Topics in Macroeconomics

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Outline

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Cross-Sectional Regressions

$$Y_i = \alpha_i + \beta X_i + \varepsilon_i$$

- Interested in β.
- Can identify β if $E(\varepsilon_i|X_i) = 0$ or suitable instrument with $E(\varepsilon_i|Z_i) = 0$ and $E(X_i|Z_i) \neq 0$.
- What's the DGP? Two views:
  1. X_i / Z_i captures quasi-random heterogenous exposure to endogenous shock(s).
  2. X_i / Z_i captures endogenous exposure to heterogenous, quasi-random shocks.

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Bartik: Canonical Example

- Structural equation:

$$y_l = \rho + \beta x_l + \varepsilon_l$$

  - y_l: wage growth in area l.
  - x_l: employment growth in area l.

- Identities:

$$x_l = \sum_k z_{l,k} g_{l,k}, \qquad g_{l,k} = g_k + \tilde{g}_{l,k}$$

  - $z_{l,k}$: employment share in area l in industry k.
  - $g_{l,k}$: employment growth in area l in industry k.
  - g_k: national employment growth in industry k.
  - $\tilde{g}_{l,k}$: idiosyncratic component of employment growth rate.

- Bartik (1991) instrument to estimate inverse labor supply elasticity:

$$B_l = \sum_k z_{l,k} g_k$$

- What is exogenous? Shares? Shocks? Product?

Special Case: 2 Industries

- Bartik instrument is proportional to industry share:

$$B_l = z_{1l}g_1 + z_{2l}g_2 = g_2 + (g_1 - g_2)z_{1l}$$

- First stage:

$$x_l = \gamma_0 + \gamma B_l + \eta_l = \gamma_0 + \gamma g_2 + \gamma(g_1 - g_2)z_{1l} + \eta_l$$

- B_l is equivalent to using z_{1l} (or z_{2l}) as instrument.
- Intution:
  - z_{1l} measures exposure, $g_1 - g_2$ the magnitude of the treatment.
  - Many cross-sectional regressions take the view $g_2 = 0$: heterogeneous exposure to single aggregate shock.
  - What endogeneity problem does the Bartik instrument (or industry shares) solve? What does it not solve?

General Case (1)

Notation:

- $Z_{lt} = (z_{l1t}, \dots, z_{lkt})$ is a $1 \times K$ vector of industry shares.
- $Z_t = (Z'_{1t}, \dots, Z'_{Lt})'$ is a $L \times K$ matrix of industry shares.
- $G_t = (g_{1t}, \dots, g_{kt})'$ is a $K \times 1$ vector of industry growth rates.
- $B_t = Z_0 G_t$ is a $L \times 1$ vector of Bartik instruments.
- $X_t = (x_{1t}, \dots, x_{Lt})'$ is a $L \times 1$ vector of endogenous variables.
- $Y_t = (y_{1t}, \dots, y_{Lt})'$ is a $L \times 1$ vector of outcomes.
- Assume X_t, Y_t previously residualized with respect to any covariates.

General Case (2)

- B is a $LT \times 1$ vector of Bartik instruments.

$$B = ZG = \begin{pmatrix} Z_0 G_1 \\ Z_0 G_2 \\ \vdots \\ Z_0 G_T \end{pmatrix} = \underbrace{\begin{pmatrix} Z_0 & 0 & \dots & 0 \\ 0 & Z_0 & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & Z_0 \end{pmatrix}}_{=Z} \underbrace{\begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_T \end{pmatrix}}_{=G}$$

- Z is a $LT \times KT$ matrix of industry shares
- G is a $KT \times 1$ vector of industry growth rates.
- $X = (X_1', ..., X_T')'$ is a $LT \times 1$ vector of endogenous variables.
- $Y = (Y_1', ..., Y_T')'$ is a $LT \times 1$ vector of outcomes.
- The Bartik and GMM estimators are

$$\hat{\beta}_{Bartik} = \frac{B'Y}{B'X}, \qquad \hat{\beta}_{GMM} = \frac{X'ZWZ'Y}{X'ZWZ'X}$$

Equivalence of GMM and Bartik

- Proposition: When $W = GG'$ then $\hat{\beta}_{Bartik} = \hat{\beta}_{GMM}$
- Proof:

$$\begin{aligned}
\hat{\beta}_{GMM} &= (X'ZGG'Z'X)^{-1}(X'ZGG'Z'Y) \\
&= (X'BB'X)^{-1}(X'BB'Y) \\
&= (B'X)^{-1}(X'B)^{-1}(X'B)(B'Y) \\
&= \hat{\beta}_{Bartik}
\end{aligned}$$

- Bartik IV is numerically equivalent to IV regression with K instruments corresponding to the industry shares in Z weighted with industry GG'.
- More notation extends to case with controls.

