12. Optimal Taxation

The Chamley-Judd Result

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Macroeconomics I, 2023

Introduction

- In the previous lecture we added fiscal policy to the standard growth model
- Both government spending and taxes were exogenous
- Today we keep government spending exogenous but we endogenize taxes
- We ask the question, what is the optimal mix of taxes that finances a given amount of government expenditure?

Lecture Outline

1. The Model
2. Ramsey Problem
3. Zero Capital Tax
4. Intuition
5. Primal Approach
6. Transition Period
7. Discussion
8. Redistribution
9. Exercises

Main Reference: Ljungqvist and Sargent, 2018, *Recursive Macroeconomic Theory*, Fourth Edition, Chapter 16

1. The Model

Introduction

- The model is the same as in the previous lecture
- We are still in a deterministic world, ie with no uncertainty
- There are three types of agents

1. Government
2. Households
3. Firms

Government

- The government purchases a stream of goods $\{g_t\}_{t=0}^{\infty}$
- g_t is exogenous; this is not about optimal public spending
- The government finances spending with three instruments
  - Flat-rate time-varying tax on capital earnings τ_t^k
  - Flat-rate time-varying tax on labor earnings τ_t^n
  - One-period government debt b_t, issued in $t-1$ and due in t
- We rule out lump-sum taxes τ_t and consumption taxes τ_t^c

Government

- The period t government budget constraint is

$$\underbrace{g_t + b_t}_{\text{Expenses}} = \underbrace{\tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t}}_{\text{Resources}}$$

- r_t is the rental rate of capital
- $R_t \equiv q_t / q_{t+1}$ is the gross rate of return on bonds held from t to $t+1$; interest earnings on bonds are exempt from tax
- Note that b_t is issued in $t-1$ and due in t, unlike the previous lecture where b_t was issued in t and due in $t+1$: this is pure choice of notation

Households

- A representative household solves

$$\max_{\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

subject to the budget constraint

$$\underbrace{c_t + k_{t+1} + \frac{b_{t+1}}{R_t}}_{\text{Expenses}} = \underbrace{(1 - \tau_t^n) w_t n_t + (1 - \tau_t^k) r_t k_t + (1 - \delta) k_t + b_t}_{\text{Resources}}$$

- This time there is no depreciation allowance on the gross rentals on capital

First-Order Conditions

- Let λ_t be the Lagrange multiplier on the constraint
- The first-order conditions are

$$
\begin{aligned}
c_t: &\quad u_c(t) = \lambda_t \\
n_t: &\quad u_\ell(t) = \lambda_t(1 - \tau_t^n)w_t \\
k_{t+1}: &\quad \lambda_t = \beta\lambda_{t+1}[(1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta] \\
b_{t+1}: &\quad \lambda_t \frac{1}{R_t} = \beta\lambda_{t+1}
\end{aligned}
$$

Rearranging the First-Order Conditions

- Combine the FOCs to substitute out for λ_t

$$\begin{aligned} u_\ell(t) &= u_c(t)(1-\tau_t^n)w_t \\ u_c(t) &= \beta u_c(t+1)[(1-\tau_{t+1}^k)r_{t+1}+1-\delta] \\ R_t &= (1-\tau_{t+1}^k)r_{t+1}+1-\delta \end{aligned}$$

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- The first FOC is the static leisure-consumption tradeoff
- The second FOC is the dynamic consumption-saving tradeoff
- The third FOC is a no-arbitrage condition between bonds and capital

Redundant Asset

- Households need only one type of asset – capital – to accomplish all intertemporal trades in a world without uncertainty
- Bonds are redundant: the no-arbitrage condition ensures that the two assets, bonds and capital, have the same return
- At the optimum, households are indifferent between investing in either one

$$\underbrace{R_t}_{\text{Return on bonds}} = \underbrace{(1-\tau_{t+1}^k)r_{t+1}+1-\delta}_{\text{Return on capital}}$$

Firms

- A representative firm maximizes profit

$$\Pi_t = F(k_t, n_t) - r_t k_t - w_t n_t$$

- The first-order conditions are

$$\begin{aligned} r_t &= F_k(t) \\ w_t &= F_n(t) \end{aligned}$$

- Inputs are employed until the marginal product of the last unit is equal to its rental price

Definitions

- A feasible allocation is a sequence $\{k_t, c_t, \ell_t, g_t\}_{t=0}^{\infty}$ that satisfies the resource constraint
$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t \quad \text{for all } t$$
- A price system is a 3-tuple of nonnegative bounded sequences $\{w_t, r_t, R_t\}_{t=0}^{\infty}$
- A government policy is a 4-tuple of sequences $\{g_t, \tau_t^k, \tau_t^n, b_t\}_{t=0}^{\infty}$

Competitive Equilibrium

A competitive equilibrium is a feasible allocation, a price system, and a government policy such that

