

3G. Preliminary assessment of the PhD thesis

To be handed in minimum 3 weeks before expected date of defense



1. PhD student

Name: **Johannes Agerskov Schiødt** UCPH user id: **Ptz888**
Or date of birth

2. Supervisor(s)

Principal Supervisor

Title & Name: **Professor Jan Philip Solovej** E-mail: **solovej@math.ku.dk**

Co-supervisor

Title & Name: E-mail:

Place of employment:

Co-supervisor

Title & Name: E-mail:

Place of employment:

3. PhD thesis

Title of PhD thesis: **One-dimensional Dilute Quantum Gases and Their Ground State Energies**

Is the thesis acceptable for defense in its present form?

Yes ☒ Agreed date for defense of the PhD thesis: **June 6th, 2023**
Max. 3 months after date of submission

When the assessment is positive the defense will take place based on the submitted thesis. Suggestions for minor corrections may well be included in the preliminary recommendation and can be discussed at the defense.

A negative preliminary assessment means that there are fundamental errors and misunderstandings in the PhD thesis. The preliminary assessment must reflect and argue for those errors. All PhD theses must be given the chance for re-submission at least one time. The PhD student must be given adequate guidance to make the corrections required for the thesis to be re-submitted for a PhD defence and the award of the PhD degree. After the first re-submission, however, the shortcomings may be too grave and the thesis is rejected.

No ☐ The committee recommends that the thesis is rewritten and resubmitted within (Min. 3 months):

No ☐ The thesis has been re-submitted after a negative preliminary assessment and the committee finds that the shortcomings of the revised thesis are so grave that the PhD thesis is rejected.

4. The committee's preliminary assessment

The preliminary assessment should include a discussion of the aim and hypothesis, results, methods and conclusions. The different parts of the thesis should be evaluated as well as the mutual relationship between different parts. The assessment should also evaluate if the state-of-the-art research is presented and understood in the context of the overall conclusion. Please note: the principal supervisor may be called in as a consultant to the committee.

Short overview of the structure of the thesis

The thesis of Johannes Agerskov, discusses the ground-state properties of one-dimensional quantum systems, of bosonic, fermionic and also anyonic type.

Chapter 1 contains a concise introduction to the subject of the thesis, and to relevant recent results in the literature. The introduction gives a brief summary of the results presented in the manuscript, and of the ideas used in their proof.

Chapter 2 introduces the necessary mathematical concepts needed to formulate the problems studied in the thesis. In particular: many-body wave functions of identical particles; the spectral theorem; the energy functional and the ground state energy; delta interactions, appearing in various exactly solvable models discussed later; the even and the odd scattering length. Then, the dilute regime is introduced, and rigorous results for the ground state energy asymptotics are reviewed, both for fermions and for bosons. Most of the available results in the literature deal with dimension greater or equal than two. A notable feature of the one-dimensional case is the existence of exactly solvable models, for special choices of the interaction potential. The thesis reviews the exact solution of the Lieb-Liniger model, for spinless bosons, via the Bethe ansatz. Then, the thesis also discusses the more complicated case of the Yang-Gaudin model, describing fermions with spin.

Chapter 3 contains a recent preprint of Agerskov, Reuvers and Solovej, about the asymptotics of the ground state energy of dilute, spinless Bose gases in one dimension. The type of interaction potential considered is quite general, and it also allows for a hard-core component. The main result of the chapter is a theorem providing a two-term asymptotics in the density for the ground state energy of the system. The two explicit terms appearing in the result display universality: the first term corresponds to the ground state energy of a non-interacting, 1d Fermi gas, while the second term only depends on the scattering length of the potential (as for the higher dimensional counterparts of such result). The physical significance of the result is drastically different from the two and three dimensional case, as it is consistent with the absence of Bose-Einstein condensation, as is expected in one dimension. Interestingly, a corollary of the result is an analogous two-term asymptotics for the ground state energy of dilute particles of anyonic type (including fermions), due to the fact that the energy of the 1d anyon gas agrees with the energy of a Bose gas modified by a suitable two-body delta interaction, depending on the statistics parameter.

The strategy of the proof is based on matching upper and lower bounds for the ground state energy of the system, that agree at next-to-leading order in the density. The upper bound is based on the choice of a suitable trial state: namely, the absolute value of the free Fermi gas with Dirichlet boundary conditions, suitably weighted by the solution of the scattering equation. The proof proceeds by restricting the trial state into boxes containing not too many particles, and relies of a precise control of the density matrices of the free Fermi gas in each box. Such control is made particularly non-trivial due to the Dirichlet boundary conditions. The lower bound is based on the comparison with the Lieb-Liniger model with Neumann boundary conditions, which appears as a reference model for the ground state energy. In order to make the comparison rigorous, a crucial role is played by Dyson's lemma. Then, a precise control of the ground state energy of the Lieb-Liniger model allows to obtain the desired expansion for the energy, for small values of the number of particles N . Larger values of N can be considered using an argument based on Legendre transform, and superadditivity of the energy. Finally, the chapter is concluded with the extension of the result to anyons, via a unitary transformation.

Chapter 4 focuses on the ground state energy of spin-1/2 dilute fermions. The main result contained in the chapter is an upper bound for the ground state energy of the system. The expression obtained exhibits universality, however it differs qualitatively from the analogous expressions obtained in dimension higher than one. Specifically, the second order here depends on both the even and the odd scattering lengths. The chapter conjectures that the upper bound is actually sharp, as a next-to-leading order asymptotics for the ground state energy. This is consistent with few explicit cases, namely the hard-core Fermi gas, and the Yang-Gaudin model.

