14. Overlapping Generations

Pareto Inefficiency

Yvan Becard
PUC-Rio

Macroeconomics I, 2023

From Immortal

- So far we have studied economies in which individuals live forever
- In the simplest case, we assumed no population growth, ie the same people stuck around eternally
- We also modeled population growth, ie new people came in but no one died

To Mortal

- We now study a more realistic world in which people have a finite life
- Each individual is born at some point in time
- Each individual lives for a number of periods
- Each individual dies and disappears at some other point in time

Overlapping Generations

- At every point in time, at least two cohorts of different age coexist
- This is why we use the term overlapping generations or OLG
- We study the simplest case with two cohorts, a young and an old cohorts
- We can extend to as many cohorts as we want, eg one cohort per year

Background

- Allais (1947) first suggested the concept of overlapping generations
- Samuelson (1958) developed the first overlapping generations model in the context of a pure exchange economy
- Diamond (1965) added neoclassical production to the OLG framework
- All three received the Nobel Prize

A Wide Range of Issues

- The OLG model is the natural framework for studying a number of issues
- Economic growth and structural change: globalization, automation, inequality, job polarization, global saving glut, secular stagnation
- Life-cycle behavior: schooling, human capital accumulation, borrowing, working life, non-employment, saving for retirement, bequest to children
- Fiscal policy: resource allocation across generations, social security, education policy, intergenerational transfers, national debt, bubbles
- Demographics: fertility decisions, demographic transition, mortality

Lecture Outline

1. Model Setup
2. Time Zero Trading
3. Example
4. Pareto Inefficiency
5. Sequential Trading
6. Money
7. Money and Infinite Lives
8. Conclusion
9. Exercises

Main Reference: Ljungqvist and Sargent, 2018, *Recursive Macroeconomic Theory*, Fourth Edition, Chapter 9

1. Model Setup

Pure Exchange Economy

- The environment is deterministic, there is no uncertainty
- We start with a pure exchange economy: no production, only endowments
- There is a single good at each date
- The good is not storable; think of perishable goods or services

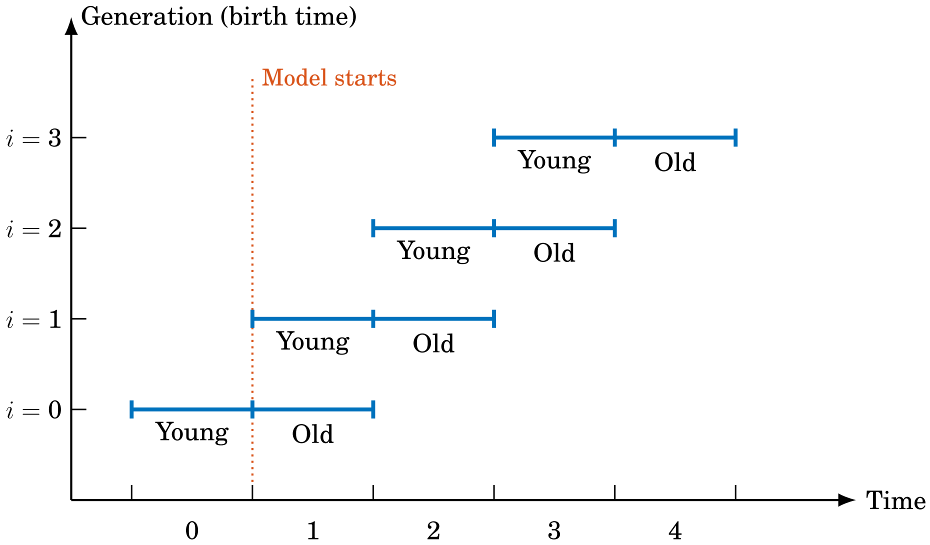
Infinite Time

- Time is discrete
- Time starts at $t = 1$ and lasts forever, $t = 1, 2, 3, \ldots$
- Notice the first period is no longer $t = 0$
- We will see why in a second

Infinite Population, Finite Lives

- There is an infinity of agents named $i = 0, 1, \ldots$
- But at every point in time the population is finite
- We can regard i as consumer i's period of birth
- Agents are born, live two periods, and then die
- Agent $i = 0$ is born at time 0, is young during period 0, is old during period 1, dies at the end of time 1; agent $i = 1$ is born in 1 and dies at the end of 2

