Options: Valuation

Reference: Bodie et al, Ch 21

Econ 457

Week 13-b

Outline

1. Embedded Options
2. Market for Options
3. Put-Call Parity
4. Option Valuation
    - Time Value
    - Black-Scholes-Merton
    - Greeks

1. Embedded Options

Callable Bonds

A callable bond is a bond that gives the issuer the right (but not the obligation) to redeem the bond before its maturity date at a predetermined price.

Key Features:

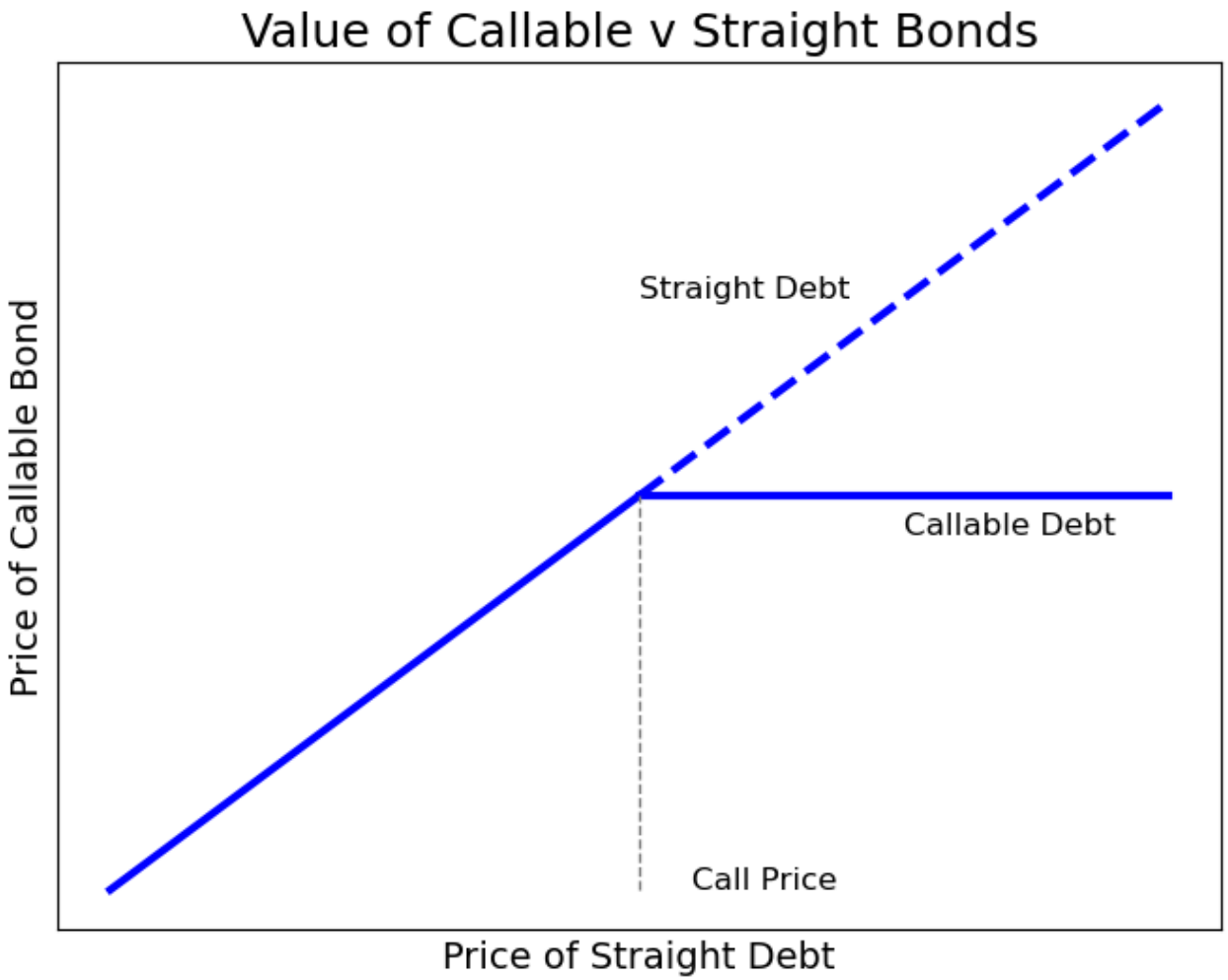
- **Call Price:** Usually set at par value ($1,000) or slightly above
- **Call Protection:** Period when bond cannot be called (e.g., first 5 years)
- **Call Schedule:** Specific dates when bond can be called
- **Higher Yield:** Callable bonds typically offer higher yields to compensate investors

Why Do Issuers Call Bonds?

