# Simplified detection for MIMO systems using diversity maximizing incremental channel partition

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***Abstract*—MIMO communication is a core technology for wireless systems. This work presents an improved fixed complexity detection technique for underdetermined and overdetermined MIMO systems, based on a low complexity, diversity optimal, incremental channel partition. This new algorithm also employs a new efficient tree search implementation of the precancellation step. Because of these improvements, this algorithm provides substantial complexity gains while maintaining quasi-optimal performance, making it attractive for practical application.**

## I. Introduction

Spatial multiplexing MIMO systems, such as V-BLAST [1], can achieve very high bandwidth efficiencies. However, the optimum maximum likelihood (ML) receiver for V-BLAST has a complexity that increases exponentially with the number of transmit antennas [2]. The fixed-complexity sphere decoder (FSD) of [3] has a lower complexity and achieves quasi-ML performance. However, the FSD algorithm is not applicable for underdetermined systems.

Recently, in [4], we proposed a fixed complexity algorithm called Sel-MMSE-OSIC which improved the "Generalized Parallel Interference Cancellation" (GPIC) algorithm of [5], and was guaranteed to provide optimal performance at high SNR. Contrary to the FSD algorithm, Sel-MMSE-OSIC is suitable for both overdetermined and underdetermined systems. However, the complexity of the Sel-MMSE-OSIC algorithm is 2 to 3 times higher than the FSD algorithm's for overdetermined systems.

In this paper, we propose a new improved algorithm over the Sel-MMSE-OSIC scheme, that uses a simpler incremental channel partition technique and employs a tree based approach to generate candidates during precancellation. We show analytically and numerically that the proposed algorithm can provide optimal performance at high SNR. Furthermore, this algorithm provides substantial complexity gains over many previously reported schemes.

## II. System model and review of the Sel-ZF/MMSE-OSIC algorithm

Consider a communication system employing $N_T$ transmit and $N_R$ receive antennas over a Rayleigh flat fading channel that is time invariant over one use. Assuming perfect synchronization, the corresponding discrete-time system model is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_R\times 1}$ is the received signal vector, $\mathbf{H} = [\mathbf{h}_1 \ \ \dots \ \ \mathbf{h}_{N_T}] \in \mathbb{C}^{N_R\times N_T}$ has independent identically distributed (i.i.d), circularly symmetric complex Gaussian (CSCG) zero mean entries, of unit variance. Furthermore, $\mathbf{n} \in \mathbb{C}^{N_R\times 1}$ is the additive noise vector whose components are i.i.d CSCG with zero mean and variance $\sigma_n^2$. The components of the transmitted signal vector $\mathbf{x}$ are equally probable and mutually independent, drawn from a symbol constellation $\mathcal{A}$ with zero mean and average symbol energy $\sigma_s^2$. The SNR is defined here as $\mathrm{SNR} = \dfrac{\sigma_s^2}{\sigma_n^2}$.

Let $\mathbf{H}$ and $\mathbf{x}$ be partitioned such that $\mathbf{H}\mathbf{x} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2$, with $\mathbf{H}_1 \in \mathbb{C}^{N_R\times N}$ composed of a subset of $N$ (where $\max\{1, N_T - N_R\} \le N \le N_T - 1$) columns of $\mathbf{H}$, and the remaining columns forming $\mathbf{H}_2 \in \mathbb{C}^{N_R\times L}$, where $L = N_T - N$. There are a total of $N_U = \binom{N_T}{N}$ such possible subsets. Let $M = |\mathcal{A}|$ be the cardinality of the set $\mathcal{A}$, and let $\{\tilde{\mathbf{x}}_1^1, \tilde{\mathbf{x}}_1^2, \dots, \tilde{\mathbf{x}}_1^K\}$ denote the elements of the set $\mathcal{A}^N$ ($K = |\mathcal{A}^N| = M^N$). Then the precancellation step of the Sel-ZF/MMSE-OSIC algorithm of [4] can be summarized as follows:

1) For each candidate $\tilde{\mathbf{x}}_1^k$ ($k \in \{1, \dots, K\}$), compute $\mathbf{y}^k = \mathbf{y} - \mathbf{H}_1\tilde{\mathbf{x}}_1^k = \mathbf{H}_2\mathbf{x}_2 + \mathbf{H}_1\big(\mathbf{x}_1 - \tilde{\mathbf{x}}_1^k\big) + \mathbf{n}$.
2) Use either ZF or MMSE based, linear or ordered, SIC detection with the model
$$\mathbf{y}^k = \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}\,, \tag{2}$$
to obtain the corresponding candidate $\mathbf{d}_2^k \in \mathcal{A}^L$ for $\mathbf{x}_2$.
3) As a final decision, choose the pair $(\tilde{\mathbf{x}}_1^{k^*}, \mathbf{d}_2^{k^*})$ where
$$k^* = \arg\min_{k\in\{1,2,\dots,K\}} \|\mathbf{y}^k - \mathbf{H}_2\mathbf{d}_2^k\|^2\,. \tag{3}$$

