Fixed Income, continued

Reference: Bodie et al, Ch 14

Econ 457

Week 9-b

Outline

Review of Bond Yields, from previous class

1. Geometric Series
2. Perpetuities and Coupon Bonds
3. Default Risk and Corporate Bonds
4. Prepayment Risk and Mortgage Bonds
5. Indexed bonds and TIPS
6. Practice

0. Yield - Review

The bond pricing formula is:

$$\text{Bond Price} = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

The bond coupons (C) and face value (FV) are determined at the time the bond is issued and generally do not change (TIPS are the exception, see last section of this lecture.)

The bond price and the yield (y) are set by the market. These can and will change frequently due to changes in investor preferences and expectations.

0. Yield - Review

- The **yield to maturity** is the single interest rate that makes the present value of a bond's payments equal to its price. It is the *expected* rate of return that will be earned on the bond if it is bought now and held to maturity, assuming all coupons are reinvested and earn the yield to maturiy.
- The **current yield** is the annual coupon payment divided by the bond price. As with yield to maturity, this is an indicator of *expected* return of the bond.
- The **holding period return** for a bond is defined in the normal way: income return plus capital gain return.
- The **realized compound return** for a bond incorporates reinvestment of coupon payments. It will equal the yield to maturity only in the case that coupon payments are reinvested at the same yield to maturity. If the reinvestment rate changes through the life of the bond (likely) it may not equal the yield.

0. Yield - Review

Example

Consider a 2-year bond issued at par that has a 10% annual coupon.

- Verify that if the first coupon is reinvested at a 10% rate, then the annualized rate of return on the bond is equal to 10%.
- Remember that if the coupon rate is equal to the yield, the price is equal to the par value.
- Now, imagine that at the end of year 1, market rates drop to 5%.
  - What is the realized compound return on this bond if the first coupon is reinvested at a 5% interest rate?
  - What is the price of the bond at the end of year 1 if the yield falls to 5%? (Note: requires Excel or financial calculator)
  - What is the holding period return on the bond in the first year? In the second year? (Note: requires Excel or financial calculator)

0. Yield - Review

Example

REALIZED COMPOUND RETURN

	Scenario 1: Yields stay at 10%				Scenario 2: Yields fall to 5%			
	Coupon Payments	FV of Coupons	Principal	Total Return	Coupon Payments	FV of Coupons	Principal	Total Return
10/23/25	$ -				$ -			
10/23/26	$ 100	$ 110			$ 100	$ 105		
10/23/27	$ 100	$ 100	$ 1,000	$ 1,210	$ 100	$ 100	$ 1,000	$ 1,205
			Return	21.0%			Return	20.5%
			Annualized	10.0%			Annualized	9.8%

HOLDING PERIOD RETURN

	Yield	Coupon	Price	1y HPR	Yield	Coupon	Price	1y HPR
10/23/25	10%		$ 1,000		10%		$ 1,000	
10/23/26	10%	$ 100	$ 1,000	10%	5%	100	$ 1,048	15%
10/23/27	10%	$ 100	$ 1,000	10%	10%	100	$ 1,000	5%

1. Geometric Series

Foundation Material

A **geometric series** is a sum of terms where each term is a constant multiple of the previous term.

General Form:

$$S = a + ar + ar^2 + ar^3 + \ldots = \sum_{n=0}^{\infty} ar^n$$

where a is the first term and r is the common ratio.

Convergence: If $|r| < 1$, the infinite series converges to:

$$S = \frac{a}{1 - r}$$

Proof of Convergence

If $|r| < 1$, then $S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

Proof: Multiply both sides by r:

$$rS = ar + ar^2 + ar^3 + \ldots$$

Subtract: $S - rS = a$ (note that all the other terms cancel)

Factor out the S on the left hand side: $S(1 - r) = a$

Therefore: $S = \frac{a}{1-r}$

2. Perpetuities and Coupon Bonds

Value of Perpetuity

The present value of perpetuity can be represented by a geometric series:

$$PV(\text{perpetuity}) = \frac{c}{(1+r)} + \frac{c}{(1+r)^2} + \frac{c}{(1+r)^3} + \dots$$

Where c is the payment made in each period and r is the appropriate discount rate.

Assume that $r > 0$ and let $d = \frac{1}{(1+r)} < 1$. Note that the first payment is discounted, so we need to start the series at $i = 1$ instead of $i = 0$.

Continued...

2. Perpetuities and Coupon Bonds

Value of Perpetuity, Continued

...Contiued.

With the expression for d and the adjustment to $i = 1$ start, we can now use the same proof as the previous slide:

$$\begin{aligned} PV(perpetuity) &= c\sum_{i=1}^{\infty} d^i \\ &= c\left(\sum_{i=0}^{\infty} d^i - 1\right) \\ &= c\left(\frac{1}{1-d} - 1\right) = c\frac{d}{(1-d)} \\ &= c\frac{(\frac{1}{1+r})}{(1-\frac{1}{1+r})} \\ &= \frac{c}{r} \end{aligned}$$

2. Perpetuities and Coupon Bonds

Value of Finite Coupon Stream

A similar approach gives the following formula for a finite geometric series (as before, write out all the terms to $(N-1)$, then multiply by r, then subtract and see what cancels.):

$$\sum_{i=1}^{N} d^i = \frac{1-d^N}{1-d}$$

We can use this finite geometric series formula to define a pricing formula for a bond that pays coupon C for N periods.

$$PV(\text{coupons}) = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^N}$$

2. Perpetuities and Coupon Bonds

Bond Valuation Formula

Again, substitute $d = \frac{1}{1+r}$ and the present value of the coupons

$$\begin{aligned} PV(\text{coupons}) &= C \cdot \frac{d(1-d^N)}{1-d} \\ &= C \cdot \frac{\frac{1}{1+r}\left(1-\frac{1}{(1+r)^N}\right)}{1-\frac{1}{1+r}} \\ &= C \cdot \frac{1-\frac{1}{(1+r)^N}}{r} \end{aligned}$$

Complete Bond Price: Coupons plus principal repayment

$$P = PV(\text{coupons}) + PV(\text{principal})$$

$$P = C \cdot \frac{1-\frac{1}{(1+r)^N}}{r} + \frac{F}{(1+r)^N}$$

2. Perpetuities and Coupon Bonds

Bond Valuation Formula

Standard Form of Bond Price Equation:

$$P = C \cdot \frac{1 - (1 + r)^{-N}}{r} + F(1 + r)^{-N}$$

This was derived using Geometric Series, along with a clever definitions of d to make it look like a geometric series.

