11. Fiscal Policy in the Growth Model

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Introduction

- Today we return to the deterministic neoclassical growth model
- We introduce fiscal policy in the form of government spending and taxes
- The goal is to understand how government policies affect the behavior of private agents – households and firms – and aggregate economic outcomes

In a Nutshell

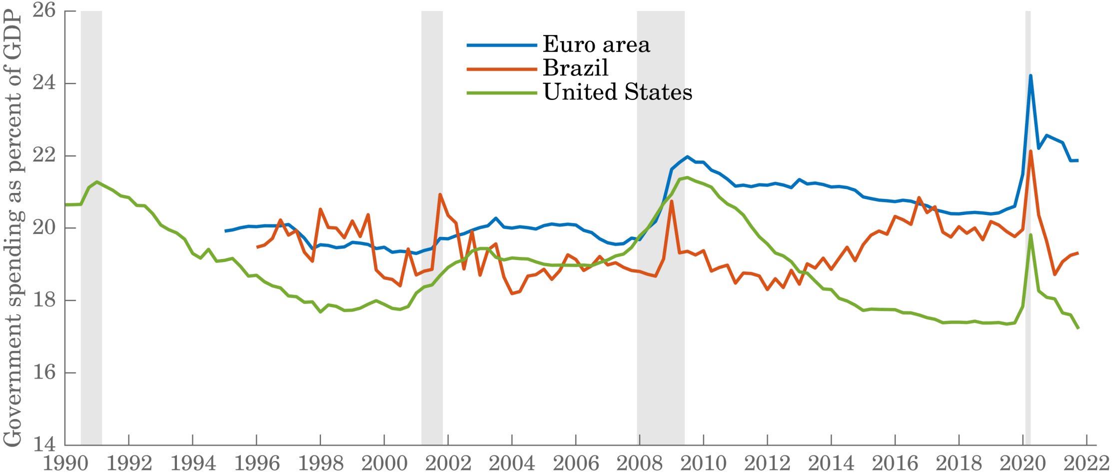
- In the basic neoclassical growth model, the resource constraint is

$$Y_t = C_t + I_t$$

- We now add an important component of GDP

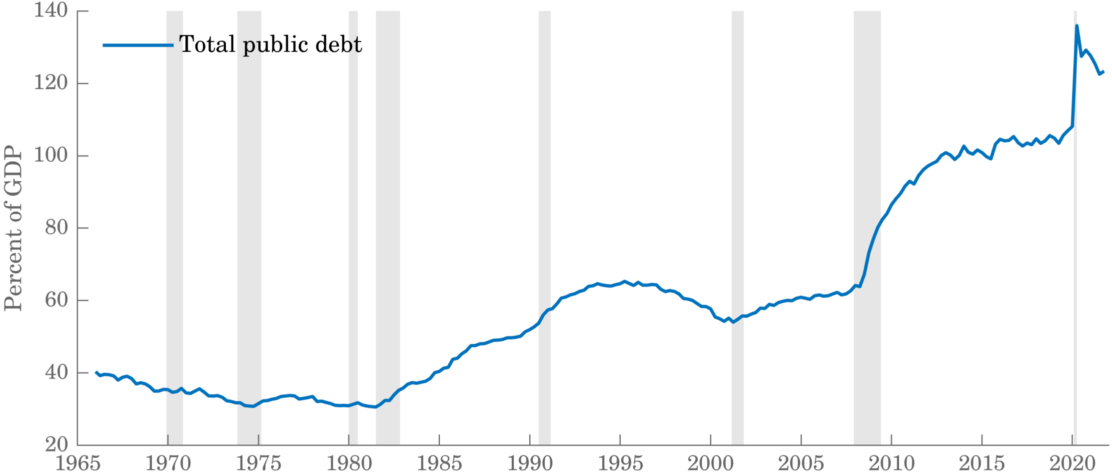
$$Y_t = G_t + C_t + I_t$$

Government Spending Share of Output



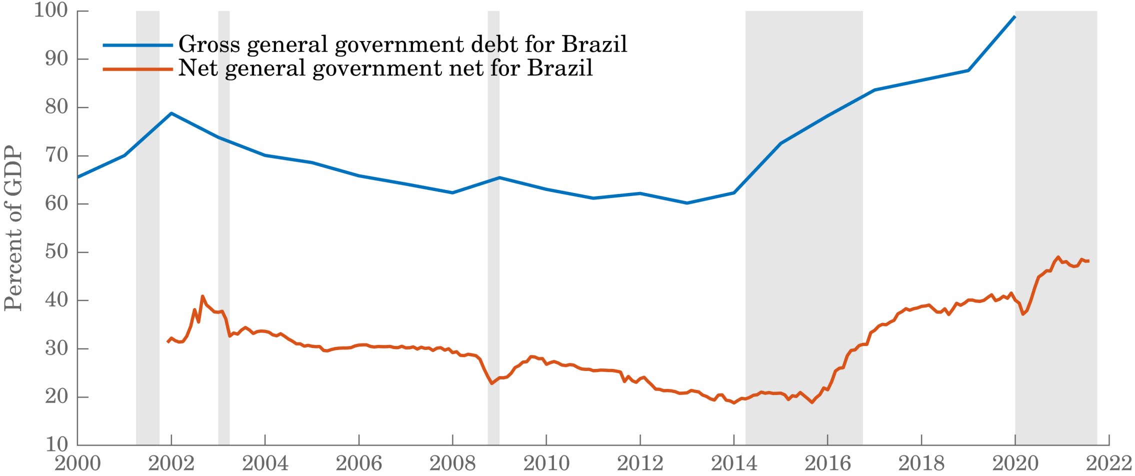
Sources: Organization for Economic Co-operation and Development and US Bureau of Economic Analysis

