Report

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1 System Model

Consider a MIMO system with N_r receive antennas and N_t transmit antennas, where N_r is large, for simplification, we assume $N_r \to \infty$, The corresponding discrete time model is given:

$$y = \mathbf{H}s + n \tag{1}$$

$s \in \mathbb{C}^{N_t \times 1}$ is the transmit symbol vector, $y \in \mathbb{C}^{N_r \times 1}$ is the receive symbol vector, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel propagation matrix has independent identity distributed(i.i.d) circularly symmetric complex Gaussian zero mean elements of unit variance(Rayleigh fading), $n \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise(AWGN) with zero mean and variance σ^2, E_s denotes the average symbol energy, hence $E(ss^H) = E_s\mathbf{I}_{Nt}$, E denotes the expectation operation, M denotes the constellation size.

2 Diversity Maximization Antenna Selection Criterion

First we consider a general case to illustrate how the diversity maximization antenna selection scheme works, $N_t - N$ denotes the number of antennas that are chosen to transmit symbols, $\mathbf{H_j} \in \mathbb{C}^{N_r \times (N_t - N)}$ denotes the propagation matrix, $j \in \left[1, \binom{N_t}{N}\right]$, $s_d \in \mathbb{C}^{Nt - N \times 1}$ denotes the the transmit symbol vector, $y_d \in \mathbb{C}^{Nr \times 1}$ denotes the receive symbol vector, $n \in \mathbb{C}^{Nr \times 1}$ denotes the Additive White Gaussian Noise(AWGN) with zero mean and variance σ^2,

$$y_d = \mathbf{H_j} s_d + n \tag{2}$$

matrix $\mathbf{G} \in \mathbb{C}^{Nt - N \times Nr}$ denotes linear equalizer(ZF/MMSE), the linear estimation after equalization $\hat{s_d}$ expressed by:

$$\hat{s_d} = \mathbf{G} y_d = \mathbf{G}\mathbf{H_j} s_d + \mathbf{G} n \tag{3}$$

From (3) we can see, the equalizer colors the noise, the detection is performed assuming $\mathbf{GH_j} = \mathbf{I}_{Nt-N}$, so that the off-diagonal elements of $\mathbf{GH_j}$ contributes to spatial inter symbol interference, vector $\mathbf{G}n$, contributes to additional noise, post processing SNR of kth sub datastream and jth selected antenna subset is defined as ρ_k^j [1], from (3) we have:

$$[\hat{s_d}]_k = (\mathbf{GH_j})_k s_d + g_k n \tag{4}$$

where $[\hat{s_d}]_k$ denotes the kth element of $\hat{s_d}$, $(\mathbf{GH_j})_k$ denotes the kth row of $\mathbf{GH_j}$, g_k denotes the kth row of $\mathbf{G}$ and h_k denotes the kth column of $\mathbf{H_j}$, from (4) we can get the expression of post processing SNR [1]

$$\rho_k^j = \frac{E_s|g_k h_k|^2}{\sigma^2 \parallel g_k \parallel^2 + E_s \sum_{j \neq k} |g_k h_j|^2} \tag{5}$$

E_s and σ are defined in section 1, $\sigma^2||g_k||^2$ is the energy of noise after equalization, which can be derived as follow:

$$E[|g_k n|^2] = E[(g_k n)(g_k n)^H] = E[g_k n n^H g_k^H] = g_k E[n n^H] g_k^H = g_k \sigma^2 I g_k^H = \sigma^2 \parallel g_k \parallel^2 \tag{6}$$

as to $(\mathbf{GH_j})_k$, we have:

$$E((\mathbf{GH_j})_k s_d s_d^H (\mathbf{GH_j})_k^H) = \mathbf{GH_j})_k E[s_d s_d^H]\mathbf{GH_j}_k^H = \mathbf{GH_j})_k E_s I \mathbf{GH_j})_k^H = E_s||\mathbf{GH_j}_k||^2 \tag{7}$$

in (7), $E_s|g_k h_k|^2$ is the post processing energy of detected symbol, $E_s \sum_{j\neq k} |g_k h_j|^2$ is the energy of spatial inter symbol interference.

