Business Forecasting

Combining Forecasts

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In theory and in practice **significant improvements can be made in forecasting accuracy** by **combining statistical and judgmental forecasts**

Models involve judgment to some degree (choice of alpha, choice of variables etc.)

Utilise the **best aspects of a statistical predictions** while **exploiting the value of knowledge and judgmental information**

Capitalises on the **experience** of top and other mangers and can **reduce judgmental bias**

Theory of Combining Forecasts

If Y_1 and Y_2 are any two **unbiased** forecasts of Y with forecast errors e_1 and e_2 such that

$$COV(e_1,e_2) \neq VAR(e_1) \text{ and } COV(e_1,e_2) \neq VAR(e_2)$$

The composite forecast

$$Y_c = k^*Y_1 + (1-k)^*Y_2$$

has the following properties:

- **It is an unbiased forecast of Y;**

- **There exists a constant k* so that**
  $Var(Y_c) < \min[VAR(Y_1), VAR(Y_2)]$

There is a composite forecast model which generates more efficient forecasts than either of the two originating models

More on the Theory

The theorem holds for any **unbiased forecasting technique**

Thus, even if you have 'the correct' model it will generally be possible to generate **more efficient forecasts by combining** it with another unbiased model

Note that the theorem states that if you take a **good model and combine it with an inferior model** you might get an **even better model**

The 'more different' the two forecasting models are, the **greater will be the efficiency gains of combining the forecasts**

What is the value of “k*”?

The theorem states that there exists a value k* which does the job, **it does not say that there is *only one* value which works**

In general, there will be a **range of values for k*** which will work

The theorem only tells us that a value k* exists, **it does not tell us what that value is**

Thus, the big remaining question is **‘how do we find a value for k (k*)** such that the composite forecast gives us an improvement in forecasting performance

Choosing k*

Combining forecasts will only improve efficiency in some cases

Only particular values of k will improve forecast efficiency

Typically, the first thought is to **average two forecasts** to produce a combined forecast

This may or may not improve efficiency **Averaging is identical to assuming $k^*=0.5$**

This value may not be optimal as it gives equal weight to both forecasts **which may be sub-optimal given one of the forecasts may be poor and the other good.**

Alternative choices of k*

Instead of giving both forecasts equal weight you could try and **weight the forecasts in the combination according to performance**

It would make sense to **weight the better performing forecast higher than the poorer performing forecast**

The weighting to use can be ascribed using various methods

One approach is to use the inverse of the MSE for both forecasts (standardised to create weights)

The forecast with the **lower MSE** would be given a **higher weight**

Inverse MSE Based Weights

Consider two forecast methods **Y_1 and Y_2** with MSE's (MSE_1 and MSE_2) calculated over some test set of data

We could determine the weighting for k* by using the formula

$$k^* = (MSE_1)^{-1} / \{(MSE_1)^{-1} + (MSE_2)^{-1}\}$$

The formula ensures that k* will be **between 0 and 1 inclusive**

Once k* (the weight for Y_1) is determined, weight for Y_2 is (1-k*)

k* can then be applied when forecasting (given Y_{1F}, Y_{2F})

$$Y_F = k^* Y_{1F} + (1-k)^* Y_{2F}$$

Example of Inverse MSE weights

Suppose we have two **unbiased forecasts** of a target variable and over a **test set of 20 observations** we have determined the MSE of both methods as **$MSE_1 = 20$ and $MSE_2 = 50$**

The weights [k* and (1-k*)] for combining forecasts (based on inverse MSE) are

$$k^* = (MSE_1)^{-1} / \{(MSE_1)^{-1} + (MSE_2)^{-1}\}$$
$$= (1/20) / \{(1/20) + (1/50)\}$$
$$= 0.05/\{0.05+0.02\}$$
$$= 0.05/0.07 = 0.714 \qquad [(1-k) = 0.286]$$

Thus the combined Forecast for period (t+1) is for $Y_{c,t+1} = 0.714 * Y_{1,t+1} + 0.286 * Y_{2,t+1}$