

SEQUENCE SPACE METHODS

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OUTLINE

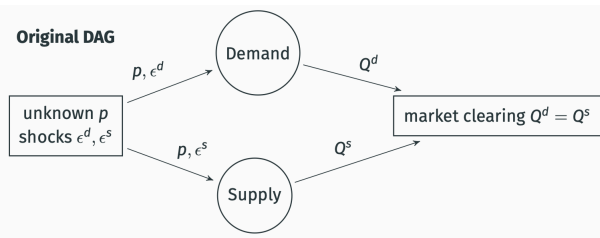
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- 2 STATE SPACE
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INTRODUCTION

- We will learn how to solve models in sequence space.
- Basic idea: organize models into “blocks” that represent behavior of (possibly heterogeneous) agents, and interact in GE via a small set of aggregates.
- We will arrange these blocks into Directed Acyclic Graph (“DAG”). Helpful to solve model, think about causality in GE, do decompositions, etc.
- Very useful for solving HA models (not today).



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STATE SPACE APPROACH

- You are likely more familiar with the state space approach.
- Solve for policies and prices as functions of the state.
- State: what is given to you today.
- Policy: what you chose based on FOC.
- Prices: those that implement the equilibrium (market clearing)
- Requires specifying a law of motion for the exogenous states.

GENERAL STATE SPACE REPRESENTATION

- State space representation of a linearized macro model (including HANK) is:

$$A \begin{pmatrix} x_{t+1} \\ E_t y_{t+1} \end{pmatrix} + B \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} \varepsilon_{t+1} \\ 0 \end{pmatrix}$$

- ▶ x_t is an $n - m \times 1$ vector of predetermined variables.
- ▶ y_t is an $m \times 1$ vector of jump/policy variables.

NEOCLASSICAL GROWTH MODEL

- Euler equation:

$$C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma} (1 - \delta + MPK_{t+1}^k)$$

- Law of motion for capital:

$$K_t = I_t + (1 - \delta)K_{t-1}$$

- Production function:

$$Y_t = Z_t K_{t-1}^\alpha$$

- Return to capital:

$$MPK_{t+1}^k = \alpha Z_{t+1} K_t^{\alpha-1}$$

- Market clearing:

$$Y_t = C_t + I_t$$

NEOCLASSICAL GROWTH MODEL LINEARIZED

- Euler equation:

$$-\gamma \hat{c}_t = -\gamma \mathbb{E}_t \hat{c}_{t+1} + (1 - \beta(1 - \delta)) \mathbb{E}_t \hat{m}pk_{t+1}$$

- Law of motion for capital:

$$\hat{k}_t = \delta \hat{i}_t + (1 - \delta) \hat{k}_{t-1}$$

- Production function:

$$\hat{y}_t = \hat{z}_t + \alpha \hat{k}_{t-1}$$

- Return to capital:

$$\hat{m}pk_{t+1} = \hat{z}_{t+1} + (\alpha - 1) \hat{k}_t$$

- Market clearing:

$$\hat{y}_t = s_c \hat{c}_t + (1 - s_c) \hat{i}_t$$

SOLVING STATE SPACE

- How to solve this?

$$\begin{aligned}\begin{pmatrix} x_{t+1} \\ E_t y_{t+1} \end{pmatrix} &= -A^{-1}B \begin{pmatrix} x_t \\ y_t \end{pmatrix} + A^{-1} \begin{pmatrix} \varepsilon_{t+1} \\ 0 \end{pmatrix} \\ &= VD^{-1}V^{-1} \begin{pmatrix} x_t \\ y_t \end{pmatrix} + A^{-1} \begin{pmatrix} \varepsilon_{t+1} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \tilde{x}_{t+1} \\ E_t \tilde{y}_{t+1} \end{pmatrix} &= D \begin{pmatrix} \tilde{x}_t \\ \tilde{y}_t \end{pmatrix} + V^{-1}A^{-1} \begin{pmatrix} \varepsilon_{t+1} \\ 0 \end{pmatrix}\end{aligned}$$

- Solve \tilde{x} backward and \tilde{y} forward. Then rotate back using

$$\begin{pmatrix} x_{t+1} \\ E_t y_{t+1} \end{pmatrix} \equiv V^{-1} \begin{pmatrix} \tilde{x}_{t+1} \\ E_t \tilde{y}_{t+1} \end{pmatrix}$$

- Unique stable solution if number of eigenvalues D inside unit circle is $n - m$.
- Dimension of the problem is $n \times n$.