- Interest rates have declined since issuance
- Can refinance debt at lower cost
- Improved credit quality allows cheaper financing

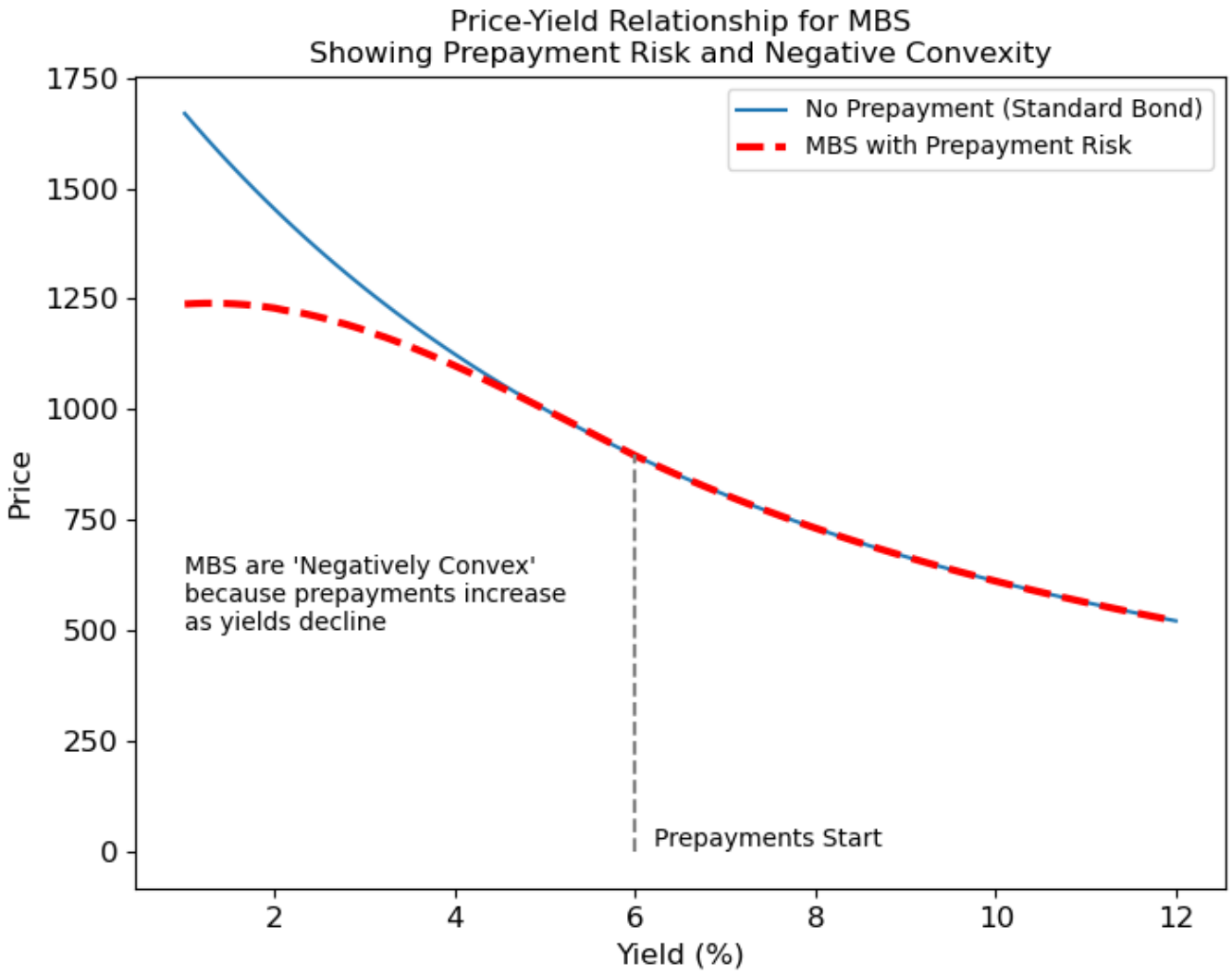
1. Embedded Options

Callable Bonds



1. Embedded Options

Callable Bonds - Mortgages



1. Embedded Options

Convertible Bonds

A convertible bond is a bond that gives the bondholder the right (but not the obligation) to convert the bond into a predetermined number of shares of the issuer's common stock.

Key Features:

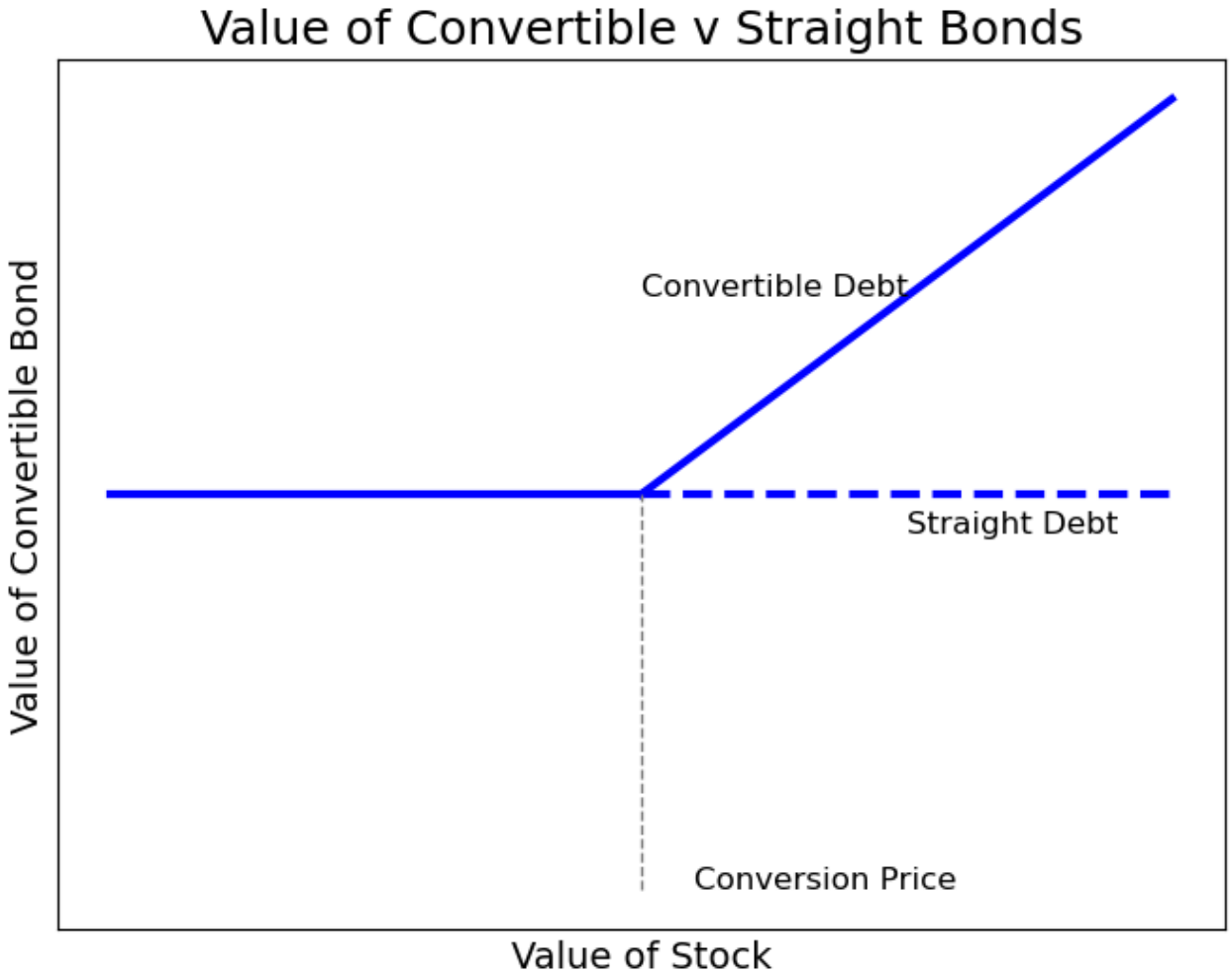
- **Conversion Ratio:** Number of shares per bond (e.g., 25 shares per $1,000 bond)
- **Conversion Price:** Effective price per share ($1,000 ÷ 25 = $40)
- **Conversion Value:** Current stock price × conversion ratio
- **Lower Yield:** Convertibles offer lower yields than straight bonds

Value Components:

- **Bond Floor:** Minimum value as a straight bond
- **Option Premium:** Value of conversion feature
- Total Value = Bond Floor + Option Premium

1. Embedded Options

Convertible Bonds



1. Embedded Options

Convertible Bond Arbitrage

Convertible arbitrage is a hedge fund strategy that exploits pricing inefficiencies between convertible bonds and their underlying stocks.

Basic Strategy:

- **Long Position:** Buy the underpriced convertible bond
- **Short Position:** Short sell the underlying stock (hedge ratio based on delta)
- **Profit Source:** Capture the difference between implied and actual volatility

Why is the convertible bond underpriced? Complexity, maybe? Liquidity, maybe?

1. Embedded Options

Warrants

A warrant is a security that gives the holder the right to purchase shares of the issuing company at a fixed price (exercise price) for a specified period of time.

Key Features:

- **Long Maturity:** Typically 3-5 years (vs. months for options)
- **Issued by Company:** Company creates new shares when exercised
- **Dilution Effect:** Exercise increases total shares outstanding

Warrants vs. Call Options:

- **Source:** Warrants issued by company; calls created by investors
- **Dilution:** Warrant exercise dilutes existing shareholders
- **Proceeds:** Exercise price goes to company (warrants) vs. option writer (calls)

1. Embedded Options

Corporate Bonds as Short Put Options

Corporate bonds can be viewed as equivalent to buying a risk-free bond and selling a put option on the firm's assets.

The Logic:

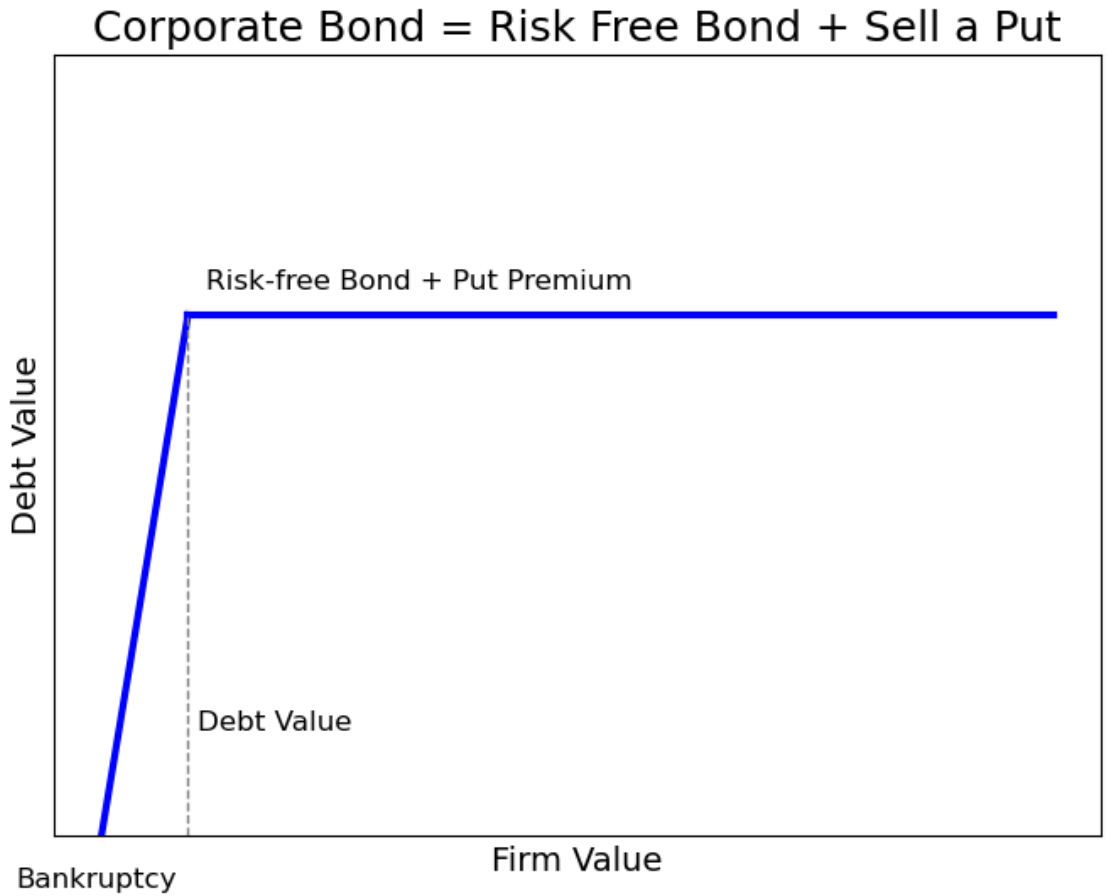
- **Default occurs when:** Firm value falls below debt obligations
- **Bondholders receive:** Min(Face Value, Firm Value)
- **This is equivalent to:** Face Value - Max(0, Face Value - Firm Value)
- **Which equals:** Risk-free bond - Put option on firm value

Implications:

- Higher firm volatility $\rightarrow$ Higher put value $\rightarrow$ Lower bond value
- Corporate bond yield = Risk-free rate + Credit spread
- Credit spread compensates for the "short put" position
- Explains why bond prices fall when default risk increases

1. Embedded Options

Corporate Bonds as Short Put Options



2. Market For Options

How Options Are Traded

Exchange-Traded Options (Listed):

- **Major Exchanges:** Chicago Board Options Exchange (CBOE), CME Group, NYSE American
- **Standardized:** Fixed expiration dates, strike prices, contract sizes
- **Clearing:** Options Clearing Corporation (OCC) guarantees all trades
- **Liquid:** High volume, tight bid-ask spreads for popular contracts
- **Examples:** SPY, QQQ, individual stock options

Over-the-Counter (OTC) Options:

- **Customized:** Tailored strike prices, expiration dates, underlying assets
- **Counterparty Risk:** No central clearing, direct bilateral agreements
- **Less Liquid:** Harder to trade before expiration
- **Examples:** Currency options, commodity options, exotic options

2. Market For Options

One month options on the S&P:

29 TABBED VIEW CALLS STRIKES SMART TRADING CLASS 100 My Chains

IV: 23.6%

STRIKE	CALLS BID x ASK	VOLUME	OPTN ...	DELTA	GAMMA	VEGA	THETA
6600	181.50 x 182.80		65.3K	0.548	0.001	7.548	-2.958
6720	109.00 x 110.40		5.37K	0.415	0.001	7.432	-2.583
6900	36.80 x 37.80		33.5K	0.204	0.001	5.403	-1.592
7120	6.30 x 6.80		300	0.049	0.000	1.945	-0.510

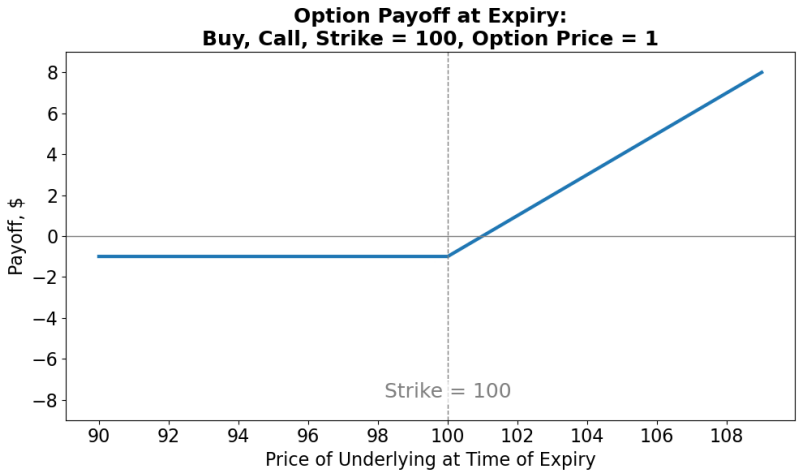
29 TABBED VIEW PUTS STRIKES SMART TRADING CLASS 100 My Chains

