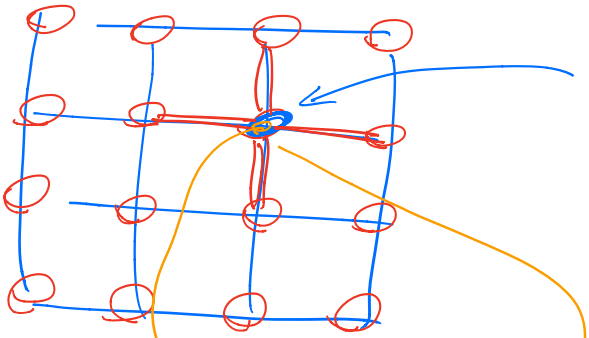
Lecture November 6

- one spin flip at each "time" in order to compute $\Delta E = E_i' - E_j'$

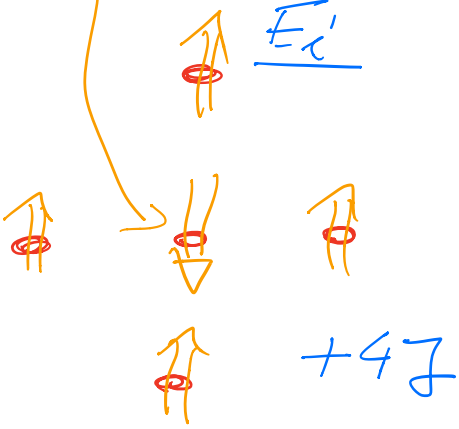


random pick of spin

E_j'

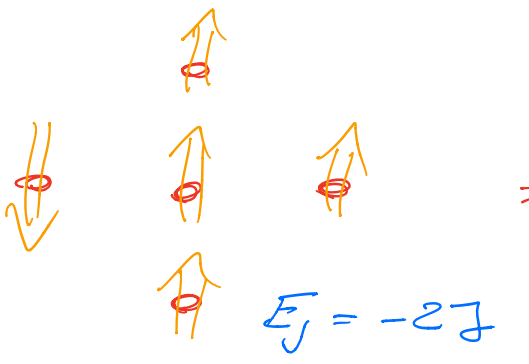
$-4J$

E_i'

