Oppg. 1

Investeringsmulighet [mill. USD]

	1	2	3	4	5	6
Estimert profitt	15	12	16	18	9	11
Kapitalbehov	38	33	39	45	23	27

Budsjett: 100 mill. USD

Begrensninger:

- 1, 2 og 3, 4 gjensidig utelukkende
- 3, 4 kan ikke startes uten 1 eller 2.

a) La $x_i \in \{0, 1\}$, $i = 1, \dots, 6$, slik at $x_i = 1$ dersom investeringsmulighet i benyttes.

Ønsker da å maksimere profitt $z = \underline{p}^T \underline{x}$

$$= 15x_1 + 12x_2 + 16x_3 + 18x_4 + 9x_5 + 11x_6$$

slik at

$$x_1 + x_2 \leq 1$$

$$x_3 + x_4 \leq 1$$

$$x_3 + x_4 \leq x_1 + x_2$$

$$\underline{c}^T \underline{x} \leq b \Rightarrow 38x_1 + 33x_2 + 39x_3 + 45x_4 + 23x_5 + 27x_6 \leq 100$$

b) Løser problemet med Solver i Excel og får følgende løsning:

	A	B	C	D	E	F	G	H	I	J	K
1											
2	Investering	1	2	3	4	5	6			Budsjett	100
3	Estimert profitt	15	12	16	18	9	11			Profitt	33
4	Kapitalbehov	38	33	39	45	23	27			Kostnad	83
5											
6	x	1	0	0	1	0	0				
7											
8	x1+x2	1									
9	x3+x4	1									
10											

Oppg. 2

	Pris	Tid	Tre
Stor figur	50 NOK	2 t	5 dm^3
Liten figur	40 NOK	3 t	2 dm^3

	Tid	Tre
Budsjett:	20 t	35 dm^3

a) La x_1 være antall store figurer og x_2 antall små.

