Antenna Selection for Spatial Multiplexing Systems Based on Minimum Error Rate

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Abstract—**Future cellular systems will employ spatial multiplexing with multiple antennas at both transmitter and receiver to take advantage of large capacity gains. In such systems it will be desirable to select a subset of available transmit antennas for link initialization, link maintenance, or handoff. In this paper we present a criteria for selecting the optimal antenna subset in terms of minimum error rate, when coherent receivers, either linear or maximum likelihood (ML), are used over a slowly varying channel. For the ML receiver we propose to pick the subset whose output constellation has the largest minimum Euclidean distance. For the linear receiver we propose use of the post-processing SNRs (signal to noise ratios) of the multiplexed streams whereby the antenna subset that induces the largest minimum SNR is chosen. Simulations demonstrate that our selection algorithms also provides diversity advantage thus making subset selection useful over fading channels.**

I. Introduction

Multiple-transmit multiple-receive antenna links are increasingly important because of their potential for extremely high spectral efficiencies [1]. One spatial modulation technique for such systems is known as spatial multiplexing [2] [3]. This modulation scheme obtains high spectral efficiencies by multiplexing the incoming data into multiple substreams and transmitting each substream on a different antenna. The substreams subsequently can be separated at the receiver by means of various different receiver algorithms [2] [3].

Mobiles in future cellular systems supporting spatial multiplexing [4] will be capable of receiving substreams from transmit antennas on one or more base stations. Selection from a plurality of transmit antennas might occur upon *link initialization*, when the mobile determines from which antennas it wishes to receive streams, for *link maintenance* when substreams are shifted to alternate antennas as the channel changes, or for *partial handoff* of some of the substreams between cells as the mobile moves. Simultaneous transmission from all available transmit antennas, however, may be difficult due to hardware costs [5]. It is therefore of interest to select a subset of available antennas for transmission.

Contributions. In this paper we propose criteria, motivated by minimizing the probability of symbol error, for selecting a subset of transmit antennas for spatial multiplexing systems that employ either the maximum likelihood (ML) receiver, the zero-forcing (ZF) or the minimum mean-square error (MMSE) linear receiver. Maximum likelihood receivers have superior performance [6] [7], can be implemented using efficient lattice code decoders [10], and are useful for lower bounding the symbol error rate of suboptimal receivers. Linear receivers offer a significant computational reduction and are more practical in systems with large numbers of transmit and receive antennas. A suboptimal switching criteria based on the minimum singular value of the channel is provided which works well for all receivers. We compare the performance of our selection criterion with baseline ML and linear systems over the flat-fading quasi-static channel in terms of SER (symbol error rate). The results are surprising - with as little as one extra transmit antenna - subset selection can dramatically improve the performance of both linear and ML receivers.

Prior work. Antenna selection has been considered in the past in the context of both transmit and receive diversity [8] [9]. Selection for multiple-transmit multiple-receive antenna systems was first presented in [5] based on an argument that it increases capacity. The selection criterion proposed therein is based on Shannon capacity and does not necessarily minimize the symbol error rate in spatial multiplexing systems with finite complexity decoding algorithms.

Organization. This paper is organized as follows. In Section 2 we present the system model under consideration. In Section 3 we present analysis of spatial multiplexing systems with either ML and linear receivers to motivate the selection criteria proposed in Section 4. In Section 5 we present Monte Carlo simulations which show that our selection can also offer diversity advantage. Finally, in Section 6, we provides some conclusions.

II. System Model

Consider a spatial multiplexing system with M_t transmit antennas, M_r receive antennas, and a $1 : M$ ($M_t > M$, $M_r \geq M$) multiplexer. The channel is flat-fading and slowly time varying. It is unknown at the transmitter but is known perfectly at the receiver. A low bandwidth, zero-delay, error-free, feedback path indicates the optimal M of M_t antennas for transmission computed using current channel state information at the receiver. Channel estimation errors, errors in the feedback path, and delay in the feedback path will reduce the benefits of antenna selection. Effects of these errors, particularly in mobile fading channels, is a topic for future work.

