



MACQUARIE  
University

# Forecasting Seasonal Time Series

Winters Exponential Smoothing





# Seasonality

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Previous smoothing models were appropriate for time series that were **horizontal** or had **trend** but not appropriate for time series with **seasonal** components

Time series may exhibit a seasonal component due to **weather, holiday periods, weekends** etc.

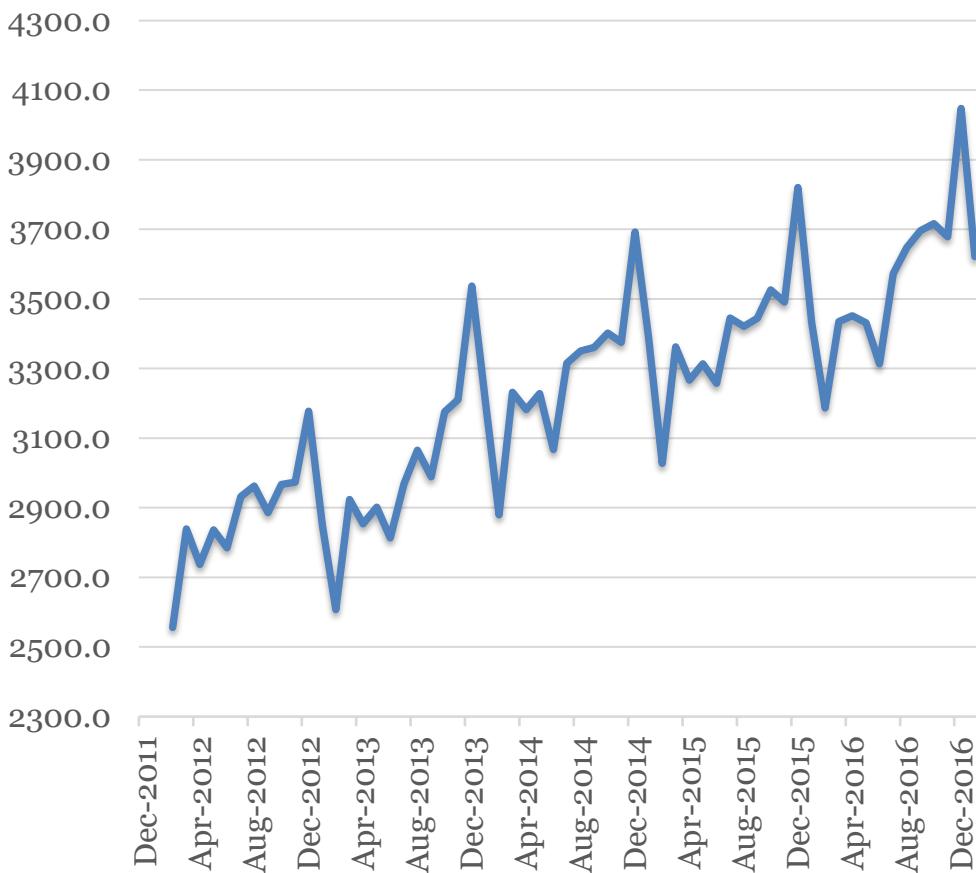
Seasonal components typically lead to **systematic fluctuations** of the level of the time series

The fluctuation pattern is typically **repeated for every seasonal cycle**



# Seasonal Data Example

## Cafe+Rest+TakeAway - Australia (\$m)



Monthly Café, Restaurant and  
Takeaway Sales (\$m) for Aust.  
Feb 12 – Jan 17

Seasonal fluctuation spiking  
in **December** each year

Due to **weather, holidays,  
Christmas**

Also **trend** and **random**  
component

Trend due to **inflation,  
population, market size**



# Broad Types of Seasonality

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Seasonality can be classified into two broad categories;  
Additive and Multiplicative

