Probabilistic Programming:

$\texttt{let}\ \ \alpha = \mathcal{N}(0, s_{large}^2)\ \texttt{in}$
$\texttt{let}\ \beta = \mathcal{N}(0, s_{large}^2)\ \texttt{in}$
$\texttt{let}\ \pi = \mathcal{N}(1, \lambda_{small})\ \texttt{in}$
$$\big((\underbrace{\alpha, \beta}_{\text{coefficients}}), \underbrace{[\ \texttt{for}\ z < \texttt{students} \rightarrow \alpha + x[z] \times \beta + \mathcal{N}(0, \frac{1}{\pi})]}_{\text{predicted values of the y variable}}\big)$$

A regression calculus $y \sim r$:

r::=
- $D(v_1, \ldots, v_n)$
- $v\{\alpha \sim r\}$
- $r + r'$
- $r \mid r'$
- $(v\alpha)r$

u, v ::=
- s
- x
- $u : v$

Pure Intercept:

$\mathbf{y} \sim 1\{\alpha\} \sim \mathcal{N}(0, s_{large}^2)$
$\texttt{let}\ \alpha = \mathcal{N}(0, s_{large}^2)\ \texttt{in}$
$\big(\alpha, [\ \texttt{for}\ z < \texttt{students} \rightarrow 1 \times \alpha]\big)$

$v\{\alpha\} \triangleq v\{\alpha \sim \mathcal{N}(0, s_{large}^2)\}$
$v \triangleq (v\alpha)v\{\alpha\}$ **for** $\alpha \notin fv(v)$

Pure Noise:

$\mathbf{y} \sim ?$
$\texttt{let}\ \pi = \mathcal{N}(1, \lambda_{small})\ \texttt{in}$
$\big((), [\ \texttt{for}\ z < \texttt{students} \rightarrow \mathcal{N}(0, \frac{1}{\pi})]\big)$

Intercept with Noise:

$\mathtt{y} \sim 1\{\alpha\}+?$
$\mathtt{let}\ \alpha = \mathcal{N}(0, \lambda_{small})\ \mathtt{in}$
$\mathtt{let}\ \pi = \mathcal{N}(1, \lambda_{small})\ \mathtt{in}$
$\big((\alpha), [\ \mathtt{for}\ z < \mathtt{students} \rightarrow \alpha + \mathcal{N}(0, \frac{1}{\pi})]\big)$

Slope and intercept:

$\mathtt{y} \sim 1\{\alpha\} + x\{\beta\}+?$
$\mathtt{let}\ \alpha = \mathcal{N}(0,^2_{large})\ \mathtt{in}$
$\mathtt{let}\ \beta = \mathcal{N}(0,^2_{large})\ \mathtt{in}$
$\mathtt{let}\ \pi = \mathcal{N}(1, \lambda_{small})\ \mathtt{in}$
$\big((\alpha), [\ \mathtt{for}\ z < \mathtt{students} \rightarrow \alpha + xz \times \beta + \mathcal{N}(0, \frac{1}{\pi})]\big)$

Intercept per School:

$\mathtt{y} \sim (1\{\alpha\} \mid s)$
$\mathtt{let}\ \alpha = [\mathtt{for}\ z < \mathtt{schools} \rightarrow \mathcal{N}(0, s^2_{large})]\ \mathtt{in}$
$\mathtt{let}\ \pi = \mathcal{N}(1, \lambda_{small})\ \mathtt{in}$
$\big((\alpha), [\ \mathtt{for}\ z < \mathtt{students} \rightarrow \alpha[s[z]] + \mathcal{N}(0, \frac{1}{\pi})]\big)$