Fixed Cost Models

Juan Herreño   Johannes Wieland

UCSD, Spring 2024

Outline

Outline

Fixed Cost Models

- Lots of behavior at the micro level is lumpy: buying houses, cars, setting prices, investment, information acquisition.
- Fixed cost models naturally give rise to such lumpiness.
- Fixed cost models are hard to solve because they are non-convex, non-differentiable, and in GE have to carry distribution around.
- But lots of interesting economics: state-dependence, size-dependence, policy targeting.
- Today we will see how to solve these models within SSJ routines.

Fixed Cost Models: Examples

- Does lumpy price setting behavior matter? lots from Alvarez-Lippi; Auclert, Rigato, Rognlie, Straub (2024).

- Do fixed cost frictions matter? Kahn-Thomas, Winberry (2021), Koby-Wolf (2020), Bailey-Blanco (2021).

- Lumpy durables and monetary transmission: McKay and Wieland (2021, 2022).

Outline

Environment

- Liquid assets a
- Durable stock d
- Nondurable consumption c
- Income y
- Real rate r
- Depreciation rate δ
- Fixed cost f

Constraints

- Budget constraint if adjusting:

$$\frac{a'}{1+r} + c' + pd' = a + (1-\delta)(1-f)pd + y \equiv x_{adj}$$

- Budget constraint if not adjusting with $d' = (1-\delta)d$

$$\frac{a'}{1+r} + c' = a + y \equiv x_{noadj}$$

- Borrowing constraint

$$a' \geq 0$$

Value Functions

- Let the value function be

$$V(y,d,a,\varepsilon) = \max\{V^{adj}(y,d,a)+\varepsilon^{adj}, V^{noadj}(y,d,a)+\varepsilon^{noadj}\}$$

  - ε^i are drawn from a Gumbell distribution with standard deviation $\sigma_V \frac{\pi}{\sqrt{6}}$,[1].

  Define the post-adjustment value function as

$$W(y,d',a') \equiv E_y \tilde{V}(y',d',a')$$

- Value from adjusting and not adjusting:

$$V^{adj}(y,d,a) = \max_{c'+d'+\frac{a'}{1+r} \leq x_{adj}} [\psi \ln c' + (1-\psi)\ln d' + W(y,d',a'),$$

$$V^{noadj}(y,d,a) = \max_{c'+\frac{a'}{1+r} \leq x_{noadj}} [\psi \ln c' + (1-\psi)\ln(1-\delta)d + W(y,(1-\delta)d$$

[1] https://eml.berkeley.edu/choice2/ch3.pdf

Adjustment Probabilities and Expected Values

- The probability of adjustment is

$$adjust(y,d,a) = \frac{\exp\{(V^{adj}(y,d,a) - V^{noadj}(y,d,a))/\sigma_V\}}{1+\exp\{(V^{adj}(y,d,a) - V^{noadj}(y,d,a))/\sigma_V\}}$$

- The distribution of V is

$$\begin{aligned} Prob[V \leq x] &= Prob[\varepsilon^{adj} \leq x - V^{adj}(y,d,a)] \\ &\quad \times Prob[\varepsilon^{noadj} \leq x - V^{noadj}(y,d,a)] \\ &= \exp\{-\exp[-(x - \sigma_V \log\{\exp[V^{adj}(y,d,a)/\sigma_V] \\ &\quad + \exp[V^{noadj}(y,d,a)/\sigma_V]\})/\sigma_V]\} \end{aligned}$$

- The expected value is,

$$\begin{aligned} \tilde{V}(y,d,a) &\equiv E_\varepsilon V(y,d,a,\varepsilon) \\ &= V^{noadj}(y,d,a) - \sigma_V \log noadjust(y,d,a) + \sigma_V \gamma \end{aligned}$$

  where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. If $\sigma_V \to 0$, then

$$\tilde{V}(y,d,a) = \max\{V^{adj}(y,d,a), V^{noadj}(y,d,a)\}$$

No Adjustment Problem

- Break up into sequential problem of choosing c', a' given d'. Then choose d'.
- For given choice of d':

$$
\begin{aligned}
V^{noadj}(y,n,m) &= \max_{c',a'}[\psi \ln c' + (1-\psi)\ln d' + \beta W(y',d',a')] \\
d' &= n \\
a' &= \left[m - c'\right] \\
a' &\geq 0
\end{aligned}
$$

where $m \geq 0$.
- First order conditions:

$$
\begin{aligned}
V_m^{noadj}(y,n,m) &= \beta W_a(y',d',a') + \zeta \\
V_n^{noadj}(y,n,m) &= \frac{(1-\psi)}{d'} + \beta W_d(y',d',a') \\
\frac{\psi}{c'} &= \beta W_a(y',d',a') + \zeta
\end{aligned}
$$

