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Low-Complexity Algorithms for Large-MIMO Detection

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Abstract—**Low-complexity near-optimal detection of large multiple-input multiple-output (MIMO) signals has attracted recent research attention. Recently, it has been shown that certain algorithms rooted in machine learning/artificial intelligence are well suited to achieve near-optimal detection performance in MIMO systems with large number (tens) of antennas at practically affordable complexities. In this paper, we present three such low-complexity algorithms that we have proposed recently, and compare their bit error rate performance and complexities in large-MIMO detection. These algorithms include two *local neighborhood search* based algorithms, namely, likelihood ascent search (LAS) and reactive tabu search (RTS) algorithms, and a *message passing* algorithm based on and belief propagation (BP). Feasibility of such low-complexity algorithms for large-MIMO detection can enable practical implementation of high-spectral efficiency (tens to hundreds of bps/Hz) large-MIMO systems.**

Keywords – *Large-MIMO signal detection, likelihood ascent search, reactive tabu search, probabilistic data association, belief propagation.*

I. Introduction

Multiple-input multiple-output (MIMO) systems with large number (e.g., tens) of transmit and receive antennas, referred to as 'large-MIMO systems,' are of interest because of the high capacities/spectral efficiencies theoretically predicted in these systems [1],[2]. Research in low-complexity receive processing (e.g., MIMO detection) techniques that can lead to practical realization of large-MIMO systems is both nascent as well as promising. For e.g., NTT DoCoMo has already field demonstrated a 12×12 V-BLAST system operating at 5 Gbps data rate and 50 bps/Hz spectral efficiency in 5 GHz band at a mobile speed of 10 Km/hr [3]. Evolution of WiFi standards (evolution from IEEE 802.11n to IEEE 802.11ac to achieve multi-gigabit rate transmissions in 5 GHz band) now considers 16×16 MIMO operation; see 16×16 MIMO indoor channel sounding measurements at 5 GHz reported in [4] for consideration in WiFi standards. Also, 64×64 MIMO channel sounding measurements at 5 GHz in indoor environments have been reported in [5]. We note that, while RF/antenna technologies/measurements for large-MIMO systems are getting matured, there is an increasing need to focus on low-complexity algorithms for detection in large-MIMO systems to reap their high spectral efficiency benefits.

In the above context, in our recent works, we have shown that certain algorithms from machine learning/artificial intelligence achieve near-optimal performance in large-MIMO systems at low complexities [6]-[13][1]. In [6]-[8], a local neighborhood search based algorithm, namely, a *likelihood ascent search* (LAS) algorithm, was proposed and shown to achieve close to maximum-likelihood (ML) performance in MIMO systems with several tens of antennas (e.g., 32×32 and 64×64 MIMO). Subsequently, in [9],[10], another local search algorithm, namely, *reactive tabu search* (RTS) algorithm, which performed better than the LAS algorithm through the use of a local minima exit strategy was presented[2]. In [11], a large-MIMO detection algorithm based on *probabilistic data association* (PDA) was shown to achieve near maximum a posteriori (MAP) performance. In [12], near-ML performance in a 50×50 MIMO system was demonstrated using a *Gibbs sampling* based detection algorithm, where the symbols take values from $\{\pm 1\}$. More recently, we, in [13], proposed a factor graph based *belief propagation* (BP) algorithm for large-MIMO detection, where we adopted a Gaussian approximation of the interference (GAI). Another BP based algorithm using Markov random field representation of large-MIMO systems is presented in [14]. In this paper, we present three of the above low-complexity algorithms suited for large-MIMO detection, and present a comparison of their bit error rate performances and complexities.

The rest of this paper is organized as follows. The MIMO system model is presented in Section II. Low-complexity algorithms for near-optimal large-MIMO detection and their performances/complexities are presented in Section III. Conclusions are presented in Section IV.

