

MACROECONOMICS OF CONSUMPTION WITH HETEROGENEITY

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UCSD, Spring 2024

OUTLINE

- 1 INTRODUCTION
- 2 AUCLERT, ROGNLIE, STRAUB (2018)
- 3 AUCLERT, ROGNLIE, STRAUB (2020)
- 4 BERGER, MILBRADT, TOURRE, VAVRA (2021)
- 5 MCKAY, WIELAND (2021)
- 6 MCKAY, WIELAND (2022)
- 7 BERAJA AND ZORZI (2024)

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BIG PICTURE

- At the heart of the NK transmission mechanism is the Euler equation.
- Empirical evidence suggests that consumer behavior is much more nuanced than PIH / intertemporal substitution behavior.
- What are the lessons from these data for policy?
- Today: explore these questions in HANK style models.

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SIMPLIFIED CONSUMER PROBLEM

- Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

- After-tax income:

$$z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it} \right)^{1-\lambda} = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}} Z_t$$

- Budget constraint:

$$c_{it} + \sum_j a_{it}^j = z_{it} + (1 + r_{t-1}) \sum_j a_{i,t-1}^j$$

- Nests: RA, TA, HA, HA-IL

OPTIMAL POLICY RULES

- Budget constraint with $Z_t = Y_t - T_t$:

$$c_{it} + \sum_j a_{it}^j = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}} (Y_t - T_t) + (1 + r_{t-1}) \sum_j a_{i,t-1}^j$$

- Idiosyncratic states: e_{it} .
- Assume aggregate variables follow a known sequence $\{Y_t - T_t, r_t\}_{t=0}^\infty$.
- Conditional on knowing these sequences, can solve for individual consumption and assets given any initial state $\{a_0^j\}, e_{i0}$:

$$c_t(\{a^j\}, e), \{a_t^j(\{a^j\}, e)\}, \quad t \geq 0$$

- Why? Distribution only enters consumers problem through effect on $\{Y_t - T_t, r_t\}_{t=0}^\infty$. No longer have infinite state space.
- Then aggregate all consumption decisions to get:

$$C_t = \int c_t(\{a^j\}, e) d\Psi_t(\{a^j\}, e) = \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^\infty)$$

NEW KEYNESIAN MODEL IN SEQUENCE SPACE

- Consumption function:

$$\begin{aligned} C_t &= \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty}) \\ &\equiv \mathbb{C}(Y - T, r) \end{aligned}$$

- NKPC:

$$\pi_t = \mathbb{S}(Y - Y^*)$$

- Interest rate rule:

$$r = \Phi_Y(Y - Y^*) + \Phi_{\pi}\pi + \varepsilon^r$$

- Excess demand:

$$ED = \mathbb{C}(Y - T, r) + G - Y = 0$$

LINEARIZE TO SOLVE WITH LINEAR ALGEBRA

- Linearized Model:

$$\hat{\mathbf{C}} = (\nabla_Y \mathbf{C})(\hat{Y} - \hat{T}) + (\nabla_r \mathbf{C})\hat{r}$$

$$\hat{\boldsymbol{\pi}} = (\nabla_Y \mathbb{S})(\hat{Y} - \hat{Y}^*)$$

$$\hat{r} = \Phi_Y(\hat{Y} - \hat{Y}^*) + \Phi_\pi \hat{\boldsymbol{\pi}} + \boldsymbol{\varepsilon}'$$

where the Jacobians of the model look like:

$$\nabla_Y \mathbf{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \dots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \dots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- Equilibrium:

$$\begin{aligned} \hat{Y} &= [\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} \{ [\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} - [I - \nabla_Y \mathbf{C}]^{-1} (\nabla_r \mathbf{C}) \}^{-1} \times \\ &\quad \times [I - \nabla_Y \mathbf{C}]^{-1} [\hat{G} - (\nabla_Y \mathbf{C}) \hat{T} + (\nabla_r \mathbf{C}) \boldsymbol{\varepsilon}'] \end{aligned}$$

DEMAND AND SUPPLY ELASTICITIES DETERMINE GE

- Define:

- ▶ Demand multiplier $\mu^D \equiv [I - \nabla_Y \mathbf{C}]^{-1}$
- ▶ Demand elasticity $\eta^D \equiv -\nabla_r \mathbf{C}$
- ▶ Supply elasticity $\eta^S \equiv [\Phi_Y + \Phi_\pi(\nabla_Y \mathbf{S})]^{-1}$

$$\hat{Y} = \underbrace{\eta^S[\eta^S + \mu^D \eta^D]^{-1}}_{\text{Demand Incidence}} \mu^D \underbrace{[\hat{G} - (\nabla_Y \mathbf{C}) \hat{T} + \eta^D \varepsilon^r]}_{\text{PE Excess Demand}} \\ + \underbrace{\mu^D \eta^D [\eta^S + \mu^D \eta^D]^{-1}}_{\text{Supply Incidence}} \underbrace{\hat{Y}^*}_{\text{PE Excess Supply}}$$

Implications:

- GE effects determined by a standard incidence formula.
- ⇒ Matrices of micro elasticities are sufficient statistics for GE effects.

INTERTEMPORAL KEYNESIAN CROSS

- Also assume no real rate change, so η^s blows up in some meaningful sense.

$$\hat{Y} = [I - \nabla_Y \mathbf{C}]^{-1} [\hat{G} - (\nabla_Y \mathbf{C}) \hat{T}]$$

- This is the Intertemporal Keynesian Cross.
- Balanced budget: $\hat{G} = \hat{T}$

$$\begin{aligned}\hat{Y} &= [I - \nabla_Y \mathbf{C}]^{-1} [I - (\nabla_Y \mathbf{C})] \hat{G} \\ &= \hat{G}\end{aligned}$$

- The balanced budget multiplier is 1.

FISCAL POLICIES

- Proposition 4: In the RA model, the G-multiplier is 1.
 - ▶ Proof: The G-multiplier is 1 when budget is balanced, and we know for the RA model the timing of T does not matter.
- Proposition 5: in the TA model with μ constrained agents,

$$\hat{Y} = \frac{1}{1-\mu} [\hat{G} - \mu \hat{T}]$$

- ▶ Deficit-financed fiscal expansion ($\hat{T}_0 < \hat{G}_0$) has impact multiplier greater than 1.

MEASURING $\nabla_Y \mathbf{C}$

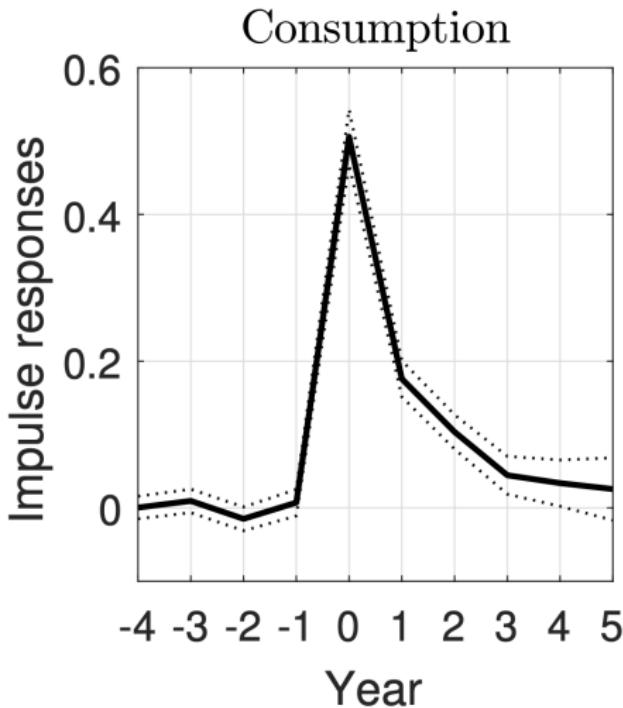
- Demand multiplier: $\boldsymbol{\mu}^D \equiv [I - \nabla_Y \mathbf{C}]^{-1}$ where

$$\nabla_Y \mathbf{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \dots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \dots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Columns are PE IRFs of average consumption to income changes at different horizons.

- What evidence do we have?

LOTTERY STUDIES

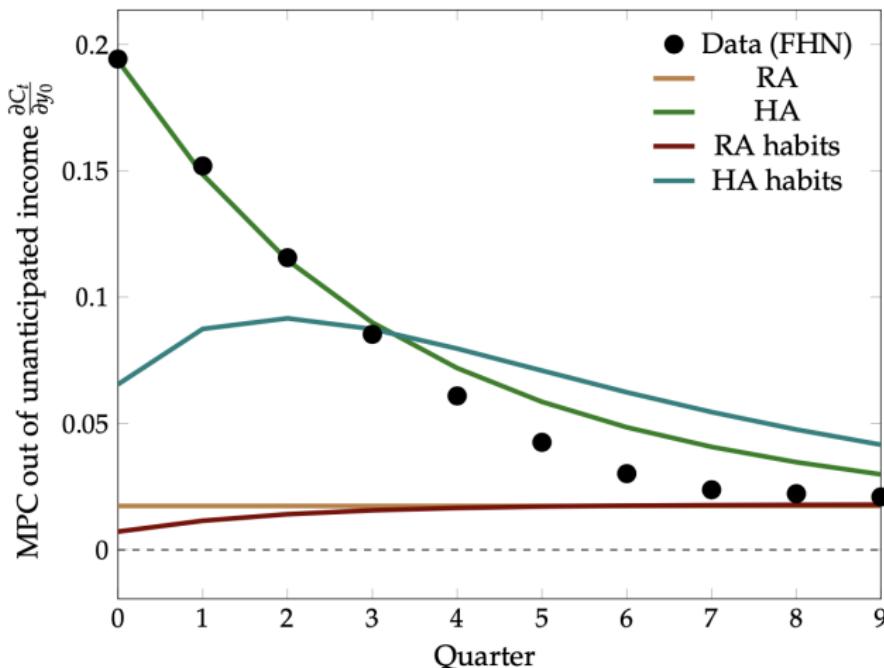


- Provides estimate of first column of $\nabla_{\mathbf{y}} \mathbf{C}$.
- Need a model to extrapolate.

Source: Fagereng, Holm, Natvik (2018)

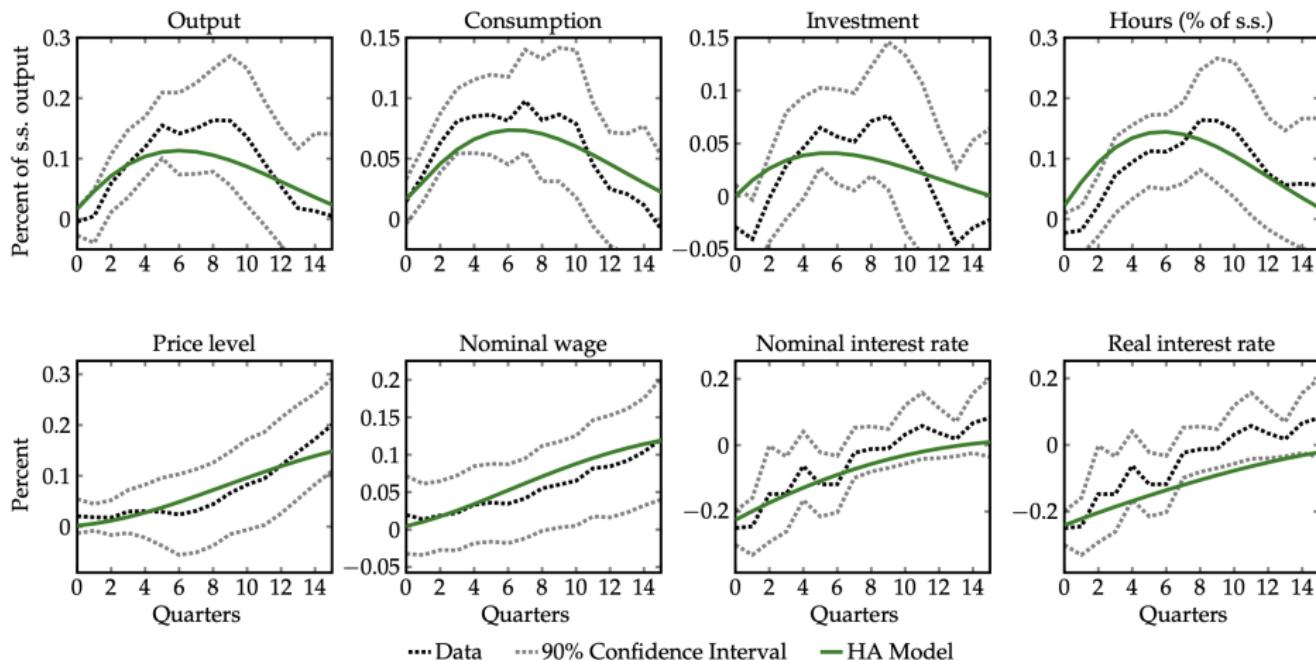
IMPCs IN THE MODELS

Figure 2: Intertemporal MPCs $\partial C_t / \partial y_0$ in models and in the data



