10. Real Business Cycles

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From Economic Growth...

- We have studied the deterministic and stochastic neoclassical growth model
- The model has limited success in explaining long-run economic growth and differences in income per capita across countries
- But it is an ideal starting point: it is microfounded, simple, and intuitive
- It serves as the bedrock framework for most modern growth theory

. . . To Business Cycles

- In this lecture, we switch gear from economic growth to business cycles
- We address the question: can a model designed to account for long-run growth also be consistent with short-run business-cycle fluctuations?

The Founding Article

- Kydland and Prescott (1982, *Econometrica*) make a seminal contribution that changes the way macroeconomic research is conducted
- On the theoretical side, there is nothing revolutionary
- They take the basic stochastic neoclassical growth model
- They assume that technology follows an exogenous AR(1) process

Methodological Leap

- The key novelty is methodological and empirical
- Kydland and Prescott measure total factor productivity in the data with Solow growth accounting tools and use it to calibrate the model's TFP shock
- They also calibrate the other model parameters using micro and macro data
- They simulate artificial data with their model and find that productivity shocks account for the bulk of US postwar business-cycle fluctuations

A Theory Is Born

- This is a striking result: a very stylized, simple model with one type of shocks can replicate aggregate fluctuations observed in the real world
- Fluctuations are the efficient response to real shocks, ie changes in the real economic environment, as opposed to policy-induced nominal shocks
- The theory is called real business cycle theory or RBC theory

Legacy

- Spoiler: the thesis that productivity shocks drive the business cycle, especially negative shocks cause recessions, has been discredited
- But today most macro models are confronted to the data in a rigorous way
- Parameters are calibrated or estimated so that statistical moments of the model match those of the data; eg mean, variance, correlation
- This is the legacy of Kydland and Prescott, who won the 2004 Nobel Prize

Lecture Outline

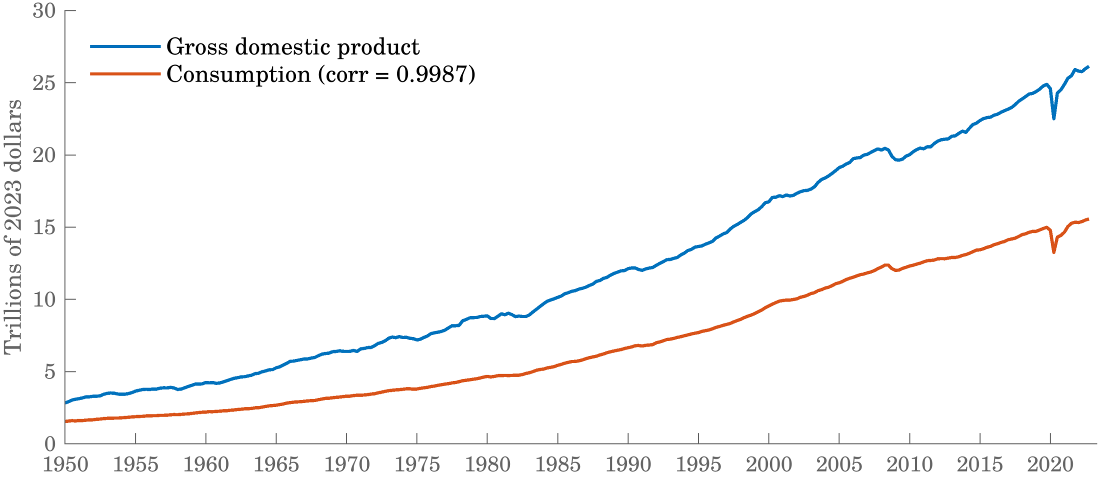
1. Stylized Facts
2. The Real Business Cycle Model
3. Functional Forms
4. Solving the Model
5. Calibration
6. Steady-State Properties
7. Impulse Response Functions
8. Stochastic Simulations
9. Criticisms

1. Stylized Facts

Data

- Establishing stylized facts about long-run phenomena is straightforward
- Collect the raw data, plot it, and compute means, ratios, growth rates
- But for stylized facts about business cycles, things are not so simple
- Let's see an example with two major macroeconomic time series

GDP and Consumption in the United States

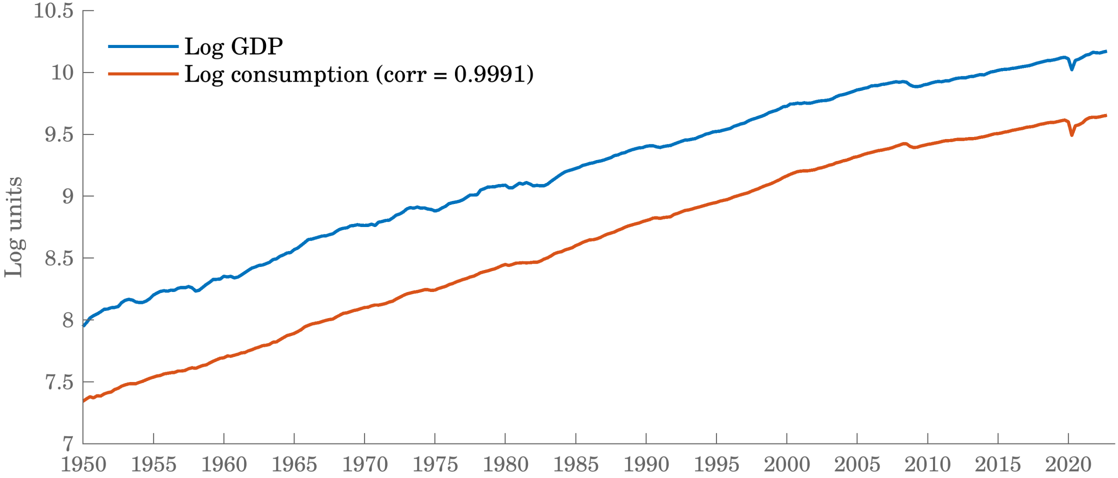


Source: US Bureau of Economic Analysis

High Correlation

- The two series grow together and appear to diverge
- But this would happen if both series had the same constant growth rate
- The correlation between the two is extremely high
- To see better how the two series evolve, we can plot their natural logarithm

Sames Series in Log



Source: US Bureau of Economic Analysis

Even Higher Correlation

- The picture is different: consumption grows faster than GDP and thus seems to be catching up, ie its share in GDP increases over time
- We see that consumption appears to be smoother than GDP
- The two series are even more highly correlated

Blurry Relationship

- As such, eyeballing the raw series does not tell us much about the short-run co-movements between GDP and consumption
- The trend trumps everything else and is the cause of the high correlation
- We need to remove this long-run component

Two Components

- Let Y_t be output and $y_t \equiv \ln Y_t$
- We can divide the time series into two components

$$y_t = y_t^{\text{trend}} + y_t^{\text{cycle}}$$

- y_t^{trend} is the long-run growth component
- y_t^{cycle} is the short-run component we are interested in

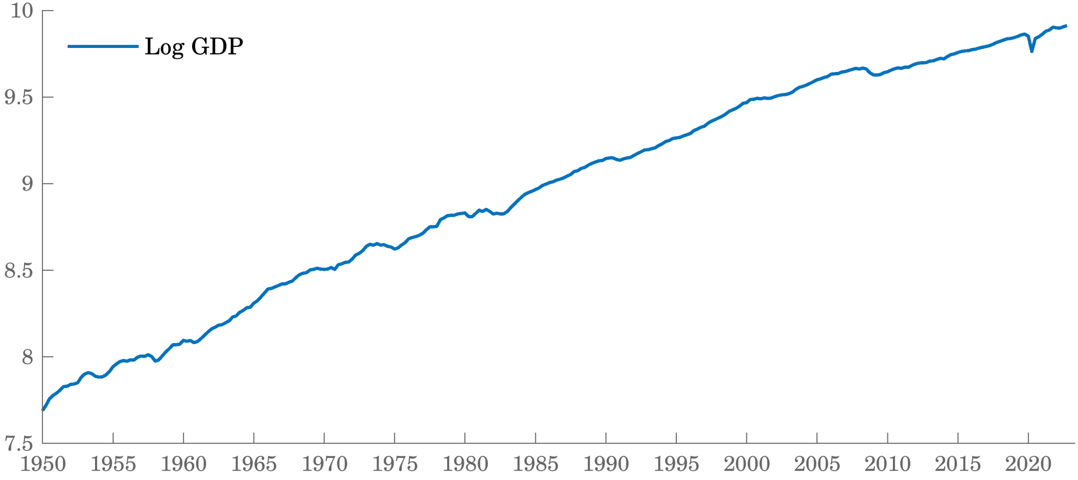
The Hodrick-Prescott Filter

- Proposed by Hodrick and Prescott (1981, 1997)
- Given data $(y_1, y_2, ..., y_T)$, we compute

$$\min_{y^{\text{trend}}} \left\{ \sum_{t=1}^{T} (y_t - y_t^{\text{trend}})^2 + \lambda \sum_{t=1}^{T} \left[(y_t^{\text{trend}} - y_{t-1}^{\text{trend}}) - (y_{t-1}^{\text{trend}} - y_{t-2}^{\text{trend}}) \right]^2 \right\}$$

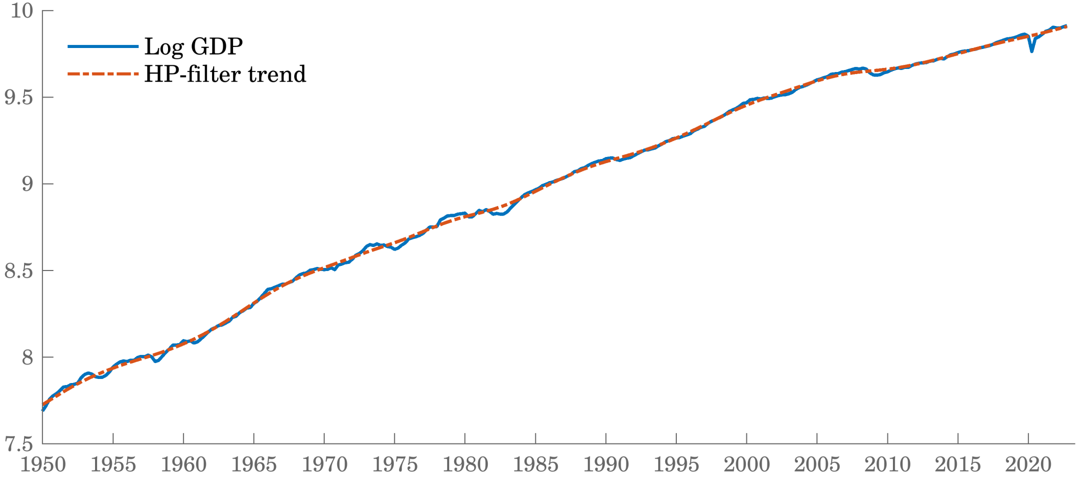
- The HP trend is the result of a tradeoff between minimizing the variance of the cyclical component and keeping the growth rate of the trend constant
- The parameter λ governs the tradeoff: if $\lambda \to 0$, the trend is the series itself $y_t = y_t^{\text{trend}}$; if $\lambda \to \infty$, the trend y_t^{trend} is linear, ie the second difference is zero
- Generally, $\lambda = 100$ for annual data, 1600 for quarterly, 14400 for monthly