Public Debt in the United States



Sources: US Office of Management and Budget and US Bureau of Economic Analysis

Public Debt in Brazil



Sources: International Monetary Fund and Central Bank of Brazil

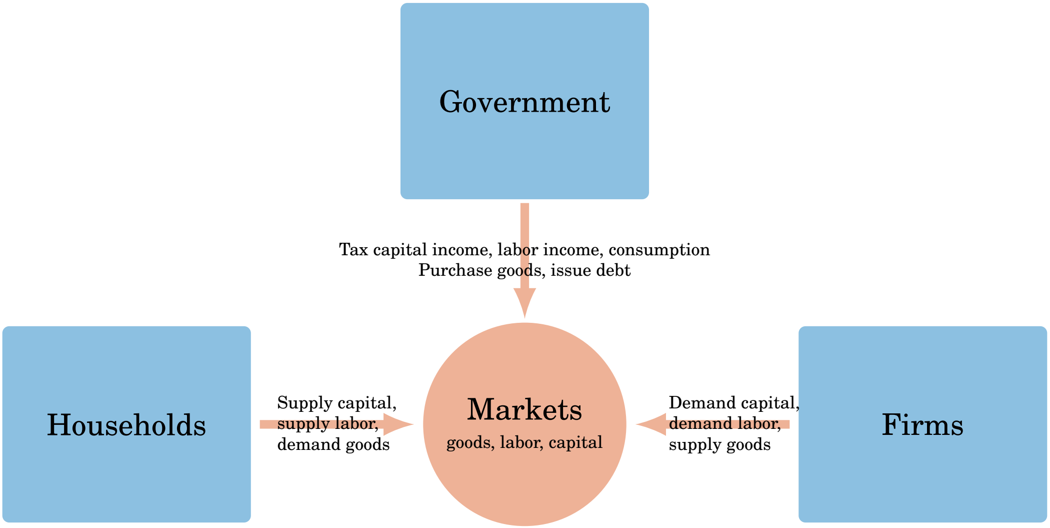
Lecture Outline

1. Model Setup
2. Sequential Budget Constraint
3. Term Structure of Interest Rates
4. Competitive Equilibrium
5. Ricardian Equivalence
6. Inelastic Labor Supply
7. Steady State
8. Equilibrium Path
9. Fiscal Policy Experiments
10. Exercises

Main Reference: Ljungqvist and Sargent, 2018, *Recursive Macroeconomic Theory*, Fourth Edition, Chapter 11

1. Model Setup

Model Diagram



Preferences

- There is no uncertainty, agents have perfect foresight
- Time is normalized to unity, $\ell_t + n_t = 1$
- A representative household enjoys consumption and leisure

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t)$$

- U is strictly increasing in consumption c_t and leisure $1 - n_t$, is twice continuously differentiable, strictly concave, satisfies the Inada conditions

Resource Constraint

- The aggregate resource constraint is

$$g_t + c_t + i_t \leq F(k_t, n_t)$$

- All produced goods in the economy are either consumed by the government g_t, consumed by households c_t, or saved by households i_t
- The production function $F(k, n)$ is linearly homogeneous, has positive and decreasing marginal products, and satisfies the Inada conditions

Capital Accumulation

- Capital evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t$$

- Plug that into the resource constraint

$$g_t + c_t + k_{t+1} \leq F(k_t, n_t) + (1 - \delta)k_t$$

Time Zero Trading

- Let's consider a competitive equilibrium with all trades occurring at time 0
- The representative household owns the capital stock, makes investment decisions, rents out capital and labor to firms, and buys goods from firms
- The representative firm finances the rental of capital and labor with sales of output goods it produces using the production function $F(k_t, n_t)$
- Even though there is no uncertainty, the household and the firm meet in time 0 and trade securities for every period from $t = 0$ to the infinite future

Price System

- A price system is a triple of sequences $\{q_t, r_t, w_t\}_{t=0}^{\infty}$
  - q_t is the time 0 pretax price of one unit of good at time t
  - r_t is the pretax rental rate of capital at time t
  - w_t is the pretax wage at time t
- q_t is expressed in terms of time 0 goods, w_t and r_t are expressed in terms of time t goods

Government

- Government behavior is exogenous
- The government finances a sequence of goods purchases $\{g_t\}_{t=0}^{\infty}$ from firms with a sequence of taxes $\{\tau_t^c, \tau_t^k, \tau_t^n, \tau_t\}_{t=0}^{\infty}$ on households
    - τ_t^c is tax rate on consumption
    - τ_t^k is tax rate on capital income
    - τ_t^n is tax rate on labor income
    - τ_t is a lump-sum tax

Lump-Sum Tax

- A lump-sum tax is a tax based on a fixed amount; eg a flat fee for all workers in the township (a head tax); or a flat fee to register a vote (a poll tax)
- It is independent of the taxpayers' actions who pay it no matter what
- Since agents cannot affect the amount of the lump-sum tax by changing their behavior, the tax implies no distortion in choice
- Lump-sum taxes play a special role in macroeconomic theory

Government Budget Constraint

- The government faces the following time 0 budget constraint

$$\sum_{t=0}^{\infty} q_t g_t \leq \sum_{t=0}^{\infty} q_t \left[\tau_t^c c_t + \tau_t^k (r_t - \delta) k_t + \tau_t^n w_t n_t + \tau_t \right] \tag{1}$$

- The lifetime market value of purchases cannot exceed that of tax revenues
- A government expenditure and tax plan is budget feasible if it satisfies the government budget constraint

Household Budget Constraint

- The household faces the time 0 budget constraint

$$\sum_{t=0}^{\infty} q_t \left\{(1+\tau_t^c)c_t + [k_{t+1} - (1-\delta)k_t]\right\} \leq \sum_{t=0}^{\infty} q_t \left\{r_t k_t - \tau_t^k (r_t - \delta)k_t + (1-\tau_t^n)w_t n_t - \tau_t\right\}$$

- Notice the positive sign of τ_t^c and negative signs of τ_t^k τ_t^n, and τ_t
- The government gives a depreciation allowance δk_t from the gross rentals on capital $r_t k_t$ and so collects $\tau_t^k (r_t - \delta)k_t$ on rentals from capital

2. Sequential Version of Government Budget Constraint

From Time Zero to Sequential Trading

- We have used the time 0 trading abstraction seen in lecture 6
- Sequential trading of one-period risk-free debt can also support the equilibrium allocations we will study today
- Let's describe the sequence of one-period government debt that is implicit in the equilibrium tax policies here

Government Budget Constraint

- Suppose the government enters period 0 with no government debt
- Define total tax collections as

$$T_t \equiv \tau_t^c c_t + \tau_t^k (r_t - \delta) k_t + \tau_t^n w_t n_t + \tau_t$$

- The government budget constraint (1) becomes

$$\sum_{t=0}^{\infty} q_t (g_t - T_t) = 0$$

Deficit Today Is Surplus Tomorrow

- The previous equation can be written as

$$q_0(g_0 - T_0) + \sum_{t=1}^{\infty} q_t(g_t - T_t) = 0$$

$$\textbf{or} \quad g_0 - T_0 = \sum_{t=1}^{\infty} \frac{q_t}{q_0}(T_t - g_t)$$

- The government deficit at time 0, $g_0 - T_0$, equals the present value of future government surpluses, $\sum_{t=1}^{\infty} \frac{q_t}{q_0}(T_t - g_t)$

Introducing Debt

- In the previous equation, $B_0 \equiv \sum_{t=1}^{\infty} \frac{q_t}{q_0}(T_t - g_t)$ is the value of government debt issued at time 0, denominated in units of time 0 goods

$$g_0 - T_0 = B_0 \quad \text{where } B_0 \equiv \sum_{t=1}^{\infty} \frac{q_t}{q_0}(T_t - g_t)$$

- Rewrite the value of debt at time 0

$$B_0 = \frac{q_1}{q_0}(T_1 - g_1) + \sum_{t=2}^{\infty} \frac{q_t}{q_0}(T_t - g_t)$$

