


# Low Complexity Near Optimal Hybrid Detectors for Large-Scale MIMO Uplink Systems Based on Complex Support Vector Regression

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## Abstract

This report describes the use of  $\text{\LaTeX}$  to format a thesis. A number of topics are covered: content and organization of the thesis,  $\text{\LaTeX}$  macros for controlling the thesis layout, formatting mathematical expressions, generating bibliographic references, importing figures and graphs, generating graphs in MATLAB, and formatting tables. The  $\text{\LaTeX}$  macros used to format a thesis (and this document) are described.

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Thesis regulations require that contributions by others in the collection of materials and data, the design and construction of apparatus, the performance of experiments, the analysis of data, and the preparation of the thesis be acknowledged.

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# List of Acronyms

16-QAM	16-point Quadrature Amplitude Modulation
3GPP	Third Generation Partnership Project
3GPP2	Third Generation Partnership Project 2
64-QAM	64-point Quadrature Amplitude Modulation
ADSL	Asymmetric Digital Subscriber Line
ARQ	Automatic Repeat Request
WPAN	Wireless Personal Area Network



# Chapter 1

## Introduction

### 1.1 Large MIMO system

One of the biggest challenges the researchers and industry practitioners are facing in wireless communication area is how to bridge the sharp gap between increasing demand of high speed communication of rich multimedia information with high level Quality of Service (QoS) requirements and the limited radio frequency spectrum over a complex space-time varying environment. The promising technology for solving this problem, Multiple Input Multiple Output (MIMO) technology has been of immense research interest over the last several tens of years is incorporated into the emerging wireless broadband standard like 802.11ac [1] and long-term evolution (LTE) [2]. The core idea of MIMO system is to use multiple antennas at both transmitting and receiving end, so that multiplexing gain (multiple parallel spatial data pipelines that can improve spectrum efficiency) and diversity gain (better reliability of communication link) are obtained by exploiting the spatial domain. Large MIMO (also called Massive MIMO) is an upgraded version of conventional MIMO technology employing hundreds of low power low price antennas at base station (BS), that serves several tens of terminals simultaneously. This technology can achieve full potential of conventional MIMO system while providing additional power efficiency as well as system robustness to both unintended man-made interference and intentional jamming. [3] [4].

### 1.2 Large MIMO detections

The price paid for large MIMO system is the increased complexities for signal processing at both transmitting and receiving ends. The uplink detector is one of the key components

in a large MIMO system. With orders magnitude more antennas at the BS, benefits and challenges coexist in designing of detection algorithms for the uplink communication of large MIMO systems. On one hand, a large number of receive antennas provide potential of large diversity gains, on the other hand, complexities of the algorithms become crucial to make the system practical.

Vertical Bell Laboratories Layered Space-Time (V-BLAST) architecture for MIMO system can achieve high spectrum efficiency by spatial multiplexing (SM), that is, each transmit antenna transmits independent symbol streams. However the optimal maximum likelihood detector (MLD) for V-BLAST systems that perform exhaustive search over the transmit symbol vector space has a complexity that increases exponentially with the number of transmitted antennas, which is prohibitive for practical applications.

In order to alleviate this problem, linear detectors (LD) such as zero-forcing (ZF) and minimum mean square error (MMSE) aided by successive interference cancellation with optimal ordering (ZF-OSIC, MMSE-OSIC) are exploited in V-BLAST architecture [5] [6] [7], although ZF-OSIC and MMSE-OSIC can provide significant improvement comparing to their LD counterparts, a common drawback of SIC aided LD is the error propagation effect, which can not be eliminated by ordering. That results in inferior performances comparing to MLD [8] [9] [10].

