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Optimal Monetary Policy in a Fixed Currency Regime: A Two-Economy Macroeconomic Model

Bachelor's Thesis

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Abstract

This paper examines the implications of having a pegged currency and, thus, forfeiting the ability to conduct independent monetary policy. We take our starting point in the simple New Keynesian model presented in Persson and Tabellini (2000) and extend it to include two economies. From this, we first analyze the peg in expectation. Here we prove that, in expectation, it can be rational for the central bank to keep the peg. However, this result rests upon the assumption that the central bank greatly prefers stabilizing output growth relative to inflation. Secondly, we analyze the effects of having a peg in the case of heterogeneous shocks. We find that benefits can arise in the case of a heterogeneous shock. This occurs as a decrease in the output growth gap is traded off for increased inflationary pressure. This result, however, requires that the shocks to the two economies are of a certain relative magnitude, either through a higher demand shock or a lower supply shock for the home country. In the inverse case, there will always be an instantaneous loss for the home central bank. However, we find that when including indirect costs connected to leaving the peg, it can still be rational for the central bank to keep the peg. Lastly, we discuss the implications of our result in light of the Danish ERM 2 participation and elucidate some of the key shortcomings of our model in capturing the complexities of a monetary relationship.

Responsibilities:
Jointly: 1, 5, 6
Aske Engmark: 2.1, 2.2, 2.3, 2.4 2.4.3, 3, 3.1, 4.1, 4.1.1, 4.3
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Contents

1 Introduction

In recent years global events on both the demand and supply side have sent most western economies, including Denmark, into a period of inflationary pressure and subsequently raised interest rates. With shocks propagating differently through Europe[1], this means the need for monetary policy is different. With Denmark's participation in the ERM 2 (exchange rate mechanism 2) program, it holds a peg to the euro, constraining Danmarks Nationalbank and its ability to conduct independent monetary policy. With the ECB (European Central Bank) conducting the monetary policy for Denmark and the rest of the Eurozone, it begs the question if this is optimal. Building on the New Keynesian theoretical framework of Persson and Tabellini (2000) chapter 15, this paper aims to add to the existing literature by exploring the intricacies of optimal monetary policy when participating, and potentially deviating from, a fixed currency regime. We expand the one-economy model to a two-economy model and only let the countries interact through the central bank, simultaneously choosing the optimal monetary policy for both economies. This is a peculiar situation, as the common central bank makes policy for both countries, but only one of the countries has a "peg" on the other. The situation could be compared to the Danish peg to the euro. The ECB makes monetary policy for the entire Eurozone, including Denmark, but Denmark can deviate from the monetary relationship. In the context of our model, this corresponds to the rest of the Eurozone area making up the foreign country. Lastly, we analyze under which conditions the home central bank would be incentivized to deviate from the monetary relationship in a game-theoretical framework.

In our analysis, we find that when central banks are adequately inclined to stabilize output-growth relative to inflation, the optimal choice for the central bank is to maintain the monetary relationship in expectation. Furthermore, we find that even though most heterogeneous shocks to either the demand or supply curve entail an instantaneous loss for the home central bank when compared to not being part of it in the first place and only a small subset of the shocks minimizes the instantaneous loss evaluated by the central bank. However, when including costs entailed with scrapping the peg, it, in many cases, is rational for the central bank to maintain the peg.

There are, however, potentially important matters not included in our analysis; even though we expand the baseline model into a two-economy model, the economies are still closed economies. We do not consider any international nor bilateral trade between the two countries.

Participating in a monetary relationship could increase inter-state trade, e.g., by removing the exchange rate risk related to having a floating currency. Further, we do not include the risk aversion of foreign investors that might be present when the country is conducting its independent monetary policy.

[1]See e.g. Claeys and Vašíček (2019) and DØRS (2000)

1.1 Literature Review

Work on New Keynesian models has grown in popularity since the first models were developed in the 1970s. Muth (1961) establish the foundation of rational expectations in the context of microeconomics, but rational expectations made their way into macroeconomic modeling, forming the foundation for the New Keynesian models. Lucas popularized this, especially with his contribution in Robert E Lucas (1972) and Robert E. Lucas (1973), where he begins his journey to formulating the Lucas Island Model, tying together money supply, prices, and output with rational expectations. His "Lucas critique" was a call for micro-founded macroeconomics, where Lucas argues that the decision rules of previous Keynesian models could not be independent of policy changes, and thus, call for rational forward-looking expectations.

Around the late 70s, work like Phelps and Taylor (1977), Fischer (1977), Taylor (1979), and Calvo (1983) was seminal to the development of models with nominal stickiness thus, providing convincing frameworks for short-term non-neutrality of money. Combined with the move from an expectation-augmented Phillips curve based on adaptive expectations to one of rationality gave us what is now the core of the New Keynesian models we know today, playing a vital role in dynamic quantitative economics. Our paper takes our starting point in a simple version of the models developed at the beginning of the 80s. This paper further analyzes the optimal monetary policy in a game theoretical framework with a two-economy expansion implemented.

This paper is not the first inquiry into the dynamics of optimal monetary policy in a setting with foreign monetary relationships. Most relevant literature utilizes modern New Keynesian Dynamic Stochastic General Equilibrium models, which gained traction in the 90s. Some of the earlier papers, such as Beetsma and Jensen (2005), lay the foundation for a two-economy DSGE framework with a monetary union where optimal policy, fiscal and monetary, is examined. Later work such as Galí and Monacelli (2008) presents a benchmark case for optimal coordination policy across authorities in a currency union, with Forlati (2009) building on this in a non-cooperative framework.

In contrast to our paper, these papers analyze monetary unions. We imitate a "peg" by providing the central bank of the home country with an option to opt out of the monetary relationship and begin conducting autonomous monetary policy. Very few models are present in the literature that aims at estimating the cost and benefits of a fixed currency regime like the Danish one, most likely due to the rare nature of such a constellation. One example of such a paper is Dam and Linaa (2005), where they assess the benefits for Denmark in the hypothetical scenario where Denmark gives up the peg to the euro. This closely resembles the analysis we conduct in this paper. Although the specific inquiries and results differ, the overarching results they find are comparable to those in this paper.

The remainder of this paper goes as follows: In Section Two, we derive the baseline model following Persson and Tabellini (2000), and in Section Three, we expand the model to include two economies. In Section Four, we use the results of the previous sections to analyze the optimal choice for the central bank. In Section Five, we discuss the key shortcomings of our model and the implications of our findings; in Section Six, we conclude. The appendices are structured such that Appendix A presents our derivations for expectations and first-order conditions for the baseline model presented in Section Two; Appendix B re-derives the parts of the model affected by our extension to two economies presented in Section Three; Appendix C derives the loss in expectation and as a result of heterogeneous shocks together with the optimal behavior for the home central bank presented in Section Four.

2 The Baseline Model

This baseline model use the framework presented in Persson and Tabellini (ibid.) chapter 15, building on Kydland and Prescott (1977), Fischer (1977) and Barro and Gordon (1983). The model is a static New Keynesian model which we use to analyze optimal monetary policy.

2.1 Supply side

Following Persson and Tabellini (2000), unions and firms negotiate nominal wages as a function of some real wage growth target, $\bar{\omega}$, and rational inflation expectations, π^e. In logs, it can be expressed as:

$$\omega = \bar{\omega} + \pi^e \tag{2.1}$$

In Persson and Tabellini (ibid.) they model unemployment as a proxy for the overall state of the economy and reference that it can be modeled analogously with the output growth rate, which is what we do. The output growth rate is assumed to follow the form below which combined with equation (2.1) gives us our expectations-augmented Phillips-curve:

$$\begin{aligned} y &= \xi - (\omega - \pi) - \varepsilon \\ &= \xi - (\bar{\omega} + \pi^e - \pi) - \varepsilon \\ &= \xi - \bar{\omega} + (\pi - \pi^e) - \varepsilon \\ &= \theta + (\pi - \pi^e) - \varepsilon, \qquad \theta \equiv \xi - \bar{\omega} \end{aligned}$$

Where y is the output growth rate, θ is the stochastic "natural" rate of growth, π is inflation, and ε is a supply shock.

2.2 Demand side

The demand side is given by:

$$\pi = m + \nu + \mu$$

Where m is a monetary policy instrument that can be interpreted as money growth, ν is a demand shock, and μ is a policy error as a result of an imperfect forecast by the central bank. It can be shown that this is the log difference of QTM (Quantitative Theory of Money) where output is omitted to simplify the problem – this does not have any critical impact on the analysis as per Persson and Tabellini (2000).

2.3 Central Bank's Monetary Policy

In line with Woodford (1999) and Persson and Tabellini (2000) we will assume that the central bank follows a conventional instantaneous loss function in macroeconomic game theory with the objective to minimize squared deviations from the steady state growth path and inflation.

$$\mathcal{L} = \frac{1}{2}\left([\pi - \bar{\pi}]^2 + \gamma\,[y - \bar{y}]^2\right), \qquad \gamma > 0$$

Where γ describes the preference for minimizing output gap relative to the inflation gap. As the loss function is quadratic and the shocks are linear, the solution to the linear-quadratic systems is linear in its parameters. Thus, the optimal policy rule for the central bank must be of the form:

$$m = \psi + \psi_\theta \theta + \psi_\nu \nu + \psi_\varepsilon \varepsilon$$

Where ψ, ψ_θ, ψ_ν, and ψ_ε are policy parameters of the central bank all with values in the set of real numbers.

2.4 Analytical Solution of the Baseline Model

To summarize, the baseline model is made up of the following set of equations

$$y = \theta + (\pi - \pi^e) - \varepsilon \tag{2.2}$$

$$\pi = m + \nu + \mu \tag{2.3}$$

$$\mathcal{L} = \frac{1}{2}\left([\pi - \bar{\pi}]^2 + \gamma\,[y - \bar{y}]^2\right) \tag{2.4}$$

$$m = \psi + \psi_\theta \theta + \psi_\nu \nu + \psi_\varepsilon \varepsilon \tag{2.5}$$

Where equation (2.5) only is in force if the central bank commits to its monetary rule.

2.4.1 Assumptions

To solve the model we will have 2 sets of assumptions. First, we will assume that all shocks, θ, ε, ν, and μ are independent normally distributed shocks with mean 0, and well defined variances of σ_θ^2, σ_ε^2, σ_ν^2, and σ_μ^2.

Secondly, we will under perfect commitment assume the timing is as follows; (1) the central bank discloses a monetary rule that they want to follow; (2) ξ and $\bar{\omega}$ are realized making θ known to the general public; (3) inflation expectations are formed. As θ is known we get $\pi^e = \mathbb{E}(\pi|\theta)$; (4) the values of ν and ε are realized; (5) the central bank decides m;

(6) μ is realized pinning down y and π.

Under discretion, the timing is assumed to be similar the only difference being that the central bank does not disclose a monetary policy rule ex-ante. Rather they are free to set m as they want in (5).

2.4.2 Optimal Policy under Commitment

We will start by solving the problem ex-ante where, under perfect commitment, the central bank follows the monetary rule disclosed to the public. Thus, inflation in the economy expressed ex-ante is:

$$\begin{aligned}\pi &= m + \nu + \mu \\ &= \psi + \psi_\theta \theta + (1 + \psi_\nu)\nu + \psi_\varepsilon \varepsilon + \mu \end{aligned} \tag{2.6}$$

As expectations are rational, the public is aware of this rule and forms expectations based on it. Expected inflation therefore is:

$$\begin{aligned}\pi^e = \mathbb{E}(\pi|\theta) &= \mathbb{E}\left(\psi + \psi_\theta \theta + (1 + \psi_\nu)\nu + \psi_\varepsilon \varepsilon + \mu\right) \\ &= \psi + \psi_\theta \theta \end{aligned} \tag{2.7}$$

Inserting (2.7), and (2.6) in (2.4) yields the ex-ante realized output growth in the economy as:

$$\begin{aligned}y &= \theta + (\pi - \pi^e) - \varepsilon \\ &= \theta + (1 + \psi_\nu)\nu - (1 - \psi_\varepsilon)\varepsilon + \mu \end{aligned} \tag{2.8}$$

Since the central bank decides its policy rule ahead of time it must be ex-ante optimal and minimize the unconditional expected loss, $\mathbb{E}(\mathcal{L})$. Ex-ante expected loss is derived in appendix A.1:

$$\begin{aligned}\mathbb{E}(\mathcal{L}) &= \mathbb{E}\left(\frac{1}{2}\left[(\pi - \bar{\pi})^2 + \gamma\,(y - \bar{y})^2\right]\right) \\ &= \frac{1}{2}\Big(\left[\psi^2 + \psi_\theta^2\sigma_\theta^2 + (1 + \psi_\nu)^2\sigma_\nu^2 + \psi_\varepsilon^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi}\right] \\ &\qquad + \gamma\left[\sigma_\theta^2 + (1 + \psi_\nu)^2\sigma_\nu^2 + (1 - \psi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{y}^2\right]\Big) \end{aligned} \tag{2.9}$$

In appendix A.2 we take the derivatives w.r.t. the central bank's policy parameters, ψ, ψ_θ, ψ_ν, and ψ_ε, and get optimum in:

$$
\begin{aligned}
\psi &= \bar{\pi} \\
\psi_\theta &= 0 \\
\psi_\nu &= -1 \\
\psi_\varepsilon &= \frac{\gamma}{1+\gamma}
\end{aligned}
$$

Inserting in equations (2.5), (2.7), (2.2), and (2.3) we get the equilibrium under commitment as:

$$m_C = \bar{\pi} - \nu + \frac{\gamma}{1+\gamma}\varepsilon \tag{2.10}$$

$$\pi_C = \bar{\pi} + \frac{\gamma}{1+\gamma}\varepsilon + \mu \tag{2.11}$$

$$\pi^e_C = \bar{\pi} \tag{2.12}$$

$$y_C = \theta - \frac{1}{1+\gamma}\varepsilon + \mu \tag{2.13}$$

Inserting in the central bank's loss function we get the loss while adhering to a credible monetary policy as:

$$\mathcal{L} = \frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon + \mu\right]^2 + \gamma\left[\theta - \frac{1}{1+\gamma}\varepsilon + \mu - \bar{y}\right]^2\right) \tag{2.14}$$

2.4.3 Optimal Policy under Discretion

Under discretion, the central bank no longer sets a monetary rule. Rather they set m to the value, minimizing $\mathcal{L}$ ex-post while still taking π^e as given, i.e.:

$$\mathbb{E}(\mathcal{L}|\theta,\nu,\varepsilon) = \mathbb{E}\left[\frac{1}{2}\left([m+\nu+\mu-\bar{\pi}]^2 + \gamma\left[\theta + m + \nu + \mu - \pi^e - \varepsilon - \bar{y}\right]^2\right)\middle|\,\theta,\nu,\varepsilon\right] \tag{2.15}$$

The derivative w.r.t m, derived in appendix A.3, is:

$$
\begin{aligned}
\frac{\partial\,\mathcal{L}}{\partial\,m} &= 0 \Leftrightarrow \\
m &= \frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}(\pi^e - \theta + \varepsilon + \bar{y}) - \nu
\end{aligned} \tag{2.16}
$$

The public will set their inflation expectations based on this optimal discretionary policy. Thus, their expectations, as derived in A.4:

$$\begin{aligned}\mathbb{E}(\pi|\theta) = \pi^e_D &= \mathbb{E}\left(\frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}\left[\pi^e - \theta + \varepsilon + \bar{y}\right] - \left[\nu + \mu\right]\right) \\ &= \bar{\pi} + \gamma\left(\bar{y} - \theta\right) \end{aligned} \tag{2.17}$$

Comparing to the result from (2.12), we see that expectation of discretionary policy results entails an inflationary bias in the public expectations of $\gamma\left(\bar{y} - \theta\right)$. Inserting in (2.16), (2.2), and (2.3) we get the discretionary equilibrium as:

$$\begin{aligned} m_D &= \bar{\pi} + \gamma(\bar{y} - \theta) + \frac{\gamma}{1+\gamma}\varepsilon - \nu \\ \pi_D &= \bar{\pi} + \gamma(\bar{y} - \theta) + \frac{\gamma}{1+\gamma}\varepsilon + \mu \\ y_D &= \theta - \frac{1}{1+\gamma}\varepsilon + \mu \end{aligned}$$

A reasonable assumption to make is that the desired output growth of the central bank is higher than its natural level, i.e., $\theta < \bar{y}$[2]. Though θ is a normally distributed stochastic variable, and thus, it is not certain, it seems sensible that the natural output growth level does not frequently exceed the output growth targets. We will therefore assume the variance of θ to be of such magnitude that this is a five-sigma event[3] and thus, negligible for our further analysis.

