Oppgave 2

$$\min z = 4x_1 + 8x_2 + 3x_3$$

Når

$$x_1 + x_2 \geq 2$$
$$2x_2 + x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

a) Utvidet form:

$$\min z = 4x_1 + 8x_2 + 3x_3$$

Når

$$x_1 + x_2 - s_1 = 2$$
$$2x_2 + x_3 - s_2 = 5$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Origo er ikke en lovlig løsning. Innfører kunstvariable $a_1$ og $a_2$, og skriver som maksimeringsproblem.

$$\max -z = -4x_1 - 8x_2 - 3x_3 - Ma_1 - Ma_2$$

Når

$$x_1 + x_2 - s_1 + a_1 = 2$$
$$2x_2 + x_3 - s_2 + a_2 = 5$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

b)

Gjøre $a_1$ og $a_2$ til basisvariable:

$$(0) \quad -z + 4x_1 + 8x_2 + 3x_3 + Ma_1 + Ma_2 = 0$$

$$(1) \quad x_1 + x_2 - s_1 + a_1 = 2$$

$$(2) \quad 2x_2 + x_3 - s_2 + a_2 = 5$$

$(0) - M(1)$: $\quad -z + (4-M)x_1 + (8-M)x_2 + 3x_3 + Ms_1 = -2M$

$(0) - M(1) - M(2)$: $\quad -z + (4-M)x_1 + (8-3M)x_2 + (3-M)x_3 + Ms_1 + Ms_2 = -7M$

Finne mulig basisløsning ved å få $a_1$ og $a_2$ ut av basis:

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | $-1$ | $(4-M)$ | $\boxed{(8-3M)}$ | $(3-M)$ | $M$ | $M$ | $0$ | $0$ | $-7M$ | |
| $a_1$ | (1) | $0$ | $1$ | $\boxed{1}$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2$ | $\frac{2}{1} = 2$ |
| $a_2$ | (2) | $0$ | $0$ | $\boxed{2}$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $5$ | $\frac{5}{2} = 2.5$ |
| $z$ | (0) | $-1$ | $(2M-4)$ | $0$ | $(3-M)$ | $\boxed{(8-2M)}$ | $M$ | $(3M-8)$ | $0$ | $-M-16$ | |
| $x_2$ | (1) | $0$ | $1$ | $1$ | $0$ | $\boxed{-1}$ | $0$ | $1$ | $0$ | $2$ | |
| $a_2$ | (2) | $0$ | $-2$ | $0$ | $1$ | $\boxed{2}$ | $-1$ | $-2$ | $1$ | $1$ | $\frac{1}{2}$ |
| $z$ | (0) | $-1$ | $4$ | $0$ | $-1$ | $0$ | $4$ | $M$ | $M-4$ | $-20$ | |
| $x_2$ | (1) | $0$ | $0$ | $1$ | $1/2$ | $0$ | $-1/2$ | $0$ | $1/2$ | $5/2$ | |
| $s_1$ | (2) | $0$ | $-1$ | $0$ | $1/2$ | $1$ | $-1/2$ | $-1$ | $1/2$ | $1/2$ | |

Fase 1 ferdig.

Mulig basisløsning:

$$x_1 = 0, \quad x_2 = \frac{5}{2}, \quad x_3 = 0, \quad z = 20$$

c) Løsningen er ikke optimal, $x_3$ kan økes for å gi en mindre verdi av $z$.

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | -1 | 4 | 0 | -1 | 0 | 4 | M | M-4 | -20 | |
| $x_2$ | (1) | 0 | 0 | 1 | 1/2 | 0 | -1/2 | 0 | 1/2 | 5/2 | 5 |
| $s_1$ | (2) | 0 | -1 | 0 | 1/2 | 1 | -1/2 | -1 | 1/2 | 1/2 | 1 |
| $z$ | (0) | -1 | 2 | 0 | 0 | 2 | 3 | M-2 | M-3 | -19 | |
| $x_2$ | (1) | 0 | 1 | 1 | 0 | -1 | 0 | 1 | 0 | 2 | |
| $x_3$ | (2) | 0 | -2 | 0 | 1 | 2 | -1 | -2 | 1 | 1 | |

Optimal løsning:

$$x_1 = 0, \quad x_2 = 2, \quad x_3 = 1, \quad z = 19$$

d)

$$z = 4x_1 + 8x_2 + 3x_3$$

$$z = 4x_1 + C_2 x_2 + 3x_3$$

Løsning: $(x_1, x_2, x_3) = (0, 2, 1)$

Sensitivitetsanalyse:

Setter $C_2^{ny} = C_2 + \Delta$

Initielt tablå, fase 1:

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | $-1$ | $(4-M)$ | $(8+\Delta-3M)$ | $(3-M)$ | $M$ | $M$ | $0$ | $0$ | $-7M$ | |
| $a_1$ | (1) | $0$ | $1$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2$ | $\frac{2}{1} = 2$ |
| $a_2$ | (2) | $0$ | $0$ | $2$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $5$ | $\frac{5}{2} = 2.5$ |
| $z$ | (0) | $-1$ | $(-4-\Delta+2M)$ | $0$ | $3-M$ | $(8+\Delta-2M)$ | $M$ | $(3M-8-\Delta)$ | $0$ | $-M-16-2\Delta$ | |
| $x_2$ | (1) | $0$ | $1$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2$ | |
| $a_2$ | (2) | $0$ | $-2$ | $0$ | $1$ | $2$ | $-1$ | $-2$ | $1$ | $1$ | $\frac{1}{2}$ |
| $z$ | (0) | $-1$ | $4$ | $0$ | $(-1-\frac{1}{2}\Delta)$ | $0$ | $4+\frac{1}{2}\Delta$ | $M$ | $(-4-\frac{1}{2}\Delta+M)$ | $-20-\frac{5}{2}\Delta$ | |
| $x_2$ | (1) | $0$ | $0$ | $1$ | $1/2$ | $0$ | $-1/2$ | $0$ | $1/2$ | $5/2$ | |
| $s_1$ | (2) | $0$ | $-1$ | $0$ | $1/2$ | $1$ | $-1/2$ | $-1$ | $1/2$ | $1/2$ | |

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | $-1$ | $4$ | $0$ | $\boxed{(-1-\frac{1}{2}\Delta)}$ | $0$ | $4+\frac{1}{2}\Delta$ | $M$ | $(-4-\frac{1}{2}\Delta+M)$ | $-20-\frac{5}{2}\Delta$ | |
| $x_2$ | (1) | $0$ | $0$ | $1$ | $1/2$ | $0$ | $-1/2$ | $0$ | $1/2$ | $5/2$ | $5$ |
| $s_1$ | (2) | $0$ | $-1$ | $0$ | $1/2$ | $1$ | $-1/2$ | $-1$ | $1/2$ | $1/2$ | $1$ |
| $z$ | (0) | $-1$ | $(2-\Delta)$ | $0$ | $0$ | $(2+\Delta)$ | $3$ | $(M-2-\Delta)$ | $(M-3)$ | $-19-2\Delta$ | |
| $x_2$ | (1) | $0$ | $1$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2$ | |
| $x_3$ | (2) | $0$ | $-2$ | $0$ | $1$ | $2$ | $-1$ | $-2$ | $1$ | $1$ | |

Krever positive koeffisienter i (0) for optimal løsning:

$$2-\Delta \geq 0 \Rightarrow \Delta \leq 2$$

$$2+\Delta \geq 0 \Rightarrow \Delta \geq -2$$

$$M-2-\Delta \geq 0 \Rightarrow \Delta \leq M-2$$

Sensitivitetsområde for $C_2$: $8-2 \leq C_2 \leq 8+2$

$$\Rightarrow 6 \leq C_2 \leq 10$$

e) Sensitivitetsområde for høyresiden til restriksjonen $x_1 + x_2 \geq 2$

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | $-1$ | $(4-M)$ | $(8-3M)$ | $(3-M)$ | $M$ | $M$ | $0$ | $0$ | $-7M$ | |
| $a_1$ | (1) | $0$ | $1$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2+\Delta$ | $\frac{2}{1} = 2$ |
| $a_2$ | (2) | $0$ | $0$ | $2$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $5$ | $\frac{5}{2} = 2.5$ |
| $z$ | (0) | $-1$ | $(2M-4)$ | $0$ | $(3-M)$ | $(8-2M)$ | $M$ | $(3M-8)$ | $0$ | $-M-16$ | |
| $x_2$ | (1) | $0$ | $1$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $2+\Delta$ | |
| $a_2$ | (2) | $0$ | $-2$ | $0$ | $1$ | $2$ | $-1$ | $-2$ | $1$ | $1-2\Delta$ | $\frac{1}{2}$ |
| $z$ | (0) | $-1$ | $4$ | $0$ | $-1$ | $0$ | $4$ | $M$ | $M-4$ | $-20$ | |
| $x_2$ | (1) | $0$ | $0$ | $1$ | $1/2$ | $0$ | $-1/2$ | $0$ | $1/2$ | $5/2$ | |
| $s_1$ | (2) | $0$ | $-1$ | $0$ | $1/2$ | $1$ | $-1/2$ | $-1$ | $1/2$ | $1/2-\Delta$ | |

| Basis. var | Ligning | $z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | RHS | Forholds-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | (0) | -1 | 4 | 0 | -1 | 0 | 4 | M | M - 4 | - 20 | |
| $x_2$ | (1) | 0 | 0 | 1 | 1/2 | 0 | -1/2 | 0 | 1/2 | 5/2 | 5 |
| $s_1$ | (2) | 0 | -1 | 0 | 1/2 | 1 | -1/2 | -1 | 1/2 | $1/2 - \Delta$ | 1 |
| $z$ | (0) | -1 | 2 | 0 | 0 | 2 | 3 | M-2 | M-3 | -19 | |
| $x_2$ | (1) | 0 | 1 | 1 | 0 | -1 | 0 | 1 | 0 | $2 - \Delta$ | |
| $x_3$ | (2) | 0 | -2 | 0 | 1 | 2 | -1 | -2 | 1 | $1 - 2\Delta$ | |

$$2 - \Delta \geq 0 \quad \Rightarrow \quad \Delta \leq 2$$

$$1 - 2\Delta \geq 0 \quad \Rightarrow \quad \Delta \leq \frac{1}{2}$$

Ser på opprinnelig restriksjon:

$$x_1 + x_2 \geq 2 + \Delta \quad , \quad x_1 = 0 \;, \quad x_2 \geq 0$$

$$\Rightarrow \quad \Delta \geq -2$$

Sensitivitetsområde $B_1$ : $\quad -2 \leq B_1 \leq \frac{1}{2}$