Classical Dichotomy
Economics 210C

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Outline

Introduction

- So far all(?) models you have seen in the core are real.
- Today we will add money and prices to a standard business cycle model.
- The key prediction from this model is the Classical Dichotomy: money is neutral, i.e., it does not affect real variables.
- This result will be useful starting point for analyzing the determinants of prices and inflation (next class).
- We will then examine evidence for / against the Classical Dichotomy.

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Money Demand: Ideas

- Money has no nominal return. If bonds pay interest, why hold money?
  - Money provides "liquidity services."
    - ⋆ Costly and time consuming to buy and sell bonds every time you want to buy something.
    - ⋆ You also don't want to seek out someone who wants to trade for exactly what you have.
  - Money provides anonymity.
- To focus on interesting questions about how money changes economy, punt on why people hold it and just put real money balances M_t/P_t in the utility function.
  - Choose convenient isoelastic form.
  - Alternative: Cash in advance constraint or New Monetarist day/night markets.

Setup: Households

- Preferences:

$$\max_{\{C_{t+s},N_{t+s},B_{t+s},M_{t+s}\}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \zeta \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\}$$

  - Discount factor $\beta \in (0,1)$, $\rho = -\log\beta$ is the discount rate.
  - $\gamma > 0$ is CRRA, $\sigma = 1/\gamma$ is IES.
  - $\nu > 0$ determines elasticity of money demand.
  - $\varphi > 0$ where $1/\varphi$ is the Frisch elasticity of labor supply.

- Notes:
  - All that matters is U being twice continuously differentiable with $U_c > 0,\ U_{cc} < 0,\ U_m > 0,\ U_{mm} < 0,\ U_n < 0,\ U_{nn} < 0$.

Setup: Households

- Budget constraint:

$$P_t C_t + B_t + M_t \leq W_t N_t + Q_{t-1} B_{t-1} + M_{t-1} + P_t (TR_t + PR_t)$$

- Real budget constraint:

$$C_t = \frac{W_t}{P_t} N_t - \frac{B_t - Q_{t-1} B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} + TR_t + PR_t$$

  - P_t is the price of output C_t.
  - B_t is holdings of a nominal bond bought at price 1 and yielding Q_t at time $t+1$.
  - Q_t is gross nominal interest rate between periods t and $t+1$.
  - W_t is the nominal wage.
  - M_t is quantity of money households hold at end of period t.
  - TR_t are real transfers and PR_t are real rebated profits.

A Note on Timing

- Instead of giving a claim to a return on capital determined at time $t+1$, nominal bonds are coupon bonds.
  - Buy at face price of 1 at t, know that will pay Q at $t+1$.
  - Bond return between t and $t+1$ now determined at time t.
  - Consequently, gross t to $t+1$ return is denoted as Q_t.
- The (ex-post) real interest rate on the bond is not known at t:

$$R_{t+1} \equiv Q_t \frac{P_t}{P_{t+1}}$$

  - Depends on the realization of inflation at $t+1$.
  - This timing helps clarify what an "expectation at time t" means and is consistent with literature.
- Note: Different timing from Gali.

HOUSEHOLD PROBLEM: THREE FOCS

- Static FOC WRT labor:

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

- Dynamic FOC WRT B_t: Euler equation

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$$

- Dynamic FOC WRT M_t:

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} + \zeta \frac{(M_t/P_t)^{-\nu}}{C_t^{-\gamma}}$$

Intertemporal Consumption Choice

- Solve the Euler equation forward:

$$
\begin{aligned}
C_t^{-\gamma} &= E_t\left\{\beta R_{t+1} C_{t+1}^{-\gamma}\right\} \\
&= \lim_{T\to\infty} E_t \prod_{s=0}^{T} (\beta R_{t+1+s}) C_{t+1+T}^{-\gamma}
\end{aligned}
$$

- Consumption today determined by:
  - Long-run consumption $C_{t+1+T} \approx$ permanent income.
  - Intertemporal substitution through the path of real interest rates.
- Very different from old Keynesian consumption function, $c_t = k + mpc \times y_t$.
  - Old Keynesian consumption behavior remerges in Heterogeneous Agent New Keynesian (HANK) models.

The Stochastic Discount Factor

- Call

$$\Lambda_{t,t+1} \equiv \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}}$$

  the household's *stochastic discount factor*.

- Pins down economy's real interest rate as Euler is:

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \equiv E_t\{\Lambda_{t,t+1} R_{t+1}\}$$

  - Very important for asset pricing, as with different assets price is determined by covariance between return and SDF.
  - Will be implicit discount rate of firms if households own firms as this is how shareholders discount cashflows.

Bonds Vs. Money

- The bonds and money FOCs can also be written as:

$$U_{C_t} = \beta Q_t E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\}$$

$$U_{C_t} = \beta E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\} + U_{M_t/P_t}$$

- Euler: MU cost of buying ε more bonds today = discounted price-level and return adjusted benefit of having ε more bonds tomorrow.
- Money: MU cost of holding ε more money today = discounted price-level adjusted benefit of having ε more money tomorrow plus liquidity benefits of holding ε more money overnight.
- Money Demand Trade-Off: Can buy bond and get return or money and get utility benefit

Money Demand

- Combining bonds and money FOCs gives money demand:

$$\frac{M_t}{P_t} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu}$$

  - Increasing in C_t : Consume more, demand more money.
  - Decreasing in Q_t: Decreasing in opportunity cost of holding money, the nominal interest rate.
  - Often summarize as reduced-form function:

$$\frac{M_t}{P_t} = \Phi(C_t, Q_t)$$

Transversality Conditions

- TVC B_t:

$$\lim_{T\to\infty} E_t \Lambda_{t,t+T} \frac{B_T}{P_T} = 0$$

- TVC M_t:

$$\lim_{T\to\infty} E_t \Lambda_{t,t+T} \frac{M_T}{P_T} = 0$$

- In words:
  - Cannot borrow exponentially more and more to repay existing debt and finance consumption (rules out < 0).
  - Sub-optimal to save exponentially more each period (rules out > 0).