1. Given the price system and government policy, the allocation solves both the household and firm problems
2. Given the allocation and price system, the policy satisfies the sequence of government budget constraints

2. Ramsey Problem

Many Policy Paths

- For any given path of government spending $\{g_t\}_{t=0}^{\infty}$, there exist many combinations of possible tax and debt policies $\{\tau_t^k, \tau_t^n, b_t\}_{t=0}^{\infty}$
- Each of these policies generates a different competitive equilibrium
- Which among these many equilibria should we choose?
- The Ramsey problem is here to provide an answer

Ramsey Problem

- The Ramsey problem consists in selecting the competitive equilibrium that maximizes agents' utility $\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$
- The Ramsey problem is different from the central planner's problem
- The central planner "forces" agents to behave – consume, invest, work, hire, produce – in a way that maximizes welfare
- The Ramsey problem selects a policy – here a tax policy – that is compatible with the competitive equilibrium and that maximizes welfare

FOCs Become Constraints

- In the the Ramsey problem, we maximize household utility subject to

1. All the constraints: the aggregate resource constraint, the household budget constraint, and the government budget constraint
2. All the optimality conditions: the two household FOCs and two firm FOCs

- The Ramsey problem thus chooses a policy that takes into account the response of the decentralized economy (households, firms) to that policy

Restrictions

- To make the problem interesting, we impose restrictions

1. There is no lump-sum tax available to the government: since they are non distortionary, the government would use only lump-sum taxes if it could
2. There is no taxation of the initial capital stock, $\tau_0^k = 0$: since k_0 is in fixed supply, taxing it via a so-called capital levy would be like a lump-sum tax

3. Zero Capital Tax

After-Tax Prices

- Following Chamley (1986, *Econometrica*), we formulate the Ramsey problem in a particular way
- Instead of choosing tax rates τ_t^k and τ_t^n, the government chooses the after-tax rental rate of capital $\tilde{r}_t$ and the after-tax wage rate $\tilde{w}_t$, defined as

$$\tilde{r}_t \equiv (1 - \tau_t^k) r_t$$
$$\tilde{w}_t \equiv (1 - \tau_t^n) w_t$$

- As long as r_t and w_t are compatible with the competitive equilibrium, choosing $\tilde{r}_t$ and $\tilde{w}_t$ is equivalent to choosing τ_t^k and τ_t^n, respectively

Government Budget Constraint

- Using the definitions of $\tilde{r}_t$ and $\tilde{w}_t$, write tax revenues as

$$\begin{aligned}\tau_t^k r_t k_t + \tau_t^n w_t n_t &= (r_t - \tilde{r}_t)k_t + (w_t - \tilde{w}_t)n_t \\ &= F_k(t)k_t + F_n(t)n_t - \tilde{r}_t k_t - \tilde{w}_t n_t \\ &= F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t\end{aligned}$$

- The second line uses the firm's FOCs; the third line uses Euler's theorem
- Substitute this into the government budget constraint

$$g_t = F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t$$

Reduction

- We just plugged the two firm's FOCs into the government budget constraint
- Also, when both the government budget constraint and resource constraint hold, the household budget constraint is redundant by Warlas's law
- We are therefore left with four constraints: 1) government budget constraint; 2) resource constraint; 3) household FOC 1; 4) household FOC 2

Lagrangian of the Ramsey Problem

- Write a Lagrangian

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t, 1-n_t) &+ \Psi_t \left[F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t - g_t \right] \\
&+ \theta_t [F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}] \\
&+ \mu_{1t} [u_\ell(t) - u_c(t) \tilde{w}_t] \\
&+ \mu_{2t} [u_c(t) - \beta u_c(t+1)(\tilde{r}_{t+1} + 1 - \delta)] \Big\}
\end{aligned}
$$

- $\Psi_t, \theta_t, \mu_{1t}, \mu_{2t}$ are Lagrange multipliers on the government budget constraint, resource constraint, and two household FOCs, respectively

Optimal Capital

- The first-order condition with respect to capital k_{t+1} is

$$\theta_t = \beta\theta_{t+1}[F_k(t+1) + 1 - \delta] + \beta\Psi_{t+1}[\underbrace{F_k(t+1) - \tilde{r}_{t+1}}_{\tau^k_{t+1} r_{t+1}}] \tag{1}$$

- θ_t is the social marginal value of a rise in investment in period t
- This rise in investment in t increases available goods at time $t+1$ by the amount $[F_k(t+1) + 1 - \delta]$, which has social marginal value $\beta\theta_{t+1}$
- It also increase tax revenues by $[F_k(t+1) - \tilde{r}_{t+1}]$, which enables the government to reduce debt or other taxes, and has social value $\beta\Psi_{t+1}$

Steady State

- Suppose that government expenditure stays constant after some period
- Suppose that the solution to the Ramsey problem converges to a steady state, ie all variables remain constant

