Optimization of Pinhole SPECT Calibration

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***Abstract*— Previously, we developed a method to determine the acquisition geometry of a pinhole camera. This information is needed for the correct reconstruction of pinhole SPECT images. The method uses a calibration phantom consisting of three point sources. Their positions in the field of view influence the accuracy of the geometry estimate. This study proposes a specific configuration of point sources for optimal image reconstruction accuracy. It also demonstrates that any remaining inaccuracies of the geometry estimate can only cause sub-resolution inaccuracies in reconstructed images. Finally, the calibration method makes use of the geometry of the point source configuration, which is only known with limited accuracy. The study demonstrates however, that, with the proposed point source configuration, the error in reconstructed images is comparable to the error on the configuration geometry.**

I. Introduction

The reconstruction of pinhole SPECT data requires a correct description of the acquisition geometry of the pinhole camera. Previously, we developed a method to determine this geometry from the SPECT acquisition of a calibration phantom consisting of three point sources [1]. The geometry is determined by a least squares fit of the estimated point source projection locations to the measured locations. Besides the acquisition geometry, the positions of the point sources are estimated as well, but not the distances between them. This implies that the geometry of the calibration phantom has to be known, but not its position in the field of view of the camera.

With noisy calibration data, the acquisition geometry can only be estimated with limited accuracy. Both the geometry of the calibration phantom and its position in the field of view influence this accuracy and errors on the phantom geometry can degrade it even further. During reconstruction, the resulting errors on the acquisition geometry propagate into the reconstructed images, causing loss of spatial resolution and/or image deformation. The aim of this study is to determine the optimal configuration of point sources for accurate image reconstruction and to study the effect of errors on the phantom geometry for this optimal configuration.

II. Method

Both noise on the calibration data and errors on the calibration phantom geometry cause errors on the calibration results. These errors, in turn, degrade the accuracy of image reconstruction by loss of spatial resolution and/or image deformation. First, the propagation of noise on the calibration data to errors on the acquisition geometry is calculated by a linear approach, and the propagation of calibration phantom errors is estimated by simulation. Then the effect on image reconstruction of these estimation errors, due to either noisy data or calibration phantom errors, is evaluated.

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A. Estimation Accuracy

The projection coordinates U_0 of the three calibration point sources can be calculated analytically [1], and for small variations ΔP in acquisition geometry and phantom position, we assume that the resulting projection coordinates U can also be approximated by a linear system

$$U = U_0 + M\,\Delta P \tag{1}$$

in which M is a matrix containing the first order derivatives of the projection coordinates to the parameters P, describing the acquisition geometry of the camera and the position of the calibration phantom in the field of view. With $\Delta U = U - U_0$, equation (1) can be rewritten as

$$\Delta P = (M^T M)^{-1} M^T \Delta U. \tag{2}$$

The noise on the projection coordinates is represented by a diagonal covariance matrix $\mathrm{cov}(U)$. This matrix is propagated through the linear estimator of equation (2), yielding the covariance matrix $\mathrm{cov}(P)$ of the estimated parameters P

$$\mathrm{cov}(P) = (M^T M)^{-1} M^T \mathrm{cov}(U)\, M (M^T M)^{-1}. \tag{3}$$

The variances of $\mathrm{cov}(P)$ provide a measure of the error on the estimated parameters, but some of the errors are correlated, because of nonzero covariances between them. To evaluate the effect of these errors on image reconstruction, they must be split up into different, uncorrelated sets of errors on the parameters P. It can be shown that this is possible. For the calibration phantom errors, the estimation accuracy is estimated by calibration simulations, since no sufficiently accurate linear model was found.

B. Reconstruction Accuracy

The reconstruction accuracy is evaluated for both loss of resolution and deformation of the reconstructed images. If a point source is reconstructed from its pinhole projections using the correct acquisition geometry, all projection rays intersect at the correct point in the field of view. With an incorrect acquisition geometry, we assume that the point source will be reconstructed at the point that is closest to all projection rays in a least squares way. The distances from that point to the projection rays are decomposed into three orthogonal components and either the maximum value or the standard deviation of each component is used as a measure of the loss of spatial resolution. The image deformation due to the incorrect acquisition geometry is calculated by 'reconstructing' a grid of point sources. After correction for a global translation and rotation of the reconstructed image, the remaining displacements of the reconstructed point sources reflect the image deformation. The displacements are again decomposed into three orthogonal components.

