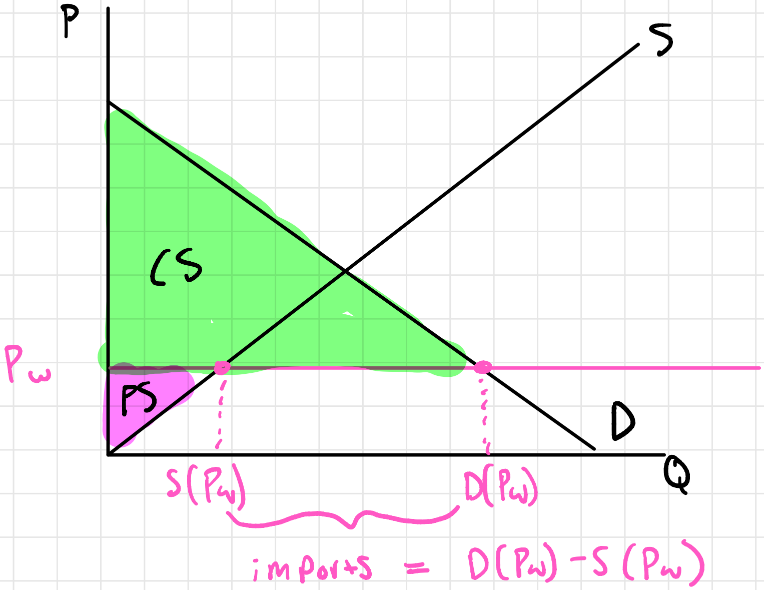
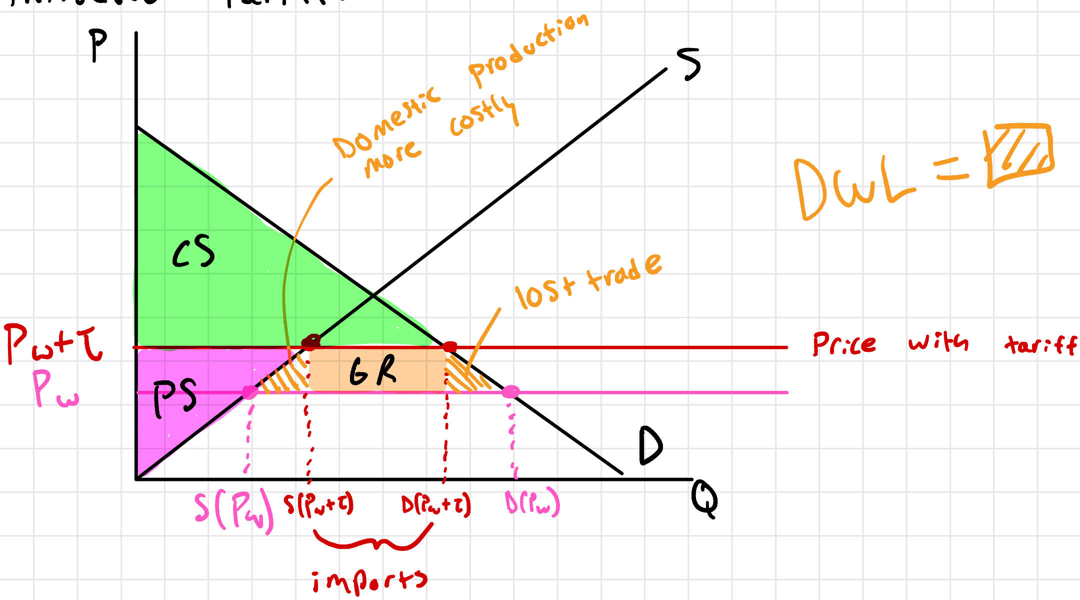
12 - 8 - 23

- open economy & tariffs
  - exogenous world price P_w



- introduce tariff:



- General equilibrium in endowment economy
  - >1 goods (2)
    >1 agents (2)
  - agents are endowed with initial quantities (y_1^a, y_1^b), (y_2^a, y_2^b)