Combining the FOCs

- Use the firm's FOC for capital $r = F_k$ to rewrite equation (1) in steady state

$$\theta = \beta[\theta(r + 1 - \delta) + \Psi(r - \tilde{r})]$$

- The household's intertemporal FOC (Euler equation) in steady state is

$$1 = \beta(\tilde{r} + 1 - \delta)$$

- Combine these two equations

$$(\theta + \Psi)(r - \tilde{r}) = 0$$

Zero Capital Tax

- The marginal social value of goods is strictly positive, $\theta > 0$
- The marginal social value of reducing debt or taxes is nonnegative, $\Psi \geq 0$
- Therefore it must be that $r - \tilde{r} = 0$ and thus

$$r = \tilde{r} = (1 - \tau^k)r \quad \text{so that} \quad \tau^k = 0$$

- The optimal capital income tax in steady state is zero

Optimal Long Run Taxation

Proposition: if there exists a steady-state Ramsey allocation, the associated limiting tax on capital is zero

- This result holds only for the limiting steady state, ie the (very) long run
- This analysis says nothing about how much we should tax capital in the transition period

4. Intuition

A Strong Result

- The Ramsey approach generates a strong result
- Capital income taxes should be zero in the long run
- This was first shown independently by Chamley (1986) and Judd (1985)
- This is surprising and goes against the intuition of many economists that capital taxes are a useful policy tool

A Surprising Result

"Economic reasoning sometimes holds its surprises. The Chamley-Judd result was not anticipated by economists' intuitions, despite a large body of work at the time on the incidence of capital taxation and on optimal tax theory more generally. It represented a major watershed from a theoretical standpoint. One may even say that the result is puzzling, as witnessed by the fact that economists have continued to take turns putting forth various intuitions to interpret it, none definitive nor universally accepted."

Ludwig Straub and Iván Werning, 2020, *American Economic Review*

Intuition

- So what is the economic intuition behind this result?
- There are at least two ways to see it

1. Tax on future consumption or tax wedge
2. Infinitely elastic capital supply in the long run

Intuition 1 – Tax on Future Consumption

- Capital is simply future consumption
- A positive capital tax in period t means the government is taxing consumption in period $t+1$ at a higher rate than consumption in t
- The difference cumulates exponentially over time: as $T \to \infty$, consumption in $t+T$ is taxed at a rate infinitely higher than consumption in t

"Such an extreme level of distortion is suboptimal"

Narayana Kocherlakota, 2010, *The New Dynamic Public Finance*

Tax Wedge

- To be concrete: invest one dollar today at annual risk-free interest rate r
- Without any capital income tax, the investment is worth

$$(1+r)^T \quad \text{after } T \text{ years}$$

- With annual capital income tax rate $\tau^k > 0$, the investment is worth

$$[1+(1-\tau^k)r]^T \quad \text{after } T \text{ years}$$

Tax Wedge

- Define a tax wedge as

$$\text{wedge} = 1 - \frac{[1 + (1 - \tau^k) r]^T}{(1 + r)^T}$$

- Set for example $r = .05$ and $\tau^k = 0.3$

$$\text{wedge} = 13.4\% \text{ for } T = 10 \qquad \text{wedge} = 43.8\% \text{ for } T = 40$$

- A constant capital income tax creates a growing tax wedge between current and future consumption, leading to suboptimal accumulation of capital

Supply-Side Economics

"A principle in Ramsey's analysis is that goods that appear symmetrically in consumer preferences should be taxed at the same rate—taxes should be spread evenly over similar goods. . . . Since capital taxation applied to new investment involves taxing later consumption at heavier rates than early consumption, this principle implies that capital is a bad thing to tax."

Robert Lucas, 1990, "Supply-Side Economics: An Analytical Review"

Intuition 2 – Elastic vs Inelastic Goods

- Take a good X_t that is in perfect inelastic supply or demand
- The equilibrium quantity of good X_t that is produced and sold does not depend on its price P_t^X
- Taxing X_t does not affect the quantity produced

Intuition 2 – Tax Inelastic Goods

- Now take a good Y_t that is in very elastic supply or demand
- The supply or demand of Y_t is very sensitive to the price P_t^Y
- Taxing Y_t reduces a lot the quantity produced

Upshot: one should tax inelastic goods (X_t) more than elastic ones (Y_t)

Relatively Inelastic Labor Supply

- In the model, labor demand is downward-sloping in the number of hours n_t
- Labor supply is limited by the curvature of the labor disutility function or by the time constraint if labor supply is perfectly inelastic
- Thus neither labor demand nor labor supply are perfectly elastic

Perfectly Elastic Capital Supply

- Capital demand is downward-sloping in the level of capital k_t
- Capital supply depends on the rental rate r_t; in steady state

$$r = \frac{\beta^{-1} - (1 - \delta)}{1 - \tau^k}$$

- The rental rate does not depend on the level of capital
- Thus capital supply is perfectly elastic in the steady state

