4. Search

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Take It Or Leave It

- Today we apply dynamic programming to a binary labor market decision
- An unemployed worker looking for a job receives take-it-or-leave-it job offers with different characteristics
- Given her perception of the job offer she must decide to accept the offer or to reject the offer and keep searching

Search Theory

- Search theory was initiated by George Stigler (1961, Nobel Prize in 1982)
- Search theory studies buyers or sellers who cannot instantly find a trading partner and must therefore search before transacting
  - A consumer looks for a high-quality, low-price good
  - A worker looks for an interesting, high-paying job
  - An investor looks for a low-risk, high-return asset
- Search theory essentially relaxes the full information assumption present in conventional models

Search Theory in Labor Economics

- Stigler (1962) applied search theory to the labor market
- It has become a widely used tool to study unemployment
- Search theory studies how workers respond to
  - Labor market tightness
  - The probability of being fired
  - Unemployment benefits
  - Information about jobs
  - New job opportunities while working
  - The riskiness of wage distributions

Lecture Outline

1. The McCall Model of Job Search
2. Waiting Time
3. Allowing Quits
4. Allowing Firing
5. Aggregating: A Lake Model
6. Exercises

Main Reference: Ljungqvist and Sargent, 2018, *Recursive Macroeconomic Theory*, Fourth Edition, Chapter 6

1. The McCall Model of Job Search

Lucas on McCall

- The McCall model was developed by John McCall (1970, *QJE*)

"Questioning a McCall worker is like having a conversation with an out-of-work friend: 'Maybe you are setting your sights too high' or 'Why did you quit your old job before you had a new one lined up?' This is real social science: an attempt to model, to understand, human behavior by visualizing the situations people find themselves in, the options they face and pros and cons as they themselves see them."

Robert Lucas, 1987, *Models of Business Cycles*

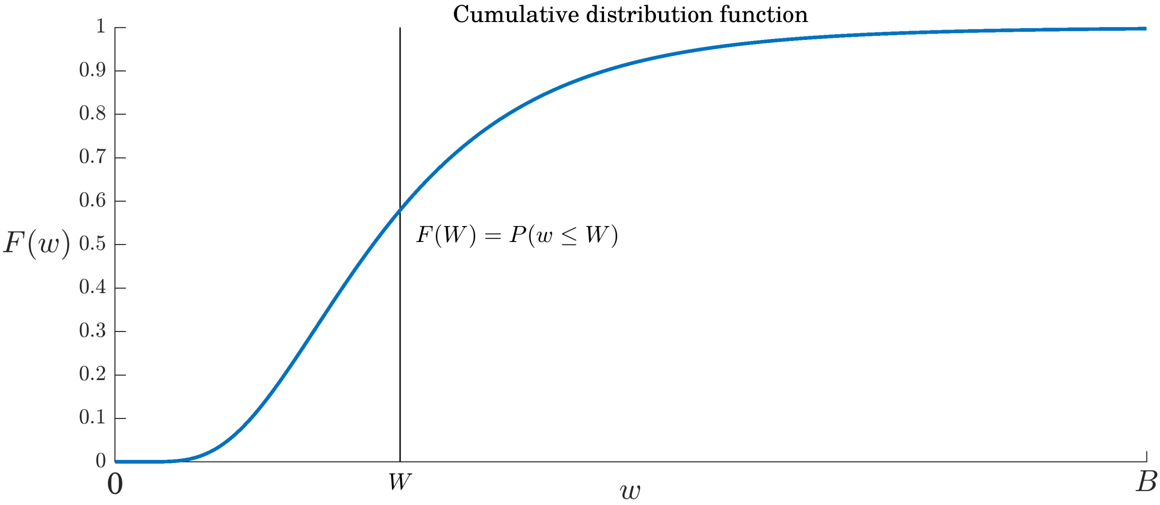
Model Setup

- Consider an unemployed worker who is searching for a job
- Each period the worker draws one offer w from the same wage distribution

$$F(W) = \text{Prob}\{w \leq W\} \quad \text{with } F(0) = 0 \text{ and } F(B) = 1 \text{ for } B < \infty$$

- w summarizes the job's characteristics, with B the maximum possible wage
- We assume that draws from F are independent and identically distributed

Wage Distribution



Undirected Search

- This is a model of random search or undirected search
- The worker has no ability to direct her search towards different types of jobs or different parts of the wage distribution
- The alternative, directed search, is more realistic (but a bit more complex)

Binary Choice

- The worker has two options

1. Accept the offer and receive a wage w per period forever
2. Reject the offer, receive unemployment compensation c, and wait until next period to draw another offer from F

- Neither quitting nor firing is permitted for now

Preferences

- Let y_t be the worker's income in period t

$$y_t = c \quad \text{if the worker is unemployed}$$
$$y_t = w \quad \text{if the worker accepts the offer and works}$$

- The worker maximizes the expected infinite sum of discounted income

$$E_0 \sum_{t=0}^{\infty} \beta^t y_t, \qquad 0 < \beta < 1$$

- Utility is linear in income, so the worker is risk neutral

Value Function

- We assume the worker behaves optimally
- Let the value function $v(w)$ be the expected optimal value of $\sum_{t=0}^{\infty} \beta^t y_t$ for a previously unemployed worker who has offer w in hand
- We want to write a Bellman equation
- But first let's study the two choices the worker faces

Accept or Reject

1. If the worker accepts the job, she receives wage w each period forever

$$\text{Value of accepting} = w + \beta w + \beta^2 w + \ldots = \sum_{t=0}^{\infty} \beta^t w = \frac{w}{1-\beta}$$

2. If she rejects the job, she receives unemployment benefit c this period and draws a new offer w' from the same distribution F next period

$$\begin{aligned}\text{Value of rejecting} &= c + \beta \int_0^B v(w')dF(w') \\ &= c + \beta \int_0^B v(w')f(w')dw'\end{aligned}$$

