

FIXED COST MODELS

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OUTLINE

- 1 INTRODUCTION
- 2 BASELINE FC MODEL
- 3 CONTINUOUS TIME

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FIXED COST MODELS

- Lots of behavior at the micro level is lumpy: buying houses, cars, setting prices, investment, information acquisition.
- Fixed cost models naturally give rise to such lumpiness.
- Fixed cost models are hard to solve because they are non-convex, non-differentiable, and in GE have to carry distribution around.
- But lots of interesting economics: state-dependence, size-dependence, policy targeting.
- Today we will see how to solve these models within SSJ routines.

FIXED COST MODELS: EXAMPLES

- Does lumpy price setting behavior matter? lots from Alvarez-Lippi; Auclert, Rigato, Rognlie, Straub (2024).
- Do fixed cost frictions matter? Kahn-Thomas, Winberry (2021), Koby-Wolf (2020), Bailey-Blanco (2021).
- Lumpy durables and monetary transmission: McKay and Wieland (2021, 2022).

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ENVIRONMENT

- Liquid assets a
- Durable stock d
- Nondurable consumption c
- Income y
- Real rate r
- Depreciation rate δ
- Fixed cost f

CONSTRAINTS

- Budget constraint if adjusting:

$$\frac{a'}{1+r} + c' + pd' = a + (1-\delta)(1-f)pd + y \equiv x_{adj}$$

- Budget constraint if not adjusting with $d' = (1-\delta)d$

$$\frac{a'}{1+r} + c' = a + y \equiv x_{noadj}$$

- Borrowing constraint

$$a' \geq 0$$

VALUE FUNCTIONS

- Let the value function be

$$V(y, d, a, \varepsilon) = \max\{V^{adj}(y, d, a) + \varepsilon^{adj}, V^{noadj}(y, d, a) + \varepsilon^{noadj}\}$$

- ε^i are drawn from a Gumbell distribution with standard deviation $\sigma_V \frac{\pi}{\sqrt{6}},^1$.

Define the post-adjustment value function as

$$W(y, d', a') \equiv E_y \tilde{V}(y', d', a')$$

- Value from adjusting and not adjusting:

$$V^{adj}(y, d, a) = \max_{c' + d' + \frac{a'}{1+r} \leq x_{adj}} [\psi \ln c' + (1 - \psi) \ln d' + W(y, d', a')],$$

$$V^{noadj}(y, d, a) = \max_{c' + \frac{a'}{1+r} \leq x_{noadj}} [\psi \ln c' + (1 - \psi) \ln(1 - \delta)d + W(y, (1 - \delta)d)],$$

¹<https://eml.berkeley.edu/choice2/ch3.pdf>

ADJUSTMENT PROBABILITIES AND EXPECTED VALUES

- The probability of adjustment is

$$\text{adjust}(y, d, a) = \frac{\exp\{(V^{\text{adj}}(y, d, a) - V^{\text{noadj}}(y, d, a))/\sigma_V\}}{1 + \exp\{(V^{\text{adj}}(y, d, a) - V^{\text{noadj}}(y, d, a))/\sigma_V\}}$$

- The distribution of V is

$$\begin{aligned} \text{Prob}[V \leq x] &= \text{Prob}[\varepsilon^{\text{adj}} \leq x - V^{\text{adj}}(y, d, a)] \\ &\quad \times \text{Prob}[\varepsilon^{\text{noadj}} \leq x - V^{\text{noadj}}(y, d, a)] \\ &= \exp\{-\exp[-(x - \sigma_V \log\{\exp[V^{\text{adj}}(y, d, a)/\sigma_V] \\ &\quad + \exp[V^{\text{noadj}}(y, d, a)/\sigma_V]\})/\sigma_V]\} \end{aligned}$$

- The expected value is,

$$\begin{aligned} \tilde{V}(y, d, a) &\equiv E_{\varepsilon} V(y, d, a, \varepsilon) \\ &= V^{\text{noadj}}(y, d, a) - \sigma_V \log \text{noadjust}(y, d, a) + \sigma_V \gamma \end{aligned}$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. If $\sigma_V \rightarrow 0$, then

$$\tilde{V}(y, d, a) = \max\{V^{\text{adj}}(y, d, a), V^{\text{noadj}}(y, d, a)\}$$

NO ADJUSTMENT PROBLEM

- Break up into sequential problem of choosing c' , a' given d' . Then choose d' .
- For given choice of d' :

$$V^{noadj}(y, n, m) = \max_{c', a'} [\psi \ln c' + (1 - \psi) \ln d' + \beta W(y', d', a')]$$

$$d' = n$$

$$a' = [m - c']$$

$$a' \geq 0$$

where $m \geq 0$.

