10-13-23

Content Review:

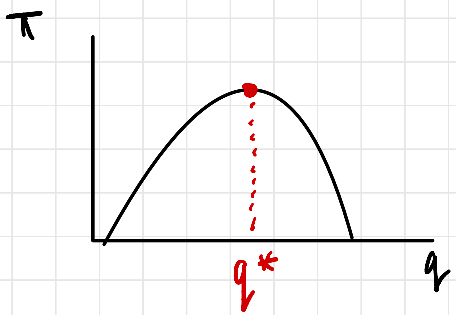
- Profit maximization

$$\pi(q) = R(q) - C(q)$$

If competive market, $R(q) = p \cdot q$

$$MR = \frac{dR}{dq} = P$$

$$MC = \frac{dC}{dq}$$



$$\pi'(q) = 0$$

$$\Leftrightarrow R'(q) - c'(q) = 0$$

$$\Leftrightarrow R'(q) = c'(q)$$

$$\boxed{MR = MC}$$

- Production functions: $F(K, L) = q$

  - Cardinal, not ordinal

① perfect substitutes

$$F(K, L) = \alpha K + \beta L$$

② perfect complements

$$F(K, L) = \min\{\alpha K, \beta L\}$$

③ Cobb - Douglas

$$F(K,L) = A K^{\alpha} L^{\beta}$$

- Isoquants

$$\bar{q} \text{ isoquant} = \{(K,L) : F(K,L) = \bar{q}\}$$

Slope of isoquant $MRTS_{KL} = \dfrac{-MPK}{MPL}$

$$MPK = \frac{\partial F}{\partial K}, \quad MPL = \frac{\partial F}{\partial L}$$

- Returns to Scale: $F(K,L)$ has

① CRS iff $\forall x > 1$

$$F(xK, xL) = x F(K,L)$$

② IRS iff $\forall x > 1$

$$F(xK, xL) > x F(K,L)$$

- assume firms can't have IRS in competitive markets

③ DRS iff $\forall x > 1$

$$F(xK, xL) < x F(K,L)$$

Practice Questions:

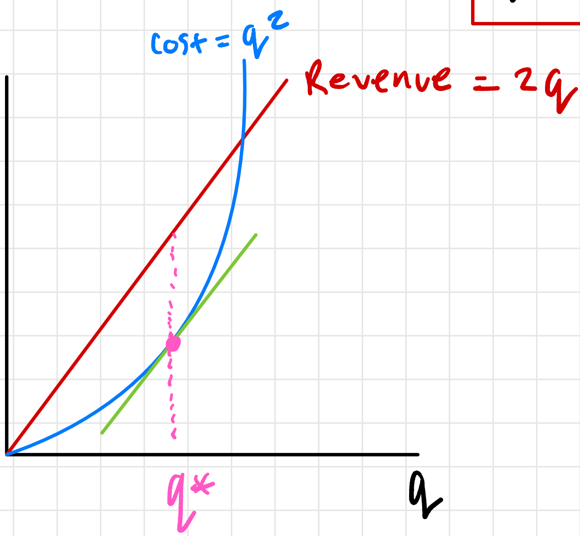
1. $c(q) = q^2$, $P = 2$, $R = p \cdot q$

$$MC = \frac{dC}{dq} = \frac{d}{dq} q^2 = \boxed{2q}$$

$$MR = \frac{dR}{dq} = p = \boxed{2}$$

$$MC = MR \iff 2q = 2$$

$$\iff \boxed{q = 1}$$



$\Pi = R - C$

2. $F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{3}}$

$$MPL = \frac{\partial F}{\partial L} = \frac{1}{3} K^{\frac{1}{2}} L^{-\frac{2}{3}} = \boxed{\frac{1}{3} \frac{K^{\frac{1}{2}}}{L^{2/3}}}$$

$\boxed{\text{DRS}}$: $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$

$\alpha + \beta < 1$

Proof.