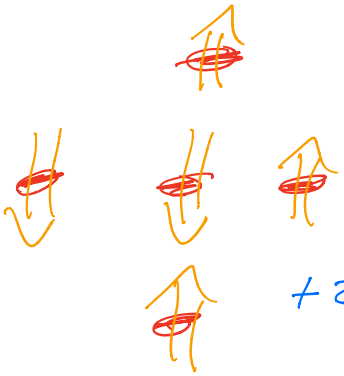


$+4J$

$E_i' - E_j' = 8J$

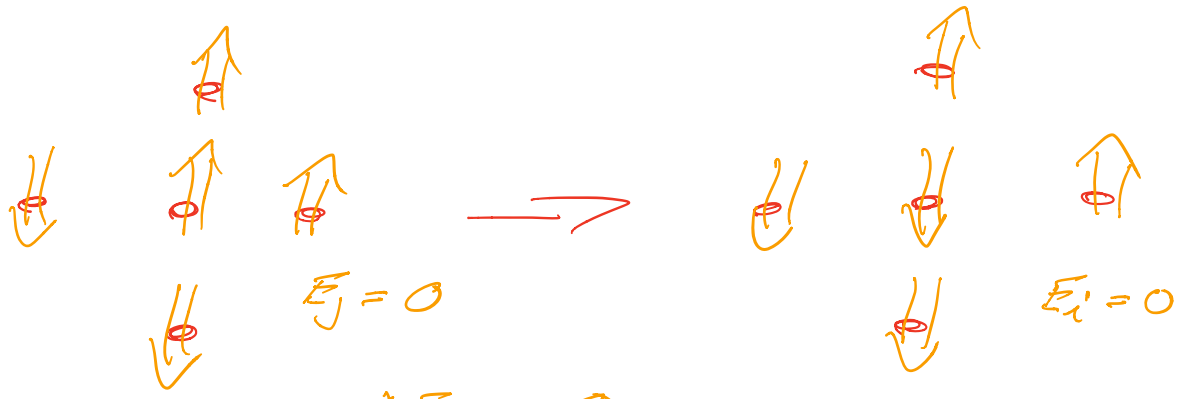


$E_j = -2J$

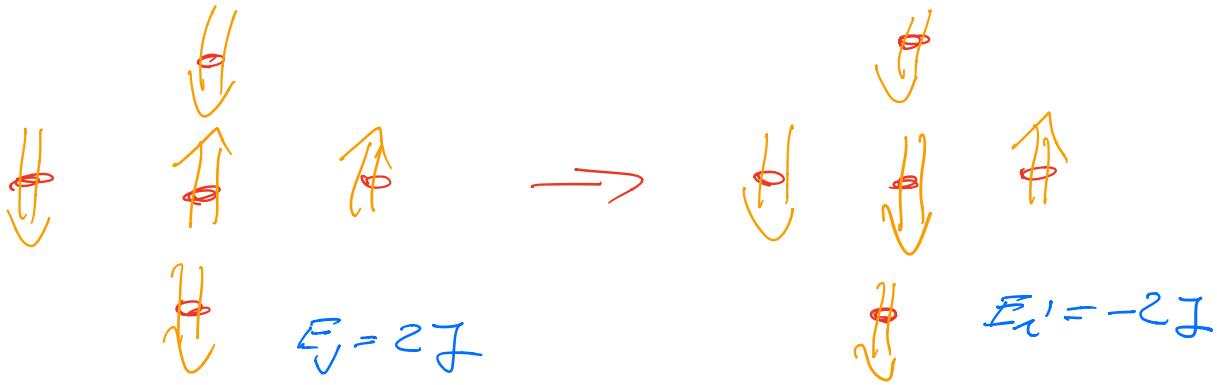


$+2J$

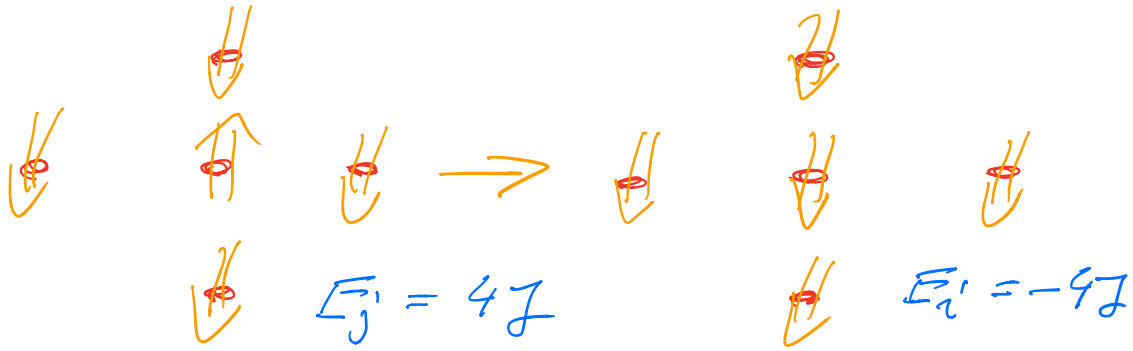
$\Delta E = 4J$



$\Delta E = 0$



$\Delta E = -4J$



$\Delta E = -8J \quad \Rightarrow$

$$\Delta E = \{-8J, -4J, 0, +4J, +8J\}$$

For every temp, precalculate

$$w = e^{-\beta \Delta E} = e^{-\Delta E / k_B T}$$

$$k_B = 1$$

In the sweep through the lattice, flipping one spin at the time gives a simple way to compute ΔE ?

$$\Delta E = E_2 - E_1 = -J \sum_{\langle kl \rangle} s_k^2 s_l^2 + J \sum_{\langle kl \rangle} s_k^1 s_l^1$$

s_k = surrounding spins

$$\Delta E = -J \sum_{\langle kl \rangle} s_k^{\mathbb{C}} \left(s_l^2 - s_l^1 \right)$$

