9-29-23

- $U(a,b) = a^{\alpha} b^{\beta}$

Maximization problem: $\max \; a^{\alpha} b^{\beta}$

s.t. $P_a \cdot a + P_b \cdot b \leq Y$

Use substitution

$$P_b \cdot b = Y - P_a \cdot a$$

$$b = \frac{Y}{P_b} - \frac{P_a \cdot a}{P_b}$$

$$\max \quad a^{\alpha}\left(\frac{Y}{P_b} - \frac{P_a}{P_b}a\right)^{\beta}$$

FOC: $$\frac{\partial\left[a^{\alpha}\left(\frac{Y}{P_b} - \frac{P_a}{P_b}a\right)^{\beta}\right]}{\partial a} = 0$$

income share on a

$$a^* = \frac{\frac{\alpha}{\alpha+\beta} Y}{P_a}$$

$$b^* = \frac{\frac{\beta}{\alpha+\beta} Y}{P_b}$$

- Elasticities

Elasticity of $D_a(P_a, P_b, Y)$ with respect to K

$$\varepsilon_K = \frac{\partial P_a}{\partial K} \cdot \frac{K}{D_a}$$

Price $\quad \varepsilon_{P_a} = \frac{\partial P_a}{\partial P_a} \cdot \frac{P_a}{D_a} \leq 0$

income $\quad \varepsilon_Y = \frac{\partial D_a}{\partial Y} \cdot \frac{Y}{D_a}$

≥ 0 "normal"

≤ 0 "inferior"

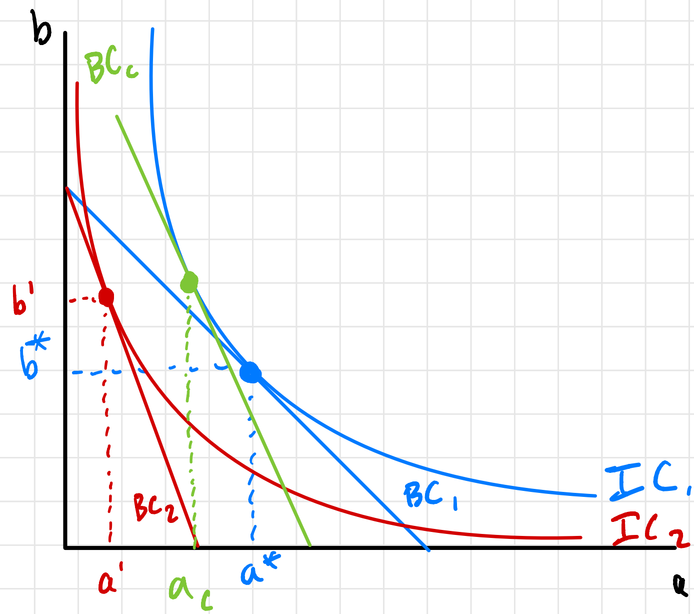
cross price $\quad \varepsilon_{P_a, P_b} = \frac{\partial D_a}{\partial P_b} \cdot \frac{P_b}{D_a}$

- income & Substitution effects:

a^*, b^*

Price of a ↑

a', b'



$a_c - a^* =$ Subtitution effect

$a' - a_c =$ income effect

1. $U(b,t) = b\cdot t \quad , Y=4, \quad P_b = P_t = 1$

$$b^* = \frac{\frac{1}{1+1}\cdot 4}{1} = \frac{\frac{1}{2}\cdot 4}{1} = 2$$

$$t^* = \frac{\frac{1}{1+1}\cdot 4}{1} = 2$$

$$\max \quad bt$$

$$\text{s.t.} \quad b+t=4$$

$$b = 4-t$$

$$\max \quad (4-t)t = 4t - t^2$$

$$\text{FOC:} \quad \frac{\partial\, 4t-t^2}{\partial t} = 0$$

$$4-2t=0$$

$$t^*=2$$

$$b^*=2$$

2. We need Y' such that

$$U(b_c, t_c) = U(t^*, b^*) = U(2,2) = 2\cdot 2 = 4$$

$$b_c = \frac{\frac{\alpha}{\alpha+\beta}\cdot Y'}{P_b'} = \frac{\frac{1}{2}\cdot Y'}{4} = \frac{Y'}{8}$$

$$t_c = \frac{\frac{\beta}{\alpha+\beta}\cdot Y'}{P_t} = \frac{\frac{1}{2}Y'}{1} = \frac{Y'}{2}$$

$$U(b_c, t_c) = \frac{Y'}{8} \cdot \frac{Y'}{2} = \frac{(Y')^2}{16} = 4$$

$$\Rightarrow (Y')^2 = 4 \cdot 16 = 64$$

$$Y' = \sqrt{64} = 8$$

$$b_c = \frac{8}{8} = 1 \quad , \quad t_c = \frac{8}{2} = 4$$

3. $b^* = 2 \quad , \quad b_c = 1$

$$b' = \frac{\frac{1}{2} \cdot Y}{P_b'} = \frac{\frac{1}{2} \cdot 4}{4} = \frac{1}{2}$$

Substitution effect is $b_c - b^* = 1 - 2 = -1$

income effect is $b' - b_c = -\frac{1}{2}$

4. $U(Y, 0) = Y$

$U(0, \frac{Y}{3}) = \frac{2}{3} Y$

$$a^* = Y, \; b^* = 0$$

5. $$\varepsilon_p = \frac{\partial D}{\partial P} \cdot \frac{P}{D} = (-1) \cdot \frac{5}{15} = -\frac{1}{3}$$

b. Prices are equal

$$\left(a^{\frac{1}{3}}\right)^3\left(b^{\frac{2}{3}}\right)^3$$

$$a \quad b^2$$