18. Search and Matching

Dynamics

Yvan Becard
PUC-Rio

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Recap of the Previous Lecture

- We presented the basic labor search and matching model
- We studied its steady-state properties
- We performed comparative statics
- We looked at how the model fits the data

Moving Forward

- Today we continue our analysis of the model
- We look at recent debates in the literature
- We analyze its dynamic properties

Lecture Outline

1. Digression on Nash Bargaining
2. Recent Debates
3. Welfare Analysis
4. Search, Matching, and Neoclassical Growth
5. Search and Matching in Continuous Time
6. Exercises

1. Digression on Nash Bargaining

Bargaining Situation

Reference: Osborne and Rubinstein, 1990, *Bargaining and Markets*, Chapter 2

- There are $N = \{1, 2\}$ bargainers or players
- Players can reach an agreement in the set A, or fail to reach an agreement, in which case disagreement event D occurs
- Each player $i \in N$ has preferences u_i over the set $A \cup \{D\}$

Set of All Possibilities

- Let P be the set of all utility pairs that can be the outcome of bargaining

$$P = \{(u_1(a), u_2(a)) : a \in A\}$$

- $P \subset \mathbb{R}^2$ is compact (ie closed and bounded) and convex
- Let d be a disagreement point

$$d = (u_1(D), u_2(D))$$

- $d \in P$ and there exists $s \in P$ such that $p_i > d_i$ for $i = 1, 2$
- Nash takes the pair (P, d) as the primitive of the problem

Bargaining Problem and Solution

- Nash (1950) defines the following objects
- A bargaining problem is a pair (P, d)
- A bargaining solution is a function f that assigns to each bargaining problem (P, d) a unique element of P

$$f(P, d) = (f_1(P, d), f_2(P, d)) \in P$$

Axiomatic Approach

- Nash does not try to build a model that captures all the details of a specific bargaining process, no bargaining procedure is explicit in his model
- Instead his approach is axiomatic

"One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely."

John Nash, 1953, *Econometrica*

Nash's Axioms

- Nash imposes four axioms on a bargaining solution f

1. Invariance to equivalent utility representations. Consider (P, d), (P', d') such that $p'_i = \alpha_i s_i + \beta_i$ and $d' = \alpha_i d_i + \beta_i$, with $\alpha_i > 0$. Then $f_i(P', d') = \alpha_i f_i(P, d) + \beta_i$ for $i = 1, 2$
2. Symmetry. (P, d) is a symmetric bargaining problem if $d_1 = d_2$ and $(p_1, p_2) \in P \iff (p_2, p_1) \in P$ imply $f_1(p, d) = f_2(p, d)$
3. Independence of irrelevant alternatives. If (P, d) and (T, d) are bargaining problems with $P \subset T$ and $f(T, d) \in P$, then $f(P, d) = f(T, d)$
4. Pareto efficiency. If (P, d) is a bargaining problem such that $p \in P$, $t \in P$, and $t_i > p_i$ for $i = 1, 2$, then $f(P, d) \neq p$

Intuition

1. Invariance to equivalent utility representations. If the utility function of one player is transformed by a linear function, then the agreement should also be transformed by the same linear function
2. Symmetry. Players are interchangeable, have the same bargaining ability; if there is any asymmetry between the players it must be captured in (P, d)
3. Independence of irrelevant alternatives. If players agree on an outcome p in P when all alternatives in T are available, then they must agree on the same outcome p if only alternatives in P are available, with $P \subset T$
4. Pareto efficiency. The agreement should represent a situation that could not be improved on to both players' advantage, otherwise they are not rational

Nash's Theorem

- Nash (1950, *Econometrica*) proves the following theorem
- There is a unique bargaining problem solution f^N satisfying the four axioms

$$f^N(P, d) = \underset{(d_1,d_2)\leq(p_1,p_2)\in P}{\arg\max} (p_1 - d_1) - (p_2 - d_2)$$

- Kalai (1977, *Econometrica*) shows that without axiom 2 (symmetry), there exists a family of solution indexed by $\phi \in (0, 1)$

$$f^N(P, d; \phi) = \underset{(d_1,d_2)\leq(p_1,p_2)\in P}{\arg\max} (p_1 - d_1)^{\phi} - (p_2 - d_2)^{1-\phi}$$

Example

- Coming back to our search and matching model

$$
\begin{aligned}
&P = \{(E, J) : E - U + J \leq S, E \geq 0, J \geq 0\} \\
&d = (U, V) = (U, 0)
\end{aligned}
$$

