The ground state energy of 1D dilute many-body quantum systems

Johannes Agerskov

QMATH
University of Copenhagen

Abstract

We study the ground state energy of a gas of 1D bosons with density ρ, interacting through a general, repulsive 2-body potential with scattering length a, in the dilute limit $\rho|a| \ll 1$. The first terms in the expansion of the thermodynamic energy density are $\pi^2\rho^3/3(1+2\rho a)$, where the leading order is the 1D free Fermi gas. This result covers the Tonks–Girardeau limit of the Lieb–Liniger model as a special case, but given the possibility that $a > 0$, it also applies to potentials that differ significantly from a delta function.

Set up and previous results

The scattering length

Theorem 1 (ref needed)

For $B_R := \{0 < |x| < R\} \subset \mathbb{R}^d$ with $R > R_0 :=$ range(v), let $\phi \in H^1(B_R)$ satisfy

$$-\Delta\phi + \frac{1}{2}v\phi = 0, \qquad \text{on } B_R, \tag{1}$$

with boundary condition $\phi(x) = 1$ for $|x| = R$. Then $\phi(x) = f(|x|)$ for some $f : (0, R] \to [0, \infty)$, and for range$(v) < r < R$, we have

$$f(r) = \begin{cases} (r-a)/(R-a) & \text{for } d = 1 \\ \ln(r/a)/\ln(R/a) & \text{for } d = 2 \\ (1-ar^{2-d})/(1-aR^{2-d}) & \text{for } d \geq 3, \end{cases} \tag{2}$$

with some constant a called the (s-wave) **scattering length**.

Main result

Theorem 2 (A., R. Reuvers, J. P. Solovej, 2022)

Consider a Bose gas with repulsive interaction $v = v_{reg} + v_{h.c.}$ as defined above. Write $\rho = N/L$. For $\rho|a|$ and ρR_0 sufficiently small, the ground state energy can be expanded as

$$E(N, L) = N\frac{\pi^2}{3}\rho^2\left(1 + 2\rho a + \mathcal{O}\left((\rho|a|)^{6/5} + (\rho R_0)^{6/5} + N^{-2/3}\right)\right), \tag{3}$$

where a is the scattering length of v.