Lecture october 9

Numerical integration
(lecture notes chap 5)

- Equal step methods
(Newton-Cotes)

$$I = \int_a^b f(x)\,dx \simeq \sum_{i=1}^{n} w_i f(x_i)$$

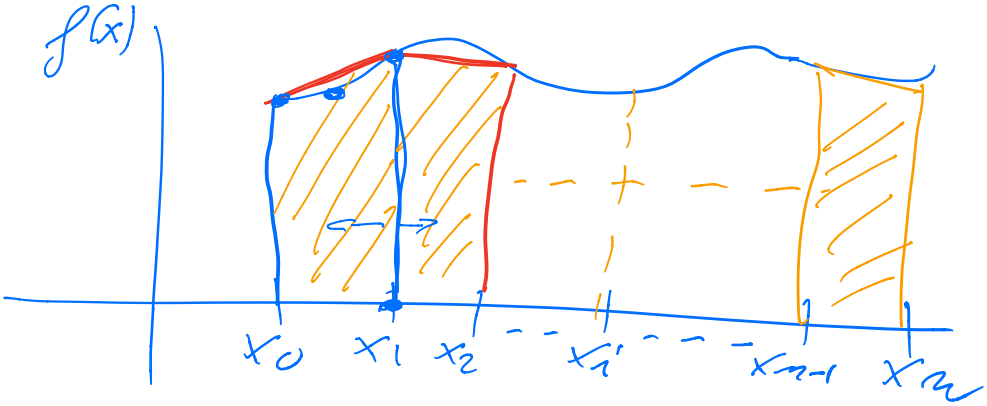
$x \to x_i = x_0 + i \cdot h \quad i = 0, 1, 2, \ldots n$

$x \in [x_0, x_n]$

x_i = integration/grid/mesh points

w_i = integration weights

Trapezoidal rule



$$I = h\left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \cdots + f(x_{m-1}) + \frac{f(x_m)}{2}\right) = \sum w_i f(x_i)$$

$$w = \left\{\frac{h}{2}, h, h, \cdots, h, \frac{h}{2}\right\}$$

Simpson's rule (RK4)

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots 4f(x_{m-1}) + f(x_m)\right]$$

$$w = \left\{\frac{h}{3}, \frac{4}{3}h, \frac{2}{3}h \cdots \frac{h}{3}\right\}$$

Gaussian Quadrature;

- with n - points, we can fit a polynomial of degree $P_{2n-1}(x)$ instead of $P_{n-1}(x)$.
- orthogonal polynomials