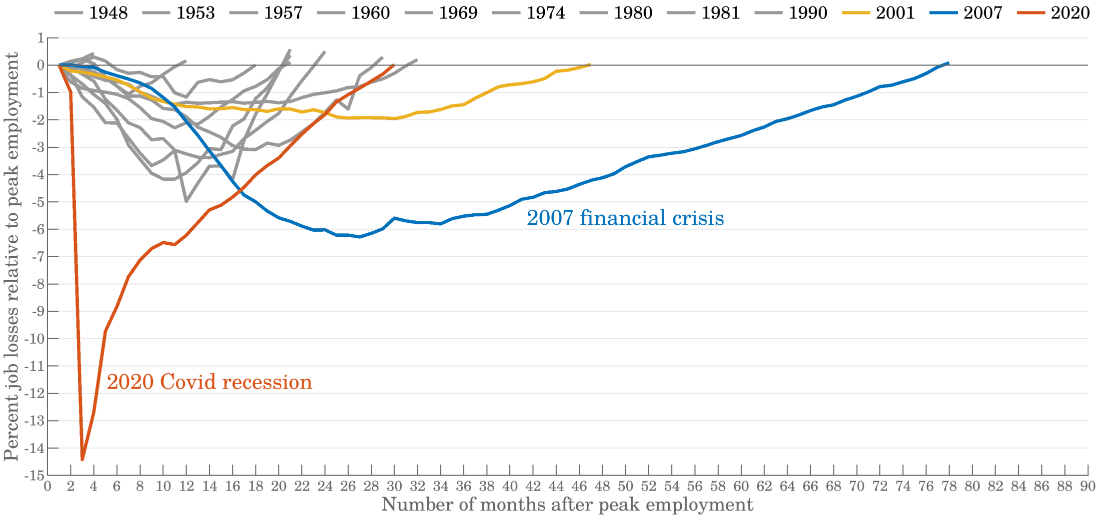
- We obtain the bargaining problem between firm and worker

2. Recent Debates

A Lively Debate

- The 2008-2009 recession in the developed world stood out by its severity but also by its recovery
- Unemployment in particular remained abnormally high for years after the end of the recession
- This jobless recovery has fueled an intense debate among labor economists: what are its causes?

Jobless Recovery



Source: US Bureau of Labor Statistics

Unemployment Benefit Extension

- One potential cause is unemployment benefit extension
- A federal law allows US states to extend benefits from the usual 26 weeks to up to 99 weeks with the objective to
  - Provide income support "for a vulnerable group after they have lost their jobs through no fault of their own"
  - Provide "needed support for the fragile economy"

No Consensus

- Is this a good policy?
- Some emphasize the potential stimulus effects of increasing transfers to unemployed people (Summers 2010, Congressional Budget Office 2021)
- Others argue they make unemployment more attractive and cause the slow labor market recovery (Barro 2010, Mulligan 2012, Hagedorn et al. 2019)

Demand and Supply Effect

- Through the lens of a search and matching model with costly search effort, increasing unemployment benefit duration has two effects

1. The first is a labor supply, micro effect: people's search intensity falls
2. The second is a labor demand, macro effect: higher benefits push wages up, firm's profit from filled jobs decline, vacancy creation falls

- Of course the model is silent about other general equilibrium effects such as higher consumption leading to higher demand and higher employment

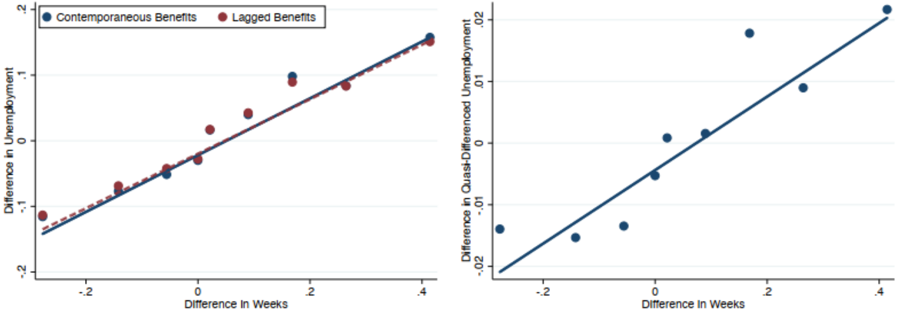
Macro Matters

- Most empirical micro studies focus on the first effect
- They find small impact of benefit extension on labor supply
- But Hagedorn, Karahan, Manovski, and Mitman (2019) argue that the second, general equilibrium macro effect matters a lot

Hagedorn, Karahan, Manovski, and Mitman (2019)

- US unemployment insurance policies are determined at the state level and apply to all locations within a state
- The authors exploit this policy discontinuity at state borders to look at differences between neighboring counties from different states
- These places arguably share the same geography, climate, access to transportation, agglomeration benefits, access to specialized labor, etc.
- They find that benefit extensions “lead to a sharp contraction in vacancy creation, employment, and a rise in unemployment”
- Without the extension, unemployment rate would have been 3.02 and 2.15 percent lower in 2010 and 2011