Sphere Decoder (SD) [11] is the most prominent algorithm that utilizes the lattice structure of MIMO systems, which can achieve optimal performance with relatively much lower complexity comparing to MLD. However, SD has two major shortages that make it problematic to be integrated into a practical systems. The first shortage is SD has various complexities under different signal to noise ratios (SNR), while a constant processing data rate is required for hardware. The second shortage is the complexity of SD still has a lower bound that increases exponentially with the number of transmit antennas and the order of modulation scheme [12]. The fixed complexity sphere decoder (FCSD) [13] makes it possible to achieve near optimal performance with a fixed complexity under different values of SNR. The FCSD inherits the principle of list based searching algorithms, which first generate a list of candidate symbol vectors and then the best candidate is chosen as the solution. The other sub optimal detectors belong to this class include Generalized Parallel Interference Cancellation (GPIC) [14] and Selection based MMSE-OSIC (sel-MMSE-OSIC) [15]. However, all these list based searching algorithms have the same shortage - their complexities increase exponentially with the number of transmit antennas and the order of modulation scheme [15]. Therefore, such algorithms are prohibitive when it comes to a

large number of antennas or a high order modulation scheme, for example in IEEE 802.11ac standard [1], the modulation scheme is 256QAM.

Besides the above detection algorithms designed for conventional MIMO systems, in the last several years, a variety of metaheuristic based local search algorithms invoked from machine learning field [16] have been proposed for large MIMO systems. These algorithms have complexities that are comparable or slightly higher comparing to MMSE detector and near-optimal performance. Such algorithms include likelihood ascend searching (LAS) algorithm and variants [17] [18] [19] and Reactive Tabu search (RTS) algorithms and variants (e.g. Layered Tabu search (LTS) [20], Random Restart Reactive Tabu search (R3TS) [21]). Additionally, there are other algorithms proposed for large MIMO systems including Message passing technique based algorithms (e.g. Belief propagation (BP) detectors based on graphic model and Gaussian Approximation (GA) [22] [23] [24] [25]), Probabilistic Data Association (PDA) based algorithms [26], Monte Carlo sampling technique based algorithms (e.g. Multiple Restart- Mixed Gibbs Sampling (MR-MGS) algorithm [27]) and Element based Lattice Reduction (LR) aided algorithms [28].

Considering MIMO detection from a Combinatorial Optimization (CO) problem viewpoint, as powerful tool for solving CO problems, metaheuristic algorithms [29] are good choices for designing large MIMO detectors, driven by demand of achieving acceptable performance with significantly lower computational complexity. Besides the metaheuristic algorithms that based on local search strategies which use trajectory methods based on single solution, another class of metaheuristic algorithms is defined as population based. The population based metaheuristic algorithms deal with a population of candidate solutions. Intrinsically the population based metaheuristic algorithms can provide wider and more efficient exploration of search space. The major population based metaheuristic algorithms includes Evolutionary Computation (EC) and Ant Colony Optimization (ACO).

Genetic algorithm (GA) is one kind of EC algorithms, which are designed for CO problems. GA mimics the natural evolution process of a population and is powerful tool in searching a solution that is close enough to global optimum [30].

### 1.3 Support Vector Regression

Firmly grounded in framework of statistical learning theory or VC (Vapnik–Chervonenkis) theory, the Support Vector (SV) technique has become a powerful tool to solve real world supervised learning problems such as classification, regression and prediction. The SV

method is a nonlinear generalization of Generalized Portrait algorithm developed by Vapnik in 1960s [31] [32], which can provide good generalization performance [33].

Interest in SV algorithms boosted since 1990s, promoted by the works of Vapnik and co-workers at AT& T Bell laboratory [34] [35] [36] [37] [38] [39]. Moreover, the kernel based methods [33] were proposed in order to extending the SV algorithms to nonlinear learning cases. The input data samples are mapped into some high dimensional feature space (also called Reproducing Kernel Hilbert Space (RKHS)) and then linear tools are applied to the feature mappings of the input data samples. This is equivalent to transforming the nonlinear learning tasks in the original space into the linear learning tasks in high dimensional feature space. The mathematical notion underlying kernel based methods is that of RKHS [33], in which the inner products of the feature mappings can be simply replaced by the computationally economical kernel functions. Because SV algorithms only deal with the inner products of the the feature mappings. Therefore, by kernel based methods, it is sufficient to use the specific kernel functions based on the RKHS discarding the actual structure of the feature space.

