Macroeconomics of Consumption with Heterogeneity

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OUTLINE

Outline

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Simplified Consumer Problem

- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_{it})$$

- After-tax income:

$$z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it}\right)^{1-\lambda} = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}} Z_t$$

- Budget constraint:

$$c_{it} + \sum_j a_{it}^j = z_{it} + (1+r_{t-1})\sum_j a_{i,t-1}^j$$

- Nests: RA, TA, HA, HA-IL

Optimal Policy Rules

- Budget constraint with $Z_t = Y_t - T_t$:

$$c_{it} + \sum_j a^j_{it} = \frac{e_{it}^{1-\lambda}}{\int e_{it}^{1-\lambda}}(Y_t - T_t) + (1 + r_{t-1}) \sum_j a^j_{i,t-1}$$

- Idiosyncratic states: e_{it}.
- Assume aggregate variables follow a known sequence $\{Y_t - T_t, r_t\}_{t=0}^{\infty}$.
- Conditional on knowing these sequences, can solve for individual consumption and assets given any initial state $\{a^j_0\}, e_{i0}$:

$$c_t(\{a^j\}, e),\ \{a^j_t(\{a^j\}, e)\}, \qquad t \geq 0$$

- Why? Distribution only enters consumers problem through effect on $\{Y_t - T_t, r_t\}_{t=0}^{\infty}$. No longer have infinite state space.
- Then aggregate all consumption decisions to get:

$$C_t = \int c_t(\{a^j\}, e) d\Psi_t(\{a^j\}, e) = \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty})$$

New Keynesian Model in Sequence Space

- Consumption function:

$$\begin{aligned} C_t &= \mathbb{C}(\{Y_{t+s} - T_{t+s}, r_{t+s}\}_{s=0}^{\infty}) \\ &\equiv \mathbb{C}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) \end{aligned}$$

- NKPC:

$$\pi_t = \mathbb{S}(\boldsymbol{Y} - \boldsymbol{Y}^*)$$

- Interest rate rule:

$$\boldsymbol{r} = \boldsymbol{\Phi}_Y(\boldsymbol{Y} - \boldsymbol{Y}^*) + \boldsymbol{\Phi}_\pi \boldsymbol{\pi} + \boldsymbol{\varepsilon}^r$$

- Excess demand:

$$\boldsymbol{ED} = \mathbb{C}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) + \boldsymbol{G} - \boldsymbol{Y} = 0$$

Linearize to Solve with Linear Algebra

- Linearized Model:

$$\hat{\boldsymbol{C}} = (\nabla_{\boldsymbol{Y}}\mathbb{C})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{T}}) + (\nabla_{\boldsymbol{r}}\mathbb{C})\hat{\boldsymbol{r}}$$

$$\hat{\boldsymbol{\pi}} = (\nabla_{\boldsymbol{Y}}\mathbb{S})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*)$$

$$\hat{\boldsymbol{r}} = \boldsymbol{\Phi}_Y(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*) + \boldsymbol{\Phi}_\pi\hat{\boldsymbol{\pi}} + \boldsymbol{\varepsilon}^r$$

where the Jacobians of the model look like:

$$\nabla_{\boldsymbol{Y}}\mathbb{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \cdots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \cdots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- Equilibrium:

$$\hat{\boldsymbol{Y}} = [\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1}\{[\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1} - [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})\}^{-1} \times$$
$$\times [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C})\boldsymbol{\varepsilon}^r]$$

Demand and Supply Elasticities Determine GE

- Define:
  - Demand multiplier $\boldsymbol{\mu}^D \equiv [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}$
  - Demand elasticity $\boldsymbol{\eta}^{\boldsymbol{D}} \equiv -\nabla_{\boldsymbol{r}}\mathbb{C}$
  - Supply elasticity $\boldsymbol{\eta}^{\boldsymbol{S}} \equiv [\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\boldsymbol{\nabla}_{\boldsymbol{Y}}\mathbb{S})]^{-1}$

$$\hat{\boldsymbol{Y}} = \underbrace{\boldsymbol{\eta}^{\boldsymbol{S}}[\boldsymbol{\eta}^{\boldsymbol{S}} + \boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}]^{-1}}_{\text{Demand Incidence}}\boldsymbol{\mu}^D \underbrace{[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + \boldsymbol{\eta}^{\boldsymbol{D}}\boldsymbol{\varepsilon}^r]}_{\text{PE Excess Demand}}$$
$$+ \underbrace{\boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}[\boldsymbol{\eta}^{\boldsymbol{S}} + \boldsymbol{\mu}^D\boldsymbol{\eta}^{\boldsymbol{D}}]^{-1}}_{\text{Supply Incidence}} \underbrace{\hat{\boldsymbol{Y}}^*}_{\text{PE Excess Supply}}$$

Implications:

- GE effects determined by a standard incidence formula.

$\Rightarrow$ Matrices of micro elasticities are sufficient statistics for GE effects.

Intertemporal Keynesian Cross

- Also assume no real rate change, so $\boldsymbol{\eta^S}$ blows up in some meaningful sense.

$$\hat{\boldsymbol{Y}} = [I - \nabla_{\boldsymbol{Y}} \mathbb{C}]^{-1} [\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}} \mathbb{C}) \hat{\boldsymbol{T}}]$$

- This is the Intertemporal Keynesian Cross.
- Balanced budget: $\hat{\boldsymbol{G}} = \hat{\boldsymbol{T}}$

$$\begin{aligned} \hat{\boldsymbol{Y}} &= [I - \nabla_{\boldsymbol{Y}} \mathbb{C}]^{-1} [\boldsymbol{I} - (\nabla_{\boldsymbol{Y}} \mathbb{C})] \hat{\boldsymbol{G}} \\ &= \hat{\boldsymbol{G}} \end{aligned}$$

- The balanced budget multiplier is 1.

Fiscal Policies

- Proposition 4: In the RA model, the G-multiplier is 1.
  - Proof: The G-multiplier is 1 when budget is balanced, and we know for the RA model the timing of T does not matter.
- Proposition 5: in the TA model with μ constrained agents,

$$\hat{\boldsymbol{Y}} = \frac{1}{1-\mu}[\hat{\boldsymbol{G}} - \mu \hat{\boldsymbol{T}}]$$

  - Deficit-financed fiscal expansion ($\hat{T}_0 < \hat{G}_0$) has impact multiplier greater than 1.

Measuring $\nabla_{\boldsymbol{Y}}\mathbb{C}$

- Demand multiplier: $\boldsymbol{\mu}^D \equiv [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}$ where

$$\nabla_{\boldsymbol{Y}}\mathbb{C} = \begin{pmatrix} \frac{\partial C_0}{\partial Y_0} & \frac{\partial C_0}{\partial Y_1} & \cdots \\ \frac{\partial C_1}{\partial Y_0} & \frac{\partial C_1}{\partial Y_1} & \cdots \\ \frac{\partial C_2}{\partial Y_0} & \frac{\partial C_2}{\partial Y_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

  Columns are PE IRFs of average consumption to income changes at different horizons.

- What evidence do we have?

