Aalto-university

Multiple Correspondence Analysis on sleeping pads

MS-E2112

Johannes Mäkinen

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1 Introduction

Finding the perfect sleeping pad to spend your nights on can be a hard choice. When hiking in nature, you want to maximise the utility of a good night's sleep. This can be done by choosing an appropriate sleeping pad, with enough comfort and warmth. If the pad is uncomfortable, the sleep you get will be hindered by waking up in the middle of the night to adjust yourself to a better position. This is especially common with thin pads, which do not support the body at the optimal spots like hips and shoulders. The warmth of the pad is crucial when temperatures are close or even under 0 °C. Most people start having trouble sleeping at this temperature, and the biggest factor in keeping you warm throughout the night is the insulation between the ground and you. The other factors are related to the sleeping bag or quilt you have, which will not be considered in this analysis.

Before knowing anything about hiking, you would probably just want to maximise both thickness and the thermal insulation of a pad. This could be a mistake, as different styles of pads weigh different amounts. The longer the hike is, the less extra weight you want to carry with you. Thus, you must balance between these three properties to find a suitable pad for your use case.

(a) Nemo Tensor (b) Therm-a-rest UberLite (c) Therm-a-rest Z Lite Sol

Figure 1: Various sleeping pads designed for different conditions

Figure 1 shows three different sleeping pads, all of which have different thermal insulation, weight, and thickness. So, how should one choose the sleeping pad for their needs? Will warm pads always be heavy due to extra material used in the insulation? Does the thickness of the pad affect the warmth of the pad? Are foam pads lighter but colder than inflatable pads? The main research question for this analysis is: **"Is there a type of pad that is the best option for hiking?"** In addition, we will be exploring the differences in sleeping pads and the correspondences between the specifications mentioned above using Multiple Correspondence Analysis (MCA).

2 Data

The data which is used in this analysis covers over 100 sleeping pads (Data source). Each pad has four values: R-value, weight, thickness, and type. The R-value of a pad describes numerically the thermal insulation of the pad. The weight and thickness of the pad are self-explanatory and they were converted from ounces and inches to the metric system. The type of the pad is a categorical variable and it can be either Foam, Self-Inflatable or Inflatable. Table 1 shows some of the pads and their corresponding values.

Make_Model	R_Value	Weight_g	Thickness_cm	Type
Therm-a-Rest NeoAir Venture	2.2	538.6	5.1	Inflatable
Exped SIM 3.8	4.6	870.3	3.8	Self-Inflating
REI Kindercamp	2.5	538.6	2.5	Self-Inflating
Therm-a-Rest Ridegrest Classic	2.0	396.9	1.6	Foam
Sea-to-Summit Camp SI	4.2	765.4	3.8	Self-Inflating
Exped SynMat Lite 5	3.4	629.4	5.1	Inflatable
Big Agnes Insulated Q Core Deluxe	4.3	708.7	8.9	Inflatable
NEMO Quasar 3D Non-Insulated	1.8	680.4	8.9	Inflatable
Big Agnes Third Degree Foam	1.5	340.2	1.3	Foam
NEMO Vector UL Non-insulated	1.6	623.7	7.6	Inflatable

Table 1: Various sleeping pads available on the market

As we are performing MCA, we want to categorise the data using expert knowledge on the variables. First, R-values are usually used to inform the buyer about what season(s) of the year the pad is preferably used and not the absolute warmness. For example, a 4-season pad is meant to be used all year round, but a 3-season pad is not suitable for winter use. Secondly, the weight is categorised to represent the weight of the pad when hiking on foot for long durations. This means that for example the really heavy pads will be put into the "Too Heavy" category, as they are sensible only when camping by car etc. The thickness of a pad is usually close to 6cm, as it is the industry standard to make the pad's packed volume as small as possible without compromising its structural integrity. Thus, that will be the baseline for normal thickness. The type of a pad is already a categorical variable and thus it is left as is. Below are presented the mappings used in the categorisation, and Table 2 shows how the variables look after the mapping. The categories are named s_class for the intended season(s), w_class for the weight, and t_class for the thickness.

