Report

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1 System Model

We consider a complex uncoded spatial multiplexing MIMO system with N_r receive and N_t transmit antennas, $N_r \geq N_t$, over a flat fading channel. Using a discrete time model, $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received symbol vector written as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ is the transmitted symbol vector, with components that are mutually independent and taken from a signal constellation $\mathbb{O}$ (4-QAM, 16-QAM, 64-QAM) of size

M. The possible transmitted symbol vectors $\mathbf{s} \in \mathbb{O}^{N_t}$, satisfy $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_t} E_s$, where E_s denotes the symbol average energy, and $\mathbb{E}[\cdot]$ denotes the expectation operation. Furthermore $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the Rayleigh fading channel propagation matrix with independent identically distributed (i.i.d) circularly symmetric complex Gaussian components of zero mean and unit variance. Finally, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise (AWGN) vector with zero mean components and $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = \mathbf{I}_{N_r} N_0$, where N_0 denotes the noise power spectrum density, and hence $\frac{E_s}{N_0}$ is the signal to noise ratio (SNR).

Assume the receiver has perfect channel state information (CSI), meaning that $\mathbf{H}$ is known, as well as the SNR. The task of the MIMO decoder is to recover $\mathbf{s}$ based on $\mathbf{y}$ and $\mathbf{H}$.

2 Modification of Orthogonality Deficiency

Original definition of orthogonality deficiency:

$$\phi_{od} = 1 - \frac{det(\mathbf{W})}{\prod_{i=1}^{N_t} ||\mathbf{h}_i||^2}, \tag{2}$$

where $\mathbf{W} = \mathbf{H}^H\mathbf{H}$ denotes Wishart matrix, $\mathbf{h}_i$ denotes the i th column of $\mathbf{H}$, $det(\cdot)$ denotes determinant operation, $||\cdot||^2$ denotes 2-norm operation. In (2), $||\mathbf{h}_i||^2 = \sum_{i=1}^{N_t} |\mathbf{H}_{ij}|^2$, $\mathbf{H}_{ij}$ denotes the component of $\mathbf{H}$ at i th row and j th column. $\mathbf{H}_{ij} \sim Rayleigh(1/\sqrt{2})$, therefore $||\mathbf{h}_i||^2 \sim \Gamma(N_r, 1)$ [1]. $\Gamma(k, \theta)$ denotes Gamma distribution, with k degrees of freedom. Furthermore, we have:

$$2||\mathbf{h}_i||^2 \sim \Gamma(N_r, 2) \sim \chi^2_{2N_r}, \tag{3}$$

χ^2_k denotes chi-square distribution with k degrees of freedom. Because $\ln(\chi^2)$ converges much faster than χ^2 [2] [3] as well as for simplicity, (2) can be changed to:

$$\phi_{om} = \frac{2^{N_t} det(\mathbf{W})}{\prod_{i=1}^{N_t} 2||\mathbf{h}_i||^2} \longrightarrow \frac{N_t \ln 2 + \ln det(\mathbf{W})}{\sum_{i=1}^{N_t} \ln 2||\mathbf{h}_i||^2}, \tag{4}$$

ϕ_{om} in (4) is defined as Orthogonality Measure. Based on Hadamard's inequality ($\prod_{i=1}^{N_t} ||\mathbf{h}_i|| \geq det(\mathbf{H})$) $\phi_{om} \in [0, 1]$. If ϕ_{om} is more closer to 1, $\mathbf{H}$ is closer to orthogonal matrix.

3 Derivation of Marginal Probability Density Functions (PDFs)

First we consider the marginal PDFs of the components in (4), define

$$V = \sum_{i=1}^{N_t} \ln 2||\mathbf{h}_i||^2, \tag{5}$$

$$U = N_t \ln 2 + \ln det\mathbf{W}, \tag{6}$$

where V is the sum of components $\ln 2||\mathbf{h}_i||^2 \quad i \in [1, N_t]$. Since each component converges to normality rapidly, it can be easily proved that V converges to normality.

Considering U, $\mathbf{W} = \mathbf{H}^H\mathbf{H}$, do QR factorization:

$$\mathbf{H} = \mathbf{Q}\mathbf{R}, \tag{7}$$

where $\mathbf{Q} \in \mathbb{C}^{N_r \times N_t}$ is a unitary matrix and $\mathbf{R} \in \mathbb{C}^{N_t \times N_t}$ is the upper triangular matrix. Using (7), we have $\mathbf{W} = \mathbf{R}^H\mathbf{R}$. r_{ii} denotes the i th diagonal component of $\mathbf{R}$, thus $\mathbf{W}$ can be rewritten as:

$$\mathbf{W} = N_t \ln 2 + \ln det \mathbf{R}^H \mathbf{R} = N_t \ln 2 + \ln det(\mathbf{R}^H) det(\mathbf{R}) = N_t \ln 2 + \ln \prod_{i=1}^{N_t} r_{ii}^H \prod_{i=1}^{N_t} r_{ii} = N_t \ln 2 + \sum_{i=1}^{N_t} \ln |r_{ii}|^2. \tag{8}$$

The next step that can be take into account is to find the distribution of $|r_{ii}|^2$.

An alternative is to consider Wishart distribution of $det(\mathbf{W})$.

4 Derivation of Probability of Orthogonality Measure

An alternative modification of (4) can be written as:

$$\phi_{om} = \frac{\prod_{i=1}^{N_t} |r_{ii}|^2}{\prod_{i=1}^{N_t} ||\mathbf{h}_i||^2}, \tag{9}$$

Take the logarithm of (9), we have

$$\log \phi_{om} = \sum_{i=1}^{N_t} \log \frac{|r_{ii}|^2}{||\mathbf{h}_i||^2}. \tag{10}$$

Notice that $\mathbf{R}$ can be viewed as the Cholesky factorization of $\mathbf{W}$. Based on Cholesky factorization, we have $||\mathbf{h}_i||^2 = \sum_{j=1}^{i-1} |r_{ji}|^2 + |r_{ii}|^2$. Thus (10) can be rewritten as:

$$\log \phi_{om} = \sum_{i=1}^{N_t} \log \frac{1}{\sum_{j=1}^{i-1} |r_{ji}|^2/|r_{ii}|^2 + 1}. \tag{11}$$

From (11), the lattice reduction we proposed should aim to reduce the component $\sum_{j=1}^{i-1} |r_{ji}|^2/|r_{ii}|^2$.

References

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