Two-Period Overlapping Generations



Preferences

- Agent i, born at time i, has utility

$$U^i(c^i) = u(c_i^i) + u(c_{i+1}^i), \quad i \geq 1$$

- The first generation, $i = 0$, born at time 0, has utility

$$U^0(c^0) = u(c_1^0)$$

- $u(c)$ is strictly increasing and concave, twice continuously differentiable

Interpretation

- Consumer i only wants goods dated i and $i+1$ when she is alive
- Cohort 0 is the initial old cohort, it only consumes at time $t = 1$
- We start in $t = 1$ in order to have two generations overlap
- Implicitly $\beta = 1$, agents do not discount the future, ie their old age

Endowment

- Each household has an endowment sequence y^i satisfying

$$y_i^i \geq 0, \qquad y_{i+1}^i \geq 0, \qquad y_t^i = 0 \quad \text{for all } t \neq i \text{ or } t \neq i+1$$

- Superscript i means we are considering agent i born in i
- Subscript i means we are considering period $t = i$
- Agents are endowed with goods only when they are alive (ie labor income)

2. Time Zero Trading

Complete Markets

- We assume complete markets
- There is no uncertainty, states are known in advance
- But still, complete markets imply that
  - Agents can trade claims to future consumption
  - There is one Arrow-Debreu security for each state
  - There is perfect information, no transaction cost, and full enforcement

Souls Trade with Souls

- We begin with time zero trading
- We imagine there is a clearinghouse active at time 0, which posts prices and aggregates demand and supply for goods in different periods
- All trading occurs in period 0, then stops forever
- Agents not born yet participate: souls trade with souls

Budget Constraint

- At date 0, there are complete markets in time t goods with date 0 price q_t^0
- Consumer i's individual budget constraint is

$$\sum_{t=1}^{\infty} q_t^0 c_t^i \leq \sum_{t=1}^{\infty} q_t^0 y_t^i$$

- The consumer cannot buy claims to consumption that together are worth more than the market value of her lifetime endowment
- At time 0, the consumer sells her entire endowment stream $\sum_{t=1}^{\infty} q_t^0 y_t^i$ in order to purchase enough consumption claims $\sum_{t=1}^{\infty} q_t^0 c_t^i$ for her lifetime

Household Problem

- Consumer i, born in i, maximizes lifetime utility $u(c_i^i) + u(c_{i+1}^i)$ subject to her individual budget constraint, which rewrites as

$$q_i^0 c_i^i + q_{i+1}^0 c_{i+1}^i = q_i^0 y_i^i + q_{i+1}^0 y_{i+1}^i$$

- The Lagrangian is

$$\mathcal{L} = u(c_i^i) + u(c_{i+1}^i) + \mu^i \left[q_i^0 y_i^i + q_{i+1}^0 y_{i+1}^i - q_i^0 c_i^i - q_{i+1}^0 c_{i+1}^i \right]$$

- μ_i is the unique, constant Lagrange multiplier on the constraint

First-Order Conditions

- The first-order conditions are

$$
\begin{aligned}
c_i^i: &\quad u'(c_i^i) = \mu^i q_i^0 \\
c_{i+1}^i: &\quad u'(c_{i+1}^i) = \mu^i q_{i+1}^0 \\
c_{t\neq\{i,i+1\}}^i: &\quad c_t^i = 0 \quad \text{if } t \notin \{i, i+1\}
\end{aligned}
$$

- Combining

$$
\frac{u'(c_{i+1}^i)}{u'(c_i^i)} = \frac{q_{i+1}^0}{q_i^0}
$$

Resource Constraint

- At each period $t \geq 1$, the aggregate resource constraint is

$$\underbrace{c_t^t}_{\text{young consumption}} + \underbrace{c_t^{t-1}}_{\text{old consumption}} \leq \underbrace{y_t^t}_{\text{young endowment}} + \underbrace{y_t^{t-1}}_{\text{old endowment}}$$

- The total consumption of young and old generations summed together in a given period cannot exceed aggregate income that period (closed economy)
- An allocation is feasible if it satisfies this constraint

Equilibrium

- A competitive equilibrium with time 0 trading is such that the following conditions hold

$$
\begin{aligned}
\text{FOC, } i \geq 1: &\quad \frac{u'(c^i_{i+1})}{u'(c^i_i)} = \frac{q^0_{i+1}}{q^0_i} \\
\text{Budget constraint, } i \geq 1: &\quad q^0_i c^i_i + q^0_{i+1} c^i_{i+1} = q^0_i y^i_i + q^0_{i+1} y^i_{i+1} \\
\text{Budget constraint, } i = 0: &\quad c^0_1 = y^0_1 \\
\text{Resource constraint, } i \geq 1: &\quad c^i_i + c^{i-1}_i \leq y^i_i + y^{i-1}_i
\end{aligned}
$$

