligible BHCs are prohibited from any dividend payments and share repurchases until the FED has approved of the capital distribution plan in writing. As mentioned above, such approval is based on the stress test performance and follows in three steps. First, the FED performs an initial set of stress tests given the BHCs' original dividend payout plan. The resulting (preliminary) stress-test results are communicated to the BHC. All BHCs, both insufficiently and sufficiently capitalized, are allowed once to submit an adjusted capital plan (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Then either the original or, if submitted, adjusted capital plan forms the base for the FED's payout policy interventions. Capital levels below the minimum tier 1 common ratio or CET1 (plus G-SIB surcharge) respectively, automatically trigger a payout ban. A violation of the capital conservation buffer automatically results in dividend payments to be restricted to a percentage of net income (see Table A.1). Sufficient capital levels do not result in automatic restrictions. The Fed, however, reserves the right to require a BHC to reduce or cease all capital distributions if it felt that the weaknesses in the BHC's capital planning warranted such a response (Federal Reserve Board, 2014). Thus BHCs may feel supervisory pressure especially when close to but not yet violating their respective minimum capital requirements.

Recent Developments In 2020, the Federal Reserve Board decided to replace the 2.5% capital conservative buffer by an individual stress test buffer for each BHC (Federal Reserve Board, 2020b,a). This falls outside our sample period.

Appendix B. Proofs for Section 2

B.1. Solving the Bank's Optimization Problem

1. We start by defining dividend payments at $t = 1$ and $t = 2$.

$$
\begin{aligned}
d_1 =& E_0 - E_1 && \text{(B.1)}\\
d_2 =& L_1 r_{l,2} - r_d D_1 + E_1 \quad \sim N(\mu, \sigma^2) && \text{(B.2)}\\
\text{where} \quad \mu =& (\mu_l + \rho_l r_{l,1})L_1 - r_d D_1 + E_1 && \text{(B.3)}\\
\text{and} \quad \sigma^2 =& \sigma_l^2 L_1^2 && \text{(B.4)}
\end{aligned}
$$

Further note that D_1 is perfectly determined by E_1 and L_1 via the budget constraint:

$$D_1 = L_1 - E_1 \tag{B.5}$$

Finally, note that plugging this into the stress-test constraint yields:

$$
\begin{aligned}
\chi L_1 \leq& E_1 + L_1(\bar{\mu}_l - \tau\sigma_l - r_d(L_1 - E_1) && \text{(B.6)}\\
(\chi - \bar{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq& 0 && \text{(B.7)}
\end{aligned}
$$

2. Using the above stated equations and standard properties of a normal distributions, allows us to reduce the bank optimization problem to:

$$U(d_1, d_2) = \max_{E_1, L_1} E_0 - E_1 + \beta \left[L_1(\mu_l + \rho_l r_{l,1} - r_d(L_1 - E_1) + E_1 - \frac{\gamma\sigma_l^2}{2} L_1^2) \right] \tag{B.8}$$

$$
\begin{aligned}
&s.t. && \\
&\lambda_1 : && \chi L_1 - E_1 \leq 0 && \text{(B.9)}\\
&\lambda_2 : && (\chi - \bar{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 && \text{(B.10)}\\
&\lambda_3 : && E_1 - E_0 \leq 0 && \text{(B.11)}\\
&\lambda_4 : && E_1 - L_1 \leq 0 && \text{(B.12)}\\
&\lambda_5 : && E_1 \geq 0 && \text{(B.13)}
\end{aligned}
$$

We denote the multipliers associated with constraints (B.9)- (B.13) with λ_1 through λ_5 respectively.

3. Before taking any first order conditions, two comments on the constraints.

3.1. Notice that multipliers λ_3 and λ_5 can never be simultaneously be positive. They describe each their own corner solution: full retainment of equity and no retainment of equity.

3.2. Depending on τ, either minimum-equity and stress-test test constraint binds first.

The other one consequently only binds in states in which the first one is already binding.

We start by rearranging the stress-test constraint:

$$\frac{(\chi - \bar{\mu_l} + \tau\sigma_l + r_d)}{(1+r_d)} L_1 \leq E_1 \tag{B.14}$$

Then notice that the multiplier in front of L_1 in the above equation is determined fully by model parameters and does not depend on equilibrium choices. Further, it enters multiplicatively into the constraint in the same fashion as χ.

Then, logically, the stress-test constraint binds first whenever:

$$\frac{(\chi - \bar{\mu_l} + \tau\sigma_l + r_d)}{(1+r_d)} \geq \chi \tag{B.15}$$

$$\tau \geq \frac{r_d\chi + \bar{\mu_l} - r_d}{\sigma_l} = \tau^* \tag{B.16}$$

And in reverse logic, the minimum equity constraint binds first, whenever $\tau < \tau^*$. This concludes the proof for *Lemma 1.*

4. The above described result of 3.2. allows us actually to combine the two supervisory constraints in the following fashion:

$$\chi(\tau) = \begin{cases} \chi & \tau < \tau^* \\ \frac{r_d\chi + \bar{\mu_l} - r_d}{\sigma_l} & \tau \geq \tau^* \end{cases} \tag{B.17}$$

