TOOLBOX

HYPOTHESIS TESTING FOR ONE SAMPLE TESTS

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Hypothesis Testing

Defined:

Hypothesis testing is a statistical method that uses sample data to evaluate a hypothesis about a population.

▶ Hypothesis testing therefore allows to use sample data to draw inferences about the population.

▶ The drawn conclusions can always be wrong, but with hypothesis testing, we can put a probability on whether the conclusion is correct and can make a decision beyond reasonable doubt.

Procedure of Hypothesis Testing

Step 1
State a hypothesis about a population

State a Hypothesis

We state two opposing hypothesis:

Null Hypothesis

*H*0 states that there **is no** change, no effect, no difference - nothing happened, hence the name null.

Alternative Hypothesis

H1 states that there is a change, a difference, or a relationship for the general population.

In symbols, we write: $H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

$\mu 0$ is just some number e.g. 3.34

Step 2
Use the hypothesis to predict the distribution of sample means – Z-test or t-test?

Type Tests (Z or t)

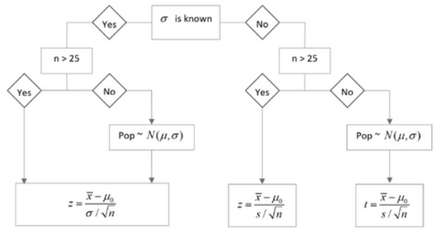
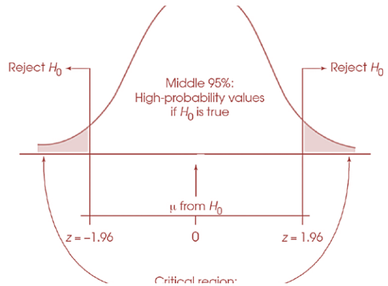


Figure: Test statistics for different cases of one sample tests

Step 3
Specify which sample means are consistent with the null hypothesis and which are not (specify critical region)

Determine the Critical Region



Step 4
Calculate the Test Statistic

Use sample data to compute the sample mean

▶ Compare sample mean with null hypothesis by using z-score

▶ Z-score describes where the sample mean is located relative to the hypothesized population mean from the null hypothesis.

▶ The Z-score formula for a sample mean is:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}},$$

where $\bar{x}$ is the sample mean,
μ is the population mean under the null hypothesis,
and σ is the population standard deviation.

Step 5
Compare and Conclude

Compare the Test Statistic to the Critical and make a conclusion.

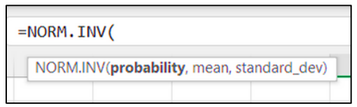
Compare the Test Statistic to the Critical and Make a Conclusion.

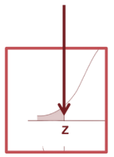
Use the Z-score to make a decision.

▶ If the Z-score is located in the critical region, reject the null hypothesis.
▶ If the Z-score does not lie in the critical region, then the data do not provide strong evidence that the null hypothesis is wrong.
▶ Sample mean is reasonably close to the population mean specified in the null hypothesis. Our conclusion is not to reject the null hypothesis.

NORM.INV

Returns the critical value from the normal distribution.





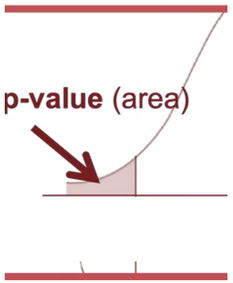
P-value

Instead of using critical values,
we can use a p-value approach.

▶ The p-value is the probability of obtaining test results at least as extreme as the observed result in the sample, when the null hypothesis is correct.

▶ p-value equal to the alpha level when the critical values are set equal to the value of the test statistic.

▶ We reject *H*0 when p-value is lower than the alpha level and do not reject H0 when p-value is higher than the alpha level.



NORM.DIST

Returns the p-value from the normal distribution.