# 13. Optimal Taxation

## Challenging Chamley-Judd

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# Introduction

- In the previous lecture we demonstrated the famous Chamley-Judd result
- The optimal long-run capital income tax is zero
- This result holds across a number of model specifications
- Today we look mostly at cases in which it does not hold

# Lecture Outline

1. Taxing Initial Capital
2. Avoidance of Tax Arbitrage
3. Heterogeneity
4. Behavioral Biases
5. Low Intertemporal Substitution
6. Incomplete Taxation
7. Commitment
8. Taxes in Practice
9. Conclusion
10. Exercises

Main Reference: Ljungqvist and Sargent, 2018, *Recursive Macroeconomic Theory*, Fourth Edition, Chapter 16

# 1. Taxing Initial Capital

# Capital Levy

- Thus far we have restricted the first-period capital tax $\tau_0^k$ to be zero
- We now suppose that the government is free to chose $\tau_0^k$
- For example, a new government is elected and decides to impose a one-off tax on capital: this is called a capital levy

# Lagrangian

- Remember the Lagrangian of the Ramsey problem

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{V(c_t, n_t, \Phi) + \theta_t[F(k_t, n_t) + (1-\delta)k_t - c_t - g_t - k_{t+1}]\}$$
$$- \Phi u_c(0)\{[(1-\tau_0^k)F_k(0) + 1 - \delta]k_0 + b_0\}$$

- $\Phi$ is the multiplier on the implementability condition, ie the time 0 budget constraint in which we have injected the FOCs

# Optimal Initial Tax

- The derivative with respect to $\tau_0^k$ is

$$\frac{\partial \mathcal{L}}{\partial \tau_0^k} = \Phi u_c(0) F_k(0) k_0$$

- $k_0 > 0$ by assumption, thus $F_k(0) > 0$ and $u_c(0) > 0$
- Thus, as long as the multiplier $\Phi > 0$, the FOC is strictly positive for all $\tau_0^k$

# Welfare Cost of Taxes

- $\Phi$ measures the utility cost of raising revenues through distorting taxes
- In this frictionless model, without distorting taxes, the competitive equilibrium attains the first-best outcome and $\Phi$ is equal to zero
- In other words, the government budget constraint does not constrain the Ramsey planner beyond the resource constraint
- By contrast, with distortionary taxation $\tau_t^k > 0$ or $\tau_t^n > 0$, $\Phi > 0$ reflects the welfare cost of distorted household decisions

# A Capital Levy Is a Lump-Sum Tax

- Since capital is fixed in period $0$, $\tau_0^k$ is non distorting, like a lump-sum tax
- By raising $\tau_0^k$, the government increases revenues and reduces its need to rely on future distortionary taxation, hence decreasing $\Phi$
- The optimal solution is to set $\tau_0^k$ high enough so as to drive $\Phi$ down to zero

# One-Off Expropriation

- The government should raise all revenues through $\tau_0^k$
- Then lend the proceeds to the private sector
- And finance expenditure using the interest from the loan
- This would enable it to set $\tau_t^n = \tau_t^k = 0$ for ever and avoid inefficient distortions

# 2. Avoidance of Tax Arbitrage

# Contested Result

- The Chamley-Judd result is highly disputed
- There are many theoretical and empirical arguments against the zero capital income tax, ie in favor of taxing capital
- We are going to see a few of them, starting with tax avoidance

# Capital or Labor Income?

- It is often difficult for a government to distinguish between capital income and labor income, as individuals can shift from one to another
- In small businesses, profits arise both from the labor of their owners and the return on assets; same thing for investors managing their portfolio
- Compensation of private equity and hedge fund managers in the United States is considered realized capital gains, but part of it is labor income

# Income Shifting

- When capital income is less taxed than labor income, there are incentives to declare labor income as capital income, ie to engage in income shifting
- Gordon and Slemrod (2000) document a shift of reported income from corporate to personal after a personal tax rate cut in the US in the 1980s
- Pirtillä and Selin (2011) present evidence of income shifting from capital to labor income by self-employed people in Finland after a tax reform in 1993
- Harris et al. (1993) find that US multinational corporations shift income from high-tax countries to low-tax countries through transfer pricing

# Taxing Capital Is Optimal

- Based on these empirical findings, some papers relax the assumption of perfect information and build theories of income shifting
- Christiansen and Tuomala (2008) develop a model in which individuals can camouflage labor income as capital income at a cost
- Reis (2010) develops a model in which the government cannot distinguish between the return from capital and the return from entrepreneurial labor
- Both studies find that a positive capital income tax becomes optimal

# 3. Heterogeneity

# Incomplete Markets and Heterogeneous Income

- The Atkinson-Stiglitz theorem and Chamley-Judd models assume complete markets and thus perfect insurance of idiosyncratic income shocks
- In reality, markets are incomplete, individuals face borrowing constraints that prevent them from insuring against adverse idiosyncratic shocks
- Aiyagari (1995), Chamley (2001), Domeij and Heathcote (2004), Conesa, Kitao, and Krueger (2009) develop models of incomplete markets
- They find that a positive capital tax is optimal

