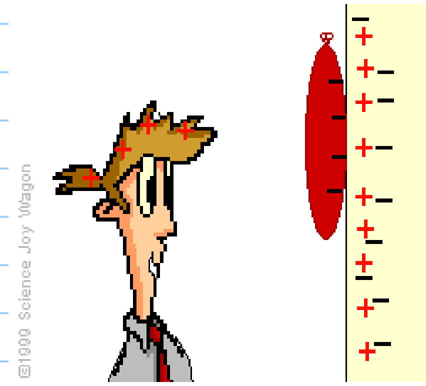
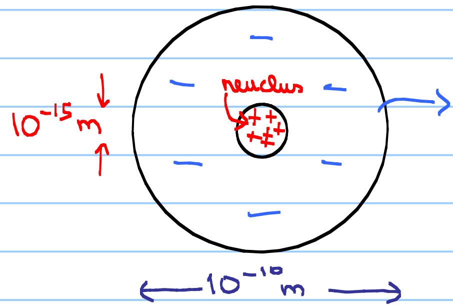
Electric Charges and Forces (Chapter 22):



Topics:
- Charges - Insulators and Conductors
    - charging by rubbing (insulators)
    - charging by contact or by induction (metals)
- Polarization
- Electric Force $\vec{F}$ between charges (Coulomb's law)
- Electric Field ($\vec{E}$) and the Electric Field Lines
- Electric Field of a point charge

Charges

- The **electron** has a negative charge and the **proton** has a positive charge.
- They are the basic charges of matter.
- They are found inside the atom. Protons are inside the tiny positive nucleus, An equal number of electrons form the much larger negative electron cloud surrounding the nucleus.
- The atom is neutral as its net charge is zero.



electron cloud
(much bigger compared to nucleus than shown)

The atom

Electron's and Proton's Charge and Mass

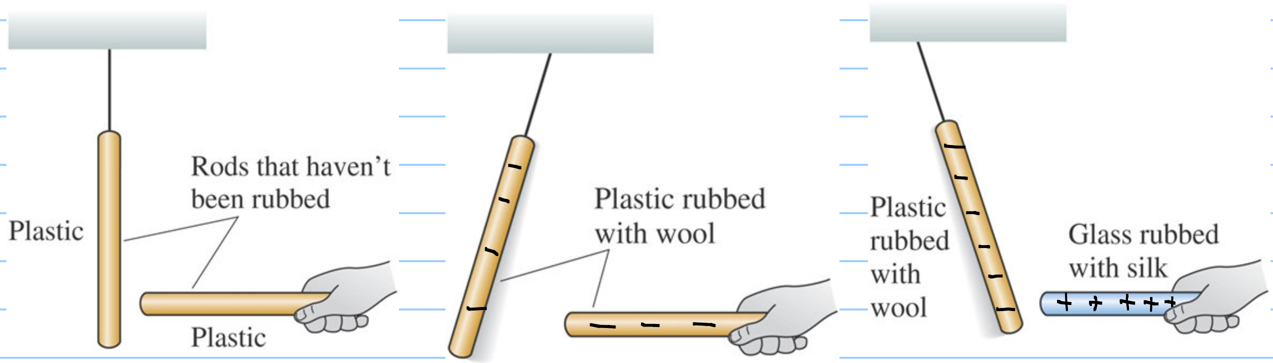
TABLE 26.1 Protons and electrons

Particle	Mass (kg)	Charge
Proton	1.67×10^{-27}	$+e$
Electron	9.11×10^{-31}	$-e$

e is taken always positive

where $e = 1.60 \times 10^{-19}$ C is the unit charge.
The units of charge is the *Coulomb* [*C*].

- In general a charge can be made up of many electrons (or many protons).
- Two **like** charges exert repulsive forces on each other, **opposite** charges attract.
- The repulsive / attractive forces are *long-range forces*.



nothing

rods repel

rods attract

Stop to think: How many electrons are there in 1 C?

$e = 1.6\times10^{-19}\,C$

a) one

$N = \frac{1\,C}{e} = \frac{1\,C}{1.6\times10^{-19}\,C} \approx 6.25\times10^{18}$

b) $\sim 10^{-6}$

c) $\sim 10^{+6}$

d) $\sim 10^{+18}$

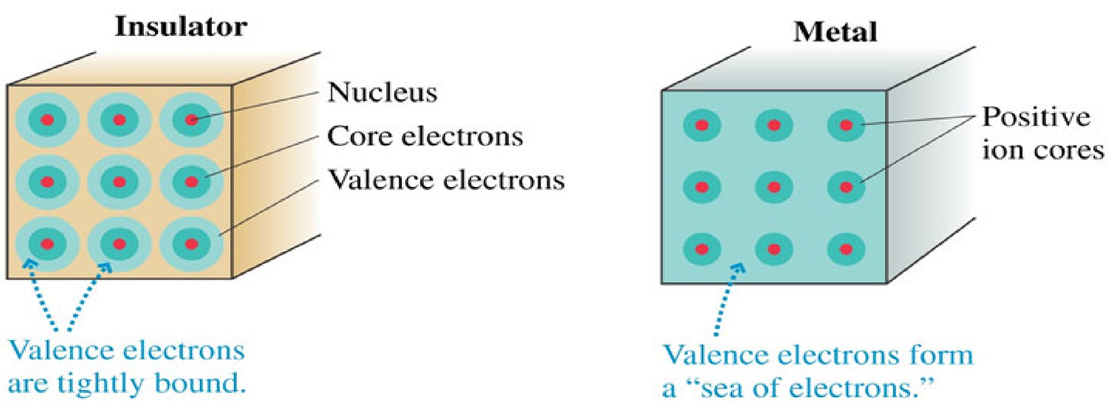
(d) ✓

Stop to think: What is net charge of the system in Coulomb?

17 protons
19 electrons

$Q_{net} = -2e = 3.2\times10^{-19}\,C$

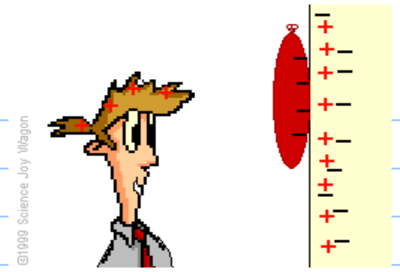
Insulators and Conductors
There are two types of materials.
- Insulators (such as the rod and glass in the previous experiments) are materials in which **charges remain fixed in place**.
- Conductors (such as metal wire, a piece of iron, aluminum, gold ....) are materials through which **charge easily moves**.
- Some very good conductors:
Beside most metals, EARTH and Human body are excellent conductors.



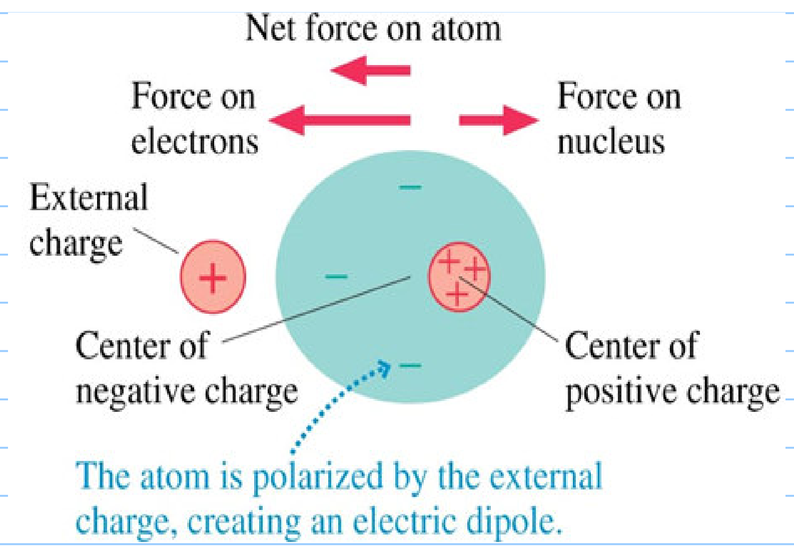
Polarization

Polarization of an insulator:

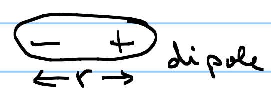
Why does a charged rod pick up neutral paper?

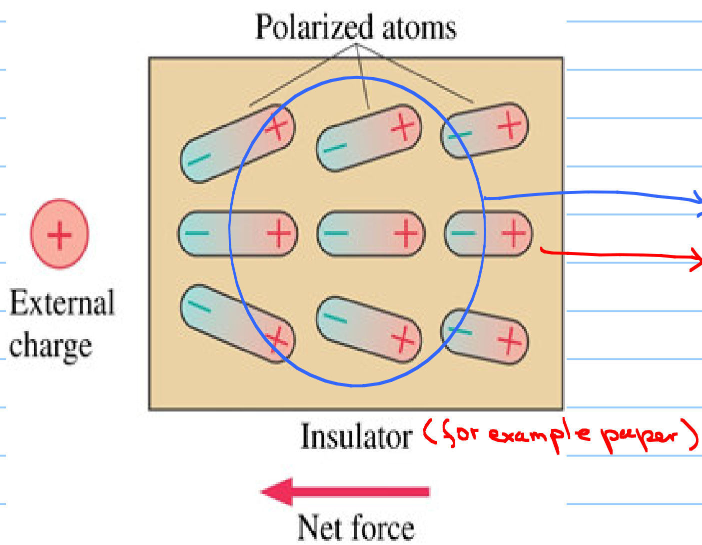


- Like charges repel, while opposite charges attract.
- Hence, a charge close to the paper causes the *nucleus of the paper atoms to shift* away from the *center of the electron cloud*. The atom now forms a DIPOLE (positive charge separated from an EQUAL negative charge), it is not neutral anymore. This is called **polarization** (separation of positive from negative charge) of the paper atoms.
- The charge then exerts a *net attractive force* on the atom, see figure, because the center of opposite charge (negative in the figure) is closer to the external charge than the center of the like-charge.



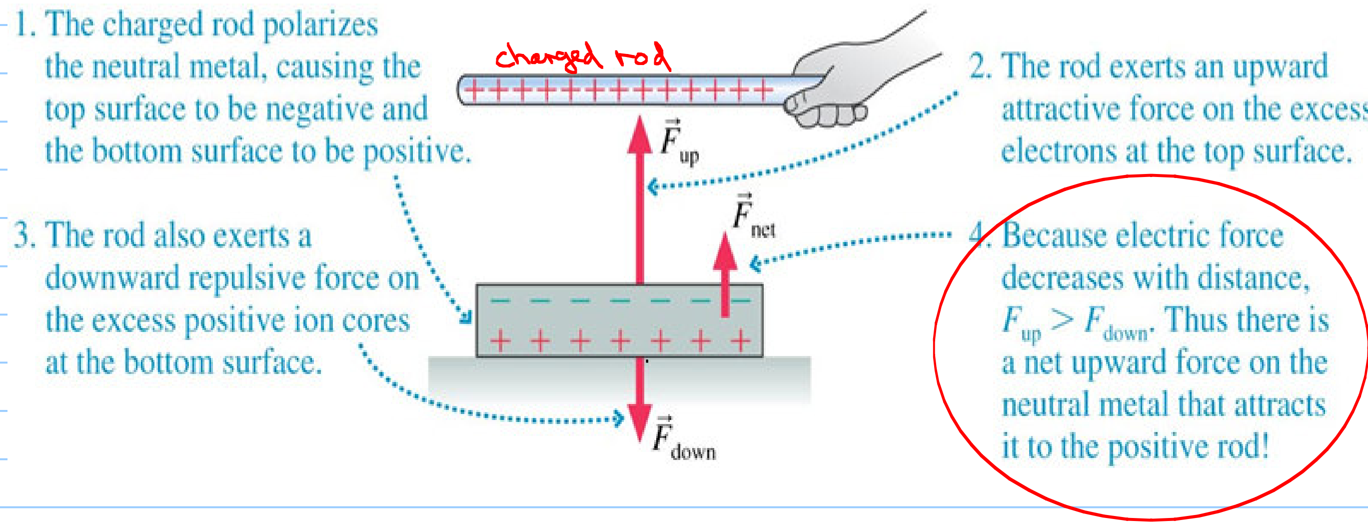
Polarization of a single atom





Polarization of an insulator
neutral inside the bulk
excess charges at the surface

Polarization of a metal:
A charged rod (i.e. an external charge) acts in a similar way on a metal, but because valence electrons are free to move, positive and negative charges are separated at opposite surfaces.



Charging:

Neutral objects have an equal mixture of both charges.

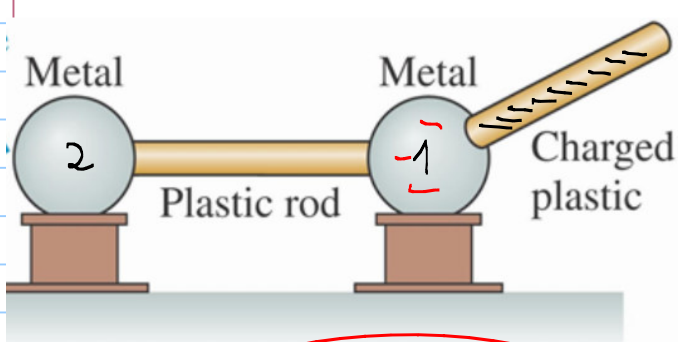
1- *You charge an **insulator** by rubbing* it with a different material.

One kind of charges is added to the rubbed object and *the opposite kind* is added to the rubbing material.

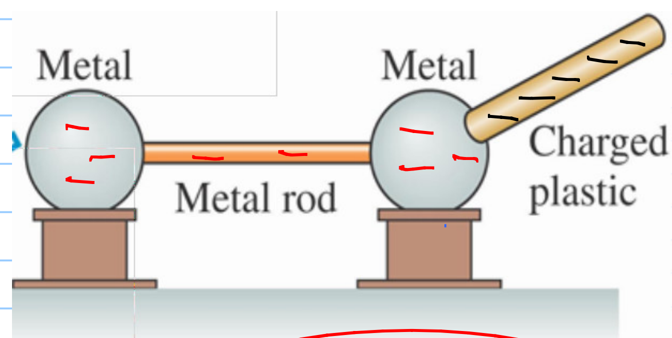
2- *You charge a **conductor*** in one of two ways:

Method 1: by contact with a *charged* rod. Some of the charge is transferred from the charged rod to the conductor. The conductor *aquires the same type of charge* of the rod.

Think: You touch metal 1 with a charged plastic rod. What happens to metal sphere 2 in each case?

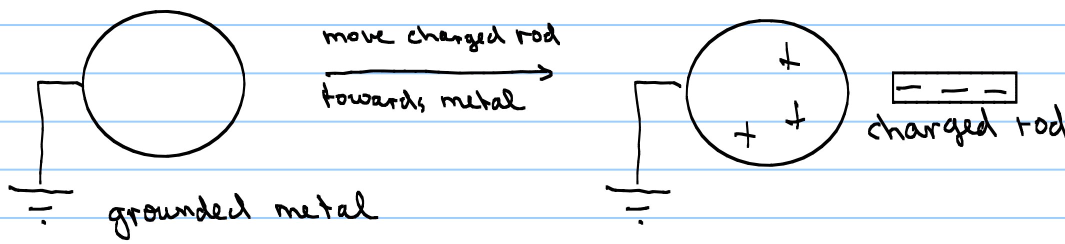


Metal 2: stays neutral

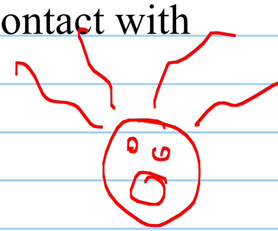


Metal 2: becomes charged

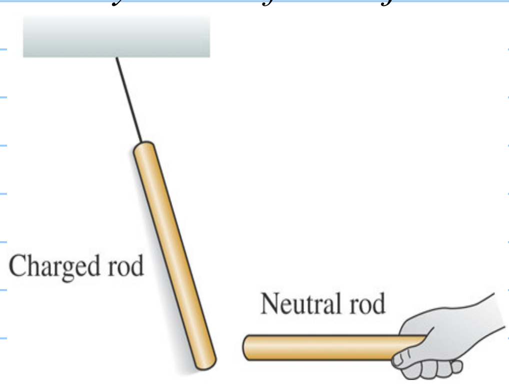
Method 2: by induction (*no direct contact* between charged rod and conductor). Induction is based on polarizing the conductor first.
If the conductor is grounded, it is left charged with the opposite charge.
Alternatively, the conductor can be divided into two parts, each will have opposite charge.

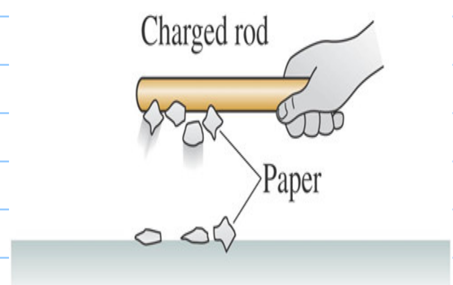


How do you discharge a conductor?
- Earth is a very good conductor, so bringing a charged object in contact with the earth will discharge it (the object is said to be *grounded*).
- If the charge is small you can touch it yourself to discharge it,
  Be careful: if the charge is large it can knock you out!!



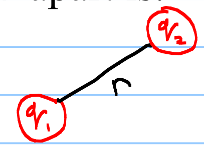
How do you test if an object is charged or not?





If a charged object attracts little peaces of neutral paper then the object is charged.

Coulomb's law:
The electric forces between two *point charges* q_1 and q_2, a distance r apart is:

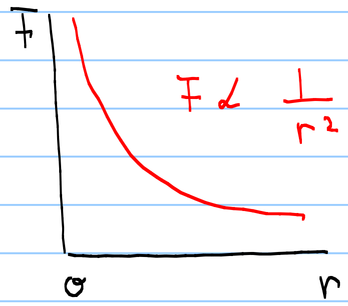


$$F_{1\,on\,2} = F_{2\,on\,1} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|\,|q_2|}{r^2} = K\frac{|q_1|\,|q_2|}{r^2}$$ Coulomb's law

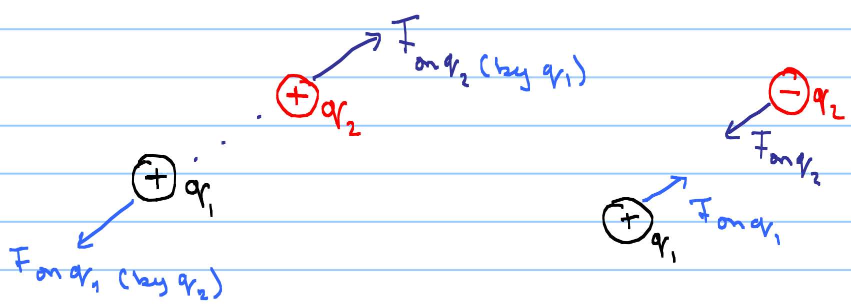
$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\,m^2$$ permittivity of free space

$$K = \frac{1}{4\pi\varepsilon_0} = \underbrace{8.99}_{9.0} \times 10^{9}\ N\,m^2/C^2$$ electric force constant

The force is a vector, its unit is the *Newton* [1 N = 1 kg m s^{-2}]

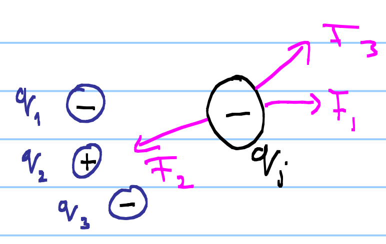


The direction of F on a charge q_2 by charge q_1 is away from q_1 for like charges, and towards q_1 for opposite charges.



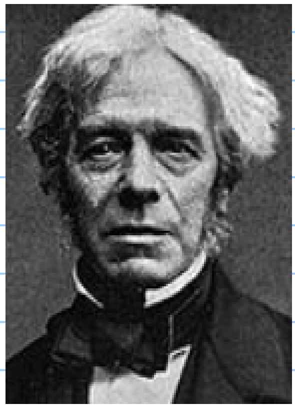
If there are more charges, then the **superposition** of all forces gives:
$\vec{F}_{net\ on\ charge\ j} = \vec{F}_{1\ on\ j} + \vec{F}_{2\ on\ j} + \vec{F}_{3\ on\ j} + \ldots$

Vector addition can be done by adding the x-components separately, similarly for the y and z-components.
$F_x = F_{1x} + F_{2x} + F_{3x} + \ldots$
$F_y = F_{1y} + F_{2y} + F_{3y} + \ldots$



The Electric Field and the Field Lines:

- Why do long range forces, like electric forces (also magnetic forces and gravity), act between objects without being in contact?
- *Faraday's* idea was that a *charged object* (source object) alters the space around it, by causing an *electric field* at every point in space around it.



- Any *other charged* particle (test charge) responds to the electric field at its location. A test charge placed in an electric field experiences an *electric force*.

The Electric Field of a Point Charge:

The electric field of a point charge q a distance r away is:

$$\boxed{\vec{E}_{\text{at } \vec{r}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\right) = \frac{Kq}{r^2}\hat{r}}$$

Electric field of a point charge q

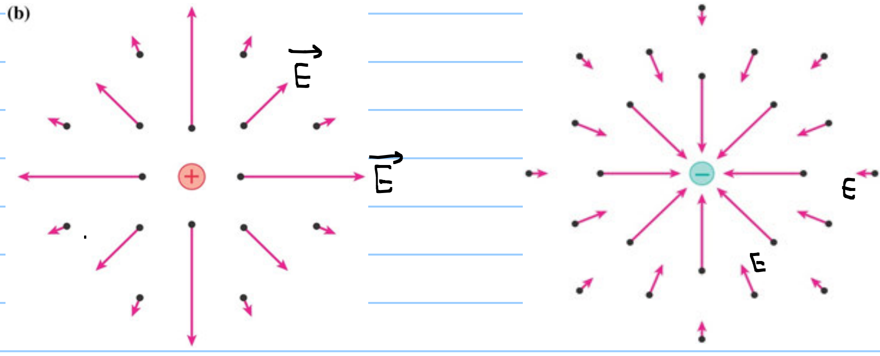
with $K = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ \text{N m}^2/\text{C}$ and $\varepsilon_0 = 8.85 \times 10^{-12} \ \text{C}^2/\text{N m}^2$

The electric field is a vector. The units of electric field is N/C.

$\hat{r}$ is the unit vector indicating direction. It points away from the source charge. It has no units and a magnitude of one.

source charge $\hat{r}$, $|\hat{r}| = 1$

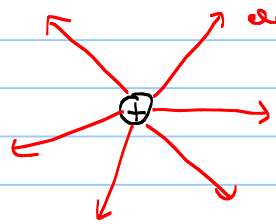
- The electric field of a positive charge at a given point points away from the charge and that of a negative charge points inwards.



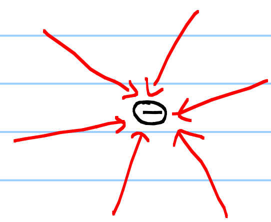
Each arrow represents the electric field at the dot, its length gives its magnitude.

The Electric field lines:
- Electric fields are most easily visualized by field lines.
- The electric field vector at any point is tangent to the electric field lines.
- The electric field magnitude is given by the density of field lines. Closely spaced lines represent a larger field and vice versa.