II. System Model

Consider a $N_t \times N_r$ V-BLAST MIMO system whose received signal vector, $\mathbf{y}_c \in \mathbb{C}^{N_r}$, is of the form

$$\mathbf{y}_c \;=\; \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c, \qquad (1)$$

where $\mathbf{x}_c \in \mathbb{C}^{N_t}$ is the symbol vector transmitted, $\mathbf{H}_c \in \mathbb{C}^{N_r \times N_t}$ is the channel gain matrix, and $\mathbf{n}_c \in \mathbb{C}^{N_r}$ is the noise vector whose entries are modeled as i.i.d $\mathcal{CN}\big(0, \sigma^2 = \frac{N_t E_s}{\gamma}\big)$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna. Assuming rich scattering, we model the entries of $\mathbf{H}_c$ as i.i.d $\mathcal{CN}(0,1)$. Each element of $\mathbf{x}_c$ is an M-PAM or M-QAM symbol. M-PAM symbols take values from $\{A_m, m = 1, 2, \cdots, M\}$, where $A_m = (2m - 1 - M)$, and M-QAM is nothing but two PAMs in quadrature. As in [7], we convert (1) into a real-valued system model, given by

$$\mathbf{y} \;=\; \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad (2)$$

where $\mathbf{H} \in \mathbb{R}^{2N_r \times 2N_t}$, $\mathbf{y} \in \mathbb{R}^{2N_r}$, $\mathbf{x} \in \mathbb{R}^{2N_t}$, $\mathbf{n} \in \mathbb{R}^{2N_r}$.

ML solution: For M-QAM, $[x_1, \cdots, x_{N_t}]$ can viewed to be from an underlying M-PAM signal set, and so is $[x_{N_t+1}, \cdots,$

[1] Similar algorithms have been reported earlier in the context of multiuser detection in large CDMA systems.

[2] In [8],[10], we compared the performance and complexities of LAS and RTS algorithms with those of the sphere decoding (SD) variants in [16] and [17], and showed that these SD variants do not scale well for the large dimensions considered.

$x_{2N_t}]$. Let $\mathbb{A}_i$ denote the M-PAM signal set from which x_i takes values, $i = 1, 2, \cdots, 2N_t$. Defining a $2N_t$-dimensional signal space $\mathbb{S}$ to be the Cartesian product of $\mathbb{A}_1$ to $\mathbb{A}_{2N_t}$, the ML solution vector, $\mathbf{x}_{ML}$, is given by

$$\mathbf{x}_{ML} = \underset{\mathbf{x} \in \mathbb{S}}{\arg\min} \ \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \quad = \underset{\mathbf{x} \in \mathbb{S}}{\arg\min} \ \underbrace{\mathbf{x}^T\mathbf{H}^T\mathbf{H}\mathbf{x} - 2\mathbf{y}^T\mathbf{H}\mathbf{x}}_{\triangleq \phi(\mathbf{x})}, \qquad (3)$$

whose complexity is exponential in N_t. The LAS algorithm in [7],[8] and RTS algorithm in [9],[10] are low-complexity algorithms which minimize the ML metric in (3) through local neighborhood search.

MAP Solution: In the real-valued system model in (2), for square M-QAM, each entry of $\mathbf{x}$ belongs to a $\sqrt{M}$-PAM constellation. Let $b_i^{(0)}, b_i^{(1)}, \cdots, b_i^{(q-1)}$ denote the $q = \log_2(\sqrt{M})$ constituent bits of the ith entry x_i of $\mathbf{x}$. We can write the value of each entry of $\mathbf{x}$ as a linear combination of its constituent bits as

$$x_i = \sum_{j=0}^{q-1} 2^j \, b_i^{(j)}, \qquad i = 0, 1, \cdots, 2N_t - 1. \qquad (4)$$