- Multiply by q_0/q_1 on both sides

$$B_0 \frac{q_0}{q_1} = T_1 - g_1 + \sum_{t=2}^{\infty} \frac{q_t}{q_1}(T_t - g_t)$$

Real Interest Rate

- Define $R_{0,1} \equiv \frac{q_0}{q_1}$ as the gross one-period real interest rate

$$B_0 R_{0,1} = T_1 - g_1 + B_1 \quad \text{where } B_1 = \sum_{t=2}^{\infty} \frac{q_t}{q_1}(T_t - g_t)$$

- B_1 is the value of one-period government debt issued at time 1 and repaid at time 2, expressed in units of time 1 goods

Flow Budget Constraint

- Express the last equation in terms of t and $t-1$ periods to get a sequence of period-by-period government budget constraints

$$g_t + R_{t-1,t} B_{t-1} = T_t + B_t \tag{2}$$

- One for each $t \geq 1$, where B_t is issued in t and repaid in $t+1$

$$R_{t-1,t} = \frac{q_{t-1}}{q_t} \quad \text{and} \quad B_t = \sum_{s=t+1}^{\infty} \frac{q_s}{q_t}(T_s - g_s)$$

- The left side of (2) is time t government expenditures including interest and principal payments; the right side is total revenues including new debt

Today's Deficit Is Tomorrow's Surplus

- Thus, embedded in the time 0 government budget constraint (1) is a sequence of one-period government debts satisfying $B_t \equiv \sum_{s=t+1}^{\infty} \frac{q_s}{q_t}(T_s - g_s)$
- The value of government debt at t is equal to the present value of government surpluses from date $t+1$ onward
- Government debt at time t signals future government budget surpluses

3. The Term Structure of Interest Rates

A Digression

- The price system $\{q_t\}_{t=0}^{\infty}$ embeds within it a term structure of interest rates
- To see this, write q_t as

$$
\begin{aligned}
q_t &= q_0 \frac{q_1}{q_0} \frac{q_2}{q_1} \dots \frac{q_t}{q_{t-1}} \\
&= q_0 m_{0,1} m_{1,2} \dots m_{t-1,t} \quad \text{where } m_{t,t+1} \equiv \frac{q_{t+1}}{q_t}
\end{aligned}
$$

- $m_{t,t+1}$ is the one-period (nonstochastic) discount factor

Gross and Net

- Represent the discount factor as

$$m_{t,t+1} = \frac{1}{R_{t,t+1}} = \frac{1}{1+\bar{r}_{t,t+1}} \approx \exp(-\bar{r}_{t,t+1})$$

- $R_{t,t+1}$ is the one-period gross interest rate between t and $t+1$
- $\bar{r}_{t,t+1}$ is the one-period net interest rate between t and $t+1$

Short and Long

- Notice q_t can also be expressed as

$$\begin{aligned} q_t &= q_0 \exp(-\bar{r}_{0,1}) \exp(-\bar{r}_{1,2}) \cdots \exp(-\bar{r}_{t-1,t}) \\ &= q_0 \exp[-(\bar{r}_{0,1} + \bar{r}_{1,2} + \cdots + \bar{r}_{t-1,t})] \\ &= q_0 \exp(-t\bar{r}_{0,t}) \end{aligned}$$

where $\bar{r}_{0,t}$ is the net t-period interest rate between 0 and t

$$\bar{r}_{0,t} = t^{-1}(\bar{r}_{0,1} + \bar{r}_{1,2} + \cdots + \bar{r}_{t-1,t})$$

Zero Coupon Bond

- q_t is the time 0 price of one unit of time t consumption
- So $\bar{r}_{0,t}$ is the yield to maturity on a zero coupon bond that matures at t
- A zero coupon bond promises no coupon before maturity and pays only the principal due at the date of maturity t

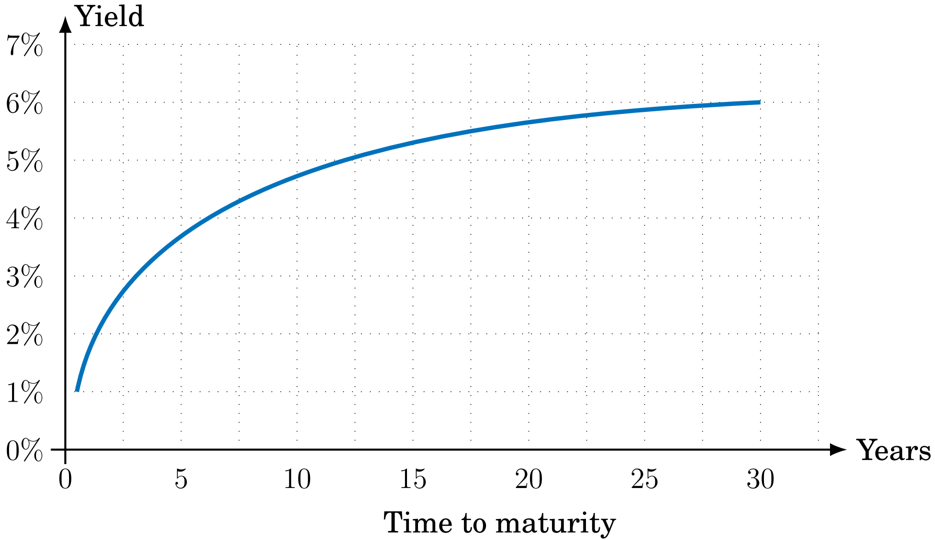
Yield Curve

- $\bar{r}_{0,t} = t^{-1}(\bar{r}_{0,1} + \bar{r}_{1,2} + \cdots + \bar{r}_{t-1,t})$ expresses the expectations theory of the term structure of interest rates
- The theory states that interest rates on long (t-period) loans are averages of rates on short (one-period) loans expected to prevail over the long horizon
- More generally, the s-period long rate at time t is

$$\bar{r}_{t,t+s} = \frac{1}{s}(\bar{r}_{t,t+1} + \bar{r}_{t+1,t+2} + \cdots + \bar{r}_{t+s-1,t+s})$$

- A graph of $\bar{r}_{t,t+s}$ against s is called the yield curve at t

Yield Curve



4. Competitive Equilibrium

Household Budget Constraint

- Back to the model: collect capital terms in the household budget constraint

$$\sum_{t=0}^{\infty} q_t \left\{ r_t k_t - \tau_t^k (r_t - \delta) k_t - [k_{t+1} - (1-\delta) k_t] \right\}$$

$$= \sum_{t=0}^{\infty} q_t \left\{ [(1-\tau_t^k)(r_t - \delta) + 1] k_t - k_{t+1} \right\}$$

$$= q_0 [(1-\tau_0^k)(r_0 - \delta) + 1] k_0 - q_0 k_1 + q_1 [(1-\tau_1^k)(r_1 - \delta) + 1] k_1 - q_1 k_2 + \dots$$
$$+ \dots + q_T [(1-\tau_T^k)(r_T - \delta) + 1] k_T - q_T k_{T+1}$$

$$= [(1-\tau_0^k)(r_0 - \delta) + 1] q_0 k_0 + \sum_{t=1}^{\infty} \left\{ [(1-\tau_t^k)(r_t - \delta) + 1] q_t - q_{t-1} \right\} k_t - \lim_{T \to \infty} q_T k_{T+1}$$