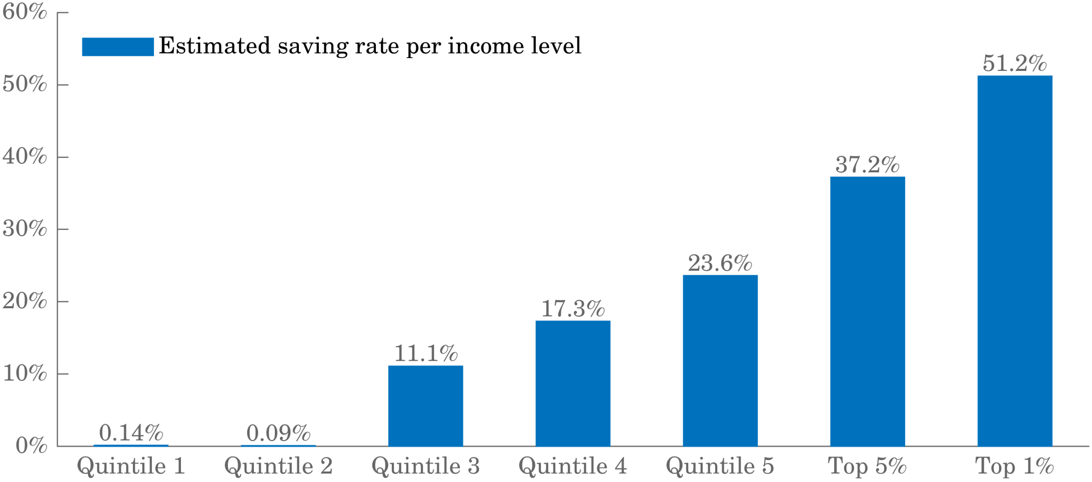
# Precautionary Saving and Capital Overaccumulation

- What is the intuition? When facing incomplete insurance markets, households engage in precautionary savings and accumulate more assets
- The possibility of being borrowing-constrained in some future periods after a sequence of bad shocks leads agents to save more than otherwise
- As a result, the economy accumulates too much capital and the return to capital falls below the rate of time preference, ie $\beta(1 + r) < 1$
- A positive capital income tax discourages agents from saving too much and can therefore solve the capital overaccumulation problem

# Heterogeneous Preferences

- Even with complete markets, there can be heterogeneity in preferences
- Wealthy people display higher saving rates, perhaps because they are more educated, care more about retirement, or simply love accumulating wealth
- Saez (2002), Diamond and Spinnewijn (2011), Golosov et al. (2013) develop models of heterogeneous skills, tastes, or discount factors
- So long as high-income/high-skill households have a higher desire for wealth accumulation, a positive tax on capital income is optimal

# Rich People Love to Save Their Money

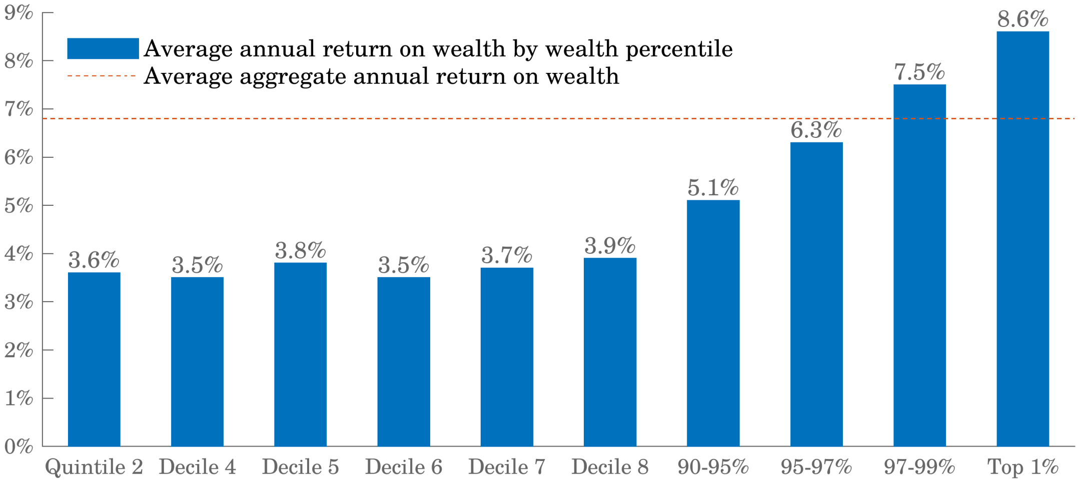



Source: Dynan, Skinner, and Zeldes (2004, *Journal of Political Economy*)

# Heterogeneous Wealth

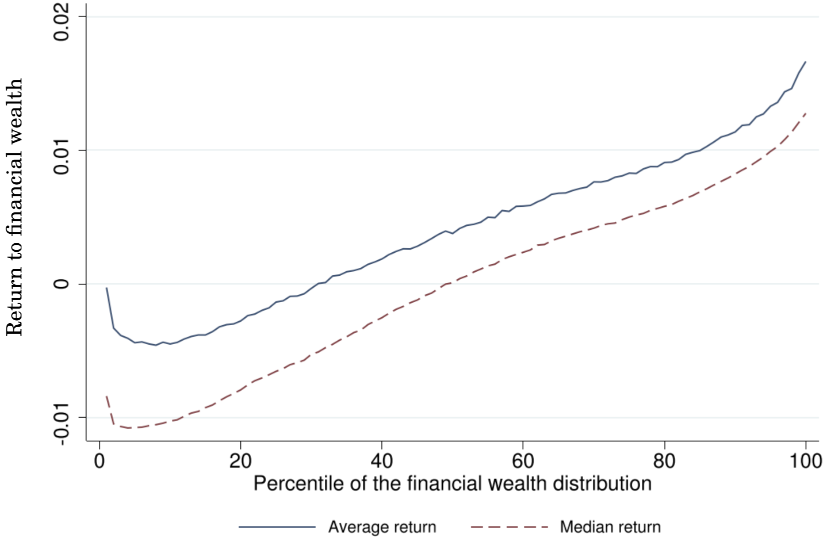
- Even with complete markets, there can be heterogeneity in wealth
- If wealth passes on from one generation to another through bequests, taxing capital income reaches inherited wealth which otherwise escapes taxation
- Cremer et al. (2003) show that a positive capital tax is optimal when inherited wealth is correlated to innate ability
- Also, rich people tend to have higher returns on their savings because they know more, can afford to take on more risk, and can hire financial advisers
- Gerritsen et al. (2020) show that in an model with heterogeneous returns to wealth, taxing capital is optimal: $\tau^k$ increases as returns differ more