Maksimere $z = 50x_1 + 40x_2$

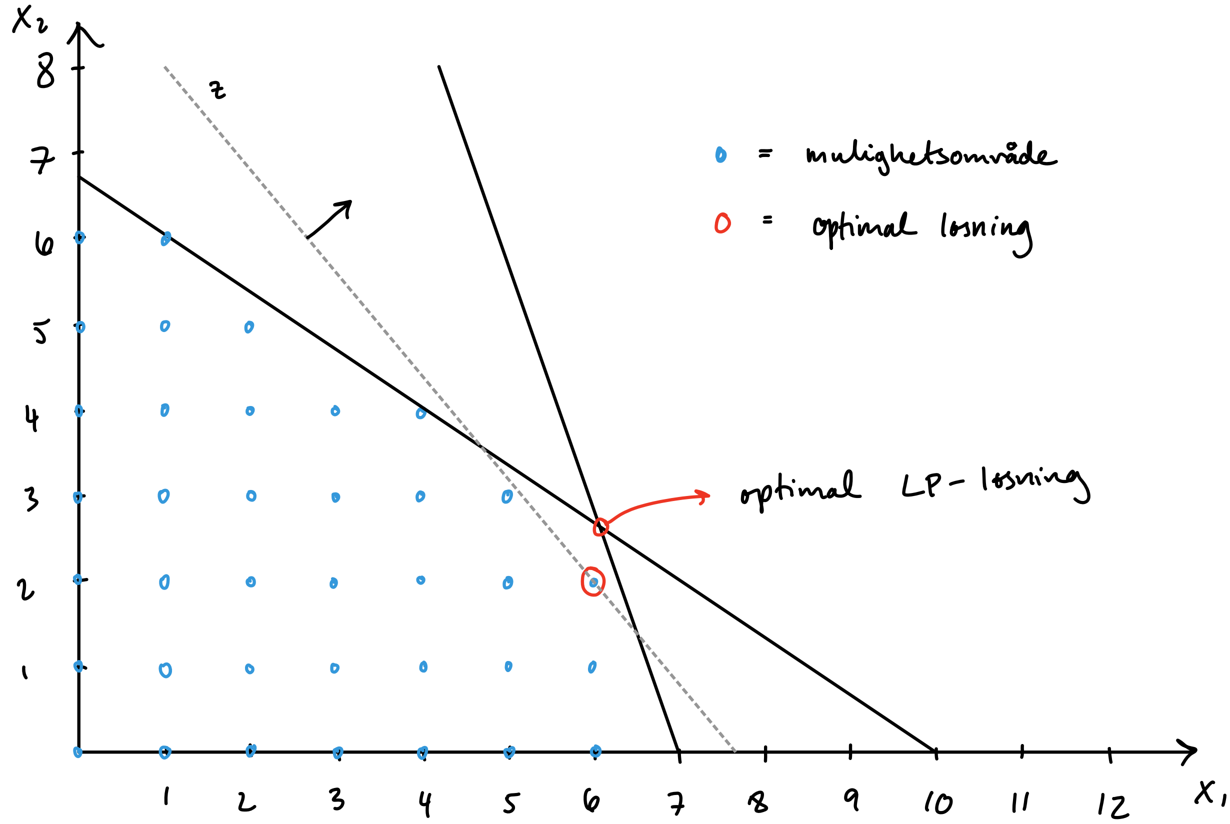
slik at

x_1, x_2 heltall,

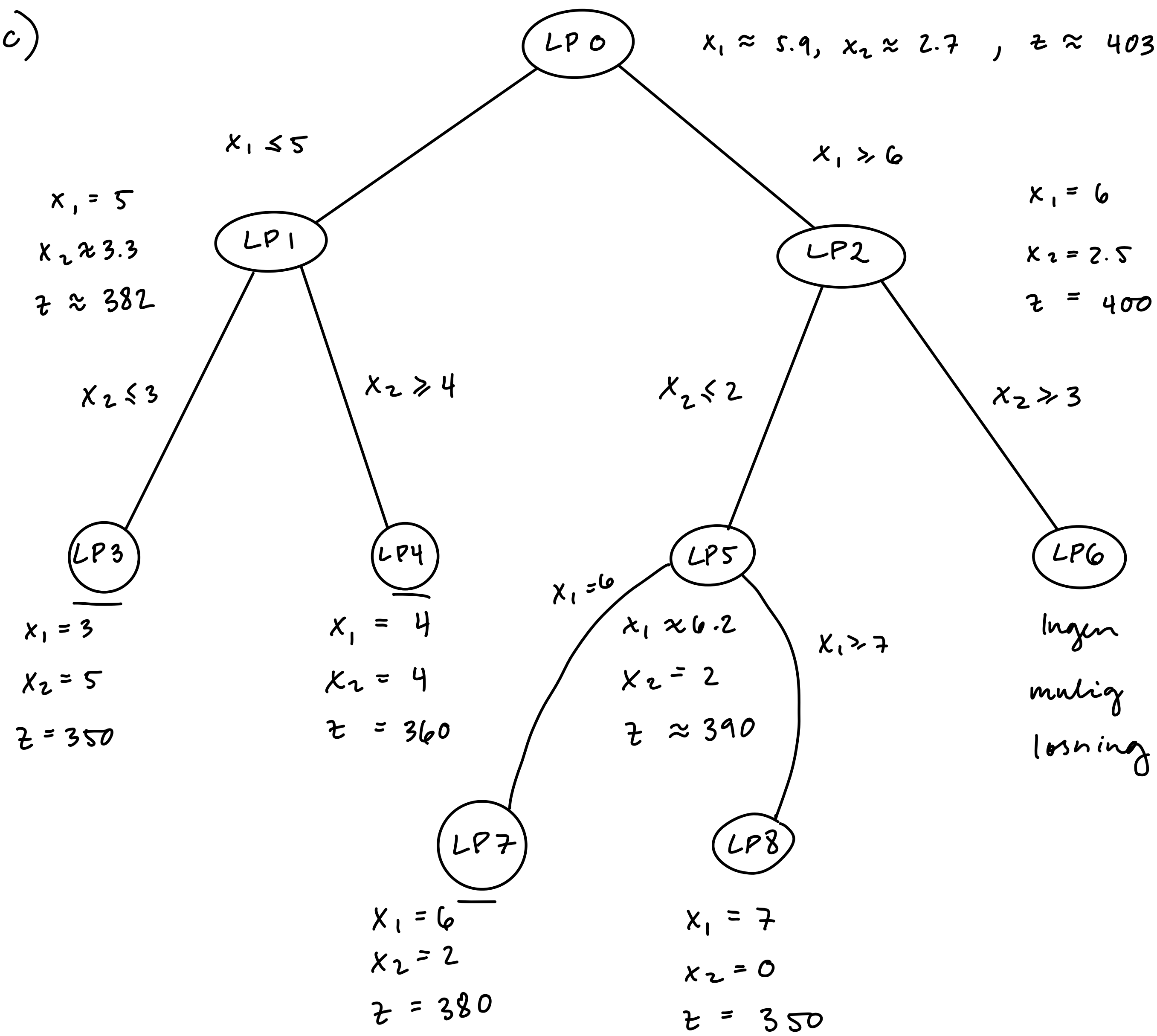
$$2x_1 + 3x_2 \leq 20$$