Let $\mathbf{b} \in \{+1, -1\}^{2qN_t}$, defined as

$$\mathbf{b} \triangleq \Big[b_0^{(0)} \cdots b_0^{(q-1)} b_1^{(0)} \cdots b_1^{(q-1)} \cdots b_{2N_t-1}^{(0)} \cdots b_{2N_t-1}^{(q-1)}\Big]^T \qquad (5)$$

denote the transmitted bit vector. Defining $\mathbf{c} \triangleq [2^0 \, 2^1 \cdots 2^{q-1}]$, we can write $\mathbf{x}$ as $\mathbf{x} = (\mathbf{I}_{2N_t} \otimes \mathbf{c})\mathbf{b}$, using which we can rewrite (2) as

$$\mathbf{y} = \underbrace{\mathbf{H}(\mathbf{I}_{2N_t} \otimes \mathbf{c})}_{\triangleq \mathbf{H}'} \mathbf{b} + \mathbf{n}, \qquad (6)$$

where $\mathbf{H}' \in \mathbb{R}^{2N_r \times 2qN_t}$ is the effective channel matrix. The MAP estimate of bit $b_i^{(j)}$ is

$$\hat{b}_i^{(j)} = \underset{a \in \{\pm 1\}}{\arg\max} \ p\big(b_i^{(j)} = a \,|\, \mathbf{y}, \mathbf{H}'\big), \qquad (7)$$

whose computational complexity is exponential in N_t. The PDA algorithm in [11] is an iterative algorithm that achieves near-MAP performance for large N_t. Likewise, for $\{\pm 1\}$ modulation alphabet, the BP algorithm in [13] also achieves near-MAP performance for large N_t. Both PDA and BP algorithms in [11] and [13] make Gaussian approximation of the multi-antenna interference which reduces complexity.

III. Low-Complexity Algorithms for Large-MIMO Detection

In this paper we present the highlights and performances/complexities of the following algorithms in the large-MIMO setting: *i)* LAS algorithm [7],[8], *ii)* RTS algorithm [9],[10], *iii)* PDA algorithm [11], and *iv)* BP algorithm [13][3]

[3]For detailed listing of these algorithms, refer to the respective references.

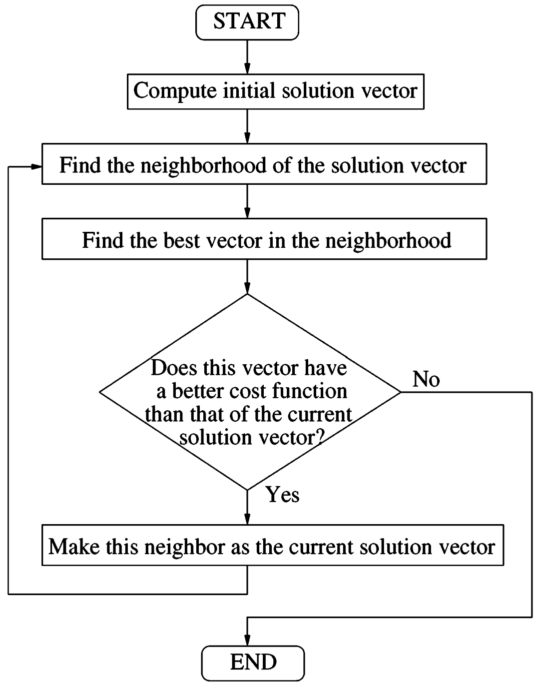

Fig. 1. Flow chart of the LAS algorithm.

A. LAS Algorithm

The LAS algorithm starts with an initial solution vector $\mathbf{x}^{(0)}$, given by $\mathbf{x}^{(0)} = \mathbf{B}\mathbf{y}$, where $\mathbf{B}$ is the initial solution filter, which can be a matched filter (MF) or zero-forcing (ZF) filter or minimum mean square error (MMSE) filter. A neighborhood around the initial vector is defined; e.g., set of all vectors which differ from the initial solution vector in one coordinate is an example of a neighborhood. For each of the vectors in the neighborhood, the algorithm computes the ML cost function $\phi(\mathbf{x})$ defined in (3). The best vector among the neighboring vectors (in terms of least ML cost among them) which also happens to have a lesser ML cost than that of the initial vector is chosen, and declared as the new solution vector $\mathbf{x}^{(1)}$; the 1 in $\mathbf{x}^{(1)}$ denotes the iteration index. This new solution vector $\mathbf{x}^{(1)}$ is passed on as the initial vector for the next iteration, where the best vector among the neighboring vectors of $\mathbf{x}^{(1)}$ is chosen as the new solution vector for the next iteration, and so on until a local minima is reached. The algorithm ends once a local minima is encountered, and the local minima is declared as the final solution vector. A flow chart illustrating the LAS algorithm is shown in Fig. 1.