Electric field lines of point charges:

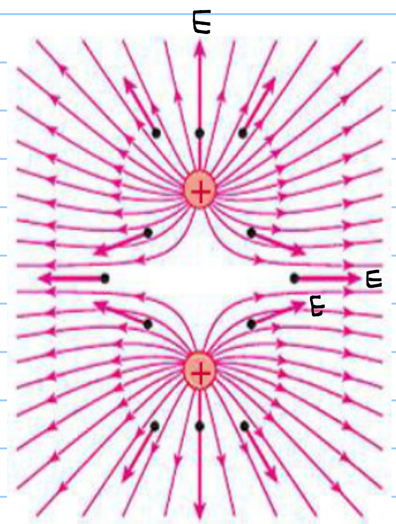


electric field lines point outwards from positive charge

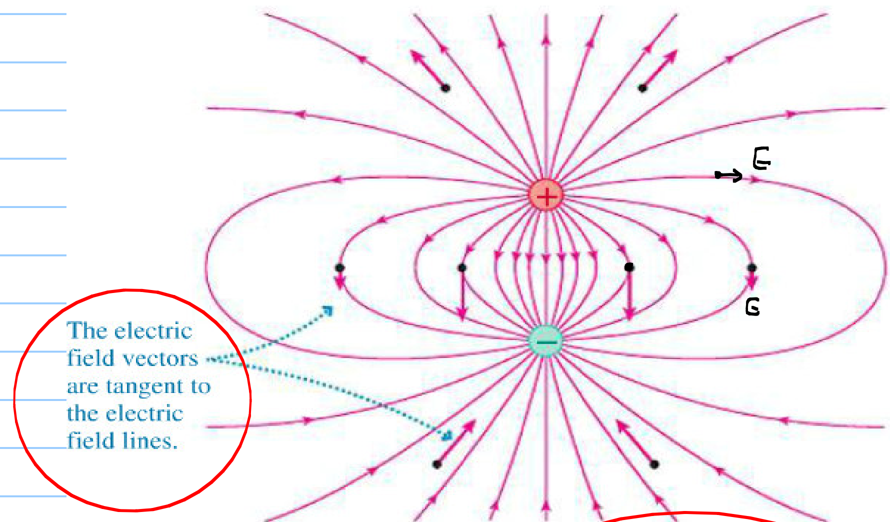


point inwards into negative charge

Two equal point charges:



Electric Dipole:



Electric Field lines never cross

They point away from positive charges and towards negative charges

closely spaced lines with larger arrows represent a larger field and vice versa

Electric Force and Electric Field:

A test charge q placed inside an electric field $\vec{E}$ will experience a force $\vec{F}_q$ given by:

$$\boxed{\vec{F}_{\text{on } q} \text{ at } (x,y,z) = q\,\vec{E}_{\text{at } (x,y,z)}}$$

along $\vec{E}$ for positive test charge q
opposite $\vec{E}$ for negative test charge q

It is very important to distinguish between source charges that cause the electric field, and a test charge - placed inside that electric field - that experiences an electric force. !!!

One way to measure the electric field at a point:

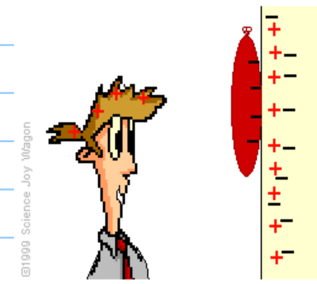
If the electric force on a charge q is known at a given point in space (say its acceleration is measured), the electric field at that point is then simply $\vec{E} = \vec{F}_{\text{on q}} / q$.

Electric Charges and Forces (Chapter 22):

1- Why do you experience the following?

After combing your hair, your comb attracts little peaces of paper (water drops).

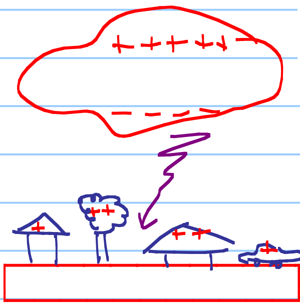
- Charging by rubbing, electrons move from one object to another
- Polarization of paper by the charged comb.



2- How do you charge a conductor?

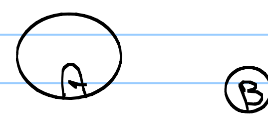
- conduction by contact
- by induction

3- Can we explain lightning?



4- Object *A* has 20 times the charge of object *B*?

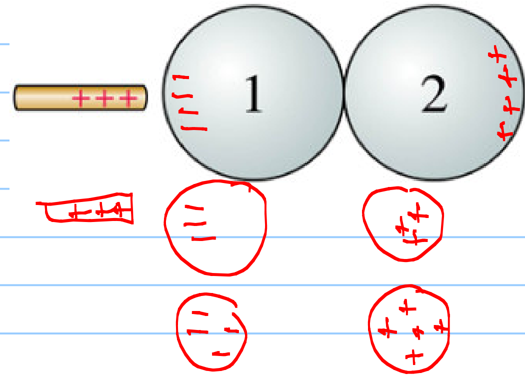
How do the forces both objects experience on each other compare?



$F_{on\,A} = F_{on\,B}$ same magnitude

$\vec{F}_{on\,A} = -\vec{F}_{on\,B}$ opposite direction.

Question 1: Metal spheres 1 and 2 are touching. Both are initially neutral.
a. The charged rod is brought near (*i.e.* not touching).
b. The spheres are separated.
c. The charged rod is then removed.
Afterward, the charges on the spheres are



A. Q1 is + and Q2 is +
B. Q1 is + and Q2 is –
C. Q1 is – and Q2 is + (c) ✓
D. Q1 is – and Q2 is –
E. Q1 is 0 and Q2 is 0

Think: In the previous example, what happens if the charged rod actually touched metal 1?

Question 2:

Identical metal spheres are initially charged as shown.
Spheres P and Q are touched together and then separated.
Then spheres Q and R are touched together and separated.
Afterward the charge on
sphere R is

7 nC -1 nC -2 nC

$\underbrace{7\ nC\ \ -1\ nC}_{6\ nC}$

3nC 3nC

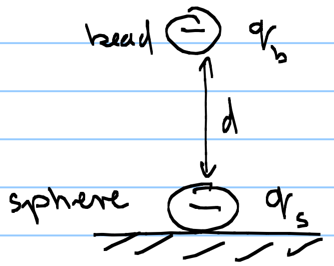
$\underbrace{3nC\ \ -2\ nC}_{1nC}$

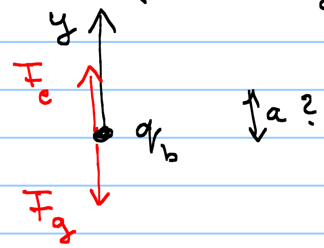
0.5nC 0.5nC

A. –1 nC or less.
B. –0.5 nC.
C. 0 nC.
D. +0.5 nC. D ✓
E. +1.0 nC or more.

Example: A bead charged to -10.0 nC is held 1.00 cm above a stationary sphere of charge -20.0 nC and then let go. The bead has 15.0 mg mass. What is the acceleration of the bead?

Model: Treat bead and sphere as point charges





$q_b = -10.0 \times 10^{-9}$ C
$q_s = -20.0 \times 10^{-9}$ C
$m_b = 15.0 \times 10^{-6}$ kg
$d = 0.01$ m
$K = 8.99 \times 10^{9}$ Nm^2/C^2

Solve:

along y, Newton's 2nd law:

$$F_{net\,y} = m a_y$$

$$F_e - F_g = m a_y$$

$$a_y = \frac{F_e - F_g}{m_b}$$

$$= \frac{1}{m_b} \frac{K|q_b||q_s|}{d^2} - \frac{1}{\cancel{m_b}} \cancel{m_b} g$$

$$= \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(10 \times 10^{-9}\text{ C})(20 \times 10^{-9}\text{ C})}{(15 \times 10^{-6}\text{ kg})\ (0.01\text{ m})^2} - 9.8 \text{ m/s}^2$$

$$= 1190 \text{ m/s}^2$$

$$\vec{a} = (1190 \text{ m/s}^2, \text{up})$$

alternatively $\vec{a} = 1190 \text{ m/s}^2\ \hat{j}$

$\vec{a} = (0, 1190 \text{ m/s}^2)$

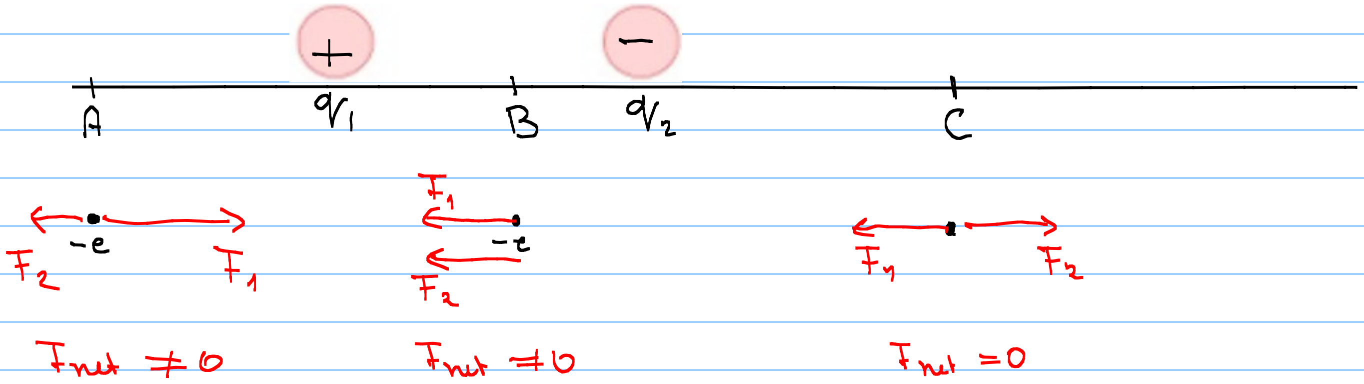
Assess: units, significant figures, directions, magnitudes?

✓ ✓ ✓ ✓

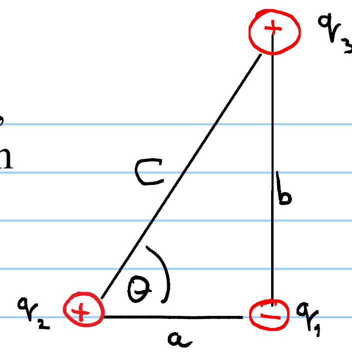
Question 1: Two opposite charges +4C and -1C are placed as shown. At what point approximately on the *x*-axis *A, B, C* (other than infinity) can you place an electron, such that it is in static equilibrium (experiences zero force)?

A) at point A
B) at point B
C) at point C   (C) ✓
D) at no points other than infinity

$q_1 = 4\ C$
$q_2 = -1\ C$



soh cah toa



Example: Three charged particles $q_1 = -50.0$ nC, $q_2 = 40.0$ nC, $q_3 = 30.0$ nC are placed on the corners of the 5.0 cm x 10.0 cm rectangle as shown.

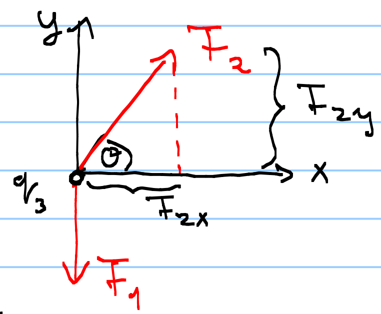
a)What is the net force on charge q_3? Give your answer as a magnitude and angle (ccw with positive x-axis).

b) What is the electric field at the location of q_3?

Model: Treat charges as point charges

$q_1 = -50.0\,nC$
$q_2 = 40.0\,nC$
$q_3 = 30.0\,nC$
$a = 0.05\,m$
$b = 0.1\,m$

Vizualize:



a) *Solve*: $c = \sqrt{a^2 + b^2} = 0.112$

$\tan\Theta = \frac{opp}{adj} = \frac{b}{a} \rightarrow \Theta = \tan^{-1}\left(\frac{b}{a}\right) = 63.4^\circ$

Forces on q_3

magnitude $\rightarrow F_1 = \frac{K\,|q_1|\,q_3}{b^2} = \frac{(8.99\times10^9)(50\times10^{-9})(30\times10^{-9})}{(0.1)^2} = 1.35\times10^{-3}\,N$

$\searrow F_2 = \frac{K\,q_2\,q_3}{c^2} = 8.64\times10^{-4}\,N$

$\vec{F_1} = (F_{1x}, F_{1y}) = (0, -F_1)$

$\vec{F_2} = (F_{2x}, F_{2y}) = (+F_2\cos\Theta, +F_2\sin\Theta)$

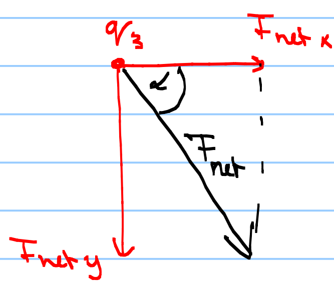
$\vec{F}_{net} = (0 + F_2\cos\Theta, -F_1 + F_2\sin\Theta)$

$= (8.64\times10^{-4}\times\cos 63.4, -1.35\times10^{-3} + 8.64\times10^{-4}\times\sin 63.4)$

$= (\underbrace{3.86\times10^{-4}\,N}_{F_{net,x}}, \underbrace{-5.80\times10^{-4}\,N}_{F_{net,y}})$

Find F_{net}:

$F_{net} = \sqrt{F_{net\,x}^2 + F_{net\,y}^2} = 7.0\times10^{-4}\,N$



$\tan\alpha = \frac{|F_{net\,y}|}{|F_{net\,x}|} \rightarrow \alpha = \tan^{-1}\frac{|F_{net\,y}|}{|F_{net\,x}|} = 56.4^\circ$

$\vec{F}_{net} = (7.0\times10^{-4}\,N, \underbrace{304^\circ}_{360^\circ - 56.4^\circ}\ ccw)$

Assess: units, significant figures, directions, magnitudes?

✓ ✓ ✓ ✓

Reading Assignment Questions:

1- Faraday come up with the concept of the electric field. Explain his motivation.

He was uncomfortable with the concept of long range forces.
He invented the concept of the electric field.
A "source" charge modifies space around it, introducing the electric field at every point. Any "test" charge interacts with that field by experiencing a force.

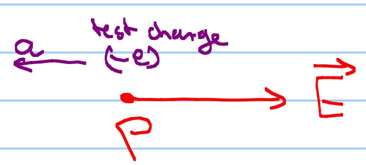
2- A point charge has an electric field E_0 of 5 N/C a distance d away.

If you double the charge, $E =$ $2E_0 = 10$ N/C

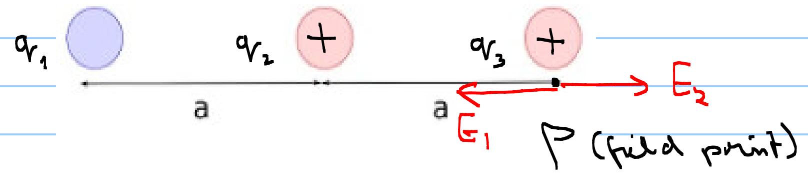
If you double the distance, $E =$ $\frac{1}{4}E_0 = 1.25$ N/C

3- You release a charged particle inside an electric field that points to the right.

If it was an electron, it will accelerate to the ..left....... (right, left, up, down)



Question 1 : In the figure three charges are placed as shown. $q_2 = 1.8$ nC. If q_3 is in static equilibrium what is q_1?



A) 7.2 nC
B) -7.2 nC (B) ✓
C) 3.6 nC
D) -3.6 nC

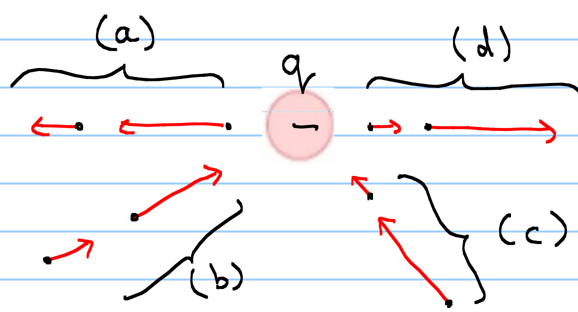
$$|E_1| = |E_2| \longrightarrow E_{net} \text{ must be zero}$$

$$\frac{K|q_1|}{(2a)^2} = \frac{K|q_2|}{a^2}$$

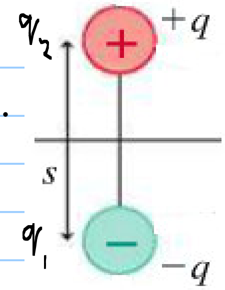
$$|q_1| = 4|q_2|$$
$$= 7.2\,\text{nC}$$

$$q_1 = -7.2\,\text{nC}$$

Question 2: The electric field caused by the negative charge is shown at several points. Choose the correct pair of electric field vectors (b) ✓



Electric field of a Dipole:

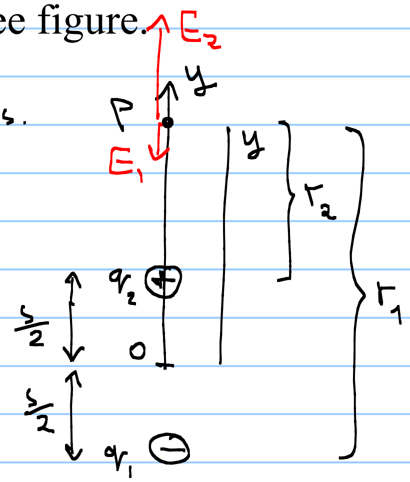


A dipole consists of 2 opposite charges $+/-q$ seperated by distance s.

Example: Two charges $+/-\ q$, where $q = 5.0$ nC, are separated by a distance $s = 4.0$ cm. Find the electric field at point P, which is $y = 2.0$ m above the center, see figure.

Model: Treat charges as point charges.

Vizualize:



$q = 5\times10^{-9}$ C
$s = 0.04$ m
$y = 2$ m

$r_1 = y + s/2$
$r_2 = y - s/2$

Solve: at point P along y

$$E_{net} = \underbrace{E_2 - E_1}_{\text{magnitude}}$$

$$= \frac{K|q_2|}{r_2^2} - \frac{K|q_1|}{r_1^2} = Kq\left(\frac{1}{(y-\frac{s}{2})^2} - \frac{1}{(y+\frac{s}{2})^2}\right)$$

$$= (8.99\times10^9)(5\times10^{-9})\left(\frac{1}{(2-0.02)^2} - \frac{1}{(2+0.02)^2}\right) = 0.45 \text{ N/C}$$

$$\vec{E} = (0.45 \text{ N/C},\ \text{up along } y)$$

Assess: units, significant figures, directions, magnitudes?
✓ ✓ ✓ ✓

If $y >> s$, you can show that $E = 2K\,(qs)\,/\,y^3$ i.e. proportional to $1/y^3$ and to qs.
The *dipole moment* of a dipole is the product *qs*,
and is taken to be a vector pointing from the negative to positive charge.

The Electric Field (chapter 23)

Electric Field of Continuous Charge Distributions:

- o Line of Charge along its axis,
- o Ring of Charge along its axis,
- o Infinite Plane of Charge
- o Parallel Plate Capacitor

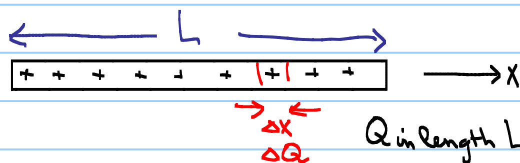
Continuous charge distribution (density)

Consider an extended body of total charge *Q* *uniformly distributed* within the body. For example, a 1D (one dimension) uniformly charged body, such as a rod. Because the charge is uniform, the charge *is proportional to the length*.
The constant of proportionality is called the *linear charge density* λ (pronounced lamda).

$$\boxed{\lambda = \frac{Q}{L} = \frac{\Delta Q}{\Delta x}}$$

a constant

Linear charge density
units: C/m



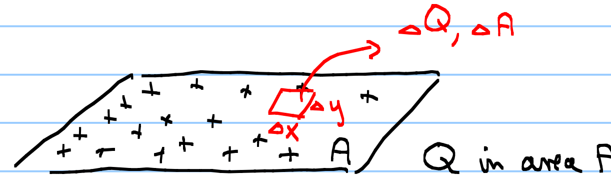
A little length Δx of the rod will hence carry a charge ΔQ given by:

$$\boxed{\Delta Q = \lambda \Delta x}$$

Similarly, a 2D uniformly charged body, such as a plane or sheet, has a *surface charge density* η (pronounced eta).

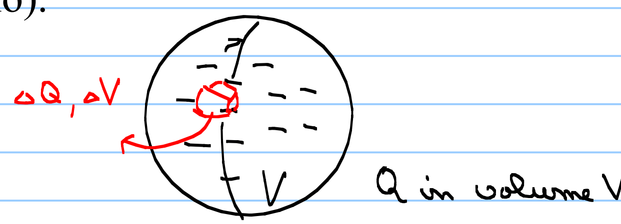
$$\boxed{\begin{aligned} \eta &= \frac{Q}{A} \\ \Delta Q &= \eta \Delta A \end{aligned}} = \eta \Delta x \Delta y$$

(in area ΔA there is ΔQ charge)

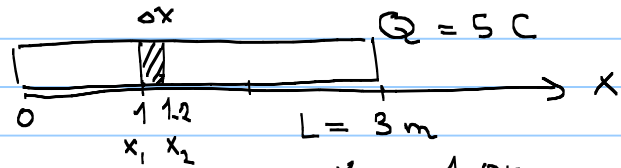


A 3D uniformly charged body, such as a solid sphere or solid cylinder, has a *volume charge density* ρ (pronounced rho).

$$\boxed{\begin{aligned} \rho &= \frac{Q}{V} \\ \Delta Q &= \rho \Delta V \end{aligned}}$$



Example: A 3.0 m long rod with one end at the origin has 5.0 C uniformly distributed along its length. Find the charge between x = 1.0 and 1.2 m.



$x_1 = 1.0\,m$

$x_2 = 1.2\,m$

$$\Delta Q = \lambda\, \Delta x$$

$$\lambda = \frac{Q}{L}$$

$$\Delta Q = \left(\frac{Q}{L}\right)(x_2 - x_1)$$

$$= \frac{5}{3} \times 0.2 = 0.33\ C$$

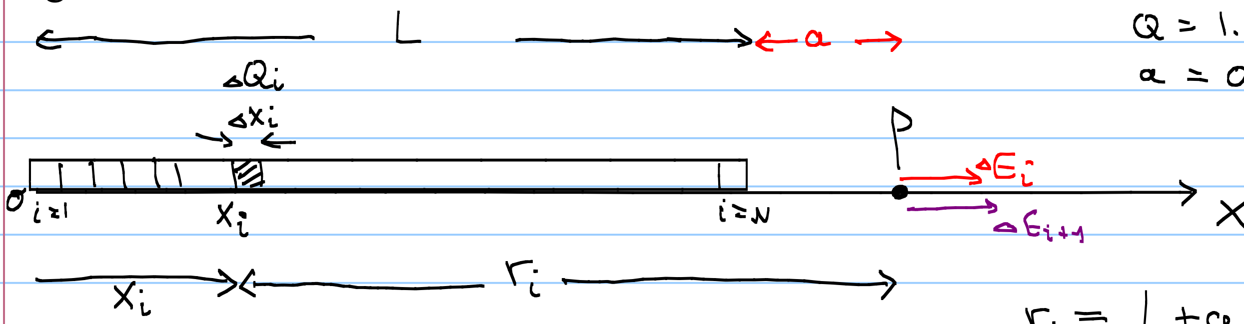
Electric field of a charged rod:

Find the electric field at point P, along the axis of a charged rod of length L=1.0 m and uniform charge Q=1.2 nC, a distance a=20.0 cm away from the right end.

$L = 1.0\,m$

$Q = 1.2\,nC$

$a = 0.2\,m$



$$r_i = L + a - x_i$$

$x_1 = 0$

$x_N = L$

1- Establish a coordinate system

2- Divide the total charge Q into N little pieces of charge ΔQ_i, and length Δx_i. Index i runs from 1 to N

3- Pick up an arbitrary piece i and treat that piece as a point charge. Find the electric field ΔE_i at point P due to charge ΔQ_i.

$$\Delta E_i = \frac{K|\Delta Q_i|}{r_i^2}$$

We will eventually integrate over dx, so write down variables in terms of x_i.

$$\Delta Q_i = \lambda\, \Delta x_i$$

with $\lambda = Q/L$

$$\Delta E_i = \frac{K}{r_i^2}\frac{Q}{L}\Delta x_i = \frac{KQ}{L}\frac{\Delta x_i}{(L+a-x_i)^2}$$

4- Look at symmetry, do we need to calculate the *x* and *y*-components of E_i?