($s_l^2 = -1$, $s_l^1 = 1$)

$$s_l^1 = \pm 1 \qquad s_l^2 = \pm 1$$

if $s_l^1 = +1 \Rightarrow s_l^2 = -1$

if $s_l^1 = -1 \Rightarrow s_l^2 = +1$

$$\boxed{\Delta E = 2J \, s_l^1 \sum s_k} \Longleftarrow$$

<k>

update of E:

$$E_i = E_j + \Delta E$$

update of M

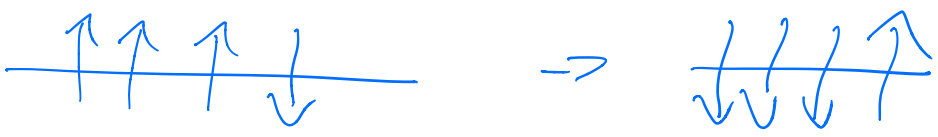
$$M_i = M_j + \Delta M$$

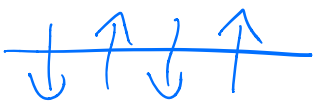
$$\Delta M = 2 \cdot s_i$$

with ΔE, you need to look up the table of $\exp(-\Delta E / k_B T)$ and then perform Metropolis Test.

- Sweep through the whole lattice, flip spins randomly, New configuration:

1 dim 4 spins

E_j 

E_i 

Now calculate $E_i - E_j'$

Need to loop over all spins and compute E_i'