Based on the principle of structural risk minimization [40], the  $\epsilon$ -SVR [36] [41] solves an original constraint optimization problem (primal objective problem) by transforming it into a Lagrange dual form (dual objective problem), which is a Quadratic Programming (QP) problem.

Decomposition method is an efficient method to solve QP problems with large scale data sets, which decomposes a large QP problem into sub QP problems and solve them in an iterative manner [?] [43]. Decomposition process is performed by sub set selection solver, which refers to a set of algorithms that separate the optimization variables into two sets S and N, S is the work set and N is the freezing set. In each iteration, only the optimization variables in the work set is updated while keeping other optimization variables fixed. Based on this principle, Sequential Minimal Optimization (SMO) algorithm [42] was proposed, which can solve the sub QP problems analytically instead of using the time consuming numerical steps. An important issue of the sub set selection solvers is the selection of the work set. One strategy is to choose Karush-Kuhn-Tucker (KKT) condition violators, ensuring the final converge [43]. The SMO algorithm restricts the size of the work set to two. A method to train SVM without offset was proposed In [44], with the comparable performance to the SVM with offset. A set of sequential single variable work set selection strategies, which require  $O(n)$  searching time are proposed. The optimal double variable work set selection strategy, which performs exhaustively searching, however,

requires  $O(n^2)$  searching time. The authors demonstrate that with the combination of two different proposed single variable work set selection strategies, convergence can be achieved by a iteration time that is as few as optimal double variable work set selection strategy.

The mathematical foundation of kernel based methods is RKHS which is defined in complex domain, however most of the practitioners are dealing with real data sets. In communication and signal processing area, the channel gains, signals, waveforms etc. are all represented in complex form. Recently, a pure complex SVR & SVM based on complex kernel was proposed in [45], which can deal with the complex data set purely in complex domain. The results in [45] demonstrate the better performance as well as reduced complexity comparing to simply split learning task into two real case by real kernels. Based on this work, we derive a complexity-performance controllable detector for large MIMO systems based on a dual channel complex SVR (CSVR). The detector can work in two parallel real SVR channels which can be solved independently. Moreover, only the real part of kernel matrix is needed in both channels. This means a large amount of computation cost saving can be achieved. Based on the discrete time MIMO channel model, in our regression model, this CSVR-detector is constructed without offset, Therefore, for each real SVR without offset, Two types of combined single optimization variable selection strategy are proposed based on the work in [44]. The proposed combined single optimization variable selection strategy can approximate optimal double optimization variable selection strategy. The former one can achieve as few as iteration time while enjoy significant speed gain in each iteration.

## 1.4 Thesis Contribution

## 1.5 Thesis Outline

## Chapter 2

# Theoretical Analysis of Channel Hardening Phenomenon

### 2.1 System Model

## Chapter 3

# Complex Support Vector Preliminary Detector (CSVDP) for Large-Scale MIMO Uplink Systems

Decomposition methods were proposed to solve this QP problem by decomposing it into sub QP problems and solving them iteratively [42]. Therefore, the computational intensive numerical methods can be avoided. Decomposition is performed by sub set selection solver, which refers to a set of algorithms that separate the optimization variables (Lagrange multipliers) into two sets S and N, S is the work set and N contains the remaining optimization variables. In each iteration, only the optimization variables in the work set is updated while keeping other optimization variables fixed. The Sequential Minimal Optimization (SMO) algorithm [42] is an extreme case of the decomposition solvers. An important issue of the sub set selection solvers is the selection of the work set. One strategy is to choose Karush-Kuhn-Tucker (KKT) condition violators, ensuring the final converge [43]. The SMO algorithm restricts the size of the work set to two. A method to train SVM without offset was proposed In [44], with the comparable performance to the SVM with offset. A set of sequential single variable work set selection strategies, which require  $O(n)$  searching time are proposed. The optimal double variable work set selection strategy, which performs exhaustively searching, however, requires  $O(n^2)$  searching time. The authors demonstrate that with the combination of two different proposed single variable work set selection strategies, convergence can be achieved by a iteration time that is as few as optimal double variable work set selection strategy.