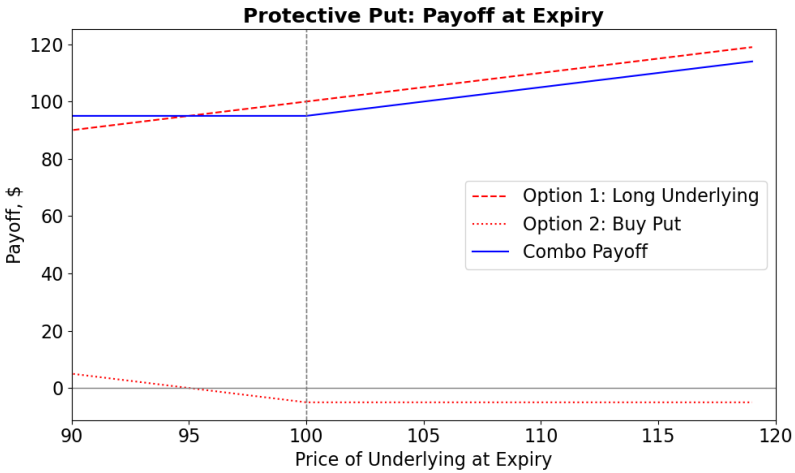
IV: 23.6%

STRIKE	PUTS BID x ASK	VOLUME	OPTN OPN I...	DELTA	GAMMA	VEGA	THETA
5600	13.50 x 14.00		29.3K	-0.048	0.000	1.916	-1.111
6000	29.70 x 30.40	18	270K	-0.112	0.000	3.638	-1.684
6480	93.90 x 95.20		288	-0.343	0.001	7.006	-2.235
6660	149.50 x 151.50		5.60K	-0.516	0.001	7.598	-1.922

3. Put-Call Parity

Notice that **buy call** strategy looks very similar to a **protective put**.





3. Put-Call Parity

In order to make them exactly equal, we need to combine the call with a zero-coupon bond that returns the value of the strike.

$$C_0 + \frac{X}{(1 + r_f)^T} = S_0 + P_0$$

Where X is the zero-coupon bond with face value equal to the strike price. One final detail is that the value of the stock (S_0) incorporates the right to dividends that are paid before the option expiry. We need to add those divdends to the left hand side to make these equal.

$$C_0 + \frac{X}{(1 + r_f)^T} + PV(\text{dividends}) = S_0 + P_0$$

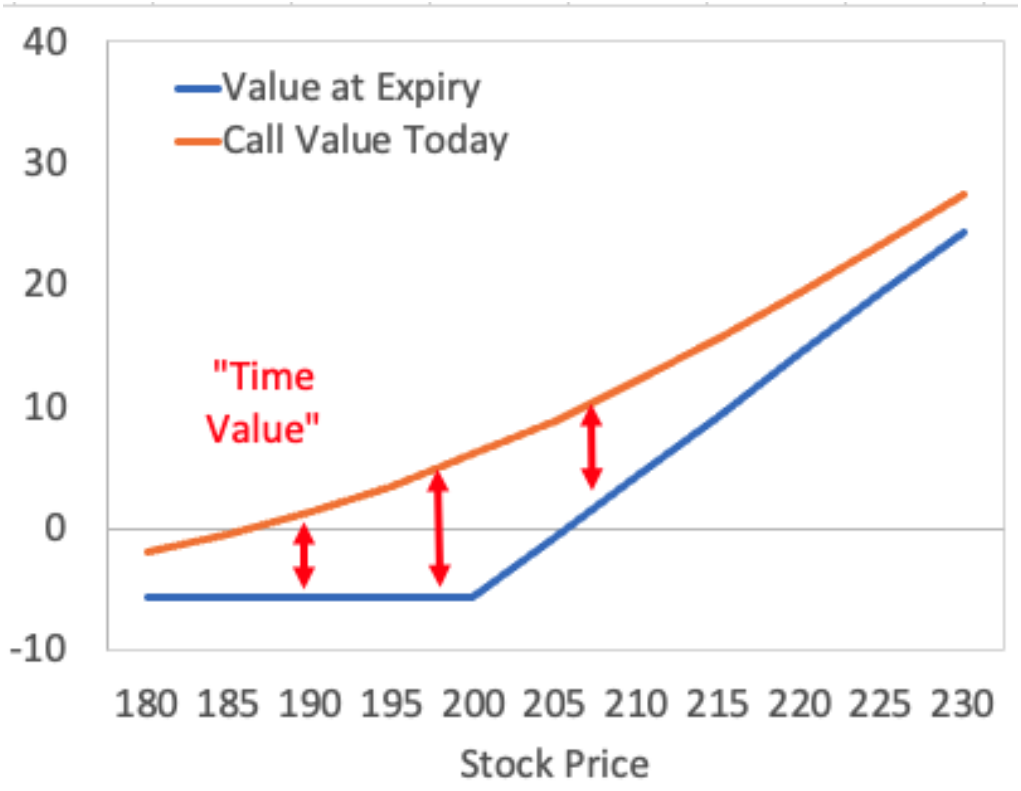
Rearranging slightly gives us the expression of **put-call parity**