The transmitter in this system, illustrated in Fig. 1 works as follows. At one symbol time, M input symbols are multiplexed to produce the $(M \times 1)$ vector symbol $\mathbf{s}_n$ for transmission over M transmit antennas. The subset of $M \leq M_t$ transmit antennas is determined by a selection algorithm operating at the receiver, which indicates to the transmitter the optimal subset $p \in P$, where P is the set of all possible $\binom{M_t}{M}$ subsets of transmit antennas.

Let $\mathbf{H}$ denote the $M_r \times M_t$ channel matrix and let $\mathbf{H}_p$ denote the $M_r \times M$ submatrix corresponding to transmit antenna subset p. The corresponding received signal, at discrete-time n, after

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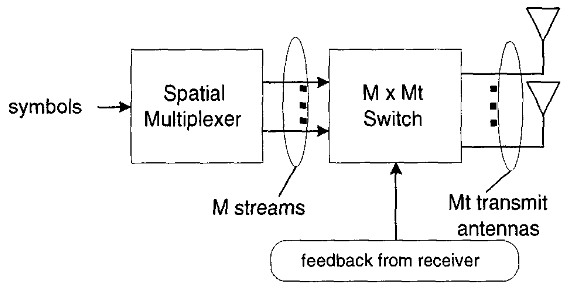

Fig. 1. Spatial multiplexing transmitter with subset selection

matched filtering and sampling is

$$\mathbf{x}_n = \sqrt{\frac{E_s}{M}}\mathbf{H}_p\mathbf{s}_n + \mathbf{v}_n \tag{1}$$

where $\mathbf{x}_n$ is $M_r \times 1$ and the noise $\mathbf{v}_n$ is $M_r \times 1$. The maximum total power transmitted on M antennas at one symbol time is E_s assuming that $\mathbf{s}_n$ is normalized such that $\mathrm{tr}(\mathrm{E}\{\mathbf{s}_n\mathbf{s}_n^H\}) = \mathrm{M}$. Symbols on all substreams are derived from the same constellation. The entries of $\mathbf{v}_n$ are i.i.d., $\mathbf{v}_n(i) \sim \aleph_c(0, N_0)$, and independent over n.

III. Performance Analysis

In this section we provide analysis of spatial multiplexing systems which employ either maximum likelihood or linear receivers. Let R denote the desired spectral efficiency and now define $M_{sm} = 2^{R/M}$ to be the number of points in the per-antenna constellation. Let $\mathbf{x}$ be the received data vector at sample time n, $\mathbf{s}$ the transmitted vector symbol, and $\mathcal{S}_{SM}$ the set of all possible transmitted vectors $\mathbf{s}$. The size of $\mathcal{S}_{SM}$ is $M_{SM} := |\mathcal{S}_{SM}| = M_{sm}^M$. In the analysis we consider the channel $\mathbf{H}_p$ which consists of the appropriate columns of $\mathbf{H}$ as dictated by the subset indicated by p. In the following we let p be the parameter of interest.

A. Maximum Likelihood Receiver

The maximum likelihood (ML) receiver is optimal when the transmitted vectors are equally likely. Recent advances in decoding [10] make ML receivers practical for even moderate spectral efficiencies and/or data rates. For large numbers of antennas or very high rate applications, however, other lower complexity receivers such as the linear receiver (for e.g., see the next section) or the Bell Labs layered space-time (BLAST) receiver [3] are generally preferred. Analysis of the ML receiver nonetheless provides a lower bound on the symbol error rate of practical spatial multiplexing systems.

From the above definitions, the ML estimate of the transmitted vector $\mathbf{s}$ is

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathcal{S}_{SM}} \left\| \mathbf{x} - \sqrt{\frac{E_s}{M}}\mathbf{H}_p\mathbf{s} \right\|^2. \tag{2}$$

This computation requires a search over all M_{SM} possible transmitted vectors.

To determine the performance of the receiver in (2) we would like to compute the probability of (vector) symbol error. The exact form can be numerically computed using a geometric approach [11] but, unfortunately, the result can not be readily expressed in closed form. We can at high SNR, however, upper-bound the performance of the receiver using the Union bound which is a function of $d^2_{min,SM}$, the squared minimum distance of the receive constellation defined as $\{\mathbf{H}_p\mathbf{s} \mid \mathbf{s} \in \mathcal{S}_{SM}\}$ [12].

