Financial Economics | Problem Set 7

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1 Risk

(a)

Stock B has a higher beta with respect to the market portfolio. This is seen from the slope (β) of the regression line, which implies greater sensitivity to market movements.

(b)

To an investor currently holding the market portfolio, Stock B is riskier. Its higher beta means it will fluctuate more in response to market movements, thus introducing more volatility to the investor's portfolio.

(c)

From the picture, it is seen how the variance of Stock A as there is a greater dis is greater dispersion of the point. This means that while Stock B is more correlated to the market, Stock A has a greater total variance and is thus the riskiest for an undiversified investor.

2 CAPM test

(a)

Using the provided data, we can calculate the average return as seen in table 1.

	Small-Low	Small-High	Big-Low	Big-High	Market
Average Return	1.129	1.449	0.778	1.213	0.822
β	1.256	1.531	0.927	1.442	1.000
α	0.096	0.190	0.016	0.027	0.000
CAPM Prediction	1.033	1.259	0.762	1.185	0.822

Table 1: Average Excess Return, β, α, and CAPM Predictions using data from 1930-1963. Small and Big indicate if stocks in the portfolio have a Small or Big market capitalisation. Low and High indicate if stocks in the portfolio have a High or Low book to market value

(b)

We calculate the β for portfolio i as

$$\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2} \tag{1}$$

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Where $\sigma_{i,m}$ is the covariance between the portfolio i's and the market portfolio's excess return and σ_m^2 variance of the market portfolio's excess return. Our results can be found in table 1

(c)

Using the regression formula, take the expectations to get

$$\begin{aligned} \mathbb{E}(R_i) - R_f = \alpha_i + \beta_i(\mathbb{E}(R_m) - R_f) \Leftrightarrow \\ \alpha_i = \mathbb{E}(R_i) - R_f - \beta_i(\mathbb{E}(R_m) - R_f) \end{aligned} \tag{2}$$

Using this, we have calculated the intercept, α_i, as presented in table 1. Interestingly we find that all portfolios (expect the market itself) have positive α_i.

(d)

Under CAPM, we have $\alpha_i = 0 \ \forall i$, and we can, therefore, calculate the CAPM predicted excess return as

$$\mathbb{E}(R_i) - R_f - \beta_i(\mathbb{E}(R_m) - R_f) \tag{3}$$

The predicted CAPM values are shown in table 1.

(e)

We see they are all exactly α_i lower than the average return. We find evidence of both the value and size effect, i.e. value stocks outperform growth, and small stocks outperform large. It is, therefore, not surprising that the portfolio that outperforms the CAPM predictions the most is the Small-High portfolio, while the Big-Low only outperforms slightly.

(f)

We have fitted an unrestricted linear trendline to minimize the squared residuals of the 5 portfolios found in table 1.

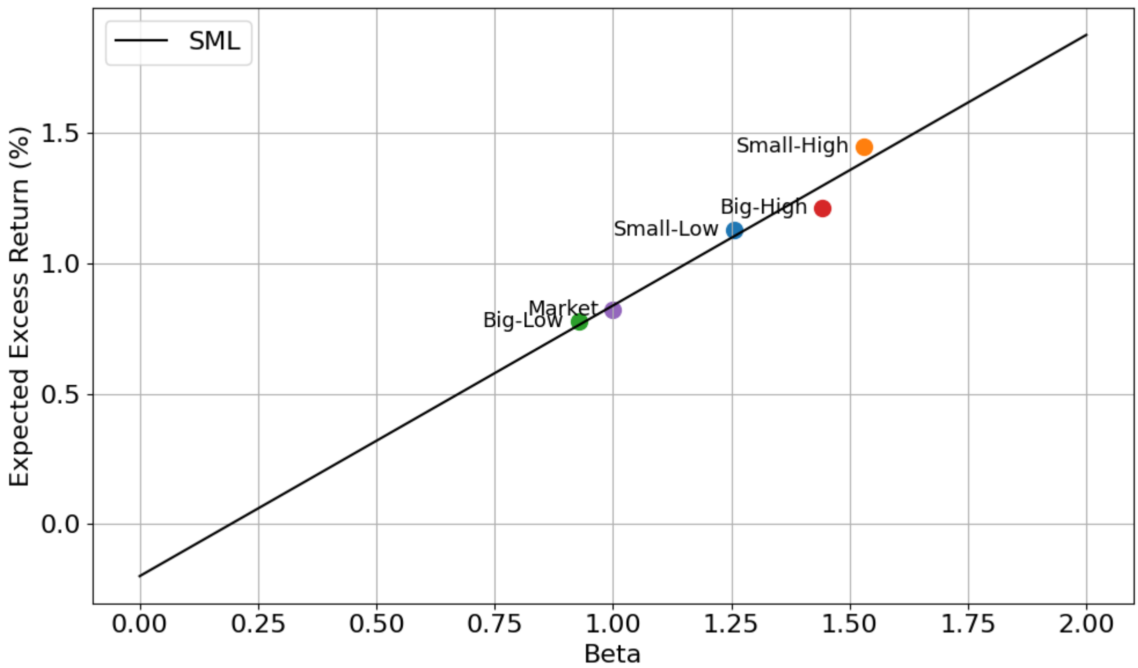

Figure 1: Expected Excess Return to Beta, with a linear trendline for using the data for years 1930-1963. Small and Big indicate if stocks in the portfolio have a Small or Big market capitalisation. Low and High indicate if stocks in the portfolio have a High or Low book to market value

We see that the portfolios align with the CAPM overall as the mean excess return increases in the beta, and the points are nicely distributed close to the trendline.

