Optimal Monetary Policy in the NK Model: Discretionary Policy

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Outline

The Three Equation Model

- For the next two classes, we will examine optimal policy in the simple three-equation NK model we have developed:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$

- Goal: Understand key principles of conduct of monetary policy.
- Because both equations are forward looking, a key question is whether the central bank can affect expectations by committing ex-ante to policy rules that they may have an ex-post incentive to break.
  - Today: Discretionary policy (no credibility to commit to rules).
  - Next Class: ZLB constraint $i_t \geq 0$ and benefits of commitment.

Outline

The Costs of Inflation

- People generally do not like high inflation.
- Many reasons why. Some commonly cited include:
  - Reduces real income.
  - "Shoe leather" costs.
  - Menu costs.
  - Inefficient allocation of resources.
  - Redistributes nominal debt from creditors to debtors.
  - Uncertainty for household and business financial planning.
  - General nuisance.

Welfare In the NK Model

- In the NK model, there are two sources of distortion.
  1. Markups: Lead to inefficiently low Y_t and N_t.
  2. Sticky Prices:
     - 2.1 Average markup varies over time and not equal to desired markup.
     - 2.2 With staggered pricing, inefficient price dispersion as prices vary in ways unwarranted by preferences and tech.

- See this in output:

$$Y_t = A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - Second order loss, so not in log-linearization.

The Central Bank Objective Function

- Take "cashless limit" as $\zeta \to 0$ (recall ζ is weight on real money balances).
  - Money in utility is shortcut; do not want in planner's problem.
- Assume steady state markup distortion is corrected with subsidy scheme.
- Take second-order approximation to losses of representative household and use first-order model approximation to find losses are proportional to:

$$\frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\vartheta (\hat{y}_t - \hat{y}_t^{flex})^2 + \hat{\pi}_{t+s}^2 \right] \right\}$$

  - See Gali Appendix 4.1.
  - Inflation comes from price dispersion cost of inflation.
  - But setup accommodates other costs of inflation if ϑ is generalized weight on output gap relative to inflation.

The Planning Problem

$$\min \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\vartheta (\hat{y}_{t+s} - \hat{y}_{t+s}^{flex})^2 + \hat{\pi}_{t+s}^2 \right] \right\}$$

subject to

$$\hat{y}_t = -\sigma E_t\{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$

The Planning Problem: Strategy

- Split problem into two parts:
- First, solve for $\{\hat{y}_{t+s}, \hat{\pi}_{t+s}\}_{s=0}^{\infty}$

$$\min \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\vartheta (\hat{y}_{t+s} - \hat{y}_{t+s}^{flex})^2 + \hat{\pi}_{t+s}^2 \right] \right\}$$

subject to

$$\hat{\pi}_t = \kappa (\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t \{\hat{\pi}_{t+1}\}$$

- Second, given $\{\hat{y}_{t+s}, \hat{\pi}_{t+s}\}_{s=0}^{\infty}$ find $\{\hat{i}_{t+s}\}_{s=0}^{\infty}$ that satisfies:

$$\hat{y}_t = -\sigma E_t \{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t \{\hat{y}_{t+1}\}$$

- Works because absent ZLB constraint, can always find such an $\{\hat{i}_{t+s}\}_{s=0}^{\infty}$.

The Divine Coincidence

- The solution is simple:

$$\hat{y}_t = \hat{y}_t^{flex}$$
$$\hat{\pi}_t = 0$$
$$\hat{i}_t = \frac{1}{\sigma} E_t\{\hat{y}_{t+1}^{flex} - \hat{y}_t^{flex}\} \equiv E_t \hat{r}_{t+1}^n$$

- $E_t \hat{r}_{t+1}^n$ is the *natural rate of interest*.
  - The real interest rate that prevails when prices are flexible.
- Lessons for optimal policy:
  - The optimal policy *stabilizes the output gap*, not output.
  - The central bank faces *no tradeoff between stabilizing output gap and inflation.* This is known as the "divine coincidence."
  - To implement the optimal policy, the nominal interest rate should track the natural rate of interest.
  - Offsets demand shocks, accommodates supply shocks.

The Divine Coincidence

- Intuition: Zero inflation simultaneously avoids relative price distortions and gives flexible price allocation, which is optimal.
- This does not rely on particulars of Calvo. General argument:
  - Flexible price equilibrium is optimal.
  - Nominal rigidities are the constraint on firms and only source of distortions.
  - No distortion if constraint does not bind, which occurs with zero inflation.
  - So price stability implies output gap stability.

Outline

Creating an Inflation-Output Tradeoff

- The divine coincidence is not very realistic.
  - Central banks see themselves as facing significant tradeoffs between stabilizing output and inflation in the short run.
- Intuition of divine coincidence suggests that adding other frictions will eliminate it.
  - Stabilizing inflation does not remove only friction in model and stabilize output.