  - a Pareto improvement over an an allocation (a_1, b_1, a_2, b_2) is another allocation which makes ≥ 1 person better off and no one worse off

  - an allocation is Pareto efficient if there are no possible pareto improvements

  - competitive equilibrium:

    an allocation (a_1, b_1, a_2, b_2) and prices (P_a, P_b) such that

    1. given prices, agent 1 maximizes utility

    $$\max_{a_1, b_1} U_1(a_1, b_1)$$

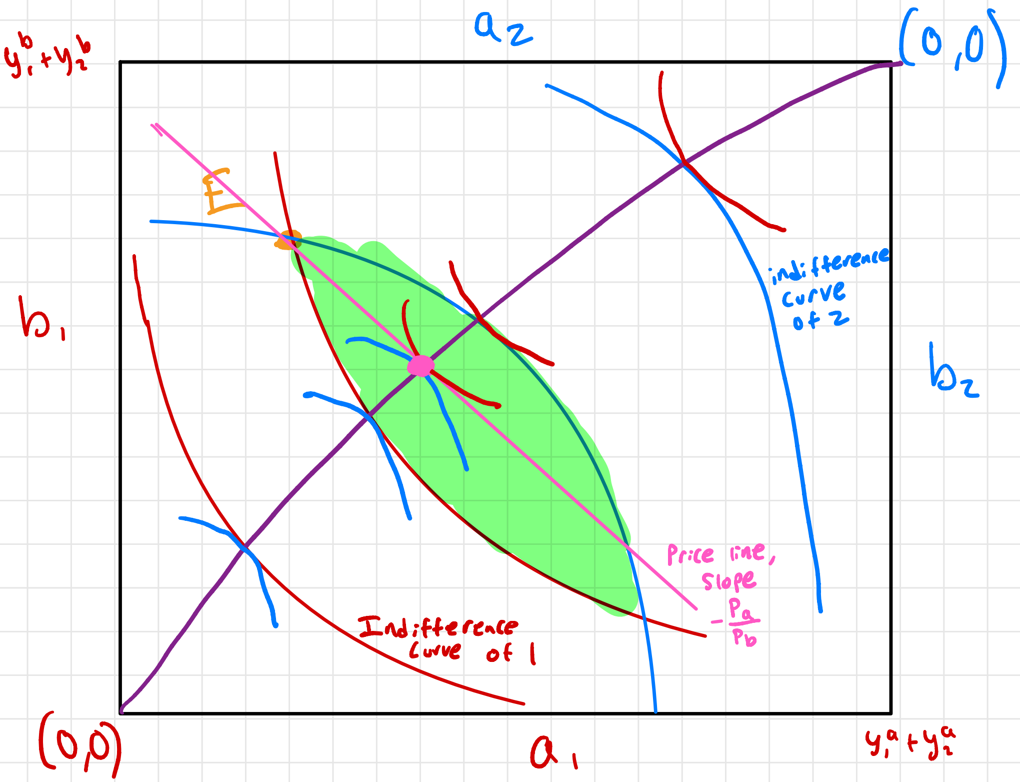
    $$\text{s.t.} \quad P_a a_1 + P_b b_1 = \underbrace{P_a y_1^a + P_b y_1^b}_{\text{"income" } Y_1}$$

    2. given prices, agent 2 maximizes utility

3. Markets clear: $a_1 + a_2 = y_1^a + y_2^a$

$$\underbrace{b_1 + b_2}_{\text{total consumption}} = \underbrace{y_1^b + y_2^b}_{\text{total endowment}}$$