Comparing our discretionary result to the result derived in section 2.4.2, it is seen how the sole expectation of discretionary monetary policy leads the public to expect higher inflation. Under discretion, it is optimal for the central bank to anticipate this, setting the money supply higher while there is no impact on output growth. Inserting the equilibrium values in the loss function yields:

$$\mathcal{L} = \frac{1}{2}\left(\left[\gamma(\bar{y} - \theta) + \frac{\gamma}{1+\gamma}\varepsilon + \mu\right]^2 + \gamma\left[\theta - \frac{1}{1+\gamma}\varepsilon + \mu - \bar{y}\right]^2\right)$$

With higher inflation and identical output growth compared to the commitment scenario, the economy is only further away from the central bank's inflation target while maintaining the distance to the targeted output growth. The loss in equilibrium, therefore, is higher under discretion, leaving this scenario undesirable to the central bank.

3 Extension of the Model

So far we have only modeled optimal policy rules for a single closed economy. We will now extend the model with a foreign country. As we will focus the analysis on the loss

[2]Similar to Persson and Tabellini (2000)

[3]Having a 0.000029% chance of happening

entailed by optimal monetary policy, we make the simplifying assumption that the foreign economy only interacts with the home country through a common central bank, making the equations derived for the single economy, (2.2) and (2.3) still hold. Equations describing the economies are:

$$
\begin{aligned}
y_H &= \theta_H + (\pi_H - \pi_H^e) - \varepsilon_H \\
\pi_H &= m + \nu_H + \mu \\
y_F &= \theta_F + (\pi_F - \pi_F^e) - \varepsilon_F \\
\pi_F &= m + \nu_F + \mu
\end{aligned}
$$

Meaning only the money supply, m, and the central bank's policy error are affecting both economies.

3.1 Common Central Bank's Monetary Policy

We will assume that the central bank has a similar squared loss function that they want to minimize, as presented in section 2.3. However, their incentive is to stabilize both economies, as opposed to a single economy. We will therefore modify the policy rule presented in section 2.3 by assuming the target is to minimize the weighted averages of the countries it conducts monetary policy for. The loss function of the extended model is therefore assumed to take the following form:

$$
\mathcal{L} = \frac{1}{2}\left([\eta\pi_H + (1-\eta)\pi_F - \bar{\pi}]^2 + b\,[\eta y_H + (1-\eta)y_F - \bar{y}]^2\right) \qquad \eta \in [0,1]
$$

Where η describes the relative size of each economy. Again due to the quadratic nature of the loss function, the optimal policy rule is of the form:

$$
m = \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + \psi_{\nu H}\nu_H + \psi_{\nu F}\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F
$$

3.2 Analytical Solution of the Extended Model

The full system of equations in the extended model is

$$
\begin{aligned}
y_H &= \theta_H + (\pi_H - \pi_H^e) - \varepsilon_H && (3.1) \\
\pi_H &= m + \nu_H + \mu && (3.2) \\
y_F &= \theta_F + (\pi_F - \pi_F^e) - \varepsilon_F && (3.3) \\
\pi_F &= m + \nu_F + \mu && (3.4) \\
\mathcal{L} &= \frac{1}{2}\left([\eta\pi_H + (1-\eta)\pi_F - \bar{\pi}]^2 + b\,[\eta y_H + (1-\eta)y_F - \bar{y}]^2\right) && (3.5) \\
m &= \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + \psi_{\nu H}\nu_H + \psi_{\nu F}\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F && (3.6)
\end{aligned}
$$

Just as in 2.4 we will assume that all shocks are independently normally distributed with mean 0. Further, we will again assume the timing of the model to be as described in 2.4.1.

3.2.1 Optimal Policy under Commitment in a Monetary Union

As in 2.4.2 the central bank follows the disclosed monetary rule thus, inflation in the home country is equal to:

$$\begin{aligned}\pi_H &= \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + \psi_{\nu H}\nu_H + \psi_{\nu F}\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \nu_H + \mu \\ &= \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + (1+\psi_{\nu H})\nu_H + \psi_{\nu F}\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu \end{aligned} \tag{3.7}$$

The expected inflation is:

$$\mathbb{E}(\pi_H|\theta) = \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F$$

Inserting in (3.1) yields output growth as:

$$y_H = \theta_H + (1+\psi_{\nu H})\nu_H + \psi_{\nu F}\nu_F - (1-\psi_{\varepsilon H})\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu \tag{3.8}$$

Similarly, for the foreign economy, we can derive:

$$\pi_F = \psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + \psi_{\nu H}\nu_H + (1+\psi_{\nu F})\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu \tag{3.9}$$

$$y_F = \theta_F + \psi_{\nu H}\nu_H + (1+\psi_{\nu F}\nu_F) + \psi_{\varepsilon H}\varepsilon_H - (1-\psi_{\varepsilon F})\varepsilon_F + \mu \tag{3.10}$$

As the central bank decides the monetary policy ex-ante, they minimize the unconditionally expected loss derived in B.1 as:

$$\begin{aligned}\mathbb{E}(\mathcal{L}) &= \mathbb{E}\left[\frac{1}{2}\left([\eta\pi_H + (1-\eta)\pi_F - \bar{\pi}]^2 + b\,[\eta y_H + (1-\eta)y_F - \bar{y}]^2\right)\right] \\ &= \frac{1}{2}\Big[\big(\psi^2 + \psi_{\theta H}^2\sigma_{\theta H}^2 + \psi_{\theta F}^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 \\ &\qquad + [(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2 + \psi_{\varepsilon H}^2\sigma_{\varepsilon H}^2 + \psi_{\varepsilon F}^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi}\big) \\ &\quad + b\left(\eta^2\sigma_{\theta H}^2 + (1-\eta)^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 + [(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2 \right. \\ &\qquad \left. + [\eta - \psi_{\varepsilon H}]^2\sigma_{\varepsilon H}^2 + [(1-\eta) - \psi_{\varepsilon F}]^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{y}^2\right)\Big]\end{aligned} \tag{3.11}$$

The first-order conditions are derived in B.2 and give the optimal parameter values as:

$$
\begin{aligned}
\psi &= \bar{\pi} \\
\psi_{\theta H} &= 0 \\
\psi_{\theta F} &= 0 \\
\psi_{\nu H} &= -\eta \\
\psi_{\nu F} &= -(1-\eta) \\
\psi_{\varepsilon H} &= \frac{b\eta}{1+b} \\
\psi_{\varepsilon F} &= \frac{(1-\eta)b}{1+b}
\end{aligned}
$$

Comparing with the optimal parameter values found in 2.4.2, we find that the central bank in the monetary union has a similar strategy. The major difference is that the response is weighted with the relative size of the economies. Thus, the equilibrium for the home country under a common central bank's commitment is:

$$m = \bar{\pi} - \eta\nu_H - (1-\eta)\nu_F + \frac{b\eta}{1+b}\varepsilon_H + \frac{(1-\eta)b}{1+b}\varepsilon_F \tag{3.12}$$

$$\pi_H = \bar{\pi} + (1-\eta)(\nu_H - \nu_F) + \frac{b}{1+b}[\eta\varepsilon_H + (1-\eta)\varepsilon_F] + \mu \tag{3.13}$$

$$\pi_H^e = \bar{\pi} \tag{3.14}$$

$$y_H = \theta_H + (1-\eta)(\nu_H - \nu_F) - \frac{1}{1+b}\varepsilon_H + \frac{(1-\eta)b}{1+b}(\varepsilon_F - \varepsilon_H) + \mu \tag{3.15}$$

Similar results for the foreign economy are derived in appendix B.3.

4 Evaluation of Optimal Policy

As long as the home economy is linked to the foreign economy, it will adhere to the monetary policies of the common central bank. However, the policy is still evaluated by the home central bank using its own loss function. Assuming the common central bank is credible the loss as evaluated by the home central bank can be found by combining (2.4) with (3.13) and (3.15) yielding

$$
\begin{aligned}
\mathcal{L}_H &= \frac{1}{2}\left([\pi_H - \bar{\pi}_H]^2 + \gamma\,[y_H - \bar{y}_H]^2\right) \\
&= \frac{1}{2}\Bigg(\left[\bar{\pi} + (1-\eta)(\nu_H - \nu_F) + \frac{b}{1+b}(\eta\varepsilon_H + [1-\eta]\varepsilon_F) + \mu - \bar{\pi}_H\right]^2 \\
&\qquad + \gamma\left[\theta_H + (1-\eta)(\nu_H - \nu_F) - \frac{1}{1+b}\varepsilon_H + \frac{(1-\eta)b}{1+b}(\varepsilon_F - \varepsilon_H) + \mu - \bar{y}_H\right]^2\Bigg)
\end{aligned} \tag{4.1}
$$

Where $\bar{\pi}_H$ is the home central banks inflation target whilst $\bar{\pi}$ is the inflation target of the common central bank and equally $\bar{y}_H$ is the output growth target for the home central bank, and μ is the policy error by the common central bank.

Comparing this to the optimal scenario under commitment derived in section 2.4.2 it is immediately apparent that if preferences are equal i.e., $b = \gamma$ and $\bar{\pi}_H = \bar{\pi}$ and both countries experience an equivalent shock the central bank would be indifferent between being in a union or not. The same would be the case if $\eta = 1$ as the common central bank would only prioritize stabilizing the home country. However, the central bank will prefer one scenario when this is not the case. In the following sections, we analyze the central bank's problem in expectations to understand when it is ex-ante rational to keep the peg as well as the effects of heterogeneous demand and supply shocks. For simplicity, we will in all future sections assume $\bar{\pi}_H = \bar{\pi}$, $\mu_H = \mu = 0$, $\eta \in [0,1)$ and that the variance of shocks is equivalent across the countries, i.e., $\sigma_{\theta H} = \sigma_{\theta F} = \sigma_\theta$, $\sigma_{\nu H} = \sigma_{\nu F} = \sigma_\nu$, $\sigma_{\varepsilon H} = \sigma_{\varepsilon F} = \sigma_\varepsilon$.

4.1 Expected Future Loss

Before the first realization of a shock, the central bank can evaluate the expected loss from keeping the peg or straying away from it. The expectation to (4.1) is derived in appendix C.1 and gives:

$$\mathbb{E}(\mathcal{L}_H) = \frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + \frac{\gamma}{1+\gamma}\sigma_\varepsilon^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 + \gamma\sigma_\theta^2 + \gamma\bar{y}_H^2\right] \tag{4.2}$$

Equation (4.2) describes the expected loss in every period for all future periods. We, therefore, expand the model with time and let the central bank evaluate the loss function in the future as well. The infinite sum of the expected losses can therefore be expressed as:

$$\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0(\mathcal{L}_H) = \frac{1}{2}\frac{1}{1-\beta}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + \frac{\gamma}{1+\gamma}\sigma_\varepsilon^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 + \gamma\sigma_\theta^2 + \gamma\bar{y}_H^2\right]$$

4.1.1 Deviating from the Monetary Union

We will analyze a situation where the home country decided to conduct an autonomous monetary policy, but when they deviate from the peg, they might face disbelief from agents in the model. Thus, we want to model a situation where the home country's central bank announces at the beginning of period 0 that it now wishes to conduct monetary policy like in the single economy commitment case from section 2.4.2, but this creates disbelief and the public, therefore, do not trust the central bank for ρ periods. Again expanding the model to incorporate time, we suppose that after announcing deviation at the beginning of period 0, in period 0 through $\rho - 1$, the central bank has to commit to "commitment policy" to convince the public that it will conduct a believable monetary policy. For all ρ periods of disbelief, the public will form inflation expectations like in the discretionary case derived in section 2.4.3. As the central bank only cares about the expectation, the intertemporal loss function will take the general form:

$$\mathcal{L}_{\mathbb{E}_0} = \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \mathcal{L}_t(e) \right)$$

Where $\mathcal{L}_t(e)$ is the period-specific loss as a function of the inflation expectations of the public, where $e \in \{c = \textit{credible}, d = \textit{disbelief}\}$, β is the discount factor and $\mathcal{L}_{\mathbb{E}_0}$ is the expectation, formed in period 0, of the intertemporal loss. We can split this up into the two following parts: 1) the loss incurred in period 0 through $\rho - 1$, where the central bank of the home country conduct committed monetary policy but the public is in disbelief, and 2) from period ρ to ∞ where there is full commitment. Therefore the expression can be rewritten as:

$$\begin{aligned} \mathcal{L}_{\mathbb{E}_0} &= \mathbb{E}_0 \left(\sum_{t=0}^{\rho-1} \beta^t \mathcal{L}_t(d) + \sum_{t=\rho}^{\infty} \beta^t \mathcal{L}_t(c) \right) \\ &= \underbrace{\mathbb{E}_0 \left(\sum_{t=0}^{\rho-1} \beta^t \mathcal{L}_t(d) \right)}_{\text{A}} + \underbrace{\mathbb{E}_0 \left(\sum_{t=\rho}^{\infty} \beta^t \mathcal{L}_t(c) \right)}_{\text{B}} \end{aligned} \tag{4.3}$$

Using the same quadratic loss function as previously, we can now solve:

$$\begin{aligned} \text{A: } \mathbb{E}_0 \left(\sum_{t=0}^{\rho-1} \beta^t \mathcal{L}_t(d) \right) &= \mathbb{E}_0 \left(\sum_{t=0}^{\rho-1} \beta^t \frac{1}{2} \left([\pi_{Ht} - \bar{\pi}]^2 + \gamma [y_{Ht} - \bar{y}_H]^2 \right) \right) \\ \text{B: } \mathbb{E}_0 \left(\sum_{t=\rho}^{\infty} \beta^t \mathcal{L}_t(c) \right) &= \mathbb{E}_0 \left(\sum_{t=\rho}^{\infty} \beta^t \frac{1}{2} \left([\pi_{Ht} - \bar{\pi}]^2 + \gamma [y_{Ht} - \bar{y}_H]^2 \right) \right) \end{aligned} \tag{4.4}$$