Suppose that $\mathbf{s}_i \in \mathcal{S}_{SM}$, $\mathbf{s}_j \in \mathcal{S}_{SM}$ s.t. $\mathbf{s}_i \neq \mathbf{s}_j$. Let the squared minimum distance of the receive constellation be defined as

$$d^2_{min,SM} := \min_{\mathbf{s}_i,\mathbf{s}_j\in\mathcal{S}_{SM}, \mathbf{s}_i\neq\mathbf{s}_j} \frac{\|\mathbf{H}_p(\mathbf{s}_i - \mathbf{s}_j)\|^2}{M}. \tag{3}$$

Due to the linear transformation by the channel, the distance properties of $\mathbf{s}_i$ are not preserved unless $\mathbf{H}_p$ is unitary.

The computation in (3) requires a search over $\mathcal{S}_{SM}(\mathcal{S}_{SM} - 1)$ vectors which can be prohibitive for larger constellations. We therefore find it useful to develop a lower bound on $d^2_{min,SM}$ to simplify computations and to develop intuition on properties of the receive constellation. An upper bound is presented in [12].

We will compute the lower bound using some properties from linear algebra. Let $\mathbf{e}_{min}$ be the right singular vector of $\mathbf{H}_p$ corresponding to the smallest singular value λ_{min}. Let $\mathbf{e}_{ij} = \mathbf{s}_i - \mathbf{s}_j$ and let the minimum squared distance of the transmit constellation be $d^2_{min,sm} = \min_{\mathbf{s}_i,\mathbf{s}_j\in\mathcal{S}_{SM}} \|\mathbf{s}_i - \mathbf{s}_j\|^2$. Observe that

$$\min_{\mathbf{s}_i,\mathbf{s}_j\in\mathcal{S}_{SM}} \|\mathbf{H}_p\mathbf{e}_{ij}\|^2 = \min_{\mathbf{s}_i,\mathbf{s}_j\in\mathcal{S}_{SM}} \frac{\|\mathbf{H}_p\mathbf{e}_{ij}\|_2^2}{\|\mathbf{e}_{ij}\|^2}\|\mathbf{e}_{ij}\|^2 \tag{4}$$

$$\geq \lambda^2_{min}(p)\, d^2_{min,sm} \tag{5}$$

where (5) follows from the Rayleigh-Ritz theorem, i.e., $\frac{\|\mathbf{H}_p\mathbf{e}_{ij}\|^2}{\|\mathbf{e}_{ij}\|^2} \geq \min_{\mathbf{x}} \frac{\|\mathbf{H}_p\mathbf{x}\|^2}{\|\mathbf{x}\|^2} = \lambda^2_{min}(p)$ [13] and recognizing that $d_{min,sm} = \min_{ij} \|\mathbf{e}_{ij}\|$. Clearly, equality occurs if there exists an $\mathbf{s}_i - \mathbf{s}_j$ which is a scalar multiple of the minimum right singular vector of $\mathbf{H}_p$. Using (3) and (4) we have the desired result that

$$d^2_{min,SM} \geq \frac{\lambda_L^2(p)\, d^2_{min,sm}}{M}. \tag{6}$$

For $M_r \times M$ channels for which $\mathrm{rank}(\mathbf{H}_p) < \min(\mathrm{M_r}, \mathrm{M})$, (6) says that spatial multiplexing may perform quite poorly since in this case $\lambda_L = 0$. This is an important observation because, as M_{sm} increases, $\mathcal{S}_{SM}$ becomes more dense and thus it is more likely to find an error vector which lies near the smallest singular value of $\mathbf{H}_p$.

Using (6) we can upper bound the probability of symbol error using the Union bound as

$$P_{SM} < (|\mathcal{S}_{SM}| - 1)\, Q\left(\sqrt{\frac{E_s}{2N_0} d^2_{min,SM}}\right). \tag{7}$$

Further substituting the lower bound in (6) into (7) we obtain

$$P_{SM} < (|\mathcal{S}_{SM}| - 1)\, Q\left(\sqrt{\frac{E_s}{2N_0}\frac{\lambda_L^2 d^2_{min,sm}}{M}}\right). \tag{8}$$

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From (8) we see that spatial multiplexing with the ML receiver is influenced by the smallest mode in the channel and is therefore sensitive to channel condition and rank. This means that "good" channels are as close to unitary as possible. Such channels have an equal effect on all transmitted symbols.