Bellman Equation

- The value function $v(w)$ satisfies the Bellman equation

$$v(w) = \max_{\text{accept,reject}} \left\{ \frac{w}{1-\beta}, \quad c + \beta \int_0^B v(w')dF(w') \right\} \tag{1}$$

- This is the functional equation discussed in the previous class
- But here the maximization is over two actions only, not a continuum

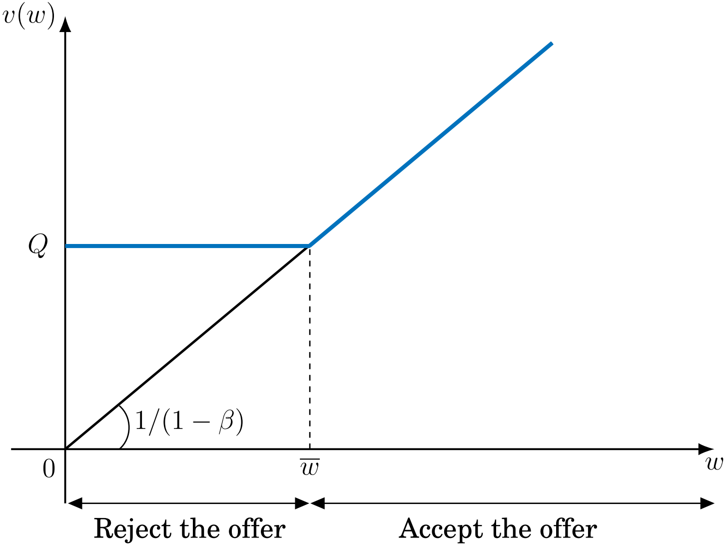
Studying the Value Function

- Let's plot the value function $v(w)$
- From the Bellman equation, we see that

$$v(w) = \max_{\text{accept,reject}} \left\{ \underbrace{\frac{w}{1-\beta}}_{\text{Increasing in } w}, \underbrace{c + \beta \int_0^B v(w')dF(w')}_{\text{Constant in } w \equiv Q} \right\}$$

- The function $v(w)$ has two parts: 1) a part constant in w, call it Q; 2) a part increasing in w with slope $\frac{1}{1-\beta}$
- Thus the value function is not linear but rather has a kink

Plotting the Value Function



Solution

- Define $\overline{w}$ such that the value of accepting equals the value of rejecting

$$\frac{\overline{w}}{1-\beta} = c + \beta \int_0^B v(w')dF(w')$$

- $\overline{w}$ is the reservation wage, ie the threshold wage at which the worker is indifferent between working and not working
- The solution is of the form

$$v(w) = \begin{cases} \dfrac{\overline{w}}{1-\beta} = c + \beta \displaystyle\int_0^B v(w')dF(w') = Q & \text{if} \quad w \leq \overline{w} \\ \dfrac{w}{1-\beta} & \text{if} \quad w \geq \overline{w} \end{cases}$$

Reservation Wage

- Using the solution, we can convert the functional equation (1) in the value function $v(w)$ into an ordinary equation in the reservation wage $\overline{w}$
- In other words we only need to know $\overline{w}$ to characterize v (we know c, β, B)
- Evaluating $v(\overline{w})$, we have

$$\begin{aligned}\frac{\overline{w}}{1-\beta} &= c + \beta \int_0^B v(w')dF(w') \\ &= c + \beta \int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') + \beta \int_{\overline{w}}^B \frac{w'}{1-\beta} dF(w')\end{aligned}$$

Simple Algebra

$$
\begin{aligned}
\frac{\overline{w}}{1-\beta} &= c + \beta \int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') + \beta \int_{\overline{w}}^{B} \frac{w'}{1-\beta} dF(w') \\
\frac{\overline{w}}{1-\beta} \int_0^{\overline{w}} dF(w') + \frac{\overline{w}}{1-\beta} \int_{\overline{w}}^{B} dF(w') &= c + \beta \int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') + \beta \int_{\overline{w}}^{B} \frac{w'}{1-\beta} dF(w') \\
\frac{\overline{w}}{1-\beta} \int_0^{\overline{w}} dF(w') - \beta \int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') - c &= \beta \int_{\overline{w}}^{B} \frac{w'}{1-\beta} dF(w') - \frac{\overline{w}}{1-\beta} \int_{\overline{w}}^{B} dF(w') \\
\overline{w} \int_0^{\overline{w}} dF(w') - c &= \frac{1}{1-\beta} \int_{\overline{w}}^{B} (\beta w' - \overline{w}) dF(w') \\
\overline{w} \int_0^{\overline{w}} dF(w') - c + \overline{w} \int_{\overline{w}}^{B} dF(w') &= \frac{1}{1-\beta} \int_{\overline{w}}^{B} (\beta w' - \overline{w}) dF(w') + \overline{w} \int_{\overline{w}}^{B} dF(w') \\
\overline{w} - c &= \frac{\beta}{1-\beta} \int_{\overline{w}}^{B} (w' - \overline{w}) dF(w')
\end{aligned}
$$

Cost Equals Benefit

- We find that the reservation wage must satisfy

$$\underbrace{\overline{w} - c}_{\text{Cost of search}} = \underbrace{\frac{\beta}{1-\beta} \int_{\overline{w}}^{B} (w' - \overline{w}) dF(w')}_{\text{Benefit of search}} \tag{2}$$

- The left side is the foregone revenue, or cost, of searching one more time when an offer $\overline{w}$ is in hand
- The right side is the expected present-value benefit of searching one more time and drawing a better offer $w' > \overline{w}$
- (2) tells the agent to set $\overline{w}$ so that the cost of searching equals the benefit

Characterizing the Reservation Wage

- We want to study further the reservation wage
- In particular how does $\overline{w}$ evolve when
  - The unemployment benefit c changes?
  - The discount factor β changes?
  - The standard deviation of F changes?