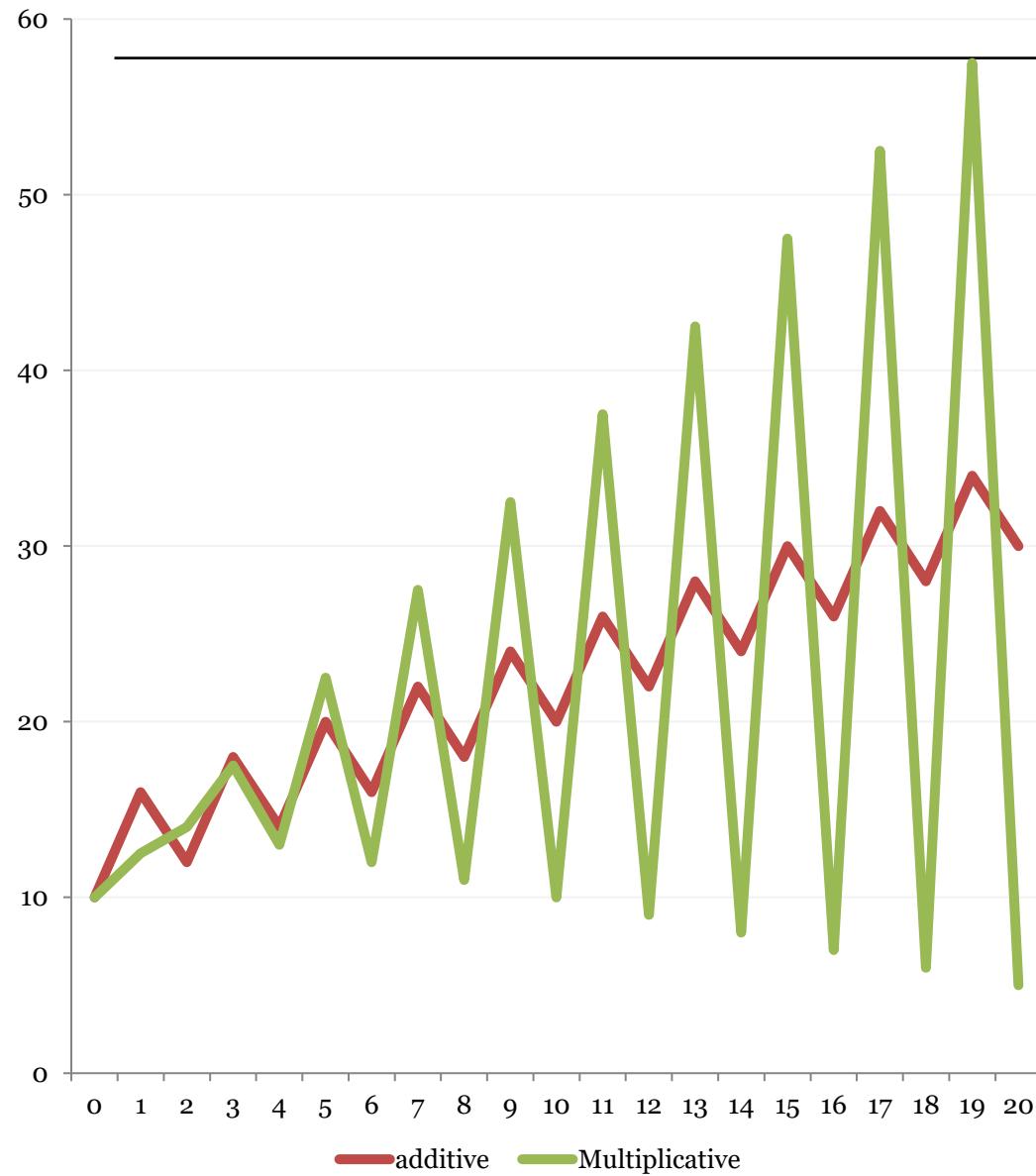
Additive is when seasonal fluctuations of the time series can be modelled by addition of a defined seasonal component

Multiplicative is when seasonal fluctuation of time series can be modelled by multiplication of a defined seasonal component

In Additive models, seasonal component size is absolute

In Multiplicative models, seasonal component is relative to the level of the time series

# Additive Vs Multiplicative Seasonal Variation



**Additive** → Seasonal changes are fixed over the time series

The magnitude of seasonal fluctuation does not vary with the level of the series)

