Fixed Income

Reference: Bodie et al, Ch 14

Econ 457

Week 8-a

Outline

1. Time Value of Money
2. Bond Characteristics
3. Bond Prices
4. Bond Yields

1. Time Value of Money

Basic Concept

A dollar today is worth more than a dollar tomorrow.

- *Opportunity Cost:* Money can be invested to earn returns
- *Risk:* Future payments are uncertain, and people value uncertain things less
- *Inflation:* Purchasing power decreases over time

$$FV = PV \times (1 + r)^t$$

Where:

- FV = Future Value
- PV = Present Value
- r = Interest/discount rate
- t = Time periods

1. Time Value of Money

Present vs. Future Value

Future Value (Compounding):

$$FV = PV \times (1 + r)^t$$

Example: $1,000 invested at 5% for 3 years

$$FV = 1,000 \times (1.05)^3 = \$1,157.63$$

Present Value (Discounting):

$$PV = \frac{FV}{(1 + r)^t}$$

Example: What is $1,157.63 received in 3 years worth today at 5%?

$$PV = \frac{1,157.63}{(1.05)^3} = \$1,000$$

1. Time Value of Money

The Rule of 72

The Rule of 72: To find how long it takes for money to double at a given rate of return:

$$\text{Years to Double} \approx \frac{72}{\text{Rate of Return (\%)}}$$

Examples:

- At 6% interest: $\frac{72}{6} = 12$ years to double
- At 8% interest: $\frac{72}{8} = 9$ years to double
- At 12% interest: $\frac{72}{12} = 6$ years to double

1. Time Value of Money

The Rule of 72

For money to double: $FV = 2 \times PV$

$$2 \times PV = PV \times (1 + r)^t$$

$$2 = (1 + r)^t$$

Taking natural logarithm of both sides:

$$\ln(2) = t \times \ln(1 + r)$$

$$t = \frac{\ln(2)}{\ln(1 + r)}$$

- $\ln(2) \approx 0.693$
- For small r: $\ln(1 + r) \approx r$
- Therefore: $t \approx \frac{0.693}{r} = \frac{69.3}{r \times 100}$
- Rule of 72 uses 72 instead of 69.3 for easier mental math

1. Time Value of Money

The Rule of 72

At 6% annually:

$$FV = 1,000 \times (1.06)^{12} = \$2,012$$

Close to exactly double.

Rule of 72 works best for small values of r, say below 15%.

2. Bond Characteristics

Principal

At maturity the bond issuer repays the principal of the bond.

Also referred to as the 'par value' or 'face value' of the bond.

Most bonds – including US Treasury bonds and corporate bonds – typically have par values of $1,000. For the purposes of examples and Excel formulas, $100 is also commonly used for the par value.

2. Bond Characteristics

Coupons

The bond issuer makes regular payments to bond holder. These payments are called 'coupon payments.' The coupon rate usually doesn't change over the life of the bond (hence the term 'fixed income').

$$\text{Coupon} = \text{Coupon Rate (\%)} \cdot \text{Par Value}$$

Bonds commonly pay coupons 'semi-annually', or twice per year.

- Semi-annual coupon payment = $\frac{\text{Annual Coupon Rate}}{2} \times$ Par Value
- Bond math often assumes semi-annual compounding.

2. Bond Characteristics

Coupons

2. Bond Characteristics

Security - US Treasury Bonds

US Treasury bonds are Backed by the full faith and credit of the US government. Often referred to as ”risk-free”.

This refers only to default risk

Still subject to other risks:

- **Interest rate risk:** Bond prices fall when rates rise
- **Inflation risk:** Real purchasing power may decline

2. Bond Characteristics

Prices

Bond prices are set in the market.

The **Clean Price** is the quoted market price and *excludes* accrued since the last coupon payment.

The **Dirty Price** is the total price paid by the buyer and *includes* the value of interest accrued since the last coupon payment.

$$\text{Dirty Price} = \text{Clean Price} + \text{Accrued Interest}$$

Bond quotes in newspapers/Bloomberg refer to the clean price.