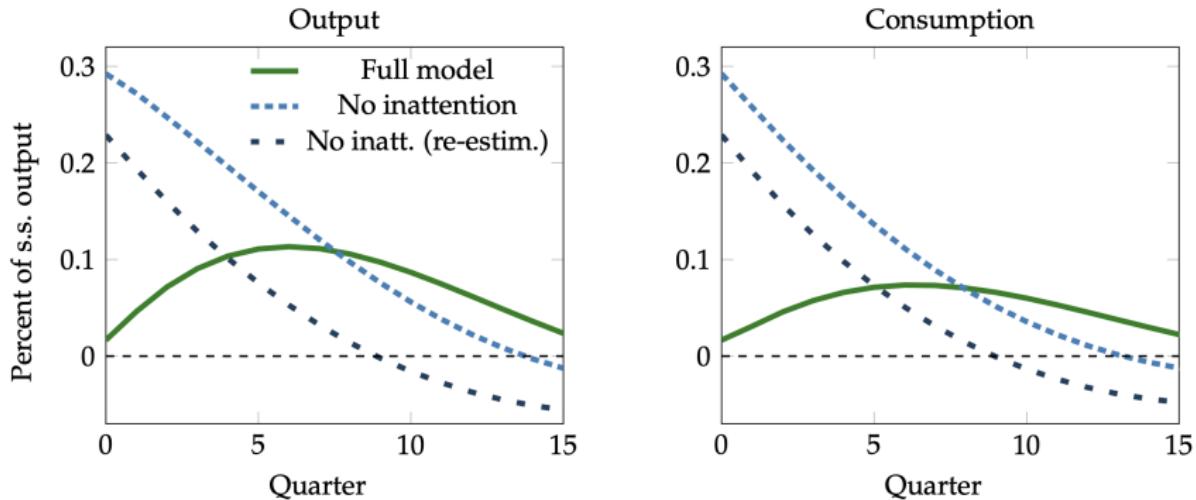
MODEL EXTRAPOLATION

Figure 3: Impulse response to a monetary policy shock vs. model fit



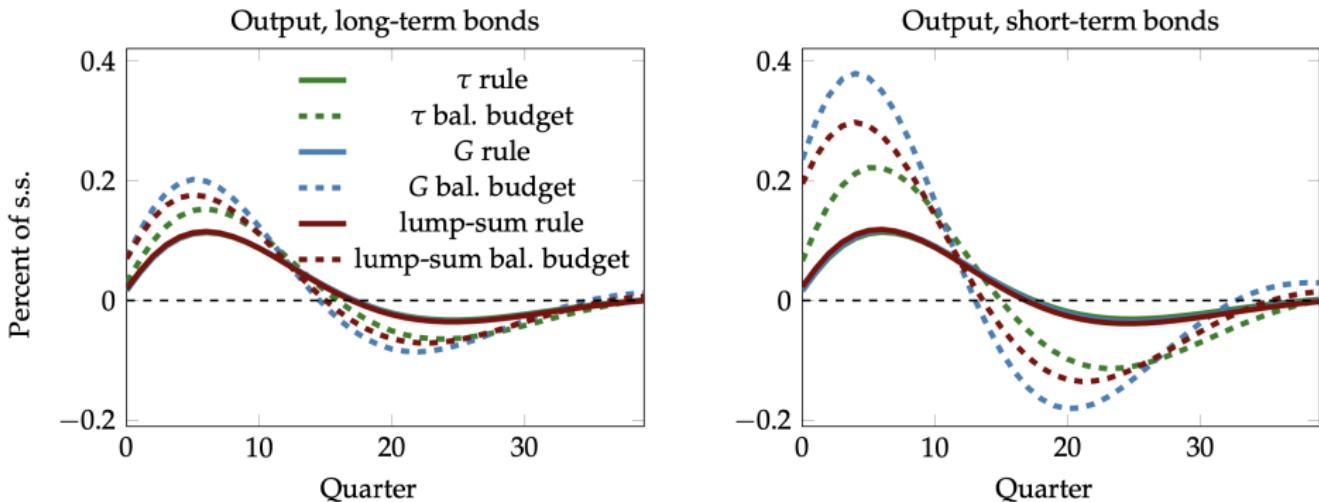
FISCAL POLICIES: ALL MODELS

Figure 4: Impulse responses with and without inattention



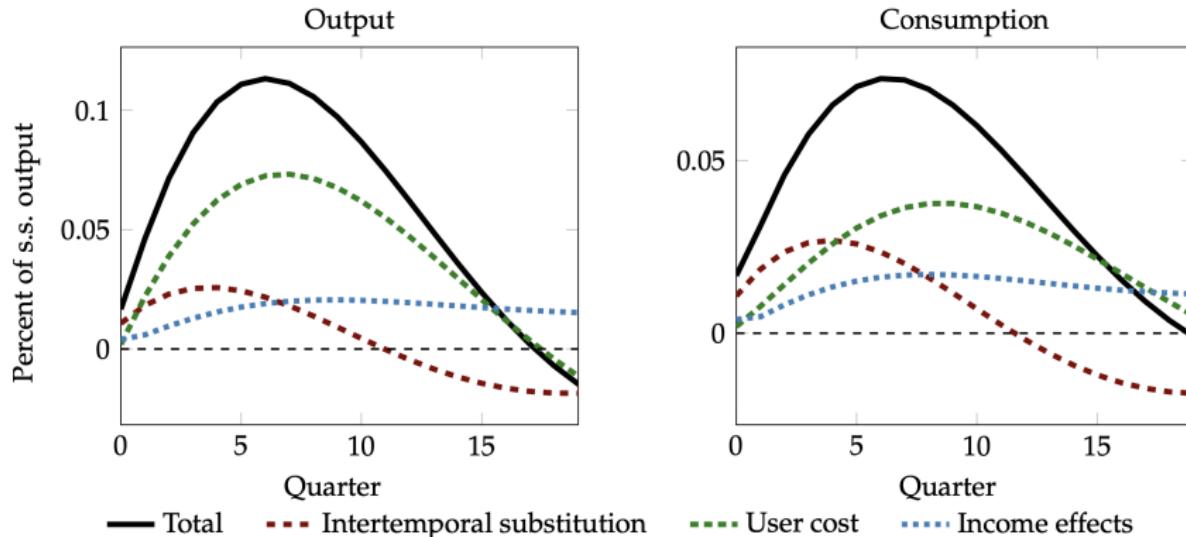
QUANTITATIVE MODEL

Figure 7: The role of fiscal policy for monetary transmission



- What does this tell us about supply elasticities?

PRIVATE DEFICITS



- Why so much amplification?

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$$\hat{r} = \Phi_Y(\hat{Y} - \hat{Y}^*) + \Phi_\pi \hat{\boldsymbol{\pi}} + \boldsymbol{\varepsilon}^r$$

- Equilibrium:

$$\begin{aligned}\hat{Y} = & [\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} \{ [\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} - [I - \nabla_Y \mathbf{C}]^{-1}(\nabla_r \mathbf{C}) \}^{-1} \times \\ & \times [I - \nabla_Y \mathbf{C}]^{-1} [\hat{\mathbf{G}} - (\nabla_Y \mathbf{C})\hat{T} + (\nabla_r \mathbf{C})\boldsymbol{\varepsilon}^r]\end{aligned}$$

- Kaplan, Moll, and Violante (2018):

- ▶ Small direct effect $\nabla_r \mathbf{C}$
- ▶ Large indirect effect $[I - \nabla_Y \mathbf{C}]^{-1}$
- ▶ Product roughly constant $[I - \nabla_Y \mathbf{C}]^{-1}(\nabla_r \mathbf{C})$

⇒ Should we care about the decomposition?

ADD INVESTMENT

- Yes:

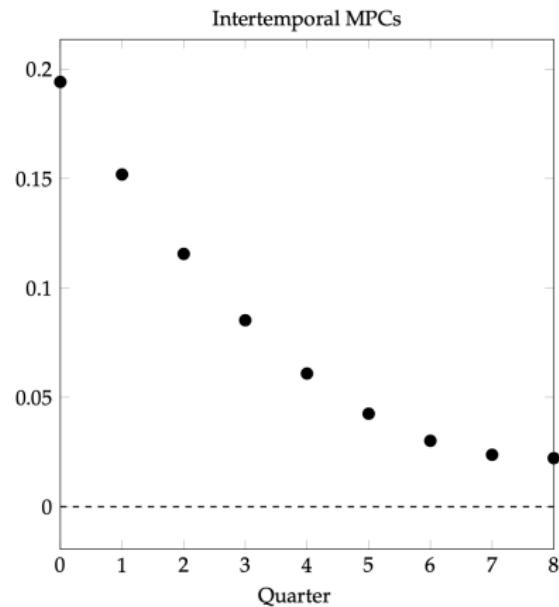
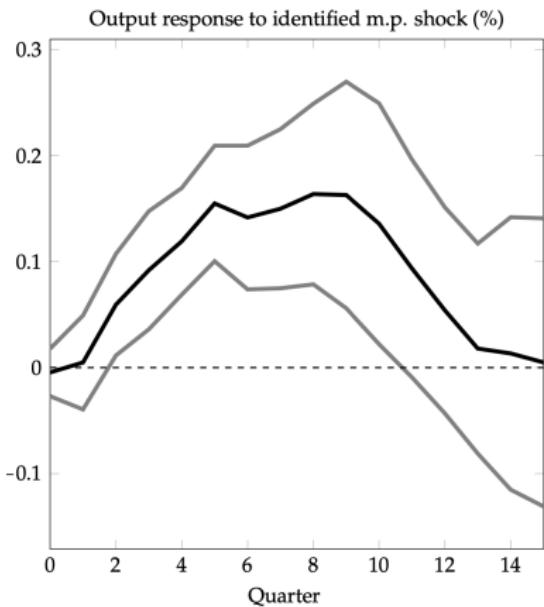
$$\begin{aligned}\hat{\mathbf{Y}} = & [\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} \times \\ & \times \{[\Phi_Y + \Phi_\pi(\nabla_Y \mathbb{S})]^{-1} - [I - \nabla_Y \mathbb{C} - \nabla_Y \mathbb{I}]^{-1} (\nabla_r \mathbb{C} + \nabla_r \mathbb{I})\}^{-1} \times \\ & \times [I - \nabla_Y \mathbb{C} - \nabla_Y \mathbb{I}]^{-1} [\hat{\mathbf{G}} - (\nabla_Y \mathbb{C}) \hat{\mathbf{T}} + (\nabla_r \mathbb{C} + \nabla_r \mathbb{I}) \boldsymbol{\varepsilon}']\end{aligned}$$

- Key challenge: How to set up model to match

- ▶ Immediate consumption response to higher income ("jump").
- ▶ Delayed consumption, investment, output response to lower real interest rate ("hump").

THE CHALLENGE

Figure 1: Macro Humps, Micro Jumps.



MODEL

- Household problem:

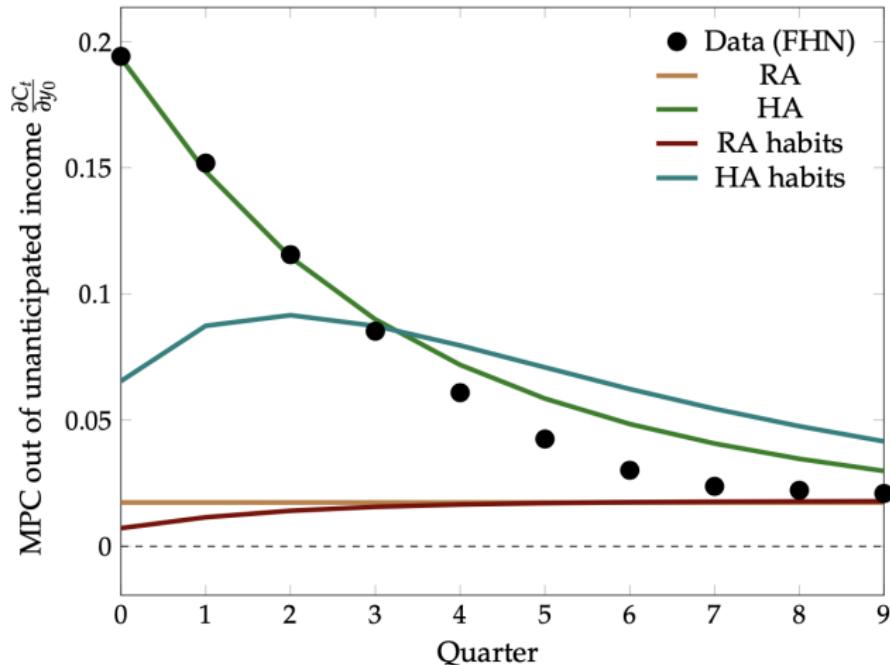
$$\begin{aligned}V_t(l, s) &= \max_{c, l'} u(c) + \beta \mathbb{E}[V_{t+1}(l', s') | s] \\c' + l' &\leq (1 + r_t)l + y_t e(s) \\l' &\geq 0\end{aligned}$$

- Standard sequence space result:

$$C_t = \mathcal{C}(\{y_s, r_s\}_{s \geq 0}), \quad t \geq 0$$

MATCHING MICRO DATA

Figure 2: Intertemporal MPCs $\partial C_t / \partial y_0$ in models and in the data



- Why does habit model have difficulty?

