

Econ 210C Homework 5

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Due: no due date.

1. Cost-push shocks

Consider the standard new Keynesian model

$$\hat{y}_t = E_t \hat{y}_{t+1} - E_t(\hat{i}_t - \hat{\pi}_{t+1}) \quad (1)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{y}_t^{eff}) + u_t \quad (2)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t, \quad \phi_\pi > 1 \quad (3)$$

- (a) Interpret each of the equations (1)-(3) (max 2 sentence each).
- (b) Assume $\hat{a}_t = 0$ and $u_t = \rho_u u_{t-1} + \epsilon_t^u$ with $\epsilon_t^u \sim N(0, \sigma_{\epsilon^u}^2)$. Solve for the equilibrium levels of \hat{y}_t , $\hat{y}_t - \hat{y}_t^{eff}$, $\hat{\pi}_t$, \hat{i}_t , and $\hat{r}_t = \hat{i}_t - E_t \hat{\pi}_{t+1}$ as a function of u_t .
- (c) Modify the Jupyter notebook “newkeynesianlinear.ipynb” to verify that your solution in (b) are correct.
- (d) Explain intuitively how a supply shock affects the output gap, inflation, the nominal interest rate, and the real interest rate. (4 sentences should suffice.)
- (e) Use your solution to express the loss function $L = \vartheta \text{var}(\hat{y}_t - \hat{y}_t^{eff}) + \text{var}(\hat{\pi}_t)$ as a function of the model parameters, where $\text{var}(\hat{y}_t - \hat{y}_t^{eff})$ is the variance of the output gap and $\text{var}(\hat{\pi}_t)$ is the variance of inflation.
- (f) Show that the optimal interest rate rule satisfies $\phi_\pi = \rho_u + \frac{\kappa(1-\rho_u)}{\vartheta(1-\beta\rho_u)}$.
- (g) Using the optimal ϕ_π , show that $\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta(1-\beta\rho_u)} \hat{\pi}_t$.
- (h) The optimal monetary policy under discretion is $\hat{y}_t - \hat{y}_t^{eff} = -\frac{\kappa}{\vartheta} \hat{\pi}_t$. Does the optimal ϕ_π deliver a better, a worse, or the same loss? Explain intuitively. (No derivation should be necessary.)

2. Estimating an Interest Rate Rule

- (a) Download quarterly data from Fred for CPI inflation rate, the output gap, and the Federal Funds Rate from 1985Q1 to 2007Q4. Plot each data series.
- (b) Estimate the Interest Rate Rule via OLS:

$$\hat{i}_t = \alpha + \rho_i \hat{i}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t$$

and report your estimates.

- (c) Explain why the OLS estimates are likely biased.
- (d) In which direction does the bias go?

3. Estimating an Interest Rate Rule in the NK model

Implement the linearized new Keynesian model in a Jupyter notebook.

$$\tilde{y}_t = -\sigma \left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\} \right) + E_t\{\tilde{y}_{t+1}\}$$

$$\hat{\pi}_t = \beta E_t\{\hat{\pi}_{t+1}\} + \kappa \hat{y}_t + \hat{u}_t$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \bar{i}_t$$

$$\bar{i}_t = \rho_i \bar{i}_{t-1} + \epsilon_t^i$$

$$\hat{u}_t = \rho_u \hat{u}_{t-1} + \epsilon_t^u$$

Parameters: $\sigma = 1, \kappa = 0.03, \beta = 0.99, \rho_i = 0.8, \rho_u = 0.8$, and use the interest rate rule parameters you estimated in the previous question. Set the standard deviation of the monetary shock to 0.1 and that of the cost-push shock to 0.01.

- (a) Plot the IRFs.
- (b) Intuitively explain how the shocks ϵ_t^r and ϵ_t^i affect $\hat{y}_t, \hat{\pi}_t, \hat{i}_t$.
- (c) Simulate a time series of length 1000 and plot it.
- (d) Estimate the interest rate rule on your simulated data.
- (e) Explain why this procedure does not recovery the true parameters.
- (f) Would you be able to identify the true parameters using a Cholesky decomposition? Explain why or why not.
- (g) What data do you need to identify the parameters of the interest rate rule?
- (h) Implement your proposed approach and show that it correctly recovers ϕ_π in the model.
- (i) Suppose the true interest rate rule was $\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \epsilon_t^i$. Would you be able to identify both ϕ_π and ϕ_y using your procedure? Explain.