LAS Algorithm Complexity: A key advantage of the LAS algorithm is its simplicity in its search operation. Much of the algorithm complexity arises from the initial vector computation (which involves matrix inversion operation in ZF and MMSE filters) and the computation of $\mathbf{H}^T\mathbf{H}$, which requires $O(N_tN_r)$ complexity per symbol. The average complexity in the search part alone is found to be $O(N_t)$ through simulations. So the overall per-symbol complexity of the LAS algorithm is $O(N_tN_r)$. This low order of complexity is well suited for scaling to large number of dimensions.

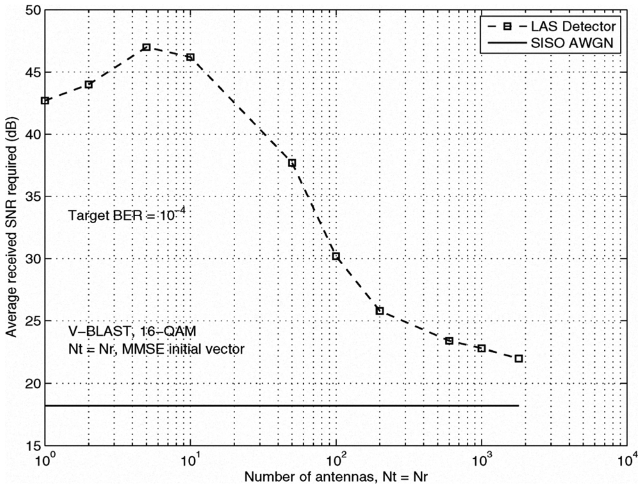

Fig. 2. Average received SNR required to achieve a target uncoded BER of 10^{-4} in V-BLAST for increasing values of $N_t = N_r$ for 16-QAM. MMSE initial vector. LAS detector performance approaches SISO AWGN performance for large $N_t = N_r$.

LAS Algorithm Performance: A even more interesting aspect of the LAS algorithm is that its bit error rate (BER) performance improves with increasing values of N_t; a behavior we refer to as the 'large system behavior' of the algorithm. Increasingly closer to ML performance is achieved for increasing number of transmit antennas. Figure 2 shows the large system effect of the LAS algorithm, where we can see that the SNR required to achieve 10^{-4} uncoded BER decreases (moves closer and closer to SISO AWGN performance) for increasing N_t. In [15], we have presented an analytical proof that the bit error performance of the LAS detection in V-BLAST with 4-QAM in i.i.d. Rayleigh fading converges to that of the ML detector as $N_t, N_r \rightarrow \infty$.

Applicability to Large Non-Orthogonal STBC MIMO Systems: The LAS detector is suboptimal in systems with small dimensions (e.g., V-BLAST with small number of transmit antennas), and becomes optimal in the large system limit (as proved in [15]). With hundreds of dimensions, LAS algorithm achieves close to optimal performance. Hundreds of dimensions in space alone (i.e., hundreds of transmit antennas) may not be practical. However, hundreds of dimensions can be created with tens of dimensions each in space and time using non-orthogonal space-time block codes (STBC) which achieve both full rate (same as that achieved in V-BLAST) as well as full transmit diversity [18]. For example, a 16×16 non-orthogonal STBC in [18] has 256 complex data symbols has 512 real dimensions; with 64-QAM and rate-3/4 turbo code, this STBC achieves a spectral efficiency of 72 bps/Hz. In [8], it has been established through extensive simulations that the LAS algorithm is very effective, both in terms of complexity as well as achieving near-ML/near capacity performance, in decoding 16×16 and 32×32 non-orthogonal STBCs, even in the presence of spatial correlation and with estimated channel matrix.