INATTENTION

- Households know current micro state $e(s)$.
- Households update info about aggregate shocks with iid probability $1 - \theta$ each period.
- Households know current r_t, Y_t but forecast r_{t+s}, Y_{t+s} using info from $t - k$.
⇒ Households always on budget constraint.
- Optimize subject to information from k periods ago:

$$V_t(l, s, k) = \max_{c, l'} u(c) + \beta \mathbb{E}_{t-k} [\theta V_{t+1}(l', s', k+1) + (1-\theta) V_{t+1}(l', s', 0)]$$

ESTIMATION

- Two-step procedure.
- Calibrate household problem to micro data, so only need to compute household sequence space Jacobians once.
- Estimate information and macro parameters to match estimated IRF to monetary policy shocks.
- Sticky info micro Jacobian at t to shock at time s :

$$\mathcal{J}_{t,s}^{o,i} = \begin{cases} \theta \mathcal{J}_{t-1,s-1}^{o,i} + (1-\theta) \mathcal{J}_{t,s}^{o,i,FI} & t > 0, s > 0 \\ \mathcal{J}_{t,s}^{o,i,FI} & s = 0 \\ (1-\theta) \mathcal{J}_{t,s}^{o,i,FI} & t = 0, s > 0 \end{cases}$$

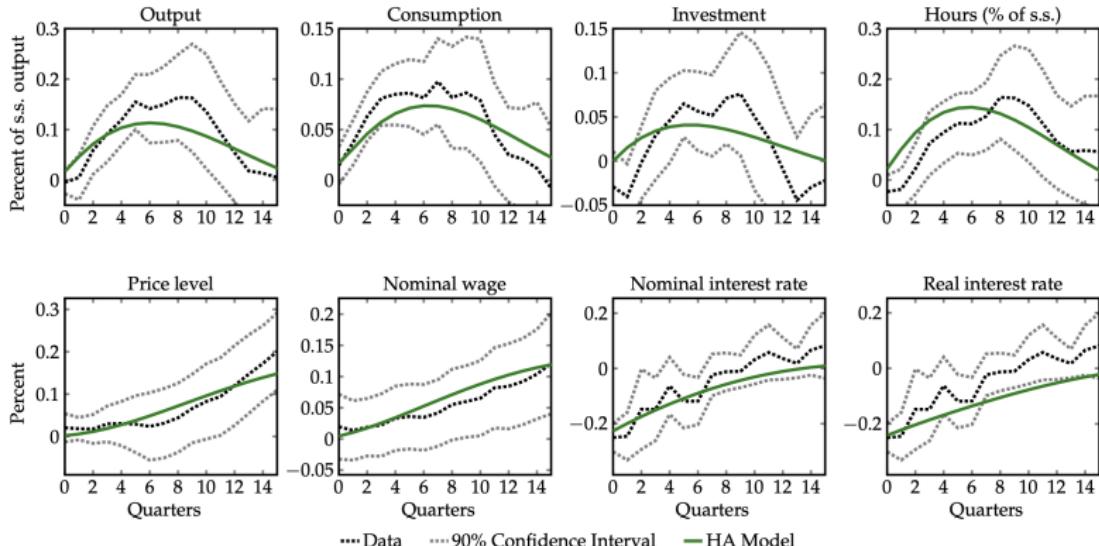
CALIBRATION

Table 2: Calibrated and estimated parameters.

Panel A: Calibrated parameters		Panel B: Estimated parameters		
Parameter		Value	Parameter	Value
ν	EIS	1	θ	Household inattention
ζ	Frisch	0.5	ϕ	Investment adj. cost parameter
β_g	Discount factors (p.a.)	Table 1	ζ_p	Calvo price stickiness
r	Real interest rate (p.a.)	0.050	ζ_w	Calvo wage stickiness
α	Capital share	0.24	ρ^m	Taylor rule inertia
δ_K	Depreciation of capital (p.a.)	0.053	σ^m	Std. dev. of monetary shock
μ_p	Steady-state retail price markup	1.06		
K/Y	Capital to GDP (p.a.)	2.23		
L/Y	Liquid assets to GDP (p.a.)	0.23		
ξ	Intermediation spread (p.a.)	0.065		
G/Y	Spending-to-GDP	0.16		
qB/Y	Government bonds to GDP (p.a.)	0.42		
$\frac{1+r}{1+r-\delta}$	Maturity of government debt (a.)	5		
ψ	Response of tax rate to debt (p.a.)	0.1		
ϕ_π	Taylor rule coefficient	1.5		

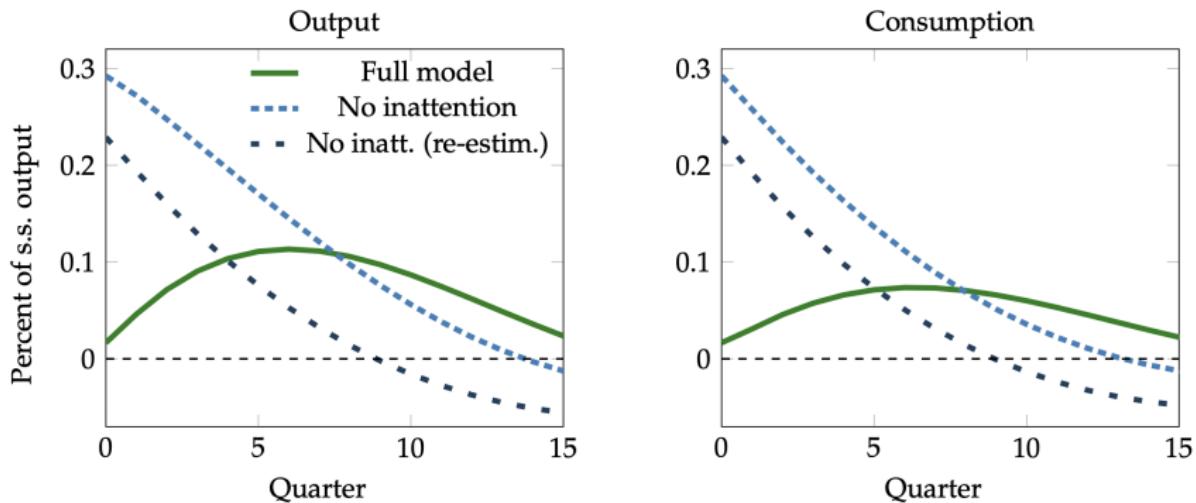
MATCHING THE HUMPS

Figure 3: Impulse response to a monetary policy shock vs. model fit



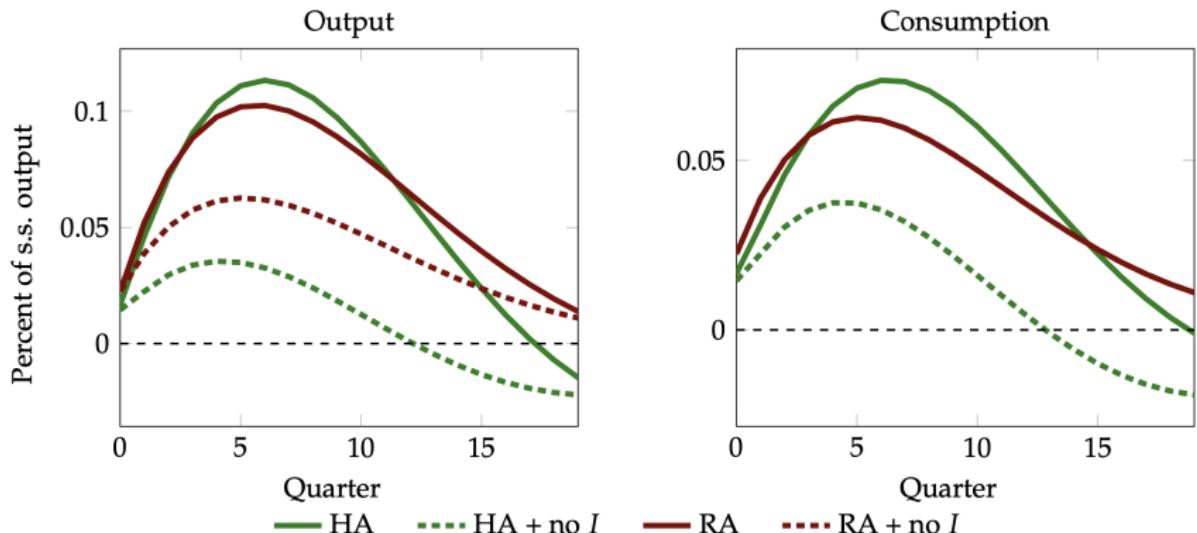
ROLE OF INATTENTION

Figure 4: Impulse responses with and without inattention



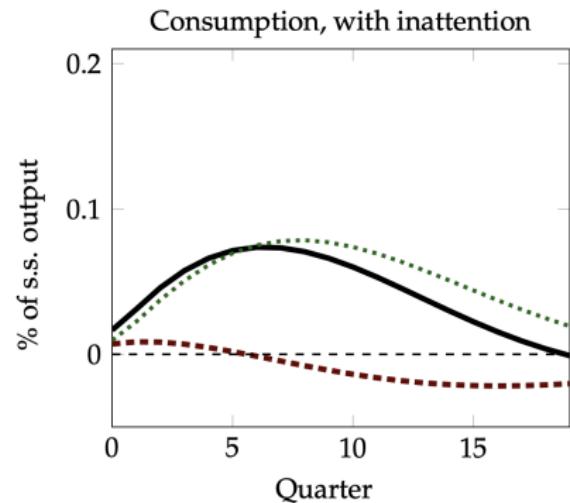
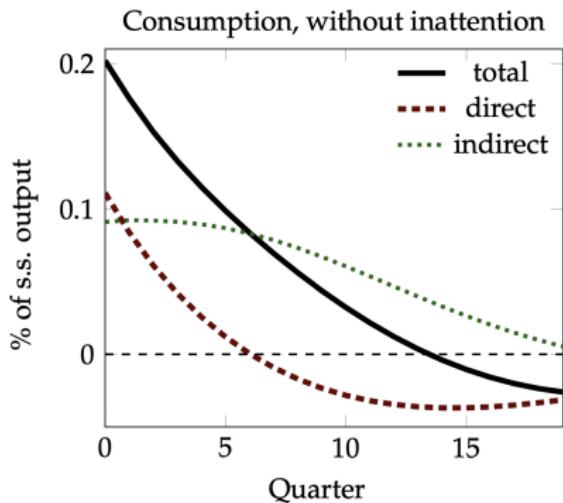
IMPORTANCE OF INVESTMENT

Figure 5: Role of investment in the transmission mechanism



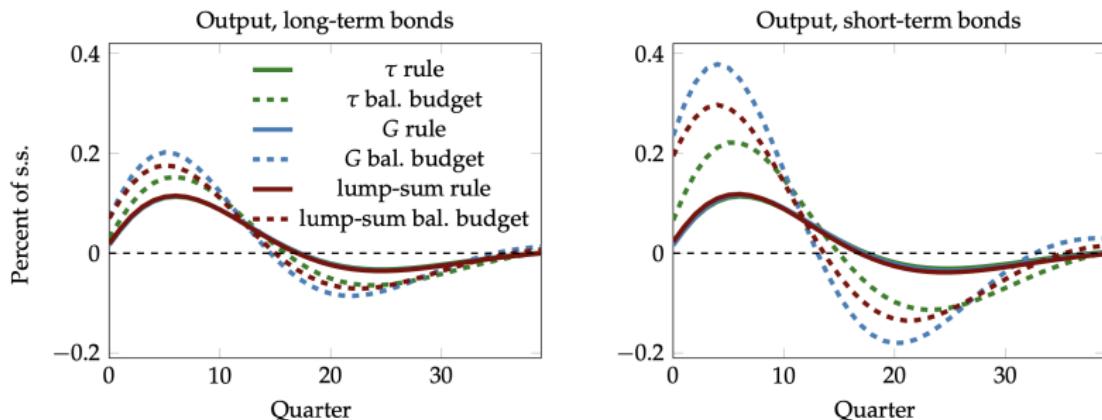
DIRECT VS INDIRECT EFFECTS

Figure 6: Decomposition of consumption

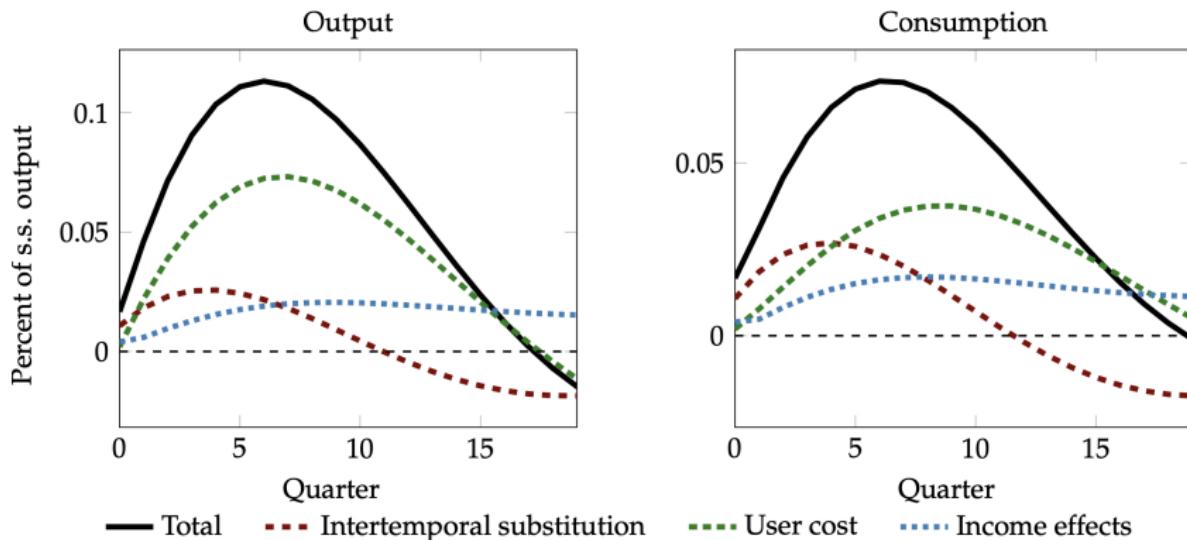


ASIDE: ROLE OF FISCAL POLICY

Figure 7: The role of fiscal policy for monetary transmission

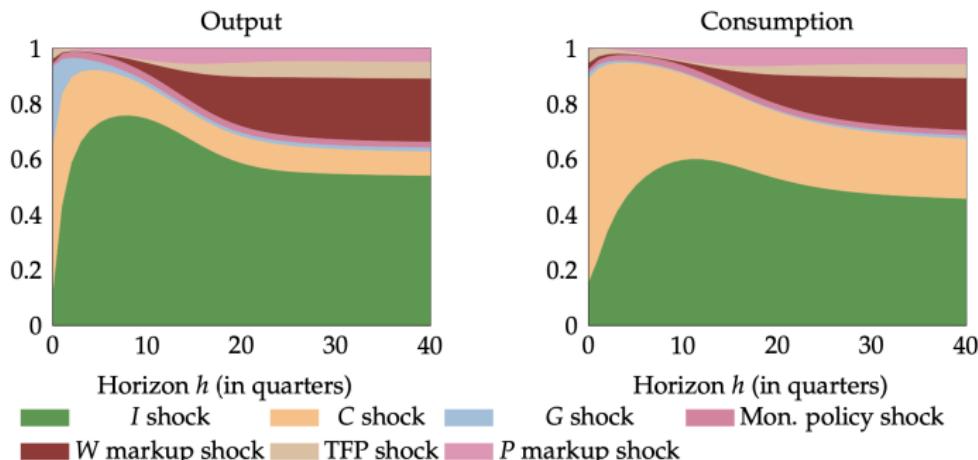


STARTING THE TRANSMISSION MECHANISM



REASSESSING THE IMPORTANCE OF INVESTMENT

Figure 12: Forecast error variance decomposition for the HA model



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POLICY SPACE

- With ZLB constraining $i_t \geq 0$ (or small negative number), central banks can only lower interest rates a finite amount.
- This means that central bank can only offset an AD shock of finite size. This is the “policy space” or “limited ammunition.”
- This paper: when lower interest rates today, then also reduce *responsiveness* to future interest rate changes (up or down).
- Means that past interest rate cuts limit future policy face as subsequent interest rate cuts less effective.