GDP



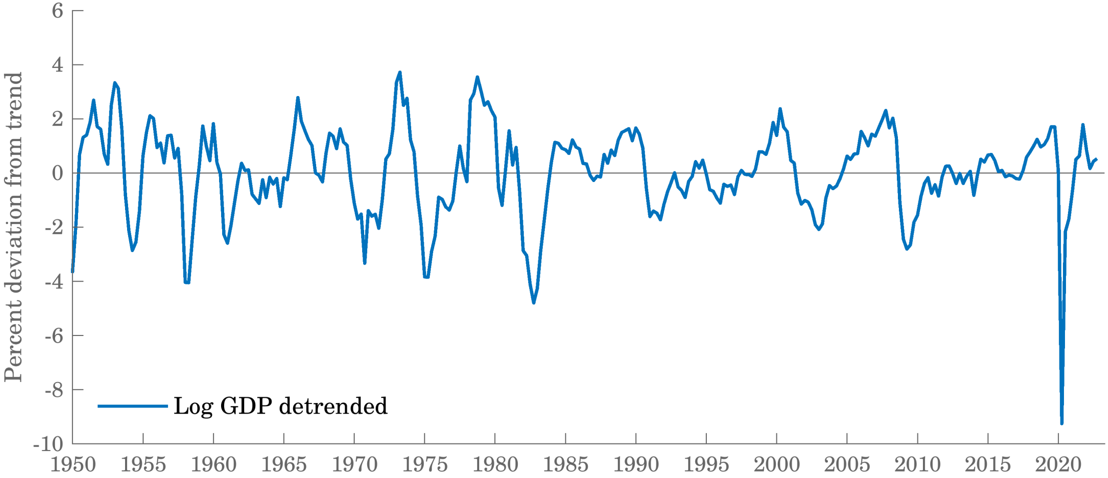
Source: US Bureau of Economic Analysis

GDP and Trend



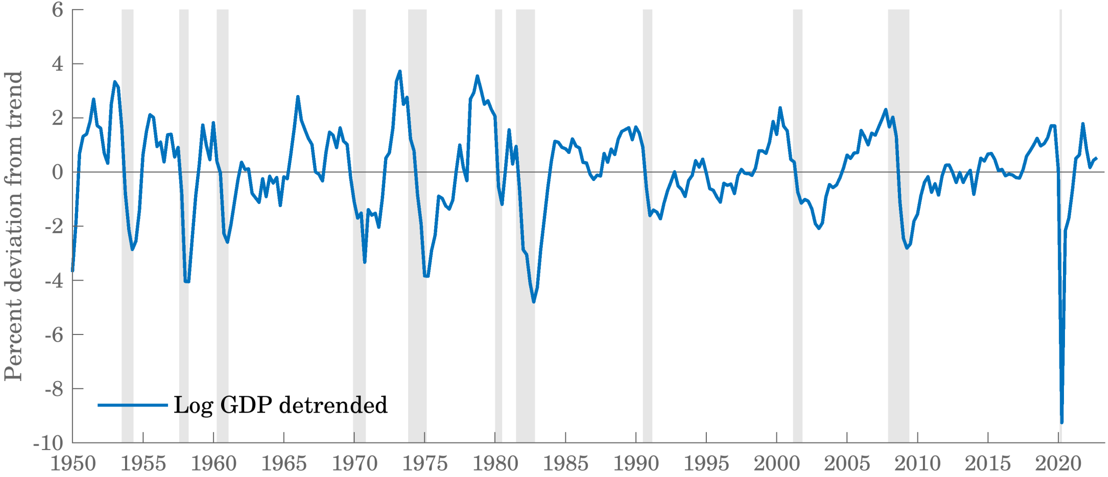
Source: US Bureau of Economic Analysis

Detrended GDP



Source: US Bureau of Economic Analysis

GDP and Recessions



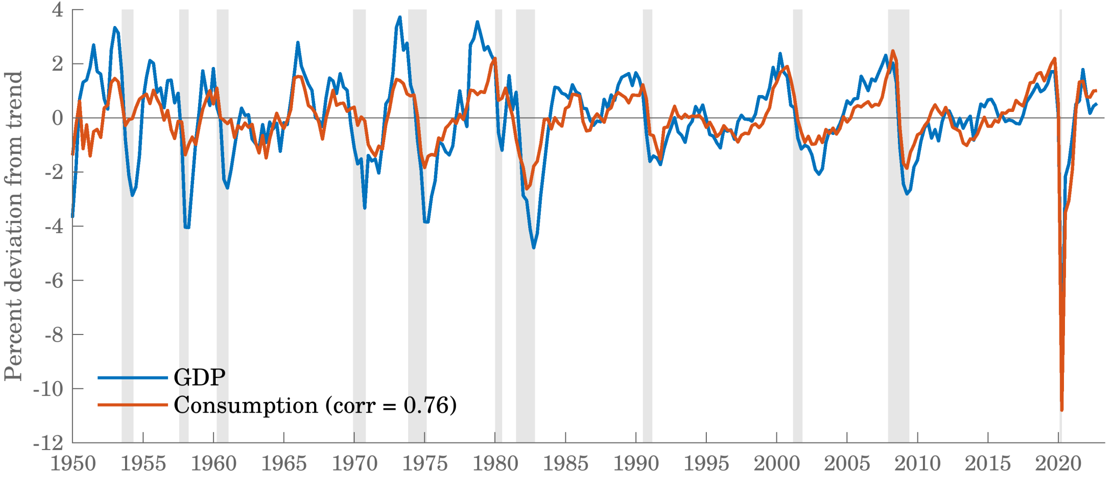
Source: US Bureau of Economic Analysis

Five Stylized Facts

- We establish five stylized facts about business cycles

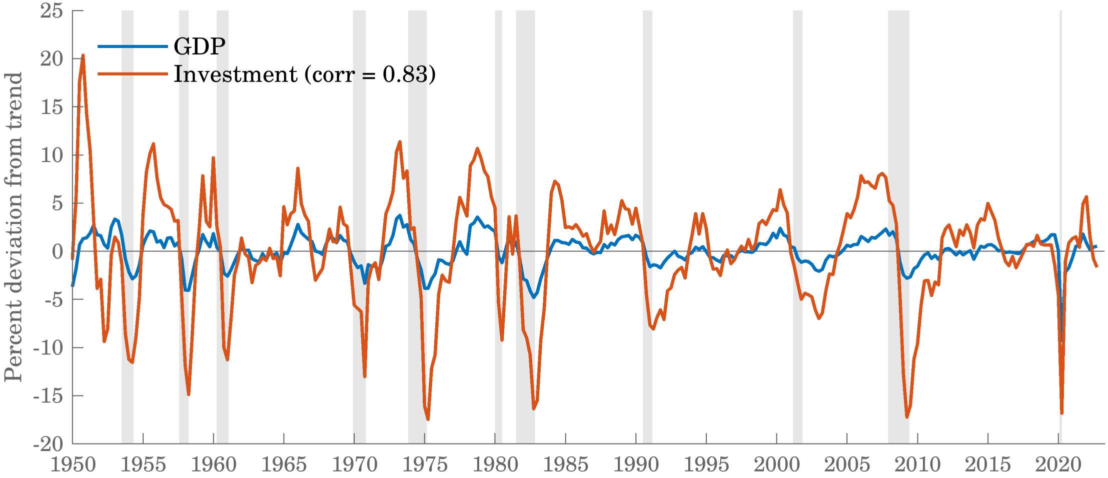
1. Consumption is procyclical and less volatile than output
2. Investment is procyclical and more volatile than output
3. Hours worked are procyclical and as volatile as output
4. Government spending is countercyclical
5. Labor productivity is mildly procyclical or acyclical

1. Consumption Is Procyclical and Less Volatile Than Output



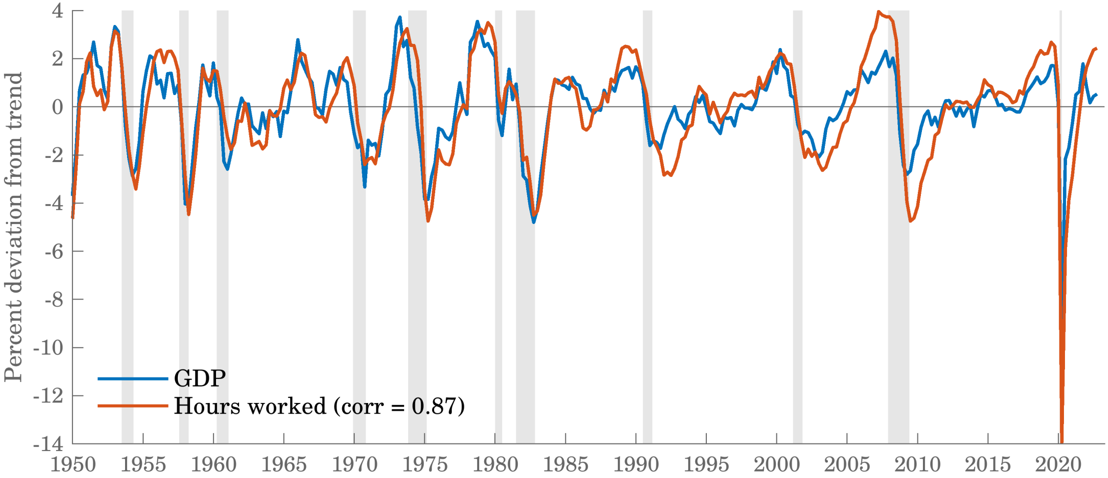
Source: US Bureau of Economic Analysis

2. Investment Is Procyclical and More Volatile Than Output



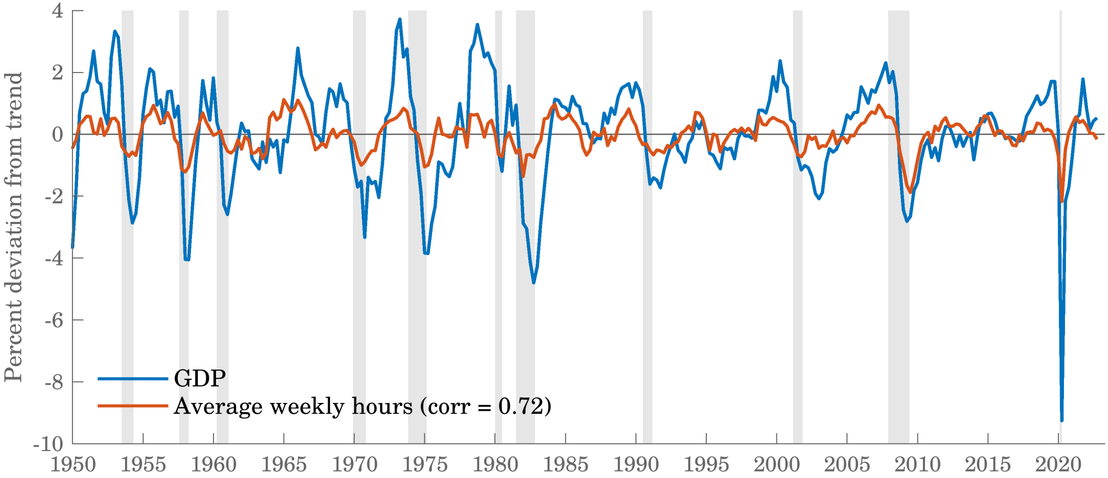
Source: US Bureau of Economic Analysis

3. Hours Are Procyclical and About as Volatile as Output



Sources: US Bureau of Labor Statistics and US Bureau of Economic Analysis

Total Hours vs Average Weekly Hours

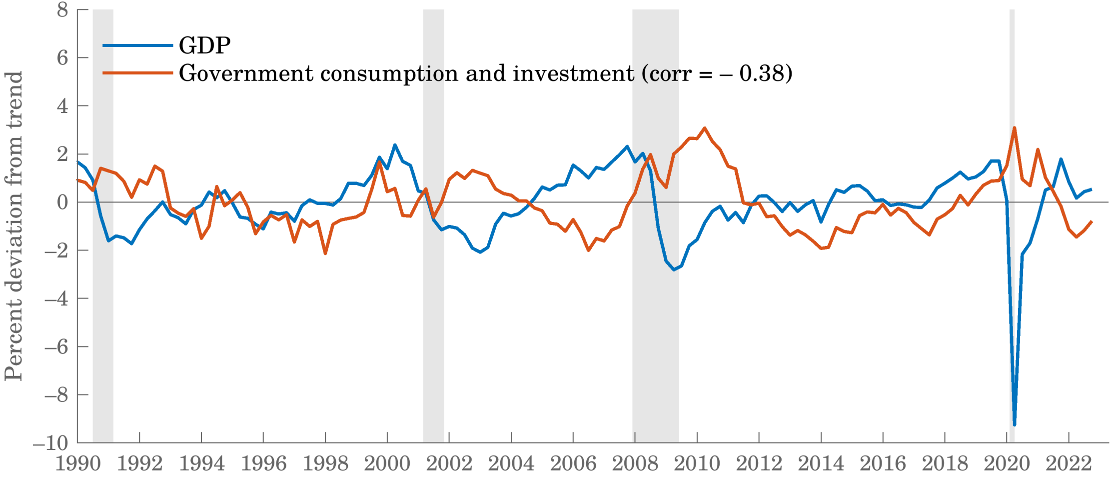


Sources: US Bureau of Economic Analysis and US Bureau of Labor Statistics

Extensive vs Intensive Margin

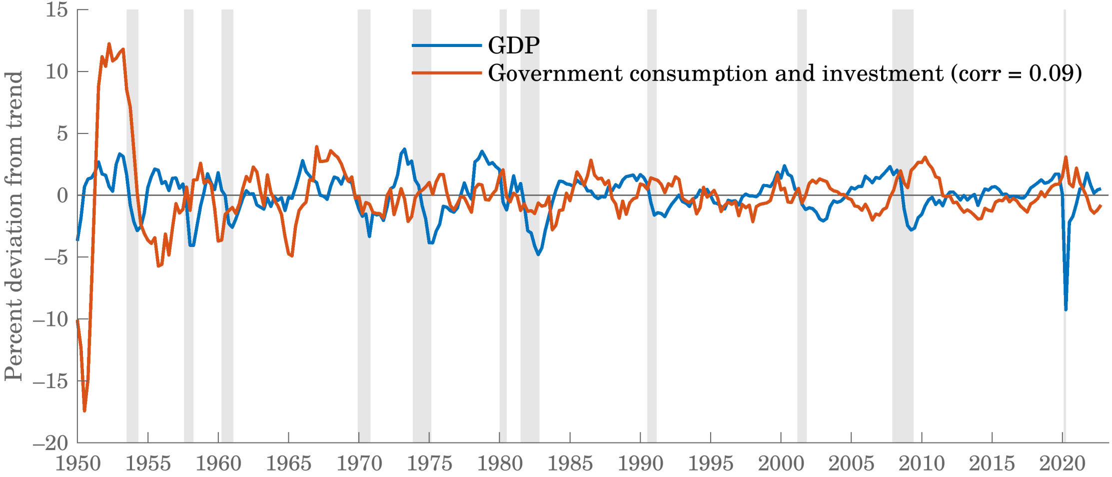
- Total hours vary about as much as GDP
- But average weekly hours are much less volatile
- This suggests that hours vary more along the extensive margin, ie the number of people employed in the economy
- By contrast, hours are more stable on the intensive margin, ie the number of hours worked per employed person