Consider zero forcing algorithm first, the spatial equalizer $\mathbf{G_{ZF}} = (\mathbf{H_j}^H \mathbf{H_j})^{-1}\mathbf{H_j}^H$, therefore from (3) $\mathbf{GH_j} = \mathbf{I}_{Nt-N}$ and

$$(\rho_{ZF}^j)_k = \frac{E_s}{\sigma^2 \parallel g_k \parallel^2} \tag{8}$$

let $(\mathbf{H_j^H H_j})_k^{-1}$ denote the kth row of $(\mathbf{H_j^H H_j})^{-1}$, we have

$$\parallel g_k \parallel^2 = g_k g_k^H = (\mathbf{H_j^H H_j})_k^{-1} \mathbf{H_j}^H \mathbf{H_j} ((\mathbf{H_j^H H_j})_k^{-1})^H \tag{9}$$

in (9) because $(\mathbf{H_j^H H_j})^{-1}\mathbf{H_j}^H\mathbf{H_j} = \mathbf{I}_{Nt-N}$ hence $(\mathbf{H_j^H H_j})_k^{-1}\mathbf{H_j}^H\mathbf{H_j} = e_k$, e_k deontes the row vector that the kth element is 1, the others are all 0, therefore (9) can be changed to:

$$\parallel g_k \parallel^2 = ((\mathbf{H_j}^H \mathbf{H_j})_{kk}^{-1})^H \tag{10}$$

$((\mathbf{H_j}^H\mathbf{H_j})^{-1}_{kk})^H$ denotes the kth diagonal element of $((\mathbf{H_j}^H\mathbf{H_j})^{-1})^H$, because $\mathbf{H_j^H H_j}$ is a Hermitian matrix, thus $((\mathbf{H_j}^H\mathbf{H_j})^{-1})^H = (\mathbf{H_j}^H\mathbf{H_j})^{-1}$ [2] thus (8) can be changed to

$$(\rho^j_{ZF})_k = \frac{E_s}{\sigma^2(\mathbf{H_j}^H\mathbf{H_j})^{-1}_{kk}} \tag{11}$$

where $(\mathbf{H_j}^H\mathbf{H_j})^{-1}_{kk}$ denotes the kth diagonal element of $(\mathbf{H_j}^H\mathbf{H_j})^{-1}$. Consider minimum mean square error(MMSE) detection, where $\mathbf{G} = \mathbf{G_{MMSE}} = (\mathbf{H_j}^H\mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}\mathbf{H_j}^H$, from [3] [4], we have the expression of the post processing SNR of MMSE:

$$(\rho^j_{MMSE})_k = \frac{E_s}{\sigma^2(\mathbf{H_j}^H\mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}_{kk}} - 1 \tag{12}$$

where $(\mathbf{H_j}^H\mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}_{kk}$ denotes the kth diagonal element of $(\mathbf{H_j}^H\mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$. The criterion of the diversity maximization selection scheme is to choose the antenna subset which can maximize the worst post-processing SNR of the sub-datastream. when $N_r \to \infty$, the column elements of $\mathbf{H_j}$ is mutually independent Gaussian random variables and under favorable propagation condition: the transmit antennas have enough spatial diversity and the column length of propagation matrix becomes sufficient long, the 2 norm of the column will become much larger than the inner product of any two different columns, that is, the columns of $\mathbf{H_j}$ becomes asymptotically orthogonal, this phenomenon can be expressed as

$$\lim_{N_r \to \infty} \frac{(\mathbf{H_j}^H \mathbf{H_j})}{N_r} = \mathbf{I}_{N_t - N} \tag{13}$$

$\mathbf{I}_{N_t-N}$ denotes an $(N_t - N) \times (N_t - N)$ identity matrix. Based on this property, for any k, $(\rho^j_{MMSE})_k$ and $(\rho^j_{ZF})_k$ in (11)(12) are all the same, means according to diversity maximization antenna selection criterion, the selection can be arbitrary.

3 Complexity Analysis of sel-MMSE and sel-MMSE-OSIC algorithm

3.1 sel-MMSE

Define the cost of complex multiplication as 6 flops and complex addition as 2 flops, the sel-MMSE algorithm can be divided into 4 steps [5]:

1. Antenna selection

Based on diversity maximization antenna selection criterion, divide the detection task into two parts:

$$y = \mathbf{H_1} s_1 + \mathbf{H_2} s_2 + n \tag{14}$$

$\mathbf{H_1} \in \mathbb{C}^{N_r \times N}$ and $\mathbf{H_2} \in C^{N_r \times (N_t - N)}$, there are $K = \binom{N_t}{N}$ possible antenna subsets.

