Econ 301 Discussion - 9/15/2023

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Content Review

- Preferences: Primitive $\succsim$ - weak preference

  $a \succsim b$ - "a is weakly preferred to b"

  $a \succ b \iff a \succsim b$ and $b \not\succsim a$ - "a is strictly preferred to b"

  $a \sim b \iff a \succsim b$ and $b \succsim a$ - "indifferent between a and b"

- Rational preferences: A preference relation $\succsim$ on set X is rational iff

  Completeness : $a \succsim b$ or $b \succsim a$ $\quad (\forall x, y \in X)$

  Transitivity : $a \succsim b$ and $b \succsim c \Rightarrow a \succsim c$ $\quad (\forall a, b, c \in X)$

- Utility representation of preferences $U: X \to \mathbb{R}$, $U(a) \geq U(b) \iff a \succsim b$

  - assuming rational preferences, U is increasing - "more is better"

  - Utility is ordinal, not cardinal - preserved under monotonic transformation

  - if $g: \mathbb{R} \to \mathbb{R}$ is increasing, then $U(x)$ and $g(U(x))$ represent the same preferences

- Marginal Utility and Marginal Rate of Substitution $U(a,b): \mathbb{R}^2 \to \mathbb{R}$

  $MU_a = \frac{\partial U}{\partial a}(a,b)$, $MU_b = \frac{\partial U}{\partial b}(a,b)$

  $MRS_{ab} = \frac{\partial b}{\partial a} = -\frac{MU_a}{MU_b}$ - willingness to trade b for a
  - slope of an indifference curve

Practice Questions

1. Let $f(x,y) = 2x^2y + 3xy^3$. Find $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$.

$$\frac{\partial f}{\partial x} = 2 \cdot 2x^{2-1} \cdot y + 3x^{1-1}y^3 = 4xy + 3y$$

$$\frac{\partial f}{\partial y} = 2x^2y^{1-1} + 3 \cdot 3xy^{3-1} = 2x^2 + 9xy^2$$

2. Consumer A has rational preferences and chooses a from budget set $B1 = \{a, b, c, d\}$ and d from budget set $B_2 = \{c, d, e\}$. Provide a possible strict preference ranking of all a, b, c, d, e.

we know $a \succ b, c, d$, $d \succ c, e$

by transitivity, $a \succ e$. Possible rankings:

① $a \succ b \succ d \succ c \succ e$
② $a \succ b \succ d \succ e \succ c$
③ $a \succ d \succ b \succ c \succ e$
④ $a \succ d \succ b \succ e \succ c$
⑤ $a \succ d \succ c \succ b \succ e$
⑥ $a \succ d \succ c \succ e \succ b$
⑦ $a \succ d \succ e \succ b \succ c$
⑧ $a \succ d \succ e \succ c \succ b$

3. Consumer B's utility function is $u(a, b) = 2a + 3b$. What is Consumer B's preferences over the bundles $(a_1, b_1) = (2, 3)$ and $(a_2, b_2) = (4, 1)$?

$$U(a_1, b_1) = 2a_1 + 3b_1 = 2\cdot 2 + 3\cdot 3 = 13$$
$$U(a_2, b_2) = 2a_2 + 3b_2 = 2\cdot 4 + 3\cdot 1 = 11$$
$$U(a_1, b_1) > U(a_2, b_2) \implies (a_1, b_1) \succ (a_2, b_2)$$

So (a_1, b_1) is strictly preferred to (a_2, b_2)

Note that $\left(u(x) \geq u(y) \iff x \succsim y\right)$ and $\left(x \succ y \iff x \succsim y \text{ and } y \not\succsim x\right) \implies \left(u(x) > u(y) \iff x \succ y\right)$

4. Prove that $u(x, y) = x^a y^b$ on $\mathbb{R}^2_+$ is equivalent to $v(x, y) = a\log(x) + b\log(y)$. Our theorem states that U and V are equivalent iff $\exists\, g: \mathbb{R} \to \mathbb{R}$ with g monotonic such that $U(x,y) = g(V(x,y))$.

Let $g(z) = e^z$. g is monotonic since $\frac{\partial g}{\partial z} = e^z > 0$.

Now, $V(x,y) = a\log(x) + b\log(y) = \log(x^a) + \log(y^b) = \log(x^a y^b)$

so $U(x,y) = x^a y^b = e^{\log(x^a y^b)} = g(\log(x^a y^b)) = g(V(x,y))$.

5. Consumer B has utility function $u(a, b) = a^2 b$. What is Consumer B's marginal rate of substitution of a for b (MRS_{ab}) at $(a, b) = (2, 1)$?

$$MU_a = \frac{\partial U}{\partial a} = 2ab, \quad MU_b = a^2$$
$$MRS_{ab} = -\frac{MU_a}{MU_b} = \frac{-2ab}{a^2} = -2\frac{a}{b}$$
$$\implies MRS_{ab}(2,1) = -2 \cdot \frac{2}{1} = -4.$$