ΔE_i always points along x.

→ no y-components needed.

5- Sum ΔE_{ix} over all segments $i = 1.. N$

$$E = \sum_{i=1}^{N} \Delta E_i = \sum_{i=1}^{N} \frac{KQ}{L} \frac{\Delta x_i}{(L+a-x_i)^2} = \frac{KQ}{L} \sum_{i=1}^{N} \frac{\Delta x_i}{(L+a-x_i)^2}$$

6- Replace the sum over *i* by an integral over the integration variable (here *dx*).

drop subscript i, hence $x_i \to x$ $\qquad \sum_{i=1}^{N} \cdots \Delta x_i \longrightarrow \int_0^L \cdots dx$

$$E = \frac{KQ}{L} \int_0^L \frac{dx}{(L+a-x)^2}$$

use usub: $u = \frac{1}{(L+a-x)} \to (L+a-x)^{-1}$, $du = -1(L+a-x)^{-2}(-1)dx = \frac{1}{(L+a-x)^2}$

$$= \frac{KQ}{L} \left. \frac{1}{(L+a-x)} \right|_0^L$$

$$= \frac{KQ}{L}\left[\frac{1}{L+a-L} - \frac{1}{L+a-0}\right] = \frac{KQ}{L}\left[\frac{1}{a} - \frac{1}{L+a}\right] = \frac{KQ}{L} \frac{(L+a)-a}{a(L+a)} = \frac{KQ}{a(L+a)}$$

$$\vec{E} = \left(\frac{KQ}{a(L+a)}, \text{ along } x\right) = \left(45\,N/C, \text{ along } x\right)$$

Check:

if $a \gg L$

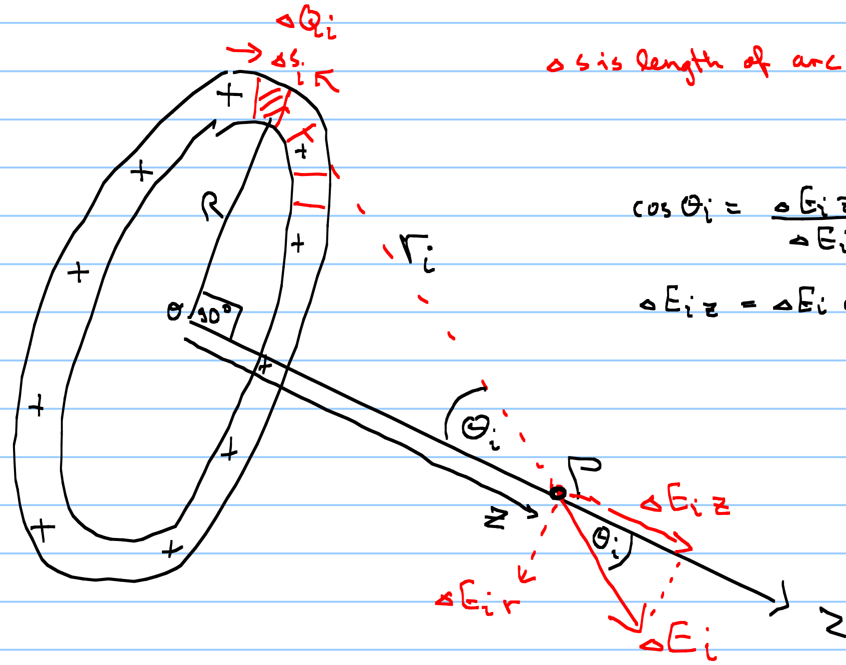
$$E = \frac{KQ}{a(\underbrace{\text{negligible } L}_{0} + a)} = \frac{KQ}{a^2}$$

E of point charge

Electric field of a ring of charge:
A charged ring of radius *R* has a uniform charge *Q*.
Find the electric field at a point on the axis of the ring a distance *z* away from the center.

$$r_i = \sqrt{R^2 + z^2}$$

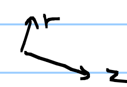
$$\cos\theta_i = \frac{z}{r_i}$$



$$\cos\theta_i = \frac{\Delta E_{iz}}{\Delta E_i}$$

$$\Delta E_{iz} = \Delta E_i \cos\theta_i$$

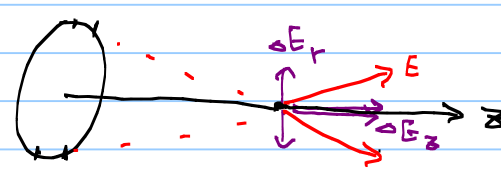
1- Use a polar coordinate system (*r, θ, z*), where the z-axis is normal to the radial axis.



2- Divide ring into pieces of charge ΔQ_i and length Δs_i. Δs is a circular arc of radius *R*.
3- Pick one piece (index *i*) of the ring. Find E_i at point *P* due to piece *i*.

$$\Delta E_i = \frac{K\,\Delta Q_i}{r_i^2}$$

4- Look at symmetry. *E* at *P* has two components, one along *z* and one pointing radially out along *r*.
The r-components of ΔE_i for segments lying on opposite sides of the ring cancel.



$$\Delta E_{iz} = \Delta E_i \cos\theta_i$$

$$= \frac{K\,\Delta Q_i}{r_i^2}\,\frac{z}{r_i} = \frac{Kz}{r_i^3}\,\Delta Q_i$$

$$= \frac{Kz}{\sqrt{R^2+z^2}^{\,3}}\,\Delta Q_i$$

5- Sum over all pieces

$$E_z = \sum_{i=1}^{N} E_{iz} = \sum_{i=1}^{N} \frac{Kz}{\underbrace{\sqrt{R^2+z^2}^{\,3}}_{\text{constant}}} \Delta Q_i = \frac{Kz}{\sqrt{R^2+z^2}^{\,3}} \underbrace{\sum_{i=1}^{N} \Delta Q_i}_{Q} = \frac{Kz}{\sqrt{R^2+z^2}^{\,3}} Q$$

Check answer:

If $z \gg R$

$$E_z = \frac{KzQ}{\sqrt{\underbrace{\text{negligible term}}_{0}{}^2 + z^2}^{\,3}} = \frac{KzQ}{z^3} = \frac{KQ}{z^2}$$ E of point charge

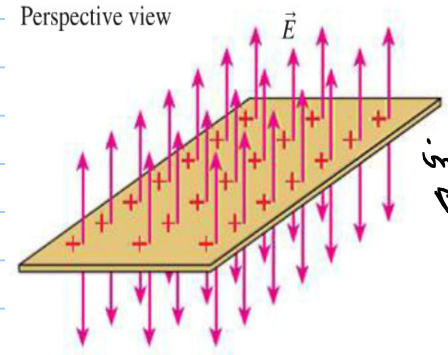
Electric field of an infinite plane of charge:

The electric field of a plane that *extends to infinity* and has uniform surface charge density η can be shown to be:

$\eta = \frac{\Delta Q}{\Delta A}$

$$\boxed{E = \frac{\eta}{2\varepsilon_0}}$$

E of infinite plane of charge



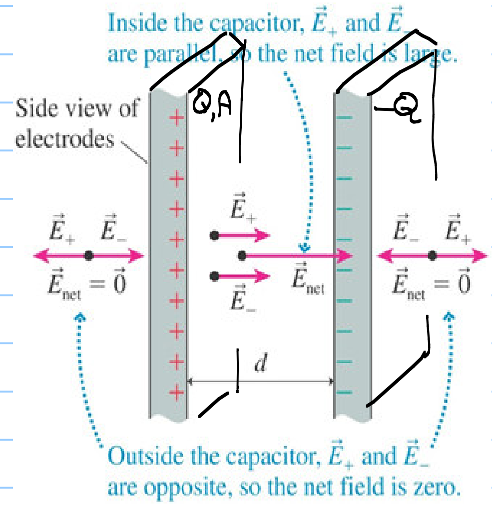
infinite plane

E points away from (towards) the plane for a positive (negative) surface charge.

Note: *E* of a charged plane that extends to infinity is *independent of distance*. The expression also holds for points whose *distance from the plane is small compared to the dimensions of the plane*.

Parallel plate capacitor (constant (uniform E within the plates):

One very common electrical device that creates a *uniform electric field* is a parallel plate capacitor, where two charged plates (electrodes) are separated in space. Both plates are charged by the *same amount of charge Q* but of *opposite sign*. Suppose the surface area of each capacitor is A. Each plate produces an electric field E between the plates that is constant and that points in the same direction.



$$E_{inside} = E_+ + E_- = 2E_+ = 2\frac{\eta}{2\varepsilon_0}$$

$$\boxed{E_{inside} = \frac{\eta}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}}$$

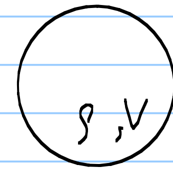
E inside capacitor points from + to - plane

$$E_{outside} = E_+ - E_- = 0$$

Inside a parallel plate capacitor *E is constant*.
E points from the *positive to negative plate*.
Outside a parallel plate capacitor $E = 0$.

The Electric Field $\vec{E}$ (chapter 23):

- A sphere of volume $V = 1.2\ \mathrm{m}^3$ carries a homogenous charge of 2.4 C. What is the charge ΔQ inside a volume $0.04\ \mathrm{m}^3$?



$\Delta Q = \rho\, \Delta V$

↖ "volume" charge density

with $\rho = \dfrac{Q}{V}$

$= \dfrac{Q}{V} \Delta V = 0.08\ \mathrm{C}$

Question 1:

If 8 nC of charge are placed on the square loop of wire, the linear charge density will be λ

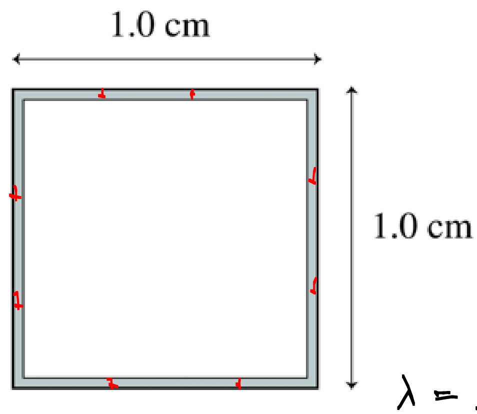
A. 800 nC/m

B. 400 nC/m

C. 200 nC/m (C) ✓

D. 8 nC/m

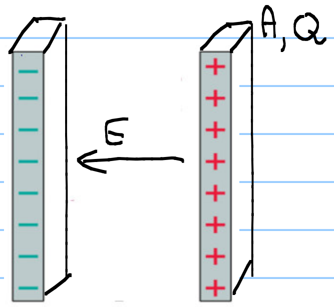
E. 2 nC/m



$\lambda = \dfrac{Q}{\text{length}} = \dfrac{8\ \mathrm{nC}}{0.04\ \mathrm{m}}$

- Mention three facts about the electric field $\vec{E}$ of a parallel plate capacitor.

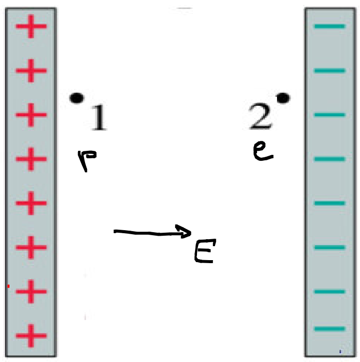
- E is constant between the plates
- E points from +ve plate to −ve plate.
- E = 0 outside the plates



surface charge density

$$E = \frac{\eta}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{Q}{A}$$

Question 2: A proton is released from rest at point 1 and an electron is released at point 2. Where will their path cross? Neglect their mutual attraction. (remember: mass of proton >> mass of electron)



A) Closer to the negative plate.
B) Closer to the positive plate. (B) ✓
C) At the center
D) Their path will never cross.

constant

$$|F_p| = |F_e| = eE$$

with $m_p \gg m_e$ ← mass

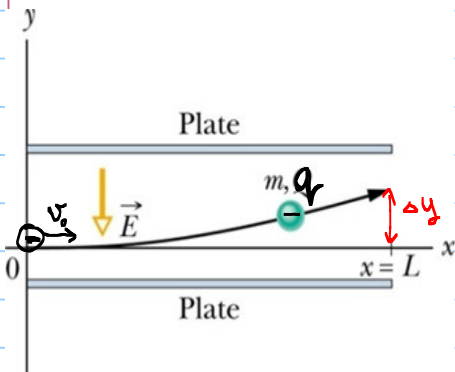
→ $a_p \ll a_e$
→ electron covers more distance in same time.

Example: **Ink Jet Printer:** An ink drop of mass 1.3×10^{-10} kg and charge $q = -1.5 \times 10^{-13}$ C enters the region between two conducting plates at 18 m/s along the x-axis. The electric field between the plates is directed downward, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates, whose length L is 1.6 cm?

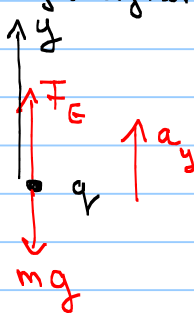
Model: Treat bead as point charge.

Kinematic equations:
$v_y = v_{0y} + a_y t$
$\Delta y = v_{0y} t + 0.5 a_y t^2$
$v_y^2 = v_{0y}^2 + 2 a_y \Delta y$

Vizualize:



Free body diagram:



$m = 1.3 \times 10^{-10}$ kg
$q = -1.5 \times 10^{-13}$ C
$L = 0.016$ m
$E = 1.4 \times 10^6$ N/C, along $-y$
for kinematics: $a_x = 0$
$v_0 = v_{0x} = 18$ m/s; $v_{0y} = 0$
find Δy

A negative charge experiences a force opposite to $\vec{E}$, hence up.

Newton's 2nd law:

along y: $F_{net\,y} = m a_y$

$$\underbrace{F_E - F_g}_{\text{magnitude}} = m a_y$$

$$\frac{|q|E - mg}{m} = a_y$$

$$a_y = \frac{|q|E}{m} - g$$

We see a_y = constant along the path of the drop.
→ use kinematics

along x, $a_x = 0$ → Time to reach the plate's end is

$$t = \frac{L}{v_{0x}} = \frac{L}{v_0}$$

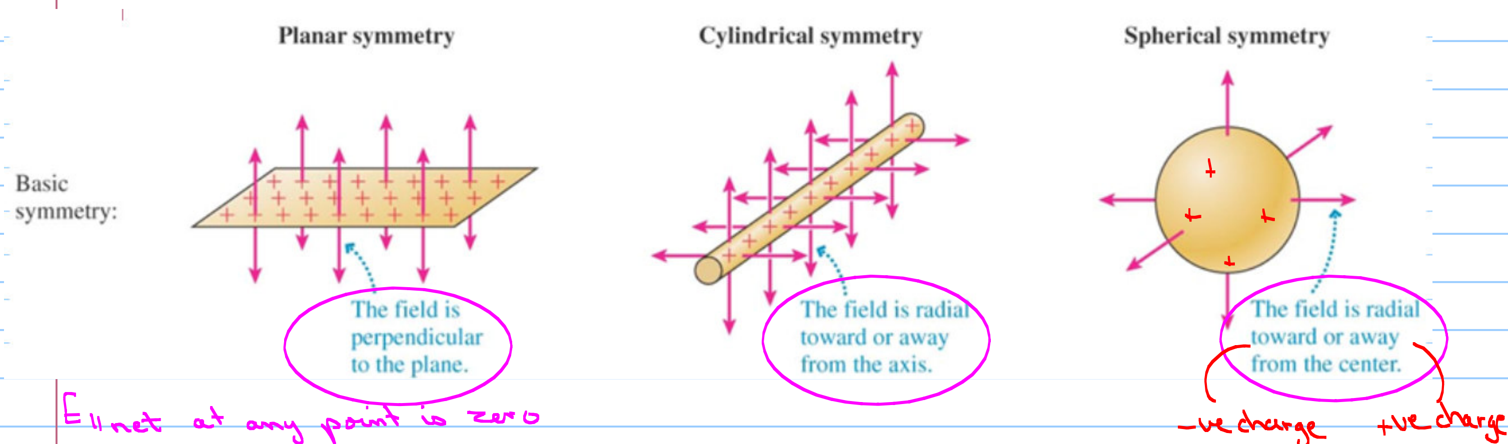
$$\Delta y = \cancel{v_{0y}}^{0} t + \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{|q E|}{m} - g\right)\left(\frac{L}{v_0}\right)^2$$

$= 6.34 \times 10^{-4}$ m (0.63 mm)

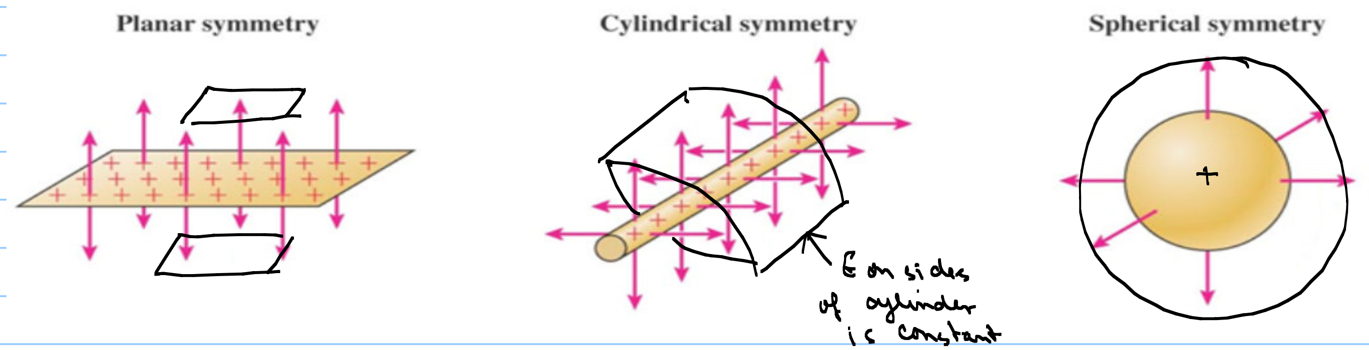
Gauss's law: (chapter 24)
- Symmetry - Electric Flux - Gauss' Law
- Conductors in Electrostatic Equilibrium.

A symmetry operation leaves the object without any physical change. Examples are *translation* along an axis, *rotation* about an axis or *reflection* in a mirror.

Electric field of some symmetrical charge distributions:
What is the direction of the electric field of an infinitely large charged plane, an infinitely long charged rod and a charged sphere?



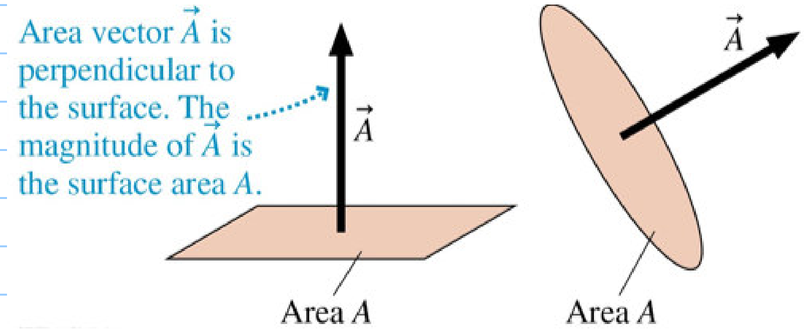
For each of the following uniform charge distributions draw a surface (flat or curved) that has a constant magnitude of the electric field at every point.



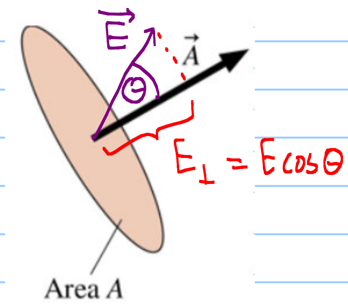
The symmetry of the electric field matches the symmetry of the charge distribution.
NOTE, that E at any point of these chosen surfaces of constant electric field are perpendicular to the surface.

The Electric flux:

The area vector $\vec{A}$ is a vector perpendicular to the surface, it has magnitude A.



Consider a flat surface of area A placed in an uniform electric field **E** that makes an angle θ with the surface vector **A**.



The *electric flux through that surface* is defined as:

$$\boxed{\begin{aligned}\Phi_e &= E_\perp A \\ &= E\cos\theta\, A \\ &= \vec{E}\cdot\vec{A}\end{aligned}}$$

dot product of 2 vectors

Electric flux through surface A for a constant electric field

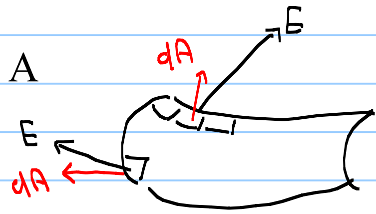
The unit of electric flux is N m^2 / C.

Note: If E is parallel to a surface, the flux is zero,
if E is pependicular to a surface the flux is EA.

For a nonuniform electric field or a curved area, we divide the area into infinitesimal area elements dA in which the electric field is nearly uniform.

$$\boxed{\Phi_e = \int_{\text{surface}} \vec{E}\cdot d\vec{A}}$$

Electric flux through surface A



Gaussian surfaces:
A *closed surface* is a real or imaginary surface that surrounds a 3D region of space, hence it divides space into an inside and an outside region.

A *Gaussian surface* is a closed surface through which an electric field passes.

Gauss's law:
Gauss's law provides an easy way to find the electric field due to some charge distributions by taking advantage of the symmetry of the problem.

Gauss's law states:
The *electric flux through a closed surface* (Gaussian surface) is *proportional to the total charge enclosed ~~within that~~ surface*.

$$\boxed{\begin{aligned} \Phi_{e\ \text{through Gaussian surface}} &= \frac{Q_{in}}{\varepsilon_0} \\ \oint_{\text{surface}} \vec{E} \cdot d\vec{A} &= \frac{Q_{in}}{\varepsilon_0} \end{aligned}}$$

Gauss's law in integral form

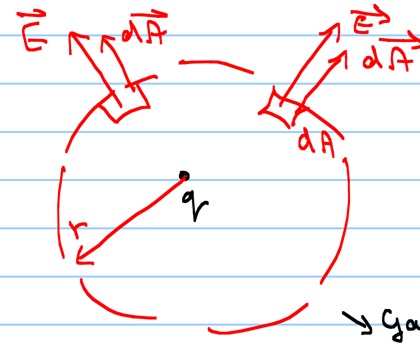
E at Gaussian surface

dA on Gaussian surface

net charge inside and on Gaussian surface

The circle on the integral sign means integration over a closed surface.

Point charge example: Let us check Gauss's law on a positive point charge. We will choose a Gaussian surface that matches the symmetry of the charge. Here a concentric sphere of some radius r around the point charge q.



↘ Gaussian surface in form of a concentric sphere of radius r

Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

RHS: $\dfrac{Q_{in}}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$

Now on chosen Gaussian surface:
E is the same everywhere and $\vec{E}$ is parallel to $d\vec{A}$.

LHS:

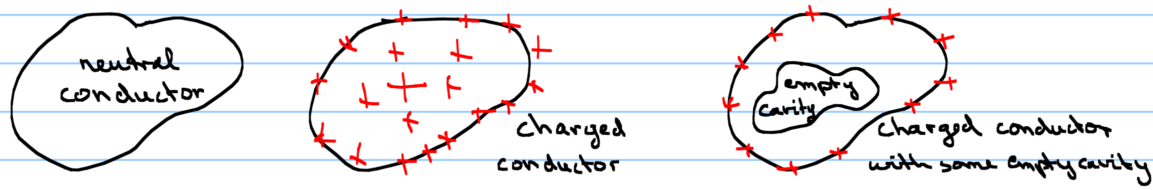
$$\oint \vec{E} \cdot d\vec{A} = \oint \underset{\text{constant}}{E}\, dA \underbrace{\cos(\theta)}_{1} = E \oint dA = E \underset{\text{surface area of Gaussian surface}}{A}$$

$$= \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}\right)(4\pi r^2)$$

$$= \frac{q}{\varepsilon_0} = \text{RHS}$$

Gauss's law would have been satisfied for any other shape of closed surface, surrounding q.