B. Linear Receiver

The linear receiver is the simplest spatial multiplexing receiver since it requires only a matrix multiply to separate the substreams. It will therefore be of interest in practical systems, particularly those with large numbers of transmit and receive antennas. The linear receiver works as follows. An $M \times M_r$ matrix equalizer $\mathbf{G}$ (computed according to some criterion) is applied to $\mathbf{x}_n$ to obtain an estimate of $\mathbf{s}_n$ as

$$\hat{\mathbf{s}}_n = \mathbf{G}\mathbf{x}_n = \mathbf{G}\mathbf{H}_p\mathbf{s}_n + \mathbf{G}\mathbf{v}_n. \tag{9}$$

Detection is performed assuming that $\mathbf{GH}_p = \mathbf{I}$ thus nonzero cancellations in the off-diagonal terms contribute to additional noise. Note that, unlike the ML receiver, the minimum distance of the received constellation is the same as that of the transmit constellation. The equalizer, however, colors the noise thus the noise power can vary as a function of channel. Therefore for the linear receiver it is the post-processing SNR that is the parameter of interest.

We derive expressions for the post-processing SNR of each of the M multiplexed streams. Let $\mathbf{g}_k'$ be the k^{th} row of $\mathbf{G}$ and $\mathbf{h}_k$ be the k^{th} column of $\mathbf{H}_p$. From (9) the post-processing SNR of the k^{th} stream is

$$SNR_k = \frac{E_s|\mathbf{g}_k'\mathbf{h}_k|^2}{MN_0||\mathbf{g}_k||^2 + E_s\sum_{j\neq k}|\mathbf{g}_k'\mathbf{h}_j|^2}. \tag{10}$$

Let $\mathbf{G}_P = \mathbf{H}_p^\dagger$ be the pseudo inverse of $\mathbf{H}_p$. We are interested in the minimum distance of the output constellation on each of the M multiplexed streams. Let $\mathbf{g}_k'$ be the k^{th} row of $\mathbf{G}$ and $\mathbf{h}_k$ be the k^{th} column of $\mathbf{H}_p$. From (9) the post-processing SNR of the k^{th} stream is

$$SNR_k = \frac{E_s|\mathbf{g}_k'\mathbf{h}_k|^2}{MN_0||\mathbf{g}_k||^2 + E_s\sum_{j\neq k}|\mathbf{g}_k'\mathbf{h}_j|^2}.$$

For the MMSE (minimum mean-square error) receiver, $\mathbf{G} = [\mathbf{H}_p^*\mathbf{H}_p + N_0/E_s\mathbf{I}_M]^{-1}\mathbf{H}_p^*$ where * denotes the Hermitian. After some computation (10) simplifies to

$$SNR_k^{(MMSE)} = \frac{E_s}{MN_0[\mathbf{H}_p^*\mathbf{H}_p + N_0/E_s\mathbf{I}_M]_{kk}^{-1}} - 1.$$

For the ZF (zero-forcing) receiver $\mathbf{G} = \mathbf{H}_p^\dagger$, the pseudoinverse of $\mathbf{H}_p$, and (10) simplifies to

$$SNR_k^{(ZF)} = \frac{E_s}{MN_0[\mathbf{H}_p^*\mathbf{H}_p]_{kk}^{-1}}. \tag{11}$$

An approximation to (11) can be obtained by using the result that

$$\begin{aligned}\max_k[\mathbf{H}_p^*\mathbf{H}_p]_{kk}^{-1} &= \max_k \mathbf{e}_k'[\mathbf{H}_p^*\mathbf{H}_p]^{-1}\mathbf{e}_k && (12)\\ &\le \max_{||x||^2=1}\mathbf{x}'[\mathbf{H}_p^*\mathbf{H}_p]^{-1}\mathbf{x} && (13)\\ &= \lambda_{max}([\mathbf{H}_p^*\mathbf{H}_p]^{-1}) && (14)\\ &= \lambda_{min}^{-2}(p)\end{aligned}$$

where $\mathbf{e}_k$ is the k^{th} column of $\mathbf{I}_M$. Equation (13) follows by recognizing that $\{\mathbf{e_k}\}_{\mathbf{k=1}}^{\mathbf{M_r}}$ is a subset of the space of complex vectors $\mathbf{x}$ such that $||\mathbf{x}|| = 1$ while (14) follows from the Rayleigh-Ritz theorem. This result can then be used to bound (11) as

$$SNR_{min}^{(ZF)} \ge \lambda_{min}^2(p)\frac{E_s}{MN_0} \tag{15}$$

which is a simple function of the channel. The expression in (15) confirms the intuition that the performance of linear receivers should improve as the smallest singular value of the channel increases. Note the similarity of (15) to that in (5)

