Øving 3 - Oper. An.

Oppgave 1:

Max $z = 3x_1 + x_2$ s. t.

$x_1 - x_2 \leq 5$ (1) $(x_2 = x_1 - 5)$

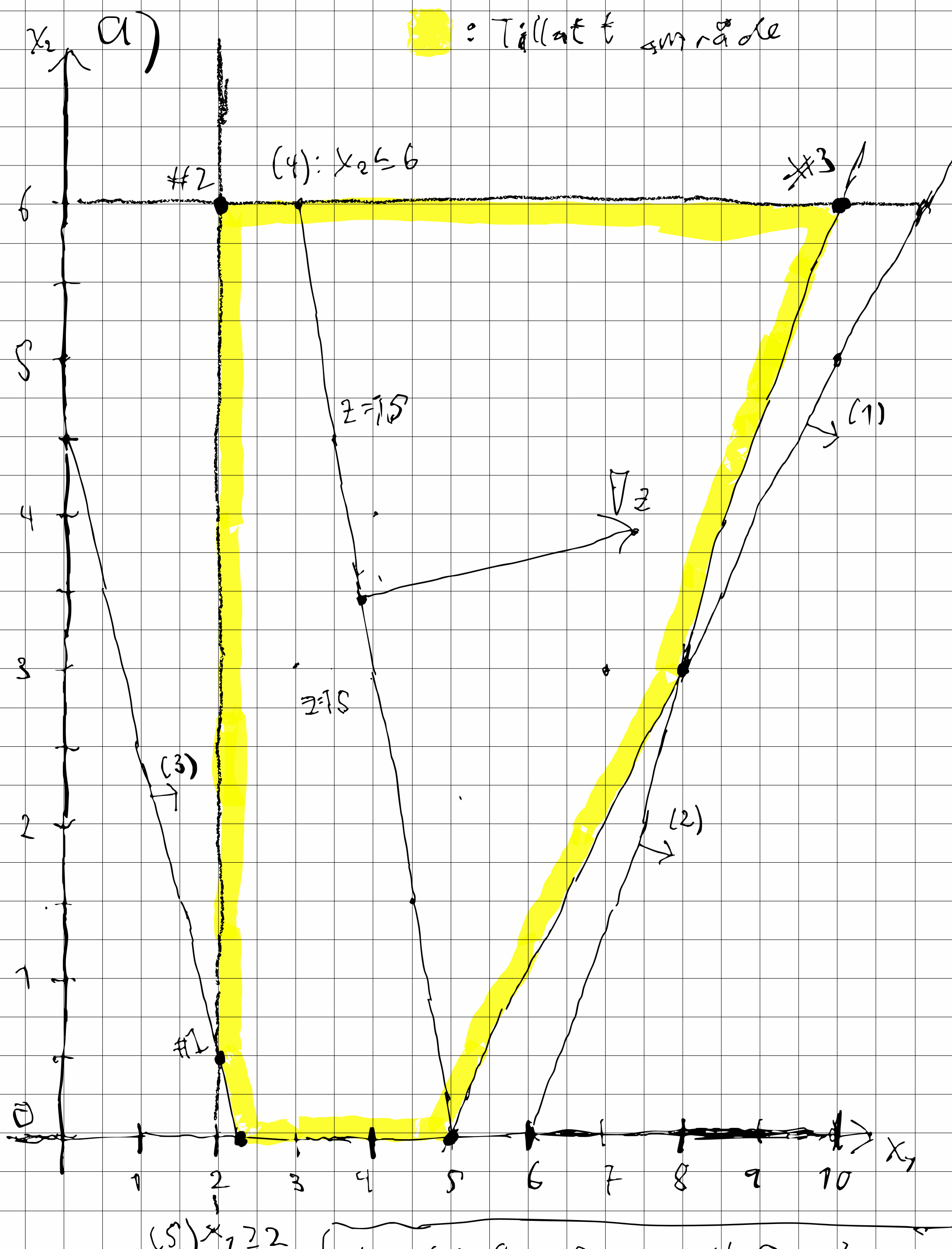
$3x_1 - 2x_2 \leq 18$ (2) $(x_2 = \frac{1}{2}(3x_1 - 18))$