Do Not Tax the Most Elastic Input

- By taxing capital, the government taxes the most elastic of the two inputs
- The capital tax limits the supply and thus the accumulation of capital, leading to lower output, lower consumption, and thus lower welfare
- To be sure, taxing labor also reduces hours, production, and consumption, but the distortion is smaller because labor is less elastic

5. Primal Approach

Primal Approach

- So far we formulated the Ramsey problem following Chamley (1986)
- We reduced two pairs of taxes (τ_t^k, τ_t^n) and prices (r_t, w_t) to just one pair of numbers $(\tilde{r}_t, \tilde{w}_t)$ using equilibrium outcomes in factor markets
- We now introduce the primal approach, the most common in the literature
- The idea is to eliminate all prices and taxes altogether (in contrast with the dual approach where we search directly for optimal taxes)

Same Concept

- The government directly chooses a feasible allocation subject to

1. Constraints that ensure prices and taxes exist
2. Optimality conditions, ie household and firm FOCs

- In this way, the chosen allocation is consistent with the optimization behavior of households and firms

Sequential Budget Constraint

- With complete markets, we can express the sequence of time t budget constraints into a single present-value constraint
- Merge two consecutive constraints by eliminating b_{t+1}

$$c_t + \frac{c_{t+1}}{R_t} + \frac{k_{t+2}}{R_t} + \frac{b_{t+2}}{R_t R_{t+1}} = (1-\tau_t^n) w_t n_t + \frac{(1-\tau_{t+1}^n) w_{t+1} n_{t+1}}{R_t}$$
$$+ \left[\frac{(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta}{R_t} - 1 \right] k_{t+1} + (1-\tau_t^k) r_t k_t + (1-\delta) k_t + b_t$$

- By arbitrage, the term multiplying k_{t+1} must equal zero

Rewriting the Sequential Budget Constraint

- Iterate the process to eliminate successive b_{t+j}

$$\sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\frac{1}{R_i}\right)c_t=\sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\frac{1}{R_i}\right)(1-\tau_t^n)w_t n_t+[(1-\tau_0^k)r_0+1-\delta]k_0+b_0$$

where we have imposed the transversality conditions

$$\lim_{T\to\infty}\left(\prod_{i=0}^{t-1}\frac{1}{R_i}\right)k_{T+1}=0 \quad \text{and} \quad \lim_{T\to\infty}\left(\prod_{i=0}^{t-1}\frac{1}{R_i}\right)\frac{b_{T+1}}{R_T}=0$$

Time Zero Budget Constraint

- Recall, q_t^0 is the Arrow-Debreu price of a claim bought in 0 and delivered in t

$$q_t^0 = q_0^0 \frac{q_1^0}{q_0^0} \frac{q_2^0}{q_1^0} \dots \frac{q_t}{q_{t-1}} = q_0^0 \frac{1}{R_0} \frac{1}{R_1} \dots \frac{1}{R_{t-1}} = \prod_{i=0}^{t-1} \frac{1}{R_i} \quad \text{with numeraire } q_0^0 = 1$$

- The present-value, time 0 budget constraint thus writes

$$\sum_{t=0}^{\infty} q_t^0 c_t = \sum_{t=0}^{\infty} q_t^0 (1 - \tau_t^n) w_t n_t + [(1 - \tau_0^k) r_0 + 1 - \delta] k_0 + b_0$$

Steps of the Primal Approach

1. Obtain the household's and firm's FOCs as well as any arbitrage pricing condition and solve for $\{q_t^0, r_t, w_t, \tau_t^k, \tau_t^n\}_{t=0}^{\infty}$ as functions of $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$
2. Substitute these tax and price expressions into the household time 0 budget constraint to obtain an "implementability condition"
3. Maximize utility subject to the aggregate resource constraint and the implementability condition derived in step 2
4. Use the formulas from step 1 to find taxes and prices

Step 1 – Derive First-Order Conditions

- The household solves

$$\max_{\{c_t\},\{n_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

$$\text{subject to } \sum_{t=0}^{\infty} q_t^0 c_t = \sum_{t=0}^{\infty} q_t^0 (1 - \tau_t^n) w_t n_t + [(1 - \tau_0^k) r_0 + 1 - \delta] k_0 + b_0$$

- Let λ be the Lagrange multiplier on the budget constraint

Household FOCs

- The first-order conditions are

$$
\begin{aligned}
c_t: &\quad \beta^t u_c(t) - \lambda q_t^0 = 0 \\
n_t: &\quad -\beta^t u_\ell(t) + \lambda q_t^0 (1-\tau_t^n) w_t = 0
\end{aligned}
$$

- With numeraire $q_0^0 = 1$, we have $\beta^0 u_c(0) = \lambda$, and thus

$$
\begin{aligned}
q_t^0 &= \beta^t \frac{u_c(t)}{u_c(0)} \\
(1-\tau_t^n) w_t &= \frac{u_\ell(t)}{u_c(t)}
\end{aligned}
$$