$$\text{s\_class} = \begin{cases} \text{Summer}, & \text{R-value} \in [0,3) \\ \text{Season\_3}, & \text{R-value} \in [3,5) \\ \text{Season\_4}, & \text{R-value} \in [5,\infty) \end{cases} \qquad \text{w\_class} = \begin{cases} \text{Ultralight}, & \text{Weight (g)} \in [0,400) \\ \text{Light}, & \text{Weight (g)} \in [400,600) \\ \text{Heavy}, & \text{Weight (g)} \in [600,800) \\ \text{Too Heavy}, & \text{Weight (g)} \in [800,\infty) \end{cases}$$

$$\text{t\_class} = \begin{cases} \text{Thin}, & \text{Thickness (cm)} \in [0,5) \\ \text{Normal}, & \text{Thickness (cm)} \in [5,8) \\ \text{Thick}, & \text{Thickness (cm)} \in [8,\infty) \end{cases}$$

Make_Model	R_Value	Weight_g	Thickness_cm	Type
Therm-a-Rest NeoAir Venture	Summer	Light	Normal	Inflatable
Exped SIM 3.8	Season_3	Too_Heavy	Thin	Self-Inflating
REI Kindercamp	Summer	Light	Thin	Self-Inflating
Therm-a-Rest Ridegrest Classic	Summer	Ultralight	Thin	Foam
Sea-to-Summit Camp SI	Season_3	Heavy	Thin	Self-Inflating
Exped SynMat Lite 5	Season_3	Heavy	Normal	Inflatable
Big Agnes Insulated Q Core Deluxe	Season_3	Heavy	Thick	Inflatable
NEMO Quasar 3D Non-Insulated	Summer	Heavy	Thick	Inflatable
Big Agnes Third Degree Foam	Summer	Ultralight	Thin	Foam
NEMO Vector UL Non-insulated	Summer	Heavy	Normal	Inflatable

Table 2: Categorised values

3 Uni- and Bivariate analysis

In this section we'll perform uni- and bivariate data analysis to describe the data and its variations. We have filtered the pads that were over 1000 grams as they are not sensible on any hiking trip, and they would just increase the variation and errors made in the analysis. Therefore we are left with 92 pads.

The distributions of the numerical values of the pads are presented in Figure 2 as a violin plot. We see that the inflatable pads have the most variation in their values in each of the categories. The foam pads are almost always the lightest and thinnest, but also the coldest. The self-inflating pads are the heaviest as expected due to the inflation mechanism and heavier material used in them. The thickest pads are the inflatable pads, which is due to the large volume of the inner tubes compared to other types of pads.

Overall, the inflatable pads have much higher variations, which is expected as there are more of those pads than any other. Besides, the industry-standard recommendation for a pad type is the inflatable, which implies that there are many more versions of inflatable pads to serve the broadest use cases.

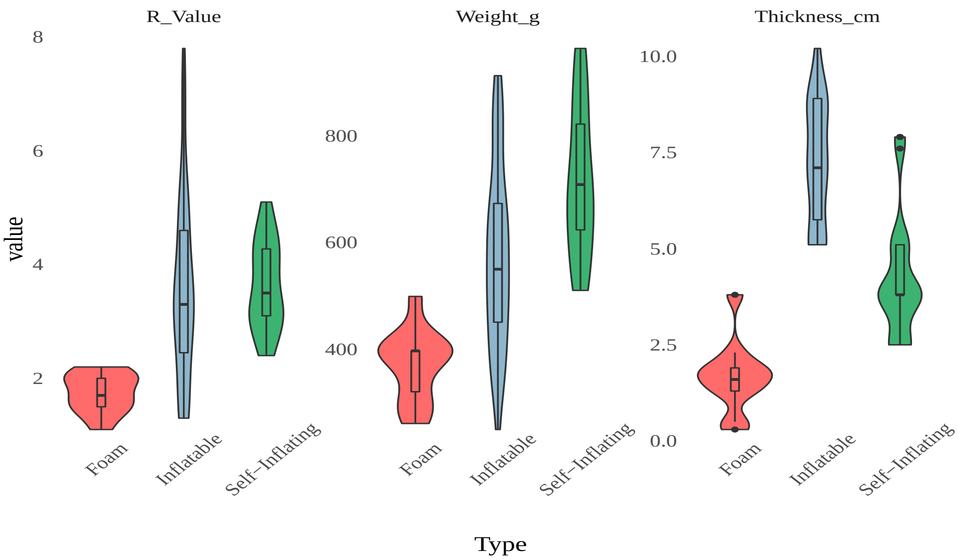

Figure 2: Distributions of the numerical variables

The summarised statistics of the data can be seen in Table 3. R-values are between 1.1 and 7.8, whereas the full range of industry-standard values is from 1 to 10. The 1st and 3rd quartile together with the mean and median of R-values indicate that most pads are suitable for three- or four-season use. The weights are at lightest about 250 grams, and at heaviest 967 grams. There's are quite a large variation in weight and the summary statistics show that the weight variable is more evenly distributed than the R-values. The thickness ranges from 0.3 to 10.2 cm, and the statistics show that the thickness is mostly concentrated between 3.8 and 7.6 cm.