Lottery Studies

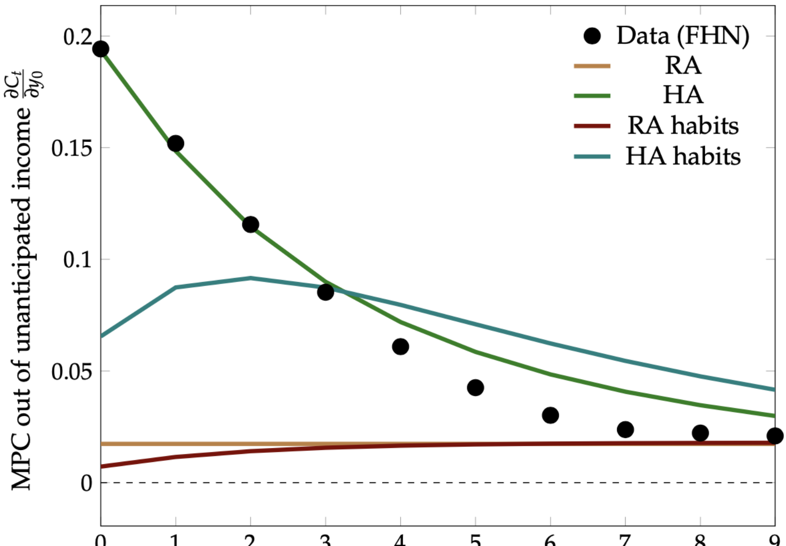
- Provides estimate of first column of $\nabla_{\boldsymbol{Y}}\mathbb{C}$.
- Need a model to extrapolate.

figures/LotteryMPCestimate.png

Source: Fagereng, Holm, Natvik (2018)

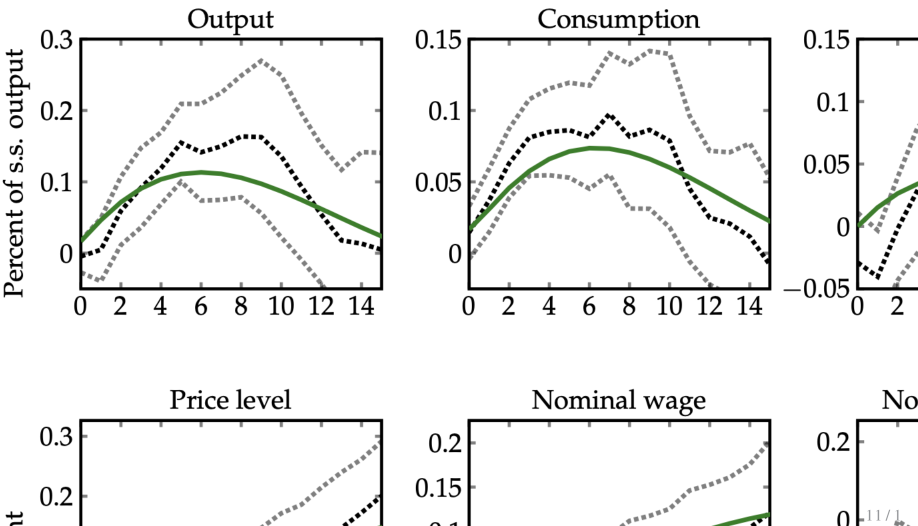
iMPCs in the Models

Figure 2: Intertemporal MPCs $\partial C_t / \partial y_0$ in models and in the data



Model Extrapolation

Figure 3: Impulse response to a monetary po



Fiscal Policies: All Models

Figure 4: Impulse responses with and witho

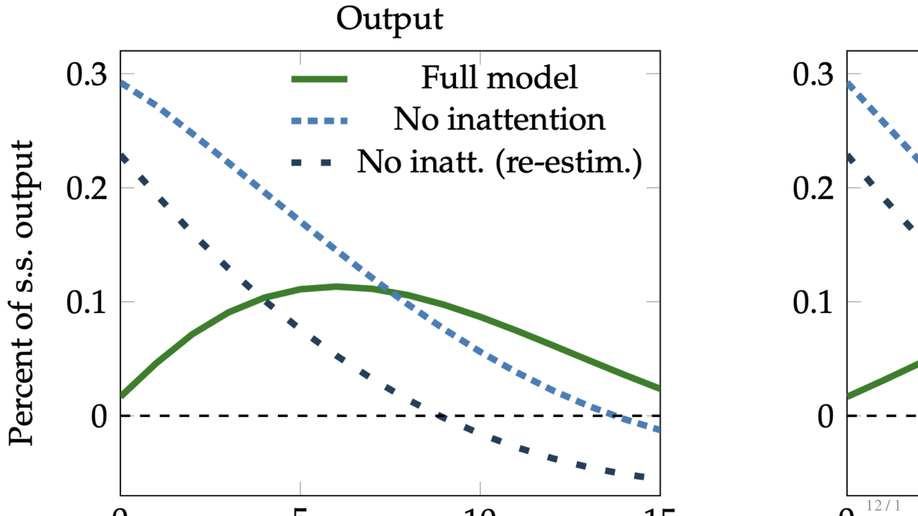
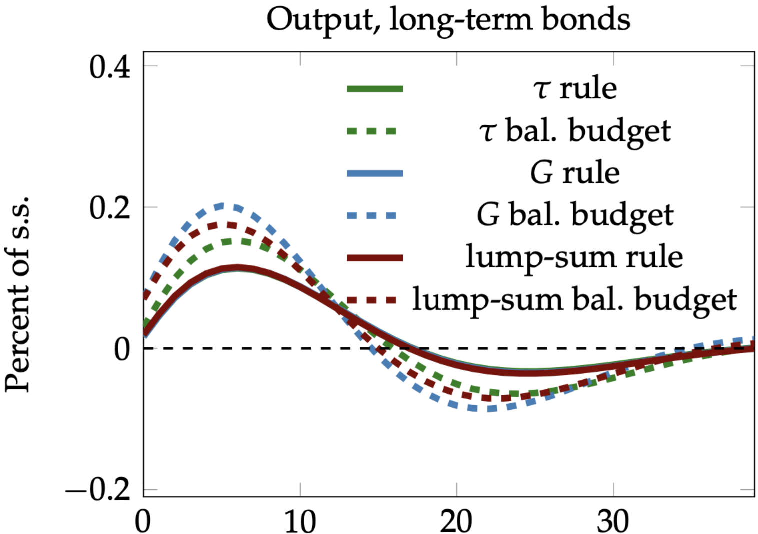


Figure 7: The role of fiscal policy for mon



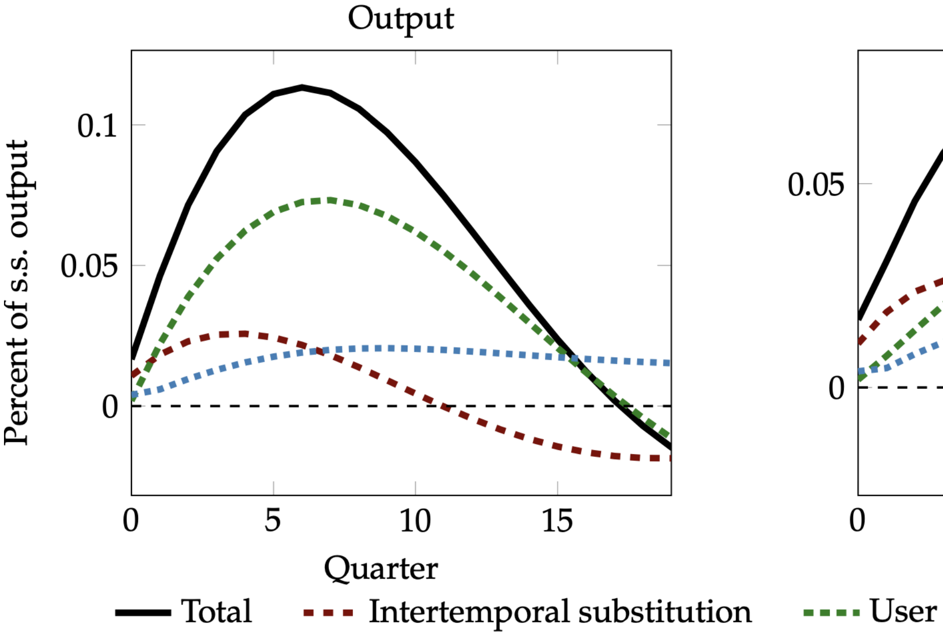
0.4

0.2

0

−0.2

Private Deficits



- Why so much amplification?

