The Elasticity of the Capital Supply Curve

Juan Herreño Johannes Wieland

May 2024

1 Setting

There is one representative household who owns a continuum of firms who produce goods, and a competitive firm that produces capital.

2 Good-producing firms

A firm operates a DRS technology of the form

$$y_{jt} = e_{jt} k_{jt}^{\theta} l_{jt}^{\nu}, \tag{1}$$

the produce undifferentiated varieties, where y, k, l denote output, capital, and labor, respectively. e is idiosyncratic productivity, and can take two values, a high (H), and a low value (L), and firms move away from each value with Poisson intensities λ_H and λ_L, respectively.

Firms accumulate capital by investment. The law of motion of capital is given by:

$$\dot{k}_{jt} = -\delta k_{jt} + i_{jt}. \tag{2}$$

Firms profits are equal to revenue, minus the wage bill, minus investment expenditures minus adjustment costs, which are convex. The relative price of capital is q, the wage rate is w, and firms receive investment subsidies equal to τ. Firms take all these aggregate quantities as given.

$$\pi_{jt} = y_{jt} - w_t l_{jt} - q_t(1-\tau_t) i_{jt} - \frac{\varphi}{2}\left(\frac{i_{jt}}{k_{jt}}\right)^2 k. \tag{3}$$

Firm's value is the discounted sum of profits in terms of utility for the household, so profits are weighted by the marginal utility of consumption of the household Λ.

$$v_{j0} = \int_0^{\infty} e^{-\rho t} \Lambda_t \pi_{jt} dt. \tag{4}$$

2.1 HJB equation

We can represent the firm's problem using an HJB equation:

$$\rho v_j(k) = \max_{i,l} \Lambda \pi(k, e_j) + \frac{\partial v_j}{\partial k}(-\delta k + i) + \lambda_j (v_{-j}(k) - v_j(k)) \ , \forall j \in (L, H) \tag{5}$$

$$\pi(k, e) = e k^{\theta} l^{\nu} - wl - q(1-\tau)i - \frac{\varphi}{2}\left(\frac{i}{k}\right)^2 k \tag{6}$$

$$\pi(k, e_j) \geq -\underline{\pi} k \ , \forall j \in (L, H) \tag{7}$$

2.2 Optimality conditions

Labor decisions of the firm are given by:

$$l = \left(\frac{\nu e k^{\theta}}{w}\right)^{\frac{1}{1-\nu}}. \tag{8}$$

And optimal firm investment is given by

$$\Lambda(\varphi\left(\frac{i}{k}\right) + q(1-\tau)) = \frac{\partial v}{\partial k}. \tag{9}$$

3 Capital Producing Firm

A competitive firm produces capital but faces an upward sloping marginal cost (presumably from adjustment costs).

The supply curve for capital of this firm takes the shape:

$$q_t = x_t^{1/\xi}, \tag{10}$$

where x is the quantity of capital goods produced, and ξ is the elasticity of the supply curve.

4 Household

The representative household has preferences that are represented by a the following utility function:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\psi}}{1+\psi}. \tag{11}$$

The household's problem consists on maximizing the stream of utility using a discount rate ρ

$$V_0 = \max \int_0^\infty e^{-\rho t} u(C_t, L_t) dt \tag{12}$$

The household's budget constraint equalizes consumption expenditures to total income coming from labor income, dividends, savings and lump-sum taxes.

$$\dot{B}_t = B_t r_t + w_t L_t + \Pi_t - C_t - T. \tag{13}$$

The problem of the household can be characterized by a Hamiltonian.

$$\mathcal{H} = e^{-\rho t} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\psi}}{1+\psi} \right) + \lambda_t \left(B_t r_t + w_t L_t + \Pi_t - C_t \right), \tag{14}$$

with first order conditions:

$$e^{-\rho t} C_t^{-\sigma} = \lambda_t \tag{15}$$

$$e^{-\rho t} L_t^{\psi} = \lambda_t w_t \tag{16}$$

$$\lambda_t r_t = -\dot{\lambda}_t \tag{17}$$

$$\lim_{t \to \infty} \lambda_t B_t = 0 \tag{18}$$

Define the current value multiplier as:

$$\mu_t = \lambda_t e^{\rho t}, \tag{19}$$

and redefine the system as

$$C_t^{-\sigma} = \mu_t \tag{20}$$

$$L_t^{\psi} C_t^{\sigma} = w_t \tag{21}$$

$$\frac{\dot{\mu}_t}{\mu_t} = \rho - r_t \tag{22}$$

$$\lim_{t \to \infty} e^{-\rho t} \mu_t B_t = 0. \tag{23}$$

As it is clear from this block $\Lambda_t = \mu_t$.

5 The government

The government maintains a balanced budget in every period, so:

$$T = q\tau \int i(k, e) dG(k, e) \tag{24}$$

6 Market clearing

In the labor market, total labor supply L is equal to total labor demand $\int l(k,e)dG(k,e)$. In the capital market, investment expenditures are equal to total new capital produced $\int i(k,e)dG(k,e) = x$. The bond is in zero net supply $B = 0$. Furthermore, in a stationary equilibrium without aggregate shocks, $r = \rho$. For simplicity, I assume that $\tau = 0$ in that stationary equilibrium, so total taxes on households $T = 0$ as well.

7 MIT shock

One day, the economy wakes up to the unexpected news that $\tau = \bar{\tau}$.

8 Inference by two Econometricians

An econometrician is interested in backing up the capital supply elasticity. She takes advantage of the exogenus increase in τ to do that. τ appears on the capital demand block of the model. Therefore, she can back out ξ by computing the ratio between the IRF of investment triggered by the shock, to the IRF of the price of capital. Formally,

$$\hat{\xi} = \frac{\frac{\int_0^\infty \%\Delta I_t}{\int_0^\infty \Delta\tau_t}}{\frac{\int_0^\infty \%\Delta q_t}{\int_0^\infty \Delta\tau_t}} \tag{25}$$

A less refined econometrician only observes a bundle of new capital purchases I and in-house expenditures on capital installation at the national level. That is:

$$\tilde{I}_t = I_t + \text{Adjustment Costs}_t. \tag{26}$$

This second econometrician also uses the exogenous tax reform to back-out the elasticity of the capital supply curve:

$$\tilde{\xi} = \frac{\frac{\int_0^\infty \%\Delta \tilde{I}_t}{\int_0^\infty \Delta\tau_t}}{\frac{\int_0^\infty \%\Delta q_t}{\int_0^\infty \Delta\tau_t}} \tag{27}$$

Is it the case that $\tilde{\xi} = \hat{\xi}$?

9 Theoretical Arguments on the shape of the demand curve from capital

House and Shapiro (2008) argue that if $\delta \approx 0$, $\rho \approx 0$, then the capital demand curve should be horizontal. That is, that $d \log q/d\tau \approx 1$. Therefore in that limit, information on q is superfluous. Only having information about the semi-elasticity of I to τ is enough to infer the elasticity of the capital supply curve.

Is our DGP close enough to this limit?