

CLASSICAL DICHOTOMY

ECONOMICS 210C

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OUTLINE

1 INTRODUCTION

2 FLEXIBLE PRICE MONETARY MODEL

- Households
- Firms
- Government
- Markets
- Equilibrium

3 CLASSICAL DICHOTOMY

4 NEXT STEPS

INTRODUCTION

- So far all(?) models you have seen in the core are real.
- Today we will add money and prices to a standard business cycle model.
- The key prediction from this model is the Classical Dichotomy: money is neutral, i.e., it does not affect real variables.
- This result will be useful starting point for analyzing the determinants of prices and inflation (next class).
- We will then examine evidence for / against the Classical Dichotomy.

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MONEY DEMAND: IDEAS

- Money has no nominal return. If bonds pay interest, why hold money?
 - ▶ Money provides “liquidity services.”
 - ★ Costly and time consuming to buy and sell bonds every time you want to buy something.
 - ★ You also don't want to seek out someone who wants to trade for exactly what you have.
 - ▶ Money provides anonymity.
- To focus on interesting questions about how money changes economy, punt on why people hold it and just put real money balances M_t/P_t in the utility function.
 - ▶ Choose convenient isoelastic form.
 - ▶ Alternative: Cash in advance constraint or New Monetarist day/night markets.

SETUP: HOUSEHOLDS

- Preferences:

$$\max_{\{C_{t+s}, N_{t+s}, B_{t+s}, M_{t+s}\}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \zeta \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\phi}}{1+\phi} \right) \right\}$$

- ▶ Discount factor $\beta \in (0,1)$, $\rho = -\log \beta$ is the discount rate.
- ▶ $\gamma > 0$ is CRRA, $\sigma = 1/\gamma$ is IES.
- ▶ $\nu > 0$ determines elasticity of money demand.
- ▶ $\phi > 0$ where $1/\phi$ is the Frisch elasticity of labor supply.

- Notes:

- ▶ All that matters is U being twice continuously differentiable with $U_c > 0$, $U_{cc} < 0$, $U_m > 0$, $U_{mm} < 0$, $U_n < 0$, $U_{nn} < 0$.

SETUP: HOUSEHOLDS

- Budget constraint:

$$P_t C_t + B_t + M_t \leq W_t N_t + Q_{t-1} B_{t-1} + M_{t-1} + P_t (TR_t + PR_t)$$

- Real budget constraint:

$$C_t = \frac{W_t}{P_t} N_t - \frac{B_t - Q_{t-1} B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} + TR_t + PR_t$$

- ▶ P_t is the price of output C_t .
- ▶ B_t is holdings of a nominal bond bought at price 1 and yielding Q_t at time $t+1$.
- ▶ Q_t is gross nominal interest rate between periods t and $t+1$.
- ▶ W_t is the nominal wage.
- ▶ M_t is quantity of money households hold at end of period t .
- ▶ TR_t are real transfers and PR_t are real rebated profits.

A NOTE ON TIMING

- Instead of giving a claim to a return on capital determined at time $t+1$, nominal bonds are coupon bonds.
 - ▶ Buy at face price of 1 at t , know that will pay Q at $t+1$.
 - ▶ Bond return between t and $t+1$ now determined at time t .
 - ▶ Consequently, gross t to $t+1$ return is denoted as Q_t .
- The (ex-post) real interest rate on the bond is not known at t :

$$R_{t+1} \equiv Q_t \frac{P_t}{P_{t+1}}$$

- ▶ Depends on the realization of inflation at $t+1$.
 - ▶ This timing helps clarify what an “expectation at time t ” means and is consistent with literature.
- Note: Different timing from Gali.