A) We now need to find the expected loss in period 0 through $\rho - 1$. In each period, the central bank announces the rule it wishes to follow, but the public does not believe it to be credible. Thus inflation expectations are set as in the discretionary case and look exactly like equation (2.17) and realized inflation is just like the credible scenario in equation (2.11). Therefore realized output gap is:

$$\begin{aligned} y_{Ht} &= \theta_{Ht} + (\bar{\pi} + \frac{\gamma}{1+\gamma} \varepsilon_{Ht} - \bar{\pi} + \gamma (\bar{y}_H - \theta_t)) - \varepsilon_t \\ &= (1-\gamma)\theta_{Ht} - \frac{1}{1+\gamma} \varepsilon_{Ht} + \gamma \bar{y}_H \end{aligned}$$

Inserting in equation (4.4), we derive loss in period 0 through $\rho - 1$ (Appendix C.2.1):

$$\mathbb{E} \left(\sum_{t=0}^{\rho-1} \beta^t \mathcal{L}_t(d) \right) = \frac{1-\beta^\rho}{1-\beta} \frac{1}{2} \gamma \left(\frac{1}{(1+\gamma)} \sigma_\varepsilon^2 + (1-\gamma)^2 \sigma_\theta^2 + (1-\gamma)^2 \bar{y}_H^2 \right)$$

Where the loss in each period is:

$$\mathcal{L}_t(d) = \frac{1}{2}\gamma\left(\frac{1}{(1+\gamma)}\sigma_\varepsilon^2 + (1-\gamma)^2\sigma_\theta^2 + (1-\gamma)^2\bar{y}_H^2\right) \tag{4.5}$$

B) After ρ periods, the home country is now in full commitment and therefore we can sum over the expectations of equation (2.14) for all periods (Appendix C.2.2):

$$\mathbb{E}_0\left(\sum_{t=\rho}^{\infty}\beta^t\mathcal{L}_t(c)\right) = \frac{\beta^\rho}{1+\beta}\frac{1}{2}\gamma\left(\frac{1}{(1+\gamma)}\sigma_\varepsilon^2 + \sigma_\theta^2 + \bar{y}_H^2\right)$$

Where the loss in each period is:

$$\mathcal{L}_t(c) = \frac{1}{2}\gamma\left(\frac{1}{(1+\gamma)}\sigma_\varepsilon^2 + \sigma_\theta^2 + \bar{y}_H^2\right)$$

We can now plug in A) and B) into equation (4.3) to find the intertemporal loss:

$$\begin{aligned}\mathcal{L}_{\mathbb{E}_0} = &\frac{1}{2}\frac{1-\beta^\rho}{1-\beta}\gamma\left(\frac{1}{1+\gamma}\sigma_\varepsilon^2 + (1-\gamma)^2\sigma_\theta^2 + (1-\gamma)^2\bar{y}_H^2\right)\\ &+\frac{1}{2}\frac{\beta^\rho}{1+\beta}\gamma\left(\frac{1}{1+\gamma}\sigma_\varepsilon^2 + \sigma_\theta^2 + \bar{y}_H^2\right)\end{aligned}$$

4.1.2 Incentive to Deviate in Expectation

Building on the previous section we derive the cost of deviating in the appendix C.2.3. In each period (for the first ρ periods) the loss from leaving is greater when the following inequality is met:

$$2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \le \gamma^2(\gamma-2)\left(\sigma_\theta^2 + \bar{y}_H^2\right)$$

From this equation, we see that a necessary requirement is for the central bank to have enough emphasis on the stabilization of output growth such that $\gamma > 2$. When the inequality holds the additional loss incurred from deviating in the first ρ periods is:

$$\Delta\mathcal{L}_H(d) = \frac{1}{2}\left[\gamma^2(\gamma-2)\left(\sigma_\theta^2 + \bar{y}_H^2\right) - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right]$$

In the following infinite periods, the central banks in both scenarios are perceived as credible. The decreased loss in these periods from being outside the union and thus unaffected by shocks from the foreign economy is:

$$\Delta\mathcal{L}_H(c) = -\frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right]$$

The infinite sum of additional expected loss therefore is:

$$\sum_{t=0}^{\infty} \Delta\mathcal{L}_H = \frac{1}{2(1-\beta)}\Bigg([1-\beta^{\rho}] \left[\gamma^2 (\gamma-2) (\sigma_\theta^2 + \bar{y}_H^2)\right] - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \Bigg) \tag{4.6}$$

It is seen from equation (4.6) how there will always be an incentive to deviate if $\rho = 0$ as the central bank can avoid the additional variance spilling over from the shocks of the other economy. In the appendix C.2.3, we derive the requirement which must be met for the home central bank not to have the incentive to deviate in expectation as:

$$\beta^{\rho} \leq 1 - \frac{2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2}{\gamma^2(\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2)} \tag{4.7}$$

To determine what this constraint means for γ and η, we will calibrate our model using estimates from comparable models and frequently employed parameter values specified in table 4.1. Following Galí (2008) we will set the quarterly discounting to $\beta = 0.99$, corresponding to a yearly discounting of 0.96. Rudebusch and Svensson (2002) is used for empirical values for the standard deviations, σ_ν, σ_ε, σ_θ. We assume the central bank generally has a target of 2% a year for the output growth target. In addition, this ensures that $\theta \geq \bar{y}$ is a five-sigma event.

Table 4.1: Calibration

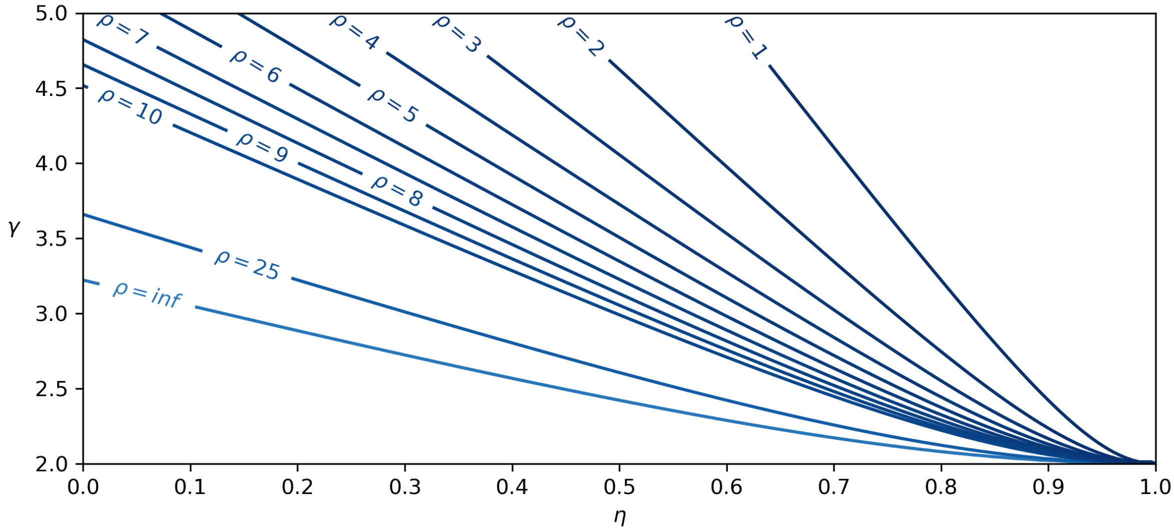
$\beta = 0.96$	Yearly discounting
$\bar{y}_H = 2.00\%$	Yearly GDP growth target
$\sigma_\theta = 0.38$	Percentage standard deviation of shocks in θ
$\sigma_\nu = 2.16$	Percentage standard deviation of shocks in ν
$\sigma_\varepsilon = 1.64$	Percentage standard deviation of shocks in ε

With this calibration in hand, we can plot the isoparametric curves showing when the inequality is met for different values of γ and η.

Figure 4.1 depicts the iso-curves characterizing the home central bank's problem for different ρ. It illustrates how the minimum required preference for output stabilization, γ, is decreasing in η and ρ. Assuming that the public is in disbelief for just five years, $\rho = 5$, and that the home and foreign economies are equal in size, $\eta = 0.5$, the required value of γ for it to be rational to stay in a monetary union in expectation is $\gamma = 3.525$. Though no empirical analysis has been made on models identical to ours, the minimum required value of γ appears to be substantially larger than what is found in the literature investigating familiar models[4]. For our further analysis, however, we will move forward assuming the home economy keeps the peg, though, parameter values of $\eta = 0.5$ and $\gamma = 3.525$ seem

[4]See, e.g., Ozlale (2003) and Debortoli and Nunes (2014)

Figure 4.1: Iso-curves for different values of ρ with $\beta = 0.96$



NOTE: The criterion specified in equation (4.7) is met to the northeast of iso-curves. 1 ρ is equivalent to 1 additional year of disbelief in the central bank.

high compared to the literature.

4.2 Heterogeneous Demand Shock

When analyzing the effects of differences in demand shocks in the economy, we will assume that both economies are hit by some demand shocks, ν_H and ν_F. Further, to only examine the effects of a demand shock we will let $\varepsilon_H = \varepsilon_F = 0$ and $\gamma = b$. These assumptions make it possible to simplify (4.1) into:

$$\begin{aligned}\mathcal{L}_H &= \frac{1}{2}\left([\pi_H - \bar{\pi}_H]^2 + \gamma\,[y_H - \bar{y}_H]^2\right) \\ &= \frac{1}{2}\left([(1-\eta)(\nu_H - \nu_F)]^2 + \gamma\,[\theta_H + (1-\eta)(\nu_H - \nu_F) - \bar{y}_H]^2\right) \qquad (4.8)\end{aligned}$$

If they the central bank were to scrap the peg this would happen after inflation expectations had been formed. As $\pi^e_H = \pi^e = \bar{\pi}_H = \bar{\pi}$, the central bank could deviate to a period of perfect commitment but independent monetary policy. The loss for the central bank in the commitment scenario without a peg stated in equation (2.14), simplifies in the case of a demand shock to:

$$\mathcal{L}_H = \frac{1}{2}\left(\gamma\,[\theta_H - \bar{y}_H]^2\right) \qquad (4.9)$$

In appendix C.3, we derive the additional loss the home central bank incurs when sticking to the peg and analyze when they have the incentive to deviate from it in a single period.

The inequality describing if the central bank should deviate is:

$$\begin{Bmatrix} \nu_H - \nu_F \leq -\frac{2\gamma(\theta_H - \bar{y}_H)}{(1+\gamma)(1-\eta)} & \text{for } \nu_H < \nu_F \\ \\ \nu_H - \nu_F \geq -\frac{2\gamma(\theta_H - \bar{y}_H)}{(1+\gamma)(1-\eta)} & \text{for } \nu_H > \nu_F \end{Bmatrix} \tag{4.10}$$

Assuming that the "natural" level of output growth does not exceed the output growth targets is a five-sigma event, it can reasonably be assumed that the right-hand side is positive.

Thus, the inequality always holds when the shock to the foreign economy is greater than the shock to the home economy, giving the central bank in this simplistic model a justification for leaving the peg when only evaluating the single period loss.

Interestingly, both sides are positive when the shock to the home economy is more significant, $\nu_H > \nu_F$. The central bank is only motivated to deviate in this scenario where the difference in shock magnitude is greater than the limit established. This result can seem counter-intuitive at first glance; however, the reason behind this is that the demand shock is not fully offset in the home economy. The home economy, thus, experiences increased output growth at the cost of increased inflation.

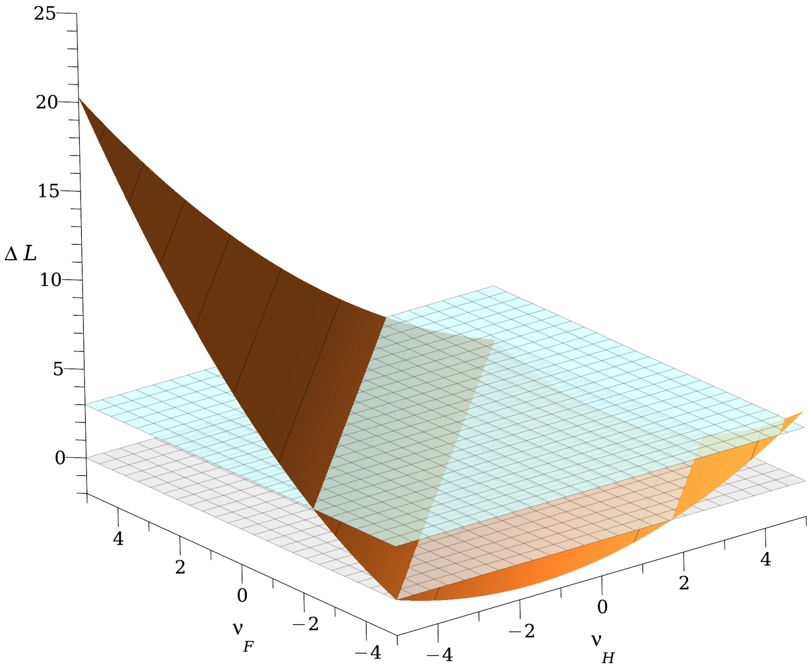
In the single economy commitment case, the benchmark we compare to, there is no incentive to decrease the output gap further because you need the public's trust. So when $\theta - \bar{y} > 0$, you would want to increase the money supply, but because θ is known to the public, this would not work. If higher m is expected, this will result in price adjustments. In the case of the peg, the home central bank, if it chooses to keep its peg, can trade off a little inflation to minimize the output gap without losing the public's trust, and that is precisely what happens.

Assuming, however, that we are not in this scenario but rather one of the inequalities in equation (4.10) hold, the single period net gain from deviating from the union is derived in appendix C.3 as:

$$\Delta\mathcal{L}_H = \frac{1}{2}\left[(1+\gamma)(1-\eta)^2(\nu_H - \nu_F)^2 + \gamma(1-\eta)(\theta_H - \bar{y}_H)(\nu_H - \nu_F)\right] \tag{4.11}$$

As described in section 4.1.1 in practice, it appears doubtful that the general public will find the home country's central bank credible when they have just deviated. However, in period 0 (when the central bank deviates), inflation expectations were already formed before the demand shock was observed. The expectations can, therefore, not be changed before the following period. In the following period, the expected loss in all future periods from deviating is the loss derived in equation (4.6). For the central bank to want to leave the union, it must hold that the benefits of leaving derived in (4.11) outweighs the loss from (4.6) discounted one period. Thus, the central bank will deviate when the following constraint holds:

Figure 4.2: Single period net gain from deviating due to a demand shock



NOTE: The orange surface illustrates the net gain from deviating as a function of ν_H and ν_H, the blue grid is the plane defined by the expected loss from deviating, and the gray grid is the plane defined by $\Delta\mathcal{L}_H = 0$. Shocks are scaled by their standard deviations.

$$\frac{1}{2}\left[(1+\gamma)(1-\eta)^2(\nu_H-\nu_F)^2 + 2\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F)\right]$$
$$\geq \frac{\beta}{2(1-\beta)}\left(\left[1-\beta^\rho\right]\left[\gamma^2\left(\gamma-2\right)\left(\sigma_\theta^2+\bar{y}_H^2\right)\right] - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right)$$

The net gain from equation (4.11) forms a surface in 3 dimensions illustrated in figure 4.2. The implications of the inequalities in (4.10) are illustrated here. It is seen how the orange surface dips below the grey plane, thus giving the central bank an instantaneous net gain from keeping the peg.