	R_Value	Weight_g	Thickness_cm
Min.	1.1	249.5	0.3
1st Quartile	2.3	442.2	3.8
Median	3.2	595.3	5.1
Mean	3.4	590.0	5.7
3rd Quartile	4.2	708.7	7.6
Max.	7.8	963.9	10.2

Table 3: Summary statistics

To see if there are any pairwise correlations, we plot the scatterplots between each pair of variables in Figure 3. We can see that there might be some dependency between the R-value and the weight of the pad as the R-value is increasing almost linearly for foam and self-inflating pads. Variation of R-value for inflatable pads is much higher, but it seems to also increase with weight. The same dependency can be seen between the R-value and thickness of the pad, though it is not as strong. The figure also shows that on average, the thicker the pad is, the more it weights. The thickness of a pad is almost a categorical value even before the categorisation of the variables. Thus, there are only a few different basic thickness types that are being manufactured.

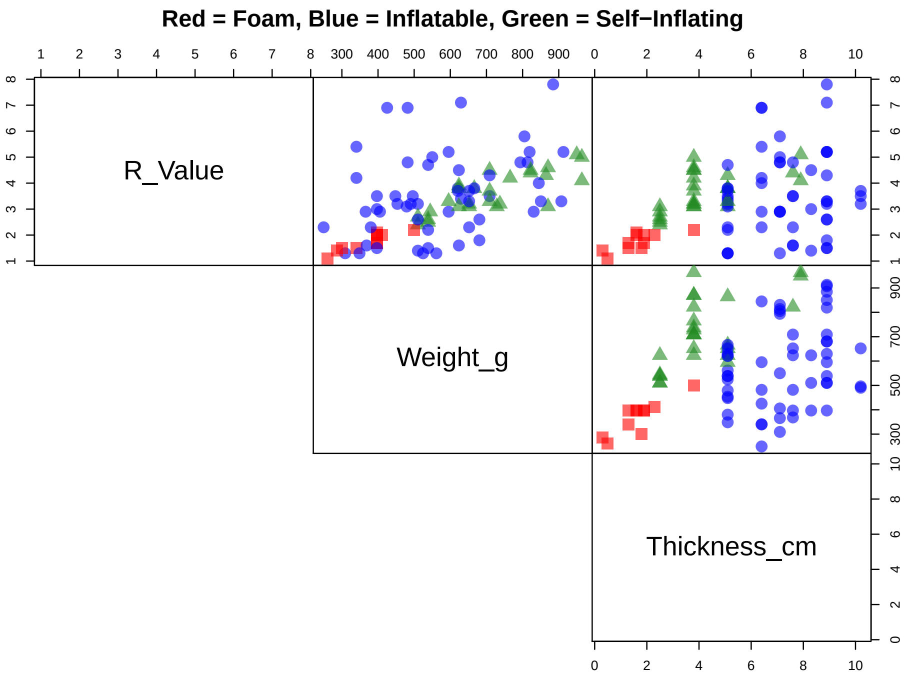

Figure 3: Pairwise scatterplot of each numerical variable

In addition, Table 4 presents the correlation matrix of the variables. The correlation coefficients seen here imply the same phenomena as previously described from Figure 3.

	R_Value	Weight_g	Thickness_cm
R_Value	1.00	0.51	0.35
Weight_g	0.51	1.00	0.28
Thickness_cm	0.35	0.28	1.00