Additional $O(N^2)$ FLOPs

$E_i - E_j' \neq \{-8J, -4J, 0, 4J, 8J\}$

could happen

$e^{-\Delta E / k_B T}$

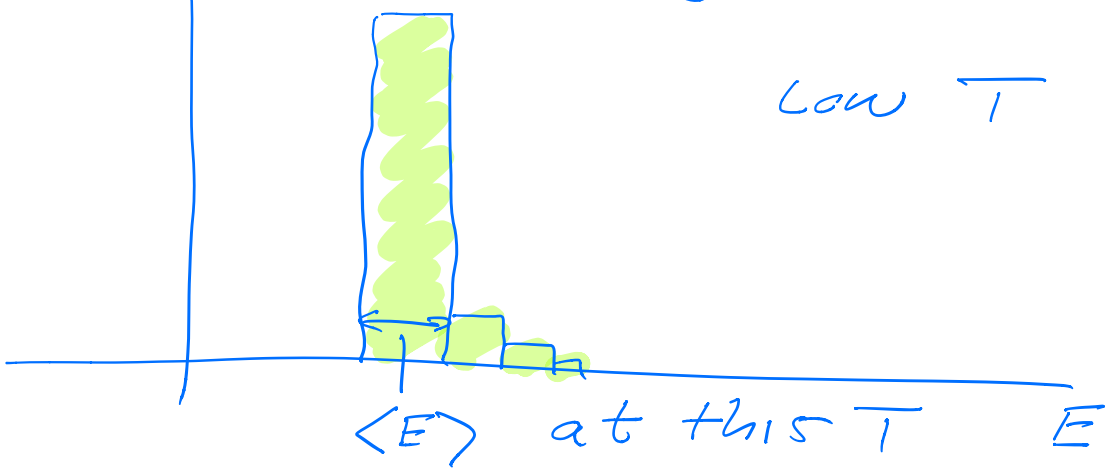
= Interpretations =

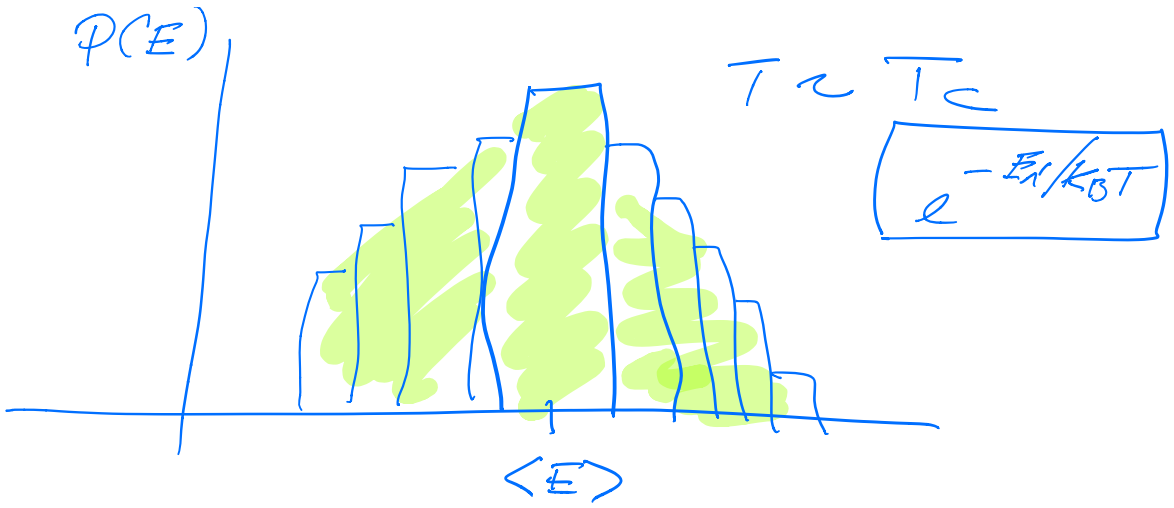
T small $T < T_c = 2.609$

$e^{-E_i'/k_B T}$ ~ most likely state with ordered spins