Household Budget Constraint

- The household budget constraint thus writes

$$\sum_{t=0}^{\infty} q_t[(1+\tau_t^c)c_t] \leq \sum_{t=0}^{\infty} q_t(1-\tau_t^n)w_t n_t - \sum_{t=0}^{\infty} q_t \tau_t + \sum_{t=1}^{\infty} \left\{[(1-\tau_t^k)(r_t-\delta)+1]q_t - q_{t-1}\right\} k_t$$
$$+ [(1-\tau_0^k)(r_0-\delta)+1]q_0 k_0 - \lim_{T\to\infty} q_T k_{T+1}$$

- All other things being equal, the household wants to increase resources (right side) to buy more consumption goods (left side)

Unbounded Profit

- The household assembles capital in $t-1$ at cost $q_{t-1}k_t$; it then rents it out in t and sells the undepreciated stock to make a return $[(1-\tau_t^k)(r_t-\delta)+1]q_t k_t$
- If $[(1-\tau_t^k)(r_t-\delta)+1]q_t - q_{t-1} > 0$, the agent makes profit and wants $k_t \to \infty$
- If $[(1-\tau_t^k)(r_t-\delta)+1]q_t - q_{t-1} < 0$, the agent short sells capital, ie it sells "synthetic" units in $t-1$ and buys these units back at a lower price in t
- That is, the agent makes profit and wants $k_t \to -\infty$

No Arbitrage

- In equilibrium, capital is nonnegative and bounded, therefore it must be that the terms multiplying k equal zero

$$\frac{q_t}{q_{t+1}} = [(1 - \tau^k_{t+1})(r_{t+1} - \delta) + 1] \quad \text{for all } t \geq 1$$

- This is the zero-profit, or no-arbitrage, condition

Transversality Condition

- A similar argument holds for the limiting term $-\lim_{T\to\infty} q_T k_{T+1}$
- The household wants this term to be as small as possible, by short selling capital in the limit, in order to buy unlimited amounts of consumption goods
- But the market would stop the household from undertaking such a short sale since there would be no party on the other side of the transaction
- Thus, as a condition of optimality, we have the transversality condition

$$-\lim_{T\to\infty} q_T k_{T+1} = 0$$

Household Problem

- To solve the household problem, we write a Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t) + \mu \sum_{t=0}^{\infty} q_t \left\{(1 - \tau_t^n) w_t n_t - \tau_t - (1 + \tau_t^c) c_t\right\}$$

- Note the budget constraint is not discounted: implicitly, μ is a present-value multiplier, which embeds discounting

First-Order Conditions

- The first-order conditions are, for each t

$$\begin{aligned} c_t: &\quad \beta^t U_c(c_t, 1-n_t) = \mu q_t (1+\tau_t^c) \\ n_t: &\quad \beta^t U_n(c_t, 1-n_t) = \mu q_t w_t (1-\tau_t^n) \end{aligned}$$

- We verify indeed that μ depends on β^t

Firm Problem

- The firm solves
$$\max_{k_t, n_t} \sum_{t=0}^{\infty} q_t [F(k_t, n_t) - w_t n_t - r_t k_t]$$
- The first-order conditions are, for each t

$$\begin{aligned} k_t : &\quad r_t = F_k(k_t, n_t) \\ n_t : &\quad w_t = F_n(k_t, n_t) \end{aligned}$$

Competitive Equilibrium

A competitive equilibrium with distorting taxes is a price system $\{q_t, r_t, w_t\}$, a budget-feasible government policy $\{g_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t\}$, and an allocation $\{c_t, n_t, k_{t+1}\}$ that solve the system of difference equations consisting of the resource constraint (or feasible allocation), the zero-profit (or no-arbitrage) condition, the household's first-order conditions, and the firm's first-order conditions, subject to the initial condition on k_0 and the terminal condition

Summary of Equilibrium Conditions

- We end up with a system of six equations for six endogenous variables $\{c_t, k_t, n_t, q_t, r_t, w_t\}_{t=0}^{\infty}$, given the exogenous variables $\{g_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t\}_{t=0}^{\infty}$

$$
\begin{aligned}
g_t + c_t + k_{t+1} &= F(k_t, n_t) + (1-\delta)k_t \\
q_t &= [(1-\tau_{t+1}^k)(r_{t+1}-\delta)+1]q_{t+1} \\
\beta^t U_c(c_t, 1-n_t) &= \mu q_t (1+\tau_t^c) \\
\beta^t U_n(c_t, 1-n_t) &= \mu q_t w_t (1-\tau_t^n) \\
r_t &= F_k(k_t, n_t) \\
w_t &= F_n(k_t, n_t)
\end{aligned}
$$

Exogenous But Constrained

- On top of the previous six equations, the budget constraint of the government must hold

$$\sum_{t=0}^{\infty} q_t g_t \leq \sum_{t=0}^{\infty} q_t \left[\tau_t^c c_t + \tau_t^k (r_t - \delta) k_t + \tau_t^n w_t n_t + \tau_t\right]$$

- Thus, one of the five exogenous variables $\{g_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t\}_{t=0}^{\infty}$ must adjust to satisfy this equation

5. Ricardian Equivalence

No Lump-Sum Tax in Equilibrium

- Notice that the lump-sum tax τ_t does not appear in any of the system's six equilibrium conditions
- But its present value $\sum_{t=0}^{\infty} q_t \tau_t$ does appear in the time 0 government budget constraint

No Lump-Sum Tax in Equilibrium

- We conclude two things

1. The path or timing of lump-sum taxes has no effect on the model's dynamics
2. Only the present value of current and future taxes matters

- Let's explore these results in detail

Two Modifications

- We consider the same model
- But to simplify we set all distorting taxes to zero, $\tau_t^k = \tau_t^c = \tau_t^n = 0$
- Also, we introduce government debt

Government Budget Constraint

- The government issues debt b_t at price q_t
- b_t is one-period debt due at t, denominated in t goods
- The government's period t budget constraint writes

$$\underbrace{q_t g_t + q_t b_t}_{\text{expenses}} = \underbrace{q_t \tau_t + q_{t+1} b_{t+1}}_{\text{resources}}$$

- Solve for τ_t

$$\tau_t = g_t + b_t - \frac{q_{t+1}}{q_t} b_{t+1}$$

Household Budget Constraint

- The household's period t budget constraint is

$$q_t c_t + q_t[k_{t+1} - (1-\delta)k_t] + q_{t+1}a_{t+1} = q_t w_t n_t + q_t r_t k_t + q_t a_t - q_t \tau_t$$