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GENERAL SEQUENCE SPACE REPRESENTATION

- Equilibrium is a solution to an equation

$$\mathbf{H}(\mathbf{U}, \mathbf{Z}) = 0$$

where

- ▶ \mathbf{U} represents the time path U_0, U_1, \dots , of unknown aggregate sequences (e.g., quantities, prices).
 - ▶ \mathbf{Z} represents the time path Z_0, Z_1, \dots , of known exogenous shocks.
- Totally differentiate and evaluate at steady state to get:

$$d\mathbf{U} = -\mathbf{H}_U(\bar{\mathbf{U}}, \bar{\mathbf{Z}})^{-1} \mathbf{H}_Z(\bar{\mathbf{U}}, \bar{\mathbf{Z}}) d\mathbf{Z}$$

- Solution requires finding the sequence-space Jacobians \mathbf{H}_U and \mathbf{H}_Z .

SEQUENCE SPACE APPROACH

- Solve for sequences of policies and prices given a known sequence of exogenous shocks.
- Aligns closely with equilibrium definition.
- Dimension of the problem is $T \times \text{number of variables}$.
- Unlike state space we do not solve for a function of the state.
 - ▶ This can be helpful when the state is very high-dimensional
- The sequence space approach assumes perfect foresight.
 - ▶ Can get around this to some extent using “MIT-shocks.”

NEOCLASSICAL GROWTH MODEL LINEARIZED

- Repeated substitution:

$$\begin{aligned} 0 = H_t(\hat{\mathbf{K}}, \hat{\mathbf{Z}}) \equiv & \gamma \left[\alpha + (1 - s_c) \frac{1 - \delta}{\delta} \right] \hat{k}_{t-1} \\ & - \gamma \left[\alpha + (1 - s_c) \frac{1}{\delta} + (1 - \alpha) s_c (1 - \beta(1 - \delta)) \right] \hat{k}_t \\ & + \gamma(1 - s_c) \frac{1}{\delta} \hat{k}_{t+1} \\ & + \gamma \hat{z}_t \\ & - [\gamma - s_c(1 - \beta(1 - \delta))] \hat{z}_{t+1} \end{aligned}$$

- The H-matrix is:

$$\mathbf{0} = \mathbf{H}(\hat{\mathbf{K}}, \hat{\mathbf{Z}}) \equiv \begin{pmatrix} H_0(\hat{\mathbf{K}}, \hat{\mathbf{Z}}) \\ H_1(\hat{\mathbf{K}}, \hat{\mathbf{Z}}) \\ \vdots \\ H_T(\hat{\mathbf{K}}, \hat{\mathbf{Z}}) \end{pmatrix}$$

EXERCISE 1: NEOCLASSICAL GROWTH MODEL

1 Derive $\mathbf{H}_{\hat{\mathbf{K}}} =$

$$\begin{pmatrix} \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{k}_0} & \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{k}_1} & \cdots & \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{k}_T} \\ \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{k}_0} & \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{k}_1} & \cdots & \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{k}_T} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{k}_0} & \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{k}_1} & \cdots & \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{k}_T} \end{pmatrix}$$

2 Derive $\mathbf{H}_{\hat{\mathbf{Z}}} =$

$$\begin{pmatrix} \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{z}_0} & \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{z}_1} & \cdots & \frac{\partial H_0(\hat{\mathbf{K}})}{\partial \hat{z}_T} \\ \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{z}_0} & \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{z}_1} & \cdots & \frac{\partial H_1(\hat{\mathbf{K}})}{\partial \hat{z}_T} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{z}_0} & \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{z}_1} & \cdots & \frac{\partial H_T(\hat{\mathbf{K}})}{\partial \hat{z}_T} \end{pmatrix}$$

- 3 Assume \hat{z}_t is an AR(1) with persistence $\rho_z = 0.9$. Compute and plot $\hat{\mathbf{k}}$.

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SOLVING IN SEQUENCE SPACE

- Organize the model in blocks.