No-Arbitrage Condition and Firm FOCs

- The no-arbitrage condition is

$$\frac{q_t^0}{q_{t+1}^0} = (1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta$$

- Firm profit maximization implies

$$\begin{aligned} r_t &= F_k(t) \\ w_t &= F_n(t) \end{aligned}$$

Step 2 – Compute the Implementability Condition

- To obtain the implementability condition, substitute the FOCs $q_t^0 = \beta^t \frac{u_c(t)}{u_c(0)}$, $(1-\tau_t^n)w_t = \frac{u_\ell(t)}{u_c(t)}$, and $r_0 = F_k(0)$ into the time 0 budget constraint

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t - u_\ell(t)n_t] - A = 0$$

$$\text{where} \quad A = A(c_0, n_0, \tau_0^k, b_0) = u_c(0)\{[(1-\tau_0^k)F_k(0) + 1 - \delta]k_0 + b_0\}$$

- The terms in A are all given at time 0, ie they are not choices

Step 3 – Solve the Ramsey Program

- The Ramsey problem is to choose an allocation that maximizes utility subject to the implementability condition and the feasibility constraint
- Let Φ be a multiplier on the implementability condition and define

$$V(c_t, n_t, \Phi) \;=\; \underbrace{u(c_t, 1-n_t)}_{\text{Utility}} + \Phi \times \underbrace{\left[u_c(t)c_t - u_\ell(t)n_t\right]}_{\text{Implementability}}$$

Lagrangian

- Form a Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \big\{ V(c_t, n_t, \Phi) + \theta_t [F(k_t, n_t) + (1-\delta)k_t - c_t - g_t - k_{t+1}] \big\} - \Phi A$$

- $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of multipliers on the sequence of resource constraints
- Φ is the unique multiplier on the unique implementability condition

First-Order Conditions

- The first-order conditions are

$$
\begin{aligned}
c_t &: \quad V_c(t) = \theta_t, \qquad t \geq 1 \\
n_t &: \quad V_n(t) = -\theta_t F_n(t), \qquad t \geq 1 \\
k_{t+1} &: \quad \theta_t = \beta\theta_{t+1}[F_k(t+1) + 1 - \delta], \qquad t \geq 0 \\
c_0 &: \quad V_c(0) = \theta_0 + \Phi A_c \\
n_0 &: \quad V_n(0) = -\theta_0 F_n(0) + \Phi A_n
\end{aligned}
$$

Final System

- Substitute out for θ_t

$$\begin{aligned}
V_c(t) &= \beta V_c(t+1)[F_k(t+1)+1-\delta], \qquad t \geq 1 \\
V_n(t) &= -V_c(t)F_n(t), \qquad t \geq 1 \\
V_c(0) - \Phi A_c &= \beta V_c(1)[F_k(1)+1-\delta] \\
V_n(0) &= [\Phi A_c - V_c(0)]F_n(0) + \Phi A_n
\end{aligned}$$

- To this system, we add the two constraints

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta)k_t$$

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t - u_\ell(t)n_t] - A = 0$$

Solving the System

- We seek an allocation $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ and a multiplier Φ that satisfies this system of difference equations
- We can solve the system iteratively
  - Fix an arbitrary Φ and solve the FOCs and resource constraint
  - Check the implementability condition
  - Raise or lower Φ depending on if the budget is in deficit or surplus
  - Iterate until convergence

Step 4 – Compute Taxes and Prices

- Once we find an allocation, we obtain $\{q_t^0, r_t, w_t, \tau_t^n, \tau_t^k\}_{t=0}^{\infty}$ using

$$q_t^0 = \beta^t \frac{u_c(t)}{u_c(0)} \qquad r_t = F_k(t) \qquad w_t = F_n(t)$$

$$(1 - \tau_t^n) w_t = \frac{u_\ell(t)}{u_c(t)}$$

$$\frac{q_t^0}{q_{t+1}^0} = (1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta$$

Steady State

- Consider the steady state where after some time, we assume $g_t = g$ and we suppose that the Ramsey problem converges to constant c, n, and k
- The capital FOC $V_c(t) = \beta V_c(t+1)[F_k(t+1) + 1 - \delta]$ in steady state becomes

$$1 = \beta(F_k + 1 - \delta)$$

- Equations $q_t^0 = \beta^t \frac{u_c(t)}{u_c(0)}$ and $q_{t+1}^0 = \beta^{t+1} \frac{u_c(t+1)}{u_c(0)}$ in steady state imply

$$\frac{q_t^0}{q_{t+1}^0} = \beta^{-1}$$

Revisiting a Zero Capital Tax

- The no-arbitrage condition $\frac{q_t^0}{q_{t+1}^0} = (1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta$ in steady state is

$$1 = \beta[(1 - \tau^k) F_k + 1 - \delta]$$

- Combine $1 = \beta(F_k + 1 - \delta)$ and $1 = \beta[(1 - \tau^k) F_k + 1 - \delta]$

$$\tau^k = 0$$

- We reach the same conclusion: the optimal capital tax is zero in steady state

6. Transition Period

The Long Run Is Too Long

- The Chamley-Judd result applies to the steady state: we interpret this result as saying that capital taxes should be driven to zero in the long run
- But when exactly is the long run?
- Political cycles are typically short (4 to 8 years), so we would like to say something about the short term, ie the transition to the steady state