Benefit Duration Increases Unemployment

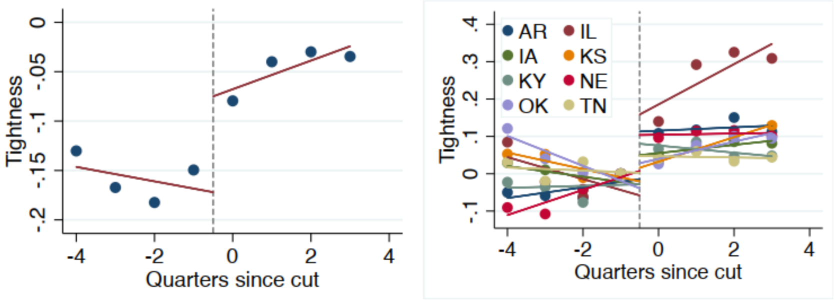


(a) Current and Lagged Benefits and Unemployment (b) Current Benefits and QD Unemployment

Figure 3: (Quasi-Differenced) Unemployment Differences between Border Counties versus Differences in Benefit Duration.

Source: Hagedorn, Karahan, Manovski, and Mitman (2019)

A Cut in Benefit Duration Increases Vacancies and Tightness



(a) All Missouri Borders.

(b) Each Missouri Border.

Figure 4: The Macro Effect of the Cut in Potential Benefit Duration in Missouri in April 2011.

Note - Panel 4(a) plots the difference in the log vacancy-unemployment ratio between border counties in Missouri and adjacent border counties across all Missouri state borders. Panel 4(b) plots the same statistic separately for each Missouri border. The dashed vertical line indicates the date when potential benefit duration was cut by 16 weeks in Missouri.

Source: Hagedorn, Karahan, Manovski, and Mitman (2019)

There Is Still Endogeneity

- Chodorow-Reich, Cogalianese, and Karabarbounis (2019, *QJE*) dispute the findings of Hagedorn et al. (2019)
- The main obstacle to measuring the effect of benefit duration on unemployment is endogeneity, in particular simultaneity
- Higher benefit duration might cause higher unemployment, yes, but higher unemployment also causes higher benefit duration

Measurement Error Approach

- The authors use a measurement error approach
- Unemployment is measured in real time
- This determines the duration policy in the state
- But real-time data is subject to measurement error
- Data revision gives the extent of that error

Benefit Duration Has Virtually No Impact

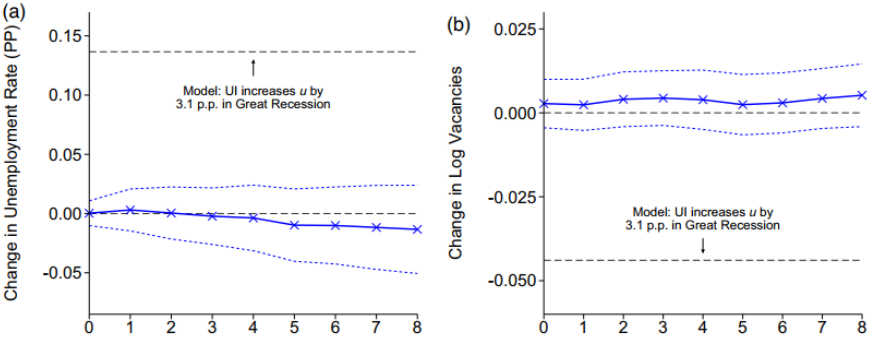


FIGURE III

Impulse Responses of Unemployment Rate and Log Vacancies

The figure plots the coefficients on $\epsilon_{s,t}$ from the regression $y_{s,t+h} = \beta(h)\epsilon_{s,t} + \sum_{j=1}^{12} \gamma_j(h) u_{s,t-j} + d_s(h) + d_t(h) + \nu_{s,t+h}$, where $y_{s,t+h} = u_{s,t+h}$ is the unemployment rate (left panel) or $y_{s,t+h} = \log v_{s,t+h}$ is log vacancies (right panel). The dashed lines denote the 90% confidence interval based on two-way clustered standard errors.

Small Impact

- Chodorow-Reich and coauthors find that benefit extensions have “limited influence on state-level macroeconomic outcomes”
- Higher benefit duration during the Great Recession “increased unemployment by at most by 0.3 percent”
- The debate is not settled

Mismatch Unemployment

- Another possible explanation for the jobless recovery is mismatch unemployment
- Idle workers are seeking in sectors, occupations, industries, or locations different from those where the vacant jobs are
- This is consistent with the fact that after the crisis the job separation rate quickly returned to its pre-crisis level while the job-finding rate stayed low

Features of the Great Recession

- Mismatch unemployment is also consistent with three facts

1. The Beveridge curve shifted outwards after the recession: u is higher than it used to be for a given v, ie matching efficiency has declined
2. Half the job losses were in construction and manufacturing: if workers do not switch sectors, occupation mismatch increases
3. House prices fell dramatically: a homeowner may choose not to move to avoid large capital losses, thus creating geographical mismatch´

Mismatch Matters

"We develop a framework where mismatch between vacancies and job seekers across sectors translates into higher unemployment by lowering the aggregate job-finding rate. We use this framework to measure the contribution of mismatch to the recent rise in US unemployment . . . Our calculations indicate that mismatch, across industries and three-digit occupations, explains at most one-third of the total observed increase in the unemployment rate. Occupational mismatch has become especially more severe for college graduates, and in the West of the United States. Geographical mismatch unemployment plays no apparent role."