The channel partition rule has a significant impact on the performance of the candidates generation process. It was shown in [6] that for both linear ZF and ZF-SIC with any ordering rule, the maximal achievable diversity gain is $d_L = (N_R - L + 1)(N_T - L + 1)$. To obtain the desired matrix $\mathbf{H}_2$, [4] adopts the diversity optimal selection rule introduced in [7], and shows that the condition

$$N \ge N_{min} \tag{4}$$

with

$$N_{min} = \left\lfloor \sqrt{N_R + \frac{1}{4}\big(N_R - N_T\big)^2} - \frac{1}{2}\big(N_R - N_T\big) \right\rfloor \tag{5}$$

is sufficient to provide optimal ML performance at high SNR. This work builds on the algorithm above, and introduces a lower complexity alternative with similar error rate performance. In the rest of this paper, we adopt the ZF criterion

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for the sake of simplicity.

## III. Proposed algorithm

### A. Simplified channel partition

The channel partition procedure used in [4] is diversity optimal, but requires the inversion of $N_U$ matrices of dimension $L$. In [7], a low complexity incremental antenna selection rule was employed to prove the lower bound on the maximal achievable diversity with antenna selection and spatial multiplexing. We propose to apply the same rule for the channel partition step. Specifically, the columns of $\mathbf{H}_2$ are selected as follows [7]:

1) The first selected antenna is $\mathbf{h}_{k_1}$ with
$$k_1 = \arg \max_{l=1,\ldots,N_T} \|\mathbf{h}_l\|^2 \tag{6}$$
2) At the $i$th step ($i = 2, \ldots, L$), $\mathbf{h}_{k_i}$ is selected if
$$k_i = \arg \max_{l \notin \{k_1,\ldots,k_{i-1}\}} \frac{1}{\left([\mathbf{H}_{i-1}\ \mathbf{h}_l]^H [\mathbf{H}_{i-1}\ \mathbf{h}_l]\right)^{-1}_{i,i}} \tag{7}$$
where $\mathbf{H}_{i-1} = [\mathbf{h}_{k_1} \ldots \mathbf{h}_{k_{i-1}}]$.

Recently, the so-called Greedy QR algorithm, which is a Householder reflections based QR implementation of the proposed algorithm was considered in [8]. This efficient implementation is adopted in this work.

Now let $\tilde{\lambda}_1 \geq \cdots \geq \tilde{\lambda}_L > 0$ be the singular values of the selected matrix $\tilde{\mathbf{H}}_2$. Similarly, let $\lambda_1 \geq \cdots \geq \lambda_{N_{RT}} > 0$ where $N_{RT} = \min\{N_R, N_T\} \geq L$ be the singular values of the original channel matrix $\mathbf{H}$. Then the following result is derived in [8]
$$\lambda_i^2 \prod_{j=1}^{i} \frac{1}{(N_R - j + 1)(N_T - j + 1)} \leq \tilde{\lambda}_i^2 \leq \lambda_i^2, \quad i = 1, \ldots, L\ .$$
In particular, we have
$$\alpha \lambda_L^2 \leq \tilde{\lambda}_L^2 \leq \lambda_L^2 \tag{8}$$
where $\alpha = \prod_{j=1}^{L} \frac{1}{(N_R - j+1)(N_T - j+1)} > 0$ is a positive constant. Using (8), we can show that this selection procedure is diversity optimal for both linear ZF and ZF-SIC detection with any ordering rule, and hence similar conclusions as in [4] apply here. In particular, if $N$ can be chosen such that $d_L = N_R$, then full diversity performance is achieved. Furthermore, $N \geq N_{min}$, with $N_{min}$ given by (5), is a sufficient condition to provide optimal performance at high SNR.