2. as to $s_1 \in \mathbb{C}^{N \times 1}$, we use brute-force method: choose every possible symbol vector s_1 in the lattice and calculate y_k, which is expressed as:

$$y_k = y - \mathbf{H_1} s_1^k \tag{15}$$

for ML detection:

$$x^{ML} = arg \min_s \parallel y - \mathbf{H} s \parallel^2 \tag{16}$$

based on (14), (16) can be changed to

$$arg \min_s \parallel y - \mathbf{H_1} s_1 - \mathbf{H_2} s_2 \parallel \tag{17}$$

we define s_1^k denotes the kth possible $N \times 1$ symbol vector that chosen in channel partition stage, at this stage sel-MMSE uses brute-force search, take every possible $s_1^k \in \mathbb{C}^{N \times 1}$ vector according to signal constellation into consideration, therefore there are M^N possible s_1^k

3. for every s_1^k calculate $y_k = y - \mathbf{H_1} s_1^k$, because there are M^N possible s_1^k thus there are the same number of y_k, using linear detection scheme or their OSIC counterparts to estimate s_2^k, the estimation is expressed as $\hat{s_2}^k$, then mapping $\hat{s_2}^k$ to the constellation to the nearest points, we use s_2^k to represent it, correspondingly there are M^N posssible s_2^k.

4. Find the solution $[s_1^k, s_2^k]$ among M^N candidates that has the minimum total Euclidean distance.

$$\hat{s} = arg \min_{k=1,2,\ldots,K} \parallel y_k - \mathbf{H_2} s_2^k \parallel^2 \tag{18}$$

According to section 2, based on diversity maximization antenna selection criterion we can choose arbitrary antenna subset, thus the cost of antenna selection is zero, the second step calculating $y_k = y - \mathbf{H_1} s_1^k$, including an matrix multiplication and vector substruction,

$$N_r N \tag{19}$$

complex multiplications and

$$N_r N \tag{20}$$

complex additions are required. Then calculate the MMSE estimation $\hat{s_2^k}$:

$$s_2^k = [\mathbf{H_2}^H \mathbf{H_2} + \frac{\sigma^2}{E_s} \mathbf{I}]^{-1} \mathbf{H_2}^H y_k \tag{21}$$

$\mathbf{H_2} \in \mathbb{C}^{N_r \times (N_t - N)}$ because of the channel hardening phenomenon, $[\mathbf{H_2}^H \mathbf{H_2} + \frac{\sigma^2}{E_s}\mathbf{I}]^{-1} = \frac{1}{N_r + \frac{\sigma^2}{E_s}}\mathbf{I}$, thus calculating s_2^k requires

$$(N_t - N)N_r + \frac{N_t - N}{3} \tag{22}$$

complex multiplications and

$$(N_t - N)(N_r - 1) \tag{23}$$

complex additions. Finally the calculation of MED in (18) requires matrix multiplication, vector substruction and the calculation of 2-norm, therefore

$$N_r(N_t - N) + \frac{N_r}{2} \tag{24}$$

complex multiplications and

$$N_r(N_t - N) + \frac{N_r - 1}{2} \tag{25}$$

complex additions are required. Totally as mentioned before there is one s_2^k corresponding to each s_1^k thus there are M^N possible solution candidates $[s_1^k, s_2^k]$, therefore the sel-MMSE requires overall:

with combination of (19)(22)(24)

$$M^N(NrN+(Nt-N)Nr+\frac{Nt-N}{3}+Nr(Nt-N)+\frac{Nr}{2}) = M^N(2N_rN_t-N_rN+\frac{N_t-N}{3}+\frac{N_r}{2}) \tag{26}$$

complex multiplications and with combination of (20)(23)(25)

$$M^N(NrN+(Nt-N)(Nr-1)+Nr(Nt-N)+\frac{Nr-1}{2}) = M^N(2N_rN_t-N_rN-N_t+N+\frac{N_r-1}{2}) \tag{27}$$

complex additions, Basing on (26)(27) and 6 flops for complex multiplication, 2 flops for complex addition, hence the total cost of sel-MMSE algorithm is:

$$\begin{aligned} f_{MMSE}(N) &= 6M^N(NrN+(Nt-N)Nr+\frac{Nt-N}{3}+Nr(Nt-N)+\frac{Nr}{2}) \quad (28)\\ &\quad +2M^N(NrN+(Nt-N)(Nr-1)+Nr(Nt-N)+\frac{Nr-1}{2}) \\ &= M^N(16N_rN_t-8N_rN+4N_r-1) \quad flops \end{aligned}$$