Outline

New Keynesian Model in Sequence Space

- Consumption function:

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- NKPC:

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- Excess demand:

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Linearize to Solve with Linear Algebra

- Linearized Model:

$$\hat{\boldsymbol{C}} = (\nabla_{\boldsymbol{Y}}\mathbb{C})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{T}}) + (\nabla_{\boldsymbol{r}}\mathbb{C})\hat{\boldsymbol{r}}$$
$$\hat{\boldsymbol{\pi}} = (\nabla_{\boldsymbol{Y}}\mathbb{S})(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*)$$
$$\hat{\boldsymbol{r}} = \boldsymbol{\Phi}_Y(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^*) + \boldsymbol{\Phi}_\pi\hat{\boldsymbol{\pi}} + \boldsymbol{\varepsilon}^r$$

- Equilibrium:

$$\hat{\boldsymbol{Y}} = [\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1}\{[\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1} - [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})\}^{-1} \times$$
$$\times [I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C})\boldsymbol{\varepsilon}^r]$$

- Kaplan, Moll, and Violante (2018):
  - Small direct effect $\nabla_{\boldsymbol{r}}\mathbb{C}$
  - Large indirect effect $[I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}$
  - Product roughly constant $[I - \nabla_{\boldsymbol{Y}}\mathbb{C}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C})$

$\Rightarrow$ Should we care about the decomposition?

Add Investment

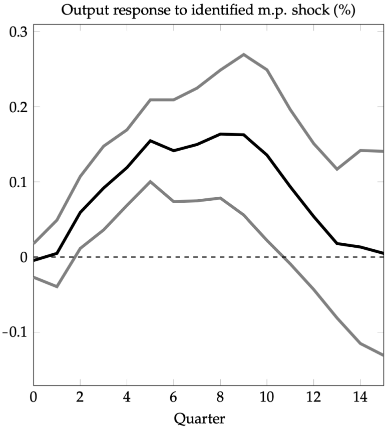
- Yes:

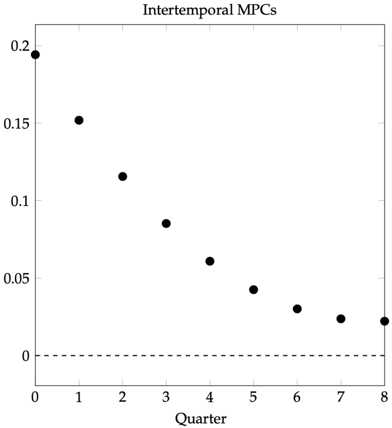
$$\begin{aligned}\hat{\boldsymbol{Y}} = {} & [\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1}\times \\ & \times \{[\boldsymbol{\Phi}_Y + \boldsymbol{\Phi}_\pi(\nabla_{\boldsymbol{Y}}\mathbb{S})]^{-1} - [I - \nabla_{\boldsymbol{Y}}\mathbb{C} - \nabla_{\boldsymbol{Y}}\mathbb{I}]^{-1}(\nabla_{\boldsymbol{r}}\mathbb{C} + \nabla_{\boldsymbol{r}}\mathbb{I})\}^{-1}\times \\ & \times [I - \nabla_{\boldsymbol{Y}}\mathbb{C} - \nabla_{\boldsymbol{Y}}\mathbb{I}]^{-1}[\hat{\boldsymbol{G}} - (\nabla_{\boldsymbol{Y}}\mathbb{C})\hat{\boldsymbol{T}} + (\nabla_{\boldsymbol{r}}\mathbb{C} + \nabla_{\boldsymbol{r}}\mathbb{I})\boldsymbol{\varepsilon}^r]\end{aligned}$$

- Key challenge: How to set up model to match
  - Immediate consumption response to higher income ("jump").
  - Delayed consumption, investment, output response to lower real interest rate ("hump").

THE CHALLENGE

Figure 1: Macro Humps, Micro Jumps.





Model

- Household problem:

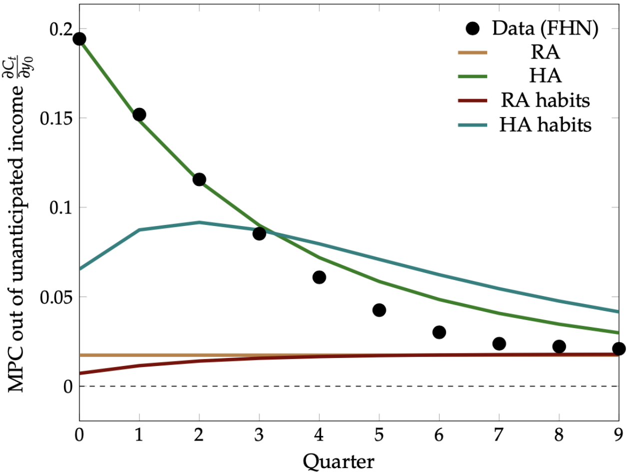
$$V_t(l,s) = \max_{c,l'} u(c) + \beta \mathbb{E}[V_{t+1}(l',s')|s]$$
$$c' + l' \leq (1+r_t)l + y_t e(s)$$
$$l' \geq 0$$

- Standard sequence space result:

$$C_t = \mathscr{C}(\{y_s, r_s\}_{s\geq 0}),\ t \geq 0$$

Matching Micro Data

Figure 2: Intertemporal MPCs $\partial C_t / \partial y_0$ in models and in the data



- Why does habit model have difficulty?

Inattention

- Households know current micro state $e(s)$.
- Households update info about aggregate shocks with iid probability $1-\theta$ each period.
- Households know current r_t, Y_t but forecast r_{t+s}, Y_{t+s} using info from $t-k$.

$\Rightarrow$ Households always on budget constraint.

- Optimize subject to information from k periods ago:

$$V_t(l,s,k) = \max_{c,l'} u(c) + \beta \mathbb{E}_{t-k}[\theta V_{t+1}(l',s',k+1) + (1-\theta) V_{t+1}(l',s',0)|$$

Estimation

- Two-step procedure.
- Calibrate household problem to micro data, so only need to compute household sequence space Jacobians once.
- Estimate information and macro parameters to match estimated IRF to monetary policy shocks.
- Sticky info micro Jacobian at t to shock at time s:

$$\mathscr{J}_{t,s}^{o,i} = \begin{cases} \theta\, \mathscr{J}_{t-1,s-1}^{o,i} + (1-\theta)\, \mathscr{J}_{t,s}^{o,i,FI} & t > 0, s > 0 \\ \mathscr{J}_{t,s}^{o,i,FI} & s = 0 \\ (1-\theta)\, \mathscr{J}_{t,s}^{o,i,FI} & t = 0, s > 0 \end{cases}$$

Calibration

Table 2: Calibrated and estimated parameters.

Panel A: Calibrated parameters

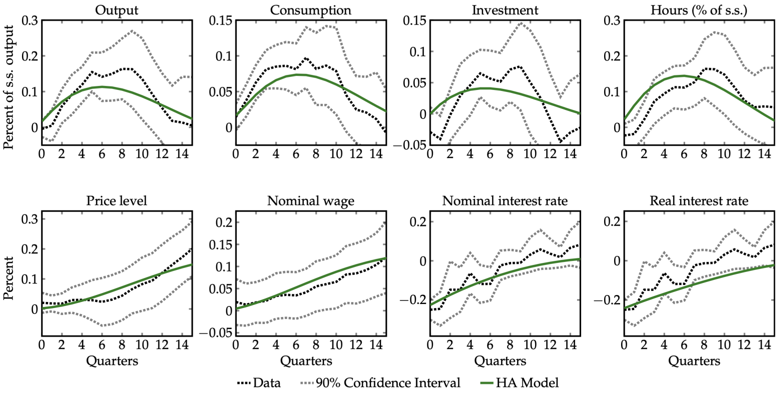
Parameter		Value
ν	EIS	1
ς	Frisch	0.5
β_g	Discount factors (p.a.)	Table 1
r	Real interest rate (p.a.)	0.050
α	Capital share	0.24
δ_K	Depreciation of capital (p.a.)	0.053
μ_p	Steady-state retail price markup	1.06
K/Y	Capital to GDP (p.a.)	2.23
L/Y	Liquid assets to GDP (p.a.)	0.23
ξ	Intermediation spread (p.a.)	0.065
G/Y	Spending-to-GDP	0.16
qB/Y	Government bonds to GDP (p.a.)	0.42
$\frac{1+r}{1+r-\delta}$	Maturity of government debt (a.)	5
ψ	Response of tax rate to debt (p.a.)	0.1
ϕ_π	Taylor rule coefficient	1.5