For the detection order, [7] adopts the following rule: the symbol corresponding to $\mathbf{h}_{k_L}$ is detected first, then the one corresponding to $\mathbf{h}_{k_{L-1}}$, etc. However, this approach yields $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_L$, where $\rho_i$ is the post processing SNR associated with the $i$-th detected layer. Hence, the first detected layer has poor performance and the detector suffers significantly from error propagation. Therefore, we choose to employ the V-BLAST ordering rule for improved performance (see. e.g. [9]). The proposed low complexity channel partition can now be summarized as follows:

1) Use the Greedy QR algorithm of [8], to obtain the desired $\tilde{\mathbf{H}}_2$, and let $\tilde{\mathbf{\Pi}}$ be the corresponding column permutation matrix such that $\mathbf{H}\tilde{\mathbf{\Pi}} = [\tilde{\mathbf{H}}_2\ \ \tilde{\mathbf{H}}_1] = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}$, where $\tilde{\mathbf{R}} = \begin{bmatrix} \tilde{\mathbf{R}}_2 & \tilde{\mathbf{R}}_3 \\ \mathbf{0} & \tilde{\mathbf{R}}_4 \end{bmatrix}$, with $\tilde{\mathbf{R}}_2 \in \mathbb{C}^{L \times L}$ an upper triangular with positive diagonal elements and $\tilde{\mathbf{Q}} \in \mathbb{C}^{N_R \times N_R}$ unitary. Also let $\tilde{\mathbf{y}} = \tilde{\mathbf{Q}}^H \mathbf{y}$. Note that we do not need to compute $\tilde{\mathbf{Q}}$ explicitly.
2) Compute $\tilde{\mathbf{R}}_2^{-1}$ and obtain $\left(\tilde{\mathbf{H}}_2^H \tilde{\mathbf{H}}_2\right)^{-1} = \tilde{\mathbf{R}}_2^{-1} \left(\tilde{\mathbf{R}}_2^{-1}\right)^H$.
3) Apply the algorithm of [9] to find $\hat{\mathbf{\Pi}}$, the column permutation matrix corresponding to the V-BLAST order associated with $\tilde{\mathbf{H}}_2$.
4) Permute the first $L$ columns of $\tilde{\mathbf{R}}$, and use Givens rotations [10] to obtain
$$\tilde{\mathbf{R}} \begin{bmatrix} \hat{\mathbf{\Pi}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N \end{bmatrix} = \hat{\mathbf{Q}} \begin{bmatrix} \bar{\mathbf{R}} \\ \mathbf{0} \end{bmatrix} \qquad \text{along with} \qquad \bar{\mathbf{y}} = \hat{\mathbf{Q}}^H \tilde{\mathbf{y}}\ ,$$
where $\bar{\mathbf{R}} = [\bar{\mathbf{R}}_1\ \ \bar{\mathbf{R}}_2] \in \mathbb{C}^{N_{RT} \times N_T}$ and $\bar{\mathbf{R}}_1 \in \mathbb{C}^{N_{RT} \times N_{RT}}$ is upper triangular with positive diagonal elements. Define the column permutation matrix $\bar{\mathbf{\Pi}} = \tilde{\mathbf{\Pi}} \begin{bmatrix} \hat{\mathbf{\Pi}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N \end{bmatrix}$. Then, we have $\mathbf{H}\bar{\mathbf{\Pi}} = [\bar{\mathbf{H}}_2\ \bar{\mathbf{H}}_1] = \bar{\mathbf{Q}} \begin{bmatrix} \bar{\mathbf{R}} \\ \mathbf{0} \end{bmatrix}$ and $\bar{\mathbf{y}} = \bar{\mathbf{Q}}^H \mathbf{y}$, where $\bar{\mathbf{Q}} = \tilde{\mathbf{Q}}\hat{\mathbf{Q}}$.

### B. Simplified precancellation

Given $\bar{\mathbf{R}}$ and $\bar{\mathbf{y}}$, the precancellation step can be efficiently implemented in a tree search fashion. The tree has $N_T$ layers, with a total of $V_l$ nodes at layer $l$ ($l = 1, \ldots, N_T$) where,
$$V_l = \begin{cases} M^{N_T - l + 1} & L + 1 \leq l \leq N_T \\ M^N & \text{otherwise} \end{cases}\ . \tag{9}$$

Similar to the FSD tree [3], our tree is fully expanded in the first $N$ layers and for the remaining layers only one child node is considered per parent node. However, contrary to [3], our algorithm does not require $N_T \leq N_R$. Denote the $V_l$ nodes at the $l$-th layer as $v_i^{(l)}$, $i = 0, \ldots, V_l - 1$. Associated with each of these nodes there is a parent node $v_{p_i}^{(l+1)}$ ($p_i \in \{0, \ldots, V_{l+1} - 1\}$) at layer $l+1$. Finally, corresponding to each node $v_i^{(l)}$ is the partial symbol vector $\mathbf{z}_i^{(l)} = [z_i^{(l)}\ \mathbf{z}_{p_i}^{(l+1),T}]^T$ where $z_i^{(l)} \in \mathcal{A}$ and $\mathbf{z}_{p_i}^{(l+1)} \in \mathcal{A}^{N_T - l}$. The precancellation step can then be described as follows:

1) For each node $v_i^{(N_T)}$ at layer $N_T$, compute
$$F_{N_T}\left(v_i^{(N_T)}\right) = \bar{y}_{N_{RT}} - \bar{r}_{N_{RT}, N_T} z_i^{(N_T)} \tag{10}$$
2) For layers $l = N_T - 1, \ldots, N_{RT} + 1$, for each node $v_i^{(l)}$, compute
$$F_l\left(v_i^{(l)}\right) = F_{l+1}\left(v_{p_i}^{(l+1)}\right) - \bar{r}_{N_{RT}, l} z_i^{(l)} \tag{11}$$
3) For each node $v_i^{(N_{RT})}$ at layer $N_{RT}$, compute
$$F_{N_{RT}}\left(v_i^{(N_{RT})}\right) = \bar{r}_{N_{RT}, N_{RT}}^{-1} F_{N_{RT}+1}\left(v_{p_i}^{(N_{RT}+1)}\right) \tag{12}$$
then
$$D_{N_{RT}}\left(v_i^{(N_{RT})}\right) = \bar{r}_{N_{RT}, N_{RT}}^2 \left|F_{N_{RT}}\left(v_i^{(N_{RT})}\right) - z_i^{(N_{RT})}\right|^2 \tag{13}$$

4) For layers $l = N_{RT}-1,\ldots,L+1$, for each node $v_i^{(l)}$, compute
$$F_l\left(v_i^{(l)}\right) = \bar{r}_{l,l}^{-1}\Big(\bar{y}_l - \sum_{j=l+1}^{N_T} \bar{r}_{l,j}\mathbf{z}_{p_i}^{(l+1)}(j-l)\Big) \quad (14)$$
then
$$D_l\left(v_i^{(l)}\right) = \bar{r}_{l,l}^{2}\big|F_l\left(v_i^{(l)}\right) - z_i^{(l)}\big|^2 + D_{l+1}\left(v_{p_i}^{(l+1)}\right) \quad (15)$$

5) For layers $l = L,\ldots,1$, for each node $v_i^{(l)}$, compute $F_l\left(v_i^{(l)}\right)$ and $D_l\left(v_i^{(l)}\right)$ as in (14) and (15) respectively, but $z_i^{(l)}$ is now given by
$$z_i^{(l)} = \mathcal{Q}\Big(F_l\left(v_i^{(l)}\right)\Big) \quad (16)$$

6) Compute
$$k^* = \arg\min_i D_1\left(v_i^{(1)}\right) \quad (17)$$
and the final decision is $\mathbf{z}_{k^*}^{(1)}$ .

If $N_{RT} = N_T$, i.e $N_T \le N_R$, then Steps 1) and 2) above are skipped and in Step 3) (12) is replaced by
$$F_{N_{RT}}\left(v_i^{(N_{RT})}\right) = \bar{r}_{N_{RT},N_{RT}}^{-1}\bar{y}_{N_{RT}} \quad (18)$$
which is independent of $v_i^{(N_{RT})}$. Note that in this case, we compute similar metrics as [3] at each node of the tree. If now $N_{RT} = N_R = L$, then in Step 3) above, we must add $z_i^{(N_{RT})} = \mathcal{Q}\Big(F_{N_{RT}}\left(v_i^{(N_{RT})}\right)\Big)$, Step 4) is then skipped and Step 5) applies now for layers $l = L-1,\ldots,1$.