2. Bond Characteristics

US Treasury Auctions

Auction Frequency:

- *Bills (4w, 8w, 13w, 26w, 52w):* Weekly
- *Notes (2y, 3y, 5y, 7y, 10y):* Monthly
- *Bonds (20y, 30y):* Monthly
- *TIPS:* Quarterly for most maturities

Auction Timeline:

- *Announcement:* 3-5 business days before auction
- *Auction day:* Bids due by 1:00 PM ET
- *Results:* Released within 30 minutes
- *Settlement:* Next business day (bills) or 2-3 days (notes/bonds)

Auction Sizes: Typically $40-60 billion per auction for popular maturities (10y, 30y).

2. Bond Characteristics

US Treasury Auctions

TREASURY NEWS

Department of the Treasury • Bureau of the Fiscal Service

Embargoed Until 08:30 A.M.
July 30, 2025

CONTACT: Treasury Auctions
202-504-3550

TREASURY OFFERING ANNOUNCEMENT [1]

Term and Type of Security	10-Year Note
Offering Amount	$42,000,000,000
Currently Outstanding	$0
CUSIP Number	91282CNT4
Auction Date	August 06, 2025
Original Issue Date	August 15, 2025
Issue Date	August 15, 2025
Maturity Date	August 15, 2035
Dated Date	August 15, 2025
Series	E-2035
Yield	Determined at Auction
Interest Rate	Determined at Auction
Interest Payment Dates	February 15 and August 15
Accrued Interest from 08/15/2025 to 08/15/2025	None
Premium or Discount	Determined at Auction
Minimum Amount Required for STRIPS	$100
Corpus CUSIP Number	912821SR7
Additional TINT(s) Due Date(s) and	None
CUSIP Number(s)	None

2. Bond Characteristics

US Treasury Auctions

TREASURY NEWS

Department of the Treasury • Bureau of the Fiscal Service

For Immediate Release
August 06, 2025

CONTACT: Treasury Auctions
202-504-3550

TREASURY AUCTION RESULTS

Term and Type of Security	10-Year Note
CUSIP Number	91282CNT4
Series	E-2035
Interest Rate	4-1/4%
High Yield [1]	4.255%
Allotted at High	52.85%
Price	99.959620
Accrued Interest per $1,000	None
Median Yield [2]	4.195%
Low Yield [3]	4.120%
Issue Date	August 15, 2025
Maturity Date	August 15, 2035
Original Issue Date	August 15, 2025
Dated Date	August 15, 2025

3. Bond Prices

Generic Pricing Formula

Generic pricing formula:

$$\text{Bond Price} = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

where C is the coupon amount and FV is the payment due at maturity.

Calculating the price requires the following inputs:

1. Number of coupon payments (T)
2. Discount rate or yield (y). Typically this the the market interest rate on the bond.
3. The par value (FV)
4. The coupon amount (C), usually Coupon Rate x Face Value.

Continued...

...continued

Things to remember:

3. Bond Prices

Zero Coupon Bonds

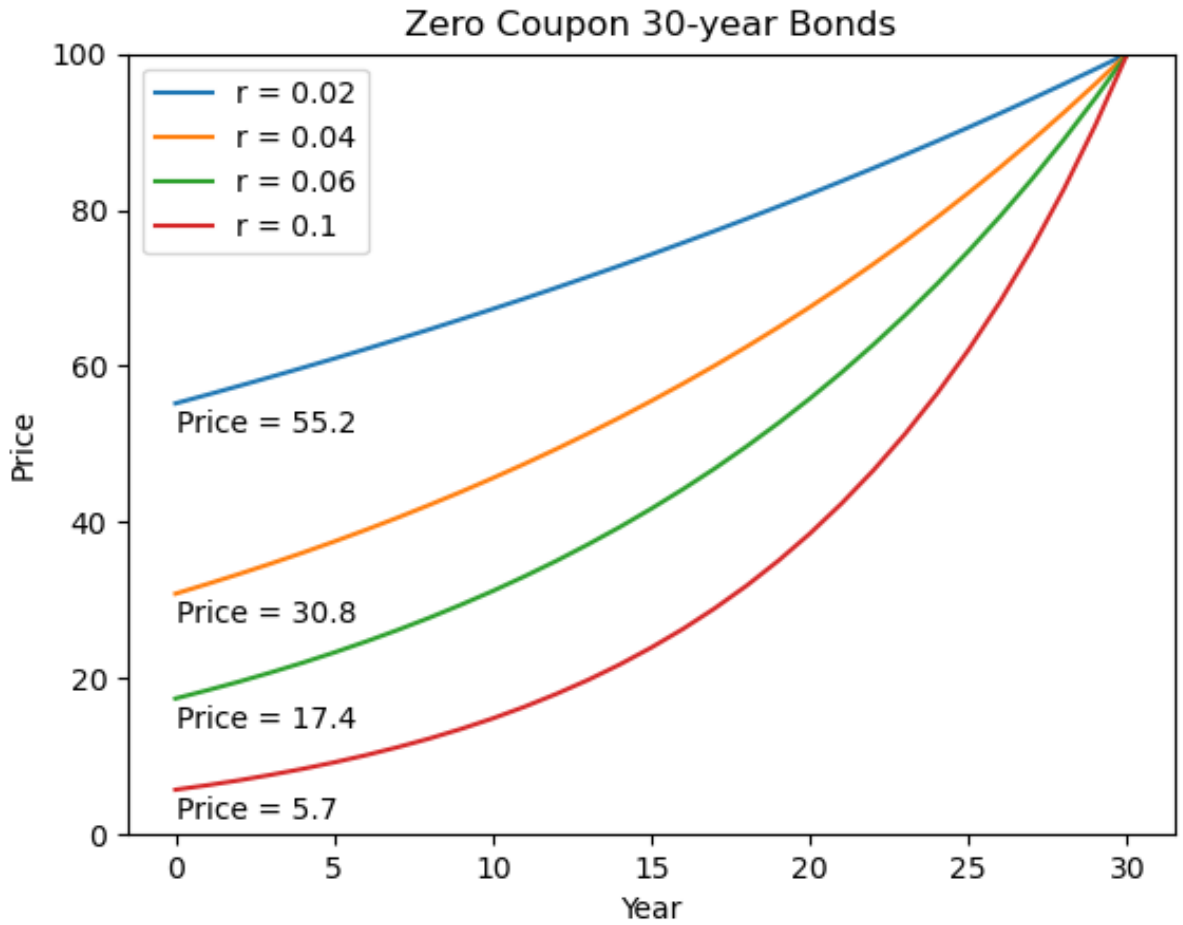
Zero coupon bonds have no coupons, and therefore easier math.

$$\text{Zero Coupon Price} = \frac{FV}{(1+r)^T}$$

Zero coupon bonds have other nice characteristics too, as we'll discuss in the lecture on duration and convexity. Zero coupon Treasury bonds are also referred to as "Ps" or as "STRIPS" because they can be created by "stripping" a normal, coupon bond into separate parts.

3. Bond Prices

Zero Coupon Bonds



3. Bond Prices

Prices and Yields are Inversely Related

Bond prices and yields are inversely related

Math: The yield (r) appears in the denominator of the bond pricing equation.

$$\text{Bond Price} = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{FV}{(1+r)^T}$$

Intuition: The price is the present value. Higher yields (discount rates) reduce the present value of the future bond payments, leading to lower bond prices today.