Sahin, Song, Topa, and Violante, 2014, *American Economic Review*

Unemployment after the Recession

- It is not clear why unemployment has fallen so slowly
- Some economists have argued that it was simply a larger recession, so it took longer to recover
- Others argue that the debt deflation phenomenon caused the slow recovery
- Yet others invoke long-run factors such as globalization, automation, job polarization

Unemployment Today

- Before the covid-19 crisis, US unemployment stood at 3.5%, a 50-year low; now it is back to its pre-pandemic levels
- So this debated had receded somewhat
- A new debate has appeared in the face of disappointing employment (not unemployment) numbers: what is behind the Great Resignation?

3. Welfare Analysis

Efficiency

- Is the search and matching model Pareto efficient?
- Let's derive the allocation chosen by a central planner

No Uncertainty

- The planner chooses an allocation that maximizes the discounted value of output and leisure net of vacancy costs
- The social problem does not involve any uncertainty
- This is because the aggregate fractions of successful and destroyed matches equal the probabilities of these events

Planner Problem

- The planner chooses the measure of vacancies v_t and next period's employment level n_{t+1}

$$\max_{\{v_t, n_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t [y n_t + z(1 - n_t) - c v_t]$$

$$\text{subject to} \quad n_{t+1} = (1 - s) n_t + q\left(\frac{v_t}{1 - n_t}\right) v_t, \quad n_0 \text{ given}$$

- The constraint is the law of motion of employment
- Remember, $u_t = 1 - n_t$

Lagrangian

- Write a Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ [y n_t + z(1 - n_t) - c v_t] + \lambda_t \left[(1 - s) n_t + q \left(\frac{v_t}{1 - n_t} \right) v_t - n_{t+1} \right] \right\}$$

- λ_t is the Lagrange multiplier on the constraint

First-Order Conditions

- The first-order conditions are

$$
\begin{aligned}
v_t: &\quad -c + \lambda_t[q'(\theta_t)\theta_t + q(\theta_t)] = 0 \\
n_{t+1}: &\quad -\lambda_t + \beta(y - z) + \beta\lambda_{t+1}[1 - s + q'(\theta_{t+1})\theta_{t+1}^2] = 0
\end{aligned}
$$

Steady State

- Let's consider the steady state
- Combine the two FOCs to get rid of λ

$$y - z = \frac{r + s - q'(\theta)\theta^2}{q'(\theta)\theta + q(\theta)} c$$

Cobb-Douglas

- With the functional form $M(u,v) = Au^{\alpha}v^{1-\alpha}$, we have

$$\frac{\partial M(u,v)}{\partial u}\frac{u}{M(u,v)} = -q'(\theta)\frac{\theta}{q(\theta)} = \alpha$$

- Substitute this into the FOC

$$y - z = \frac{r + s + \alpha\theta q(\theta)}{(1-\alpha)q(\theta)}c$$

Competitive Versus Social

- The social optimum is

$$y - z = \frac{r + s + \alpha\theta q(\theta)}{(1-\alpha)q(\theta)}c$$

- The solution of the decentralized equilibrium is

$$y - z = \frac{r + s + \phi\theta q(\theta)}{(1-\phi)q(\theta)}c$$

- The competitive equilibrium is efficient if and only if $\phi = \alpha$

Hosios Condition

- Hosios (1990, *Restud*) was the first to show the efficiency condition

$$\phi = \alpha$$

- This is known as the Hosios condition
- The Hosios condition requires the bargaining power of a factor to be equal to the elasticity of the matching function with respect to that factor
- What is the intuition?

Intuition

- ϕ is the bargaining strength of workers and α is the elasticity of the matching function $M(u, v)$ with respect to unemployment u
- A high bargaining strength $\phi > \alpha$ means firms get only a small share of the surplus and thus don't enter enough, ie do not create enough vacancies
- A low bargaining strength $\phi < \alpha$ implies the opposite: too many firms enter, there is excess entry, ie the equilibrium job supply is too high
- The planner wants to increase the number of vacancies to the point where the marginal benefit in terms of additional matches is equal to the cost
- Note Shimer (2005) sets $\phi = \alpha$; Hagedorn and Manovskii (2008) set $\phi < \alpha$

4. Search, Matching, and Neoclassical Growth

Partial to General

- The search and matching model is a partial-equilibrium model
- The labor market is in equilibrium, the wage is endogenous
- But the goods market does not clear, output y is exogenous
- It is possible to embed the search and matching framework into a general-equilibrium neoclassical growth model