Note that if the coupon rate is equal to the yield (i.e. $C = cFV$ and $c = r$) then $P = FV$.

3. Default Risk and Corporate Bonds

Security

In a bankruptcy proceeding corporate bondholders are paid before equity holders.

Security Hierarchy:

1. *Senior Secured Debt* - backed by specific collateral
2. *Senior Unsecured Debt* - general claim on assets
3. *Subordinated Debt* - paid after senior debt
4. *Preferred Stock* - hybrid security
5. *Common Stock* - residual claim

3. Default Risk and Corporate Bonds

Credit Ratings

Major Rating Agencies: Moody's, S&P, Fitch

Rating Categories:

Moody's	S&P	Quality	Description
Aaa	AAA	Investment	Highest quality
Aa	AA		High quality
A	A		Upper medium grade
Baa	BBB		Medium grade
Ba	BB	Speculative	Lower medium grade
B	B		Speculative
Caa	CCC		Poor standing
Ca	CC		Highly speculative
C	C/D		Default

Investment Grade vs. High Yield:

- BBB-/Baa3 and above = Investment Grade
- BB+/Ba1 and below = High Yield ("Junk Bonds")

3. Default Risk and Corporate Bonds

Credit Ratings

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S&P Strips U.S. of Top Credit Rating

Unprecedented Downgrade Comes After Last-Minute Standoff; Treasury Says Decision Is 'Flawed by a $2 Trillion Error'

By Damian Paletta And Matt Phillips

Updated August 7, 2011

S&P said the downgrade "reflects our opinion that the fiscal consolidation plan that Congress and the administration recently agreed to falls short of what, in our view, would be necessary to stabilize the government's medium-term debt dynamics." It also blamed the weakened "effectiveness, stability, and predictability" of U.S. policy making and political institutions at a time when challenges are mounting.

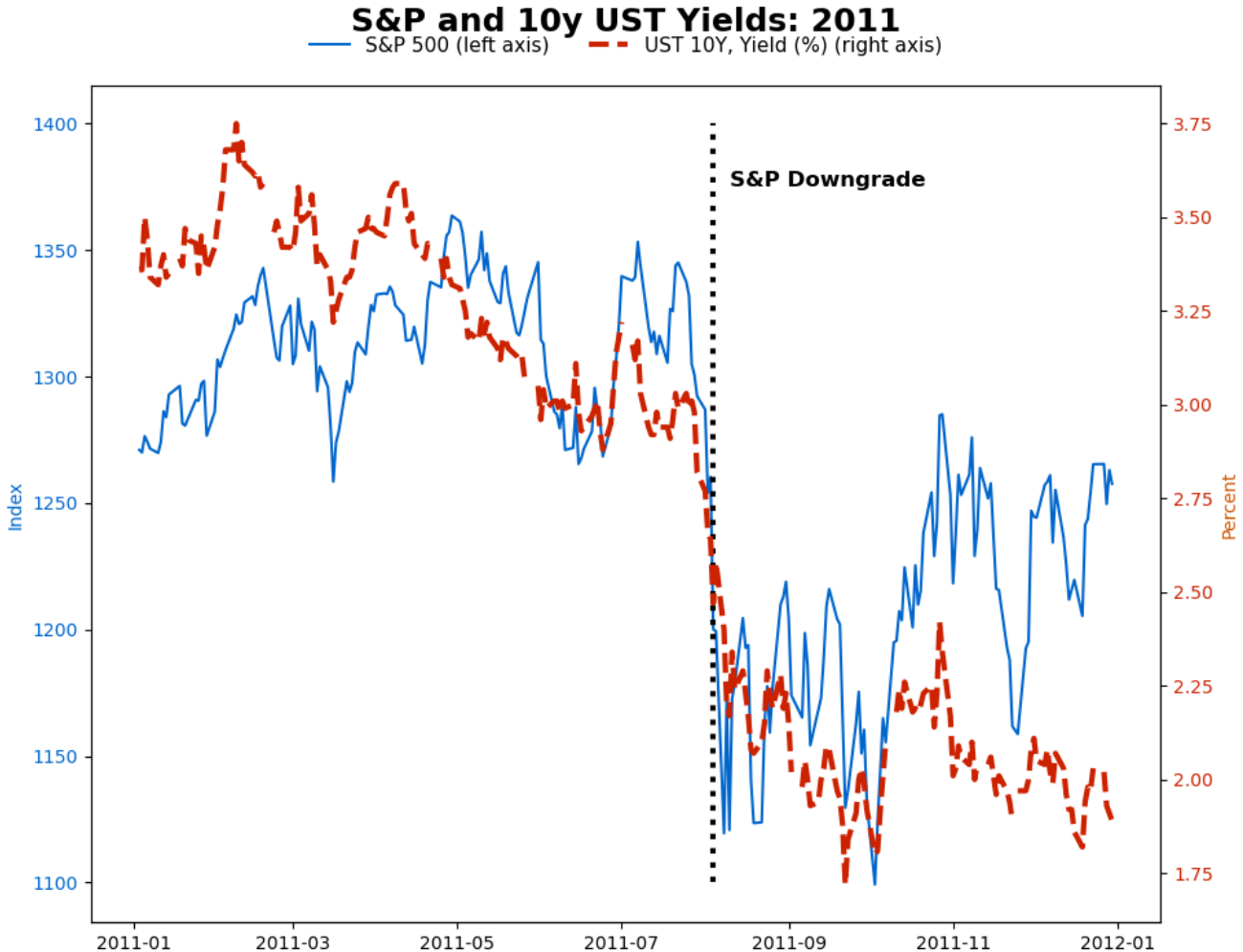
A senior Treasury official Saturday called S&P's move a "$2 trillion mistake" and said there was no justifiable rationale for the decision.