Panel B: Estimated parameters

Parameter		Value	std. dev.
θ	Household inattention	0.935	(0.01)
ϕ	Investment adj. cost parameter	9.639	(2.428)
ζ_p	Calvo price stickiness	0.926	(0.012)
ζ_w	Calvo wage stickiness	0.899	(0.016)
ρ^m	Taylor rule inertia	0.890	(0.01)
σ^m	Std. dev. of monetary shock	0.057	(0.005)

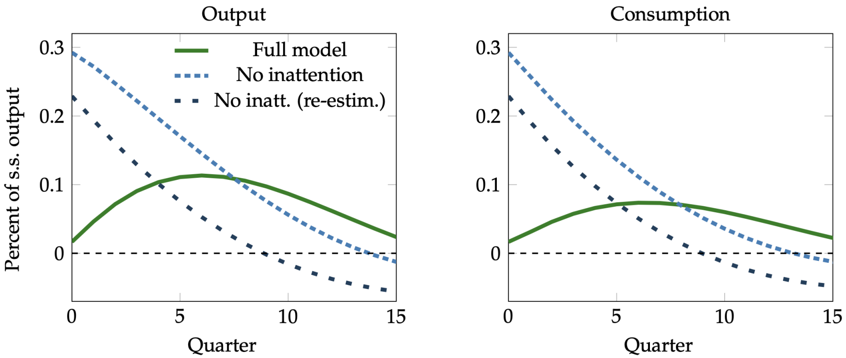
Matching the Humps

Figure 3: Impulse response to a monetary policy shock vs. model fit



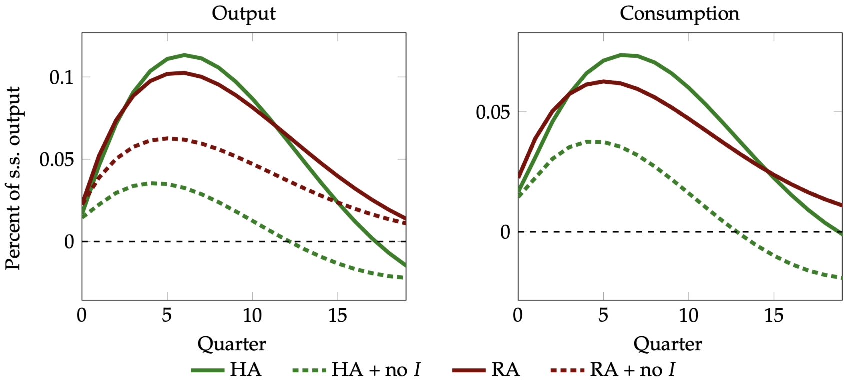
Role of Inattention

Figure 4: Impulse responses with and without inattention



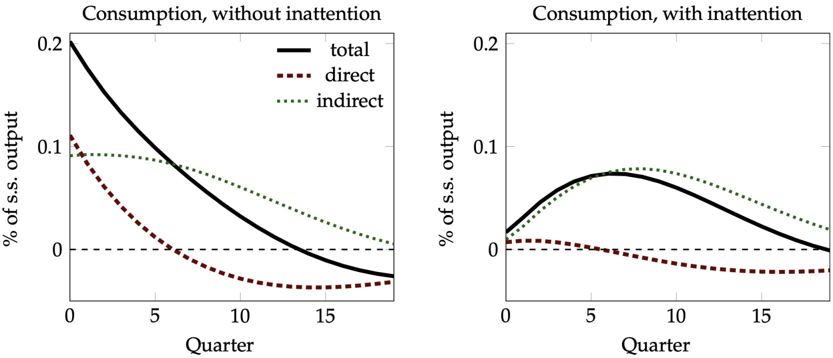
Importance of Investment

Figure 5: Role of investment in the transmission mechanism



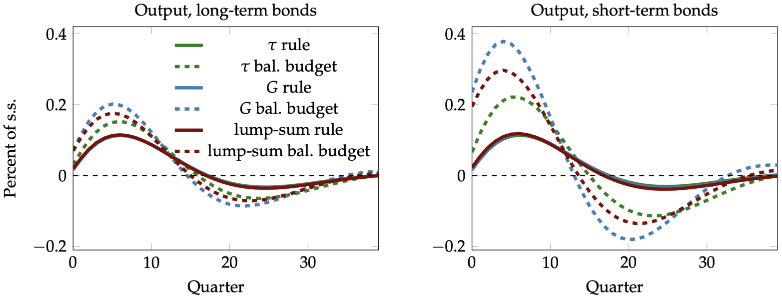
Direct vs Indirect Effects

Figure 6: Decomposition of consumption

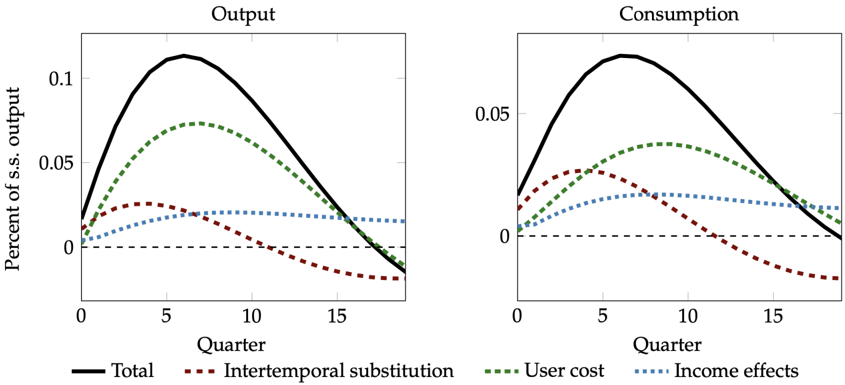


Aside: Role of Fiscal Policy

Figure 7: The role of fiscal policy for monetary transmission

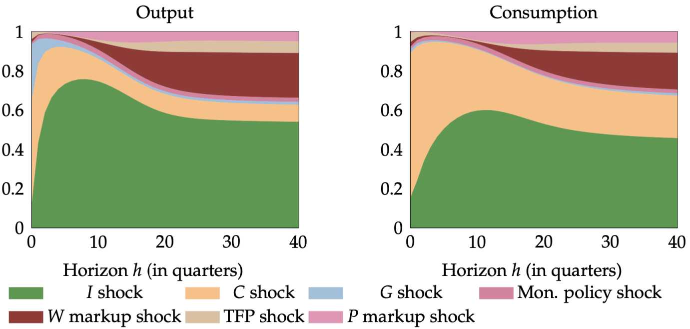


Starting the Transmission Mechanism



Reassessing the Importance of Investment

Figure 12: Forecast error variance decomposition for the HA model



OUTLINE

Policy Space

- With ZLB constraining $i_t \geq 0$ (or small negative number), central banks can only lower interest rates a finite amount.

- This means that central bank can only offset an AD shock of finite size. This is the "policy space" or "limited ammunition."

- This paper: when lower interest rates today, then also reduce *responsiveness* to future interest rate changes (up or down).

- Means that past interest rate cuts limit future policy face as subsequent interest rate cuts less effective.

Refinancing (Pre-payment)

- Estimate effect of rate gap on refinancing probability.

$$prepay_{i,j,t} = \beta_{gapbin} 1(gapbin)_{j,t} + \beta_X X_{i,j,t} + \delta_i + \varepsilon_{i,j,t}$$

- Identification assumption?