$4x_1 + 2x_2 \geq 9$ (3) $(x_2 = \frac{1}{2}(9 - 4x_1))$

$x_2 \leq 6$ (4)

$x_1 \geq 2$ (5)

$x_2 \geq 0$ (6).



c) Vi gå Rut til Simplex

b) max z s.t.

$$z - 3x_1 - x_2 = 0$$

$$x_1 - x_2 + s_1 = 5, \quad s_1 \geq 0,$$

$$3x_1 - 2x_2 + s_2 = 18, \quad s_2 \geq 0 \quad (2)$$

$$4x_1 + 2x_2 - s_3 + \bar{x}_3 = 9, \quad \bar{x}_3, s_3 \geq 0,$$

$$x_2 + s_4 = 6, \quad s_4 \geq 0,$$

$$x_1 - s_5 + \bar{x}_4 = 2, \quad \bar{x}_4, s_5 \geq 0.$$

$$x_1, x_2, \{s_i\}_{i=1}^{5}, \bar{x}_3, \bar{x}_4 \geq 0.$$

Solve in two phases.

first: min $\bar{x}_3 + \bar{x}_4$ s.t. conditions hold.

Second: max z s.t. conditions hold.

Phase 1:

c) $\min \bar{z} = \bar{x}_3 + \bar{x}_4$

$$x_1 - x_2 + s_1 = 5 \quad (1)$$

$$3x_1 - 2x_2 + s_2 = 18 \quad (2)$$

$$4x_1 + 2x_2 - s_3 + \bar{x}_3 = 9, \quad (3)$$

$$x_2 + s_4 = 6, \quad (4)$$

$$x_1 - s_5 + \bar{x}_4 = 2, \quad (5)$$

$$x_1, x_2, \{s_i\}_{i=1}^{5}, \bar{x}_3, \bar{x}_4 \geq 0.$$

Initialize at: $x_1 = x_2 = 0$,

$s_1 = 5$, $s_2 = 18$, $s_3 = 0$, $s_4 = 6$, $s_5 = 0$,

$\bar{x}_3 = 9$, $\bar{x}_4 = 2$.

$$\bar{z} = (9 + s_3 - 4x_1 - 2x_2) + (2 + s_5 - x_1)$$

$$\bar{z} = -5x_1 - 2x_2 + s_3 + s_5 + 11$$

$$\max(-\bar{z}) = 5x_1 + 2x_2 - s_3 - s_5 - 11,$$

$$(-\bar{z}) - 5x_1 - 2x_2 + s_3 + s_5 = -11$$

First artificial BF solution:

$(x_1, x_2, \bar{x}_3, \bar{x}_4, s_1, s_2, s_3, s_4, s_5)$
$= (0, 0, 9, 2, 5, 18, 0, 6, 0)$

Iteration 1: Coeff. of:

Bas. vars.:	x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
max $(-\bar{Z})$	-5	-2	0	0	0	0	+1	0	+1	-11	
s_1	1	-1			1					5	5
s_2	3	-2				1				18	6.
$\bar{x}_3$	4	2	1				-1			9	9/4
s_4		1						1		6	—
$\bar{x}_4$	1			1					-1	2	2*

Entering basic variable: x_1,
Leaving Basic var.: $\bar{x}_4 = 0$.

Solve for new values: $x_1 = 2$, $\bar{x}_4 = 0$, Iteration 2:

Bas. vars.:		x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
max	$(-Z)$	0	-2	0	5	0	0	+1	0	-4	-1	
	s_1	0	-1		-1	1				1	3	–
	s_2	0	-2		-3		1			3	12	–
	$\bar{x}_3$	0	2	1	-4			-1		4	1	1/2
	s_4	0	1						1		6	6
	x_1	1			1					-1	2	–

Entering Basic var.: x_2, leaving: $\bar{x}_3$.

Iteration 3:

Bas. vars.:		x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
				x	x			x		x		
max	$(-Z)$	0	0	0	1	0	0	0	0	0	0	
	s_1	0	0	1/2	-3	1	0	-1/2	0	3	3,5	
	s_2	0	0	0	-7	0	1	-1	0	7	13	
	x_2	0	1	1/2	-2	0	0	-1/2	0	2	1/2	
	s_4	0	0	-1/2	2	0	0	+1/2	1	-2	5,5	
	x_1	1	0	0	1	0	0	0	0	-1	2	

↑

Optimal solution. No negative coeff's in the zero'th row.