HOUSEHOLD PROBLEM: THREE FOCs

- Static FOC WRT labor:

$$\frac{W_t}{P_t} = \frac{\chi N_t^\varphi}{C_t^{-\gamma}}$$

- Dynamic FOC WRT B_t : Euler equation

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$$

- Dynamic FOC WRT M_t :

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} + \zeta \frac{(M_t/P_t)^{-\nu}}{C_t^{-\gamma}}$$

INTERTEMPORAL CONSUMPTION CHOICE

- Solve the Euler equation forward:

$$\begin{aligned}C_t^{-\gamma} &= E_t \left\{ \beta R_{t+1} C_{t+1}^{-\gamma} \right\} \\&= \lim_{T \rightarrow \infty} E_t \prod_{s=0}^T (\beta R_{t+1+s}) C_{t+1+T}^{-\gamma}\end{aligned}$$

- Consumption today determined by:
 - ▶ Long-run consumption $C_{t+1+T} \approx$ permanent income.
 - ▶ Intertemporal substitution through the path of real interest rates.
- Very different from old Keynesian consumption function,
 $c_t = k + mpc \times y_t$.
 - ▶ Old Keynesian consumption behavior reemerges in Heterogeneous Agent New Keynesian (HANK) models.

THE STOCHASTIC DISCOUNT FACTOR

- Call

$$\Lambda_{t,t+1} \equiv \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}}$$

the household's *stochastic discount factor*.

- Pins down economy's real interest rate as Euler is:

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \equiv E_t \{ \Lambda_{t,t+1} R_{t+1} \}$$

- ▶ Very important for asset pricing, as with different assets price is determined by covariance between return and SDF.
- ▶ Will be implicit discount rate of firms if households own firms as this is how shareholders discount cashflows.

BONDS VS. MONEY

- The bonds and money FOCs can also be written as:

$$U_{C_t} = \beta Q_t E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\}$$

$$U_{C_t} = \beta E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\} + U_{M_t/P_t}$$

- Euler: MU cost of buying ε more bonds today = discounted price-level and return adjusted benefit of having ε more bonds tomorrow.
- Money: MU cost of holding ε more money today = discounted price-level adjusted benefit of having ε more money tomorrow plus liquidity benefits of holding ε more money overnight.
- Money Demand Trade-Off: Can buy bond and get return or money and get utility benefit

MONEY DEMAND

- Combining bonds and money FOCs gives money demand:

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu}$$

- Increasing in C_t : Consume more, demand more money.
- Decreasing in Q_t : Decreasing in opportunity cost of holding money, the nominal interest rate.
- Often summarize as reduced-form function:

$$\frac{M_t}{P_t} = \Phi(C_t, Q_t)$$

TRANSVERSALITY CONDITIONS

- TVC B_t :

$$\lim_{T \rightarrow \infty} E_t \Lambda_{t,t+T} \frac{B_T}{P_T} = 0$$

- TVC M_t :

$$\lim_{T \rightarrow \infty} E_t \Lambda_{t,t+T} \frac{M_T}{P_T} = 0$$

- In words:

- ▶ Cannot borrow exponentially more and more to repay existing debt and finance consumption (rules out < 0).
- ▶ Sub-optimal to save exponentially more each period (rules out > 0).

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SETUP: FIRMS

- Firms produce output Y_t CRS with labor N_t (no capital).

$$Y_t = A_t N_t$$

- Firms maximize profits:

$$\max_{N_t} PR_t = Y_t - \frac{W_t}{P_t} N_t$$

- FOC:

$$MPL_t = A_t = \frac{W_t}{P_t}$$

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SETUP: GOVERNMENT

- The government budget constraint:

$$B_t + M_t = P_t Tr_t + Q_{t-1} B_{t-1} + M_{t-1}$$

- In real terms:

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} = Tr_t + \frac{Q_{t-1} B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t}$$

- Government issues bonds or prints money to finance transfers and pay off past debt.
- Taxes are captured as negative transfers.

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SETUP: MARKETS

- Four markets

- ▶ Labor: $N_t^{firms} = N_t^{households}$
- ▶ Bond: $B_t^{government} = B_t^{households}$
- ▶ Money: $M_t^{government} = M_t^{households}$
- ▶ Output: $Y_t = C_t$.

