Financial Economics | Problem Set 5

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1 Efficient Markets

(a)

True. The stock reacted to the earnings and not before, suggesting that there had been no leakage of sensitive information from the company to the market. The negative reaction in the market might be due to the fact that Good News Inc. did not meet the expected earnings or that they might forecast lower-than-expected future earnings.

(b)

False. In the semi-strong form, the market only has knowledge of all publicly available information, and thus, they would be surprised by any private information disclosed by the company.

(c)

True. The EMH does not imply that all stocks have the same return. Riskier stocks often have higher expected returns to compensate for the higher risk, which is consistent with the EMH.

(d)

False. The hypothesis relies on the market as a whole to reflect all available information in stock prices, not on the rationality of an individual investor.

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2 A Biotech Company

(a)

The game tree can be illustrated as:

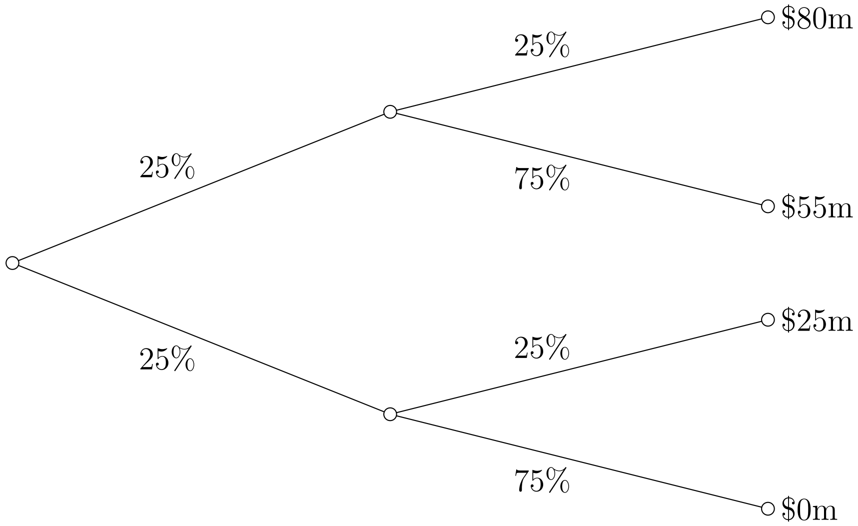

Figure 1: Value of the patent illustrated in a game tree

(b)

Conditional on the success of the first trial, the value of the patent can be calculated as:

$$\mathbb{E}(X|\text{Trial 1} = \text{Success}) = 25\% \cdot \$80\text{m} + 75\% \cdot \$55\text{m} = \$61.25\text{m} \tag{1}$$

Conditional on the failure of the first trial, the value of the patent can be calculated as:

$$\mathbb{E}(X|\text{Trial 1} = \text{Fail}) = 25\% \cdot \$25\text{m} + 75\% \cdot \$0\text{m} = \$6.25\text{m} \tag{2}$$

(c)

Using the calculations from (b) we can calculate the unconditional rational expectation of the drug patent as

$$\mathbb{E}(X) = 25\% \cdot \$61.25\text{m} + 75\% \cdot \$6.25\text{m} = \$20\text{m} \tag{3}$$

The unconditional expected value of the drug patent is $20m.

(d)

We start by showing that $\mathbb{E}_0(X) = \mathbb{E}_0(\mathbb{E}_1(X))$. We start by looking at $\mathbb{E}_0(X)$.

$$\begin{aligned}
\mathbb{E}_0(X) &= \frac{1^2}{4^2} X_{\text{Trial 1 and 2 = Success}} + \frac{1 \cdot 3}{4^2} X_{\text{Trial 1 = Success, Trial 2 = Fail}} \\
&\quad + \frac{3 \cdot 1}{4^2} X_{\text{Trial 1 = Fail, Trial 2 = Success}} + \frac{3^2}{4^2} X_{\text{Trial 1 and 2 = Fail}} \\
&= \frac{1}{16} X_{\text{Trial 1 and 2 = Success}} + \frac{3}{16} X_{\text{Trial 1 = Success, Trial 2 = Fail}} \\
&\quad + \frac{3}{16} X_{\text{Trial 1 = Fail, Trial 2 = Success}} + \frac{9}{16} X_{\text{Trial 1 and 2 = Fail}} \\
&= \frac{1 \cdot 80}{16} + \frac{3 \cdot 55}{16} + \frac{3 \cdot 25}{16} + \frac{9 \cdot 0}{16} = 20
\end{aligned} \tag{4}$$