3. Bond Prices

Coupon Bonds

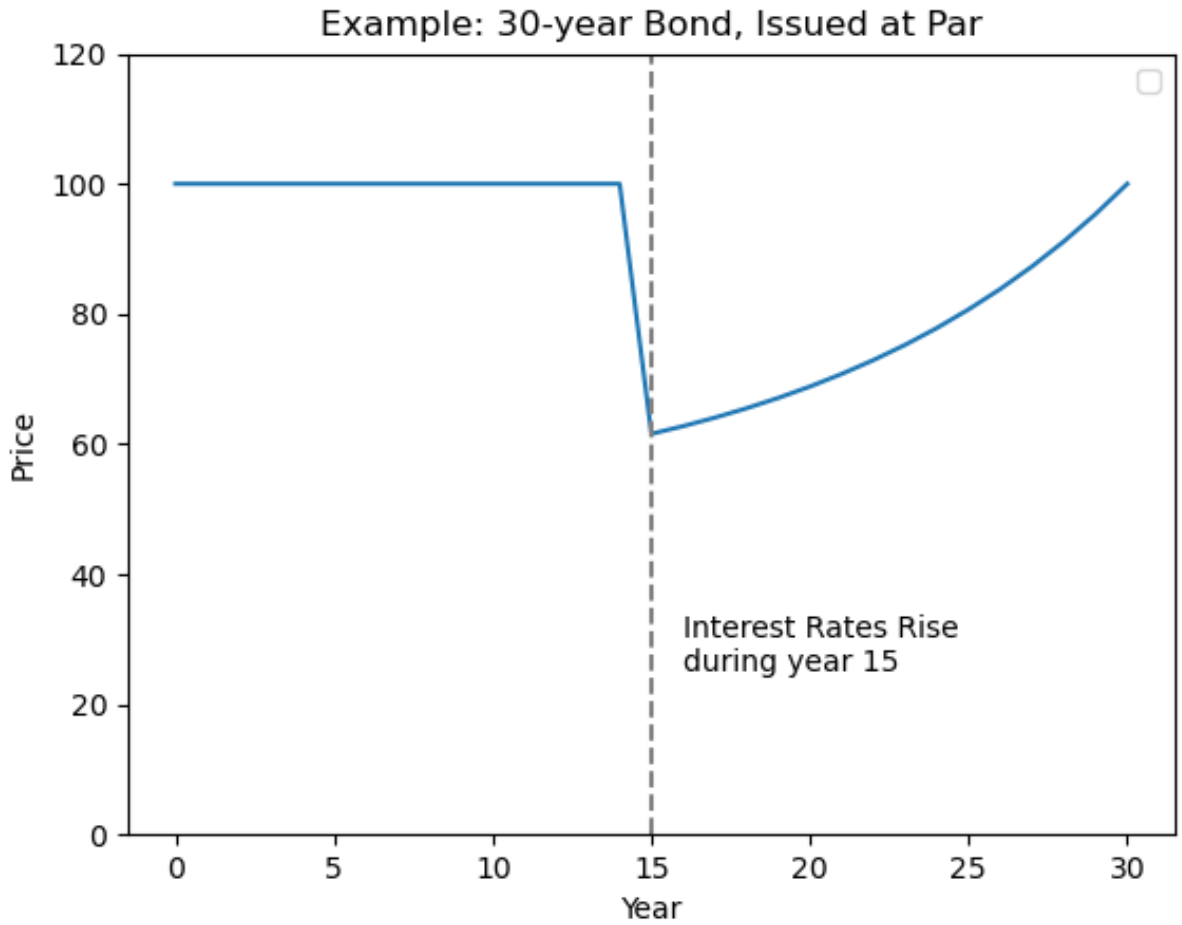
When the yield is equal to the coupon rate, the bond price is equal to the par value.

Proof used geometric series.

The intuition is that when the coupons equal the yield, the discount rate and the coupon payments (roughly) cancel eachother out.