Multistage LAS: A more general version of LAS algorithm, termed as multistage LAS (MLAS), was reported in [8]. The MLAS algorithm executes an escape mechanism when it encounters a local minima, by changing the neighborhood definition: it considers vectors which differ in two or more coordinates (as opposed to only one coordinate in the basic neighborhood definition in LAS) as neighbors. On escaping from a local minima, the algorithm reverts back to the basic neighborhood definition till the next local minima is encountered and stops when no escape from a local minima is possible. The performance gain due to this escape strategy is found to be not very significant [8].

LAS Algorithm Implementation in Hardware: An FPGA implemenation of the LAS algorithm in [7] for detection of 32×32 V-BLAST with 4-QAM/16-QAM/64-QAM on Xilinx Virtex 5 330 LX FPGA has been recently reported in [19].

B. RTS Algorithm

A more promising local neighborhood search is reactive tabu search reported in [9],[10]. Like the LAS algorithm, the RTS algorithm also starts with an initial solution vector, defines a neighborhood around it (i.e., defines a set of neighboring vectors based on a neighborhood criteria), and moves to the best vector among the neighboring vectors (*even if the best neighboring vector is worse, in terms of ML cost, than the current solution vector*); this allows the algorithm to escape from local minima. This process is continued for a certain number of iterations, after which the algorithm is terminated and the best among the solution vectors in all the iterations is declared as the final solution vector. In defining the neighborhood of the solution vector in a given iteration, the algorithm attempts to avoid cycling by making the moves to solution vectors of the past few iterations as 'tabu' (i.e., prohibits these moves), which ensures efficient search of the solution space. The number of these past iterations is parametrized as the 'tabu period.' The search is referred to as fixed tabu search if the tabu period is kept constant. If the tabu period is dynamically changed (e.g., increase the tabu period if more repetitions of the solution vectors are observed in the search path), then the search is called reactive tabu search. Reactive tabu search is preferred because of its robustness (choice of a good fixed tabu period can be tedious). A flow chart of the RTS algorithm is shown in Fig. 3.

RTS versus LAS: The basic definition of neighborhood is the same in both LAS and RTS. However, RTS differs from LAS in the following aspects: i) while the definition of neighborhood is static in LAS for all iterations, in RTS, in addition to the basic neighborhood definition, there is also a dynamic aspect to the neighborhood definition by way of prohibiting certain vectors from being included in the neighbor list (implemented through repetition checks/tabu period), and ii) while LAS gets trapped in the local minima that it first encounters and declares this minima to be the final solution vector, RTS can potentially find better minimas because of the escape strategy embedded in the algorithm (by way of allowing to pick and move to the best neighbor even if that neighbor has a lesser ML cost than the current solution vector). The RTS algorithms involves several parameters to choose like stopping criterion parameters (e.g., minimum and maximum number of iterations), initial tabu period, maximum number