Looking at the next term $\mathbb{E}_0(\mathbb{E}_1(X))$ gives:

$$\begin{aligned}
\mathbb{E}_0(\mathbb{E}_1(X)) &= \frac{1}{4}\cdot \mathbb{E}_1(X|\text{Trial 1} = \text{Success}) + \frac{3}{4}\cdot \mathbb{E}_1(X|\text{Trial 1} = \text{Fail}) \\
&= \frac{1}{4}\left(\frac{1}{4}X_{\text{Trial 1 and 2 = Success}}\right) + \frac{1}{4}\left(\frac{3}{4}X_{\text{Trial 1 = Success, Trial 2 = Fail}}\right) \\
&\qquad + \frac{3}{4}\left(\frac{1}{4}X_{\text{Trial 1 = Fail, Trial 2 = Success}}\right) + \frac{3}{4}\left(\frac{3}{4}X_{\text{Trial 1 and 2 = Fail}}\right) \\
&= \frac{1}{16}X_{\text{Trial 1 and 2 = Success}} + \frac{3}{16}X_{\text{Trial 1 = Success, Trial 2 = Fail}} \\
&\qquad + \frac{3}{16}X_{\text{Trial 1 = Fail, Trial 2 = Success}} + \frac{9}{16}X_{\text{Trial 1 and 2 = Fail}} \\
&= \frac{1\cdot 80}{16} + \frac{3\cdot 55}{16} + \frac{3\cdot 25}{16} + \frac{9\cdot 0}{16} = 20 = \mathbb{E}_0(X)
\end{aligned} \tag{5}$$

Thus, we see that LIE holds. Moving on to showing that $\mathbb{E}_0(X - \mathbb{E}_1(X)) = 0$ we will use our result from (5):

$$\mathbb{E}_0(X - \mathbb{E}_1(X)) = \mathbb{E}_0(X) - \mathbb{E}_0(\mathbb{E}_1(X)) = \mathbb{E}_0(X) - \mathbb{E}_0(X) = 0 \tag{6}$$

The results make sense because any new information that arises at time 1 was already accounted for in the original expectation at time 0 on average, reflecting that the market efficiently incorporates information into prices.

(e)

We can calculate the individual variances as:

$$\begin{aligned}
\mathbf{Var}_0(X) &= \mathbb{E}_0(X^2) - \mathbb{E}_0(X)^2 \\
&= \frac{1}{16}X^2_{\text{Trial 1 and 2 = Success}} + \frac{3}{16}X^2_{\text{Trial 1 = Success, Trial 2 = Fail}} \\
&\qquad + \frac{3}{16}X^2_{\text{Trial 1 = Fail, Trial 2 = Success}} + \frac{9}{16}X^2_{\text{Trial 1 and 2 = Fail}} - \mathbb{E}_0(X)^2 \\
&= \frac{1}{16}\cdot(80\cdot 10^6)^2 + \frac{3}{16}\cdot(55\cdot 10^6)^2 + \frac{3}{16}\cdot(25\cdot 10^6)^2 + \frac{9}{16}\cdot 0 - 20^2 = 684.375\cdot 10^{12}
\end{aligned} \tag{7}$$

$$\begin{aligned}
\mathbf{Var}_0(\mathbb{E}_1(X)) &= \mathbb{E}_0(\mathbb{E}_1(X)^2) - \mathbb{E}_0(\mathbb{E}_1(X))^2 \\
&= \frac{1}{4}\cdot \mathbb{E}_1(X|\text{Trial 1} = \text{Success})^2 + \frac{3}{4}\cdot \mathbb{E}_1(X|\text{Trial 1} = \text{Fail})^2 - \mathbb{E}_0(X)^2 \\
&= \frac{1}{4}\cdot(61.25\cdot 10^6)^2 + \frac{3}{4}\cdot(6.25\cdot 10^6)^2 - 20^2 = 567.1875\cdot 10^{12}
\end{aligned} \tag{8}$$

$$\mathbf{Var}_0(\mathbb{E}_0(X)) = \mathbb{E}_0(\mathbb{E}_0(X)^2) - \mathbb{E}_0(\mathbb{E}_0(X))^2 = \mathbb{E}_0(X)^2 - \mathbb{E}_0(X)^2 = 20^2 - 20^2 = 0 \tag{9}$$

Thus we see $\mathbf{Var}_0(X) > \mathbf{Var}_0(\mathbb{E}_1(X)) > \mathbf{Var}_0(\mathbb{E}_0(X))$. This result makes sense as there is an increased uncertainty in the outcome, which in turn increases the variance.