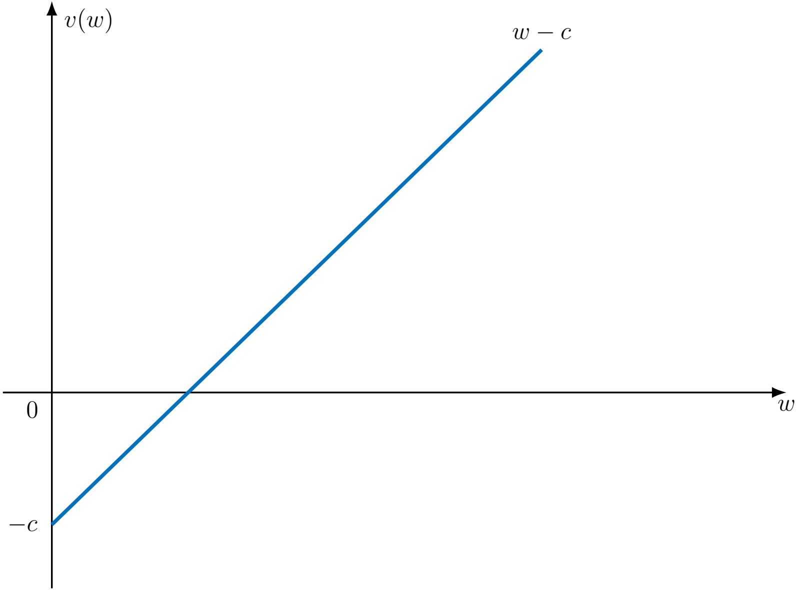
Left Side

- Define the function of the left side of the reservation wage equation (2) as

$$j(w) \equiv w - c$$

- Notice that $j(0) = -c$ and $j'(w) = 1$
- So $j(w)$ is a straight line increasing in w with a slope of 1 and intercept $-c$

Increasing Cost of Search



Right Side

- Define the function on the right side of (2) as

$$h(w) \equiv \frac{\beta}{1-\beta}\int_{w}^{B}(w'-w)dF(w')$$

- Notice that $h(0) = \frac{\beta}{1-\beta}\int_0^B w' dF(w') = \frac{\beta}{1-\beta}E(w)$ and $h(B) = 0$
- To find $h'(w)$ and $h''(w)$, let's differentiate $h(w)$ using Leibniz's rule

Reminder on Leibniz's Rule

- The Leibniz integral rule helps differentiate integrals
- Named after German mathematician Gottfried Leibniz
- Consider the following integral

$$\int_{a(z)}^{b(z)} f(x,z)dx$$

- The derivative of this integral with respect to z is

$$\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z}dx + f(b(z),z)\frac{\partial b}{\partial z} - f(a(z),z)\frac{\partial a}{\partial z}$$

Differentiating

- Apply Leibniz's rule to compute the first derivative of $h(w)$

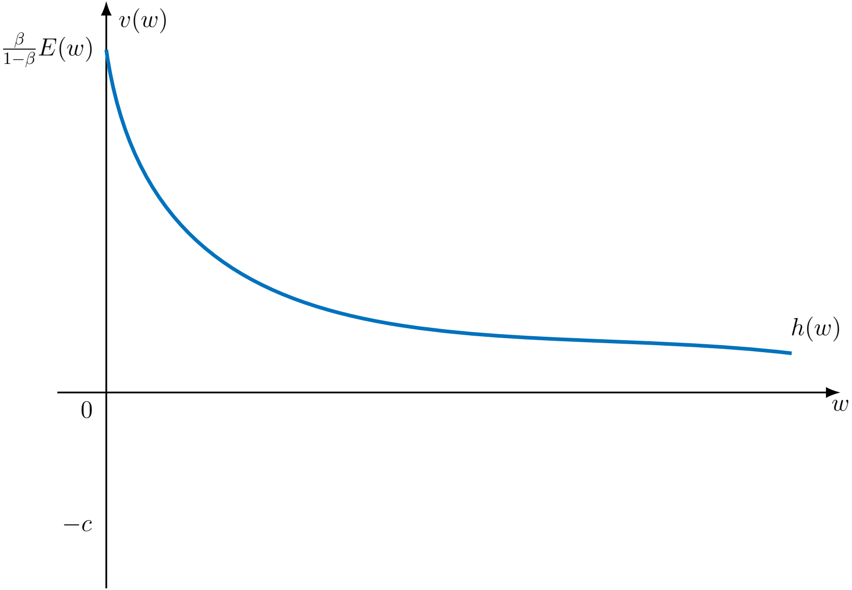
$$h'(w) = -\frac{\beta}{1-\beta}\int_w^B dF(w') = -\frac{\beta}{1-\beta}[1 - F(w)] < 0$$