Conductors in electrostatic equilibrium
- A conductor has charges that are *free* to move.
- In electrostatic equilibrium charges on average *don't move*.

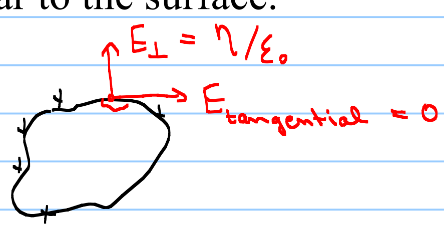


It follows that for a conductor **in electrostatic equilibrium**:

1- The *electric field inside the conductor is zero* (or charges would move).
It is also zero *inside any cavity* withing a conductor, *unless* a charge is in that cavity.

2- All *excess charges* on a conductor remain entirely *on its surface*.

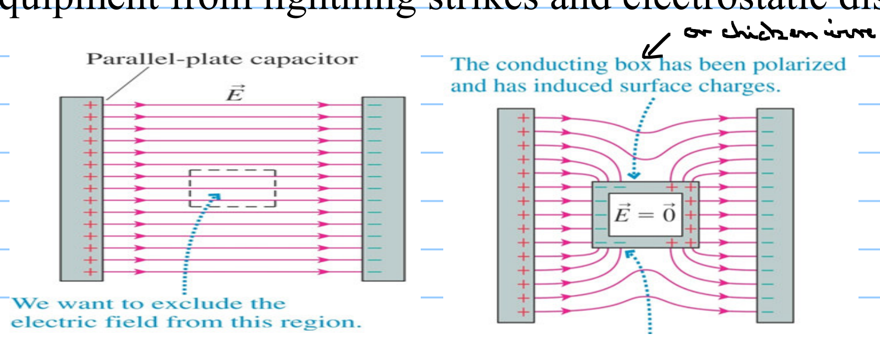
3- The electric field at the surface is always perpendicular to the surface.
- $E_{perpendicular} = \eta / \varepsilon_o$,
where $\boxed{\eta \text{ is the surface charge density}}$ at that point.
- $E_{tangential\ at\ the\ surface} = 0$.



A Faraday cage:
The main task of this metal chicken wire used in the power plant in the picture is not to keep intruding persons or rats off the power plant, but to keep away an 'intruding' external electric field. How does it work?

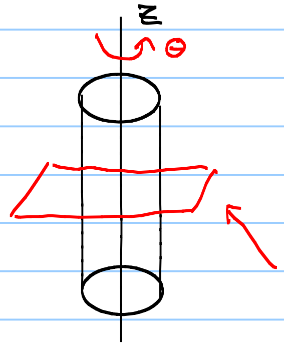

Faraday shield at Art Nouveau power plant in Heimbach, Germany (from wikipedia)

An external static electrical field causes the electric charges within the cage's conducting material to be distributed such that they cancel the field's effect in the cage's interior. This phenomenon is used, for example, to protect electronic equipment from lightning strikes and electrostatic discharges.



Gauss's law: (chapter 24)

1- Mention 3 symmetry operations of an infinite uniformly charged cylinder.



1- Rotation along the axis by any angle

2- Translation along the axis

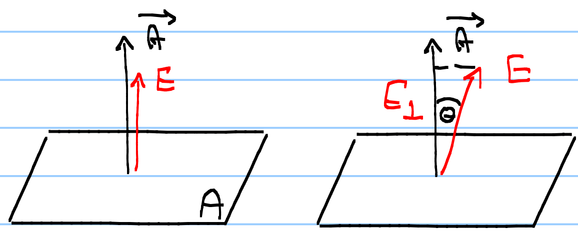
3- Mirror plane of reflection

2- An electric field E points perpendicular to area A. What is the net electric flux ϕ_e through that area?

$$\Phi_e = E A$$

If you tilt E by 10°, what is the ϕ_e ?

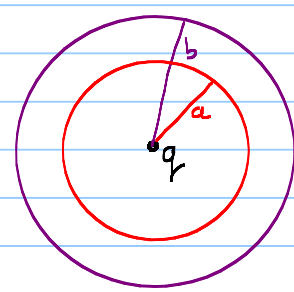
$$\Phi_e = E_\perp A = E A \cos(10^\circ)$$



3- Point charge Q is surrounded by two concentric spheres of radii a and b, ($a < b$).

-Compare the electric flux through the spheres a and b.

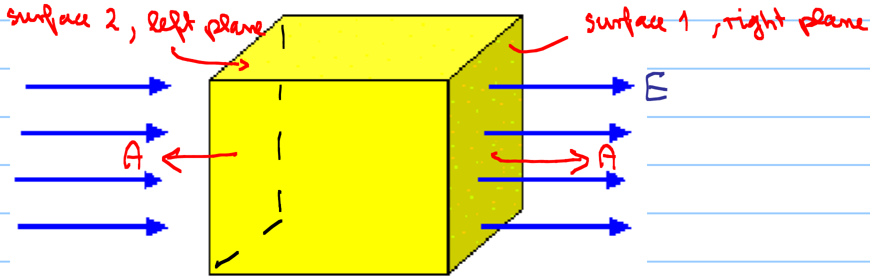
$$\Phi_a = \Phi_b = \frac{Q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0} \qquad (\text{Gauss's law})$$



-Compare the electric field magnitudes at radii a and b.

$$E_a > E_b \qquad \left(E_{\text{point charge}} = \frac{K q}{r^2}\right)$$

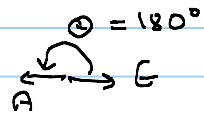
Example: In the figure a uniform electric field $E = 5$ N/C is aligned along x. A cubic box of side length 2.0 m is placed inside that field. Find the flux through its various surfaces.



The flux through surface 1 (right plane) is:

$$\Phi_{e_1} = EA\cos\Theta = EA\underbrace{\cos\Theta}_{1} = EA = 5 \times 2^2 = 20\ \frac{N\,m^2}{C}$$

The flux through surface 2 (left plane) is:



$$\Phi_{e_2} = EA\underbrace{\cos 180^\circ}_{-1} = -20\ \frac{N\,m^2}{C}$$

The flux through each other surface (top, bottom, front, back) is:

E is parallel to all other surface $\rightarrow$ $\Phi_e = 0$

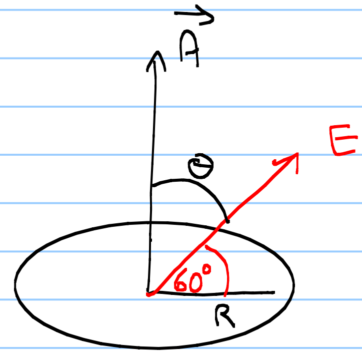
The total flux through all six surfaces of the box is: zero

$\rightarrow$ no net charge inside.

Question 1: The electric flux through the circle of radius 5 cm, placed in a uniform electric field of strength 4 N/C as shown is:

A) 0.027 Nm^2/C
B) 0.016 Nm^2/C
C) 6.8 x 10^{-3} Nm^2/C
D) 3.9 x 10^{-3} Nm^2/C

(A) ✓

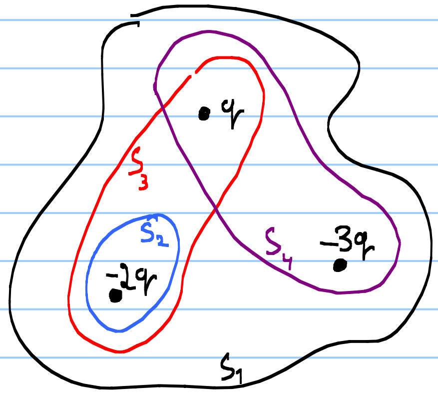


$$\phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$
$$= EA\cos 30^\circ$$
$$= E(\pi R^2)\cos 30^\circ$$

Question 2: Which Gaussian surfaces have an electric flux of $-2q/\varepsilon_0$?

A) S_1
B) S_2
C) S_3
D) S_4
E) S_2 and S_4
F) S_2 and S_1

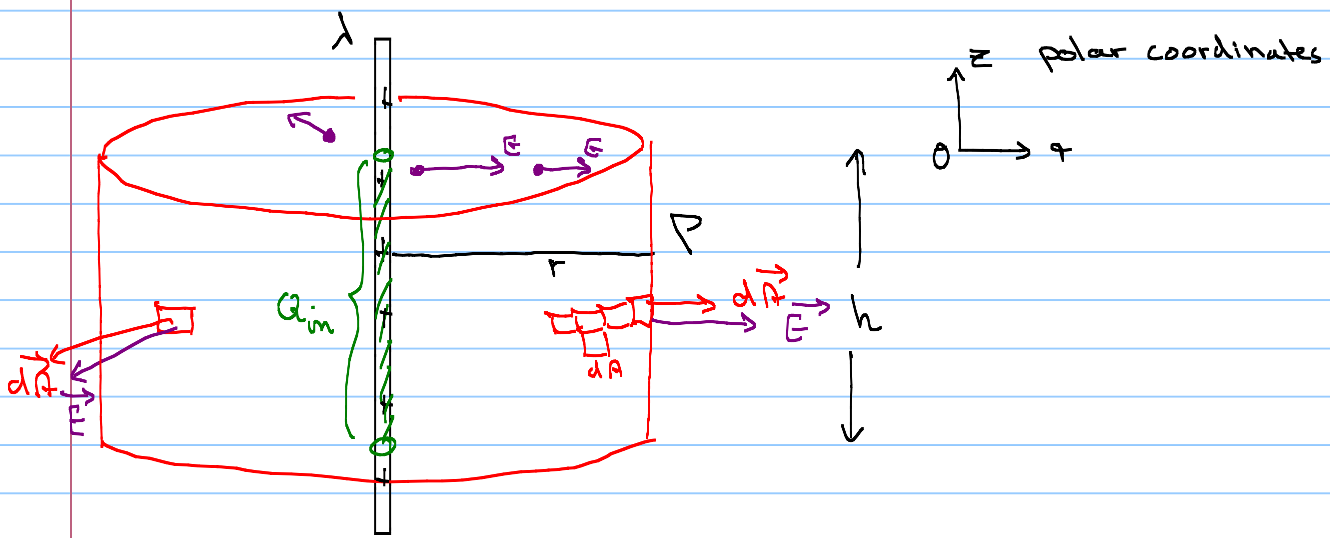
(E) ✓



Example: Using Gauss's law, find the electric field at a distance r of an infinitely long wire with a positive linear charge density λ.

- E has cylindrical symmetry around the charged rod. At any point, it points radially out from a positively charged wire.

1- Draw a concentric Gaussian surface (cylinder) of radius r and height h. $\vec{E}$ on that surface points tangent or perpendicular to the surface, show.



2- Find the flux Φ_e through the Gaussian surface.

$$\Phi_{e\ \text{gauss}} = \Phi_{\text{curved surface}} + \underbrace{\Phi_{\text{top}} + \Phi_{\text{bottom}}}_{\Phi_e = 0,\ \text{because } E \text{ everywhere on those surface is parallel to the area.}}$$

$$= \int_{\text{curved surface}} \vec{E}\cdot d\vec{A} = \int \underset{\text{constant}}{E}\, dA\, \underbrace{\cos\theta}_{1} = E\int dA = E\underset{\text{surface area of curved cylinder surface}}{A}$$

$$= E\, 2\pi r h$$

3- Find Q_{in}.

$$\Delta q = \lambda \Delta y$$

$$Q_{in} = \lambda h$$

4- Use Gauss`s law.

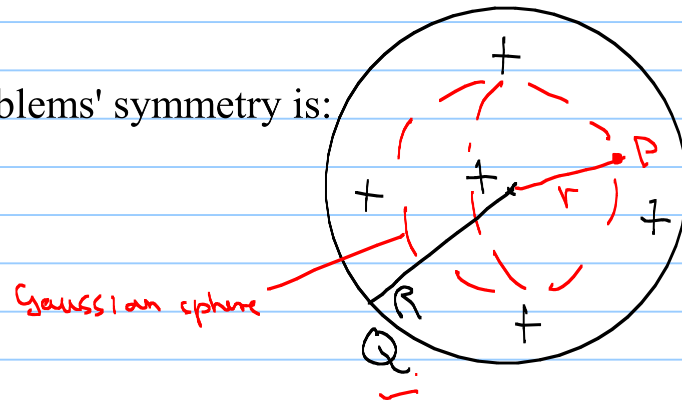
$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$E\, 2\pi r \cancel{h} = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{r}}$$

Example: Find the electric field of a uniformly charged sphere of radius R and charge Q at a distance r from the center ($r < R$) by answering the following:



The Gaussian surface representing the problems' symmetry is:

A) a sphere   (A) ✓

B) a cylinder

C) a rectangle

The charge inside that Gaussian surface is

A) $Q\, r^3/R^3$

B) $\rho\, V_{\text{Gaussian}}$

C) $\rho\, 4\pi r^3/3$

D) B and C

E) A, B and C   (E) ✓

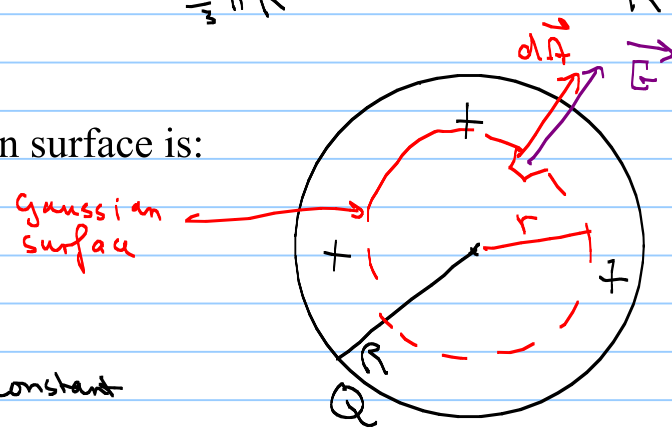
Here ρ is the sphere's charge density.

$$Q_{in} = \rho V_{in} = \rho V_{gaussian} \quad \text{(B) ✓}$$

$$= \rho \frac{4}{3}\pi r^3 \quad \text{(C) ✓}$$

$$\rho = \frac{Q}{V_R} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$Q_{in} = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3} \quad \text{(A) ✓}$$



The electric flux ϕ_E through the Gaussian surface is:

A) $E\, 4\, \pi\, r^2$ (A) ✓

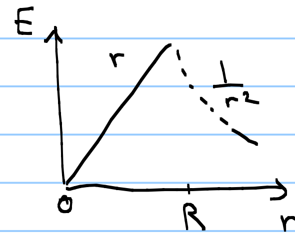
B) $E\, 4\, \pi\, R^2$

Here E is the electric field at r.

$$\Phi_{e_{gauss}} = \oint \underbrace{\vec{E}\cdot d\vec{A}}_{\vec{E} \text{ and } \vec{A} \text{ are parallel}} = \oint \overset{\text{constant}}{E}\, dA\, \underbrace{\cos\theta}_{1} = E\int dA = E\underbrace{A}_{\text{Surface Area of gauss. sphere}}$$

$$= E\, 4\pi r^2$$

Putting all together into Gauss's law:



$$\Phi_e = \frac{Q_{in}}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3}$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3}\, \hat{r} \qquad r \leq R}$$

E inside charged sphere

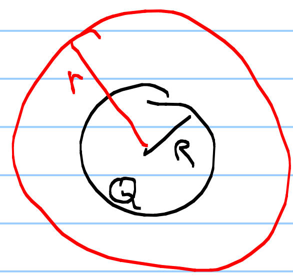
Homework: Using Gauss's law, show that $\vec{E}$ outside the uniformly charged sphere (r > R) is that of a point charge of charge *Q* centered at the center.

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}\, \hat{r} \qquad r \geq R}$$

E outside charged sphere

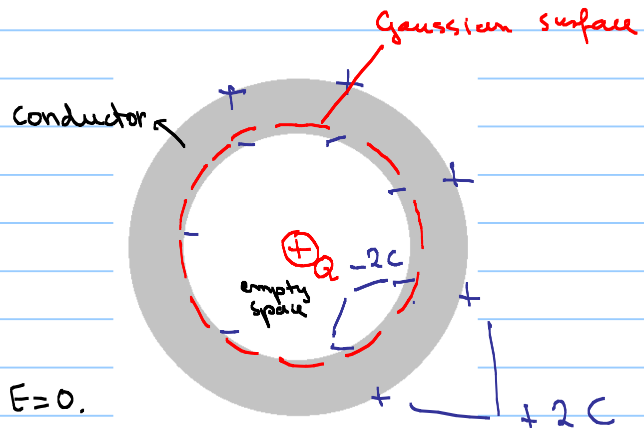
This is a very important result: *E* outside a homogeneously charged sphere is same as E of point charge of same charge.

$$Q_{in} = Q$$



Example: A positive charge $Q = 2\ C$ is placed at the center of a hollow *conductor*. Draw a sketch of how charge redistributes on the initially neutral conductor.

The charge (electrons) inside the conductor redistributes in such a way as to make the electric field zero everywhere inside the conductor.
Place a Gaussian surface (a sphere) inside the conductor just touching the inner sphere and apply Gauss's law:



Gauss's law:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Φ_e: flux through Gaussian surface

On that Gaussian surface $E = 0$, because that surface is chosen inside the metal, where $E = 0$.

$\Phi_e = 0 \rightarrow Q_{in} = 0$

But there is already the 2 C charge at the center

$$\rightarrow Q_{in} = \underbrace{2\ C}_{\text{at center}} + \boxed{\underbrace{(-2\ C)}_{\text{at inner surface of conductor.}}} = 0$$

Conductor started off neutral $\rightarrow$ an equal but opposite amount of charge $+2\ C$ collects on the outer surface (bare nuclei).

Electric potential *V*, Potential energy $U_{electric}$, Capacitors & Capacitance *C*:
(chapter 25 and 26)

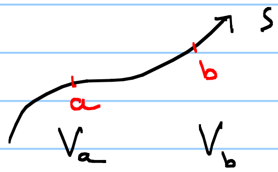
1- Electric potential *V*:

- Electric potential *V* from electric field *E* and *vice versa*
- *V* of a point charge and of multiple point charges
- *V* of a parallel plate capacitor
- *V* of a continuous charge distribution (a charged rod)

The electric potential *V* from E and vice versa:
We defined the electric field $\vec{E}$ to be a modification of space due to source charges. A test charge q in that electric field experiences a force

$$\vec{F} = q\vec{E}.$$

Similarly, we define the electric potential to be a modification of space due to source charges. A test charge q moving through a potential difference ΔV aquires potential energy ΔU.



$$\boxed{\Delta U = q\Delta V = q(V_b - V_a)}$$

The potential is a scalar quantity.
Its units are the Volt [1 V = 1 J /C].

"Potential, same as potential energy, is always determined with respect to a reference, that can be taken arbitrary. Hence it is the difference of *V* between two points in space that is uniquely defined."

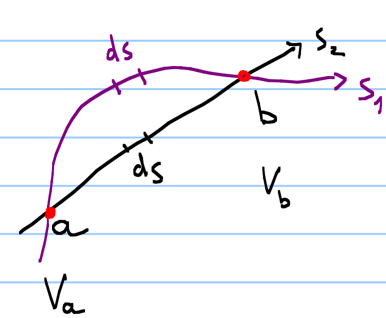
Relation between *V* and *E*:

***V* from $\vec{E}$**: Consider a charge *q* moving between points *a* to *b*.
Because the electrical force is a *conservative force,* the same amount of work is done to move *q* from *a* to *b* along different paths.
Hence we can assign an electric potential energy U_{elec} to *q*.

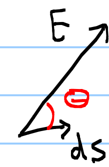
The *change in electric potential energy ΔU* by definition is *negative the work done by an electric force* to move the charge from point *a* to *b*.

$$\Delta U = -\int_a^b \vec{F}\cdot d\vec{s}$$

$$\frac{\Delta U}{q} = -\int_a^b \underbrace{\frac{\vec{F}}{q}}_{\vec{E}}\cdot d\vec{s}$$



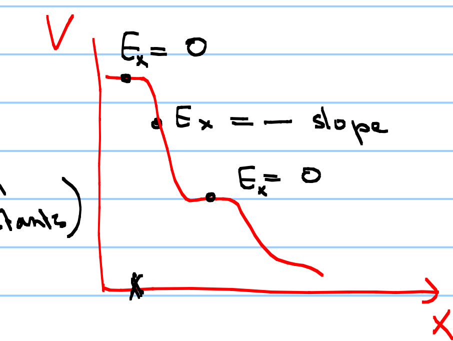
$$\boxed{\Delta V = V_b - V_a = -\int_a^b \underbrace{\vec{E}\cdot d\vec{s}}_{\text{component of E along s}} = -\int_a^b E\, ds \cos\Theta}$$



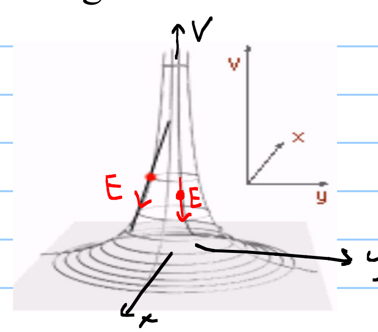
$\vec{E}$ from *V*: The reverse of an integral is a derivative. If *V* is known everywhere in space, *E* can be found everywhere.

$$\boxed{E_x = -\frac{\partial V}{\partial x}\;;\quad E_y = -\frac{\partial V}{\partial y}\;;\quad E_z = -\frac{\partial V}{\partial z}}$$

partial derivative of V w.r.t x (treat y and z as constants)

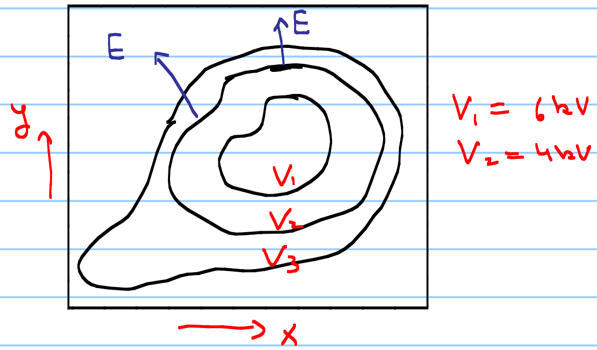


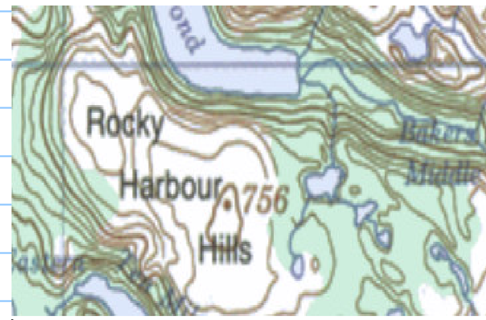
$\vec{E}$ is negative the gradient of *V*.
It points along direction of steepest descent.
The absolute value of the slope along that direction gives the magnitude of *E*.



Equipotential surfaces and lines:
They are surfaces or lines at a constant potential V.
A test charge will move freely (zero force) along these lines/surfaces.
- The *Electric field* points *perpendicular to equipotential lines/surfaces* and $E_{\parallel} = 0$.
- A *conductor in equilibrium is everywhere at the same potential.*

You can represent them similar to topographic maps.



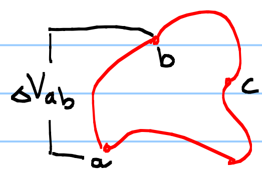


topographic map

Potential difference around a closed loop:
The potential difference around a closed path is zero.
(Think of the analogy of the topographic map. To return to your starting point on your mountain hike, you need to descend as much as you have ascended).