The mathematical foundation of kernel based methods is RKHS which is defined in

### 3 Complex Support Vector Preliminary Detector (CSVPD) for Large-Scale MIMO Uplink Systems

~~complex domain, however most of the practitioners are dealing with real data sets. In~~ communication and signal processing area, the channel gains, signals, waveforms etc. are all represented in complex form. Recently, a pure complex SVR & SVM based on complex kernel was proposed in [45], which can deal with the complex data set purely in complex domain. The results in [45] demonstrate the better performance as well as reduced complexity comparing to simply split learning task into two real case by real kernels. Based on this work, we derive a complexity-performance controllable detector for large MIMO systems based on a dual channel complex SVR (CSVSR). The detector can work in two parallel real SVR channels which can be solved independently. Moreover, only the real part of kernel matrix is needed in both channels. This means a large amount of computation cost saving can be achieved. Based on the discrete time MIMO channel model, in our regression model, this CSVSR-detector is constructed without offset, Therefore, for each real SVR without offset, Two types of combined single optimization variable selection strategy are proposed based on the work in [44]. The proposed combined single optimization variable selection strategy can approximate optimal double optimization variable selection strategy. The former one can achieve as few as iteration time while enjoy significant speed gain in each iteration.

#### 3.1 The Algorithm



## Chapter 4

# Mathematical Layout Styles

T<sub>E</sub>X does a marvelous job of setting mathematical formulas, most often choosing pleasing spacing. However, on occasion one should intercede to improve the layout. This chapter defines a few such occasions. In addition, this chapter documents some features of the `amsmath` package which overcome difficulties in typesetting some mathematical forms. The `amsmath` package is documented in *The L<sup>A</sup>T<sub>E</sub>X Companion* [?].

The modified setup is typeset as

$$G(z) = \begin{cases} \frac{P(z)}{1+z^{-1}} & \text{for } p \text{ even,} \\ P(z) & \text{for } p \text{ odd.} \end{cases} \quad (4.1)$$

With the modified definitions, we get the following.

$$\begin{aligned} \mathbf{d}^{(i)} &= \hat{\mathbf{v}}^{(i)} - \hat{\tilde{\mathbf{v}}}^{(i)} \\ \mathbf{n}^{(i)} &= \mathbf{u}^{(i)} - \tilde{\mathbf{v}}^{(i)} \end{aligned} \quad (4.2)$$

# Chapter 5

## Tables

### 5.1 Tables in L<sup>A</sup>T<sub>E</sub>X

Tables of many different sorts can be made with L<sup>A</sup>T<sub>E</sub>X. This chapter gives suggestions on producing tables, along with a number of examples.

To illustrate these rules, here is a table and the L<sup>A</sup>T<sub>E</sub>X input which was used to generate it.

**Table 5.1** Filter specifications

Taps ( $N$ )	Transition Band	Stopband Weighting ( $\alpha$ )	Passband Ripple dB	Stop-band Rejection dB	Ultimate Stop Band dB
8	A	1	0.06	31	31
12			0.025	48	50
16			0.008	60	75
12	B	1	0.04	33	36
16			0.02	44	48
24			0.008	60	78
16	C	1	0.07	30	36
24		1	0.02	44	49
32		2	0.009	51	60
48		2	0.006	50	66
24	D	1	0.1	30	38
48		2	0.006	50	66
64		5	0.002	65	80
48	E	2	0.07	32	46
64		5	0.025	40	51

Transition Code Letter	Normalized Transition Band
A	0.14
B	0.10
C	0.0625
D	0.043
E	0.023

The normalized transition band is the width of the transition band normalized to  $2\pi$ ; that is,  $(\omega_s - \pi/2)/(2\pi)$ .

# Appendix A

## L<sup>A</sup>T<sub>E</sub>X Macros

The L<sup>A</sup>T<sub>E</sub>X commands and macros used in formatting the title page for this document are shown in this appendix.

### A.1 Thesis Preamble

The commands used to create the title page for a thesis are shown below. The McGill University crest is brought in via a macro `McGillCrest` which allows for setting the size and colour of an imported PostScript file which contains the actual crest. The title page also includes a red separator line.

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