When including the loss from deviating, we see how the orange surface sits well below the blue plane for most plausible shocks, where the central bank has no incentive to deviate as the costs of leaving are too high. The shock to the foreign economy can be slightly above 2.5 standard deviations larger than the home shock, while it remains rational for the central bank to stay within the union as the cost of leaving is too high. For shocks where the foreign shock is smaller than the home shock, the difference must be almost ten standard deviations before it is optimal for the home country to leave.

4.3 Heterogeneous Supply Shock

Here we want to analyze the effects of differences in supply shocks. We will assume that some supply shock hits both economies, ε_H and ε_F. Furthermore, we will assume no demand shocks, $\nu_H = \nu_F = 0$, and $\gamma = b$. We can now simplify (4.1) to

$$\begin{aligned}\mathcal{L}_H &= \frac{1}{2}\left([\pi_H - \bar{\pi}_H]^2 + \gamma\,[y_H - \bar{y}_H]^2\right)\\ &= \frac{1}{2}\Bigg(\left[\frac{\gamma}{1+\gamma}(\eta\varepsilon_H + [1-\eta]\varepsilon_F)\right]^2\\ &\qquad + \gamma\left[\theta_H - \frac{1}{1+\gamma}\varepsilon_H + \frac{(1-\eta)\gamma}{1+\gamma}(\varepsilon_F - \varepsilon_H) - \bar{y}_H\right]^2\Bigg)\end{aligned} \tag{4.12}$$

Which is the loss of staying in the monetary union. We want to compare this to the alternative of leaving, where the loss will be like equation (2.14), which simplifies to:

$$\mathcal{L}_H = \frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_H\right]^2 + \gamma\left[\theta_H - \frac{1}{1+\gamma}\varepsilon_H - \bar{y}_H\right]^2\right) \tag{4.13}$$

For it to be beneficial to conduct an independent monetary policy, it must hold that equation (4.13) is smaller than equation (4.12). In appendix C.4, we derive the conditions:

$$\left\{\begin{array}{ll}\varepsilon_H - \varepsilon_F \leq \frac{2(\theta_H - \bar{y})}{1-\eta} & \text{for } \varepsilon_H < \varepsilon_F\\ \\ \varepsilon_H - \varepsilon_F \geq \frac{2(\theta_H - \bar{y})}{1-\eta} & \text{for } \varepsilon_H > \varepsilon_F\end{array}\right\} \tag{4.14}$$

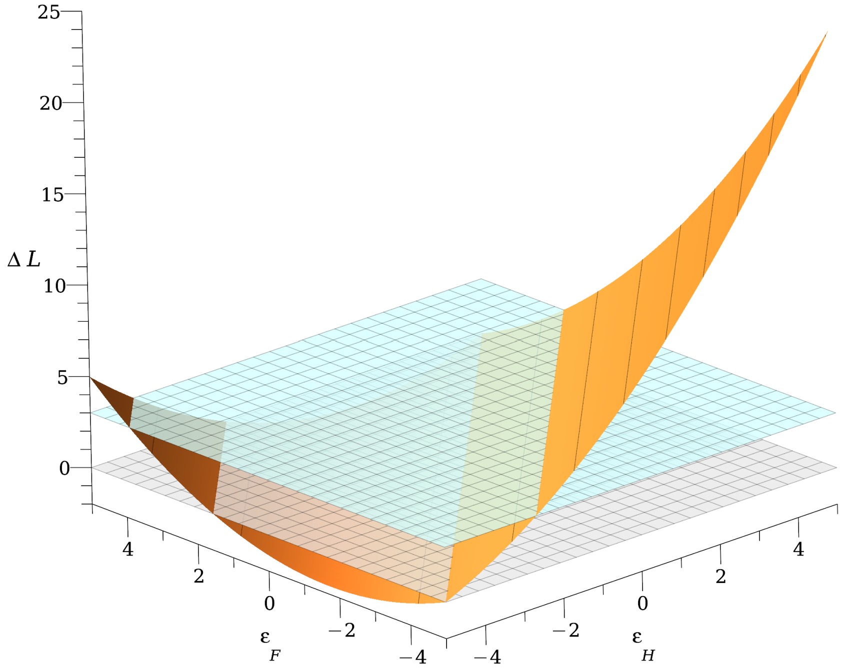
Where we still assume that the "natural" level of output growth exceeding the output growth targets, $\theta > \bar{y}$, is a five-sigma event. This means that we reasonably can assume that the right-hand side of the inequalities is negative.

In the case of a supply shock, it is now clear that if $\varepsilon_H > \varepsilon_F$, the inequality always holds. Therefore, when the shock to the home economy is greater than that in the foreign economy, the central bank should always deviate and conduct independent monetary policy when only evaluating in 1 period.

More interesting is the second result. Suppose the shock to the home economy is smaller than the shock to the foreign economy. In that case, it is only worth deviating if the shock is numerically more significant than the abovementioned threshold. This means there is a scenario where asymmetric shocks lead to the monetary union being the optimal choice. This result closely resembles the result found in section 4.2. When the shock to the foreign economy is slightly larger than at home, the joint central bank increases m slightly, increasing inflation and output growth in the home country while the policy remains credible. When the central bank at home evaluates this compared to its own credible policy, where it cannot fool the public for multiple periods, it is impossible to fool the public into further minimizing the output gap and remain credible. Thus, a deviation is not desirable.

Assuming, however, that we are not in this scenario but rather one of the inequalities

Figure 4.3: Single period net gain from deviating due to a supply shock



NOTE: The colored surface illustrates the net gain from deviating as a function of ε_H and ε_F, the blue grid is the plane defined by the expected loss from deviating, and the gray grid is the plane defined by $\Delta\mathcal{L}_H = 0$. Shocks are scaled by their standard deviations.

in equation (4.14) hold, the single period net gain from deviating from the union is derived in appendix C.4 as:

$$\Delta\mathcal{L}_{\mathcal{H}} = \frac{\gamma}{2+2\gamma}\left[(1-\eta)^2(\varepsilon_H - \varepsilon_F)^2 - (1-\eta)2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F)\right] \tag{4.15}$$

As argued for demand shocks in section 4.2, it seems reasonable to assume that people would have trouble trusting the central bank after it deviates. Therefore, to determine if deviation makes sense when considering future expectations, we will compare the gain, lower loss, in period 0 calculated above to the extra loss incurred in the future calculated in equation (4.6). The additional loss incurred in the future happens from period $t = 1$ and not period $t = 0$, equation (4.6) is discounted one period:

$$\begin{aligned}
&\frac{\gamma}{2+2\gamma}\left[(1-\eta)^2(\varepsilon_H - \varepsilon_F)^2 - (1-\eta)2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F)\right] \\
&\quad \geq \frac{\beta}{2(1-\beta)}\left(\left[1-\beta^\rho\right]\left[\gamma^2(\gamma-2)\left(\sigma_\theta^2 + \bar{y}_H^2\right)\right] - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right)
\end{aligned}$$

Like the previous section, the surface defined by equation (4.15) can be depicted together with the expected loss from deviating as a 3-dimensional graph with the x- and y-axis as the shocks and the loss on the z-axis.

It is seen in figure 4.3 that for some combinations of ε_H and ε_F, where $\varepsilon_H < \varepsilon_F$, the one period net gain from deviating is negative – below the gray plane – and thus, the central bank would have no incentive to deviate even before looking at potential loss related to disbelief from agents in the future.

When we include the loss of deviating, it can be seen how the potential loss from keeping the peg once again sits well below for most of the probable outcomes. We see how when the foreign shock is smaller than the shock to the home economy, the difference between the two can be just shy of 2.5 standard deviations. In the other case, where the shock to the home economy is greater, the difference between the two can be close to 10 standard deviations before it is optimal for the home economy to scrap the peg.

5 Discussion

The overarching theme of this thesis is an inquiry into the implications of monetary policy when a country's currency is pegged. In this section, we first address some of the critical shortcomings of our analysis. Following this, we will discuss our results' implications for the debate on the benefits and disadvantages of monetary unions and pegs.

Modeling the Economies

The first and most obvious place to begin is the model we use. Generally speaking, the model is very simplistic and obviously does not imitate the real world. Even within macroeconomic models for optimal policy, this model is one of the simplest models that will function and give you exciting insights into some aspects of optimal monetary policy. In the paper Persson and Tabellini (2000), the authors call the model "a workhorse model", and that is exactly what it is. It is a practical model with a reasonably simple analytical solution that still provides interesting and relevant insight into the central bank's problem when setting monetary policy. For this exact reason, the model was a good fit for this paper, providing a simple framework to analyze the implications of specific considerations a policymaker might find relevant when evaluating monetary policy, despite the simplistic nature, the lack of a complete micro-foundation, and potential obsolescence. The literature has moved past models like the one used for this paper. Still, nevertheless, the model produces valuable insights into the central banker's problem.

The Extension of the Model

In section 3, we expand upon the baseline model by introducing a new economy. It is unrealistic to assume that economies do not trade with each other, and the negligence of this aspect in the model is indeed problematic. If you introduced an element of trade, it is not unrealistic to think positive shocks in the foreign economy might have a positive spillover on output in the home economy and vice versa, which could make keeping the peg seem more attractive for the home country - if the peg incentivizes trade. Furthermore, we need to address the difficulty that the instantaneous benefits that can arise for the home country from keeping the peg only arise when either $\nu_H > \nu_F$ or $\varepsilon_H < \varepsilon_F$. When evaluating only a single period, a paradox arises since, in the event of a heterogeneous shock, at least one of the countries will experience a greater instantaneous loss. This we mitigate by assuming a loss from exiting. However, this effect heavily depends on the central bank's preference for stabilizing output growth which makes an longterm equilibrium where both parties stay seem more plausible.

Model calibration

We calibrated the model using estimates from the literature. This provides us with insights that have a slight degree of real-world relevance. Mostly, this does not present any pressing issues throughout the paper. However, some of our results, specifically related to the two-economy expansion, do not fit well with realistic parameter estimates. Figure 4.1 shows how, for given parameter values, there are certain situations where the peg is the optimal choice in expectation. While this makes the subsequent analysis more interesting because it can be shown that the central bank might choose the peg in expectation before observing any shocks, the requirement for this to be true yields some quite unrealistic scenarios. Specifically, γ, which represents the relative weight the central bank puts on minimizing output gap as opposed to the inflation gap, needs to be just above 3.5 for the peg to be the optimal choice if $\rho = 5, \eta = 0.5$. While it is exciting that the solution exits for the extension we created, $\gamma = 3.525$ is, to say the least, very high compared to what you might find in the real world. Ozlale (2003) runs a regression for the weights for the US in the period 1970-1999 and finds a relative weight of around $\gamma = 0.67$. It does not seem unlikely that an institution like the Federal Reserve, with its dual mandate, would have about equal weight on its two mandates. Additionally, institutions like the ECB do not even have an official mandate of maximum employment – minimizing output gap – like the Federal Reserve, and thus, officially, only care about the inflation gap, implying $\gamma = 0$ for the ECB.

Furthermore, this is given the assumption of $\eta = 0.5$, which in the case of Denmark and its part in ERM 2 seems unreasonably high. A more realistic assumption for η in the case of Denmark would be $\eta = 0$, thus, making the required γ even higher. The assumption of $\rho = 5$ years, intuitively, does not seem that high. If the public loses faith in the central bank, it seems reasonable that it takes considerable time to restore credibility. However, this still does not change the finding that $\gamma > 2$ is required to make it a rational decision to keep the peg in expectation.

We will shortly comment on some of the assumptions we made early on when solving the model. We assume that shocks to the home and foreign economies have the same variance. This is not a critical assumption but simplifies the math. We also assume that there is no covariance between shocks in the single economy nor the two-economy framework. This assumption is, to some degree, critical for the results produced, especially with regard to the feasibility of the peg. It is not unreasonable to think that the home country would be pegged to a country with similar characteristics and, therefore, potentially also similar shocks, which makes this assumption less critical. If the countries look similar, the potential positive single-period effect from small heterogeneous shocks can make the peg more attractive. Furthermore, the peg would most likely also become more attractive in expectation as the monetary policy in the two-country scenario would more closely resemble the policy of the credible single-economy scenario.

Perspective

Following our optimal policy analysis, our model shows that for large heterogeneous shocks, the home central bank is incentivized to deviate from the peg as the policy conducted by the common central bank is sub-optimal, especially if agents do not form disbelief after the deviation. Interestingly there are also scenarios where the central bank finds the policy of the common central bank optimal - a result we did not predict going into the analysis. Although our results are based on assumptions of a high value of γ, similar results would appear if a constant exogenous cost of deviating – or similarly benefit from being in the union - not modeled in this paper exists. For one, it seems likely that there could be political benefits of participation in a monetary relationship like the one presented in this paper. In the case of Denmark, participation in the ERM 2 seems likely to have a political impact and help cement Danish participation in the EU.

The result that asymmetric shocks can lead to staying in the monetary relationship as the optimal choice, even without future disbelief, seems more like a prediction of the model than an insight into the workings of the actual economy. In DØRS (2000), the Danish Economic Council examines the implication of Danish participation in the ERM 2 and finds specifically that asymmetric shocks are undesirable in the context of a peg and one of the more problematic aspects of participating in the ERM 2. This fits well with our prediction that very heterogenous shocks can make deviation more attractive. Our results, therefore, are interesting in the question of optimality when participating in ERM 2 in a time of turbulence. The shocks required for it to be interesting to deviate, as described in sections 4.2 and 4.3, must be of a great magnitude, as leaving might come at a high cost.

6 Conclusion

This paper has sought insights into the central bank's optimal policy when pegging one's currency. To analyze the optimal behavior for central banks, we extended a workhorse New Keynesian model to include a simplistic monetary relationship between two countries. We assumed that the central bank has a quadratic loss function to evaluate deviation away from inflation and output growth targets.

First, we analyze the effects of having a currency peg on a country with a perfectly credible central bank in expectation and evaluate the optimal behavior of a home central bank. We find that the home central bank will be incentivized to deviate if it could deviate to a scenario perceived as credible. Imposing several periods of public distrust in the home central bank after deviating, the central bank can have an incentive to keep the peg in expectation. However, the results show that, as generally assumed in the literature, when $\gamma < 2$, the central bank is incentivized to deviate even for $\rho \to \infty$. Our results, therefore, show that in the expectation, it is only rational to keep the peg if their preference to stabilize output growth far exceeds their preference for stabilizing inflation.

Under the assumption that the central bank does not leave in expectation, we examine the effects of heterogeneous demand and supply shocks. In the case of a demand shock, we find that when the foreign country's shock exceeds that of the home country, the home central bank always has an incentive to deviate when only evaluating the single period loss. This result appears as the optimum for the common central bank to decrease the money supply more than the home central bank desires. Thus, inflation and output growth are brought down further from the home central bank's target. When the demand shock is greater in the home country, however, we find that for some combination of shocks, the home central bank benefits from having a peg. This happens as the common central bank does not completely cancel out the demand shock in the home country, leading to higher inflation. Though inflation rises, so does output growth, and the home central bank trades a small amount of inflation to lower output gap. However, this can only continue until a certain point, after which the home central bank is again incentivized to deviate.