REFINANCING (PRE-PAYMENT)

- Estimate effect of rate gap on refinancing probability.

$$prepay_{i,j,t} = \beta_{gapbin} 1(gapbin)_{j,t} + \beta_X X_{i,j,t} + \delta_i + \varepsilon_{i,j,t}$$

- Identification assumption?

REFINANCING (PRE-PAYMENT)

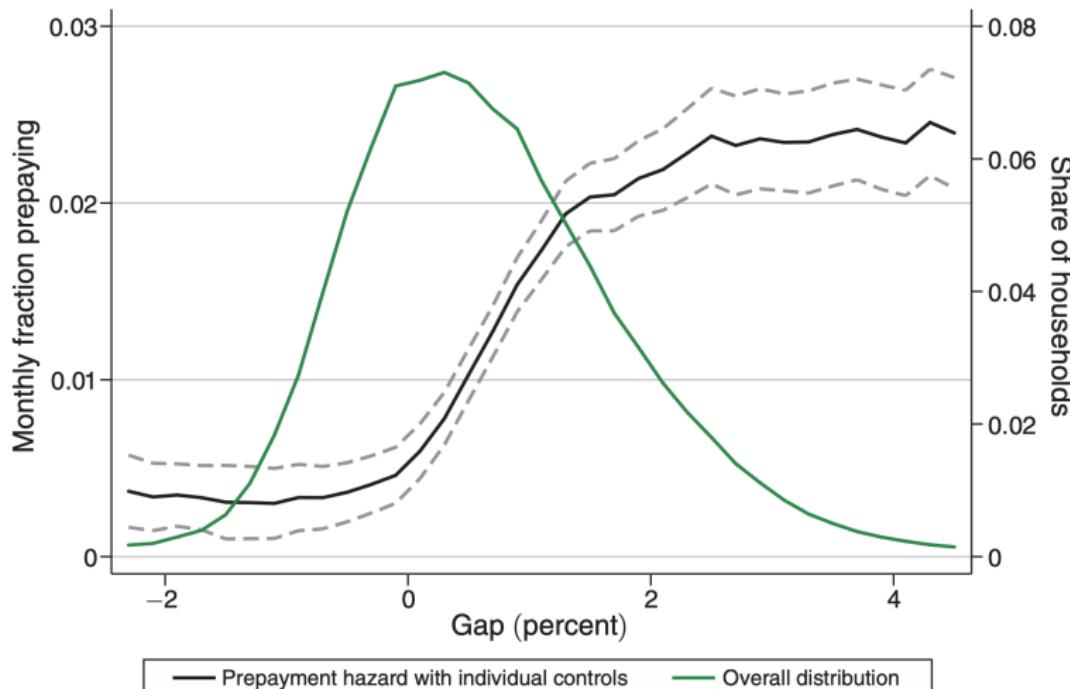


FIGURE 1. PREPAYMENT HAZARD WITH INDIVIDUAL CONTROLS

MOTIVES

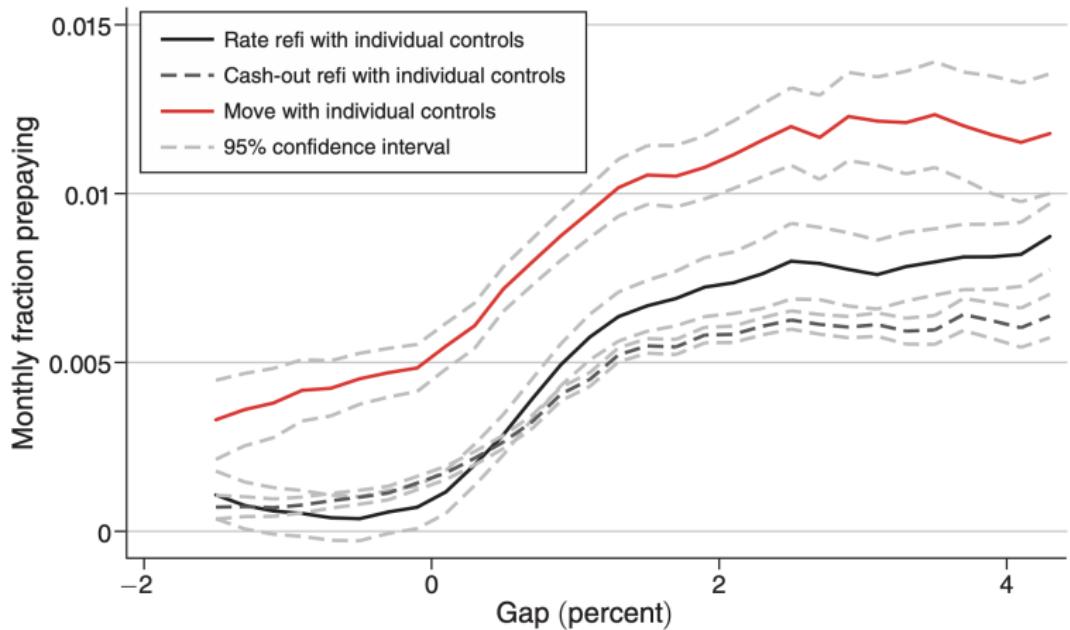


FIGURE 3. PREPAYMENT HAZARD DECOMPOSITION WITH INDIVIDUAL CONTROLS

AGGREGATE CORRELATION

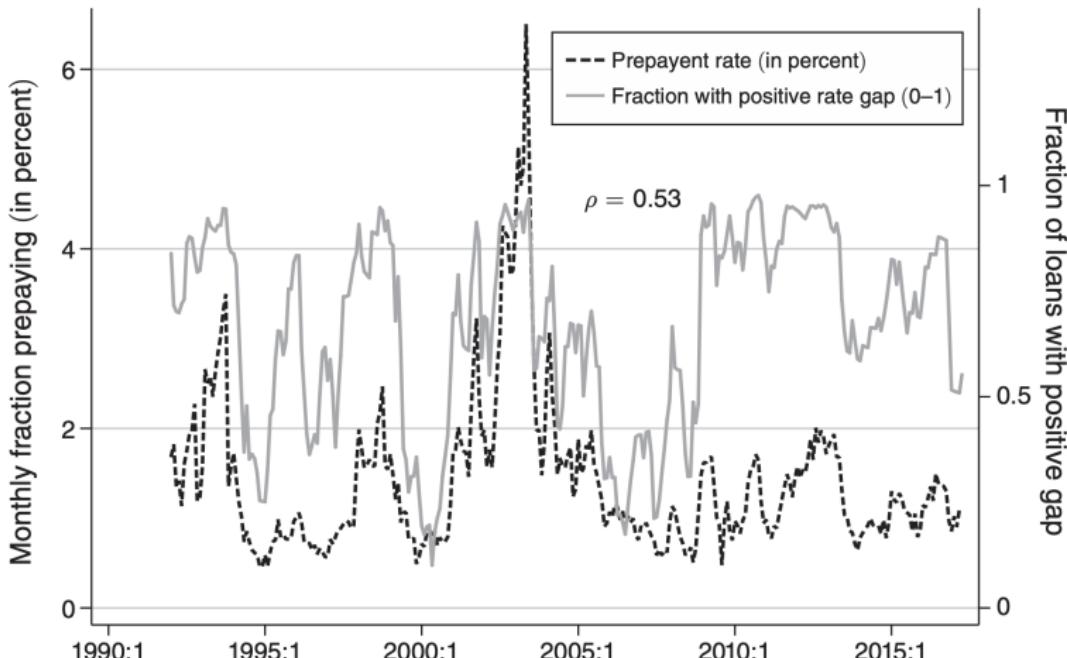


FIGURE 5. PREPAYMENT VERSUS FRACTION WITH POSITIVE RATE GAP TIME SERIES

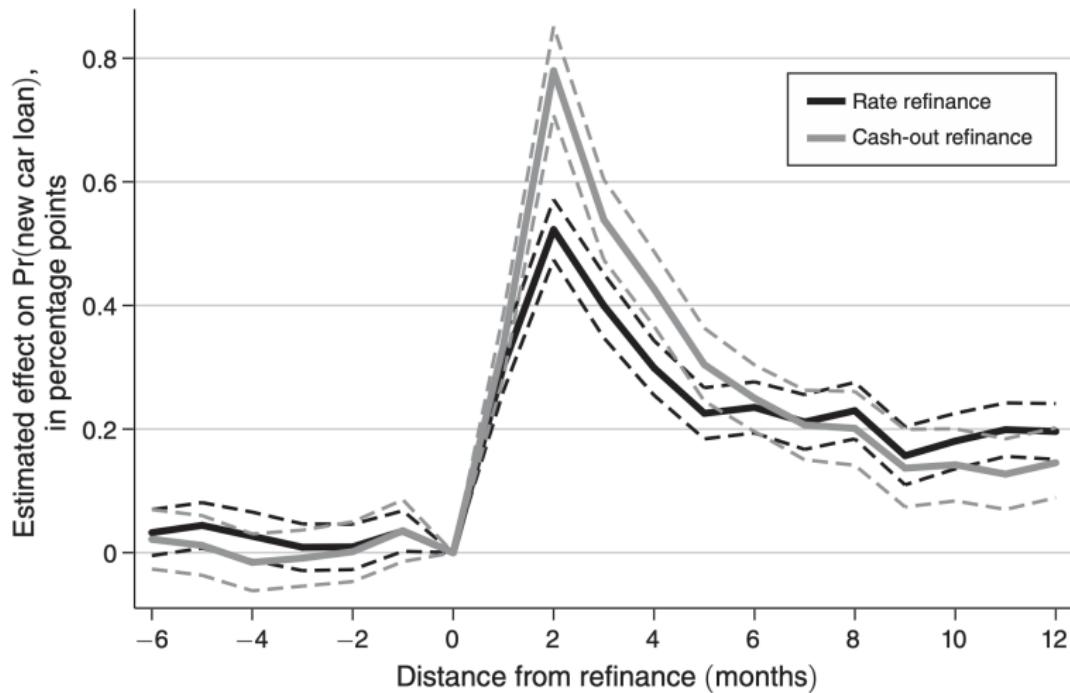
SPENDING

- Estimate effect of refinancing probability on spending.

$$1(\text{carloan})_{i,t} = \sum_k 1(\text{refinanced})_{i,t-k} + \delta_i + \delta_t + \varepsilon_{i,t}$$

- Identification assumption?

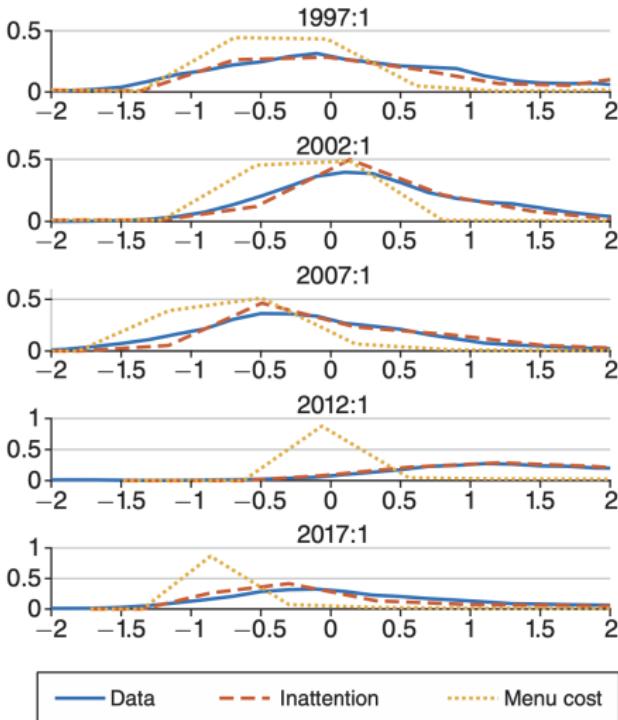
SPENDING



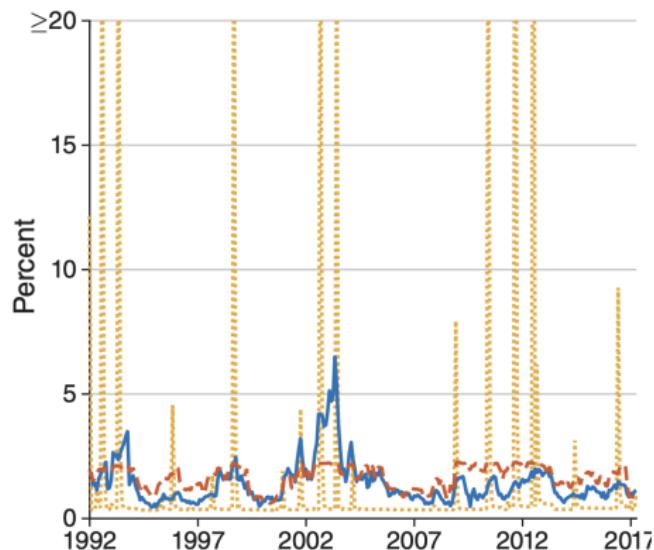
WHY DO WE NEED A MODEL?