"The magnitude of this mistake—and the haste with which S&P changed its principal rationale for action when presented with this error—raise fundamental questions about the credibility and integrity of S&P's ratings action," John Bellows, acting assistant secretary for economic policy, said in a blog on the Treasury's Web site.

The downgrade will force traders and investors to reconsider what has been an elemental assumption of modern finance. Since July 14, when Standard & Poor's warned it could downgrade the credit rating, analysts have struggled to determine how such a move could affect the financial landscape, given how Treasurys permeate Wall Street and the economy.

3. Default Risk and Corporate Bonds

Credit Ratings



3. Default Risk and Corporate Bonds

Default Risk - Example

Example 2: Start with a 10-year 9% coupon corporate bond with a price of \$750. Yield to maturity = 12%. What if there is a 20% chance of the final payment not being made?

- The *expected value* of the final payment is again only \$800.
- Instead of substituting $\mathbb{E}[FV]$ for FV in the pricing equation, subtract $\frac{\$100}{(1+y)^{20}}$ from the bond price.
- Recalculate the yield using the new bond price and $FV = \$1,000$.
- Yield to maturity is now approx 14%.

Continued...

3. Default Risk and Corporate Bonds

Default Risk - Example

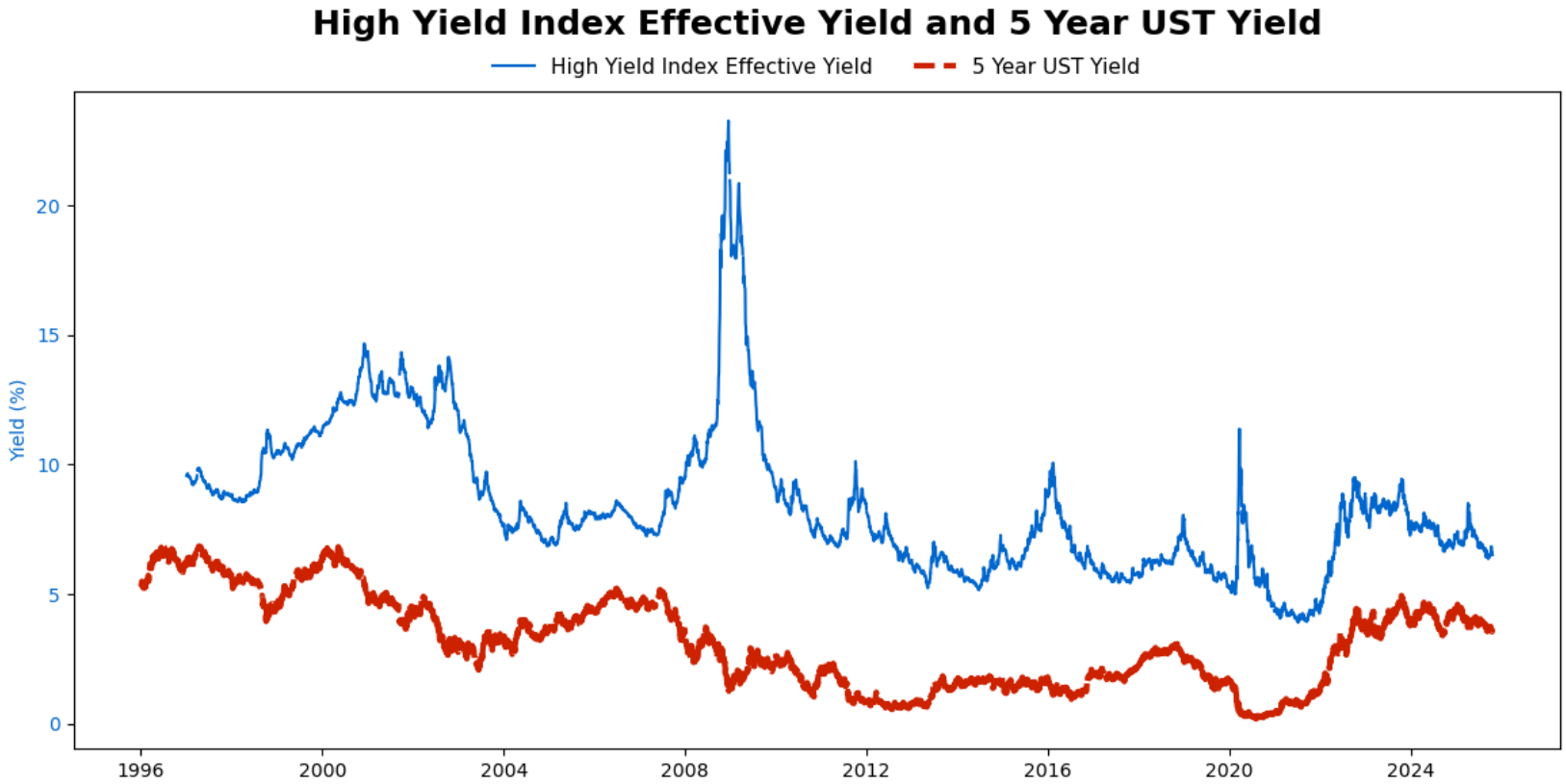
...Continued. What if there is a 20% chance of no payment on the final payment *and* all the coupons?

- Subtract `PRICE(today,10y,0.009,14,10,2)` from the bond price.
- Recalculate the yield using the new bond price and $FV = \$1,000$
- Yield to maturity is now approx 15.5%.

This example illustrates how default risk actually works: default risk causes the expected value of payments to decline, which leads to lower bond prices, and therefore higher yields.

3. Default Risk and Corporate Bonds

Spreads



3. Default Risk and Corporate Bonds

Spread

The **spread** on a corporate bond is defined to be the difference in yields between a corporate bond and a Treasury bond of similar maturity.

While the spread is primarily due to the default risk, it also may reflect differences in liquidity. Corporate bonds are typically less liquid than US Treasuries (i.e. harder to sell when you need cash). Investors may require a discount (lower price, higher yield) to be compensated for lower liquidity.

