Multifactor Models, Fama-French Factors

Reference: Bodie et al, Ch 10

Econ 457

Week 8-a

Outline

1. Multifactor Models
   - Macroeconomic factors
   - Arbitrage Pricing Theory
   - Factor Portfolios and Risk Premiums
2. Fama and French Factors
3. Multifactor Models in Practice (Smart Beta)

1. Multifactor Models

Motivation

One takeaway from CAPM was that *only systematic risk would be priced*. A security with higher than average idiosyncratic risk will not have a higher than average expected return. Only to the extent that the security's risk is correlated with systematic risk will it influence the expected return.

Up to now we defined "systematic risk" as broad market risk. Idiosyncratic, or company specific, risk could be diversified away and therefore should not be rewarded with higher returns. But the risk of the broad market can not be diversified away, which is why investors should receive higher returns for bearing this risk.

Today we are going to extend these ideas slightly and introduce *multiple factors*.

1. Multifactor Models

The Index model said that security returns were determined by one market factor and then idiosyncratic risk. Let's augment that by adding two additional factors reflecting the macroeconomic environment.

For example, add linear terms to make the risk premium of security i a function of macro surprises (GDP, CPI):

$$R_{i,t} = \alpha_i + \beta_{m,i} \cdot R_{m,t} + \beta_{GDP,i} \cdot GDP_t + \beta_{CPI,i} \cdot CPI_t + \epsilon_{i,t}$$

Where GDP_t is the surprise, relative to expectations, of the GDP announcement.

1. Multifactor Models

Application 1

Suppose this is your model for a security's returns:

$$R_{i,t} = \alpha_i + \beta_{m,i} \cdot R_{m,t} + \beta_{GDP,i} \cdot GDP_t + \beta_{CPI,i} \cdot CPI_t + \epsilon_{i,t}$$

Where *GDP* and *CPI* are *surprises*.

You estimate that $\beta_{GDP,i} = 1.2$.

Your initial estimate is that the excess return of the security will be 10%. If GDP comes in 1 percentage point lower than expected, how should you revise your estimate of the security's expected return?

1. Multifactor Models

This isn't CAPM

While this may look similar to CAPM, we are not yet able to make the same conclusions with regards to equilibrium pricing. CAPM was motivated by (strong) assumptions about complete markets, shared investor preferences, etc. And the market return was, by assumption in CAPM, the return on the complete portfolio of all investable assets.

In contrast, here we just chose a few macro factors to include in the model. While GDP and Inflation are reasonable choices, we could have chosen anything. Because these factors aren't based on the same set of assumptions, and as a consequence can't make the same claims to characterizing the equilibrium, multifactor models need to rely on something else to make statements about pricing.

1. Multifactor Models

Arbitrage Pricing Theory

Instead of relying on a economic model with equilibrium, these multifactor models instead rely on an *arbitrage* argument to make a statement about expected pricing. This is **Arbitrage Pricing Theory (APT)**

An *arbitrage* opportunity is when an investor can make a riskless profit without making a net investment (or exposing herself to risk). Arbitrages should be quickly closed by traders. So if you can find two trades with the exact same risk, it's usually safe to assume the expected returns on the two trades should be equal.

Obviously it's rare that the risk are exactly the same. We'll see these arguments again in the classes on options and derivatives.

1. Multifactor Models

Security Market Line

In multifactor models, we can argue that a no-arbitrage condition leads to the following conclusion:

Portfolios with equal betas should have equal expected returns, for any beta in a multifactor model.

This is similar to saying that all securities must lie on the Security Market Line (SML), but now we are talking about a multidimensional SML (harder to visualize).

1. Multifactor Models

Security Market Line

To see why there must be a single SML, consider the following chart where the x-axis is the realization (surprise) of factor F.
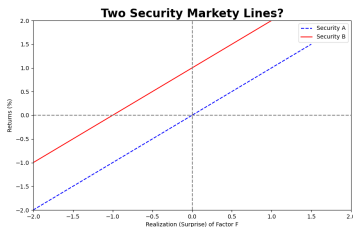
In this case, security A is always preferred to security B. An arbitrage strategy of selling B and buying A provides a riskless profit. As this arbitrage will be immediately closed by traders, prices will adjust so that both A and B eventually lie on the same line.

