10-27-23

Content Review:

- Inverse demand, residual demand

$$\underbrace{D(p)}_{q} = \underbrace{140 - 2p}_{f(p)}$$

$q = 140 - 2p$

$\Rightarrow 2p = 140 - q$

$\Rightarrow p = \frac{140 - q}{2}$

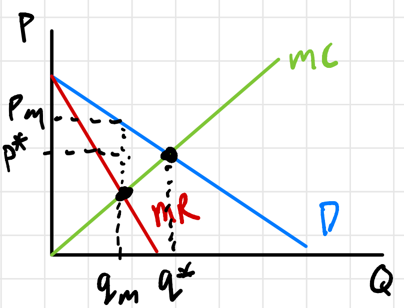
$$P(q) = D^{-1}(q) = \frac{140 - q}{2}$$

firm i, not firm i

residual demand: $P(q_i ; q_{-i}) = \frac{140 - (q_i + q_{-i})}{2}$

use $q = q_i + q_{-i}$

- Monopoly: $\max_q \; P(q) \cdot q - c(q)$



$MR = MC$

$$P = \underbrace{\frac{1}{1 + \frac{1}{\varepsilon}}}_{\text{monopoly markup}} MC$$

- Nash Equilibrium: a profile of strategies such that each player is best responding to others' strategies

	Q	S
Q	-1, -1	-3, 0
S	0, -3	-2, -2

- no incentive to deviate in equilibrium.

- Check for an intersection of best responses of all players.

- Cournot competion:

  - firms compete on quantity

$$\max_{q_i} \; P(q_i\,;q_{-i})\cdot q_i - C_i(q_i)$$

number of firms

$$\varepsilon_i = N\varepsilon - (N-1)\eta_{-i}$$

firm i demand elasticity

total demand elasticity

elasticity of supply of other firms

Note $P \neq MC$

- Bertrand competition:

  - firms compete on price
  - Compete down to $P = MC$

Practice Questions:

1. $C(q) = \frac{1}{2}q^2$, $D(P) = 12 - P$

if $P = MC$

$MC = \frac{\partial C}{\partial q} = q$

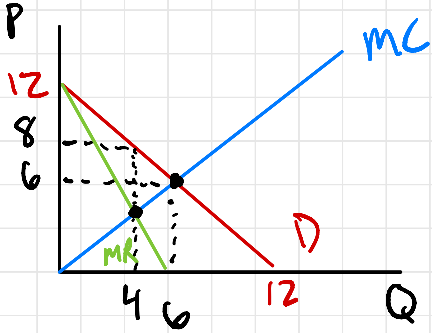
System of linear equations

$$\begin{cases} ① \; q = 12 - P \\ ② \; P = q \end{cases}$$

use substitution or elimination

$\Rightarrow P = 12 - P$

$\Rightarrow 2P = 12 \Rightarrow P = 6$



$q = 12 - p \Rightarrow q = 6$

2. $D(p) = 12 - p$

$\Rightarrow q = 12 - p$

$\Rightarrow p = 12 - q$

$\Rightarrow P(q) = D^{-1}(q) = 12 - q$

$$\max_q \quad P(q) \cdot q - c(q) = (12 - q) \cdot q - \frac{1}{2} q^2$$
$$= 12q - q^2 - \frac{1}{2} q^2$$

FOC: $\frac{\partial}{\partial q} \left[12q - q^2 - \frac{1}{2} q^2 \right] = 0$

$\Rightarrow 12 - 2q - q = 0$

$\Rightarrow q = 4$

$P(q) = 12 - q = 12 - 4 = 8$

3. $N = 2$, $c_i(q_i) = 10 q_i$, $D(p) = 140 - 2p$

$q = 140 - 2p \Rightarrow 2p = 140 - q \Rightarrow P(q) = 70 - \frac{q}{2}$

$\Rightarrow P(q_1; q_2) = 70 - \frac{(q_1 + q_2)}{2}$

Firm 1: $\max_{q_1} \; P(q_1 ; q_2) \cdot q_1 - C_1(q_1)$

$$= \max_{q_1} \left[70 - \frac{q_1}{2} - \frac{q_2}{2}\right] \cdot q_1 - 10q_1$$

F.O.C.: $\frac{\partial}{\partial q_1} \left[\left(70 - \frac{q_1}{2} - \frac{q_2}{2}\right) q_1 - 10 q_1 \right] = 0$

$$\left(-\frac{1}{2}\right) q_1 + (1)\left[70 - \frac{q_1}{2} - \frac{q_2}{2}\right] - 10 = 0 \Rightarrow 70 - q_1 - \frac{q_2}{2} - 10 = 0$$

Firm 1's Best Response: $\Rightarrow q_1 = 60 - \frac{q_2}{2}$

Use Symmetry: $q_1 = q_2 \Rightarrow q_1 = 60 - \frac{q_1}{2}$

$$\Rightarrow \frac{3}{2} q_1 = 60$$

$$\Rightarrow q_1 = 40$$

$$q_2 = 40$$

$$P(q) = \frac{140 - q}{2} = \frac{140 - 40 - 40}{2}$$

$$= \frac{60}{2} = 30$$

Bertrand competition: $P = MC = 10$

4. $P(q_1 ; q_2, q_3) = \frac{140 - q_1 - q_2 - q_3}{2}$

Firm 1: $\max_{q_1} \left(70 - \frac{q_1}{2} - \frac{q_2}{2} - \frac{q_3}{2}\right) \cdot q_1 - 10 q_1$

FOC: $-\frac{1}{2} q_1 + 70 - \frac{q_1}{2} - \frac{q_2}{2} - \frac{q_3}{2} - 10 = 0$

solve for q_1: $$\underbrace{q_1 = 60 - \frac{(q_2+q_3)}{2}}_{\text{Firm 1's best response to firms 2\&3}}$$

Use Symmetry: $q_1 = q_2 = q_3$

$$\Rightarrow q_1 = 60 - \frac{q_1+q_1}{2}$$

$$\Rightarrow 2q_1 = 60$$

$$\Rightarrow q_1 = 30$$

$$q_2 = q_3 = 30$$

$$P(q) = 70 - \frac{q_1+q_2+q_3}{2} = 70 - \frac{90}{2} = 25$$

Bertrand: $P = MC = 10$