

# Simulating lower-dose scans from an available CT scan

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## ABSTRACT

Low-dose CT scans can be obtained by reducing the radiation dose to the patient; however, lowering the dose results in a lower signal-to-noise ratio and therefore also in a reduced image quality. In this research, we aim to develop a tool to simulate a reduced-dose scan from an existing standard-dose scan. The motivation for simulating a reduced-dose scan is to determine how much the dose can be reduced without losing the relevant information required for proton treatment planning. The method estimates the noise equivalent number of photons in the sinogram and applies a thinning to reduce that number. The method accounts for the bowtie filter, for the noise correlation between neighboring detector elements and for the fact that for the same image intensity, a harder beam has fewer photons and therefore a higher variance. The proposed model shows a close agreement between the variance in the observed and in the simulated lower-dose scans. Simulations of low-dose scans of a 21 cm and a 6 cm water phantom in a range from 300 to 20 mAs show that the noise variance of the reconstructed images matches the reconstructions from the real scans with less than 5% error.

**Keywords:** low-dose simulation, X-ray computed tomography, noise modeling, noise correlation, beam hardening

## 1. INTRODUCTION

Proton therapy is an advanced form of radiotherapy which utilizes proton beams to destroy cancer cells. Protons might be a better option over photons due to their maximum dose delivery to the tumor cells and significant dose reduction to healthy tissue.<sup>1</sup> However, any uncertainty on the range of the protons may have dramatic consequences for the patient. Consequently, it is important to do adaptive planning to make sure that the compositions and densities of the tissues are the same as those assumed during treatment planning. This requires a CT scan in each treatment session which involves a substantial additional dose to the patients. Therefore, we aim to develop a tool to simulate a lower-dose CT scan from an existing standard-dose scan which can be used to determine the lowest possible radiation dose that still produces sufficient information for the treatment planning.

Lower-dose scans simulation techniques can be classified into two categories: those based on reconstructed CT images<sup>2,3</sup> and those based on raw projection data.<sup>4-7</sup> The main limitations of image-based approaches arise from the lack of an appropriate noise model in image space. The methods of the second category simulate a lower-dose scan using raw projection data by adding synthetic noise to the higher dose scan. Our method falls into the second category. The advantage of this category is the existence of appropriate noise models in projection space. The methods of this category mainly use a monochromatic noise model and consider synthetic noise as a Poisson, normal, or combination of Poisson and normal distribution to model the two principal sources of noise in CT transmission data, quantum noise and electronic noise.

Previous studies have shown that the bowtie filter is one of the main components affecting noise characteristics of CT transmission data. The measurement noise is often considered to be uncorrelated<sup>5</sup> although the noise of neighboring detector channels is actually correlated due to crosstalk between detector elements.<sup>8</sup> The noise characteristics are also affected by beam hardening, and this effect has, to our knowledge, not been addressed in previous studies. In this paper, a new strategy is presented to simulate reduced-dose scans from an existing standard dose scan. The bowtie filter, the crosstalk between detector channels, and the beam hardening effect have been included in the proposed model.

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## 2. METHODOLOGY

For polychromatic X-ray generation, the CT transmission data  $I_i$ , observed in detector element  $i$ , can be modeled as a combination of a compound Poisson distribution and a normal distribution,<sup>8</sup> representing the two principal sources of noise in CT transmission data, quantum noise and electronic noise. This model can describe the polychromatic nature of X-ray generation; however, it is inconveniently complicated. The compound Poisson distribution, therefore, can be approximated as a scaled Poisson distribution.

### 2.1 Thinning Technique

Reducing the X-ray tube current is the most obvious way of minimizing radiation dose to the patients. This can be modeled by estimating the (noise equivalent) number of photons in the high exposure scan and applying a thinning technique to it. Assuming a monochromatic beam, the mean number of detected photons in the high dose scan can be written as

$$I^{(\beta)} = I_0^{(\beta)} \exp(-\rho) \quad (1)$$

where the superscript  $(\beta)$  identifies tube current level in mAs.  $I_0$  and  $\rho$  represent incident X-ray intensity and log-converted raw data, respectively. The log-converted raw data should first be converted into the form of transmission data which requires knowledge of  $I_0^{(1)}$ , a noise free estimate of the incident X-ray intensity for a unit tube current. The total number of incident X-ray photons is linearly proportional to the tube current which means  $I_0^{(\alpha)} = \alpha I_0^{(1)}$ . The variance of an air scan without any attenuating object can be used to estimate these parameters. The variance of the transmission data in an air scan is given by