GAPS AND FREQUENCY

Panel A. Distribution of gaps



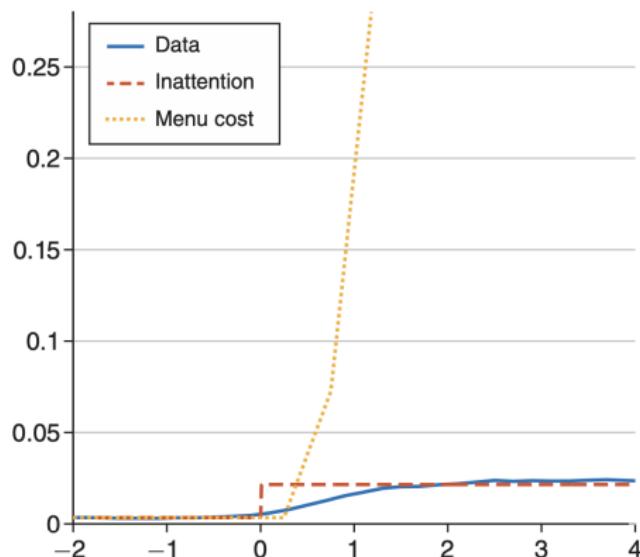
Panel B. Frequency



- Monthly prepayment frequency: data
- - - Monthly prepayment frequency: inattention
- Monthly prepayment frequency: menu cost

HYBRID MODEL

Panel A. Inattention and menu cost versus data



Panel B. Hybrid versus data

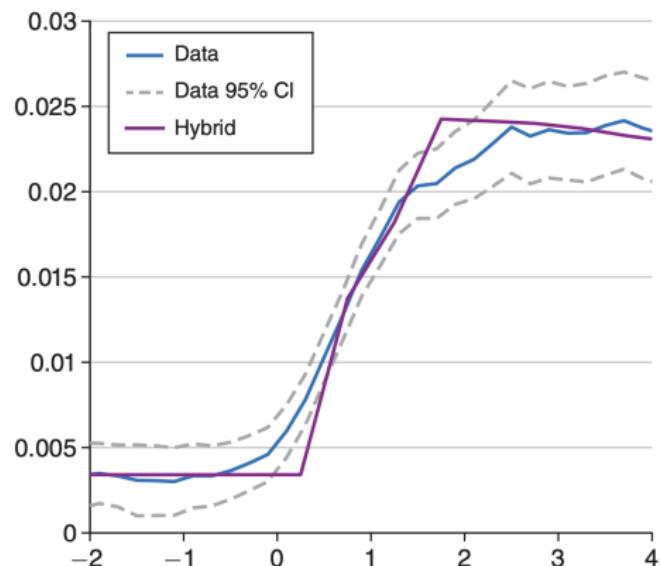
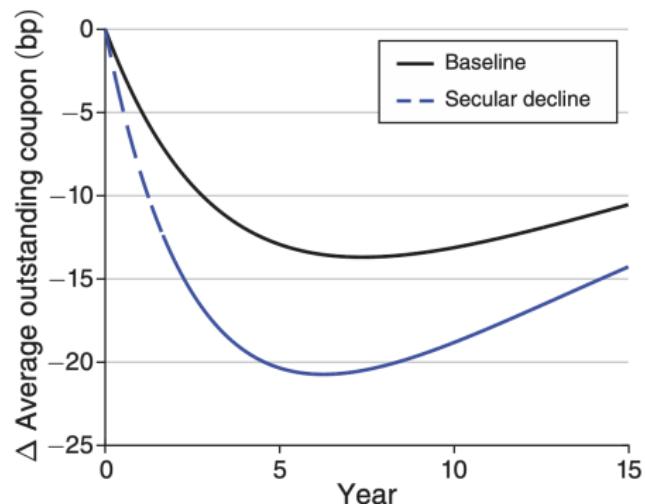


FIGURE 10. PREPAYMENT HAZARDS

STATE DEPENDENCE

Panel A. Average coupon m^*



Panel B. Monthly prepayment flows

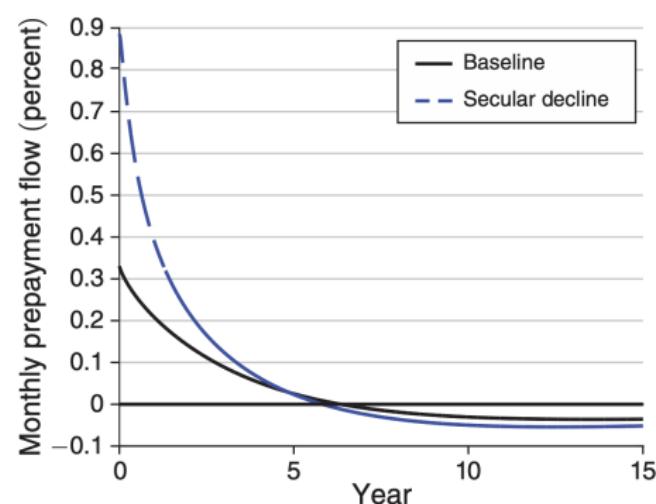
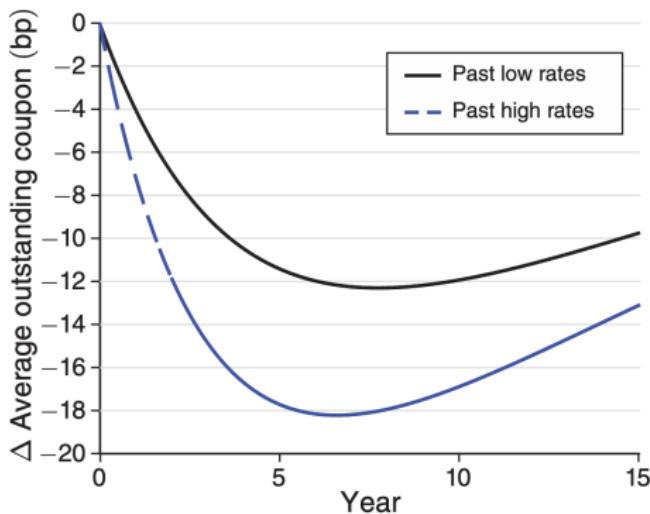


FIGURE 12. IMPULSE RESPONSE FUNCTIONS TO 100 BP DECLINE IN r

STATE DEPENDENCE

Panel A. Average coupon m^*



Panel B. Monthly prepayment flows

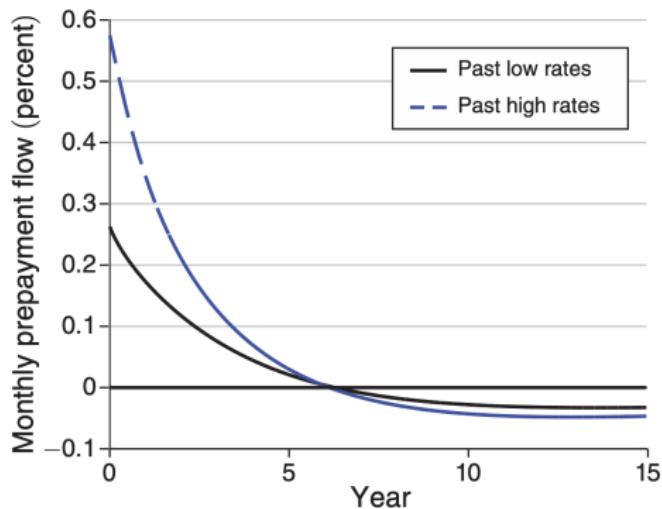
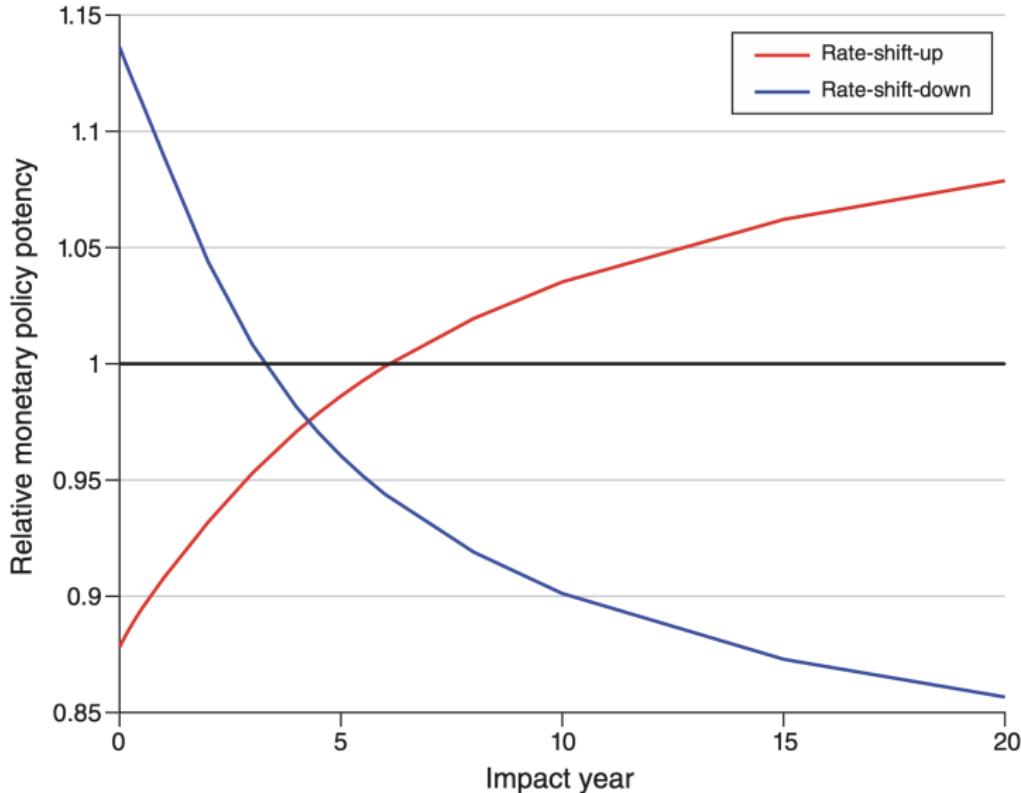


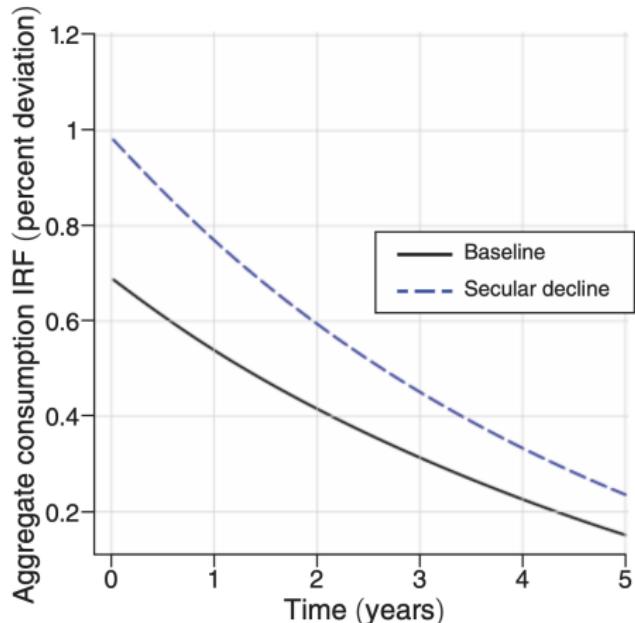
FIGURE 14. IRF OF AVERAGE COUPON m^* TO 100 BP DECLINE IN r

SHIFT IN POLICY STANCE



CONSUMPTION EFFECTS

Panel A. 100 bp shock



Panel B. Max shock

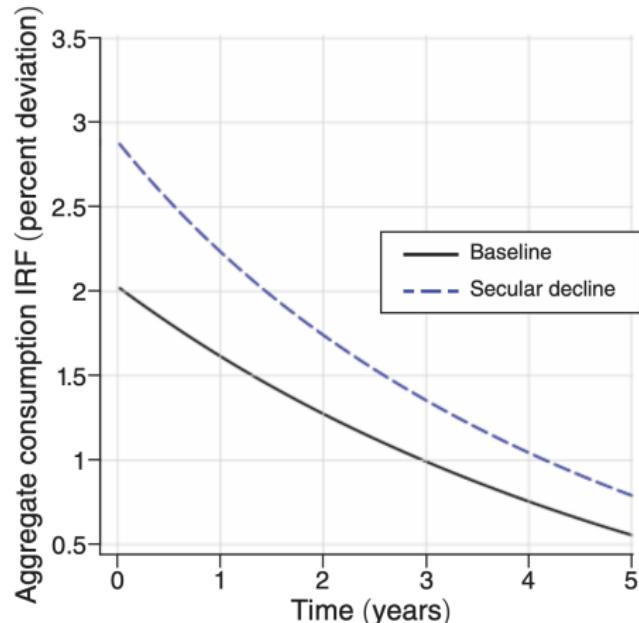


FIGURE 17. IRF OF CONSUMPTION—BASELINE VERSUS SECULAR DECLINE

QUESTIONS

- Convincing?