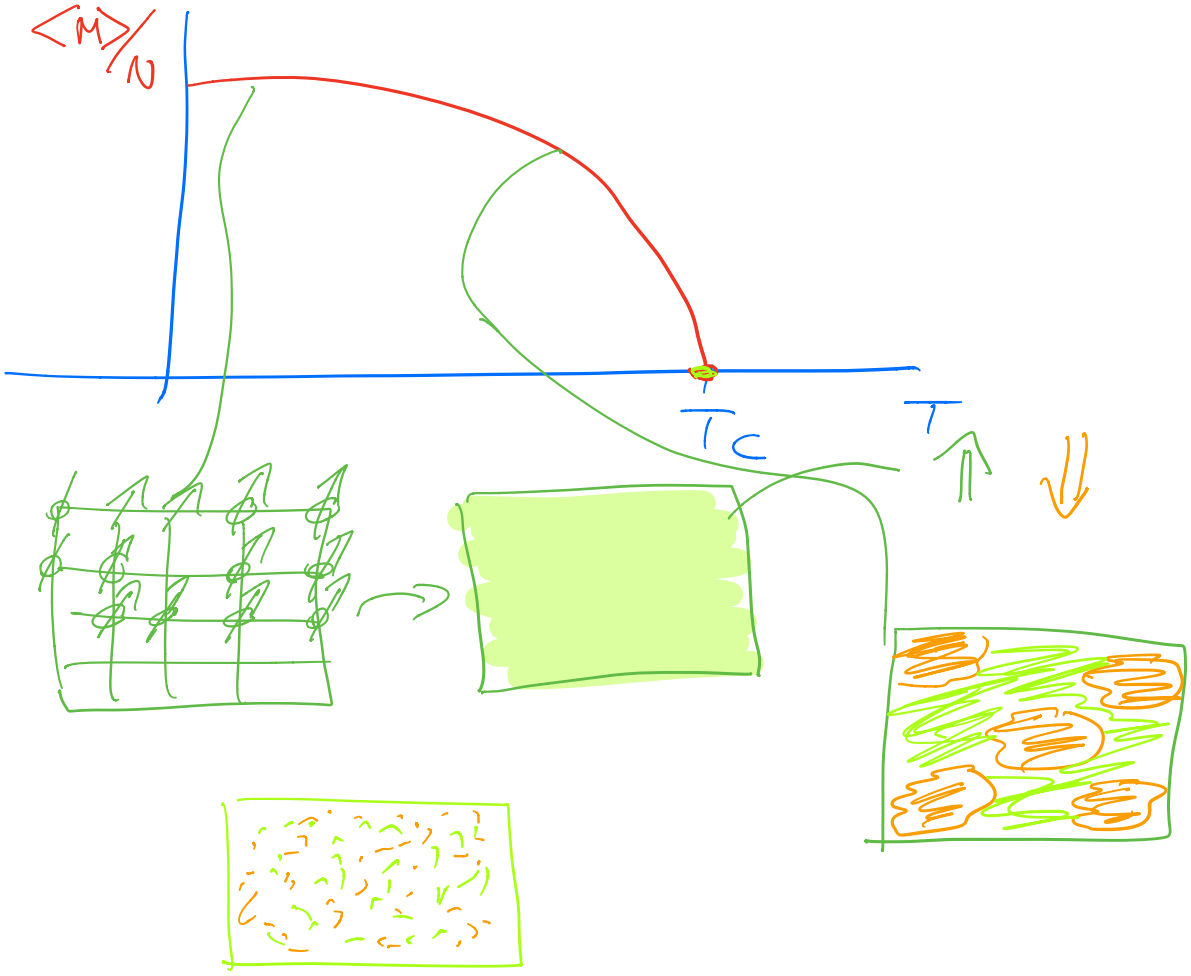
P(E) = # times a given energy appears

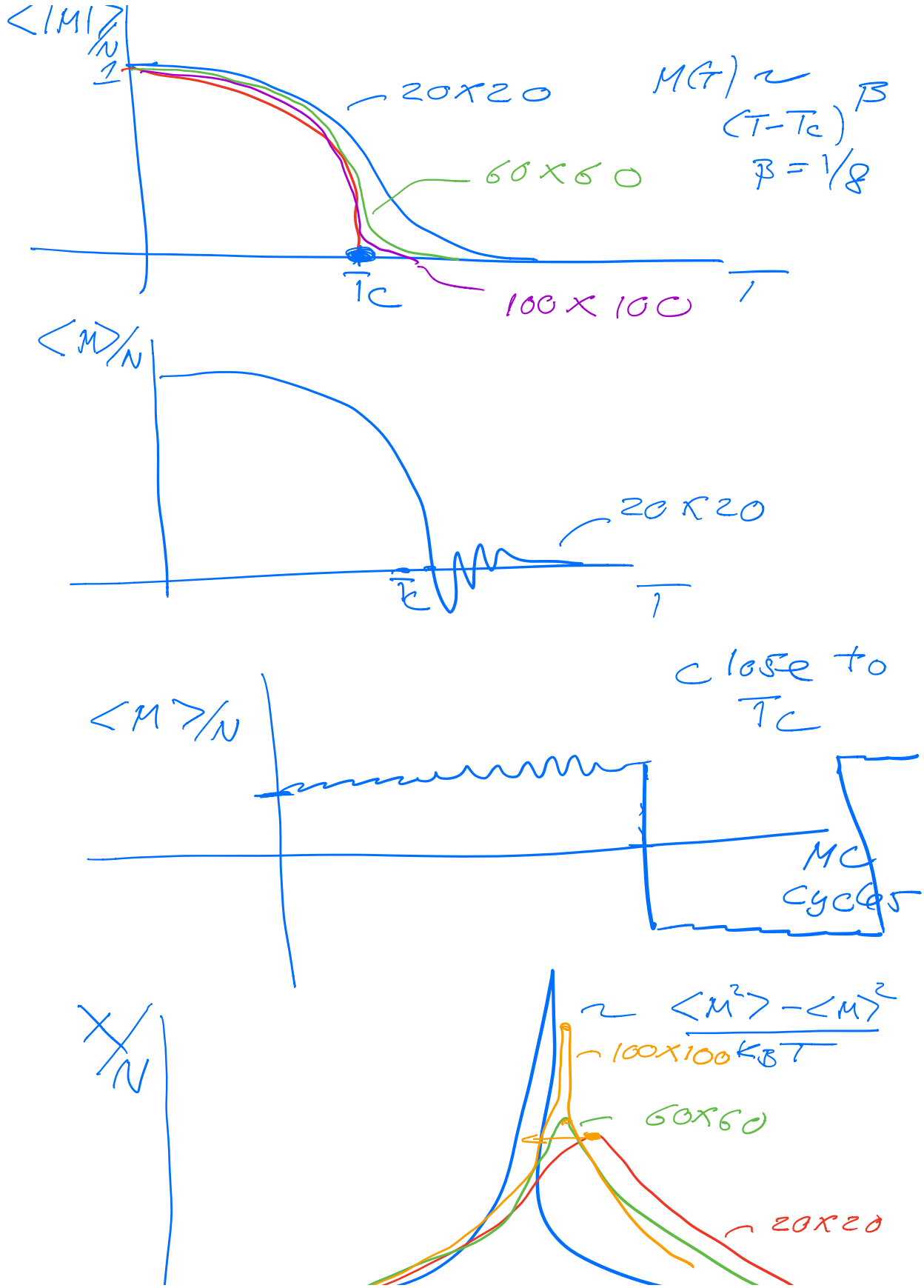
low T

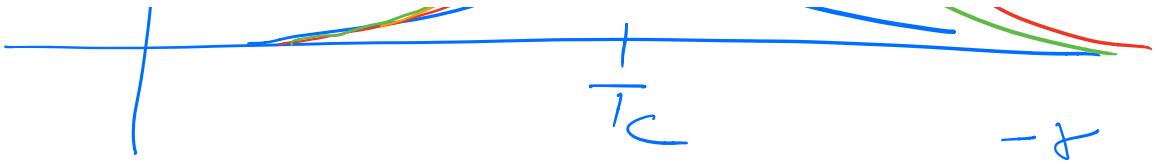




$E[M] = \langle M \rangle$ or $\langle |M| \rangle$




$\langle |M| \rangle / N$
1
20x20
60x60
100x100
T_c
T
$M(T) \sim (T - T_c)^{\beta}$
$\beta = 1/8$
$\langle M \rangle / N$
20x20
T_c
T
close to T_c
$\langle M \rangle / N$
MC cycles
χ / N
$\sim \frac{\langle M^2 \rangle - \langle M \rangle^2}{100 \times 100 K_B T}$
60x60
20x20