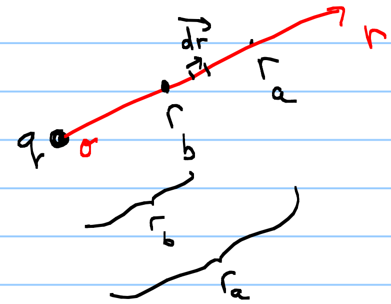
This is *Kirchoff's loop law*. It states:

$$\boxed{\begin{array}{l} \Delta V_{loop} = \sum_i \Delta V_i = 0 \\ \Delta V_{ab} + \Delta V_{bc} + \Delta V_{ca} = 0 \end{array}}$$



The Potential of a Point Charge:

Consider a charge *q* placed at the origin, the potential difference between two points *a* and *b* is then :



$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_{\vec{r}_a}^{\vec{r}_b}\left(\frac{Kq}{r^2}\hat{r}\right)\cdot(dr\,\hat{r})$$

$$\hat{r}\cdot\hat{r} = 1$$

$$= -Kq\int_{r_a}^{r_b}\frac{dr}{r^2} = -Kq\left(-\frac{1}{r}\right)\Bigg|_{r_a}^{r_b}$$

$$= Kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

If point a is taken at ∞, then $\frac{1}{r_a} = 0$, and by dropping subscript b:

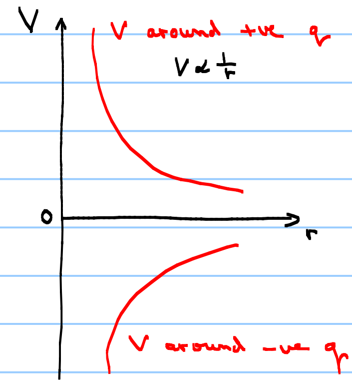
$$V_b = \frac{Kq}{r_b} + V_\infty \longrightarrow V = \frac{Kq}{r} + \underbrace{V_\infty}$$

let's agree on taking V_∞ as our zero potential reference $\quad V_\infty = 0$

The potential *V* of a source charge *q* a distance *r* away becomes:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{Kq}{r}}$$

Electric potential of a point charge (with the reference $V_{r=\text{infinity}} = 0$).



V is *positive around a positive charge*, and *negative around a negative charge*.

The same expression is true for the potential *outside a charged sphere* carrying total charge *Q*.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{KQ}{r}}$$

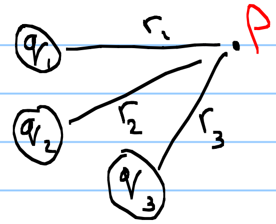
Potential outside a *charged sphere*.

for $r \geq R$

For more charges q_i the superposition principle holds:

$$\boxed{V = K \sum_i \frac{q_i}{r_i}}$$

$$= K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right)$$

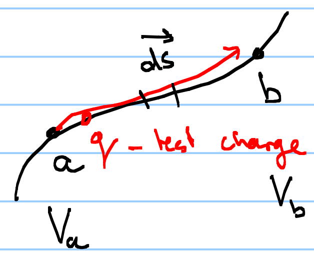


V is a scalar quantity, it does not have any x or y-components!!

Review:

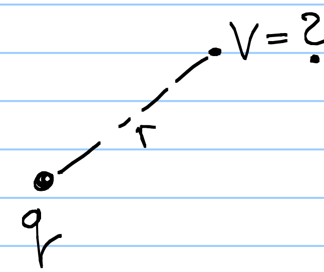
- $\Delta U_e = q\,\Delta V = q(V_b - V_a)$

change in potential energy of test charge q as it moves from point a to b



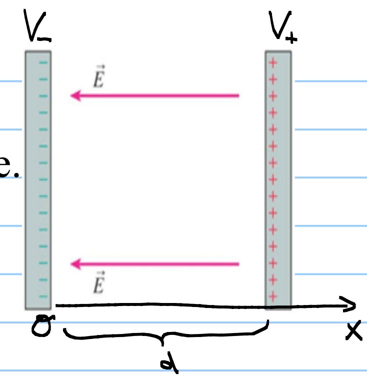
- $\Delta V = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E}\cdot d\vec{s}$; $E_x = -\frac{\partial V}{\partial x}$

- $V_{\text{of a source (point) charge}} = \frac{kq}{r}$



$$E = \left|\frac{\partial V}{\partial x}\right|$$

constant $\rightarrow V = Ex$



Potential inside a parallel plate capacitor:
Consider a parallel plate capacitor with a plate seperation d.
Let V_- and V_+ be the potential of the negative and positive plate.

You can show that the potential at any point inside the capacitor is:

$$V_{at\ x} = Ex + V_-$$

where the *x-axis starts at the negative plate* and points towards the positive plate.
Start from $V_{at\ x} - V_- = -\int_0^x \vec{E} \cdot d\vec{x}$

If we take the *reference of potential* ($V_- = 0$) *to be at the negative plate* (x = 0), then:

$$\boxed{V = Ex}$$

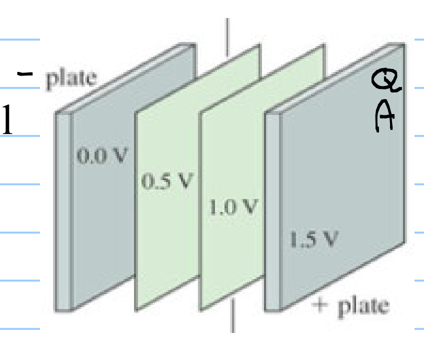
The potential difference between the two plates is:

$$\boxed{V_c = Ed}$$

Note: E inside can be determined by either of the two relations:

$$\boxed{E = \frac{\overset{Q/A}{\eta}}{\varepsilon_0} = \frac{V_c}{d}}$$

Note: Inside the parallel plate capacitor, planes parallel to the capacitor plates are equipotential surfaces (constant potential).

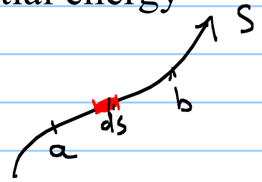


2- Electric potential energy:
- Potential energy $U_{electric}$ of two and more point charges
- Conservation of mechanical energy

We found that the change in potential energy ΔU of a charge q as it moves between two points having a potential difference ΔV between them is:

$$\boxed{\Delta U_{electric} = q\,\Delta V}$$

Change in electric potential energy



Units of energy is the Joule [1 J = 1 N.m]

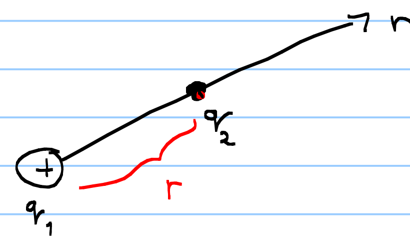
If we agree on the zero reference of potential energy to coincide with the zero reference of potential we get:

$$\boxed{U_{electric} = q\,V}$$

electric potential energy

The Potential energy of two point charges:
U_e is actually always shared between the test charge and the electric field (source charges) - same as gravitational potential energy, which is shared between the mass and planet.
Consider two point charges q_1 and q_2 a distance r apart.
The potential energy shared by the two point charges is the external work needed to bring q_2 from infinity to a distance r away from q_1 and vice versa.

$$U_{\text{electric}} = q_2 \, V_{\substack{\text{at location of } q_2 \\ \text{due to source charge } q_1}} = q_2 \left(\frac{K q_1}{r} \right)$$



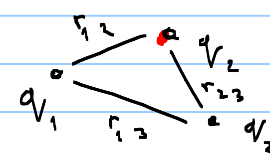
$$\rightarrow \boxed{U_{\text{electric}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}}$$

Electric potential energy of two point charges with the zero potential energy reference at infinity.

Here, *negative potential energy* means *attraction*, positive means repulsion.
For example, a molecule having a negative potential energy is stable.

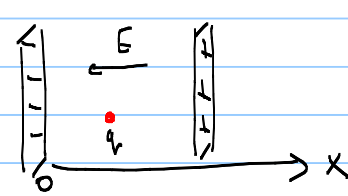
Similarly the **potential energy stored between multiple charges** is the external work needed to bring the charges together from infinity.

$$\boxed{U_{\text{electric}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}}$$



The potential energy inside a capacitor,
If we take $V_- = 0$, and the origin of coordinates at the negative plate, then we can define the potential energy U of a test charge q (shared between q and E) as:

$$\boxed{\begin{aligned} U &= qV \\ &= qEx \end{aligned}}$$



Conservation of mechanical energy:
Mechanical (kinetic + potential) energy is conserved in an *isolated system* of particles that interact with each other through *conservative forces*.

no dissipative forces (friction)

no external forces

$$E_{mech(f)} = E_{mech(i)}$$

final

initial

$$K_f + U_f = K_i + U_i$$

kinetic energy

potential energy

f for final, i for initial

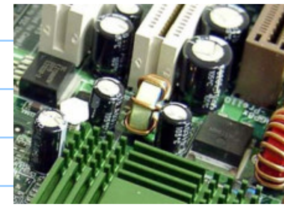
$$\Delta E_{mech} = 0$$

$$\Delta K + \Delta U = 0$$

equivalent statements

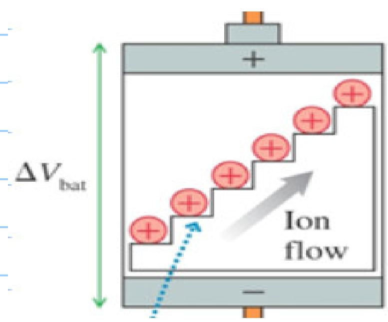
3- Capacitors and capacitance:

- Batteries and *emf* ε
- Capacitance C.
- Energy stored inside a capacitor
- Series and parallel capacitors



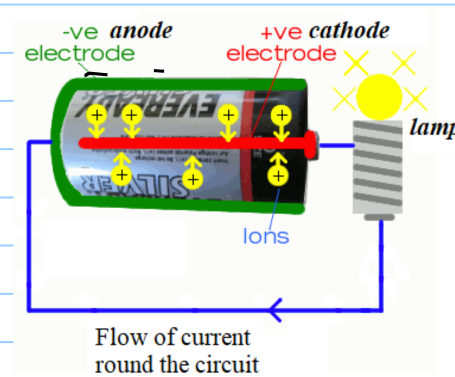
Batteries and *emf* ε :

- The most commonly known source of electric potential is a battery.
- Two electrodes made of different metals are separated inside an electrolyte.
- Chemical work done W_{chem} (chemical reactions) in the electrolyte transports ions of opposite charge q in opposite directions to the electrodes. The emf $\varepsilon = W_{chem}/q$.
- This separation of charge creates a potential difference ΔV_{bat} between the two electrodes. In an ideal battery $\Delta V_{bat} = \varepsilon$, however due to internal resistance it might be less.
- We can visualize the battery as a charge escalator. It lifts a positive charge from the negative electrode to the positive electrode, raising its potential energy.



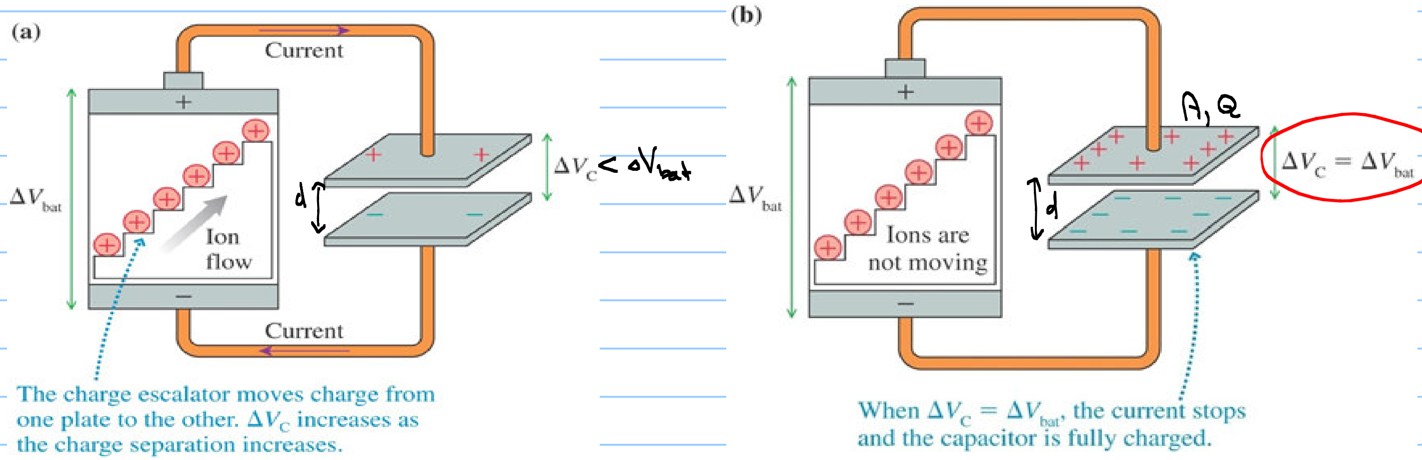
Positive electrode (cathode)

Negative electrode (anode)

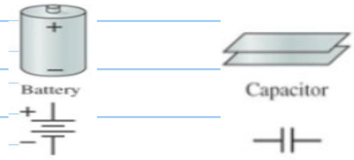


Capacitors and capacitance:

- A capacitor is a passive component that *stores energy*.
- It consists of *two metal plates* separated in space.
- When a battery is connected to the plates, the plate connected to the positive electrode gradually accumulates a charge $+Q$, the plate connected to the negative electrode accumulates a charge $-Q$.
- When $\Delta V_c = \Delta V_{battery}$ the charges stop moving. The capacitor is fully charged.



In circuits, a battery and capacitor are represented as:



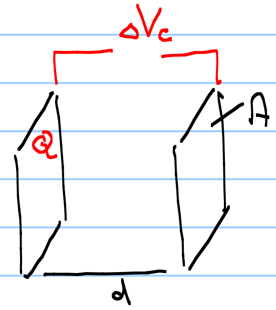
The capacitance C is the constant of proportionality that relates Q to ΔV_c.

$$\boxed{Q = C\,\Delta V_c}$$ Capacitance C

It depends only on the geometry of the capacitor and on the material between its plates. C has units in farad [1 farad = 1 F = 1 C / V].

For a parallel plate capacitor:

$$C = \frac{Q}{\Delta V_c} \quad \text{with } \Delta V_c = E d = \frac{\eta}{\varepsilon_0} d = \frac{1}{\varepsilon_0}\frac{Q}{A} d$$



$$C = \cancel{Q} \cdot \frac{\varepsilon_0 A}{\cancel{Q}\, d}$$

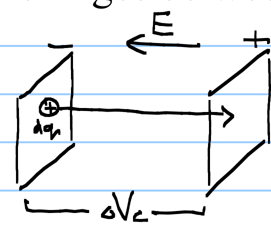
with $$\boxed{C = \frac{\varepsilon_0 A}{d}}$$ Capacitance of a parallel plate capacitor

Some applications require a large capacitance. By inserting a *dielectric material (an insulator)* between the plates *of dielectric constant* $\kappa = \varepsilon / \varepsilon_0$ (> 1), the capacitance C increases to κC_{vac}.

- ε is the *permittivity* of the dielectric material.
- ε_0 is the *permittivity* of free space (~ that of air) and C_{vac} is the capacitance of the air-filled capacitor.

Energy stored in a capacitor:
Capacitors store energy U_c and release it when they discharge. The energy stored is the work done against the electric field to separate the charges between a positive and a negative plate:

$$U_c = \int_0^Q dq\, V = \int_0^Q dq \frac{q}{C} = \frac{1}{C}\int q\, dq = \left.\frac{q^2}{2C}\right|_0^Q$$



$$\boxed{\begin{aligned} U_c &= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C (\Delta V_c)^2 \\ &= \frac{1}{2} Q \Delta V_c \end{aligned}}$$

Energy stored inside a parallel plate capacitor

using $Q = C \Delta V_c$

Energy stored in an electric field:
More generally, the energy density (energy per unit volume) stored in

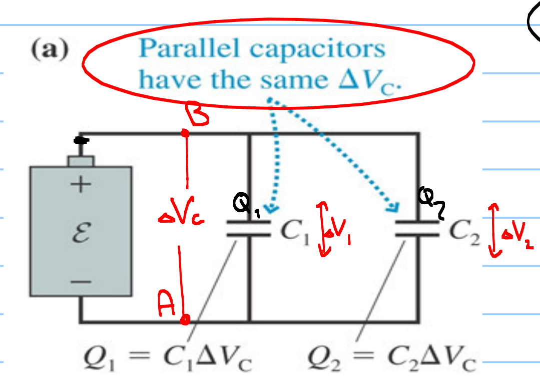
$$u_{electric} = \frac{U_{electric}}{Volume} = \frac{1}{2}\varepsilon_0 E^2$$

Parallel and series capacitors:

Parallel capacitors: (potential difference accross each is the same)

Two parallel capacitors have capacitances C_1 and C_2.

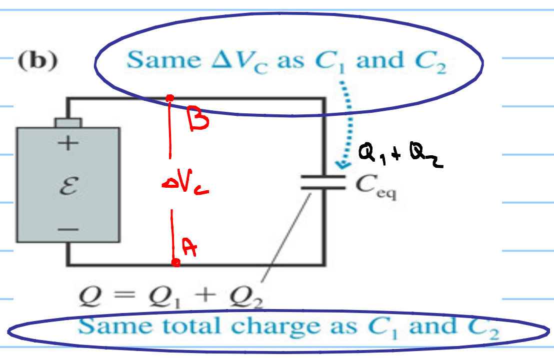
- Describe the potentials and charges.
- What is the equivalent capacitance.



$\Delta V = 0$ on the metal wires (equipotential surfaces)

$$\Delta V_c = \Delta V_1 = \Delta V_2$$

Replace these capacitors by one equivalent capacitor that accumulates charge $Q = Q_1 + Q_2$ under a potential difference ΔV_c.



$$Q = Q_1 + Q_2$$

$$C_{eq}\Delta V_c = C_1 \Delta V_1 + C_2 \Delta V_2$$

$$= C_1 \Delta V_c + C_2 \Delta V_c$$

$$= \Delta V_c (C_1 + C_2)$$

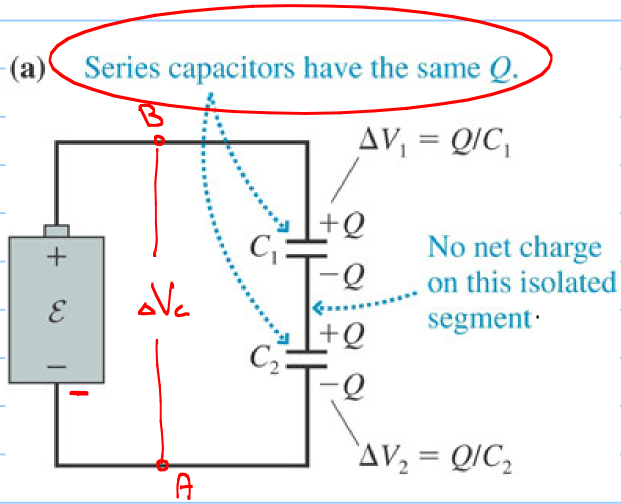
$$C_{eq} = C_1 + C_2$$

Parallel capacitors have an equivalent capacitance:

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \ldots}$$

Capacitors in series: (a single wire connecting the two capacitors, with no junction in the wire.)
Two capacitors in series have capacitances C_1 and C_2.
Describe the potentials and charges and find the equivalent capacitance.

$$Q_1 = Q_2 = Q$$



$$Q = C\,\Delta V_c$$

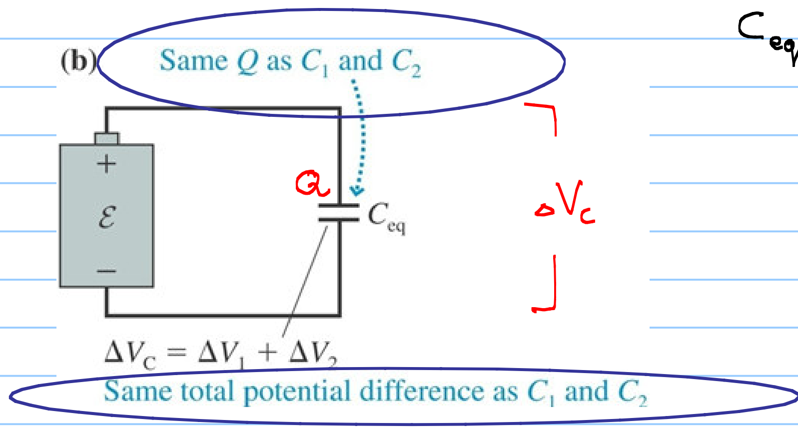
Replace these capacitors by one equivalent capacitor that accumulates same charge Q under a potential difference $\Delta V_c = \Delta V_1 + \Delta V_2$.

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



Capacitors in series have an equivalent capacitance:

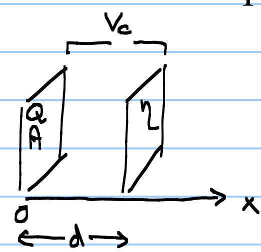
$$\boxed{\frac{1}{C_{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\right)}$$ (series capacitors)

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\right)^{-1}$$

Review: Some properties of parallel plate capacitor:

1- The electric field inside a parallel plate capacitor is constant and points from the positive to negative plate:

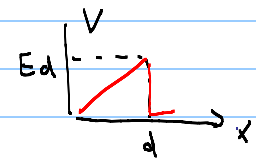
$$\boxed{\begin{array}{l} E = \eta / \varepsilon_o \qquad \text{with } \eta = Q / A \\ also: \; E = \Delta V_C / d \end{array}}$$



2- The potential inside a parallel plate capacitor varies linearly according to

$V(x) = V_- + Ex$, x is measured from the negative plate.

We usually take the reference $V_- = 0$, so $\boxed{V(x) = Ex}$



3- The potential difference between the two plates $\boxed{\Delta V_c = V_+ - V_- = Ed.}$
d being the plate separation.

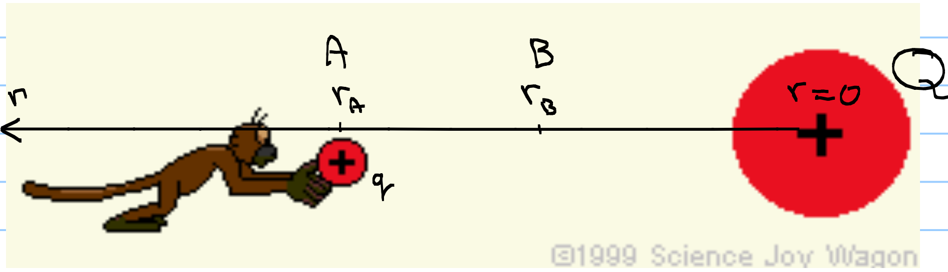
4- By definition $\boxed{Q = C\,\Delta V_c.}$
The capacitance of a parallel plate capacitor is $\boxed{C = \varepsilon_0 A / d.}$

5- The energy stored inside a capacitor is $\boxed{U_c = Q\,\Delta V_c / 2}$
$= Q^2 / (2C) = C\,(\Delta V_c)^2 / 2.$

6- Capacitors in series: $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots..$

7- Capacitors in parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots..$

Electric Potential *V*, Electric Potential Energy U_e: (chapter 25 and 26)



1- As charge *q* moves from points A to B in an electric field (that of charge *Q*), its potential energy changes by ΔU_e. Explain ΔU_e in terms of work done.

