Computational Physics - Project 1

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1 Introduction to Project 1

In physics, we often have to deal with differential equations o second order, which can be generally written in the from

$$\frac{d^2y}{dx12} + k^2(x)y = f(x) \quad , \tag{1}$$

where we call f the inhomgeneous term and $k^2(x)$ is a real function. A special case of these cases is Poisson's equation, which reads in the one-dimensional, spherical case

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -4\pi\rho(\vec{r}) \quad . \tag{2}$$

Doing some substitutions, we can write this in the following, more general form:

$$-u''(x) = f(x) \tag{3}$$

In this project, we try to solve eq.(3) numerical. Therefore, we have to discretize f and u''. We approximate $u(x)$ as v_i, using a grid of n gridpoints $x_i = i \cdot h$. h is our

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & & -1 & 2 & -1 \\ 0 & \dots & & 0 & -1 & 2 \end{pmatrix} \tag{4}$$

$$\mathbf{A} = \begin{pmatrix} b_1 & c_1 & 0 & \dots & \dots & \dots \\ a_2 & b_2 & c_2 & \dots & \dots & \dots \\ & a_3 & b_3 & c_3 & \dots & \dots \\ & \dots & \dots & \dots & \dots & \dots \\ & & & a_{n-2} & b_{n-1} & c_{n-1} \\ & & & & a_n & b_n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ \dots \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \dots \\ \dots \\ \dots \\ \tilde{b}_n \end{pmatrix}. \tag{5}$$

2 Rewrite the set of equations in matrix-form