3. Default Risk and Corporate Bonds

Option Adjusted Spread

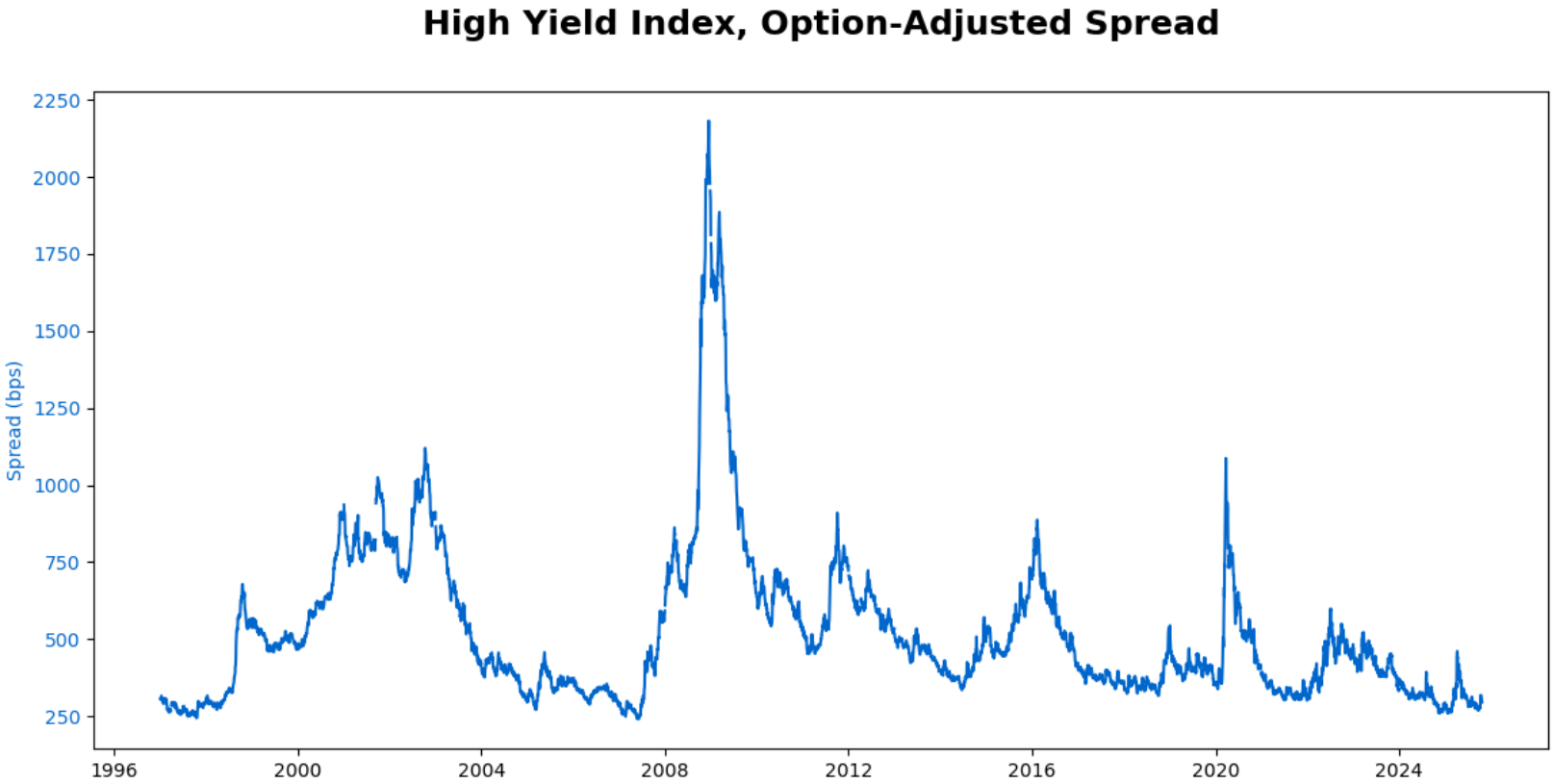
Spreads on corporate bonds could also be due to embedded options. Some corporate bonds have embedded call options. The issuer can repay before the maturity date. This option is valuable to the bond issuer, and therefore requires a lower price (higher yield) for the bond purchaser.

Option Adjusted Spread (OAS):

- Removes the value of embedded options from the spread
- Shows the ”pure” credit and liquidity risk premium
- Calculated using interest rate trees and Monte Carlo simulation
- Formula: Nominal Spread - Option Cost = OAS

3. Default Risk and Corporate Bonds

Spreads



4. Prepayment Risk and Mortgages

Conforming Mortgages

In the United States the standard mortgage has the following characteristics:

- 30-year term
- 20% downpayment, 80% loan-to-value ratio (LTV)
- Fixed-rate, self-amortizing (i.e. fixed monthly payments, no balloon payment at maturity)
- Ability to refinance (prepay at par) anytime

Mortgages that meet these characteristics are referred to as 'conforming' mortgages. Conforming mortgages can be purchased by the GSEs (Fannie Mae and Freddie Mac).