Stationary Allocation

- Denote subscripts y for young and o for old
- An allocation is stationary if all old generations consume equally and all young generations consume equally

$$c_{i+1}^{i} = c_o, \qquad c_i^i = c_y, \qquad \text{for all } i \geq 1$$

- A stationary equilibrium is an equilibrium with a stationary allocation

3. Example

Rich Young, Poor Old

- We normalize lifetime endowment to one and consider an empirically plausible example in which the endowments vary across young and old age
- Let ϵ be a constant such that $\epsilon \in (0, 0.5)$; the endowments are

$$\begin{aligned} y_i^i &= 1 - \epsilon \quad \text{for all } i \geq 1 \\ y_{i+1}^i &= \epsilon \quad \text{for all } i \geq 0 \\ y_t^i &= 0 \quad \text{otherwise} \end{aligned}$$

- The young have a larger endowment than the old: think of active workers earning a wage versus retirees receiving a relatively smaller pension

Multiple Equilibria

- This economy has many equilibria
- Let's describe two stationary equilibria

1. A high-interest-rate equilibrium H
2. A low-interest-rate equilibrium L

High Equilibrium – Guess

- We use a guess-and-verify method
- We guess that H is a high-interest-rate equilibrium with

$$
\begin{aligned}
q_t^0 &= 1 \quad \text{for all } t \geq 1 \\
c_i^i &= c_{i+1}^i = 0.5 \quad \text{for all } i \geq 1 \\
c_1^0 &= \epsilon
\end{aligned}
$$

- The price of the Arrow-Debreu security is such that consumption of young and old is equal, except for the initial old who consume their income

Verify

- Is H really an equilibrium? Let's check, recall $\epsilon \in (0, 0.5)$

$$
\begin{aligned}
\text{FOC, } i \geq 1: &\quad \frac{u'(0.5)}{u'(0.5)} = \frac{1}{1} \\
\text{Budget constraint, } i \geq 1: &\quad 0.5 + 0.5 = 1 - \epsilon + \epsilon \\
\text{Budget constraint, } i = 0: &\quad \epsilon = \epsilon \\
\text{Resource constraint, } t = 1: &\quad 0.5 + \epsilon < 1 - \epsilon + \epsilon \\
\text{Resource constraint, } t > 1: &\quad 0.5 + 0.5 = 1 - \epsilon + \epsilon
\end{aligned}
$$

- All equations are satisfied, H is an equilibrium

Consumption Smoothing

- What is going on in the background?
- The young, who are relatively rich, want to save for the future, ie delay consumption for when they will be old and relatively poor
- They do this by purchasing Arrow-Debreu securities

A Way to Save

- The young buy $0.5 - \epsilon$ securities, each worth $q_t^0 = 1$, that promise to pay $0.5 - \epsilon$ units in the next period
- After buying the securities, the young are left with

$$\underbrace{1 - \epsilon}_{\text{endowment}} - \underbrace{(0.5 - \epsilon) \times 1}_{\text{claims}} = 0.5 \text{ units to consume in the current period}$$

- When they are old, they enjoy

$$\underbrace{\epsilon}_{\text{endowment}} + \underbrace{(0.5 - \epsilon) \times 1}_{\text{payout}} = 0.5 \text{ units to consume in their old age}$$

Intergenerational Trade

- Who is at the other end of the transaction?
- The young generation buys from the next not-yet-born generation
- When the young turn old, they receive the payout from the new young
- Since all trade is done in at time 0, all generations participate at time 0, except for the initial old which do not take part in trading
- Thus, in this high-interest-rate equilibrium, extensive intergenerational trade occurs at time 0 at the equilibrium price $q_t^0 = 1$

Low Equilibrium – Guess

- We now consider a second stationary equilibrium
- We guess that L is a low-interest-rate equilibrium with

$$
\begin{aligned}
q_1^0 &= 1 \\
\frac{q_{t+1}^0}{q_t^0} &= \frac{u'(\epsilon)}{u'(1-\epsilon)} \equiv \alpha > 1 \\
c_t^i &= y_t^i \quad \text{for all } i, t
\end{aligned}
$$

- The price of Arrow-Debreu claims starts at 1 but then grows without bound
- Both old and young consume only their endowment at each period