(g)

We have recalculated the mean excess return, β_i, α_i and CAPM predictions for the data from 1964-2004. The results can be found in table 2

	Small-Low	Small-High	Big-Low	Big-High	Market
Average Return	0.499	1.094	0.430	0.697	0.461
β	1.374	1.003	1.037	0.856	1.000
α	-0.136	0.631	-0.048	0.302	0.000
CAPM Prediction	0.634	0.463	0.478	0.395	0.461

Table 2: Average Excess Return, β, α, and CAPM Predictions using data from 1964-2004. Small and Big indicate if stocks in the portfolio have a Small or Big market capitalisation. Low and High indicate if stocks in the portfolio have a High or Low book to market value

We now find that only the value stocks, Small-High and Big-High, outperform the CAPM Predictions and have consistently had a positive α_i. Again, the Small-High is the one with the highest α_i, but the portfolio with the worst performance now is the Small-Low. We have also plotted these points and fit an unrestricted linear trendline as seen in figure 2 We now see the trendline has a negative slope, i.e. higher risk, β_i, leads to lower expected excess return. Furthermore, the data points are spread out more than in figure 1. This shows that over the more recent period, CAPM does *not* seem to hold any longer.

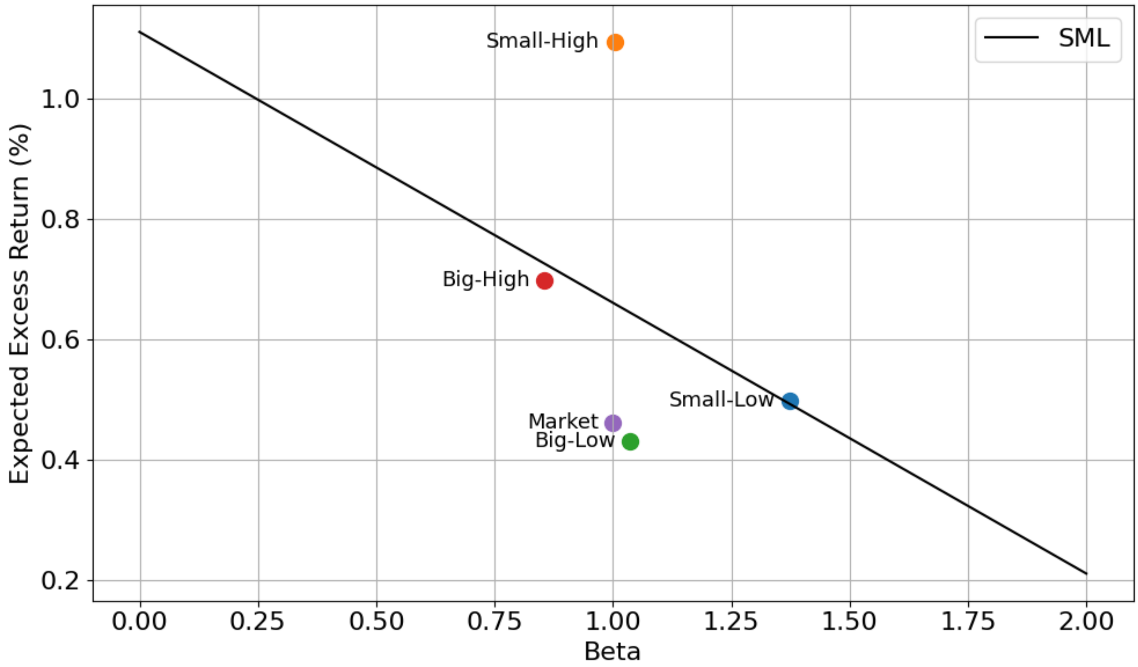

Figure 2: Expected Excess Return to Beta, with a linear trendline for using the data for years 1964-2004. Small and Big indicate if stocks in the portfolio have a Small or Big market capitalisation. Low and High indicate if stocks in the portfolio have a High or Low book to market value

3 Derivatives

(a)

The forward price is given as

$$F = S \cdot e^{r \cdot T} \tag{4}$$

Where F is the forward price, S is the current spot price, r is the annual risk-free interest rate compounded continuously, and T is the time into the future in years. Using this we can calculate the spot forward prices as

$$\begin{aligned} F_{Today} &= \$12 \cdot e^{0.03 \cdot 0} = \$12.00 \\ F_{6m} &= \$12 \cdot e^{0.03 \cdot 0.5} = \$12.18 \\ F_{12m} &= \$12 \cdot e^{0.03 \cdot 1} = \$12.37 \end{aligned} \tag{5}$$

Unsurprisingly, the price of a forward contract with delivery today is exactly the spot price today, hence $F_{today} = \$12$. Further, we find the price for the 6-month forward contract as $F_{6m} = \$12.18$ and the price for the 12-month forward contract as $F_{12m} = \$12.18$.

(b)

The forward price of a stock in a no-arbitrage environment is determined by the spot price and the cost of carry. In this case, since neither ABC nor XYZ pays dividends during the life of the contract, the cost of carry is the risk-free interest rate, and the price is given by equation (4). Hence, the forward prices for ABC and XYZ will, in this setting, be the same.

The volatility of the stocks and their systematic risk factors are relevant to the risk premium that investors might require for holding the stock. This premium influences the spot price, but once the spot price is set, the forward price is determined solely by the aforementioned no-arbitrage relationship.

4 Using derivatives

(a)

If we do not want our strategy to cost anything but want to limit our exposure, we could use a Zero-Cost Collar strategy. With this strategy, we should buy put options in our stocks and sell call options at the same price. This will limit our exposure both up and down as it guarantees we cannot lose more than the strike of the put, while we will never gain more than the strike of the call — at zero cost.

(b)

If we do not mind spending some money on insurance to limit our downside, we should just buy the puts. This will limit our downside exposure to the difference between the current value and the strike plus the costs of the puts. However, should the market rise there is no limit on our upside.