And the revised constraint, which nests both cases, is:

$$\chi(\tau)L_1 \leq E_1 \tag{B.18}$$

5. Then, we start solving the simplified maximization problem by assuming the bank has choosen a feasible level $E_1 \in [0, E_0]$. Taking E_1 as given reduces the bank optimization problem to:

$$U(E_0 - E_1, d_2) = \quad E_0 - E_1 + \beta E_1(1+r_d) + \max_{L_1} \beta \left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2 \right] \tag{B.19}$$

$$s.t.$$

$$\lambda_{1+2}: \quad \chi(\tau)L_1 - E_1 \leq 0 \tag{B.20}$$

$$\lambda_4: \quad -L_1 + E_1 \leq 0 \tag{B.21}$$

Then, the FOC wrt to L_1 becomes:

$$(\mu_l + \rho_{1,l} r_{l,1}) - r_d - \gamma\sigma_l^2 L_1 - \lambda_{1+2}\chi(\tau) + \lambda_4 = 0 \tag{B.22}$$

6. We now discuss the different cases for the multipliers. Here, notice that λ_{1+2} and λ_4 can never bind simultaneously: one would bind if the bank would like to set significantly lower L_1 than E_1 and one would bind if the bank would like set significantly higher than E_1/χ.

6.1. With this in mind, we start with (temporarily) ignoring both constraints. Then, the optimal loan level is:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{B.23}$$

6.2. Then for a given E_1, logically there exists a lower threshold level $r^*_{l,1}$ for which investing $L_1 = E_1$ is optimal. And for all lower levels, the bank would like to set $L_1 < E_1$ but cannot due to its constraint choice.

Following a similar logic there exist a second threshold $r^{**}_{l,1}$, for which the bank would like to invest E_1/χ units into loans. And for any higher level, it would like to invest more, but cannot due to the minimum equity constraint.

6.3. However, as we will see later, these two thresholds are not really playing a core role, because E_1 is chosen by the bank and not taken as given. Here, it is important to take away from Equation (B.23) that any interior solution of L_1 without either constraints binding is independent of the level of equity E_1.

7. Lets start with assuming that $\lambda_{1+2} = \lambda_4 = 0$. This implies that the bank indeed finances some loans, but that these loans are more equity-financed than strictly required.

7.1. Recall then that L_1 is independent of E_1 and thus, the optimal level of E_1 can be chosen by the following optimization problem:

$$\begin{aligned} U(d_1, d_2) = \quad & \max_{E_1} E_0 - E_1 + \beta(1 + r_d)E_1 && \text{(B.24)} \\ s.t. \quad & && \\ \lambda_3 : \quad & E_1 - E_0 \leq 0 && \text{(B.25)} \end{aligned}$$

Abstracting for now from constraint λ_3 this implies a FOC wrt E_1:

$$-1 + \beta(1 + r_d) \tag{B.26}$$

Relying on parameter assumptions, it can be shown that this FOC is always negative:

$$-1 + \beta(1 + r_d) < 0 \tag{B.27}$$

$$(1 + r_d) \leq \frac{1}{\beta} \quad \text{True by assumption} \tag{B.28}$$

Hence, any interior solution with only partial debt-financing cannot be sustained. Any solution with positive loan levels is characterized by $E_1 = \chi(\tau)L_1$.

8. With this in mind, we can now derive the optimal equity level E_1 by solving the following maximization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta \left[\frac{E_1}{\chi(\tau)}(\mu_l + \rho_l r_{l,1}) - \frac{\gamma\sigma_l^2}{2\chi(\tau)^2} E_1^2 - r_d \frac{1-\chi(\tau)}{\chi(\tau)} E_1 + E_1 \right] \tag{B.29}$$

$$s.t.$$

$$\lambda_4 : \quad E_1 - E_0 \leq 0 \tag{B.30}$$

$$\lambda_5 : \quad - E_1 \leq 0 \tag{B.31}$$

8.1. Again, we will for now ignore the two feasibility constraints. Then the FOC wrt E_1:

$$-1 + \beta \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{\chi(\tau)^2} E_1 - r_d \frac{1-\chi(\tau)}{\chi} + 1 \right] = 0 \tag{B.32}$$

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2} \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta} \right] \tag{B.33}$$

8.2. Now recall that an constraint solution requires $E_1 \leq E_0$. This holds up until:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2} \left[\frac{\mu_l + \rho r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta} \right] \geq E_0 \tag{B.34}$$

$$r_{l,1} \geq \frac{1}{\rho_l} \left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + \chi(\tau) \left(\frac{1}{\beta} - 1 \right) + r_d(1-\chi(\tau)) - \mu_l \right] = \overline{r_l} \tag{B.35}$$

Or in other words, for any level of $r_{l,1}$ exceeding the threshold $\overline{r_l}$ equity is fully retained and invested in loans. The optimal bank choices and (expected) dividends are thus:

$$E_1^* = E_0 \tag{B.36}$$

$$L_1^* = \frac{E_0}{\chi(\tau)} \tag{B.37}$$

$$d_1^* = 0 \tag{B.38}$$

$$\mathbf{E}[D_1^*] = E_0 \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi\tau} - r_d \frac{(1-\chi(\tau))}{\chi(\tau)} + 1 \right] \tag{B.39}$$