4. Government Spending Is Countercyclical



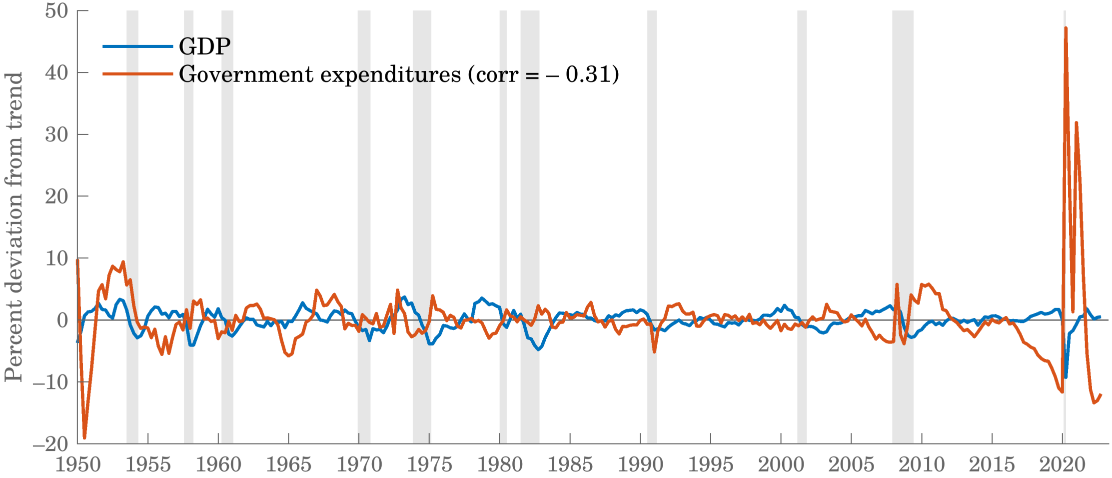
Source: US Bureau of Economic Analysis

This Was Not Always the Case



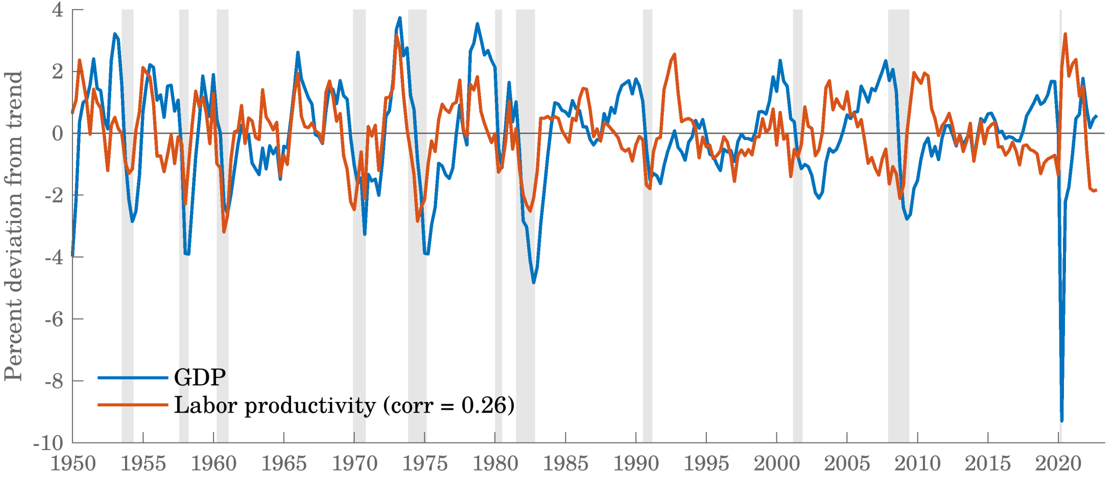
Source: US Bureau of Economic Analysis

Same Picture but with Total Government Expenditures



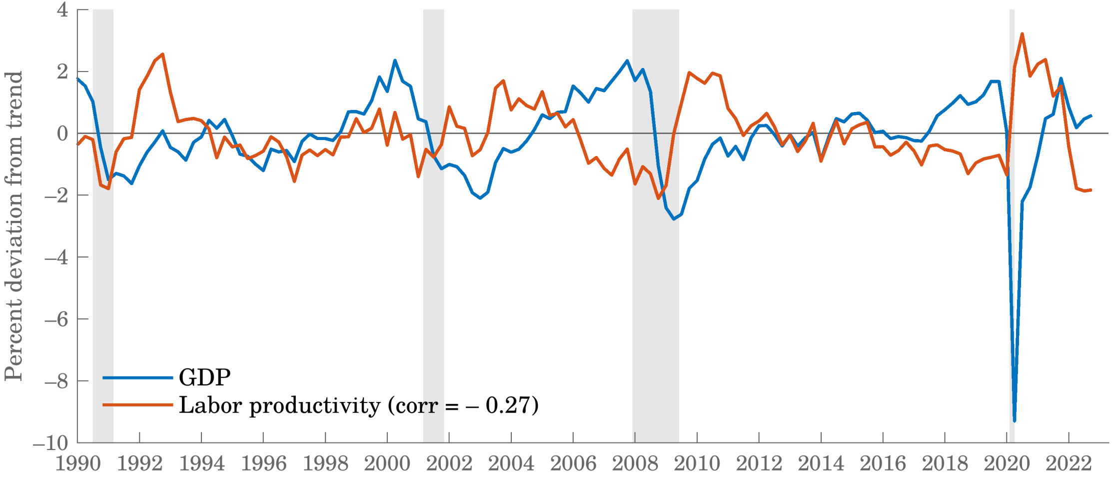
Source: US Bureau of Economic Analysis

5. Productivity Used to Be Procyclical...



Sources: US Bureau of Economic Analysis and US Bureau of Labor Statistics

. . . But Not Anymore



Sources: US Bureau of Economic Analysis and US Bureau of Labor Statistics

Productivity Puzzle

- Labor productivity, or output per hour, is defined as

$$\text{Labor productivity} = \frac{\text{Real output}}{\text{Total hours worked}}$$

- There is an ongoing debate on the causes of the change in the cyclicality of productivity, see Fernald and Wang (2016) for a review

2. The Real Business Cycle Model

Stochastic Growth Model

- The real business cycle (RBC) model is simply a standard neoclassical growth model with stochastic technology
- We have just seen this model in the last lecture
- Let's review it briefly

Households

- A representative household is composed of L_t members
- The household's preferences are

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t L_t u \left(\frac{C_t}{L_t}, 1 - \frac{N_t}{L_t} \right) \right\}$$

- C_t is aggregate consumption
- L_t is aggregate population
- N_t is aggregate hours worked

Constraints

- The household faces a budget constraint

$$C_t + I_t = W_t N_t + r_t K_t$$

- The aggregate resource constraint is

$$Y_t = C_t + I_t$$

- The law of motion of capital is

$$K_{t+1} = (1 - \delta) K_t + I_t$$

Firms

- A representative firm maximizes profit

$$Y_t - r_t K_t - W_t N_t$$

- The firm has the following technology

$$Y_t = a_t F(A_t, K_t, N_t)$$

- A_t is the aggregate level of technology, a_t is a technology shock

Growth

- There are two sources of growth in the model
- The first one is population growth

$$L_t = (1+\eta)^t L_0 = (1+\eta)^t, \quad \eta \geq 0$$

- The second one is productivity growth

$$A_t = (1+\mu)^t A_0 = (1+\mu)^t, \quad \mu \geq 0$$

- Both L_t and A_t are exogenous state variables

Total Factor Productivity

- a_t is the stochastic component of productivity
- a_t is stationary, this is to capture transitory changes
- a_t is called total factor productivity (TFP)

TFP Shock

- TFP follows an exogenous AR(1) process

$$\ln a_t = \rho_a \ln a_{t-1} + (1 - \rho_a) \ln a + \varepsilon_t^a$$

- $\rho_a \in (0, 1)$ is an autocorrelation parameter
- a is TFP in the steady state
- ε_t^a is the TFP shock, also called technology shock
- ε_t^a is a Gaussian random variable, $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$

Functional Forms

- Before we solve the model, we have to specify concrete functional forms for two important functions

1. The production function of firms
2. The utility function of households

3. Functional Forms

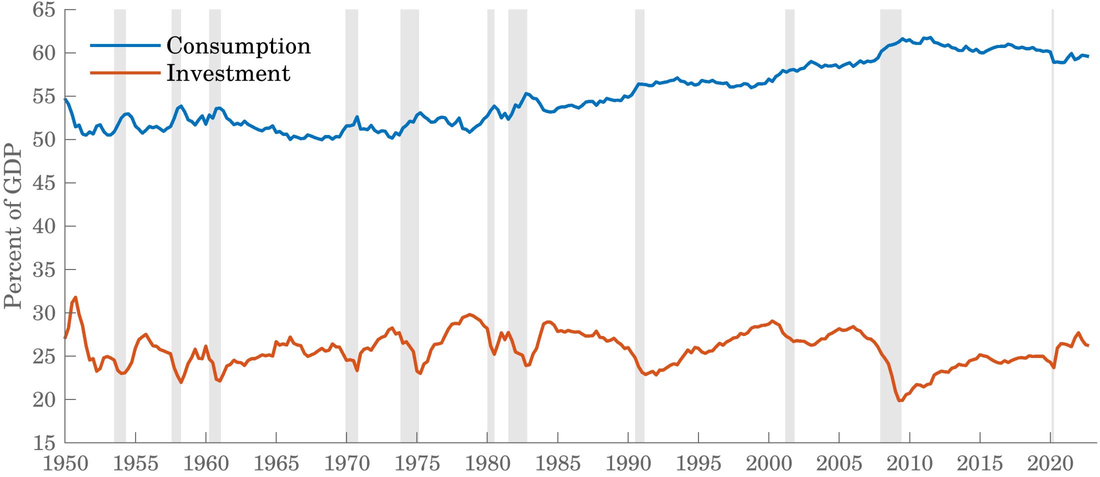
Parametric Functions

- How do we specify functional forms for F and u?
- We look at the data and establish facts about the long run
- We come up with functions that can best match these facts

Balanced Growth

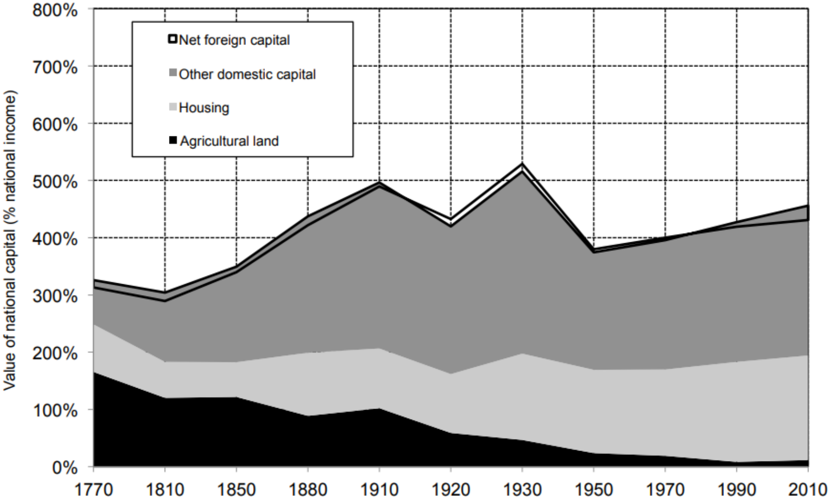
- The first type of evidence is balanced growth
- Balanced growth means that most macro variables (output, consumption , investment, capital, wages) grow at the same rate in the long run
- Labor is an exception: hours per capita are stable in the long run

Shares of Consumption and Investment in Output



Source: US Bureau of Economic Analysis

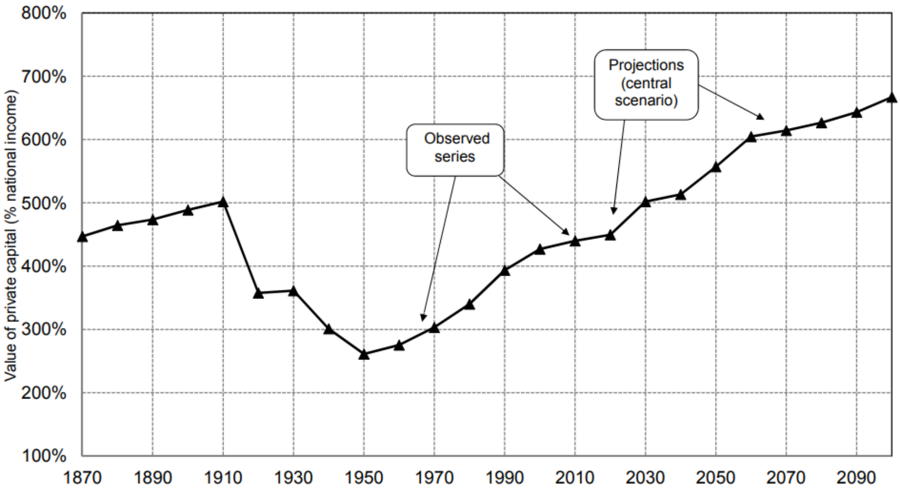
US Capital Stock



National capital is worth 3 years of national income in the United States in 1770 (incl. 1.5 years in agricultural land). Sources and series: see piketty.pse.ens.fr/capital21c.