Partial to General

- Assuming the Hosios condition, we use the planner's problem to solve for

$$V(k,n,z) = \max_{x^e,x^u,l,k',v} \{nU^e(x^e, 1-l) + (1-n)U^u(x^u, 1) + \beta EV(k', n', z')\}$$

$$\text{subject to} \quad \begin{aligned} nx^e + (1-n)x^u + k' + cv &= e^z F(k, nl) + (1-\delta)k \\ n' &= (1-s)n + M(v, 1-n) \\ z' &= \rho z + \varepsilon \end{aligned}$$

- If $s = 0$ and $n = 1$ we are back to the standard RBC model

Improving the Fit of the RBC Model

- Andolfatto (1996, *AER*) solves a variant of this problem
- He shows that with this more realistic labor market structure, some undesirable, counterfactual properties of the RBC model are overturned

1. Hours become more volatile than wages
2. Hours and productivity become less correlated
3. GDP growth autocorrelation increases

Hours Fluctuate More than Wages

	U.S. economy $\sigma(y) = 1.58$			RBC economy $\sigma(y) = 1.22$			Search economy $\sigma(y) = 1.45$		
Variable (x)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.56	0.74	0	0.34	0.90	0	0.32	0.91	0
Investment	3.14	0.90	0	3.05	0.99	0	2.98	0.99	0
Total hours	0.93	0.78	+1	0.36	0.98	0	0.59	0.96	0
Employment	0.67	0.73	+1	0.00	0.00	0	0.51	0.82	+1
Hours/worker	0.34	0.66	0	0.36	0.98	0	0.22	0.66	0
Wage bill	0.97	0.76	+1	1.00	1.00	0	0.94	1.00	0
Labor's share	0.68	−0.38	−3	0.00	0.00	0	0.10	−0.62	−1
Productivity	0.64	0.43	−2	0.64	0.99	0	0.46	0.94	0
Real wage	0.44	0.04	−4	0.64	0.99	0	0.39	0.95	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y. Column (3) is the phase shift in x relative to y: $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Source: Andolfatto (1996)

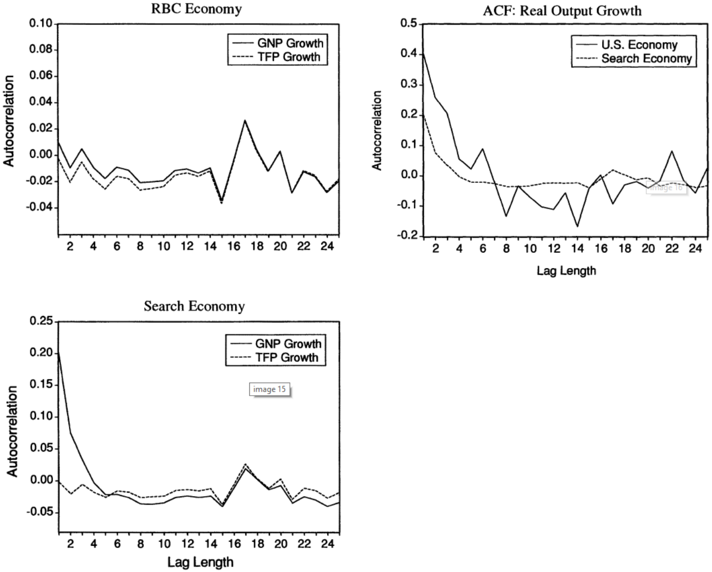
Hours and Productivity Are Less Correlated

- Cross-correlations of hours with productivity and wages

Variable (x)	$x(t-4)$	$x(t-3)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$
U.S. economy:									
Productivity	0.47	0.43	0.31	0.10	−0.22	−0.25	−0.27	−0.31	−0.36
Real wage	−0.25	−0.10	−0.01	0.01	−0.13	0.10	0.19	0.18	0.09
RBC economy:									
Real wage	−0.10	0.09	0.33	0.61	0.95	0.77	0.59	0.42	0.25
Search economy:									
Productivity	0.14	0.38	0.63	0.82	0.81	0.56	0.40	0.28	0.18
Real wage	0.05	0.30	0.57	0.80	0.84	0.66	0.52	0.40	0.29

Source: Andolfatto (1996)

GDP Growth Autocorrelation Increases



Source: Andolfatto (1996)

5. Search and Matching in Continuous Time

Dynamics

- So far we have studied the model in steady state
- We are now going to study the dynamics of the model
- To do so we solve the model in continuous time
- It is the same model as before, with the same notation

Why Continuous Time?