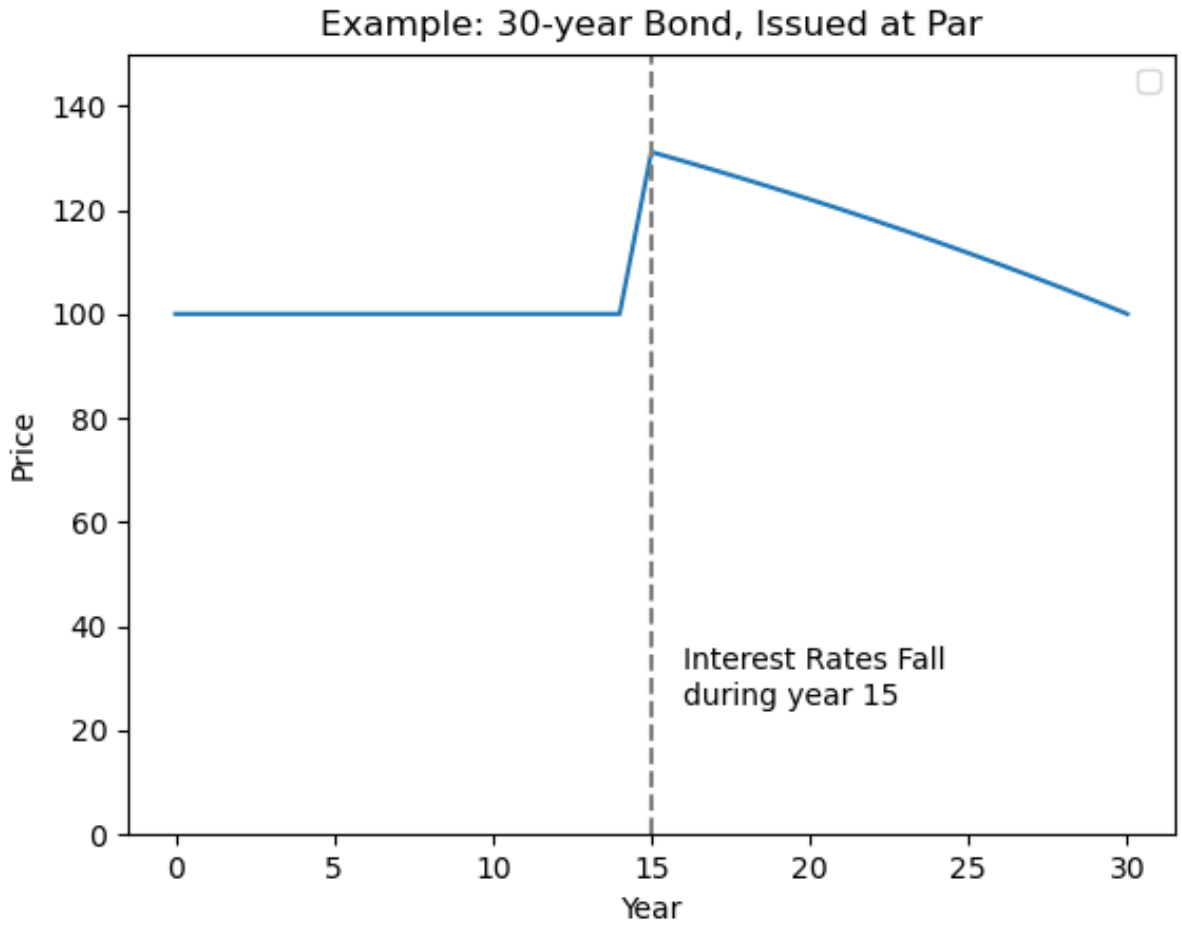
3. Bond Prices

Coupon Bonds



3. Bond Prices

Coupon Bonds



3. Bond Prices

Pull to par

At maturity, the bond price is equal to the face value.

The tendency of the bond price to approach face value as the bond approaches maturity is the *pull to par*. Note that tendency in the graphs on prevous slides.