# Rich People Have Higher Returns to Wealth




Source: Xavier (2022)

# Rich People Have Higher Returns to Wealth




Source: Fagereng, Guiso, Malacrino, and Pistaferri (2020, *Econometrica*)

# Heterogeneous Skills

- Golosov, Kocherlakota, Tsyvinski (2003), Golosov, Tsyvinski, Werning (2007) and Kocherlakota (2010) develop the so-called "new dynamic public finance"
- Workers have heterogeneous unobservable skills that evolve stochastically
- In choosing its optimal taxation, the government faces a tradeoff

1. On the one hand, the government wants to insure its citizens against the skill risk and thus favors high taxes on income
2. On the other hand, it wants to motivate high-skill workers to produce more income than low-skilled workers and thus favors taxes on income

# Capital Taxes Are Good

- The main finding is that positive capital taxes are optimal
- With uncertain future skills, risk-averse people tend to save too much
- Once they have accumulated wealth, they also want to enjoy more leisure and thus work less, which is bad for the economy
- Taxing capital discourages wealth accumulation and encourages high-skill individuals to work more and produce more, which increases welfare

# 4. Behavioral Biases

# Finite Lives

- The Chamley-Judd result holds for infinitely-lived agents
- But we human beings have finite lives!
- Diamond (1973), Atkinson and Sandmo (1980), Erosa and Gervais (2002) and Abel (2005) develop models in which agents die and leave no bequest
- They find that in these setups, long-run positive capital taxes are optimal
- Intuitively, as people enjoy more leisure late in life, they also consume more, so capital taxation is a way to tax this old-age consumption

# Dynastic Linkages

- One way to interpret infinite lives is to think of family dynasties that link successive generations through bequest and altruism
- In an overlapping generations model with bequest, Atkeson, Chari, and Kehoe (1999) show that the Chamley-Judd result holds
- The intuition is that people save not only for retirement but also for their children, and therefore capital taxation discourages capital accumulation
- This finding requires the assumption that rational intertemporal decision making 1) hold for entire lifetimes and 2) extend across dynasties
- The empirical literature challenges both assumptions

# Behavior

- Behavioral economics show that people's savings decisions are influenced by
  - Psychological elements: self-control, procrastination
  - Information: ambiguity aversion, anecdotal evidence
  - Biases: status quo, default effects
  - Reference dependence: choice bracket, framing effects
- See eg *Nudge* by Sunstein and Thaler (2008), *Thinking Fast and Slow* by Kahneman (2011), *Phishing for Phools* by Akerlof and Shiller (2015)
- Akerlof, Kahneman, Shiller, and Thaler all received the Nobel Prize

# Bequests and Gifts

- Empirical studies on bequests show that a lot is unintended due to lack of annuitization or to love for wealth accumulation per se (eg Hurd 1987)
- The literature usually argues that accidental bequests should be taxed heavily because they do not affect donors (see Cremer and Pestieau 2006)
- Empirical studies on gifts show that 1) people are altruistic; 2) people have warm glow preferences, ie they derive selfish pleasure from “doing good”

# Rational?

- Taken together, these findings suggest that we humans are not rational, at least not in the sense our models assume: we are not *homo economicus*
- Therefore, positive capital taxation may not discourage savings and harm welfare as much as rational-expectations models imply

# 5. Low Intertemporal Substitution

# Revisiting Chamley-Judd

- A recent paper by Straub and Werning (2020, *American Economic Review*) revisits the Chamley-Judd result
- Straub and Werning do not relax any particular assumption
- They show that the result no longer holds when the elasticity of intertemporal substitution is low, that is equal or less than one
- Let's see their argument in the Judd (1985) model

# Households

- There are two types of agents, workers and capitalists
- Workers don't save and supply one unit of inelastic labor
- Capitalists don't work, accumulate capital, and derive all their income from returns to capital, ie they are rentiers

# Preferences

- Workers (lower case) have the following preferences

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

- Capitalists (upper case) have the following preferences

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \qquad \text{with} \quad U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

- $1/\sigma$ is the elasticity of intertemporal substitution

# Technology

- The production function is standard
- Normalize hours $n_t = 1$ and define $f(k) = F(k, 1)$
- The resource constraint in period $t$ is

$$c_t + C_t + g + k_{t+1} \leq f(k_t) + (1 - \delta)k_t$$

# Government

- The government taxes only capital
- It spends on expenditures $g \geq 0$ and transfers $s_t$ to workers
- It cannot issue debt or save, ie it runs a balanced budget

$$b_t = 0 \quad \text{for all } t$$

# Government Budget Constraint

- The government budget constraint is

$$g + s_t = \tau_t^k r_t k_t$$

- As before we define the after tax return on capital as

$$\tilde{r}_t = (1 - \tau_t^k) r_t$$

# Hand-to-Mouth Workers

- Workers' budget constraint is

$$c_t = w_t + s_t$$

- Workers live hand to mouth: consumption equals disposable income
- There is no maximization choice for the workers: labor supply is fixed, workers don't save, and wages and transfers are beyond their control
- Workers simply eat what they earn, hence the hand-to-mouth term, also known as rule-of-thumb consumers

# Capitalists' Problem

- Capitalists solve

$$\max_{C_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{subject to} \quad C_t + k_{t+1} = \tilde{r}_t k_t + (1-\delta) k_t$$

# First-Order Condition

- The first-order condition is

$$U'(C_t) = \beta U'(C_{t+1})(\tilde{r}_{t+1} + 1 - \delta)$$

- Express the FOC one period earlier and plug in the budget constraint to eliminate $\tilde{r}_t$

$$U'(C_{t-1})k_t = \beta U'(C_t)(C_t + k_{t+1})$$

# Ramsey Problem

- We are ready to set up the Ramsey problem
- We again assume the government only cares about the workers, $\gamma = 0$
- So we maximize the utility of workers subject to the resource constraint and the capitalists' FOC, which includes their budget constraint

