



# REGRESSION



The focus this week is on **regression**, which is a statistical method used to model and analyse the relationships between a *dependent* variable and one or more *independent* variables

## Correlation

Measures the strength & direction of the relationship between two variables.

**Pearson correlation ( $r$ ):** for sample data.

**Population correlation ( $\rho$ ):** for entire population.

Values range from -1 to +1, with values closer to  $\pm 1$  indicating a stronger relationship.

## Regression Model

$$\hat{y} = b_0 + b_1 x$$

Intercept ( $b_0$ ):

- Value of  $y$ , when  $x = 0$
- Where the line crosses the  $y$  axis.

slope ( $b_1$ ):

- Defines how steep the line is
- Which direction the line goes (positive/negative).

## Sources of Variation

**R<sup>2</sup>**

**Definition:** R<sup>2</sup> otherwise known as R-squared represents the proportion of variability in the outcome variable that is explained by the regression model.

## Assumptions of Linear Regression

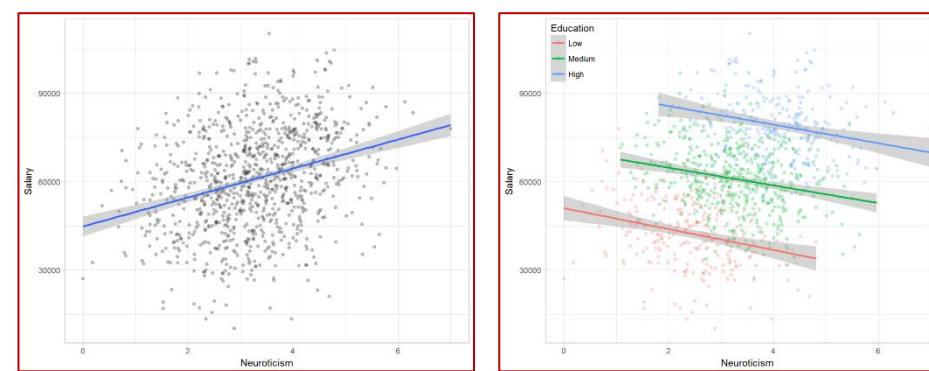


**Linearity** of the relationship between  $x$  and  $y$ .

**Independence** of residuals (errors).

**Normality** of residuals.

**Equality** of variance of residuals (homoscedasticity)

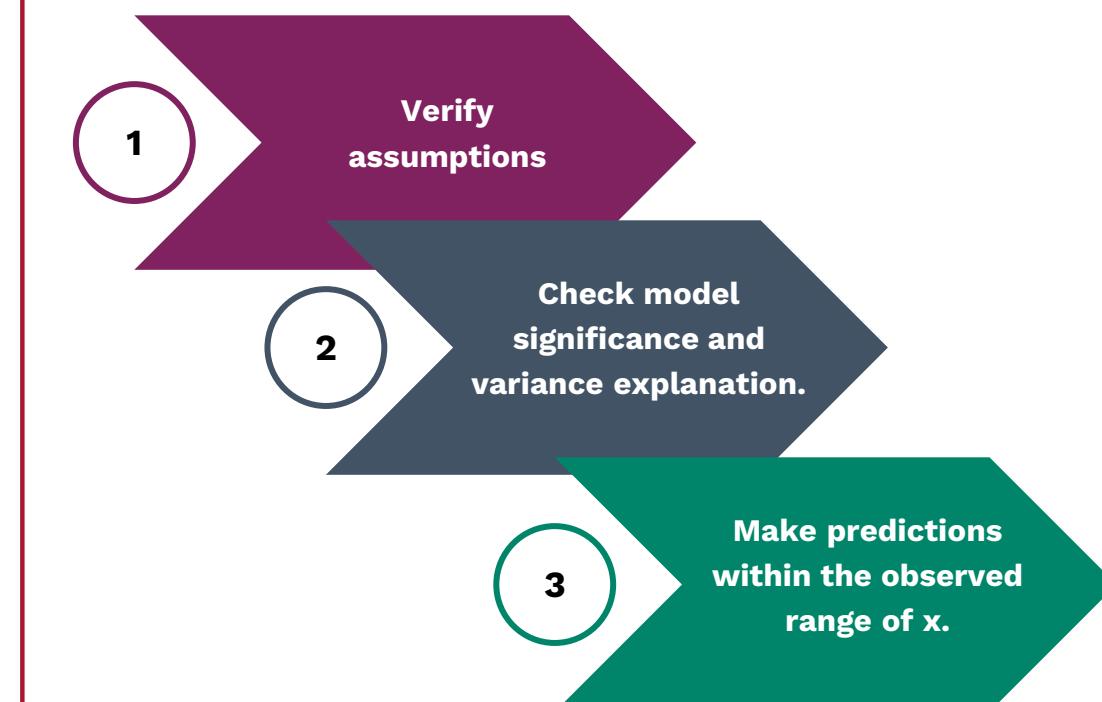


## Simple Example for Regression

- Regression line predicts a score of  $Y$  for any given value of  $X$
- The closer the observed scores are to the predicted scores, the better the model predicts  $y$ , the less variability around the line there is.
- The further away from the line (predicted scores) the observed scores are, the worse the model predicts  $y$ , the more variability around the line there is.
- Difference between predicted  $Y$  and observed  $Y$  for any given value of  $X$  is called a **residual**.

## Prediction

**Steps:**



The Coefficient of Determination is the proportion of variation explained by the model.

**Range: 0 (no explanatory power) to 1 (full explanatory power).**

## Hypothesis Testing in Regression

Focuses on the slope:

Null Hypothesis (no relationship).

$$H_0: \beta_1 = 0$$

Alternative Hypothesis (relationship exists).

$$H_1: \beta_1 \neq 0$$

Significance level ( $\alpha$ ): Often 5% threshold.

Tests for normal distribution, linear relationship, and constant variance of residuals.