1. Multifactor Models

Factor Portfolios and Risk Premiums

When we choose the macro factors, we should be careful to choose factors that are difficult to diversify. Idiosyncratic risk is easy to diversify, and therefore should not be rewarded with extra return. Factors that are more difficult to diversify, in contrast, could be rewarded with extra return.

If we choose the factors appropriately, then we can argue that investors bearing exposure to these macro factors will be rewarded with extra return. A different way to say that is that the there will be a risk premium associated with the macro factors.

1. Multifactor Models

Factor Portfolios and Risk Premiums

To implement the APT in a way that is similar to the CAPM, we next construct factor portfolios. These portfolios may track the evolution of the macrovariable of interest, or perhaps some other aspect of systematic risk that should be rewarded. (We'll have some specific examples in a moment)

The risk premium associated with the factor is then the excess return of the factor portfolio: $\mathbb{E}[r_F - r_f]$ where r_F is the return on the factor portfolio.

1. Multifactor Models

Two Models

We now have two different multifactor models. Be careful because they look similar, but they have slightly different uses.

1. Returns are determined by surprises in macroeconomic factors:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \epsilon$$

where F_1 and F_2 are *surprises* in these factors, relative to expectations.

2. The fair return on a security is determined by the risk-premiums associated with macroecomonic factors:

$$E[r_i] = r_f + \beta_{i1}\mathbb{E}[r_1 - r_f] + \beta_{i2}\mathbb{E}[r_2 - r_f]$$

where $\mathbb{E}[r_1 - r_f]$ is the risk premium associated with a factor portfolio.

1. Multifactor Models - Macro Factors

Application 2

Consider the following APT model of returns for a particular stock:

Factor	Factor Beta	Factor Risk Premium
Inflation	1.2	6%
Industrial Production	0.5	8
Oil Prices	0.3	3

If T-Bills currently yield 6%, find the expected rate of return of this stock if the market views it as fairly valued.

Note that the expected return may subsequently be adjusted due to *surprises* in these variables.

2. Fama-French Factors

The Model

The most famous multifactor model is the Fama-French model:

$$R_{i,t} = \alpha_i + \beta_{m,i} \cdot R_{m,t} + \beta_{SMB,i} \cdot SMB_t + \beta_{HML,i} \cdot HML_t + \epsilon_{i,t}$$

SML = return of small stocks minus return of large stocks
HML = return of value stocks minus return of growth stocks

Procedure for creating factor portfolios: Sort firms by size or book-to-market. Create portfolios of the bottom and top deciles. The SML factor portfolio is the return of small companies (bottom decile) minus the return of the large companies (top decile). The HML factor portfolio is the return of the value companies (top decile in book-to-market) minus the growth portfolios (bottom decile in book-to-market).

2. Fama-French Factors

Book value v. Market value - Definitions

Book Value: The net worth of a common equity according to the firm's balance sheet. Can be calculated as Assets - Liabilities. (also referred to as 'Book Equity' or BE)

Market Value: The net worth of a common equity according to the market price. Can be calculated by Shares Outstanding X Market Price. (also referred to as 'Market Equity' or ME)

Value Stock: A stock with a **high** BE/ME ratio. Equivalently, a stock with a **low** ME/BE ratio.

Growth Stock: A stock with a **low** BE/ME ratio. Equivalently, a stock with a **high** ME/BE ratio.

2. Fama-French Factors

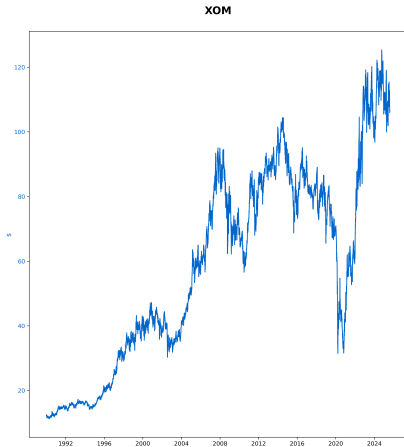Book value v. Market value - Examples

Table: Book Equity and Market Equity

	XOM	MSFT	KHC
Assets ($bn)	451.90	562.20	90.27
Liabilities ($bn)	182.10	240.70	40.60
Shares Outstanding (bn)	4.30	7.30	1.18
Price ($ as of 7-7-25)	112.00	494.40	426.50
Book Equity, BE ($bn)	269.80	321.50	49.67
Market Equity, ME ($bn)	481.60	3609.12	503.27
BE/ME	0.56	0.09	0.10
High or Low	High	Low	Low
Price to Book	1.79	11.23	10.13
Value or Growth	Value	Growth	Growth