For a collection of uncorrelated sets of parameter estimates, obtained from the decomposition of a covariance matrix, the above procedure is applied to each set individually and the results are quadratically added. This approach is based on the assumption that the uncorrelated sets of errors each have an independent effect on the image reconstruction accuracy.

III. Experiments

A. Phantom Geometry & Position

This first experiment evaluates the effect of the phantom geometry and position on the estimation and reconstruction accuracy for noisy calibration data. It can be shown that the field of view of a regular pinhole SPECT camera, can be well approximated by a sphere. The positions of the three point sources of the calibration phantom are systematically varied within this sphere to cover all possible geometries and positions of the calibration phantom. For each configuration, the estimation and reconstruction accuracy is calculated as described in section II. More than 23.000 configurations are tested for a sphere of 4 cm in diameter, which is the field of view of a pinhole camera with 24 cm focal length, 4 cm radius of rotation and a 60 deg acceptance angle. The noise of 0.3 mm standard deviation is worse than the noise of real point source measurements.

B. Incorrect Phantom Geometry

This experiment evaluates the effect of calibration phantom errors on the estimation and reconstruction accuracy for the optimal phantom from the first experiment. The effect on the parameter estimation is evaluated by 250 calibration simulations. In each realization, the calibration point sources are randomly displaced in three orthogonal directions with a normal distribution of 0.1 mm standard deviation. For each of the resulting acquisition geometry estimates, the effect on image reconstruction is evaluated as described in section II.B.

IV. Results

A. Phantom Geometry & Position

Different phantom geometries and positions optimize the accuracy of different acquisition parameter estimates and no solution was found minimizing the errors on all parameters simultaneously. However, a large number of point source configurations can be found, yielding maximum point source displacements or maximum resolution degradation of less than 0.05 mm, while the attainable image resolution is expected to be 0.5 mm or higher. Since the errors are much smaller than the attainable resolution, each of these configurations can be considered to be equally good. A single solution was chosen, that also reduces the standard deviation of each acquisition parameter estimate to less than 1.11 times the minimum value found for all possible point source configurations. It is a triangular configuration of point sources, all located on the edge of the field of view. One side of the triangle is parallel to the axis of rotation and the third point source lies at the opposite side of the rotation axis and in the mid plane of the field of view. The linear system assumption of section II.A was verified for the optimal point source configuration and yielded excellent agreement with the true calibration method.

B. Incorrect Phantom Geometry

The effect of calibration phantom errors on the estimation accuracy differs for the different parameters. Some of the parameter estimates are hardly affected, while the standard deviations of other parameters are surprisingly large in comparison with those obtained in the first experiment. However, the resulting image deformation is of the same magnitude as the errors on the calibration phantom. The displacements of the reconstructed point sources located at the same place in the field of view as the calibration point sources, are generally less than the errors on the calibration point sources. The resolution degradation is limited to 0.05 mm or less and is again negligible in comparison with the expected attainable resolution.

V. Discussion

The method we developed to determine the acquisition geometry of a pinhole camera [1], is based on the assumption that the calibration phantom is the representation of an image, which can be thought of as an infinite collection of point sources. Stated in this way, the best calibration phantom, is the one best representing the entire image. From this point of view the best way to improve the phantom clearly consists in adding additional point sources to it. It also suggests that the point sources should be distributed to cover the entire field of view, although not necessarily in a uniform way. Our results indicate that the theoretically minimal number of point sources [1], placed at well chosen locations in the field of view, is already sufficient for practical purposes. The above concept also explains how the use of an incorrect phantom geometry during calibration, results in an equivalent deformation of the reconstructed image. The calibration minimizes the inconsistency between the fixed, but incorrect phantom geometry and the measured projections by modifying the projector. During reconstruction, the same projector is then used to generate an image, which is consistent with its projections.

VI. Conclusion

A specific configuration of three point sources has been proposed as a pinhole SPECT calibration phantom. The phantom yields optimal results for both the accuracy of the estimated pinhole acquisition geometry and the accuracy of image reconstruction. With the proposed calibration phantom, errors on the phantom geometry cause image deformations of the same magnitude.

References

[1] D. Bequé, J. Nuyts, G. Bormans, P. Suetens, and P. Dupont, "Characterization of Pinhole SPECT Acquisition Geometry, " accepted for publication in *IEEE Trans. Med. Imag.*