8.3. A similar logic can be applied for the lower bound such that:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2} \left[\frac{\mu_l + \rho r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta} \right] \leq 0 \tag{B.40}$$

$$r_{l,1} \leq \frac{1}{\rho_l} \left[\chi(\tau) \left(\frac{1}{\beta} - 1 \right) + r_d(1-\chi(\tau)) - \mu_l \right] = \underline{r_l} \tag{B.41}$$

Or put differently, for any realized stated $r_{l,1}$ weakly below $\underline{r_l}$ no equity is retained. The bank's equilibrium choices and (expected) dividends are thus:

$$L_1^* = E_1^* = D_1^* = 0 \tag{B.42}$$
$$d_1 = E_0 \tag{B.43}$$

8.4. For intermediate levels $r_{l,1} \in (\underline{r_l},\ \overline{r_l})$ and interior solution exists with:

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1 - \frac{1}{\beta}\right] \tag{B.44}$$
$$L_1^* = \frac{E_1^*}{\chi(\tau)} \tag{B.45}$$
$$d_1^* = E_0 - E_1^* \tag{B.46}$$
$$\mathbf{E}[D_1^*] = E_1^*\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1\right] \tag{B.47}$$

B.2. Comparative Statics Over τ

We now compare an environment where $\tau < \tilde{\tau}$ such that $\chi(\tau) = \chi$ with an environment, where $\tau > \tilde{\tau}$ such that $\chi(\tau \geq \tau) > \chi$.

1. We start by showing that that $\underline{r_l^s} < \underline{r_l^{n,e}}$.

$$\underline{r_l^s} < \underline{r_l^{n,e}}\chi\left(\frac{1}{\beta} - 1 - r_d\right) < \qquad \chi(\tau)\tilde{\tau} < \tau \tag{B.48}$$

2. Further, we can show that $\overline{r_l^s} > \overline{r_l^{n,e}}$:

$$\overline{r_l^s} > \overline{r_l^{n,e}} \tag{B.49}$$
$$\frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) > \frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.50}$$
$$\gamma\sigma_l^2 E_0\left(\frac{1}{\chi} - \frac{1}{\chi(\tau)}\right) > (\chi(\tau) - \chi)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.51}$$

Notice that the right hand side is a term very close to zero, and thus the inequality holds true under the assumption that $E_0 >> 0$.

3. With this, we know the upper and lower feasibility implied thresholds for equity and thus lending. Now, we turn to the slope of the optimal equity and lending policies.

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \tag{B.52}$$

$$\frac{\partial^2 E_1}{\partial r_{l,1} \partial \chi(\tau)} = \frac{\rho_l}{\gamma \sigma_l^2} > 0 \tag{B.53}$$

3.1. It can be shown that E_1^* increases linearly in $r_{l,1}$:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma \sigma_l^2} \rho_l \tag{B.54}$$

And confirming the relative return state bounds, it can be shown that the slope is steeper, the higher is τ:

$$\frac{\partial^2 E_1}{\partial r_{l,1} \partial \chi(\tau)} = \frac{\rho_l}{\gamma \sigma_l^2} > 0 \tag{B.55}$$

This implies that under a stress-test constraint, the bank starts to retain equity only in relatively higher states, but once started, it reaches full retainment earlier. Naturally, there exists a threshold $\tilde{r}$ for which the two equity functions intersect.

3.2. Turning to the loans, one can show that $L_1^{*,s} < L_1^{*,e}$. Her,e we first start with the loan rates implying $E_1 < E_0$. Then:

$$L_1^{*,s} < L_1^{*,e} \tag{B.56}$$

$$-\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) < \chi\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.57}$$

$$\chi < \chi(\tau) \tag{B.58}$$

$$\tilde{\tau} < \tau \tag{B.59}$$

Now, we consider the high return states inducing $E_1^* = E_0$:

$$L_1^{*,s} < L_1^{*,e} \tag{B.60}$$

$$\frac{E_0}{\chi(\tau \geq \tau)} < \frac{E_0}{\chi} \tag{B.61}$$

$$\chi < \chi(\tau) \tag{B.62}$$

$$\tilde{\tau} < \tau \tag{B.63}$$

We omit the proof for the variance of lending here due to its complexity here, and discuss it in detail during the supervisory problem. We would nevertheless like to highlight here, that lending L_1^* follows a rectified normal distribution with a lower and an upper bound. By increasing τ (above $\tilde{\tau}$), we bring the bounds closer together, thus reducing the variance of the overall distribution.