- a_t is financial wealth excluding capital holdings
- Divide by q_t

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{q_{t+1}}{q_t}a_{t+1} = w_t n_t + r_t k_t + a_t - \tau_t$$

Merging the Two Budget Constraints

- Plug the government budget constraint into the household budget constraint to substitute out for τ_t

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{q_{t+1}}{q_t}a_{t+1} = w_t n_t + r_t k_t + a_t - g_t - b_t + \frac{q_{t+1}}{q_t}b_{t+1}$$

- In equilibrium, demand of bonds equals supply, $a_t = b_t$, and hence we obtain

$$c_t + k_{t+1} - (1-\delta)k_t = w_t n_t + r_t k_t - g_t$$

- Taxes τ_t and debt b_t no longer appear in the household budget constraint
- The household only cares about government spending g_t

More Government Spending Makes Me Poorer

- From the previous equation, we see that an increase in government spending g_t directly reduces the household's resources
- Why? The reason is that the household is rational and infinitely-lived and therefore internalizes the budget constraint of the government
- The household knows that an increase in government expenditure g_t must be financed either by an increase in taxes τ_t or by an increase in debt b_t

Irrelevant Tax-Debt Mix

- But to repay debt in the future, taxes will have to go up
- Higher debt is simply future higher taxes
- Thus the choice is between higher taxes today or higher taxes tomorrow
- For a rational and infinitely-lived household, that is equivalent
- Thus it does not matter how the government finances its spending: the tax-debt mix is irrelevant

Ricardian Equivalence

- The way the government finances its expenditures – through lump-sum taxes or debt – has no impact on the dynamics of the economy
- This is the Ricardian equivalence proposition, discussed first by Ricardo (1820) and shown formally by Barro (1974) in a seminal paper
- An increase in government spending makes households feel poorer today
- Again, this is because they realize that taxes will have to go up, either today or in the future, to finance the increase in spending

Alternative Way

- We can show Ricardian equivalence in another way
- Solve for b_t in the government's period t budget constraint

$$b_t = \tau_t - g_t + \frac{q_{t+1}}{q_t} b_{t+1}$$

- Solve this equation forward: $b_t = \tau_t + g_t + \frac{q_{t+1}}{q_t}(\tau_{t+1} + g_{t+1}) + \frac{q_{t+2}}{q_t} b_{t+2}$ or

$$b_t = \sum_{s=0}^{\infty} \frac{q_{t+s}}{q_t}(\tau_{t+s} - g_{t+s}) + \underbrace{\lim_{T \to \infty} \frac{q_{t+T+1}}{q_T} b_{t+T+1}}_{=0}$$

- The transversality condition rules out a government-induced Ponzi scheme

Household Budget Constraint

- Do the same for assets a_t in the household's period t budget constraint

$$a_t = \sum_{s=0}^{\infty} \frac{q_{t+s}}{q_t} \left(c_{t+s} + k_{t+1+s} - (1-\delta)k_{t+s} + \tau_{t+s} - w_{t+s}n_{t+s} - r_{t+s}k_{t+s}\right)$$

- Since $c_t + k_{t+1} - (1-\delta)k_t = F(k_t, n_t) - g_t$ and since zero profit implies $F(k_t, n_t) - w_t n_t - r_t k_t = 0$, we have

$$a_t = \sum_{s=0}^{\infty} \frac{q_{t+s}}{q_t} \left(\tau_{t+s} - g_{t+s}\right)$$

Spending Is Taxing

- We conclude for all t

$$a_t = \sum_{s=0}^{\infty} \frac{q_{t+s}}{q_t} \left(\tau_{t+s} - g_{t+s} \right) = b_t$$

- Suppose that in period t, government debt is zero, $a_t = b_t = 0$

$$\sum_{t=0}^{\infty} \frac{q_{t+s}}{q_t} g_{t+s} = \sum_{s=0}^{\infty} \frac{q_{t+s}}{q_t} \tau_{t+s}$$

- The present value of government spending equals the present value of taxes

Conditions for Ricardian Equivalence

- The conditions for Ricardian equivalence to hold are threefold

1. Rational and infinitely-lived households
2. Complete markets
3. Lump-sum taxes

- With uncertainty, Ricardian equivalence holds
- But with distorting taxes, Ricardian equivalence breaks down

6. Inelastic Labor Supply

Inelastic Labor Supply

- Let's consider the simplifying case

$$U(c, 1-n) = u(c)$$

- No utility from leisure, so constant labor supply $n = 1$
- Also, define $f(k) = F(k, 1)$, so $F_k(k, 1) = f'(k)$ and $F_n(k, 1) = f(k) - f'(k)k$

Inelastic Labor Supply

- Rewrite the previous system with $n_t = n = 1$

$$g_t + c_t + k_{t+1} = f(k_t) + (1-\delta)k_t \tag{3}$$
$$q_t = [(1-\tau^k_{t+1})(r_{t+1}-\delta)+1]q_{t+1} \tag{4}$$
$$\beta^t u'(c_t) = \mu q_t(1+\tau^c_t) \tag{5}$$
$$r_t = f'(k_t) \tag{6}$$
$$w_t = f(k_t) - f'(k_t)k_t \tag{7}$$

- The household's FOC for labor drops out

Non-Distorting Labor Income Tax

- The labor income tax τ_t^n no longer appears in the equilibrium conditions
- Thus when the labor supply is inelastic, ie constant, the labor income tax is non distortionary
- Intuitively, since workers have no marginal disutility of work, taxing their labor income will not affect their willingness to work

Reducing the System

- Combine equations (4), (5), and (6) of the system

$$u'(c_t) = \beta u'(c_{t+1}) \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left[(1-\tau_{t+1}^k)[f'(k_{t+1}) - \delta] + 1 \right]$$

- The resource constraint (3) is

$$c_t = f(k_t) + (1-\delta)k_t - k_{t+1} - g_t$$

- We are down to a system of two difference equations for two endogenous variables, c_t and k_t, given exogenous variables g_t, τ_t^k, and τ_t^c

Non-Distorting Constant Consumption Tax

- Suppose the consumption tax is constant, $\tau_t^c = \tau_{t+1}^c = \bar{\tau}^c$
- With an inelastic labor supply, a constant consumption tax is non distortionary

$$u'(c_t) = \beta u'(c_{t+1}) \left[(1 - \tau_{t+1}^k)[f'(k_{t+1}) - \delta] + 1 \right]$$

7. Steady State

Constant Fiscal Policy

- We assume that fiscal policy is eventually constant

$$\lim_{t\to\infty} g_t = \bar{g} \qquad \lim_{t\to\infty} \tau_t^k = \bar{\tau}^k \qquad \lim_{t\to\infty} \tau_t^c = \bar{\tau}^c$$

- Given this assumption and those on the production and utility functions, we know that the system converges to a unique steady state