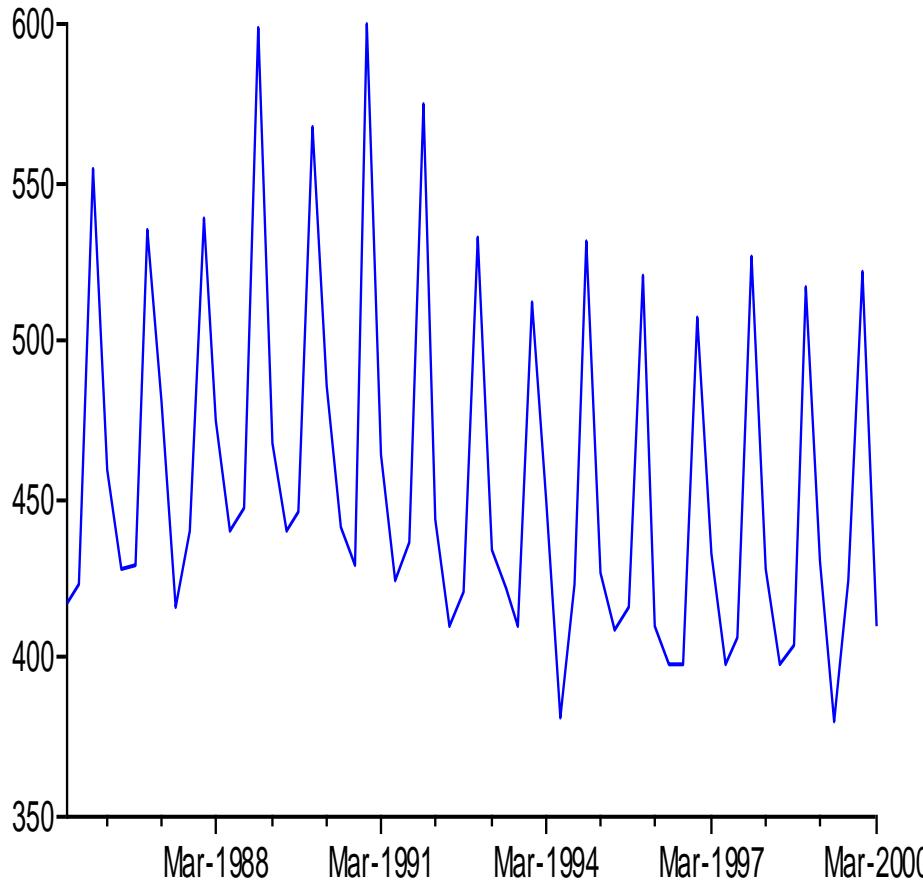
**Multiplicative** → Seasonal changes are a fixed percentage of the time series

The magnitude of seasonal fluctuation varies with the level of the time series



# Additive Seasonality

Original: Production: Beer (incl ale & stout, excl beer < 1.15% alcohol)  
Megalitres



Quarterly Beer Production Aust.  
(megalitres)