"In the long run we are all dead"

John Maynard Keynes, 1923, *A Tract on Monetary Reform*

Extra Assumption

- Same framework but we make one extra assumption
- We assume a CRRA, ie power, utility function

$$u(c_t, 1 - n_t) = \frac{c_t^{1-\sigma}}{1 - \sigma} + v(n_t), \qquad \sigma > 0$$

- With this form, it turns out we get a much more policy-relevant result

Derivatives

- With CRRA utility, we have $\frac{u_c(t)}{u_c(t+1)} = \frac{c_t^{-\sigma}}{c_{t+1}^{-\sigma}}$
- Recalling $V(c_t, n_t, \Phi) = u(c_t, 1-n_t) + \Phi[u_c(t)c_t - u_\ell(t)n_t]$, we derive

$$\frac{V_c(t)}{V_c(t+1)} = \frac{c_t^{-\sigma}(1-\Phi\sigma+\Phi)}{c_{t+1}^{-\sigma}(1-\Phi\sigma+\Phi)} = \frac{c_t^{-\sigma}}{c_{t+1}^{-\sigma}}$$

- Thus we have

$$\frac{V_c(t)}{V_c(t+1)} = \frac{u_c(t)}{u_c(t+1)} \quad \text{for all } t \geq 1$$

A Much Stronger Result

- From the household's problem, we derived

$$\frac{q_t^0}{q_{t+1}^0} = \frac{u_c(t)}{\beta u_c(t+1)} = (1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta$$

- From the primal approach, we derived

$$\frac{V_c(t)}{\beta V_c(t+1)} = r_{t+1} + 1 - \delta \quad \text{for all } t \geq 1$$

- Mixing both by using the result $\frac{V_c(t)}{V_c(t+1)} = \frac{u_c(t)}{u_c(t+1)}$, we obtain

$$\tau_{t+1}^k = 0 \quad \text{for all } t \geq 1 \quad \text{or} \quad \tau_t^k = 0 \quad \text{for all } t \geq 2$$

Optimal Taxation in the Transition

- With a power utility function, assuming the economy converges to a steady state, the optimal tax rate on capital income is zero for all periods $t \geq 2$
- One could still tax capital in period $t = 1$
- Agents willingly accumulate capital in period 1 that is taxed because they know that eventually (from period 2 onward) capital is free of taxes

7. Discussion

Don't Tax Capital

- The Chamley-Judd result of zero-capital-tax is often associated to the Atkinson-Stiglitz theorem
- Together, these two theoretical findings are the bedrock of the academic arguments against capital taxation
- Let's describe briefly the Atkinson-Stiglitz theorem

Different Abilities But Homogeneous Preferences

- Seminal paper by Atkinson and Stiglitz (1976, *Journal of Public Economics*)
- The authors build a two-period model in which heterogeneous workers work in the first period and retire in the second; the model assumes that
  - Workers have different earning abilities
  - All consumers have the same consumption subutility
  - Preferences are separable between consumption and labor
  - The government can use nonlinear income taxation

Atkinson-Stiglitz Theorem

- The Atkinson-Stiglitz theorem says that under the previous assumptions, differential taxation of first- and second-period consumption is not optimal
- In other words, if nonlinear income taxation is an option, the government should use only this option and should not employ indirect taxes
- Note that in a model with only two periods, differential consumption taxation is the same thing as capital taxation

Intuition

- Workers are motivated by their after-tax earnings
- With nonlinear income taxation, the government can discriminate between workers with high earning abilities and workers with low earning abilities
- Differential consumption taxation cannot yield any further distinction beyond what the income tax already achieves
- But it has an added efficiency cost from distorting spending choices
- Conclusion: it is not optimal

A Hot Debate

- The Chamley-Judd and Atkinson-Stiglitz results have generated a great deal of controversy in the literature
- Many researchers have opposed or defended these findings
- Two broad research communities study optimal taxation
  1. The public economics or public finance community
  2. The macro or general-equilibrium community
- As of today the debate is not settled

Assumptions

- The Chamley-Judd result rests on a number of (unrealistic) assumptions
  - Identical households
  - Infinitely-lived households
  - Time-additively separable utility
  - Constant returns-to-scale production function
  - Competitive markets
  - Complete set of linear, ie flat-rate, taxes
  - Steady-state growth not affected by taxes
  - Closed economy
- Which of these assumptions can be relaxed? The literature is mixed