Steady State

- The system in steady state is

$$
\begin{aligned}
1 &= \beta\left[(1-\bar{\tau}^k)(f'(\bar{k})-\delta)+1\right] \\
\bar{c} &= f(\bar{k})-\delta\bar{k}-\bar{g}
\end{aligned}
$$

- In a steady state with inelastic labor supply, only the capital income tax is distorting, ie changes the behavior of agents

8. Equilibrium Path

System

- Our system is

$$u'(c_t) = \beta u'(c_{t+1}) \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left[(1-\tau_{t+1}^k)[f'(k_{t+1}) - \delta] + 1 \right] \tag{8}$$

$$c_t = f(k_t) + (1-\delta)k_t - k_{t+1} - g_t \tag{9}$$

Optimal Path

- We know the initial condition k_0
- We have the terminal steady state $\bar{k}$ and $\bar{c}$
- We know the path of exogenous variables g_t, τ_t^c, τ_t^k
- We want to compute the equilibrium path that starts from the initial condition and ends at the terminal value

Shooting Algorithm

- We are going to use a shooting algorithm, a numerical method for solving a two-point boundary value problem
- We "shoot" out trajectories in different directions until we find a trajectory that has the desired boundary value
- We search for an initial c_0 that makes the two equations (8) and (9) imply $k_S \approx \bar{k}$, where S is a finite but large time index

Steps

1. Solve the steady state
2. Select a large time index S and guess an initial consumption c_0; compute $u'(c_0)$ and solve (9) for k_1
3. Given c_t and k_{t+1}, use (8) and (9) to compute c_{t+1} and k_{t+2}
4. Iterate on step 3 to compute candidate values $\hat{k}_t$, $t = 1, \ldots, S$
5. Compute $\hat{k}_S - \bar{k}$; if $\hat{k}_S > \bar{k}$, raise c_0 and compute a new $\hat{k}_t$; if $\hat{k}_S < \bar{k}$, lower c_0 and compute a new $\hat{k}_t$
6. In this way search for a c_0 that makes $\hat{k}_S \approx \bar{k}$

After Convergence

- If S is not big enough the algorithm might not converge
- We can implement the shooting algorithm in Matlab
  - Dynare implements the shooting algorithm
- Once we solve for the equilibrium $\{k_t\}$ sequence, we recover all variables using the system's other equations

Balanced Government Budget

- How do we ensure the time 0 government budget constraint is satisfied?
- Two cases

1. The government can impose lump-sum taxes
2. The government cannot impose lump-sum taxes

With Lump-Sum Taxes

- With lump-sum taxes, we assume a path for $\{g_t, \tau_t^k, \tau_t^c\}_{t=0}^{\infty}$
- We solve for all equilibrium variables with the algorithm
- We find a value for $\sum_{t=0}^{\infty} q_t \tau_t$ that balances the budget
- In other words, lump-sum taxes are the residual variable that satisfies the government budget constraint

No Lump-Sum Taxes

- If lump-sum taxes are not available, we need an extra loop
- Compute a candidate equilibrium for an arbitrary tax mix
- Change some elements of the expenditure or tax sequence until the government budget constraint is satisfied
- There is a huge number of possibilities

9. Fiscal Policy Experiments

Experiments Using Models

"One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost."

Robert Lucas, 1980, *Journal of Money, Credit, and Banking*

Fiscal Policy Experiments

- We are going to run some fiscal policy experiments
- At time $t = 0$, the economy is in steady state
- At time $t = 1$, the government announces a policy change that will be implemented at time $t = 10$
- Our goal is to analyze how the economy responds to these changes

Functional Forms

- We use the following utility function

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

- We use the following production function

$$f(k_t) = k_t^{\alpha}$$

Calibration

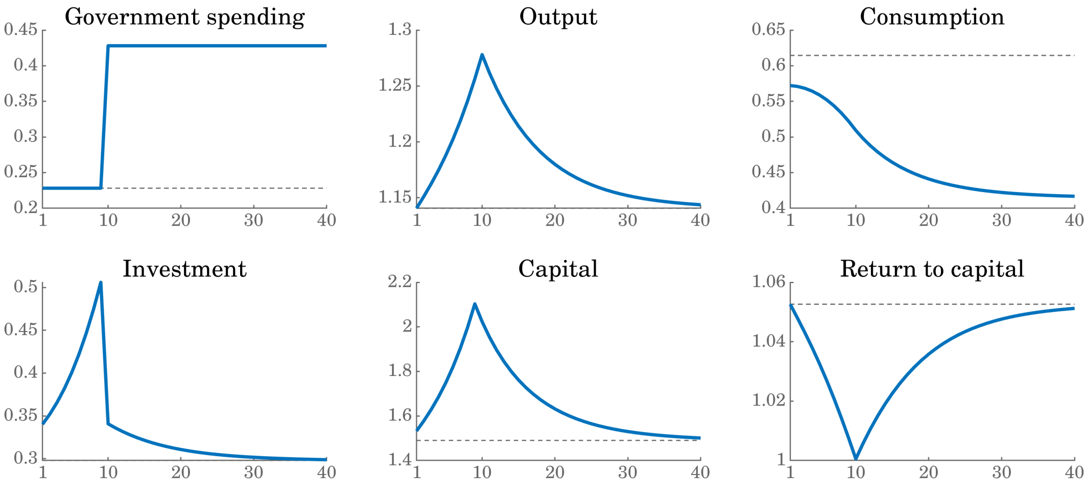
- We calibrate the model at annual frequency, one period is a year

Parameter	Description	Value
α	Capital share in production	0.33
β	Discount factor	0.95
δ	Depreciation rate of capital	0.2
σ	Inverse elasticity of substitution	2.0
$\bar{g}$	Steady-state government spending	0.2
$\bar{\tau}^c$	Steady-state consumption tax	0
$\bar{\tau}^k$	Steady-state capital income tax	0
$\bar{\tau}^n$	Steady-state labor income tax	0

Permanent Increase in Government Spending

- The first experiment is an increase in government spending
- In period $t = 1$, the government announces it will double g_t in $t = 10$
- Financed only with the lump-sum tax, no distorting tax
- This is a once-and-for-all change: unique and permanent
- The change is perfectly foreseen, ie no uncertainty

Permanent Increase in Government Spending



Permanent Drop in Income

- Consumers are forward-looking: they react immediately to the policy shock even though it takes effect only in ten periods
- Consumers are Ricardian: they know that an increase in government expenditure means an increase in taxes, now or later
- The permanent increase in g_t means a permanent negative wealth effect
- Households consume less and save more to compensate (hours are fixed)
- This leads to a gradual build-up of capital until $t = 10$

Crowding Out of Investment

- In period $t = 10$, the policy change takes place
- Taxes increase, households become poorer, they dissave, and capital gradually falls back to its steady-state value
- Notice how the increase in g_t is matched by a decrease in investment: this is the crowding out effect of public spending on private investment
- Variation in capital allows households to smooth consumption; changes in the return to capital r_t, inversely related to k_t, help accomplish this