① Firm block:

$$\begin{aligned}\hat{y}_t &= \hat{z}_t + \alpha \hat{k}_{t-1} \\ \hat{m}pk_{t+1} &= \hat{z}_{t+1} + (\alpha - 1) \hat{k}_t\end{aligned}$$

② Household block:

$$\begin{aligned}\hat{k}_t &= \delta \hat{l}_{t-1} + (1 - \delta) \hat{k}_t \\ -\gamma \hat{c}_t &= -\gamma \hat{c}_{t+1} + (1 - \beta(1 - \delta)) \hat{m}pk_{t+1}\end{aligned}$$

③ Market clearing block:

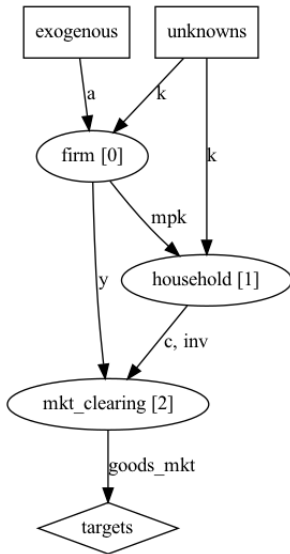
$$0 = s_c \hat{c}_t + (1 - s_c) \hat{l}_t - \hat{y}_t$$

SEQUENCE SPACE ALGORITHM HEURISTICS

- ➊ Start with an initial guess of the sequences $(\hat{\mathbf{k}}) = \{\hat{k}_t\}_{t=0}^{\infty}$.
 - ▶ $\hat{\mathbf{a}} = \{a_t\}_{t=0}^{\infty}$ and \hat{k}_{t-1} are given to us.
- ➋ Solve the firm block for $\hat{\mathbf{y}}$ and $\hat{\mathbf{m}}\hat{\mathbf{p}}\mathbf{k}$.
- ➌ Solve the household block for $\hat{\mathbf{c}}, \hat{\mathbf{i}}$.
- ➍ Check that market clears. If not update guess $\hat{\mathbf{k}}$
 - ▶ It turns out we do not need to guess, but can solve the entire system with linear algebra in one step.
 - ▶ Approach follows equilibrium definition: find sequences such that everyone optimizes and markets clear.

DAG REPRESENTATION

- Effectively what we have done is organized our model in a Directed Acyclical Graph (DAG).



DAG RULES

- There are no cycles in a DAG—we travel in one direction only.
- A model has many DAG representations.
 - ▶ One representation is to treat every endogenous variable as unknown and have single block.
 - ▶ Another representation is to treat each equation as an individual block.
 - ▶ These are often not the most useful representation.

DAG SUGGESTIONS

- A good DAG minimizes the number of unknowns.
- Generally useful to organize blocks by agent: household, firm, union, government, market clearing.
- Blocks make updating the model easy. Often we change just one problem (e.g., firm for NK model) and leave others untouched.
- Logical check for each block: are the number of variables to solve for equal to the number of equations?
- The computer can organize our model in a DAG and substitute for us.
- Today we will see what the computer does under the hood.

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SOLVING IN SEQUENCE SPACE

- We want to clear markets at all points in time:

$$\mathbf{H} = \begin{pmatrix} s_c \hat{c}_0 + (1 - s_c) \hat{l}_0 - \hat{y}_0 \\ \vdots \\ s_c \hat{c}_T + (1 - s_c) \hat{l}_T - \hat{y}_T \end{pmatrix} = (\Phi_{gm,c} \hat{\mathbf{c}} + \Phi_{gm,l} \hat{\mathbf{l}} + \Phi_{gm,y} \hat{\mathbf{y}}) = \mathbf{0}$$

where

$$\Phi_{gm,c} = s_c I_T$$

$$\Phi_{gm,l} = (1 - s_c) I_T$$

$$\Phi_{gm,y} = -I_T$$

and I_T is the $T \times T$ identity matrix.