We now observe that for any layer $l \in \{1,\ldots,N_{RT}-1\}$, and any two nodes $v_a^{(l)}$ and $v_e^{(l)}$ $\left(\{a,e\} \in \{0,\ldots,V_l-1\}^2\right)$, we have:
$$F_l\left(v_a^{(l)}\right) = F_l\left(v_e^{(l)}\right) + \bar{r}_{l,l}^{-1}\sum_{j=l+1}^{N_T}\bar{r}_{l,j}\mathbf{\Delta}_{a,e}^{(l+1)}(j-l) \quad (19)$$
where
$$\mathbf{\Delta}_{a,e}^{(l+1)} = \mathbf{z}_{p_e}^{(l+1)} - \mathbf{z}_{p_a}^{(l+1)}. \quad (20)$$
Then, letting $\mathcal{I}_{a,e}^{(l+1)} = \left\{j : \mathbf{\Delta}_{a,e}^{(l+1)}(j) \neq 0\right\}$, (19) becomes
$$F_l\left(v_a^{(l)}\right) = F_l\left(v_e^{(l)}\right) + \bar{r}_{l,l}^{-1}\sum_{j\in\mathcal{I}_{a,e}^{(l+1)}}\bar{r}_{l,j+l}\mathbf{\Delta}_{a,e}^{(l+1)}(j)\ . \quad (21)$$
Note that $|\mathcal{I}_{a,e}^{(l+1)}| \le N_T - l$, therefore given $\mathbf{\Delta}_{a,e}^{(l+1)}$ and $F_l\left(v_e^{(l)}\right)$, (21) is more efficient than (14). If $N > 1$, then the special structure of the tree can be exploited to provide a more efficient implementation through (21).

Let $\mathcal{A} = \{a_0,\ldots a_{M-1}\}$. For each node $v_i^{(l)}$, $l = N_T,\ldots,L+1$, let $z_i^{(l)} = a_{i \bmod M}$. Then if $N > 1$, for each node $v_i^{(l)}$, $l = N_{RT}-1,\ldots,1$, we compute $F_l\big(v_i^{(l)}\big)$ as follows

1) $F_l\big(v_0^{(l)}\big)$ is computed from (14)
2) Let $(m_{J,i}\ldots m_{0,i})$ be the base-$M$ representation of $i$, i.e $i = m_{J,i}M^J + \cdots + m_{1,i}M + m_{0,i}$ with $m_{j,i} \in \{0,\ldots,M-1\}$ and
$$J = \begin{cases} N_T - l & \text{if } l > L \\ N-1 & \text{otherwise}\end{cases} \quad (22)$$

Then all $F_l\big(v_i^{(l)}\big)$ are computed from $F_l\big(v_{w(i)}^{(l)}\big)$ through (21) with
$$w(i)=\begin{cases}(0\ldots0) & i<(0\ldots10)\\ (0\ldots0) & i=(0\ldots10)\\ \quad\vdots & \quad\vdots\\ (0\ldots100) & (0\ldots100)<i<(0\ldots110)\\ (0\ldots100) & i=(0\ldots110)\\ \quad\vdots & \quad\vdots\\ (M-1\ldots M-1\,0) & (M-1\ldots M-1\,0)<i\le(M-1\ldots M-1)\end{cases}.$$
For layers $l = N_{RT}-1,\ldots,L+1$, it can be easily verified that with this strategy, (21) simplifies to:
$$F_l\left(v_i^{(l)}\right) = \begin{cases}F_l\left(v_{w(i)}^{(l)}\right) & \text{if } m_{0,i}\neq 0\\ F_l\left(v_{w(i)}^{(l)}\right) + \bar{r}_{l,l}^{-1}\,\bar{r}_{l,N_T-k_i+1}\left(a_0 - a_{m_{k_i,i}}\right) & \text{otherwise}\end{cases}$$
where $k_i$ is the index of the last nonzero element in $(m_{J,i}\ldots m_{1,i}\,m_{0,i})$. For layer $l = L$, we find that (21) simplifies to:
$$F_L\left(v_i^{(L)}\right) = F_L\left(v_{w(i)}^{(L)}\right) + \bar{r}_{L,L}^{-1}\,\bar{r}_{L,N_T-k_i+1}\left(a_0 - a_{m_{k_i,i}}\right).$$
As for the remaining layers, we obtain:
$$F_l\left(v_i^{(l)}\right) = F_l\left(v_{w(i)}^{(l)}\right) + \bar{r}_{l,l}^{-1}\Big(\bar{r}_{l,N_T-k_i+1}\left(a_0 - a_{m_{k_i,i}}\right) + \sum_{j\in\mathcal{I}_{i,w(i)}^{(l+1)}}\bar{r}_{l,j+l}\mathbf{\Delta}_{i,w(i)}^{(l+1)}(j)\Big)\ ,\ l = L-1,\ldots,1\ . \quad (23)$$
Now we note that for these layers, the parent node $v_{p_i}^{(l+1)}$ of node $v_i^l$ is node $v_i^{(l+1)}$, $\forall i$. Therefore, from (20), we get:
$$\mathbf{\Delta}_{i,w(i)}^{(l+1)} = [z_{w(i)}^{(l+1)} - z_i^{(l+1)}\ ,\ \mathbf{\Delta}_{i,w(i)}^{(l+2),T}]^T \quad (24)$$
with
$$\mathbf{\Delta}_{i,w(i)}^{(L+1)} = \mathbf{0}\ . \quad (25)$$
Hence,
$$|\mathcal{I}_{i,w(i)}^{(l+1)}| = \begin{cases}\left|\mathcal{I}_{i,w(i)}^{(l+2)}\right| & \text{if } z_{w(i)}^{(l+1)} = z_i^{(l+1)}\\ 1 + \left|\mathcal{I}_{i,w(i)}^{(l+2)}\right| & \text{otherwise}\end{cases} \quad (26)$$
with
$$|\mathcal{I}_{i,w(i)}^{(L+1)}| = 0\ . \quad (27)$$

