Problem 3:

Full problem:

$$\min Z = 5x_1 + 6x_2 + 7x_3 + 8x_4 + 9x_5$$

s.t.:

$$-3x_1 + x_2 - x_3 - x_4 + 2x_5 \leq -2, \quad (1)$$

$$-x_1 - 3x_2 + x_3 + 2x_4 - x_5 \leq 0, \quad (2)$$

$$+x_1 + x_2 - 3x_3 - x_4 - x_5 \leq -1 \quad (3),$$

$$(Ax \leq b), \qquad x = [x_1, \ldots, x_5]^T$$

$$x_j \in \{0, 1\}, \; j = 1, \ldots, 5,$$

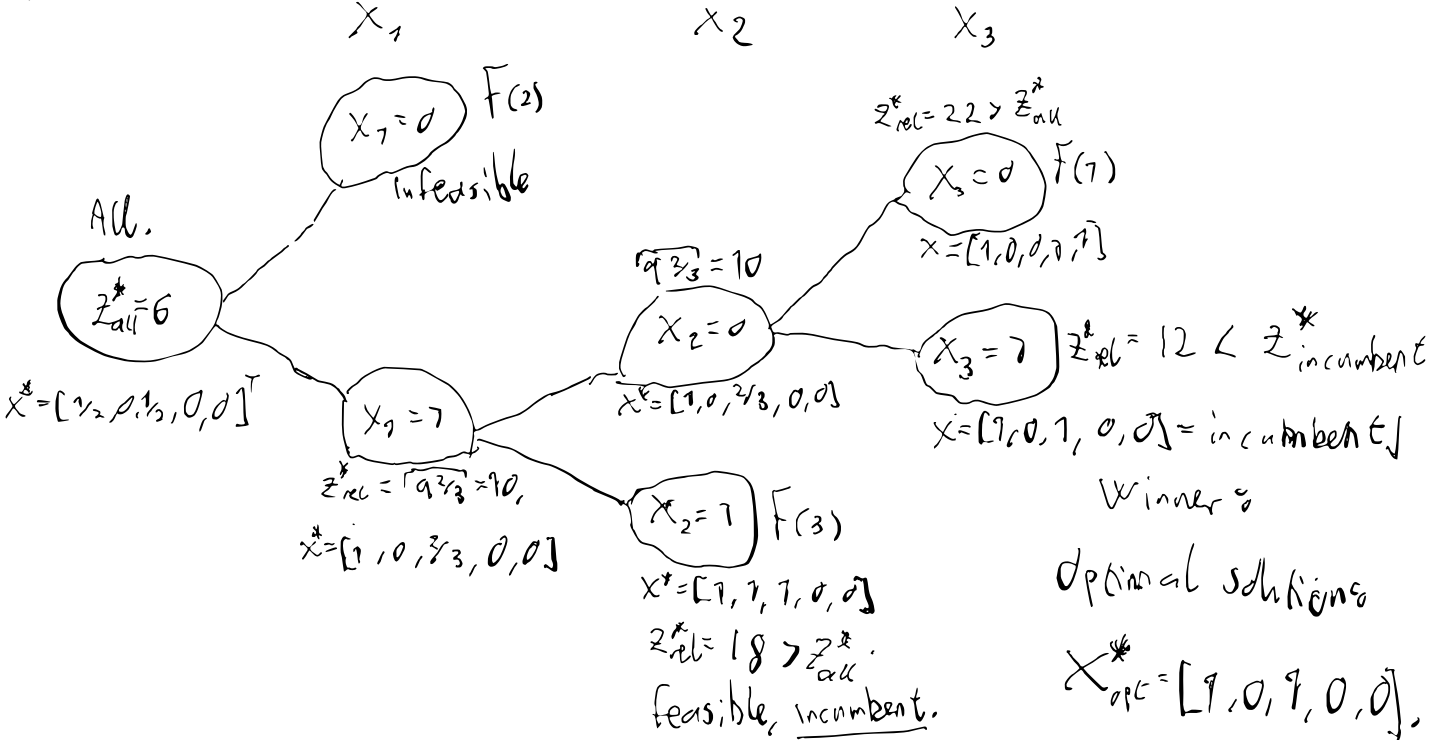
Relaxed problem: $0 \leq x_i \leq 1 \quad \forall i \in 1, \ldots, 5$

Solution: $Z^*_{rel} = 6.0,$

$$x^* = [1/2, 0, 1/2, 0, 0]^T.$$

Branch-and-Bound

Vars: x_1 x_2 x_3



Optimal solution:

$x^*_{opt} = [1, 0, 1, 0, 0]$.

$z^*_{opt} = 12$.

Oppgave 4:

$$\min \quad z = c^T x$$

$$\text{s.t.} \quad Ax = b$$

$$x_j \leq 1, \quad j = 1, 2,$$

$$x_j \leq 5, \quad j \neq 1, 2.$$

$$x_j \geq 0, \; x_j \in \mathbb{N}_0 \; \forall \; j \in \{1, \dots, 7\}.$$

a) Hvis $3x_2 + 2x_4 \leq 15$,

må $x_5 + 2x_6 \leq 10$.

reformulere: $3x_2 + 2x_4 + s_1 = 15, \quad (1^*) \; s_1 \in \mathbb{R}$

$x_5 + 2x_6 + s_2 = 10, \quad (2^*) \; s_2 \in \mathbb{R},$

Hvis $s_1 \geq 0 \Rightarrow s_2 \geq 0.$

$y_1 \in \{0, 1\}$ $\quad y_1 = \begin{cases} 0, & s_1 < 0, \\ 1, & s_2 \geq 0. \end{cases}$

Til pass uliksheter: $M >> 0$.