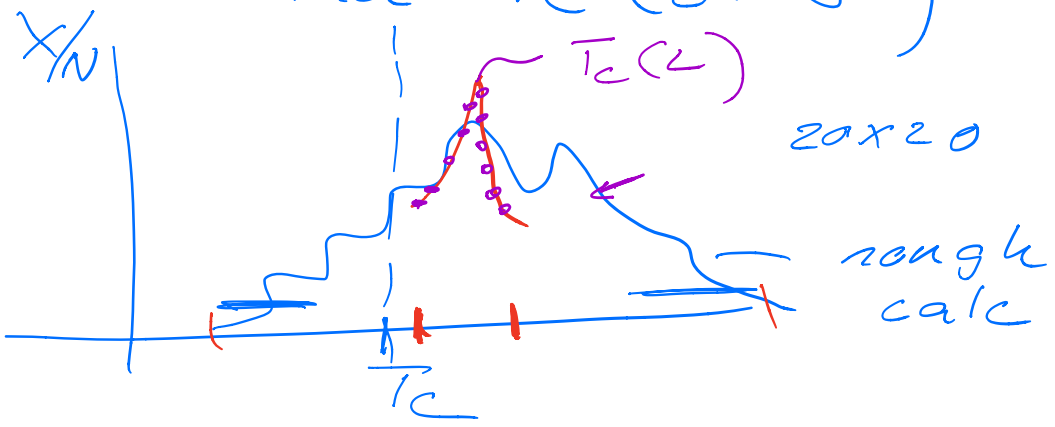
$\chi \sim (T_c - T)^{-\gamma}$

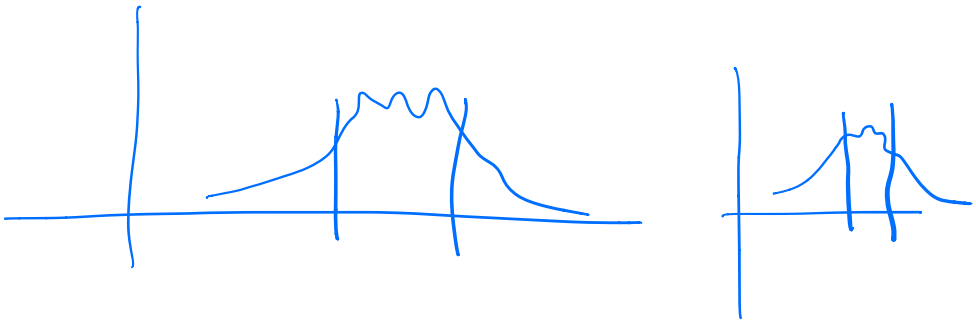
$\gamma = -7/4$

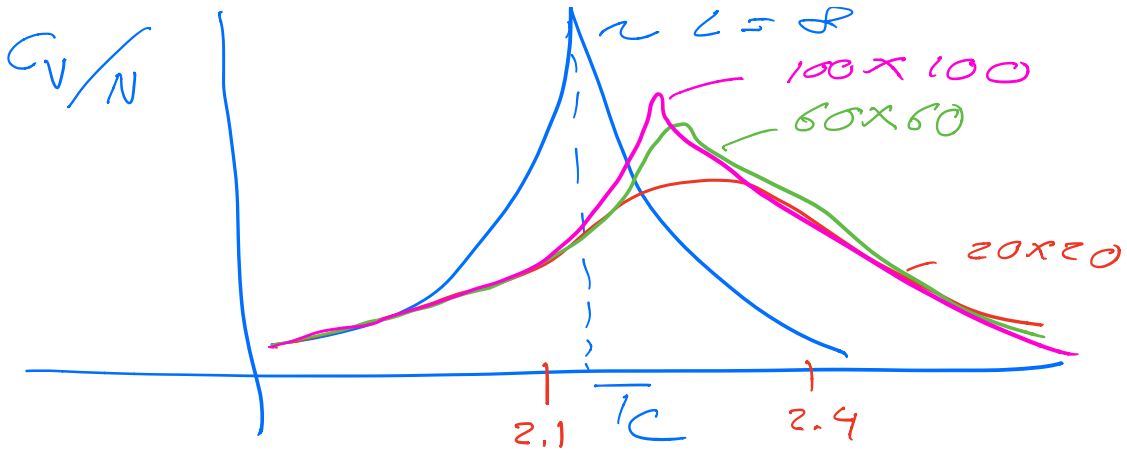
$T_c(L) - T_c(L=\infty) = aL^{-1/\nu}$

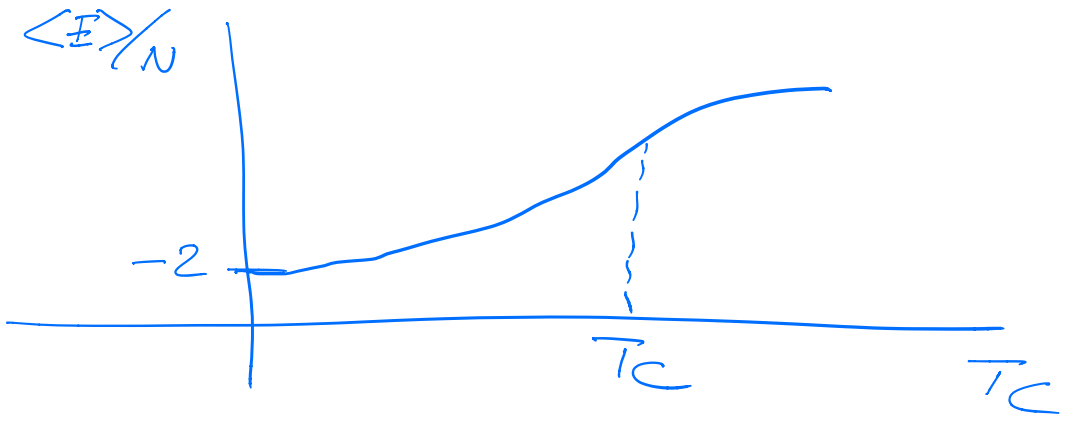
$\nu = \underline{1}$ in Ising class.

can use $\chi(L)$, find the top for different L values and use this to estimate a and $T_c(L=\infty)$






C_V/N
$L = \infty$
100 x 100
60 x 60
20 x 20
2.1
T_c
2.4


$\langle E \rangle / N$
-2
T_c
T

C_V : $T_c(L)$