Also, the chapter contains a nice check of the conjecture using perturbation theory, via the Feynman-Hellman argument. The proof of the rigorous upper bound is based on a judicious choice of a trial state, that now comes with a non-trivial spin dependence. The most interesting aspect of the proof is, in my view, the fact that the energy of the trial state turns out to be related to the ground state energy of the antiferromagnetic Heisenberg chain, another exactly solvable 1d system. The result has several corollaries, among which an improvement of an old upper bound for the ground state energy of the Lieb-Liniger-Heisenberg (LLH) model due to Girardeau. The proof of the lower bound remains open. The thesis proposes to follow the strategy adopted in Chapter 3: in the present case, Dyson's lemma would allow to bound below the ground state energy using the Lieb-Liniger-Heisenberg model. However, the main problem in concluding the proof is that the Lieb-Liniger-Heisenberg is not exactly solvable via Bethe ansatz. Thus, this comparison, in general, does not allow yet to get an explicit expansion for the ground state energy as function of the density. The chapter is concluded with the discussion of a few special cases, in which the LLH ground state energy can be bounded below using the Lieb-Liniger energy, where a more explicit control on the energy can be achieved.

Finally, Chapter 5 nicely summarizes the content of the thesis, and discusses the open problems that can be studied via an extension of the methods introduced in the thesis.

Objectives of the thesis' research

Are the objectives clearly expressed in the thesis, and is the state-of-the-art research well expressed with relevant review of recently published results

The objectives of the research are clearly laid out in the introduction. Further details and state-of-the-art are well-presented in the overview in Chapter 2.

Summary and quality of main findings and scientific achievements

The thesis presents interesting new results on the ground state properties of many-body systems. In the last years there has been a lot of activity in dimensions two and three, but relatively little has been done in dimension one. Therefore, the thesis of Johannes Agerskov introduces substantial progress in the field by both establishing the two-terms asymptotics of the ground state energy in the spinless case and by breaking ground towards the spin $\frac{1}{2}$ -case.

The one-dimensional world has special features, among which the existence of several exactly solvable systems. These are of course non-generic situations, however in physics it is often argued that they are the representative of universality classes: systems belonging to the same universality class share the same qualitative features, despite being very different at the microscopic level. The present thesis introduces a strategy that allows to make rigorous use of exactly solvable models, to obtain predictions about nonsolvable one-dimensional systems, in a nonperturbative way. This insight is particularly interesting and noteworthy. The writing of the thesis, is generally very clear and shows considerable overview and mastery of the technical details and fundamental ideas.

Quality of all papers (unsubmitted, submitted as well as published)

The thesis contains a preprint as Chapter 3. This preprint is clearly publishable in a high quality journal of mathematical physics as it contains the main result on the energy of the 1-dimensional gas.

Other relevant factors

Preliminary conclusion

All together, this is a very good thesis, that contains interesting new results in a highly competitive and quickly evolving research field. The material presented in the thesis is certainly sufficient to warmly recommend acceptance for the final PhD defense.

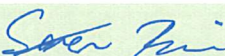
5. Signatures

Or confirmation by e-mail

Member 1 (SCIENCE, chairman)

Name: Professor, Søren Fournais
Department of Mathematical Sciences,
University of Copenhagen

E-mail: sofo@math.ku.dk

Date: 14/5 '23 Signature: 

Member 2 (External)

Name: Professor, Marcello Porta
Sissa (International School for Advanced Studies), Trieste

Date: Signature:

Member 3 (External)

Name: Professor, Simona Rota Nodari
Université Côte d'Azur

Date: Signature:

Once you have completed the application form, please save it as a Word or an OCR-readable (editable) PDF file and e-mail it to the PhD secretary at your department. You can find your PhD secretary here:
www.science.ku.dk/phd/

Nina Weisse

From: Simona Rota Nodari <Simona.ROTANODARI@univ-cotedazur.fr>
Sent: 14. maj 2023 23:12
To: Marcello Porta
Cc: Søren Fournais <EKSTERN>; Nina Weisse
Subject: Re: PhD thesis of Johannes Agerskov Schiødt

Dear all,

I fully agree with the content of the report too.

Best regards,
Simona

Simona ROTA NODARI

Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur
Parc Valrose
28, avenue Valrose
06108 Nice Cedex 2
FRANCE

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Le 14 mai 2023 à 20:16, Marcello Porta <mporta@sissa.it> a écrit :

Dear Soeren, Nina and Simona,

I fully agree with the content of the report.
Thank you.

With best regards,

Marcello

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Marcello Porta

SISSA
Mathematics Area
Via Bonomea 265
34136 Trieste, Italy
<https://sites.google.com/view/marcelloporta/home>

> On 14 May 2023, at 15:49, Søren Fournais <EKSTERN> <sofo@math.ku.dk> wrote:
>

> Dear Nina,
> Please find enclosed our common pre-defense report on the thesis of Johannes Agerskov. As you will see the report recommends the thesis for defense.
>
> I have signed the (pdf-file of the) attached report. The two other members of the committee will by email confirm that this is our common report (instead of a signature).
>
> Best regards,
> Soeren Fournais
>

<3G.pdf><3G. Johannes Agerskov Schiødt Preliminary assessment of the PhD thesis .docx>

Nina Weisse

From: Marcello Porta <mporta@sissa.it>
Sent: 14. maj 2023 20:16
To: Søren Fournais <EKSTERN>
Cc: Nina Weisse; Simona Rota Nodari
Subject: Re: PhD thesis of Johannes Agerskov Schiødts
Attachments: 3G.pdf; 3G. Johannes Agerskov Schiødts Preliminary assessment of the PhD thesis .docx

Dear Soeren, Nina and Simona,

I fully agree with the content of the report.
Thank you.

With best regards,

Marcello

--

Marcello Porta

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Mathematics Area
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