Econ 210C Homework 5

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Due: no due date.

1. Cost-push shocks

Consider the standard new Keynesian model

$$\hat{y}_t = E_t\hat{y}_{t+1} - E_t(\hat{i}_t - \hat{\pi}_{t+1}) \tag{1}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{y}_t^{eff}) + u_t \tag{2}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t, \qquad \phi_\pi > 1 \tag{3}$$

(a) Interpret each of the equations (1)-(3) (max 2 sentence each).

(b) Assume $\hat{a}_t = 0$ and $u_t = \rho_u u_{t-1} + \epsilon_t^u$ with $\epsilon_t^u \sim N(0, \sigma^2_{\epsilon^u})$. Solve for the equilibrium levels of $\hat{y}_t$, $\hat{y}_t - \hat{y}_t^{eff}$, $\hat{\pi}_t$, $\hat{i}_t$, and $\hat{r}_t = \hat{i}_t - E_t\hat{\pi}_{t+1}$ as a function of u_t.

(c) Modify the Jupyter notebook "newkeynesianlinear.ipynb" to verify that your solution in (b) are correct.

(d) Explain intuitively how a supply shock affects the output gap, inflation, the nominal interest rate, and the real interest rate. (4 sentences should suffice.)

(e) Use your solution to express the loss function $L = \vartheta var(\hat{y}_t - \hat{y}_t^{eff}) + var(\hat{\pi}_t)$ as a function of the model parameters, where $var(\hat{y}_t - \hat{y}_t^{eff})$ is the variance of the output gap and $var(\hat{\pi}_t)$ is the variance of inflation.

(f) Show that the optimal interest rate rule satisfies $\phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$.

(g) Using the optimal ϕ_π, show that $\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta(1-\beta\rho_u)}\hat{\pi}_t$.

(h) The optimal monetary policy under discretion is $\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta}\hat{\pi}_t$. Does the optimal ϕ_π deliver a better, a worse, or the same loss? Explain intuitively. (No derivation should be necessary.)

2. Estimating an Interest Rate Rule

(a) Download quarterly data from Fred for CPI inflation rate, the output gap, and the Federal Funds Rate from 1985Q1 to 2007Q4. Plot each data series.

(b) Estimate the Interest Rate Rule via OLS:

$$i_t = \alpha + \rho_i i_{t-1} + \phi_\pi\pi_t + \phi_y\tilde{y}_t + v_t$$

and report your estimates.

(c) Explain why the OLS estimates are likely biased.

(d) In which direction does the bias go?

3. Estimating an Interest Rate Rule in the NK model

Implement the linearized new Keynesian model in a Jupyter notebook.

$$
\begin{aligned}
\tilde{y}_t &= -\sigma\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) + E_t\{\tilde{y}_{t+1}\} \\
\hat{\pi}_t &= \kappa\tilde{y}_t + \beta E_t\{\hat{\pi}_{t+1}\} + \hat{u}_t \\
\hat{i}_t &= \phi_\pi\hat{\pi}_t + \bar{i}_t \\
\bar{i}_t &= \rho_i\bar{i}_{t-1} + \epsilon_t^i \\
\hat{u}_t &= \rho_u\hat{u}_{t-1} + \epsilon_t^u
\end{aligned}
$$

Parameters: $\sigma = 1, \kappa = 0.03, \beta = 0.99, \rho_i = 0.8, \rho_u = 0.8$, and use the interest rate rule parameters you estimated in the previous question. Set the standard deviation of the monetary shock to 0.1 and that of the cost-push shock to 0.01.

(a) Plot the IRFs.

(b) Intuitively explain how the shocks ϵ_t^r and ϵ_t^i affect $\tilde{y}_t, \tilde{\pi}_t, \hat{i}_t$.

(c) Simulate a time series of length 1000 and plot it.

(d) Estimate the interest rate rule on your simulated data.

(e) Explain why this procedure does not recovery the true parameters.

(f) Would you be able to identify the true parameters using a Cholesky decomposition? Explain why or why not.

(g) What data do you need to identify the parameters of the interest rate rule?

(h) Implement your proposed approach and show that it correctly recovers ϕ_π in the model.

(i) Suppose the true interest rate rule was $\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t + \epsilon_t^i$. Would you be able to identify both ϕ_π and ϕ_y using your procedure? Explain.