$$\boxed{P_0 = C_0 - S_0 + PV(X) + PV(\text{dividends})}$$

4. Option Valuation

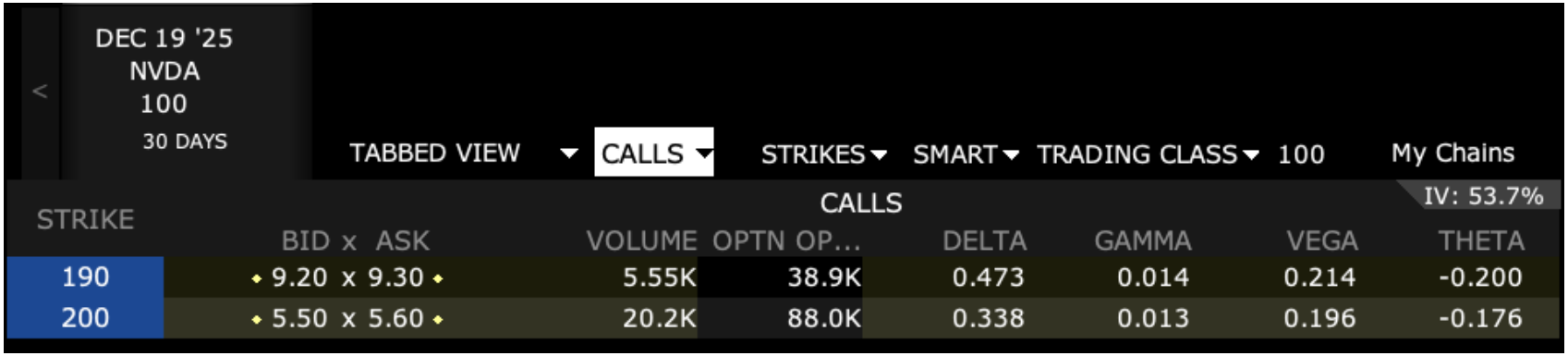
Time Value

The **intrisic value** of an option is the value of the option at the time of expiry. The **time value** of the option is defined as the difference between the value today and the intrinsic value of the option.