- The second derivative is

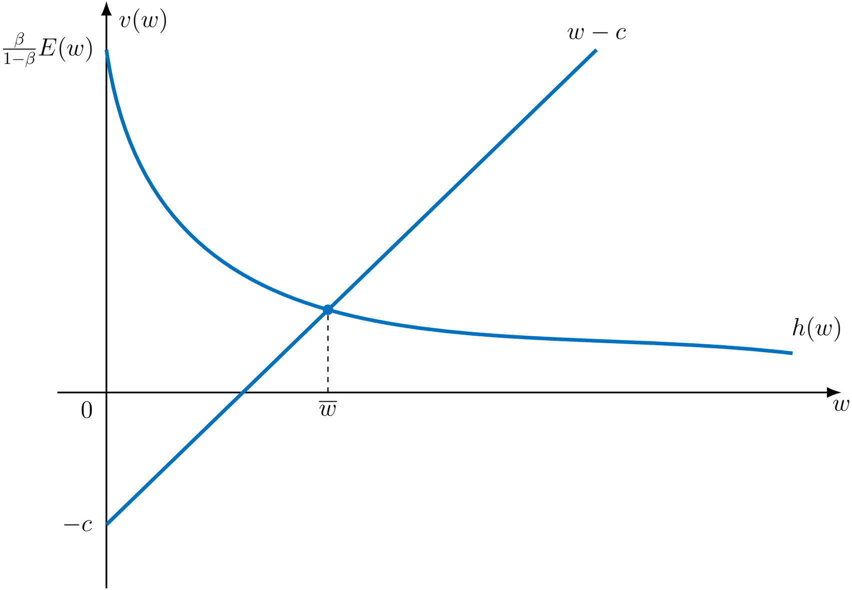
$$h''(w) = \frac{\beta}{1-\beta}F'(w) > 0$$

- We conclude that $h(w)$ is strictly decreasing and convex

Decreasing Benefit of Search



Plotting Both Functions Together



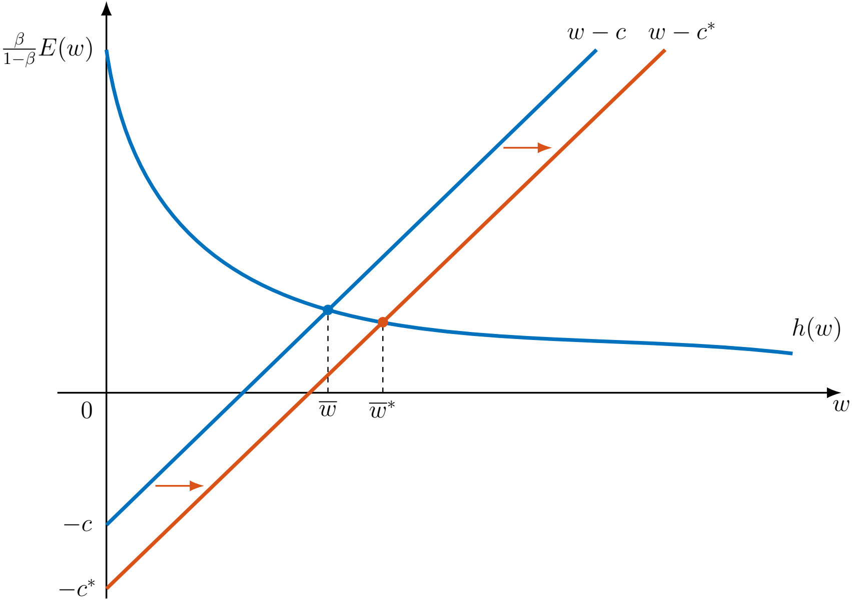
Comparative Statics – Unemployment Benefit

- What is the effect of an increase in unemployment benefit c?

Comparative Statics – Unemployment Benefit

- What is the effect of an increase in unemployment benefit c?
- An increase in c shifts the straight line $j(w) = w - c$ downward
- This leads to an increase in the reservation wage $\overline{w}$
- The worker has a better outside option and allows herself to be more picky

Increase in Unemployment Benefit



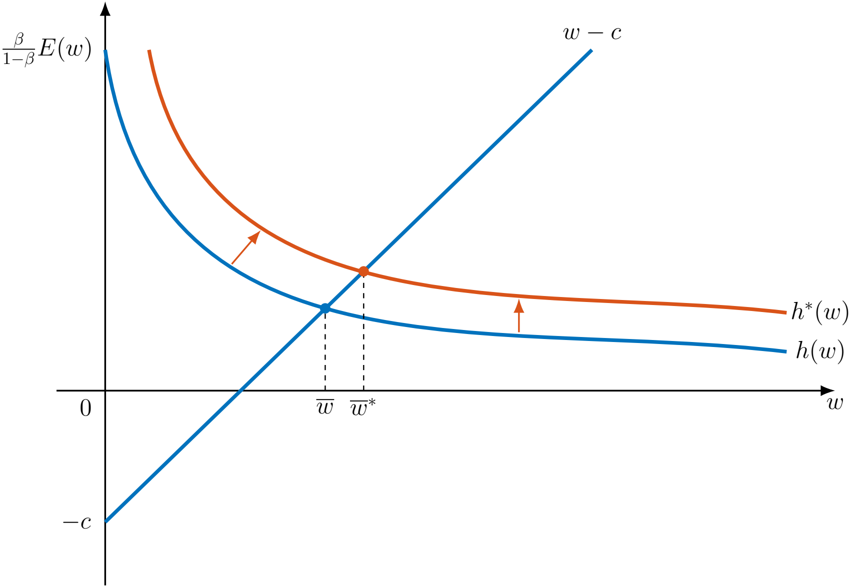
Comparative Statics – Discount Factor

- What is the effect of an increase in the discount factor β?