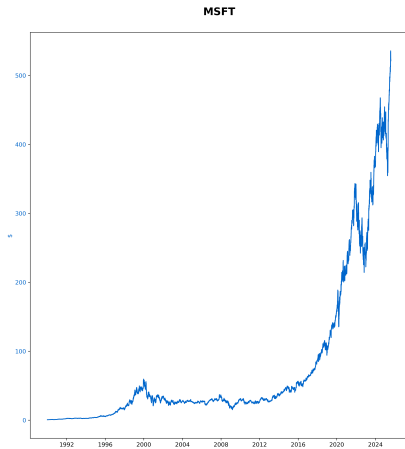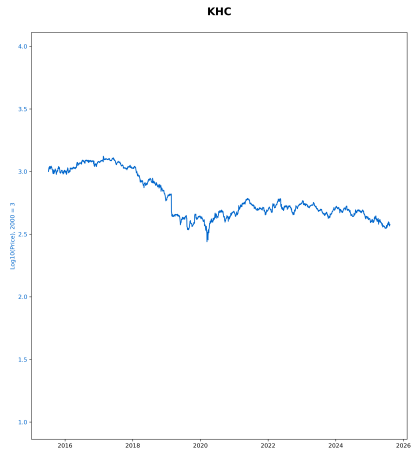Data Source: Yahoo Finance as of 7-7-2025

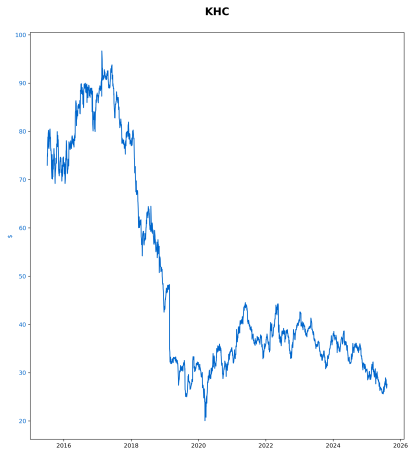# 2. Fama-French Factors
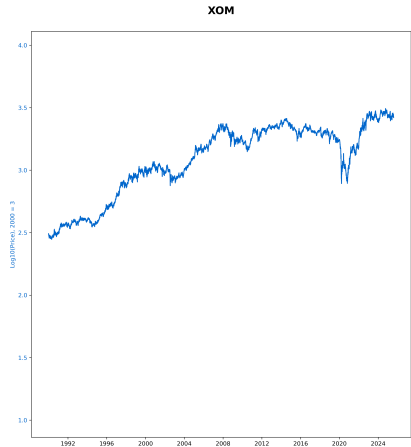
Book value v. Market value - Examples

2. Fama-French Factors

Book value v. Market value - Examples

2. Fama-French Factors

Book value v. Market value - Examples

KHC

KHC
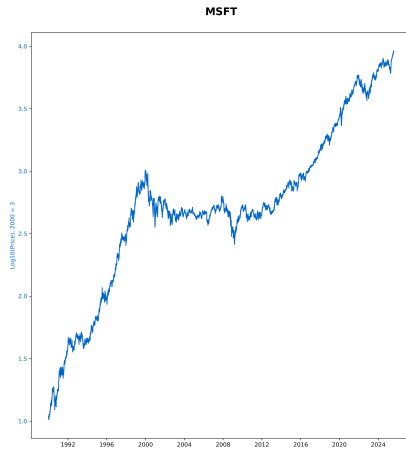
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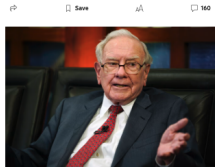
Kraft-Heinz (KHC)

