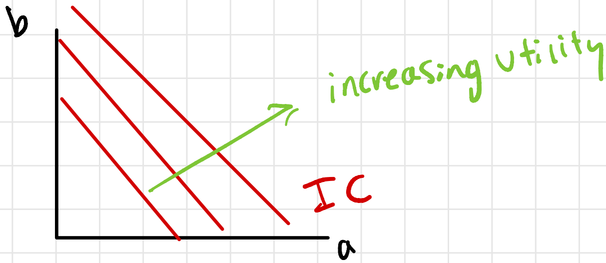
9-22-23
Friday

- Utility functions

① perfect substitutes – linear

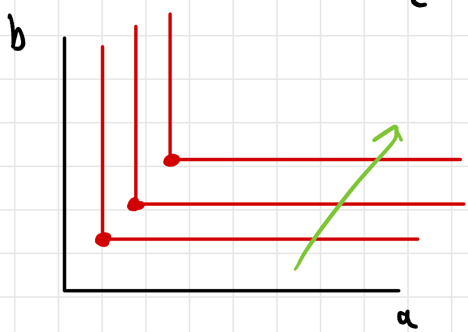
$$U(a,b) = \alpha a + \beta b$$

$$\left.\begin{aligned} MU_a &= \alpha \\ MU_b &= \beta \end{aligned}\right\} \quad MRS_{ab} = -\frac{\alpha}{\beta}$$



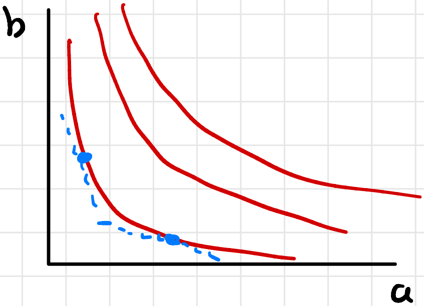
② Perfect complements – Leontief

$$U(a,b) = \min\{\alpha a, \beta b\}$$



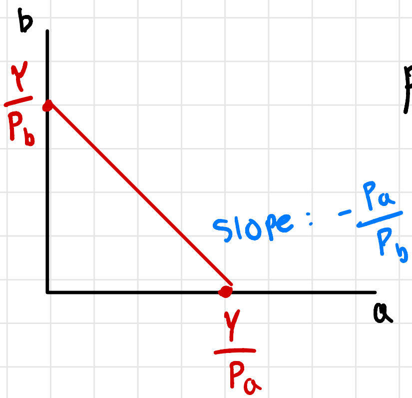
③ Cobb-Douglas $\quad U(a,b) = a^{\alpha} b^{\beta}$

$$MRS_{ab} = \frac{-MU_a}{MU_b} = -\frac{\alpha a^{\alpha-1} b^{\beta}}{\beta a^{\alpha} b^{\beta-1}}$$



$$= - \frac{\alpha b}{\beta a}$$

- Budget Constraint: given P_a, P_b, Y

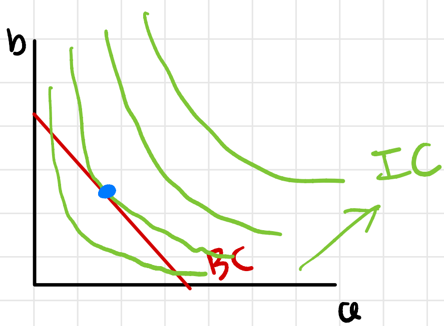


$$P_a \cdot a + P_b \cdot b \leq Y$$

- Utility maximization problem:

$$\max \quad U(a,b)$$

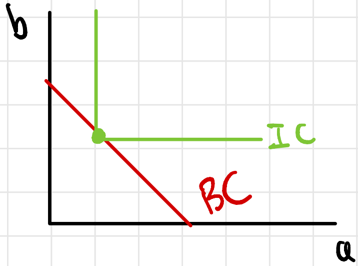
$$\text{s.t.} \quad P_a \cdot a + P_b \cdot b = Y, \quad a \geq 0, \; b \geq 0$$



① linear:
consume all a or all b

② Perfect complements
consume in the optimal ratio

③ Cobb-Douglas
$MRS_{ab} = -\frac{P_a}{P_b}$

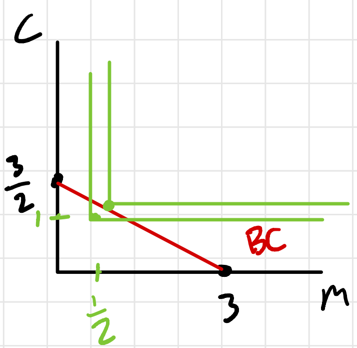


1. $U(m,c) = \min\{2m, c\}$. $\qquad 2m + 4c = 6$

$$\max \quad \min\{2m, c\}$$
$$\text{s.t.} \quad 2m + 4c = 6$$

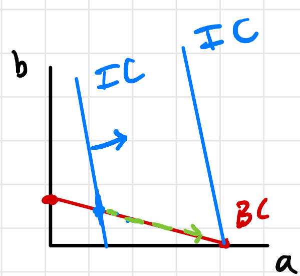
optimal ratio: $2m = c \iff m = \frac{c}{2}$

$$c + 4c = 6$$
$$c = \frac{6}{5}$$
$$\left(2m + 4 \cdot \frac{6}{5} = 6\right) \cdot 5$$
$$\Rightarrow 10m + 24 = 30$$
$$10m = 6$$
$$m = \frac{3}{5}$$



2. $MRS_{ab} = 2$

$P_a = 1, P_b = 3$



consume less	more of	a b.

3. $U(a,b) = a^{\frac{1}{2}} + b^{\frac{1}{2}}$

$IC_1: \quad U(a,b) = 1$

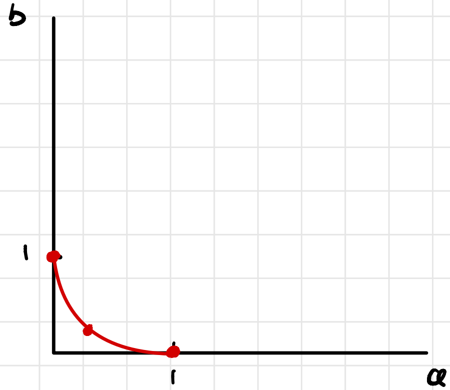
$a^{\frac{1}{2}} + b^{\frac{1}{2}} = 1$

$a^{\frac{1}{2}} + 0 = 1$

$\Rightarrow a = 1$

$b^{\frac{1}{2}} = 1$

$b = (1 - a^{\frac{1}{2}})^2$



$a = \frac{1}{2}$

$b = (1 - .707)^2$

$= .08$

4. $U(a,b) = ab^2$, $Y \geq 0$

consumes $a = 0$

$Y > 0$

$a = \frac{Y}{2}$, $b = \frac{Y}{2}$