3.2 sel-MMSE-OSIC

First we use a general case to illustrate how MMSE-OSIC works, first consider a MIMO system as mentioned in section 1:

$$y = \mathbf{H_j} s + n \tag{29}$$

$\mathbf{G}_{MMSE} = (\mathbf{H_j}^H \mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}\mathbf{H_j}^H$, after equalization using $\mathbf{G}_{MMSE}$ (29) can be changed to

$$\hat{s} = \mathbf{G}y = \mathbf{G}\mathbf{H_j} s + \mathbf{G}n \tag{30}$$

$\hat{s}$ denotes the estimation of s, from(30) we can express kth estimation of sub-datastream $\hat{s_k}$:

$$\hat{s_k} = g_k y \tag{31}$$

where g_k denotes the kth row of $\mathbf{G}$, different from MMSE, MMSE-OSIC algorithm detects the s_k symbol by symbol, the detection order at every iteration is basing on the following principle: the $\hat{s_k}$ is detected based on their post-processing SNR at each iteration, $\hat{s_k}$ with the highest SNR will be detected , according to (12), this element is associated to the diagonal elements of $(\mathbf{H_j}^H \mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$, smaller the kth diagonal element of $(\mathbf{H_j}^H \mathbf{H_j} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$ is, higher the post-processing SNR of $\hat{s_k}$ is. Hence MMSE-OSIC algorithm will detect different s_k for N_t times, in the ith ($i \in [0, N_t - 1]$) iteration, it majorly has 3 steps:

1.Ordering the channel based on the principle mentioned above, choose the sub-datastream with highest SNR(smallest diagonal element of $(\mathbf{H_j^i}^H \mathbf{H_j^i} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$) to detect, $H_j^i \in \mathbb{C}^{Nr \times Nt-N-i}$ denotes the propagation matrix in ith step.

2.Find the estimation

$$\hat{s_{i+1}} = g_{i+1} y \tag{32}$$

slicing $\hat{s_{i+1}}$ to $\bar{s_{i+1}}$ according to corresponding symbol constellation.

3.Cancel $\bar{s}_i$ from received symbol vector y_i, $h_k \quad k \in [1, N_t]$ denotes the kth column of $\mathbf{H_j^i}$, (29) can be expressed as:

$$y = \sum_{k=1,2,\ldots Nt} h_k s_k + n \tag{33}$$

y can be viewed as a summation of all the s_k and their channel h_k with AWGN. therefore the cancellation works by remove $\bar{s_{i+1}}$ and corresponding channel column h_{i+1}

$$y_i = y_{i-1} - h_{i+1} \bar{s_{i+1}} \tag{34}$$

then refresh propagation matrix $\mathbf{H_j^i}$ by remove the column h_{i+1} from $\mathbf{H_j^{i-1}}$.

Consider the computational complexity of sel-MMSE-OSIC, the only difference between sel-MMSE-OSIC and sel-MMSE is in the linear detection step to detect s_2 in $y = \mathbf{H_1}s_1 + \mathbf{H_2}s_2 + n$, so that their computation cost except this step are all the same, in MMSE-OSIC detection to s_2, when $N_r \to \infty$, with channel hardening phenomenon, at ith iteration($i \in [0, \quad N_t - N - 1]$) in step 1, the diagonal elements of $(\mathbf{H_j^i}^H \mathbf{H_j^i} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$ are all the same, where $\mathbf{H_j^i} \in \mathbb{C}^{Nr \times Nt-N-i}$ denotes the propagation matrix in the ith step after channel selection, therefore the post-processing SNR of $\hat{s_k}$ are all the same, detection order can be arbitrary, this step can be ignored. in the second step, based on (34) and channel hardening phenomenon, $G_{MMSE} = \frac{1}{N_r + \frac{\sigma^2}{E_s}} \mathbf{H_j^i}^H$, $g_k = \frac{1}{N_r + \frac{\sigma^2}{E_s}} (\mathbf{H_j^i})_{\mathbf{k}}{}^H$, where $(\mathbf{H_j^i})_{\mathbf{k}}{}^H$ denotes the kth row of $\mathbf{H_j^i}^H$, hence (32) requires a vector multiplication, the cost is N_r complex multiplications and $N_r - 1$ complex additions, (34) requires N_r complex multiplications and N_r complex additions. MMSE-OSIC algorithm has totally $N_t - N$ iteration, hence the cost of getting $\hat{s_2}$ is:

$$\sum_{i=0}^{N_t-N-1} 2N_r = 2N_r(N_t - N) \tag{35}$$

complex multiplications and

$$\sum_{i=0}^{N_t-N-1} 2N_r - 1 = (2N_r - 1)(N_t - N) \tag{36}$$

complex additions.

In conclusion, as same as sel-MMSE algorithm there are M^N possible $[s_1^k, s_2^k]$, the other steps of sel-MMSE-OSIC except MMSE-OSIC are all the same, calculating $y_k = y - \mathbf{H_1} s_1^k$, including an matrix multiplication and vector substruction, $N_r N$ complex multiplications and $N_r N$ complex additions are required, the calculation of (18) requires $N_r(N_t - N) + N_r/2$ complex multiplications and $N_r(N_t - N) + (N_r - 1)/2$ complex additions. the total cost:

with combination of (19)(24)(35)

$$M^N(N_r N + 2N_r(N_t - N) + N_r(N_t - N) + \frac{N_r}{2}) = M^N(3N_r N_t - 2N_r N + \frac{N_r}{2}) \tag{37}$$

complex multiplications and with combination of (20)(25)(36)

$$M^N(N_r N + (2N_r - 1)(N_t - N) + N_r(N_t - N) + \frac{(N_r - 1)}{2}) = M^N(3N_r N_t - 2N_r N + N - N_t + \frac{N_r - 1}{2}) \tag{38}$$

The computation cost in flops, based on (37)(38)

$$\begin{aligned} f_{MMSE-OSIC}(N) &= 6M^N(3N_r N_t - 2N_r N + \frac{N_r}{2}) + \\ &\quad 2M^N(3N_r N_t - 2N_r N + N - N_t + \frac{N_r - 1}{2}) \\ &= M^N(24N_r N_t - 16N_r N + 2N + 4N_r - 2N_t - 1) \quad \textit{flops} \end{aligned} \tag{39}$$

4 Minimization of the Complexity

4.1 sel-MMSE

Proposition 1: $f_{MMSE}(N)$ increases when N increases, $N \in [1, Nt - 1]$

Proof: According to (29), the computational cost of sel-MMSE algorithm can be expressed as:

$$f_{MMSE}(N) = M^N(16N_rN_t - 8N_rN + 4N_r - 1) \quad flops \tag{40}$$

In this section a discussion of the relation between N and $f(N)$, firstly relax N to a real number and make the derivative of $f(N)$,

$$\frac{\partial f_{MMSE}(N)}{\partial N} = M^N[(16N_rN_t - 8N_rN + 4N_r - 1)\ln M - 8N_r] \tag{41}$$

solve $\frac{\partial f_{MMSE}(N)}{\partial N} = 0$, N_{zero} denotes the solution, we have

$$\begin{aligned} & [(16N_rN_t - 8N_rN + 4N_r - 1)\ln M - 8N_r] = 0 \\ \Rightarrow \quad & (16N_rN_t - 8N_rN + 4N_r - 1) = \frac{8N_r}{\ln M} \\ \Rightarrow \quad & N_{zero} = N_t + N_t - 1/\ln M + 1/2 - 1/8N_r \end{aligned} \tag{42}$$

because $N_r > 1$ thus $1/8N_r < 1/8$, thus $1/2 - 1/8N_r > 3/8$, $M \geq 2$, thus $-1/\ln M \geq -1.4427$, $N_t \geq 2$, in conclusion $N_t - 1/\ln M + 1/2 - 1/8N_r > 0.9323$ therefore in (42) $N_{zero} > N_t + 0.9323$, according to (41), $\frac{\partial f_{MMSE}(N)}{\partial N}$ decreases when N increases, N is the number of antennas chosen in channel partition stage, therefore $N \in [1, N_t - 1] \Rightarrow \frac{\partial f(N)}{\partial N} > 0$, proposition 1 is proved.