3. Bond Prices

Pull to par

4. Bond Yields

Formula and calculations

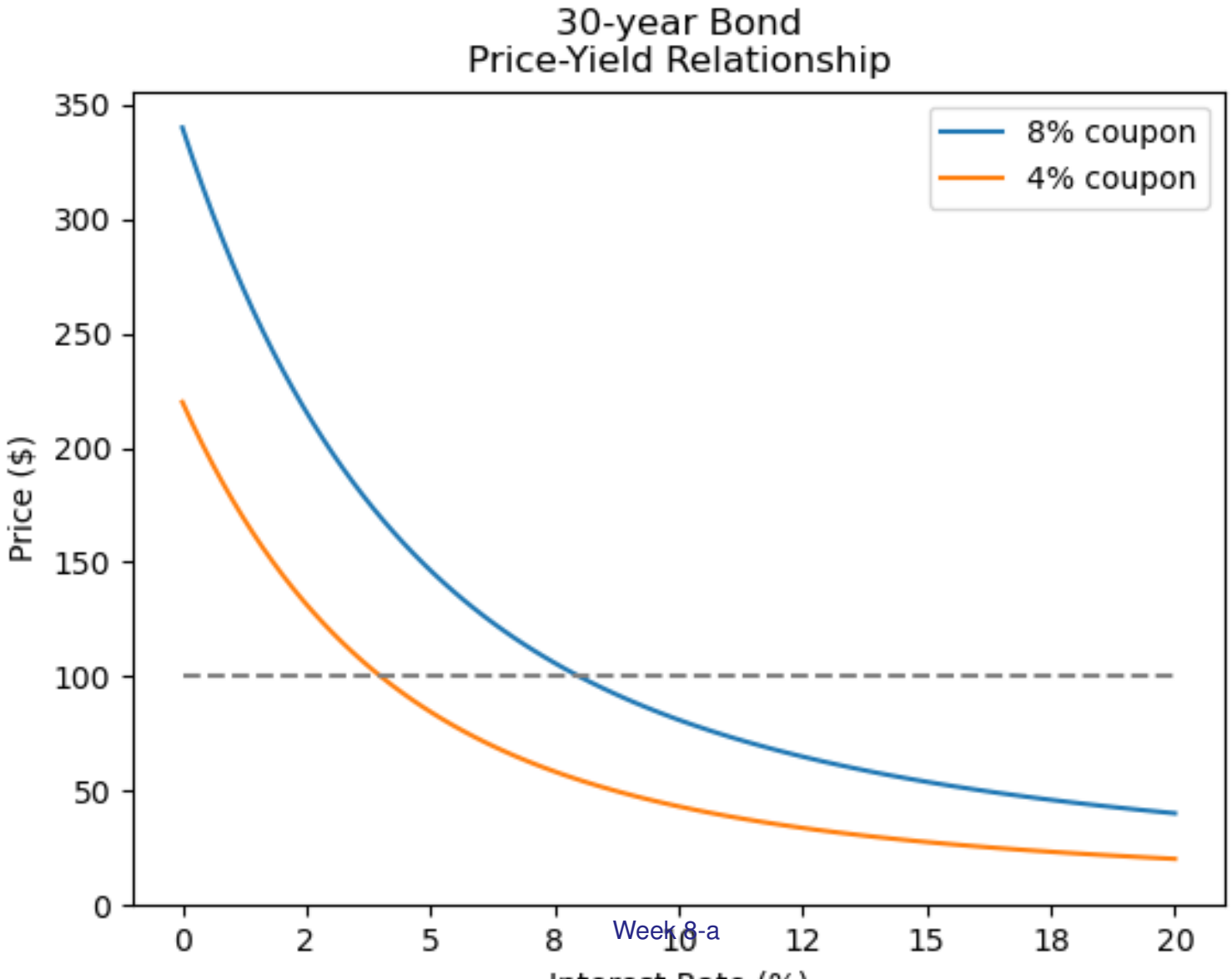
If the bond price is known, the bond yield can be calculated using the pricing equation and solving for y:

$$\text{Bond Price} = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

In most cases, there is no closed form expression for y. Numerical solving methods must be used instead.