# Ramsey Problem

- The Ramsey problem writes

$$\max_{\{c_t, C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [u(c_t) + 0 \times U(C_t)]$$

subject to

$$\begin{aligned} c_t + C_t + g + k_{t+1} &= f(k_t) + (1-\delta)k_t \\ U'(C_{t-1})k_t &= \beta U'(C_t)(C_t + k_{t+1}) \end{aligned}$$

# Lagrangian

- The Lagrangian writes

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + \theta_t [f(k_t) + (1-\delta)k_t - c_t - C_t - g - k_{t+1}] \\ + \mu_t [\beta U'(C_t)(C_t + k_{t+1}) - U'(C_{t-1})k_t] \Big\}$$

- $\theta_t$ is the multiplier on the resource constraint
- $\mu_t$ is the multiplier on the capitalists' FOC + budget constraint

# First-Order Conditions

- The first-order conditions are

$$
\begin{aligned}
c_t:&\quad u'(c_t) = \theta_t \\
C_t:&\quad \theta_t = \beta U''(C_t)[\mu_t(C_t + k_{t+1}) - \mu_{t+1}k_{t+1}] + \mu_t \beta U'(C_t) \\
k_{t+1}:&\quad \theta_t = \beta\theta_{t+1}[f'(k_{t+1}) + 1 - \delta] - \beta U'(C_t)[\mu_{t+1} - \mu_t] \\
C_{-1}:&\quad \mu_0 = 0
\end{aligned}
$$

# Rewriting the First-Order Conditions

- Recall, $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, so $U'(C_t) = C_t^{-\sigma}$ and $U''(C_t) = -\sigma C_t^{-\sigma-1}$
- Rewrite the FOCs by substituting out $\theta_t$

$$\mu_{t+1} = \mu_t \left( \frac{\sigma - 1}{\sigma} \frac{C_t}{k_{t+1}} + 1 \right) + \frac{1}{\beta\sigma} \frac{u'(c_t)}{U'(C_t)} \frac{C_t}{k_{t+1}} \tag{1}$$

$$\frac{u'(c_{t+1})}{u'(c_t)} [f'(k_t) + 1 - \delta] = \frac{1}{\beta} + \frac{U'(C_t)}{u'(c_t)} [\mu_{t+1} - \mu_t] \tag{2}$$

# The Classic Steady-State Result

- Let's derive the Chamley-Judd result
- Suppose quantities and multipliers converge to an interior steady state, ie $c_t$, $C_t$, $k_{t+1}$, and $\mu_t$ become constant after some time
- Equation (2) implies that

$$f'(k) + 1 - \delta = \frac{1}{\beta} \iff r = \frac{1}{\beta} - 1 + \delta$$

# The Classic Steady-State Result

- The first-order condition of the capitalists implies

$$\tilde{r} = \frac{1}{\beta} - 1 + \delta$$

- Combining the previous two equations we get

$$r = \tilde{r} = (1 - \tau^k)r \quad \textbf{or} \quad \tau^k = 0$$

# Exploding Multiplier

- The classic Chamley-Judd result holds if we suppose that quantities and multipliers converge
- But do they? Take equation (2) in steady state

$$\mu_{t+1} = \mu_t \left( \frac{\sigma - 1}{\sigma} \frac{C}{k} + 1 \right) + \text{constant}$$

- If $\sigma < 1$, $\mu_t$ does indeed converge: the result holds
- If $\sigma = 1$, $\mu_t$ explodes: $\mu_{t+1} = \mu_t + \text{constant}$
- If $\sigma > 1$, $\mu_t$ explodes even faster

# Knife Edge

- For any $\sigma \geq 1$, the multiplier goes to infinity
- It turns out we can circumvent the problem with the particular case $\sigma = 1$
- Before we do this, let's review a bit the role of $\sigma$ in the model

# Euler Equation

- Take the capitalists' FOC, $\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(\tilde{r}_{t+1} + 1 - \delta)$, and express it in logs

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = \frac{1}{\sigma}\ln\beta + \frac{1}{\sigma}\ln(\tilde{r}_{t+1} + 1 - \delta)$$

- Compute how much the growth rate of consumption varies when the real interest rate changes

$$\frac{d\ln\left(C_{t+1}/C_t\right)}{d\ln(\tilde{r}_{t+1} + 1 - \delta)} = \frac{1}{\sigma}$$

- This is exactly the elasticity of intertemporal substitution (EIS)

# Elasticity of Intertemporal Substitution

- The formal definition of the intertemporal elasticity of substitution is

$$\text{EIS} = -\frac{d\ln\left(C_{t+1}/C_t\right)}{d\ln\left[U'(C_{t+1})/U'(C_t)\right]} = -\frac{d\ln\left(C_{t+1}/C_t\right)}{d\left[-\sigma\ln\left(C_{t+1}/C_t\right)\right]} = \frac{1}{\sigma}$$

- It measures the degree of consumption smoothing, ie how the growth rate of consumption changes when the growth rate of marginal utility changes
- It also measures the responsiveness of the growth rate of consumption to a change in the real interest rate, via the Euler equation (previous slide)

# Two Effects

- What happens when the interest rate increases by 1 percent?
- There are two effects at work

1. Income effect: capitalists feel richer, they consume more and save less
2. Substitution effect: capital becomes more attractive, capitalists save more and consume less

# Income vs Substitution

1. When $\sigma < 1$, the EIS $> 1$, the substitution effect dominates
   - Capitalists save a larger share of their income
   - The growth rate of consumption increases by more than 1 percent
2. When $\sigma > 1$, the EIS $< 1$, the income effect dominates
   - Capitalists increase consumption, the saving rate falls
   - The growth rate of consumption increases by less than 1 percent
3. When $\sigma = 1$, the EIS $= 1$, the two effects cancel out
   - The consumption-savings share does not change
   - The growth rate of consumption increases by exactly 1 percent