ΔU_e is minus the work done by the electric field.

$$\Delta U_e = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r} = -\int_{r_A}^{r_B} q \vec{E} \cdot d\vec{r}$$

$$= q\left[-\int_{r_A}^{r_B} \vec{E} \cdot d\vec{r}\right]$$

$$\boxed{= q \quad \Delta V}$$

difference in potential between points B and A

$$\boxed{\Delta V = V_B - V_A = -\int_{r_A}^{r_B} \vec{E} \cdot d\vec{r}}$$

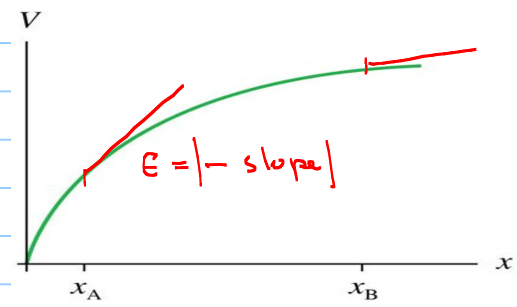
Question 1: At which point is the strength (magnitude) of the electric field stronger?

A) There's not enough information to tell.

B) At x_A.

C) At x_B.

(B) ✓



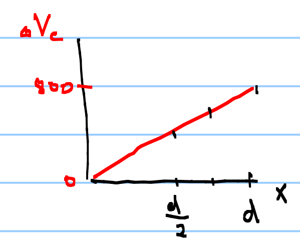
$|\text{slope at } x_A| > |\text{slope at } x_B|$

Example: The potential in Volts in a region of space is given as $V = 5x^2 + 2y$, where x and y are in meters. Find the electric field in vector form at the point (x = 2m, y = 3m).

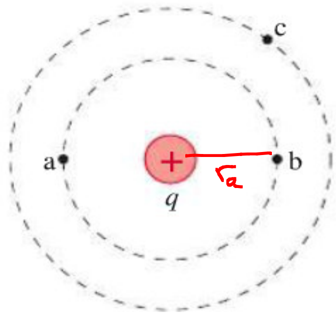
Answer: $\vec{E} = -10x\,\hat{i} - 2\,\hat{j}$

At (2m, 3m)

$\vec{E} = (-20\,\hat{i} - 2\,\hat{j})\ \text{V/m}$



STOP TO THINK 28.5 Rank in order, from largest to smallest, the potential differences ΔV_{ab}, ΔV_{ac}, and ΔV_{bc} between points a and b, points a and c, and points b and c.



A) $|\Delta V_{ca}| > |\Delta V_{cb}| > |\Delta V_{ba}|$
B) $|\Delta V_{ca}| = |\Delta V_{cb}| > |\Delta V_{ba}|$ (B)
C) $|\Delta V_{cb}| > |\Delta V_{ca}| > |\Delta V_{ba}|$
D) $|\Delta V_{ca}| > |\Delta V_{cb}| = |\Delta V_{ba}|$

$$V_a = V_b = \frac{Kq}{r_a}$$

$$\Delta V_{ba} = V_b - V_a = 0$$

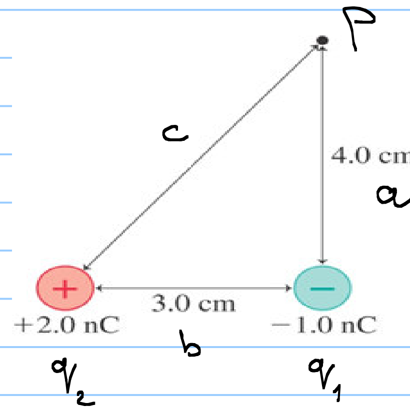
$$|\Delta V_{ca}| = V_c - V_a$$

$$|\Delta V_{cb}| = V_c - V_b$$

$$V_a = V_b$$

$$|\Delta V_{ca}| = |\Delta V_{cb}|$$

Example: What is the electric potential at the point P indicated in figure.



$$V_{at\,P} = V_1 + V_2$$

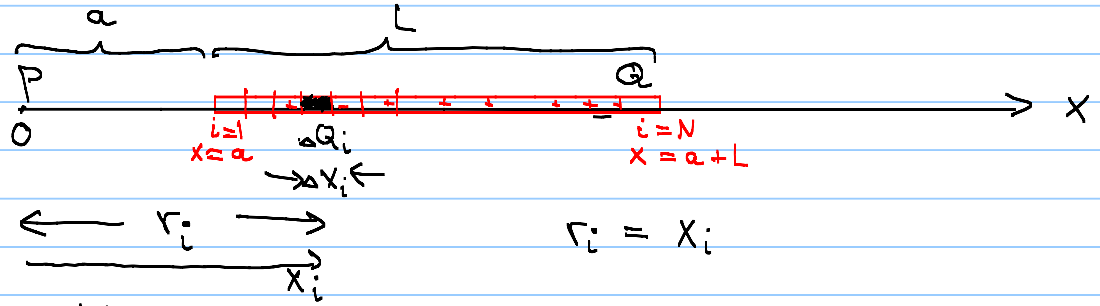
$$= \frac{Kq_1}{a} + \frac{Kq_2}{c} \quad ; \quad c = \sqrt{a^2+b^2} = 5\text{ cm} = 0.05\text{ m}$$

$$= K\left(\frac{q_1}{a} + \frac{q_2}{c}\right)$$

$$= (8.99\times10^{9})\left(\frac{-1\times10^{-9}}{0.04} + \frac{2\times10^{-9}}{0.05}\right)$$

$$= 135\text{ V}$$

Example: Find the potential of a uniformly charged rod of length L and charge Q at the origin, which is a distance a away from one end, see figure.



$$r_i = x_i$$

$$\Delta V_i = \frac{K\,\Delta Q_i}{r_i}$$

$$\Delta Q_i = \lambda\,\Delta x_i$$

$$\lambda = \frac{Q}{L}$$

$$\Delta Q_i = \frac{Q}{L}\,\Delta x_i$$

$$= K\,\frac{Q}{L}\,\frac{\Delta x_i}{x_i}$$

$$V = \sum_{i=1}^{N} \Delta V_i = \frac{KQ}{L} \sum_i \frac{\Delta x_i}{x_i}$$

$$\sum_{i=1} \cdots \Delta x_i \longrightarrow \int_{x=a}^{a+L} \cdots\, dx$$

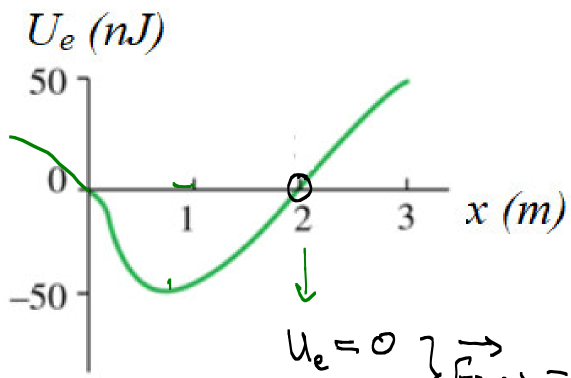
$$V = \frac{KQ}{L} \int_a^{a+L} \frac{dx}{x} = \frac{KQ}{L} \Big[\ln|x|\Big]\Big|_a^{a+L}$$

$$= \frac{KQ}{L}\Big[\ln(a+L) - \ln(a)\Big] \quad , \ln a - \ln b = \ln \frac{a}{b}$$

$$= \frac{KQ}{L} \ln\left(\frac{a+L}{a}\right)$$

Question 3: A charge inside an electric field is released from rest at $x = 2$ m. Its potential energy is shown by the graph. What is its kinetic energy when it reaches 1 m?

A) 0 nJ
B) ~ 100 nJ
C) ~ 50 nJ   (c) ✓
D) ~ -50 nJ



$U_e = 0$, $K = 0$ $\Big\}$ → $E_{mech} = 0$

At $x = 2$ m

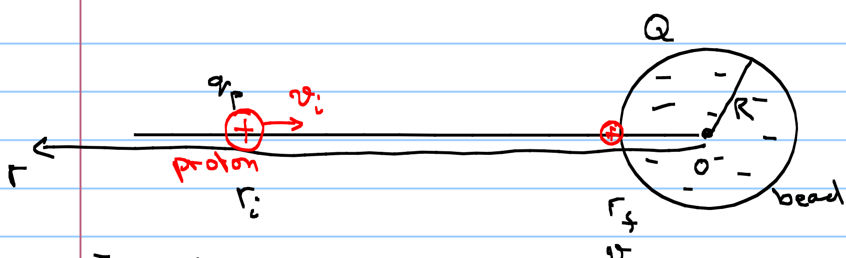
$$U_{mech} = \underbrace{U_e}_{=0 \text{ from graph}} + \underbrace{K}_{\text{charge is at rest}} = 0 + 0 = 0$$

At 1 m, the charge lost 50 nJ of potential energy, and hence it gained + 50 nJ of kinetic energy.

$$U_e + K = 0$$
$$-50 + (50) = 0$$

from ∞

Example: A 2.0 mm diameter stationary plastic bead is charged to -1.0 nC. A proton is fired at the bead from far away with a speed of 1.0×10^6 m/s, and it collides head-on. What is the impact speed? Note: the potential outside a charged bead is the same as the potential of a point charge of same charge located at the center.



$q_p = 1.6\times10^{-19}$ C

$m = 1.67\times10^{-27}$ kg

$Q = -1.0\times10^{-9}$ C

$R = \frac{2\times10^{-3}}{2} = 1\times10^{-3}$ m

$v_i = 1\times10^{6}$ m/s to right.

find v_f

$r_i = \infty$

$r_f = R$

The system of proton and bead is

1. Isolated
2. Non-dissipative (no friction)

$\rightarrow$ Emech is conserved

$$K_f + U_f = K_i + U_i$$

mass of proton

proton moves in the potential of the bead.

$$\frac{1}{2}mv_f^2 + q_p \underbrace{V_f}_{\text{of bead}} = \frac{1}{2}mv_i^2 + q_p V_i$$

$$\frac{1}{2}mv_f^2 + q_p\frac{KQ}{R} = \frac{1}{2}mv_i^2 + q_p \cancel{\frac{KQ}{\infty}}^{\,0}$$

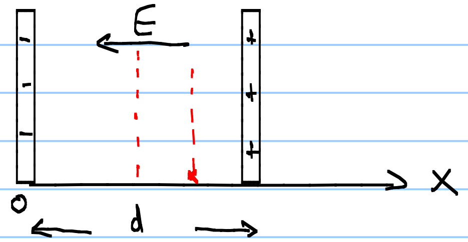
$$v_f^2 = \frac{2}{m}\left[\frac{1}{2}mv_i^2 - q_p\frac{KQ}{R}\right]$$

$$v_f = \sqrt{v_i^2 - \frac{2K}{m}\frac{q_p Q}{R}}$$

$$= 1.65\times10^{6}\ \text{m/s}$$

Proton looses potential energy and gains kinetic energy as it approaches the negative bead.

Example: The plates of a parallel plate capacitor are separated by 2.0 mm. The plates have surface charge density $\eta = 3.54\ \mu\text{C/m}^2$.
a) What is the potential difference between the two plates?
b) What is the electric potential at 1.0 and 1.5 mm from the negative plate?



$d = 2 \times 10^{-3}$ m

$\eta = 3.54 \times 10^{-6}$ C/m^2

$\eta = \frac{Q}{A}$

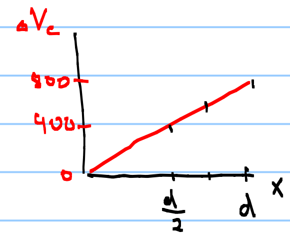
a) $V_c = E \cdot d$

with $E = \frac{\eta}{\varepsilon_0}$

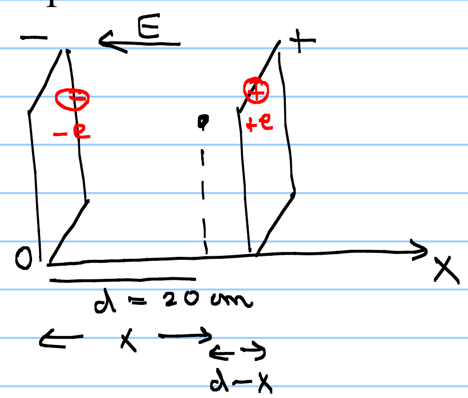
$$V_c = \frac{\eta}{\varepsilon_0} \cdot d = \underbrace{\frac{3.54 \times 10^{-6}}{8.85 \times 10^{-12}}}_{4 \times 10^5\ N/C} \cdot 2 \times 10^{-3} = 800\ V$$

b) $V_c = E\,x = (4 \times 10^5)(1 \times 10^{-3}) = 400\ V \quad \left(= \frac{1}{2} V_c\right)$

$V_c = E\,x = (4 \times 10^5)(1.5 \times 10^{-3}) = 600\,V \quad \left(= \frac{3}{4} V_c\right)$



Example from module 2: Two metal plates are placed 20 cm apart.
An electron is released from rest from the negative plate,
and a proton is released from rest from the positive plate.
Where, calculated from the negative plate, do their paths cross?



for electron

$$x = \cancel{v_{0e}}\,t + \frac{1}{2} a_e t^2$$

$$t^2 = \frac{2x}{a_e} \qquad (1)$$

for proton

$$d - x = \cancel{v_{0p}}\,t + \frac{1}{2} a_p t^2 \qquad (2)$$

$$F_e = F_p = eE = \text{same}$$

$$m_e a_e = m_p a_p \longrightarrow a_p = \frac{m_e}{m_p} a_e \qquad (3)$$

plug (1) and (3) into (2)

$$d - x = \frac{1}{\cancel{2}} \left(\frac{m_e}{m_p} \cancel{a_e} \right) \left(\frac{\cancel{2} x}{\cancel{a_e}} \right)$$

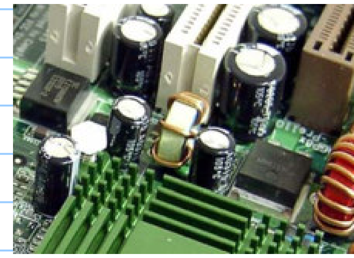
$$d - x = \frac{m_e}{m_p} x$$

$$d = \left(\frac{m_e}{m_p} + 1 \right) x$$

$$x = \frac{d}{\frac{m_e}{m_p} + 1} = \ldots$$

PHY1020, winter 2020
Electric Potential V, Potential Energy Ue and Field E
- part 2 - Problems

1- What is a capacitor?



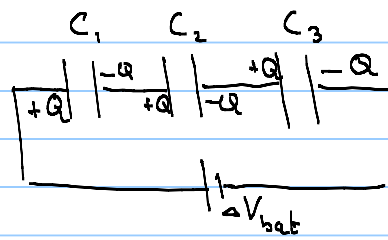
Passive object made of 2 conducting terminals.
They store charge and energy.

2- A capacitor is characterized by its capacitance C. What does large C mean?

$$C = \frac{Q}{\Delta V_c}$$

The larger C, the more charge the capacitor stores under same ΔV_c

3- Three capacitors are connected to a battery as shown :
Capacitor 1 draws a charge Q. What is Q_2 and Q_3?

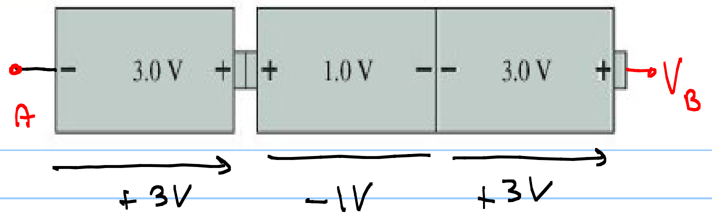


Capacitors in series have the same charge.
$$Q_2 = Q_3 = Q_1 = Q$$

Question 1:

$V_B - V_A$

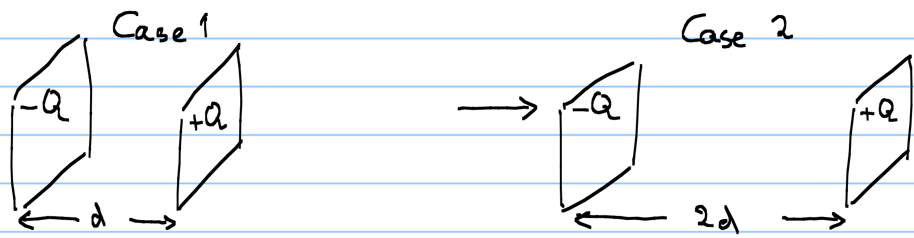
STOP TO THINK 29.1 What total potential difference is created by these three batteries?



A) 7 V
B) 1 V
C) 5 V
D) 3 V

(C) ✓

Question 2: Equal but opposite charges *Q* are placed on the square plates of a parallel-plate capacitor of capacitance *C*, this is referred to case 1.
The plates are then pulled apart to twice their original separation (case 2).



The capacitance C_2 after pulling the plates apart is:
A) $C_2 = 2C_1$ B) $C_2 = C_1 / 2$ C) $C_2 = C_1$

(B) ✓

$C = \frac{\varepsilon_0 A}{d}$

The energy stored in the capacitor U_2 is:
A) $U_2 = 2U_1$ B) $U_2 = U_1 / 2$ C) $U_2 = U_1$

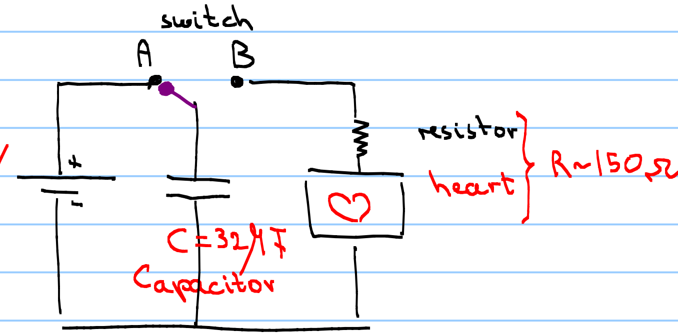
(A) ✓

$U = \frac{1}{2} \frac{Q^2}{C}$

Be careful: If the plates were connected to a battery during the pulling, your answers would differ.

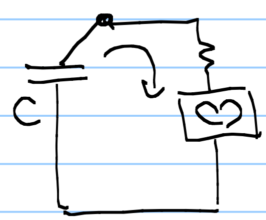
How does a ***defibrillator*** work?
A simplified circuit of a defibrillator (a device used to deliver a strong shock to the heart to regulate its functioning) is shown. The capacitor stores energy while charging, then it discharges quickly through the heart, delivering a strong electric pulse.

Example: In the figure, the switch is in position *A* for a long time as shown.
a) What is the charge on the capacitor?
b) The switch is then switched to position *B*, the heart circle position, and the capacitor discharges its energy to the heart. What is the energy delivered to the body/heart once the capacitor has totally discharged?



a) With switch in position A
the capacitor starts charging, $Q = C\Delta V_c$
Its charge increases until
$\Delta V_c = \Delta V_{bat}$
$Q = C\Delta V_c = C\Delta V_{bat}$
$= (32 \times 10^{-6} F)(5000V) = 0.16\ C$

b) With switch in position B, the capacitor discharges all of its energy through the heart/resistance.



$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{0.16^2}{32 \times 10^{-6}} = 400 J$

Question 3: In the following circuit, which two capacitors are in series?

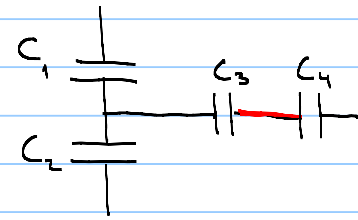
A) C_1 and C_2

B) C_1 and C_3

C) C_2 and C_3

D) C_3 and C_4 (D) ✓

E) both (A) and (D)



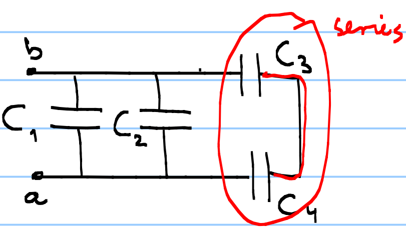
Question 4: All four capacitors have capacitance C, find the equivalent capacitance C_{eq} between points a and b.

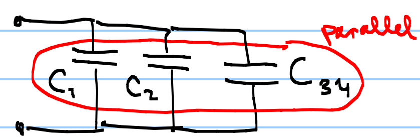
A) 5/2 C (A) ✓

B) 2/5 C

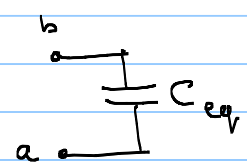
C) 4 C

D) 1/4 C





$$C_{34} = \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \left(\frac{2}{C}\right)^{-1} = \frac{C}{2}$$



$$C_{eq} = C_1 + C_2 + C_{34} = C + C + \frac{C}{2} = 2.5C$$

Example: a) What is the equivalent capacitance across the battery.
b) Find the charge on and the potential difference across each of the four capacitors.

Diagram I

a)

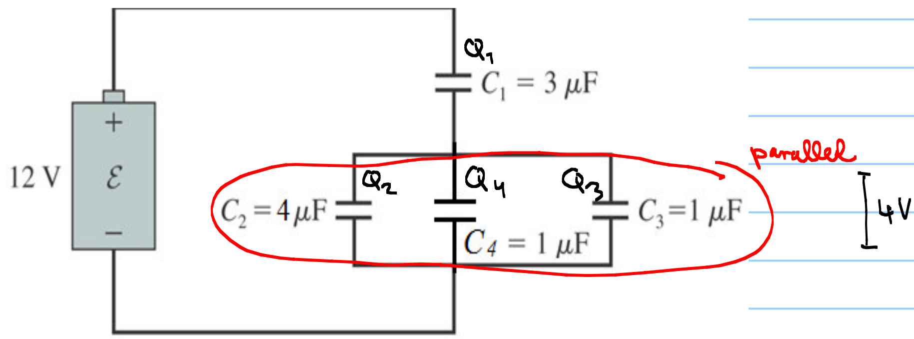
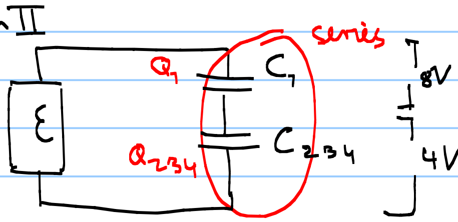
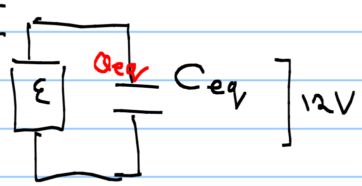


Diagram II



$$C_{234} = C_2 + C_3 + C_4 = 4+1+1 = 6\,\mu F$$

Diagram III



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{234}}\right)^{-1} = \left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = \left(\frac{2+1}{6}\right)^{-1}$$
$$= 2\,\mu F$$

From Diagram III

$$Q_{eq} = C_{eq}\,\mathcal{E} = 2\,\mu F \times 12\,V = 24\,\mu C$$

From Diagram II

series capacitors have the same charge, which is also the same as Q_{eq}

$$Q_1 = Q_{234} = Q_{eq} = \boxed{24\,\mu C = Q_1}$$

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{24\,\mu C}{3\,\mu F} = 8\,V$$

Also $\mathcal{E} = \Delta V_1 + \Delta V_{234}$

$$\Delta V_{234} = \mathcal{E} - \Delta V_1 = 12 - 8 = 4\,V$$

From Diagram I

C_2, C_3, C_4 are in parallel, and have the same potential, which is the same as ΔV_{234}

$$\rightarrow \boxed{\Delta V_2 = \Delta V_3 = \Delta V_4 = 4\,V}$$

$$Q_2 = C_2 \Delta V_2 = 4\mu F \times 4V = \boxed{16\mu C = Q_2}$$

$$Q_3 = C_3 \Delta V_3 = 1\mu F \times 4V = \boxed{4\mu C = Q_3}$$

$$Q_4 = C_4 \Delta V_4 = 1\mu F \times 4V = \boxed{4\mu C = Q_4}$$

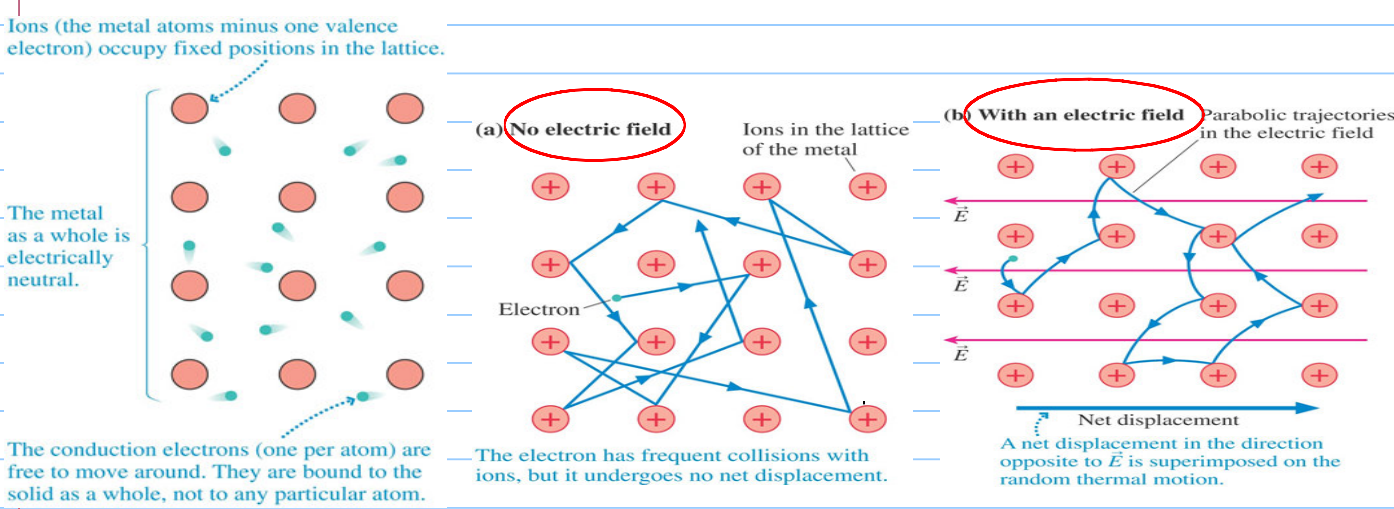
check : $Q_2 + Q_3 + Q_4 = \underbrace{Q_{234}}_{\text{in series with } Q_1}$

$$Q_2 + Q_3 + Q_4 = 24\mu F = Q_1$$

Current and Resistance (chapter 27)
- The electron current i_e and the current I
    - drift velocity - conductivity - resistivity
- The current density J
- Ohm's Law

An electric current flows when an electric field is maintained inside a conductor.
- How does current relate to electric field?
- How does current relate to the potential difference across the conductor?