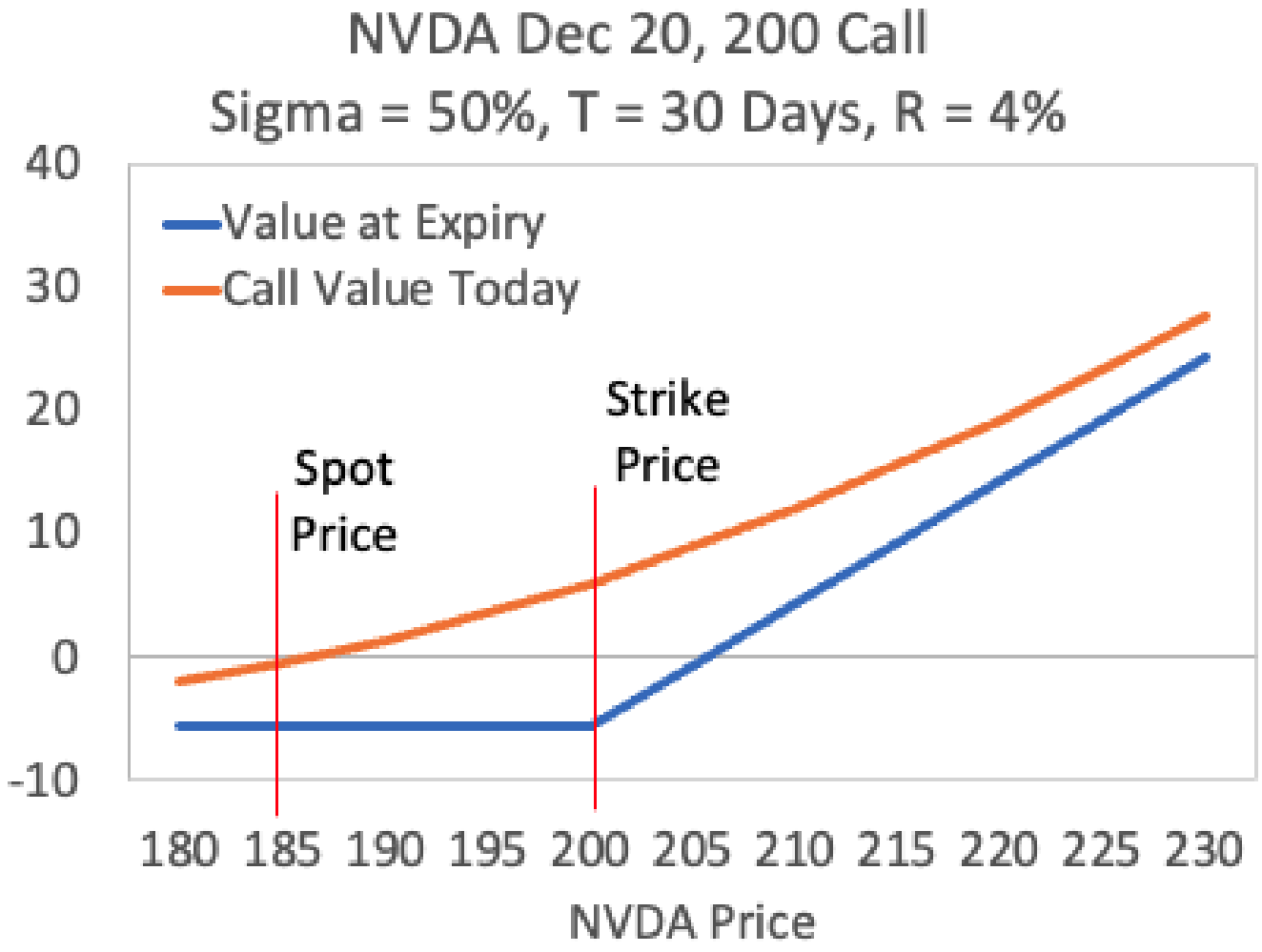
4. Option Valuation

NVDA calls



4. Option Valuation

NVDA calls



4. Option Valuation

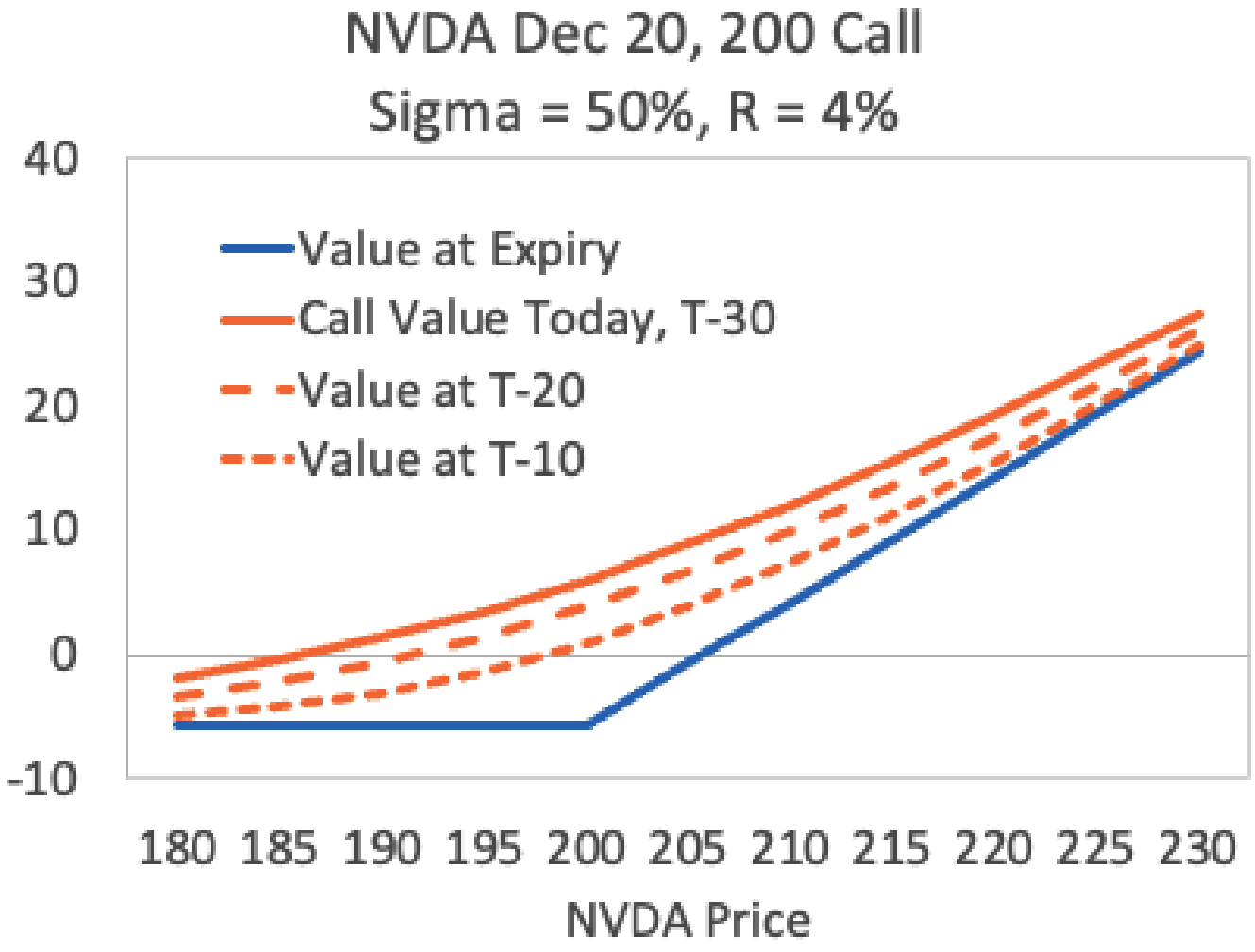
Determinants of Call Option Value

Table: Determinants of Call Option Value

If this Variable Increases...	The Call Option...
Stock Price, S_0	Increases
Exercise Price, X	Decreases
Volatility, σ	Increases
Time to Expiration, T	Increases

