Optimal Lending by Stress-Tested Banks*

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Abstract

Stress tests constrain bank balance sheets: Equity must be sufficient to maintain current lending also tomorrow, even after absorbing severe loan losses. We study such forward looking stress-test constraints in a three-period representative bank model, and show that they lead to lower dividends, higher equity levels, and universally lower, albeit less volatile, lending. Subsequently, we compare stress tests with several policy alternatives, such as the Covid-19 dividend ban, the counter-cyclical capital buffer (CCyB), and the dividend prudential target (DPT): While the first two perform well as complementary policies, a DPT is not welfare improving for a supervisor seeking stable lending levels.

JEL Classification: E61, G18, G21, G32

Keywords: bank stress-tests, forward-looking balance sheet constraints, dividend policy, optimal lending

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1. Introduction

The financial crisis 2008-09 has highlighted how crucial bank health is for economic stability and growth. To promote a safe and sound financial system going forward, supervisory authorities around the world have since introduced a wide range of new regulatory measures. As part of this policy package, the Federal Reserve (Fed), the European Central Bank (ECB), and many other authorities have begun to subject banks to regular stress tests. The objective of stress tests is to ensure that banks are sufficiently capitalized to maintain their current lending activities even under severely adverse macroeconomic conditions in the future.[1] Banks found to be insufficiently capitalized in a hypothetical downturn, are consequently restricted in their dividend payments: depending on the severity of violation, an increasing amount of net-income must be retained to boost equity levels.[2]

This regulatory pressure on dividend payments clashes with the banks' apparent objective to generate stable dividends that compensate shareholders for their investments (Koussis and Makrominas, 2019; Larkin et al., 2017).[3] These dividends are paid from both accumulated equity and returns on assets that are financed via equity capital and debt. To keep dividends smooth across the business cycle, banks deplete capital reserves when facing negative earning shocks (see Figure 1). Unregulated, simultaneously maintaining stable dividend levels and minimum capital ratio requirements may lead to asset shrinkage during crisis periods. Thus, intuitively, supervisory restrictions on dividend payments via stress tests seem warranted to maintain equity capital and thereby to ensure lending to viable firms.

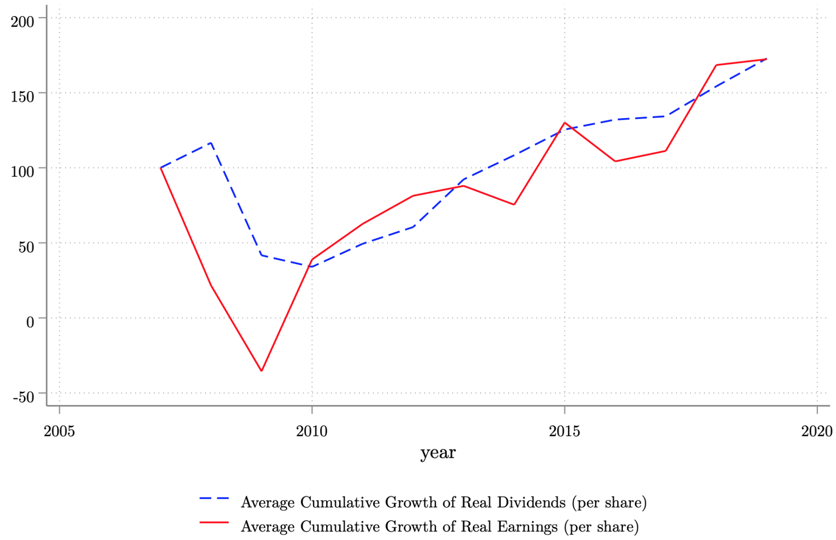
This argument, however, ignores how banks might change their behavior in anticipation of stress-test constrained dividend payments. To the banks' risk averse shareholders, a safe payment today is worth more than an expected equal amount tomorrow that is subject to uncertainty. To pass the stress tests, banks therefore may avoid cutting dividends and instead reduce lending levels. Hence, one must account for the bank's margin of adjustment when evaluating the efficiency of stress tests. Thus far, the existing stress test literature provides little insights on *ex ante* dividend and lending choices by stress-tested banks, as it focuses mainly on the announcement effect of bank stress test results and the subsequent immediate stock-price responses (Beck et al., 2020; Goldstein and Leitner, 2018; Sahin et al.,

[1]Thus, stress tests extend the existing macro-prudential framework by going beyond point-in-time-estimates.

[2]A detailed description of the U.S. regulatory framework can be found in Appendix A. Similar restrictions exist in the European Union and China (Svoronos and Vrbaski, 2020).

[3]There is no shortage of potential explanations for banks' dividend smoothing policies, ranging from investor interests to managerial pay-out schemes directly linked to dividend stability (Lambrecht and Myers, 2012; Wu, 2018). In this paper we do not take a stand on the cause of this behaviour but rather take it as a given bank objective.

Figure 1: Cumulative Growth of BHC Shareholder Payouts (2007=100)



Note: Sample includes all banks that were registered as Bank Holding Companies (BHCs) in 2007 and were at any subsequent point subject to CCAR stress tests.

2020).

Research Agenda In this paper, we therefore study the effect of a forward-looking stress-test constraint on banks' dividend policies, equity levels, and lending activities. To answer this, we build a partial equilibrium framework that characterizes these three bank choices given different realized states of the world and varying tightness of the stress-test constraint. We then derive the optimal tightness of the stress-test constraint for a supervisor that seeks to maximize lending levels while avoiding lending volatility. Here, we partially rely on a calibration of our model for a quantitatively meaningful discussion. Finally, we investigate how stress tests perform relative to other policies, such as the Covid-19 dividend ban, the countercyclical capital buffer, and the dividend prudential target by Muñoz (2020) (banks must pay a punishment fee when dividends deviate from a regulatory target).

Theoretical Framework To illustrate the effects of a stress-test constraint on bank balance sheet choices, we propose a three-period, partial equilibrium framework. The model is populated by a supervisor with mean-variance welfare over bank lending and a representative investor with mean-variance preferences over dividends received from investments in said bank loans.[4] The objective of the supervisor is, thus, in conflict with objective of the investor: while the investor prefers high and stable dividends, the supervisor prefers high and stable lending. The environment is characterized by a single source of uncertainty:

[4]Assuming mean-variance preferences introduces the above described bank preference for smooth dividends. Lambrecht and Myers (2012) provide a micro-foundation for such objective function.

loan returns evolve over all three periods following an AR(1) process. The parameter space additionally contains an initial bank equity endowment, an interest rate on bank-deposits, and an exogenously given minimum equity-to-loan ratio requirement.

In period 0, an initial loan return state realizes and the representative investor is endowed with the equity holding in the bank. Observing both, the supervisor decides on the tightness of the forward looking stress-test constraint, our key novelty in this paper, with the objective to maximize welfare. The stress-test constraint will apply in period 1 and requires that the bank's retained equity is sufficient to absorb (simulated) severely adverse losses from the chosen lending levels without violating the minimum equity-to-loan ratio. In period 1, the bank observes the initial equity and an evolved loan return state. With the objective to maximize the shareholder's total expected dividends, the bank first decides how much equity to retain versus to pay out as period 1 dividends. The retained equity and additional external debt are used to invest in risky loans. Here, the degree of debt financing of loans is constrained by both the stress-test and the minimum equity-to-loan ratio constraint. In period 2, a further evolved loan return rate realizes and, together with last period's equity, lending, and debt choices, determines period 2 dividends. After paying out such to the investor, the bank seizes to exist.

Bank Choices First, we show that any meaningful stress test scenario results in a de facto increased minimum equity-to-asset ratio requirement. Hence, the forward looking stress-test constraint always binds before the minimum equity-to-loan ratio constraint. Moreover, the bank always lends as much as the stress-test constraint allows given the level of optimal equity. The optimal equity follows a step function in return states: in bad return states no equity is retained as loans are very risky and investments not profitable; in medium states a portion of equity is retained for risky investments and a portion is paid out as dividends; only in high return states all inherited equity is retained to be fully invested in loans. Performing comparative statics over the stress-test constraint tightness (the severity of the adverse scenario) highlights the core supervisory trade-off: An increase in tightness leads to higher retained equity in (almost) all states of the world, but always reduces lending levels. At the same time, however, a tighter stress-test constraint leads to less volatility lending.

Optimal Tightness The underlying stochastic process together with the kinks in optimal lending and equity, however, do not allow for a fully analytical expression of the optimal stress test tightness. To nevertheless provide a quantitative estimate we calibrate the model parameters using the balance sheet data of U.S. bank holding companies subject to the Fed's CCAR stress tests.[5] We, then, numerically derive the ex-ante optimal tightness of the stress-test constraint that maximizes the supervisor's mean-variance preferences over

[5]See Appendix A for a detailed description of the regulatory environment.

expected lending. We find that the optimal tightness typically leads to additional capital buffers of $1\% - 9\%$, depending on the different initial return states and welfare weights: a supervisor more (less) concerned about the level than the volatility of lending imposes a looser (tighter) stress-test scenario; a supervisor in a higher (lower) initial return state imposes a looser (stricter) stress-test scenario. This numerical result closely matches the Federal Reserves' recently announced stress-test buffers for 2021 that are reported to lie between 2.5% to 7.5% (Federal Reserve Board, 2021), indicating that we are able to capture well the magnitude of bank balance sheet choices under stress tests.

Policy Extensions Utilizing the calibrated model, we first study bank choices when compliance with the stress-test constraint is voluntary, showing that for stress-tested U.S. banks voluntary violation would often be optimal indeed. We further use the model to evaluate several other policies in their ability to maintaining stable lending levels, acting both as complements and substitutes to stress tests. First, we investigate how a blanket dividend ban, as many supervisory agencies introduced at the beginning of the Covid-19 pandemic, impacts the lending of stress-tested banks. Here, we find that a ban successfully increases lending, but banks refrain from using as much debt financing as the stress-test constraint allows. Subsequently, we show that relaxing a counter-cyclical capital buffer (CCyB) increases lending in bad states. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects.[6] We conclude by comparing the performance of the dividend prudential target (DPT) of Muñoz (2020) with that of stress tests. Here, we find that a DPT is a useful policy instrument to maximise lending levels. However, if a supervisor also cares about the volatility of lending, even an optimal DPT policy leads to substantially lower welfare than the stress-test constraint.

Literature Our paper primarily contributes to the stress test literature. Thus far, the bulk of papers in this literature is empirical and studies the information revealing mechanism of stress-tests and their immediate impact on stock-prices (Bird et al., 2020; Morgan et al., 2014; Petrella and Resti, 2013; Quijano, 2014). Even though the outcomes of stress tests are to a large extent predictable (Ahnert et al., 2020), a range of studies has shown that the release of stress test results nonetheless provides valuable information. Among others, Flannery et al. (2017) and more recently Fernandes et al. (2020) identify positive abnormal equity returns and negative responses of CDS spreads in response to stress-test disclosure. However, this effect is heterogeneous across the business cycle (Sahin et al., 2020), across banks' risk-exposure (Flannery et al., 2017), and between those banks passing and those failing the stress-test (Sahin et al., 2020). As a result, the optimal disclosure policy

[6]Here, we are thus able to provide an explanation for the current policy puzzle of unused CCyB buffers during the Covid-19 crisis (FSB, 2021).

of stress test results is not trivial: it depends non-linearly on a bank's capital gap (Goldstein and Leitner, 2018) and it is subject to a time inconsistency problem (Parlasca, 2021).

A small but growing empirical literature furthermore studies the change in lending levels following stress test announcements, thus going beyond the immediate disclosure effects of stress tests. Using U.S. loan level data, Acharya et al. (2018); Cortés et al. (2020) and Doerr (2021) document that stress-tested banks reduce credit supply, especially to risky borrowers. However, it remains unclear whether this results in an aggregate decrease of credit supply or whether the decrease of stress-tested banks is offset by unaffected banks. Cappelletti et al. (2019) argue that the 2016 stress-testing exercise in the euro area similarly has led banks to increase their capital ratios by reducing their lending and risk-taking. Finally, Cornett et al. (2020) find that the banks subject to stress tests lower dividends significantly compared to non-tested banks. However, this behavior reverses completely afterwards, suggesting that stress-tested banks may be managing financial performance. Our paper provides the theoretical counterpart to these empirical analyses by rationalizing these findings in a partial equilibrium framework.

To the best of our knowledge, we are the first to explicitly model the forward looking stress-test constraint and thus theoretically study its impact on banks' joined decision over lending, equity, and dividend payments. Most closely to our paper are Shapiro and Zeng (2019), who study how banks optimally risk-adjust their portfolio in response to stress tests, holding dividends, equity, and debt levels fixed. We complement their work by endogenising the banks' balance sheet choices while abstracting from portfolio risk-adjustments. For this purpose, we extend the banking model by Gollier et al. (1997), borrowing several elements from the dynamic banking literature. For our objective function, we rely on Lambrecht and Myers (2012), who provide a micro-foundation for the dividend smoothing behavior of banks. Further, we extend the uncertainty of the asset to span all three periods, by utilizing the AR(1) process describing loan returns in Bolton et al. (2020). To maintain tractability in the face of an evolving return state, we abstract from bank default originally studied in (Gollier et al., 1997). The result is an easily extendable model that not only highlights the effect of stress tests, but allows us to study a range of complementary and substitute policy measures.

