CSCI 430: Homework 7

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1 6.1-1

Minimum: 2^h
Maximum: $2^{h+1} - 1$

2 6.1-3

If the n^{th} element is the root of the sub-tree then its children will be $\leq$ to it; the same holds for their children as well, all descendants will be $\leq$ the root, thus the root is the largest value.

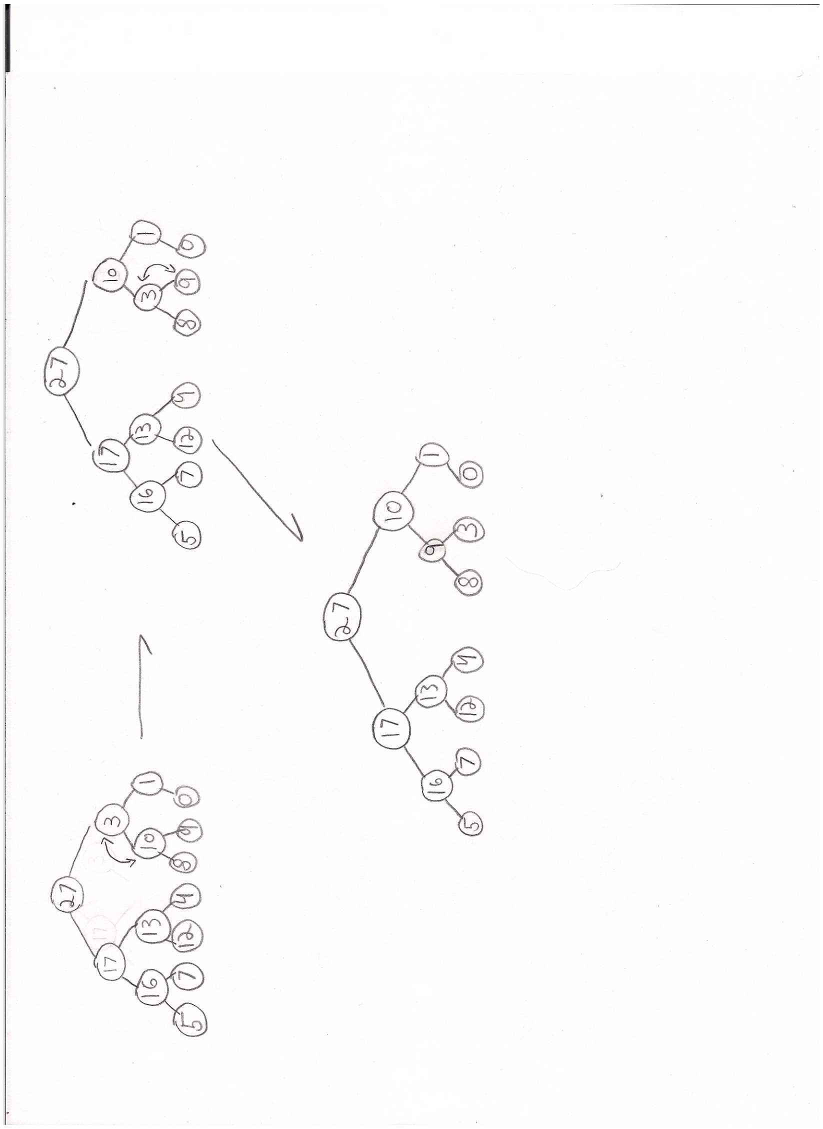
3 6.1-4

The smallest element would reside in one of the leafs, indexed as such: $\frac{n}{2} + 1$

4 6.1-5

Sorted array A has the following property for any two indexes i¡j: $A[i] \leq A[j]$
Any array is a mini-heap if $A[i] \leq A[left(i)]$ and $A[i] \leq A[right(i)$ where $i < left(i)$ and $i < right(i)$
Thus a sorted array is a mini-heap.

5 6.2-1



6 6.2-3

There would not be an effect, and since A[i] would be found to be larger than it would simply just return.

7 6.2-6

If we were to put the smallest element at the root and was to put the largest elements at the bottom of the left most path in the heap, then MAX-HEAPIFY will be called at every level in the heap in order to move the minimum element to the left most leaf. Thus, since the height of a heap is lgn then the the worst-case running time must be $\Omega(lgn)$

8 6.3-3

Proof: by mathematical induction:
Base case: We know that there are $\frac{n}{2}$ leaves in a heap, and nodes of height 0 give us: $\frac{n}{2^{0+1}}$ leaves in an n-element heap. Thus, this holds for the base case of h=0.
Inductive step: Suppose h=k, s.t. there are $\frac{n}{2^{k+1}}$ nodes of height k in an n-element heap. Since heap closely resembles a binary tree, this means that nearly every two nodes with a height of k share a parent at height k+1. This tells us that there are at most $\frac{\frac{n}{2^{k+1}}}{2} = \frac{n}{2^{(k+1)+1}}$ nodes at height k+1. This tells us that this proposition holds for the case when h=k+1.
Therefore, by mathematical induction, there are at most $\frac{n}{2^{h+1}}$ nodes of height h in any n-element heap.

9 6.4-2

Initialization: Prior to the first iteration of the loop: i=n, the sub-array A[1...i] is a max-heap, and the sub-array A[i+1...n] is empty.
Maintenance: A[1] is the largest element of A[1...i], A[1] is also smaller than the elements of A[i+1...n]. When we put A[1] in the ith position, then A[i...n] will contain the largest elements in sorted order. Calling MAX-HEAPIFY turns A[1...i-1] into a max-heap. Lastly, decrementing i will set it up for the next iteration.
Termination: The loop terminates when i=1. A[2...n] is sorted and A[1] is the smallest element in the array, thus the array is sorted.