4. Prepayment Risk and Mortgages

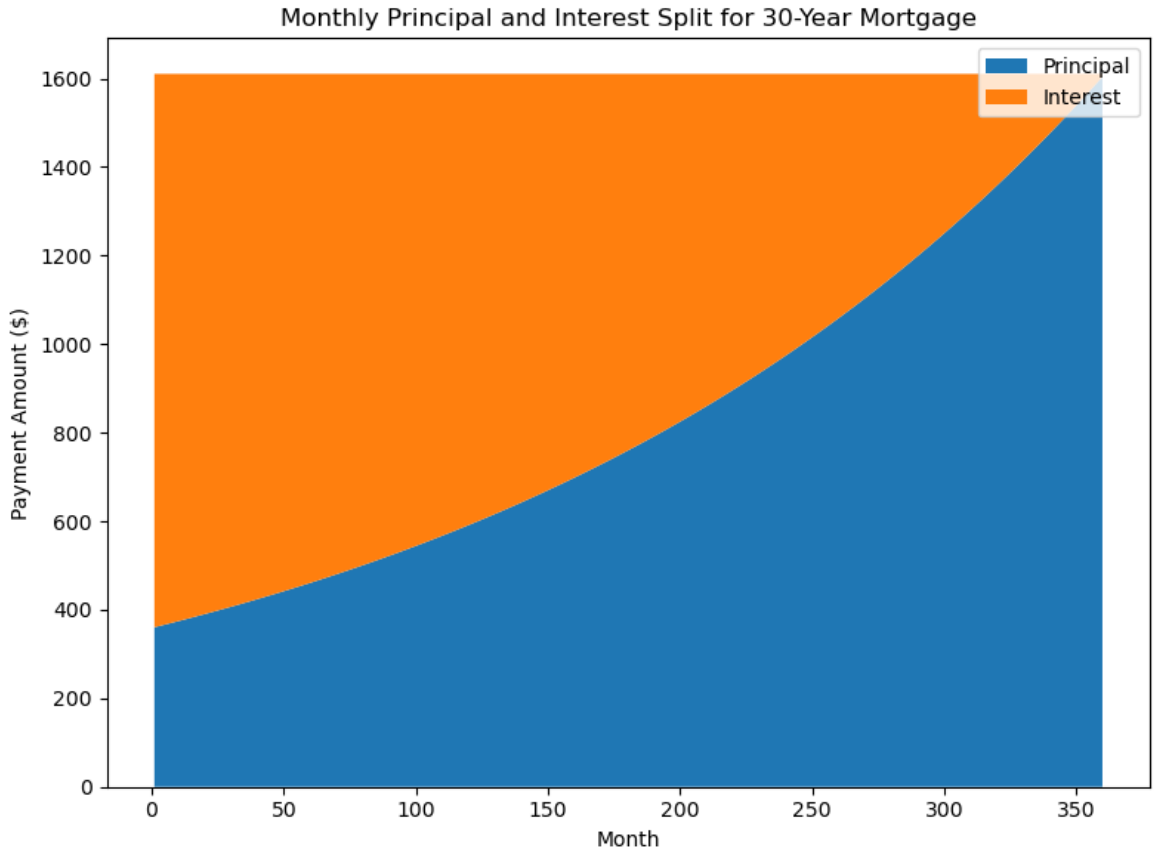
Mortgage Debt Outstanding

The median home price in the United States $419,000 at the end of 2024. The average sales price of new homes was $500,000 in June 2025.

4. Prepayment Risk and Mortgages

Mortgage Payments

Fixed-rate mortgages repay principal each month, instead of requiring a balloon payment at the end of the loan.

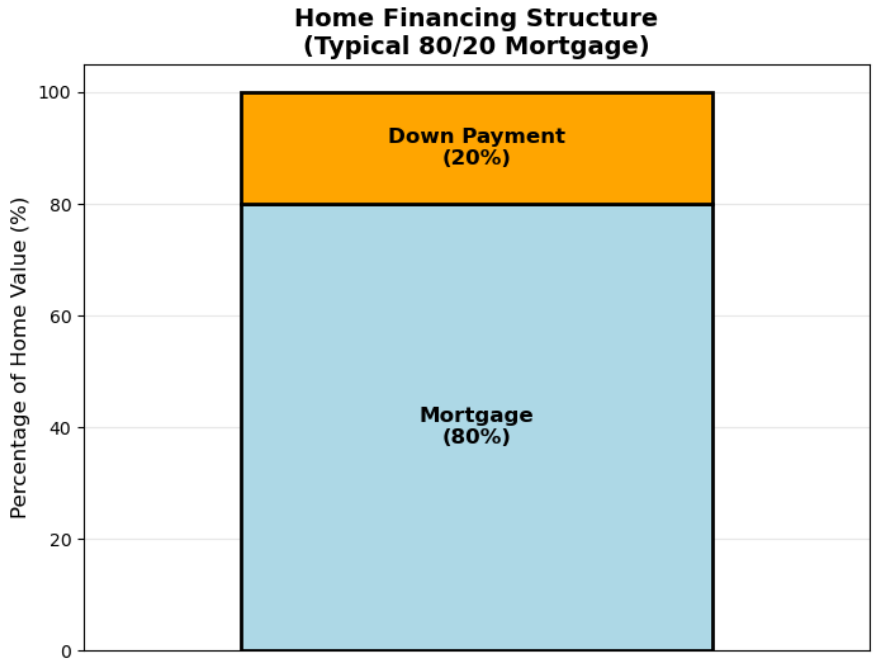


4. Prepayment Risk and Mortgages

Mortgage Leverage

The typical mortgage requires a 20% downpayment. The remaining 80% of the cost of the house is borrowed.

The leverage ratio is 5:1 (total cost / amount of equity).



4. Prepayment Risk and Mortgages

Mortgage Leverage

Example:

- You buy a house for $1,000,000.
- You make a 20% downpayment ($200,000)
- You finance the remainding $800,000 with a 7% mortgage.

Table: Example Home Ownership Returns

Scenario	Price Change	Interest Cost	Return on Equity
HPI +10%	$100,000	$56,000	22.0%
HPI +20%	$200,000	$56,000	72.0%
HPI -10%	-$100,000	$56,000	-78.0%