4.2 sel-MMSE-OSIC

Proposition 2: $f_{MMSE-OSIC}(N)$ increases when N increases, when $N \in [1, N_t - 1]$

Proof: according to (40), relax N from a integer to a real number,

$$\frac{\partial f_{MMSE-OSIC}(N)}{\partial N} = M^N(\ln M(24N_rN_t - 16N_rN + 2N + 4N_r - 2N_t - 1) - 16N_r + 2) \quad (43)$$

$\frac{\partial f_{MMSE-OSIC}(N)}{\partial N}$ decreases when N increases, solve $\frac{\partial f_{MMSE-OSIC}(N)}{\partial N} = 0$, based on (43)

$$\begin{aligned}
& \ln M(24N_rN_t - 16N_rN + 2N + 4N_r - 2N_t - 1) - 16N_r + 2 = 0 \quad (44) \\
\Rightarrow \quad & 24N_rN_t - 16N_rN + 2N + 4N_r - 2N_t - 1 = \frac{16N_r - 2}{\ln M} \\
\Rightarrow \quad & N_{zero} = \frac{24N_rN_t + 4N_r - 2N_t - 1}{16N_r - 2} - \frac{1}{\ln M}
\end{aligned}$$

$$
\begin{aligned}
N_{zero} &= \frac{24N_rN_t + 4N_r - 2N_t - 1}{16N_r - 2} - \frac{1}{\ln M} \\
&> \frac{24N_rN_t + 4N_r - 2N_t - 1}{16N_r} - \frac{1}{\ln M} \\
&= \frac{3N_t}{2} + \frac{1}{4} - \frac{N_t}{8N_r} - \frac{1}{16N_r} - \frac{1}{\ln M} \\
&= N_t + (\frac{1}{2} - \frac{1}{8N_r})N_t + \frac{1}{4} - \frac{1}{16N_r} - \frac{1}{\ln M}
\end{aligned} \tag{45}
$$

because $N_r > 1$, $M \geq 2$ therefore $-\frac{1}{8N_r} > -\frac{1}{8}$, $\quad -\frac{1}{16N_r} > -\frac{1}{16}$, $\quad -\frac{1}{\ln M} \geq -1.4427$ thus

$$
N_{zero} = N_t + (\frac{1}{2} - \frac{1}{8N_r})N_t + \frac{1}{4} - \frac{1}{16N_r} - \frac{1}{\ln M} \tag{46}
$$

$$
> \quad N_t + \frac{3N_t}{8} + \frac{3}{16} - 1.4427
$$

because $N_t > 1$, thus $N_{zero} > N_t + \frac{3N_t}{8} + \frac{3}{16} - 1.4427 > N_t - 0.5052 > N_t - 1$, because N is the number of antennas chosen at channel partition stage therefore $N \in [1, \quad N_t - 1]$, because $\frac{\partial f_{MMSE-OSIC}(N)}{\partial N}$ decreases when N increases and $N < N_{zero}$, therefore $\frac{\partial f_{MMSE-OSIC}(N)}{\partial N} > 0$, $N \in [1, \quad N_t - 1]$, proposition 2 is proven.

5 Trade off between Complexity and diversity

let P_e denotes the average probability of detection error of sel-MMSE and sel-MMSE-OSIC, P_e^{ML} denotes the average probability of detection error of Maximum-Likelihood detection, $(P_e)_2$ denotes the average probability of detection error of s_2^k after channel partition, P_e is bounded by [5]:

$$\max[(P_e)_2, P_e^{ML}] \leq P_e \leq P_e^{ML} + (P_e)_2 \tag{47}$$

The diversity gain of sel-MMSE and its OSIC counterpart are expressed by d_{MMSE} and $d_{MMSE-OSIC}$ diversity maximization selection, they can be expressed as [5]

$$d_{MMSE} = (N+1)(N_r - N_t + N + 1) \tag{48}$$

$$d_{MMSE-OSIC} = (N+1)(N_r - N_t + N + 1) \tag{49}$$

(48) is proven in [6], in [5], $d_{MMSE-OSIC} \geq (N+1)(N_r - N_t + N + 1)$ is proven however strictly proven of (49) is still lacking. therefore we take (49) as a conjecture here.