Source: Piketty (2014)

World Capital Stock



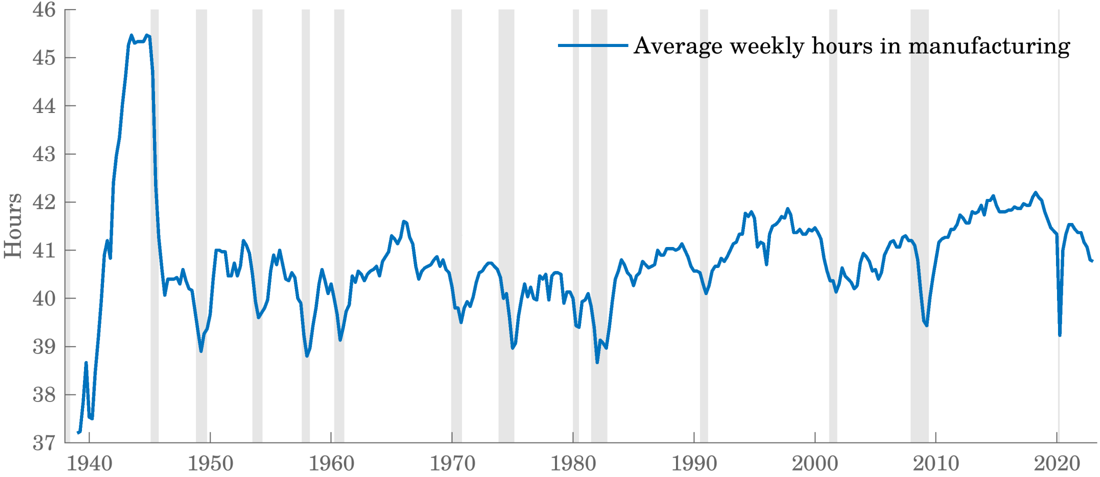
According to simulations (central scenario), the world capital/income ratio could be near to 700% by the end of the 21st century. Sources and series: see piketty.pse.ens.fr/capital21c.

Source: Piketty (2014)

Not So Balanced Growth

- National accounting: GNP (or GNI) is equal to GDP plus net factor income; national income equals GNP minus capital depreciation
- It turns out that consumption is growing faster than GDP
- Capital is also growing faster than GDP
- But for the purpose of the exercise we are going to pretend these variables are growing at the same rate

Hours Per Capita Are Stable



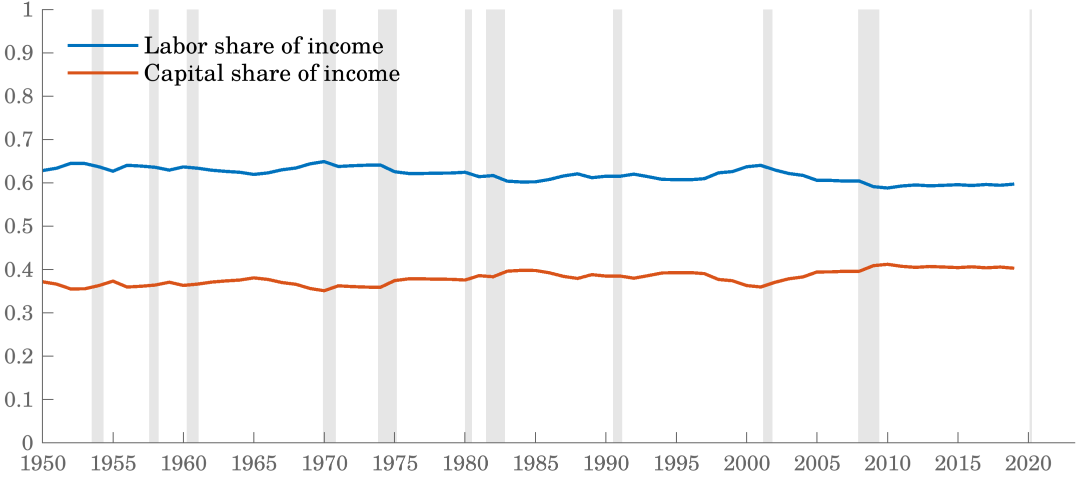
Source: Federal Reserve Board

Labor-Augmenting Technological Change

- Output, consumption, investment, capital are driven by the same force and grow at approximately the same rate
- Hours however are relatively constant in the long run
- Labor-augmenting technological progress is consistent with these balanced growth facts

$$F(A_t, K_t, N_t) = F(K_t, A_t N_t), \quad A_t = (1+\mu)A_{t-1}$$

Capital and Labor Share



Source: Penn World Table

Labor-Augmenting Technological Change

- The share of capital income in output is somewhat stable
- In fact it has ticked up in recent years
- But again let's proceed as if it were stable
- A Cobb-Douglas production function is therefore adequate

$$F(K_t, A_t N_t) = K_t^{\alpha} (A_t N_t)^{1-\alpha}$$

Digression

- The increase in the capital share α is a big deal
- Each percentage point represents \$200 billion in the US
- The trend is not limited to the US

A Global Trend

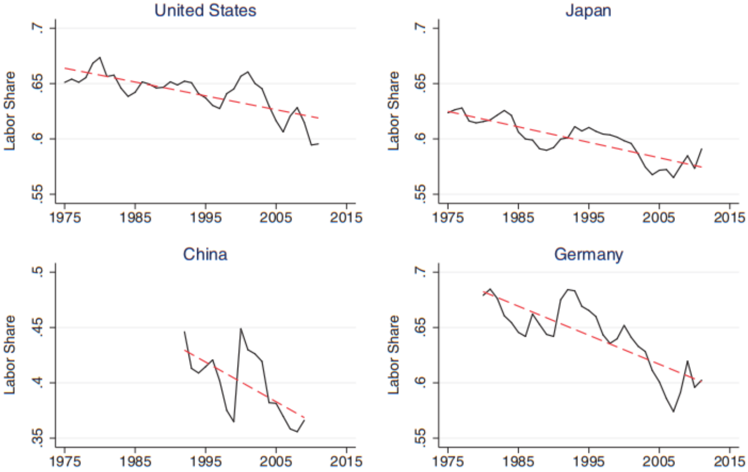


FIGURE II

Declining Labor Share for the Largest Countries

The figure shows the labor share and its linear trend for the four largest economies in the world from 1975.

Source: Karabarbounis and Neiman (2014, *QJE*)

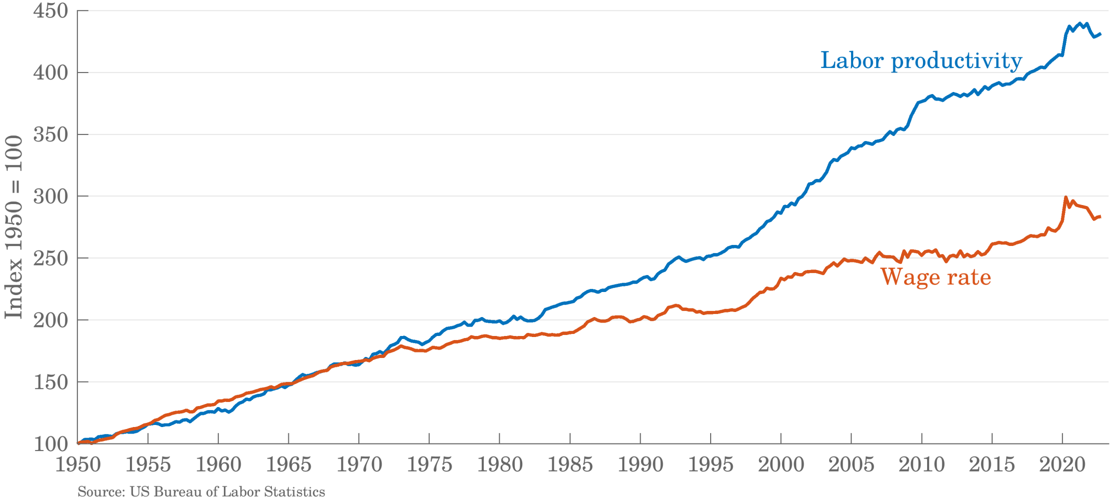
Digression

- A more reliable production function would be

$$F(K_t, A_tN_t) = K_t^{\alpha_t}(A_tN_t)^{1-\alpha_t}$$

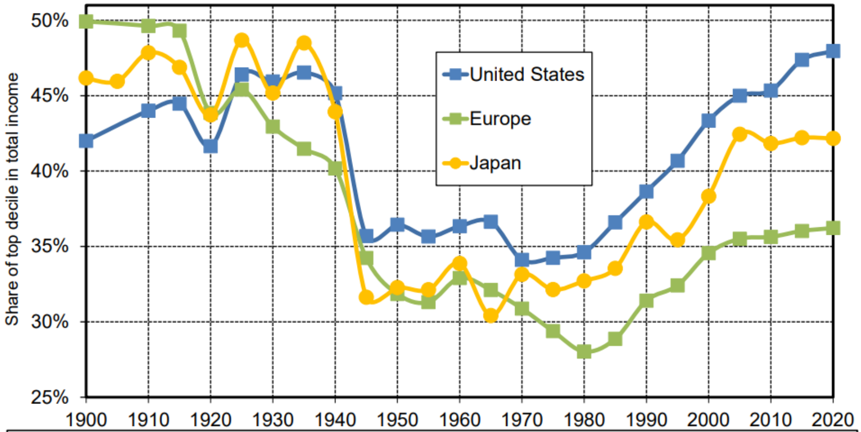
- What causes α_t to increase? What are the consequences?
- These questions are being hotly debated today
- Probable cause: stagnating wages relative to productivity
- Probable consequence: rising inequality

Productivity and Wages



Income Inequality in the US

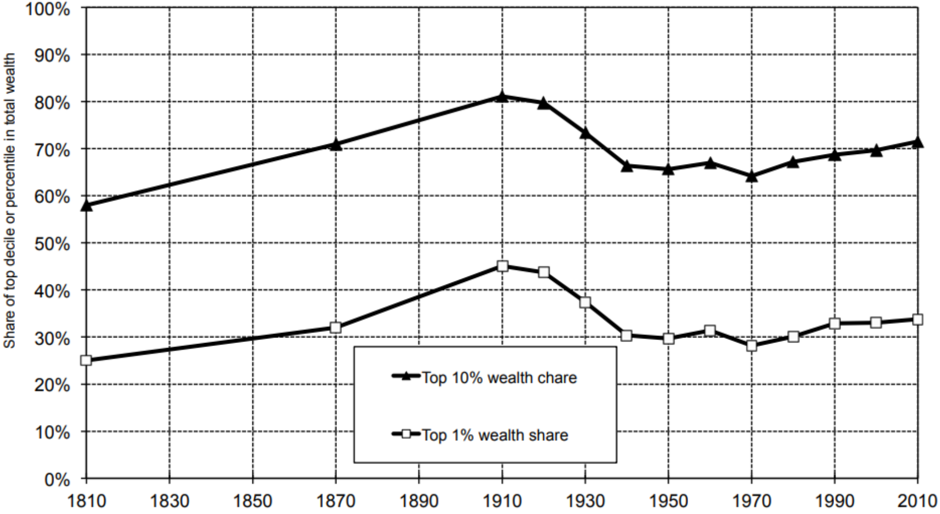
Figure 0.6. Inequality, 1900-2020: Europe, United States, Japan



Interpretation. The share of the top decile (the top 10% highest incomes) in total national income was about 50% in Western Europe in 1900-1910, before decreasing to about 30% in 1950-1980, then rising again to more than 35% in 2010-2020. Inequality grew much more strongly in the United States, where the top decile share approached 50% in 2010-2020, exceeding the level of 1900-1910. Japan was in an intermediate position. **Sources and series**: see piketty.pse.ens.fr/ideology.

Source: Piketty (2019)

Wealth Inequality in the US



The top 10% wealth holders own about 80% of total wealth in 1910, and 75% today.
Sources and series: see piketty.pse.ens.fr/capital21c.