Verify

- We verify that L is an equilibrium, recall $\epsilon \in (0, 0.5)$

$$\begin{aligned} \text{FOC, } i \geq 1: &\quad \frac{u'(\epsilon)}{u'(1-\epsilon)} = \frac{q^0_{t+1}}{q^0_t} > 1 \\ \text{Budget constraint, } i \geq 0: &\quad q^0_i(1-\epsilon) + q^0_{i+1}\epsilon = q^0_i(1-\epsilon) + q^0_{i+1}\epsilon \\ \text{Resource constraint, } t \geq 1: &\quad 1 - \epsilon + \epsilon = 1 - \epsilon + \epsilon \end{aligned}$$

- All conditions are satisfied, equilibrium L is indeed an equilibrium

Autarchy

- Equilibrium L is autarkic, ie not a single Arrow-Debreu security is traded
- Prices are set to eradicate all trade
- Agents are hand-to-mouth, they consume their own income each period
- There is no consumption smoothing
- The old consume less than the young

4. Pareto Inefficiency

High Interest Rate

- Define the interest rate $R_{t,t+1} \equiv \frac{q_t^0}{q_{t+1}^0}$ as the one-period gross rate of return between period t and $t+1$
- In the high equilibrium, we have $q_t^0 = q_{t+1}^0$ so $R_{t,t+1} = 1$ for all $t \geq 1$
- The interest rate is high enough so as to induce intergenerational trade

Low Interest Rate

- In the low equilibrium, we have $q^0_{t+1} > q^0_t$ so $R_{t,t+1} < 1$ for all $t \geq 1$
- Prices are too high to induce agents to trade
- Equivalently, the interest rate is too low to induce agents to trade
- Consumers do not buy or sell any security from each other and thus behave as if they were in autarchy

Unspent Resources

- Consider the high equilibrium
- In period 1, old agent $i = 0$ eats ϵ and young agent $i = 1$ eats 0.5
- Thus the feasibility constraint $c_t^t + c_t^{t-1} \leq y_t^t + y_t^{t-1}$ holds with strict inequality in period 1, then with equality in all subsequent periods $t \geq 2$
- Some of the period 1 consumption good is not consumed

Not Pareto Efficient

- This leaves room for a giveaway program to the initial old
- The program would make the initial old generation better off but cost nothing to future generations
- We conclude that H is a competitive equilibrium that is not Pareto optimal

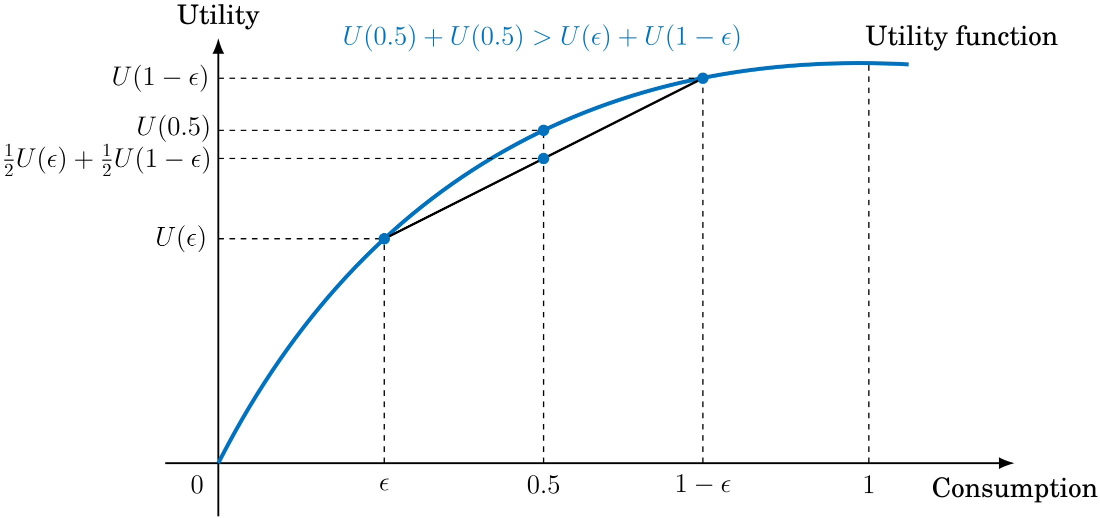
Pareto Dominance

- With $\epsilon \in (0, 0.5)$ and a concave utility function, $u' > 0$, and $u'' < 0$, we have

$$u(0.5) + u(0.5) > u(1 - \epsilon) + u(\epsilon)$$

- In equilibrium H, every generation after the initial old is better off than in equilibrium L, and no generation is worse off
- Thus equilibrium H Pareto dominates equilibrium L
- Note that if $\epsilon = 0.5$, equilibria H and L coincide and are Pareto efficient

Risk-Averse Agents Prefer Smooth Consumption



First Welfare Theorem

- Equilibrium H is a competitive equilibrium that is not Pareto optimal since we can improve one agent's welfare without worsening others'
- Equilibrium L is a competitive equilibrium that is not Pareto optimal since it is Pareto dominated by equilibrium H
- The first fundamental welfare theorem fails to hold

Missing Assumption

- Which assumption needed to deliver the first welfare theorem is missing?