Comparative Statics – Discount Factor

- What is the effect of an increase in the discount factor β?
- A higher β increases $\frac{\beta}{1-\beta}$ and hence $h(0)$ and shifts the curve $h(w)$ upward
- This leads to an increase in the reservation wage $\overline{w}$
- The worker is more patient, ie she values more the future and waits more

Increase in the Discount Factor



Another Characterization of the Reservation Wage

- We can rewrite (2) as follows

$$\bar{w} - c = \frac{\beta}{1-\beta}\int_{\overline{w}}^{B}(w' - \overline{w})dF(w') + \frac{\beta}{1-\beta}\int_{0}^{\overline{w}}(w' - \overline{w})dF(w') - \frac{\beta}{1-\beta}\int_{0}^{\overline{w}}(w' - \overline{w})dF(w')$$

$$= \frac{\beta}{1-\beta}Ew - \frac{\beta}{1-\beta}\overline{w} - \frac{\beta}{1-\beta}\int_{0}^{\overline{w}}(w' - \overline{w})dF(w')$$

$$\text{or} \quad \overline{w} - (1-\beta)c = \beta Ew - \beta\int_{0}^{\overline{w}}(w' - \overline{w})dF(w')$$

- Apply integration by parts, $\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx$, to the last integral and rearrange

$$\overline{w} - c = \beta(Ew - c) + \beta\int_{0}^{\overline{w}} F(w')dw'$$

Studying the Expression

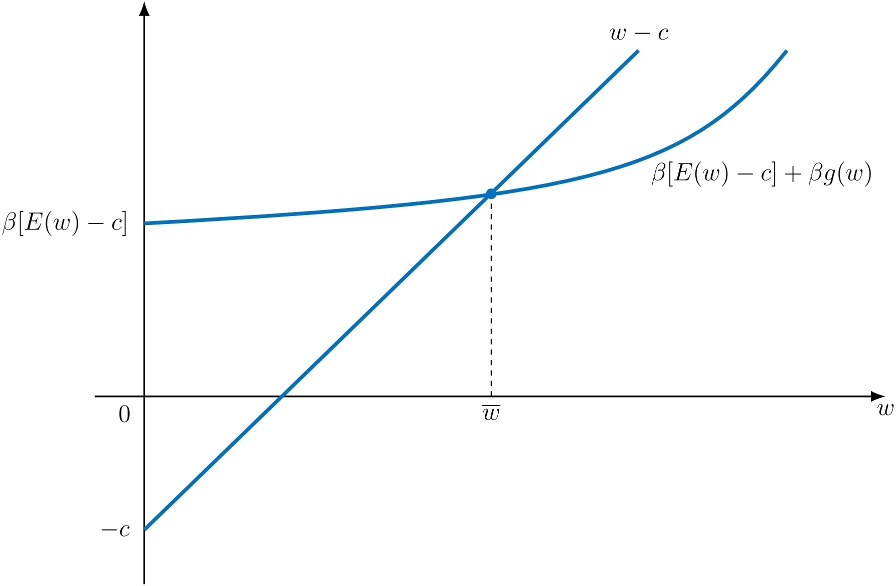
- The previous equation can be represented as

$$\overline{w} - c = \beta(Ew - c) + \beta g(\overline{w}) \quad \text{where we define } g(s) \equiv \int_0^s F(p)dp$$

- The function g has the characteristics that

$$g(0) = 0; \quad g(s) \geq 0; \quad g'(s) = F(s) > 0; \quad g''(s) = F'(s) > 0 \quad \text{for } s > 0$$

Plotting g



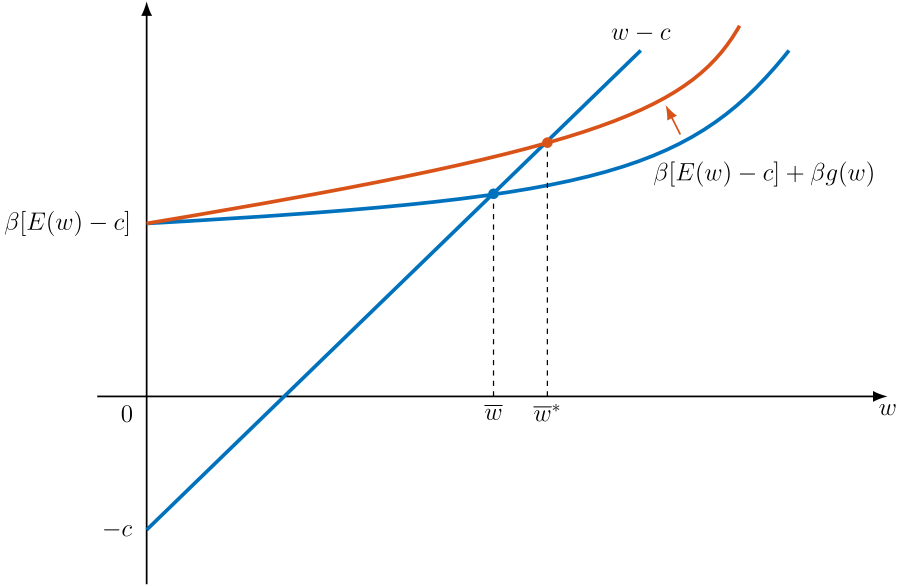
Mean-Preserving Spread

- What happens if the wage offer is drawn from a distribution $\tilde{F}$ that has the same mean as F but a higher standard deviation?

Mean-Preserving Spread

- What happens if the wage offer is drawn from a distribution $\tilde{F}$ that has the same mean as F but a higher standard deviation?
- A mean-preserving increase in risk shifts $\beta(Ew - c) + \beta g(w)$ upwards and hence increases the reservation wage $\overline{w}$
- Higher risk raises the chances to receive both very good and very bad offers
- Bad offers don't matter, since the worker won't accept them
- But a higher incidence of good offers raises the value of searching

Mean-Preserving Spread



Like an Option

- This echos a result in option pricing theory that the value of an option is an increasing function of the variance in the price of the underlying asset
- This is so because the option holder chooses to accept payoffs only from the right tail of the distribution (an option gives the right, not the obligation)
- In our context, the unemployed worker has the option to accept a job and the asset value of a job offering wage w is equal to $w/(1-\beta)$
- Higher risk increases the value of searching (right tail) while not being detrimental since the option to work at low wages will not be exercised