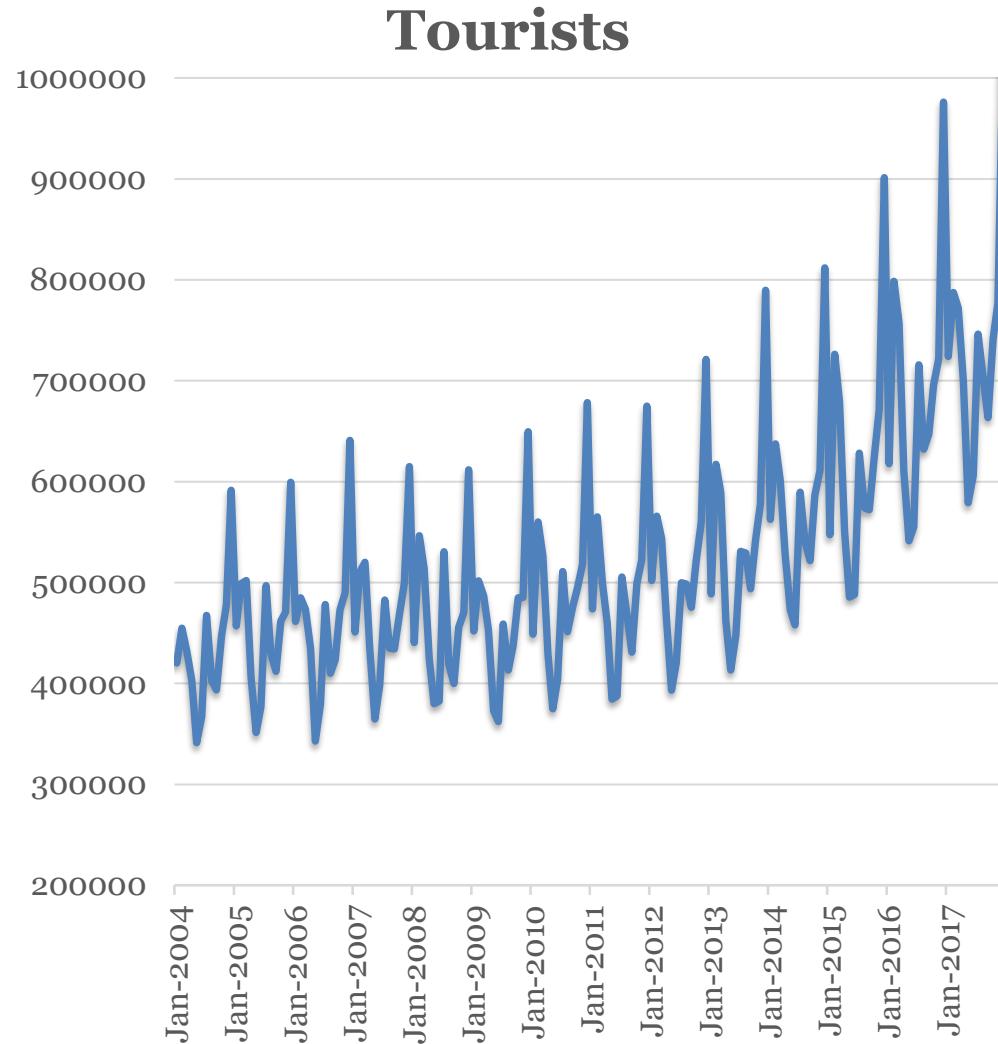
Data appears seasonal with  
some trend (downward) and  
possibly cycle

Due to **Christmas**, **weather**,  
**consumer taste** changes

Seasonal fluctuation appears  
constant and not dependent  
on level of time series

Peaks & troughs **contained**  
**within a parallel band**

# Multiplicative Seasonality



Monthly visitor arrivals  
Australia-number

Data appears seasonal with  
trend (upward) and cycle

Due to: **Christmas**, weather,  
**globalisation**, tourism  
marketing, **Olympics**