Permanent Drop in Consumption

- The permanent increase in government spending g_t has no permanent effect on capital k_t: remember that in steady state, $\bar{g}$ does not affect $\bar{k}$
- But $\bar{g}$ does affect $\bar{c}$ in steady state: $\bar{c} = f(\bar{k}) - \delta\bar{k} - \bar{g}$
- Thus the permanent increase in government spending makes consumers permanently poorer and, as a result, they consume less forever

Permanent Income Hypothesis

- Consumption smoothing illustrates the permanent income hypothesis put forward by Milton Friedman (1957)
- Changes in permanent income (assets, wage), not temporary income, are what drive changes in a person's consumption patterns
- The key determinant is the individual's lifetime income; here the present value of disposable income, ie capital and labor income net of taxes

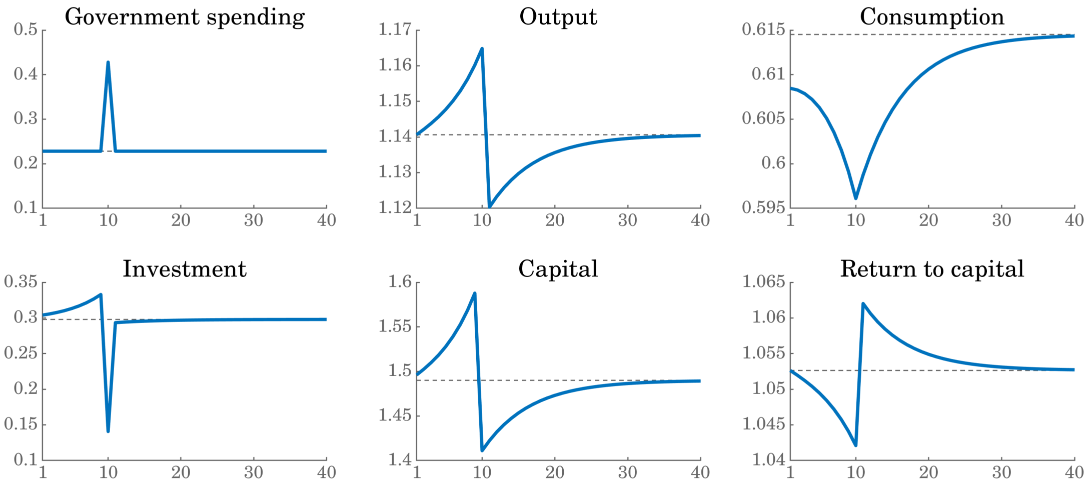
Dynamics

- Before $t = 10$, the response of each variable in the economy is entirely due to expectations about future policy changes
- After $t = 10$, there is a purely transient response to a new stationary level of exogenous variables, the forcing function
- Before $t = 10$ the forcing function changes, after $t = 10$ the policy vector is constant and the sources of dynamics are transient

Temporary Increase in Government Spending

- Let's now simulate a temporary increase in g_t
- In period $t = 0$ the government announces it will double g_t but only in $t = 10$, after that g_t returns to its steady state
- Again the change is foreseen, ie anticipated

Temporary Increase in Government Spending



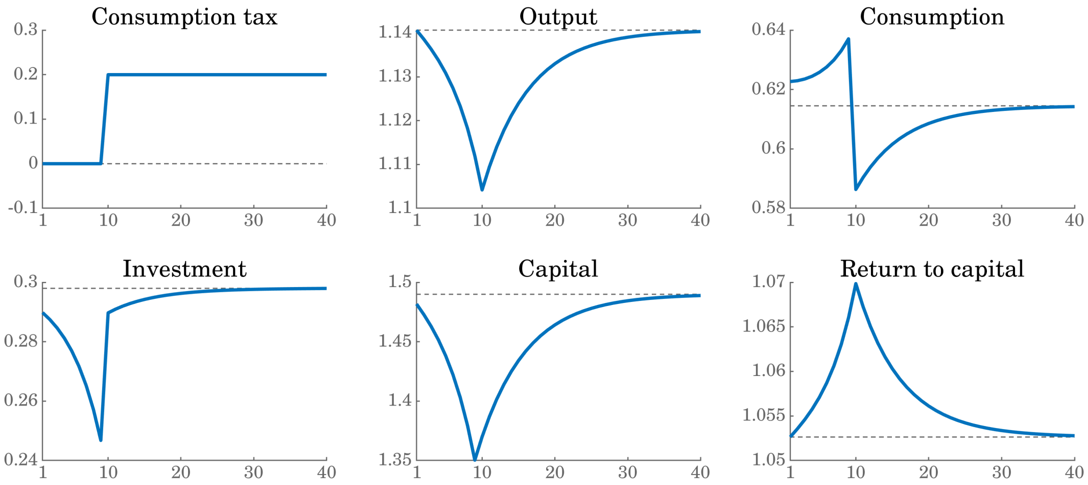
Temporary Increase in Government Spending

- Same mechanism, a negative wealth effect
- Consumption drops immediately and falls further in anticipation of the increase in taxes
- At time $t = 10$, capital and investment jump downward because the government consumes mores and raises taxes, making households poorer
- Consumption slowly returns to its steady-state value

Permanent Increase in Consumption Tax

- We now simulate an increase in the consumption tax
- In period $t = 0$, the government announces it will raise τ_t^c from zero to 20 percent in period $t = 10$ for ever, ie a permanent and anticipated shock
- The increase in τ_t^c is accompanied by a reduction in the present-value of lump-sum taxes that leaves the government budget balanced

Permanent Increase in Consumption Tax



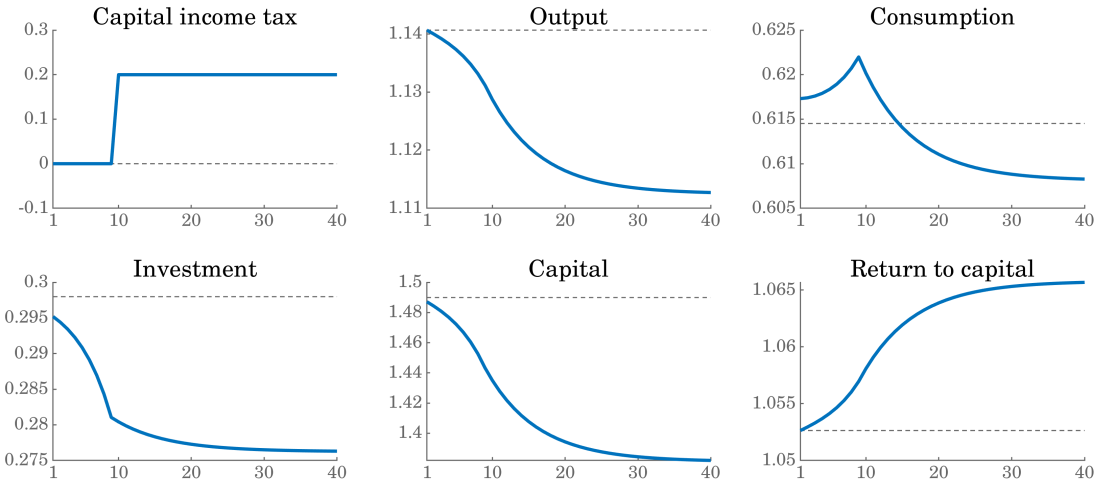
Permanent Increase in Consumption Tax

- Consumers know they are not taxed today but will be taxed later: consumption jumps on impact and a consumption binges ensues
- Consumers finance that binge by saving less, capital de-accumulates
- At time $t = 10$ the party is over, consumption plunges and households turn to saving instead
- Capital returns slowly to steady state at the cost of austerity