Outline

Setup: Firms

- Firms produce output Y_t CRS with labor N_t (no capital).

$$Y_t = A_t N_t$$

- Firms maximize profits:

$$\max_{N_t} PR_t = Y_t - \frac{W_t}{P_t} N_t$$

- FOC:

$$MPL_t = A_t = \frac{W_t}{P_t}$$

Outline

Setup: Government

- The government budget constraint:

$$B_t + M_t = P_t Tr_t + Q_{t-1} B_{t-1} + M_{t-1}$$

- In real terms:

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} = Tr_t + \frac{Q_{t-1} B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t}$$

- Government issues bonds or prints money to finance transfers and pay off past debt.

- Taxes are captured as negative transfers.

Outline

1. Introduction
2. Flexible Price Monetary Model
   - Households
   - Firms
   - Government
   - Markets
   - Equilibrium
3. Classical Dichotomy
4. Next Steps

Setup: Markets

- Four markets
  - Labor: $N_t^{firms} = N_t^{households}$
  - Bond: $B_t^{government} = B_t^{households}$
  - Money: $M_t^{government} = M_t^{households}$
  - Output: $Y_t = C_t$.

Outline

EQUILIBRIUM DEFINITION

An equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}, B_{t+s}\}_{s=0}^{\infty}$, a set of prices $\{W_{t+s}, P_{t+s}, Q_{t+s}\}_{s=0}^{\infty}$, an exogenous processes $\{A_{t+s}, Tr_{t+s}, M_{t+s}\}_{s=0}^{\infty}$ and initial conditions for bonds and capital B_{t-1}, K_{t-1} such that:

1. Households maximize utility subject to budget constraints.
2. Firms maximize profits given their technology.
3. The government satisfies its budget constraint.
4. Markets clear:
    - 3.1 Labor demanded equals labor supplied.
    - 3.2 Bond issuance by the government equals bond holding by households.
    - 3.3 Money issuance by the government equals money holdings by households.
    - 3.4 Output equals consumption plus investment.

Equilibrium Equations

$$Y_t = A_t N_t$$

$$\frac{W_t}{P_t} = A_t$$

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$Y_t = C_t$$

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$$

Outline

Classical Dichotomy: The Real Block

- Fist four equations in the "real block" pin down output, employment, the real wage, and consumption.

$$\text{Labor Supply: } \frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$\text{Labor Demand: } \frac{W_t}{P_t} = A_t N_t$$

and $C_t = Y_t = A_t N_t$

- Yields one equation in N_t:

$$\frac{\chi N_t^{\varphi}}{(A_t N_t)^{-\gamma}} = A_t \Rightarrow N_t = \left(\frac{1}{\chi} A_t^{1-\gamma}\right)^{\frac{1}{\varphi+\gamma}}$$

- Monetary Neutrality: Real outcomes are independent of the price level and unaffected by nominal variables.
- Monetary Neutrality implies the Classical Dichotomy: can analyze real and nominal variables independently.

How General Is The Neutrality Result?

- Look at real block imposing $Y_t = C_t$:

$$C_t = F(N_t; A_t)$$
$$\frac{W_t}{P_t} = F_N(N_t; A_t)$$
$$\frac{W_t}{P_t} = \frac{U_{N_t}}{U_{C_t}}$$

- Money cannot show up in aggregate resource constraint or production function.
- From labor supply, see key condition is *separability between money and consumption / labor in utility function.*
  - M_t/P_t does not affect U_{N_t} or U_{C_t} and thus does not affect MRS or labor supply curve.

Non-Separable Money In Utility

- We can generate non-neutrality from creating a cross-partial between M_t/P_t and N_t or C_t in the utility function.
  - See Gali book.
- I find this to be a fairly unsatisfying way to obtain non-neutrality.
  - Money in the utility function is a short cut.
  - What does it mean for liquidity services to increase when labor or consumption changes?
  - I used to think that the liquidity services effect was unimportant, but recent work has convinced me that it is important in emerging economies.
    - Chodorow-Reich, Gopinath, Mishra, Narayanan (2019) in India
    - Alvarez and Argente (2019) in Mexico

What About Capital?

- Assuming away capital is not driving neutrality.
- Non-capital equations and resource constraint:

$$
\begin{aligned}
Y_t &= F(K_{t-1}, N_t; A_t) \\
\frac{U_{N_t}}{U_{C_t}} &= F_N(K_{t-1}, N_t; A_t) \\
Y_t &= C_t + K_t - (1-\delta)K_{t-1}
\end{aligned}
$$

- Household Euler is in terms of real rate, as is firm capital FOC:

$$
\begin{aligned}
1 &= E_t\left\{\frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}} R_{t+1}\right\} \\
R_{t+1} &= F_K(K_t, N_{t+1}; A_{t+1}) + (1-\delta)
\end{aligned}
$$

- Real side pinned down independent of nominal side.

Outline

Next Steps

- For now we embrace the Classical Dichotomy and will learn how to solve the model, and analyze how prices and inflation are determined in our economy.
  - Classical Dichotomy makes our life simpler here, without meaningfully changing conclusions.
- Then we will at evidence for / against monetary neutrality.