# Constant Saving Rate

- With log utility, $\sigma = 1$, capitalists have a constant saving rate $s$
- Guess that $k_{t+1} = s(\tilde{r}_t + 1 - \delta)k_t$, use the budget constraint of capitalists to obtain $C_t = (1-s)(\tilde{r}_t + 1 - \delta)k_t$, and plug that into the FOC of capitalists

$$\frac{C_t}{C_{t-1}} = \beta(\tilde{r}_t + 1 - \delta) \implies \frac{(1-s)(\tilde{r}_t + 1 - \delta)\overbrace{s(\tilde{r}_{t-1} + 1 - \delta)k_{t-1}}^{k_t}}{(1-s)(\tilde{r}_{t-1} + 1 - \delta)k_{t-1}} = \beta(\tilde{r}_t + 1 - \delta) \implies s = \beta$$

- We confirm that with log utility the saving rate $s$ is constant and equal to $\beta$

$$k_{t+1} = \underbrace{\beta}_{\text{Saving rate}} \times \underbrace{(\tilde{r}_t + 1 - \delta)k_t}_{\text{Income of capitalists}} \quad \text{and} \quad C_t = \underbrace{(1-\beta)}_{\text{Consumption rate}} \times \underbrace{(\tilde{r}_t + 1 - \delta)k_t}_{\text{Income of capitalists}}$$

# Ramsey Problem

- With $\sigma = 1$, the Ramsey problem becomes

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{subject to} \quad c_t + \frac{1}{\beta} k_{t+1} + g = f(k_t) + (1-\delta)k_t$$

- This amounts to an optimal neoclassical growth problem
- The only difference is the price of capital which is equal to $1/\beta > 1$ instead of $1$ in the standard case

# Convergence

- The Ramsey problem is equivalent to a standard neoclassical growth model solved by the central planner
- We know that in this framework there exists a unique, globally stable, interior steady state
- Thus with log utility all quantities and prices converge

# Unit Elasticity of Intertemporal Substitution

- Combine (1), (2), and the capitalists' budget constraint

$$r = \frac{1}{\beta} - (1-\delta) + \frac{1}{\beta}[(1-\tau^k)r - \delta]$$

- The first-order condition of the capitalists still implies

$$\tilde{r} = (1-\tau^k)r = \frac{1}{\beta} - (1-\delta)$$

- Combine these two equations

$$\tau^k = \frac{1-\beta}{1-\beta^2(1-\delta)}$$

# Taxing Capital Is Optimal

- Since $\beta < 1$ and $\delta < 1$, we have $\tau^k > 0$
- The optimal long-run tax on capital is positive

Proposition: If $\sigma = 1$ and $\gamma = 0$, the solution to the planning problem converges monotonically to a unique steady state with a positive tax on capital given by

$$\tau^k = \frac{1-\beta}{1-\beta^2(1-\delta)} > 0$$

# Low Elasticity of Substitution

- Straub and Werning also study the case of $\sigma > 1$, ie the EIS lower than one
- They find that the multiplier $\mu$ converges to a negative value
- But $\mu_0 = 0$ (FOC for $C_{-1}$) and $\mu_t \geq 0$ for all $t = 0, 1, \ldots$
- This contradicts $\mu_t \to \mu < 0$

# Positive Capital Tax

Proposition: If $\sigma > 1$ and $\gamma = 0$, no solution to the Ramsey problem converges to the zero-tax steady state, or any other interior steady state

Proposition: If $\sigma > 1$ and $\gamma = 0$, any solution to the planning problem converges to $c_t \to 0$ and a positive capital tax $\tau^k > 0$

# Limit on Taxes

- Let's assume there is an upper bound on the capital tax
  - Evasion: above a certain level citizens don't pay
  - Political economy: the tax is not implementable, ie will not be voted
- If the constraint is binding, Straub and Werning (2020) show that the optimal tax converges to positive consumption $c$ and positive tax $\tau^k$

# Intuition

- What is the intuition behind this result?
- Start with a constant tax on capital
- Suppose the government announces higher future taxes
- How do capitalists react today?

# Two Effects

- The two same effects are at work

1. Income effect: capitalists feel poorer, they consume less today to match the drop in consumption tomorrow
2. Substitution effect: capital becomes less attractive, capitalists save less and consume more

# Income vs Substitution Effects

1. When $\sigma < 1$, the EIS $> 1$, the substitution effect dominates
   - Capitalists increase consumption and save less
2. When $\sigma > 1$, the EIS $< 1$, the income effect dominates
   - Capitalists decrease consumption and save more
   - Capital rises in the short run and falls in the long run
3. When $\sigma = 1$, the EIS $= 1$, the two effects cancel out
   - Current consumption and savings are unaffected

# Workers Save Indirectly Through Capitalists

- The government cares only about the workers
- For workers, lowering capitalists' consumption is good
- Why? Capitalists save more, capital accumulates faster, wages increase and workers are able to consume more

# Optimal Policy Changes Depending on $\sigma$

1. When $\sigma < 1$ the government induces capitalists to save more by promising lower tax rates in the future: the path for taxes is declining until $\tau^k = 0$
2. When $\sigma > 1$ the government induces capitalists to save more by promising higher taxes in the future: the path for taxes is increasing until $\tau^k = 1$
3. When $\sigma = 1$ the incentives are absent: taxes converge to a constant $0 < \tau^k < 1$

# Conclusion

- The Chamley-Judd result only holds when $\sigma < 1$
- That is, when the intertemporal elasticity of substitution is high, when the substitution effect dominates the income effect
- When $\sigma \geq 1$ taxing capital in the long run is optimal
- So what does the data say about $\sigma$?