Time-Varying Efficient Output Gap

- In our model so far, the outcome under flexible prices, $\hat{y}_t^{flex}$, is also the efficient (first-best) outcome $\hat{y}_t^{eff}$.
- The central bank will face a trade-off and the divine coincidence will break once these are no longer the same.
- The welfare function will now penalize deviations of output from the efficient level of output $\hat{y}_t - \hat{y}_t^{eff}$. The Phillips Curve is:

$$\begin{aligned}
\hat{\pi}_t &= \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} \underbrace{-\kappa(\hat{y}_t^{flex} - \hat{y}_t^{eff})}_{\equiv u_t} \\
&= \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + u_t
\end{aligned}$$

- We call u_t a "cost-push shock."
  - Exogenous increase in marginal costs.

Cost Push and the Labor Wedge

- What are cost push shocks?
  - Anything that moves the labor wedge beyond sticky prices.
  - From medium-scale NK models, accounts for a lot of inflation.
- Let μ_t^W be the log of a time-varying exogenous wage markup:

$$\hat{w}_t - \hat{p}_t = \hat{\mu}_t^W + \varphi \hat{n}_t + \gamma \hat{c}_t$$

- Then the Phillips curve becomes:

$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + \beta E_t\{\hat{\pi}_{t+1}\} + \lambda \hat{\mu}_t^W$$

- Intuition:
  - Higher mark-ups mean higher inflation and lower output.
  - Central bank wants to offset this inefficient shock, but can only move output and inflation in the same direction.

The Planning Problem With A Tradeoff

$$\frac{1}{2}E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\vartheta(\hat{y}_{t+s}-\hat{y}_{t+s}^{eff})^2+\hat{\pi}_{t+s}^2\right]\right\}$$

subject to

$$\hat{y}_t=-\sigma E_t\{\hat{i}_t-\hat{\pi}_{t+1}\}+E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t=\kappa(\hat{y}_t-\hat{y}_t^{eff})+\beta E_t\{\hat{\pi}_{t+1}\}+u_t$$

- First stage is optimizing objective function subject to NKPC.
  - Cost push shock increases $\hat{\pi}_t$.
  - To offset it, can push down output relative to efficient output $\hat{y}_t-\hat{y}_t^{eff}$.
  - Thus there is now a tradeoff.
  - Intuitively, monetary policy shifts aggregate demand but u_t is a shock to aggregate supply, so there is a tradeoff.

The Planning Problem: Rules vs. Discretion

- Standard quadratic loss function with linear constraints.
  - But targets depend on expectations of future policy.
  - To see this, iterate forward to get:

$$\hat{\pi}_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s [\kappa(\hat{y}_{t+s} - \hat{y}^{eff}_{t+s}) + u_{t+s}] \right\}$$

$$\hat{y}_t = E_t \left\{ \sum_{s=0}^{\infty} \left[-\sigma(\hat{i}_{t+s} - \hat{\pi}_{t+s+1}) + g_{t+s} \right] \right\}$$

- This raises issues of credibility and time consistency of policy.
  - Central bank can influence outcomes today by "promising" outcomes tomorrow.
  - But are those promises credible?
  - See Gali 5.3 for the solution to the commitment case. Commitment will become important when we study the ZLB.

The Discretionary Problem

- For today, we assume that the central bank follows a *discretionary optimal policy*.
  - Cannot make credible commitments about future actions (will talk about why next class).
  - So optimize *taking expectations of future actions as given*.
- Solve

$$\min_{\hat{\pi}_t, \hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

where

$$F_t = \frac{1}{2} E_t \left\{ \sum_{s=1}^{\infty} \beta^s \left[\vartheta(\hat{y}_{t+s} - \hat{y}_{t+s}^{eff})^2 + \hat{\pi}_{t+s}^2 \right] \right\}$$

$$f_t = \beta \hat{\pi}_{t+1} + u_t$$

are functions of expectations of future actions.

"Lean Against the Wind" Policy

$$\min_{\hat{\pi}_t, \hat{y}_t} \frac{1}{2}[\vartheta(\hat{y}_t - \hat{y}_t^{eff})^2 + \hat{\pi}_t^2] + F_t \text{ s.t. } \hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{eff}) + f_t$$

- The First order condition is

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta}\hat{\pi}_t$$

- "Lean Against The Wind" Policy.
  - In face of inflationary pressures from cost push shocks, *drive output below its efficient level to dampen rise in inflation*.
  - Extent to which it does so depends on:
    - κ, which determines reduced inflation per unit of output loss.
    - ϑ, the relative weight placed on output loss.
- Flip from "Old Keynesian" logic where stabilizing output at cost of inflation.