BUSINESS

Warren Buffett: I Overpaid for Kraft Heinz

Berkshire Hathaway doesn't plan to change its ownership stake in Kraft Heinz despite the struggles, he says

By Nicole Friedman
Updated Feb. 25, 2019 9:23 pm ET   WSJ PRO

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Berkshire Hathaway Chairman and CEO Warren Buffett said he wouldn't buy more Kraft Heinz, even after its share prices slumped last week, because 'it isn't worth as much." PHOTO: NATI HARNIK/ASSOCIATED PRESS

Warren Buffett said Berkshire Hathaway Inc. BRK.A 0.37% ▲ and 3G Capital overpaid in 2015 when they helped form Kraft Heinz Co., KHC -1.32% ▼ but he has no plans to sell.

The acknowledgment, made during a CNBC interview Monday, followed a disclosure Saturday that Kraft Heinz contributed a $1.7 billion loss to Berkshire in 2018. Kraft Heinz last week wrote down the value of some of its biggest brands, disclosed an investigation by federal securities regulators and slashed its dividend.

Berkshire owns 27% of Kraft Heinz. Mr. Buffett said Berkshire doesn't plan to change its stake.

Berkshire's Quarterly Earnings Drop on Insurance Results, Currency Moves

Warren Buffett's conglomerate continued to stockpile cash and refrain from buybacks during market's climb to records

By Heather Gillers Follow
Updated Aug. 2, 2025 at 11:47 am ET

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Berkshire Hathaway Chairman Warren Buffett. PHOTO: REUTERS

≡ Quick Summary   ⌄
• Berkshire Hathaway's quarterly earnings decreased by 4% after results from its insurance businesses weakened.
View more

Warren Buffett's Berkshire Hathaway reported a 4% drop in quarterly earnings after results from the conglomerate's insurance businesses weakened from a year ago.

In a period that saw the S&P 500 index reach record highs, Berkshire added to its cash stockpile and refrained from repurchasing its own shares. The company, which maintains a large investment portfolio, was a net seller of stocks for an 11th straight quarter.

Berkshire, whose businesses include insurer Geico and railroad BNSF Railway, posted a net income of $12.37 billion, or $8,601 per Class A share equivalent, for the second quarter. That compared with a net income of $30.3 billion, or $21,122 a share, in the year-earlier period.

Berkshire wrote down its investment in Kraft Heinz by $3.8 billion, after taxes. Berkshire's representatives stepped down from the food company's board in May, limiting what information Berkshire now gets about Kraft. Kraft has been buffeted by inflation and changing consumer tastes, and is now exploring possible strategic moves, including a plan to break itself up. Those factors led Berkshire to restate the holding at fair market value, the company said.
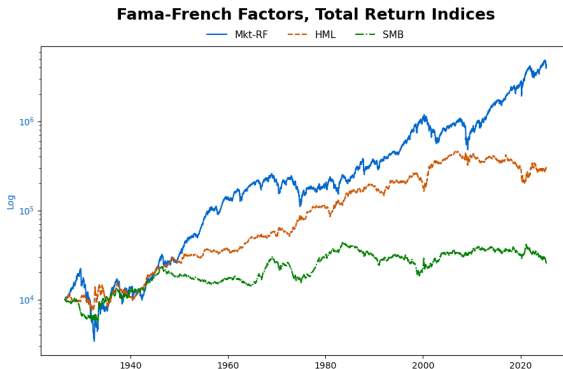
2. Fama-French Factors

Factor Portfolios

Table: Average Returns for Fama-French BE/ME Portfolios, 1926-2025

decade	Lo 10	Hi 10	Difference
1926-2025	0.91	1.30	0.39
1920	1.73	1.27	-0.46
1930	0.49	1.14	0.64
1940	0.62	1.73	1.12
1950	1.57	1.69	0.12
1960	0.74	0.94	0.20
1970	0.27	1.36	1.09
1980	1.10	1.90	0.80
1990	1.64	1.45	-0.19
2000	-0.16	0.85	1.01
2010	1.29	0.85	-0.44
2020	1.55	0.94	-0.61

2. Fama-French Factors

Factor Performance

Fama-French Factors, Total Return Indices

Conclusion: Both SML and HML are factor portfolios that have significant risk premiums associated with them. Investors have been rewarded for bearing exposure to these factor portfolios.