Missing Assumption

- Which assumption needed to deliver the first welfare theorem is missing?
- It is the assumption that the value of the aggregate endowment at the equilibrium prices is finite
- Let's see this in more detail

Finite Endowment

- From the example, we have lifetime endowment $y_i^i + y_i^{i-1} = 1$ for all $i \geq 0$
- Therefore, a finite time 0 aggregate endowment implies

$$\sum_{t=0}^{\infty} q_t^0 \left(\sum_i y_t^i \right) < \infty \quad \Longrightarrow \quad \sum_{t=1}^{\infty} q_t^0 < \infty$$

- Recall, $\frac{u'(\epsilon)}{u'(1-\epsilon)} = \frac{q_{t+1}^0}{q_t^0} = \alpha$, and both in H and L $q_1^0 = 1$, thus

$$q_t^0 = \alpha q_{t-1}^0 = \alpha^2 q_{t-2}^0 = \cdots = \alpha^{t-1} q_1^0 = \alpha^{t-1}$$

Three Cases

1. Rich young, $\epsilon \in (0, 0.5)$: implies $\alpha > 1$ and thus $\sum_{t=1}^{\infty} q_t^0 = \infty$
   - The value of the aggregate endowment is infinite
   - Both H and L are not Pareto optimal
2. Equal income, $\epsilon = 0.5$: this implies $\alpha = 1$ and thus $\sum_{t=1}^{\infty} q_t^0 = \infty$
   - The value of the aggregate endowment is infinite
   - H $=$ L, the competitive equilibrium is Pareto optimal
   - Proof: $\arg\max_x [u(1-x) + u(x)] = 0.5$
3. Rich old, $\epsilon \in (0.5, 1)$: this implies $\alpha < 1$ and thus $\sum_{t=1}^{\infty} q_t^0 = \frac{1}{1-\alpha}$
   - The value of the aggregate endowment is finite
   - H ceases to exist, L is Pareto optimal

Intuition

- What is the intuition behind these results?

1. With $\epsilon \in (0, 0.5)$, the young are richer than the old
   - If at all periods t, the young transfer $0.5 - \epsilon$ to the old, then everyone is better off including initial agent $i = 0$: there is Pareto improvement
2. With $\epsilon = 0.5$, young and old earn and consume equally, this is efficient
3. With $\epsilon \in (0.5, 1)$, the old are richer than the young
   - If at all periods t, the old transfer $0.5 - \epsilon$ to the young, then agent $i = 0$ who is already old is worse off: there is no Pareto improvement

Summary

- To sum up

1. If the young are richer than the old, H and L are not Pareto optimal
2. If the young are as rich as the old, both H and L are Pareto optimal
3. If the old are richer than the young, H no longer exists and L is Pareto optimal

Taking Stock

- We have presented the simplest possible OLG model
- The model features complete markets and time zero trading
- We have reached a surprising conclusion: the competitive equilibrium is not necessarily Pareto efficient
- The invisible hand fails to work properly
- This leaves room for improving policies

5. Sequential Trading

Sequential Trading

- We abandon the time zero trading arrangement
- We switch to a sequential trading arrangement
- We maintain the assumption of complete markets

Durable Asset

- To adopt sequential trading, we need some durable good or asset
- This asset is traded by agents and passed from old to young generations
- This asset can take many forms
  - Money
  - Government bond
  - One-period Arrow security
  - A Lucas tree, ie a dividend stream like a “fruit” falling from a “tree” (Lucas 1978)

6. Money

Fiat Money

- We introduce fiat money as in the original OLG model of Samuelson (1958)
- Fiat money is a currency that has no intrinsic value
- *Fiat* is the Latin for “let it be done” (order, decree)
- Examples include the US dollar, the euro, the Brazilian real

Unbacked Currency

- Fiat money is inconvertible, ie it is not backed by any government promise to redeem it for gold or another good
- Its main role is to serve as a medium of exchange
- It is typically issued and declared legal tender by the state
- It has value only because people trust it, agree on its value

Inconvertible and Intrinsically Useless

"There are two widely accepted defining characteristics of fiat money: inconvertibility and intrinsic uselessness. Inconvertibility means that the issuer, if there is one, does not promise to convert the money into anything else—gold or wheat for example. Intrinsic uselessness means that fiat money is never wanted for its own stake; it is not legitimate to take fiat money to be an argument of anyone's utility or any engineering production function."