- Continuous time simplifies the dynamic equations

Continuous time	Discrete time
$rJ = y - w + s[V - J] + \dot{J}$	$rJ_t = (y - w)(1 + r) + s[V_{t+1} - J_{t+1}] + J_{t+1} - J_t$
$rV = -c + q(\theta)[J - V] + \dot{V}$	$V_t = -c(1 + r) + q(\theta_t)[J_{t+1} - V_{t+1}] + V_{t+1} - V_t$
$rE = w + s[U - E] + \dot{E}$	$rE_t = (1 + r)w + s[U_{t+1} - E_{t+1}] + E_{t+1} - E_t$
$rU = z + \theta q(\theta)[E - U] + \dot{U}$	$rU_t = (1 + r)z + \theta_t q(\theta_t)[E_{t+1} - U_{t+1}] + U_{t+1} - U_t$

Variation

- Take the worker's value of being unemployed between t and $t+\Delta$, where $\Delta \in (0,1)$; multiply by $1+r\Delta$ on both sides; re-arrange and divide by Δ

$$
\begin{aligned}
U_t &= z\Delta + \frac{1}{1+r\Delta}[\theta_t q(\theta_t)\Delta E_{t+\Delta} + (1-\theta_t q(\theta_t)\Delta)U_{t+\Delta}] \\
(1+r\Delta)U_t &= z\Delta(1+r\Delta) + \theta_t q(\theta_t)\Delta[E_{t+\Delta} - U_{t+\Delta}] + U_{t+\Delta} \\
rU_t &= z(1+r\Delta) + \theta_t q(\theta_t)[E_{t+\Delta} - U_{t+\Delta}] + \frac{U_{t+\Delta} - U_t}{\Delta}
\end{aligned}
$$

Infinitesimal Variation

- $\Delta \to 0$ implies
$$rU_t = z + \theta_t q(\theta_t)[E_t - U_t] + \dot{U}_t$$
- $\dot{U}_t \equiv dU_t/dt$ is the time derivative, ie the rate of change of the value of being unemployed U

Value Functions

- Proceeding in the same way, we write the firms' and workers' value functions in continuous time

$$
\begin{aligned}
rU &= z + \theta q(\theta)[E - U] + \dot{U} \\
rE &= w + s[U - E] + \dot{E} \\
rV &= -c + q(\theta)[J - V] + \dot{V} \\
rJ &= y - w + s[V - J] + \dot{J}
\end{aligned}
$$

Nash Bargaining

- Recall the stationary solution to Nash bargaining

$$E - U = \frac{\phi}{1-\phi}(J - V)$$

- We now assume that wages can be renegotiated at any point in time, eg when new information arrives

$$\dot{E} - \dot{U} = \frac{\phi}{1-\phi}(\dot{J} - \dot{V})$$

- Free entry implies $V = 0$ at any point in time, and thus

$$\dot{V} = 0$$

Unemployment Dynamics

- The law of motion of unemployment is

$$u_{t+\Delta} = u_t[1 - \theta_t q(\theta_t)\Delta] + (1 - u_t)s\Delta$$
$$\frac{u_{t+\Delta} - u_t}{\Delta} = -u_t\theta_t q(\theta_t) + (1 - u_t)s$$

- $\Delta \to 0$ implies

$$\dot{u} = -u\theta q(\theta) + (1 - u)s$$

Beveridge Curve

- In steady state, unemployment is constant, $\dot{u} = 0$
- The Beveridge curve is thus

$$u^* = \frac{s}{s + \theta^* q(\theta^*)}$$

- Superscript $*$ denotes the steady-state value

Recap of the Model

- Firms' problem with $V = \dot{V} = 0$

(1) $$rJ = y - w + s[V - J] + \dot{J} \implies J = \frac{y - w + \dot{J}}{r + s}$$

(2) $$rV = -c + q(\theta)[J - V] + \dot{V} \implies J = \frac{c}{q(\theta)}$$

- Workers' problem

(3) $$rE = w + s[U - E] + \dot{E} \implies E = \frac{w + sU + \dot{E}}{r + s}$$

(4) $$rU = z + \theta q(\theta)[E - U] + \dot{U} \implies E - U = \frac{rU - z - \dot{U}}{\theta q(\theta)}$$

- Nash bargaining

(5) $$E - U = \frac{\phi}{1 - \phi} J$$

(6) $$\dot{E} - \dot{U} = \frac{\phi}{1 - \phi} \dot{J}$$

Equilibrium

- Combine (1) and (2) to get the job creation condition

$$w = y - \frac{(r+s)c}{q(\theta)} + \dot{J} \tag{7}$$

- Combine (1), (3), (5), and (6)

$$w = rU + \phi(y - rU) - (1-\phi)\dot{U} \tag{8}$$

- Combine (2), (4), and (5)

$$rU = z + \frac{\phi\theta c}{1-\phi} + \dot{U} \tag{9}$$

Wage Curve

- Combine the last two equations, (8) and (9), to obtain the wage curve

$$w = z + \phi(y - z + \theta c)$$

- This is the same wage curve as before
- The equation holds in and out of steady state

Job Creation

- If $\dot{J} = 0$, the job creation condition (7) becomes

$$w = y - \frac{(r+s)c}{q(\theta)}$$

- This is the same job creation condition as before
- The equation holds in and out of steady state