Identifying assumptions

- TSLS estimator:

$$\hat{\beta} - \beta_0 = \frac{\sum_{t=1}^{T} \sum_{k=1}^{K} g_{kt} \sum_{l=1}^{L} z_{lk0} \varepsilon_{lt}}{\sum_{t=1}^{T} \sum_{k=1}^{K} g_{kt} \sum_{l=1}^{L} z_{lk0} x_{lt}}$$

- Identifying assumption (conditional on observables):

$$E\left[\frac{1}{L} \sum_{l=1}^{L} \varepsilon_{lt} z_{zlk0}\right] = 0, \qquad \forall k, t$$

  What are the asymptotics?

- KT moment conditions in GMM.
- In words: the differential effect of higher exposure of one industry (compared to another) only affects the change in the outcome (y_{lt}) through the endogenous variable of interest, and not through any potential confounding channel.

Rotemberg Weights

- In principle, must make exogeneity claim for every industry $k = 1, \ldots, k$. Very difficult to do in practice.
- GPSS: focus on select industries that are most influential in determining $\hat{\beta}_{Bartik}$.

$$\hat{\beta}_{Bartik} = \sum_k \hat{\alpha}_k \hat{\beta}_k$$

where

$$\hat{\beta}_k = (Z_k' X)^{-1}(Z_k' Y), \qquad \hat{\alpha}_k = \frac{G_k' Z_k' X}{\sum_k G_k' Z_k' X} = \frac{G_k' Z_k' X}{B' X}$$

- $\hat{\beta}_k$ is the just-identified IV estimate from using only the industry shares of industry k, Z_k.
- $\hat{\alpha}_k$ are the *Rotemberg Weights*, which sum to 1 (can be negative).
    - Contribution of industry k to Bartik first stage covariance. (Not the same as F-stat.)
    - Measure the sensitivity to bias in instrument k.

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BHJ Identification

- Bartik = TSLS estimator:

$$\hat{\beta} - \beta_0 = \frac{\sum_{t=1}^{T} \sum_{k=1}^{K} g_{kt} \sum_{l=1}^{L} z_{lk0} \varepsilon_{lt}}{\sum_{t=1}^{T} \sum_{k=1}^{K} g_{kt} \sum_{l=1}^{L} z_{lk0} x_{lt}}$$

- Moment condition for identification:

$$E\left[\frac{1}{L}\sum_{l=1}^{L} b_{lt}\varepsilon_{lt}\right] = E\left[\frac{1}{L}\sum_{l=1}^{L}\left(\sum_{t=1}^{T}\sum_{k=1}^{K} g_{kt} z_{lk0}\right)\varepsilon_{lt}\right] = 0$$

  - GPSS: quasi-random shares $E(\frac{1}{L}\sum_l z_{lk0}\varepsilon_{lt} | g_{kt}) = 0$
  - BHJ approach: quasi-random shocks

BHJ Moment Condition

- Moment condition in terms of shocks:

$$
\begin{aligned}
E\left[\frac{1}{L}B'\varepsilon\right] &= E\left[\frac{1}{L}\sum_{l=1}^{L}\left(\sum_{t=1}^{T}\sum_{k=1}^{K} g_{kt} z_{lkt}\right)\varepsilon_{lt}\right] \\
&= E\left[\sum_{k=1}^{K}\sum_{t=1}^{T}\underbrace{\left(\frac{1}{L}\sum_{l=1}^{L} z_{lkt}\right)}_{z_{kt}} g_{kt} \underbrace{\left(\frac{\frac{1}{L}\sum_{l=1}^{L} z_{lkt}\varepsilon_{lt}}{\frac{1}{L}\sum_{l=1}^{L} z_{lkt}}\right)}_{\bar{\varepsilon}_{kt}}\right] \\
&= E\left[\sum_{k=1}^{K}\sum_{t=1}^{T} z_{kt} g_{kt} \bar{\varepsilon}_{kt}\right] = E\left[(\check{Z}G)'\bar{\varepsilon}\right]
\end{aligned}
$$

where $\check{Z} = diag(z_{10}, ..., z_{KT})$.

- Interpretation:
  - z_{kt} is average exposure to industry k.
  - $\bar{\varepsilon}_{kt}$ is exposure-weighted average of shocks to wage growth.

BHJ Proposition 1

- Claim: Bartik IV is equivalent to TSLS with KT excluded instruments g_{kt} and weights z_{kt} in the second-stage regression:

$$\bar{y}_{kt} = \alpha + \beta \bar{x}_{kt} + \bar{\varepsilon}_{kt}$$

where $\bar{v}_{kt} = \frac{\frac{1}{L}\sum_{l=1}^{L} z_{lkt} v_{lt}}{\frac{1}{L}\sum_{l=1}^{L} z_{lkt}}$.

- Proof:

$$\begin{aligned}
\hat{\beta}_{Bartik} &= (B'X)^{-1}(B'Y) \\
&= \left(\sum_{k=1}^{K} \sum_{t=1}^{T} z_{kt} g_{kt} \bar{x}_{kt} \right)^{-1} \left(\sum_{k=1}^{K} \sum_{t=1}^{T} z_{kt} g_{kt} \bar{y}_{kt} \right) \\
&= (\check{Z} G' \bar{X})^{-1} (\check{Z} G' \bar{Y})
\end{aligned}$$

Which case?