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- 6 MCKAY, WIELAND (2022)
- 7 BERAJA AND ZORZI (2024)

DOES CENTRAL BANK CREATE OR BORROW DEMAND?

- In standard NK model:

$$\begin{aligned}y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\&= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T \rightarrow \infty} y_{t+1}\end{aligned}$$

- Central bank creates demand and output by changing r_t .
 - Past r_t has no bearing on future AD.
- ⇒ Central bank can create AD subject to ZLB constraint.
- This paper: with durable goods, central bank is much more in the business of borrowing demand than creating demand.

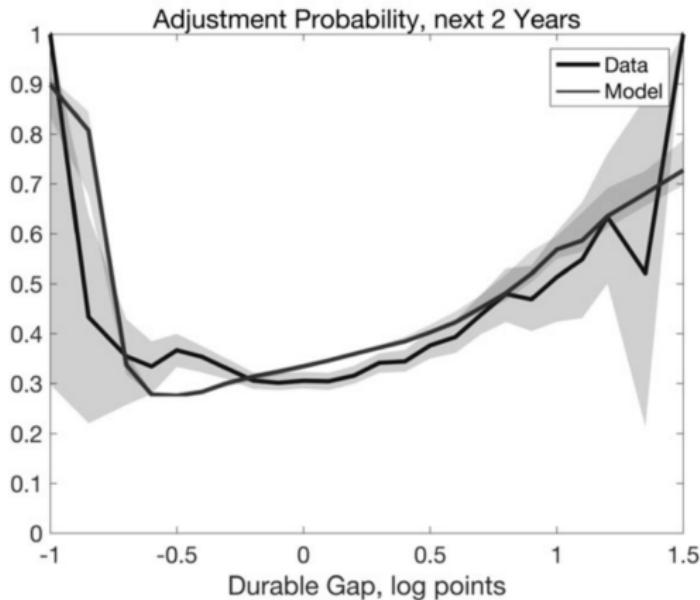
MODEL: HOUSEHOLDS

- Households consume non-durables and durables.

$$\max E_{j0} \int_{t=0}^{\infty} e^{-\rho t} u(c_{it}, q_{it} d_{it}) dt$$

- Durables subject to:
 - ▶ fixed adjustment cost (if adjust) $a'_{it} + p_t d'_{it} = a_{it} + (1-f)p_t d_{it}$
 - ▶ depreciation and maintenance cost $dtd_{it} = -(1-\chi)\delta d_{it}$
 - ▶ operating cost (e.g. household utilities, gas, taxes) $v d_{it}$
 - ▶ match-quality shocks $q_{it} = 1$, drops to zero with intensity θ
- Idiosyncratic labor income risk
$$y_{it} = (1 - \tau_t) Y_t z_{it} \quad d \ln z_{it} = -\rho_z \ln z_{it} + \sigma_z d \mathcal{W}_{it}$$
- Save in liquid assets
$$dta_{it} = r_t a_{it} + r_t^b a_{it} I_{\{a_{it} < 0\}} - c_{it} + y_{it} - (\chi \delta p_t + v) d_{it}$$
- Collateralized borrowing $a_{it} \geq -\lambda(1-f)p_t d_{it}$
- Sticky information (Carrol et al, 2018; Auclert et al, 2020).

HAZARD RATE



INTERTEMPORAL SHIFTING IN THE DATA

LUMPY DURABLE CONSUMPTION DEMAND

2727

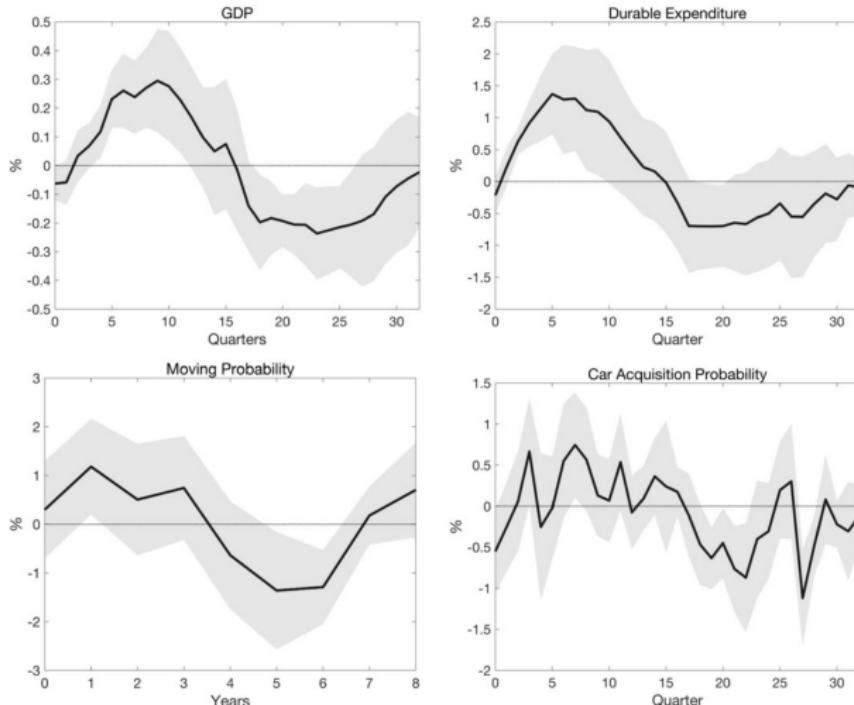


FIGURE 2.—Impulse response function of real GDP (top-left panel), real durable expenditure (top-right), probability of moving house (bottom-left), and probability of buying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence bands.

INTERTEMPORAL SHIFTING IN THE DATA

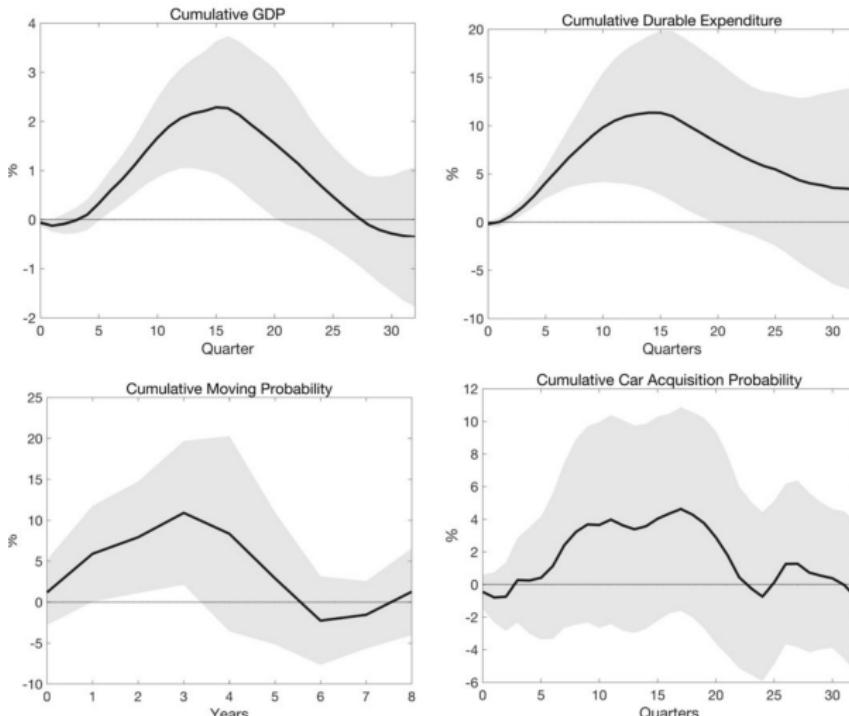


FIGURE 3.—Impulse response function of cumulative real GDP (top-left panel), cumulative real durable expenditure (top-right), cumulative probability of moving house (bottom-left), and cumulative probability of buying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence bands.

MODEL VS DATA

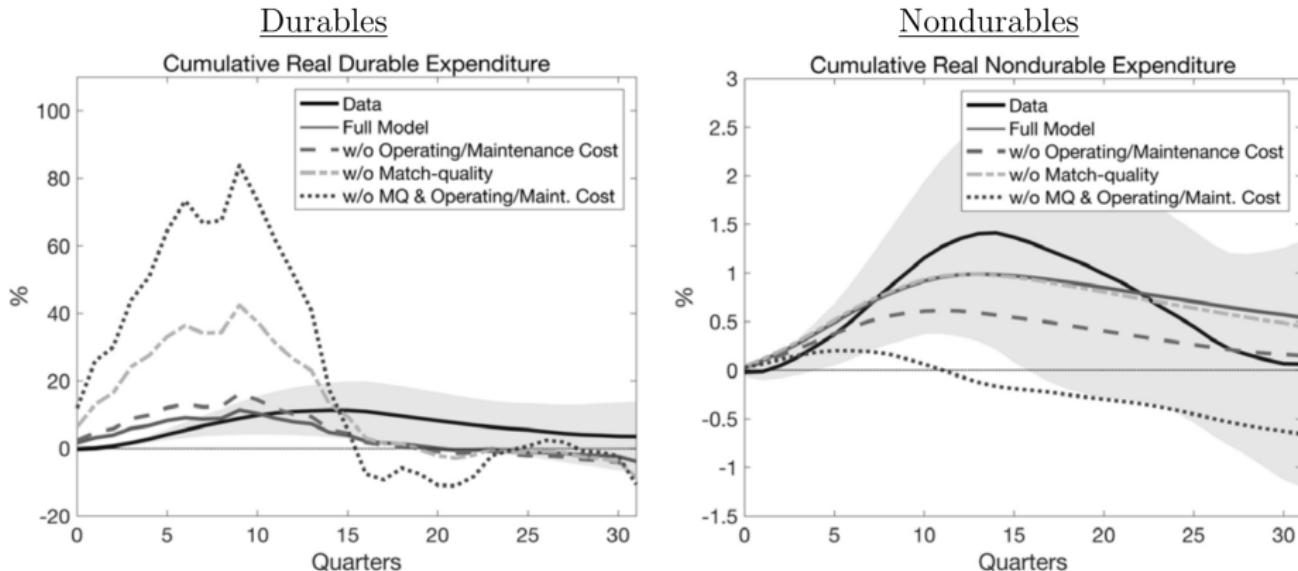
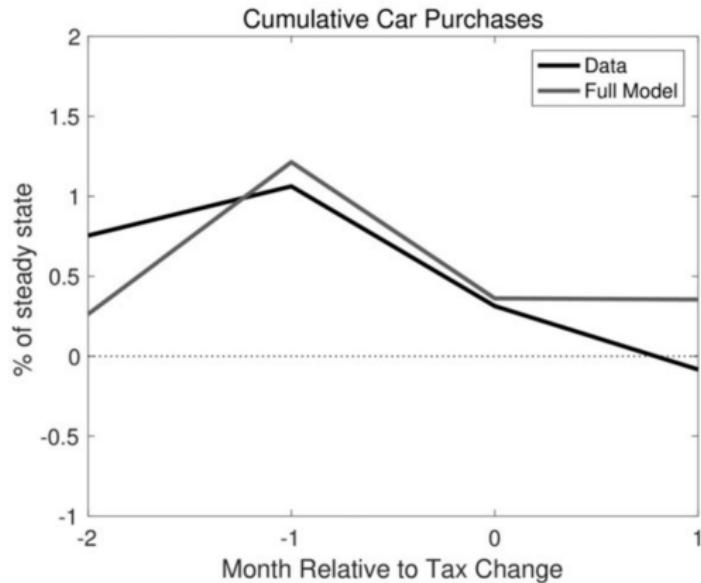


FIGURE 4.—Cumulative response of durable (left) and nondurable (right) expenditure to a simulated monetary policy shock. Model simulations feeding in the estimated impulse responses for (Y_t, r_t, p_t) . Each panel shows the full model as well as the models that omit one or both of match-quality shocks and operating and maintenance costs.

MODEL vs DATA



THE MONETARY TRANSMISSION MATRIX

- Solve model with sequence space methods.
- The “monetary transmission matrix”:

$$\mathcal{M} = \begin{pmatrix} \frac{d\hat{Y}_0}{dr_0} & \frac{d\hat{Y}_0}{dr_1} & \frac{d\hat{Y}_0}{dr_2} & \dots \\ \frac{d\hat{Y}_1}{dr_0} & \frac{d\hat{Y}_1}{dr_1} & \frac{d\hat{Y}_1}{dr_2} & \dots \\ \frac{d\hat{Y}_2}{dr_0} & \frac{d\hat{Y}_2}{dr_1} & \frac{d\hat{Y}_2}{dr_2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- GE effect of monetary news on output gap.
 - ▶ Below-diagonal elements capture intertemporal-shifting effects.
 - ▶ Above-diagonal elements capture forward guidance effects.