Now performance in terms of symbol error rate can be computed as follows. Define SNR_{min}:=$\min_k SNR_k$ and k_{min}:=$\arg\min_k SNR_k$. Depending on the input constellation, the exact probability of symbol error P_k can be computed from published formulas [14] for a given SNR_k. We write the probability of vector symbol error (probability that at least one substream symbol is in error) as

$$\begin{aligned}P_{SM} &= 1 - \prod_{k=1}^{M}(1 - P_k) && (16)\\ &\le 1 - (1 - P_{k_{min}})^M && (17)\\ &\approx MP_{k_{min}} && (18)\\ &\le MN_eQ\left(\sqrt{SNR_{min}\frac{d_{min}^2}{2}}\right) && (19)\end{aligned}$$

where d_{min}^2 is the squared minimum distance of the transmit constellation. In (16) we rewrite the probability of vector symbol error as one minus the probability of no errors and in (17) we upper-bound this quantity by $P_{k_{min}} := \min_k P_k$. For low values of $P_{k_{min}}$, we get the approximation in (18) and apply the NNUB (nearest neighbor union bound) to obtain (19). In (19) N_e is the number of nearest neighbors in the per-antenna constellation.

Computation of SNR_{min} requires a search over all equalizers $\mathbf{g}_k$ in (10) for all channel subsets $p \in P$ in (1). In (15) we provide a lower bound for SNR_{min} that only requires searching over all subsets $p \in P$.

IV. Antenna Selection Criteria

Performance in a spatial multiplexing system depends on the receiver type. For the optimum ML receiver, performance depends on the minimum Euclidean distance of the received constellation as shown in (3). This value is a function of both the constellation and the channel $\mathbf{H}_p$. A natural switching criteria is therefore obtained as follows.

SC-ML. *Maximum minimum Euclidean distance: For every subset of transmit antennas $p \in P$ compute $d_{min,SM}^2$, choose the subset with the largest $d_{min,SM}^2$.*

For the linear receiver, performance is influenced by the SNR_{min} induced by the particular subset of transmit antennas as shown in (19). The logical antenna subset selection criterion (SC) for the linear receiver is therefore the following.

SC-LN. *Maximum post-processing SNR: For every subset of transmit antennas $p \in P$ compute $\mathbf{G}$ and the corresponding SNR_{min}, choose the subset with the largest SNR_{min}.*

Unfortunately, computation of SC-ML requires an extensive search while computation of SC-LN requires computing the linear receiver corresponding to every possible subset. It is therefore of interest to develop alternative selection criteria independent of the receiver. To this end, note that the lower bound on the $d^2_{min,SM}$ in (6) is a function of $\lambda^2_L(p)$ as is a lower bound on SNR_{min} in (15) for the ZF receiver. We therefore obtain the following approximate selection criterion which is based solely on the channel.

SC-SV. *Maximum minimum singular value: For every subset of transmit antennas $p \in P$ compute λ_{min} corresponding to $\mathbf{H}_p$. Choose the subset with the largest λ_{min}.*

Finally, for reference we provide the capacity-based selection criterion proposed in [5].

SC-C. *Maximum capacity: For every subset of transmit antennas $p \in P$ compute $C_p := \log\det(\mathbf{I}_M + E_s/N_0 \mathbf{H}'_p\mathbf{H}_p)$. Choose the subset with the largest C_p.*

To clarify that the selection criteria presented above do not give the same results for all channels we provide the following example.

Example. Consider 4-QAM transmit constellation, $SNR = 10dB$, and

$$\mathbf{H} = \begin{bmatrix} 4 & 1 & -1 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}. \quad (20)$$

SC-ML picks the first two antennas, SC-LN and SC-SV pick the last two antennas, and SC-C picks the first and third antennas.

We remark that SC-C is based on a general capacity formula and is not specialized to a specific receiver structure. Therefore we expect that for certain channels, optimal selection in terms of capacity will yield suboptimal performance particularly for suboptimal receivers.