Refinancing (Pre-payment)

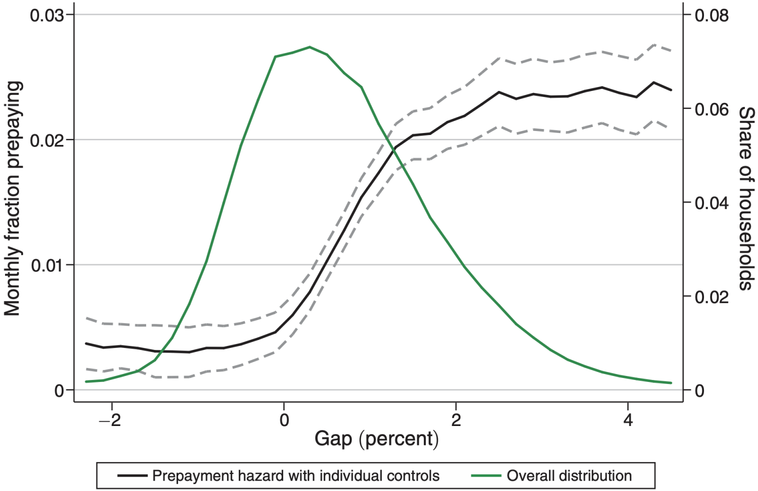


Figure 1. Prepayment Hazard with Individual Controls

Motives

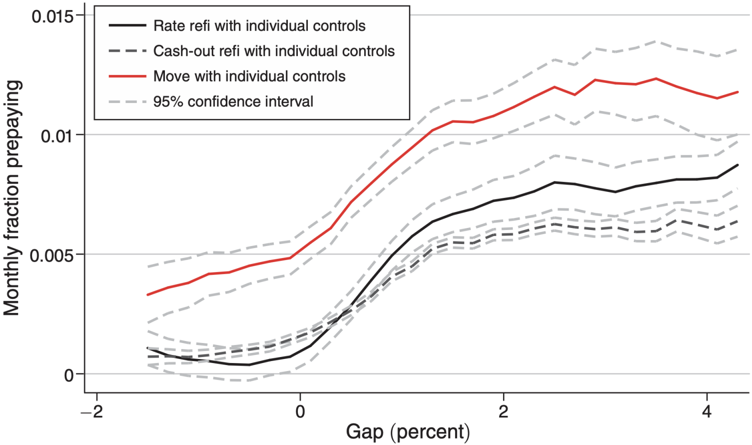


Figure 3. Prepayment Hazard Decomposition with Individual Controls

Aggregate Correlation

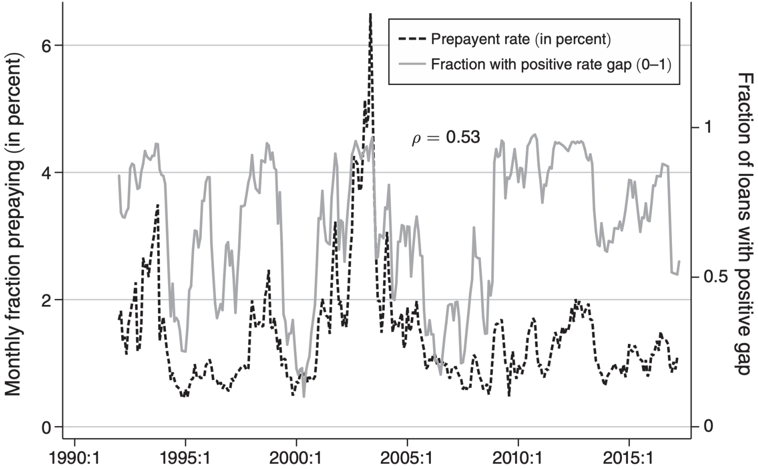


Figure 5. Prepayment versus Fraction with Positive Rate Gap Time Series

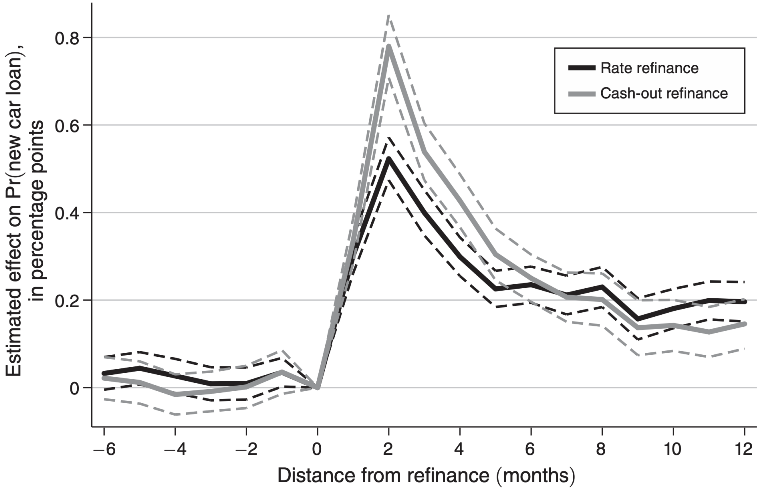
Spending

- Estimate effect of refinancing probability on spending.

$$1(carloan)_{i,t} = \sum_k 1(refinanced)_{i,t-k} + \delta_i + \delta_t + \varepsilon_{i,t}$$

- Identification assumption?

SPENDING

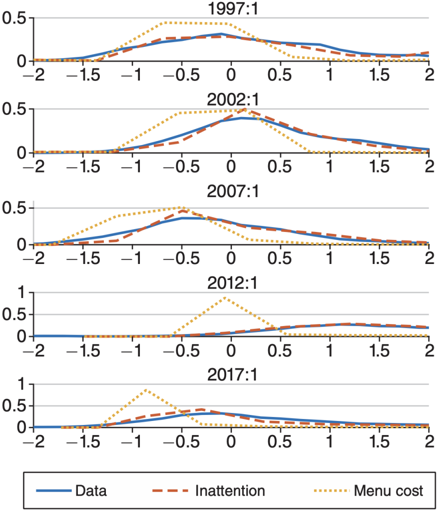


Rate refinance
Cash-out refinance
Estimated effect on Pr(new car loan), in percentage points
0.8
0.6
0.4
0.2
0
−6
−4
−2
0
2
4
6
8
10
12
Distance from refinance (months)

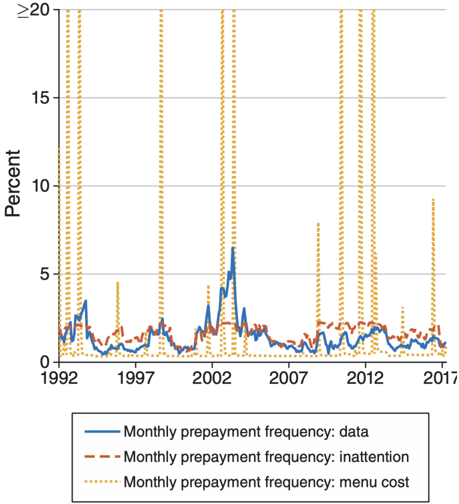
Why Do We Need a Model?