4. Prepayment Risk and Mortgages

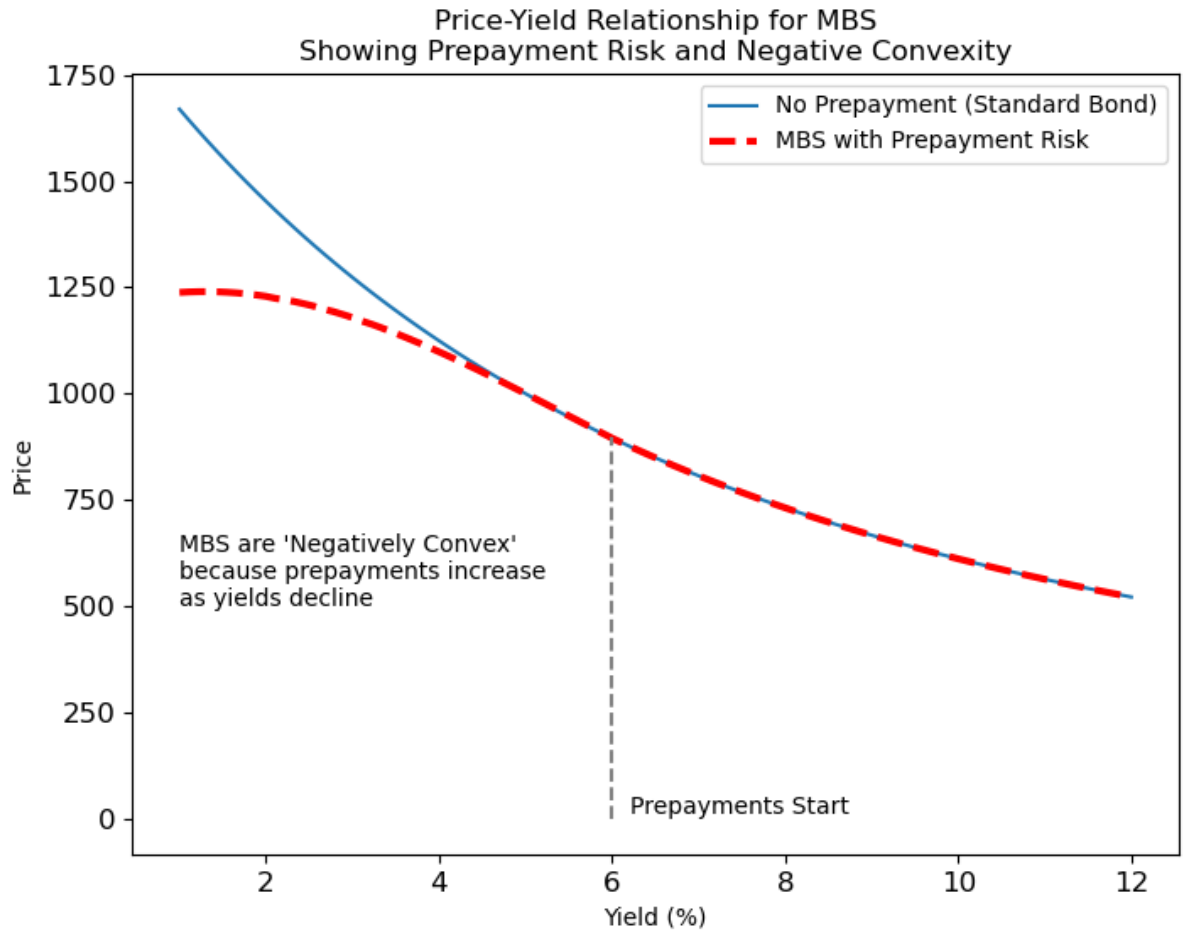
Mortgage - Refinancing and Prepayments

Mortgages can be prepaid at any time. In fact, when interest rates fall it is very common for homeowners to 'refinance' their mortgages. Essentially, they are prepaying the old lender and taking out a replacement loan from a new lender.

While this is very advantageous for the homeowner (they get a lower interest rate), it is equally *disadvantagous* for the lender. They are losing an asset that was paying a high interest rate, which they are unable to replace because market rates are now lower. This is know as **prepayment risk** for the lender.