Appendix C. The Optimal Tightness τ

In this section, we derive the optimal supervisory choice under two different objective functions. To maintain tractability, we will assume that the realization of return states above $r_{l,1}^{f,s}$ are very low probability events for large banks with sufficient equity stocks. Thus, loan levels are fully characterized. Let us denote the optimal lending in the absence of feasibility constraints with L_1^x, where:

$$L_1^x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{C.1}$$

$$L_1^x \sim N(\mu_x, \sigma_x^2) \tag{C.2}$$

$$\mu_x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l(\mu_l + \rho_l r_{l,0}) - r_d - \chi(\tau)(1/\beta - 1 - r_d)\right] \tag{C.3}$$

$$\sigma_x^2 = \left(\frac{\rho_l}{\gamma\sigma_l}\right)^2 \tag{C.4}$$

The optimal bank lending L_1^* thus takes the following step-function.

$$L_1^* = \begin{cases} 0 & L_1^x < 0 \\ L_1^x & 0 \geq L_1^x \geq \frac{E_0}{\chi(\tau)} \\ \frac{E_0}{\chi(\tau)} & L_1^x > \frac{E_0}{\chi(\tau)} \end{cases} \tag{C.5}$$

Appendix D. Additional Proofs

D.1. *Proofs for Voluntary Violation*

Voluntary violation of the stress-test constraint implies a ban on dividends and, thus, the following equalities:

$$d_1 = 0 \tag{D.1}$$

$$E_1 = E_0 \tag{D.2}$$

$$D_1 = L_1 - E_0 \tag{D.3}$$

With this, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{D.4}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{D.5}$$

Here note that the upper feasibility limit is now determined by χ and not anymore $\chi(\tau)$.

Ignoring the two feasibility constraints for now, the FOC and the consequent optimal lending level are:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{D.6}$$

$$L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.7}$$

Recall that L_1^{*V} is bounded above by the minimum asset-to-equity ratio constraint which allows us to derive a threshold $\overline{r_l^V}$. Similarly, in this business model L_1 can never be below E_0, allowing us a lower threshold $\underline{r_l^V}$

$$\overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{D.8}$$

$$\underline{r_l^V} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{D.9}$$

With this in mind, it remains to be shown when the total utility exceeds the one of complying to the stress-test constraint. The resulting total utility from violation is:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{D.10}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + (1 + r_d)\, E_0\right] \tag{D.11}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.12}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0\,(1 + r_d)\right] \tag{D.13}$$

This, we have to compare to the following aggregate utilities from complying:

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{D.14}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right] \tag{D.15}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2} \tag{D.16}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{2}\left(\frac{E_0}{\chi(\tau)}\right)^2 + E_0(1 + r_d)\right] \tag{D.17}$$

To derive when violation would be optimal, one must compare the appropriate utilities given the return state $r_{l,1}$. A challenge here is that $\underline{r_l^V} \lesseqgtr \underline{r_l}$ and $\overline{r_l^V} \lesseqgtr \overline{r_l}$, depending on E_0:

$$\underline{r_l^V} \lesseqgtr \underline{r_l} \tag{D.18}$$

$$\frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \lesseqgtr \frac{1}{\rho_l}\left[\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{D.19}$$

$$E_0 \lesseqgtr \frac{\chi(\tau)}{\gamma\sigma_l^2}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{D.20}$$

$$\overline{r_l^V} \lesseqgtr \overline{r_l} \tag{D.21}$$

$$\frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \lesseqgtr \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{D.22}$$

$$E_0 \lesseqgtr \frac{\chi\chi(\tau)^2}{(\chi(\tau) - \chi)\gamma\sigma_l^2}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{D.23}$$

Without further restrictions on E_0, a closed-form proof is a cumbersome comparison of all possible combinations for the different functional forms that the utilities may take. As this provides little additional insight without restricting the parameter space, we refrain from doing so. Instead, we show when voluntary violation is optimal for the above calibrated parameters and several different values of E_0. Please refer to the main text for results.

D.2. Covid-19 Dividend Ban

Sketch of proof for Proposition 2.

1. A ban on bank dividend payments implies the following equalities:

$$d_1 = 0 \tag{D.24}$$

$$E_1 = E_0 \tag{D.25}$$

$$D_1 = L_1 - E_0 \tag{D.26}$$

2. As the stress-test constraint is still binding, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{D.27}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{D.28}$$

3. Temporarily ignoring the two feasibility constraints, taking the FOC and equating it to zero yields the following optimal lending level:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{D.29}$$

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{D.30}$$

4. Now, we turn to the upper feasibility limit on L_1^{*B} determined by the stress-test-implied minimum asset-to-equity ratio constraint. This allows us to derive a threshold $\overline{r_l^B}$:

$$L_1^{*B} \geq \frac{E_0}{\chi(\tau)} \tag{D.31}$$

$$r_{l,1} \geq \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l\right] = \overline{r_l^B} \tag{D.32}$$

Similarly, in this business model L_1 can never be lower than E_0, allowing us to define the lower threshold $\underline{r_l^B}$

$$L_1^{*B} \leq E_0 \tag{D.33}$$

$$r_{l,1} \leq \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] = \underline{r_l^B} \tag{D.34}$$

5. Then, the total utility under the Covid-19 dividend ban, denoted with $U^B(d_1, d_2)$, becomes:

$$r_{l,1} < \underline{r_l}: \quad U^B(d_1,d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{D.35}$$

$$r_{l,1} \in [\underline{r_l}, \overline{r_l}]: \quad U^B(d_1,d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^{*B} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*B}\right)^2 + (1 + r_d)E_0\right] \tag{D.36}$$

$$\text{where} \quad L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.37}$$

$$r_{l,1} > \overline{r_l}: \quad U^B(d_1,d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi(\tau)^2} + E_0(1 + r_d)\right] \tag{D.38}$$