Overview The remainder of the paper is organized as follows. In Section 2, we describe the baseline model environment and state the banks optimal dividend, equity, and lending choices. In Section 3, we calibrate the model to obtain a numerical value, consequently quantify the marginal responses of equity and lending to changes in the stress-test tightness, and finally numerically establish the optimal stress-test tightness. Section 4 addresses the possibility for banks to voluntary violate stress tests. In Section 5, we discuss several policy extensions, such as the Covid-19 dividend ban, the CCyB, and the dividend prudential target. Section 6 concludes and puts the theoretical and calibration exercise in perspective. The appendix contains a detailed description of the regulatory framework and all proofs.

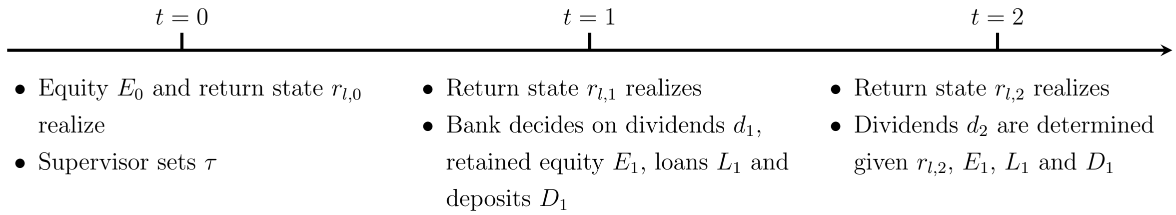
2. Theoretical Analysis

The following section contains the representative bank problem and is structured in the two following sub-sections: Section 2.1 describes the baseline partial equilibrium framework that was inspired by the dynamic banking models of Bolton et al. (2020) and Lambrecht and Myers (2012), but modified to a three-period environment to allow for a tractable introduction of bank stress tests;[7] Section 2.2 subsequently derives the lending and equity choices by a stress-tested bank and, relying on this, Section 2.3 performs comparative statistics to study the response of equity and lending to the introduction of a stress test.

2.1. Three-Period Model

The model is populated by a representative risk-averse investor owning a bank, or a representative bank for short, and a welfare maximizing supervisor. Both agents live for three periods, denoted with $t = \{0, 1, 2\}$ respectively, and share a common discount factor β. Each period t is characterized by the return on loans $r_{l,t}$ which follows an AR(1) process (more below). In period $t = 0$, an initial bank equity endowment $E_0 > 0$ and initial return state $r_{l,0}$ realize. Observing these, the supervisor decides on the optimal stress-test tightness τ. In period $t = 1$, the representative bank observes an evolved loan return $r_{l,1}$ and E_0, and decides how much of the inherited equity to pay out as dividends versus to retain for loan investments. Here, the additional deposit financing of loans is constrained by both the stress test and a minimum equity-to-asset ratio requirement. In period $t = 2$, a further evolved loan return state $r_{l,2}$, together with inherited loan, deposit, and equity levels, determines the final dividend payment by the bank to the investor.

[7] We rely on the serially auto-correlated loan returns from Bolton et al. (2020), but abstract from bank default and investments in risk-free bonds for tractability, as these play a subordinate role in a three-period model, where the choice is only between consuming today versus tomorrow. Similarly to Lambrecht and Myers (2012), we further assume that deposit rates are fixed and we rely on their Proposition 1 that provides a micro-foundation for the here proposed bank objective function. Here, we utilize that normally distributed future loan returns simplify their exponential utility function to mean-variance utility. We additionally include a supervisor constraining bank choices via stress tests.



The Investor There exists a representative investor that is hand-to-mouth and subject to mean-variance utility $u(\cdot)$ from received time t dividends d_t.[8] We denote the resulting aversion to risk with γ, such that:

$$u(d_t) = \mathbb{E}[d_t] - \frac{\gamma}{2}\mathbb{VAR}[d_t] \tag{1}$$

The Bank Balance Sheet The investor dividends are financed through an initial equity endowment E_0 in a representative bank. At time $t = 1$, the bank observes E_0, a loan return state $r_{l,1}$, and the two regulatory constraints (more below). Given these states, the bank first decides of how much initial dividends d_1 to pay versus how much equity E_1 to retain.

$$d_1 = E_0 - E_1 \tag{2}$$

Subsequently, the bank additionally sources costly deposits D_1, with interest rate r_d, to finance investments in the risky loans L_1:

$$L_1 = E_1 + D_1 \tag{3}$$

In period $t = 2$, a new loan return $r_{l,2}$ realizes, where we assume that the loan returns follow an AR(1) process:

$$r_{l,t} = \mu_l + \rho_l r_{l,t-1} + \sigma_l \epsilon_t \qquad \text{where} \quad \epsilon_t \sim \mathcal{N}(0,1), \quad \mu_l > r_d, \quad \rho_l \in (0,1) \tag{4}$$

Then the combined choices of equity E_1, deposits D_1, and lending L_2 determine dividends d_2. Accounting for the underlying AR(1) process and the the loan return state $r_{l,1}$, this implies:

$$d_2 = r_{l,2}L_1 - r_d D_1 + E_1 \qquad \text{where} \quad d_2 \sim \mathcal{N}\left(\left(\mu_l + \rho_l r_{l,1}\right) L_1 - r_d D_1 + E_1,\ L_1^2 \sigma_l^2\right) \tag{5}$$

Supervisory Constraints The choices of E_1, D_1, and L_1 are restricted by two supervisory constraints: a minimum equity-to-asset ratio constraint and a stress-test

[8] This assumption is micro-founded by Lambrecht and Myers (2012), who show that payout smoothing naturally arises when insiders are risk averse and/or subject to habit formation. Here, we rely on their result from Proposition 1 and directly model an objective function over dividends rather than over managerial rents subject to investor participation constraints.

constraint. The first defines a minimum equity-to-asset ratio χ that effectively restricts the bank's debt financing of loans. Here, we assume that the minimum ratio χ is given exogenously.[9] For the choices E_1 and L_1 this implies:

$$\frac{E_1}{L_1} \geq \chi \tag{6}$$

The stress-test constraint is forward looking instead, and requires that the bank's available equity at time $t = 2$ cannot drop below χ even under a severely adverse loan return state realization $r_{l,2}$. Here, the expected available equity is the sum of the retained equity E_1 and next period profits $\Pi_2(\tau)$ simulated for stress-test scenario τ:

$$\Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1 \qquad \text{where} \quad \overline{\mu}_l = \frac{\mu_l}{1-\rho_l} \tag{7}$$

Here, $\overline{\mu}_l$ denotes the unconditional mean of the AR(1) process and τ defines the number of standard deviations below $\overline{\mu}_l$ that describe the adverse scenario of $r_{l,2}$. As τ defines the severity of the adverse scenario, we will refer to it as stress-test constraint tightness throughout the paper. For now, tightness $\tau > 0$ is taken as given and can be interpreted as a model parameter. In Section 3, we relax this assumption and explicitly determine the optimal τ. With the definition of $\Pi_2(\tau)$ in mind, the stress-test constraint thus takes the following shape:

$$\frac{E_1 + \Pi_2(\tau)}{L_1} \geq \chi \tag{8}$$

Bank Optimization Problem The above described constraints complete the model environment and we now turn to the bank optimization problem. For this, we denote the investor's total utility from d_1 and d_2 with $U(d_1, d_2)$. The bank's optimization problem is thus:

$$U(d_1, d_2) = \max_{E_1,\, L_1} \quad d_1 + \beta\left[\mathbb{E}[d_2] - \frac{\gamma}{2}\mathbb{VAR}(d_2)\right] \tag{9}$$

$s.t.$

$$d_1 = E_0 - E_1 \tag{10}$$

$$L_1 = E_1 + D_1 \tag{11}$$

$$d_2 = r_{l,2}L_1 - r_d D_1 + E_1 \qquad \sim \mathcal{N}\Big((\mu_l + \rho_l r_{l,1})L_1 - r_d D_1 + E_1,\ \sigma_l^2 L_1^2\Big) \tag{12}$$

$$E_1 \geq \chi L_1 \tag{13}$$

$$E_1 + \Pi_2(\tau) \geq \chi L_1 \qquad \text{where} \quad \Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1 \tag{14}$$

$$L_1 \geq 0 \tag{15}$$

[9] This follows the narrative that global minimum capital standards, such at the Basel III requirements, are not quickly and easily adjustable by a national authority without severe costs.

$$E_1 \in [0, E_0] \tag{16}$$

Here, Equations (10) - (12) are the bank's balance sheet constraints, Inequalities (13) and (14) denote the two supervisory constraints on equity, and Constraints (15) and (16) are the feasibility constraints on lending and equity.[10]

Parameter Restrictions For the AR(1) process on loan returns, we assume that $\mu_l > 0$, $\rho_l \in (0, 1)$ and $\sigma_l > 0$. For the supervisory constraints, we assume $\chi \in (0, 1)$ and $\tau > 0$. For the risk-aversion we assume that $\gamma > 0$. For the initial equity endowment, we assume that $E_0 >> 0$, reflecting that we are dealing with large banks. Finally, for the deposit rate, we assume that $r_d < \mu_l$ and $1 + r_d < 1/\beta$, jointly ensuring that debt financing of loans is desirable.[11]

2.2. The Bank's Optimal Choices

We now turn to solving the bank optimization, starting with simplifying the two supervisory constraints: the minimum equity-to-asset ratio (13) and the stress-test constraint (14). First, we use the budget constraint in (11) and the definition of $\Pi_2(\tau)$ to rearrange the stress-test constraint:

$$E_1 + (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d(L_1 - E_1) \geq \chi L_1 \tag{17}$$

$$E_1 \geq \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} L_1 \tag{18}$$

Comparing this to the minimum equity-to-asset ratio constraint in (13), it is easy to see that, for sufficiently large τ, the stress-test constraint always binds first:

$$\frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} \geq \chi \tag{19}$$

$$\tau \geq \frac{\overline{\mu} - r_d(1 + \chi)}{\sigma_l} = \tilde{\tau} \tag{20}$$

And for τ below $\tilde{\tau}$, the minimum equity-to-asset ratio constraint binds first. In either case, the second constraint is binding exclusively in states, where the first one is binding too.

***Lemma* 1.** *There exists a stress-test tightness threshold $\tilde{\tau}$, such that :*

(i) If $\tau < \tilde{\tau}$, the minimum equity-to-asset ratio constraint always binds first.
(ii) If $\tau \geq \tilde{\tau}$, the stress-test constraint always binds first.

[10] Constraint (15) implies that the bank cannot short-sales loans. In (16), the lower bound implies that the bank cannot debt finance dividends and the upper bound rules out additional equity injections.

[11] The latter implies that shareholders are less patient than depositors and thus have a preference for debt-financing of loans. As (Gollier et al., 1997) discuss, this is a necessary assumption for this type of banking models and thus commonly found. The alternatives with $1/\beta = 1 + r_d$ and $1/\beta < 1 + r_d$ would respectively imply that the Modigliani Miller theorem holds or that the bank exclusively equity finances loans.

The results from *Lemma 1* allow us to generalize the bank optimization problem to nest both supervisory constraints in a single equity constraint:

$$E_1 \geq \chi(\tau) L_1 \qquad \text{where} \qquad \chi(\tau) = \begin{cases} \chi & \tau < \tilde{\tau} \\ \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} & \tau \geq \tilde{\tau} \end{cases} \tag{21}$$

Relying on this, we then derive the banks optimal equity, dividend, and lending choices as a function of $\chi(\tau)$. The proof is described in detail in Appendix B, but follows a few very intuitive steps. First, it can be shown that, given the parameter assumptions, equity financing loans is never desirable. And thus, the revised minimum equity constraint is always binding at the optimum. Denote the optimal loan level with L_1^*. Then this implies:

$$L_1^* = \frac{E_1}{\chi(\tau)} \tag{22}$$

This result can be substituted into the bank optimization problem to simplify it further. Temporarily ignoring the feasibility constraints on equity, equating the first-order-condition with respect to retained equity with zero, yields the following optimal equity level E_1^*:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{23}$$

However, E_1 is feasibility constrained from below at zero and from above at E_0. Inserting these bounds in the above Equation (23) and rearranging allows us to derive two thresholds $\underline{r_l}$ and $\overline{r_l}$:

$$\underline{r_l} = \frac{1}{\rho_l}\left[r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{24}$$

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{25}$$

Here, threshold $\underline{r_l}$ denotes the return state $r_{l,1}$ below which no equity is retained and $d_1^* = E_0$. $\overline{r_l}$ denotes the return threshold above which which equity is fully retained and $E_1^* = E_0$. With this, the optimal choices are fully characterized for a given $\chi(\tau)$, and summarized in *Proposition 1.*

***Proposition* 1.** *A given constraint tightness τ, equity endowment E_1, and return state $r_{l,1}$ imply the following optimal bank choices:*

(i) If $r_{l,1} \leq \underline{r_l}$ all initial equity is paid out, such that:

$$d_1^* = E_0 \tag{26}$$

$$E_1^* = L_1^* = d_2^* = 0 \tag{27}$$

(ii) If $r_{l,1} \in (\underline{r_l},\ \overline{r_l})$, some equity is paid out and some retained, such that:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{28}$$

$$d_1^* = E_0 - E_1^* \tag{29}$$

$$L_1^* = \frac{E_1^*}{\chi(\tau)} \tag{30}$$

$$d_2^* = \frac{E_1^*}{\chi(\tau)}(r_{l,2} - r_d) + E_1^*(1 + r_d) \tag{31}$$

(iii) If $r_{l,1} \geq \overline{r_l}$, the initial equity is fully retained, such that:

$$E_1^* = E_0 \tag{32}$$

$$d_1^* = 0 \tag{33}$$

$$L_1^* = \frac{E_0}{\chi(\tau)} \tag{34}$$

$$d_2^* = \frac{E_0}{\chi(\tau)}(r_{l,2} - r_d) + E_0(1 + r_d) \tag{35}$$

Finally, note that for an initial equity level equal to the optimal equity level at the unconditional mean of the return process (i.e. $E_0 = E^{ss}(\tau)$), the full-retainment return level is exactly equal to the unconditional mean of the the return process. To see this, first define the steady state equity level for a given stress-test tightness τ

$$E^{ss}(\tau) = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{36}$$

and substitute it into the full-retainment return level

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]\right) + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{37}$$

which simplifies to

$$\overline{r_l} = \frac{1}{\rho_l}\left(\bar{\mu}_l - \mu_l\right) = \bar{\mu}_l \tag{38}$$

Therefore, the bank will retain all its initial equity for all return states equal or larger than the unconditional mean of the return process.