Zero Capital Tax Is Optimal

- The following papers find zero long-run capital tax is optimal when we have
- Heterogeneous agents: Atkeson, Chari, and Kehoe (1999), Werning (2007), Greulich, Laczo, and Marcet (2022)
- Overlapping generations: Atkeson, Chari, and Kehoe (1999), Garriga (2019)
- Endogenous growth or human capital: Lucas (1990), Jones et al. (1997)
- Uncertainty: Zhu (1992), Judd (1993), Chari, Christiano, and Kehoe (1994)
- An open economy: Razin and Sadka (1995)
- A rich set of taxes: Chari, Nicolini, and Teles (2020)

Policy Recommendation

"This approach has produced a substantive lesson for policymakers: In the long run, in a broad class of environments, the optimal tax on capital income is zero. With further restrictions on our model, this result applies to the short run as well. Theoretically, that is, our result concurs with that of Chamley (1986): taxing capital income is a bad idea."

Andrew Atkeson, V. V. Chari, and Patrick Kehoe, 1999, "Taxing Capital Income: A Bad Idea", *Quarterly Review*

8. Redistribution

Redistribution

- Let's extend the model to include heterogeneous agents
- On top of financing government spending, there is now another motive for taxes: to redistribute resources from rich households to poor households

Households

- There are N classes of agents indexed by $i = 1, \ldots, N$
- Each class has utility u^i

$$\sum_{t=0}^{\infty} \beta^t u^i(c_t^i, 1 - n_t^i)$$

- Each class receives a lump-sum transfer $s_t^i \geq 0$ from the government that plays the role of redistribution
- The budget constraint of class i is

$$c_t^i + k_{t+1}^i = (1 - \tau_t^n) w_t n_t^i + (1 - \tau_t^k) r_t k_t^i + (1 - \delta) k_t^i + s_t^i$$

Government

- The government attributes a weight $\alpha^i \geq 0$ to each of its taxpaying class i
- We assume the government cannot issue debt or save

$$b_t = 0 \qquad \text{for all } t$$

- The government's flow budget constraint is

$$g_t + \sum_{i=1}^{N} s_t^i = \sum_{i=1}^{N} \tau_t^k r_t k_t^i + \sum_{i=1}^{N} \tau_t^n w_t n_t^i$$

Ramsey Problem

- We solve the Ramsey problem as in Chamley (1986)

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \sum_{i=1}^{N} \alpha^i u^i(c_t^i, 1 - n_t^i) &+ \Psi_t \left[F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t - g_t - s_t \right] \\
&+ \theta_t [F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}] \\
&+ \sum_{i=1}^{N} \varepsilon_t^i [\tilde{w}_t n_t^i + \tilde{r}_t k_t^i + (1-\delta) k_t^i + s_t^i - c_t^i - k_{t+1}^i] \\
&+ \sum_{i=1}^{N} \mu_{1t}^i [u_\ell^i(t) - u_c^i(t) \tilde{w}_t] \\
&+ \sum_{i=1}^{N} \mu_{2t}^i [u_c^i(t) - \beta u_c^i(t+1)(\tilde{r}_{t+1} + 1 - \delta)] \Bigg\}
\end{aligned}
$$

Optimal Capital

- The FOC with respect to capital k_{t+1}^i is

$$\theta_t + \varepsilon_t^i = \beta \left\{ \theta_{t+1}[F_k(t+1) + 1 - \delta] + \Psi_{t+1}[F_k(t+1) - \tilde{r}_{t+1}] + \varepsilon_{t+1}^i(\tilde{r}_{t+1} + 1 - \delta) \right\}$$

- Compared to the representative-agent case, a new term ε_t^i appears, which represents the social marginal value of redistribution

Steady State

- Suppose fiscal policy is constant after a certain period and the Ramsey solution converges to a steady state
- The previous equation in steady state writes

$$\theta + \varepsilon^i = \beta[\theta(r + 1 - \delta) + \Psi(r - \tilde{r}) + \varepsilon^i(\tilde{r} + 1 - \delta)]$$

- The household's intertemporal FOC in steady state is

$$1 = \beta(\tilde{r} + 1 - \delta)$$

Zero Capital Tax

- Combine the last two equations

$$(\theta + \Psi)(r - \tilde{r}) = 0 \quad \text{so that} \quad \tau^k = 0$$

- The optimal capital income tax in steady state is zero
- Even with a redistribution motive, the result holds

Intuition

- By taxing capital, we make the economy smaller
- Therefore, all agents are worse off
- Even if we want to redistribute from rich to poor, we should not tax capital
- Let's see an extreme example of this: capitalists vs workers

Workers and Capitalists

- We take the original model of Judd (1985, *Journal of Public Economics*)
- There are two classes of agents: workers and capitalists
- Workers do not own capital, they just earn labor income
- Capitalists do not work, they just enjoy capital income (rentiers)