Inflation and Output Under Discretion

- Plug policy into Phillips curve:

$$\hat{\pi}_t = \frac{\vartheta\beta}{\vartheta+\kappa^2} E_t\hat{\pi}_{t+1} + \frac{\vartheta}{\vartheta+\kappa^2} u_t$$

- And iterate forward to get:

$$\hat{\pi}_t = \frac{\vartheta}{\vartheta(1-\beta\rho_u)+\kappa^2} u_t$$

- Combine with optimality condition to get

$$\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta(1-\beta\rho_u)+\kappa^2} u_t$$

- So central bank lets output gap and inflation fluctuate in proportion to current value of cost push shock.
  - Intuition: Cost push increases inflation, central bank wants to smooth both so trades some inflation for output.

Interest Rate Under Discretion

- Plugging into dynamic IS:

$$\hat{y}_t = -\sigma E_t\{\hat{i}_t - \hat{\pi}_{t+1}\} + E_t\{\hat{y}_{t+1}\} + g_t$$

obtains

$$\hat{i}_t = E_t\{\hat{r}^e_{t+1}\} + \phi_\pi \hat{\pi}_t + \frac{g_t}{\sigma}$$

where

$$\hat{r}^e_{t+1} = \frac{1}{\sigma} E_t\{\hat{y}^{eff}_{t+1} - \hat{y}^{eff}_t\}, \qquad \phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\sigma[\vartheta(1-\beta\rho_u) + \kappa^2]}$$

- Central bank implements the optimal outcome with what looks like an interest rate (Taylor) rule.

Outline

Monetary Policy In Practice

- How does the Fed know everything it needs to know to set:

$$\hat{i}_t = E_t\{\hat{r}^e_{t+1}\} + \phi_\pi \hat{\pi}_t + \frac{g_t}{\sigma}$$

  - It doesn't!
- Because of quadratic loss function and linear constraints, *everything holds in certainty equivalents*.
  - Fed should do the best it can using available information.
  - As long as on average correct, pursue optimal policy as if certain of $E_t\{\hat{r}^e_{t+1}\}$, π_t, g_t, etc.
  - Welfare is lower than case in which central bank knows variables, but cannot do better *ex ante*.
- However, motivates policy instrument choice.
  - What matters for economy is i_t not money supply.
  - Use i_t as policy instrument so shocks to money demand do not cause fluctuations.

What About If Policy Has Lagged Effect?

- In data, we found that monetary policy has lagged effect on real economy, and even longer lag on inflation.
  - Suppose takes j periods for shift in interest rate to affect output and another k periods to impact inflation.
  - Let Fed's information set be Ω_t. Assume private agents have same info.
- Then

$$E_t\{\hat{y}_{t+j} - \hat{y}^{eff}_{t+j} | \Omega_t\} = -\frac{\kappa}{\vartheta} E_t\{\hat{\pi}_{t+j+k} | \Omega_t\}$$

  - Generalization of previous result: Certainty equivalence still holds, just now j and $j+k$ periods ahead.
- Similar results for inertial inflation.

Policy Persistence and Parameter Uncertainty

- What about if the imperfect information is not about the state of the economy, but the parameters of the model?
- Assume $\kappa_t = \kappa + \hat{\kappa}_t$ and $\sigma_t = \sigma + \hat{\sigma}_t$ where $\hat{\kappa}_t, \hat{\sigma}_t$ are iid normal random variables of mean zero and known variance $\sigma_\kappa^2, \sigma_\sigma^2$.
- Can show (with robust control):

$$\hat{y}_t - \hat{y}_t^{eff} = \frac{-\kappa}{\vartheta + \kappa^2 \sigma_\kappa^2} \hat{\pi}_t + (\vartheta + \kappa^2) \frac{\sigma_\sigma^2}{\sigma} (\hat{i}_t - E_t\{\hat{\pi}_{t+1}\})$$

- Parameter uncertainty reduces the response of the policy instrument (relative to $\hat{x}_t = -\frac{\kappa}{\vartheta}\hat{\pi}_t$), motivating a smoother path for the interest rate.
  - Contraction of output below potential raises variability of inflation, so moderate doing this.
  - Also adjusting interest rate raises variability of output, moderating extent to which it is adjusted.

Conduct of Monetary Policy in Practice

- If look at central bank statements, can see them interpreting economic indicators through lens of model.
  - Gauging inflationary pressures relative to output to get sense of whether facing tech shock, demand shock, or cost push shock.
  - Indicating they will respond aggressively to inflation when it arises. (At least until recently.)
  - Referencing dual concern for inflation and output (in U.S., full employment by law).
  - Adjusting things slowly given uncertainty about how economy will respond.

Example: January 30, 2019 Statement

Information received since the Federal Open Market Committee met in December indicates that the labor market has continued to strengthen and that economic activity has been rising at a solid rate. Job gains have been strong, on average, in recent months, and the unemployment rate has remained low. Household spending has continued to grow strongly, while growth of business fixed investment has moderated from its rapid pace earlier last year. On a 12-month basis, both overall inflation and inflation for items other than food and energy remain near 2 percent. Although market-based measures of inflation compensation have moved lower in recent months, survey-based measures of longer-term inflation expectations are little changed.