Table 4: Correlation coefficients

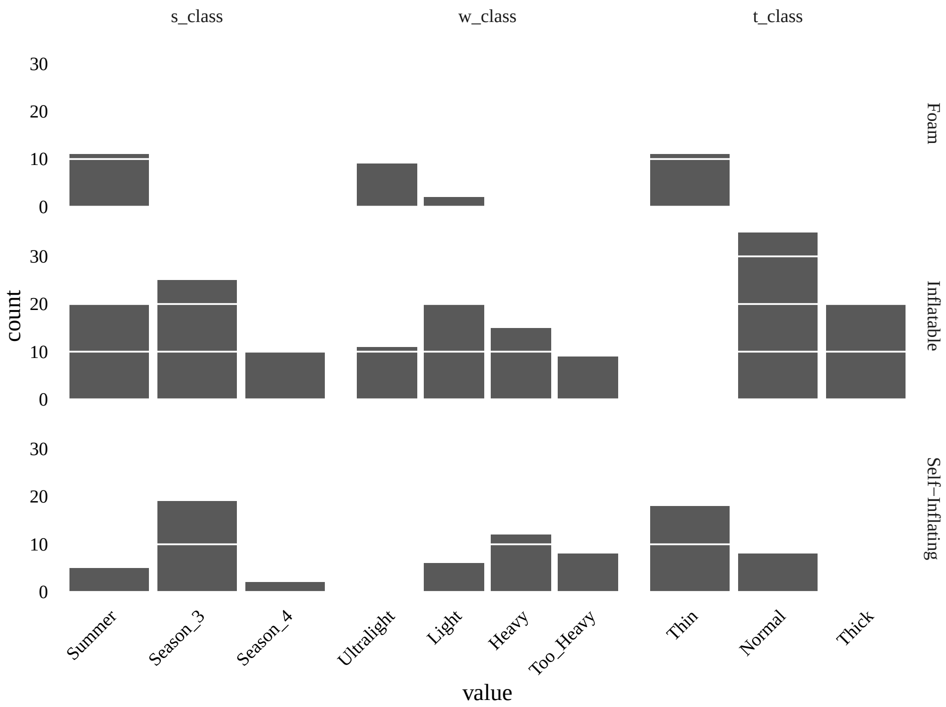

Figure 4: Data after categorisation

Figure 4 presents the data after the categorisation described in Section 2 is done. Here we see some details that can be problematic in the MCA. For example, there are much fewer foam pads than other types of pads. There are also only thin summer foam pads, no thin inflatable pads, and no thick self-inflating pads. There are also some category combinations that have only a few observations, and this can affect the results of MCA, as the method is not robust to outliers. Overall, there are 11 foam, 55 inflatable, and 26 self-inflatable pads, and the most common pad is a light inflatable three-season pad with normal thickness.

4 MCA

Multiple Correspondence Analysis (MCA) is a statistical method to detect and represent underlying structures in data. This is done by projecting the multidimensional categorical data points into a lower dimension. In our analysis, this is a way to see what kind of repulsions and attractions there are between different categories and variables. Principal Component Analysis could have been used had all the variables been numerical, but as the Type of the pad is not, we chose MCA. We will be using the method as presented on the MS-E2112 course, using the indicator matrix, also known as the complete disjunctive table. The algorithm uses the chi-square distances between different categories of the variables and between the individual pads to uncover the associations between the variables. As we have already processed the data into a suitable format, as seen in Table 2, we can simply apply MCA to it using R.

The R output can be seen in Appendix A.1. The % of variation explained by the principal components are presented in Figure 5. The first two principal components explain approximately 45% of the variation in the data, which is sufficient to identify basic underlying structures in the data. From A.1 we also see that the qualities of representations of each categorical data point are quite varied. The different types of pads are well represented by the first two principal components. The thickness class "Thin" is also well represented. The values: 4 Season, Light, Too Heavy, and Normal thickness are poorly represented by the first two components. This can be due to insufficient observations of pads with these specifications, or too high within-group variations.