2. Waiting Time

Waiting Time

- We may ask the following question: in this model, what is the probability distribution of the waiting time until a job offer is accepted?
- Let N be the random variable "length of time until a successful offer comes"

$$
\begin{aligned}
N &= 1 && \text{if the first job offered is accepted} \\
N &= 2 && \text{if the second job offered is accepted} \\
&\vdots && \qquad\qquad\vdots \\
N &= n && \text{if the } n\text{th job offered is accepted}
\end{aligned}
$$

Geometric Distribution

- Let $\lambda = \int_0^{\overline{w}} dF(w') = F(\overline{w})$ be the probability that a job offer is rejected

$$
\begin{aligned}
\text{Prob}\{N = 1\} &= 1 - F(\overline{w}) = 1 - \lambda \\
\text{Prob}\{N = 2\} &= [1 - F(\overline{w})]F(\overline{w}) = (1 - \lambda)\lambda \\
\text{Prob}\{N = 3\} &= [1 - F(\overline{w})]F(\overline{w})^2 = (1 - \lambda)\lambda^2 \\
\text{Prob}\{N = j\} &= [1 - F(\overline{w})]F(\overline{w})^{j-1} = (1 - \lambda)\lambda^{j-1}
\end{aligned}
$$

- The waiting time is geometrically distributed

Convenient Formula

- To get the mean waiting time of unemployment, we use the formula

$$\text{Expected duration} = \sum_{j=1}^{\infty} (\text{Duration} = j) \times (\text{Probability of duration} = j)$$

Mean Waiting Time

- The mean waiting time $\overline{N}$ is

$$\overline{N} = \sum_{j=1}^{\infty} j \times \text{Prob}\{N = j\} = \sum_{j=1}^{\infty} j(1-\lambda)\lambda^{j-1} = (1-\lambda)\sum_{j=1}^{\infty}\sum_{k=1}^{j} \lambda^{j-1}$$

$$= (1-\lambda)\sum_{k=0}^{\infty}\sum_{j=1}^{\infty} \lambda^{j-1+k} = (1-\lambda)\sum_{k=0}^{\infty} \lambda^{k}(1-\lambda)^{-1} = \frac{1}{1-\lambda}$$

Explanation for third equality

$$\sum_{j=1}^{\infty}\sum_{k=1}^{j} \lambda^{j-1} = (\lambda^{1-1}) + (\lambda^{2-1} + \lambda^{2-1}) + (\lambda^{3-1} + \lambda^{3-1} + \lambda^{3-1}) + \cdots = 1 + 2\lambda + 3\lambda^2 + \cdots = \sum_{j=1}^{\infty} j \times \lambda^{j-1}$$

- The mean waiting time is the reciprocal of the probability of accepting an offer on a single trial

Alternative Method

- We can also compute the mean waiting time in a different way, as follows
- We know that the search environment is stationary: there is a constant reservation wage and a constant probability of escaping unemployment
- Thus in any period, the remaining mean waiting time is $\overline{N}$ regardless of how long the worker has been unemployed

Alternative Method

- The mean waiting time equals the weighted sum of two possible outcomes
  1. Either accept a job next period with probability $(1 - \lambda)$
  2. Or remain unemployed next period with probability λ
- Hence, the mean waiting time must satisfy

$$\overline{N} = \underbrace{(1-\lambda) \times 1}_{\substack{\text{Accept and end U} \\ \text{after one period}}} + \underbrace{\lambda \times (1 + \overline{N})}_{\substack{\text{Refuse and face} \\ \text{the same remaining time}}}$$

- Solve for $\overline{N}$

$$\overline{N} = \frac{1}{1-\lambda}$$

Higher Unemployment Benefit

- How does a higher unemployment benefit c affect the mean waiting time $\overline{N}$?

Higher Unemployment Benefit

- How does a higher unemployment benefit c affect the mean waiting time $\overline{N}$?
- A higher unemployment benefit c
  - Increases the reservation wage $\overline{w}$
  - Increases the probability to reject an offer λ
  - Increases the mean waiting time $\overline{N}$

Extensions

- That's it for the basic McCall model
- In what follows we are going to see three extensions to the basic model

1. Allowing quits
2. Allowing firing
3. Aggregating to economy-wide totals

3. Allowing Quits

Allowing Quits

- The worker is now allowed to quit her job and find another one
- If she quits she must become unemployed for one period
- In other words we rule out job-to-job transitions

The Setup

- Remember, the reservation wage $\overline{w}$ satisfies

$$v(\overline{w}) = \frac{\overline{w}}{1-\beta} = c + \beta \int_0^B v(w')dF(w')$$

- Suppose the unemployed worker has in hand an offer to work at wage w
- Let's compute the lifetime utility associated with three alternative ways of responding to that offer

Three Possibilities

1. Accept the wage and keep the job forever

$$v_1 = \frac{w}{1-\beta}$$

2. Accept the wage but quit after t periods

$$v_2 = \sum_{s=0}^{t-1} \beta^s w + \beta^t \left(c + \beta \int_0^B v(w')dF(w') \right) = w\frac{1-\beta^t}{1-\beta} + \beta^t \frac{\bar{w}}{1-\beta} = \frac{w}{1-\beta} - \beta^t \frac{w - \overline{w}}{1-\beta}$$

3. Reject the wage

$$v_3 = c + \beta \int_0^B v(w')dF(w') = \frac{\overline{w}}{1-\beta}$$

Low-Wage Offer

- Suppose the worker draws an offer $w < \overline{w}$
- Then for any $0 < \beta < 1$

$$v_1 < v_2 < v_3$$

- The third option strictly dominates
- The worker rejects the offer

High-Wage Offer

- Now suppose the worker draws an offer $w > \overline{w}$
- Then for any $0 < \beta < 1$

$$v_1 > v_2 > v_3$$

- The first option strictly dominates
- The worker accepts the offer and never quits

Never Quit

- The conclusion is straightforward: v_2 is always dominated
- Even if she has the option to quit, the worker never quits
- What if the worker draws $w = \overline{w}$?

