Fixed Income Derivatives - Curve fitting Exam Preparation

Assume you have the following market data available where the interest rate swaps pay 6M EURIBOR semiannually against a fixed rate paid annually.

EURIBOR	Fixing	FRA	Midquote	IRS	Midquote
6M	0.03478	1X7	0.03743	2Y	0.04377
		2X8	0.03871	3Y	0.04625
		3X9	0.03989	4Y	0.04777
		4X10	0.04098	5Y	0.04875
		5X11	0.04198	7Y	0.04992
		6X12	0.04289	10Y	0.05081
		7X13	0.04374	15Y	0.05148
		8X14	0.04452	20Y	0.05181
		9X15	0.04524	30Y	0.05211

Problem 1 - Fitting the yield curve

a) Discuss what properties the term structure of the short rate and the term structure of the instantaneous forward rate should have after fitting a term structure to market data.

b) Fit a zero coupon bond spot rate curve to the market data above using different techniques and discuss which fit is better in terms of the criteria from a).

c) Plot the term structures of spot rates, instantaneous forward rates and par swap rates implied by the fitted ZCB term structure.

Problem 2 - Bumping spot rates and finding the DV01 of a swap

Next, we will use the ZCB term structure for risk management and compute the DV01 of a 10Y payer swap when spot rates are bumped by 1 bps up and down.

a) Bump each of the spot rates for $T \in [1, 2, ..., 10]$ by 1 bp up and down.

b) Bump all of the spot rates for $T \in [1, 2, ..., 10]$ by 1 bp up and down.

c) Discuss which of the spot rates that influences the DV01 of the 10Y swap the most and interpret your result.

d) Redo a)-c) for the 5Y interest rate swap.

Problem 3 - Bumping market rates and finding the DV01 of a swap

Finally, we will investigate the sensitivity of a 10 Y payer swap to changes in market rates by bumping these up and down also by one bps.

a) Bump each of the market rates for the Euribor fixing, the FRAs and the interest rate swaps by 1 bp up and down.

b) Bump all of the market rates by 1 bp up and down.

c) Discuss which of the market rates that influences the DV01 of the 10Y swap the most and interpret your result.

d) Redo a)-c) for the 5Y interest rate swap.

Problem 4 - Mark-to-market

In this problem, you will compute the mark-to-market value of three different swap related derivatives and compute the PnL of each position since inception.

a) Compute the market value of a 7Y swap with a notional of 1 entered into a year ago at the swap rate 0.048. Also, compute the PnL of this position.

b) Compute the market value of a 1Y6Y payer swaption also with a notional of one entered into exactly one year ago at the then at-the-money strike of 0.048. Also compute the PnL of this position if the initial cost of the swaption was 38 bps.

c) Compute the market value of a 5Y5Y forward receiver swap with a notional of 1 entered into exactly two years ago at the then 5Y5Y par forward swap rate of 0.051. Also compute the PnL of this position.

Problem 5 - Fitting a Vasicek model to the yield curve

In this problem, we will fit the Vasicek model to our yield curve data

a) Use the initial values $r_0 = 0.03$, $a = 0.5$, $b = 0.04$ and $\sigma = 0.03$ to fit a Vasicek model to your ZCB spot rate curve.

b) Plot the fitted spot rates in your Vasicek model in the same plot as your ZCB spot and forward rates.

c) Based on the quality of your fit, can you conclude that the short rate much follow a Vasicek model?