$$5x_1 + 2x_2 \leq 35$$

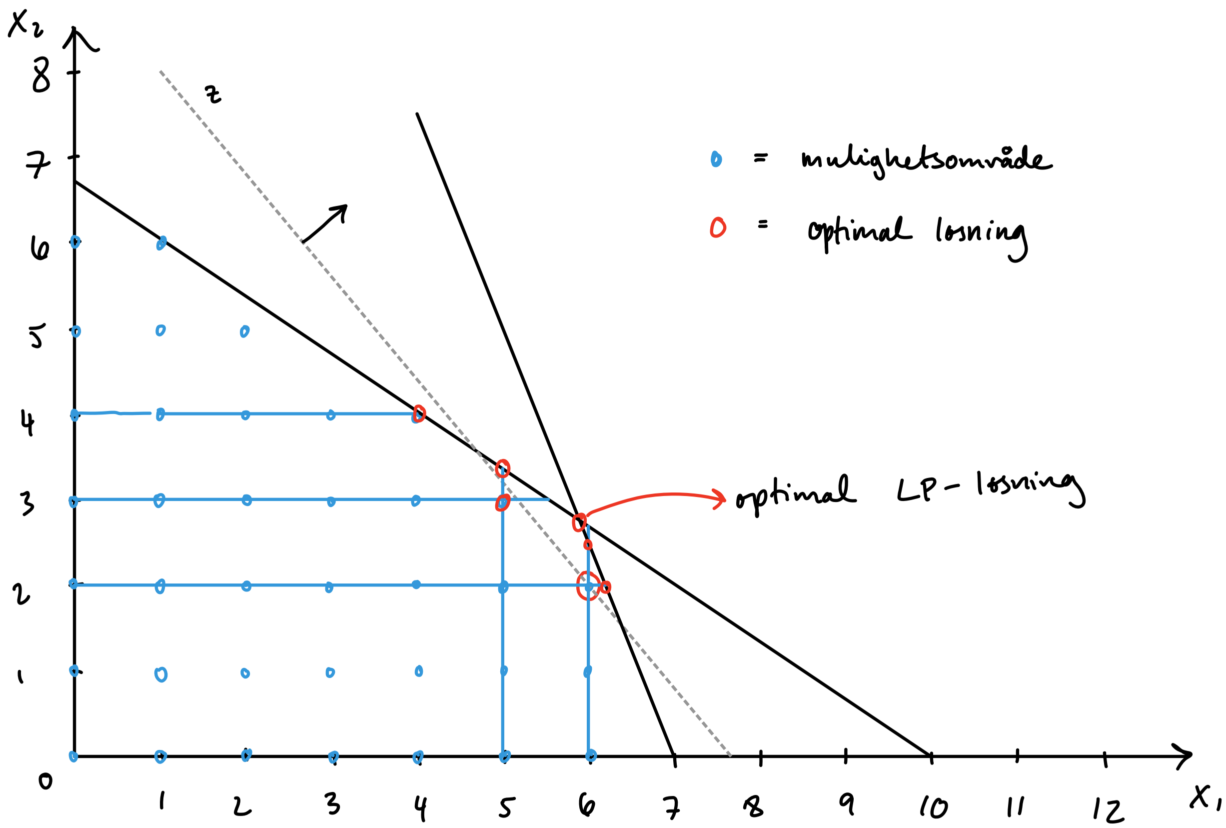
b)


x_2
8
7
6
5
4
3
2
1
z
= mulighetsområde
= optimal løsning
optimal LP-løsning
1
2
3
4
5
6
7
8
9
10
11
12
x_1

c)