For a BF solution:

$$(x_1, x_2, s_1, s_2, s_3, s_4, s_5)$$
$$= \left(2,\ 1/2,\ \tfrac{7}{2},\ 13,\ 0,\ \tfrac{11}{2},\ 0\right) = \vec{x}_0.$$

Now begin phase two, with $\bar{x}_3 = \bar{x}_4 = 0$.

max Z s.t.

$$Z - 3x_1 - x_2 = 0, \quad (0)$$
$$x_1 - x_2 + s_1 = 5, \quad (1)$$
$$3x_1 - 2x_2 + s_2 = 18, \quad (2)$$
$$4x_1 + 2x_2 - s_3 = 9, \quad (3)$$
$$x_2 + s_4 = 6, \quad (4)$$
$$x_1 - s_5 = 2, \quad (5)$$

$$x_1, x_2, \{s_i\}_{i=1}^{5} \geq 0.$$

$$x_1 = 2 - s_5, \quad 2x_2 = 9 + s_3 - 4x_1$$
$$= 9 + s_3 - 4(2 - s_5)$$
$$x_2 = \tfrac{1}{2}(9 + s_3) - 4 + 2s_5$$
$$= 4.5 + \tfrac{1}{2}s_3 - 4 + 2s_5,$$
$$= 1/2 + 1/2\, s_3 + 2s_5,$$

$$Z - 3(2 - S_5) - \left(\tfrac{1}{2} + \tfrac{1}{2} S_3 + 2 S_5\right) = 0$$

$$Z - 6 + 3 S_5 - \tfrac{1}{2} - \tfrac{1}{2} S_3 - 2 S_5 = 0$$

$$Z + S_5 - \tfrac{1}{2} S_3 = 13/2$$

	Bas. Vars.	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
max	Z	0	0	0	0	$-1/2$	0	1	$13/2$	
(1)		1	-1	1	0	0	0	0	5	
(2)		3	-2		1				18	
(3)		4	2			-1			9	
(4)			1				1		6	
(5)		1						-1	2	

Use final Phase I Tableau with Z:

S_3 Entering Basic Var.

Bas. Vars.:	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
Z	0	0	0	0	$-1/2$	0	1	$13/2$	
S_1	0	0	1	0	$-1/2$	0	3	3.5	—
S_2	0	0	0	1	-1	0	7	13	—
X_2	0	1	0	0	$-1/2$	0	2	$1/2$	—
S_4	0	0	0	0	$+1/2$	1	-2	5.5	11
X_1	1	0	0	0	0	0	-1	2	—

S_4 Leaving.

Iteration 2:

Entering: S_5

	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
Z	0	0	0	0	0	1	-1	12	
S_1	0	0	1	0	0	1	1	9	9
S_2	0	0	0	1	0	2	3	24	8
X_2	0	1	0	0	0	1	0	6	—
S_3	0	0	0	0	1	2	-4	11	—
X_1	1	0	0	0	0	0	-1	2	—

Leaving: S_2

Iteration 3:

	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	
Z	0	0	0	1/3	0	5/3	0	20	+1
S_1	0	0	1	-1/3	0	1/3	0	1	(-1)
S_5	0	0	0	1/3	0	2/3	1	8	×
X_2	0	1	0	0	0	1	0	6	
S_3	0	0	0	-4/3	1	10/3	0	43	+4
X_1	1	0	0	1/3	0	2/3	0	10	+1

Optimal solution found! $(x_1, x_2) = (10, 6)$

d) Hvis vi setter høyresiden i betingelse (7) til 4.

(7) $x_1 - x_2 \leq 5 \Rightarrow (7)' \; x_1 - x_2 \leq 4$

vil vi en degenerert løsning da tre betingelser er gyldige i det optimale punktet (10, 6), istedet for vanlig antall 2.

Vi avdekker degenerasjon ved en BF løsning ved at det er to ratioer med like minimums-verdi i minimums-ratio testen, eller ekvivalent at en basis-variabel får verdien 0.

e) For å få flere optimale løsninger, spesifiser en betingelse ortogonalt på ∇z.