6. We are left with showing that $L_1^* < L_1^{*,B}$:

6.1. Assume a realized $r_{l,1}$ in the range $(-\infty,\ \min\{\underline{r_l};, \underline{r_l^B}\}]$. Then:

$$L_1^* < L_1^{*B} \tag{D.39}$$

$$0 < E_0 \tag{D.40}$$

6.2. Assume a realized return in the range $(\underline{r_l},\ \underline{r_l^B}]$. Then:

$$L_1^* < L_1^{*B} \tag{D.41}$$
$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < E_0 \tag{D.42}$$
$$r_{l,1} < \frac{1}{\rho_l}\left(\gamma\sigma_l E_0 - \mu_l + r_d + \chi(\tau)(1/\beta - 1 - r_d)\right) \tag{D.43}$$
$$< \underline{r_l^B} + \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) \tag{D.44}$$

Which holds true by assumption.

6.3. Assume a realized return $r_{l,1}$ in the range $(\underline{r_l}^B,\ \underline{r_l}]$. Then:

$$L_1^* < L_1^{*B} \tag{D.45}$$
$$0 < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.46}$$
$$\underline{r_l^B} - \frac{\gamma\sigma_l^2 E_0}{\rho_l} < r_{l,1} \tag{D.47}$$

Which holds true by assumptions.

6.4. Assume a realized $r_{l,1}$ in the range $\Big(\max\{\underline{r_l}, \underline{r_l^B}\},\ \min\{\overline{r_l}, \overline{r_l^B}\}\Big]$. Then:

$$L_1^* < L_1^{*B} \tag{D.48}$$
$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.49}$$
$$-\chi(\tau)(1/\beta - 1 - r_d) < 0 \tag{D.50}$$

Which holds true by parameter assumption.

6.5. Assume a realized $r_{l,1}$ in the range $(\overline{r_l},\ \overline{r_l^B})$. Then:

$$L_1^* < L_1^{*B} \tag{D.51}$$
$$\frac{E_0}{\chi(\tau)} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.52}$$
$$\overline{r_l} - \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) < r_{l,1} \tag{D.53}$$

Which holds true by assumption.

6.6. Assume a realized $r_{l,1}$ in the range $(\overline{r_l^B},\ \overline{r_l})$. Then:

$$L_1^* < L_1^{*B} \tag{D.54}$$
$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < \frac{E_1}{\chi(\tau)} \tag{D.55}$$
$$r_{l,1} < \overline{r_l} \tag{D.56}$$

This holds true by assumption.

6.7. Finally, assume a realized return state $r_{l,1} \in [\max\{\overline{r_l},\ \overline{r_l^B}\},\ +\infty$. Then:

$$L_1^* = L_1^{*B} \tag{D.57}$$

$$\frac{E_1}{\chi(\tau}) = \frac{E_1}{\chi(\tau)} \tag{D.58}$$

D.3. Proof for CCyB

Proof omitted due to its triviality. Please see the main-text for results.

D.4. Dividend Prudential Target

The steady state of our model is characterized by the unconditional mean $\bar{\mu}_l$ and implies a dividend of:

$$\begin{aligned} d_1^{SS} =& E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) - E_1^{SS} && \text{(D.59)} \\ =& \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) && \text{(D.60)} \\ =& \left[\frac{\bar{\mu} - r_d}{\chi} + r_d\right]\frac{\chi}{\gamma\sigma_l^2}\left[\bar{\mu} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right]. && \text{(D.61)} \end{aligned}$$

Given this, a state-dependend dividend prudential targest is introduced:

$$d_1^T = \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} \tag{D.62}$$

Any deviations from the target are punished with the following fine:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2 \tag{D.63}$$

$$\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \tag{D.64}$$

This results in the following revised optimization problem:

$$\begin{aligned} U(E_0 - E_1, d_2) = \quad & \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \\ & + \beta E_1(1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2\right] && \text{(D.65)} \end{aligned}$$

$$s.t.$$

$$\lambda_1 : \qquad L_1 \in \left[E_1, \frac{E_1}{\chi}\right] \tag{D.66}$$

$$\lambda_2 : \qquad E_1 \in [0, E_0] \tag{D.67}$$

1. We start by ignoring the feasibility constraints on L_1 and derive the optimal equity.

1.1. The FOC with respect to equity yields the following optimal equity levels:

$$\frac{\partial U(d_1, d_2)}{\partial E_1} = -1 - \frac{\kappa}{2}\left(-2E_0 + 2\frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS} + 2E_1\right) + \beta(1+r_d) = \tag{D.68}$$

$$E_1 = \frac{1}{\kappa}\left(\beta(1+r_d) - 1\right) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS} \tag{D.69}$$

1.2. The equity in equation (D.69) is the unconstrained equity level and decreases in $r_{l,1}$. Hence, we know that for low $r_{l,1}$ below a threshold r_l^*, the upper feasibility limit binds:

$$E_1 \geq E_0 \tag{D.70}$$

$$r_{l,1} \leq r_l^* = \frac{\bar{\mu}_l}{d_1^{SS}}\frac{1}{\kappa}(\beta(1+r_d) - 1). \tag{D.71}$$

1.3. Similarly, the equity level is constrained below at zero:

$$E_1 \leq 0 \tag{D.72}$$

$$r_{l,1} \geq= r_l^{**} = \frac{\bar{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d) - 1) + E_0\right]. \tag{D.73}$$

2. The above derived thresholds on equity ignore that the equity choice may relax feasbility constraints on lending. They are nevertheless necessary for a complete proof.