$S_1 - My_1 \leq \frac{1}{M}$. $S_1 < 0 \Rightarrow y = 0, 1$.

$S_1 - My_1 \leq \frac{1}{M}$, $S_1 \geq 0 \Rightarrow y = 1$.

$M(S_1 - My_1) \leq 1$, if $S_1 > \frac{1}{M}$, $y = 1$ ✓,

if $S < 0$, $y = 0$ or 1 ✓

And: $S_2 + M(1 - y_1) \geq 0$, $y_1 = 1 \Rightarrow S_2 \geq 0$. ✓

$\Updownarrow$

$S_2 - My_1 \geq -M$, $y_1 = 0 \Rightarrow S_2 \geq -M$, ✓

$y_1 \in \{0, 1\}$.

Want to avoid the feasible point

$S_1 = 0$, $y_1 = 0$, $My_1 - S_1 \leq 1$?

So do: $My_1 + S_1 \geq 1$, $S_1 = 0 \Rightarrow y_1 = 1$,

$S_1 > 0$ not possible...

$S_1 < 0 \Rightarrow y_1 = 1$,

$S_1 = 0 \Rightarrow y_1 = 1$,

$S_1 \geq 1 \Rightarrow y_1 = 0, 1$,

New Idea

Check with a binary variable whether or not one of the two constraints are satisfied, and then we impose logical conditions on those binary vars.

So: If: $3x_2 + 2x_4 \leq 15$, (1^*)

then: $x_5 + 2x_6 \leq 10$. (2^*)

$3x_2 + 2x_4 - My_1 \leq 15$, $\quad y_1 = \begin{cases} 0 & \text{if } (1^*) \text{ satisfied} \\ 1 & \text{if } (1^*) \text{ not sat.} \end{cases}$

$x_5 + 2x_6 - My_2 \leq 10$.

Now logical requirement on y_1, y_2.

Either $y_1 = 0$ and $y_2 = 0$,

or $y_1 = 1$, $y_2 = 0, 1$.

$\Rightarrow \quad y_2 \leq y_1$

In total:

Extend the MIP model by:

Constraints and vars:

$$\bar{A}\bar{x} = \bar{b} \begin{cases} 3x_2 + 2x_4 - My_1 + s_1 = 15, \\ x_5 + 2x_6 - My_2 + s_2 = 10, \\ y_2 - y_1 \leq 0 \end{cases}$$

$y_1, y_2 \in \{0, 1\}$, $s_1, s_2 \geq 0$.

b) Add $10 x_2 \cdot x_7$ to the objective function, whilst keeping the MIP model.

Find $z_9 = x_2 \cdot x_7$ & x_2 Integer!

Have to multiply something, how to do it quadratically?

non

$$\max z = c^T x + 10 z_1,$$

If $x_2 = 1$, $z_1 = x_7$, otherwise $z_1 = 0$.

If $x_2 = 0$, we have big stack. $\left\{ \begin{array}{l} -M(1 - x_2) + z_1 - x_7 \leq 0 \\ M(1 - x_2) + z_1 - x_7 \geq 0 \end{array} \right\}$ $x_2 = 1 \Rightarrow$ $z_1 \leq x_7$, $z_1 \geq x_7$, $\Rightarrow z_1 = x_7$, ✓

And $z_1 \leq M x_2$, $(M > \max x_7)$

$z_1 - M x_2 \leq 0$. ✓✓

In folgt

$$\max \quad z = c^T x + 10 z_1,$$

$$\text{s.t.} \quad Ax = b$$

$$\left.\begin{aligned} -M(1-x_2) + z_1 - x_7 &\leq 0, \\ +M(1-x_2) + z_1 - x_7 &\geq 0, \\ z_1 - M x_1 &\leq 0, \\ z_1 &\in \{0, \dots, 5\}. \end{aligned}\right\} \text{Addiend.}$$

c) $x_8 = 0$ eller $x_8 \geq 3$.

$x_8 \in \mathbb{R}$, ikke heltall.

$x_8 \geq 3$, $x_8 - s_3 = 3$, $s_3 \in \mathbb{R}$,

Use binary variable to check if $s_3 \geq 0$:

$$s_3 - M y_3 \leq 0,$$

$$y_3 = \begin{cases} 1 \text{ if } s_3 \geq 0,\ x_8 \geq 3, \\ 0 \text{ if } s_3 < 0,\ x_8 < 3. \end{cases}$$

Allows for $s_3 = 0$, $y_3 = 0$, but but. Then $x_8 = 3$, ok!

Enforce that $x_8 = 0$ if $y_3 = 0$:

$$\left.\begin{aligned} x_8 &\leq M y_3 \\ x_8 &\geq -M y_3 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} x_8 - M y_3 &\leq 0 \\ -x_8 - M y_3 &\leq 0. \end{aligned}$$