MIMO systems with antenna selection - an overview

Andreas F. Molisch, *Senior Member, IEEE*

***Abstract*— We consider multiple-input - multiple-output (MIMO) systems with reduced complexity. Either one, or both, link ends choose the "best" L out of N available antennas. This implies that only L instead of N transceiver chains have to be built, and also the signal processing can be simplified. We show that in ideal channels, full diversity can be achieved, and also the number of independent data streams for spatial multiplexing can be maintained if certain conditions on L are fulfilled. We then discuss the impact of system nonidealities, like noisy channel estimation, correlations of the received signals, etc.**

I. Introduction

MIMO (multiple-input - multiple output) wireless systems are those that have antenna arrays at both transmitter and receiver. First simulation studies that reveal the potentially large capacities of those systems were already done in the 1980s [1], and later papers explored the capacity analytically [2], [3]. Since that time, interest in MIMO systems has exploded. Layered space-time (ST) receiver structures and coding strategies allow to approach the theoretical capacities; such systems have become known as "spatial multiplexing" or "BLAST" systems [4]. An alternative way for exploiting the multiple antenna elements is the use of transmit and receive diversity purely for link-quality improvement, exploiting the diversity effect. It has been shown that with N_t transmit and N_r receive antennas, a diversity degree of $N_t N_r$ can be achieved [5].

Regardless of the use as "BLAST" or as "diversity" system, the main problem of any MIMO system is the increased complexity, and thus cost, due to the requirement of N_t (N_r) complete RF chains. There are numerous situations where this high degree of hardware complexity is undesirable - this is especially important for the mobile station (MS). Additional antenna elements (patch or dipole antennas) are usually cheap, and the additional digital signal processing is becoming less of a burden as digital processing becomes ever more powerful. However, RF elements like low-noise amplifiers, downconverters, and analog-to-digital converters are a significant cost factor. Due to the reason, there is now great interest in so-called hybrid-selection schemes, where the "best" L out of N antennas are chosen (either at one, or at both link ends), downconverted, and processed. This reduces the number of required RF chains from N to L, and thus leads to significant savings; this comes at the price of a (usually small) performance loss compared to the full-complexity system. In the case that the multiple antennas are used for diversity purposes, the approach is called "hybrid-selection - *maximum ratio combining* (HS-MRC), or sometimes also "generalized selection combining" [6]; if they are used for spatial multiplexing, the scheme is called hybrid-selection/MIMO (HS-MIMO).

[0] A. F. Molisch is with Mitsubishi Electric Research Labs, Cambridge, MA, USA, and the Department of Electroscience, Lund University, Sweden. Email: Andreas.Molisch@ieee.org.

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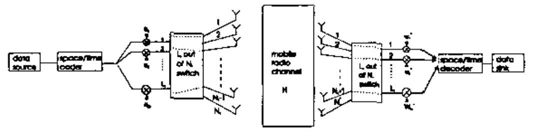

Fig. 1. Blockdiagram of the considered system.

In this paper, we describe the performance that can be achieved with such a system. We furthermore describe how the "best" antennas can be selected in an efficient manner, and what nonidealities have a significant effect on the performance. The paper gives an overview of the results in the literature; more details can be found in the cited papers.

Notation: in this paper, a vector is denoted by an arrow, $\overrightarrow{x}$, a matrix by underline $\underline{A}$. Superscript * denotes complex conjugation; superscript ^{H} denotes the Hermitian transpose.

II. System Model

Figure 1 shows the generic system that we are considering. A bit stream is sent through a vector encoder and modulator. This encoder converts a single bitstream into L_t parallel streams of complex symbols. These streams can contain all the same information (e.g., for a simple transmit diversity system with channel knowledge), can all have independent symbol streams (e.g., in V-BLAST spatial multiplexing), or have partially correlated data streams. Each modulated symbol stream is multiplied by a complex weight u whose actual value depends on the current channel realization. If the channel is unknown at the transmitter, all weights are set to unity. Subsequently, a multiplexer switches the modulated signals to the best L_t out of N_t available antennas.

In a real system, the signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. For the performance computations, these stages, as well as their corresponding stages at the receiver, are usually omitted, and the whole problem is treated in equivalent baseband. Note, however, that exactly these stages are the most expensive and make the use of antenna selection desirable.

Next, the signal is sent over a quasi-static flat-fading channel. We denote the $N_r \times N_t$ matrix of the channel as $\underline{H}$. The output of the channel is polluted by additive white Gaussian noise. At the receiver, the best L_r of the available N_r antenna elements are selected, and downconverted for further processing (note that only L_r receiver chains are available). This further processing can consist of weighting with complex weights $\overrightarrow{w}^*$ (where) and linear combining (if the transmitter uses simple transmit diversity), or space-time-processing and -decoding.

