Oppg. 3

primal problem:

$$\max \quad z = x_1 + 2x_2 + x_3 + x_4$$

$$\begin{aligned}
\text{st.} \quad & 2x_1 + x_2 + 5x_3 + x_4 \leq 8 \\
& 2x_1 + 2x_2 + 4x_4 = 12 \\
& 3x_1 + x_2 + 2x_3 \geq 18 \\
& x_1, x_2, x_4 \geq 0 \\
& x_3 \leq 0
\end{aligned}$$

First write primal on standard form:

$$\max \quad z = x_1 + 2x_2 + x_3 + x_4$$

$$\begin{aligned}
\text{st.} \quad & 2x_1 + x_2 + 5x_3 + x_4 \leq 8 \\
& 2x_1 + 2x_2 + 4x_4 \leq 12 \\
& -2x_1 - 2x_2 - 4x_4 \leq -12 \\
& -3x_1 - x_2 + 2x_3 \leq -18 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

This is equivalent to

$$\begin{aligned} \max \quad & z = \underline{c}^T \underline{x} \\ \text{st.} \quad & \underline{\underline{A}}\,\underline{x} \leq \underline{b} \\ & \underline{x} \geq \underline{0} \end{aligned}$$

with

$$\underline{c} = [\,1 \quad 2 \quad 1 \quad 1\,]^T$$

$$\underline{b} = [\,8 \quad 12 \quad -12 \quad -18\,]^T$$

$$\underline{\underline{A}} = \begin{bmatrix} 2 & 1 & 5 & 1 \\ 2 & 2 & 0 & 4 \\ -2 & -2 & 0 & -4 \\ -3 & -1 & 2 & 0 \end{bmatrix}$$

Dual problem

$$\min \quad w = 8y_1 + 12y_2 - 12y_3 - 18y_4$$

$$\begin{aligned} 2y_1 + 2y_2 - 2y_3 - 3y_4 &\geq 1 \\ y_1 + 2y_2 - 2y_3 - y_4 &\geq 2 \\ 5y_1 + 2y_4 &\geq 1 \\ y_1 + 4y_2 - 4y_3 &\geq 1 \\ y_1, y_2, y_3, y_4 &\geq 0 \end{aligned}$$

Oppg. 4

$$
\begin{aligned}
\min \quad z = \; & 2x_1 + 2x_2 + x_3 - x_4 + x_5 \\
\text{st.} \quad & x_1 + 2x_2 - x_3 + 2x_4 = 6 \\
& 2x_1 + x_3 + x_5 \geq 4 \\
& 3x_2 - 2x_3 + 3x_4 = 7 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}
$$

Assuming the dual solution $\underline{v} = [1 \quad 0 \quad -1]$

a) Determine if the dual solution is feasible.

$$
\begin{aligned}
\max \quad w = \; & 6y_1 + 4y_2 + 7y_3 \\
\text{s.t.} \quad & y_1 + 2y_2 \leq 2 \\
& 2y_1 + 3y_3 \leq 2 \\
& -y_1 + y_2 - 2y_3 \leq 1 \\
& 2y_1 + 3y_3 \leq -1 \\
& y_2 \leq 1 \\
& y_1 \;, \; y_2 \geq 0 \;, \; y_3
\end{aligned}
$$

Check if $y = [1 \quad 0 \quad -1]$ is feasible:

LHS	RHS	slack	Satisfied?
1	2	1	✓
-1	2	3	✓
1	1	0	✓
-1	-1	0	✓
0	1	1	✓

The dual solution is feasible.

b) Corresponding primal solution.

Dual w/ SOB method:

$$\begin{aligned}
\min \quad z = \; & 2x_1 + 2x_2 + x_3 - x_4 + x_5 \\
& x_1 + 2x_2 - x_3 + 2x_4 = 6 \\
& 2x_1 + x_3 + x_5 \geq 4 \\
& 3x_2 - 2x_3 + 3x_4 = 7 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

Basisvariable: S_1, S_2, S_5, X_3, X_4

$$-x_3 + 2x_4 = 6$$

$$-2x_3 + 3x_4 = 7$$

$$x_3 = 2x_4 - 6 = 4$$

$$-2(2x_4 - 6) + 3x_4 = 7$$

$$-4x_4 + 12 + 3x_4 = 7$$

$$x_4 = 5$$

Corresponding primal solution:

$$(0, 0, 4, 5, 0)$$

c) Is the dual solution optimal?

Yes, if the primal solution is feasible.

LHS	RHS	"Constraint"	Satisfied?
6	6	=	✓
4	4	$\geq$	✓
-8	7	=	✗

No, the given dual solution is not optimal.