### C. Complexity analysis

In this section, we provide a FLOPS count of the proposed algorithm, where a complex addition/substraction costs 2 flops and a complex multiplication/division costs 6 flops.
We can show that the channel partition step costs
$$N_{RT}\Big(24N_TN_R + 8N_{RT}^2 - 12N_{RT}N_R - 12N_{RT}N_T + 29N_R - 8N_T - \frac{21}{2}N_{RT} - \frac{15}{2}\Big) - L\Big(8N_TN_R - 4LN_R - 16LN_T + \frac{20}{3}L^2 + 15N_R - 11N_T - 44L - \frac{68}{6}\Big) + 4N_TN_R - 9N_T - 15 \quad \text{flops}\ . \quad (28)$$
As for the precancellation step, if $N = 1$, then it costs
$$\begin{cases} M\left(4N_T^2 + (5+q)N_T - q - 2\right) + 2N_T + 1 \quad \text{flops} & \text{if } N_{RT} = N_T\\ M\left(4N_T^2 + (5+q)N_T - q - 1\right) + 2N_T - 3 \quad \text{flops} & \text{otherwise}\ .\end{cases}$$
If $N > 1$ then layers $l = N_T,\ldots,N_{RT}+1$ cost $8\sum_{l=N_{RT}+1}^{N_T} M^{N_T-l+1}$ flops. Layer $N_{RT}$ costs $2 + 8M^{N_T-N_{RT}+1}$ flops. The total cost associated with layers $l = N_{RT}-1,\ldots,L+1$ is

TABLE I
FLOPS COUNT OF VARIOUS ALGORITHMS RELATIVE TO ISTP

| System \ $M$ | | 4 | 16 | 64 |
|---|---|---|---|---|
| $N_T = 8$, $N_R = 8$ | Sel-MMSE-OSIC [4] | 6.78 | 4.03 | 3.59 |
| | FSD [3] | 1.14 | 1.50 | 1.57 |
| | SFSD [11] | 0.95 | 0.56 | - |
| $N_T = 4$, $N_R = 2$ | Sel-MMSE-OSIC [4] | 1.73 | 2.02 | 2.07 |

$$2(M-1)+2(N_{RT}-1)(4N_T-2N_{RT}-3)-2L(4N_T-2L-5) + (7M+10)\sum_{l=L+1}^{N_{RT}-1} M^{N_T-l} \quad \text{flops}\,. \tag{29}$$

Layer $L$ requires

$$(17+q)M^N + 8N - 6 \quad \text{flops}\ . \tag{30}$$

The remaining layers cost

$$(L-1)\Big((19+q)M^N + 8(N_T-1) - 4L\Big) + 8\sum_{l=1}^{L-1}\sum_{i=1}^{M^N-1} |\mathcal{I}_{i,w(i)}^{(l+1)}| \text{ flops}\ . \tag{31}$$

Now using (26), we can show

$$0 \le \sum_{l=1}^{L-1}\sum_{i=1}^{M^N-1} |\mathcal{I}_{i,w(i)}^{(l+1)}| \le \frac{1}{2}(L-1)L(M^N-1)\ . \tag{32}$$

Obtaining the final decision requires $M^N - 1$ flops. If $N > 1$ and $N_{RT} = N_T$, then there are no layers above layer $N_{RT}$, and using (18) the new cost associated with this layer is $6M + 4$ flops. If now $N > 1$ and $N_{RT} = N_R = L$, then the new cost of layer $N_{RT}$ is $2 + (8+q)M^N$ flops, (29) and (30) are skipped, and the $2(M-1)$ flops required to construct $\mathcal{A}'$ must be added to (31) if $L > 1$.