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EQUILIBRIUM DEFINITION

An equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}, B_{t+s}\}_{s=0}^{\infty}$, a set of prices $\{W_{t+s}, P_{t+s}, Q_{t+s}\}_{s=0}^{\infty}$, an exogenous processes $\{A_{t+s}, Tr_{t+s}, M_{t+s}\}_{s=0}^{\infty}$ and initial conditions for bonds and capital B_{t-1}, K_{t-1} such that:

1. Households maximize utility subject to budget constraints.
2. Firms maximize profits given their technology.
3. The government satisfies its budget constraint.
4. Markets clear:
 1. Labor demanded equals labor supplied.
 2. Bond issuance by the government equals bond holding by households.
 3. Money issuance by the government equals money holdings by households.
 4. Output equals consumption plus investment.

EQUILIBRIUM EQUATIONS

$$Y_t = A_t N_t$$

$$\frac{W_t}{P_t} = A_t$$

$$\frac{W_t}{P_t} = \frac{\chi N_t^\phi}{C_t^{-\gamma}}$$

$$Y_t = C_t$$

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$$

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CLASSICAL DICHOTOMY: THE REAL BLOCK

- First four equations in the “real block” pin down output, employment, the real wage, and consumption.

$$\text{Labor Supply: } \frac{W_t}{P_t} = \frac{\chi N_t^\varphi}{C_t^{-\gamma}}$$

$$\text{Labor Demand: } \frac{W_t}{P_t} = A_t N_t$$

$$\text{and } C_t = Y_t = A_t N_t$$

- Yields one equation in N_t :

$$\frac{\chi N_t^\varphi}{(A_t N_t)^{-\gamma}} = A_t \Rightarrow N_t = \left(\frac{1}{\chi} A_t^{1-\gamma} \right)^{\frac{1}{\varphi+\gamma}}$$

- Monetary Neutrality: Real outcomes are independent of the price level and unaffected by nominal variables.
- Monetary Neutrality implies the Classical Dichotomy: can analyze real and nominal variables independently.

HOW GENERAL IS THE NEUTRALITY RESULT?

- Look at real block imposing $Y_t = C_t$:

$$C_t = F(N_t; A_t)$$

$$\frac{W_t}{P_t} = F_N(N_t; A_t)$$

$$\frac{W_t}{P_t} = \frac{U_{N_t}}{U_{C_t}}$$

- Money cannot show up in aggregate resource constraint or production function.
- From labor supply, see key condition is *separability between money and consumption / labor in utility function*.
 - ▶ M_t/P_t does not affect U_{N_t} or U_{C_t} and thus does not affect MRS or labor supply curve.

NON-SEPARABLE MONEY IN UTILITY

- We can generate non-neutrality from creating a cross-partial between M_t/P_t and N_t or C_t in the utility function.
 - ▶ See Gali book.
- I find this to be a fairly unsatisfying way to obtain non-neutrality.
 - ▶ Money in the utility function is a short cut.
 - ▶ What does it mean for liquidity services to increase when labor or consumption changes?
 - ▶ I used to think that the liquidity services effect was unimportant, but recent work has convinced me that it is important in emerging economies.
 - ★ Chodorow-Reich, Gopinath, Mishra, Narayanan (2019) in India
 - ★ Alvarez and Argente (2019) in Mexico

WHAT ABOUT CAPITAL?

- Assuming away capital is not driving neutrality.
- Non-capital equations and resource constraint:

$$\begin{aligned}Y_t &= F(K_{t-1}, N_t; A_t) \\ \frac{U_{N_t}}{U_{C_t}} &= F_N(K_{t-1}, N_t; A_t) \\ Y_t &= C_t + K_t - (1 - \delta)K_{t-1}\end{aligned}$$

- Household Euler is in terms of real rate, as is firm capital FOC:

$$\begin{aligned}1 &= E_t \left\{ \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}} R_{t+1} \right\} \\ R_{t+1} &= F_K(K_t, N_{t+1}; A_{t+1}) + (1 - \delta)\end{aligned}$$

- Real side pinned down independent of nominal side.

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NEXT STEPS

- For now we embrace the Classical Dichotomy and will analyze how prices and inflation are determined in our economy.
 - ▶ Classical Dichotomy makes our life simpler here, without meaningfully changing conclusions.
- Then we will at evidence for / against monetary neutrality.