We find the same result for a supply shock with the opposite sign. Thus only if the supply shock in the foreign country exceeds that of the home country will there be a point where it is, in fact, beneficial to keep the peg. However, when including the costs of deviating, we find that for most plausible shocks, the expected costs outweigh the benefits of scrapping the peg.

Our analysis suggests that having a peg to a perfectly credible central bank in most cases results in a sub-optimal monetary policy when evaluated by the home country, thus, suggesting Danmarks Nationalbank and the Danish economy would be better off outside ERM 2. However, as costs are connected to scrapping the peg, the costs will often outweigh the possible benefits, thus trapping Denmark in ERM 2 in our model. However, recalibrating with a more realistic value of γ challenges this prediction, making scraping the peg the optimal choice. As we only analyze the monetary policy, there could be benefits not included in this model that could make keeping the peg seem more attractive.

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A Appendix for Baseline Model

A.1 Central Bank's ex-ante Expected Loss

Inserting equations (2.6) and (2.8) into the loss function, (2.5), and taking the unconditional expectation yields

$$
\begin{aligned}
\mathbb{E}(\mathcal{L}) &= \mathbb{E}\left(\frac{1}{2}\left[(\pi-\bar{\pi})^2+\gamma\,(y-\bar{y})^2\right]\right) \\
&= \frac{1}{2}\mathbb{E}\left(\left[(\psi+\psi_\theta\theta+(1+\psi_\nu)\nu+\psi_\varepsilon\varepsilon+\mu-\bar{\pi})^2\right.\right. \\
&\qquad\qquad \left.\left.+\gamma\left(\theta+(1+\psi_\nu)\nu-(1-\psi_\varepsilon)\varepsilon+\mu-\bar{y}\right)^2\right]\right) \\
&= \frac{1}{2}\left[\mathbb{E}\left(A\right)+\gamma\mathbb{E}\left(B\right)\right] \qquad \text{(A.1)}
\end{aligned}
$$

Where A and B are defined as

$$
A = [\psi+\psi_\theta\theta+(1+\psi_\nu)\nu+\psi_\varepsilon\varepsilon+\mu-\bar{\pi}]^2 \qquad \text{(A.2)}
$$

$$
B = [\theta+(1+\psi_\nu)\nu-(1-\psi_\varepsilon)\varepsilon+\mu-\bar{y}]^2 \qquad \text{(A.3)}
$$

First deriving $\mathbb{E}(A)$

$$
\begin{aligned}
\mathbb{E}(A) &= \mathbb{E}\left([\psi+\psi_\theta\theta+(1+\psi_\nu)\nu+\psi_\varepsilon\varepsilon+\mu-\bar{\pi}]^2\right) \\
&= \mathbb{E}\left[\psi^2+\psi_\theta^2\theta^2+(1+\psi_\nu)^2\nu^2+\psi_\varepsilon^2\varepsilon^2+\mu^2+\bar{\pi}^2+2\psi\psi_\theta\theta+2\psi(1+\psi_\nu)\nu+2\psi\psi_\varepsilon\varepsilon\right. \\
&\qquad +2\psi\mu-2\psi\bar{\pi}+2\psi_\theta(1+\psi_\nu)\theta\nu+2\psi_\theta\psi_\varepsilon\theta\varepsilon+2\psi_\theta\theta\mu-2\psi_\theta\theta\bar{\pi}+2(1+\psi_\nu)\psi_\varepsilon\nu\varepsilon \\
&\qquad\quad \left.+2(1+\psi_\nu)\nu\mu-2(1+\psi_\nu)\nu\bar{\pi}+2\psi_\varepsilon\varepsilon\mu-2\psi_\varepsilon\varepsilon\bar{\pi}-2\mu\bar{\pi}\right] \qquad \text{(A.4)}
\end{aligned}
$$

Using the assumption that all shocks are independent normally distributed with mean zero, we can reduce equation (A.4) to

$$
\mathbb{E}(A) = \psi^2+\psi_\theta^2\sigma_\theta^2+(1+\psi_\nu)^2\sigma_\nu^2+\psi_\varepsilon^2\sigma\varepsilon^2+\sigma_\mu^2+\bar{\pi}^2-2\psi\bar{\pi} \qquad \text{(A.5)}
$$

Similarly, for B we can derive the ex-ante expected loss

$$
\begin{aligned}
\mathbb{E}(B) &= \mathbb{E}\left([\theta+(1+\psi_\nu)\nu-(1-\psi_\varepsilon)\varepsilon+\mu-\bar{y}]^2\right) \\
&= \mathbb{E}\left[\theta^2+(1+\psi_\nu)^2\nu^2+(1-\psi_\varepsilon)^2\varepsilon^2+\mu^2+\bar{y}^2+2(1+\psi_\nu)\theta\nu-2(1-\psi_\varepsilon)\theta\varepsilon\right. \\
&\qquad +2\theta\mu-2\theta\bar{y}-2(1+\psi_\nu)(1-\psi_\varepsilon)\nu\varepsilon+2(1+\psi_\nu)\nu\mu+2(1+\psi_\nu)\nu\bar{y} \\
&\qquad\quad \left.-2(1-\psi_\varepsilon)\varepsilon\mu+2(1-\psi_\varepsilon)\varepsilon\bar{y}-2\mu\bar{y}\right] \\
&= \sigma_\theta^2+(1+\psi_\nu)^2\sigma_\nu^2+(1-\psi_\varepsilon)^2\sigma_\varepsilon^2+\sigma_\mu^2+\bar{y}^2 \qquad \text{(A.6)}
\end{aligned}
$$

Thus, the total ex-ante expected loss is:

$$\begin{aligned}\mathbb{E}(\mathcal{L}) = \frac{1}{2}\big(&\left[\psi^2 + \psi_\theta^2\sigma_\theta^2 + (1+\psi_\nu)^2\sigma_\nu^2 + \psi_\varepsilon^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi}\right] \\ &+\gamma\left[\sigma_\theta^2 + (1+\psi_\nu)^2\sigma_\nu^2 + (1-\psi_\varepsilon)^2\sigma_\varepsilon^2 + \sigma_\mu^2 + \bar{y}^2\right]\big)\end{aligned} \tag{A.7}$$

A.2 First Order Conditions under Commitment

Finding the FOC's of equation (2.9) with respect to the central bank's policy parameters yields:

$$\begin{aligned}\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi} &= 0 \Leftrightarrow \\ 0 &= \psi - \bar{\pi} \Leftrightarrow \\ \psi &= \bar{\pi} \qquad (A.8)\\ \frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_\theta} &= 0 \Leftrightarrow \\ 0 &= \psi_\theta\sigma_\theta^2 \Leftrightarrow \\ \psi_\theta &= 0 \qquad (A.9)\\ \frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_\nu} &= 0 \Leftrightarrow \\ 0 &= (1+\psi_\nu)\sigma_\nu^2 + \gamma(1+\psi_\nu)\sigma_\nu^2 \Leftrightarrow \\ \psi_\nu &= -1 \qquad (A.10)\\ \frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_\varepsilon} &= 0 \Leftrightarrow \\ 0 &= \psi_\varepsilon\sigma_\varepsilon^2 - \gamma(1-\psi_\varepsilon)\sigma_\varepsilon^2 \Leftrightarrow \\ \psi_\varepsilon &= \frac{\gamma}{1+\gamma} \qquad (A.11)\end{aligned}$$

A.3 First Order Conditions under Discretion

Using dominated convergence theorem, i.e. $\frac{\partial}{\partial\ x}\mathbb{E}(g(x)) = \mathbb{E}\left(\frac{\partial}{\partial\ x}g(x)\right)$, we can take the derivative of (2.15) w.r.t m to find optimal policy.

$$\begin{aligned}\frac{\partial\ \mathbb{E}(\mathcal{L}|\theta,\nu,\varepsilon)}{\partial\ m} &= 0 \Leftrightarrow \\ 0 &= \mathbb{E}\left[(m+\nu+\mu-\bar{\pi}) + \gamma\left(\theta+m+\nu+\mu-\pi^e-\varepsilon\bar{y}\right)|\theta,\nu,\varepsilon\right] \\ &= (m+\nu-\bar{\pi}) + \gamma\left(\theta+m+\nu-\pi^e-\varepsilon\bar{y}\right) \Leftrightarrow \\ m &= \frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}(\pi^e-\theta+\varepsilon+\bar{y}) - \nu\end{aligned} \tag{A.12}$$

A.4 Expected Inflation under Discretion

Using the results from A.3 we can derive the expected inflation formed by the public given θ as

$$\begin{aligned}
\mathbb{E}(\pi|\theta) = \pi^e &= \mathbb{E}\left(\frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}\left[\pi^e - \theta + \varepsilon + \bar{y}\right] - [\nu + \mu]\right) \Leftrightarrow \\
\left(1 - \frac{\gamma}{1+\gamma}\right)\pi^e &= \frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}\left[\bar{y} - \theta\right] \Leftrightarrow \\
\frac{1}{1+\gamma}\pi^e &= \frac{1}{1+\gamma}\bar{\pi} + \frac{\gamma}{1+\gamma}\left[\bar{y} - \theta\right] \Leftrightarrow \\
\pi^e &= \bar{\pi} + \gamma\left[\bar{y} - \theta\right] \qquad \text{(A.13)}
\end{aligned}$$

B Appendix for Extended Model

B.1 Central Bank's ex-ante Expected Loss

Inserting equations (3.7), (3.9), (3.8) and (3.10) into the extended loss function, (3.5) yields

$$\begin{aligned}
\mathbb{E}(\mathcal{L}) &= \mathbb{E}\left[\frac{1}{2}\left([\eta\pi_H + (1-\eta)\pi_F - \bar{\pi}]^2 + b\,[\eta y_H + (1-\eta)y_F - \bar{y}]^2\right)\right] \qquad \text{(B.1)}\\
&= \frac{1}{2}\mathbb{E}\,[(\eta\,[\psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + (1+\psi_{\nu H})\nu_H + \psi_{\nu F}\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu]\\
&\quad +[1-\eta]\,[\psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + \psi_{\nu H}\nu_H + (1+\psi_{\nu F})\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu]\\
&\quad -\bar{\pi})^2 + b\,(\eta\,[\theta_H + (1+\psi_{\nu H})\nu_H + \psi_{\nu F}\nu_F - (1-\psi_{\varepsilon H})\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu]\\
&\quad +[1-\eta]\,[\theta_F + \psi_{\nu H}\nu_H + (1+\psi_{\nu F}\nu_F) + \psi_{\varepsilon H}\varepsilon_H - (1-\psi_{\varepsilon F})\varepsilon_F + \mu] - \bar{y})^2]\\
&= \frac{1}{2}\mathbb{E}\,[(\psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + [\eta + \psi_{\nu H}]\nu_H + [(1-\eta) + \psi_{\nu F}]\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F\\
&\quad +\mu - \bar{\pi})^2 + b\,(\eta\theta_H + (1-\eta)\theta_F + [\eta + \psi_{\nu H}]\nu_H + [(1-\eta) + \psi_{\nu F}]\nu_F\\
&\quad -[\eta - \psi_{\varepsilon H}]\varepsilon_H - [(1-\eta) - \psi_{\varepsilon F}]\varepsilon_F + \mu - \bar{y})^2]\\
&= \frac{1}{2}\,[\mathbb{E}(A) + b\mathbb{E}(B)] \qquad \text{(B.2)}
\end{aligned}$$

Where A and B are defined as

$$\begin{aligned}
A &= (\psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + [\eta + \psi_{\nu H}]\nu_H\\
&\quad +[(1-\eta) + \psi_{\nu F}]\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu - \bar{\pi})^2 \qquad \text{(B.3)}\\
B &= (\eta\theta_H + (1-\eta)\theta_F + [\eta + \psi_{\nu H}]\nu_H\\
&\quad +[(1-\eta) + \psi_{\nu F}]\nu_F - [\eta - \psi_{\varepsilon H}]\varepsilon_H - [(1-\eta) - \psi_{\varepsilon F}]\varepsilon_F + \mu - \bar{y})^2 \qquad \text{(B.4)}
\end{aligned}$$

First deriving $\mathbb{E}(A)$

$$\begin{aligned}
\mathbb{E}(A) = \mathbb{E}\,&[(\psi + \psi_{\theta H}\theta_H + \psi_{\theta F}\theta_F + [\eta + \psi_{\nu H}]\nu_H \\
&+[(1-\eta) + \psi_{\nu F}]\nu_F + \psi_{\varepsilon H}\varepsilon_H + \psi_{\varepsilon F}\varepsilon_F + \mu - \bar{\pi})^2\big] \\
= \mathbb{E}\,&\big[\psi^2 + \psi_{\theta H}^2\theta_H^2 + \psi_{\theta F}^2\theta_F^2 + [\eta + \psi_{\nu H}]^2\nu_H^2 + [(1-\eta) + \psi_{\nu F}]^2\nu_F^2 + \psi_{\varepsilon H}^2\varepsilon_H^2 \\
&+\psi_{\varepsilon F}^2\varepsilon_F^2 + \mu^2 + \bar{\pi}^2 + 2\psi\psi_{\theta H}\theta_H + 2\psi\psi_{\theta F}\theta_F + 2\psi[\eta + \psi_{\nu H}]\nu_H \\
&+2\psi[(1-\eta) + \psi_{\nu F}]\nu_F + 2\psi\psi_{\varepsilon H}\varepsilon_H + 2\psi\psi_{\varepsilon F}\varepsilon_F + 2\psi\mu - 2\psi\bar{\pi} \\
&+2\psi_{\theta H}\psi_{\theta F}\theta_H\theta_F + 2\psi_{\theta H}[\eta + \psi_{\nu H}]\theta_H\nu_H + 2\psi_{\theta H}[(1-\eta) + \psi_{\nu F}]\theta_H\nu_F \\
&+2\psi_{\theta H}\psi_{\varepsilon H}\theta_H\varepsilon_H + 2\psi_{\theta H}\psi_{\varepsilon F}\theta_H\varepsilon_F + 2\psi_{\theta H}\theta_H\mu - 2\psi_{\theta H}\theta_H\bar{\pi} \\
&+2\psi_{\theta F}[\eta + \psi_{\nu H}]\theta_F\nu_H + 2\psi_{\theta F}[(1-\eta) + \psi_{\nu F}]\theta_F\nu_F + 2\psi_{\theta F}\psi_{\varepsilon H}\theta_F\varepsilon_H \\
&+2\psi_{\theta F}\psi_{\varepsilon F}\theta_F\varepsilon_F + 2\psi_{\theta F}\theta_F\mu - 2\psi_{\theta F}\theta_F\bar{\pi} + 2[\eta + \psi_{\nu H}][(1-\eta) + \psi_{\nu F}]\nu_H\nu_F \\
&+2[\eta + \psi_{\nu H}]\psi_{\varepsilon H}\nu_H\varepsilon_H + 2[\eta + \psi_{\nu H}]\psi_{\varepsilon F}\nu_H\varepsilon_F + 2[\eta + \psi_{\nu H}]\nu_H\mu \\
&-2[\eta + \psi_{\nu H}]\nu_H\bar{\pi} + 2[(1-\eta) + \psi_{\nu F}]\psi_{\varepsilon H}\nu_F\varepsilon_H + 2[(1-\eta) + \psi_{\nu F}]\psi_{\varepsilon F}\nu_F\varepsilon_F \\
&+2[(1-\eta) + \psi_{\nu F}]\nu_F\mu - 2[(1-\eta) + \psi_{\nu F}]\nu_F\bar{\pi} + 2\psi_{\varepsilon H}\psi_{\varepsilon F}\varepsilon_H\varepsilon_F + 2\psi_{\varepsilon H}\varepsilon_H\mu \\
&-2\psi_{\varepsilon H}\varepsilon_H\bar{\pi} + 2\psi_{\varepsilon F}\varepsilon_F\mu - 2\psi_{\varepsilon F}\varepsilon_F\bar{\pi} - 2\mu\bar{\pi}]
\end{aligned} \tag{B.5}$$