Data Source: Ken French data library

2. Fama-French Factors

Application

In order to estimate the expected return for stock *i*, estimate the following model:

$$r_i = \alpha_i + \beta_{im}r_m - r_f + \beta_{i,SML}SML + \beta_{i,HML}HML + \epsilon$$

Then use the estimated risk premiums for *SML* and *HML*, along with the estimated risk premium of the market, to calculate the expected return of security r_i.

Note that just because there has been a risk premium on a factor in the past doesn't guarantee there will be a risk premium on the factor in the future.

3. Smart Beta

Blackrock iShares Factors

https://www.ishares.com/us/strategies/smart-beta-investing

3. Smart Beta

Blackrock iShares Factors

	EQUITY **iShares MSCI USA Value Factor ETF** VLUE	EQUITY **iShares MSCI USA Quality Factor ETF** QUAL	EQUITY **iShares MSCI USA Momentum Factor ETF** MTUM	EQUITY **iShares MSCI USA Size Factor ETF** SIZE
REMOVE ALL 🗑				
∧ OVERVIEW				
Asset Class	Equity	Equity	Equity	Equity
Sub Asset Class	Large/Mid Cap	Large/Mid Cap	Large/Mid Cap	Large/Mid Cap
Investment Objective	The iShares MSCI USA Value Factor ETF seeks to track the investment results of an index composed of U.S. large- and mid-capitalization stocks with value characteristics and relatively lower valuations.	The iShares MSCI USA Quality Factor ETF seeks to track the investment results of an index composed of U.S. large- and mid-capitalization stocks with quality characteristics as identified through certain fundamental metrics.	The iShares MSCI USA Momentum Factor ETF seeks to track the investment results of an index composed of U.S. large- and mid-capitalization stocks exhibiting relatively higher price momentum.	The iShares MSCI USA Size Factor ETF seeks to track the investment results of an index composed of U.S. large- and mid-capitalization stocks with relatively smaller average market capitalization.
Benchmark Index	MSCI USA Enhanced Value Index (USD)	MSCI USA Sector Neutral Quality Index (USD)	MSCI USA Momentum SR Variant Index (USD)	MSCI USA Low Size Index
As of	Aug 07, 2025	Aug 07, 2025	Aug 07, 2025	Aug 07, 2025
Net Assets	$6,686,957,733	$52,865,260,490	$17,824,846,363	$357,856,145
As of	Current Prospectus	Current Prospectus	Current Prospectus	Current Prospectus
Gross Expense Ratio	0.15%	0.15%	0.15%	0.15%
Net Expense Ratio	0.15%	0.15%	0.15%	0.15%

3. Smart Beta

Blackrock iShares Factors

	EQUITY iShares MSCI USA Value Factor ETF `VLUE`	**EQUITY** iShares MSCI USA Quality Factor ETF `QUAL`	**EQUITY** iShares MSCI USA Momentum Factor ETF `MTUM`	**EQUITY** iShares MSCI USA Size Factor ETF `SIZE`
REMOVE ALL 🗑	🗑	🗑	🗑	🗑

⌃ PERFORMANCE QUARTER-END

As of	Jun 30, 2025	Jun 30, 2025	Jun 30, 2025	Jun 30, 2025
YTD Cumulative NAV Return	8.69%	3.14%	16.75%	4.95%
1 Yr Avg Annual NAV Return	12.62%	8.33%	24.72%	14.15%
5 Yr Avg Annual NAV Return	12.42%	15.26%	14.14%	13.70%
10 Yr Avg Annual NAV Return	8.47%	12.99%	14.29%	10.83%
Incept Avg Annual NAV Return	9.60%	13.20%	14.82%	11.47%
YTD Cumulative Price Return	8.59%	3.12%	16.68%	4.80%
1 Yr Avg Annual Price Return	12.50%	8.19%	24.60%	14.14%
5 Yr Avg Annual Price Return	12.42%	15.27%	14.14%	13.72%
10 Yr Avg Annual Price Return	8.47%	12.99%	14.27%	10.83%
Incept Avg Annual Price Return	9.59%	13.20%	14.82%	11.48%