



UNIVERSITY OF  
COPENHAGEN

PHD SCHOOL OF SCIENCE  
UNIVERSITY OF COPENHAGEN

## Principal Supervisor's Final Report

### 1. General information

Document date: 31/03/2023  
Name PhD student: Johannes Agerskov Schiødt  
Department: Department of Mathematical Sciences  
KU ID: ptz888  
Principal Supervisor: Jan Philip Solovej  
Co-supervisor:

### 2. PhD Programme

5+3 PhD programme: 5+3 PhD programme  
Subcategory: Please choose  
Subcategory: Please choose  
Subcategory: Please choose  
Subcategory: Please choose  
Full-time/part-time: Full-time

### 3. Start date and expected end date

Start date: 01/10/2019  
Expected end date: 31/03/2023

### 4. Thesis

Title of thesis: One-dimensional Dilute Quantum Gases and Their Ground State Energies  
Expected date of submission:  
Expected date of defence:

### 5. Supervisor's statement

Short description of main  
accomplishments:

Johannes Agerskov has in his Phd thesis studied Bose and Fermi gases in one dimension. In recent years there has been great progress in the mathematical understanding of the ground states of Bose gases in the dilute limit in two and three dimensions. In these dimensions Bose gases form a condensate and their ground states are well described by the theory of superfluidity. In one dimension the situation is very different as there is no condensation. In one dimension there are several systems that are exactly solvable, e.g., the celebrated Lieb-Liniger model. There was, however, until the work of Agerskov no theory that described the dilute limit for general interactions. With the Phd work of Agerskov, which gives a two term asymptotic formula for the ground state energy in the dilute limit, we now have a complete picture of dilute Bose gases in one, two, and three dimensions. The thesis is thus a very important and groundbreaking contribution to our general understanding of Bose gases. The thesis also contains work on Fermi gases in the dilute limit. The case of spin-polarized Fermi gases turns out to be very similar to the bosonic case. The non-polarized case is, however, much more

difficult. The thesis gives a rigorous upper bound on the ground state energy in the dilute limit, which is conjectured to be also a correct lower bound and several arguments are presented to support this. This is particularly interesting because it relates the spin-1/2 Fermi gas to the antiferromagnetic Heisenberg model and may be the first time that this connection has been addressed rigorously.

The work on Bose gases is ready for submission. It is a preprint on the ArXiv. It is joint work with Jan Philip Solovej (PhD Advisor) and Robin Reuvers. Johannes Agerskov played a dominant role in the research and was the driving force in all aspects from details of the mathematical proof to the writing of the manuscript.

The work on spin-1/2 Fermi gases will be a separate publication to be completed later.

The thesis work of Agerskov is simply outstanding and of the highest quality. It has already been recognized internationally and already cited in recent preprints. It addresses many issues in the rigorous study of many-body quantum mechanics and opens up several new lines of research.

Has a publication agreement been made: Yes

The co-author statements are satisfactory: Yes

Subject area: Mathematics

## 6. Supervisor's overall evaluation of the PhD programme

I confirm that the PhD programme has been completed satisfactorily

## 7. Student's overall evaluation of the PhD programme

I confirm that I have completed the PhD programme elements and read the statement.

## 8. Mandatory seminars

Activity	Start date	End date	Progress
Presentation of Moser and Seiringer point interactions.	04/11/2019	04/11/2019	Completed
QLunch talk Title: The ground state energy of dilute 1d many-body quantum systems Abstract: Dilute Bose and Fermi gases are well-studied subjects in the mathematical physics literature. In two and three dimensions, asymptotic formulas for the bosonic ground state energy are known to leading and next-to-leading order in the diluteness parameters respectively. I will in this talk present joint work with Robin Reuvers and Jan Philip Solovej, in which we derive asymptotic formulas for the one dimensional dilute gas consisting of either fermions or bosons.	12/01/2022	12/01/2022	Completed

## 9. Change of scientific environment

Institution	Country	Start date	End date	Progress	Status
Stay at the Institute of Science and Technology in Austria	AT Austria	04/10/2021	03/12/2021	Completed	

## 10. Knowledge dissemination and/or teaching activities

Activity	Contribution	Number of hours
Analyse 1 assistant	Making weekly problem sheets, mandatory assignments, and helping with administrative tasks on the course "Analyse 1".	150
Functional analysis.	TA in the master level course Functional Analysis.	160
Grading exams in MatAn.	Grading exams.	20
TA in Functional Analysis (FunkAn)	TA in exercise class and exam grading.	200

## 11. PhD Course Portfolio

Title of course	Start date	End date	ECTS suggested	ECTS completed	Progress	Status
Responsible Conduct of Research						

(generic course)	12/11/2019	12/11/2019	1.0	1.0	Completed	
Advanced Mathematical Physis (AdvMathPhys)	20/04/2020	19/06/2020	7.5	7.5	Completed	
Introduction to University Pedagogy - English Spoken Team 1	11/05/2020	15/05/2020	3.0	3.0	Completed	
Computability and Complexity	08/02/2021	18/04/2021	7.5	7.5	Completed	09 The course/conference is approved
Differential Operators and Function Spaces (DiffFun)	08/02/2021	16/04/2021	7.5	7.5	Completed	
Qmath 15 - Conference at UC Davis 2022	12/09/2022	16/09/2022	2.0	2.0	Completed	15 The activity is approved

## 12. Comments

Student's comments:	
Supervisor's comments:	The thesis is outstanding and is a major contribution to current research. It opens up several new lines of research and will be a motication fo rmuch research to come.
PhD administration's comments:	