# Measuring the EIS

- Typically we estimate
$$\Delta C_t = \alpha_i + \frac{1}{\sigma} R_{i,t} + \varepsilon_{i,t}$$
- $R_{i,t}$ is the gross return on asset $i$

# Which Value for the Elasticity of Substitution?

- Estimates vary considerably
  - Halls (1988) finds $1/\sigma$ close to 0 using macro data
  - Blundell et al. (1994) find close to 1 using micro data
- Havranek et al. (2015) provide a meta analysis
  - 169 studies, 2735 estimates, 104 countries
  - Report a mean elasticity $1/\sigma$ of 0.5
- So in most cases, the EIS is low, substitution effects are weak, ie $\sigma > 1$

# Failure of the Literature

- Why has this simple finding been missed by the literature?
- Lansing (1999) and Reinhorn (2002) had shown it for $\sigma = 1$ but not $\sigma > 1$
- Straub and Werning show that the twist is about the convergence of the endogenous Lagrange multipliers

# An Assumed Result

"[Previous papers] assume that the endogenous multipliers associated with the planning problem converge. Although this seems natural, we have shown that this is not necessarily true at the optimum. In fact, on closer examination it is evident that presuming the convergence of multipliers is equivalent to the assumption that the intertemporal rates of substitution of the planner and the agent are equal. This then implies that no intertemporal distortion or tax is required. Consequently, analyses based on these assumptions amount to little more than assuming zero long-run taxes."

Ludwig Straub and Iván Werning, 2020, *American Economic Review*

# 6. Incomplete Taxation

# Incomplete Taxation

- The result that the limiting capital tax should be zero hinges on a complete set of linear taxes
- Let's see the consequences of incomplete taxation
- This was first done by Correia (1996, *Journal of Public Economics*)

# Fixed Factor

- We assume there is an additional factor in production $z_t$
- The factor is in fixed supply, $z_t = Z$
- The factor cannot be taxed, $\tau_t^z = 0$
- The new production function $F(k_t, n_t, z_t)$ exhibits constant returns to scale in all of its inputs

# Interpretation

- The factor $z_t$ might represent different things
  - Public goods
  - Inframarginal profits from decreasing returns to scale
  - Returns to monopolistic rents
  - Positive or negative productivity spillovers
  - Labor or capital of specific types
  - Goods devoted to human-capital accumulation

# Modifications

- Firm profit maximization implies that the rental price of $z_t$ is equal to its marginal product
$$p_t^z = F_z(t)$$
- There is a new stream of revenues in the household's present-value budget constraint

$$\sum_{t=0}^{\infty} q_t^0 c_t = \sum_{t=0}^{\infty} q_t^0 (1-\tau_t^n) w_t n_t + [(1-\tau_0^k) r_0 + 1 - \delta] k_0 + b_0 + \sum_{t=0}^{\infty} q_t^0 p_t^z Z$$

# Ramsey Plan

- Let's solve the Ramsey problem with the primal approach
- Step 1, deriving the FOCs, yields one more FOC: $p_t^z = F_z(t)$
- Step 2, substituting the FOCs into the time 0 budget constraint, yields the following implementability condition

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)(c_t - F_z(t)Z) - u_\ell(t)n_t] - A = 0$$

where $A = u_c(0)\{[(1-\tau_0^k)F_k(0) + 1 - \delta]k_0 + b_0\}$ is the same as in the standard case with two factors

# Ramsey Plan

- In Step 3 we formulate

$$V(c_t, n_t, k_t, \Phi) = u(c_t, 1 - n_t) + \Phi[u_c(t)(c_t - F_z(t)Z) - u_\ell(t)n_t]$$

- $k_t$ now enters as an argument in $V$ because of the presence of the marginal product of factor $Z$

# Ramsey Plan

- Except for changes on $F$ and $V$ the Lagrangian is the same

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ V(c_t, n_t, k_t, \Phi) + \theta_t [F(k_t, n_t, Z) + (1-\delta)k_t - c_t - g_t - k_{t+1}] \} - \Phi A$$

- The first-order condition with respect to $k_{t+1}$ is

$$\theta_t = \beta V_k(t+1) + \beta \theta_{t+1} [F_k(t+1) + 1 - \delta]$$

# Steady State

- In the steady state the previous equation becomes

$$1 = \beta(F_k + 1 - \delta) + \beta\frac{V_k}{\theta}$$

- The no-arbitrage condition is the same

$$1 = \beta[(1 - \tau^k)F_k + 1 - \delta]$$

- Combine the two

$$\tau^k = -\frac{V_k}{\theta F_k} = \frac{\Phi u_c Z}{\theta F_k} F_{zk}$$

# Nonzero Capital Tax

- $u_c > 0, Z > 0, \theta > 0, F_k > 0$, and with distorting tax $\Phi > 0$
- Thus the sign of $\tau^k$ depends on the sign of $F_{zk}$
  - If $k$ and $Z$ are complements, $F_{zk} > 0$, $\tau^k$ is positive
  - If $k$ and $Z$ are substitutes, $F_{zk} < 0$, $\tau^k$ is negative
- Conclusion: when some factors cannot be taxed, the optimality of the zero tax on capital income disappears

# Intuition

- What's the intuition? Remember in steady state

$$\tilde{r} = (1 - \tau^k)r = \frac{1}{\beta} - (1 - \delta)$$

- The after-tax real interest rate $\tilde{r}$ depends only on $\beta$ and $\delta$
- Thus the tax policy cannot affect the steady-state after-tax real interest rate, it can only after the pre-tax rate $r$

# Intertemporal and Intratemporal Margins

- It follows that the growth rate of consumption and the intertemporal marginal rate of substitution are also independent of the tax policy
- It is when the tax policy cannot affect the intertemporal margin that it is optimal to set the capital tax rate to zero
- So the optimal zero capital tax must be related to minimizing distortions at the intratemporal margins

# Intuition

- At the intratemporal margin taxing capital is more distortionary than taxing other factors such as labor
- So the burden of taxation shifts to these factors
- But when these factors cannot be taxed directly, it becomes optimal to tax capital