The Electron Current:
- The electrons in a metal are not sitting idle, but undergo *random thermal motion*. The net velocity is however zero. This is *static equilibrium*.
- When applying an *external electric field* $\vec{E}$, all free electrons start moving together with a net *drift speed* v_d opposite to the direction of $\vec{E}$, like a liquid in a pipe.
- This controlled motion of electric charge is called *the current*.
- The charge carriers in metals are electrons. In semiconductors or solutions they can be different.



- Beside being accelerated by $\vec{E}$, the electrons undergo frequent collisions with the nuclei, which slows them down. If the average time between an electron-nuclei collision is τ, and electrons mass is m, the drift (net) velocity is:

$$\boxed{v_d = \frac{\tau e}{m} E}$$

drift velocity of electrons in field E

Question: Is there a dilemma between Newton's second law, and the fact that the electrons move at constant speed v_d instead of accelerating in the field E?

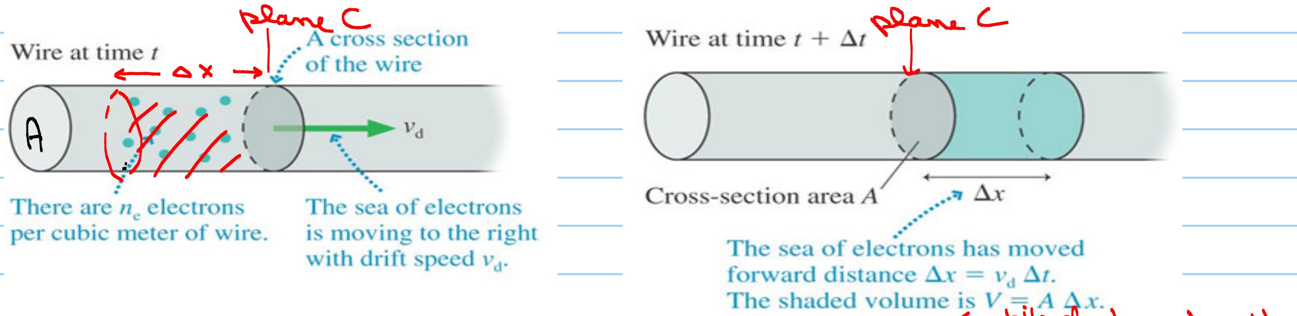
The Electron Current i_e and current I:

- The *electron current i_e* is the *number of electrons per second* that pass through a cross section of a conductor (or wire). [unit s^{-1}]
- The current I is the *rate* at which *charge* moves through a wire.

$$\boxed{I = \frac{dQ}{dt} = e\, i_e}$$ electrical current

where e is the magnitude of the elementary charge (1.6×10^{-19} C).
Unit of current is the Ampere : [1 A = 1 C / s = 1 Coulomb per second].
1 A corresponds to 6.3×10^{18} electrons per sec passing a cross section of the wire.

Consider a conducting wire of cross section area A that has an *electron density* n_e (number of mobile electrons per unit volume). The wire is subject to an electric field, so electrons will move at drift velocity v_d.
Consider a section Δx of the wire, see figure.



- The amount of electrons in that section = n_e . volume = $n_e\, \Delta x\, A$. mobile electron density
- In time $\Delta t = \Delta x / v_d$, all electrons in length Δx will have crossed plane C.
- The electron current through the wire is hence:

$$i_e = \frac{\text{\# of electrons in that volume moving through plane } C}{\text{time it takes the last one to pass}} = \frac{n_e\, \Delta x\, A}{\Delta x / v_d}$$

$$i_e = n_e\, A\, v_d$$

The current $e\, i_e$ is hence:

$$\boxed{I = e\, n_e\, A\, v_d}$$ current

NOTE:

- The direction of the current is actually taken in the opposite direction the electrons move. It is *the direction positive charges would move*, the **same direction of the electric field E**.
- The current *is the same through a given wire*, even if its cross sectional area changes. However the current density may vary.

The current density *J*:

A large current *I* through a thin wire will cause it to melt, while the same *I* through a wide wire will only cause slight heating. It is hence useful to define the current density *J*.

$$\boxed{J = \frac{I}{A}} = \frac{n_e e \cancel{A} v_d}{\cancel{A}} = n_e e \frac{\tau e E}{m} = \frac{n_e e^2 \tau}{m} E$$

$$\boxed{J = \sigma E}$$

current density

Ohm's law in microscopic form

units of current density is A m^{-2}.

where **the conductivity** σ of the material is:

$$\boxed{\sigma = \frac{n_e e^2 \tau}{m}}$$

electrical conductivity of the material

units of conductivity is the ohm^{-1} m^{-1} or Ω^{-1} m^{-1}.

where 1 Ω = 1 V A^{-1}.

The inverse of conductivity is the **resistivity**:

$$\boxed{\rho = \frac{1}{\sigma}} \quad \text{in } \Omega\, m$$

electrical resistivity of the material

- The resistivity depends only on the conducting material, not its shape.
- The resistivity in metals increases with temperature, because the atoms vibrate more heavily, increasing the collisions of the electrons.

TABLE 31.2 Resistivity and conductivity of conducting materials

Material	Resistivity (Ω m)	Conductivity (Ω^{-1} m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires.

How many electrons per unit volume n_e are there in metals? Copper, for example, has a nearest neighbor distance of 2.56×10^{-10} m and one free electron per atom. So, roughly we can expect a density of the order of 10^{28} electrons/m^3.

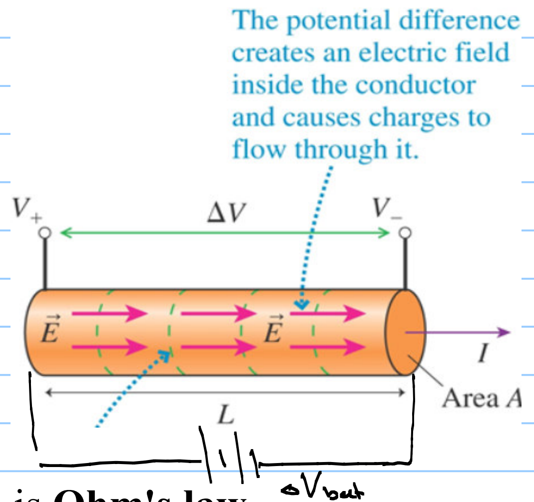
TABLE 31.1 Conduction-electron density in metals n_e

Metal	Electron density (m^{-3})
Aluminum	6.0×10^{28}
Copper	8.5×10^{28}
Iron	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Ohm's law and resistors:

A potential difference ΔV applied to a conductor causes a *constant electric field* inside the wire.

$$\boxed{E = \frac{\Delta V}{L}} \qquad E = -\frac{\partial V}{\partial x}$$



As a result, a current I flows.

$$I = JA = \sigma E A = \sigma \frac{\Delta V}{L} A = \Delta V\left(\frac{\sigma A}{L}\right)$$

$$\boxed{I = \frac{\Delta V}{R}}$$

which is **Ohm's law**

Ohm's law in macroscopic form

It states a linear relation between current and potential difference.

The **resistance** of the conductor:

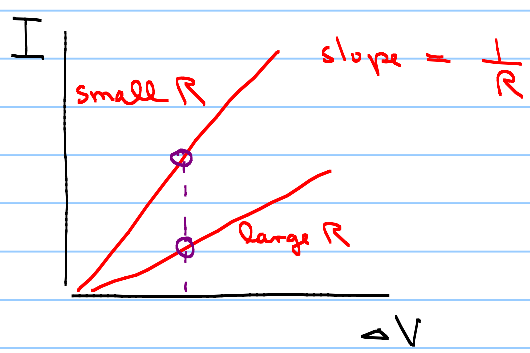
$$\boxed{R = \frac{L}{\sigma A} = \frac{\rho L}{A}}$$

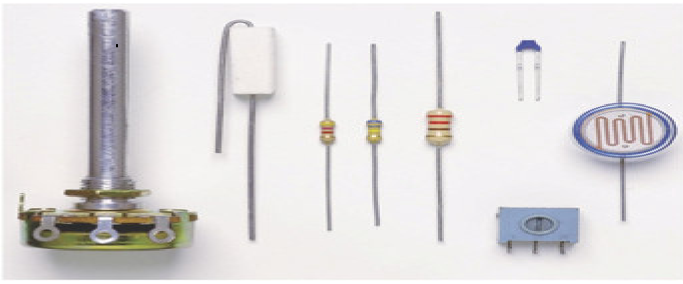
resistance of a conducting wire

where σ is the conductivity and ρ is the resistivity.

[units of R is the ohm (Ω): $1\ \Omega = 1\ \mathrm{V\ A^{-1}}$].

R depends both on the material (ρ) and shape (length L, area A).





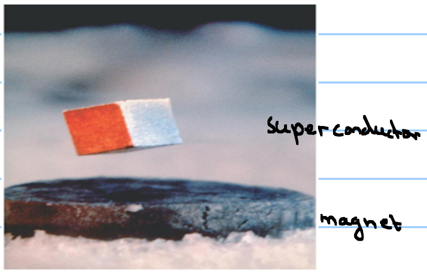
various shapes of resistors (conductors)

Note: *NOT* all materials obey Ohm's

In a circuit diagram a resistance is shown as: 

A word on Superconductivity

In 1911 K. Onnes studied the conductivity of mercury at very low temperatures. He expected its resistivity to go down with decreasing temperature. What he found instead is a sudden and complete loss of resistance when cooled below 4.2 K. This is great! Charge will continue moving through the 'frictionless' superconductor without an electric field.



Many applications: especially where otherwise there would be large dissipation of heat, for example in superconducting electromagnets.

Current and Resistance (chapter 27)

1- Electrons randomly move around at extremely high speeds (thermal motion). How does the electron current differ from this random motion?

$v_{thermal} \sim 10^6$ m/s , $v_{drift} = 10^{-4}$ m/s

↓ net velocity = 0 ↓ net velocity is opposite to electric field E

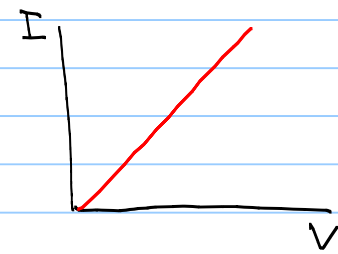
Temperature is a measure of thermal motion I is a measure of v drift .

2- If you connect a resistor of length *L* between the positive and negative poles of a battery, an *electric field is set up* inside the resistor *E* = *ΔV* / *L*, hence charges are accelerated. Why then is the current through the circuit constant and not increasing constantly? Rephrased, why is the electron drift velocity constant?

Electrons accelerate between collisions with the nuclei , these collisions dampen the motion → we observe a constant v_{drift}.

3- What is a Ohmic material. Mention 2 materials that are non-ohmic.

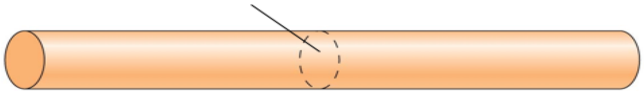
$R = \dfrac{\Delta V}{I}$ is constant over a range of ΔV



Air and Semiconductors and superconductors

Question 1:

Every minute, 120 C of charge flow through this cross section of the wire.



The wire's current is

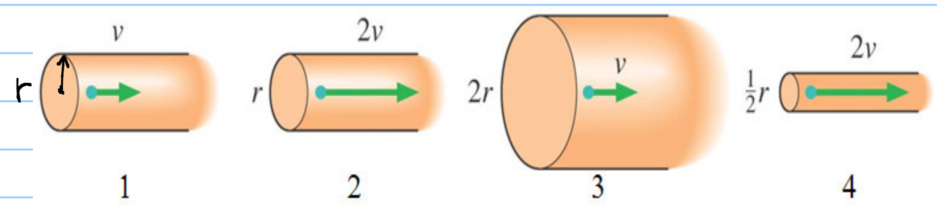
A. 240 A
B. 120 A
C. 60 A
D. 2 A (D)
E. Some other value.

constant I

$$I = \frac{dQ}{dt} \equiv \frac{\Delta Q}{\Delta t} = \frac{120\ C}{(1 \times 60)\ s}$$

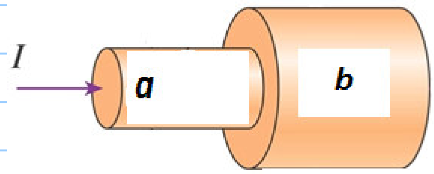
Question 2: Rank in order, from largest to smallest, the current.

A) (2), (4), (3), (1)
B) (2)=(4), (1)=(3)
C) (3), (2), (1), (4)



Question 3: A current I_a flows through section *a* of the wire. Compare currents *I* and current densities *J* in both sections *a* and *b*.

A) $I_b > I_a$ and $J_b > J_a$
B) $I_b < I_a$ and $J_b < J_a$
C) $I_b = I_a$ and $J_b > J_a$
D) $I_b = I_a$ and $J_b < J_a$ (D)
E) none of the above

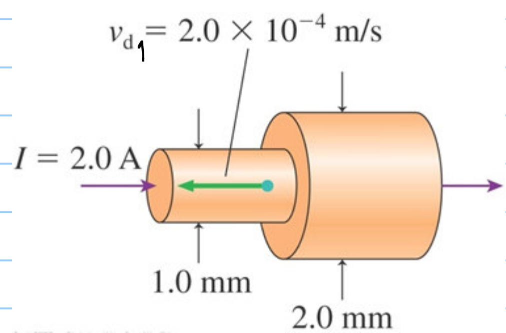


$J = \frac{I}{A}$

The rate at which charge flows (current) is everywhere the same.

Question 4: The two wires below are made of the same material. What is the electron drift speed in the 2.0 mm diameter segment of the wire?

A) 2.0 x 10^{-4} m/s
B) 8.0 x 10^{-4} m/s
C) 1.0 x 10^{-4} m/s
D) 5.0 x 10^{-5} m/s (D) ✓



segment 1 segment 2

$I = n_e e A v_d$

$I_1 = I_2$

$\cancel{n_e} \cancel{e} A_1 v_{d-1} = \cancel{n_e} \cancel{e} A_2 v_{d-2}$

$v_{d-2} = \frac{A_1}{A_2} v_{d-1}$

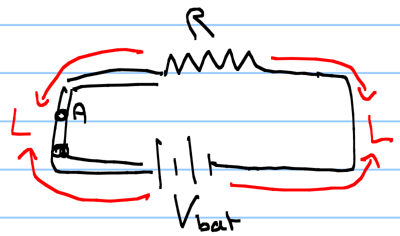
$= \frac{\cancel{\pi} r_1^2}{\cancel{\pi} r_2^2} v_{d-1} = \frac{1}{4} v_{d-1}$

radius r_1, r_2

$r_1 = \frac{1\,mm}{2} = 0.5 \times 10^{-3}\,m$; $A_1 = \pi r_1^2$

$r_2 = \frac{2\,mm}{2} = 1 \times 10^{-3}\,m$; $A_2 = \pi r_2^2$

Example: In an electric circuit, a 100 ohm resistor is connected through two 1.5 m long, 1.0 mm in diameter copper wires to a battery [resistivity of copper = 1.7 x 10^{-8} ohm.m].
a) What is the resistance of each copper wire? Compare it to that of the resistor.
b) If the battery output is 10 V, what is the current through the resistor?



$R = 100\,\Omega$

$$\begin{cases} L = 1.5\,m \\ \underset{radius}{r} = \frac{1}{2}\,mm = 0.5 \times 10^{-3}\,m \\ \rho = 1.7 \times 10^{-8}\,\Omega m \end{cases}$$

$V_{bat} = 10\,V$

a) $$R_{wire} = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$$= 0.033\,\Omega$$

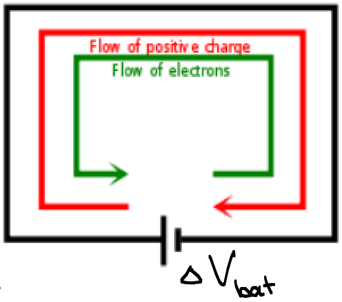
2 wires $\rightarrow R_{2wires} = 2R_{wire} = 0.066\,\Omega \ll R = 100\,\Omega$

b) Ohm's law

$$I = \frac{\Delta V}{R_{total}} = \frac{V_{bat}}{R + \underbrace{R_{2wires}}_{neglect}} = \frac{10\,V}{100\,\Omega} = 0.1\,A$$

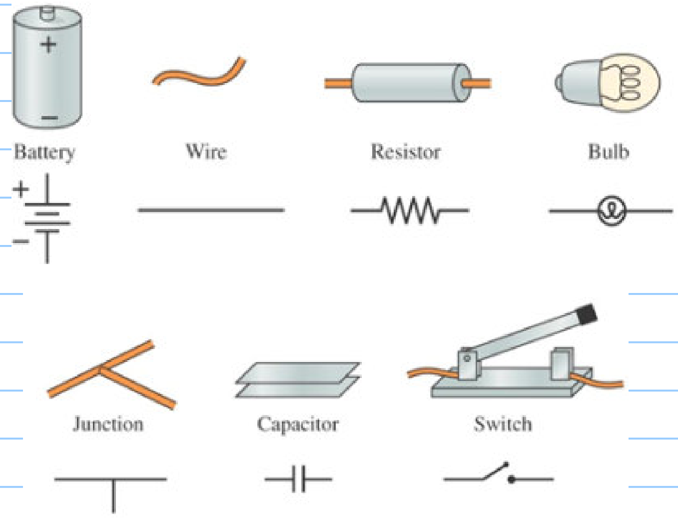
Fundamentals of Circuits (chapter 28)
- Kirchoff's laws
- Resistors in series and in parallel
- Power dissipated in a resistance and power delivered by battery
- RC-circuits (charging and discharging capacitors)

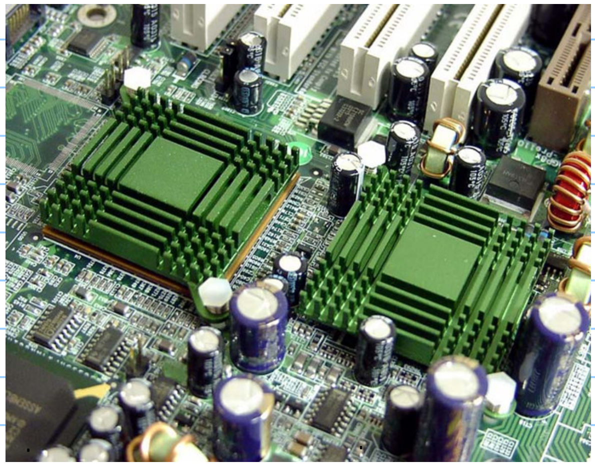
Basic Circuits and Kirchoff's Laws:
Batteries, resistors, wires and potential difference:
- A battery is a source of potential difference ΔV_{bat}, ideally (if the internal resistance can be neglected) the emf ε.
- If connected to a resistor (or resistive wire) of length L, then $\Delta V_R = \Delta V_{bat}$.
- ΔV_R causes an electric field $E = \Delta V_R / L$ in the resistor.
- E establishes a current $I = JA = \sigma AE$ in the resistor.
- The current describes the motion of positive charge carriers.



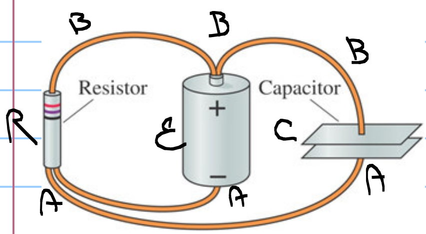
- A resistor is a poor conductor (large R). A wire (metal) is a very good conductor.
- Ideally in a circuit with resistors $R_{resistor}$, R_{wire} is very small (~0) compared to $R_{resistor}$, so that ΔV_{wire} is assumed to be zero.

Some common circuit components are:

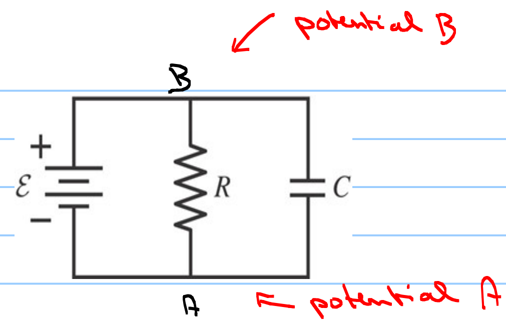




Very nice online circuit simulator from the university of colorado: the circuit construction icon at:
http://phet.colorado.edu/en/simulations/category/physics/electricity-magnets-and-circuits



= equivalent to

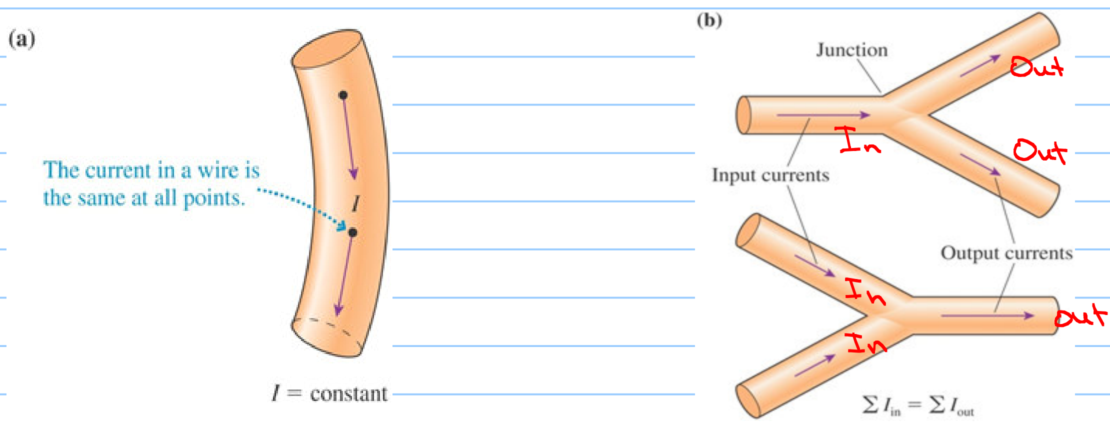


Kirchhoff's laws:

Kirchoff's junction law: The total current into the junction must equal the total current leaving the junction.

$$\boxed{\sum I_{in} = \sum I_{out}}$$

Kirchoff 's junction law

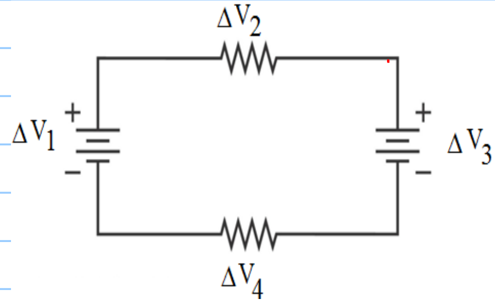


Kirchoff 's loop law: The sum of the potential differences around any loop (closed path) is zero.

$$\boxed{\Delta V_{loop} = \sum_i (\Delta V_i) = 0}$$

Kirchoff 's loop law

Remember ΔV_{wire} is usually taken to be zero, because $R_{wire} << R_{circuit}$.