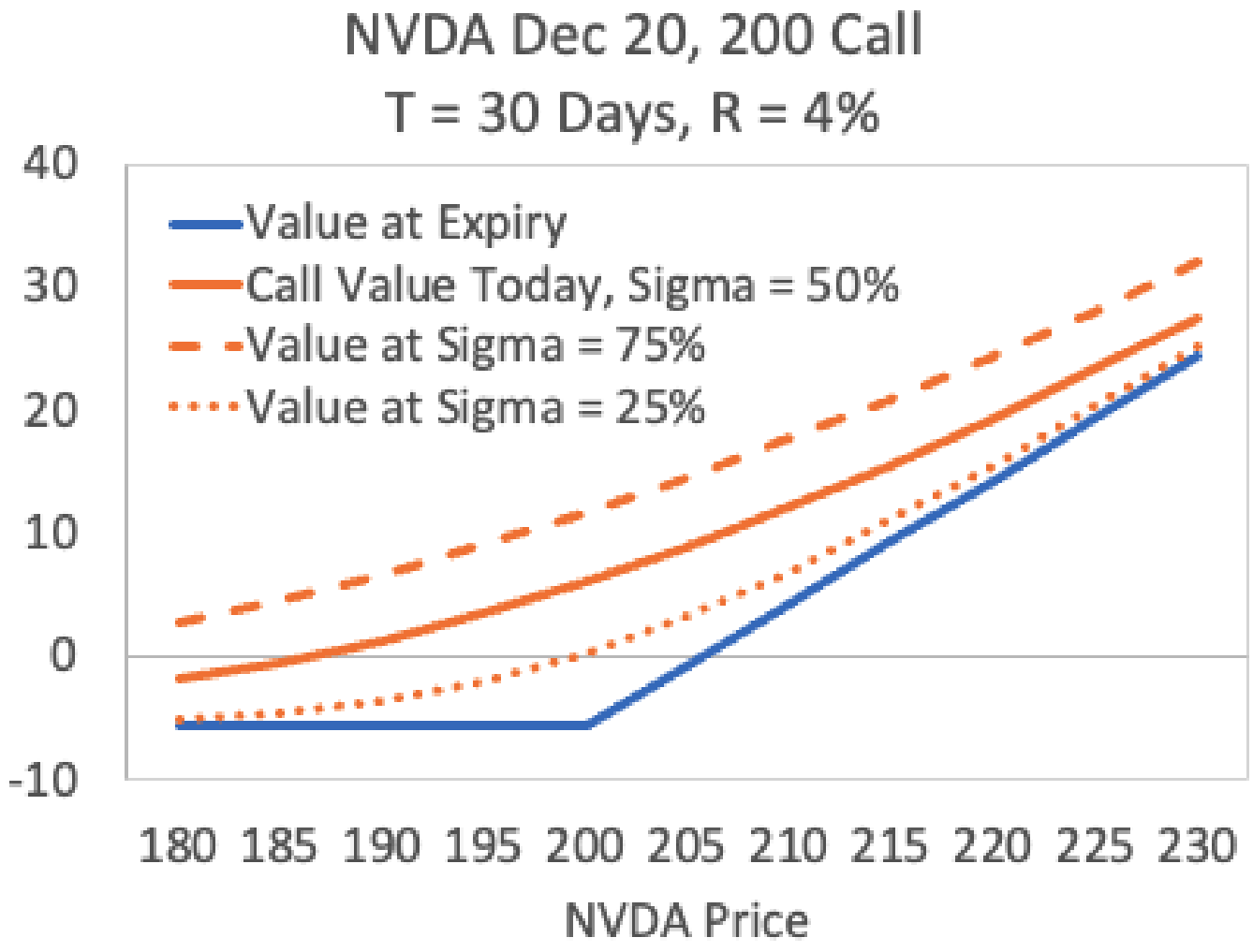
4. Option Valuation

NVDA calls - Change in Time to Expiry



4. Option Valuation

NVDA calls - Change in Volatility



4. Option Valuation

Black-Scholes-Merton

The value of a call option is given by the **Black-Scholes-Merton** formula:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Where

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

And S_0 is the spot price, K is the strike price, r is the risk-free rate, T is time to expiry, and σ is the implied volatility. $N()$ is the cdf of the normal distribution.

4. Option Valuation

Greeks

The **greeks** of an option are how much the option value changes for a change in one of the input paramaters.

Table: Option Greeks for **Call** Options

Greek	Measures Sensitivity to	Sign
Delta (δ)	Stock price (S_0)	Positive
Gamma (γ)	Delta	Positive
Theta (θ)	Time to Expiry (T)	Negative
Vega (ν)	Implied Volatility	Positive

Note: For puts, delta is negative while gamma and vega have the same positive sign. Theta is typically negative for both calls and puts (time decay)

5. Practice

Practice Questions

In each of the following questions you are asked to compare two options. The risk free interest rate for all cases is 4%. Assume the stocks on which these options are written pay no dividends.

1. Which put option is written on the stock *with the lower price*? A, B, or not enough information given.

Put	T	X	σ	Price
A	0.5	50	0.2	\$10
B	0.5	50	0.25	\$10

2. Which put option is written on the stock *with the lower price*? A, B, or not enough information given.

Put	T	X	σ	Price
A	0.5	50	0.2	\$10
B	0.5	50	0.2	\$12

5. Practice

Practice Questions

3. Which call option must have the *lower time to expiration*? A, B, or not enough information given.

Call	S	X	σ	Price
A	50	50	0.2	$12
B	50	50	0.2	$10

4 . Which call option is written on the stock *with higher volatility*? A, B, or not enough information given.

Call	T	S	X	Price
A	0.5	50	55	$10
B	0.5	50	50	$7

5. Practice

Practice Questions

5. Would you expect a $1 increase in a call option's exercise price to lead to a decrease in the call option's value of more or less than $1?
6. Is a put option on a high-beta stock worth more than one on a low-beta stock? The stocks have identical firm specific risk.
7. If the time to expiration falls and the put price rises, what has happened to the put option's implied volatility?
8. What will happen to the delta of a convertible bond as the stock price becomes very large?