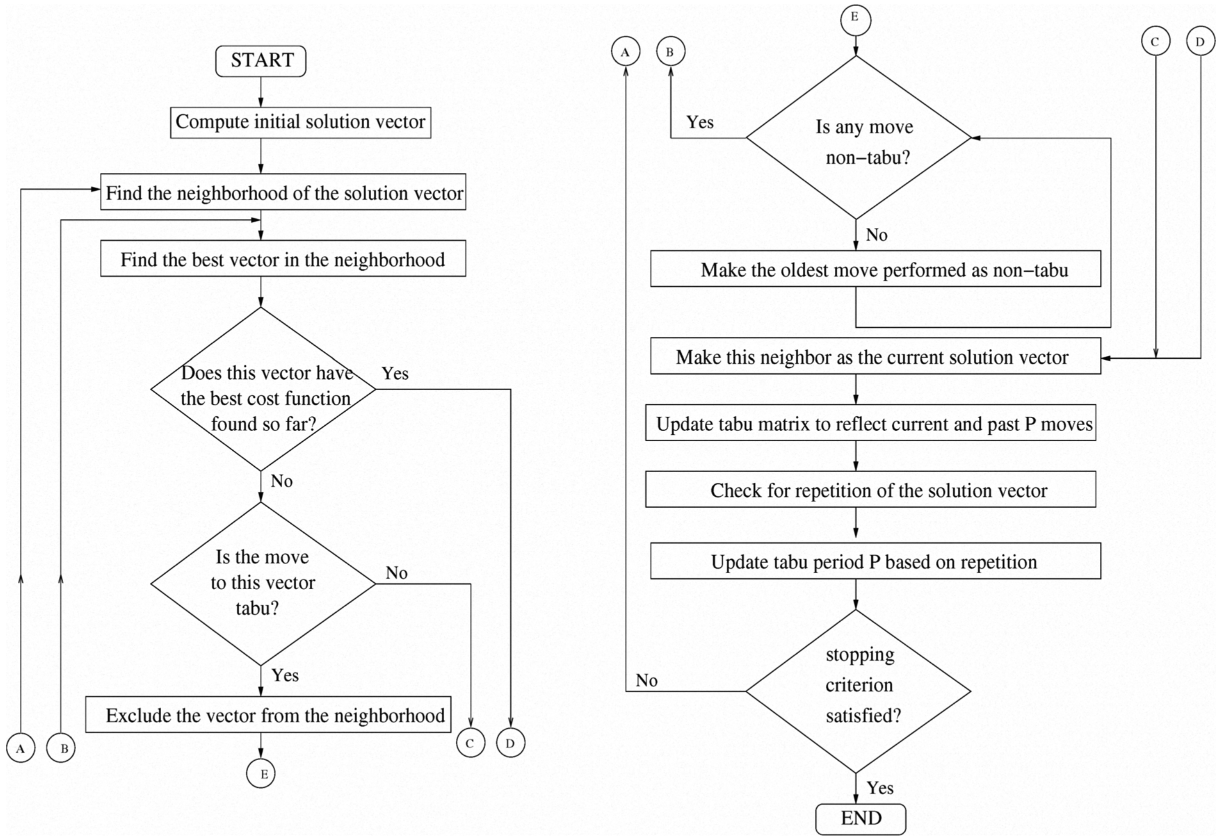

Fig. 3. Flow chart of the RTS algorithm.

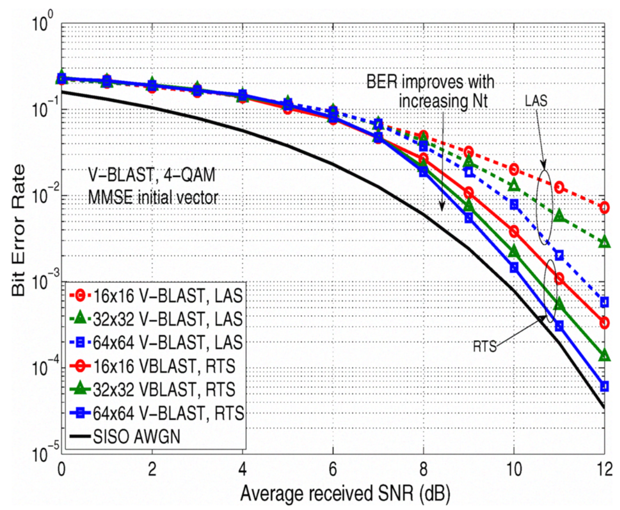

Fig. 4. Uncoded BER performance comparison between RTS and LAS algorithms in 16×16, 32×32 and 64×64 V-BLAST with 4-QAM.