IRF TO r_0 AND NEWS

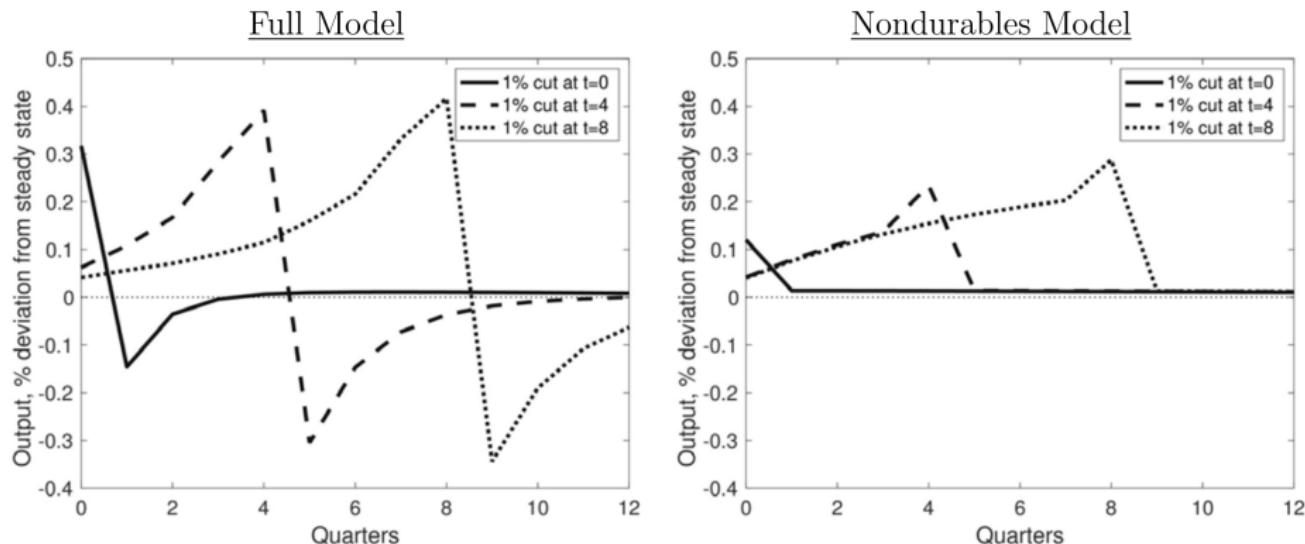
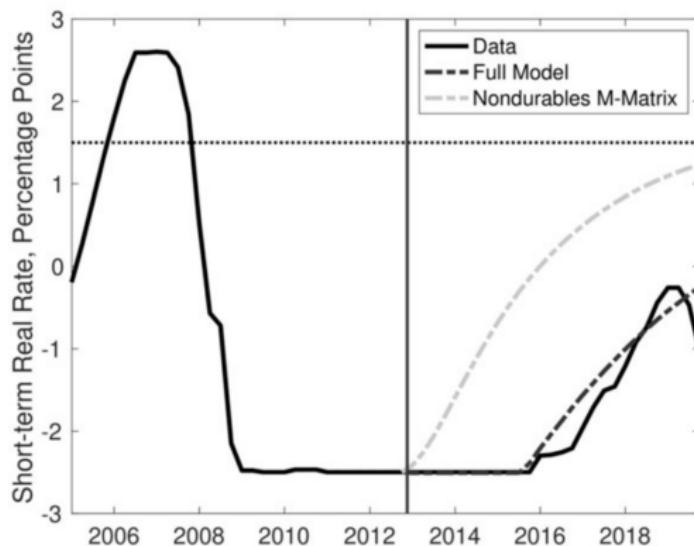


FIGURE 7.—Percentage change in output following a 1% reduction in the real interest rate at horizons $t = 0$ (solid line), $t = 4$ (dashed line), and $t = 8$ (dotted line). The left panel shows the effect in the full model, and the right panel shows the effect in the nondurables model.

GREAT RECESSION—FILTERING THE DATA

- Construct IRFs for shocks to Z_t, G_t, r_t^b, η_t .
- Match time series:
 - ▶ output gap,
 - ▶ durable spending,
 - ▶ real interest rate,
 - ▶ mortgage-treasury spread.
- Unique sequence of shocks that fit data exactly given initial condition.
(Kalman filter with no observational error.)
 - ▶ No need for state transition matrix.
- Account for zero lower bound using monetary news shocks.

MODEL PREDICTS SLOW NORMALIZATION OF REAL RATE



CONTRIBUTION OF INTERTEMPORAL SHIFTING TO r^*

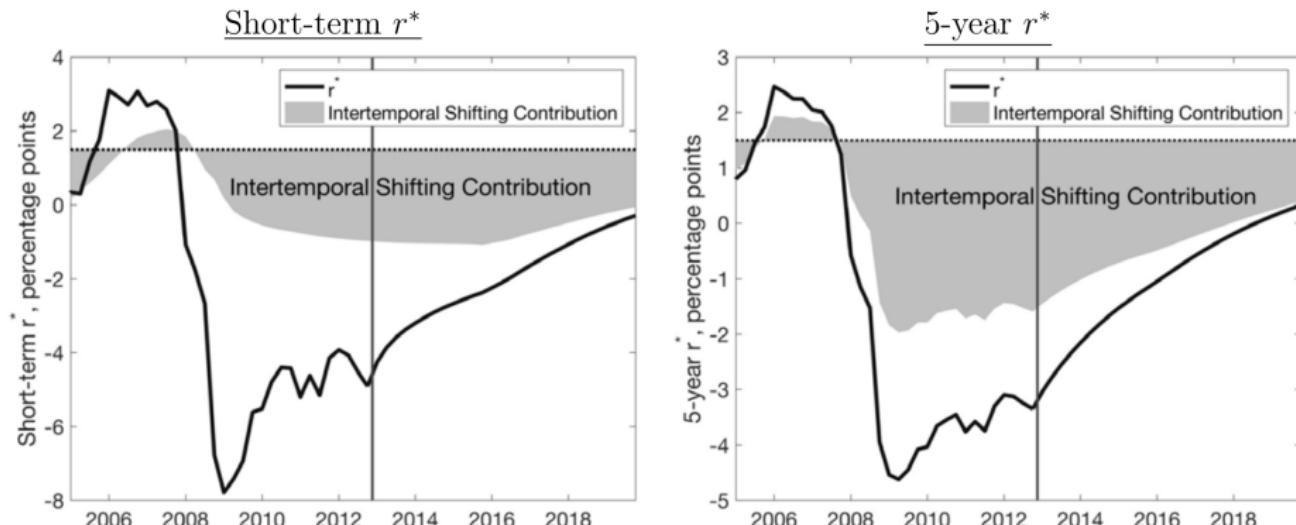
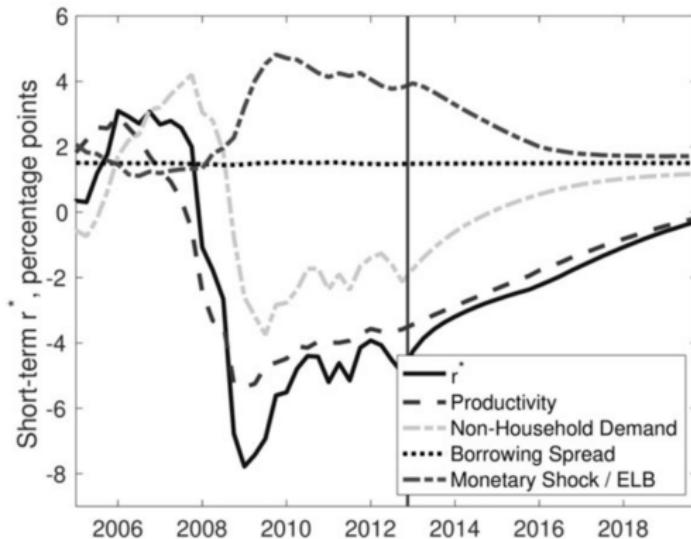


FIGURE 10.—Time series of the short-term natural rate of interest (left panel) and the 5-year natural rate of interest (right panel). The shaded area shows the contribution of intertemporal shifting effects of previous real interest rates to r^* . Forecast as of 2012Q4. The dotted horizontal line is the steady state real interest rate, equal to 1.5%.

CONTRIBUTION SHOCKS TO r^*



QUESTIONS

- Convincing?
- What role does the micro evidence play?

OUTLINE

- 1 INTRODUCTION
- 2 AUCLERT, ROGNLIE, STRAUB (2018)
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FORWARD GUIDANCE PUZZLE

- In standard NK model:

$$\begin{aligned}y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\&= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T \rightarrow \infty} y_{t+1}\end{aligned}$$

- Credible future real rate change has the same effect on current output regardless of how distant it is.

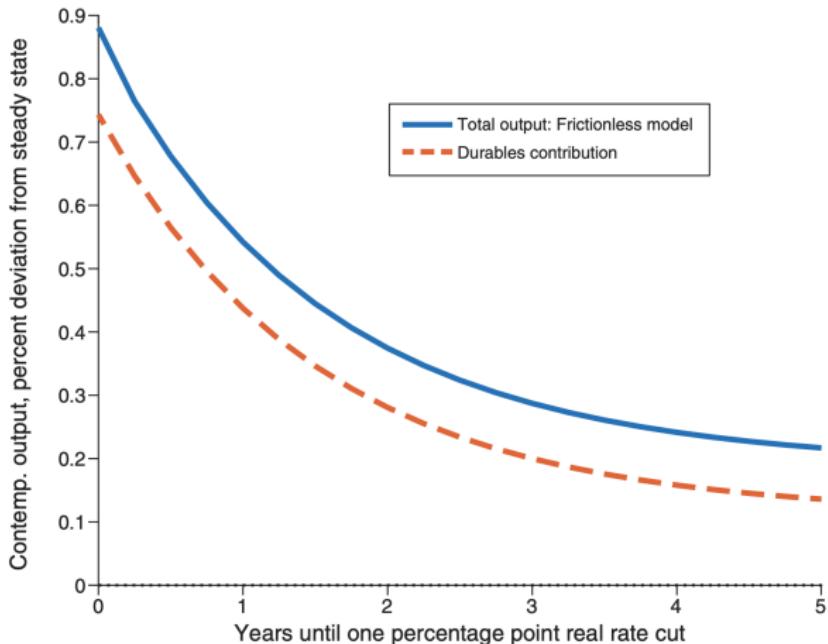
WITH DURABLE GOODS

- MRS between durable and nondurable good is equal to user cost:

$$\left(\frac{\psi}{1-\psi} \frac{c_{it}}{d_{it}} \right)^{\frac{1}{\xi}} = p_t(r_t + v + \delta) - \dot{p}_t \equiv r_t^d$$

- Special role for contemporaneous real rate in durable goods demand.

FRICTIONLESS DURABLE MODEL



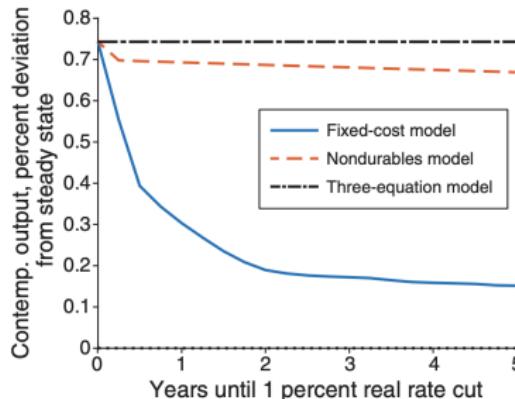
LUMPY DURABLES

- Frictionless model inconsistent with lumpy nature of durable adjustment.

⇒ How credible are the results?

FIXED COST DURABLE MODEL

Panel A. Power of forward guidance in the fixed-cost model and alternative models



Panel B. Fixed-cost model: Contributions from the extensive and intensive margins

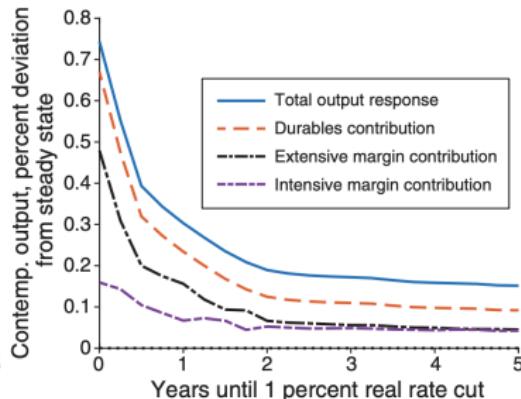


FIGURE 2. CONTEMPORANEOUS OUTPUT RESPONSE TO FORWARD GUIDANCE IN THE FIXED-COST MODEL

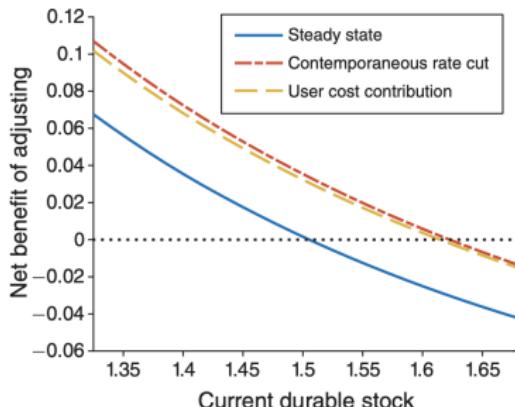
EXTENSIVE MARGIN

$$\begin{aligned} \frac{u(c_t^*, d_t^*) - u(c_t, d)}{V_{a,t}(a_t^*, d_t^*, z)} &= r_t^d (d_t^* - d) + [r_t^d - (v + \delta\chi)p_t] fd + (c_t^* - c_t) \\ &+ \frac{\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1}{1 - \lambda(1-f)} \left\{ \frac{a}{p_t} [r_t^d - (v + \delta\chi)p_t] + z(1 - \tau_t)Y_t - c_t - (v + \delta\chi)p_t c \right\} \end{aligned}$$

- Intuition: adjust now vs a little bit later.