$$\text{var}[\exp(-\rho_{air}^{(\alpha)})] = \text{var}\left[\frac{I_{air}^{(\alpha)}}{I_0^{(\alpha)}}\right] = \frac{\alpha I_0^{(1)} + \sigma_e^2}{(\alpha I_0^{(1)})^2} \quad (2)$$

where  $\alpha$  represents the tube current. Performing some air scans at different levels of tube current, the system is overdetermined and  $I_0^{(1)}$  and  $\sigma_e^2$  can be estimated. Knowing the number of incident photons, the number of photons in the transmission scan with tube current  $\beta$  can be estimated from Eq. 1. Thinning can then be applied to produce a (more) noisy low-dose scan of  $\alpha$  mAs. Thinning means the random elimination of some of the detected photons with survival probability of  $\frac{\alpha}{\beta}$ , where the chance of not being eliminated is the same for each photon. Thinning of a known number of counts  $n$  with survival probability  $p$  produces a binomial distribution  $\mathcal{B}(n; p)$ . Modeling the electronic noise as a shifted Poisson distribution instead of a zero mean Gaussian represents its contribution as an effective number of photons. The thinning is then applied to all photons. This also reduces the contribution of the electronic noise, which can be restored by adding a (noisy) number of additional photons. After that, the final Poisson distribution is shifted back. This yields the following equation:

$$I_{sim}^{(\alpha)} = \mathcal{B}\left(\beta I_0^{(1)} \exp(-\rho) + \sigma_e^2, \frac{\alpha}{\beta}\right) + \mathcal{P}\left\{\left(1 - \frac{\alpha}{\beta}\right)\sigma_e^2\right\} - \sigma_e^2 \quad (3)$$

The first term reduces the number of photons observed in the converted raw transmission data, which itself is a random value because  $\rho$  is obtained from a measurement. The electronic noise is modeled as additional photons, which are also thinned. The second term adds a noisy number of photons to compensate for the thinning of the electronic noise. The last term subtracts the mean number of electronic noise photons, to ensure that the electronic noise has zero mean.

### 2.2 Noise Correlation

The thinning noise, added by the thinning procedure, is an uncorrelated noise. Turning to the experimental evidence, we found that the noise of neighboring CT detector pixels is correlated with an almost fixed correlation matrix  $r$  which is due to crosstalk between detector pixels.<sup>8</sup> We can assume that the expected number of photons in neighboring detector pixels  $i$  and  $j$  both equal  $\bar{A}$  such that their covariance  $C_{ij} = r_{ij}\bar{A}$ . It can be proved that the covariance between the detector pixels  $i$  and  $j$  is decreased to  $\eta^2 C_{ij}$  after applying the thinning algorithm with the survival probability of  $\eta$ . The problem is that in realistic data, the covariance should equal  $\eta C_{ij}$ , not  $\eta^2 C_{ij}$ . It is important to restore this noise correlation in the reduced-dose simulated scan because the noise propagation through the image reconstruction is different for correlated and uncorrelated noise. Therefore, we

have to create an additional covariance of  $(\eta - \eta^2)C_{ij}$ , without changing the mean or the variance of the noise, and also without changing the correlations in the data, which should be correctly inherited from the original high count data. We therefore propose to convolve the thinning noise with a smoothing matrix  $w$  where the convolution with mask  $w$ , first, has no effect on the mean and variance of the noise obtained after thinning, and second, restores the desired correlation. It can be proved that convolving the thinning noise with the matrix  $w$  creates a covariance of  $\eta\bar{A}(1 - \eta)[w \otimes w]_{i-j}$  which should be the same as the required additional covariance,  $(\eta - \eta^2)r_{ij}\bar{A}$ . Consequently, the smoothing matrix  $w$  should satisfy  $[w \otimes w]_{i-j} = r_{ij}$ . The convolution can be then written as the multiplication with a circulant matrix and convolution mask  $w$  can be computed as the square root of matrix  $r$ .

### 2.3 Beam Hardening Effect

In the above, we have derived for each detector the relation between beam intensity and the effective number of photons using blank scans. However, in patient scans, the photon beams reaching the detector are harder than in blank scans, because of the beam hardening effect. A harder beam contains fewer photons for the same intensity and is therefore subject to a higher amount of quantum noise.<sup>8</sup> This effect is not negligible, beam hardening by 20 cm of water produces 12 percent more variance than that of an air scan with the same intensity. Knowing the attenuation value in each beam path and assuming that all attenuation is due to water, the effect of beam hardening on the prediction of the effective number of surviving photons can be compensated.