Example: January 30, 2019 Statement

Consistent with its statutory mandate, the Committee seeks to foster maximum employment and price stability. In support of these goals, the Committee decided to maintain the target range for the federal funds rate at 2-1/4 to 2-1/2 percent. The Committee continues to view sustained expansion of economic activity, strong labor market conditions, and inflation near the Committee's symmetric 2 percent objective as the most likely outcomes. In light of global economic and financial developments and muted inflation pressures, the Committee will be patient as it determines what future adjustments to the target range for the federal funds rate may be appropriate to support these outcomes.

Example: January 30, 2019 Statement

In determining the timing and size of future adjustments to the target range for the federal funds rate, the Committee will assess realized and expected economic conditions relative to its maximum employment objective and its symmetric 2 percent inflation objective. This assessment will take into account a wide range of information, including measures of labor market conditions, indicators of inflation pressures and inflation expectations, and readings on financial and international developments.

Example: March 3, 2020 Statement

The fundamentals of the U.S. economy remain strong. However, the coronavirus poses evolving risks to economic activity. In light of these risks and in support of achieving its maximum employment and price stability goals, the Federal Open Market Committee decided today to lower the target range for the federal funds rate by 1/2 percentage point, to 1 to 1-1/4 percent. The Committee is closely monitoring developments and their implications for the economic outlook and will use its tools and act as appropriate to support the economy.

Example: March 15, 2020 Statement

The coronavirus outbreak has harmed communities and disrupted economic activity in many countries, including the United States. Global financial conditions have also been significantly affected. Available economic data show that the U.S. economy came into this challenging period on a strong footing. Information received since the Federal Open Market Committee met in January indicates that the labor market remained strong through February and economic activity rose at a moderate rate. Job gains have been solid, on average, in recent months, and the unemployment rate has remained low. Although household spending rose at a moderate pace, business fixed investment and exports remained weak. More recently, the energy sector has come under stress. On a 12-month basis, overall inflation and inflation for items other than food and energy are running below 2 percent. Market-based measures of inflation compensation have declined; survey-based measures of longer-term inflation expectations are little changed.

Example: March 15, 2020 Statement

Consistent with its statutory mandate, the Committee seeks to foster maximum employment and price stability. The effects of the coronavirus will weigh on economic activity in the near term and pose risks to the economic outlook. In light of these developments, the Committee decided to lower the target range for the federal funds rate to 0 to 1/4 percent. The Committee expects to maintain this target range until it is confident that the economy has weathered recent events and is on track to achieve its maximum employment and price stability goals. This action will help support economic activity, strong labor market conditions, and inflation returning to the Committee's symmetric 2 percent objective.

Example: September 22, 2021 Statement

The Committee seeks to achieve maximum employment and inflation at the rate of 2 percent over the longer run. With inflation having run persistently below this longer-run goal, the Committee will aim to achieve inflation moderately above 2 percent for some time so that inflation averages 2 percent over time and longer-term inflation expectations remain well anchored at 2 percent. The Committee expects to maintain an accommodative stance of monetary policy until these outcomes are achieved. The Committee decided to keep the target range for the federal funds rate at 0 to 1/4 percent and expects it will be appropriate to maintain this target range until labor market conditions have reached levels consistent with the Committee's assessments of maximum employment and inflation has risen to 2 percent and is on track to moderately exceed 2 percent for some time.

Example: May 4, 2022 Statement

The Committee seeks to achieve maximum employment and inflation at the rate of 2 percent over the longer run. With appropriate firming in the stance of monetary policy, the Committee expects inflation to return to its 2 percent objective and the labor market to remain strong. In support of these goals, the Committee decided to raise the target range for the federal funds rate to 3/4 to 1 percent and anticipates that ongoing increases in the target range will be appropriate. In addition, the Committee decided to begin reducing its holdings of Treasury securities and agency debt and agency mortgage-backed securities on June 1, as described in the Plans for Reducing the Size of the Federal Reserve's Balance Sheet that were issued in conjunction with this statement.

Example: September 21, 2022 Statement

The Committee seeks to achieve maximum employment and inflation at the rate of 2 percent over the longer run. In support of these goals, the Committee decided to raise the target range for the federal funds rate to 3 to 3-1/4 percent and anticipates that ongoing increases in the target range will be appropriate. In addition, the Committee will continue reducing its holdings of Treasury securities and agency debt and agency mortgage-backed securities, as described in the Plans for Reducing the Size of the Federal Reserve's Balance Sheet that were issued in May. The Committee is strongly committed to returning inflation to its 2 percent objective.