Source: Piketty (2014)

Utility Function

- Which form for the utility function should we choose?
- Hours are constant but wages and consumption increase over time
- Thus we need a utility function that insulates the marginal utility of leisure from the marginal utility of consumption
- In other words we need an additively separable utility function

Nonseparable Preferences

- Let $c_t \equiv C_t/L_t$ and $n_t \equiv N_t/L_t$
- Consider the following CRRA utility function

$$u(c_t, n_t) = \frac{1}{1-\sigma}\left(c_t - \frac{n_t^{1+\varphi}}{1+\varphi}\right)^{1-\sigma}$$

- $\sigma \geq 0$ is the inverse elasticity of intertemporal substitution
- $\varphi \geq 0$ is the inverse Frisch elasticity of labor supply
- Consumption and hours are not additively separable

Separable Preferences

- An example of separable preferences is

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi}$$

- Let us set $\sigma = 1$ to have log utility in consumption

$$u(c_t, n_t) = \ln c_t - \frac{n_t^{1+\varphi}}{1+\varphi}$$

- This functional form is parsimonious in parameters

4. Solving the Model

Balanced Growth Path

- With population growth and technological progress, the model variables will converge to a balanced growth path
- Once there, the economy will grow forever at a constant rate
- Thus the steady state in this economy is not stationary

Stationary Variables

- To solve the model, we need to make transformations such that all variables are stationary in the steady state
- The two sources of growth are A_t and L_t
- We define the following stationary variables

$$\hat{y}_t \equiv \frac{Y_t}{A_t L_t} \quad \hat{c}_t \equiv \frac{C_t}{A_t L_t} \quad \hat{i}_t \equiv \frac{I_t}{A_t L_t} \quad \hat{k}_t \equiv \frac{K_t}{A_t L_t} \quad \hat{w}_t \equiv \frac{W_t}{A_t} \quad n_t \equiv \frac{N_t}{L_t}$$

Household Problem

- The problem of the representative household becomes

$$\max_{\hat{c}_t, n_t, \hat{i}_t, \hat{k}_{t+1}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t (1+\eta)^t \left[\ln \hat{c}_t + \ln[(1+\mu)^t] - \frac{n_t^{1+\varphi}}{1+\varphi} \right] \right\}$$

subject to

$$\hat{c}_t + \hat{i}_t = \hat{w}_t n_t + r_t \hat{k}_t$$

$$(1+\mu)(1+\eta)\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \hat{i}_t$$

First-Order Conditions

- The first-order conditions are

$$\begin{aligned} \text{Labor supply:} \quad & n_t^{\varphi} \hat{c}_t = \hat{w}_t \\ \text{Euler equation:} \quad & (1+\mu)\frac{1}{\hat{c}_t} = \beta E_t \frac{1}{\hat{c}_{t+1}} \left(\alpha \frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta \right) \end{aligned}$$

Firm Problem

- The problem of the representative firm becomes

$$\max_{K_t, N_t} \{a_t K_t^{\alpha} (A_t N_t)^{1-\alpha} - r_t K_t - W_t N_t\} = \max_{\hat{k}_t, \hat{n}_t} \{A_t L_t [a_t \hat{k}_t^{\alpha} n_t^{1-\alpha} - r_t \hat{k}_t - \hat{w}_t n_t]\}$$

- The first-order conditions are

$$\text{Capital demand}: \quad r_t = \alpha \frac{\hat{y}_t}{\hat{k}_t}$$

$$\text{Labor demand}: \quad \hat{w}_t = (1-\alpha) \frac{\hat{y}_t}{n_t}$$

Equilibrium

- A competitive equilibrium is a set of prices $(r_t, \hat{w}_t)$ and allocations $(\hat{c}_t, \hat{i}_t, n_t, \hat{k}_{t+1}, \hat{y}_t)$ such that
  - $\hat{k}_t$ and a_t are given
  - The household solves its problem
  - The firm solves its problem
  - Goods, capital, and labor markets clear
  - The transversality condition holds

Equilibrium Conditions

$$
\begin{aligned}
\text{Euler equation:} \quad & 1 = \frac{\beta}{1+\mu} E_t \frac{\hat{c}_t}{\hat{c}_{t+1}} \left(r_{t+1} + 1 - \delta\right) && (1) \\
\text{Labor supply:} \quad & n_t^{\varphi} \hat{c}_t = \hat{w}_t && (2) \\
\text{Capital demand:} \quad & r_t = \alpha \hat{y}_t / \hat{k}_t && (3) \\
\text{Labor demand:} \quad & \hat{w}_t = (1-\alpha) \hat{y}_t / n_t && (4) \\
\text{Resource constraint:} \quad & \hat{y}_t = \hat{c}_t + \hat{i}_t && (5) \\
\text{Production function:} \quad & \hat{y}_t = a_t \hat{k}_t^{\alpha} n_t^{1-\alpha} && (6) \\
\text{Capital accumulation:} \quad & (1+\mu)(1+\eta) \hat{k}_{t+1} = (1-\delta) \hat{k}_t + \hat{i}_t && (7) \\
\text{Technology:} \quad & \ln a_t = \rho_a \ln a_{t-1} + (1-\rho_a) \ln a + \varepsilon_t^a && (8)
\end{aligned}
$$

Steady State

- Denote a steady-state variable by removing its subscript t
- The nonstochastic steady state is such that there is no no uncertainty, ie TFP is at its mean $a = 1$, and all variables are constant, eg $\hat{c}_t = \hat{c}_{t+1} = \hat{c}$

Steady State

$$\text{Euler equation:} \quad 1 = \frac{\beta}{1+\mu}(r+1-\delta) \tag{1}$$

$$\text{Labor supply:} \quad n^{\varphi}\hat{c} = \hat{w} \tag{2}$$

$$\text{Capital demand:} \quad r = \alpha\hat{y}/\hat{k} \tag{3}$$

$$\text{Labor demand:} \quad \hat{w} = (1-\alpha)\hat{y}/n \tag{4}$$

$$\text{Resource constraint:} \quad \hat{y} = \hat{c} + \hat{i} \tag{5}$$

$$\text{Production function:} \quad y = a\hat{k}^{\alpha}n^{1-\alpha} \tag{6}$$

$$\text{Capital accumulation:} \quad (1+\mu)(1+\eta)\hat{k} = (1-\delta)\hat{k} + \hat{i} \tag{7}$$

Steady State

- From (1) we get the rental rate

$$r = \frac{1+\mu}{\beta} - (1-\delta)$$

- From (3) we deduce the output-to-capital ratio

$$\frac{\hat{y}}{\hat{k}} = \frac{r}{\alpha}$$

- From (6) we obtain the capital-to-labor ratio

$$\frac{\hat{k}}{n} = \left(\frac{\hat{y}}{\hat{k}}\right)^{\frac{1}{\alpha-1}}$$

Steady State

- Next, using (6) again we get the output-to-labor ratio

$$\frac{\hat{y}}{n} = \left(\frac{\hat{k}}{n}\right)^{\alpha}$$

- From (7) we get the investment-to-labor ratio

$$\frac{\hat{i}}{n} = [(1+\mu)(1+\eta) - (1-\delta)]\,\frac{\hat{k}}{n}$$

- From (5) we get the consumption-to-labor ratio

$$\frac{\hat{c}}{n} = \frac{\hat{y}}{n} - \frac{\hat{i}}{n}$$

Steady State

- Continuing, from (4) we get the wage

$$\hat{w} = (1-\alpha)\frac{\hat{y}}{n}$$

- From (2) we find hours

$$n = \left[\left(\frac{\hat{c}}{n}\right)^{-1}\hat{w}\right]^{\frac{1}{1+\varphi}}$$

- Finally, with n we can deduce all other variables

$$\hat{c} = \frac{\hat{c}}{n}n; \quad \hat{i} = \frac{\hat{i}}{n}n; \quad \hat{k} = \frac{\hat{k}}{n}n; \quad \hat{y} = \frac{\hat{y}}{n}n$$

Nonlinear System

- We are facing a system of nonlinear difference equations
- This system cannot be solved analytically (unless $\delta = 1$)
- Thus we use numerical methods

Linear System

- The most common way is to use perturbation techniques
- We linearize the model, that is we compute a first-order approximation of the solution around the steady state
- Then we study "small" deviations away from steady state

Future Class

- Solving dynamic stochastic general equilibrium (DSGE) models is the subject of a vast literature
- The Macro III course has a whole part dedicated to this
  - Theory: log-linearization, vector system, stability conditions
  - Practice: coding in Python/Matlab, in Dynare

5. Calibration

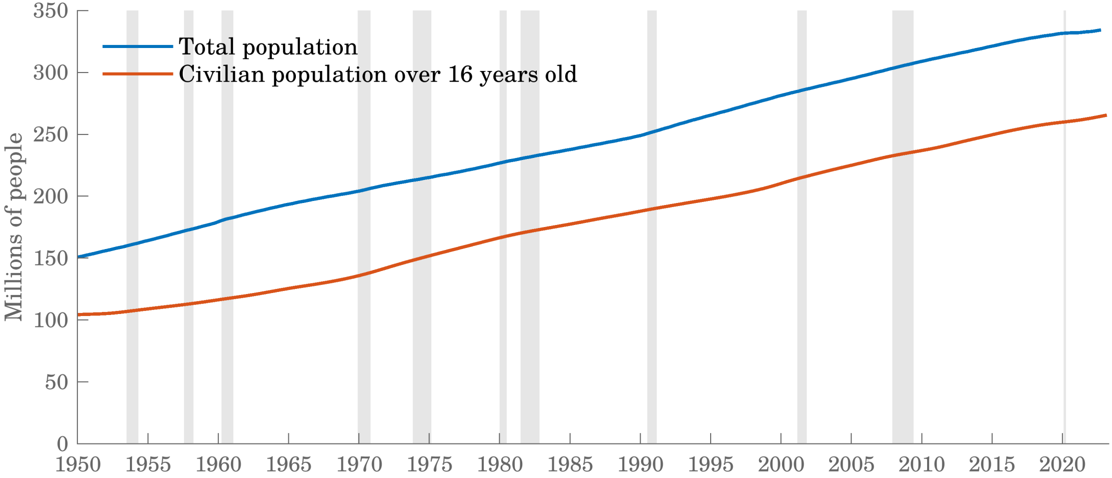
Data-Fitting Exercise

- We are now going to calibrate the model
- That is, we assign numeric values to each parameter
- The objective is to fit the data as well as possible

One Period is a Quarter

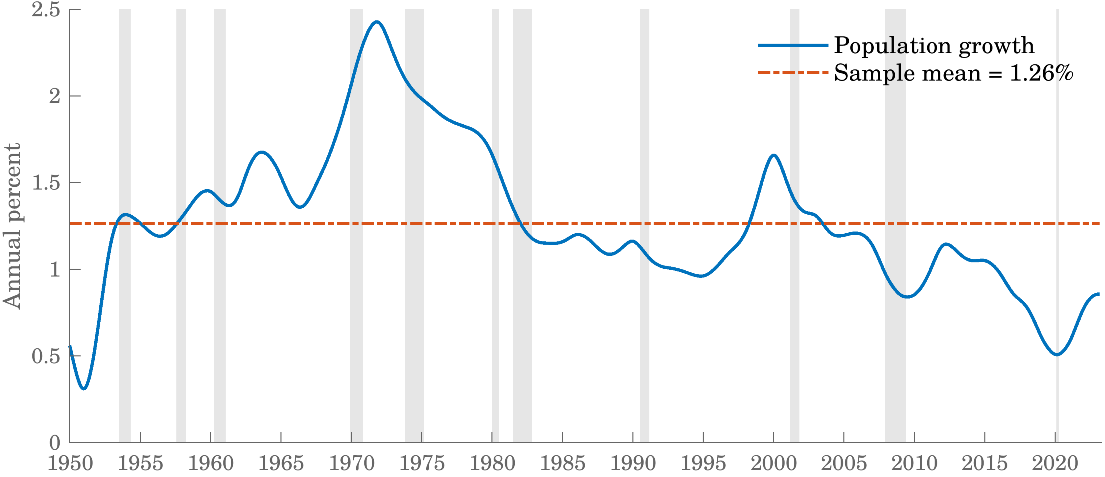
- There are 8 parameters: $(\eta, \mu, \alpha, \beta, \delta, \varphi, \rho_a, \sigma^a)$
- We calibrate the model at quarterly frequency
- Our data spans the period 1950Q1–2021Q1
- Let's see each parameter in turn

Population Over 16



Source: US Bureau of Economic Analysis

Population Growth

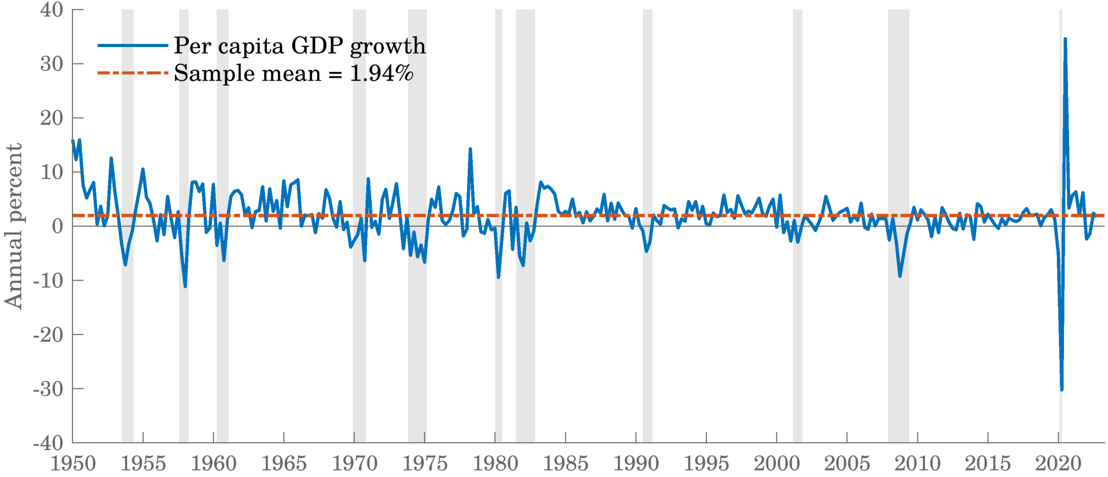


Source: US Bureau of Economic Analysis

Population Growth

- Working-age population grows at 1.27% on average
- In quarterly terms this gives us 0.32%
- We set $\eta = 0.0032$