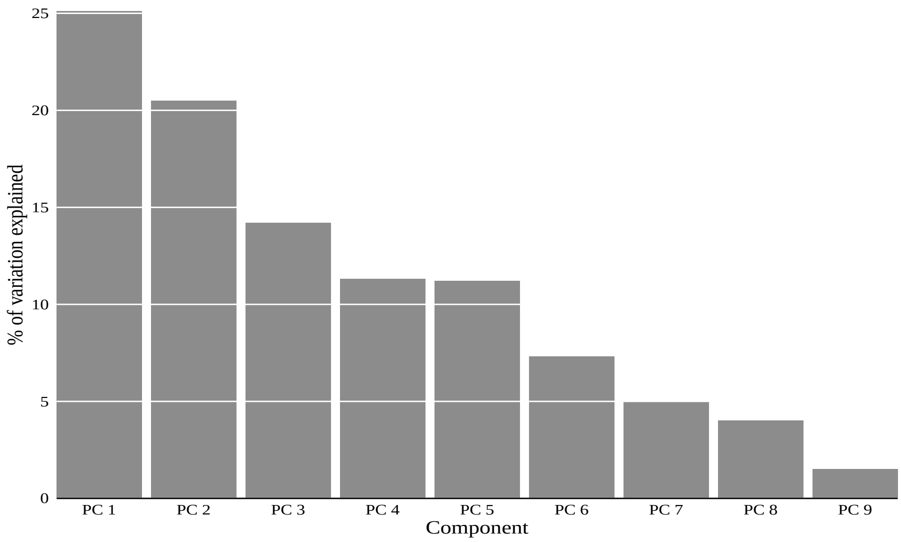

Figure 5: Scree plot of the MCA components

Using MCA, we can represent the data in two dimensions, as opposed to the four original dimensions. Figure 6 shows the data using the two first principal components. The arrows in the figure are used to represent the correspondences between the values. If the angle between two arrows is less than 90°, the two categorical values are attracted to each other. Conversely, if the angle is greater than 90°, there is repulsion between them. The category Type is colored red and other categories blue to emphasise the difference between the pad types.

According to the MCA plot, foam pads are likely to be thin, ultralight, and suited for summer use. The inflatable pads are usually normal to thick but lightweight four-season pads. The third type of pad is self-inflating, which is attracted to being Heavy to Too Heavy and also only suited to three-season use. Though, there are also some thin self-inflatable pads that are lighter.

In addition to differences between the types, we can also analyse the other variables and their attractions to each other. Three-season pads are attracted to being heavy or too heavy for hiking. The pads meant for summer use are usually ultralight, which makes sense as they do not need to be as warm or thick as three- or four-season pads. The pads used in four-season hiking are either thick or normal thickness, but not thin. This implies that to get enough insulation between the ground and the user, there needs to be a sufficient amount of air inside the pad. We can also see that the lightweight pads are mostly for four-season use, which can be explained by the fact that people who hike during winter usually also want to optimise their weight, and thus manufacturers had to create innovations to save weight for these demanding customers.

In addition to the points stated above, we can also detect that the arrows for "Type:Self-Inflating", "Type:Foam" and "t_class:Thin" are quite long. This can be a sign of atypical rows, which will have a negative effect on the analysis as MCA is not a robust method. For example, foam pads are only $\frac{11}{92} \approx 12\%$ of all observations and thus they are comparably rare.

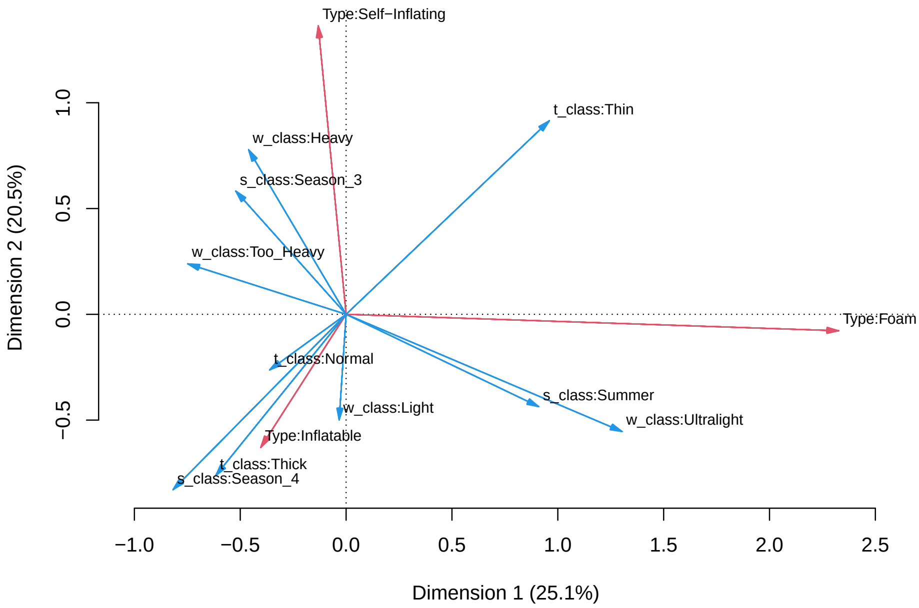

Figure 6: MCA plot with respect to two components that explain most of the variation

5 Conclusions

The analysis performed in Section 4 provides answers to some of the questions raised at the beginning in Section 1. According to Figure 6, there is no one single pad type that would be the optimal solution for every condition. This is simply due to differences between each hiker and the conditions that they're going to be venturing in. That being said, one can observe how the self-inflating pads are usually much heavier than inflatable pads, but still not as warm. On the other hand, foam pads are even lighter but do not give enough insulation to withstand the below zero conditions of winter. The warmth of the pad seems not to contribute to the weight of the pad as much as one would think. There are several light and even some ultralight pads that are rated for four-season usage.

