Øving 3 - Oper. An.

Oppgave 1:

Max $z = 3x_1 + x_2$ s.t.

$x_1 - x_2 \leq 5$ (1) $(x_2 = x_1 - 5)$

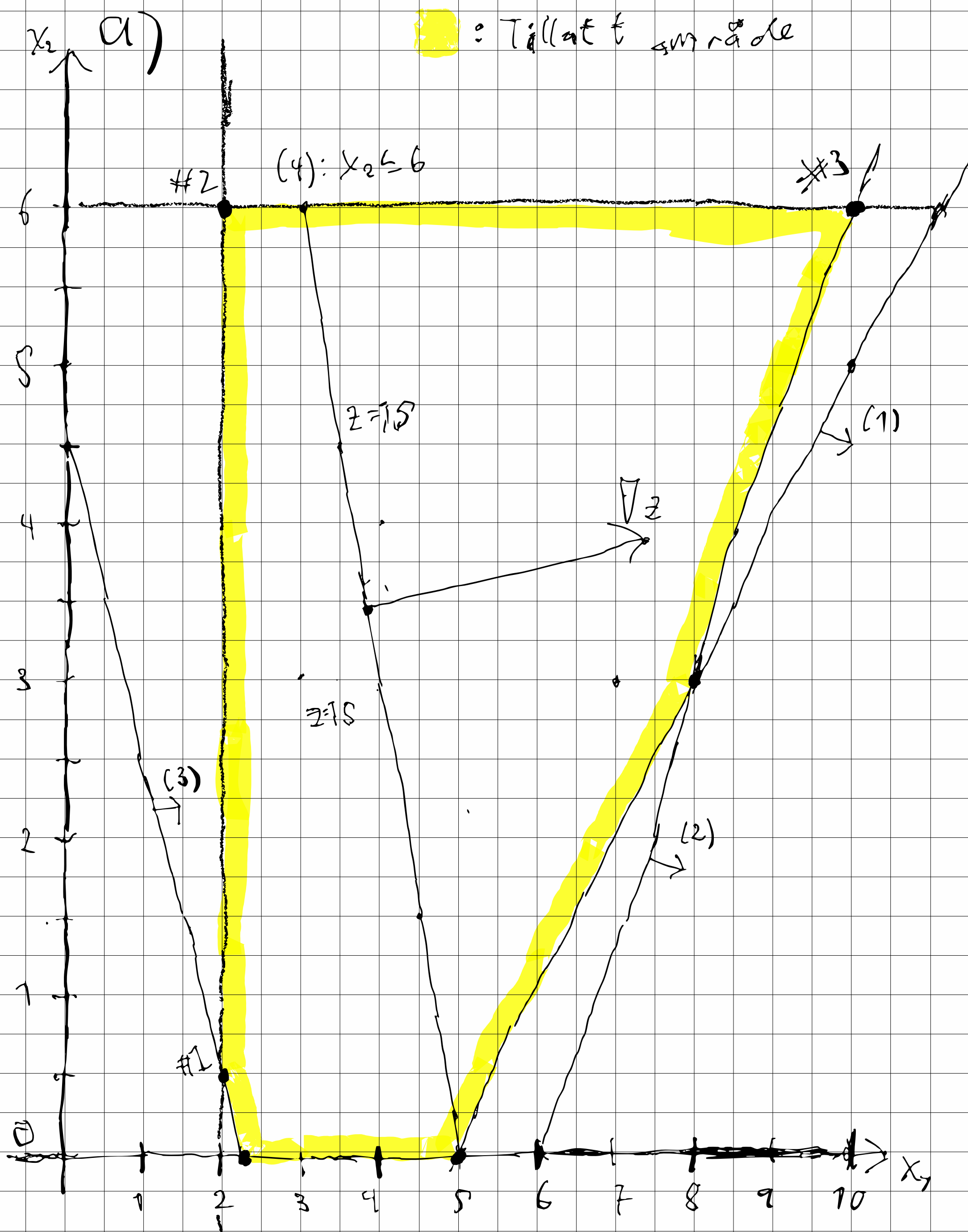
$3x_1 - 2x_2 \leq 18$ (2) $\left(x_2 = \frac{1}{2}(3x_1 - 18)\right)$

$4x_1 + 2x_2 \geq 9$ (3) $\left(x_2 = \frac{1}{2}(9 - 4x_1)\right)$

$x_2 \leq 6$ (4)

$x_1 \geq 2$ (5)

$x_2 \geq 0$ (6).



c) Vi går Rakt till Simplex

b) $\max z$ s.t.

