Aiyagari 1994

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Outline

1. Idiosyncratic risk
2. Task
3. Tracing supply and demand

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Our problem - Individual Households

$$\max_{\{c_{jt}\}} \int_0^\infty e^{-\rho t} u(c_{jt}) \tag{1}$$

$$u(c_{jt}) = \log(c_{jt}) \tag{2}$$

$$\dot{a}_{jt} = r a_{jt} + w_t e_{jt} - c_{jt} \tag{3}$$

$$e_{jt} = \begin{cases} 1 \\ 0 \end{cases} \tag{4}$$

$$\text{with Poisson intensities } \lambda_0, \lambda_1 \tag{5}$$

Very similar to our previous problem! No aggregate shocks.

Representative Firm

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{6}$$

$$\tag{7}$$

- Competitive input factors
  - for capital $r_t = \frac{\partial Y}{\partial K} - \delta = \alpha \frac{Y_t}{K_t} - \delta$
  - for labor $w_t = \frac{\partial Y}{\partial L} = (1-\alpha)\frac{Y_t}{L_t}$

Aggregation in the labor market

- Super simple given that labor supply is inelastic

$$L_t^s = \int e dG(a, e)$$

- Labor demand given by the Cobb-Douglas structure

$$L_t^d = \left(\frac{1 - \alpha}{w} \right)^{1/\alpha} K_t$$

- Wage equates labor supply to labor demand:

$$w_t = \frac{K_t^{\alpha}(1 - \alpha)}{\int e dG_t(a, e)}$$

- We will come back to this equation

Aggregation in the capital market

- On one side labor demand is given by:

$$K_t^d = \left(\frac{\alpha}{r_t + \delta}\right)^{1/(1-\alpha)} L_t$$

- but we know what equilibrium labor must be equal to:

$$K_t^d = \left(\frac{\alpha}{r + \delta}\right)^{1/(1-\alpha)} \int e dG_t(a, e)$$

- On the other side, capital supply is just savings demand

$$K_t^s = \int a dG_t(a, e)$$

- and in equlibrium r_t is such that

$$K_t^s = K_t^d$$

Revisit labor market clearing

- In principle one can clear the markets for labor and capital looking for the right r and w numerically

- Turns out to be unnecessary

- Imposing capital demand on the equation for w_t yields:

$$w_t = (1-\alpha)\left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- For a given guess of r you can impose a **consistent** guess for w

How you should think of tackling these problems

- Usually we write in our papers an *Equilibrium Definition* subsection. For the Aiyagari model it would read like this:
  - An equilibrium is a sequence of sequences for c, a, L, K, r, and w such that:
    1. Sequences for a and c solve the household problem **taking prices as given**
    2. Sequences for K and L solve the firm problem **taking prices as given**
    3. Sequences for r and w **clear the markets** for capital and labor
    4. **given sequences** for e and initial values for a.

  Rule of thumb: If this is how we write the equilibrium definitions, that is how you should write your code

  1. A function that takes r and w as inputs and solves the *HJB* of the household
  2. A function that takes r and w as inputs and solves the firm's problem
  3. A function that takes policy functions and prices as given and produces excess demand for labor and capital
  4. A function that takes initial guesses and sets r, w to clear the markets.

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Obvious task

Write code that solves the Aiyagari model

- Some notes:
  1. I give you some codes on exercise1 so we all work with the same parameters
  2. I could give you the algorithm, but it is better if you come up with it
  3. You are welcome to recycle your old codes, or my old codes from last class

  You can have 60-90 minutes from now to complete the task. Tell me when you are done.

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How to compute this figure?

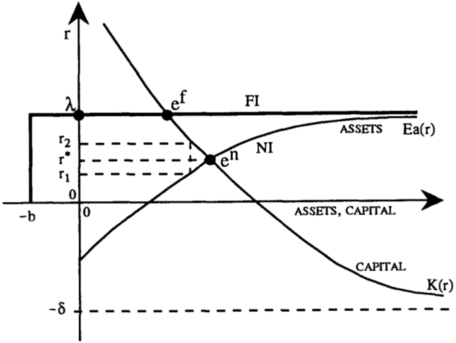


Figure IIb
Steady-State Determination

What would you need to recreate this figure with your code?

Two approaches

- In general if you want to trace the K^d equation, must shift the K^s; and vice-versa. Complication. What to do with GE objects other than r? (in this case w).

1. Set w at its steady state value. In that case the market for labor is not clearing.

2. Let prices w, r to adjust in equilibrium

    - Which is the right choice?
    - Analogy of this exercise to an IV
    - Examples of supply and demand shifters?

Task 2.1

- Change the production function of firms to

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$

- Solve the Aiyagari model for a vector $\vec{Z} \in [0.1, 1]$ with steps of 0.02.
- Collect the equilibrium capital stock and interest rates $\vec{r}, \vec{K}$.
- Plot them.
- Save the array $\vec{r}, \vec{K}$ in a file called z_{loop}
- Do this for the case in which you let wages to adjust, or when you set them to the initial equilibrium value.
- Is this the capital supply curve or the capital demand curve?

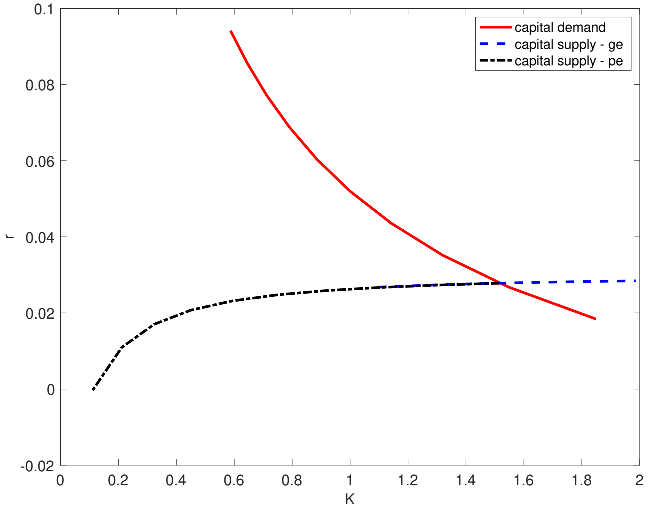
Task 2.2

- Solve the Aiyagari model for a vector $\vec{\rho} \in [0.02, 0.1]$ with steps of 0.02.
- Collect the equilibrium capital stock and interest rates $\vec{r}, \vec{K}$.
- Plot them.
- Save the array $\vec{r}, \vec{K}$ in a file called ρ_{loop}
- Is this the capital supply curve or the capital demand curve?

Task 2.3

Plot together the combinations of $\vec{r}, \vec{K}$ from the files ρ_{loop}, z_{loop}

My solution


0.1
0.08
0.06
0.04
0.02
0
-0.02
r
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
K
capital demand
capital supply - ge
capital supply - pe

Homework

- Keep working on your Aiyagari solution if you did not quite get it in class. Just submit it if you got it to work.
- Extend the Aiyagari model to a case where on top of the Poisson income shocks, the shocks not only affect income, but also returns (Google the large literature on heterogeneous returns). In particular.

$$e_{jt} = \begin{cases} 0.1 \\ 0.2 \end{cases}$$

$$r_{jt} = \begin{cases} 1.05 \times r_t \text{ if } e = 0.2 \\ 0.95 \times r_t \text{ if } e = 0.1 \end{cases}$$

with Poisson intensities λ_0, λ_1