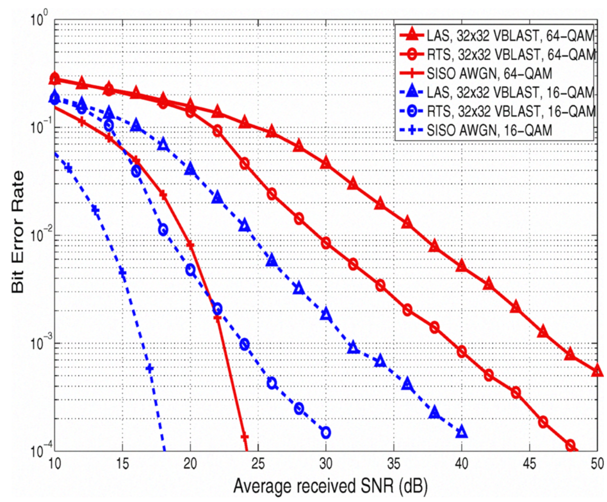

Fig. 5. Uncoded BER performance comparison between RTS and LAS algorithms in 32×32 V-BLAST with 16-QAM and 64-QAM.

of repetitions. The LAS algorithm, on the other hand, is free of such parameters. The additional features in RTS compared to LAS results in some increased complexity; however, the order of complexity of RTS remains the same as that of LAS, i.e., $O(N_t N_r)$ per-symbol complexity. Also, because of the additional features in it, RTS is able to achieve better performance than LAS, which is highlighted next.

Performance Comparison Between RTS and LAS: Figure 4 presents a BER comparison between RTS and LAS algorithms in 16×16, 32×32 and 64×64 V-BLAST with 4-QAM. It can be seen that for the number of dimensions (i.e., N_t) considered, RTS performs better than LAS; e.g., LAS requires 128 real dimensions (i.e., 64×64 V-BLAST with 4-QAM) to achieve performance close to within 1.8 dB of SISO AWGN performance at 10^{-3} BER, whereas RTS is able to achieve even better closeness to SISO AWGN performance with just 32 real dimensions (i.e., 16×16 V-BLAST with 4-QAM). Also, in 64×64 V-BLAST, RTS achieves 10^{-3} BER at an SNR of just 0.4 dB away from SISO AWGN performance. RTS is able to achieve this better performance because the inherent escape strategy in RTS is more effective in moving out of local minimas and move towards better solutions. Figure 5 shows that RTS performs better than LAS by about 6 dB at 10^{-3} uncoded BER in 32×32 V-BLAST with 16-QAM and 64-QAM.

C. BP Algorithm

A detection algorithm based on BP on factor graphs of MIMO systems is presented in [13]. In (2), each entry of the vector $\mathbf{y}$ is treated as a function node (observation node), and each symbol, $x_i \in \{\pm 1\}$, as a variable node. A key ingredient in the BP algorithm in [13], which contributes to its low complexity, is the Gaussian approximation of interference (GAI), where the interference plus noise term, z_{ik}, in

$$y_i \;=\; h_{ik}x_k + \underbrace{\overbrace{\sum_{j=1, j\neq k}^{2N_t} h_{ij}x_j}^{interference} + n_i}_{\triangleq\; z_{ik}}, \qquad (8)$$

is modeled as $\mathbb{C}\mathcal{N}(\mu_{z_{ik}}, \sigma^2_{z_{ik}})$ with $\mu_{z_{ik}} = \sum_{j=1, j\neq k}^{N_t} h_{ij}\mathbb{E}(x_j)$, and $\sigma^2_{z_{ik}} = \sum_{j=1,j\neq k}^{2N_t} |h_{ij}|^2 \,\text{Var}(x_j) + \frac{\sigma^2}{2}$, where h_{ij} is the (i,j)th element in $\mathbf{H}$. With x_i's $\in \{\pm 1\}$, the log-likelihood ratio (LLR) of x_k at observation node i, denoted by Λ_i^k, is

$$\begin{aligned} \Lambda_i^k &= \log \frac{p(y_i|\mathbf{H}, x_k = 1)}{p(y_i|\mathbf{H}, x_k = -1)} \\ &= \frac{2}{\sigma^2_{z_{ik}}} \Re\left(h^*_{ik}(y_i - \mu_{z_{ik}})\right). \end{aligned} \qquad (9)$$