$$z - 3x_1 - x_2 = 0$$

$$x_1 - x_2 + s_1 = 5, \quad s_1 \geq 0,$$

$$3x_1 - 2x_2 + s_2 = 18, \quad s_2 \geq 0 \quad (2)$$

$$4x_1 + 2x_2 - s_3 + \bar{x}_3 = 9, \quad \bar{x}_3, s_3 \geq 0,$$

$$x_2 + s_4 = 6, \quad s_4 \geq 0,$$

$$x_1 - s_5 + \bar{x}_4 = 2, \quad \bar{x}_4, s_5 \geq 0.$$

$$x_1, x_2, \{s_i\}_{i=1}^{5}, \bar{x}_3, \bar{x}_4 \geq 0.$$

Solve in two phases.

first: $\min \bar{x}_3 + \bar{x}_4$ s.t. conditions hold.

Second: $\max z$ s.t. conditions hold.

Phase 1:

c) $\min \bar{z} = \bar{x}_3 + \bar{x}_4$

$$x_1 - x_2 + s_1 = 5 \qquad (1)$$

$$3x_1 - 2x_2 + s_2 = 18 \qquad (2)$$

$$4x_1 + 2x_2 - s_3 + \bar{x}_3 = 9, \qquad (3)$$

$$x_2 + s_4 = 6, \qquad (4)$$

$$x_1 - s_5 + \bar{x}_4 = 2, \qquad (5)$$

$$x_1, x_2, \{s_i\}_{i=1}^{5}, \bar{x}_3, \bar{x}_4 \geq 0.$$

Initialize at: $x_1 = x_2 = 0,$
$s_1 = 5,\ s_2 = 18,\ s_3 = 0,\ s_4 = 6,\ s_5 = 0,$
$\bar{x}_3 = 9,\ \bar{x}_4 = 2.$

$$\bar{z} = (9 + s_3 - 4x_1 - 2x_2) + (2 + s_5 - x_1)$$

$$\bar{z} = -5x_1 - 2x_2 + s_3 + s_5 + 11$$

$$\max\ (-\bar{z}) = 5x_1 + 2x_2 - s_3 - s_5 - 11,$$

$$(-\bar{z}) - 5x_1 - 2x_2 + s_3 + s_5 = -11$$

First artificial BF solution:

$(x_1, x_2, \bar{x}_3, \bar{x}_4, s_1, s_2, s_3, s_4, s_5)$
$= (0, 0, 9, 2, 5, 18, 0, 6, 0)$

Iteration 1: Coeff. of:

Bas. vars.:	x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
max $(-\bar{Z})$	-5	-2	0	0	0	0	+1	0	+1	-11	
s_1	1	-1			1					5	5
s_2	3	-2				1				18	6.
$\bar{x}_3$	4	2	1				-1			9	9/4
s_4		1						1		6	—
$\bar{x}_4$	1			1					-1	2	2*

Entering basic variable: x_1,
leaving Basic var.: $\bar{x}_4 = 0$.

Solve for new values: $x_1 = 2$, $\bar{x}_4 = 0$, Iteration 2:

Bas. vars.:		x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
max	$(-Z)$	0	-2	0	5	0	0	+1	0	-4	-1	
	s_1	0	-1		-1	1				1	3	-
	s_2	0	-2		-3		1			3	12	-
	$\bar{x}_3$	0	2	1	-4			-1		4	1	1/2
	s_4	0	1						1		6	6
	x_1	1			1					-1	2	-

Entering Basic var.: x_2, leaving: $\bar{x}_3$.

Iteration 3:

Bas. vars.:		x_1	x_2	$\bar{x}_3$	$\bar{x}_4$	s_1	s_2	s_3	s_4	s_5	RHS	Ratio
				x	x			x		x		
max	$(-Z)$	0	0	0	1	0	0	0	0	0	0	
	s_1	0	0	1/2	-3	1	0	-1/2	0	3	3.5	
	s_2	0	0	0	-7	0	1	-1	0	7	13	
	x_2	0	1	1/2	-2	0	0	-1/2	0	2	1/2	
	s_4	0	0	-1/2	2	0	0	+1/2	1	-2	5.5	
	x_1	1	0	0	1	0	0	0	0	-1	2	

↑

Optimal solution. No negative coeff's in the zeroth row.