Constant Job Value

- Why is $\dot{J} = 0$?
- Since $J = \frac{c}{q(\theta)}$ at any point in time, then $\dot{J} > 0$ implies $\dot{\theta} > 0$
- Combine the wage curve and job creation condition

$$\dot{J} = \frac{r + s + \phi\theta q(\theta)}{q(\theta)}c + (1 - \phi)(z - y)$$

- $\dot{\theta} > 0$ implies $d\dot{J}/dt > 0$ and thus $d\dot{\theta}/dt > 0$
- This means J and θ follow an explosive path

Constant Job Value

- Since v is limited by the number of firms, $\theta = v/u \to \infty$ means $u \to 0$
- But this contradicts a nonzero steady-state $u^* = \frac{s}{s+q(\theta^*)\theta^*} = \frac{s}{s+M(1,v^*/u^*)}$
- The same reasoning holds for $\dot{J} < 0$
- We conclude that the only rational expectations solution is

$$\dot{J} = \dot{\theta} = 0$$

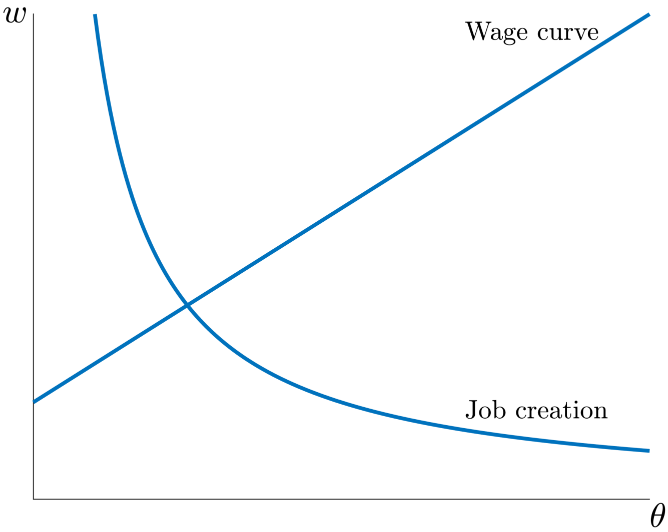
Unemployment Dynamics

- Recall the law of motion of unemployment

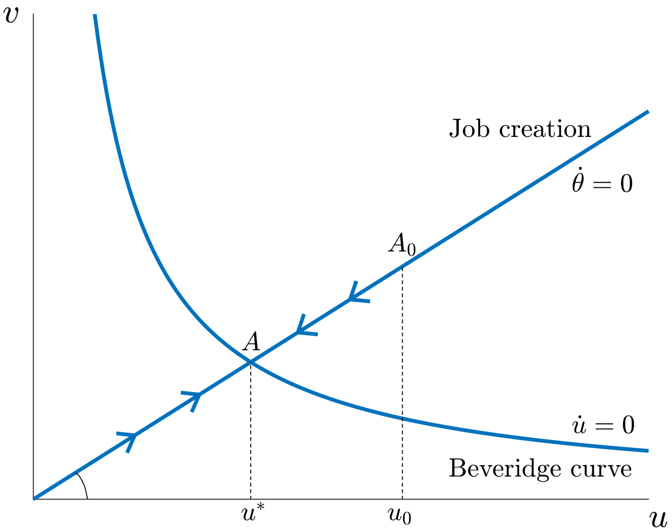
$$\dot{u} = -u\theta q(\theta) + (1-u)s$$

- w and θ are constant, u is predetermined and slow moving
- If $u > \frac{s}{s+\theta q(\theta)} = u^*$ then $\dot{u} < 0$
- If $u < \frac{s}{s+\theta q(\theta)} = u^*$ then $\dot{u} > 0$
- u converges to its unique steady state u^*

Tightness–Wage Space



Vacancy–Unemployment Space



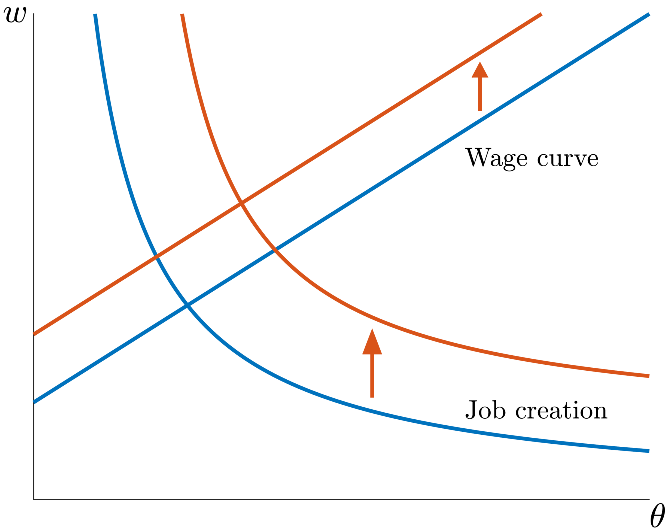
Equilibrium Path

- The equilibrium trajectory is unique
- If we start in A_0 with $u_0 > u^*$, unemployment gradually falls along the job creation line until it reaches u^*, ie A
- u and v are negatively correlated in steady state but positively correlated over the transition

Experiment

- Let's run an experiment
- We consider a sudden increase in productivity y
- How does the dynamic model respond?