Utilizing the assumption that all shocks are independent and normally distributed with mean 0 we can reduce equation (B.5) to:

$$\begin{aligned}
\mathbb{E}(A) = {} & \psi^2 + \psi_{\theta H}^2\sigma_{\theta H}^2 + \psi_{\theta F}^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 \\
& + [(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2 + \psi_{\varepsilon H}^2\sigma_{\varepsilon H}^2 + \psi_{\varepsilon F}^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi}
\end{aligned} \tag{B.6}$$

Similarly for we can derive $\mathbb{E}(B)$ as:

$$\begin{aligned}
\mathbb{E}(B) = \mathbb{E}\,&[(\eta\theta_H + (1-\eta)\theta_F + [\eta + \psi_{\nu H}]\nu_H \\
&+[(1-\eta) + \psi_{\nu F}]\nu_F - [\eta - \psi_{\varepsilon H}]\varepsilon_H - [(1-\eta) - \psi_{\varepsilon F}]\varepsilon_F + \mu - \bar{y})^2\big] \\
= \mathbb{E}\,&\big[\eta^2\theta_H^2 + (1-\eta)^2\theta_F^2 + [\eta + \psi_{\nu H}]^2\nu_H^2 + [(1-\eta) + \psi_{\nu F}]^2\nu_F^2 + [\eta - \psi_{\varepsilon H}]^2\varepsilon_H^2 \\
&+[(1-\eta) - \psi_{\varepsilon F}]^2\varepsilon_F^2 + \mu^2 + \bar{y}^2 + 2\eta(1-\eta)\theta_H\theta_F + 2\eta[\eta + \psi_{\nu H}]\theta_H\nu_H \\
&+2\eta[(1-\eta) + \psi_{\nu F}]\theta_H\nu_F - 2\eta[\eta - \psi_{\varepsilon H}]\theta_H\varepsilon_H - 2\eta[(1-\eta) - \psi_{\varepsilon F}]\theta_H\varepsilon_F \\
&+2\eta\theta_H\mu - 2\eta\theta_H\bar{y} + 2(1-\eta)[\eta + \psi_{\nu H}]\theta_F\nu_H + 2(1-\eta)[(1-\eta) + \psi_{\nu F}]\theta_F\nu_F \\
&-2(1-\eta)[\eta - \psi_{\varepsilon H}]\theta_F\varepsilon_H - 2(1-\eta)[(1-\eta) - \psi_{\varepsilon F}]\theta_F\varepsilon_F + 2(1-\eta)\theta_F\mu \\
&-2(1-\eta)\theta_F\bar{y} + 2[\eta + \psi_{\nu H}][(1-\eta) + \psi_{\nu F}]\nu_H\nu_F - 2[\eta + \psi_{\nu H}][\eta - \psi_{\varepsilon H}]\nu_H\varepsilon_H \\
&-2[\eta + \psi_{\nu H}][(1-\eta) - \psi_{\varepsilon F}]\nu_H\varepsilon_F + 2[\eta + \psi_{\nu H}]\nu_H\mu - 2[\eta + \psi_{\nu H}]\nu_H\bar{y} \\
&-2[(1-\eta) + \psi_{\nu F}][\eta - \psi_{\varepsilon H}]\nu_F\varepsilon_H - 2[(1-\eta) + \psi_{\nu F}][(1-\eta) - \psi_{\varepsilon F}]\nu_F\varepsilon_F \\
&+2[(1-\eta) + \psi_{\nu F}]\nu_F\mu - 2[(1-\eta) + \psi_{\nu F}]\nu_F\bar{y} + 2[\eta - \psi_{\varepsilon H}][(1-\eta) - \psi_{\varepsilon F}]\varepsilon_H\varepsilon_F \\
&-2[\eta - \psi_{\varepsilon H}]\varepsilon_H\mu + 2[\eta - \psi_{\varepsilon H}]\varepsilon_H\bar{y} - 2[(1-\eta) - \psi_{\varepsilon F}]\varepsilon_F\mu \\
&+2[(1-\eta) - \psi_{\varepsilon F}]\varepsilon_F\bar{y} - 2\mu\bar{\pi}] \\
= \eta^2&\sigma_{\theta H}^2 + (1-\eta)^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 + [(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2 \\
&+ [\eta - \psi_{\varepsilon H}]^2\sigma_{\varepsilon H}^2 + [(1-\eta) - \psi_{\varepsilon F}]^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{y}^2
\end{aligned} \tag{B.7}$$

Combining we get the ex-ante expected loss for the central bank under commitment in the monetary union as:

$$\begin{aligned}
\mathbb{E}(\mathcal{L}) = \frac{1}{2}\,&\big[\big(\psi^2 + \psi_{\theta H}^2\sigma_{\theta H}^2 + \psi_{\theta F}^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 \\
&\quad+[(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2 + \psi_{\varepsilon H}^2\sigma_{\varepsilon H}^2 + \psi_{\varepsilon F}^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{\pi}^2 - 2\psi\bar{\pi}\big) \\
&+b\left(\eta^2\sigma_{\theta H}^2 + (1-\eta)^2\sigma_{\theta F}^2 + [\eta + \psi_{\nu H}]^2\sigma_{\nu H}^2 + [(1-\eta) + \psi_{\nu F}]^2\sigma_{\nu F}^2\right. \\
&\quad\left.+[\eta - \psi_{\varepsilon H}]^2\sigma_{\varepsilon H}^2 + [(1-\eta) - \psi_{\varepsilon F}]^2\sigma_{\varepsilon F}^2 + \sigma_\mu^2 + \bar{y}^2\right)\big]
\end{aligned} \tag{B.8}$$

B.2 First Order Conditions under Commitment

Taking the first derivatives of equation (3.11) w.r.t the policy parameters yields

$$\begin{aligned}
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi} &= 0 \Leftrightarrow \\
0 &= \psi - \bar{\pi} \Leftrightarrow \\
\psi &= \bar{\pi} && \text{(B.9)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\theta H}} &= 0 \Leftrightarrow \\
0 &= \psi_{\theta H}\sigma^2_{\theta H} \Leftrightarrow \\
\psi_{\theta H} &= 0 && \text{(B.10)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\theta F}} &= 0 \Leftrightarrow \\
0 &= \psi_{\theta F}\sigma^2_{\theta F} \Leftrightarrow \\
\psi_{\theta H} &= 0 && \text{(B.11)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\nu H}} &= 0 \Leftrightarrow \\
0 &= \left([\eta + \psi_{\nu H}]\sigma^2_{\nu H}\right) + b\left([\eta + \psi_{\nu H}]\sigma^2_{\nu H}\right) \Leftrightarrow \\
\psi_{\nu H} &= -\eta && \text{(B.12)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\nu F}} &= 0 \Leftrightarrow \\
0 &= \left([(1-\eta) + \psi_{\nu F}]\sigma^2_{\nu F}\right) + b\left([(1-\eta) + \psi_{\nu F}]\sigma^2_{\nu F}\right) \Leftrightarrow \\
\psi_{\nu F} &= -(1-\eta) && \text{(B.13)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\varepsilon H}} &= 0 \Leftrightarrow \\
0 &= \left(\psi_{\varepsilon H}\sigma^2_{\varepsilon H}\right) - b([\eta - \psi_{\varepsilon H}]\sigma^2_{\varepsilon H}) \Leftrightarrow \\
\psi_{\varepsilon H} &= \frac{b\eta}{1+b} && \text{(B.14)} \\
\frac{\partial\ \mathbb{E}(\mathcal{L})}{\partial\ \psi_{\varepsilon F}} &= 0 \Leftrightarrow \\
0 &= \left(\psi_{\varepsilon F}\sigma^2_{\varepsilon F}\right) - b([(2-\eta) - \psi_{\varepsilon F}]\sigma^2_{\varepsilon F}) \Leftrightarrow \\
\psi_{\varepsilon F} &= \frac{(1-\eta)b}{1+b} && \text{(B.15)}
\end{aligned}$$

B.3 Equilibrium in the Foreign Economy

Using the optimum derived in B.2 combined with equations (3.9) and (3.10) the equilibrium in the foreign economy can be derived as

$$\pi_F = \bar{\pi} + \eta(\nu_F - \nu_B) + \frac{b}{1+b}[\eta\varepsilon_H + (1-\eta)\varepsilon_F] + \mu \tag{B.16}$$

$$y_F = \theta_F + \eta(\nu_F - \nu_B) - \frac{1}{1+b}\varepsilon_F + \frac{\eta b}{1+b}(\varepsilon_H - \varepsilon_F) + \mu \tag{B.17}$$

C Appendix for Analysis

C.1 Expected Loss for Home Central Bank

Taking the expectation to the home central bank's loss function when evaluating the loss in the monetary union, equation (4.1), we get:

$$\begin{aligned}
\mathbb{E}(\mathcal{L}_H) &= \mathbb{E}\left[\frac{1}{2}\left([\pi_H - \bar{\pi}_H]^2 + \gamma\,[y_H - \bar{y}_H]^2\right)\right] \\
&= \frac{1}{2}\mathbb{E}\left(\left[(1-\eta)(\nu_H - \nu_F) + \frac{\gamma}{1+\gamma}(\eta\varepsilon_H + [1-\eta]\varepsilon_F)\right]^2\right. \\
&\quad \left. +\gamma\left[\theta_H + (1-\eta)(\nu_H - \nu_F) - \frac{1+(1-\eta)\gamma}{1+\gamma}\varepsilon_H + \frac{(1-\eta)\gamma}{1+\gamma}\varepsilon_F - \bar{y}_H\right]^2\right) \\
&= \frac{1}{2}\mathbb{E}(A + \gamma B)
\end{aligned} \tag{C.1}$$

Where A and B are equal to

$$A = \left[(1-\eta)(\nu_H - \nu_F) + \frac{\gamma}{1+\gamma}(\eta\varepsilon_H + [1-\eta]\varepsilon_F)\right]^2 \tag{C.2}$$

$$B = \left[\theta_H + (1-\eta)(\nu_H - \nu_F) - \frac{1+(1-\eta)\gamma}{1+\gamma}\varepsilon_H + \frac{(1-\eta)\gamma}{1+\gamma}\varepsilon_F - \bar{y}_H\right]^2 \tag{C.3}$$

First deriving $\mathbb{E}(A)$

$$\begin{aligned}
\mathbb{E}(A) &= \mathbb{E}\left(\left[(1-\eta)(\nu_H - \nu_F) + \frac{\gamma}{1+\gamma}(\eta\varepsilon_H + [1-\eta]\varepsilon_F)\right]^2\right) \\
&= \mathbb{E}\left([1-\eta]^2[\nu_H - \nu_F]^2 + \left[\frac{\gamma}{1+\gamma}\right]^2[\eta\varepsilon_H + (1-\eta)\varepsilon_F]^2\right. \\
&\quad \left. +2[1-\eta]\frac{\gamma}{1+\gamma}[\nu_H - \nu_F][\eta\varepsilon_H + (1-\eta)\varepsilon_F]\right) \\
&= \mathbb{E}\left([1-\eta]^2[\nu_H^2 + \nu_F^2 - 2\nu_H\nu_F] + \left[\frac{\gamma}{1+\gamma}\right]^2[\eta^2\varepsilon_H^2 + (1-\eta)^2\varepsilon_F^2 + 2\eta(1-\eta)\varepsilon_H\varepsilon_F]\right. \\
&\quad \left. +2[1-\eta]\frac{\gamma}{1+\gamma}[\nu_H - \nu_F][\eta\varepsilon_H + (1-\eta)\varepsilon_F]\right)
\end{aligned} \tag{C.4}$$

Utilizing the fact that shocks have mean 0 and are i.i.d the expression reduces to

$$\mathbb{E}(A) = (1-\eta)^2(\sigma_\nu^2 + \sigma_\nu^2) + \left(\frac{\gamma}{1+\gamma}\right)^2(\eta^2\sigma_\varepsilon^2 + [1-\eta]^2\sigma_\varepsilon^2) \tag{C.5}$$

Equally we can derive $\mathbb{E}(B)$

$$\begin{aligned}
\mathbb{E}(B) = \mathbb{E}&\left(\left[\theta_H + (1-\eta)(\nu_H - \nu_F) - \frac{1+(1-\eta)\gamma}{1+\gamma}\varepsilon_H + \frac{(1-\eta)\gamma}{1+\gamma}\varepsilon_F - \bar{y}_H\right]^2\right) \\
= \mathbb{E}&\left(\theta_H^2 + (1-\eta)^2(\nu_H^2 + \nu_F^2 - 2\nu_H\nu_F)^2 + \left(\frac{1+(1-\eta)\gamma}{1+\gamma}\right)^2 \varepsilon_H^2 + \left(\frac{(1-\eta)\gamma}{1+\gamma}\right)^2 \varepsilon_F^2\right. \\
&+\bar{y}_H^2 + 2(1-\eta)\theta_H(\nu_H - \nu_F) - 2\left(\frac{1+(1-\eta)\gamma}{1+\gamma}\right)\theta_H\varepsilon_H + \frac{2(1-\eta)\gamma}{1+\gamma}\theta_H\varepsilon_F \\
&-2\theta_H\bar{y}_H - 2(1-\eta)\left(\frac{1+(1-\eta)\gamma}{1+\gamma}\right)(\nu_H - \nu_F)\varepsilon_H + \frac{2(1-\eta)^2\gamma}{1+\gamma}(\nu_H - \nu_F)\varepsilon_F \\
&-2(1-\eta)(\nu_H - \nu_F)\bar{y}_H - 2\left(\frac{(1-\eta)\gamma + (1-\eta)^2\gamma^2}{(1+\gamma)^2}\right)\varepsilon_H\varepsilon_F \\
&\left.+\left(\frac{1+(1-\eta)\gamma}{1+\gamma}\right)\varepsilon_H\bar{y}_H - \frac{(1-\eta)\gamma}{1+\gamma}\varepsilon_F\bar{y}_H\right) \\
= \sigma_\theta^2 &+ (1-\eta)^2(\sigma_\nu^2 + \sigma_\nu^2) + \left(\frac{1+(1-\eta)\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2 + \left(\frac{(1-\eta)\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2 + \bar{y}_H^2 \qquad \text{(C.6)}
\end{aligned}$$

Gathering and rearranging the terms gives the expected loss in each period when staying in the monetary union as:

$$\begin{aligned}
\mathbb{E}(\mathcal{L}_H) &= \frac{1}{2}\mathbb{E}(A + \gamma B) \\
&= \frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + \frac{\gamma}{1+\gamma}\sigma_\varepsilon^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 + \gamma\sigma_\theta^2 + \gamma\bar{y}_H^2\right] \qquad \text{(C.7)}
\end{aligned}$$