LP 0
$x_1 \approx 5.9$, $x_2 \approx 2.7$, $z \approx 403$
$x_1 \leq 5$
$x_1 \geq 6$
LP1
$x_1 = 5$
$x_2 \approx 3.3$
$z \approx 382$
LP2
$x_1 = 6$
$x_2 = 2.5$
$z = 400$
$x_2 \leq 3$
$x_2 \geq 4$
$x_2 \leq 2$
$x_2 \geq 3$
LP3
$x_1 = 3$
$x_2 = 5$
$z = 350$
LP4
$x_1 = 4$
$x_2 = 4$
$z = 360$
LP5
$x_1 \approx 6.2$
$x_2 = 2$
$z \approx 390$
LP6
Ingen mulig løsning.
$x_1 = 6$
$x_1 \geq 7$
LP7
$x_1 = 6$
$x_2 = 2$
$z = 380$
LP8
$x_1 = 7$
$x_2 = 0$
$z = 350$

Skisse brukt over:



Optimal løsning: $x_1 = 6$, $x_2 = 2$, $z = 380$

Problem 3:

Full problem:

$$\min\ Z = 5x_1 + 6x_2 + 7x_3 + 8x_4 + 9x_5$$

s.t.:

$$-3x_1 + x_2 - x_3 - x_4 + 2x_5 \leq -2, \quad (1)$$

$$-x_1 - 3x_2 + x_3 + 2x_4 - x_5 \leq 0, \quad (2)$$

$$+x_1 + x_2 - 3x_3 - x_4 - x_5 \leq -1 \quad (3),$$

$$(Ax \leq b), \qquad x = [x_1, \ldots, x_5]^T$$

$$x_j \in \{0, 1\}, \ j = 1, \ldots, 5.$$

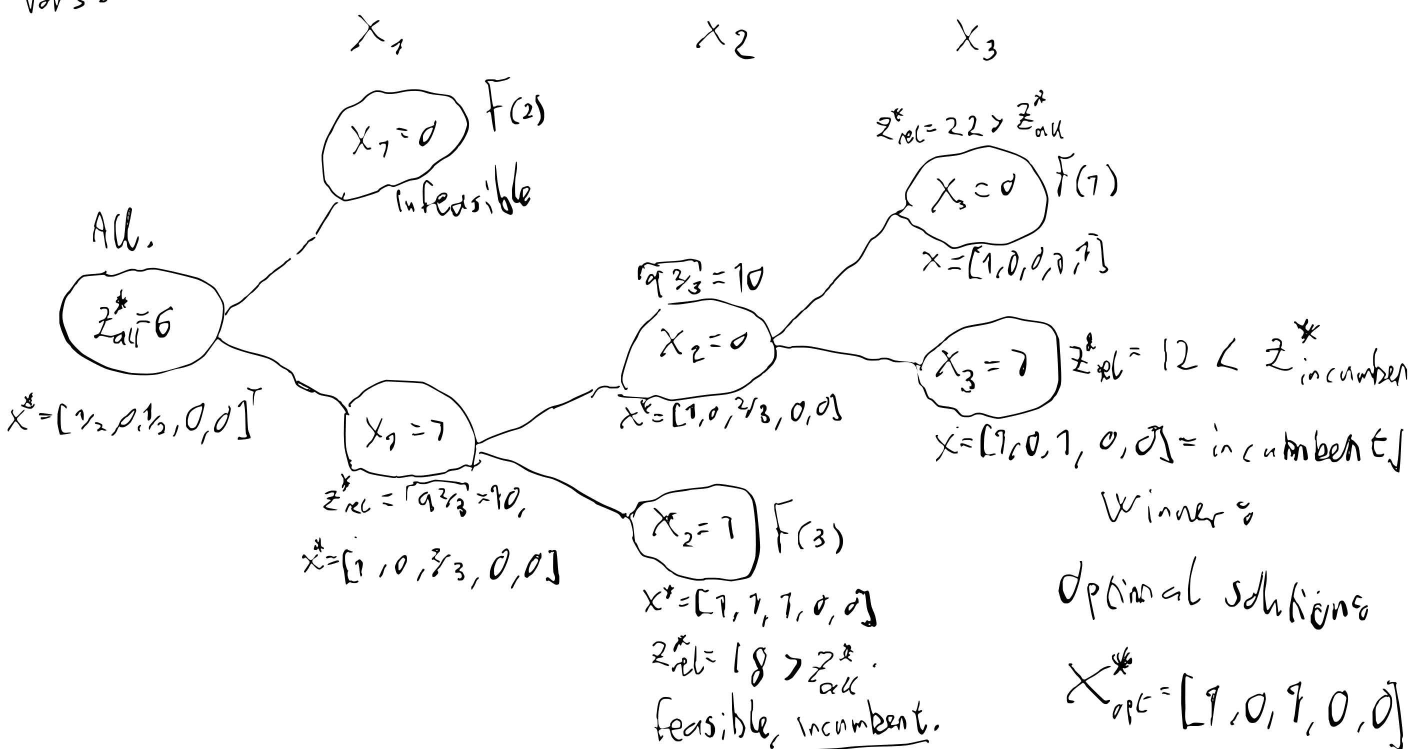
Relaxed problem: $0 \leq x_i \leq 1 \quad \forall\ i \in 1, \ldots, 5.$

Solution: $Z^*_{rel} = 6.0,$

$$x^* = [1/2, 0, 1/2, 0, 0]^T.$$

Branch-and-Bound:

Vars:



Winner:

Optimal solution:

$X^*_{opt} = [1, 0, 1, 0, 0],$

$z^*_{opt} = 12.$

Oppgave 4:

$$\min \quad z = c^T x$$

$$\text{s.t.} \quad Ax = b$$

$$x_j \leq 1, \quad j = 1, 2,$$

$$x_j \leq 5, \quad j \neq 1, 2.$$

$$x_j \geq 0, \; x_j \in \mathbb{N}_0 \; \forall \; j \in \{1, \dots, 7\}.$$

d) Hvis: $3x_2 + 2x_4 \leq 15,$

må: $x_5 + 2x_6 \leq 10.$

reformulert: $3x_2 + 2x_4 + S_1 = 15, \quad (1^*) \; S_1 \in \mathbb{R}$

$x_5 + 2x_6 + S_2 = 10, \quad (2^*) \; S_2 \in \mathbb{R},$

Hvis $S_1 \geq 0, \Rightarrow S_2 \geq 0.$

$$y_1 \in \{0, 1\} \quad y_1 = \begin{cases} 0, & S_1 < 0, \\ 1, & S_2 \geq 0. \end{cases}$$

Til pass ulikheter: $M >> 0$.

$S_1 - My_1 \leq \frac{1}{M}$. $S_1 < 0 \Rightarrow y = 0, 1$.

$S_1 - My_1 \leq \frac{1}{M}$, $S_1 \geq 0 \Rightarrow y = 1$.

$M(S_1 - My_1) \leq 1$, If $S_1 > \frac{1}{M}$, $y = 1$ ✓,

If $S < 0$, $y = 0$ or 1 ✓

$$\boxed{\begin{aligned} &\text{And:} \quad S_2 + M(1-y_1) \geq 0, \quad y_1 = 1 \Rightarrow S_2 \geq 0. \checkmark \\ &\qquad\qquad \Updownarrow \\ &\qquad\quad S_2 - My_1 \geq -M, \quad y_1 = 0 \Rightarrow S_2 \geq -M, \checkmark \end{aligned}}$$

$y_1 \in \{0, 1\}$.