Unless otherwise stated, we assume in the following that

1) The fading at the different antenna elements is independent, identically distributed Rayleigh fading.
2) The fading is frequency flat.
3) The receiver have perfect knowledge of the channel.
4) The channel is quasi-static. The capacity thus becomes a random variable, rendering the concept of a "capacity cumulative distribution function" and "outage capacity [2] a meaningful measure.

The input-output relationship can thus be written as

$$\vec{y} = \underline{H}\vec{s} + \vec{n} = \vec{x} + \vec{n} \tag{1}$$

where $\vec{s}$ is the transmit signal vector, and $\vec{n}$ is the noise vector.

III. Performance Results

A. Diversity

For the case of pure transmit diversity with channel knowledge, $\vec{s} = \vec{u} \cdot s$, where s is a (scalar) symbol. This means that we are just transmitting a single symbol, differently weighted, over the different antenna elements. Similarly, at the receiver, we obtain a "soft" symbol estimate as $\vec{y}\vec{w}^*$, which is then processed (decoded and demodulated) in the usual way.

References [7], [8] analyze the case where there is antenna selection only at one link end, while the other one uses maximum-ratio combining. Define a set of matrices $\widetilde{H}$, where $\widetilde{H}$ is created by striking $N_t - L_t$ columns from H, and $S(\widetilde{H})$ denotes the set of all possible $\widetilde{H}$, whose cardinality is $\binom{N_t}{L_t}$. The achievable SNR γ of the reduced-complexity system is now

$$\gamma = \max_{S(\widetilde{H})} \left(\max_i (\widetilde{\lambda}_i^2) \right) \tag{2}$$

where the $\widetilde{\lambda}_i$ are the singular values of $\widetilde{H}$. The papers give analytical expressions for upper and lower bounds on the SNR, as well as Monte Carlo simulations of the exact results for the SNR as well as the BER and capacity derived from it. The *mean* SNR $E\{\gamma\}$ is computed in [9].

Figure 2 shows the cumulative distribution of the capacity for different values of L_t, with mean SNR $\overline{\Gamma} = 20$ dB, $N_r = 2$. $N_t = 8$. The capacity obtained with $L_t = 3$ is already very close to the capacity of a full-complexity scheme. For comparison, we also show the capacity with pure MRT. Actually, it can be shown that the diversity *degree* obtained with antenna selection is proportional to N, not to L. Also for a space-time-coded system, where the transmitter has no channel knowledge, and the receiver performs antenna selection, the achievable diversity is $N_t N_r$, while a coding gain decreases by up to $10log(N_r/L)$, see [10].

In a highly correlated channel, no diversity gain can be achieved, but all gain is due to improvment of the mean SNR. Thus antenna selection is ineffective, and the SNR gain is only influenced by the number of actually used antenna elements.

B. Spatial Multiplexing

For spatial multiplexing, different data streams are transmitted from the different antenna elements; in the following, we consider the case where the TX, which has no channel knowledge, uses all antennas, while the receiver uses antenna selection [11]; all (linear) weights $\vec{u}$, $\vec{w}$ in Fig. 1 are set equal to unity. The receiver now selects those antennas that allow a maximization of the capacity. As shown in Ref. [2], the capacity is linearly proportional to $\min(N_r, N_t)$. Any further increase of either N_r or N_t while keeping the other one fixed only increases the diversity degree, and consequently allows a *logarithmic* increase of the capacity. Thus, if the number of antennas at one link end is limited e.g. due to space restrictions, a further increase in the antenna number at the other link end does not allow to add statistically independent transmission channels (which would imply linear increase in system capacity), but only provides additional diversity. But we have already seen that antenna selection gives good diversity degree. We can thus anticipate that a hybrid scheme with $N_r > L_r = N_t$ to give good performance. This line of argument can be quantified by performance bounds [11], [12].

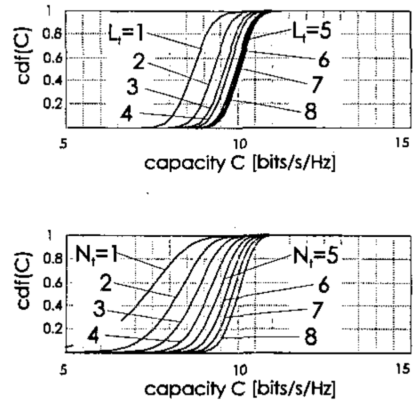

Fig. 2. Upper figure: Capacity of a system with H-S/MRT at the transmitter and MRC at the receiver for various values of L_t with $N_t = 8$, $N_r = 2$, $SNR = 20$ dB. Lower figure: capacity of a system with MRT at transmitter and MRC at receiver for various values of N_t and $N_r = 2$, $SNR = 20$ dB. From [8].