## 3. RESULTS

A phantom study was performed to evaluate the performance of the proposed framework. All the blank and phantom scans have been acquired on a Siemens SOMATOM Force at seven different levels of tube current, from 300 down to 20. The reduced-dose scans were simulated from the high-dose scan of 300 mAs and compared with corresponding real lower-dose scans. Fig. 1 represents the profile of  $I_0^{(1)}$  for one detector pixel row. The bell shape profile of  $I_0^{(1)}$  is due to the bowtie filter, which aims to reduce radiation dose to peripheral parts of the patient's cross-section, resulting in a non-uniform incident X-ray intensity. Because  $I_0^{(1)}$  is determined for every detector element, the effect of the bowtie filter is automatically accounted for.

The Fig. 2 compares the noise variance of the reconstructed images in real and simulated lower-dose scans for a small (6 cm) and a large (21 cm) water phantom. Considering a correlated noise model, in contrast to previous studies, has increased the noise level in image space while it did not have any effect on the noise variance in projection data. Additionally, compensating the effect of beam hardening effect improved the results in the large phantom where the beam hardening effect was not negligible. The results indicated that the proposed strategy is able to properly predict the noise variance of the lower-dose scan, where the noise variance of the reconstructed images matched the reconstructions from the real scans with less than 5% error.

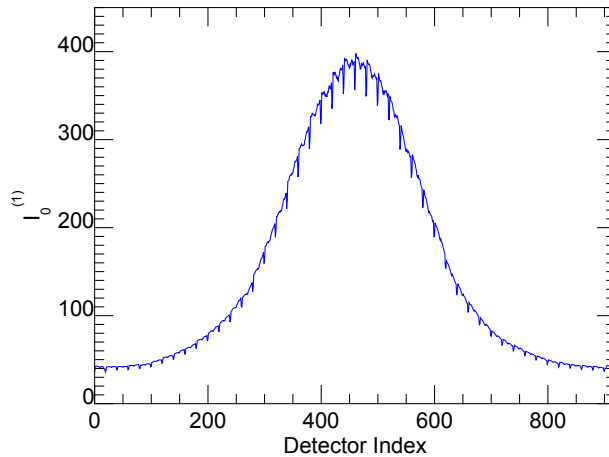


Figure 1. An illustration of estimated  $I_0^{(1)}$  for one detector pixel row as a function of detector index.

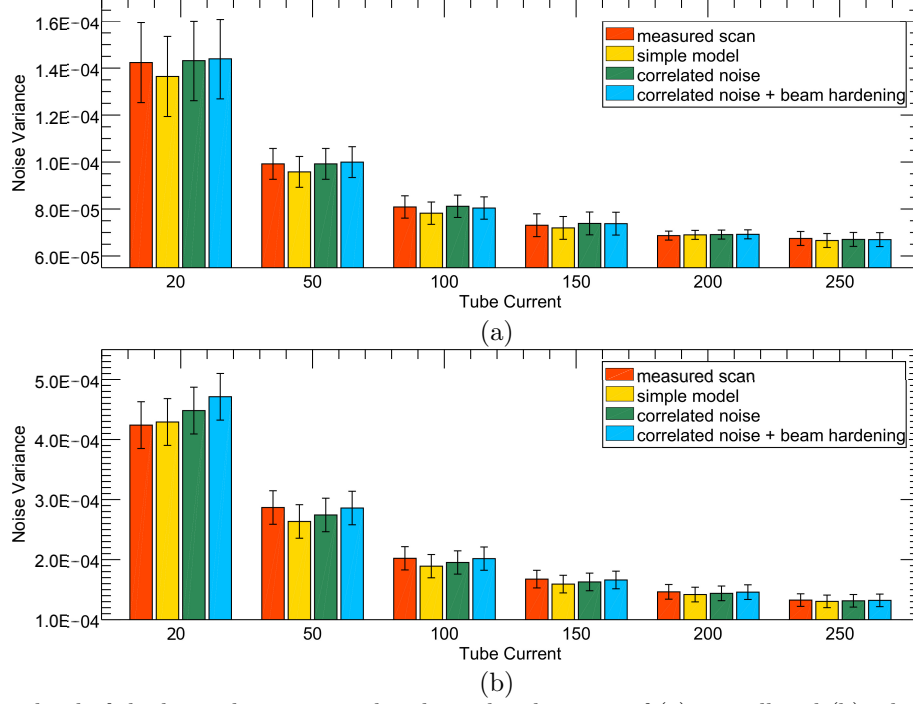


Figure 2. The noise level of the lower-dose measured and simulated images of (a) a small and (b) a large water phantom for different models at different levels of tube current.