4. Prepayment Risk and Mortgages

Mortgage - Refinancing and Prepayments



4. Prepayment Risk and Mortgages

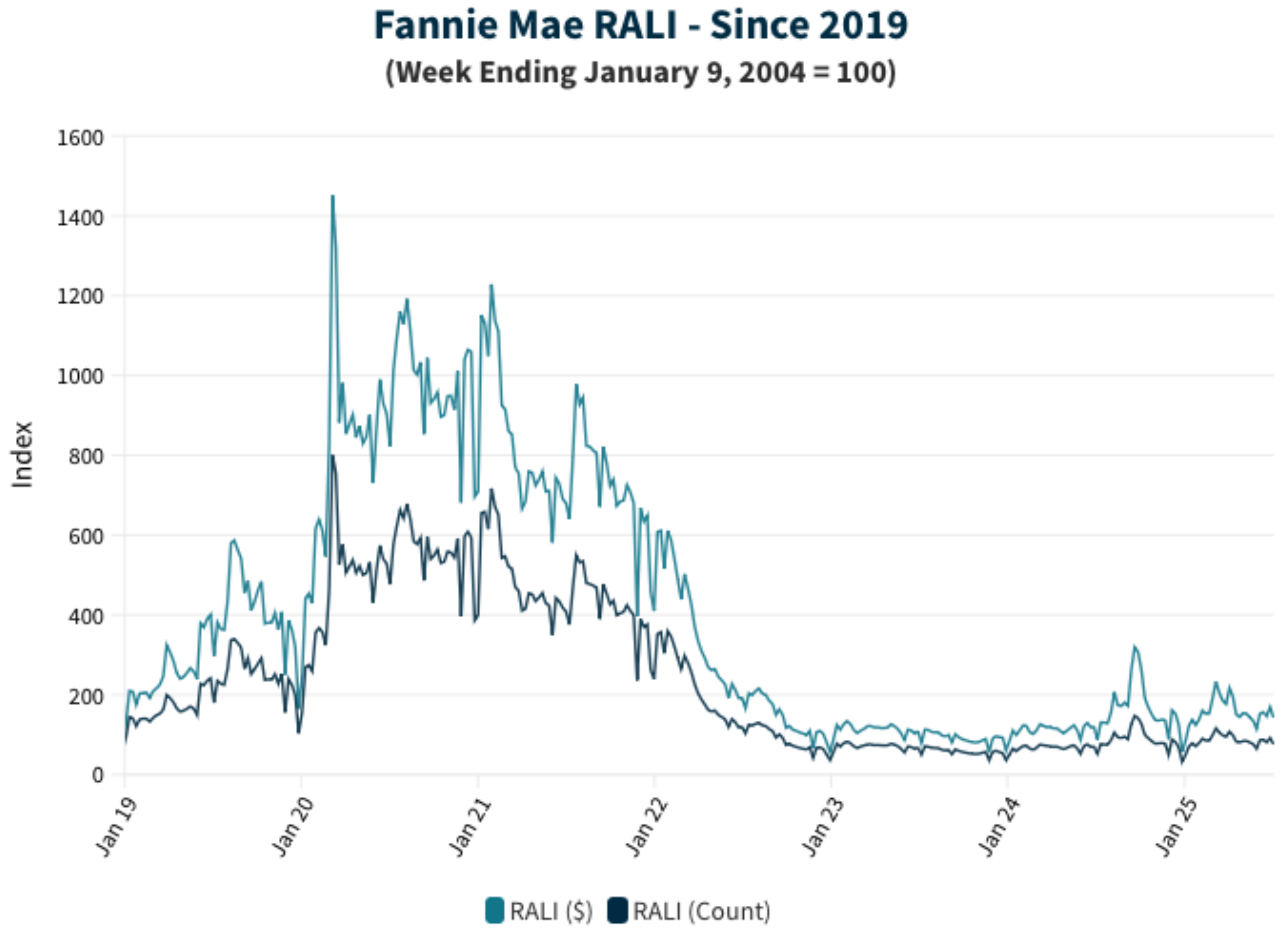
Mortgage - Refinancing and Prepayments

Because this is *disadvantagous* to the lender, the lender will pay a lower price for this bond. As a consequence, the bond will have a higher yield.

Mortgages are an example of a **callable** bond, where the homeowner has the option to call the bond at par. Some corporate bonds also have embedded call options, which give the issuer the right to repay the bond at a specified price. Even though the price is usually somewhat above par, this option is still valuable to the issuer (and disadvanteagous to the lender.)

4. Prepayment Risk and Mortgages

Mortgage - Refinancing and Prepayments



Source: Fannie Mae Refinance Application-Level Index (RALI)

Fannie Mae

5. Indexed Bonds and TIPS

Treasury Inflation-Protected Securities (TIPS)

Definition: TIPS are U.S. Treasury bonds that adjust both principal and interest payments for inflation

Key Features:

- **Principal Adjustment:** Face value increases with Consumer Price Index (CPI)
- **Fixed Real Rate:** Coupon rate is fixed, but applied to adjusted principal
- **Inflation Protection:** At maturity, receive greater of original or adjusted principal
- **Maturities:** Available in 5, 10, and 30-year terms

5. Indexed Bonds and TIPS

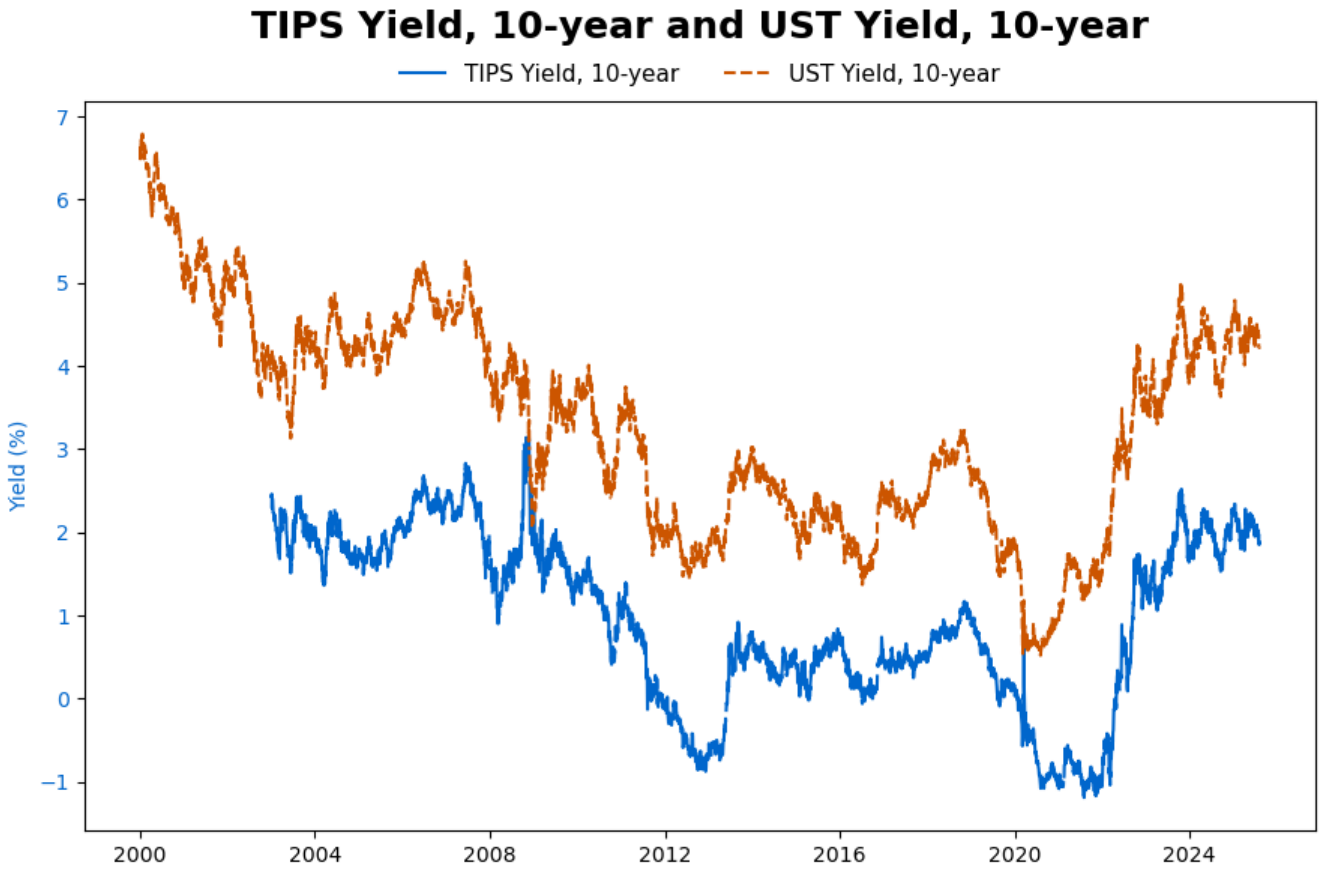
Treasury Inflation-Protected Securities (TIPS)

Example:

- $1,000 TIPS with 2% coupon, 3% inflation
- Year 1: Principal = $1,030, Interest = $20.60
- Year 2: Principal = $1,061, Interest = $21.22

5. Indexed Bonds and TIPS

TIPS - Yields over time



5. Indexed Bonds and TIPS

TIPS - Real Yields

Intuition for TIPS yields:
The return on TIPS will be comprised of the interest payments, the principal payment at maturity, AND the expected increase in the principal value.