3 Covariance and Correlation

(a)

We start by calculating the mean for each asset as

$$\mu = \frac{1}{3}\sum_{i=1}^{3} R_i \tag{10}$$

Next, we can use the mean to calculate the variances for each asset

$$\sigma^2 = \frac{1}{3}\sum_{i=1}^{3}(R_i - \mu)^2 \tag{11}$$

And lastly, we calculate the standard deviation as the square root of the variance, $\sigma = \sqrt{\sigma^2}$. All the results can be found in table 1 below.

	Asset 1	Asset 2	Equal-weighted Portfolio
State 1	8%	-6%	1%
State 2	-4%	14%	5%
State 3	11%	7%	9%
Expected Return (μ)	5%	5%	5%
Variance (σ^2)	0.0042	0.0069	0.0011
Std. Dev. (σ)	0.0648	0.0829	0.0327
Utility ($\mu - 1/2A\sigma^2$)	$0.05 - 0.0021A$	$0.05 - 0.00345A$	$0.05 - 0.00055A$

Table 1: State Return, Expected Return, Variancea, Standard Deviation, and Utility of each asset and the equal-weighted portfolio

(b)

We calculate the covariance between the two assets as

$$\begin{aligned}
\sigma_{A_1,A_2} &= \frac{1}{3}\sum_{i=1}^{3}(R_i^{A_1} - \mu^{A_1})(R_i^{A_2} - \mu^{A_2}) \\
&= \frac{1}{3}[(0.08 - 0.05)(-0.06 - 0.05) + (-0.04 - 0.05)(0.14 - 0.05) \\
&\qquad + (0.11 - 0.05)(0.07 - 0.05)] \\
&= -0.0034
\end{aligned} \tag{12}$$

Next, we can calculate the correlation as

$$\rho_{A_1,A_2} = \frac{\sigma_{A_1,A_2}}{\sigma_{A_1} \cdot \sigma_{A_2}} = \frac{-0.0034}{0.0648 \cdot 0.0829} = -0.6329 \tag{13}$$

(c)

We have calculated the return in each state as

$$R_i^P = \frac{1}{2}(R_i^{A_1} + R_i^{A_2}) \tag{14}$$

Next, we have calculated the mean return, variance and standard deviation as in (a). See table 1 for the results.

(d)

The mean variance optimising investor is seeking to optimise utility following

$$U = \mu_p - \frac{1}{2}A\sigma_p^2 \tag{15}$$

Where μ_p is the expected return of the port folio and σ_p^2 is the portfolio variance. We have evaluated the utility for each portfolio – see table 1 for results – and find that the mean-variance optimising investor will prefer the equal-weighted portfolio, and have the portfolio of solely asset 1 as the second while the portfolio of solely asset 2 is ranked last. This is true $\forall\ A > 0$ as there is no variance in the expected returns and thus the mean-variance optimising investor will choose prefer whatever portfolio has the lowest variance. If $A = 0$ the investor is indifferent to risk and thus indifferent of the portfolios.

4 Historical Returns

(a)

	Big Growth	Big Value	Small Growth	Small Value	Market
(a) mean	11.11%	14.63%	13.92%	18.83%	11.56%
(a) rank by mean return	5	2	3	1	4
(b) log-return	0.0871	0.1049	0.0888	0.1319	0.0907
(c) cumulative log-return	7.40	8.92	7.55	11.21	7.71
(d) value of $1 invested	$ 1,641.42	$ 7,474.69	$ 1,900.87	$ 74,225.77	$ 2,234.49
(f) standard deviation	20.47	27.65	33.05	32.62	20.51

Table 2: Results from excel for exercises (a)-(f)

As seen from table table 2 the best-performing portfolios have been the value portfolios – as we also saw in lecture 17 - with the small value portfolio taking the lead with an average net simple return of 18.83% compared to 14.63% for the big value portfolio. Again, as we saw in lecture 17, we also here find that the small portfolios outperform their bigger counterparts – this intuitively makes sense as the small companies most likely are more risky to invest in and thus require a higher expected return. Evidence of this can be seen in table 2 and will be discussed in (f).

Thus, we, with this data, find evidence of both the value effect and the size effect.

(b)

The calculated average log return for each of the 5 portfolios can be found in table 2. As the log transformation is monotonic, the portfolios preserve their respective ranks.

(c)

We have computed the cumulative log-return for the period 1927-2011 for the 5 portfolios in table table 2. As this is just a sum of all the underlying components for the average log return calculated in (b), it should be no surprise that the performance ranking stays the same.

(d)

We have calculated the 2011 nominal value of a dollar invested in each portfolio in 1927 in table table 2. As already pointed out in the previous questions, the relative ranking of each portfolio stays the same.