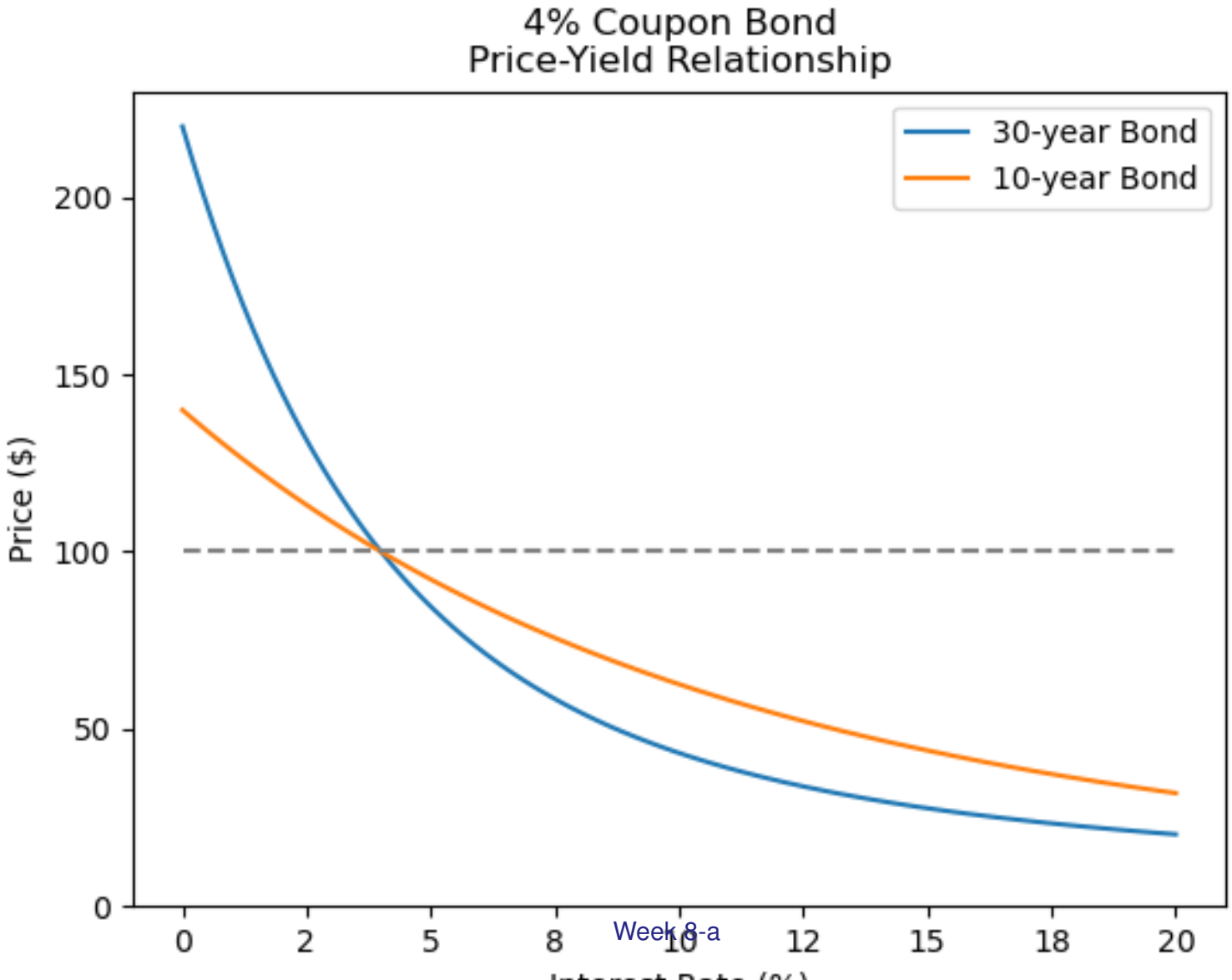
4. Bond Yields

Yields v Prices



4. Bond Yields

Yields v Prices



4. Bond Yields

Yields v Prices

Bond yields and prices are inversely related.
Intuition:

- Yield in denominator
- Reinvestment risk
- Yield is similar expected return, one way to get higher E[r] is lower (current) prices