Symbolic Computation in Algebraic Topology

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Doctoral Programme CICM 2010, Paris

Introduction

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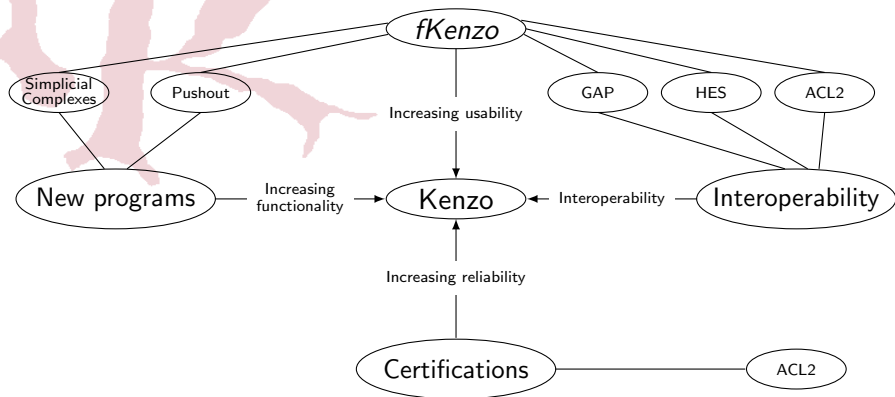 - Common Lisp Package

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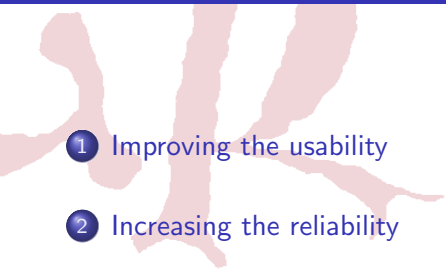 - Works with the main mathematical structures

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  - Homology groups unreachable by any other means

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Publications:

J. Heras, V. Pascual, J. Rubio, *Mediated Access to Symbolic Computation Systems*, Lecture Notes in Computer Science 5144 (2008) 446–461.

J. Heras, V. Pascual, J. Rubio, *Using Open Mathematical Documents to Interface Computer Algebra and Proof Assistant Systems*, Lecture Notes in Computer Science 5625 (2009) 467–473.

J. Heras, V. Pascual, J. Rubio, *Applying Generative Communication to Symbolic Computation in Common Lisp*, In proceedings of the 2nd European Lisp Symposium, 2009.

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- Aim: Verification of (some fragments) of Kenzo code

Deduction System

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- Both Kenzo and ACL2 are Common Lisp systems
  - Certification in ACL2 of real Kenzo code

Representation of Mathematical Structures in Kenzo/ACL2

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Publication:

J. Heras, V. Pascual, J. Rubio, *Content Dictionaries for Algebraic Topology*, In Proceedings of the 22nd OpenMath Workshop, 2009.

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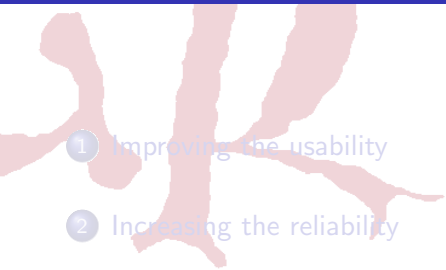Publication:

J. Heras, V. Pascual, J. Rubio, *Proving with ACL2 the correctness of simplicial sets in the Kenzo system*. To appear in Proceedings of 20th International Symposium on Logic-Based Program Synthesis and Transformation, 2010.

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Publications:

📄 J. Heras, V. Pascual, *ACL2 verification of Simplicial Complexes programs for the Kenzo system*. Proceedings of Algebraic computing, soft computing, and program verification Workshop, 2010.

📄 J. Heras, *Effective Homology of the Pushout of Simplicial Sets*. To appear in Proceedings of XII Encuentro de Álgebra Computacional y Aplicaciones, 2010.

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Publication (MKM conference):

J. Heras, V. Pascual, J. Rubio *Integrating multiple sources to answer questions in Algebraic Topology*, Lecture Notes in Computer Science 6167 (2010) 331–335.

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