Neil Wallace, 1980, *The Overlapping Generations Model of Fiat Money*

Money for the Old

- We assume that at date $t = 1$, the initial old agents are endowed in the aggregate with $M > 0$ units of currency
- Think of some inheritance from the previous generation
- No one has promised to redeem the currency for goods
- The currency is intrinsically worthless

Backed by Expectations

- Despite the currency being intrinsically worthless, there is a system of expectations that gives the currency some value
- Currency is valued today if people expect it to be valued tomorrow
- Currency is backed by expectations without promises

The Young Want Money

- At each period $t \geq 1$, young agents purchase m_t^i units of currency at a price of $1/p_t$ units of the time t consumption good
- $p_t \geq 0$ is the time t price level
- At each period $t \geq 1$, each old consumer sells her money, ie exchanges her holdings of currency for the time t consumption good

Budget Constraint

- The budget constraints of a young agent born in $i \geq 1$ are

$$c_i^i + \frac{m_i^i}{p_i} \leq y_i^i$$

$$c_{i+1}^i \leq \frac{m_i^i}{p_{i+1}} + y_{i+1}^i$$

$$m_i^i \geq 0$$

- The last constraint forbids people from borrowing money

Equivalence

- Combine the two budget constraints of agent i

$$c_i^i + c_{i+1}^i \frac{p_{i+1}}{p_i} \leq y_i^i + y_{i+1}^i \frac{p_{i+1}}{p_i}$$

- Suppose we set

$$\frac{p_{i+1}}{p_i} = \alpha_i = \frac{q_{i+1}^0}{q_i^0}$$

- Then this budget constraint is identical to the one in the time 0 trading model without money, $q_i^0 c_i^i + q_{i+1}^0 c_{i+1}^i = q_i^0 y_i^i + q_{i+1}^0 y_{i+1}^i$

Household Problem

- Consumer $i \geq 1$ solves the problem

$$\max_{c_i^i, c_{i+1}^i, m_i^i} u(c_i^i) + u(c_{i+1}^i)$$

$$\text{subject to} \quad c_i^i + \frac{m_i^i}{p_i} \leq y_i^i$$

$$\text{and} \quad c_{i+1}^i \leq \frac{m_i^i}{p_{i+1}} + y_{i+1}^i$$

First-Order Condition

- Substitute the constraints into the objective

$$\max_{m_i^i} u\left(y_i^i - \frac{m_i^i}{p_i}\right) + u\left(\frac{m_i^i}{p_{i+1}} + y_{i+1}^i\right)$$

- The first-order condition with respect to m_i^i is

$$\frac{1}{p_i}u'\left(y_i^i - \frac{m_i^i}{p_i}\right) = \frac{1}{p_{i+1}}u'\left(\frac{m_i^i}{p_{i+1}} + y_{i+1}^i\right)$$

Monetary Equilibrium

A monetary equilibrium is a feasible allocation and a positive nominal price sequence $\{p_i\}_{i \geq 1}$ with $p_t < +\infty$ for all t such that given the price sequence, the allocation solves the household's problem for each $i \geq 1$

- A monetary equilibrium means that fiat money is valued
- If $p_t = +\infty$ we call it a nonmonetary equilibrium

Example

- Consider the same example as before with $\epsilon \in (0, 0.5)$, ie the young are richer than the old

$$
\begin{aligned}
y_i^i &= 1 - \epsilon, \quad \text{for all } i \geq 1 \\
y_{i+1}^i &= \epsilon, \quad \text{for all } i \geq 0 \\
y_t^i &= 0 \quad \text{otherwise}
\end{aligned}
$$

- We guess that a stationary equilibrium is to set

$$
p_i = p \quad \text{for all } i
$$

- Let's verify that this is an equilibrium

Two Solutions

- From the first-order condition

$$\frac{1}{p}u'\left(1-\epsilon-\frac{M}{p}\right)=\frac{1}{p}u'\left(\frac{M}{p}+\epsilon\right)$$