Per Capita GDP Growth

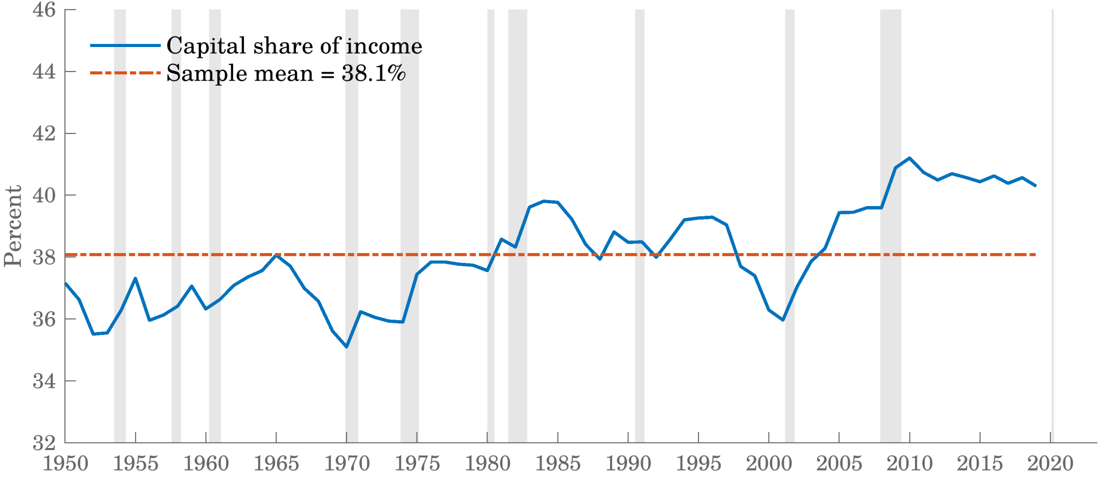


Sources: US Bureau of Economic Analysis and US Bureau of Labor Statistics

Per Capita Output Growth

- Real GDP per capita grew at an average rate of 1.86% over the time period
- In quarterly terms this gives us 0.47%
- We set $\mu = 0.0047$

Capital Share



Source: Penn World Table

Capital Share

- The capital share in GDP is

$$\text{Capital share} = \frac{\text{Capital income}}{\text{Output}} = \frac{rK}{Y} = \alpha \frac{Y}{K} \frac{K}{Y} = \alpha$$

- The average capital share in GDP is 0.38
- We set $\alpha = 0.38$

Depreciation Rate

- In the model δ is the depreciation rate of capital

$$\text{Depreciation rate}_t = \frac{\text{Depreciation}_t}{\text{Gross stock}_t}$$

- In the RBC literature people typically use an annual 10% depreciation rate, ie $\delta = 0.025$ in quarterly terms
- Let's use $\delta = 0.02$

Discount Factor

- How should we calibrate the discount factor? Since $R = (1 + \mu)/\beta$, we could look at the return on riskfree short-term government bonds
- But in the model, the returns on bonds and capital are equal, by arbitrage
- In the data, the return to capital is higher, recall the equity premium
- Let's follow the literature and set $\beta = 0.995$

Solow Accounting

- The exogenous TFP process a_t is governed by
  - The autocorrelation parameter ρ_a
  - The standard error σ_a of the shock ε_t^a
- To back out these parameters, we use Solow accounting

Solow Residual

- First, define the Solow residual as the part of output growth not explained by changes in capital and labor

$$\Delta \ln \mathbf{SR}_t = \Delta \ln Y_t - \alpha \Delta \ln K_t - (1-\alpha)\Delta \ln N_t$$

- Next, detrend the Solow residual to recover a_t

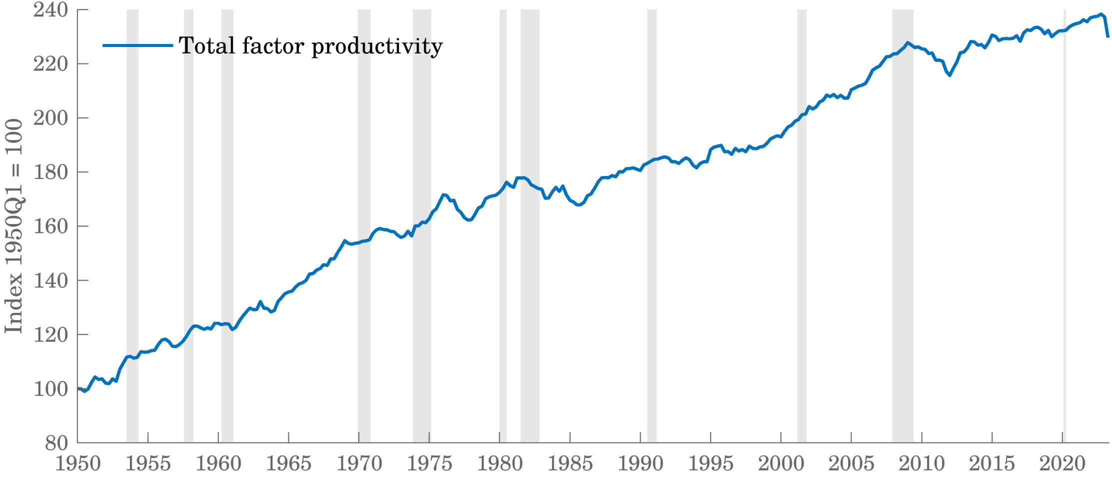
$$\ln a_t = \ln \mathbf{SR}_t - (1-\alpha) \ln X_t$$

- Finally, estimate a first-order autoregressive process

$$\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$$

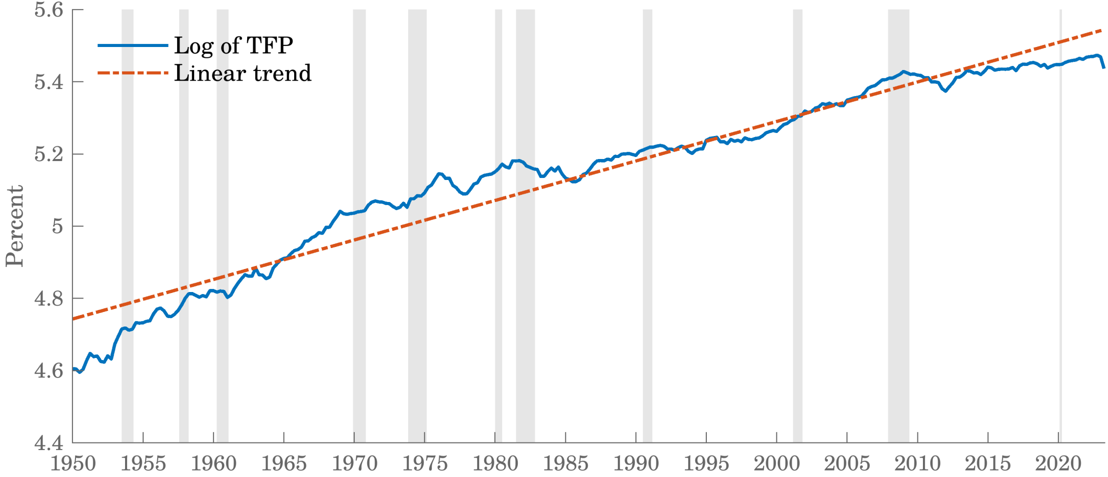
- We find $\hat{\rho}_a = 0.986$ and $\hat{\sigma}_a = 0.0086$

Solow Residual



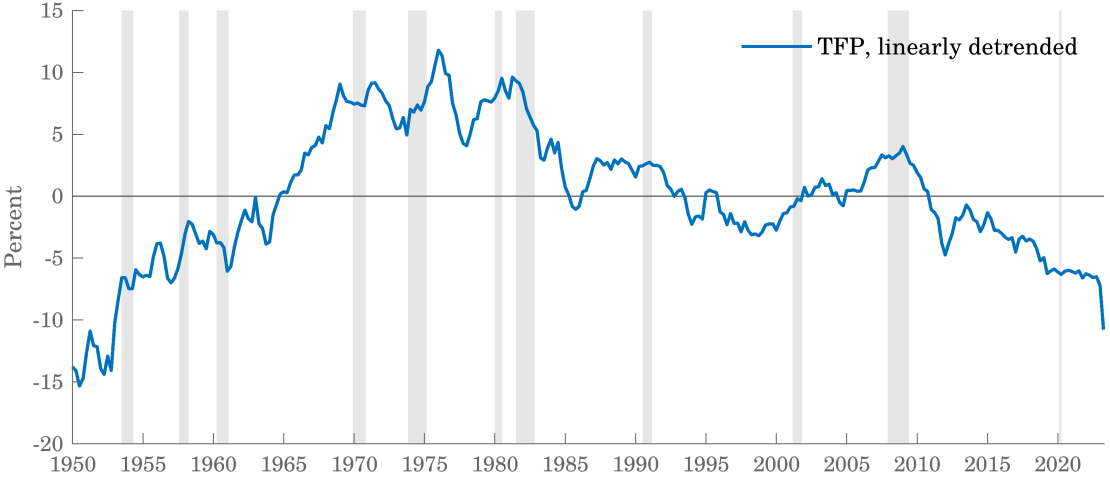
Source: John Fernald

Solow Residual in Log



Data source: John Fernald

Solow Residual, Detrended



Data source: John Fernald

Calibration

Parameter	Description	Value
η	Population growth	0.0033
μ	Productivity growth	0.0047
α	Capital share in production	0.38
δ	Depreciation rate of capital	0.02
β	Discount factor	0.995
φ	Labor supply elasticity	2
ρ_a	TFP shock AR(1)	0.986
σ_a	TFP shock standard dev.	0.0086

6. Steady-State Properties

Steady-State Properties

- We are now ready to analyze the properties of the model
- We adopt a quantitative approach
- Before looking at the dynamic properties, we study the static properties, ie the model's implication in steady state

Steady-State Properties, Model Versus Data

Variable	Description	Model	Data
c/y	Consumption-to-GDP ratio	0.64	0.55
i/y	Investment-to-GDP ratio	0.36	0.26
g/y	Government-to-GDP ratio	0	0.19
$k/(4y)$	Capital-to-GDP ratio	3.19	3.54
$4i/k$	Investment-to-capital ratio	0.11	0.07
$(1+r-\delta)^4 * 100$	Real interest rate	3.96	??

Note: Data values represent the mean over 1950Q1–2020Q1

Consumption and Investment

- There is no government in the model
- Thus, both the consumption- and investment-to-GDP ratios are too high with respect to the data
- But their respective proportion is OK

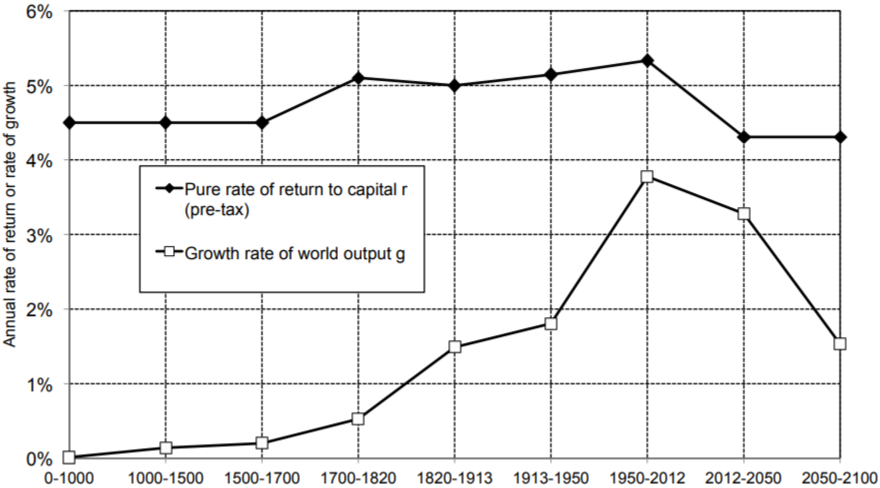
Capital

- The capital-to-GDP ratio matches the data fairly well
- The investment-to-capital ratio is a bit low, but not too bad

Real Interest Rate

- The model's real interest rate is the return on capital
- It is hard to find data on the average return on capital

World Return to Capital



The rate of return to capital (pre-tax) has always been higher than the world growth rate, but the gap was reduced during the 20th century, and might widen again in the 21st century.
Sources and series: see piketty.pse.ens.fr/capital21c

Source: Piketty (2014)

Steady-State Properties

- Overall the fit with the data is pretty good
- Especially given that the model is very stylized
- One can adjust the parameters to match the steady-state ratios more closely