- How does adjusting the sequences $\mathbf{U} = (\hat{\mathbf{k}})$ change the target?

$$\mathbf{H}_U = \left(\Phi_{gm,c} \frac{\partial \hat{\mathbf{c}}}{\partial \hat{\mathbf{k}}} + \Phi_{gm,l} \frac{\partial \hat{\mathbf{l}}}{\partial \hat{\mathbf{k}}} + \Phi_{gm,y} \frac{\partial \hat{\mathbf{y}}}{\partial \hat{\mathbf{k}}} \right)$$

SOLVING IN SEQUENCE SPACE

- A simpler and equivalent expression to work with is

$$\mathbf{H}_{\mathbf{U}} = \begin{pmatrix} \Phi_{gm,c} & \Phi_{gm,l} & \Phi_{gm,y} & \mathbf{0}_T \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{c}}{\partial \hat{\mathbf{k}}} \\ \frac{\partial \hat{l}}{\partial \hat{\mathbf{k}}} \\ \frac{\partial \hat{\mathbf{y}}}{\partial \hat{\mathbf{k}}} \\ \frac{\partial \hat{\mathbf{m}}\mathbf{p}\mathbf{k}}{\partial \hat{\mathbf{k}}} \end{pmatrix} \equiv \frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{U}}$$

- Now we move along using the chain rule:

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{U}} = \begin{pmatrix} \frac{\partial(c,l)}{\partial \mathbf{U}} \\ \frac{\partial(\hat{\mathbf{y}},(\hat{\mathbf{w}}-\hat{\mathbf{p}}))}{\partial \mathbf{U}} \end{pmatrix}$$

where we partitioned based on the two blocks we looked at earlier.

SOLVING IN SEQUENCE SPACE

- We start with the firm block:

$$\hat{y}_t = \hat{a}_t + \alpha \hat{k}_{t-1}$$

- In matrix notation:

$$\hat{\mathbf{y}} = \hat{\mathbf{z}} + \Phi_{y,k} \hat{\mathbf{k}} + \Phi_{y,k-1} \hat{k}_{-1}$$

- The matrices are:

$$\Phi_{y,k} = \alpha \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \quad \Phi_{y,k-1} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

- Note timing!

SOLVING IN SEQUENCE SPACE

- The second equation in the firm block:

$$\hat{m}pk_{t+1} = \hat{z}_{t+1} + (\alpha - 1)\hat{k}_t$$

- In matrix notation:

$$\hat{m}pk = \Phi_{mpk,z}\hat{z} + \Phi_{mpk,z}\hat{k}$$

- The matrices are:

$$\Phi_{mpk,z} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

$$\Phi_{mpk,z} = (\alpha - 1)I_T$$

- Note timing!

SOLVING IN SEQUENCE SPACE

- With $\hat{\mathbf{m}}\mathbf{p}\mathbf{k}$ and $\hat{\mathbf{k}}$ we solve for $\hat{\mathbf{c}}$ and $\hat{\mathbf{i}}$ using the household FOC.
- Start with investment:

$$\hat{k}_t = \delta \hat{i}_t + (1 - \delta) \hat{k}_{t-1}$$

- In matrix notation:

$$\hat{\mathbf{i}} = \Phi_{l,k} \hat{\mathbf{k}} + \Phi_{l,k-1} \hat{k}_{-1}$$

- The matrix is:

$$\Phi_{l,k} = \delta^{-1} \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ \delta - 1 & 1 & \dots & 0 & 0 \\ 0 & \delta - 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & 0 & \dots & \delta - 1 & 1 \end{pmatrix}$$

SOLVING IN SEQUENCE SPACE

- Next consumption:

$$-\gamma \hat{c}_t = -\gamma \hat{c}_{t+1} + (1 - \beta(1 - \delta)) \hat{m} \hat{p} k_{t+1}$$

- In matrix notation:

$$\hat{\mathbf{c}} = \Phi_{c,mpk} \hat{\mathbf{m}} \hat{\mathbf{p}} \mathbf{k}$$

- The matrices are:

$$\Phi_{c,mpk} = -\gamma^{-1}(1 - \beta(1 - \delta)) \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}^{-1}$$

SOLVING IN SEQUENCE SPACE

- From the firm block we have:

$$\frac{\partial(\hat{\mathbf{y}}, (\hat{\mathbf{m}}\mathbf{p}\mathbf{k}))}{\partial \mathbf{U}} = \begin{pmatrix} \Phi_{y,k} \\ \Phi_{mpk,k} \end{pmatrix}$$