# Summary

- When there are two factors that can be taxed, say capital and labor, it is efficient to tax the less distorting one
  - Tax labor (more inelastic), don't tax capital (elastic)
- When there are more than two factors and one or more cannot be taxed, taxing taxable factors becomes efficient
  - Tax capital to accommodate the missing market

# 7. Commitment

# Reoptimizing

- So far we have imposed through the Ramsey problem that the government optimize only in the initial period $t = 0$
- Now suppose the government can reoptimize in $t = s$
- Since $k_s$ is fixed, the government wants to set a high $\tau_s^k$, just like $\tau_0^k$
- Again this is like a lump-sum tax, ie non distortionary

# Time Inconsistency

- If the government could reoptimize, it would change its initial plan
- The Ramsey problem does not satisfy Bellman's optimality principle
- This is a classic problem of time inconsistency, or dynamic inconsistency
- See the classic paper by Kydland and Prescott (1977) for an application of time inconsistency to monetary policy

# Commitment

- The result of zero capital tax in the long run rests on the crucial assumption of commitment
- The government promises to stick with its policy forever, never reoptimize
- The government is credible, agents believe in it

# Can We Commit?

- Commitment is a strong assumption
- In practice policymakers renege on their promises
- Government change existing plans, new governments are elected
- Surveys show that people don't fully trust their government

# Recursive Ramsey Problem

- One can study optimal taxation without commitment
- Express the Ramsey problem recursively and add an appropriate "forward-looking" state variable such that re-optimizing is costly
- If the government is not credible, rational agents don't believe the announced policy and respond differently to it
- They forecast the future optimal response of the government before making their choice, resulting in a complicated game

# Optimal Policy Without Commitment

- Chari and Kehoe (1990) and Stockey (1991) study limited commitment
- Farhi et al. (2012) study optimal policy in a model without commitment and find that optimal taxes on capital are positive and progressive

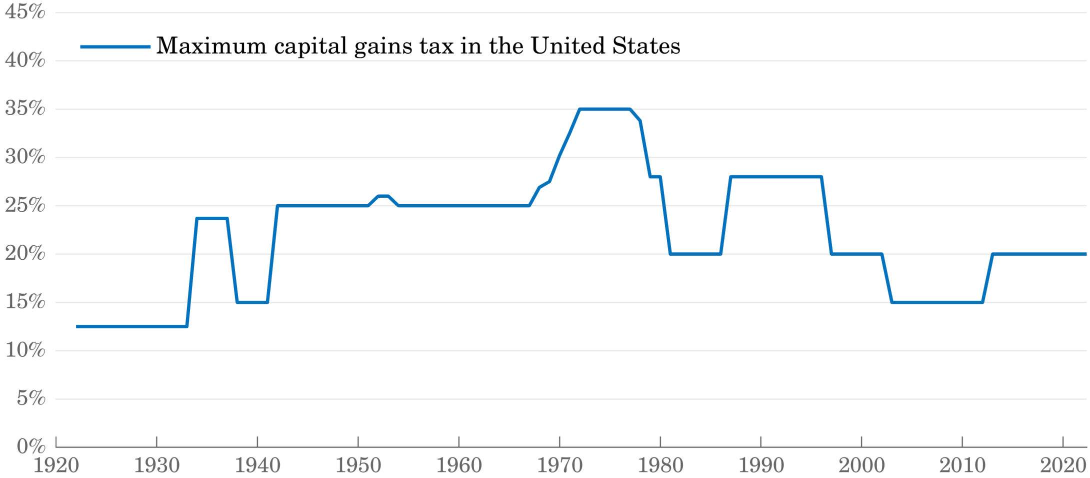
# 8. Taxes in Practice

# Actual Taxation

- There are five main types of taxes on capital

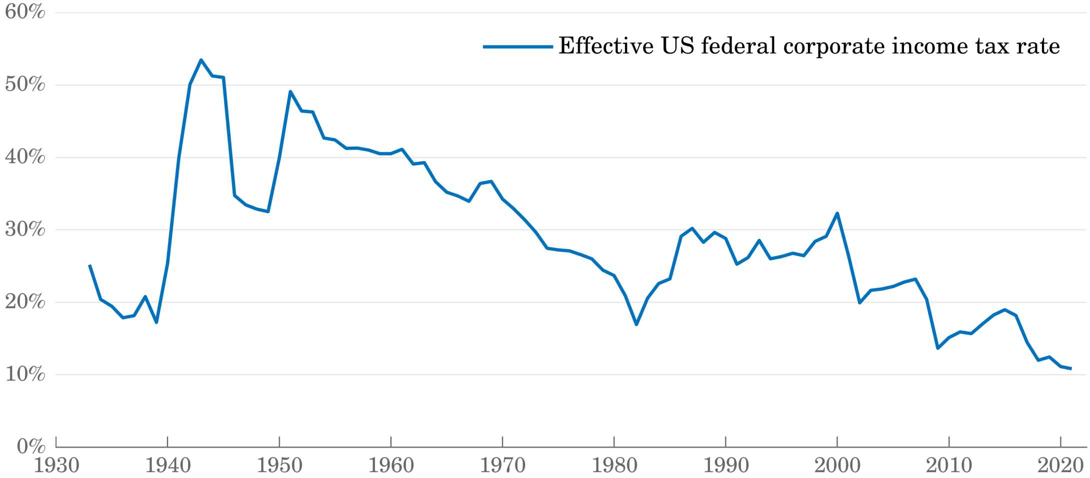
1. Personal capital income tax
2. Corporate income tax
3. Property tax
4. Inheritance tax
5. Wealth tax

