Econ 210C Homework 4

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Due: 06/5/2024, 11:59PM PST. Submit pdf write-up and zipped code packet on Github.

1. Productivity Shocks in the Three Equation Model

The log-linearized NK model boils down to three equations:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

with $\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$.

For this part assume that $v_t = 0$ and that $\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t$.

(a) Using the method of undetermined coefficients, solve for $\hat{y}_t$ and $\hat{\pi}_t$ as a function of $\hat{a}_t$.

(b) Plot the impulse response function for $\hat{y}_t, \hat{\pi}_t, \hat{y}_t^{flex}, \hat{y}_t - \hat{y}_t^{flex}, \hat{i}_t, \mathbb{E}_t\hat{r}_{t+1}, \hat{n}_t, \hat{a}_t$ to a one unit shock to $\hat{a}_t$. Use the following parameter values:

$\beta = 0.99, \sigma = 1, \kappa = 0.1, \rho_a = 0.8, \phi_\pi = 1.5$

(c) Intuitively explain your results.

(d) Use the Jupyter notebook "newkeynesianlinear.ipynb" to check that your plots in (b) are correct.

2. Non-linear NK model in Jupyter

Implement the standard new Keynesian model in Jupyter. We will write all conditions recursively and let the Sequence-Space Jacobian (SSJ) routines do the differentiation for us. Note that the first order conditions for firms and households are exactly as we have written in the lectures.

(a) The real reset price equation for the firm is,

$$p_t^* \equiv \frac{P_t^*}{P_t} = (1+\mu)E_t\left\{\sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} Y_{t+s}(P_{t+s}/P_t)^{\epsilon-1}}{\sum_{k=0}^{\infty}\theta^k \Lambda_{t,t+k} Y_{t+k}(P_{t+k}/P_t)^{\epsilon-1}} \frac{W_{t+s}/P_t}{A_{t+s}}\right\}$$

Explain why this expression is not recursive.

(b) We next show that we can write $B_t = E_t(F_{1t}/F_{2t})$, where both F_{1t}, F_{2t} are recursive. First, show that the denominator can be recursively written as,

$$\begin{aligned} F_{2t} &\equiv \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} (P_{t+k}/P_t)^{\epsilon-1} \\ &= Y_t + \theta \Pi_{t+1}^{\epsilon-1} \Lambda_{t,t+1} F_{2,t+1} \end{aligned}$$

noting that $\Lambda_{t,t+k} = \Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

(c) Second, show that the numerator can be recursively written as,

$$\begin{aligned} F_{1t} &\equiv (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\epsilon-1} \frac{W_{t+s}/P_t}{A_{t+s}} \\ &= (1+\mu) Y_t \frac{W_t/P_t}{A_t} + \theta \Pi_{t+1}^{\epsilon} \Lambda_{t,t+k} F_{1,t+1} \end{aligned}$$

noting that $\Lambda_{t,t+k}\Lambda_{t,t+1}\Lambda_{t+1,t+k}$ for all $k \geqslant 1$.

(d) Show that (gross) inflation can implicitly be written as

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta) p_t^{*1-\epsilon}$$

(e) Explain intuitively how when $p_t^* > 1$, then $\Pi_t > 1$.

(f) Implement the non-linear NK using your recursive equations in Python using the Sequence Space Jacobian toolbox. For now, ignore the dispersion of labor in production and write the aggregate production function as $Y_t = A_t N_t$. Use the following parameters: $\beta = 0.99, \gamma = 1, \varphi = 1, \chi = 1, \epsilon = 10, \rho_a = 0.8, \phi_\pi = 1.5, \phi_y = 0$ where $A_t = (A_{t-1})^{\rho_a} e^{\epsilon_t^a}$. Productivity is the only shock. Price stickiness is specified below.

(g) Compute IRFs for $\theta \in \{0.0001, 0.25, 0.5, 0.75, 0.9999\}$ using a first order approximation to your non-linear equations.

Report the IRFs for consumption, the output gap, the level of output, employment, inflation, the markup, the nominal interest rate, and the ex-ante real interest rate. Your graph for each variable should contain all cases for θ, appropriately labelled.

(h) Intuitively explain how the impulse response functions depend on the value of θ.

(i) What would you expect to see from the same shock in an RBC model without capital? (No derivation should be necessary.)

3. (Optional) Price Dispersion

Answering this question is optional.

In question 2, we ignored the labor dispersion term, $\Delta_t = \left[\int_0^1 \left(\frac{N_t(i)}{N_t}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$. In this question you will walk through the steps of writing price dispersion recursively and incorporating it in your model.

(a) Use the firm's demand curve to write labor dispersion in terms of relative firm prices $P_t(i)/P_t$.

(b) Note that everyone resetting prices at time t sets the same price P_t^*. Write Δ_t in terms of P_t^* and $P_{t-1}(i)/P_t = \frac{P_{t-1}(i)}{P_{t-1}\Pi_t}$.

(c) Finally, use Δ_{t-1} to substitute out for the integral over $P_{t-1}(i)/P_{t-1}$.

(d) Explain why the expression you derived is recursive.

(e) Add the recursive expression to your Python code in question 2, with the production function now equal to $Y_t = A_t N_t^{1-\alpha}\Delta_t$. One a single graph, plot the IRF for a technology shock in the model with price dispersion and the model without price dispersion. (Use the baseline parameters only with $\theta = 0.75$.)

(f) Interpret your results from the previous part.