- From the household block we have:

$$\begin{aligned} \frac{\partial(\hat{\mathbf{c}}, \hat{\mathbf{l}})}{\partial \mathbf{U}} &= \begin{pmatrix} \Phi_{c,mpk} \frac{d\hat{\mathbf{m}}\mathbf{p}\mathbf{k}}{d\hat{\mathbf{k}}} \\ \Phi_{l,k} \end{pmatrix} \\ &= \begin{pmatrix} \Phi_{c,mpk} \Phi_{mpk,k} \\ \Phi_{l,k} \end{pmatrix} \end{aligned}$$

- We solved for \mathbf{H}_U :

$$\begin{aligned} \mathbf{H}_U &= \begin{pmatrix} \Phi_{gm,c} & \Phi_{gm,l} & \Phi_{gm,y} & \mathbf{0}_T \end{pmatrix} \times \begin{pmatrix} \Phi_{c,mpk} \Phi_{mpk,k} \\ \Phi_{l,k} \\ \Phi_{y,k} \\ \Phi_{mpk,k} \end{pmatrix} \\ &= \begin{pmatrix} \Phi_{gm,c} \Phi_{c,mpk} \Phi_{mpk,k} + \Phi_{gm,l} \Phi_{l,k} - \Phi_{y,k} \end{pmatrix} \end{aligned}$$

SOLVING IN SEQUENCE SPACE

- Solving for \mathbf{H}_Z is a bit more straightforward:

$$\mathbf{H}_Z = \frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}}$$

We already know the first derivative.

- From the firm block we have:

$$\frac{\partial \hat{\mathbf{y}}}{\partial \mathbf{Z}} = \mathbf{I}_T$$

and

$$\frac{\partial(\hat{\mathbf{m}}\mathbf{p}\mathbf{k})}{\partial \mathbf{Z}} = \Phi_{mpk,z} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

SOLVING IN SEQUENCE SPACE

- From the household block we have:

$$\frac{\partial \hat{\mathbf{c}}, \hat{t}}{\partial \mathbf{Z}} = \begin{pmatrix} \Phi_{c,wp} \frac{\partial \hat{\mathbf{m}}_{pk}}{\partial \hat{\mathbf{z}}} \\ \mathbf{0}_T \end{pmatrix} = \begin{pmatrix} \Phi_{c,mpk} \Phi_{mpk,z} \\ \mathbf{0}_T \end{pmatrix}$$

- We solved for \mathbf{H}_Z :

$$\begin{aligned} \mathbf{H}_Z &= \begin{pmatrix} \Phi_{gm,c} & \Phi_{gm,l} & \Phi_{gm,y} & \mathbf{0}_T \end{pmatrix} \times \begin{pmatrix} \Phi_{c,mpk} \Phi_{mpk,z} \\ \mathbf{0}_T \\ I_T \\ \Phi_{mpk,z} \end{pmatrix} \\ &= (\Phi_{gm,c} \Phi_{c,mpk} \Phi_{mpk,z} - I_T) \end{aligned}$$

- We now have the solution to the model:

$$d\mathbf{U} = -\mathbf{H}_U^{-1} \mathbf{H}_Z d\mathbf{Z}$$

and calculate the remaining sequences

$$d\mathbf{Y} = \frac{\partial \mathbf{Y}}{\partial \mathbf{U}} d\mathbf{U} + \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} d\mathbf{Z} = \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{U}} \mathbf{H}_U^{-1} \mathbf{H}_Z + \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} \right) d\mathbf{Z}$$

LESSONS

- Can solve any linearized dynamic model using linear algebra.
- Extremely fast once matrices are created.
 - ▶ If we tell the computer that the matrices are sparse (mostly 0s).
- Replaced substitution with matrix multiplication.
- Strategy follows equilibrium definition: looking for sequence such that everyone optimizes and markets clear.
- Everyone should do this once by hand. Then let the computer do the work for you.

SOLVING IN SEQUENCE SPACE

- Program this.
- Do you get the same answer as before?

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MORE SEQUENCE SPACE

- ① Write a DAG for the RBC model without capital.
- ② Write a DAG for the RBC model with capital.
- ③ Write a DAG for the New Keynesian model with capital.
- ④ For each model, derive the H_U and H_Z matrices.
 - ▶ You know you will solve three models, so try to set up your code in such a way that you can easily exchange / add blocks.
- ⑤ Compute the IRF to a one-time productivity shock of 1% for each model.
- ⑥ How much of the IRF in each model is due to the endogenous investment response?