# Personal Capital Income Tax in the United States




Source: Tax Foundation

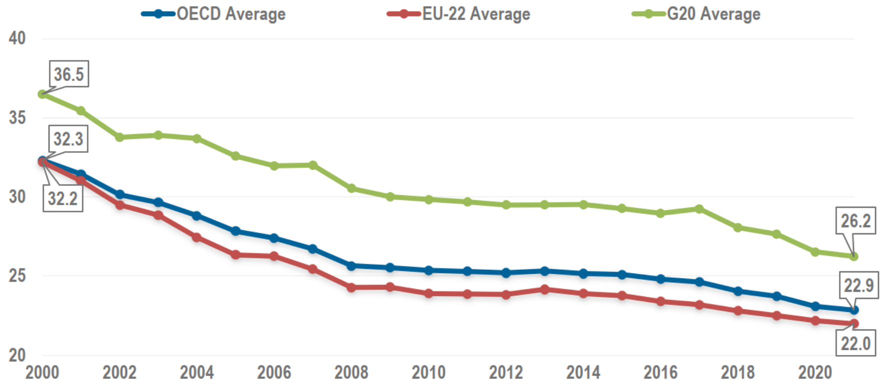
# Corporate Tax in the United States




Source: US Bureau of Economic Analysis; Effective rate = federal corporate income taxes paid / corporate profit

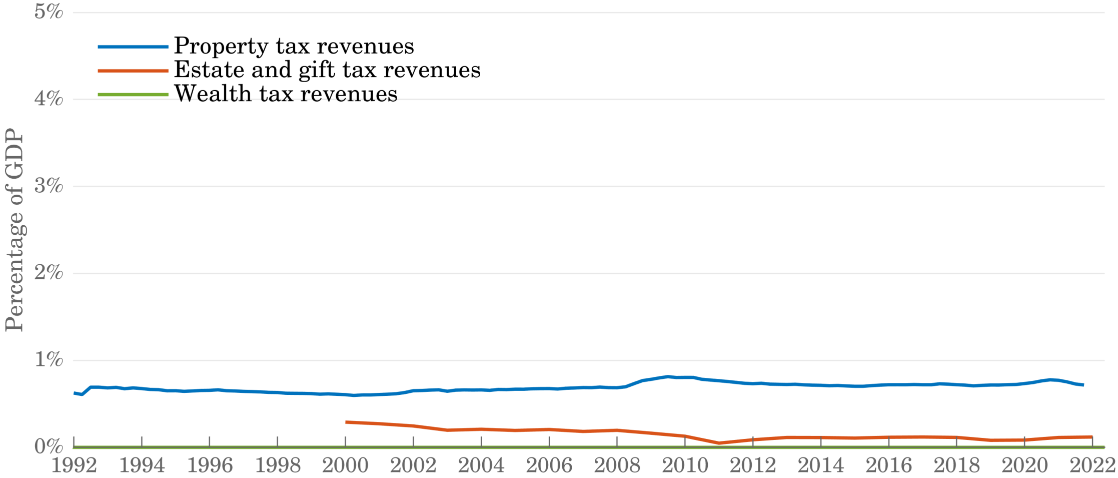
# Corporate Tax in OECD Countries

## Combined statutory CIT rates (%), 2000-2021




*Note:* The averages are unweighted averages. The EU-22 average includes all EU countries that are members of the OECD. The G20 average includes all G20 countries, excluding the EU.
*Source:* OECD (2021), OECD Tax Database. https://oe.cd/tax-database.

# Other Taxes on the Capital Stock in the United States




Sources: US Census Bureau, Congressional Budget Office, US Bureau of Economic Analysis

# Actual Taxation

- Capital income tax rates have been decreasing over the past decades
- They are now significantly lower than taxes on labor in most countries
- Taxes on stocks (inheritance, property, wealth) raise very little revenue
- Is it the time for an international wealth tax?

# 9. Conclusion

# Conclusion

- The zero capital-income-tax result has been attacked on many fronts
- It does not hold when
  - The elasticity of substitution is low
  - People face uninsurable idiosyncratic shocks
  - People have different abilities, preferences, returns to wealth
  - One factor in production cannot be taxed
  - The government cannot commit or is not credible
  - Taxes are nonlinear

# 10. Exercises

# Exercise 1 – Consumption Taxes

Consider the model in the course, but instead of labor and capital taxation assume that the government sets labor and consumption taxes, $\{\tau_t^n, \tau_t^c\}$. Thus, the household's present-value budget constraint is

$$\sum_{t=0}^{\infty} q_t^0(1+\tau_t^c)c_t = \sum_{t=0}^{\infty} q_t^0(1-\tau_t^n)w_t n_t + [r_0 + 1 - \delta]k_0 + b_0$$

1. Solve for the Ramsey plan.
2. Suppose that the solution to the Ramsey problem converges to a steady state. Characterize the optimal limiting sequence of consumption taxes.
3. In the case of capital taxation, we imposed an exogenous upper bound on $\tau_0^k$. Explain why a similar exogenous restriction on $\tau_0^c$ is needed to ensure an interesting Ramsey problem. *Hint:* explore the implications of setting $\tau_t^c = \tau^c$ and $\tau_t^n = -\tau^c$ for all $t \geq 0$, where $\tau^c$ is positive and large.

# Exercise 2 – Specific Utility Function

Chamley (1986). Consider the model in the course, and assume the period utility function is given by

$$u_t(c_t, \ell_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + v(\ell_t)$$

where $\sigma > 0$. When $\sigma = 1$, $c_t^{1-\sigma}/(1-\sigma)$ is replaced by $\ln c_t$.

Show that the optimal policy is to set capital taxes equal to zero in period $2$ and from there on, ie $\tau_t^k = 0$ for $t \geq 2$. *Hint:* given the preference specification, evaluate and compare the no-arbitrage condition, $\frac{q_t^0}{q_{t+1}^0} = (1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta$, and the dynamic FOC, $V_c(t) = \beta V_c(t+1)[F_k(t+1) + 1 - \delta]$.