V. Simulations

In this section the three selection criteria are compared via Monte Carlo simulations. Performance is measured in terms of SER (symbol error rate) for a frame of 100 symbols from QAM constellations averaged over 10,000 frames. The coefficients of channel $\mathbf{H}$ are i.i.d. (independent, identically distributed) circular complex Gaussian random variables with variance 0.5 in each dimension, i.e. $h_{i,j} \sim \aleph_c(0,1)$. Channel realizations are i.i.d. from frame to frame. In the following, let $X \times Y$ denote a spatial multiplexing system with X transmit antennas and Y receive antennas. We will consider the effect of selecting Y transmit antennas out of the X antennas available at the transmitter.

A. ML Receiver

Consider a 3×2 system with an ML receiver. For reference we compare the three selection criteria proposed above with 2×2 ML and 1×1 ML systems. In Fig. 2 we plot the SER

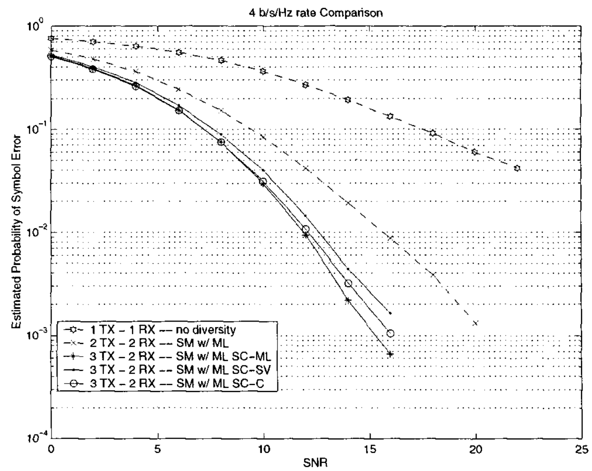

Fig. 2. ML: 3×2 System

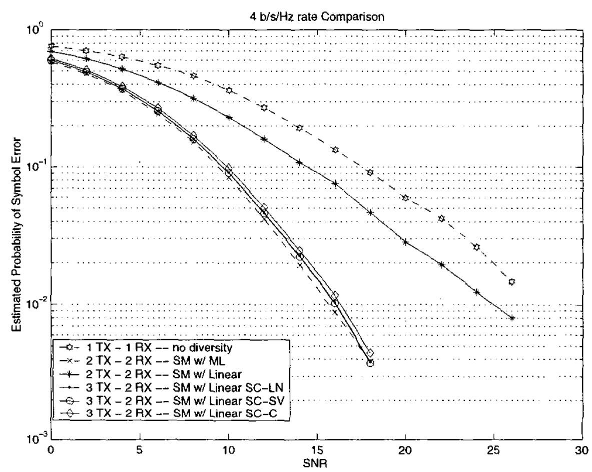

Fig. 3. ZF: 3×2 Comparison

for a 3×2 system at a spectral efficiency of 4 bits/s/Hz. Without additional transmit antennas to choose from, the 2×2 ML system obtains a second order diversity advantage over the 1×1 system. With selection, however, the 3×2 system obtains a third order diversity advantage with an improvement of greater than $3dB$ at an SER of 10^{-2}. Note from the figure that SC-ML gives the best performance with SC-C slightly worse. Capacity based switching is therefore nearly optimal for ML receivers. Singular value switching, which requires a much lower computation complexity than $d^2_{min,SM}$, loses less than $1dB$ of performance while still giving the diversity advantage making this criterion exceptional for practical applications.

B. ZF receiver

Now consider the practically relevant ZF receiver. We consider the following pairs: 3×2, 4×2, and 4×3 to fully grasp the impact of selection in this system. For reference we include the performance of three baseline techniques (1×1, 2×2 ML, and 2×2 ZF).