Regarding the cost of quantization $q$, we assume the following: $q = 0$ for BPSK or QPSK, since only the sign of the real (and imaginary) part of the decision statistic is required, and $q = 4$ for square $M$-QAM with $M > 4$. We also use the upper bound in (32) to get the maximum cost of the proposed algorithm, which will now be referred to as "Incremental Selection with Tree based Precancellation (ISTP)". In Table I, we present the ratio of the number of flops required by Sel-MMSE-OSIC [4], FSD [3] and SFSD [11] to that required by ISTP for various system configurations. For the $N_T = N_R = 8$ system, ISTP has lower complexity than both Sel-MMSE-OSIC and FSD for all constellation sizes. Indeed, Sel-MMSE-OSIC increases the complexity relative to ISTP by a factor of 3 to 7, and FSD has a complexity increase of 57% and 14% with respect to ISTP for 64QAM and QPSK respectively. The SFSD algorithm on the other hand is 5% and 44% less complex than ISTP for QPSK and 16QAM respectively. It is however worthwhile noting that for each constellation and each system configuration, the appropriate values of the parameters $n_j$ required in the SFSD algorithm can only be found through time consuming computer simulations. In addition, in [11], no results on the robustness of SFSD to the channel model are presented. Furthermore, contrary to ISTP, high SNR optimality is no longer guaranteed for SFSD since it does not necessarily find the FSD output. For the undetermined $N_T = 4$, $N_R = 2$ system, Sel-MMSE-OSIC is about 2 times more complex than ISTP. Neither FSD nor SFSD are applicable for this system configuration.

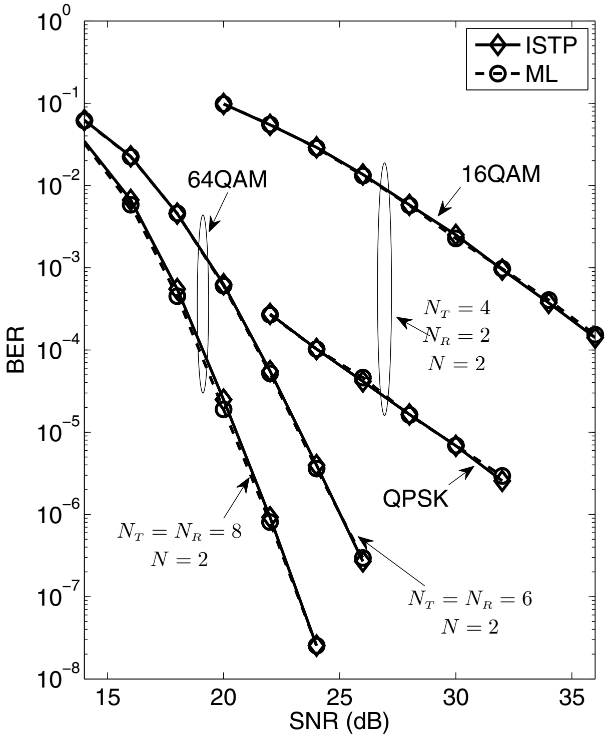


Fig. 1. Performance of the proposed algorithm with $N = N_{min}$

## IV. SIMULATION RESULTS

We consider uncoded systems with QPSK, 16QAM and 64QAM with Gray code labeling. For each channel use, a new vector of $N_T$ randomly generated symbols is transmitted, and the channel varies randomly and independently from one use to another. As a performance measure, we use the average bit error rate (BER). We compare the performance of our algorithm to the optimal ML performance.

We first confirm that with $N = N_{min}$, ISTP achieves ML performance at high SNR. Fig. 1 considers 64QAM with $N_T = N_R = 6$ and $N_T = N_R = 8$, as well as QPSK and 16QAM with $N_T = 4$ and $N_R = 2$. In all cases, we set $N = N_{min} = 2$ for the ISTP algorithm. We observe that the performance of ISTP is nearly indistinguishable from ML's.

Next we consider scenarios where $N$ can be chosen such that $d_L = N_R$. Fig 2 presents the results for the $N_T = N_R = 4$ with $N = 1 < N_{min}$ and the $N_T = N_R = 9$ with $N = 2 < N_{min}$ cases. In both cases, 16QAM is used. For the former, the ISTP algorithm has a performance very similar to ML's, and for the latter the performance loss is within 0.6 dB.