3. Next, assume that a feasible E_1 has been chosen and thus the bank is left with the optimal lending choice. Here, we can rely on results from the bank section and now for a given level E_1, the bank chooses:

$$L_1 = E_1 \qquad \forall r_{l,1} \leq r_l^l = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1 + r_d - \mu_l\right] \tag{D.74}$$

4. Notice that, unlike equity, lending increases in $r_{l,1}$. Hence, for low return states bank would lend out less than feasible and vice versa. Unconstrained, optimal lending is:

$$L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{D.75}$$

5. Let us start with the upper feasibility limit. When is lending larger than optimal E_1/χ.

5.1. First, we assume that L_1 is already constrained below r_l^{**}:

$$L_1 \geq \frac{E_1^*}{\chi}, \tag{D.76}$$

$$r_{l,1} \geq r_l^h = \frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\frac{\gamma\sigma_l^2}{\chi}\left(\frac{1}{\kappa}\left(\beta(1+r_d)-1\right)+E_0\right)+r_d-\mu_l\right]. \tag{D.77}$$

5.2. Next, we verify that indeed $r_h^{<} r_l{**}$:

$$r_l^h \leq r_l^{**} \tag{D.78}$$

$$\frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\frac{\gamma\sigma_l^2}{\chi}\left(\frac{1}{\kappa}\left(\beta(1+r_d)-1\right)+E_0\right)+r_d-\mu_l\right] < \frac{\bar{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d)-1)+E_0\right] \tag{D.79}$$

$$0 < \frac{\chi\rho_l\bar{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d)-1)+E_0+\frac{\mu_l-r_d}{\chi}\right]. \tag{D.80}$$

5.3. we can then conclude that for all levels above r_l^h retaining more equity relaxes the upper feasibility constraint on lending.

6. Taking this into account, we define an alternative optimization problem for high return states above r_l^h, where $L_1 = E_1/chi$,

6.1. Next, we derive the revised FOC wrt. E_1 that assumes $L_1 = E_1/\chi$:

$$-1-\frac{\kappa}{2}\left(-2E_0+2\frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS}+2E_1\right)+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi}-\beta\frac{\gamma\sigma_l^2}{\chi^2}E_1=0, \tag{D.81}$$

$$\kappa E_1+\beta\frac{\gamma\sigma_l^2}{\chi^2}E_1=-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi}, \tag{D.82}$$

$$E_1 = \frac{\chi^2}{\chi^2\kappa+\beta\gamma\sigma_l^2}\left[-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi}\right] \tag{D.83}$$

The optimal equity level E_1 above r_l^h is strictly decreasing in $r_{l,1}$. Eventually, as $r_{l,1}$ increases it will meet the lower feasibility limit on E_1 of zero once again. The threshold return state r_l^{hh} is:

$$0 = \frac{\chi^2}{\chi^2\kappa+\beta\gamma\sigma_l^2}\left[-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi}\right], \tag{D.84}$$

$$\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}-\frac{\beta\rho_l}{\chi}r_{l,1} = \left[-1+\kappa E_0+\beta(1+r_d)+\beta\frac{\mu_l-r_d}{\chi}\right], \tag{D.85}$$

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi-\bar{\mu}_l\beta\rho_l}\left[-1+\kappa E_0+\beta(1+r_d)+\beta\frac{\mu_l-r_d}{\chi}\right]. \tag{D.86}$$

7. Next, we turn to the lower feasibility limit on lending. Here we can distinguish two cases: L_1 intersects with E_1 below and above r_l^*. These two cases are determined by a threshold on E_0:

$$
\begin{aligned}
L_1^* \leq & E_1 * && \text{(D.87)}\\
\frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \leq & \frac{1}{\kappa}\big(\beta(1+r_d) - 1\big) + E_0 - \frac{r_{l,1}}{\bar{\mu_l}} d_1^{SS}, && \text{(D.88)}\\
r_{l,1} \leq & r_l^l = \frac{\bar{\mu_l}}{\rho_l \bar{\mu_l} + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\kappa}(\beta(1+r_d) - 1) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right], && \text{(D.89)}
\end{aligned}
$$

$$
\begin{aligned}
r_l^l \leq & r_l^*, && \text{(D.90)}\\
E_0 \geq & \frac{\rho_l \bar{\mu_l}}{\gamma\sigma_l^2 d_1^{SS}} \frac{1}{\kappa}(\beta(1+r_d) - 1) + \frac{\mu_l - r_d}{\gamma\sigma_l^2} = \overline{E_0}. && \text{(D.91)}
\end{aligned}
$$

8. We first study the case, where $r_l^l \geq r_l^*$ as $E_0 \geq \overline{E_0}$. Here, any reduction in equity allows the bank to relax the lower feasibility limit.