$\nabla z = (3, 1)$, la betingelse (4) være gitt ved $g(x_1, x_2) \leq 0$.

Da trenger vi $\nabla g = \nabla z$, og

$g(10, 6) = 0$. $\quad g(x_1, x_2) = 3x_1 + x_2 - k$

$\Rightarrow 3 \cdot 10 + 6 - k = 0$,

$k = 36$.

Altså, nye

$(4) \to (4)'$: $3x_1 + x_2 \leq 36$. og alle optimale løsninger blir på formen

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}(w) = w \begin{pmatrix} 0 \\ 36 \end{pmatrix} + (1-w) \begin{pmatrix} 10 \\ 6 \end{pmatrix}, \quad w \in [0, 1].$$

Oppgave 2

$$\min z = 4x_1 + 8x_2 + 3x_3$$

Når

$$x_1 + x_2 \geq 2$$

$$2x_2 + x_3 \geq 5$$

$$x_1, x_2, x_3 \geq 0$$

a) Utvidet form:

$$\min z = 4x_1 + 8x_2 + 3x_3$$

Når

$$x_1 + x_2 - s_1 = 2$$

$$2x_2 + x_3 - s_2 = 5$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Origo er ikke en lovlig løsning. Innfører kunstvariable a_1 og a_2, og skriver som maksimeringsproblem.

$$\max -z = -4x_1 - 8x_2 - 3x_3 - Ma_1 - Ma_2$$

Når

$$x_1 + x_2 - s_1 + a_1 = 2$$

$$2x_2 + x_3 - s_2 + a_2 = 5$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

b)

Gjøre a_1 og a_2 til basisvariable:

$$(0) \quad -z + 4x_1 + 8x_2 + 3x_3 + Ma_1 + Ma_2 = 0$$

$$(1) \quad x_1 + x_2 - s_1 + a_1 = 2$$

$$(2) \quad 2x_2 + x_3 - s_2 + a_2 = 5$$

$(0) - M(1)$: $\quad -z + (4-M)x_1 + (8-M)x_2 + 3x_3 + Ms_1 = -2M$

$(0) - M(1) - M(2)$: $\quad -z + (4-M)x_1 + (8-3M)x_2 + (3-M)x_3 + Ms_1 + Ms_2 = -7M$

Finne mulig basisløsning ved å få a_1 og a_2 ut av basis:

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	$(4-M)$	$(8-3M)$	$(3-M)$	M	M	0	0	$-7M$	
a_1	(1)	0	1	1	0	-1	0	1	0	2	$\frac{2}{1} = 2$
a_2	(2)	0	0	2	1	0	-1	0	1	5	$\frac{5}{2} = 2.5$
z	(0)	-1	$(2M-4)$	0	$(3-M)$	$(8-2M)$	M	$(3M-8)$	0	$-M-16$	
x_2	(1)	0	1	1	0	-1	0	1	0	2	
a_2	(2)	0	-2	0	1	2	-1	-2	1	1	$\frac{1}{2}$
z	(0)	-1	4	0	-1	0	4	M	$M-4$	-20	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2$	

Fase 1 ferdig.

Mulig basisløsning:

$$x_1 = 0, \quad x_2 = \frac{5}{2}, \quad x_3 = 0, \quad z = 20$$

c) Løsningen er ikke optimal, x_3 kan økes for å gi en mindre verdi av z.

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	4	0	-1	0	4	M	$M-4$	-20	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	5
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2$	1
z	(0)	-1	2	0	0	2	3	$M-2$	$M-3$	-19	
x_2	(1)	0	1	1	0	-1	0	1	0	2	
x_3	(2)	0	-2	0	1	2	-1	-2	1	1	

Optimal løsning:

$$x_1 = 0, \quad x_2 = 2, \quad x_3 = 1, \quad z = 19$$

d)

$$z = 4x_1 + 8x_2 + 3x_3$$

$$z = 4x_1 + C_2 x_2 + 3x_3$$

Løsning: $(x_1, x_2, x_3) = (0, 2, 1)$

Sensitivitetsanalyse:

Setter $C_2^{ny} = C_2 + \Delta$

Initielt tablå, fase 1:

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	$(4-M)$	$\boxed{(8+\Delta-3M)}$	$(3-M)$	M	M	0	0	$-7M$	
a_1	(1)	0	1	1	0	-1	0	1	0	2	$\frac{2}{1} = 2$
a_2	(2)	0	0	2	1	0	-1	0	1	5	$\frac{5}{2} = 2.5$
z	(0)	-1	$(-4-\Delta+2M)$	0	$3-M$	$\boxed{(8+\Delta-2M)}$	M	$(3M-8-\Delta)$	0	$-M-16-2\Delta$	
x_2	(1)	0	1	1	0	-1	0	1	0	2	
a_2	(2)	0	-2	0	1	2	-1	-2	1	1	$\frac{1}{2}$
z	(0)	-1	4	0	$(-1-\frac{1}{2}\Delta)$	0	$4+\frac{1}{2}\Delta$	M	$(-4-\frac{1}{2}\Delta+M)$	$-20-\frac{5}{2}\Delta$	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2$	

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	4	0	$\boxed{(-1-\frac{1}{2}\Delta)}$	0	$4+\frac{1}{2}\Delta$	M	$(-4-\frac{1}{2}\Delta+M)$	$-20-\frac{5}{2}\Delta$	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	5
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2$	1
z	(0)	-1	$(2-\Delta)$	0	0	$(2+\Delta)$	3	$(M-2-\Delta)$	$(M-3)$	$-19-2\Delta$	
x_2	(1)	0	1	1	0	-1	0	1	0	2	
x_3	(2)	0	-2	0	1	2	-1	-2	1	1	

Krever positive koeffisienter i (0) for optimal løsning:

$$2-\Delta \geq 0 \quad\Rightarrow\quad \Delta \leq 2$$

$$2+\Delta \geq 0 \quad\Rightarrow\quad \Delta \geq -2$$

$$M-2-\Delta \geq 0 \quad\Rightarrow\quad \Delta \leq M-2$$

Sensitivitetsområde for C_2: $\quad 8-2 \leq C_2 \leq 8+2$

$$\Rightarrow \quad 6 \leq C_2 \leq 10$$

e) Sensitivitetsområde for høyresiden til restriksjonen $x_1 + x_2 \geq 2$

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	$(4-M)$	$(8-3M)$	$(3-M)$	M	M	0	0	$-7M$	
a_1	(1)	0	1	1	0	-1	0	1	0	$2+\Delta$	$\frac{2}{1} = 2$
a_2	(2)	0	0	2	1	0	-1	0	1	5	$\frac{5}{2} = 2.5$
z	(0)	-1	$(2M-4)$	0	$(3-M)$	$(8-2M)$	M	$(3M-8)$	0	$-M-16$	
x_2	(1)	0	1	1	0	-1	0	1	0	$2+\Delta$	
a_2	(2)	0	-2	0	1	2	-1	-2	1	$1-2\Delta$	$\frac{1}{2}$
z	(0)	-1	4	0	-1	0	4	M	$M-4$	-20	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2-\Delta$	

Basis. var	Ligning	z	x_1	x_2	x_3	s_1	s_2	a_1	a_2	RHS	Forholds-test
z	(0)	-1	4	0	-1	0	4	M	$M-4$	-20	
x_2	(1)	0	0	1	$1/2$	0	$-1/2$	0	$1/2$	$5/2$	5
s_1	(2)	0	-1	0	$1/2$	1	$-1/2$	-1	$1/2$	$1/2 - \Delta$	1
z	(0)	-1	2	0	0	2	3	$M-2$	$M-3$	-19	
x_2	(1)	0	1	1	0	-1	0	1	0	$2-\Delta$	
x_3	(2)	0	-2	0	1	2	-1	-2	1	$1-2\Delta$	

$$2-\Delta \geq 0 \quad \Rightarrow \quad \Delta \leq 2$$

$$1-2\Delta \geq 0 \quad \Rightarrow \quad \Delta \leq \frac{1}{2}$$

Ser på opprinnelig restriksjon:

$$x_1 + x_2 \geq 2 + \Delta \quad , \quad x_1 = 0 \; , \; x_2 \geq 0$$

$$\Rightarrow \quad \Delta \geq -2$$

Sensitivitetsområde B_1 : $\quad -2 \leq B_1 \leq \frac{1}{2}$