On average, people should not be choosing a self-inflating pad, if they are to follow the analysis done here. For winter use, one should choose an inflatable pad with a high R-value, low weight and close to normal thickness, such as the Therm-a-Rest NeoAir XTherm with its R-value of 6.9, 425g weight, and 6.5cm thickness. This pad is not only according to our analysis but also according to the thousands of reviews on the internet, one of the best pads in the market. For summer use only, one should choose any of the foam pads, as they do not differ that much from each other, or just choose an inflatable pad for their budget as there's no such thing as "a too warm pad", and an inflatable pad is always much more comfortable Though, some people might oppose choosing a foam pad altogether due to them being usually hard or even impossible to pack inside a hiking bag.

6 Critical evaluation

The original continuous data was first categorised. Then the original and the categorised data were described using uni- and bivariate analysis. Afterward, Multiple Correspondence Analysis was performed on the categorised data, and then the results were visualised. We could have used some other statistical methods to explore the differences and similarities between the types of sleeping pads, but MCA seemed most intuitive due to the categorical Type variable. Overall, the analysis was straightforward and clearly presented.

As for the improvements and possible biases: The analysis was conducted from the author's personal perspective, who like to hike long distances and travel light. This is why pads that weighted over 1000g were filtered out from the data, as they are not sensible in their use case. Other analyses could be done from other perspectives, for example looking at sleeping pads for car camping.

Improvements concerning the data can also be done. First, the 92 pads used in this analysis can be too little of a sample to fully represent the sleeping pad markets. There also were some categories that had only a few or no observations, which leads to unstable MCA results, one example being the foam type pads. We could try to avoid the instability by grouping rare modalities in a different way. To improve the analysis even further, one could add in categorical variables for the price, packed size, and durability of the pad, as they too are key specifications that customers base their decisions on. The price of the pad can be a deciding factor between an inflatable pad and a self-inflatable pad due to the latter being much cheaper, but worse according to this analysis. The packed size is important when trying to minimise the gear used in long hikes, and this would probably make the foam pads more undesirable as they do not pack as small as other types. The durability of a pad could be one of the most important factors when buying a pad that you'd expect to last five to ten years. Though, this specification is hard to categorise, as there are lots of different kinds of fabrics used, with all of them having different pros and cons.

A Appendix

A.1 R output

```
> pad.mca <- mjca(categorical_data,lambda="indicator",reti = T)
> summary(pad.mca)

 Principal inertias (eigenvalues):

  dim    value      %    cum%   scree plot
  1      0.564900  25.1  25.1   ******
  2      0.460553  20.5  45.6   *****
  3      0.318652  14.2  59.7   ****
  4      0.254899  11.3  71.1   ***
  5      0.252306  11.2  82.3   ***
  6      0.163479   7.3  89.5   **
  7      0.112695   5.0  94.6   *
  8      0.089463   4.0  98.5   *
  9      0.033053   1.5 100.0
         --------  -----
  Total: 2.250000 100.0


Columns:
                    name   mass  qlt  inr    k=1 cor ctr    k=2 cor ctr
1  |           Type:Foam |   30  736  123 | 2327 735 286 |   -78   1   0 |
2  |     Type:Inflatable |  149  830   55 | -403 242  43 |  -629 588 128 |
3  |  Type:Self-Inflating |   71  739   86 | -132   7   2 |  1363 732 285 |
4  |      s_class:Summer |   98  655   76 |  910 532 143 |  -436 122  40 |
5  |    s_class:Season_3 |  120  561   59 | -521 249  58 |   583 312  88 |
6  |    s_class:Season_4 |   33  203   81 | -817 100  39 |  -829 103  49 |
7  |  w_class:Ultralight |   54  557   94 | 1304 472 163 |  -554  85  36 |
8  |       w_class:Light |   76  110   58 |  -33   0   0 |  -499 109  41 |
9  |       w_class:Heavy |   73  339   70 | -461  88  28 |   778 251  96 |
10 |   w_class:Too_Heavy |   46  140   77 | -748 127  46 |   239  13   6 |
11 |        t_class:Thin |   79  810   96 |  960 424 129 |   915 385 143 |
12 |      t_class:Normal |  117  175   50 | -361 114  27 |  -263  61  17 |
13 |       t_class:Thick |   54  267   75 | -616 105  36 |  -762 161  69 |
```

A.2 Data

For precaution, if hyperref is not working:
Data is from: https://sectionhiker.com/sectionhiker-gear-guide/sleeping-pad-r-values-of-2021/