The LLR values computed at the observation nodes are passed to the variable nodes (as shown in Fig. 6). Using these LLRs, the variable nodes compute the probabilities

$$p_i^{k+} \;\triangleq\; p_i(x_k = +1|\mathbf{y}) \;=\; \frac{\exp(\sum_{l\neq i} \Lambda_l^k)}{1 + \exp(\sum_{l\neq i}\Lambda_l^k)}, \qquad (10)$$

and pass them back to the observation nodes (Fig. 6). This message passing is carried out for a certain number of iterations. At the end, x_k is detected as

$$\widehat{x}_k \;=\; \text{sgn}\Big(\sum_{i=1}^{2N_r} \Lambda_i^k\Big). \qquad (11)$$

It has been shown in [13] that this BP algorithm with GAI, like LAS and RTS algorithms, exhibits 'large-system behavior,' where the bit error performance improves with increasing number of dimensions. Figure 7 shows the large-system behavior exhibited of the BP algorithm in V-BLAST with 4-QAM for $N_t = N_r = 8, 16, 24, 32, 64$.

BP Algorithm Complexity: In terms of complexity, the BP algorithm has the advantage of no need to compute an initial

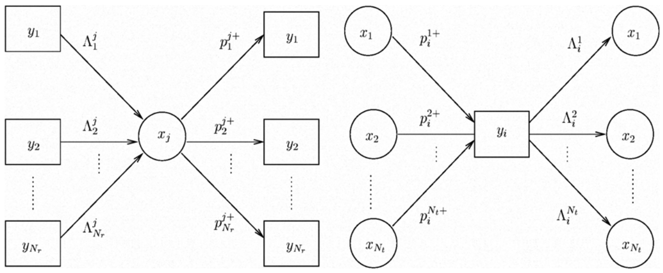

Fig. 6. Message passing between variable nodes and observation nodes.

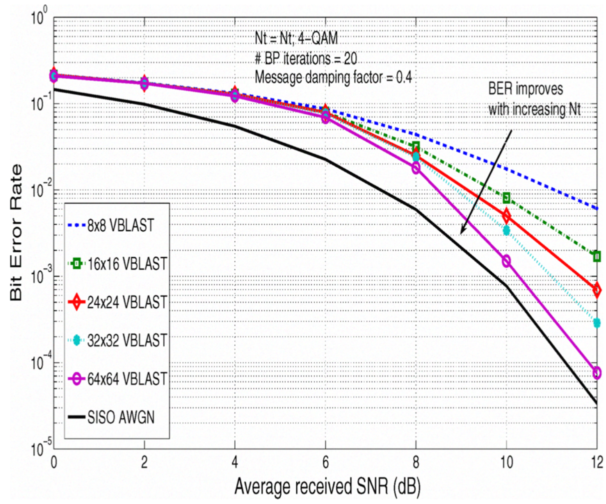

Fig. 7. Uncoded BER performance of the factor graph based BP algorithm with Gaussian approximation of interference in large V-BLAST MIMO systems. $N_t = N_r = 8, 16, 24, 32, 64$, 4-QAM, # BP iterations = 20.

solution vector and $\mathbf{H}^T\mathbf{H}$, which is required in RTS and LAS. The per-symbol complexity of the above BP algorithm for detection in V-BLAST is $O(N_t)$.

IV. Conclusions

We presented low-complexity detection algorithms that are shown to achieve near-optimal performance in large-MIMO systems, and compared their performances and complexities. These algorithms are based on local neighborhood search and belief propagation. The feasibility of such low-complexity large-MIMO signal detection algorithms can allow practical implementation of high spectral efficiency (tens to hundreds of bps/Hz) large-MIMO systems.

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