2.3. The Effect of Stress Tests

In this section, we analyze how E_1^* and L_1^* change when a the supervisor decides to introduce a stress-test constraint by raising τ above $\tilde{\tau}$. For this purpose, we introduce two additional superscripts ^{e} and ^{s}, denoting the equilibrium outcomes under a binding minimum equity-to-asset ratio and a binding stress-test constraint respectively.

First it can be shown that raising τ implies a higher $\chi(\tau) > \chi$, which consequently results in a higher no-retainment state $\underline{r_l}$. We thus have that:

$$\underline{r^s_{l,1}} > \underline{r^e_{l,1}} \tag{39}$$

An introduction of a τ above $\tilde{\tau}$ also implies that the full-retainment state is reached earlier:

$$\overline{r^s_{l,1}} < \overline{r^e_{l,1}} \tag{40}$$

This implies that at the low end of the return distribution, a stress-test constraint incentivizes banks to retain equity only in relatively better states. At the high end of the the return state distribution, full retainment is reached already at relatively worse states. Complementing this, it can be shown that for all $r_{l,1}$ above $\underline{r_l}$ and below $\overline{r_l}$, the optimal retained equity E_1^* increases linearly in $r_{l,1}$ but with a steeper slope, the higher the τ:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \qquad \frac{\partial^2 E_1}{\partial r_{l,1}\partial\tau} = \frac{\rho_l}{(1+r_d)\gamma\sigma_l} > 0 \tag{41}$$

Therefore, there exists a return state $\tilde{r} \in (\underline{r^e_{l,1}}, \overline{r^e_{l,1}})$, below (above) which a stress-test constrained bank retains less (more) equity than if it was constrained by the minimum-equity constraint only. Using Equation (23) we can characterize this threshold $\tilde{r}$ as:

$$\tilde{r}_l = \frac{1}{\rho_l}\left[r_d - \mu_l + \left(\chi(\tau) + \chi\right)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{42}$$

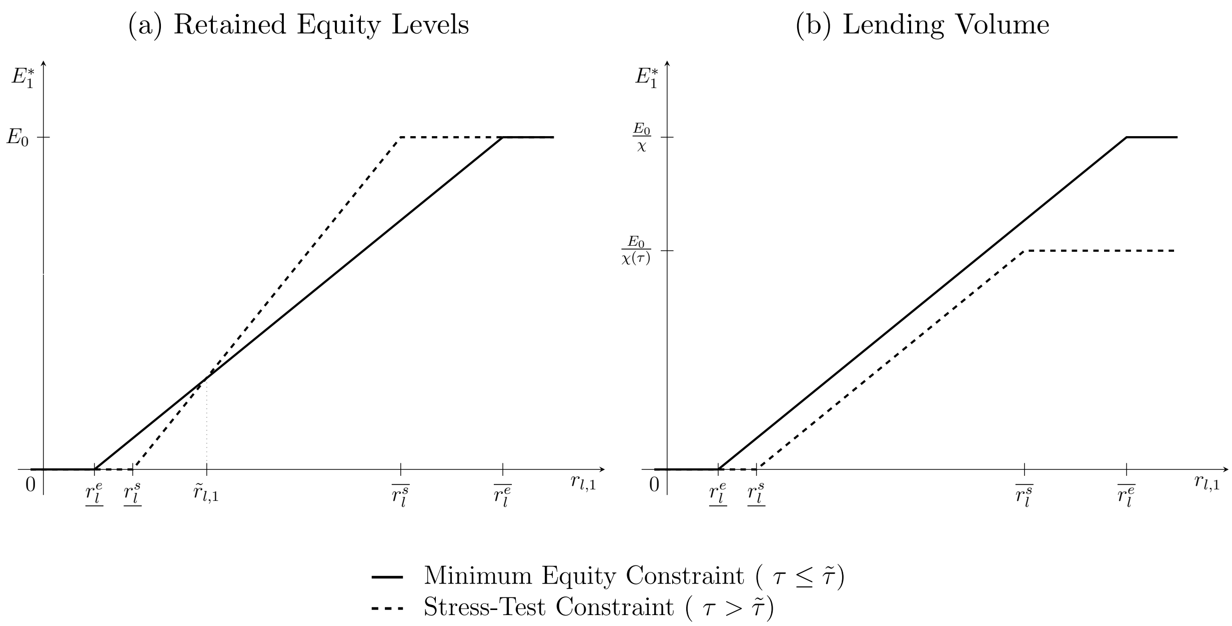
$$= \underline{r^s_{l,1}} + \frac{\chi}{\rho_l}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{43}$$

Here, Equation (43) rearranges $\tilde{r}_l$ as a function of the no-retainment state, showing it to be only marginally higher. Thus, in most return states (and definitely the positive states) more equity is retained under stress tests. Figure 2a below illustrates this effect of a stress-test constraint on retained equity.

Corollary 1. *Raising τ above $\tilde{\tau}$ leads to more retained equity in almost all states of the world.*

Figure 2b complements the comparison, by illustrating the effect of the stress-test constraint on lending. Here, we can see that the higher retained equity levels between $\tilde{r}_{l,1}$ and $r^{f,s}_{l,1}$ never translate in higher lending volumes. The extra equity is lower than the equity level that would be required to maintain the same level of lending under the tighter equity ratio constraint which is implied by the stress-test constraint. And thus:

Figure 2: Minimum Equity-to-Asset Ratio Versus Stress-Test Constraint



$$L_1^{*,s} < L_1^{*,e} \quad \forall r_{l,1} > \underline{r_{l,1}^e} \tag{44}$$

Furthermore, the volatility of lending also decreases under the stress-test constraint, given that equity retainment starts only at a relatively better state but the full-retainment state is reached earlier

Corollary 2. *Raising τ above $\tilde{\tau}$ implies strictly lower, but less volatile lending.*

3. Calibration & Optimal Stress Test Tightness τ

We now turn to the supervisory choice of τ in period 0 and the resulting impact on lending and equity levels. Since this analysis requires a realistic model calibration, we start by discussing our model calibration in Section 3.1. We then use the calibrated model to quantify the marginal effects of adjusting the stress-test tightness τ on lending and equity in Section 3.2. In a final step, we compute the optimal choice of τ in Section 3.3.

3.1. Model Calibration

To provide a quantitative estimate of the optimal τ, we calibrate our model with three sets of parameters (see Table 1). First, we set the discount factor β equal to 0.99, which corresponds to an annualized real interest rate of 1%. We take the risk aversion parameter from Eisfeldt et al. (2020) and set it to 4.37. Furthermore, we take a minimum equity-to-asset ratio of 7% as given. We then use balance sheet data of U.S. Bank Holding Companies with more than $10bn in assets between 2009 - 2019 (i.e. banks subject to CCAR stress tests) to calibrate the parameters of the loan return process as well as the return on deposits.

First, to calibrate the return process, we follow De Nicolò et al. (2014) and estimate an AR(1) process on the mean excess return on assets, given by the ratio of the interest and non-interest revenues to lagged assets (items *bhcp4000* and *bhck2170* respectively in the FR Y–9C reports) minus the 1-year Treasury rate. We then add this excess return to our implied risk free rate $1/\beta - 1$ to arrive at the mean of the return process. The calibrated return process has a mean of 1.02% with a standard deviation of 0.52% and a autocorrelation of $\rho_l = 0.62$, which implies an unconditional mean return of 2.66%.

Table 1: Calibration

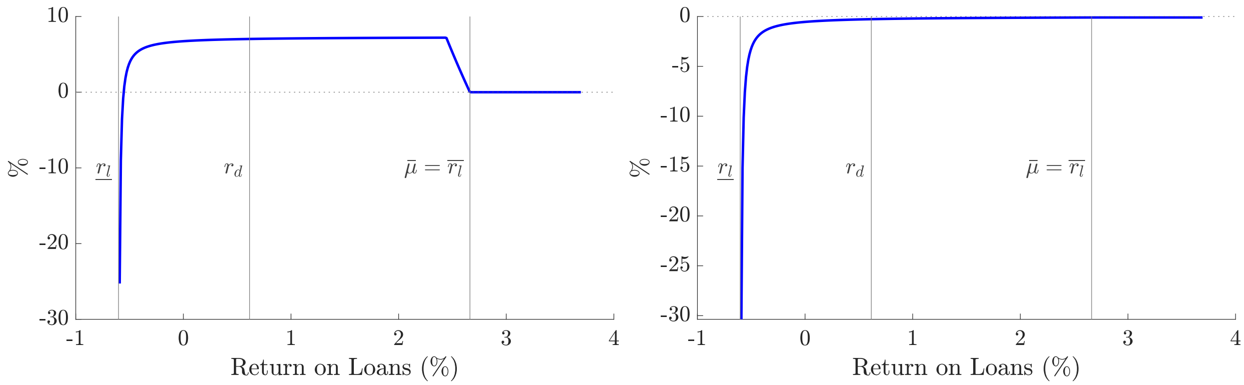
Description	Parameter	Value
Discount Factor	β	0.99
Risk Aversion	γ	4.370
Minimum equity-to-asset ratio	χ	0.07
Mean Return of Risky Asset (%)	μ_l	1.02
AR(1) of Risky Asset	ρ_l	0.62
SD of Risky Asset (%)	σ_l	0.52
Lending Spread (%)	$1/\beta - 1 - r_d$	0.39
Return on Deposits (%)	r_d	0.62

Second, to calibrate the deposit rate r_d, we estimate the deposit spread as the mean difference between the 1-year Treasury rate and the mean deposit rate, given by the ratio of interest paid on deposits (the sum of items *bhckhk03*, *bhckhk04*, *bhck6761*, and *bhck4172*) to lagged deposits (the sum of items *bhdm6631*, *bhdm6636*, *bhfn6631*, *bhfn6636*). We then subtract this deposit spread from our implied risk free rate $1/\beta - 1$ to arrive at the deposit rate. Over our sample period, bank deposits yielded on average 0.39 percentage points less than the 1-year Treasury rate, yielding a return on deposits of 0.62% for our implied risk free rate of 1%.

3.2. Effect of Stress Tests on Equity and Lending

To illustrate the effect of stress tests, we use the calibrated model and plot the marginal responses of equity and loan levels (in %) to a unit increase in the tightness of the stress-test constraint τ in Figure 3. It is clear that the effect of a higher stress-test constraint is highly non-linear in the state of the business cycle, i.e. the return state.

Figure 3: Marginal Response of Equity and Lending to a Unit Increase in τ



(a) Marginal Response of Equity (%)

(b) Marginal Response of Lending (%)

Following an increase of the stress-test tightness τ, equity (left panel) is lower for very bad states of the world due to an increased no-retainment threshold (see Equation 43). However, for most of the return realizations below the unconditional mean return $\bar{\mu}_l$ equity is higher following the increase of τ. For return realizations above $\bar{\mu}_l$ the increase of τ does not lead to higher equity retainment since banks retain all of their equity either way. This demonstrates that in all but very bad states of the world the increase of τ can weakly enhance the safety of banks, but this unequivocally comes at the cost of lower lending levels, as the right panel shows. This reduction in lending, however, approaches zero as the return realisations increase.

3.3. The Supervisory Choice of τ

We now investigate how a supervisor would optimally set τ with the the objective to ensure stable lending levels.[12] Here, Corollary 2 highlights the supervisory trade-off between reduced, but consequently less volatile lending. To capture this trade-off, we assign the welfare weight $\omega \geq 0$ to the expected variance of optimal bank lending L_1^*. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \omega \mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0] \tag{45}$$

s.t.

$$\chi(\tau) \in [\underline{\chi}, 1) \tag{46}$$

where

$$r_{l,1} \leq \underline{r_l} : \quad L_1^* = 0 \tag{47}$$

[12] Note that this supervisory objective is taken directly from the Federal Reserve Board (2020c).

$$r_{l,1} \in (\underline{r_l},\, \overline{r_l}): \quad L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2} \tag{48}$$

$$r_{l,1} \geq \overline{r_l}: \quad L_1^* = \frac{E_0}{\chi(\tau)} \tag{49}$$

Equations (47) to (49) show that the supervisor anticipates a rectified normally distributed L_1^* with upper and lower bounds: (47) states that below $\underline{r_l}$, lending L_1^* is set to zero; (48) implies that between $\underline{r_l}$ and $\overline{r_l}$ lending is normally distributed with $\mathcal{N}\left(\mu_{L_1}, \sigma_{L_1}^2\right)$; (49) states that above $\overline{r_l}$, lending is set to $E_0/\chi(\tau)$.