1. First solution, equilibrium 1

$$1-\epsilon-\frac{M}{p}=\frac{M}{p}+\epsilon \quad\Longrightarrow\quad p=\frac{2M}{1-2\epsilon}$$

2. Second solution, equilibrium 2

$$\frac{1}{p}=0 \quad\Longrightarrow\quad p=+\infty$$

Equilibrium 1 – Trade

- Equilibrium 1 is

$$p = \frac{2M}{1 - 2\epsilon}$$

- With $\epsilon < 0.5$ we have $p > 0$
- The budget constraints become for all i

$$\begin{aligned} c_i^i + \frac{M}{p} = y_i^i \quad &\Longrightarrow \quad c_i^i = 0.5 \\ c_{i+1}^i = \frac{M}{p} + y_{i+1}^i \quad &\Longrightarrow \quad c_{i+1}^i = 0.5 \end{aligned}$$

- Equilibrium 1 implies perfect consumption smoothing

Pareto Efficiency

- In equilibrium 1, money is valued by agents who hold it and trade it
- In period $t = i$, young agents i buy money from old agents $i - 1$
- In period $t + 1$, now-old agents i sell their money to new young agents $i + 1$
- Money is the asset that allows consumption smoothing
- This results in a Pareto efficient allocation

Equilibrium 2 – Autarchy

- Equilibrium 2 implies an infinite price level $p = \infty$
- Money has no value, agents don't want to hold it
- The budget constraints become

$$c_i^i = y_i^i, \qquad c_{i+1}^i = y_{i+1}^i, \qquad M = 0$$

- We are back to the autarkic case with no trade
- We know that this nonmonetary equilibrium is inefficient

Summary

- To sum up, with $\epsilon \in (0, 0.5)$ we characterized two equilibria

1. Monetary equilibrium: $p < \infty$
   - Money has value, it is used as an asset or storable good
   - Money restores Pareto efficiency
2. Nonmonetary equilibrium or autarchy: $p = +\infty$
   - Money has no value in equilibrium, agents don't hold it
   - The equilibrium is not efficient

7. Money and Infinite Lives

Flash Back

- Fiat money can have positive value in the OLG model
- What about in a world with infinitely-lived agents?
- We return to the competitive economy with infinite lives, complete markets, and time 0 trading of Arrow-Debreu securities, ie we wind back to lecture 6

Adding Money

- Agent i is endowed with $y_t^i(s^t)$ as before
- On top of that, agent i is endowed with a quantity of fiat money $m_t^i(s^t) \geq 0$ that also depends on the history of s^t
- The aggregate quantity of money in circulation is

$$\sum_i m_t^i(s^t) = M_t(s^t) > 0 \qquad \text{for all } t,\ s^t$$

Budget Constraint

- The individual budget constraint of agent i is

$$\sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t)c_t^i(s^t) \leq \sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t)y_t^i(s^t) + \sum_{t=0}^{\infty}\sum_{s^t}\frac{1}{p_t^0(s^t)}m_t^i(s^t)$$

- Summing over all agents $i = 1, 2, \ldots, I$

$$\sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t)\sum_{i}^{I} c_t^i(s^t) \leq \sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t)\sum_{i}^{I} y_t^i(s^t) + \sum_{t=0}^{\infty}\sum_{s^t}\frac{1}{p_t^0(s^t)}\sum_{i}^{I} m_t^i(s^t)$$

Infinite Price

- A feasible allocation is still

$$\sum_{i}^{I} c_t^i(s^t) = \sum_{i}^{I} y_t^i(s^t) \qquad \text{for all } t \text{ and } s^t$$

- Therefore it must be that

$$\sum_{t=0}^{\infty} \sum_{s^t} \frac{1}{p_t^0(s^t)} \sum_{i}^{I} m_t^i(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} \frac{1}{p_t^0(s^t)} M_t(s^t) = 0$$

- Since $M_t(s^t) > 0$, then we have $p_t^0(s^t) = +\infty$

Complete Markets and Finite Endowment

- The price level is infinite, money has no value
- The number of agents is finite, $i = 1, 2, \ldots, I$, and thus the value of the aggregate endowment is also finite
- In a competitive world with complete markets and a finite number of agents, fiat money has no value in equilibrium

Money and Finite Population

- For money and monetary policy to have a nontrivial role, we must move away from this framework, for example

1. Infinite number of agents, as in today's overlapping generations model
2. Intrinsic value for money, eg with money or wealth in the utility function
3. Incomplete markets, eg cash-in-advance constraints, transaction costs

Money Facilitates Exchange

"If inconvertibility and intrinsic uselessness are taken seriously, there is an immediate and long-standing problem: the devices usually invoked to prove that an object has value in equilibrium—basically, that supply is limited and that utility is increasing in the amount consumed—cannot be used for fiat money. Since getting fiat money to have value is necessary for any nontrivial theory of it, three options seem available: first, one can abandon inconvertibility and intrinsic uselessness; second, one can impose legal restrictions that give fiat money value; and third, one can attempt to model explicitly the notion that fiat money facilitates exchange. For good reasons, monetary theorists are almost unanimous in pursuing the third option."