Never Quit

- The conclusion is straightforward: v_2 is always dominated
- Even if she has the option to quit, the worker never quits
- What if the worker draws $w = \overline{w}$?
- The three alternatives yield the same lifetime utility

4. Allowing Firing

Allowing Firing

- Each period after the first period on the job, the worker faces probability α of being fired, where $0 < \alpha < 1$
- We assume the probability is independent of tenure
- A worker who is fired becomes unemployed and receives c
- The worker must stay unemployed for one period before drawing a new wage from the same distribution F

Bellman Equation

- Let $\hat{v}(w)$ be the expected present value of income of an unemployed worker who has offer w in hand
- The Bellman equation writes

$$\hat{v}(w) = \max_{\text{accept,reject}} \left\{ \underbrace{w + \beta(1-\alpha)\hat{v}(w) + \beta\alpha \left[c + \beta \int_0^B \hat{v}(w')dF(w') \right]}_{\text{Accept and work; next period may continue working or get fired}}, \underbrace{c + \beta \int_0^B \hat{v}(w')dF(w')}_{\text{Reject and stay unemployed}} \right\}$$

A Different Problem

- Note that $\hat{v}(w)$ appears on the right side
- Indeed, since F, α, c are all fixed, the problem is stationary, so $\hat{v}(w)$ is also the continuation value associated with retaining the job next period
- How do we solve this problem? We proceed in two steps

First Step – Guess

- We make the guess that $\hat{v}(w)$ is nondecreasing in w
- This implies that there exists a threshold $\overline{w}$ such that

$$\hat{v}(w) = \begin{cases} c + \beta \displaystyle\int_0^B \hat{v}(w')dF(w') & \text{if} \quad w \leq \overline{w} \\ \dfrac{w + \beta\alpha \left[c + \beta \int_0^B \hat{v}(w')dF(w')\right]}{1 - \beta(1-\alpha)} & \text{if} \quad w \geq \overline{w} \end{cases}$$

Reservation Wage

- The reservation wage $\overline{w}$ is such that the value of accepting the offer equals the value of rejecting

$$c + \beta \int_0^B \hat{v}(w')dF(w') = \frac{\overline{w} + \beta\alpha \left[c + \beta \int_0^B \hat{v}(w')dF(w')\right]}{1 - \beta(1-\alpha)}$$

- This can be rearranged as

$$\frac{\overline{w}}{1-\beta} = c + \beta \int_0^B \hat{v}(w')dF(w')$$

Second Step – Verify

- We verify that $\hat{v}(w)$ is nondecreasing in w

$$\hat{v}(w) = \begin{cases} c + \beta \displaystyle\int_0^B \hat{v}(w')dF(w') & \text{if} \quad w \leq \overline{w} \\ \dfrac{w + \beta\alpha \left[c + \beta \int_0^B \hat{v}(w')dF(w')\right]}{1 - \beta(1-\alpha)} & \text{if} \quad w \geq \overline{w} \end{cases}$$

- Indeed, as w increases, $\hat{v}(w)$ is either constant or increases

Different Reservation Wage

- In the model without firing, the reservation wage satisfies

$$\frac{\overline{w}}{1-\beta} = c + \beta \int_0^B v(w')dF(w')$$

- In the model with firing, the reservation wage satisfies

$$\frac{\overline{w}}{1-\beta} = c + \beta \int_0^B \hat{v}(w')dF(w')$$

- The two reservation wages differ because $v(w) \neq \hat{v}(w)$

No Firing vs Firing

- Remember, it is never optimal for a worker to quit
- It follows directly that $\hat{v}(w) < v(w)$ for all w
- Thus, the reservation wage $\overline{w}$ is strictly lower with firing

Intuition

- There is less reason to reject a job offer in hope of a better one when a job is expected to last for a shorter period of time
- Unemployed workers optimally invest less in search when the payoff associated with wage offers goes down due to the probability of being fired

5. Aggregating: A Lake Model

Continuum of Workers

- Consider an economy with a continuum of *ex ante* identical workers living in the previous environment with firing
- Normalize the total number of workers to 1
- Workers move in and out of unemployment and in and out of employment

$$
\begin{aligned}
\text{Probability of being fired:} \quad & \alpha \\
\text{Probability of being hired:} \quad & 1-\lambda = 1-F(\overline{w}) \\
\text{Mean duration of employment:} \quad & \alpha^{-1} \\
\text{Mean duration of unemployment:} \quad & (1-\lambda)^{-1} = [1-F(\overline{w})]^{-1}
\end{aligned}
$$

Unemployment Rate

- The average unemployment rate U_t across all workers is

$$U_{t+1} = \alpha \underbrace{(1 - U_t)}_{\substack{\text{employed}\\ \text{workers}}} + F(\overline{w})U_t$$

- α is the hazard rate of escaping employment
- $F(\overline{w})$ is the probability of remaining unemployed, so $1 - F(\overline{w})$ is the hazard rate of escaping unemployment

Convergence

- From the previous equation, we see that unemployment, a state variable, is governed by a simple first-order linear difference equation
- Unemployment depends only on its previous values and parameters
- Since $F(\overline{w}) < 1$, we have that unemployment is asymptotically stable
- In order words, unemployment will converge to a unique steady-state level of unemployment U

Stationary State

- Steady-state unemployment is such that $U_{t+1} = U_t = U$

$$U = \frac{\alpha}{\alpha + 1 - F(\overline{w})}$$

- Multiply numerator and denominator by $1/(\alpha[1 - F(\overline{w})])$

$$U = \frac{[1 - F(\overline{w})]^{-1}}{[1 - F(\overline{w})]^{-1} + \alpha^{-1}}$$