Since loans follow a two-sided rectified distribution, no closed form expression for the optimal stress-test tightness could be derived. Instead we utilize our parameterization and identify the optimal τ^* by computationally maximizing the supervisor's welfare subject to the respective constraints directly. To examine the supervisor's decision in more detail, we compute the optimal τ^* for different relative welfare weights ω and return realizations $r_{l,0}$. Since the results depend to a large degree on the amount of initial equity E_0, we first define the steady state level E_1^{ss} in the absence of stress tests as

$$E_1^{ss} = \frac{\chi}{\gamma \sigma_l^2} \left[\bar{\mu}_l - r_d - \chi \left(\frac{1}{\beta} - 1 - r_d \right) \right] \tag{50}$$

and fix the initial equity endowment E_0 at this level to ensure comparable results. Table 2 below states the resulting optimal τ^*, the implied $\chi(\tau)^*$, and the associated supervisory welfare for the different environments. In particular, we consider four possible scenarios for $r_{l,0}$: one severe crisis scenario where $r_{l,0} = -0.43\%$ (a $6 * \sigma_l$ movement), one mild slowdown where $r_{l,0} = 1.62\%$ (a $2 * \sigma_l$ movement), one median case where $r_{l,0} = 2.66\%$, and one upswing scenario where $r_{l,0} = 3.69\%$ (a $2 * \sigma_l$ movement). For each of these initial return realizations we compute the optimal stress-test tightness for a supervisor that does not care about lending volatility (i.e. $\omega = 0$), a supervisor that cares as much about lending volatility as about lending levels (i.e. $\omega = 1$), a supervisor that that dislikes lending volatility as much as the investor (i.e. $\omega = \gamma/2$), and a supervisor that that dislikes lending volatility twice as much as the investor (i.e. $\omega = \gamma$).

Based on the implied welfare for the respective τ^* it is clear that the supervisory welfare function is increasing in the initial return realization $r_{l,0}$ and decreasing in the weight given to the variance of loans. The supervisor therefore optimally sets $\tau^* = 4.05$ such that $\chi(\tau^*) = \chi$ when she does not derive any disutility from the variance of loans (i.e. when $\omega = 0$) in order to maximize the level of loans. However, as ω increases and she derives more disutility from the variance of loans, she optimally sets a higher τ^* to reduce that variance. We furthermore note that τ^* increases less in ω for higher realisations of $r_{l,0}$.

In general, a supervisor who cares about both the level and the volatility of lending,

Table 2: Optimal Stress Tests and Welfare

Welfare Weight	Optimal Tightness τ^*	$\chi(\tau^*)$	Welfare
	$r_0 = \bar{\mu} - 6 * \sigma_l = -0.43\%$		
$\omega = 0$	4.05	0.07	72.05
$\omega = 1$	104.91	0.75	1.96
$\omega = \gamma/2$	153.88	0.84	-15.34
$\omega = \gamma$	185.57	1.00	-42.53
	$r_0 = \bar{\mu} - 2\sigma_l = 1.63\%$		
$\omega = 0$	4.05	0.07	138.52
$\omega = 1$	16.31	0.13	80.71
$\omega = \gamma/2$	19.06	0.15	72.78
$\omega = \gamma$	21.80	0.16	66.11
	$r_0 = \bar{\mu} = 2.66\%$		
$\omega = 0$	4.05	0.07	162.96
$\omega = 1$	9.16	0.10	115.93
$\omega = \gamma/2$	10.53	0.10	108.25
$\omega = \gamma$	11.81	0.11	101.84
	$r_0 = \bar{\mu} + 2\sigma_l = 3.69\%$		
$\omega = 0$	4.05	0.07	172.49
$\omega = 1$	5.14	0.08	150.60
$\omega = \gamma/2$	5.96	0.08	143.02
$\omega = \gamma$	6.70	0.08	136.70

finds stress test capital buffers in the range of 1% to 9% to be optimal. This matches well the Federal Reserves' publicly announced stress-test buffers reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

4. Voluntary Stress Test Violation

In our baseline model environment, banks can neither violate the minimum equity-to-asset ratio nor the stress-test constraint. The U.S. stress test framework, however, allows for voluntary violation of the stress-test constraint, albeit automatically triggering a (partial) ban on dividend payments (see Appendix A for details). This violation allows the bank to invest up to a binding minimum-equity-to-asset ratio constraint instead. In this section, we investigate when a bank might find it optimal to purposely violate the stress-test constraint. For simplicity, we assume that this immediately triggers a total ban on dividend payments in that period. Then, voluntary violation implies the following equalities:

$$d_1 = 0 \tag{51}$$

$$E_1 = E_0 \tag{52}$$

$$D_1 = L_1 - E_0 \tag{53}$$

Inserting these equalities in the original maximization problem results in the following revised bank objective:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{54}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{55}$$

Because full retainment implies sub-optimally high equity levels, the bank no longer chooses to equity finance as little as possible. The upper feasibility limit in (55) reflects this, where now χ applies instead of $\chi(\tau)$. Quite intuitively, the upper feasibility limit is binding in very high returns states above a threshold $\overline{r_{l,1}^V}$, where the bank would like to invest more in loans than the minimum equity requirements allow. And hence:

$$L_1^{*V} = \frac{E_0}{\chi} \qquad \forall r_{l,1} \geq \overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{56}$$

On the contrary, the lower feasibility limit is binding in bad return states, where the bank would like to invest nothing but must at least invest E_0. This applies to all return states below threshold $\underline{r_{l,1}^V}$:

$$L_1^{*V} = 0 \qquad \forall r_{l,1} \leq \underline{r_l^V} = \frac{1}{\rho_l}\left[\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{57}$$

In between the two return thresholds, the bank equity finances loans with a share strictly above χ but below one. The optimal loan level is determined by the first-order-condition of the objective function (54), when both feasibility constraint multipliers are zero. For $r_{l,1}$ above $\underline{r_{l,1}^V}$ and below $\overline{r_{l,1}^V}$, this implies an optimal lending:

$$L_1^{*V} = \frac{\mu_l - \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_{l,1}^V}, \overline{r_{l,1}^V}\right) \tag{58}$$

To derive when voluntary violation is optimal, we must compare the total shareholder utility from voluntary violation, denoted with $U^V(d_1, d_2)$, to the one from the baseline analysis, denoted $U(d_1, d_1)$.

Total utility under voluntary violation:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{59}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\Big(\mu_l + \rho_l r_{l,1} - r_d\Big) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + (1 + r_d)\, E_0\right] \tag{60}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{61}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\Big(\mu_l + \rho_l r_{l,1} - r_d\Big)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0\,(1 + r_d)\right] \tag{62}$$

Total utility under compliance (baseline):

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{63}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\Big(\mu_l + \rho_l r_{l,1} - r_d\Big) L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right] \tag{64}$$

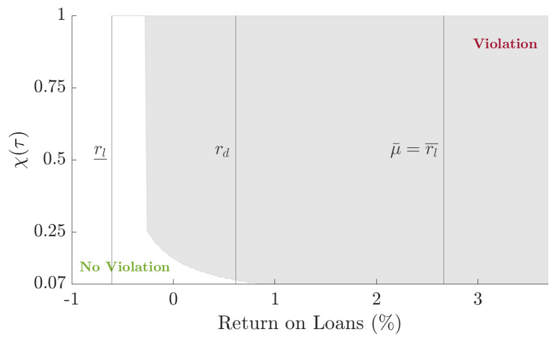
$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2} \tag{65}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta\left[\Big(\mu_l + \rho_l r_{l,1} - r_d\Big)\frac{E_0}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{2}\left(\frac{E_0}{\chi(\tau)}\right)^2 + E_0(1 + r_d)\right] \tag{66}$$

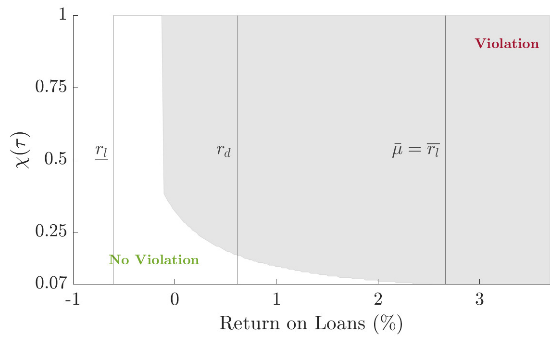
To prove when $U^V(d_1, d_2)$ exceeds $U(d_1, d_1)$ is cumbersome, as the sizes of return thresholds $\underline{r_l^V}$ and $\overline{r_l^V}$ relative to $\underline{r_l}$ and $\overline{r_l}$ strongly depend on the initially inherited equity E_0 relative to other model parameters. And hence a large number of different utility functions would have to be compared to cover all cases. Instead, we provide insights for a meaningful parameter space and numerically study the voluntary violation decision for large US banks given our calibration. Figure 4 (below) illustrates when a bank violates the stress tests voluntarily for the above presented calibration and three levels of initial equity as function of the steady state equity level E_1^{ss}.

Each of the Panels 4a - 4c has the continuum of loan returns $r_{l,1}$ on the x-axis and the range of possible stress test implied minimum equity-to-loan ratio requirements on the y-axis. The gray shaded areas indicate when the bank finds it optimal to voluntarily violate the stress-test constraint. Here, we can see that this is generally the case for higher $\chi(\tau)$ and higher return states $r_{l,1}$. This should come as no surprise: The higher $\chi(\tau)$, the lower the total loans a stress-test compliant bank may issue and the more it can increase the loan capacity by voluntarily violating. Further, expanding loan capacity is more attractive in good states of the world, where risky loan investment is desirable. On the contrary, exposing

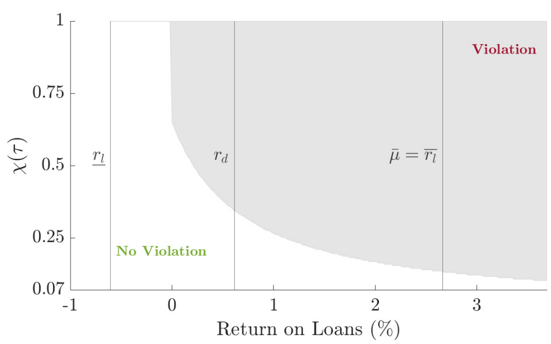
Figure 4: Optimal Choice of Stress-Test Violation



(a) $E_0 = 0.5 \cdot E_1^{SS}$



(b) $E_0 = E_1^{SS}$



(c) $E_0 = 2 \cdot E_1^{SS}$

(sub-optimally high) equity levels to risky loans in bad states by violating the stress-test constraint is not desirable. Therefore, the desirability of violation also decreases in the initial equity endowment.

Remark 1. *For U.S. stress tested banks, violation is optimal for higher tightness* τ*, higher loan return states* $r_{l,1}$*, and the lower the initial equity* E_0*.*

It should be noted, however, that the voluntary violation of the stress test constraint in our model does not incur any costs over and beyond the restriction on dividend payouts, such as financial market stigma or increased supervisory scrutiny. This explains why in reality, unlike our model would predict, large banks almost never violate the stress-test constraint.

5. Policy Extensions

Bank stress tests are typically seen as a complementary measure to a rich set of additional prudential policies. To understand their relative effectiveness in stabilizing lending, we extend the model to include two currently utilized policy tools: the Covid-19 dividend ban and the counter-cyclical capital buffer (CCyB). Additionally, we provide a welfare comparison of stress tests to the dividend prudential target proposed by Muñoz (2020). In the latter, dividends are regulated directly, but less intensely than in an outright ban.

5.1. *Covid-19 Dividend Restrictions*

At the onset of the Covid-19 crisis, several jurisdictions introduced either an outright ban on dividend payments or a strong recommendation to stop payments temporarily (Beck et al., 2020). The goal was to boost equity and thereby counteract the pro-cyclically of lending. Here, we abstract from any moral suasion frictions between supervisors and banks, and analyze the effect of a dividend ban on bank lending levels.[13] An outright ban on dividends implies full equity retainment, such that:

$$d_1 = 0 \tag{67}$$

$$E_1 = E_0 \tag{68}$$

$$D_1 = L_1 - E_0 \tag{69}$$

Inserting these into the bank original optimization problem results in a revised maximization not unlike the one under voluntary violation:

$$U^B(d_1, d_2) = \max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{70}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{71}$$

However, here the stress-test constraint still applies and determines the upper bound of loan investments in (71). It thus acts as a feasibility constraint for the revised bank maximization problem. Again, the lower and upper feasibility bounds on L_1 imply two return thresholds denoted with $\underline{r_l^B}$ and $\overline{r_l^B}$ respectively:

$$\underline{r_l^B} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \qquad \overline{r_l^B} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + r_d - \mu_l\right] \tag{72}$$

[13] This is without loss of generality. As Beck et al. (2020) show, most European banks did indeed stop dividend payments following the ECB's recommendation.