Gaps and Frequency

Panel A. Distribution of gaps



Panel B. Frequency



HYBRID MODEL

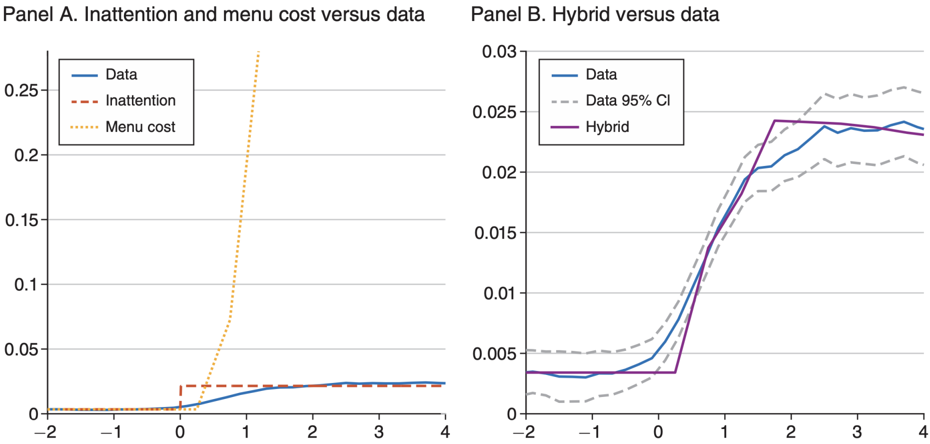


FIGURE 10. PREPAYMENT HAZARDS

State Dependence

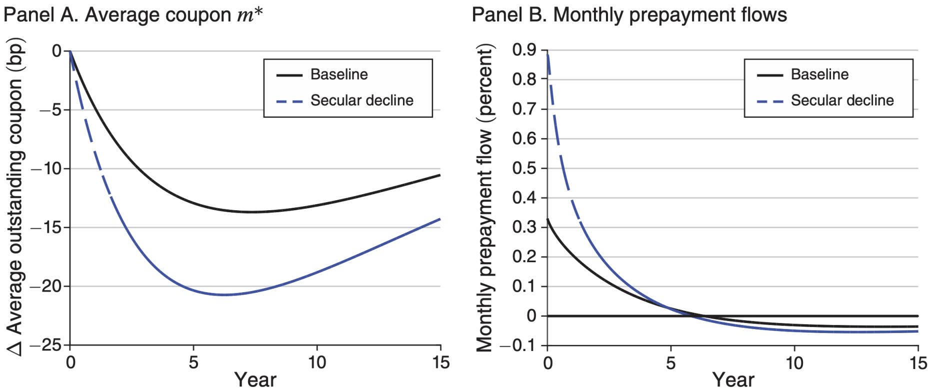


Figure 12. Impulse Response Functions to 100 bp Decline in r

State Dependence

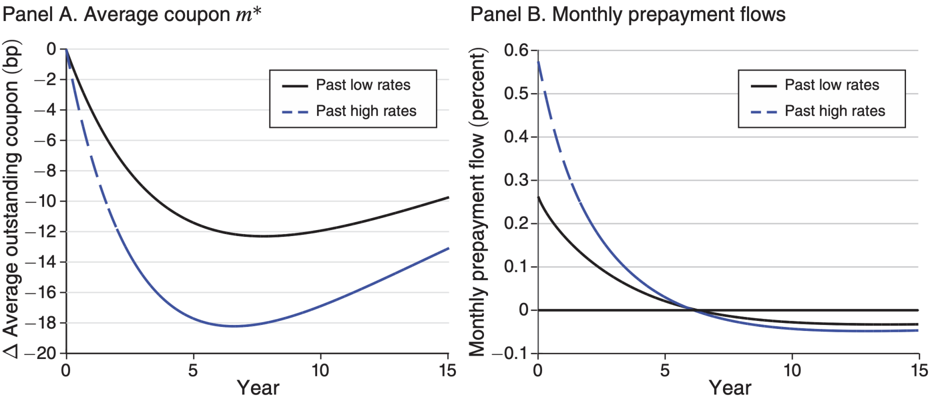
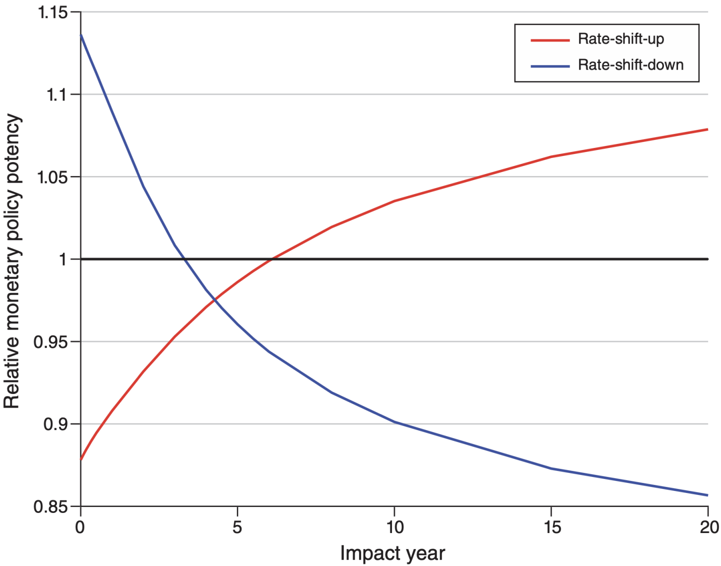


Figure 14. IRF of Average Coupon m^* to 100 bp Decline in r

Shift in Policy Stance



Consumption Effects

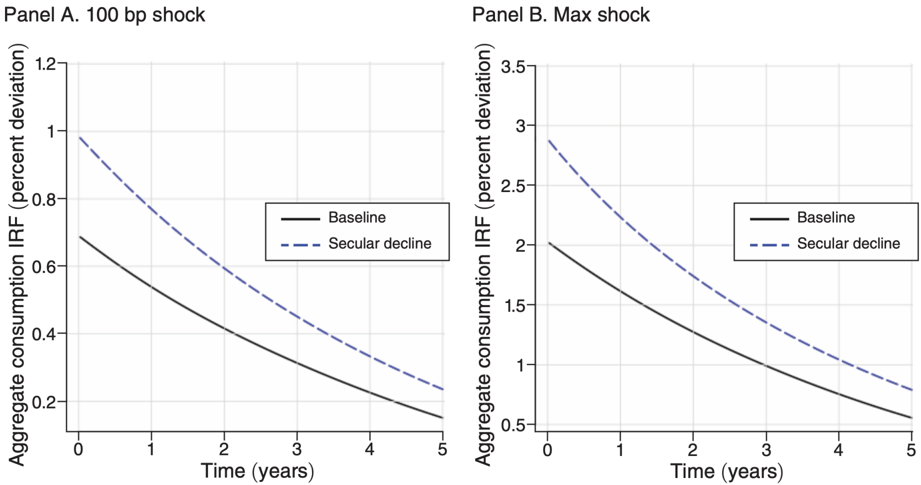


Figure 17. IRF of Consumption—Baseline versus Secular Decline

Questions

- Convincing?

OUTLINE

Does Central Bank Create or Borrow Demand?

- In standard NK model:

$$\begin{aligned} y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\ &= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T\to\infty} y_{t+1} \end{aligned}$$

- Central bank creates demand and output by changing r_t.
- Past r_t has no bearing on future AD.

$\Rightarrow$ Central bank can create AD subject to ZLB constraint.

- This paper: with durable goods, central bank is much more in the business of borrowing demand than creating demand.

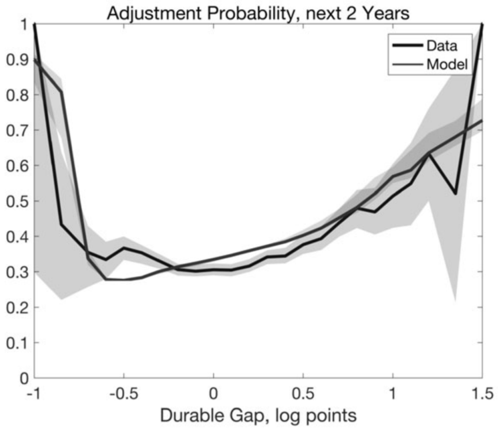
Model: Households

- Households consume non-durables and durables.

$$\max E_{i0} \int_{t=0}^{\infty} e^{-\rho t} u(c_{it}, q_{it} d_{it})\, dt$$

- Durables subject to:
  - fixed adjustment cost (if adjust) $a'_{it} + p_t d'_{it} = a_{it} + (1-f) p_t d_{it}$
  - depreciation and maintenance cost $dtd_{it} = -(1-\chi)\delta d_{it}$
  - operating cost (e.g. household utilities, gas, taxes) νd_{it}
  - match-quality shocks $q_{it} = 1$, drops to zero with intensity θ
- Idiosyncratic labor income risk
  $y_{it} = (1-\tau_t) Y_t z_{it}$ $\qquad$ $d \ln z_{it} = -\rho_z \ln z_{it} + \sigma_z d\mathscr{W}_{it}$
- Save in liquid assets
  $dta_{it} = r_t a_{it} + r_t^b a_{it} I_{\{a_{it}<0\}} - c_{it} + y_{it} - (\chi \delta p_t + \nu) d_{it}$
- Collateralized borrowing $a_{it} \geq -\lambda(1-f) p_t d_{it}$
- Sticky information (Carrol et al, 2018; Auclert et al, 2020).

