



Forecasting Trend Time Series

Incorporating Steps and Trends





Trend Fitting

Where the time series has a trend, a **trend fitting and extrapolation** approach may be used for prediction

Trend fitting is a method where the time series is linked to **some function of a time index**

$$Y_t = f(\text{time})$$

A **linear trend** equation is typically assumed although it depends on the trend observed

Prediction is based on extrapolation by **substitution of the appropriate value for the time index**



Trend Fitting (cont)

Assuming a linear trend the equation is

$$Y_t = a + b * t$$

where t = a time index and α and β are constants

The values of α and β typically **estimated by regression** of the time series (Y) against time index (t)

EXCEL has **numerous alternative ways of estimating the above equation** and/or trend fitting/extrapolation including the **regression routine**



Holts Exponential Smoothing (HES)

The two general methods already studied (MA, SES) are useful when the time series is **predominantly horizontal** but will **not be good predictors** when the **time series has other systematic components**

If the time series has a trend then MA and SES will be poor predictors

A simple **extension of the SES model (Holt's Model)** which incorporates a **trend component** can be used for better prediction

Like SES, **Holts Exponential Smoothing (HES)** uses a smoothing algorithm to **remove random influences** from the time series revealing the underlying systematic components.



HES Equations

HES is characterised by three equations;

$$1. L_t = \alpha Y_t + (1 - \alpha) (L_{t-1} + T_{t-1})$$

$$2. T_t = \beta (L_t - L_{t-1}) + (1 - \beta)(T_{t-1})$$

$$3. F_{t+m} = L_t + mT_t$$

The first equation is for level, the second for trend and the third is the forecasting equation for “m” periods into the future



HES Equations (cont)

L_t = Smoothed level at period (t)

Y_t = Actual time series value at period t

α = Smoothing constant for level

T_t = Trend estimate at period t

**β = Smoothing constant for the trend
($0 <= \beta <= 1$)**

m = Number of periods ahead to be forecast

F_{t+m} = Holt's forecast value for period $t + m$



HES (cont.)

The values of α , β are **arbitrarily determined**

Typically **between 0 and 1 inclusive** although some programs (eg MINITAB) ignore this restriction

Try different α , β to determine the “optimum” combination (as assessed by error criteria **(MSE, MAE, MAPE)**)

SOLVER in EXCEL can also be used to find the optimum by minimising a chosen error criterion

Initialisation of the model requires initial estimates for L_t and T_t . L_t is usually the **initial time series** value (Y_1)

T_t is usually the **average** of the **increase/decrease** in the first few periods (use either **zero** or $(Y_2 - Y_1)$ or $((Y_3 - Y_1)/2)$)