The infinite sum of this loss can be expressed as

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0(\mathcal{L}_H) &= \sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + \frac{\gamma}{1+\gamma}\sigma_\varepsilon^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 + \gamma\sigma_\theta^2 + \gamma\bar{y}_H^2\right] \\
&= \frac{1}{2}\frac{1}{1-\beta}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + \frac{\gamma}{1+\gamma}\sigma_\varepsilon^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 + \gamma\sigma_\theta^2 + \gamma\bar{y}_H^2\right]
\end{aligned}$$

C.2 Central Bank's ex-ante Expected Loss under temporary disbelief

C.2.1 Derivation A

$$
\begin{aligned}
\mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\mathcal{L}_t(d)\right) &= \mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\left(\left[\bar{\pi}+\frac{\gamma}{1+\gamma}\varepsilon_{Ht}-\bar{\pi}\right]^2\right.\right.\\
&\quad\left.\left.+\gamma\left[(1-\gamma)\theta_{Ht}-\left(\frac{1}{1+\gamma}\right)\varepsilon_{Ht}+\gamma\bar{y}_H-\bar{y}_H\right]^2\right)\right)\\
&= \mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_{Ht}\right]^2\right.\right.\\
&\quad\left.\left.+\gamma\left[(1-\gamma)\theta_{Ht}-\left(\frac{1}{1+\gamma}\right)\varepsilon_{Ht}-(1-\gamma)\bar{y}_H\right]^2\right)\right)\\
&= \sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\left(\mathbb{E}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_{Ht}\right]^2\right)\right.\\
&\quad\left.+\mathbb{E}\left(\gamma\left[(1-\gamma)\theta_{Ht}-\left(\frac{1}{1+\gamma}\right)\varepsilon_{Ht}-(1-\gamma)\bar{y}_H\right]^2\right)\right)\\
&= \sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\left(\mathbb{E}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\varepsilon_{Ht}^2\right)\right.\\
&\quad+\mathbb{E}\left(\gamma\left[(1-\gamma)^2\theta_{Ht}^2+\left(\frac{1}{1+\gamma}\right)^2\varepsilon_{Ht}^2+(1-\gamma)^2\bar{y}_H^2\right.\right.\\
&\quad-2(1-\gamma)\theta_{Ht}\left(\frac{1}{1+\gamma}\right)\varepsilon_{Ht}-2(1-\gamma)\theta_{Ht}(1-\gamma)\bar{y}_H\\
&\quad\left.\left.\left.+2\left(\frac{1}{1+\gamma}\right)\varepsilon_{Ht}(1-\gamma)\bar{y}_H\right]\right)\right)\\
&= \sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2\right.\\
&\quad\left.\gamma\left[(1-\gamma)^2\sigma_\theta^2+\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2+(1-\gamma)^2\bar{y}_H^2\right]\right)
\end{aligned}
$$

Assuming that all variances are constant we can rewrite to:

$$\begin{aligned}
\mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\mathcal{L}_t(d)\right) &= \sum_{t=0}^{\rho-1}\beta^t\frac{1}{2}\Bigg(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2 \\
&\qquad +\gamma\left[(1-\gamma)^2\sigma_\theta^2+\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2+(1+\gamma)^2\bar{y}_H^2\right]\Bigg) \\
&= \frac{1}{2}\Bigg(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2 \\
&\qquad +\gamma\left[(1-\gamma)^2\sigma_\theta^2+\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2+(1+\gamma)^2\bar{y}_H^2\right]\Bigg)\sum_{t=0}^{\rho-1}\beta^t
\end{aligned}$$

$$\mathcal{K}\equiv\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2+\gamma\left[(1-\gamma)^2\sigma_\theta^2+\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2+(1+\gamma)^2\bar{y}_H^2\right]\right)$$

$$\begin{aligned}
\mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\mathcal{L}_t(d)\right) &= \mathcal{K}\sum_{t=0}^{\rho-1}\beta^t \\
&= \mathcal{K}\left(\sum_{t=0}^{\mathcal{Q}}\beta^t-\sum_{t=\rho}^{\mathcal{Q}}\beta^t\right), \qquad \mathcal{Q}>\rho
\end{aligned}$$

$$\begin{aligned}
\lim_{\mathcal{Q}\to\infty}\mathcal{K}\left(\sum_{t=0}^{\mathcal{Q}}\beta^t-\sum_{t=\rho}^{\mathcal{Q}}\beta^t\right) &= \mathcal{K}\left(\frac{1}{1-\beta}-\frac{\beta^\rho}{1-\beta}\right) \\
&= \mathcal{K}\frac{1-\beta^\rho}{1-\beta}
\end{aligned}$$

Inserting $\mathcal{K}$ we get:

$$\begin{aligned}
\mathbb{E}\left(\sum_{t=0}^{\rho-1}\beta^t\mathcal{L}_t(d)\right) &= \frac{1}{2}\Bigg(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2 \\
&\qquad +\gamma\left[(1-\gamma)^2\sigma_\theta^2+\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2+(1+\gamma)^2\bar{y}_H^2\right]\Bigg)\frac{1-\beta^\rho}{1-\beta} \\
&= \frac{1}{2}\left(\frac{1+\gamma}{(1+\gamma)^2}\gamma\sigma_\varepsilon^2+\gamma\left[(1-\gamma)^2\sigma_\theta^2+(1+\gamma)^2\bar{y}_H^2\right]\right)\frac{1-\beta^\rho}{1-\beta} \\
&= \frac{1-\beta^\rho}{1-\beta}\frac{1}{2}\gamma\left(\frac{1}{(1+\gamma)}\sigma_\varepsilon^2+(1-\gamma)^2\sigma_\theta^2+(1+\gamma)^2\bar{y}_H^2\right)
\end{aligned}$$

C.2.2 Derivation B

Here, we can simply take the expectation for the loss in the commitment case from equation (2.14) and sum over the periods:

$$\begin{aligned}
\mathbb{E}_0\left(\mathcal{L}_t(c)\right) &= \mathbb{E}_0\left(\frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_{Ht}\right]^2+\gamma\left[\theta_{Ht}-\frac{1}{1+\gamma}\varepsilon_{Ht}-\bar{y}_H\right]^2\right)\right)\\
&= \mathbb{E}_0\left(\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\varepsilon_{Ht}^2+\gamma\left[\theta_{Ht}^2-\left(\frac{1}{1+\gamma}\right)^2\varepsilon_{Ht}^2\right.\right.\right.\\
&\qquad -\bar{y}_H^2-2\theta_{Ht}\frac{1}{1+\gamma}\varepsilon_{Ht}-2\theta_{Ht}\bar{y}_H\\
&\qquad \left.\left.\left.+2\frac{1}{1+\gamma}\varepsilon_{Ht}\bar{y}_H\right]\right)\right)\\
&= \frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2+\gamma\left[\sigma_\theta^2-\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2-\bar{y}_H^2\right]\right)
\end{aligned}$$

We can now sum over the periods:

$$\begin{aligned}
\mathbb{E}_0\left(\sum_{t=\rho}^{\infty}\beta^t\mathcal{L}_t(c)\right) = \sum_{t=\rho}^{\infty}\beta^t\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2\right.\\
\left.+\gamma\left[\sigma_\theta^2-\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2-\bar{y}_H^2\right]\right)
\end{aligned}$$

$$\begin{aligned}
&\lim_{t\to\infty}\sum_{t=\rho}^{\infty}\beta^t\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2+\gamma\left[\sigma_\theta^2-\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2-\bar{y}_H^2\right]\right)\\
&\quad=\frac{\beta^\rho}{1+\beta}\frac{1}{2}\left(\left(\frac{\gamma}{1+\gamma}\right)^2\sigma_\varepsilon^2+\gamma\left[\sigma_\theta^2-\left(\frac{1}{1+\gamma}\right)^2\sigma_\varepsilon^2-\bar{y}_H^2\right]\right)\\
&\quad=\frac{\beta^\rho}{1+\beta}\frac{1}{2}\gamma\left(\frac{1}{(1+\gamma)}\sigma_\varepsilon^2+\sigma_\theta^2+\bar{y}_H^2\right)
\end{aligned}$$

C.2.3 Incentive to Deviate in Expectation

We want to compare the loss from deviating to the loss from staying in the union. In the first ρ periods where there is disbelief, the loss from equation (4.5) is greater than the loss from staying in the union from equation (4.2), when the following is true:

$$
\begin{aligned}
&\frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2+\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2+\gamma\sigma_\theta^2+\gamma\bar{y}_H^2\right]\\
&\qquad\le\frac{1}{2}\left(\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+\gamma(1-\gamma)^2\sigma_\theta^2+\gamma(1-\gamma)^2\bar{y}_H^2\right)\Leftrightarrow\\
&2(1+\gamma)(1-\eta)^2\sigma_\nu^2+\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2+\gamma\sigma_\theta^2+\gamma\bar{y}_H^2\\
&\qquad\le\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+\gamma(1-\gamma)^2\sigma_\theta^2+\gamma(1-\gamma)^2\bar{y}_H^2\Leftrightarrow\\
&2(1+\gamma)(1-\eta)^2\sigma_\nu^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\le\gamma\left[(1-\gamma)^2-1\right]\sigma_\theta^2+\gamma\left[(1-\gamma)^2-1\right]\bar{y}_H^2\Leftrightarrow\\
&2(1+\gamma)(1-\eta)^2\sigma_\nu^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\le\gamma^2\left[\gamma-2\right]\sigma_\theta^2+\gamma^2\left[\gamma-2\right]\bar{y}_H^2\Leftrightarrow\\
&2(1+\gamma)(1-\eta)^2\sigma_\nu^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\le\gamma^2\left(\gamma-2\right)\left(\sigma_\theta^2+\bar{y}_H^2\right)
\end{aligned}
\tag{C.8}
$$

Assuming $\gamma > 2$ the loss from deviating is greater for some parameter values. The net loss sustained each period in the first ρ periods therefore is (loss in with disbelief - loss from staying monetary union - loss with disbelief $\mathcal{L}_H(d)$):

$$
\Delta\mathcal{L}_H(d)=\frac{1}{2}\left[\gamma^2\left(\gamma-2\right)\left(\sigma_\theta^2+\bar{y}_H^2\right)-2(1+\gamma)(1-\eta)^2\sigma_\nu^2-2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right]
\tag{C.9}
$$

In the infinite periods after period ρ both the scenario in which the home economy stays in the monetary union as well as the scenario in which the leave the central bank is perceived as credible. The decrease in loss from being out of the union is found by subtracting the expected loss when in the union while credible derived in C.1 from the expected loss when outside the union and credible derived C.2.2

$$
\begin{aligned}
\Delta\mathcal{L}_H(c)&=\frac{1}{2}\left[\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+\gamma\sigma_\theta^2+\gamma\bar{y}_H^2\right]\\
&\qquad-\frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2+\frac{\gamma}{1+\gamma}\sigma_\varepsilon^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2+\gamma\sigma_\theta^2+\gamma\bar{y}_H^2\right]\\
&=-\frac{1}{2}\left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2+2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2\right]
\end{aligned}
\tag{C.10}
$$

Due to increased volatility when in the union as both economies are hit by uncorrelated shocks this term is always negative.

The infinite sum of losses and gains therefore is

$$\begin{aligned}
\sum_{t=0}^{\infty} \Delta\mathcal{L}_H &= \sum_{t=0}^{\rho-1} \Delta\mathcal{L}_H(d) + \sum_{t=\rho}^{\infty} \Delta\mathcal{L}_H(c) \\
&= \frac{1}{2}\left(\frac{1-\beta^\rho}{1-\beta} \left[\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2) - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \right] \right. \\
&\quad \left. - \frac{\beta^\rho}{1-\beta} \left[2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \right] \right) \\
&= \frac{1}{2(1-\beta)} \left([1-\beta^\rho] \left[\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2) \right] \right. \\
&\quad \left. - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \right)
\end{aligned} \tag{C.11}$$

Solving for $0 \leq \sum_{t=0}^{\infty} \Delta\mathcal{L}_H$ gives us the requirement for the central bank not to deviate in expectation

$$\begin{aligned}
&0 \leq \sum_{t=0}^{\infty} \Delta\mathcal{L}_H \Leftrightarrow \\
&0 \leq \frac{1}{2(1-\beta)} \left[(1-\beta^\rho)\left(\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2) \right) - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \right] \Leftrightarrow \\
&0 \leq (1-\beta^\rho)\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2) - 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 - 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2
\end{aligned} \tag{C.12}$$

Using our assumption of $\gamma > 2$ and isolating for β^ρ

$$\begin{aligned}
(1-\beta^\rho)\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2) &\geq 2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2 \Leftrightarrow \\
1-\beta^\rho &\geq \frac{2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2}{\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2)} \Leftrightarrow \\
\beta^\rho &\leq 1 - \frac{2(1+\gamma)(1-\eta)^2\sigma_\nu^2 + 2\frac{(1-\eta)^2\gamma^2}{1+\gamma}\sigma_\varepsilon^2}{\gamma^2 (\gamma-2)(\sigma_\theta^2 + \bar{y}_H^2)}
\end{aligned} \tag{C.13}$$

C.3 Heterogeneous Demand Shock

The problem for the central bank on whether to leave or not is simplified into whether (4.9) is smaller than (4.8)

$$\begin{aligned}
\frac{1}{2}\left(\gamma\left[\theta_H-\bar{y}_H\right]^2\right) &\leq \frac{1}{2}\left(\left[(1-\eta)(\nu_H-\nu_F)\right]^2+\gamma\left[(\theta_H-\bar{y}_H)+(1-\eta)(\nu_H-\nu_F)\right]^2\right) \Leftrightarrow \\
\gamma(\theta_H-\bar{y}_H)^2 &\leq (1-\eta)^2(\nu_H-\nu_F)^2+\gamma(\theta_H-\bar{y}_H)^2+\gamma(1-\eta)^2(\nu_H-\nu_F)^2 \\
&\quad +2\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F) \Leftrightarrow \\
0 &\leq (1+\gamma)(1-\eta)^2(\nu_H-\nu_F)^2+2\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F) \Leftrightarrow \\
-(1+\gamma)(1-\eta)^2(\nu_H-\nu_F)^2 &\leq 2\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F) \Leftrightarrow \\
-(1+\gamma)(1-\eta) &\leq \frac{2\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F)}{(1-\eta)(\nu_H-\nu_F)^2} \Leftrightarrow \\
-(1+\gamma)(1-\eta) &\leq \frac{2\gamma(\theta_H-\bar{y}_H)}{\nu_H-\nu_F} \Leftrightarrow \\
-\frac{(1+\gamma)(1-\eta)}{2\gamma} &\leq \frac{(\theta_H-\bar{y}_H)}{\nu_H-\nu_F} \Leftrightarrow \\
-\frac{(1+\gamma)(1-\eta)}{2\gamma(\theta_H-\bar{y}_H)} &\geq \frac{1}{\nu_H-\nu_F} \qquad \text{(C.14)}
\end{aligned}$$

The inequality changes direction in the last line as we assume $\theta_H < \bar{y}_H$. As θ_H is a stochastic variable with mean 0 while $\bar{y}_H$ is a constant greater than 0 this assumption is not always correct. However, it seems reasonable to assume that the natural level of output growth does not exceed the output growth targets on a frequent basis. We will therefore here assume that this is a three-sigma event, thus having a probability of 0.13%. This edge case will therefore be ignored in our analysis.