Hazard Rate


Adjustment Probability, next 2 Years
Data
Model
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1
1.5
Durable Gap, log points

Intertemporal Shifting in the Data

LUMPY DURABLE CONSUMPTION DEMAND 2727

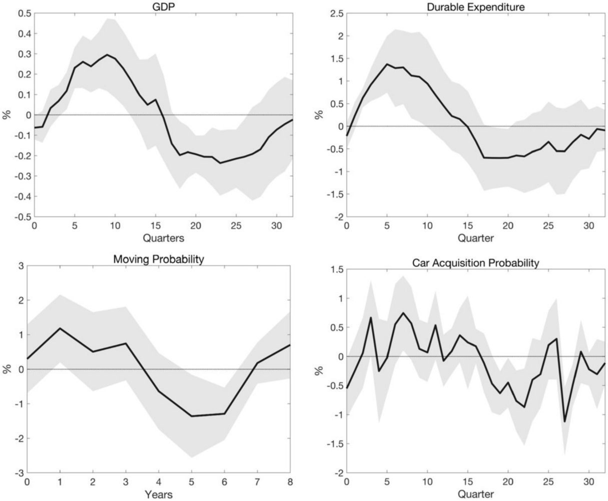


FIGURE 2.—Impulse response function of real GDP (top-left panel), real durable expenditure (top-right), probability of moving house (bottom-left), and probability of buying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence bands.

Intertemporal Shifting in the Data

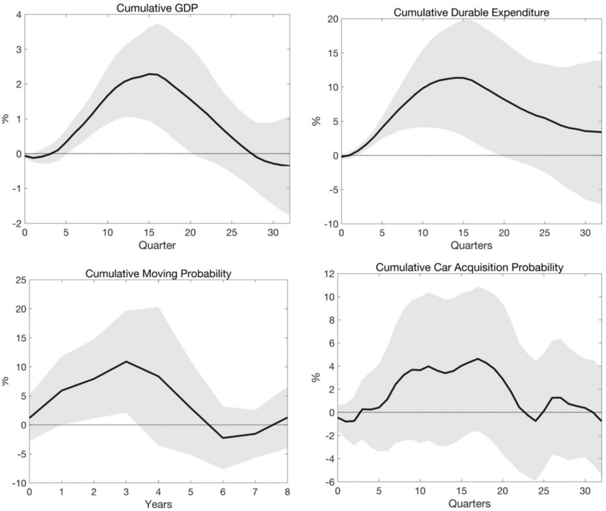


FIGURE 3.—Impulse response function of cumulative real GDP (top-left panel), cumulative real durable xpenditure (top-right), cumulative probability of moving house (bottom-left), and cumulative probability of uying a car (bottom-right) to a Romer and Romer monetary policy shock. Shaded areas are 95% confidence ands.

Durables　　　　Nondurables

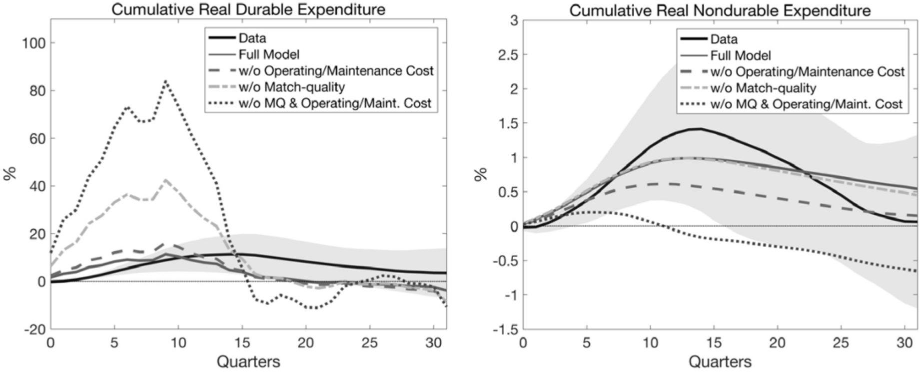
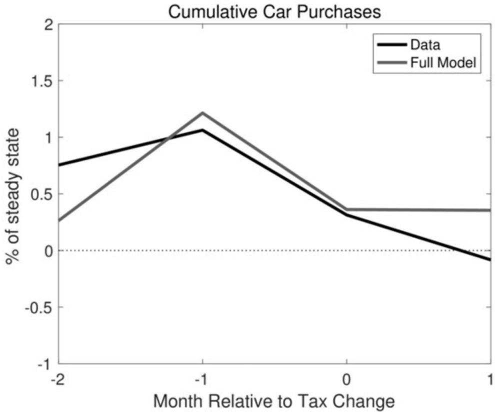


FIGURE 4.—Cumulative response of durable (left) and nondurable (right) expenditure to a simulated monetary policy shock. Model simulations feeding in the estimated impulse responses for (Y_t, r_t, p_t). Each panel shows the full model as well as the models that omit one or both of match-quality shocks and operating and maintenance costs.

Model vs Data



The Monetary Transmission Matrix

- Solve model with sequence space methods.
- The "monetary transmission matrix":

$$\mathcal{M} = \begin{pmatrix} \frac{d\hat{Y}_0}{dr_0} & \frac{d\hat{Y}_0}{dr_1} & \frac{d\hat{Y}_0}{dr_2} & \cdots \\ \frac{d\hat{Y}_1}{dr_0} & \frac{d\hat{Y}_1}{dr_1} & \frac{d\hat{Y}_1}{dr_2} & \cdots \\ \frac{d\hat{Y}_2}{dr_0} & \frac{d\hat{Y}_2}{dr_1} & \frac{d\hat{Y}_2}{dr_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- GE effect of monetary news on output gap.
  - Below-diagonal elements capture intertemporal-shifting effects.
  - Above-diagonal elements capture forward guidance effects.

IRF TO r_0 AND NEWS

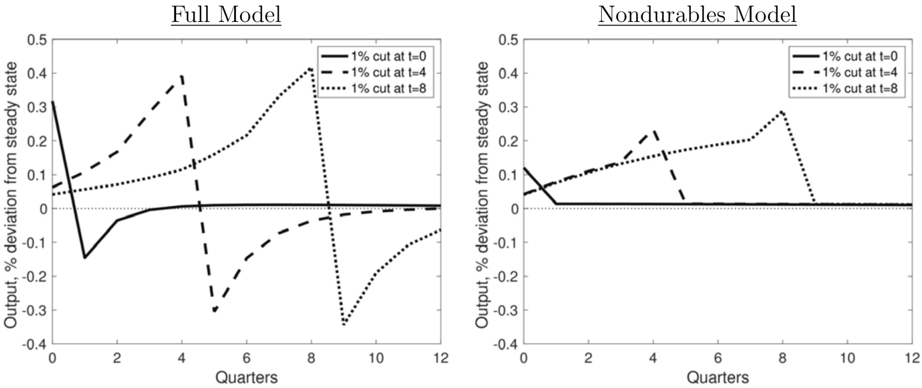
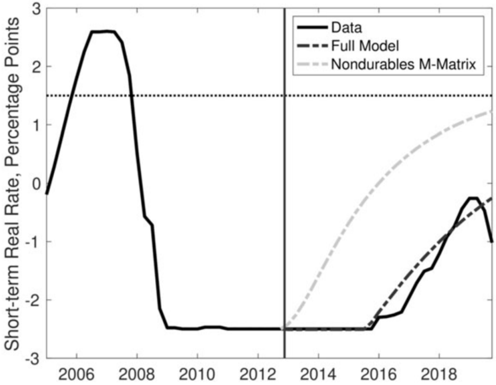


FIGURE 7.—Percentage change in output following a 1% reduction in the real interest rate at horizons $t = 0$ (solid line), $t = 4$ (dashed line), and $t = 8$ (dotted line). The left panel shows the effect in the full model, and the right panel shows the effect in the nondurables model.