Permanent Increase in Capital Income Tax

- We now simulate an increase in the capital income tax
- In period $t = 0$, the government announces it will raise τ_t^k from zero to 20 percent in period $t = 10$ for ever, ie a permanent and anticipated shock
- Again, the value of the lump-sum tax is reduced

Permanent Increase in Capital Income Tax



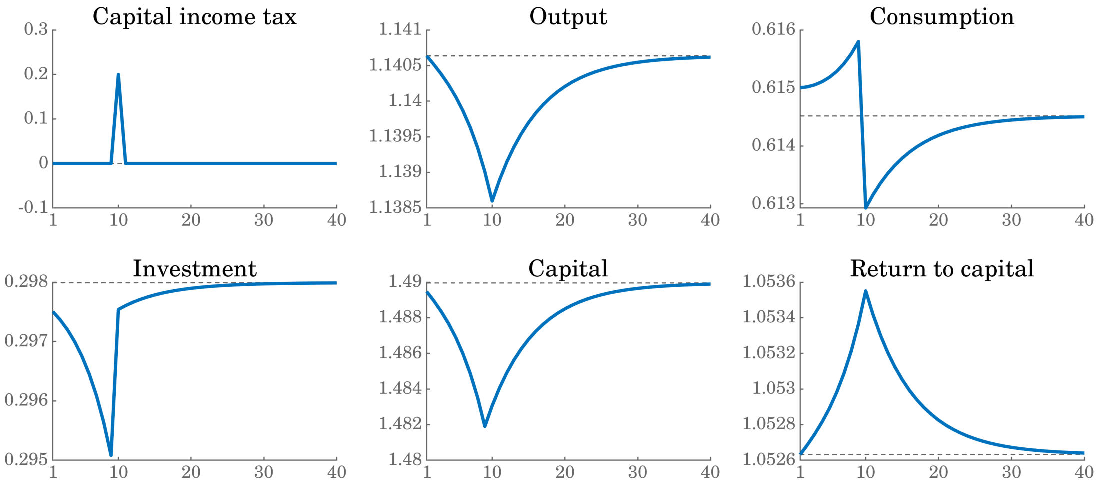
Permanent Increase in Capital Income Tax

- Capital is taxed tomorrow but loses value already today
- Households save less and consume more
- But eventually less capital produces less output and consumption falls
- The new steady state is a permanently smaller economy with lower capital, lower output, lower consumption, and a higher return to capital

Temporary Increase in Capital Income Tax

- We now simulate a temporary increase in the capital income tax
- In period $t = 0$, the government announces it will raise τ_t^k from zero to 20 percent in period $t = 10$ only

Temporary Increase in Capital Income Tax



Permanent Increase in Capital Income Tax

- Same mecanism, capital loses value today due to the future tax
- Households dissave until the day of the policy change arrives
- This sustains a temporary consumption binge from periods 1 to 9
- After the shock, all variables gradually return to steady state

Taking Stock

- In this simple model, we can study the effects of temporary and permanent policy changes
- We can also study anticipated and unanticipated policy changes
- We can extend the model to account for more realistic features of the economy and the tax system

Conclusion

- We have introduced fiscal policy in the standard growth model
- We have done that by adopting a positive approach, ie we have described the mechanisms at play without judgment
- Next class we will adopt a normative approach
- We will ask the question: what is the optimal tax policy?

10. Exercises

Exercise 1 – Tax Reform 1

Consider the nonstochastic growth model studied above. The government only levies a consumption tax τ_t^c and a lump-sum tax τ_t. Utility depends only on consumption.

1. Define a competitive equilibrium.
2. Suppose the government has unlimited access to the lump-sum tax and sets $g_t = \bar{g}$ and $\tau_t^c = 0$. This situation is expected to go on forever. Tell how to find the steady-state capital-labor ratio for this economy.
3. Prove that the timing of taxes is irrelevant.

Exercise 1 – Continued

4. Suddenly lump-sum taxes are declared illegal. Starting at time $t = 0$ the government must finance expenditures with τ_t^c. Policy advisor 1 proposes the following tax policy: find a constant $\bar{\tau}^c$ that satisfies the time 0 government budget constraint, and impose it from time 0 onward. Compute the new steady-state value of k_t under this policy. Analyze the transition path from the old to the new steady state.
5. Policy advisor 2 proposes an alternative policy. Instead of imposing the increase suddenly, he proposes to ease the pain by postponing the increase for 10 years: $\tau_t^c = 0$ for $t = 0, \ldots, 9$ and then $\tau_t^c = \bar{\tau}^c$ for $t \geq 10$. Compute the steady-state level of capital under this policy. Can you say anything about the transition path to the new steady state?
6. Which policy is better, the first or the second one?

Exercise 2 – Tax Reform 2

Consider the nonstochastic growth model studied above. The government only levies a consumption tax τ_t^c and a capital income tax τ_t^k. Utility depends only on consumption.

1. Define a competitive equilibrium.
2. Assume an initial condition in which the government finances constant expenditures $\bar{g}$ entirely with a constant $\bar{\tau}^k$ and zero τ^c. Tell how to find the steady-state levels of capital, consumption, and the rate of return on capital.

Exercise 2 – Continued

3. Suddenly the capital tax is repealed, $\tau^k = 0$ for ever. Now the government must use τ^c to finance $\bar{g}$. Tell what happens to the new steady-state values of capital, consumption, and the return on capital.
4. Compare the two alternative policies of (1) relying completely on the taxation of capital or (2) relying completely on the consumption tax, by looking at the discounted utilities of consumption in steady state, ie $\frac{1}{1-\beta}u(\bar{c})$ in the two equilibria. Is this a good way to measure the costs or gains of one policy vis-a-vis the other?

Exercise 3 – Shooting Algorithm

1. Write a Matlab code to simulate the nonstochastic growth model without fiscal policy using a shooting algorithm.
2. Write another code to simulate the nonstochastic growth model with exogenous spending and lump-sum taxes.
3. Compare the effect of a temporary increase in technology in the two models using impulse response functions.
4. Repeat the procedure using Dynare.