Problem Set 6

Econ 136, Spring 2024

This problem set is due on April 15th, by 5PM (on Gradescope).

1. **Mean-variance analysis with two risky assets**

   The spreadsheet "returns.xls" contains annual return data for big growth stocks, big value stocks, small growth stocks, and small value stocks. We are going to use Excel to explore the efficient frontier of value and growth stocks using these data. Please turn in a printout of your two Excel graphs with your solution.

   (a) Download "returns.xls", and clear the contents of columns F - Z (you will only need the data contained in columns A - E). In columns G and H, compute the annual net simple return of value and growth stocks in natural units by taking an weighted average of big value and small value for value and a weighted average of big growth and small growth for growth, with of weight of 0.9 on big and 0.1 on small (e.g., in G3, put =(B3*0.9+D3*0.1)/100, and in H3, put =(C3*0.9+E3*0.1)/100). Also compute the average annual return, variance and standard deviation for both value and growth, using the average, var, and stdev functions in Excel. These are our estimates of the expected return, variance and standard deviation.

   (b) Calculate the covariance of value stocks with growth stocks using the covar function. If value had a high return in a year, what would you expect happened to growth? What is the correlation coefficient?

   (c) Now we are going to look at portfolios of value and growth stocks. Find four empty columns next to each other. Put -0.5 in the first cell of the first column, and -0.45 in the cell below it. Select both cells, grab the lower right corner of the selection box, and drag down until you have created a column that runs from -0.5 to 1.5 in increments of 0.05 – this column contains the possible portfolio weights of growth stocks. In the first cell of the second column, calculate the expected return of the associated portfolio. Fill the third column with variances based on the weights in the first column. Use the fourth column to take the square roots of these variances (the standard deviations).

   (d) Now plot the portfolio standard deviation as a function of weight on growth stocks.

   (e) Now plot the mean-standard deviation frontier for value and growth in the mean-stdev diagram using a similar procedure. Make sure that the standard deviation is on the horizontal axis. Where is the efficient frontier? What is (approximately) the global minimum variance portfolio? What is the mean and standard deviation of the return for that portfolio? How does it compare with the mean and standard deviation of value and growth?

(f) Now we will construct the efficient frontier when there also exists a safe asset. Assume that the annual riskfree rate of return is $r_f = 2\%$. In a fifth column next to the previous four columns, compute the Sharpe-ratio of the various portfolios of value and growth. Which portfolio has the highest Sharpe-ratio? What is the mean and standard deviation of this (approximate) tangency portfolio? Where is the efficient frontier now?

(g) Suppose that you would like to invest in a portfolio of value stocks, growth stocks, and the riskfree asset. If your coefficient of risk aversion is $A = 5$, what is the composition of your optimal portfolio (what are the shares of your wealth invested in value, growth and the riskfree asset)? What is the mean and standard deviation of the annual portfolio return?

(h) Now suppose that you wish to hold a portfolio of value, growth and the riskfree asset such that the standard deviation of your annual return is equal to 16%. What is the composition of your optimal portfolio? What is the mean and standard deviation of the annual portfolio return?

2. **True or false**

Are the following statements true or false? Explain your answer in no more than two sentences.

(a) According to the logic of mean-variance optimization, highly risk-averse investors should hold no risky assets at all, because the positive expected excess return of stocks will not be enough compensation for the associated risk.

(b) In a CAPM equilibrium, no portfolio has a higher Sharpe-ratio than the market portfolio.

(c) Suppose that Xing's portfolio consists of 40% in 90-day Treasuries (riskfree), 20% in growth stocks and 40% in value stocks, while Sung Bin's portfolio consists of 60% in 90-day Treasuries, 10% in growth stocks and 30% in value stocks. The mutual fund theorem is violated in this example.

(d) If CAPM holds, a stock whose expected rate of return is less than the riskfree rate is never included in any investor's portfolio because one is always better off choosing the riskfree asset.

3. **CAPM**

Suppose that CAPM holds, the riskfree rate is $r_f = 2\%$ and the expected return on the market portfolio is $\mathbb{E}[r_m] = 12\%$. The standard deviation of the market return is 0.2 or 20%.

(a) What is the expected return on an asset that has $\beta = 1.5$ with respect to the market return?

(b) What is the beta of a portfolio that has $\mathbb{E}[r_p] = 11\%$?

(c) A security return r_i has a covariance of $\sigma_{i,m} = 0.1$ with the market return. What is the expected excess return $\mathbb{E}[r_i] - r_f$?

4. **Stock pricing**

The market price of a security is \$20. Its expected rate of return is 10%. The risk-free rate is 2% and the expected excess return on the market portfolio is 6%. What will be the market price of the security if the correlation coefficient with the market portfolio doubles (and all other variables remain unchanged)? Assume that the stock is expected to pay a constant dividend in perpetuity, and that the CAPM holds.

To answer this question, let us go through the following steps.

(a) Consider the security before the correlation coefficient doubles. Given the price, the expected return (discount rate) and the fact that $g = 0$, what are the expected dividend payments for this security?

(b) What was the security's beta before the change in the correlation coefficient?

(c) What happens to beta when the correlation coefficient doubles?

(d) What happens to the expected excess return of the security? What will be the new expected total return of the security?

(e) Using the Gordon model, the fact that dividends are constant, and your result from part (d) about the expected return (i.e., the discount rate), what will be the new market price of the security? In which direction did the stock price change? Why?