In Fig. 3, QPSK with the following underdetermined systems is considered: $N_T = 3$, $N_R = 2$ with $N = 1 < N_{min}$, $N_T = 5$, $N_R = 3$ with $N = 2 < N_{min}$ and $N_T = 7$, $N_R = 4$ with $N = 3 < N_{min}$. Although theoretically, only full diversity gain can be guaranteed in these cases, the ISTP

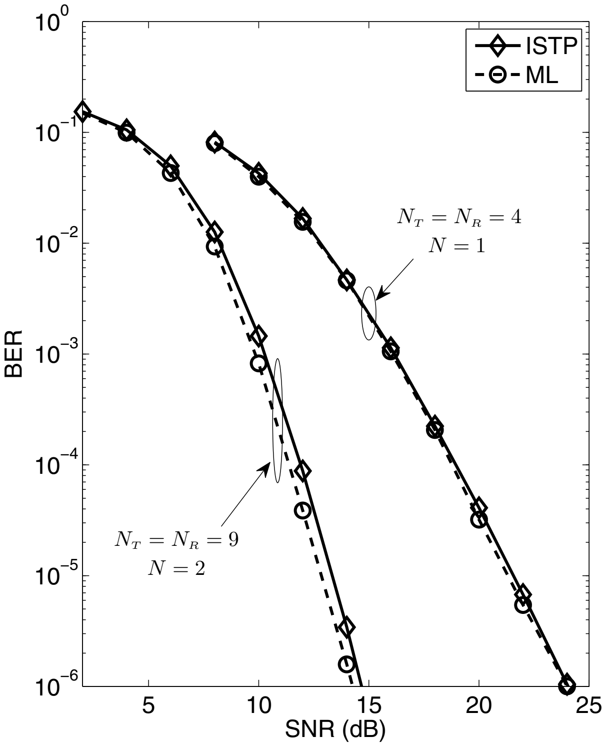


Fig. 2. Performance of the proposed algorithm with $d_L = N_R$ for overdetermined systems with 16QAM

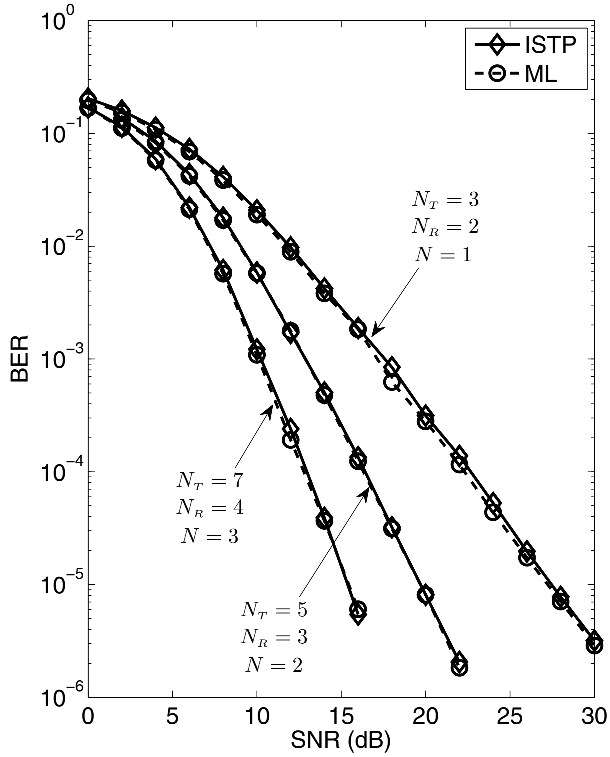


Fig. 3. Performance of the proposed algorithm with $d_L = N_R$ for underdetermined systems with QPSK

algorithm has a performance very similar to ML's.

## V. Conclusion

In this work, building on the technique of [4], we proposed a new fixed complexity algorithm called ISTP, that uses a simpler channel partition scheme based on the Greedy QR antenna selection rule of [8]. Similar to the algorithm of [4], the condition $N \geq N_{min}$ with $N_{min}$ given by (5), is sufficient to guarantee optimal performance at high $\mathrm{SNR}$. Furthermore, the precancellation step is implemented by using a tree similar to that of the FSD algorithm of [3]. A very efficient implementation that exploits the special structure of this tree was then proposed.

Computer simulations results confirmed the excellent performance of the ISTP algorithm. In addition, this work showed that ISTP lowers the complexity with respect to both the Sel-MMSE-OSIC [4] and the FSD [3] algorithms, with the latter being suitable for overdetermined systems only. The SFSD algorithm of [11] can lower the complexity with respect to ISTP, but for each constellation and system configuration, it requires time consuming computer simulations to find suitable values for a certain set of parameters that have to be set properly for its operation. Furthermore, contrary to ISTP, high $\mathrm{SNR}$ optimality is no longer guaranteed for SFSD.

This work considered only uncoded systems for the purpose of comparing with other hard outputs detection schemes. However, the ISTP algorithm can readily be extended to a list based soft outputs detector similar to those proposed in [12] and the references therein.

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