1. Exogenous shares: $E\left[\frac{1}{L}\sum_{l=1}^{L} z_{zlk0}\varepsilon_{lt}\right] = 0$
   - Ex ante exposure in location l uncorrelated with unobserved shocks to outcome.

2. Exogenous shocks (shifters): $E\left[\frac{1}{KT}\sum_{k=1}^{K}\sum_{t=1}^{T} z_{kt} g_{kt}\bar{\varepsilon}_{kt}\right] = 0$
   - Shocks to industry k uncorrelated with unobserved industry shocks when weighted by industry size.

- What are asymptotics?
- Which is more plausible? When?

Outline

General Specification Tests

1. Estimated coefficients sensitive to inclusion of covariates?
2. Pre-trends?
3. Placebo tests?
4. Overidentification tests.
5. Subsample analysis: drop influential observations.

Leave-one-out

- Typically construct leave-one-out Bartik instrument:

$$B_l = \sum_k z_{l,k} g_{-l,k}$$

  - $g_{-l,k}$ is national employment growth in industry k excluding area l.

- Removes finite sample correlation between idiosyncratic industry growth rate $\tilde{g}_{l,k}$ and Bartik instrument B_l.

- Often unimportant in practice. Why?

Standard Errors

- Adão, Kolesár, Morales (QJE, 2019): regions with similar exposure are not iid.
- Example DGP:

$$y_l = \alpha + \beta_0 x_l + \varepsilon_l, \qquad x_l = \sum_k z_{lk}(g_k^1 + g_k^2)$$

- Want to know impact of g^1 (e.g., China shock):

$$y_l = \alpha + \beta_0 \sum_k z_{lk} g_k^1 + (\sum_k z_{lk} g_k^2 + \varepsilon_l)$$

- Identified if $Cov(\sum_k z_{lk} g_k^1, \sum_k z_{lk} g_k^2 + \varepsilon_l) = 0$.
- But residuals correlated because of industry structure $\Rightarrow$ need to adjust standard errors. Severity depends on importance of g_k^2.
- BHJ solve this issue by clustering the industry-level regression.

Borusyak, Hull: Non-Random Exposure

$$y_i = \alpha + \beta x_i + \varepsilon_i$$
$$z_i = f_i(g, w)$$

- Even if shocks are exogenous, $g \perp \varepsilon | w$, due to non-random exposure.

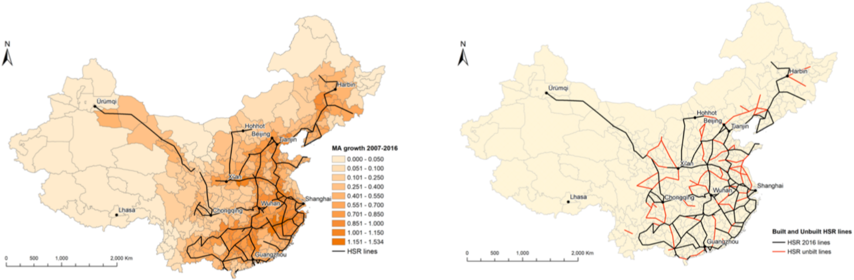
$$E\left[\frac{1}{N}\sum_i z_i \varepsilon_i\right] = E\left[\frac{1}{N}\sum_i \mu_i \varepsilon_i\right]$$

  where $\mu_i = E[f_i(g, w)]$.

- Intuition: some areas systematically get higher / lower treatment due to non-random assignment (exposure).
- Solution: simulate draws of shocks, compute μ_i, and recenter instrument to $z_i - \mu_i$.
- A problem for Bartik (inner product) instruments?

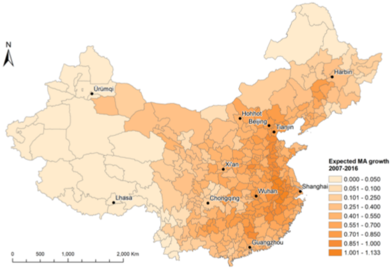
Figure 1: Chinese High Speed Rail and Market Access Growth, 2007-2016

A. Completed Lines and MA Growth B. All Planned Lines

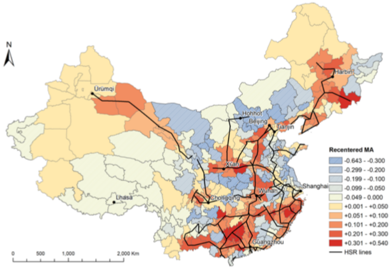


Notes: Panel A shows the completed China high-speed rail network by the end of 2016, with shading indicating MA growth (i.e. log-change in MA) relative to 2007. Panel B shows the network of all HSR lines, including those planned but not yet completed as of 2016 (in red).

A. Expected Market Access Growth



B. Recentered Market Access Growth



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