8.1. Accounting for $E_1 = L_1$ in the optimization problem, we obtain the following FOC fir equity:

$$
\begin{aligned}
-1 - \frac{\kappa}{2}\left(-2E_0 + 2\frac{r_{l,1}}{\bar{\mu_l}} d_1^{SS} + 2E_1\right) + \beta(1+r_d) + \beta(\mu_l + \rho_l r_{l,1} - r_d) - \beta\gamma\sigma_l^2 E_1 = 0 && \text{(D.92)}\\
\kappa E_1 + \beta\gamma\sigma_l^2 E_1 = -1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1}) && \text{(D.93)}\\
E_1 = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] && \text{(D.94)}
\end{aligned}
$$

8.2. Mirroring this, for low $r_{l,1}$, the upper feasibility limit of E_1 not exceeding E_0 applies:

$$
E_0 \geq \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \qquad \text{(D.95)}
$$

$$
r_{l,1} \leq = r_{l,1}^{ll} = \frac{\bar{\mu_l}}{\beta\rho_l\bar{\mu_l} - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \qquad \text{(D.96)}
$$

9. Next, we study the case where $E_0 \leq \overline{E_0}$ and thus, $r_l^l \leq r_l^*$. Here, again the bank could relax the feasbility limit on L_1 by retaining more in equity states below r_l^l. That this is not optimal can easily be shown by the fact that:

$$
r_l^l \leq r_l^{ll} \qquad \text{(D.97)}
$$

$$
\frac{\bar{\mu_l}}{\rho_l\bar{\mu_l} + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\kappa}(\beta(1+r_d) - 1) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \leq \frac{\bar{\mu_l}}{\beta\rho_l\bar{\mu_l} - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \qquad \text{(D.98)}
$$

$$
E_0 \leq \frac{\rho_l\bar{\mu_l}}{\gamma\sigma_l^2 d_1^{SS}}\frac{1}{\kappa}(\beta(1+r_d) - 1) + \frac{\mu_l - r_d}{\gamma\sigma_l^2} = \overline{E_0} \qquad \text{(D.99)}
$$

Because the above inequality (D.99) holds by assumption, we have that the bank never finds it optimal to pay out more equity to reduce lending.

10. For a given κ, assume that:

10.1.

$$E_0 \geq \overline{E_0} \tag{D.100}$$

Then, whenever we are in a very low return state $r_{l,1} \leq r_l^{ll}$, we have:

$$r_l^{ll} = \frac{\bar{\mu}_l}{\beta\rho_l\bar{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{D.101}$$

$$E_1 = E_0 = L_1 \tag{D.102}$$

For low return states, where $r_{l,1} \in (r_l^{ll}, r_l^l]$, whe have:

$$r_l^l = \frac{\bar{\mu}_l}{\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}}\left[\gamma\sigma_l^2\left(\frac{1}{\kappa}\left(\beta(1+r_d) - 1\right) + E_0\right) + r_d - \mu_l\right] \tag{D.103}$$

$$E_1 = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{D.104}$$

$$L_1 = E_1 \tag{D.105}$$

For intermediate return states $r_{l,1} \in (r_l^l, r_l^h]$, we have that :

$$r_l^h = \frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\chi}\left(\frac{1}{\kappa}\left(\beta(1+r_d) - 1\right) + E_0\right) + r_d - \mu_l\right] \tag{D.106}$$

$$E_1 = \frac{1}{\kappa}\left(\beta(1+r_d) - 1\right) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS} \tag{D.107}$$

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.108}$$

For high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1+r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{D.109}$$

$$E_1 = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}\right] \tag{D.110}$$

$$L_1 = \frac{E_1}{\chi} \tag{D.111}$$

And finally, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1+r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{D.112}$$

$$E_1 = L_1 = 0 \tag{D.113}$$

10.2. If we have $E_0 \leq \overline{E_0}$, then the bank no longer retains less equity in low return states.

For very low return states $r_{l,1} \leq r_l^l$, the optimal lending is thus:

$$r_l^l = \frac{\gamma \sigma_l^2 E_0 - \mu_l + r_d}{\rho_l} \tag{D.114}$$

$$L_1 = E_0. \tag{D.115}$$

For intermediate return states, $r_{l,1} \in [r_l^l, r_l^h]$, the lending choice is unrestricted and:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2}. \tag{D.116}$$

