Fixed Income Derivatives - Curve fitting Exam Preparation

Assume you have the following market data available where the interest rate swaps pay 6M EURIBOR semiannually against a fixed rate also paid semiannually.

EURIBOR	Fixing	FRA	Midquote	IRS	Midquote
6M	0.03478	1X7	0.03743	2Y	0.04377
		2X8	0.03871	3Y	0.04625
		3X9	0.03989	4Y	0.04777
		4X10	0.04098	5Y	0.04875
		5X11	0.04198	7Y	0.04992
		6X12	0.04289	10Y	0.05081
		7X13	0.04374	15Y	0.05148
		8X14	0.04452	20Y	0.05181
		9X15	0.04524	30Y	0.05211

a) Fitting the zero coupon curve.

i) Discuss what properties the term structure of the short rate and the term structure of the instantaneous forward rate should have after fitting a term structure to market data.

ii) Fit a zero coupon bond spot rate curve to the market data above using different techniques and discuss which fit is better in terms of the criteria from i).

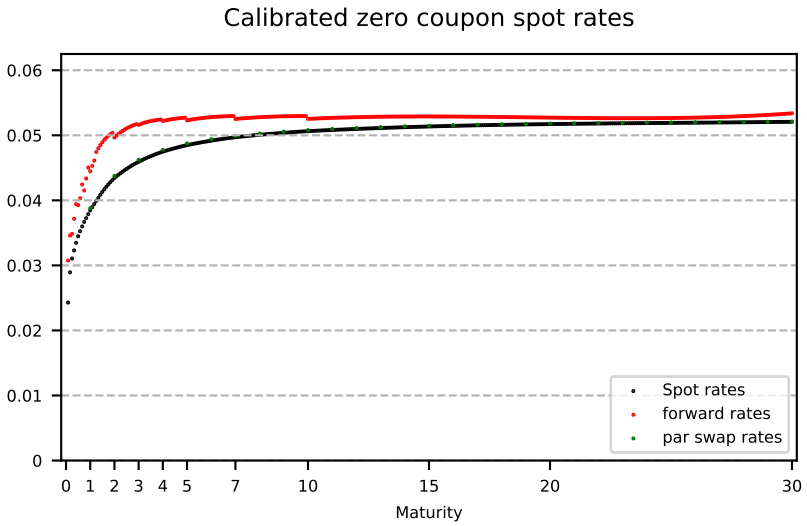
iii) Plot the term structures of spot rates, instantaneous forward rates and par swap rates implied by the fitted ZCB term structure.

Answers:

i) The fitted term structure should be such that instantaneous forward rates are positive and continuous as a function of maturity. In addition, the term structure of instantaneous forward rates should also look realistic and it's derivative should not change sign frequently.

ii) To achieve a continuous instantaneous forward rate term structure, a Hermite polynomial of high degree should be chosen. However, choosing a polynomial of too high degree will make the forward rate term structure 'swerve' at the far end an choosing a Hermite polynomial of degree around 3 seems to provide a reasonable compromise.

iii) A plot of the term structures of spot rates, instantaneous forward rates and par swap rates can be found below when fitting is performed using a Hermite polynomial of degree 3.



b) Next, we will use a ZCB term structure for risk management and compute the DV01 of a 10Y interest rate swap if we

i) bump the each of the 2Y, 5Y or 10Y spot rates by 1 bp up or down,

ii) bump all spotrates simultaneously by 1 bp up or down,

iii) bump each of the 2Y, 5Y or 10Y market swap rates by 1 bp up or down.

Answers:

i) Bumping the 2Y, 5Y or 10Y spot rates up by 1 bp increases the value of the 10Y payer swap by 0.0525, 0.1056 and 6.1825 bps respectively. Bumping the 2Y, 5Y or 10Y spot rates down by 1 bp decreases the value of the 10Y payer swap by 0.0406, 0.0938 and 6.1768 bps respectively. The 10Y interest rate swap is thus almost solely exposed to changes in the 10Y spot rate. Also, we notice that up and down moves of the spot rate affects the 10Y swap almost symmetrically but not quite.

ii) Bumping all the spot rates up by 1 bp increases the value of the 10Y payer swap by 7.9441 and bumping the spot rates down by 1 bp decreases the value of the 10Y payer swap by 7.9395. Again, the effect of up versus down moves are almost symmetrical but not quite. [iii)] Bumping the 2Y, 5Y or 10Y market swap rates up by 1 bp increases the value of 10Y payer swap by 0.0029, 0.0194 and 7.8354 bps respectively. Bumping the 2Y or 5Y market swap rates down by 1 bp increases the value of 10Y payer swap by 0.0117 and 0.0007 respectively and finally bumping the 10Y swap market rate down by 1 bp decreases the values of the 10Y payer swap by 7.8581 bps.

c) In this problem, you will compute the mark-to-market value of three different swap related derivatives and compute the PnL of each position since inception.

i) Compute the market value of a 7Y swap with a notional of 1 entered into a year ago at the swap rate 0.048. Also, compute the PnL of this position.

ii) Compute the market value of a 1Y6Y payer swaption also with a notional of one entered into exactly one year ago at the then at-the-money strike of 0.048. Also compute the PnL of this position if the initial cost of the swaption was 38 bps.

iii) Compute the market value of a 5Y5Y forward receiver swap with a notional of 1 entered into exactly two years at the then 5Y5Y par forward swap rate of 0.051. Also compute the PnL of this position.

Answers:

i) The 7Y payer swap entered into one year ago has now become a 6Y swap and the spot 6Y par swap rate is now 0.0494. The 7Y had by construction a value of zero at inception but the value has now risen to 0.007287 or roughly 72.87 bps.

ii) The 1Y6Y payer swaption entered into exactly one year ago has now reached it's exercise date and since the par swap rate has now risen above the strike of 0.048, the swaption has expired in the money and has a value equal to that of the payer swap from i). The Pnl of the swaption is therefore $72.87 - 38 = 34.87$. The 7Y payer swap entered into one year ago and the 1Y6Y payer swaption entered into one year ago have the same underlying asset and thus the same value now at the exercise of the swaption. Now, the payer swaption had an initial cost and the payer swap did not yet they now are of equal value. However, bear in mind that the swaption will not be exercised if not in-the-money and the holder of the swaption is thus insulated from a negative cashflow at maturity whereas the owner of the swap itself could have seen the value of his position become negative.

iii) The 5Y5Y forward receiver swap entered into two years ago has now become a 3Y5Y forward receiver swap. The 3Y5Y forward par swap rate is now 0.0532 and the value of the position in

the forward receiver swap is now -84.6288 which, since the forward swap was initially written at par and had an initial value of 0, is also the PnL of the position.