ACL2 verification of Simplicial Complexes programs for the Kenzo system

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1 Scope of the work

The notion of simplicial complex is the most elementary method to settle a connection between common "general" topology and homological algebra. The notion of topological space is too "abstract" in order to perform computations. A triangulation, by means of simplicial complexes, can be provided for "sensible" spaces, so that an algebraic topologist can decide every space is a simplicial complex, making the computations easier.

But many common constructions in topology are difficult to make explicit in the framework of simplicial complexes. It soon became clear in the forties the notion of simplicial set is much better. The reference [6] remains the basic reference in this subject.

The Kenzo system [3] is a Common Lisp program, developed by F. Sergeraert and devoted to Algebraic Topology. Kenzo works with the main mathematical structures used in Simplicial Algebraic Topology, namely it is able to work with simplicial sets, however the notion of simplicial complex is not included.

In addition, this system was written mainly as a research tool and has got relevant results which have not been confirmed nor refuted by any other means. Then, the question of Kenzo reliability (beyond testing) arose in a natural way. Several works (see [1], [2] and [5]) have focussed on studying the correctness of first order *fragments* of Kenzo with the ACL2 theorem prover [4]. The full verification of the Kenzo system is not possible with ACL2, since the ACL2 logic is first-order but Kenzo uses intensively higher order functional programming.

We have undertaken two task: on the one hand, the development of new tools which integrate the notion of simplicial complexes into the Kenzo system. On the other hand, a certification of the correctness of these programs using the ACL2 theorem prover.

References

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