Similar to case 10.2, for high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l \beta \rho_l} \left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta \frac{\mu_l - r_d}{\chi} \right] \tag{D.117}$$

$$E_1 = \frac{\chi^2}{\chi^2 \kappa + \beta \gamma \sigma_l^2} \left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + r_d) + \beta \frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} \right] \tag{D.118}$$

$$L_1 = \frac{E_1}{\chi} \tag{D.119}$$

And again, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l \beta \rho_l} \left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta \frac{\mu_l - r_d}{\chi} \right] \tag{D.120}$$

$$E_1 = L_1 = 0, \tag{D.121}$$

D.5. Alternative Welfare Function

As a sensitivity analysis we investigate how a supervisor would optimally set the severity of the stress test if he also takes into account the welfare of the bank's shareholders. To capture this trade-off, we assign the welfare weight $\phi \geq 0$ to the expected utility of the bank's shareholder. We, furthermore, assume that both the supervisor and the bank shareholder assign the same welfare weight γ to the expected variance of loans and dividends, respectively. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\begin{aligned} \max_{\tau} \quad & \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \frac{\gamma}{2} \mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0] \\ & + \phi \left(\mathbb{E}[d_1^* \mid r_{l,0}, E_0] + \beta \mathbb{E}[d_2^* \mid r_{l,0}, E_0] \right) \\ & - \phi \frac{\gamma}{2} \left(\mathbb{VAR}[d_1^* \mid r_{l,0}, E_0] + \beta \mathbb{VAR}[d_2^* \mid r_{l,0}, E_0] \right) \end{aligned} \tag{D.122}$$

s.t.

$$\chi(\tau) \in [\chi, 1) \tag{D.123}$$

where

$$r_{l,1} \leq \underline{r_l}: \quad L_1^* = 0 \tag{D.124}$$

$$r_{l,1} \in (\underline{r_l},\ \overline{r_l}): \quad L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2} \tag{D.125}$$

$$r_{l,1} \geq \overline{r_l}: \quad L_1^* = \frac{E_0}{\chi(\tau)} \tag{D.126}$$

As is the case for optimal loan levels L_1^*, the supervisor anticipates a rectified normally distributed d_1^* and d_2^* with lower and upper bounds. In period $t = 1$, dividends are set to $d_1^* = E_0$ for return states below $\underline{r_{l_1}}$ (no retainment); dividends are set to $d_1^* = 0$ for return states above $\overline{r_{l,1}}$ (full retainment; between $\underline{r_{l_1}}$ and $\overline{r_{l,1}}$ dividends are normally distributed with $N(\mu_{d_1}, \sigma_{d_1}^2)$:

$$d_1^x = E_0 - \frac{\chi(\tau)}{\gamma \sigma_l^2}\left(\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\right) \tag{D.127}$$

$$d_1^x \sim N(\mu_{d_1}, \sigma_{d,1}^2) \tag{D.128}$$

$$\sigma_{d,1}^2 = \left(\frac{\chi(\tau)}{\gamma \sigma_l}\rho_l\right)^2 \tag{D.129}$$

The optimal bank dividends d_1^* thus take the following step-function.

$$d_1^* = \begin{cases} E_0 & d_1^x > E_0 \\ d_1^x & 0 \geq d_1^x \geq E_0 \\ 0 & d_1^x < 0 \end{cases} \tag{D.130}$$

Consequently, in period $t = 2$, dividends are equal to $d_2^* = E_0\left(\frac{r_{l,2} - r_d}{\chi(\tau)} + 1 + r_d\right)$ if $r_{l,1} > \overline{r_{l,1}}$ with variance $(1 + \rho_l^2)(\frac{E_0}{\chi(\tau)}\sigma_l)^2$; if $r_{l,1} \in (\underline{r_{l,1}}, \overline{r_{l,1}})$ dividends are normally distributed with $N(\mu_{d_2}, \sigma_{d_2}^2)$:

$$d_2^x = E_1^*\left(\frac{r_{l,2} - r_d}{\chi(\tau)} + 1 + r_d\right) \tag{D.131}$$

$$= \frac{1}{\gamma \sigma_l^2}\left(\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\right)\left(r_{l,2} - r_d + \chi(\tau)(1 + r_d)\right) \tag{D.132}$$

$$d_2^x \sim N(\mu_{d_2}, \sigma_{d,2}^2) \tag{D.133}$$

$$\mu_{d,2} = \frac{1}{\gamma \sigma_l^2}\left(\mu_l(1 + \rho_l) + \rho_l^2 r_{l,0} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\right) \tag{D.134}$$

$$\cdot \left(1 + \rho_l)\mu_l + \rho_l^2 r_{l,0} - r_d + \chi(\tau)(1 + r_d)\right) \tag{D.135}$$

$$\sigma_{d,2}^2 = \mathbb{E}[d_2^{*2}] - \mathbb{E}[d_2^*]^2 = \left(\frac{\rho_l}{\gamma \sigma_l}\right)^2 b^2 + (1 + \rho_l^2)\sigma_l^2 a^2 + \frac{\rho_l^4}{\gamma^2} + 4ab\frac{\rho_l^2}{\gamma} \tag{D.136}$$

where

$$a = \frac{1}{\gamma\sigma_l^2}\left(\mu_l(1+\rho_l) + \rho_l^2 r_{l,0} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\right) \tag{D.137}$$

$$b = \left((1+\rho_l)\,\mu_l + \rho_l^2 r_0 - r_d + \chi\,(\tau)\,(1+r_d)\right) \tag{D.138}$$

Conditional on $r_{l,1} \in (\underline{r_{l,1}}, \overline{r_{l,1}})$, the optimal bank dividends d_2^* thus take the following step-function.

$$d_2^* = \begin{cases} d_2^x & d_2^x \geq E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) \\ E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) & d_2^x < E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) \end{cases} \tag{D.139}$$