Stationary Unemployment

- The steady-state (or stationary) unemployment rate is therefore

$$U = \frac{\text{Mean duration of unemployment}}{\text{Mean duration of unemployment} + \text{Mean duration of employment}}$$

- U is an average across workers at each moment, it reflects the average outcome experienced by workers across time

Lakes and Rivers

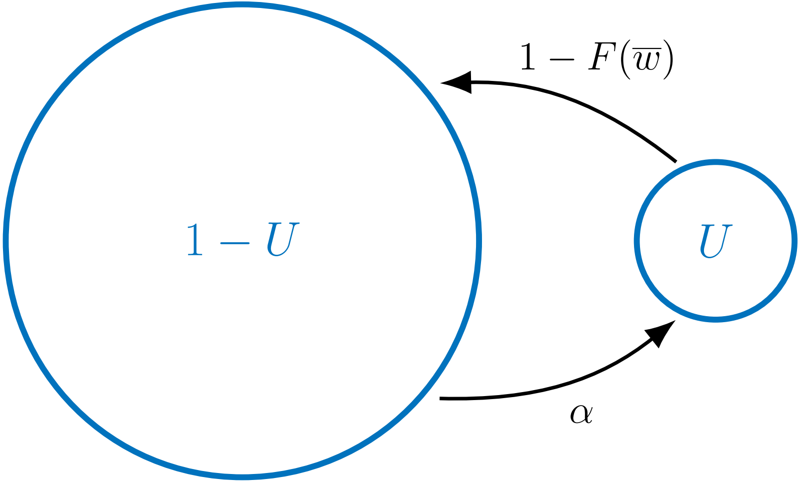
- This model is called lake model because it has two lakes

1. Lake U for the volume of unemployed workers
2. Lake $1-U$ for the volume of employed workers

- The two lakes are connected by two streams, or rivers

1. Stream α from the $1-U$ lake to the U lake
2. Stream $1-F(\overline{w})$ from the U lake to the $1-U$ lake

Flows in a Lake Model



A Markov Chain

- Notice that flows in and out of employment are governed by a stationary Markov chain with transition matrix

$$\boldsymbol{P} = \begin{bmatrix} 1-\alpha & \alpha \\ 1-F(\overline{w}) & F(\overline{w}) \end{bmatrix}$$

- Question: what is the unconditional probability distribution of this Markov chain? See exercise 4

Conclusion

- We have applied dynamic programming to a simple labor market decision
- An unemployed worker chooses to remain voluntarily unemployed today by refusing an offer to work in hope of better job prospects tomorrow
- Unemployment is like an investment incurred to improve a future situation
- This theory of voluntary unemployment enables us to analyze some of the forces impinging on the choice to work or remain unemployed

Extensions

- We can extend the model by adding many realistic features: unknown offer distributions, chances of not getting any offer, multiple offers per period, expiring unemployment benefits, search effort, directed search, life cycle
- We can also model the decisions faced by the firm: wage posting, two-sided search, uncertain quality of the match, endogenous job destruction, etc.
- In lectures 17 and 18 we will return to labor market issues by studying search and matching models

6. Exercises

Exercise 1 – Expiring Benefits and Search Effort

1. Consider the basic McCall model of Section 1 with the same assumptions (no quits, no firing). Write the Bellman equation of a unemployed worker.
2. Consider the following modification. The worker receives unemployment benefit c for only T periods. What are the state variables? Write the Bellman equation of the unemployed worker.
3. Go back to the basic model and consider the following modification. The period is divided into two subperiods. In the first subperiod the agent chooses a search effort $e \in [0, 1]$. In the second subperiod the agent receives a job offer with probability $p(e)$. Write the Bellman equation of the unemployed worker. *Hint:* Write a value function for the start of each subperiod.

Exercise 2 – Searching for a Plant Location

Consider a firm that is searching for a location to set up its only plant. Once installed, the plant produces an output with production function $Af(L_t)$. $L_t \in [0, \bar{L}]$ is the amount of labor remunerated at constant wage w. $A \in [0, \bar{A}]$ is the constant productivity associated with the plant's location. Suppose that $f : [0, \bar{L}] \to \mathbb{R}^+$ is continuous and bounded.

Each period t the firm draws a location A from a distribution $\phi(A)$. The firm can decide to install its plant in A or keep searching for another location. If it decides to keep searching it draws a new location A'. If it chooses to set up its plant it pays the fix cost C only in this period. The plant setup is immediate and irreversible.

Exercise 2 – Continued

1. The objective of the firm is to maximize the discounted sum of profits, discounted at constant rate β. Write the Bellman equation associated to the search problem of a firm that has just drawn location A.
2. Suppose there exists a value function that satisfies the previous Bellman equation. Show that there exists a threshold $\tilde{A}$ such that for all $A < \tilde{A}$ the firm chooses not to install its plant. It is not necessary to characterize $\tilde{A}$.

Exercise 3 – Promotion

Consider the basic McCall model of Section 1, with the exact same assumptions (no quits, no firing), except that the worker maximizes $E_0 \sum_{t=0}^{\infty} \beta^t y_t$. Consider the following variant. Once employed, a worker with wage w in the previous period can receive a salary increase at the beginning of the current period. With probability $\alpha \in [0, 1]$ the worker receives wage γw, with $\gamma > 1$. Pay raises are cumulative; if the worker receives another promotion her wage would become $\gamma^2 w$. Assume $\gamma\beta < 1$.

Exercise 3 – Continued

1. Let $V^e(w)$ be the value function of being employed. Write the Bellman equation of a unemployed worker.
2. Compute the value of being employed $V^e(w)$.
3. Describe the decision rule of the unemployed worker.
4. How does an increase in α affect the reservation wage?

Exercise 4 – Lake Model

Compute the unconditional probability distribution of the Markov chain in the Lake model seen in class.