EGM

- We are given an initial guess $W_a(y', d', a')$.
- First, assume borrowing constraint is not binding and solve for c' in

$$\frac{\psi}{c'} = \beta W_a(y', d', a')$$

- Then calculate implied cash on hand

$$m = c' + a'$$

- The problem is not necessarily concave, so there may be multiple combinations of (c', a') that map into the same grid point m.
  - In this case we have to check which solution yields highest utility and discard the others.
  - The "func_upper_envelop.py" function performs this task.

EGM

- Interpolate the decision rules from the m grid onto the grid for a.
- Then check if implied solution for a' violates the borrowing constraint. If so implement borrowing constrained solution $a' = 0$ and $c' = m$.
- Combining the two solutions yields the policy functions

$$c^{noadj}(y,n,m), a^{noadj}(y,n,m)$$

- The no adjustment solutions use $n = (1-\delta)d$ and $m = a + y$

$$c^{noadj}(y,(1-\delta)d,m)$$
$$a^{noadj}(y,(1-\delta)d,m)$$
$$d^{noadj} = (1-\delta)d$$

Adjustment Problem

- Given some amount of cash on hand x:

$$V^{adj}(y,x) = \max_{d'} V^{noadj}(y,d',m')$$
$$m' = x + y - pd'$$
$$m' \geq 0$$

- FOC

$$V_x^{adj}(y,x) = V_m^{noadj}(y,d',m') + \kappa$$
$$V_n^{noadj}(y,d',m') = pV_m^{noadj}(y,d',m') + \kappa$$

- Constraint will never be binding so $\kappa = 0$.
- Combine with keep FOC to get

$$p\frac{\psi}{c^{noadj}(y,d',m')} = \frac{1-\psi}{d'} + \beta W_d(y,d',a')$$

- Again must solve for upper envelope given non-concavity.

Adjustment Solution

- This yields adjustment solutions

$$c^{adj}(y,x), a^{adj}(y,x), d^{adj}(y,x)$$

  and a value function

$$V^{adj}(y,x) = \psi \ln c^{adj}(y,x) + (1-\psi) \ln d^{adj}(y,x) \\ + \beta W(y', d^{adj}(y,x), a^{adj}(y,x))$$

- We interpolate onto the existing grid using $x = a + (1-\delta)(1-f)pd$ to get

$$c^{adj}(y,d,a), a^{adj}(y,d,a), d^{adj}(y,d,a)$$

  and value function

$$V^{adj}(y,d,a) = \psi \ln c^{adj}(a,d,y) + (1-\psi) \ln d^{adj}(a,d,y) + \beta W(y', d^{adj}(a,$$

Combined Problem

- Get policy functions by combining adjust and no-adjust solutions.

$$c'(y,d,a) = adjust(a,d,y)c^{adj}(a,d,y) + (1 - adjust(a,d,y))c^{noadj}(a,d,y$$
$$a'(y,d,a) = adjust(a,d,y)a^{adj}(a,d,y) + (1 - adjust(a,d,y))a^{noadj}(a,d,y$$
$$d'(y,d,a) = adjust(a,d,y)d^{adj}(a,d,y) + (1 - adjust(a,d,y))d^{noadj}(a,d,y$$

Outline

1. Introduction
2. Baseline FC Model
3. Continuous Time

Continuous Time

- Alternatively can formulate the problem in continuous time.
- Continuous time generally very good for handling stopping time problems, such as when to adjust.
- With discretized value function can write problem as

$$\min\{\rho v - u(v) - A(v)v, v - v^*(v)\} = 0$$

  - First term is the no-adjustment utility flow.
  - Second term is the adjustment choice.

- Intuition: if $v < v^*(v)$ then do not adjust and get flow value. If adjust $v = v^*(v)$ since the flow value $u(v) + A(v)v < \rho v$.

Considerations

- To solve continuous time problem see Ben Moll's code on stopping time problems.
- In principle continuous time better: do not require sufficiently large noise shocks to make value function differentiable.
- But harder to then apply discrete time sequence space methods. Even after aggregation, the distribution of states jumps after a shock so cannot use the fake news algorithm as implemented. (See McKay-Wieland 2021.)
- Possible that continuous time SSJ could be used. But need to handle the immediate jumps in the distribution.