Unlike in the baseline model however, the two thresholds determine the share of debt financing instead of the degree of equity retainment: For return states below $\underline{r_{l,1}^C}$, the bank fully equity finances L_1 now equal to E_0. Intuitively, in these bad return states, the shareholder would prefer to liquidate the bank but this is prevented by the dividend ban. Thus the only remaining option is to invest the existing equity in loans.

$$L_1^{*B} = E_0 \qquad \forall\, r_{l,1} \leq \underline{r_l^B} \tag{73}$$

For intermediate return states, the bank sets an optimal loan level L_1^{*B} that requires a share of equity financing strictly below one but strictly above $\chi(\tau)$. Intuitively, in these return states the shareholder would actually prefer some dividends in period 1 but this is prevented by the dividend ban. At the same time, the loans are still relatively risky, limiting the attractiveness of investing in them. Thus the bank utilizes all its equity, but does not lever up as much as it could. In this case, the level of lending is:

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_l^B}, \overline{r_l^B}\right) \tag{74}$$

For high return states above $\overline{r_l^B}$, the bank debt finances as much as possible given E_0 and $\chi(\tau)$, where the stress-test constraint now becomes the upper feasibility limit:

$$L_1^{*B} = \frac{E_0}{\chi(\tau)} \qquad \forall r_{l,1} \geq \overline{r_l^B} \tag{75}$$

Comparing the optimal lending of a bank with free reign over the dividend payments with the one subject to a ban, we show that for all return states below $\overline{r_l}$ the lending is higher under the latter. Only for return states above $\overline{r_l}$ is the feasibility constraint on total lending binding under both regimes and and thus lending identical.[14]

***Proposition* 2.** *A dividend ban leads to strictly higher lending during crises.*

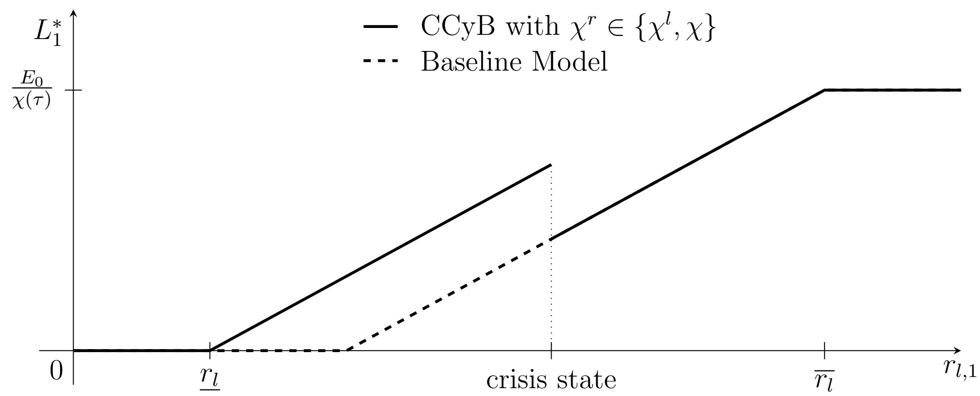
5.2. Counter-Cyclical Capital Buffer

A complementary policy tool to the dividend ban is the relaxation of the counter-cyclical capital buffer (CCyB) during times of crises. In the baseline model, we have assumed a constant χ that is state-independent. Instead, a CCyB implies a state-dependent χ^r that takes on a value $\chi^l < \chi$ for low return states. This relaxes the stress-test constrain in bad states via a reduction in $\chi(\tau)$. Relying on insights from Section 2.2, we know that this

[14]Note here that for the formal proof, we account for the fact that the thresholds $\underline{r_l}$ may be above or below and $\underline{r_l^B}$. However, $\overline{r_l^B}$ is always below $\overline{r_l}$.

triggers an increase in lending and lowers the return thresholds below which no equity is retained. Figure 5 below illustrates this.

Figure 5: The Impact of CCyBs on Lending



Relaxing a CCyB is often combined with other crisis measures, such as the above discussed Covid-19 dividend ban. Two natural question are thus, how both compare in their ability to increase lending during crisis, but also how effective a joint introduction of both is. Here, it can be shown that lending under a dividend ban is strictly higher than under a relaxed CCyB. Intuitively, the main driver of lower loan levels in bad return states is equity withdrawal, which is not adequately addressed by relaxing the CCyB. Furthermore, the CCyB actually has no additional effect once a dividend ban is put in place. A bank subject to a ban already holds sub-optimally high equity and debt finances less than allowed. Therefore, a relaxed CCyB does not change the optimal loan levels when activated on top of a dividend ban during crisis.

***Proposition* 3.** *Introduced individually, a relaxed CCyB increases lending during crises. However, the CCyB is less effective than a dividend ban and, if introduced additionally, has no further effect.*

We are thus able to provide an explanation for the recent policy puzzle regarding banks not using their CCyB buffers to finance lending during the Covid-19 crisis (FSB, 2021): The additionally relaxing of CCyBs simply does not impact lending choices of already dividend restricted banks.

5.3. Dividend Prudential Target

Finally, we discuss a substitute regulatory approach to stress tests: the dividend prudential target (DPT). Initially suggested by (Muñoz, 2020), the DPT restricts dividends directly, unlike stress tests, but less severely than an outright ban. In a first step, a DPT rule defines a target dividend pay-out – usually the pay-out made by an unrestricted bank in steady-state. We follow this tradition and evaluate our baseline model at the unconditional mean of the AR(1) process. This implies a dividend target d_1^T equal to:

$$d_1^T = E_0 - E^{ss}, \tag{76}$$

$$= E_0 - \frac{\chi}{\gamma\sigma_l^2}\left[\frac{\mu_l}{1-\rho_l} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{77}$$

Here note that we have set $\chi(\tau) = \chi$, i.e. we have assumed no further stress testing. Consequently, a DPT defines a quadratic punishment fee for deviating dividend payments from such target:

$$-\kappa(d_1 - d_1^T)^2. \tag{78}$$

This fee is financed directly by the representative investor and subtracted from any dividend payments at $t = 1$. Already accounting for the different budget constraints, a bank subject to the DPT solves the following optimization problem:

$$U^T(d_1, d_2) = \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E^{ss} - E_1\right)^2 + \beta\left[E_1(1 + r_d) + L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2}L_1^2\right] \tag{79}$$

$s.t.$

$$L_1 \in \left[E_1, \frac{E_1}{\chi}\right] \tag{80}$$

$$E_1 \in [0, E_0] \tag{81}$$

Here, retaining equity is no longer just a way to finance $t = 2$ dividends via lending, but also a way to reduce the DPT fee at $t = 1$. Thus for intermediate states above a threshold r_l^l and below a second threshold r_l^h, the bank finds it no longer beneficial to only equity finance the absolute minimum. Instead it retains a fixed equity level E_1^{*T}:

$$E_1^{*T} = E^{ss} - \frac{\beta}{\kappa}\left(\frac{1}{\beta} - 1 - r_d\right) \qquad \forall r_{l,1} \in (r_l^l, r_l^h) \tag{82}$$

where

$$r_l^l = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1^{*T} + r_d - \mu_l\right] \tag{83}$$

$$r_l^h = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2 E_1^{*T}}{\chi} + r_d - \mu_l\right] \tag{84}$$

Because the equity in (82) is state-independent, the level of lending L_1^{*T} is equivalent to lending under an outright ban:

$$L_1^{*T} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \qquad \forall r_{l,1} \in (r_l^l, r_l^h) \tag{85}$$

Here, the threshold r_1^l denotes the return state below which lending in (85) is lower than equity in (82). It thus violates the lower lending feasibility constraint as specified in Equation (80). Intuitively, r_l^l denotes the threshold below which the utility losses on lending exceed the costs of paying a higher DPT fee. The only way to reduce lending, while complying with the budget constraint $L_1 = E_1$, is to gradually retain less equity:

$$E_1^{*T} = \frac{\beta}{\kappa + \beta\gamma\sigma_l^2}\left[\frac{\kappa}{\beta}E^{ss} + \mu_l + \rho_l r_{l,1} + 1 - \frac{1}{\beta}\right] \qquad \forall r_{l,1}(\underline{r_l^T}, r_l] \tag{86}$$

$$L_1^{*T} = E_1^* \qquad \forall r_{l,1}(\underline{r_l^T}, r_l^l] \tag{87}$$

Equations (86) and (87) describe the optimal bank choices between r_l^l and the no-retainment threshold $\underline{r_l^T}$. As before, $\underline{r_l^T}$ denotes the return threshold below which the bank does not engage in lending:

$$L_1^{*T} = E_1^{*T} = 0 \qquad \forall r_{l,1} \le \underline{r_l^T} \tag{88}$$

$$\underline{r_l^T} = \frac{1}{\rho_l}\left(\frac{1}{\beta} - 1 - \frac{\kappa}{\beta}E^{ss} - \mu_l\right) \tag{89}$$

A similar logic can be applied to return states above r_l^h. Here, the cost of deviating from the DPT are lower than the utility benefits from increased lending. And thus, the bank gradually increases equity to relax the upper feasibility limit on lending as stated in (80) for all return states between r_l^h and the full-retainment threshold $\overline{r_l^T}$.

$$E_1^{*T} = \frac{1}{\frac{\kappa\chi}{\beta} + \frac{\gamma\sigma_l^2}{\chi}}\left[\frac{\chi\kappa}{\beta}E^{ss} + \mu_l + \rho_l r_{l,1} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] \qquad \forall r_{l,1} \in [r_l^h, \overline{r_l^T}) \tag{90}$$

$$L_1^{*T} = \frac{E_1^*}{\chi} \qquad \forall r_{l,1} \in [r_l^h, \overline{r_l^T}) \tag{91}$$

$$\overline{r_l^T} = \frac{1}{\rho_l}\left[\frac{\kappa\chi}{\beta}(E_0 - E^{ss}) + \frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{92}$$

Above $\overline{r_l^T}$ the bank always retains all initial equity E_0 to invest in lending:

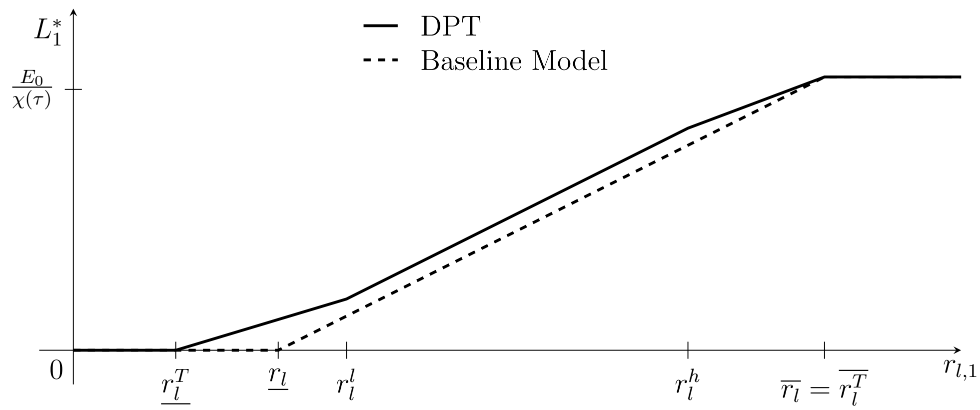
$$E_1^{*T} = E_0 \qquad \forall r_{l,1} \ge \overline{r_l^T} \tag{93}$$

$$L_1 = \frac{E_0}{\chi} \qquad \forall r_{l,1} \ge \overline{r_l^T} \tag{94}$$

Performing comparative statics we can show that the full-retainment threshold $\overline{r_l^T}$ is above (below) the baseline full-retainment threshold $\overline{r_l}$ when initial equity E_0 is above (below) the steady state equity level E^{ss}. Furthermore, whether the no-retainment threshold $\underline{r_l^T}$ increases or decreases under a DPT relative to baseline $\underline{r_l}$ depends on the punishment fee κ

set by the supervisor. Figure 6 below summarizes the just derived optimal lending under the DPT for the case with $E_0 = E^{ss}$ and κ sufficiently large, and compares it to lending under a stress test regime.

Figure 6: Optimal Bank Lending under a DPT



It can be seen in Figure 6 that a DPT results in a smoother lending function over the state-space, where κ influences both the mean and variance of lending. Recall that the supervisory authority sets the punishment parameter κ in period 0 with the objective to stabilize lending, putting welfare weight ω on the expected lending variance. Unfortunately, a full closed-form characterization of the mean and variance of lending is cumbersome and provides little general insights. We therefore again immediately rely on the calibrated model (assuming $E_0 = E^{ss}$) to derive the optimal κ and resulting supervisory welfare. The results are summarized in Table 3 below.

As the comparison of supervisory welfare between stress tests and the dividend prudential target show, the latter is in a better position to maximize lending when this is all the supervisor cares about (i.e. $\omega = 0$). In this case the optimal punishment for deviations of dividends from target is hump shaped and highest at the mean return $\bar{\mu}$. However, the dividend prudential target navigates the trade-off between the level and variance of lending much worse than stress tests and leads to much lower supervisory welfare whenever $\omega > 0$. In these cases the supervisor optimally sets κ at very low levels, which are decreasing (increasing) in the welfare weight ω (initial return state $r_{l,0}$).

