TIØ 4120 Operasjonsanalyse Øving 9

Oppg. 1

17.4-5

La T_1, T_2, T_3 være tiden det tar de tre behandlerene å betjene kundene som er der nå.

La $T_1' = (T_1 - 10 \mid T_1 > 10)$ osv.

Pga Markovegenskapen:

$$P(T_1' > t) = P(T_1 > t + 10 \mid T_1 > 10) = P(T_1 > t)$$

Ser at T_1' og T_1 da har samme fordeling. (Tilsvarende for T_2, T_3)

Tiden til neste kunde betjenes er $T = \min\{T_1', T_2', T_3'\}$.

Fordi T_1', T_2', T_3' er eksponentialfordelt gjelder:

$$T \sim \exp(\lambda_1 + \lambda_2 + \lambda_3) \sim \exp(2+3+4) \sim \exp(9)$$

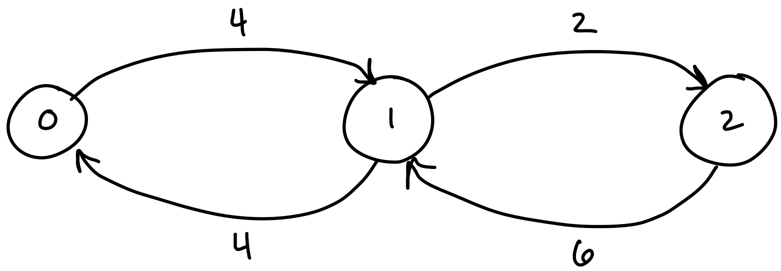
$$E[T] = \frac{1}{9} \approx 0.111$$

Forventet ventetid er 6,7 min.

Oppg. 2

Tilstand	Fodselsrate	Dodsrate
0	4	-
1	2	4
2	0	6

a)



b)

1. $4P_0 - 4P_1 = 0 \Rightarrow P_0 - P_1 = 0$

2. $4P_0 + 6P_2 - 4P_1 - 2P_1 = 0 \Rightarrow 2P_0 + 3P_2 - 3P_1 = 0$

3. $2P_1 - 6P_2 = 0 \Rightarrow P_1 - 3P_2 = 0$

c) Normalisering: $P_0 + P_1 + P_2 = 1$

1. $P_0 = P_1$

3. $P_1 = 3P_2$

4. $3P_2 + 3P_2 + P_2 = 1 \Rightarrow P_2 = \frac{1}{7}$

$P_1 = \frac{3}{7}$

$P_0 = \frac{3}{7}$

d) Generelle formler:

$$C_0 = 1, \quad C_1 = \frac{\lambda_0}{\mu_1} = \frac{4}{4} = 1 \quad C_2 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} = \frac{2 \cdot 4}{6 \cdot 4} = \frac{1}{3}$$

$$P_0 = (C_0 + C_1 + C_2)^{-1} = (1 + 1 + \tfrac{1}{3})^{-1} = \underline{\underline{\frac{3}{7}}}$$

$$P_1 = 1 \cdot P_0 = \underline{\underline{\frac{3}{7}}}$$

$$P_2 = \frac{1}{3} P_0 = \underline{\underline{\frac{1}{7}}}$$

$$L = 0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 = \frac{3}{7} + 2 \cdot \frac{1}{7} = \underline{\underline{\frac{5}{7}}}$$

Antar minst én servicestasjon ($s \geq 1$):

$$L_q = 0 \cdot P_0 + 0 \cdot P_1 + (2 - s) P_2 = (2-s) P_2 = \begin{cases} \frac{1}{7} & , s = 1 \\ 0 & , s = 2 \end{cases}$$

$$W = \frac{1}{\bar{\lambda}} L \quad , \qquad \bar{\lambda} = 4P_0 + 2P_1 + 0 \cdot P_2$$

$$= \frac{7}{18} \cdot \frac{5}{7} = \frac{5}{18} \approx \underline{\underline{0.278}} \qquad = 4 \cdot \frac{3}{7} + 2 \cdot \frac{3}{7} = \frac{18}{7}$$

$$W_q = \frac{1}{\bar{\lambda}} L_q = \frac{7}{18} \cdot L_q = \begin{cases} \frac{1}{18} & , s = 1 \\ 0 & , s \geq 2 \end{cases}$$