For a BF solution:

$$(x_1, x_2, s_1, s_2, s_3, s_4, s_5)$$
$$= \left(2, \tfrac{1}{2}, \tfrac{7}{2}, 13, 0, \tfrac{11}{2}, 0\right) = \vec{x}_0.$$

Now begin phase two, with $\bar{x}_3 = \bar{x}_4 = 0$.

max Z s.t.

$$Z - 3x_1 - x_2 = 0, \quad (0)$$
$$x_1 - x_2 + s_1 = 5, \quad (1)$$
$$3x_1 - 2x_2 + s_2 = 18, \quad (2)$$
$$4x_1 + 2x_2 - s_3 = 9, \quad (3)$$
$$x_2 + s_4 = 6, \quad (4)$$
$$x_1 - s_5 = 2, \quad (5)$$

$$x_1, x_2, \{s_i\}_{i=1}^{5} \geq 0,$$

$$x_1 = 2 - s_5, \quad 2x_2 = 9 + s_3 - 4x_1$$
$$= 9 + s_3 - 4(2 - s_5)$$
$$x_2 = \tfrac{1}{2}(9 + s_3) - 4 + 2s_5$$
$$= 4.5 + \tfrac{1}{2}s_3 - 4 + 2s_5,$$
$$= \tfrac{1}{2} + \tfrac{1}{2}s_3 + 2s_5,$$

$$Z - 3(2 - S_5) - (\tfrac{1}{2} + \tfrac{1}{2} S_3 + 2 S_5) = 0$$
$$Z - 6 + 3 S_5 - \tfrac{1}{2} - \tfrac{1}{2} S_3 - 2 S_5 = 0$$
$$Z + S_5 - \tfrac{1}{2} S_3 = 13/2$$

	Bas. Vars.	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
max	Z	0	0	0	0	$-1/2$	0	1	$13/2$	
(1)		1	-1	1	0	0	0	0	5	
(2)		3	-2		1				18	
(3)		4	2			-1			9	
(4)			1				1		6	
(5)		1						-1	2	

Use final Phase I Tableau with Z:

S_3 Entering Basic Var.

Bas. Vars.:	X_1	X_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
Z	0	0	0	0	$-1/2$	0	1	$13/2$	
S_1	0	0	1	0	$-1/2$	0	3	3,5	–
S_2	0	0	0	1	-1	0	7	13	–
X_2	0	1	0	0	$-1/2$	0	2	$1/2$	–
S_4	0	0	0	0	$+1/2$	1	-2	5.5	11
X_1	1	0	0	0	0	0	-1	2	–

S_4 Leaving.

Iteration 2:

Entering: S_5

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	RHS	Ratio
z	0	0	0	0	0	1	-1	12	
S_1	0	0	1	0	0	1	1	9	9
S_2	0	0	0	1	0	2	3	24	8
x_2	0	1	0	0	0	1	0	6	—
S_3	0	0	0	0	1	2	-4	11	—
x_1	1	0	0	0	0	0	-1	2	—

Leaving: S_2

Iteration 3:

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	RHS
z	0	0	0	$1/3$	0	$5/3$	0	20
S_1	0	0	1	$-1/3$	0	$1/3$	0	1
S_5	0	0	0	$1/3$	0	$2/3$	1	8
x_2	0	1	0	0	0	1	0	6
S_3	0	0	0	$-4/3$	1	$10/3$	0	43
x_1	1	0	0	$1/3$	0	$2/3$	0	10

$\times$ (-1) $+1$ $+4$ $+1$

Optimal solution found! $(x_1, x_2) = (10, 6)$

d) Hvis vi setter høyresiden i betingelse (7) til 4.

(7) $x_1 - x_2 \leq 5 \Rightarrow (7)' \; x_1 - x_2 \leq 4$

vil vi en degenerert løsning da tre betingelser er gyldige i det optimale punktet (10, 6), istedet for vanlig antall 2.

Vi avdekker degenerasjon ved en BF løsning ved at det er to ratioer med like minimums-verdi i minimums-ratio testen, eller ekvivalent at en basis-variabel får verdien 0.

e) For å få flere optimale løsninger, spesifiser en betingelse ortogonalt på ∇z.

$\nabla z = (3, 1)$, la betingelse

(4) være gitt ved $g(x_1, x_2) \leq 0$.

Da trenger vi $\nabla g = \nabla z$, og

$g(10, 6) = 0$. $\quad g(x_1, x_2) = 3x_1 + x_2 - k$

$\Rightarrow 3 \cdot 10 + 6 - k = 0$,

$k = 36$.

Altså, nye

(4) → $(4)'$: $3x_1 + x_2 \leq 36$. og alle

optimale løsninger blir på formen

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}(w) = w \begin{pmatrix} 0 \\ 36 \end{pmatrix} + (1-w)\begin{pmatrix} 10 \\ 6 \end{pmatrix}, \quad w \in [0, 1].$$