- First order conditions:

$$V_m^{noadj}(y, n, m) = \beta W_a(y', d', a') + \zeta$$

$$V_n^{noadj}(y, n, m) = \frac{(1 - \psi)}{d'} + \beta W_d(y', d', a')$$

$$\frac{\psi}{c'} = \beta W_a(y', d', a') + \zeta$$

EGM

- We are given an initial guess $W_a(y', d', a')$.
- First, assume borrowing constraint is not binding and solve for c' in

$$\frac{\psi}{c'} = \beta W_a(y', d', a')$$

- Then calculate implied cash on hand

$$m = c' + a'$$

- The problem is not necessarily concave, so there may be multiple combinations of (c', a') that map into the same grid point m .
 - ▶ In this case we have to check which solution yields highest utility and discard the others.
 - ▶ The "func_upper_envelop.py" function performs this task.

EGM

- Interpolate the decision rules from the m grid onto the grid for a .
- Then check if implied solution for a' violates the borrowing constraint. If so implement borrowing constrained solution $a' = 0$ and $c' = m$.
- Combining the two solutions yields the policy functions

$$c^{noadj}(y, n, m), a^{noadj}(y, n, m)$$

- The no adjustment solutions use $n = (1 - \delta)d$ and $m = a + y$

$$c^{noadj}(y, (1 - \delta)d, m)$$

$$a^{noadj}(y, (1 - \delta)d, m)$$

$$d^{noadj} = (1 - \delta)d$$

ADJUSTMENT PROBLEM

- Given some amount of cash on hand x :

$$V^{adj}(y, x) = \max_{d'} V^{noadj}(y, d', m')$$

$$m' = x + y - pd'$$

$$m' \geq 0$$

- FOC

$$V_x^{adj}(y, x) = V_m^{noadj}(y, d', m') + \kappa$$

$$V_n^{noadj}(y, d', m') = pV_m^{noadj}(y, d', m') + \kappa$$

- Constraint will never be binding so $\kappa = 0$.
- Combine with keep FOC to get

$$p \frac{\psi}{c^{noadj}(y, d', m')} = \frac{1 - \psi}{d'} + \beta W_d(y, d', a')$$

- Again must solve for upper envelope given non-concavity.

ADJUSTMENT SOLUTION

- This yields adjustment solutions

$$c^{adj}(y, x), a^{adj}(y, x), d^{adj}(y, x)$$

and a value function

$$V^{adj}(y, x) = \psi \ln c^{adj}(y, x) + (1 - \psi) \ln d^{adj}(y, x) \\ + \beta W(y', d^{adj}(y, x), a^{adj}(y, x))$$

- We interpolate onto the existing grid using $x = a + (1 - \delta)(1 - f)pd$ to get

$$c^{adj}(y, d, a), a^{adj}(y, d, a), d^{adj}(y, d, a)$$

and value function

$$V^{adj}(y, d, a) = \psi \ln c^{adj}(a, d, y) + (1 - \psi) \ln d^{adj}(a, d, y) + \beta W(y', d^{adj}(a,$$

COMBINED PROBLEM

- Get policy functions by combining adjust and no-adjust solutions.

$$c'(y, d, a) = \text{adjust}(a, d, y)c^{adj}(a, d, y) + (1 - \text{adjust}(a, d, y))c^{noadj}(a, d, y)$$

$$a'(y, d, a) = \text{adjust}(a, d, y)a^{adj}(a, d, y) + (1 - \text{adjust}(a, d, y))a^{noadj}(a, d, y)$$

$$d'(y, d, a) = \text{adjust}(a, d, y)d^{adj}(a, d, y) + (1 - \text{adjust}(a, d, y))d^{noadj}(a, d, y)$$

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CONTINUOUS TIME

- Alternatively can formulate the problem in continuous time.
- Continuous time generally very good for handling stopping time problems, such as when to adjust.
- With discretized value function can write problem as

$$\min\{\rho v - u(v) - A(v)v, v - v^*(v)\} = 0$$

- ▶ First term is the no-adjustment utility flow.
 - ▶ Second term is the adjustment choice.
- Intuition: if $v < v^*(v)$ then do not adjust and get flow value. If adjust $v = v^*(v)$ since the flow value $u(v) + A(v)v < \rho v$.

CONSIDERATIONS

- To solve continuous time problem see Ben Moll's code on stopping time problems.
- In principle continuous time better: do not require sufficiently large utility shocks to make value function differentiable.
- But harder to then apply discrete time sequence space methods. Even after aggregation, the distribution of states jumps after a shock so cannot use the fake news algorithm as implemented. (See McKay-Wieland 2021.)
- Possible that continuous time SSJ could be used. But need to handle the immediate jumps in the distribution.