Providing intuition for these results proves challenging, as an increase in κ affects the four return thresholds $\underline{r_l^T}$, r_l^l, r_l^h, and $\overline{r_l^T}$ non-linearly and in different directions. Therefore, unlike an increase in τ, an increase in κ does not naturally translate into a decrease in both mean and lending. This results in a relatively flat supervisory welfare function, where large changes in κ have little impact. Further, the existence of a pronounced lower tail of the lending function due to equity retainment starting at lower return states (see Figure 6) leads

Table 3: Welfare Comparison of Stress Tests and DPT

Welfare Weight	Stress Test		DPT	
	Tightness	Welfare	Tightness	Welfare
	$r_0 = \bar{\mu} - 6 * \sigma_l = -0.43\%$			
$\omega = 0$	$\tau^* = 4.05$	$W = 72.05$	$\kappa^* = 74.50$	$W = 67.12$
$\omega = 1$	$\tau^* = 104.91$	$W = 1.96$	$\kappa^* = 1e^{-7}$	$W = -538.21$
$\omega = \gamma/2$	$\tau^* = 153.88$	$W = -15.34$	$\kappa^* = 1e^{-7}$	$W = -1,262.34$
$\omega = \gamma$	$\tau^* = 185.57$	$W = -42.53$	$\kappa^* = 1e^{-7}$	$W = -2,597.56$
	$r_0 = \bar{\mu} - 2 * \sigma_l = 1.63\%$			
$\omega = 0$	$\tau^* = 4.05$	$W = 138.52$	$\kappa^* = 1,075.28$	$W = 140.62$
$\omega = 1$	$\tau^* = 16.31$	$W = 80.71$	$\kappa^* = 11e^{-4}$	$W = -442.88$
$\omega = \gamma/2$	$\tau^* = 19.06$	$W = 72.78$	$\kappa^* = 11e^{-4}$	$W = -1,133.53$
$\omega = \gamma$	$\tau^* = 21.80$	$W = 66.11$	$\kappa^* = 11e^{-4}$	$W = -2,407.01$
	$r_0 = \bar{\mu} = 2.66\%$			
$\omega = 0$	$\tau^* = 4.05$	$W = 162.96$	$\kappa^* = 1,783.86$	$W = 164.11$
$\omega = 1$	$\tau^* = 9.16$	$W = 115.93$	$\kappa^* = 38e^{-4}$	$W = -65.05$
$\omega = \gamma/2$	$\tau^* = 10.53$	$W = 108.25$	$\kappa^* = 37e^{-4}$	$W = -336.31$
$\omega = \gamma$	$\tau^* = 11.81$	$W = 101.84$	$\kappa^* = 37e^{-4}$	$W = -836.47$
	$r_0 = \bar{\mu} + 2 * \sigma_l = 3.69\%$			
$\omega = 0$	$\tau^* = 4.05$	$W = 172.49$	$\kappa^* = 644.37$	$W = 172.72$
$\omega = 1$	$\tau^* = 5.14$	$W = 150.60$	$\kappa^* = 0.99$	$W = 147.37$
$\omega = \gamma/2$	$\tau^* = 5.96$	$W = 143.02$	$\kappa^* = 0.04$	$W = 117.34$
$\omega = \gamma$	$\tau^* = 6.70$	$W = 136.70$	$\kappa^* = 0.03$	$W = 61.97$

to a much higher expected variance under the dividend prudential target compared to the stress test regime. The supervisor has to set very low values of κ to shift $\underline{r_l^T}$ to the right but thereby decreases expected lending much more than it would be the case under a stress testing regime. Further evidence to this is that the two regimes are most similar in terms of welfare for very good return realizations, where the lower end of the lending function is less relevant: this implies a much lower supervisory welfare once the expected variance of lending is taken into account. Even when only caring about the mean of lending, i.e. $\omega = 0$, welfare improves only marginally with a DPT. Overall, stress tests are a much better tool to generate stable lending than a DPT.

6. Conclusion

Bank stress tests have become an increasingly important policy tool designed with the intend to ensure stable lending and, thereby, to foster financial stability. In this paper, we derive the optimal bank balance sheet choices anticipating subsequent stress testing. Here, we explicitly model the forward-looking constraint that stress tests place on the bank's degree of debt financing: Equity capital levels should be sufficient to maintain current lending tomorrow, even after absorbing severe losses from said lending. We find that stress tests influence the banks' joined decision over (retained) equity, dividends, and lending. Here, we document the core supervisory trade-off: The more severe the assumed losses, the lower are both expected lending and lending volatility.

To quantitatively assess how such trade-off plays out in practice, we calibrate our model to the U.S. banks subject to the CCAR stress tests. We derive the optimal stress-test tightness (severity of the adverse scenario) and the implied stress test capital buffer. We find that a supervisor that prefers to maximize lending levels while minimize lending volatility finds stress test capital buffers in the range of 1% to 9% to be optimal. This matches well the Federal Reserves' publicly announced stress-test buffers reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

Next, we turn to placing the stress test framework in the wider net of prudential policies. Here, we highlight in particular how stress tests may be complemented with a dividend ban and/or the relaxation of the CCyB in a crisis period. We find that separately introduced, both relax lending of stress-tested banks in bad states of the world. They can, thus, be utilized to dampen the stress-test induced decrease in lending during downturns. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects. We are, thus, able to rationalize why the relaxation of the CCyB during the on-set of the Covid19 pandemic had no measurable effect of lending by stress-tested banks subject to the dividend bans (FSB, 2021).

We conclude the paper by studying a hypothetical substitute policy: the dividend prudential target. Contrary to stress tests regulating equity levels, the dividend prudential target directly regulates bank dividend payments by introducing a quadratic cost function for deviations from a target. This discourages both equity extraction in bad states and excessive leveraging in good states. We find however, that such policy mainly increases the mean of lending but fails to reduce volatility. It is thus not welfare improving for a supervisor seeking stable lending levels.

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Appendix A. Regulatory Framework

Following the financial crisis 08/09, the Federal Reserve Board (FED) was mandated to perform two complementary stress tests: the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act stress testing (DFAST). The CCAR is a forward looking exercise and assesses bank holding companies' (BHC) capital adequacy accounting for individual dividend payment plans. Banks with assets of 10$bn and above are required to take part in the CCAR. The DFAST takes the last three quarters' dividend policy as given and mainly focuses on the sufficiency of loss-absorbing capital (Federal Reserve Board, 2020c). Banks with assets of 250$bn and above are required to take part in the DFAST. For the purpose of this study (apart from the calibration), we focus on the CCAR stress test framework, which is described in detail in the following paragraphs.

CCAR Stress Test As part of the CCAR stress test, the FED calculates the individual BHCs' capitalization under three scenarios: baseline, supervisory adverse, supervisory severely adverse. Here, they account for the BHCs' proposed future dividend payments and capital repurchase plans. Subsequently, the FED decides whether to approve a BHC's planned capital actions by compare the post-stress capital levels under the severely adverse scenarios to the minimum capital requirements plus surcharges (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Minimum Capital Requirements From 2009-2013, all stress-test eligible BHCs were subject to a minimum tier 1 common ratio of 5%. In 2014, all banks with at least $250 billion total assets or more than $10 billion foreign asset exposure were subject to a 4% minimum common equity tire 1 ratio (CET1) instead. The remaining banks continued to be subject to the 5% minimum tier 1 common ratio for one more year. From 2015 onward, all BHCs were subject to a 4.5% minimum common equity tier 1 ratio (Federal Reserve Board, 2015, 2016). This change in minimum capital measures was part of the phase-in of the Basel III framework, which also introduced additional capital surcharges.

Capital Surcharge BHCs identified as globally systemically important banks (G-SIB) are subject to additional minimum risk-adjusted capital measures of 1%-3.5%. From 2014 the 2016, Basel Committee on Banking Supervisisons capital add-on is applied. Since 2017, the maximum of the surcharges calculated under the Basel capital framework and the Federal Reserve Board's assessment methodology titled "Method II" applies (Office of Financial Research, 2021). Additionally, a 2.5% conservation buffer was phased in from 2016-2019 (Federal Reserve Board, 2013, 2014). For our sample period, the banks are not subject to any countercyclical capital buffer (Federal Reserve Board, 2019a).

Table 4: Maximum Dividend to Net-Income Ratio Given CET1

CET1	Maximum Pay-Out Ratio
$< 5.125\%$	0%
$5.125\% - 5.75\%$	20%
$5.75\% - 6.375\%$	40%
$6.375\% - 7\%$	60%
$> 7\%$	no limitations

Source: BIS (2019)

Supervisory Power over Dividend Payments Stress-test eligible BHCs are prohibited from any dividend payments and share repurchases until the FED has approved of the capital distribution plan in writing. As mentioned above, such approval is based on the stress test performance and follows in three steps. First, the FED performs an initial set of stress-test given the BHCs' original dividend payout plan. The resulting (preliminary) stress-test results are communicated to the BHC. All BHCs, both insufficiently and sufficiently capitalized, are allowed once to submit an adjusted capital plan (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Then either the original or, if submitted, adjusted capital plan form the base for the FED's payout policy interventions. Capital levels below the minimum tier 1 common ratio or CET1 (plus G-SIB surcharge) respectively, automatically triggers a payout ban. A violation of the capital conservation buffer automatically results in dividend payments to be restricted to a percentage of net income (see Table 4). Sufficient capital levels do not result in automatic restrictions. The Fed however, reserves the right to require a BHC to reduce or cease all capital distributions if it felt that the weaknesses in the BHC's capital planning warranted such a response (Federal Reserve Board, 2014). And thus BHCs may feel supervisory pressure especially when close to but not yet violating their respective minimum capital requirements.

Recent Developments In 2020, the Federal Reserve Board decided to replace the 2.5% capital conservative buffer by an individual stress test buffer for each BHC (Federal Reserve Board, 2020b,a). This falls outside our sample period.

Appendix B. Proofs For Section 2

B.1. Solving The Bank Optimization Problem

1. Lets start with defining dividend payments at $t = 1$ and $t = 2$.

$$
\begin{aligned}
d_1 =& E_0 - E_1 && (95)\\
d_2 =& L_1 r_{l,2} - r_d D_1 + E_1 \quad \sim N(\mu, \sigma^2) && (96)\\
\text{where} \quad \mu =& (\mu_l + \rho_l r_{l,1}) L_1 - r_d D_1 + E_1 && (97)\\
\text{and} \quad \sigma^2 =& \sigma_l^2 L_1^2 && (98)
\end{aligned}
$$

Further note that D_1 is perfectly determined by E_1 and L_1 via the budget constraint:

$$D_1 = L_1 - E_1 \tag{99}$$

Finally, note that plugging this into the stress-test constraint yields:

$$
\begin{aligned}
\chi L_1 \leq& E_1 + L_1(\bar{\mu}_l - \tau\sigma_l - r_d(L_1 - E_1) && (100)\\
(\chi - \bar{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq& 0 && (101)
\end{aligned}
$$

2. Using the above stated equations and standard properties of a normal distributions, allows us to reduce the bank optimization problem to:

$$
\begin{aligned}
U(d_1, d_2) = \quad & \max_{E_1, L_1} E_0 - E_1 + \beta\left[L_1(\mu_l + \rho_l r_{l,1} - r_d(L_1 - E_1) + E_1 - \frac{\gamma\sigma_l^2}{2}L_1^2)\right] && (102)\\
s.t. \quad & && \\
\lambda_1: \quad & \chi L_1 - E_1 \leq 0 && (103)\\
\lambda_2: \quad & (\chi - \bar{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 && (104)\\
\lambda_3: \quad & E_1 - E_0 \leq 0 && (105)\\
\lambda_4: \quad & E_1 - L_1 \leq 0 && (106)\\
\lambda_5: \quad & E_1 \geq 0 && (107)
\end{aligned}
$$

We denote the multipliers associated with constraints (103)- (107) with λ_1 through λ_5 respectively.

3. Before taking any first order conditions two comments on the constraints.

3.1. Notice that multipliers λ_3 and λ_5 can never be simultaneously be positive. They describe each their own corner solution: full retainment of equity and no retainment of equity.

3.2. Depending on τ, either minimum-equity and stress-test test constraint binds first. The other one consequently only binds in states, where the first one is already binding.

We start by rearranging the stress-test constraint:

$$\frac{(\chi - \bar{\mu_l} + \tau\sigma_l + r_d)}{(1+r_d)} L_1 \leq E_1 \tag{108}$$

Then notice that the multiplier in front of L_1 in the above equation is determined fully by model parameters and does not depend on equilibrium choices. Further, it enters multiplicatively into the constraint in the same fashion as χ.

Then, logically, the stress-test constraint binds first whenever:

$$\frac{(\chi - \bar{\mu_l} + \tau\sigma_l + r_d)}{(1+r_d)} \geq \chi \tag{109}$$

$$\tau \geq \frac{r_d\chi + \bar{\mu_l} - r_d}{\sigma_l} = \tau^* \tag{110}$$

And in reverse logic, the minimum equity constraint binds first, whenever $\tau < \tau^*$. This concludes the proof for *Lemma 1.*

4. The above described result of 3.2. allows us actually to combine the two supervisory constraints in the following fashion:

$$\chi(\tau) = \begin{cases} \chi & \tau < \tau^* \\ \frac{r_d\chi + \bar{\mu_l} - r_d}{\sigma_l} & \tau \geq \tau^* \end{cases} \tag{111}$$

And the revised constraint, that nests both cases, is:

$$\chi(\tau) L_1 \leq E_1 \tag{112}$$

5. Then, we start solving the simplified maximization problem by assuming the bank has chooses a feasible level $E_1 \in [0, E_0]$. Taking E_1 as given reduces the bank optimization problem to:

$$U(E_0 - E_1, d_2) = \quad E_0 - E_1 + \beta E_1(1+r_d) + \max_{L_1} \beta \left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2 \right] \tag{113}$$

$$s.t.$$

$$\lambda_{1+2}: \qquad \chi(\tau)L_1 - E_1 \leq 0 \tag{114}$$

$$\lambda_4: \qquad -L_1 + E_1 \leq 0 \tag{115}$$

Then, the FOC wrt to L_1 becomes:

$$(\mu_l + \rho_{1,l} r_{l,1}) - r_d - \gamma\sigma_l^2 L_1 - \lambda_{1+2}\chi(\tau) + \lambda_4 = 0 \tag{116}$$

6. We know discuss the differenct cases for the multipliers. Here, notice that λ_{1+2} and λ_4 can never bind simultaneously: one would bind if the bank would like to set significantly lower L_1 than E_1 and one would bind, if the bank would like set significantly higher than E_1/χ.