- Note: prices are nominal $\Rightarrow$ can normalize $P_a = 1$



endowment E

Pareto improvements over E

Pareto efficient: $MRS_1 = MRS_2$ & markets clear

competitive equilibrium

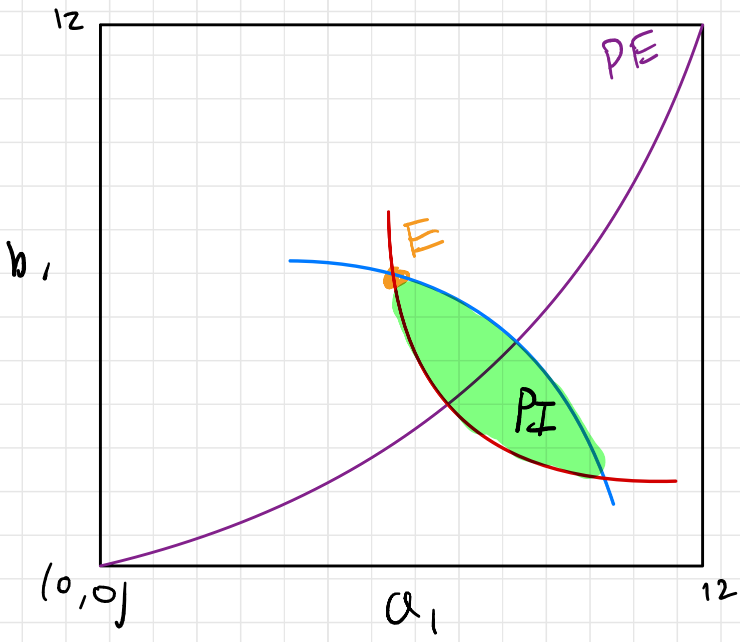
Note: find the competitive equilibrium by drawing a price line through the endowment that is also tangent to both indifference curves

Practice Questions

1. Preferences: $U_1(a,b) = a^2b$, $U_2(a,b) = ab^2$

endowments: $(6,6)$, $(6,6)$

Endowments are not pareto efficient. The intuition here is that agent 1 values good a higher than agent 2 does and vice versa.



- Example of a pareto improving allocation:

$$(a_1, b_1, a_2, b_2) = (7, 5, 5, 7)$$

Utilities at endowment:

$$U_1(6,6) = 6^2 \cdot 6 = 216$$
$$U_2(6,6) = 6 \cdot 6^2 = 216$$

Utilities at $(7,5,5,7)$:

$$U_1(7,5) = 7^2 \cdot 5 = 245$$
$$U_2(5,7) = 5 \cdot 7^2 = 245$$

Thus, we can transfer 1 unit of a from 2 to 1 and 1 unit of b from 1 to 2 and have higher utilities for both people. So the endowment is not efficient.

- given $P_a = P_b \Rightarrow \frac{P_a}{P_b} = 1$

  we can normalize $P_a = 1 \Rightarrow P_b = 1$

  agent 1's problem: $\max_{a_1, b_1} a_1^2 b_1$

  s.t. $a_1 + b_1 = 1 \cdot 6 + 1 \cdot 6 = 12$

Cobb-Douglas demand formulas

$$\begin{cases} a_1^* = \dfrac{\frac{\alpha}{\alpha+\beta} Y_1}{P_a} = \dfrac{\frac{2}{2+1} \cdot 12}{1} = \frac{2}{3} \cdot 12 = 8 \\ b_1^* = \dfrac{\frac{\beta}{\alpha+\beta} Y_1}{P_b} = \frac{1}{3} \cdot 12 = 4 \end{cases}$$

agent 2's problem: $\max_{a_2, b_2} a_2 b_2^2$

s.t. $a_2 + b_2 = 12$

$$a_2^* = \frac{\frac{\alpha}{\alpha+\beta} Y_2}{P_a} = 4, \qquad b_2^* = \frac{\frac{\beta}{\alpha+\beta} Y_2}{P_b} = 8$$

Check market clearing to verify equilibrium:

✓ $a_1^* + a_2^* = 8+4 = 6+6 = y_1^a + y_2^a$

✓ $b_1^* + b_2^* = 4+8 = 6+6 = y_1^b + y_2^b$

Competitive equilibrium:

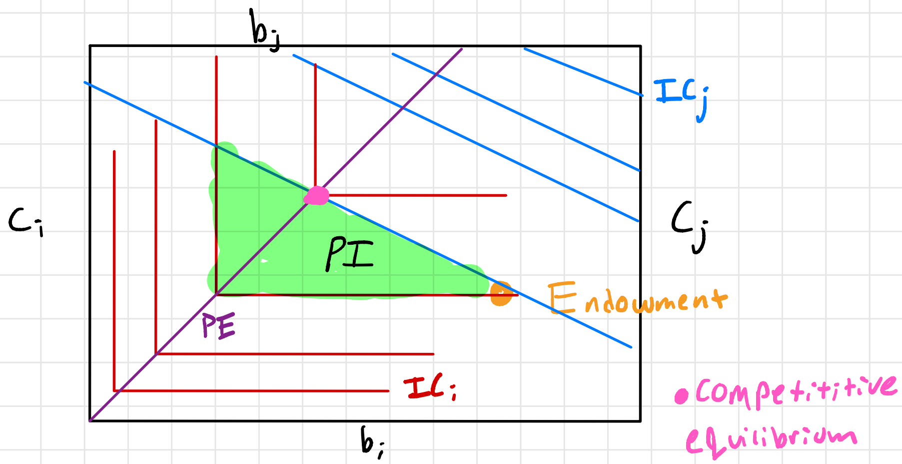
$$(a_1, b_1, a_2, b_2) = (8, 4, 4, 8)$$

2. $U_i(b, c) = \min\{b, c\}$

$U_j(b, c) = b + 2c$

endowments: $(y_i^b, y_i^c) = (6, 3)$

$(y_j^b, y_j^c) = (2, 4)$



Sinle Isaac has perfect complements utility, he gets 0 marginal utility from consuming any more of just b or c outside the line where $b_i = c_i$.

Claim: the pareto efficient allocations are all the feasible (market clearing) allocations in which $b_i = c_i$

$$PE = \{(b_i, c_i, b_j, c_j) : b_i + b_j = 8, \; c_i + c_j = 7, \; b_i = c_i\}$$

Though this is not necessary, here is a proof if you are interested:

assume for contradiction that there exists a pareto efficient allocation in which $b_i \neq c_i$.

Case 1: $b_i > c_i$ then $U_i(b_i, c_i) = c_i$

we can reduce b_i by $(b_i - c_i)$, and leave Isaac's utility unaffected. However, this increases Joe's utility by $b_i - c_i$. Thus, we have found a pareto improvement. This is a contradiction against the definition of pareto efficiency.

Case 2: $c_i > b_i$. Proof is symmetric. reduce c_i by $c_i - b_i$ and we get a pareto improvement. Contradiction.

Now, we note that the price ratio in a competitive equilibrium must equal Joe's MRS. Otherwise, Joe's optimal choice of b_j and c_j will be a corner solution since he has perfect substitutes utility. He would choose to consume where $b_j = 0$ or $c_j = 0$. But Isaac would not agree to this because he wants to consume where $b_i = c_i$.

So $$\frac{P_b}{P_c} = MRS_j = \frac{MU_b}{MU_c} = \frac{1}{2}.$$

Normalize $P_b = 1 \Rightarrow P_c = 2.$

Isaac's utility maximization problem:

$$\max_{b_i, c_i} \min\{b_i, c_i\}$$

$$\text{s.t.} \quad b_i + 2c_i = 6 + 2 \cdot 3 = 12$$

Since $b_i = c_i$, $c_i + 2c_i = 12$

$\Rightarrow c_i = 4$

$\Rightarrow b_i = 4$

Since Joe is indifferent to any allocation on his budget constraint, his consumption is given by market clearing.

$b_i + b_j = 8 \Rightarrow 4 + b_j = 8$

$\Rightarrow b_j = 4$

$c_i + c_j = 7 \Rightarrow 4 + c_j = 7 \Rightarrow c_j = 3.$

CE: $(b_i, c_i, b_j, c_j) = (4, 4, 4, 3)$, $(P_b, P_c) = (1, 2)$.