In Fig. 3 we plot the SER for a 3×2 system at a spectral effi-

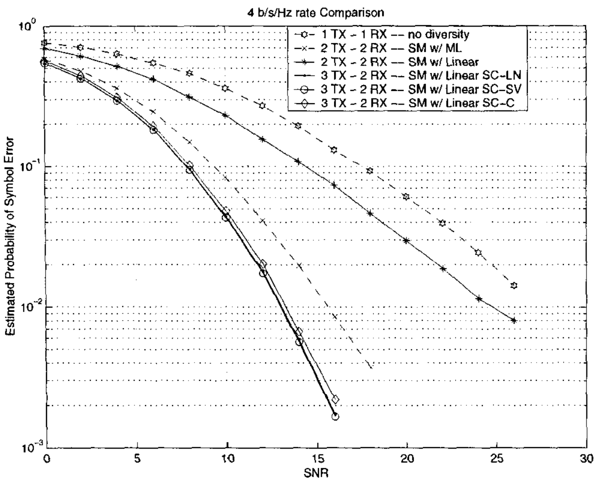

Fig. 4. ZF: 4×2 Comparison

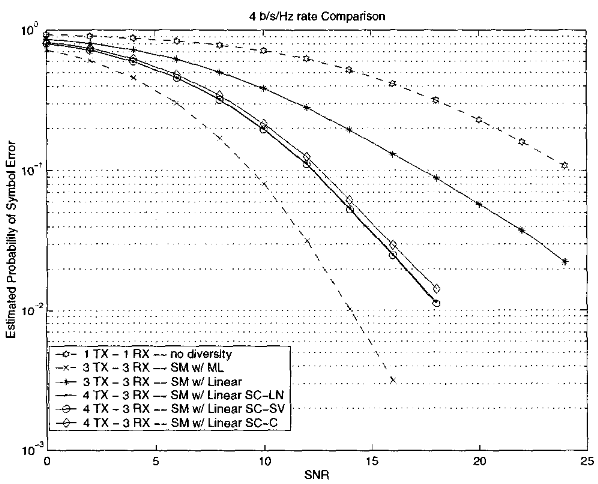

Fig. 5. ZF: 4×3 Comparison

ciency of 4 bits/s/Hz. Without additional transmit antennas from which to choose, the 2×2 ZF system does not obtain any diversity advantage over the 1×1 system. With optimal selection, however, the 3×2 linear system obtains a diversity advantage rivalling that of the 2×2 ML receiver. At an SER of 10^{-2}, the 3×2 system is $1dB$ short of the 2×2 ML and a significant 9dB better than the 2×2 linear receiver. Among the three selection criteria, SC-LN and SC-SV give the best performance while SC-C is about $1dB$ worse. As expected SC-LN has better performance (although slight) than SC-C verifying the earlier claim that capacity-based selection is not optimal in terms of SER for the linear receiver.

In Fig. 4 we repeat the above experiment for a 4×2 system. Note that the extra two transmit antennas give an effective third order diversity advantage outperforming the 2×2 ML receiver by $3dB$ at an SER of 10^{-2}.

In Fig. 5 we plot the performance of antenna selection in a 4×3 system with spectral efficiency $R = 6$ bits/s/Hz. The linear receiver with selection obtains a second order diversity advantage. Observe for the 3×3 system, as predicted by [7], the ML receiver gives a diversity advantage of $M_r = 3$ while the linear receiver obtains no diversity advantage without selection.

Selection with an extra transmit antenna appears to give an additional degree of diversity advantage. We conjecture that with optimal selection, the diversity advantage for the maximum likelihood receiver with $M_r = M$ streams and M_t transmit antennas is $M_r + M_t - M$ while for the zero-forcing receiver with $M = M_r$ streams and M_t transmit antennas it is $M_t - M + 1$. When $M_t = M_r$ (e.g., no selection) this agrees with the observations in [7]. Proof of this conjecture when multiple transmit antennas are available is currently under investigation.

VI. Conclusions

In this paper we derived criteria for selecting the an optimal subset of transmit antennas, for a spatial multiplexing system, motivated by minimizing the probability of symbol error. For the maximum likelihood receiver, it was shown that for high SNR, the desired metric is to choose the subset whose channel maximizes the minimum Euclidean distance of the received constellation. Alternatively, for the linear receiver, we showed that it is better to choose the subset whose channel maximizes the minimum post-processing SNR. The difference is due to the fact that the linear receiver colors the noise and, in the MMSE case, does not fully decouple the transmitted substreams. We showed via simulations that antenna selection is also an inexpensive way to obtain diversity advantage a multiple antenna fading channel. Availability of additional transmit antennas for selection can therefore improve performance in spatial multiplexing systems.

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