Overall, incorporating the effect of noise correlation along with beam hardening brings the noise value of the simulated image closer to the real low-dose image, except for the 20 mAs scan of the large phantom when the signal intensity becomes very small. In such cases, a low-pass filter is usually applied<sup>9</sup> to avoid negative values to be passed to the log, and consequently, the variance of very low-dose real scans is found to be less than predicted. Fig. 3 shows a high-dose scan of 300 mAs and its corresponding measured and simulated reduced-dose scans of 20, 50, 100 mAs.

## 4. DISCUSSION

In this research, we developed a strategy to simulate a reduced-dose CT scan from an existing standard-dose scan. The lower-dose scan was achieved by thinning the number of photons in the high exposure scan. The method incorporated the effect of the bowtie filter, the noise correlation between neighboring detector elements and beam hardening effect. The experimental data revealed a close agreement of the variance in the observed and the simulated lower-dose scans.

## ACKNOWLEDGMENTS

This project is supported by Fonds Baillet-Latour. The authors would like to warmly thank Edmond Sterpin and Xavier Geets for the initial design of the research, Walter Coudyzer for his contribution in data collection, and Karl Stierstorfer and Frederik Noo for their valuable comments and suggestions.

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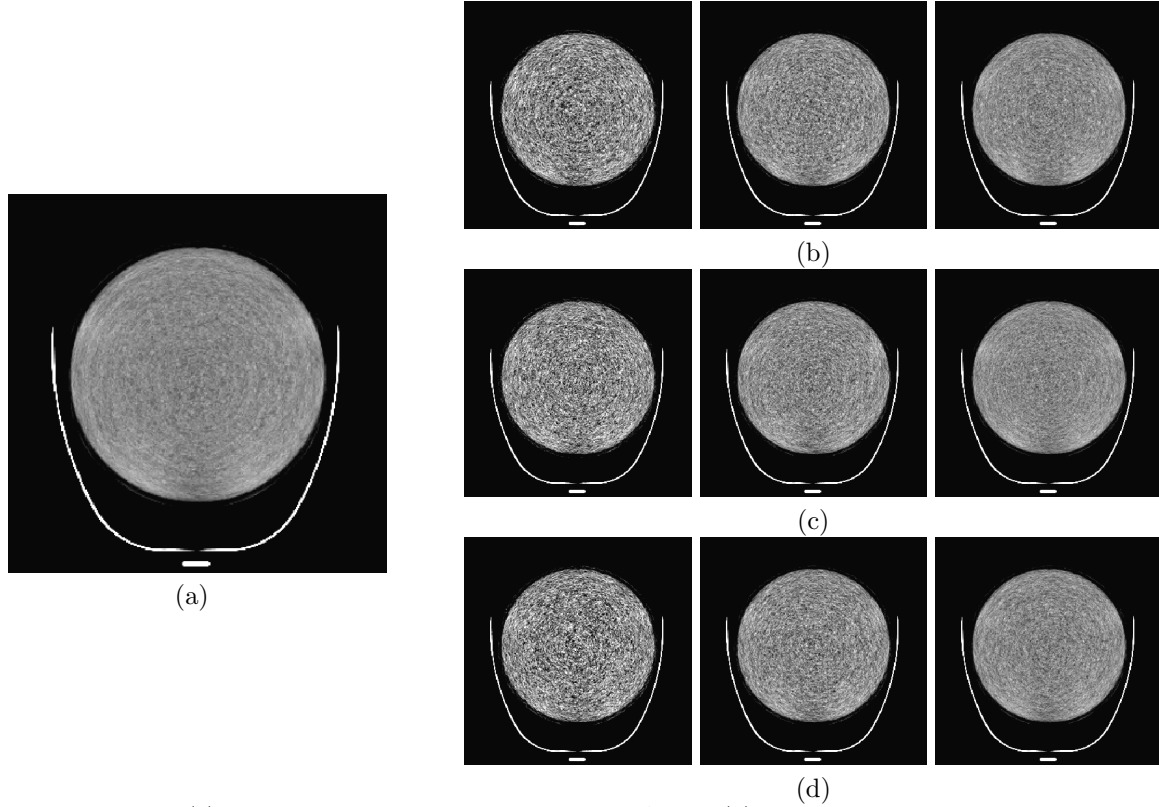


Figure 3. Presentation of (a) the measured high-dose image of 300 mAs and (b) corresponding acquired lower-dose images. (c) and (d) represent simulated images of the simple model and the model with noise correlation restoration and beam hardening compensation, respectively, for scans of 20, 50, and 100 mAs (from left to right).

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