20 October 2023, Econ 301

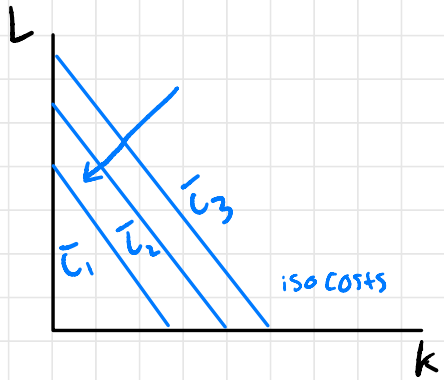
Content Review:

- Cost function $\pi(q) = R(q) - C(q)$

want to derive output cost

$C(K,L) = r \cdot k + w \cdot L$

cost from inputs; interest rate; wage



$\text{isocost}(\bar{C}) = \{(k,l): C(K,L) = \bar{C}\}$

$\text{Slope} = -\frac{r}{w}$

- Cost minimization:

$C(q) = \min C(K,L) = r \cdot k + w \cdot L$

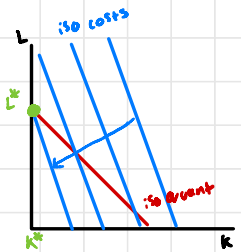
$\text{s.t. } F(K,L) = q$

isoquant fixed
find lowest iso cost

1. Cobb - Douglas: $MRTS_{KL} = -\frac{r}{w}$

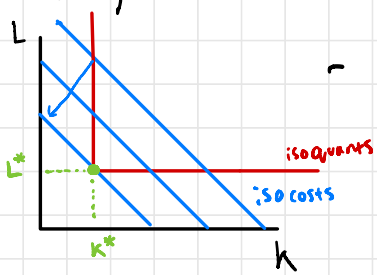
Slope of isoquant = slope of isocost

2. Perfect Substitutes: $F(K, L) = \alpha K + \beta L$



- use all capital or all labor, whichever is cheaper

3. Perfect complements: $F(K, L) = \min\{\alpha K, \beta L\}$



- use K & L in optimal ratio

$\alpha K = \beta L$

$\Rightarrow$ gives you $K^*(q)$, $L^*(q)$

$\Rightarrow$ $c(q) = C(K^*(q), L^*(q))$

$= r \cdot K^*(q) + w \cdot L^*(q)$

- Supply Curve: use that $MR = MC$
  - Solve for q

example: $c(q) = q^4$, $MC = 4q^3$

$P = MC = 4q^3 \Rightarrow q = \left(\frac{P}{4}\right)^{\frac{1}{3}}$

$S(p) = \left(\frac{p}{4}\right)^{\frac{1}{3}}$

Supply elasticity:

$$\eta = \frac{\partial S(p)}{\partial p} \cdot \frac{p}{S(p)} = \frac{1}{3}\left(\frac{p}{4}\right)^{-\frac{2}{3}} \cdot \frac{p}{\left(\frac{p}{4}\right)^{\frac{1}{3}}}$$

"for a 1% increase in P, Supply increases by η %"

Practice Problems:

1. $r = 100$, $w = 25$ $\quad F(k,L) = 5k + L$

i) $C(k,L) = 100k + 25L$

ii) $\min \; 100k + 25L$

s.t. $5k + L = q$

is using capital or labor cheaper?

if the firm uses L,

plug into constraint $\quad k = 0 \Rightarrow 5 \cdot 0 + L = q$

$\Rightarrow L^*(q) = q$

$C(q) = C(k^*, L^*) = 100 \cdot 0 + 25 \cdot q$

$= 25q$

② if they use K, $L=0$

$\Rightarrow 5k + 0 = q \Rightarrow k^* = \frac{q}{5}$

$c(q) = C\left(\frac{q}{5}, 0\right) = 100 \cdot \frac{q}{5} + 25 \cdot 0$

$= 20q$

Since $20q < 25q$, use capital

2. $F(K,L) = \min\{3K, L\}$, $r=6$, $w=5$

$c(q) = \min \; 6 \cdot k + 5 \cdot L$

s.t. $\min\{3K, L\} = q$

$\alpha K = \beta L$

$3K^* = L^* = q \Rightarrow k^*(q) = \frac{q}{3}$

$L^*(q) = q$

$c(q) = C(K^*, L^*) = 6 \cdot \frac{q}{3} + 5 \cdot q$

$= 2q + 5q = 7q$

if the firm is producing in a competitive market,

$mc = P$. $mc = \frac{\partial}{\partial q} c(q) = 7$

$\Rightarrow \quad p = 7$

3. $C(q) = 4q^2 \qquad S(p)?$

$MC = MR = p$

$MC = \frac{\partial}{\partial q} 4q^2 = 8q$

$p = 8q \Rightarrow q = \frac{p}{8}$

$S(p) = \frac{p}{8}$

$\eta = \frac{\partial S(p)}{\partial p} \cdot \frac{p}{S(p)}$

$= \frac{1}{8} \cdot \frac{p}{p/8} = \frac{8}{8} \cdot \frac{p}{p} = 1$

4. $\min \quad r \cdot k + w \cdot L \qquad MRTS_{KL} = -\frac{r}{w}$

$s.t. \quad F(K,L) = q$

$= \min \quad 4k + L$

$s.t. \quad k^{\frac{1}{4}} L^{\frac{1}{4}} = q$

Solve for k: $k^{\frac{1}{4}} = \frac{q}{L^{1/4}} \Rightarrow k = \frac{q^4}{L}$

Plug into objective function

$$= \min_L \; 4\left(\frac{q^4}{L}\right) + L$$

F.O.C.

$$\frac{\partial}{\partial L}\left(\frac{4q^4}{L} + L\right) = 0$$

$$= -\frac{4q^4}{L^2} + 1 = 0$$

$$\Rightarrow \quad L^2 = 4q^4$$

$$\Rightarrow \quad L^* = 2q^2$$

Plug L^* into rule for K

$$\Rightarrow K^* = \frac{q^4}{2q^2} = \frac{q^2}{2}$$

$$C(q) = C(K^*, L^*) = 4 \cdot \frac{q^2}{2} + 2q^2$$

$$= \boxed{4q^2}$$