The yield on TIPS assumes no increase in principal. This is referred to as the 'real yield'. If the realized increase in principal is equal to the breakeven inflation rate, then the return on TIPS will be exactly equal to the return on nominal Treasury bonds. If the increase in principal is above (below) the breakeven inflation rate, then TIPS will outperform (underperfom) nominal Treasury bonds.

5. Indexed Bonds and TIPS

TIPS - Breakeven inflation rate

$$\text{Breakeven Inflation} = \text{Nominal Treasury Yield} - \text{TIPS Real Yield}$$

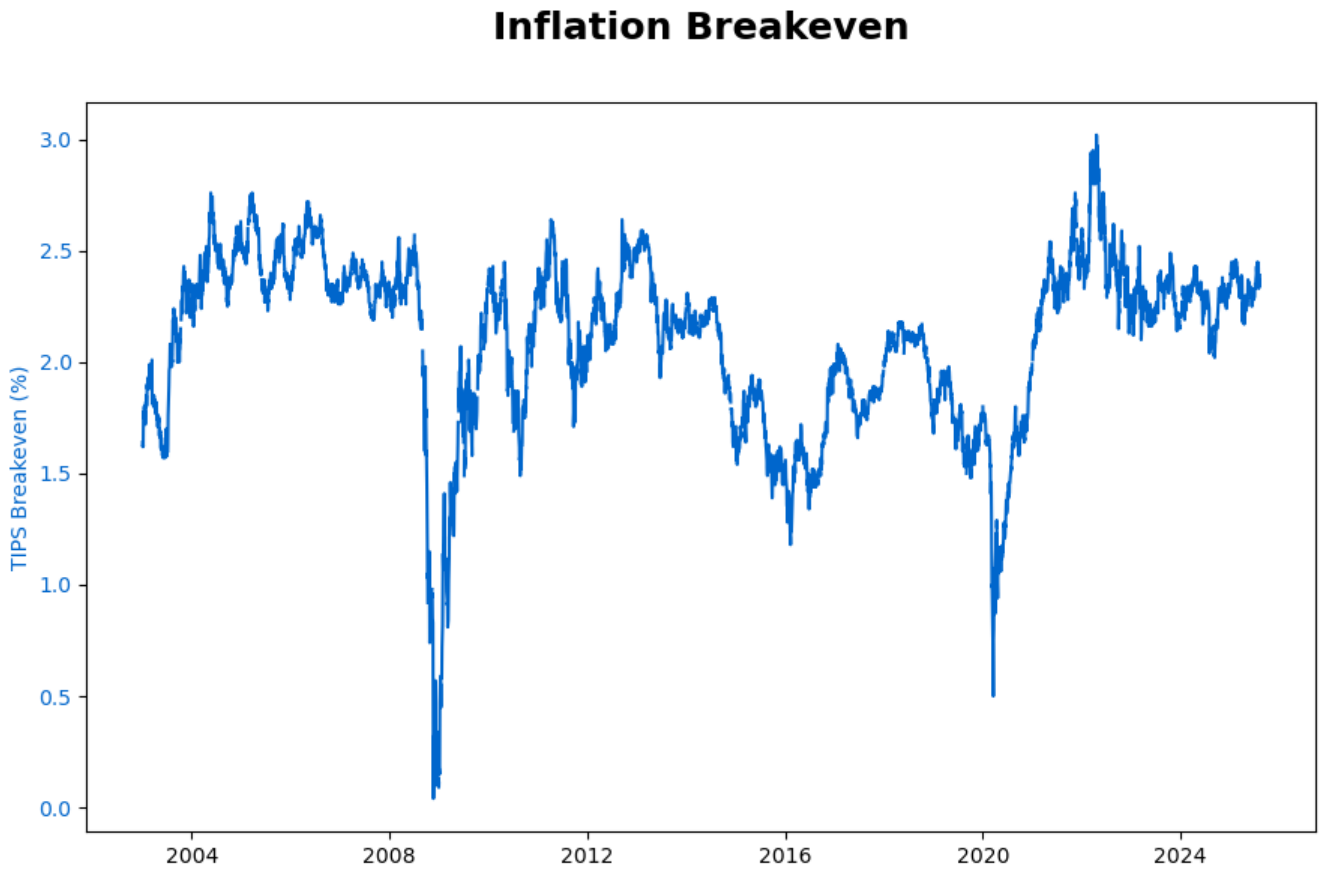
Interpretation:

- Market's expectation of average inflation over the bond's life
- If actual inflation is greater than breakeven: TIPS outperform nominal bonds
- If actual inflation is less than breakeven: Nominal bonds outperform TIPS
- Indifference point between TIPS and nominal Treasury investing

Example: 10-year Treasury yield: 4.5%, 10-year TIPS yield: 2.0%, Breakeven inflation: 4.5% - 2.0% = 2.5%

5. Indexed Bonds and TIPS

TIPS - Breakevens over time



6. Practice

Practice Questions

1. Two bonds have identical times to maturity and coupon rates. One is callable at 105, the other at 110. Which has the higher yield to maturity? Why?
2. Consider a bond with a 10% coupon and a yield to maturity of 8%. If the bond's yield to remains constant, then in one year will the bond price be higher, lower, or unchanged? Why?
3. Consider a bond paying a coupon rate of 10% per year semiannually when the market interest rate is only 4% per half-year. The bond has three years until maturity. (a) Find the bond's price today, (b) find the bond's price six months from now (assuming now change in market interest rate), (b) What is the total (6-month) return on the bond?

6. Practice

Practice Questions

4 Fill in the table below for the following zero-coupon bonds, all of which have par values of $1,000

Price	Maturity	Yield to Maturity
$400	20	-
$500	20	-
$500	10	-
-	10	10%
-	10	8%
$400	-	8%