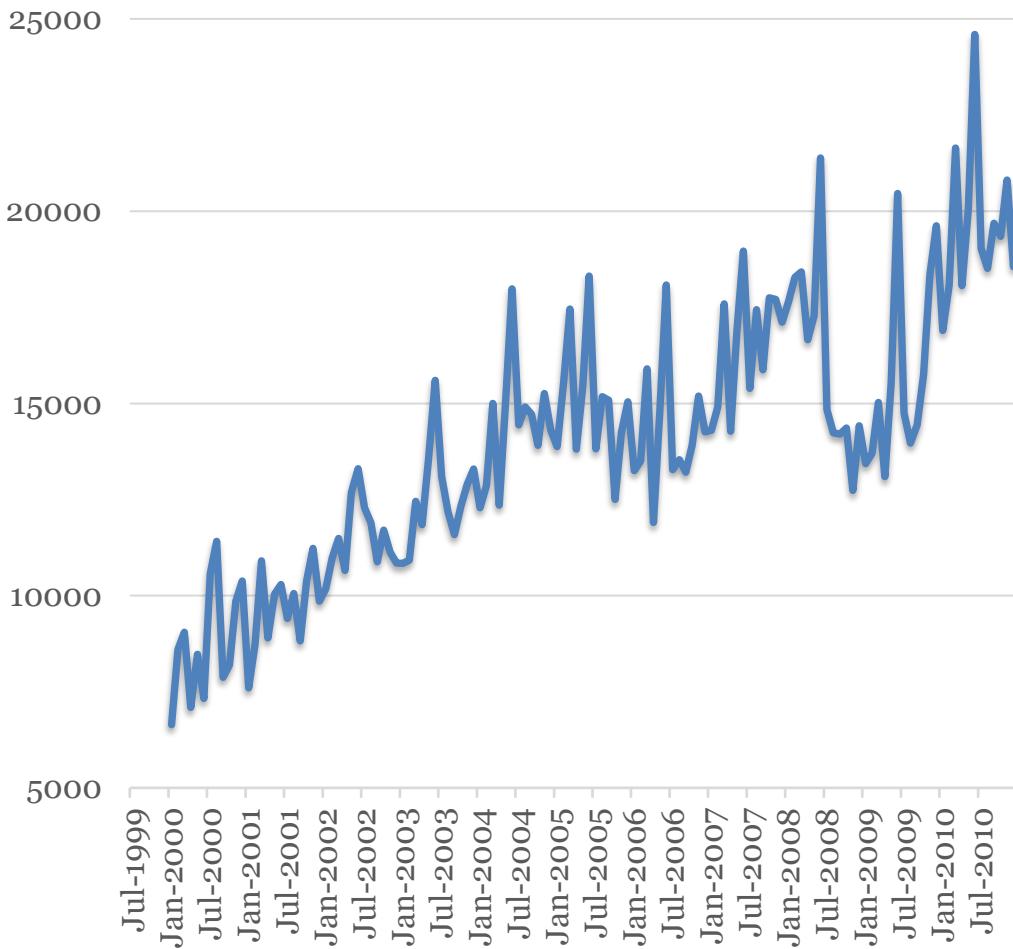
Seasonal fluctuation appears  
relative and dependent on  
level of time series (**but**  
**constant %**)

Peaks, troughs not contained  
within a parallel band

# Identifying Seasonality



SUV



It should be reasonably clear from a **time series graph** if seasonality is present

However it isn't always obvious eg. monthly SUV Sales (Jan 00 – Dec - 10)

Check the **ACF** and **PACF** for evidence of seasonality (**spikes** at **seasonal** values)

The time series may have to be **detrended** first for the seasonal spikes to appear clearly on **ACF** and **PACF**



# Seasonal Models

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If the time series has **seasonal components**, none of the previous models studied so far will be adequate

We will need to include a seasonal component in our models or adjust for seasonality when forecasting

An **extension of** Holt's method can accommodate seasonal effects

This model is the **Winters Exponential Smoothing** model

WES includes a **seasonality equation** and is a 3-parameter model with smoothing constants for **level:  $\alpha$**  (alpha), **trend:  $\beta$**  (beta) and one for **seasonality:  $\gamma$**  (gamma)



# WES (cont)

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In both models, the first equation adjusts the actual value of the time series by a seasonal estimate

The second equation is a trend equation (as per Holt's)

The third equation provides an updated estimate of the seasonal estimate at each time period

The forecast equation allows for prediction “**m**” **periods** into the future.

The first part of the equation is a trend projection “m” periods ahead. The result is adjusted by an estimate of seasonality for that specific season (added or multiplied)



# More on the WES Equations

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The values of  $\alpha, \beta, \gamma$  are all theoretically between 0 and 1 inclusive.

Choosing the values of the smoothing parameters is not easy. As a default, low values of  $\alpha, \beta, \gamma$  (**0.2, 0.2, 0.2**) are used as a preliminary estimate.

The model also needs initialization values for  $L_1, T_1$  and  $S_1 - S_p$ . Can use  $S_1 - S_p = \underline{0}$  (for Additive) or  $\underline{1}$  (for Multiplicative)

Changes to the smoothing parameters may **reduce error levels** and **improve accuracy** (usually based on an error criterion like MSE)

As with HES and SES, **SOLVER** can be used to derive the **“optimum combination”** of the smoothing parameters