7. Impulse Response Functions

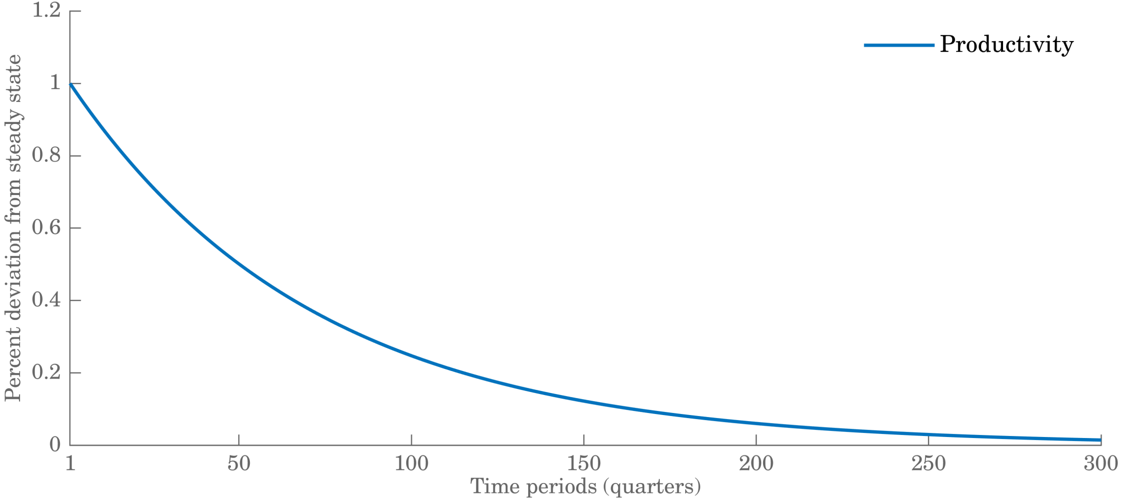
Impulse Response Functions

- A way to study the model dynamics is to plot impulse response functions
- Set an arbitrary number of periods T
- Impose a sequence of innovations, typically $\varepsilon_1 > 0$ then $\varepsilon_2 = 0, \ldots, \varepsilon_T = 0$
- Get the corresponding path for the endogenous variables by simulating the equilibrium equations; eg for output obtain $y_1, y_2, \ldots, y_T$
- Plot this sequence against time

Productivity Shock

- Suppose that, in period 0, the economy is in steady state
- In period 1, a TFP shock hits, $\varepsilon_1^a = 1$
- The shock raises productivity a_t by 1 percent above its steady-state value
- In all subsequent periods, $\varepsilon_{t+j}^a = 0$ for $j > 0$, ie the shock is zero for ever

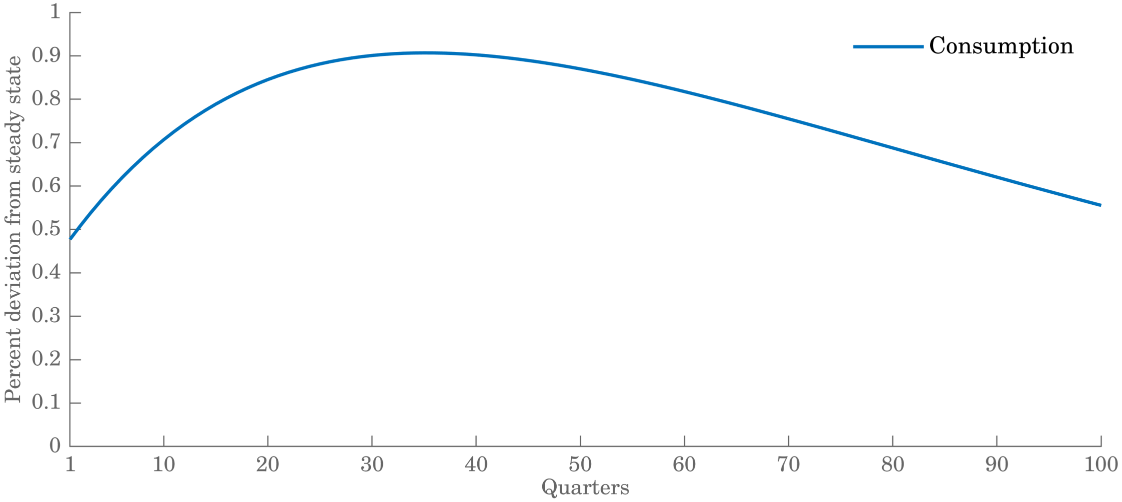
Productivity



Productivity

- The shock at time 1 has a persistent effect on productivity
- a_t returns to its steady state after only 700 periods
- This is due to the high autocorrelation that characterizes the Solow residual, $\rho_a = 0.986$

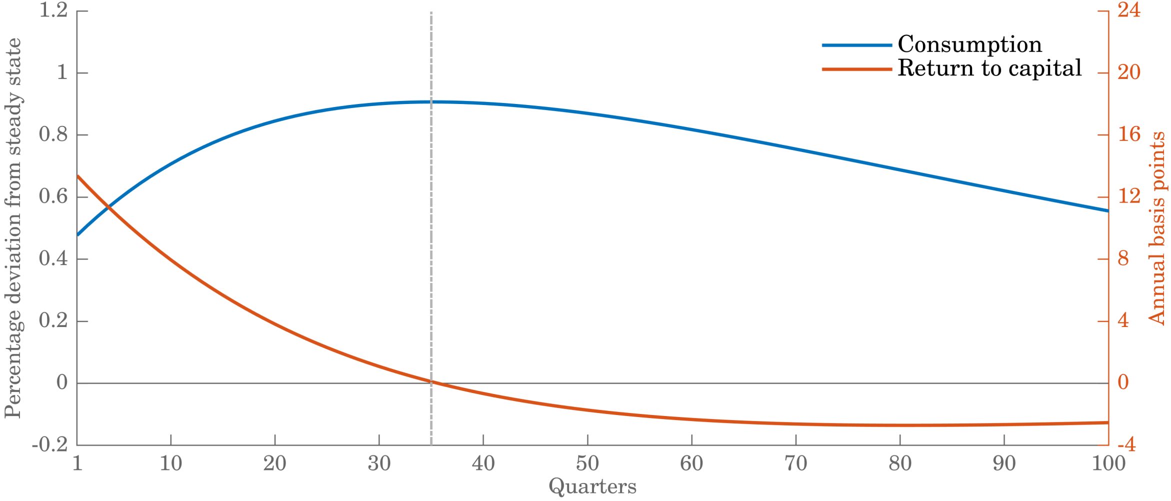
Consumption



Consumption

- Consumption jumps on impact and then keeps increasing
- This is due to a positive wealth effect, or income effect
- Higher productivity leads to 1) higher marginal product of labor and hence higher wage; 2) higher marginal product of capital and hence higher return
- Households are richer and consume more

Consumption and Return on Capital



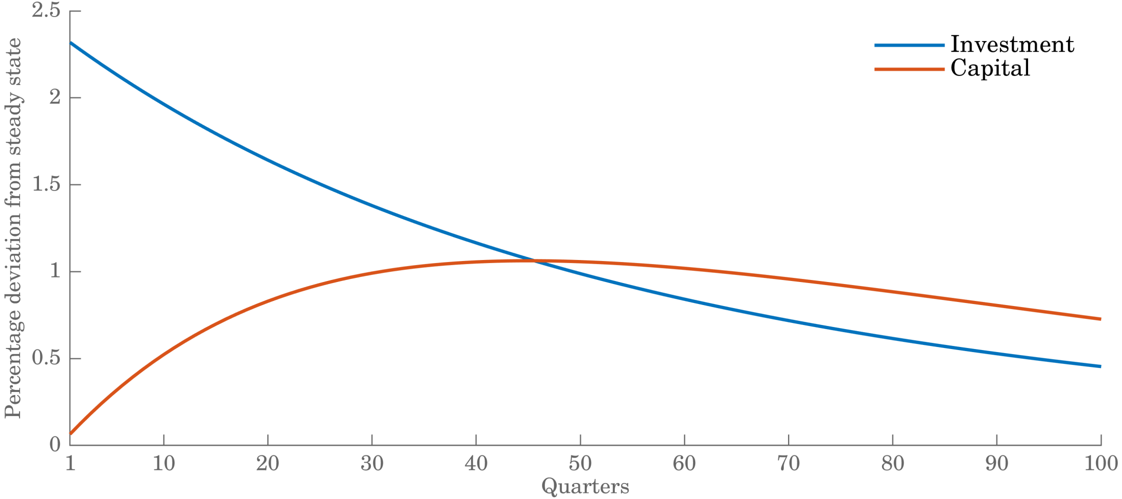
Consumption and Savings

- The profile of consumption is determined only by the expected return to capital

$$1 = \frac{\beta}{1+\mu} E_t \frac{c_t}{c_{t+1}} \left(r_{t+1} + 1 - \delta\right)$$

- Higher TFP leads to a higher expected return to capital $E_t r_{t+1}$
- This induces households to substitute future consumption for present consumption, ie to save by investing in the capital stock
- As long as r_t is above its steady-state value, consumption increases; as soon as r_t falls below its steady-state value, consumption decreases

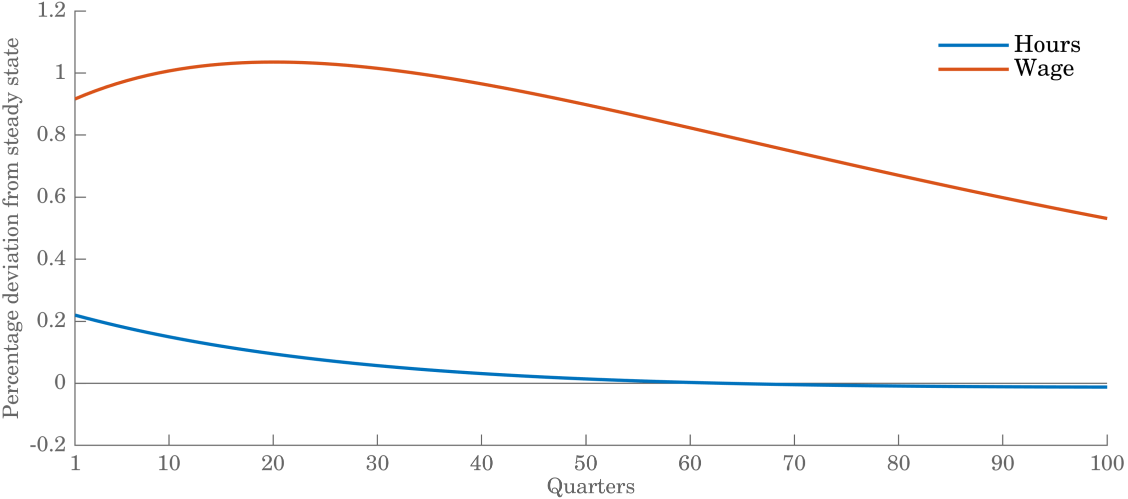
Investment and Capital



Investment and Capital

- Investment jumps on impact due to a higher return to capital
- This slowly increases the capital stock
- As capital accumulates, the marginal product of capital decreases and investment slowly returns to steady state
- At some point, too much capital makes it less productive, and capital starts to decline, ie investment is not enough to compensate for depreciation

Hours and Wage



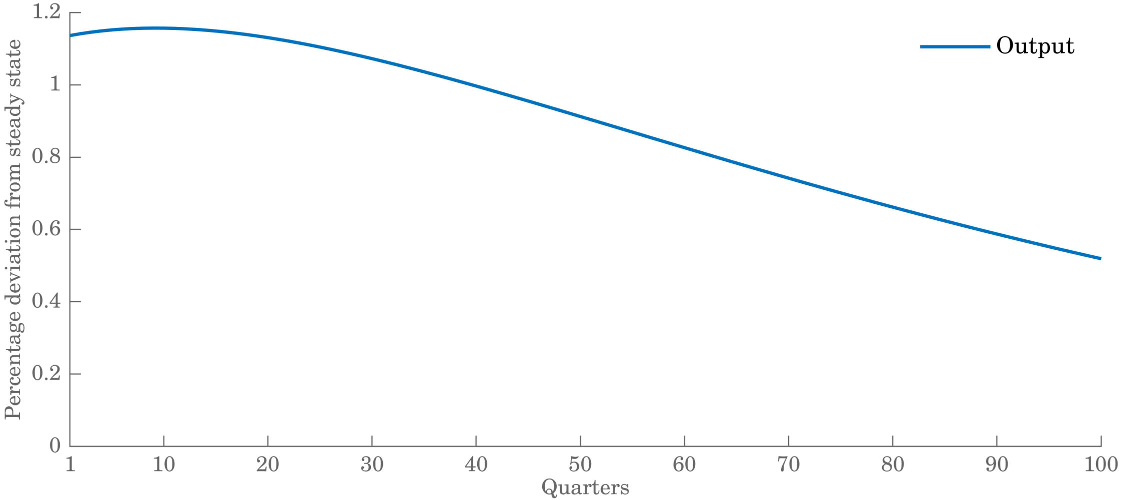
Hours and Wage

- Higher TFP increases the marginal product of labor and hence the wage
- Households face two opposing effects

1. Substitution effect: households want to postpone leisure in order to take advantage of the temporarily higher wage
2. Wealth effect: households are richer and want more leisure