A Socialist Government

- Workers' (type 1) budget constraint is

$$c_t^1 = \tilde{w}_t n_t^1 + s_t^1$$

- Capitalists' (type 2) budget constraint is

$$c_t^2 + k_{t+1}^2 = \tilde{r}_t k_t^2 + (1-\delta)k_t^2 + s_t^2$$

- We assume the government is from the worker's party and cares only about the workers

$$\alpha^1 = 1 \quad \textbf{and} \quad \alpha^2 = 0$$

Ramsey Problem

- The Lagrangian writes

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & u^1(c_t^1, 1 - n_t^1) + 0 \times u^2(c_t^2) \\
& + \Psi_t \left[F(k_t, n_t) - \tilde{r}_t k_t - \tilde{w}_t n_t - g_t - s_t^1 - s_t^2 \right] \\
& + \theta_t [F(k_t, n_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}] \\
& + \varepsilon_t^1 [\tilde{w}_t n_t^1 + s_t^1 - c_t^1] \\
& + \varepsilon_t^2 [\tilde{r}_t k_t^2 + (1-\delta) k_t^2 + s_t^2 - c_t^2 - k_{t+1}^2] \\
& + \mu_{1t}^1 [u_\ell(t) - u_c(t) \tilde{w}_t] \\
& + \mu_{2t}^2 [u_c^2(t) - \beta u_c^2(t+1)(\tilde{r}_{t+1} + 1 - \delta)] \Big\}
\end{aligned}
$$

Optimal Capital

- The FOC with respect to capital k^2_{t+1} is

$$\theta_t + \varepsilon^2_t = \beta \left\{\theta_{t+1}[F_k(t+1) + 1 - \delta] + \Psi_{t+1}[F_k(t+1) - \tilde{r}_{t+1}] + \varepsilon^2_{t+1}(\tilde{r}_{t+1} + 1 - \delta)\right\}$$

- In steady state, this becomes

$$\theta + \varepsilon^2 = \beta[\theta(r + 1 - \delta) + \Psi(r - \tilde{r}) + \varepsilon^2(\tilde{r} + 1 - \delta)]$$

- The capitalists' intertemporal FOC in steady state is

$$1 = \beta(\tilde{r} + 1 - \delta)$$

No Expropriation

- The Chamley-Judd result holds

$$\tau^k = 0$$

- The government should only tax the workers (!) to finance its spending

$$s^1 + s^2 + g = \tau^n w n^1$$

- Even with a government that redistributes fully to workers, the optimal long-run capital income tax is zero, ie capitalists are not expropriated

To Be Continued

- More about optimal taxation in lecture 13

9. Exercises

Exercise 1 – Labor Income Taxation

Consider the following problem of optimal taxation with commitment. The representative household has preferences

$$\sum_{t=0}^{\infty} \beta^t [c_t + \kappa g_t - v(n_t)]$$

where $g_t > 0$ is government spending, $\kappa > 1$ and $v(n)$ is thrice continuous-differentiable, with $v' \geq 0$, $v'' > 0$, $v''' > 0$, $v'(0) = 0$ and $v'(1) = \infty$ (these assumptions guarantee an interior solution for n). In a competitive equilibrium the household takes as given the sequence $\{g_t, w_t, \tau_t\}_{t=0}^{\infty}$ and faces a time 0 budget constraint

$$\sum_{t=0}^{\infty} q_t^0 [c_t - (1 - \tau_t) w_t n_t] = 0$$

Exercise 1 – Continued

The government maximizes households' utility by choosing a sequence $\{\tau_t, g_t\}$ subject to a time 0 budget constraint

$$\sum_{t=0}^{\infty} q_t^0 [\tau_t w_t n_t - g_t] = 0$$

The production function is linear in labor, the only input. The resource constraint is

$$c_t + g_t = n_t$$

1. Use the primal approach to characterize the optimal allocation $\{c_t^*, n_t^*, g_t^*\}_{t=0}^{\infty}$ that is compatible with a competitive equilibrium.
2. What happens with the optimal labor tax rate when $\kappa \to 1$?
3. Is the optimal policy time-consistent? Give some intuition.

Exercise 2 – A Small Open Economy

Razin and Sadka (1995). Consider the model in the course, but assume that the economy is a small open economy that cannot affect the international rental rate on capital, r_t^*. Domestic firms can rent any amount of capital at this price, and the households and the government can choose to go short or long in the international capital market at this rental price. There is no labor mobility across countries. We retain the assumption that the government levies a tax τ_t^n on each household's labor income, but households no longer have to pay taxes on their capital income. Instead, the government levies a tax $\hat{\tau}_t^k$ on domestic firms' rental payments to capital regardless of the capital's origin (domestic or foreign). Thus, a domestic firm faces a total cost of $(1 + \hat{\tau}_t^k) r_t^*$ on capital rented in period t.

1. Solve for the optimal capital tax $\hat{\tau}_t^k$.
2. Compare the optimal tax policy of this small open economy to that of the closed economy.