Let $x > 1$. $F(xK, xL) = (xK)^{\frac{1}{2}} (xL)^{\frac{1}{3}}$

$= x^{\frac{5}{6}} K^{\frac{1}{2}} L^{\frac{1}{3}}$

$= x^{\frac{5}{6}} F(K, L)$

$< x F(K, L)$

3. $F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{3}}$

$$MRTS_{KL}(3,2) = -\left.\frac{MPK}{MPL}\right|_{(3,2)} =$$

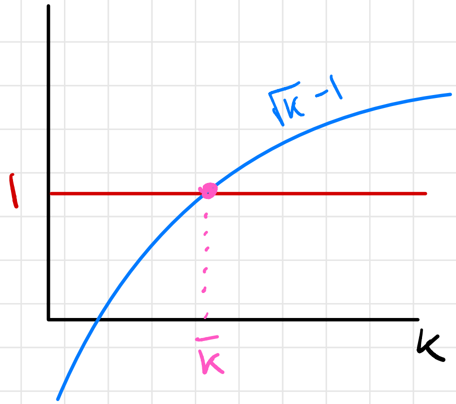
$MPK = \frac{\partial F}{\partial K} = \frac{1}{2} \frac{L^{\frac{1}{3}}}{K^{\frac{1}{2}}}$

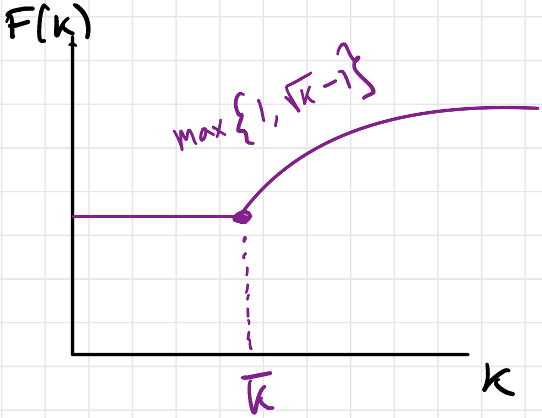
$$= \left.\frac{-\frac{1}{2}\frac{L^{\frac{1}{3}}}{K^{\frac{1}{2}}}}{\frac{1}{3}\frac{K^{\frac{1}{2}}}{L^{2/3}}}\right|_{(3,2)}$$

$$= -\frac{3}{2}\left.\frac{L}{K}\right|_{(3,2)} = -\frac{3}{2} \cdot \frac{2}{3}$$

$$= -1$$

4. $F(K) = \max\{1, \sqrt{K} - 1\}$





$$1 = \sqrt{\bar{k}} - 1 \implies \sqrt{\bar{k}} = 2$$
$$\implies \bar{k} = 4$$

$$F(k) = \begin{cases} 1, & k < 4 \\ \sqrt{k}-1, & k \geq 4 \end{cases}$$

F is piecewise
$\Rightarrow$ F' is piecewise

$$\boxed{mpk = \frac{\partial F}{\partial k} = \begin{cases} 0, & k < 4 \\ \frac{1}{2\sqrt{k}}, & k > 4 \end{cases}}$$

$F(xk)$, $x F(k)$

① $k < 4$, $x > 1$ =

$F(xk) = 1$

$< x F(k) = x$ DRS

② $k \geq 4$, $F(xk) = \sqrt{x}\sqrt{k} - 1$

$x F(k) = x\sqrt{k} - x$

DRS iff $\forall x > 1$,

$$\sqrt{x}\sqrt{k} - 1 < x\sqrt{k} - x$$

$$\iff \underbrace{\frac{x-1}{x-\sqrt{x}}}_{<2} < \underbrace{\sqrt{k}}_{>2}$$

$$\boxed{\begin{array}{l} \text{Since } F \text{ is DRS for } k \geq 4 \text{ and} \\ F \text{ is DRS for } k < 4, \text{ then} \\ F \text{ is DRS everywhere} \end{array}}$$