- In the beginning, a high wage induces households to work more; but as the shock fades, hours fall progressively toward their steady state

Output

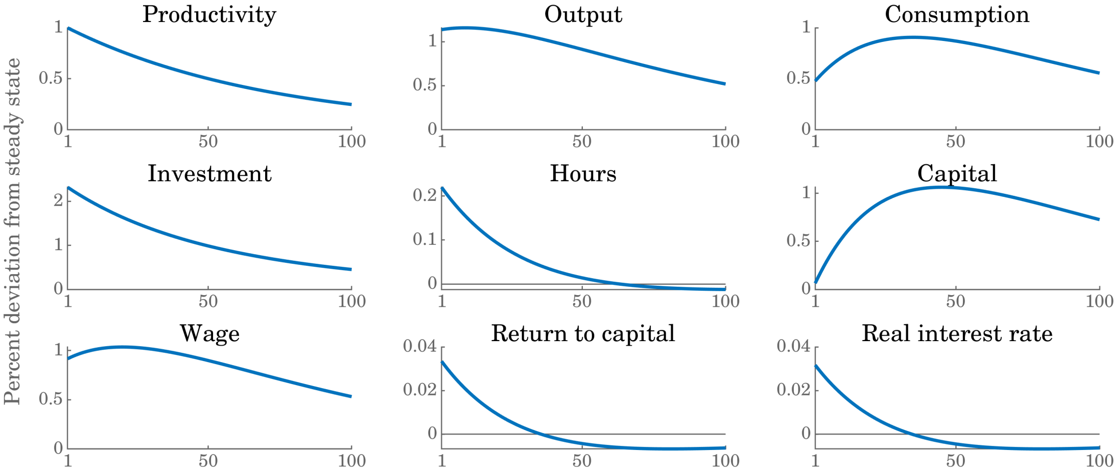


Output
Percentage deviation from steady state
0
0.2
0.4
0.6
0.8
1
1.2
1
10
20
30
40
50
60
70
80
90
100
Quarters

Output

- Higher consumption and higher investment lead to higher output
- Equivalently, higher labor and capital inputs lead to higher output
- The increase in output is higher than the increase in productivity
- This is because higher levels of labor and capital inputs amplify the initial effect of a higher productivity

All Variables Increase



Labor Supply Shock

- Let us now simulate a temporary increase households' desire for leisure
- This is equivalent to adding a shock ζ_t in the utility function

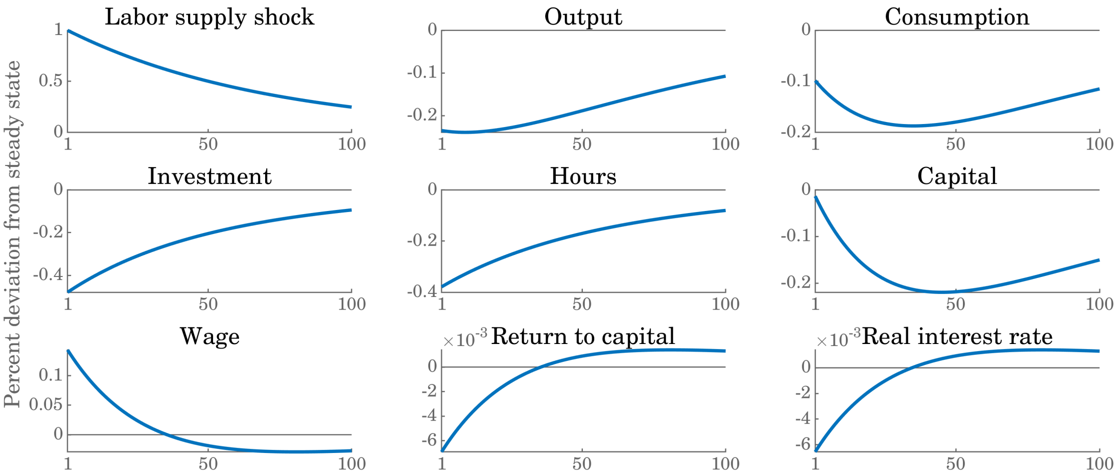
$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t L_t \left(\ln c_t - \zeta_t \frac{n_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

- Like a_t, suppose ζ_t follows an AR(1) process

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon_t^\zeta, \quad \varepsilon_t^\zeta \sim \mathcal{N}(0, \sigma_\zeta^2)$$

- We set $\varepsilon_1^\zeta = 1$ and $\varepsilon_{t+j}^\zeta = 0$ for $j > 0$, with $\rho_{\zeta 1} = 0.9$

Labor Supply Shock



Labor Supply Shock

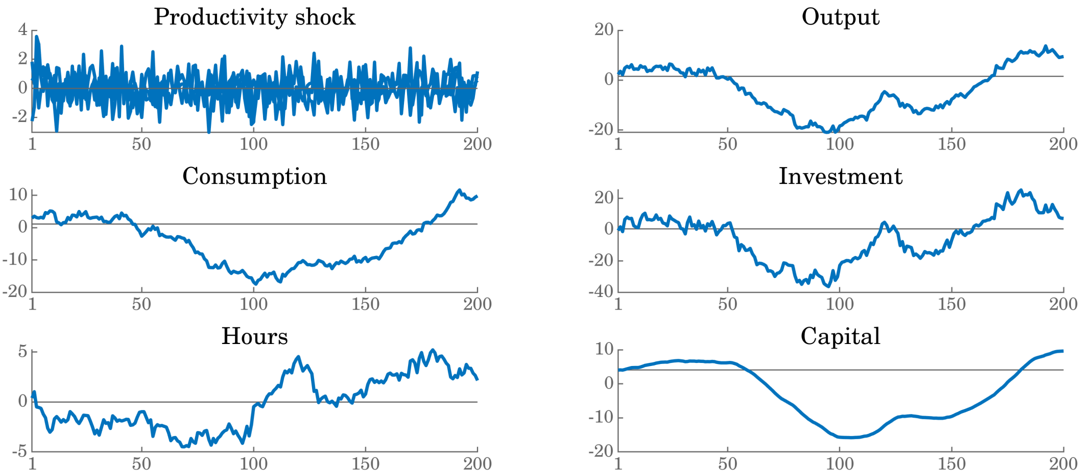
- People dislike working more, ie their marginal utility of leisure goes up
- Hours fall, wages increase because workers are relatively more productive
- Investment falls because capital is less useful now
- With fewer hours and lower capital returns, households suffer a drop in income and hence reduce consumption
- The shock creates a recession

8. Stochastic Simulations

Stochastic Simulations

- The idea is to create artificial data from the model and compare its properties to the properties of actual data
- Draw T productivity innovations ε_t^a from a normal distribution with mean zero and standard deviation σ_a
- Given these productivity shocks, compute the evolution of each endogenous variable for $t = 1, 2, \ldots, T$

Stochastic Simulations, One Draw



Moments

- Next, filter both the artificial data and the actual data, for example with a HP filter or with a bandpass filter (6,32)
- Compute the second moments: standard deviation, correlation (with output), serial correlation
- Repeat these operation n times and compute the theoretical moments by averaging over the n moments

Dynamic Properties, Model Versus Data

Variable	Standard deviation		Correlation with GDP		Autocorrelation	
	Model	Data	Model	Data	Model	Data
Output	1.46	1.54	1.00	1.00	0.90	0.91
Consumption	0.61	0.80	0.98	0.77	0.91	0.70
Investment	3.41	5.91	0.996	0.87	0.90	0.86
Hours	0.43	1.78	0.99	0.90	0.90	0.73
Productivity	1.03	0.96	0.998	0.45	0.90	0.64
Wage	1.03	0.90	0.998	0.18	0.90	0.24

All variables are detrended using a BP filter. Data values are computed over the period 1950Q1–2020Q1

Volatility

- Productivity shocks alone produce a model economy that is nearly as volatile as the US economy: the variance ratio is $(1.46/1.54)^2 = 0.90$
- In other words TFP shocks explain 90% of business cycle fluctuations
- The RBC model amplifies productivity shocks: output is $1.46/1.03 = 1.41$ as volatile as productivity
- Productivity and wages have almost the same volatility as in the data

Volatility

- Consumption is smoother than output (model ratio is .42, data is .52)
- Investment is more volatile than output, but not enough (2.34 vs 3.84)
- Hours are way too smooth compared to the data (0.43 vs 1.78)

Comovements

- The RBC model implies strong positive comovements among all variables
- These comovements are a feature of the data, but in some cases they are exaggerated by the model
  - Correlation productivity-GDP: model 0.998, data 0.45
  - Correlation wage-GDP: model 0.998, data 0.18

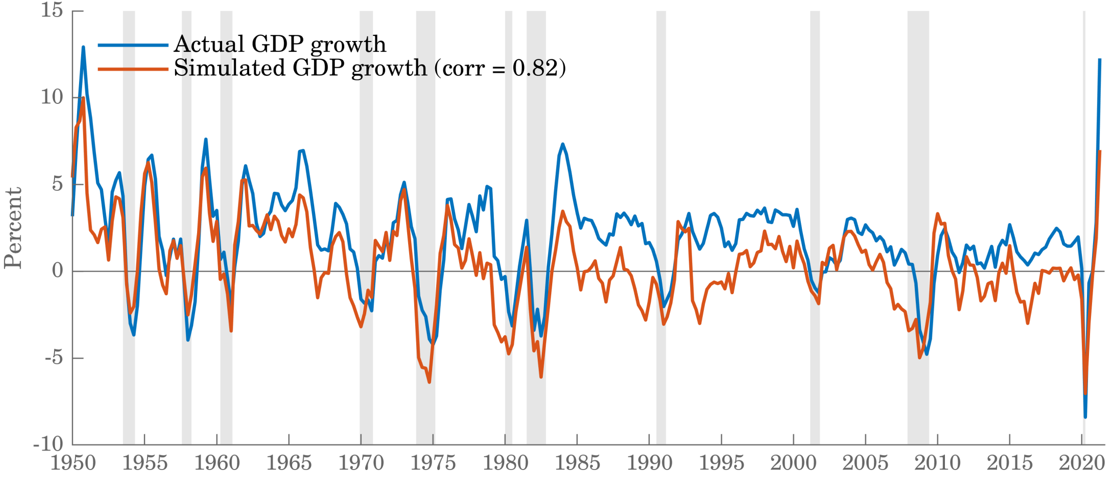
Persistence

- Business cycles are persistently high or low levels of economic activity
- A measure of this persistence is the first-order serial correlation
- Model serial correlations are high as in the data
- In some case they are too high
  - Hours: model 0.90, data 0.73
  - Productivity: model 0.90, data 0.64
  - Wage: model 0.90, data 0.24

Feeding the Solow Residual

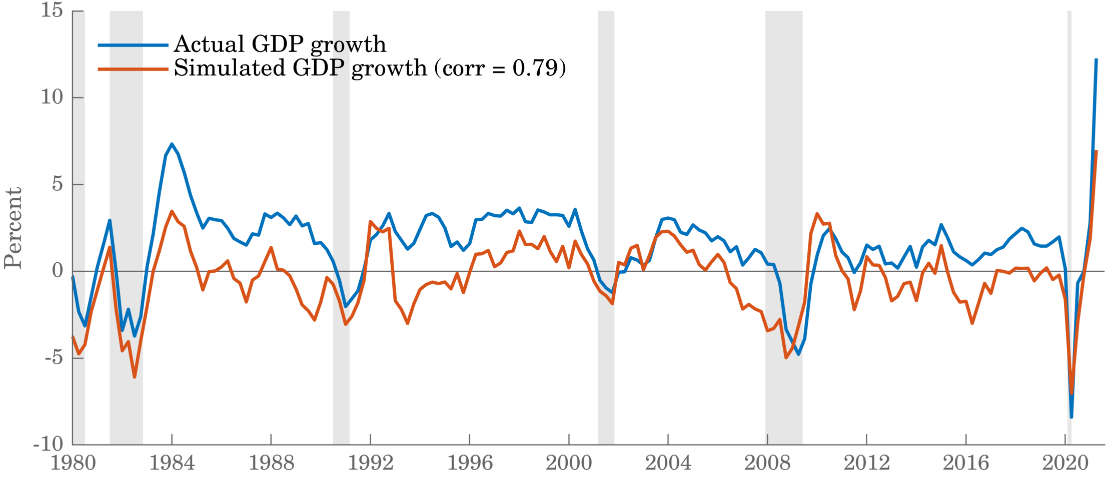
- So far we have simulated the model with random productivity shocks with parameters (ρ_a, σ_a)
- We can simulate the model with the innovations to the actual Solow residual

Feeding the Solow Residual



Data source: US Bureau of Economic Analysis

Feeding the Solow Residual



Data source: US Bureau of Economic Analysis

Feeding the Solow Residual

- The correlation between model-implied GDP and actual GDP is very high
- The two series have the same volatility (ratio 1.03)
- The model captures well the recessions

Good Results

"The match between theory and observation is excellent, but far from perfect."

Edward Prescott, 1986

"To many economists, the whole idea that such a simple model with no government, no money, no market failures of any kind, rational expectations, no adjustment costs and identical agents could replicate actual experience this well is very surprising."

Charles Plosser, 1989

9. Criticisms

Not Today

- Come to Macro III !