Want to avoid the feasible point

$S_1 = 0$, $y_1 = 0$, $My_1 - S_1 \leq 1$?

So do: $My_1 + S_1 \geq 1$, $S_1 = 0 \Rightarrow y_1 = 1$,

$S_1 > 0$ not possible...

$S_1 < 0 \Rightarrow y_1 = 1$,

$S_1 = 0 \Rightarrow y_1 = 1$,

$S_1 \geq 1 \Rightarrow y_1 = 0, 1$,

New Idea:

Check with a binary variable whether or not one of the two constraints are satisfied, and then we impose logical conditions on those binary vars.

So: if: $3x_2 + 2x_4 \leq 15$, (1*)

then: $x_5 + 2x_6 \leq 10$. (2*)

$3x_2 + 2x_4 - My_1 \leq 15$, $y_1 = \begin{cases} 0 \text{ if } (1^*) \text{ satisfied} \\ 1 \text{ if } (1^*) \text{ dsat.} \end{cases}$

$x_5 + 2x_6 - My_2 \leq 10$.

Now logical requirement on y_1, y_2.

Either: $y_1 = 0$ and $y_2 = 0$,

or $y_1 = 1$, $y_2 = 0, 1$.

$\Rightarrow \; y_2 \le y_1$

<u>In total:</u>

Extend the MIP model by:

constraints and vars:

$$\tilde{A}x = \tilde{b} \begin{cases} 3x_2 + 2x_4 - My_1 + s_1 = 15, \\ x_5 + 2x_6 - My_2 + 2 = 10, \\ y_2 - y_1 \le 0 \end{cases}$$

$y_1, y_2 \in \{0, 1\}$, $s_1, s_2 \ge 0$.

b) Add $10x_2 \cdot x_7$ to the objective function, whilst keeping the MIP model.

Find $z_1 = x_2 \cdot x_7$? x_2 Integer!

Have to multiply something, how to do it non quadratically?

$$\text{Max } z = c^T x + 10 z_1,$$

If $x_2 = 1$, $z_1 = x_7$, otherwise $z_1 = 0$.

If $x_2 = 0$, we have big slack.

$$-M(1-x_2) + z_1 - x_7 \leq 0$$
$$M(1-x_2) + z_1 - x_7 \geq 0$$

$x_2 = 1 \Rightarrow$ $z_1 \leq x_7$, $z_1 \geq x_7$, $\Rightarrow z_1 = x_7$, ✓,

And: $z_1 \leq M x_2$, ($M > \max x_7$)

$z_1 - M x_2 \leq 0$. ✓✓

In total:

$$\max \quad z = c^T x + 10 z_1,$$

$$\text{s.t.} \quad Ax = b$$

$$\left.\begin{aligned} -M(1-x_2) + z_1 - x_7 &\leq 0, \\ +M(1-x_2) + z_1 - x_7 &\geq 0, \\ z_1 - M x_1 &\leq 0, \\ z_1 \in \{0, \dots, 5\}. \end{aligned}\right\} \text{Additional.}$$

c) $x_8 = 0$ eller $x_8 \geq 3$.

$x_8 \in \mathbb{R}$, ikke heltall.

$x_8 \geq 3$, $x_8 - s_3 = 3$, $s_3 \in \mathbb{R}$,

Use binary variable to check if $s_3 \geq 0$:

$s_3 - My_3 \leq 0$,

$y_3 = \begin{cases} 1 & \text{if } s_3 \geq 0,\ x_8 \geq 3, \\ 0 & \text{if } s_3 < 0,\ x_8 < 3. \end{cases}$

Allows for $s_3 = 0$, $y_3 = 0$, but but. Then $x_8 = 3$, ok!

Enforce that $x_8 = 0$ if $y_3 = 0$:

$$\left.\begin{array}{l} x_8 \leq My_3 \\ x_8 \geq -My_3 \end{array}\right\} \Rightarrow \begin{array}{l} x_8 - My_3 \leq 0 \\ -x_8 - My_3 \leq 0. \end{array}$$