Neil Wallace, 1980, *The Overlapping Generations Model of Fiat Money*

The Need for Some Friction

"In order to pursue the notion that fiat money facilitates exchange, one must abandon the costless multilateral market clearing implicit in the Walrasian (or Arrow-Debreu) general equilibrium model. Since exchange works perfectly in that model, there can be no role for a device that is supposed to facilitate exchange. In order to get a theory of fiat money, one must generalize the Walrasian model by including in it some sort of friction, something that will inhibit the operation of markets. On that there is agreement."

Neil Wallace, 1980, *The Overlapping Generations Model of Fiat Money*

8. Conclusion

Trade, Autarchy, and Pareto Inefficiency

- In this lecture we have presented the simplest OLG model
- We have derived two competitive stationary equilibria: trade and autarky
- Even with complete markets, both equilibria are generally not Pareto efficient, ie the first welfare theorem breaks down

1. With trade: the first generation could be better off
2. Autarky: all agents could be better off, aggregate endowment is infinite

Money Restores Efficiency

- Next, we have introduced fiat money
- We have shown that money can restore Pareto efficiency
- Intrinsically worthless money acquires value as an asset and is traded
- This contrasts with a world in which agents are infinitely-lived: there, fiat money has no value in equilibrium

9. Exercises

Exercise 1 – Fiat Currency

At each date $t \geq 1$, an economy consists of overlapping generations of a constant number N of two-period-lived agents. Young agents born in t have preferences over consumption streams of a single good that are ordered by $u(c_t^t) + u(c_t^{t+1})$, where $u(c) = c^{1-\gamma}/(1-\gamma)$ and where c_t^i is the consumption of an agent born at i in time t. It is understood that $\gamma > 0$, and that when $\gamma = 1$, $u(c) = \ln c$. Each young agent born at $t \geq 1$ has identical preferences and endowment pattern (w_1, w_2), where w_1 is the endowment when young and w_2 is the endowment when old. Assume $0 < w_2 < w_1$. In addition, there are some initial old agents at time 1 who are endowed with w_2 of the time 1 consumption good, and who order consumption streams by c_1^0. The initial old (ie the old at $t = 1$) are also endowed with M units of unbacked fiat currency. The stock of currency is constant over time.

Exercise 1 – Continued

1. Find the saving function of a young agent.
2. Define an equilibrium with valued fiat currency.
3. Define a stationary equilibrium with valued fiat currency.
4. Compute a stationary equilibrium with valued fiat currency.
5. Describe how many equilibria with valued fiat currency there are. (You are not being asked to compute them.)
6. Compute the limiting value as $t \to +\infty$ of the rate of return on currency in each of the nonstationary equilibria with valued fiat currency. Justify your calculations.

Exercise 2 – Heterogeneous Endowments

Consider an economy with overlapping generations of a constant population of an even number N of two-period-lived agents. New young agents are born at each date $t \geq 1$. Half of the young agents are endowed with w_1 when young and 0 when old. The other half are endowed with 0 when young and w_2 when old. Assume $0 < w_2 < w_1$. Preferences of all young agents are as in Exercise 1, with $\gamma = 1$. Half of the N initial old are endowed with w_2 units of the consumption good and half are endowed with nothing. Each old person orders consumption streams by c_1^0. Each old person at $t = 1$ is endowed with M units of unbacked fiat currency. No other generation is endowed with fiat currency. The stock of fiat currency is fixed over time.

Exercise 2 – Continued

1. Find the saving function of each of the two types of young person for $t \geq 1$.
2. Define an equilibrium without valued fiat currency. Compute all such equilibria.
3. Define an equilibrium with valued fiat currency.
4. Compute all the (nonstochastic) equilibria with valued fiat currency.
5. Argue that there is a unique stationary equilibrium with valued fiat currency.
6. How are the various equilibria with valued fiat currency ranked by the Pareto criterion?