Figure 3 shows the cumulative distribution function (cdf) of capacity for $N_r = 8$, $N_t = 3$, and various L_r. With full exploitation of *all* available elements, an outage capacity of 21.8 bit/s/Hz can be achieved at 20dB SNR. This number decreases gradually as the number of selected elements L_r is decreased, reaching 18.2 bit/s/Hz at $L_r = 3$. For $L_r < N_t$, the capacity decreases drastically, since a sufficient number of antennas to provide N_t independent transmission channels is no longer available.

At low SNRs, diversity can give higher capacities than spatial multiplexing when antenna selection is employed, [13], similar results also hold in the case of strong interference [14].

C. Space-time coded systems

Next, we consider the problem of space-time coded systems with transmit and receive antenna selection, where the transmitter has knowledge about the statistics of the fading. The channelshows correlation, and the correlation matrices are known at the TX. Then, we introduce a modified correlation matrix $\underline{\widetilde{R}}$ which is the submatrix of the total correlation matrix $\underline{R}$ corresponding to the selected antennas. The pairwise error probability (i.e., confusing codeword $\underline{S}^{(i)}$ with codeword $\underline{S}^{(j)}$) for a

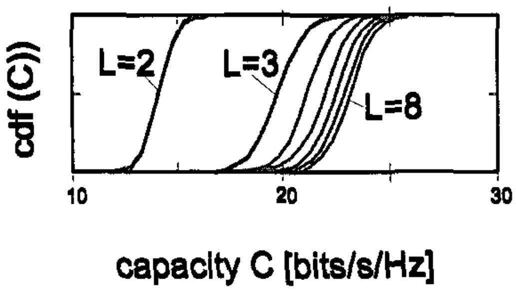

Fig. 3. Capacity for a spatial multiplexing system with $N_r = 8$, $N_t = 3$, SNR= 20 dB, and $L = 2, 3, \ldots 8$.

space-time coded system is derived in [9], [15], [16]. The optimum antenna selection is thus the one that maximizes the determinants of $\tilde{\underline{R}}$. Assume further that the so-called "Kronecker-model" [17], [18] is valid, so that the total correlation can be described by the correlation matrices at TX and RX, $\tilde{\underline{R}}_t$ and $\tilde{\underline{R}}_r$. In that case, the selection at the transmitter and the receiver can be done independently.

IV. Antenna Selection Algorithms

The only mechanism for a truly optimum selection of the antenna elements is an exhaustive search of all possible combinations for the one that gives the best SNR (for diversity) or capacity (for spatial multiplexing). However, for HS-MIMO, this requires on the order of $\binom{N_t}{L_t}\binom{N_r}{L_r}$ computations of determinants, which quickly becomes impractical. For this reason, various simplified selection algorithms have been proposed. Most of them are intended for systems where the selection is done at only one link end.

The simplest selection algorithm is the one that chooses the antenna elements with the largest power, i.e., the largest Frobenius column (or row) norm. For the diversity case, this algorithm is quite effective. However, for spatial multiplexing, this approach breaks down. Only in about 50% of all channel realizations does the power-based selection give the same result as the capacity-based selection. This behavior can be interpreted in geometric terms because the phase shifts between the antenna elements are the decisive factors for capacity, and are far more important than the instantaneous SNR [11].

An alternative class of algorithms has been suggested by [19]. Suppose there are two rows of the H which are identical. Clearly only one of these rows should be selected in $\tilde{H}$. Since these two rows carry the same information we can delete any row of these two rows without losing any information about the transmitted vector. In addition if they have different powers (i.e. magnitude square of the norm of the row), we delete the lower power row. When there are no identical rows we choose next two rows for the deletion whose mutual information is the next highest. In this manner we can have the channel matrix $\tilde{H}$ whose rows have minimum mutual information and have maximum powers. This method achieves capacities within a few tenths of a bit/s/Hz of the capacities with ideal selection. Other algorithms are derived in Ref. [20] and [21].

V. Effect of Nonidealities

A. Low-rank channels

Previously, we have assumed that the channel is i.i.d. complex Gaussian, or shows some correlation at the transmitter and/or receiver. However, in all of those cases is the channel matrix full-rank, and the goal of the antenna selection is to decrease complexity, while keeping the performance loss as small as possible. There are, however, also propagation channels where the matrix $\underline{H}$ has reduced rank [22]. Under those circumstances, antenna selection can actually *increase* the capacity of the channel [23].