Great Recession–Filtering the Data

- Construct IRFs for shocks to Z_t, G_t, r_t^b, η_t.
- Match time series:
  - output gap,
  - durable spending,
  - real interest rate,
  - mortgage-treasury spread.
- Unique sequence of shocks that fit data exactly given initial condition. (Kalman filter with no observational error.)
  - No need for state transition matrix.
- Account for zero lower bound using monetary news shocks.

Model predicts slow normalization of real rate



CONTRIBUTION OF INTERTEMPORAL SHIFTING TO r^*

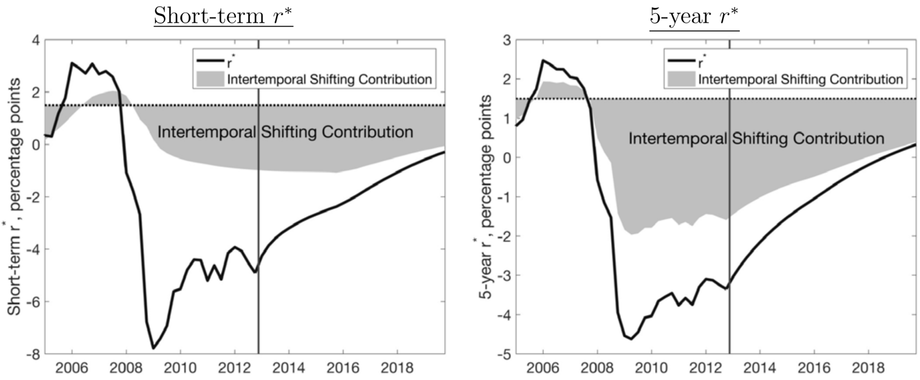
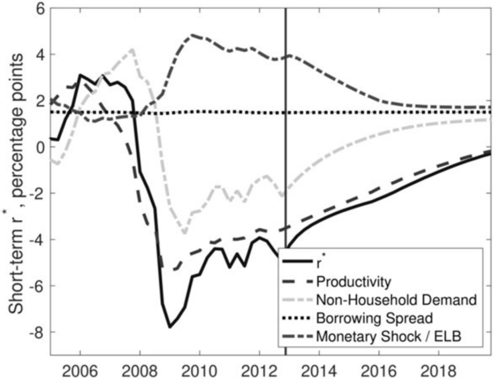


FIGURE 10.—Time series of the short-term natural rate of interest (left panel) and the 5-year natural rate of interest (right panel). The shaded area shows the contribution of intertemporal shifting effects of previous real interest rates to r^*. Forecast as of 2012Q4. The dotted horizontal line is the steady state real interest rate, equal to 1.5%.

CONTRIBUTION SHOCKS TO r^*



Questions

- Convincing?

- What role does the micro evidence play?

OUTLINE

Forward Guidance Puzzle

- In standard NK model:

$$
\begin{aligned}
y_t &= -\frac{1}{\sigma} r_t + y_{t+1} \\
&= -\frac{1}{\sigma} \sum_{s=0}^{\infty} r_{t+s} + \lim_{T \to \infty} y_{t+1}
\end{aligned}
$$

- Credible future real rate change has the same effect on current output regardless of how distant it is.

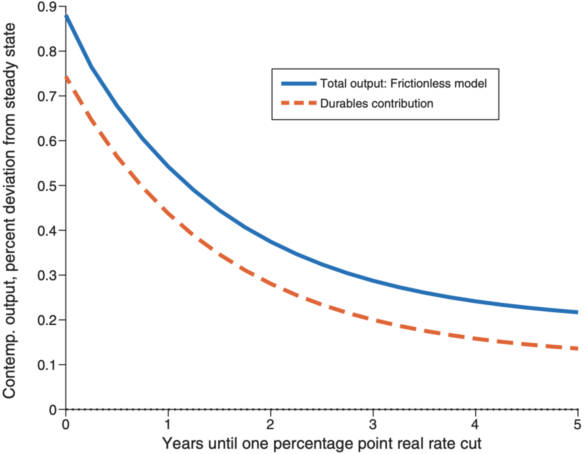
With Durable Goods

- MRS between durable and nondurable good is equal to user cost:

$$\left(\frac{\psi}{1-\psi}\frac{c_{it}}{d_{it}}\right)^{\frac{1}{\xi}} = p_t(r_t+\nu+\delta) - \dot{p}_t \equiv r_t^d$$

- Special role for contemporaneous real rate in durable goods demand.

Frictionless Durable Model



Lumpy durables

- Frictionless model inconsistent with lumpy nature of durable adjustment.

$\Rightarrow$ How credible are the results?

Fixed Cost Durable Model

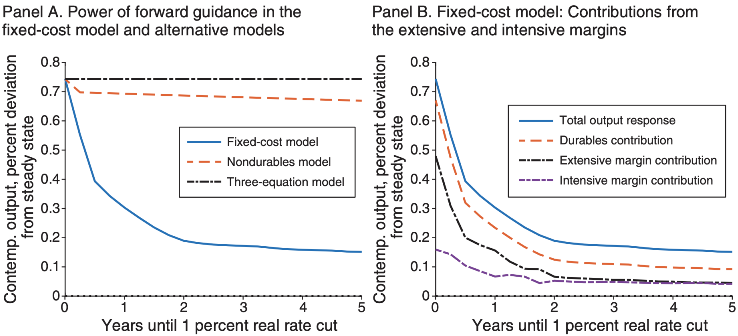


Figure 2. Contemporaneous Output Response to Forward Guidance in the Fixed-Cost Model

Extensive Margin

$$\frac{u(c_t^*, d_t^*) - u(c_t, d)}{V_{a,t}(a_t^*, d_t^*, z)} = r_t^d (d_t^* - d) + \left[r_t^d - (\nu + \delta\chi) p_t \right] fd + (c_t^* - c_t)$$

$$+ \frac{\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1}{1 - \lambda(1 - f)} \left\{ \frac{a}{p_t} \left[r_t^d - (\nu + \delta\chi) p_t \right] + z(1 - \tau_t) Y_t - c_t - (\nu + \delta\chi) p_t d \right.$$

- Intuition: adjust now vs a little bit later.

Importance of Contemporaneous User Cost

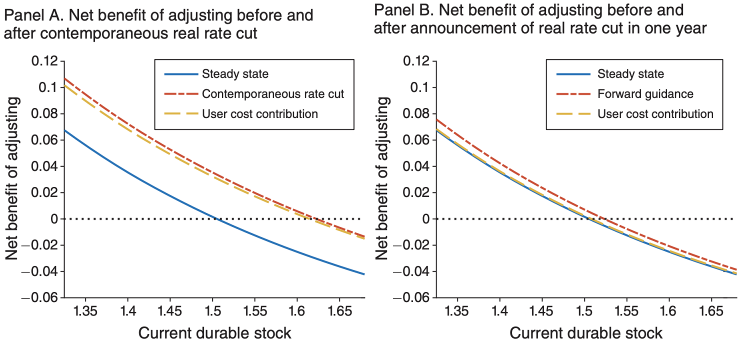


Figure 3. The Net Benefit of Making a Durable Adjustment

Intensive Margin

$$
\begin{aligned}
&\mathbb{E}_t \int_0^{\tau} e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d) ds \\
&\quad = \mathbb{E}_t e^{-\rho\tau} V^{adj}_{x,t+\tau} \left[r^d_{t,t+\tau} + e^{-\delta(1-\chi)\tau} p_{t+\tau} f \right] \\
&\qquad + \mathbb{E}_t \int_0^{\tau} e^{-\rho s} \Psi_{t+s} \left[r^d_{t,t+s} + (1-\lambda(1-f)) e^{-\delta(1-\chi)s} p_{t+s} \right] ds
\end{aligned}
$$

- Intuition: survival probability decreasing in time.

Long-term financing

- FOC for extensive margin the same when durable financed with long-term debt.

- Intuition: adjust now vs a little bit later, so expected change in financing cost matters for adjustment.

Questions

- Convincing?

- What role does the micro evidence play?