However, from this, it becomes really apparent how, though there might just be a total difference of around 8% annually between the best-performing and the worst-performing portfolio, the cumulative effect of this is massive. A dollar invested in 1927 in the small value portfolio is close to 10 times more valuable in 2011 as the second best-performing portfolio, the big value portfolio, while it is 45 times as valuable in 2011 as the worst performing of the 5 portfolios, the big growth portfolio.

(e)

Utilising the calculated values of 1 dollar invested form (d), we can plot these to see the development of the value over time.

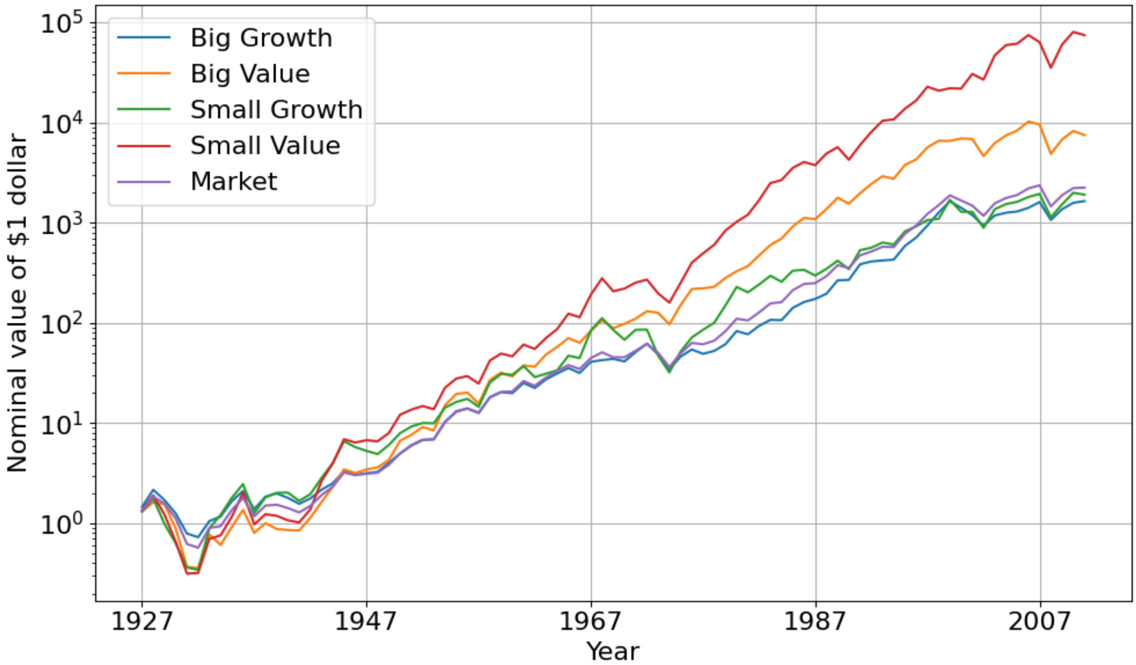

Figure 2: Nominal value of $1 invested in 1927

(f)

The calculated standard deviations can be found in table 2. Here, we unsurprisingly see that the smaller portfolios, indeed, are more volatile than the big portfolios, and thus, investors are compensated for this.

However, what is interesting is that the *Big Value* portfolio has a lower standard deviation but higher average return than the *Small Growth* portfolio. Thus, the Big Value portfolio is a strictly better investment for any investor as the Small Growth stocks do not compensate for their risk accordingly.

(g)

In an Equal-Weighted portfolio of the 4 options, the return for any year, i, will be

$$R_i^{\text{Equal-Weighted}} = \frac{1}{4} \cdot R_i^{\text{Big Growth}} + \frac{1}{4} \cdot R_i^{\text{Big Value}} + \frac{1}{4} \cdot R_i^{\text{Small Growth}} + \frac{1}{4} \cdot R_i^{\text{Small Value}} \tag{16}$$

We see that this is just the simple average, and we can, therefore, calculate every year as such and then calculate the mean net simple return for all years as well as the standard deviation.

	Big Growth	Big Value	Small Growth	Small Value	Equal-Weighted
mean	11.11%	14.63%	13.92%	18.83%	14.62%
standard deviation	20.47	27.65	33.05	32.62	26.59

Table 3: Average net simple return and standard deviation of an equal-weighted portfolio and the 4 portfolios it is based on

From table 3 we see that the Equal-Weighted portfolio performs very similarly to the Big Value with a difference in the mean return of less than 0.01%-point (calculated as 0.0083%-points in excel); however, the Equal-Weighted does have a slightly lower variance indicating that we do gain some benefits from diversifying in this simple manner.