From here we can rewrite it to be in terms of differences in shock size

$$\begin{Bmatrix} \nu_H-\nu_F \leq -\frac{2\gamma(\theta_H-\bar{y}_H)}{(1+\gamma)(1-\eta)} & \text{for } \nu_H<\nu_F \\ \\ \nu_H-\nu_F \geq -\frac{2\gamma(\theta_H-\bar{y}_H)}{(1+\gamma)(1-\eta)} & \text{for } \nu_H>\nu_F \end{Bmatrix} \qquad \text{(C.15)}$$

The net gain of deviating is (loss in the monetary union - loss of single economy under commitment):

$$\begin{aligned}
\Delta\mathcal{L}_H &= \frac{1}{2}\left(\left[(1-\eta)(\nu_H-\nu_F)\right]^2+\gamma\left[(\theta_H-\bar{y}_H)+(1-\eta)(\nu_H-\nu_F)\right]^2\right)-\frac{1}{2}\left(\gamma\left[\theta_H-\bar{y}_H\right]^2\right) \\
&= \frac{1}{2}(1+\gamma)(1-\eta)^2(\nu_H-\nu_F)^2+\gamma(1-\eta)(\theta_H-\bar{y}_H)(\nu_H-\nu_F) \qquad \text{(C.16)}
\end{aligned}$$

C.4 Heterogeneous Supply Shock

We can now use equation (4.13) and equation (4.12) to set up the inequality:

$$\underbrace{\frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_H\right]^2+\gamma\left[\theta_H-\frac{1}{1+\gamma}\varepsilon_H-\bar{y}_H\right]^2\right)}_{\text{A}} \leq$$

$$\underbrace{\frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}(\eta\varepsilon_H+[1-\eta]\varepsilon_F)\right]^2+\gamma\left[\theta_H-\frac{1}{1+\gamma}\varepsilon_H+\frac{(1-\eta)\gamma}{1+\gamma}(\varepsilon_F-\varepsilon_H)-\bar{y}_H\right]^2\right)}_{\text{B}}$$

Let's first reduce A:

$$\begin{aligned}
A &= \frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}\varepsilon_H\right]^2+\gamma\left[\theta_H-\frac{1}{1+\gamma}\varepsilon_H-\bar{y}_H\right]^2\right) \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2}{2\,(1+\gamma)^2}+\frac{\gamma\left(\theta_H-\frac{\varepsilon_H}{1+\gamma}-\bar{y}_H\right)^2}{2} \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2}{2\,(1+\gamma)^2}+\frac{\gamma\left(\frac{1+\gamma}{1+\gamma}\theta_H+\left(-\frac{\varepsilon_H}{1+\gamma}\right)-\frac{1+\gamma}{1+\gamma}\left(\bar{y}_H\right)\right)^2}{2} \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2}{2\,(1+\gamma)^2}+\frac{\gamma\left(\frac{(1+\gamma)\theta_H}{1+\gamma}+\frac{1(-\varepsilon_H)}{1+\gamma}-\frac{(1+\gamma)(\bar{y}_H)}{1+\gamma}\right)^2}{2} \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2}{2\,(1+\gamma)^2}+\frac{\gamma\frac{((1+\gamma)\theta_H-\varepsilon_H-(1+\gamma)\bar{y}_H)^2}{(1+\gamma)^2}}{2} \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2}{2\,(1+\gamma)^2}+\frac{\gamma\left((1+\gamma)\,\theta_H-\varepsilon_H-(1+\gamma)\,\bar{y}_H\right)^2}{2\,(1+\gamma)^2} \Leftrightarrow \\
&= \frac{\gamma^2\varepsilon_H^2+\gamma\left(\gamma\theta_H-\gamma\bar{y}_H-\varepsilon_H+\theta_H-\bar{y}_H\right)^2}{2\,(1+\gamma)^2} \Leftrightarrow \\
&= \frac{\gamma\,(1+\gamma)\left(\gamma\,\theta_H^2-2\gamma\theta_H\bar{y}_H+\gamma\bar{y}_H^2+\varepsilon_H^2-2\varepsilon_H\theta_H+2\varepsilon_H\bar{y}_H+\theta_H^2-2\theta_H\bar{y}_H+\bar{y}_H^2\right)}{2\,(1+\gamma)^2} \Leftrightarrow \\
&= \frac{\gamma\left(\gamma\,\theta_H^2-2\gamma\theta_H\bar{y}_H+\gamma\bar{y}_H^2+\varepsilon_H^2-2\varepsilon_H\theta_H+2\varepsilon_H\bar{y}_H+\theta_H^2-2\theta_H\bar{y}_H+\bar{y}_H^2\right)}{2+2\gamma}
\end{aligned}$$

We can now reduce B:

$$B = \frac{1}{2}\left(\left[\frac{\gamma}{1+\gamma}(\eta\varepsilon_H + [1-\eta]\varepsilon_F)\right]^2 + \gamma\left[\theta_H - \frac{1}{1+\gamma}\varepsilon_H + \frac{(1-\eta)\gamma}{1+\gamma}(\varepsilon_F - \varepsilon_H) - \bar{y}_H\right]^2\right) \Leftrightarrow$$

$$= \frac{\gamma^2\left(\eta\varepsilon_H + (1-\eta)\,\varepsilon_F\right)^2}{2\,(1+\gamma)^2} + \frac{\gamma\left(\theta_H - \frac{\varepsilon_H}{1+\gamma} + \frac{(1-\eta)\gamma(\varepsilon_F-\varepsilon_H)}{1+\gamma} - \bar{y}_H\right)^2}{2} \Leftrightarrow$$

$$= \frac{\gamma^2\left(\eta\varepsilon_H + (1-\eta)\,\varepsilon_F\right)^2}{2\,(1+\gamma)^2} + \frac{\gamma\left(\frac{(1+\gamma)\theta_H}{1+\gamma} - \frac{\varepsilon_H}{1+\gamma} + \frac{(-(-1+\eta)\gamma(\varepsilon_F-\varepsilon_H))}{1+\gamma} + \frac{(1+\gamma)(-\bar{y}_H)}{1+\gamma}\right)^2}{2} \Leftrightarrow$$

$$= \frac{\gamma^2\left(\eta\varepsilon_H + (1-\eta)\,\varepsilon_F\right)^2}{2\,(1+\gamma)^2} + \frac{\gamma\frac{((1+\gamma)\theta_H - \varepsilon_H - (-1+\eta)\gamma(\varepsilon_F-\varepsilon_H) - (1+\gamma)\bar{y}_H)^2}{(1+\gamma)^2}}{2} \Leftrightarrow$$

$$= \frac{\gamma^2\left(\eta\varepsilon_H + (1-\eta)\,\varepsilon_F\right)^2}{2\,(1+\gamma)^2} + \frac{\gamma\left((1+\gamma)\,\theta_H - \varepsilon_H - (-1+\eta)\,\gamma\,(\varepsilon_F - \varepsilon_H) - (1+\gamma)\,\bar{y}_H\right)^2}{2\,(1+\gamma)^2} \Leftrightarrow$$

$$= \frac{\gamma^2\left((-1+\eta)\,\varepsilon_F - \eta\varepsilon_H\right)^2 + \gamma\left(\varepsilon_F\eta\gamma - \varepsilon_H\eta\gamma - \varepsilon_F\gamma + \gamma\varepsilon_H - \gamma\theta_H + \gamma\bar{y}_H + \varepsilon_H - \theta_H + \bar{y}_H\right)^2}{2\,(1+\gamma)^2} \Leftrightarrow$$

$$= \frac{\gamma}{2+2\gamma}\begin{pmatrix} \varepsilon_F^2\eta^2\gamma - 2\varepsilon_F\varepsilon_H\eta^2\gamma + \varepsilon_H^2\eta^2\gamma - 2\varepsilon_F^2\eta\gamma + 4\varepsilon_F\varepsilon_H\eta\gamma \\ -2\varepsilon_F\eta\gamma\theta_H + 2\varepsilon_F\eta\gamma\bar{y}_H - 2\varepsilon_H^2\eta\gamma + 2\varepsilon_H\eta\gamma\theta_H - 2\varepsilon_H\eta\gamma\bar{y}_H \\ +\varepsilon_F^2\gamma - 2\varepsilon_F\varepsilon_H\gamma + 2\varepsilon_F\gamma\theta_H - 2\varepsilon_F\gamma\bar{y}_H + \varepsilon_H^2\gamma - 2\varepsilon_H\gamma\theta_H + 2\varepsilon_H\gamma\bar{y}_H \\ +\gamma\,\theta_H^2 - 2\gamma\theta_H\bar{y}_H + \gamma\bar{y}_H^2 + \varepsilon_H^2 - 2\varepsilon_H\theta_H + 2\varepsilon_H\bar{y}_H + \theta_H^2 - 2\theta_H\bar{y}_H + \bar{y}_H^2 \end{pmatrix}$$

We can now solve the inequality:

$$
\begin{aligned}
& A \leq B \\
\frac{\gamma}{2+2\gamma}\begin{pmatrix} \gamma\theta_H^2 - 2\gamma\theta_H\bar{y}_H + \gamma\bar{y}_H^2 \\ +\varepsilon_H^2 - 2\varepsilon_H\theta_H + 2\varepsilon_H\bar{y}_H \\ +\theta_H^2 - 2\theta_H\bar{y}_H + \bar{y}_H^2 \end{pmatrix} &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} \varepsilon_F^2\eta^2\gamma - 2\varepsilon_F\varepsilon_H\eta^2\gamma + \varepsilon_H^2\eta^2\gamma \\ -2\varepsilon_F^2\eta\gamma + 4\varepsilon_F\varepsilon_H\eta\gamma + \gamma\theta_H^2 \\ -2\varepsilon_F\eta\gamma\theta_H + 2\varepsilon_F\eta\gamma\bar{y}_H + \bar{y}_H^2 \\ -2\varepsilon_H^2\eta\gamma + 2\varepsilon_H\eta\gamma\theta_H - 2\varepsilon_H\eta\gamma\bar{y}_H \\ +\varepsilon_F^2\gamma - 2\varepsilon_F\varepsilon_H\gamma + 2\varepsilon_F\gamma\theta_H \\ -2\varepsilon_F\gamma\bar{y}_H + \varepsilon_H^2\gamma - 2\varepsilon_H\gamma\theta_H + 2\varepsilon_H\gamma\bar{y}_H \\ -2\gamma\theta_H\bar{y}_H + \gamma\bar{y}_H^2 + \varepsilon_H^2 \\ -2\varepsilon_H\theta_H + 2\varepsilon_H\bar{y}_H + \theta_H^2 - 2\theta_H\bar{y}_H \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} \varepsilon_F^2\eta^2\gamma - 2\varepsilon_F\varepsilon_H\eta^2\gamma + \varepsilon_H^2\eta^2\gamma \\ -2\varepsilon_F^2\eta\gamma + 4\varepsilon_F\varepsilon_H\eta\gamma \\ -2\varepsilon_F\eta\gamma\theta_H + 2\varepsilon_F\eta\gamma\bar{y}_H - 2\varepsilon_H^2\eta\gamma \\ +2\varepsilon_H\eta\gamma\theta_H - 2\varepsilon_H\eta\gamma\bar{y}_H + \varepsilon_F^2\gamma \\ -2\varepsilon_F\varepsilon_H\gamma + 2\varepsilon_F\gamma\theta_H - 2\varepsilon_F\gamma\bar{y}_H \\ +\varepsilon_H^2\gamma - 2\varepsilon_H\gamma\theta_H + 2\varepsilon_H\gamma\bar{y}_H \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} \varepsilon_F^2\eta^2 - 2\varepsilon_F\varepsilon_H\eta^2 + \varepsilon_H^2\eta^2 \\ -2\varepsilon_F^2\eta + 4\varepsilon_F\varepsilon_H\eta \\ -2\varepsilon_F\eta\theta_H + 2\varepsilon_F\eta\bar{y}_H - 2\varepsilon_H^2\eta \\ +2\varepsilon_H\eta\theta_H - 2\varepsilon_H\eta\bar{y}_H + \varepsilon_F^2 \\ -2\varepsilon_F\varepsilon_H + 2\varepsilon_F\theta_H - 2\varepsilon_F\bar{y}_H \\ +\varepsilon_H^2 - 2\varepsilon_H\theta_H + 2\varepsilon_H\bar{y}_H \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} \eta^2(\varepsilon_H - \varepsilon_F)^2 + (\varepsilon_H - \varepsilon_F)^2 \\ -2(\varepsilon_F^2\eta + \varepsilon_H^2\eta - 2\varepsilon_F\varepsilon_H\eta) \\ +2(\varepsilon_H - \varepsilon_F)(\theta_H - \bar{y}_H)\eta \\ -2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F) \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} \eta^2(\varepsilon_H - \varepsilon_F)^2 - 2\eta(\varepsilon_H - \varepsilon_F)^2 \\ +2(\varepsilon_H - \varepsilon_F)(\theta_H - \bar{y}_H)\eta \\ +(\varepsilon_H - \varepsilon_F)^2 - 2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F) \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} (\eta^2 + 1 - 2\eta)(\varepsilon_H - \varepsilon_F)^2 \\ -(1-\eta)2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F) \end{pmatrix} \Leftrightarrow \\
0 &\leq \frac{\gamma}{2+2\gamma}\begin{pmatrix} (1-\eta)^2(\varepsilon_H - \varepsilon_F)^2 \\ -(1-\eta)2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F) \end{pmatrix} \Leftrightarrow \\
\frac{2(\theta_H - \bar{y}_H)}{\varepsilon_H - \varepsilon_F} &\leq 1 - \eta
\end{aligned}
\tag{C.17}
$$

Which is the condition that must be satisfied. This yields:

$$
\begin{Bmatrix} \varepsilon_H - \varepsilon_F \leq \frac{2(\theta_H - \bar{y}_H)}{1-\eta} & \text{for } \varepsilon_H < \varepsilon_F \\ \\ \varepsilon_H - \varepsilon_F \geq \frac{2(\theta_H - \bar{y}_H)}{1-\eta} & \text{for } \varepsilon_H > \varepsilon_F \end{Bmatrix}
$$

The inequality changes direction in the last line as we assume $\theta_H < \bar{y}_H$. As θ_H is a stochastic variable with mean 0 while $\bar{y}_H$ is a constant greater than 0 this assumption is not always correct. However, it seems reasonable to assume that the natural level of output growth does not exceed the output growth targets on a frequent basis. We will therefore here assume that this is a three-sigma event, thus having a probability of 0.13%. This edge case will therefore be ignored in our analysis.

From the former derivation, the net gain of deviating is equation (C.17) (loss in the monetary union - loss of single economy under commitment):

$$\Delta\mathcal{L}_{\mathcal{H}} = \frac{\gamma}{2+2\gamma}\big((1-\eta)^2(\varepsilon_H - \varepsilon_F)^2 - (1-\eta)2(\theta_H - \bar{y}_H)(\varepsilon_H - \varepsilon_F)\big)$$