$$\lim_{SNR \to \infty} \frac{\log P_e^{ML}}{\log SNR} = -N_r \tag{50}$$

$$\lim_{SNR\to\infty} \frac{\log (P_e)_2}{\log SNR} = -d \tag{51}$$

$(P_e)_2^{LD}$ denotes the average probability of detection error of s_2^k using linear detection method. if $d \geq N_r$, sel-MMSE can achieve optimal performance asymptotically $\lim_{SNR\to\infty} P_e = P_e^{ML}$, if $d < N_r$, $\lim_{SNR\to\infty} \frac{\log P_e}{\log SNR} = d$ [5]. let x denotes the lower bound of the diversity gain required, $d \geq x \quad x > 0$

$$g(N) = d - x = (N+1)(N_r - N_t + N + 1) - x \quad 1 \leq N \leq N_t - 1 \tag{52}$$

Based on the results in section 4, we seek the minimum number of antennas N_{min} chosen at channel selection stage under certain detection performance constraint ($g(N) \geq 0$). so that the complexity is minimized. $g(N)$ is a quadratic function of N, the two zero points of $g(N)$ are:

$$N_1 = \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) - 1 \tag{53}$$

$$N_2 = -\sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) - 1 \tag{54}$$

Assume $N_r \geq 2 \; N_t \geq 2$

Consider the following conditions:

1. $N_r \geq 2N_t - 3$, the asymptotically optimal performance and minimum complexity can be reached at the same time, $N_{min} = 1 \leftrightarrow d \geq N_r$ [5]

2. $N_t \leq N_r \leq 2N_t - 4$, because $N_r \geq N_t$, according to (53) $N_2 < 0$, considering N_1, when x is viewed as a variable of N_1, in (53), when $1/2(N_r - N_t) + 1 > 0 \Rightarrow N_r \geq N_t - 2$, zero point x of $N_1(x)$ exists, zero point is $x = N_r - N_t + 1$, N_1 increases when x increases, because $N_r \geq N_t$, it is obvious zero point exits

if $x > N_r - N_t + 1 \rightarrow N_1 > 0$, therefore

$$N_{min} = \lfloor N_1 + 1 \rfloor = \lfloor \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) \rfloor \tag{55}$$

$\lfloor m \rfloor$ denotes the largest integer smaller than m, $N_{min} \leq N_t - 1$, based on (**??**) $x < N_r N_t$.

if $x \leq N_r - N_t + 1 \rightarrow N_1 \leq 0$, therefore $N_{min} = 1$.

3. $N_r < N_t$

consider x as an variable of $N_2(x)$, the zero point $x_{zero} = N_r - N_t + 1$ exists when $-1/2(N_r - N_t) - 1 > 0 \Rightarrow N_r \leq N_t - 2$, x andN_2 are negatively correlated.

if $N_r \leq N_t - 2$, then zero point of $N_2(x)$ exists, $N_r - N_t + 1 < 0$, because $x > 0$, therefore $x > N_r - N_t + 1$, $N_2 < 0$, because $N_r \leq N_t - 2 \Rightarrow 1/2(N_r - N_t) + 1 < 0$, based on (53) $N_1 = \sqrt{x + 1/4(N_r - N_t)^2} - (1/2(N_r - N_t) + 1) > 0$, so $N_{min} = \lfloor N_1 + 1 \rfloor = \lfloor \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) \rfloor$, because $N_{min} \leq N_t - 1$, according to (55) $x < N_r N_t$.

if $N_r = N_t - 1$, $-1/2(N_r - N_t) - 1$ in (54) equals to $-\frac{1}{2}$, thus $N_2 < 0$, $N_r \geq N_t - 2$, thus zero point of $N_1(x)$ exists, $x > 0 = N_r - N_t + 1$, based on above discussion $N_1 > 0$,

$N_{min} = \lfloor N_1 + 1 \rfloor = \lfloor \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) \rfloor$.

Conclusion

1. $N_r \geq 2N_t - 3$, $d_{total} = N_r$ $N_{min} = 1$

2. $N_t \leq N_r \leq 2N_t - 4$, $d_{total} = x$

$$N_{min} = \begin{cases} \lfloor \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) \rfloor & x > N_r - N_t + 1 \\ 1 & x \leq N_r - N_t + 1 \end{cases}$$

3. $N_r \leq N_t$, $d_{total} = x$, $N_{min} = \lfloor \sqrt{x + 1/4(N_r - N_t)^2} - 1/2(N_r - N_t) \rfloor$

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