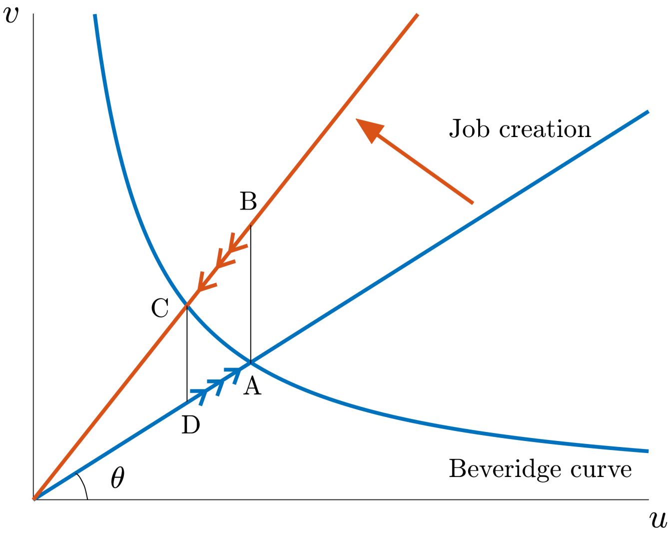
Productivity Increase



Productivity Increase

- A rise in y shifts both the wage and job creation curves up
- This causes an immediate rise in both θ and w
- The two variables jump to their new equilibrium, ie there are no adjustment dynamics

Productivity Increase



Productivity Increase

- Suppose the initial equilibrium is A
- After the positive productivity shock y, the equilibrium jumps to B as firms open more vacancies to take advantage of the higher productivity
- This sets in motion unemployment dynamics, which move the economy down the new job creation line
- The economy slowly moves towards the new steady state C
- If y falls now from C, we jump down to D and then move slowly to A

Conclusion

- The search and matching framework and its many variants is the dominant paradigm in labor economics
- It is a useful framework to think about employment and unemployment dynamics and study labor market policies
- But it has been criticized on many fronts and needs to be extended and improved to be of any quantitative use

6. Exercises

Exercise 1 – Skill-Biased Technological Change

Consider a matching model in discrete time with infinitely lived and risk-neutral workers who are endowed with different skill levels. A worker of skill type i produces h_i goods in each period that she is matched to a firm, where $i \in \{1, 2, \dots, N\}$ and $h_{i+1} > h_i$. Each skill type has its own but identical matching function $M(u_i, v_i) = A u_i^{\alpha} v_i^{1-\alpha}$, where u_i and v_i are the measures of unemployed workers and vacancies in skill market i. Firms incur a vacancy cost ch_i in every period that a vacancy is posted in skill market i; that is, the vacancy cost is proportional to the worker's productivity. All matches are exogenously destroyed with probability $s \in (0, 1)$ at the beginning of a period. An unemployed worker receives unemployment compensation b. Wages are determined in Nash bargaining between matched firms and workers. Let $\phi \in [0, 1)$ denote the worker's bargaining weight in the Nash product, and we adopt the standard assumption that $\phi = \alpha$.

Exercise 1 – Continued

1. Show analytically how the unemployment rate in a skill market varies with the skill level h_i.
2. Explain how the results would change if unemployment benefits are proportional to a worker's productivity.

Exercise 2

In the months following an economic crisis, we observe that the index of unfilled vacancies in the labor market is positively correlated with the unemployment rate. Is this phenomenon consistent with the search and matching model? Justify using graphs.

Exercise 3

Consider the standard search and matching model studied in class. We propose the following extension.

In Economy A there are N types of workers indexed by $i \in \{1, 2, \ldots, N\}$. The probability to be from type i is $\pi_i = 1/N$. There is one labor market for each type i, with matching function $M(u_i, v_i) = u_i^{\alpha} v_i^{1-\alpha}$. The output produced by worker i is y_i. Suppose that $y_{i+1} > y_i$ for all i.

In Economy B there is only one type of workers, which produces $\bar{y} = \sum_{i=1}^{N} (1/N) y_i$ once matched to a firm.

Exercise 3 – Continued

1. Show how the unemployment rate in Economy A varies with the types of labor markets, ie how u_i varies with y_i.
2. Suppose that in equilibrium u_i is a convex function of y_i (which is true numerically for a reasonable calibration of the model). Is unemployment higher in Economy A or B?
3. Instead of having a fixed cost c per vacancy, we assume that the cost is $c_i = \tilde{c} y_i$ where $\tilde{c} \in (0, 1)$. Similarly, we assume that the unemployment benefit evolves according to $z_i = \tilde{z} y_i$, where $\tilde{z} \in (0, 1)$. How does the unemployment rate vary with the different types of labor markets? Now, is unemployment higher in Economy A or B?