6.1. With this in mind, we start with (temporarily) ignoring both constraints. Then, the optimal loan level is:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{117}$$

6.2. Then for a given E_1, logically there exists a lower threshold level $r^*_{l,1}$ for which investing $L_1 = E_1$ is optimal. And for all lower levels, the bank would like to set $L_1 < E_1$ but cannot due to its constraint choice.

Following a similar logic there exist a second threshold $r^{**}_{l,1}$, for which the bank would like to invest E_1/χ units into loans. And for any higher level, it would like to invest more, but cannot due to the minimum equity constraint.

6.3. However, as we will see later, these two thresholds are not really playing a core role, because E_1 is chosen by the bank and not taken as given. Here, it is important to take away from Equation (117) that any interior solution of L_1 without either constraints binding is independent of the level of equity E_1.

7. Lets start with assuming that $\lambda_{1+2} = \lambda_4 = 0$. This implies that the bank indeed finances some loans, but that these loans are more equity financed than strictly required.

7.1. Recall then that L_1 is independent of E_1 and thus, the optimal level of E_1 can be chosen by the following optimization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta(1 + r_d)E_1 \tag{118}$$

$$s.t.$$

$$\lambda_3 : \quad E_1 - E_0 \leq 0 \tag{119}$$

Abstracting for now from constraint λ_3 this implies a FOC wrt E_1:

$$-1 + \beta(1 + r_d) \tag{120}$$

Relying on parameter assumptions, it can be shown that this FOC is always negative:

$$-1 + \beta(1 + r_d) < 0 \tag{121}$$

$$(1 + r_d) \leq \frac{1}{\beta} \quad \text{True by assumption} \tag{122}$$

Hence, any interior solution with only partial debt financing cannot be sustained. Any solution with positive loan levels is characterized by $E_1 = \chi(\tau)L_1$.

8. With this in mind, we can now derive the optimal equity level E_1 by solving the following maximization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta \left[\frac{E_1}{\chi(\tau)}(\mu_l + \rho_l r_{l,1}) - \frac{\gamma \sigma_l^2}{2\chi(\tau)^2} E_1^2 - r_d \frac{1 - \chi(\tau)}{\chi(\tau)} E_1 + E_1 \right] \tag{123}$$

$$s.t.$$

$$\lambda_4 : \quad E_1 - E_0 \leq 0 \tag{124}$$

$$\lambda_5 : \quad -E_1 \leq 0 \tag{125}$$

8.1. Again, we will for now ignore the two feasibility constraints. Then the FOC wrt E_1:

$$-1+\beta\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi(\tau)}-\frac{\gamma\sigma_l^2}{\chi(\tau)^2}E_1-r_d\frac{1-\chi(\tau)}{\chi}+1\right]=0 \tag{126}$$

$$E_1^*=\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi(\tau)}-r_d\frac{1-\chi(\tau)}{\chi}+1-\frac{1}{\beta}\right] \tag{127}$$

8.2. Now recall that an constraint solution requires $E_1 \leq E_0$. This holds up until:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho r_{l,1}}{\chi(\tau)}-r_d\frac{1-\chi(\tau)}{\chi}+1-\frac{1}{\beta}\right]\geq E_0 \tag{128}$$

$$r_{l,1}\geq\frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0+\chi(\tau)\left(\frac{1}{\beta}-1\right)+r_d(1-\chi(\tau))-\mu_l\right]=\overline{r_l} \tag{129}$$

Or in other words, for any level of $r_{l,1}$ exceeding the threshold $\overline{r_l}$ equity is fully retained and invested in loans. The optimal bank choices and (expected) dividends are thus:

$$E_1^*=E_0 \tag{130}$$

$$L_1^*=\frac{E_0}{\chi(\tau)} \tag{131}$$

$$d_1^*=0 \tag{132}$$

$$\mathbf{E}[D_1^*]=E_0\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi\tau}-r_d\frac{(1-\chi(\tau))}{\chi(\tau)}+1\right] \tag{133}$$

8.3. A similar logic can be applied for the lower bound such that:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho r_{l,1}}{\chi(\tau)}-r_d\frac{1-\chi(\tau)}{\chi}+1-\frac{1}{\beta}\right]\leq 0 \tag{134}$$

$$r_{l,1}\leq\frac{1}{\rho_l}\left[\chi(\tau)\left(\frac{1}{\beta}-1\right)+r_d(1-\chi(\tau))-\mu_l\right]=\underline{r_l} \tag{135}$$

Or put differently,for any realized stated $r_{l,1}$ weakly below $\underline{r_l}$ no equity is retained. The bank's equilibrium choices and (expected) dividends are thus:

$$L_1^*=E_1^*=D_1^*=0 \tag{136}$$

$$d_1=E_0 \tag{137}$$

8.4. For intermediate levels $r_{l,1}\in(\underline{r_l},\ \overline{r_l})$ and interior solution exists with:

$$E_1^*=\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi(\tau)}-r_d\frac{1-\chi(\tau)}{\chi(\tau)}+1-\frac{1}{\beta}\right] \tag{138}$$

$$L_1^*=\frac{E_1^*}{\chi(\tau)} \tag{139}$$

$$d_1^*=E_0-E_1^* \tag{140}$$

$$\mathbf{E}[D_1^*] = E_1^* \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi(\tau)} + 1 \right] \tag{141}$$

B.2. Comparative Statics Over τ

We now compare environment where $\tau < \tilde{\tau}$ such that $\chi(\tau) = \chi$ with an environment, where $\tau > \tilde{\tau}$ such that $\chi(\tau \geq \tau) > \chi$.

1. We start by showing that that $\underline{r_l^s} < \underline{r_l^{n,e}}$.

$$\underline{r_l^s} < \underline{r_l^{n,e}} \chi \left(\frac{1}{\beta} - 1 - r_d \right) < \qquad \chi(\tau)\tilde{\tau} < \tau \tag{142}$$

2. Further, we can show that $\overline{r_l^s} > \overline{r_l^{n,e}}$:

$$\overline{r_l^s} > \overline{r_l^{n,e}} \tag{143}$$

$$\frac{\gamma \sigma_l^2}{\chi} E_0 + \chi \left(\frac{1}{\beta} - 1 - r_d \right) > \frac{\gamma \sigma_l^2}{\chi(\tau)} E_0 + \chi(\tau) \left(\frac{1}{\beta} - 1 - r_d \right) \tag{144}$$

$$\gamma \sigma_l^2 E_0 \left(\frac{1}{\chi} - \frac{1}{\chi(\tau)} \right) > (\chi(\tau) - \chi) \left(\frac{1}{\beta} - 1 - r_d \right) \tag{145}$$

Notice that the right hand side is a term very close to zero, and thus the inequality holds true under the assumption that $E_0 >> 0$.

3. With this, we know the upper and lower feasibility implied thresholds for equity and thus lending. Now, we turn to the slope of the optimal equity and lending policies.

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma \sigma_l^2} \rho_l \tag{146}$$

$$\frac{\partial^2 E_1}{\partial r_{l,1} \partial \chi(\tau)} = \frac{\rho_l}{\gamma \sigma_l^2} > 0 \tag{147}$$

3.1. It can be shown that E_1^* increases linearly in $r_{l,1}$:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma \sigma_l^2} \rho_l \tag{148}$$

And confirming the relative return state bounds, it can be shown that the slope is steeper, the higher is τ:

$$\frac{\partial^2 E_1}{\partial r_{l,1} \partial \chi(\tau)} = \frac{\rho_l}{\gamma \sigma_l^2} > 0 \tag{149}$$

This implies that under a stress-test constraint, the bank starts to retain equity only in relatively higher states, but once started, it reaches full retainment earlier. Naturally, there exists a threshold $\tilde{r}$ for which the two equity functions intersect.

3.2. Turning to the loans, one can show that $L_1^{*,s} < L_1^{*,e}$. Her,e we first start with the loan rates implying $E_1 < E_0$. Then:

$$L_1^{*,s} < L_1^{*,e} \tag{150}$$

$$-\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) < \chi\left(\frac{1}{\beta} - 1 - r_d\right) \tag{151}$$

$$\chi < \chi(\tau) \tag{152}$$

$$\tilde{\tau} < \tau \tag{153}$$

Now, we consider the high return states inducing $E_1^* = E_0$:

$$L_1^{*,s} < L_1^{*,e} \tag{154}$$

$$\frac{E_0}{\chi(\tau \geq \tau)} < \frac{E_0}{\chi} \tag{155}$$

$$\chi < \chi(\tau) \tag{156}$$

$$\tilde{\tau} < \tau \tag{157}$$

We omit the proof for the variance of lending here due to its complexity here, and discuss it in detail during the supervisory problem. We would nevertheless like to highlight here, that lending L_1^* follows a rectified normal distribution with a lower and an upper bound. By increasing τ (above $\tilde{\tau}$), we bring the bounds closer together, thus reducing the variance of the overall distribution.

Appendix C. The Optimal Tightness τ

In this section, we derive the optimal supervisory choice under two different objective functions. To maintain tractability, we will assume that the realization of return states above $r_{l,1}^{f,s}$ are very low probability events for large banks with sufficient equity sticks. And thus, loan levels are fully characterized. Let us denote the optimal lending in the absence of feasibility constraints with L_1^x, where:

$$L_1^X = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{158}$$

$$L_1^x \sim N(\mu_x, \sigma_x^2) \tag{159}$$

$$\mu_x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l(\mu_l + \rho_l r_{l,0}) - r_d - \chi(\tau)(1/\beta - 1 - r_d)\right] \tag{160}$$

$$\sigma_x^2 = \left(\frac{\rho_l}{\gamma\sigma_l}\right)^2 \tag{161}$$

The optimal bank lending L_1^* thus takes the following step-function.

$$L_1^* = \begin{cases} 0 & L_1^x < 0 \\ L_1^x & 0 \geq L_1^x \geq \frac{E_0}{\chi(\tau)} \\ \frac{E_0}{\chi(\tau)} & L_1^x > \frac{E_0}{\chi(\tau)} \end{cases} \tag{162}$$

Appendix D. Additional Proofs

D.1. Proofs for Voluntary Violation

Voluntary violation of the stress-test constraint implies a ban on divididends and, thus, the following equalities:

$$d_1 = 0 \tag{163}$$

$$E_1 = E_0 \tag{164}$$

$$D_1 = L_1 - E_0 \tag{165}$$

With this, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{166}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{167}$$

Here note that the upper feasibility limit is now determined by χ and not anymore $\chi(\tau)$.

Ignoring the two feasibility constraints for now, the FOC and the consequent optimal lending level are:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{168}$$

$$L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{169}$$

Recall that L_1^{*V} is bounded above by the minimum asset-to-equity ratio constraint which allows us to derive a threshold $\overline{r_l^V}$. Similarly, in this business model L_1 can never be below E_0, allowing us a lower threshold $\underline{r_l^V}$

$$\overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{170}$$

$$\underline{r_l^V} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{171}$$

With this in mind, it remains to be shown when the total utility exceeds the one of complying to the stress-test constraint. The resulting total utility from violation is:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{172}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) L_1^{*V} - \frac{\gamma \sigma_l^2}{2} \left(L_1^{*V} \right)^2 + (1 + r_d) E_0 \right] \tag{173}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{174}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) \frac{E_0}{\chi} - \frac{\gamma \sigma_l^2}{2} \frac{E_0^2}{\chi^2} + E_0 (1 + r_d) \right] \tag{175}$$

This, we have to compare to the following aggregate utilities from complying:

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{176}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l} \right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) L_1^* - \frac{\gamma \sigma_l^2}{2} \left(L_1^* \right)^2 + E_1^* (1 + r_d) \right] \tag{177}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma \sigma_l^2} \tag{178}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) \frac{E_0}{\chi(\tau)} - \frac{\gamma \sigma_l^2}{2} \left(\frac{E_0}{\chi(\tau)} \right)^2 + E_0 (1 + r_d) \right] \tag{179}$$

To derive, when violation would be optimal, one must compare the appropriate utilities given the return state $r_{l,1}$. A challenge here is that $\underline{r_l^V} \lesseqqgtr \underline{r_l}$ and $\overline{r_l^V} \lesseqqgtr \overline{r_l}$, depending on E_0:

$$\underline{r_l^V} \lesseqqgtr \underline{r_l} \tag{180}$$

$$\frac{1}{\rho_l} \left[\gamma \sigma_l^2 E_0 + r_d - \mu_l \right] \lesseqqgtr \frac{1}{\rho_l} \left[\chi(\tau) \left(\frac{1}{\beta} - 1 - r_d \right) + r_d - \mu_l \right] \tag{181}$$

$$E_0 \lesseqqgtr \frac{\chi(\tau)}{\gamma \sigma_l^2} \left(\frac{1}{\beta} - 1 - r_d \right) \tag{182}$$

$$\overline{r_l^V} \lesseqqgtr \overline{r_l} \tag{183}$$

$$\frac{1}{\rho_l} \left[\frac{\gamma \sigma_l^2}{\chi} E_0 + r_d - \mu_l \right] \lesseqqgtr \frac{1}{\rho_l} \left[\frac{\gamma \sigma_l^2}{\chi(\tau)} E_0 + \chi(\tau) \left(\frac{1}{\beta} - 1 - r_d \right) + r_d - \mu_l \right] \tag{184}$$

$$E_0 \lesseqqgtr \frac{\chi \chi(\tau)^2}{(\chi(\tau) - \chi) \gamma \sigma_l^2} \left(\frac{1}{\beta} - 1 - r_d \right) \tag{185}$$

Without further restrictions on E_0, a closed form proof is a cumbersome comparison of all possible combinations for the different functional forms that the utilities may take. As this provides little additional insight without restriction the parameter space, we refrain from doing so. Instead, we show when voluntary violation is optimal for the above calibrated parameters and several different values of E_0. Please refer to the main text for results.