IMPORTANCE OF CONTEMPORANEOUS USER COST

Panel A. Net benefit of adjusting before and after contemporaneous real rate cut



Panel B. Net benefit of adjusting before and after announcement of real rate cut in one year

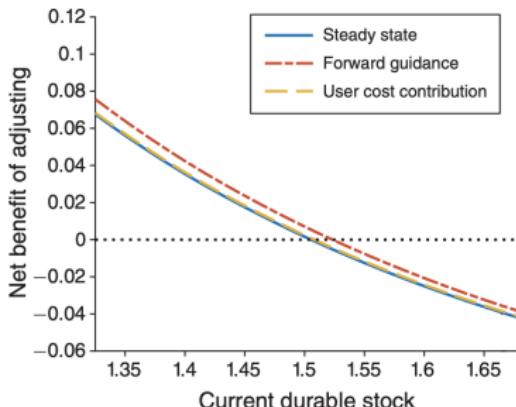


FIGURE 3. THE NET BENEFIT OF MAKING A DURABLE ADJUSTMENT

INTENSIVE MARGIN

$$\begin{aligned} \mathbb{E}_t \int_0^\tau e^{-(\rho + \delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d) ds \\ = \mathbb{E}_t e^{-\rho \tau} V_{x,t+\tau}^{\text{adj}} \left[r_{t,t+\tau}^d + e^{-\delta(1-\chi)\tau} p_{t+\tau} f \right] \\ + \mathbb{E}_t \int_0^\tau e^{-\rho s} \Psi_{t+s} \left[r_{t,t+s}^d + (1 - \lambda(1-f)) e^{-\delta(1-\chi)s} p_{t+s} \right] ds \end{aligned}$$

- Intuition: survival probability decreasing in time.

LONG-TERM FINANCING

- FOC for extensive margin the same when durable financed with long-term debt.
- Intuition: adjust now vs a little bit later, so expected change in financing cost matters for adjustment.

QUESTIONS

- Convincing?
- What role does the micro evidence play?

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- 7 BERAJA AND ZORZI (2024)

QUESTION

- How large a stimulus to close the output gap?
- Want to think about durables since important in determining MPCs as well as size and state-dependence.
- Why a model?

MODEL

- Value function:

$$V_t(x, \varepsilon) = \max\{V_t^{adj}(x) - \varepsilon, V_t^{noadj}(x)\}$$

where $x = (d, m, y)$.

- Adjustment utility:

$$V_t^{adj}(x) = \max_{c, d', a'} u(x, d') + \beta \int V_{t+1}(d', m', y', \varepsilon') d\mathcal{E}(\varepsilon) \Gamma(dy', y)$$

$$\text{s.t. } \theta d' + c + m' = \mathcal{Y}(x, T_t) + ((1 - \delta) - (1 - \theta))d$$

$$m' \geq 0$$

- No adjustment utility:

$$V_t^{noadj}(x) = \max_{c, a'} u(x, d') + \beta \int V_{t+1}(d', m', y', \varepsilon') d\mathcal{E}(\varepsilon) \Gamma(dy', y)$$

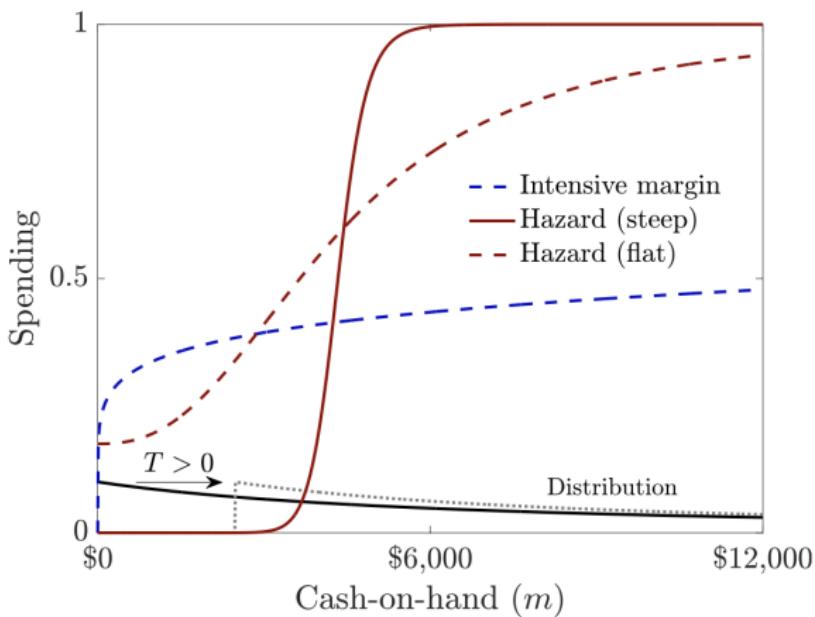
$$\text{s.t. } c + m' = \mathcal{Y}(x, T_t) - \iota \delta d - (1 - \theta)(d - d')$$

$$d' = (1 - (1 - \iota)\delta)d$$

$$m' \geq 0$$

IMPORTANCE OF THE HAZARD

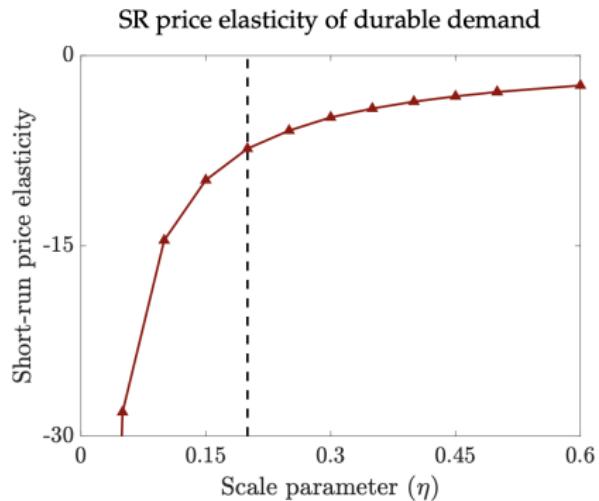
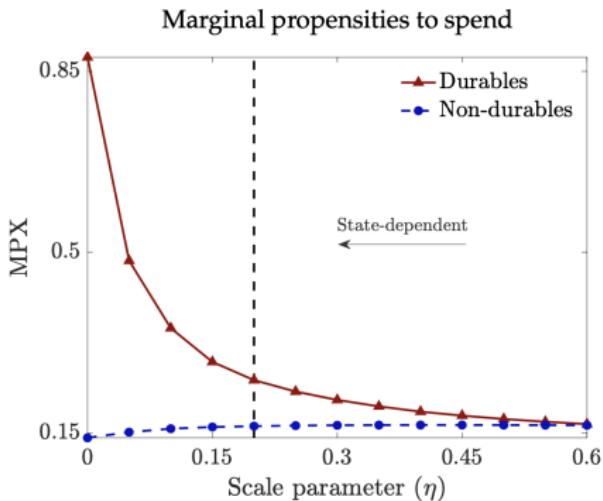
Figure 2.2: Hazard and intensive margin (fixing d and y)



WHY IS THIS A GOOD MODEL?

- How convincing did you find section 3?

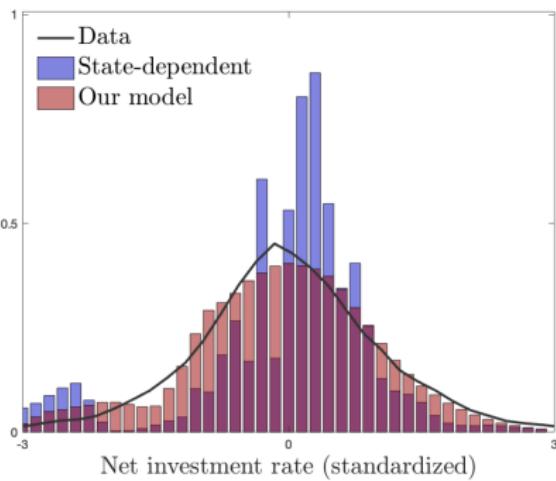
CALIBRATING THE HAZARD (1)



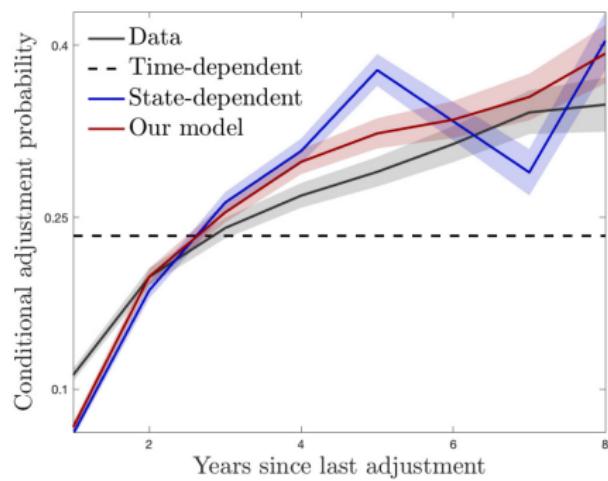
CALIBRATING THE HAZARD (2)

Figure 3.2: Distribution of adjustments and probability of adjustment

Distribution of net investment rates

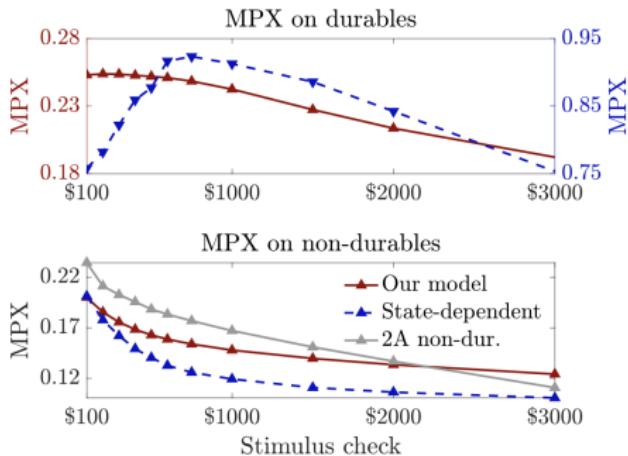


Conditional adjustment probability

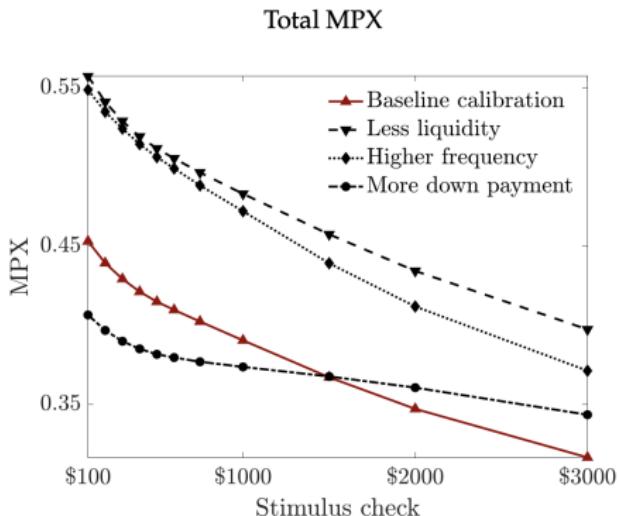


MPX IN THE MODEL VS ALTERNATIVES

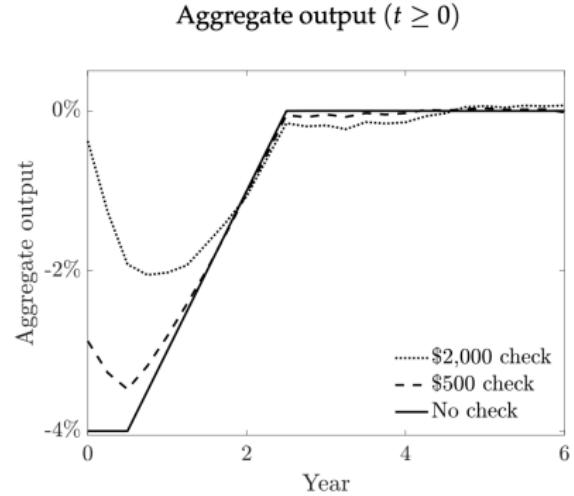
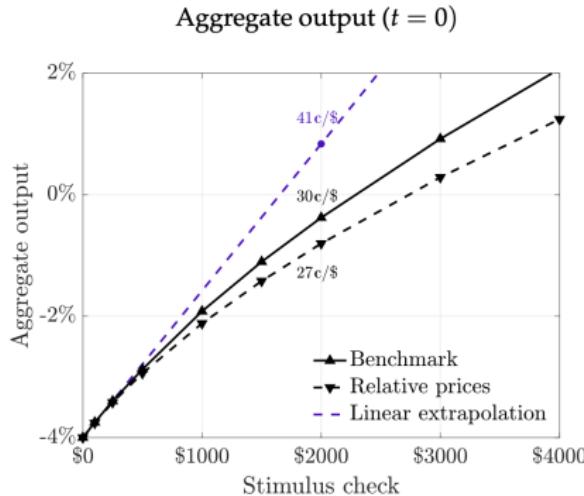
MPX on durables and non-durables



Total MPX



GE MPCs



Notes: The left panel plots aggregate output in the first quarter in deviations from steady state as a function of the size of stimulus checks. The solid curve is our benchmark model (Sections 5.1–5.2). The dashed black curve is a version with relative price movements between durables and non-durables ($\zeta \equiv 1/0.049$). The purple line extrapolates the response out of a \$300 check. We also indicate the output increase (in cents per dollar sent) after a \$2,000 stimulus check. The right panel reports the dynamic response of aggregate output in our benchmark model for stimulus checks of various sizes.

WHAT DID YOU THINK?