B. Frequency-selective channel

In frequency-selective channels, the effectiveness of antenna selection is considerably reduced. For different (uncorrelated) frequency bands, different sets of antenna elements are optimum. Thus, in the limit that the system bandwidth is much larger than the coherence bandwidth of the channel, and if the number of resolvable multipath components is large, all possible antenna subsets become equivalent. This can also be interpreted by the fact that such a system has a very high diversity degree, so that any additional diversity from antenna selection would be ineffective anyway. However, for moderately frequency-selective channels, antenna selection still gives significant benefits. A precoding scheme for CDMA that achieves such benefits is described in [24].

C. Channel estimation errors

We next investigate the influence of erroneous antenna selection on the capacity of the system [25]. We assume that in a first stage, the complete channel transfer matrix is estimated. Based on that measurement, the antennas that are used for the actual data transmission are selected, and the antenna weights are determined. Consider now the following cases: (i) perfect choice of the antennas and the antenna weights, (ii) imperfect antenna selection, but perfect knowledge of the antenna weights, (iii) imperfect choice of the antennas, as well as of the antenna weights at the transmitter, and perfect antenna weights at the receiver, and (iv) imperfect choice of the antenna weights at transmitter and receiver. The errors in the transfer functions are assumed to have a complex Gaussian distribution with SNR_{pilot}, which is the SNR during the transmission of the pilot tones. In our example, the capacity starts to decrease significantly only when the pilot tone SNR is smaller than the SNR for the actual data transmission, see Figure 4.

Another type of channel estimation error can be caused by a limit on the number of bits for the feedback of antenna weights to the TX. This problem is especially important for the W-CDMA standard. Attempts to send the full transmit weight information then has to result either in a very coarse quantization, or the feedback information has to be sent of many slots, so that - in a time-variant environment - the feedback information might be outdated by the time it arrives at the transmitter. Thus, the attempt of getting full channel state information to the transmitter carries a penalty of its own. The use of hybrid antenna selection might give better results in this case, since it reduces the number of antennas for which channel information has to be transmitted. An algorithm for optimizing the "effective" SNR is discussed in [26].

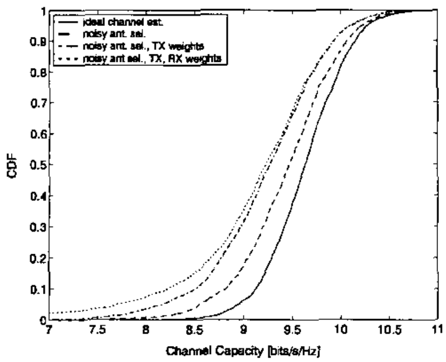

Fig. 4. Impact of errors in the estimation of transfer function matrix H. Cdf of the capacity for (i) ideal channel knowledge at TX and RX (solid), (ii) imperfect antenna selection, but perfect antenna weights (dashed), (iii) imperfect antenna weights at TX only (dotted), and (iv) imperfect antenna weights at TX and RX (dash-dotted). $SNR_{\rm pilot} = 5$ dB. From [25].

D. Hardware aspects

Finally, we consider the effects of the hardware on the performance. In all the previous sections, we had assumed "ideal" RF switches with the following properties:

- they do not cause any attenuation or additional noise in the receiver
- they are capable of switching instantaneously
- they have the same transfer function irrespective of the output and input port, and should be linear

Obviously, those conditions cannot be completely fulfilled in practice. The attenuation by the switches is the most critical issue. In the TX, the attenuation by the switch must be compensated by using a power amplifier with higher output power. At the receiver, the attenuation of the switch plays a minor role only if the switch is placed *after* the low-noise receiver amplifier (LNA). However, that implies that N_r instead of L_r receive amplifiers are required, eliminating a considerable part of the savings of antenna selection.

VI. Summary and Conclusions

This paper presented an overview of MIMO systems with antenna selection. Either the transmitter, the receiver, or both use only the signals from a subset of the available antennas. This allows considerable reductions in the hardware expense. We found that antenna selection retains the diversity degree (compared to the full-complexity system), both for linear diversity systems with complete channel knowledge, and for space-time coded systems. However, there is a penalty with respect to the average SNR. For spatial multiplexing systems (BLAST), antenna selection at the receiver gives a capacity comparable to the full-complexity system as long as $L_r \geq N_t$ (and similarly for the selection at the transmitter). Thus antenna selection is an extremely attractive scheme for reducing the hardware expense in MIMO systems.

Acknowledgement: The author would like to thank Prof. Moe Win, Dr. Makoto Miyake, Dr. Jin Zhang, Ms. Xinying Zhang, Prof. S. Y. Kung, and Dr. Jack Winters for helpful discussions. Part of this work was supported by the "double-directional channel model" INGVAR project of the Swedish Strategic Research Foundation.

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