D.2. Covid-19 Dividend Ban

Sketch of proof for Proposition 2.

1. A ban on bank dividend payments implies the following equalities:

$$d_1 = 0 \tag{186}$$
$$E_1 = E_0 \tag{187}$$
$$D_1 = L_1 - E_0 \tag{188}$$

2. As the stress-test constraint is still binding, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{189}$$

$s.t.$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{190}$$

3. Temporarily ignoring the two feasibility constraints, taking the FOC and equating it to zero yields the following optimal lending level:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{191}$$
$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{192}$$

4. Now, we turn to the upper feasibility limit on L_1^{*B} determined by the stress test implied minimum asset-to-equity ratio constraint. This allows us to derive a threshold $\overline{r_l^B}$:

$$L_1^{*B} \geq \frac{E_0}{\chi(\tau)} \tag{193}$$
$$r_{l,1} \geq \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l\right] = \overline{r_l^B} \tag{194}$$

Similarly, in this business model L_1 can never be lower than E_0, allowing us to define the lower threshold $\underline{r_l^B}$

$$L_1^{*B} \leq E_0 \tag{195}$$
$$r_{l,1} \leq \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] = \underline{r_l^B} \tag{196}$$

5. Then, the total utility under the Covid19 dividend ban, denoted with $U^B(d_1, d_2)$ become:

$$r_{l,1} < \underline{r_l}: \quad U^B(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{197}$$
$$r_{l,1} \in [\underline{r_l}, \overline{r_l}]: \quad U^B(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^{*B} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*B}\right)^2 + (1 + r_d)E_0\right] \tag{198}$$
$$\text{where} \quad L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{199}$$

$$r_{l,1} > \overline{r_l}: \quad U^B(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) \frac{E_0}{\chi} - \frac{\gamma \sigma_l^2}{2} \frac{E_0^2}{\chi(\tau)^2} + E_0 \left(1 + r_d\right) \right] \tag{200}$$

6. We are left with showing that $L_1^* < L_1^{*,B}$:

6.1. Assume a realized $r_{l,1}$ in the range $(-\infty,\ \min\{\underline{r_l};, \underline{r_l^B}\}]$. Then:

$$L_1^* < L_1^{*B} \tag{201}$$

$$0 < E_0 \tag{202}$$

6.2. Assume a realized return in the range $(\underline{r_l},\ \underline{r_l^B}]$. Then:

$$L_1^* < L_1^{*B} \tag{203}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2} < E_0 \tag{204}$$

$$r_{l,1} < \frac{1}{\rho_l} \left(\gamma \sigma_l E_0 - \mu_l + r_d + \chi(\tau)(1/\beta - 1 - r_d \right) \tag{205}$$

$$< \underline{r_l^B} + \frac{1}{\rho_l} \chi(\tau)(1/\beta - 1 - r_d) \tag{206}$$

Which holds true by assumption.

6.3. Assume a realized return $r_{l,1}$ in the range $(\underline{r_l}^B,\ \underline{r_l}]$. Then:

$$L_1^* < L_1^{*B} \tag{207}$$

$$0 < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{208}$$

$$\underline{r_l^B} - \frac{\gamma \sigma_l^2 E_0}{\rho_l} < r_{l,1} \tag{209}$$

Which holds true by assumptions.

6.4. Assume a realized $r_{l,1}$ in the range $\left(\max\{\underline{r_l}, \underline{r_l^B}\},\ \min\{\overline{r_l}, \overline{r_l^B}\} \right]$. Then:

$$L_1^* < L_1^{*B} \tag{210}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{211}$$

$$-\chi(\tau)(1/\beta - 1 - r_d) < 0 \tag{212}$$

Which holds true by parameter assumption.

6.5. Assume a realized $r_{l,1}$ in the range $(\overline{r_l},\ \overline{r_l^B})$. Then:

$$L_1^* < L_1^{*B} \tag{213}$$

$$\frac{E_0}{\chi(\tau)} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{214}$$

$$\overline{r_l} - \frac{1}{\rho_l} \chi(\tau)(1/\beta - 1 - r_d) < r_{l,1} \tag{215}$$

Which holds true by assumption.

6.6. Assume a realized $r_{l,1}$ in the range $(\overline{r_l^B},\ \overline{r_l})$. Then:

$$L_1^* < L_1^{*B} \tag{216}$$
$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < \frac{E_1}{\chi(\tau)} \tag{217}$$
$$r_{l,1} < \overline{r_l} \tag{218}$$

This holds true by assumption.

6.7. Finally, assume a realized return state $r_{l,1} \in [\max\{\overline{r_l},\ \overline{r_l^B}\},\ +\infty$. Then:

$$L_1^* = L_1^{*B} \tag{219}$$
$$\frac{E_1}{\chi(\tau}) = \frac{E_1}{\chi(\tau)} \tag{220}$$

D.3. Proof for CCyB

Proof omitted due to its triviality. Please see main-text for results.

D.4. Dividend Prudential Target

The steady state of our model is characterized by the unconditional mean $\bar{\mu}_l$ and implies a DPT of:

$$d_1^T = E_0 - E_1^T \tag{221}$$
$$= E_0 - \frac{\chi}{\gamma\sigma_l^2}\left[\frac{\mu_l}{1-\rho_l} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{222}$$

Any deviations from the target are punished with the following fine:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2 \tag{223}$$
$$\frac{\kappa}{2}\left(E_0 - E_1 - d_1^T\right)^2 \tag{224}$$
$$\frac{\kappa}{2}\left(E_1^T - E_1\right)^2 \tag{225}$$

This results in the following revised optimization problem:

$$U(E_0 - E_1, d_2) = \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_1^T - E_1\right)^2 + \beta E_1(1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2}L_1^2\right] \tag{226}$$

$$s.t.$$

$$\lambda_1: \qquad L_1 \in \left[E_1, \frac{E_1}{\chi}\right] \tag{227}$$

$$\lambda_2: \qquad E_1 \in [0, E_0] \tag{228}$$

Again, we start by assuming a feasible E_1 has been chosen and thus the bank is left with the optimal lending choice. Here, we can rely on results from the bank section and now for a given level E_1, the bank chooses:

$$L_1 = E_1 \qquad \forall r_{l,1} \leq r_l^l = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1 + r_d - \mu_l\right] \tag{229}$$

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(r_l^l,\ r_l^h\right) \tag{230}$$

$$L_1 = \frac{E_1}{\chi} \qquad \forall r_{l,1} \geq r_l^h = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2 E_1}{\chi} + r_d - \mu_l\right] \tag{231}$$

With these levels, we are left with determining the feasible level of E_1 given the DPT and punishment fees. Here, we simply start by assuming the feasibility limits are not binding and thus the choice of E_1 is not affecting lending. Then the FOC wrt. E_1 becomes:

$$-1 - \frac{\kappa}{2}\left(-2E_1^T + 2E_1\right) + \beta(1 + r_d) = 0 \tag{232}$$

$$E_1 = \frac{1}{\kappa}\left(\beta(1 + r_d) - 1\right) + E_1^T \tag{233}$$

Then we turn to return states above r_l^h. Here we know that any unit increase of E_1 above (233) relaxes the upper feasibility limit on lending and thereby allows for an increase in lending by $\frac{1}{\chi}$ units. To show that this is optimal, we use a revised FOC wrt. E_1 that assumes $L_1 = E_1/\chi$:

$$-1 + \kappa(E_1^T - E_1) + \beta(1 + r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} - \beta\frac{\gamma\sigma_l^2}{\chi^2}E_1 \tag{234}$$

If expanding feasibility is ever optimal, this FOC must be positive in the states above $r_{l,1}^h$ for the above derived E_1 in (233). Inserting (233) in (234) yields:

$$\beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} - \beta\frac{\gamma\sigma_l^2}{\chi^2}E_1 > 0 \tag{235}$$

$$r_{l,1} > \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2 E_1}{\chi} + r_d - \mu_l\right] = r_l^h \tag{236}$$

Because the latter holds true by assumption, we know increasing equity above (233) is optimal. Utilizing the FOC assuming $L_1 = E_1/\chi$, we can easily determine the appropriate equity levels in these high states:

$$E_1 = \frac{1}{\frac{\kappa\chi}{\beta} + \frac{\gamma\sigma_l^2}{\chi}} \left[\frac{\kappa\chi}{\beta} E_1^T + \mu_l + \rho_l r_{l,1} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{237}$$

However, this solution is valid only as long as $E_1 \leq E_0$. For high $r_{l,1}$ the feasibility constraint binds:

$$E_1 = E_0 \tag{238}$$

$$\frac{1}{\frac{\kappa\chi}{\beta} + \frac{\gamma\sigma_l^2}{\chi}} \left[\frac{\kappa\chi}{\beta} E_1^T + \mu_l + \rho_l r_{l,1} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] = E_0 \tag{239}$$

$$\frac{\kappa\chi}{\beta} E_1^T + \mu_l + \rho_l r_{l,1} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right) = E_0\left(\frac{\kappa\chi}{\beta} + \frac{\gamma\sigma_l^2}{\chi}\right) \tag{240}$$

$$\frac{\kappa\chi}{\beta}(E_0 - E_1^T) + \frac{\gamma\sigma_l^2}{\chi} E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) = \mu_l + \rho_l r_{l,1} - r_d \tag{241}$$

$$\frac{1}{\rho_l}\left[\frac{\kappa\chi}{\beta}(E_0 - E_1^T) + \frac{\gamma\sigma_l^2}{\chi} E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] = r_{l,1} = \overline{r_l} \tag{242}$$

Having fully explored the optimal equity and lending for high return states, we now apply a similar logic to the low return states. Here, for $L_1 = E_1$ the FOC becomes:

$$-1 + \kappa(E_1^T - E_1) + \beta(\mu_l + \rho_l r_{l,1} + 1) - \beta\gamma\sigma_l^2 E_1 < 0 \tag{243}$$

$$\beta(\mu_l + \rho_l r_{l,1} - r_d) - \beta\gamma\sigma_l^2 E_1 < 0 \tag{244}$$

$$r_{l,1} < \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1 - \mu_l + r_d\right] = r_l^l \tag{245}$$

Again the latter inequality holds true by assumption and thus, the bank has an incentive to lower equity beyond E_1^*, thereby also lowering lending. Here, the optimal level of equity and lending is:

$$-1 + \kappa(E_1^T - E_1) + \beta(\mu_l + \rho_l r_{l,1} + 1) - \beta\gamma\sigma_l^2 E_1 = 0 \tag{246}$$

$$-\frac{1}{\beta} + \frac{\kappa}{\beta}(E_1^T - E_1) + (\mu_l + \rho_l r_{l,1} + 1) - \gamma\sigma_l^2 E_1 = 0 \tag{247}$$

$$E_1 = \frac{1}{\frac{\kappa}{\beta} + \gamma\sigma_l^2}\left[\frac{\kappa}{\beta} E_1^T + \mu_l + \rho_l r_{l,1} + 1 - \frac{1}{\beta}\right] \tag{248}$$

Again, this value holds until the lower feasibility limit of $E_1 = 0$ binds. This is the case for any state below:

$$0 = \frac{1}{\frac{\kappa}{\beta} + \gamma\sigma_l^2}\left[\frac{\kappa}{\beta} E_1^T + \mu_l + \rho_l r_{l,1} + 1 - \frac{1}{\beta}\right] \tag{249}$$

$$r_{l,1} = \frac{1}{\rho_l}\left(\frac{1}{\beta} - 1 - \frac{\kappa}{\beta} E_1^T - \mu_l\right) = \underline{r_l} \tag{250}$$

From there on, we label everything related to the DPT with an additional superscript ^{T}. We then conclude with several comparative statistics:

$$\underline{r_l^T} < \underline{r_l} \tag{251}$$

$$\frac{1}{\rho_l}\left(\frac{1}{\beta} - 1 - \frac{\kappa}{\beta}E_1^T - \mu_l\right) < \frac{1}{\rho_l}\left[r_d - \mu_l + \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{252}$$

$$\frac{1}{\beta} - 1 - \frac{\kappa}{\beta}E_1^T < r_d + \chi\left(\frac{1}{\beta} - 1 - r_d\right) \tag{253}$$

$$(1-\chi)\left(\frac{1}{\beta} - 1 - r_d\right) < \frac{\kappa}{\beta}E_1^T \tag{254}$$

$$\frac{\beta(1-\chi)}{E_1^T}\left(\frac{1}{\beta} - 1 - r_d\right) < \kappa \tag{255}$$

Whether the no retainment state increases or decreases under a DPT thus depends on the κ set by the supervisor.

$$\overline{r_l^T} > \overline{r_l} \tag{256}$$

$$\frac{1}{\rho_l}\left[\frac{\kappa\chi}{\beta}(E_0 - E_1^T) + \frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right] > \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{257}$$

$$\frac{\kappa\chi}{\beta}(E_0 - E_1^T) > 0 \tag{258}$$

$$\kappa > 0 \tag{259}$$

The latter is a necessary condition for the optimization to apply and hence the solution to be correct.