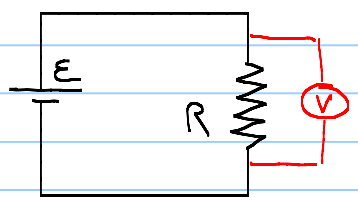
$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

Voltmeters and Ammeters:

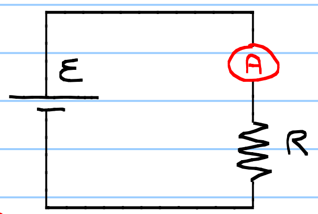
You want to place a voltmeter or an ammeter inside the circuit *without* changing the *potential difference* across or *current* through any component.

A voltmeter is placed in parallel with the resistor and is chosen with much higher resistance than that of the resistor measured.

An ammeter is placed in series with the resistor and is chosen with much lower resistance than that of the resistor measured.



$R_{voltmeter}$ must be $\gg R$



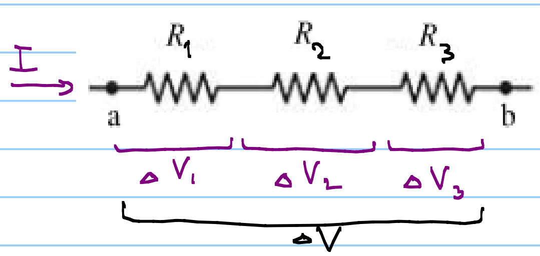
$R_{voltmeter}$ must be $\ll R$

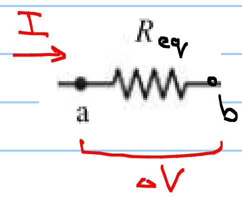
Series and Parallel Resistors:

Resistors in series have the same current.

The equivalent resistor will have that same current.

The potential difference ΔV on the equivalent resistor is the sum of ΔV_i.





$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$
$$= I R_1 + I R_2 + I R_3$$
$$\cancel{I} R_{eq} = \cancel{I} (R_1 + R_2 + R_3)$$

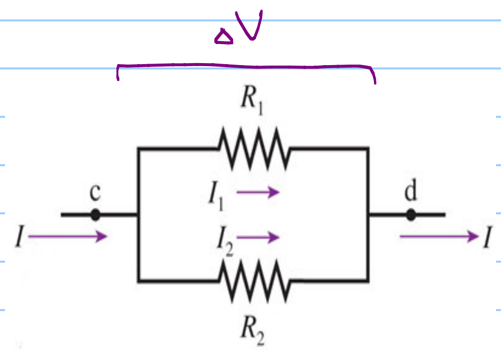
$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots}$$

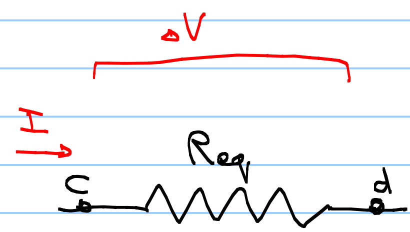
resistors in series

Resistors in parallel have the same potential difference.

The equivalent resistor will have that same potential difference.

The current I through the equivalent resistor is the sum I_i.





$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$
$$\frac{\Delta V}{R_{eq}} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

$$\boxed{R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \right)^{-1}}$$

resistors in parallel

Power:

1- The *power* ***delivered by an emf*** is the potential energy transferred to the charges per unit time as it moves them up the charge escalator.

$$\boxed{P = \frac{dU}{dt}} = \frac{d}{dt}(q\mathcal{E}) = \mathcal{E}\frac{dq}{dt}$$

constant for a battery

$$\boxed{P_{emf} = I\mathcal{E}}$$

2- As the electrons move through the resistor potential energy is transferred into kinetic energy, and as they collide with the atoms kinetic energy is transferred into thermal energy heating up the resistor.

The *power dissipated* in the resistor is $\boxed{P = I\,\Delta V_R}$ and use Ohm's law $\Delta V_R = IR$

$$= I^2 R = \frac{\Delta V_R^2}{R}$$

3- $P_{emf} = \sum_i P_i$ *(all resistors in circuit)*.

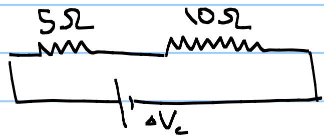
Units of power is the Watt [1W = 1 J / s = 1 V A].

Test yourself: Which resistor dissipates more power?

a) The 5 ohm resistor.

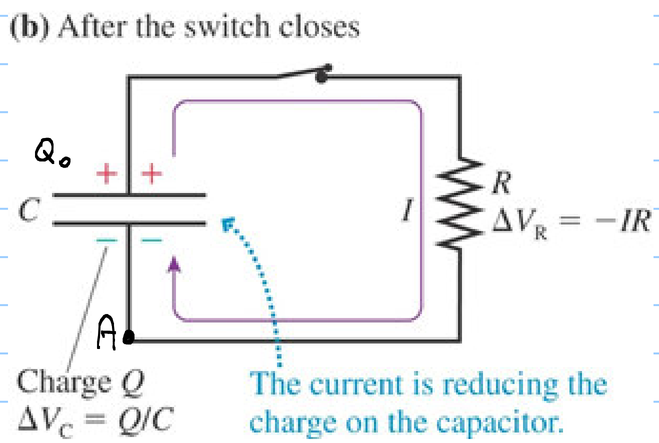
b) The 10 ohm resistor. (b) ✓ $P = I^2 R$

at same I, P is proportional to R



RC Circuits:
A capacitor is a device for storing charge.
It acts like a ***temporary battery*** once charged.
So, when connetcted to a resistor, a current starts to flow and decays gradually.

Discharging a capacitor:
A capacitor initially charged with charge Q_0 is connected at time $t = 0$ to a resistor (resistance R) and discharges.
1- Find the *charge* Q as a function of time:



Apply Kirchoff's loop law. Starting at A and going clockwise:

$$\Delta V_c + \Delta V_R = 0$$

$$\frac{Q}{C} - IR = 0 \qquad I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

$$\frac{dQ}{Q} = -\frac{1}{RC}\,dt$$

$$\int_{Q_0}^{Q}\frac{dQ}{Q} = \int -\frac{1}{RC}\,dt = -\frac{1}{RC}\int_0^t dt$$

$$\ln Q\Big|_{Q_0}^{Q} = -\frac{1}{RC}\,t\Big|_0^t$$

$$\ln Q - \ln Q_0 = \frac{-1}{RC}(t-0)$$

$$\ln\left(\frac{Q}{Q_0}\right) = \frac{-1}{RC}t$$

$$\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$$

$$\boxed{\begin{aligned} Q &= Q_0 e^{-t/RC} \\ Q &= Q_0 e^{-t/\tau} \end{aligned}}$$ discharging a capacitor in an RC circuit

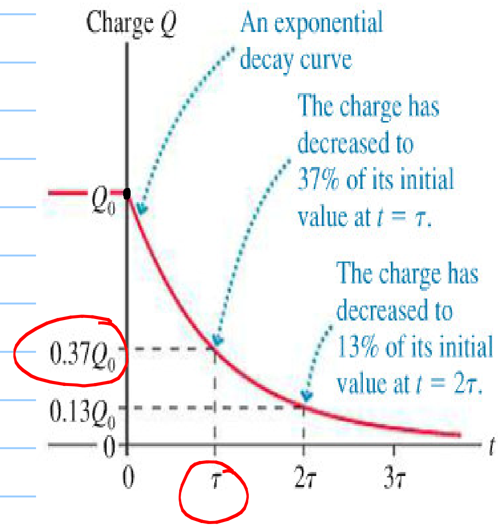
with $\boxed{\tau = RC}$ time constant of the RC circuit

Meaning of time constant:

when t reaches τ

$$Q = Q_0 e^{-1} = \frac{Q_0}{e} = 0.37\, Q_0$$

Q decreases to about $\frac{1}{3}$ of its initial value Q_0



2- Find the *current* during discharge:

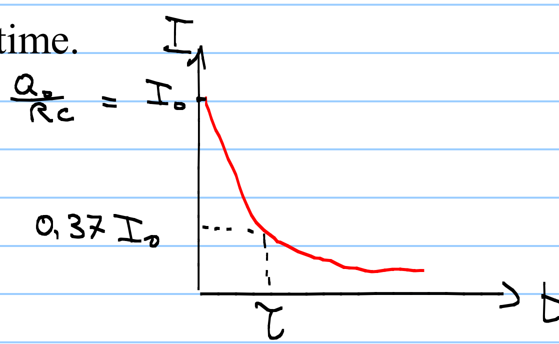
$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\left(Q_0 e^{-t/RC}\right)$$

$$= -Q_0\left(-\frac{1}{RC} e^{-t/RC}\right) = \underbrace{\frac{Q_0}{RC}}_{I_{max}} e^{-t/RC}$$

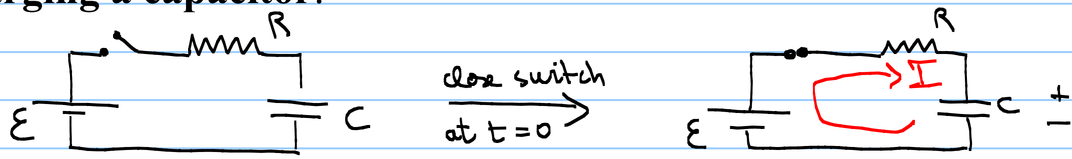
$$= I_{max}\, e^{-t/RC}$$

with $I_{max} = \frac{Q_0}{RC}$ (max. current at $t = 0$)

Both I and Q decrease exponentially with time.



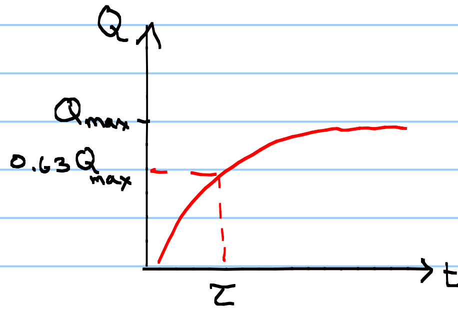
Charging a capacitor:



You can similarly show that charging a capacitor follows the equation:

$$\boxed{Q = Q_{max}\left(1 - e^{-t/RC}\right)}$$

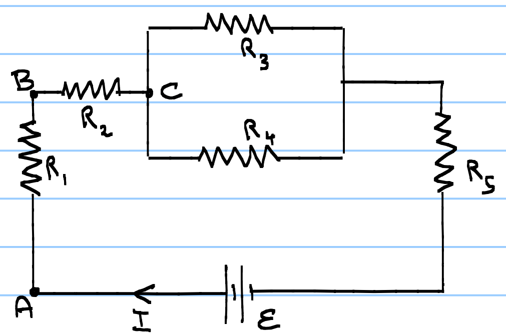
where $Q_{max} = C\,\mathcal{E}$

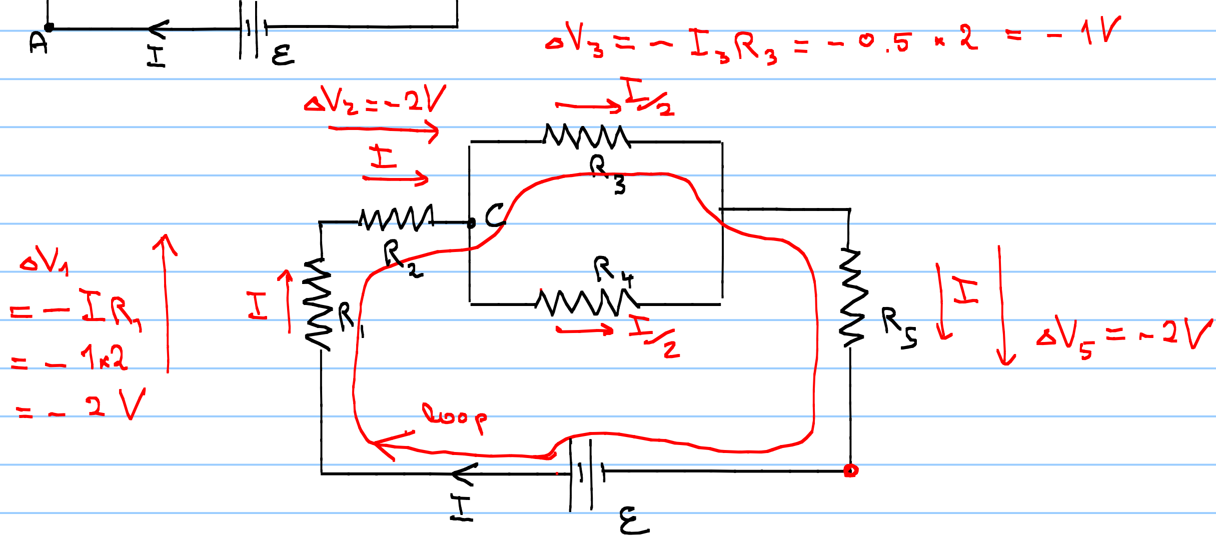


Fundamentals of Circuits (chapter 28)

1- In the circuit shown, all resistances have the same resistance $R = 2.0$ ohm and the current $I = 1.0$ A.

- Apply Kirchoff 's junction rule at point C to find I_3.
- Apply Kirchoff 's loop rule to find the battery's voltage ΔV_{bat} (let's call it ε).





- At junction C : Apply kirchoff's junction law

$$I_2 = I_3 + I_4$$
$$I = 2 I_3 \longrightarrow I_3 = \frac{I}{2}$$

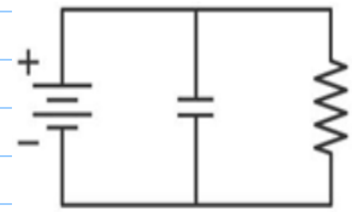
- Loop Law applied on loop : (going clockwise)

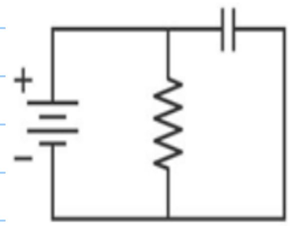
$$\sum_i \Delta V_i = 0$$
$$\varepsilon + \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_5 = 0$$
$$\varepsilon - 2 - 2 - 1 - 2 = 0 \longrightarrow \varepsilon = 7\ V$$

2- Resistors in series have same ....I....... (I, ΔV, R).

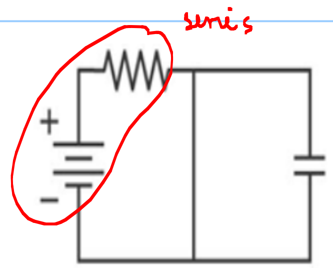
Resistors in parallel have same ....V....... (I, ΔV, R).

Question 1: Which of the circuits in the second row are equivalent to the circuit in the first row?

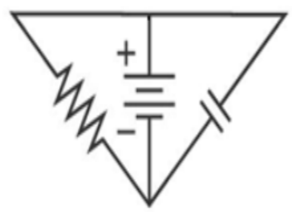




(1)



(2)

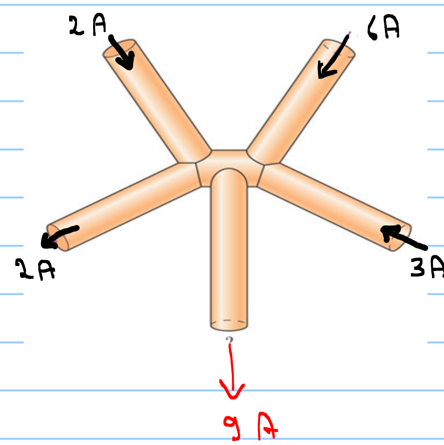


(3)

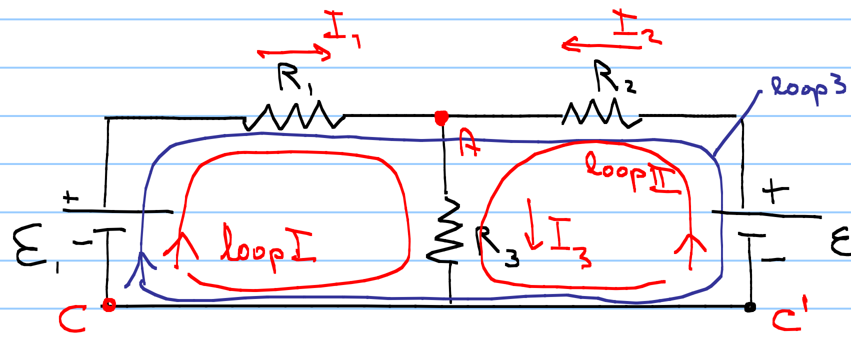
A) only (1)
B) only (2)
C) only (3)
D) (1) and (2)
E) (1) and (3) (E) ✓
F) (2) and (3)
G) all three

Question 2: What are the magnitudes and direction of the current in the fifth wire.

A) 9 A in
B) 9 A out (B)
C) 2 A in
D) 2 A out
E) 3 A in
F) 3 A out



Example: In the circuit shown the *emf* of the ideal batteries is $\varepsilon_1 = 3.0$ V and $\varepsilon_2 = 2.0$ V. They are connected to the 5.0 ohm resistors as shown. Find the current and the potential difference across each resistor.



$\mathcal{E}_1 = 3.0V$
$\mathcal{E}_2 = 2.0V$
$R_1 = R_2 = R_3 = 5.0\Omega = R$

Also
$+\mathcal{E}_1 - I_1R_1 + I_2R_2 - \mathcal{E}_2 = 0$

Apply both Kirchoff's loop law and junction law:
To use Kirchoff's *loop law*:
1- Assign a cw or ccw direction in each loop.
2- Assign a current direction in each resistor.
2- Travel around the loop in the direction you chose.
   For a battery, going from – to +, $\Delta V = +\varepsilon$
   For a battery, going from + to –, $\Delta V = -\varepsilon$
   For a resistor, going along the current, $\Delta V = -IR$
   For a resistor, going opposit the current, $\Delta V = +IR$
3- Apply Kirchoff's loop law.

Apply Kirchoff's loop law around loop I
Starting at C going cw in loop I

$$+\mathcal{E}_1 - I_1R_1 - I_3R_3 = 0$$

same $R_i = R$

$$I_3 = \frac{\mathcal{E}_1 - I_1R_1}{R_3} = \frac{\mathcal{E}_1}{R} - I_1 \qquad (1)$$

Loop law around loop II
Starting at C' and going ccw
$$+\mathcal{E}_2 - I_2R_2 - I_3R_3 = 0$$

$$I_2 = \frac{\mathcal{E}_2 - I_3R_3}{R_2} = \frac{\mathcal{E}_2}{R} - I_3 = \frac{\mathcal{E}_2}{R} - \left(\frac{\mathcal{E}_1}{R} - I_1\right) = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R} + I_1 \qquad (2)$$

At junction A apply junction law.
$$I_1 + I_2 = I_3$$

$$I_1 = I_3 - I_2 \qquad (3)$$

from (1) and (2)

$$I_1 = \frac{\mathcal{E}_1}{R} - I_1 - \left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{R} + I_1\right)$$

$$= \frac{2\mathcal{E}_1 - \mathcal{E}_2}{R} - 2I_1$$

$$3I_1 = \frac{2\mathcal{E}_1 - \mathcal{E}_2}{R}$$

$$\boxed{I_1 = \frac{2\mathcal{E}_1 - \mathcal{E}_2}{3R} = \frac{2 \times 3 - 2}{3 \times 5} = \frac{4}{15} = 0.267\ A = I_1} \qquad (4)$$

$$\Delta V_1 = I_1 R_1 = \frac{4}{15} \times 5 = \frac{4}{3} = 1.33\ V$$

Solve for I_2 from (2) and (4)

$$I_2 = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R} + I_1 = \frac{2-3}{5} + \frac{4}{15} = \frac{-3+4}{15} = \frac{1}{15}\ A = 0.067\ A$$

$$\Delta V_2 = I_2 R_2 = \frac{1}{3}\ V = 0.333\ V$$

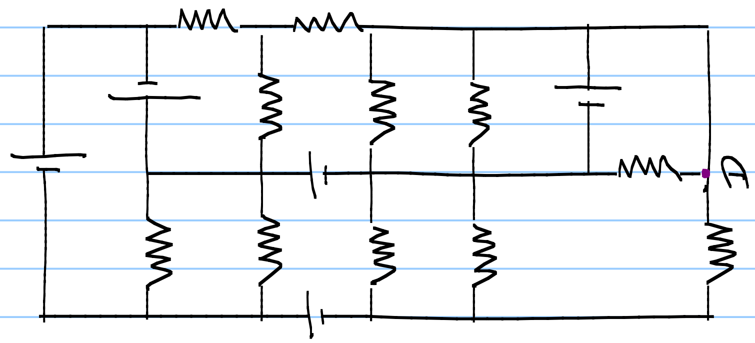
Similarly find I_3 from (1) ΔV_3.

$$\rightarrow I_3 = \frac{1}{3} = 0.33\ V$$

$$\Delta V_3 = 1.67\ V$$

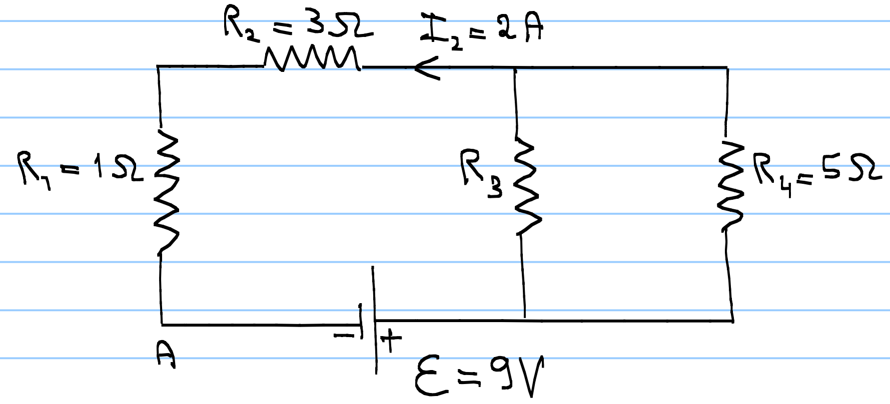
Challenge Problem:

In the circuit each resistor is 2 ohms and each battery is 5 V. What are the three currents at junction A?



Question 3: What is $|\Delta V_3|$ in the following circuit?

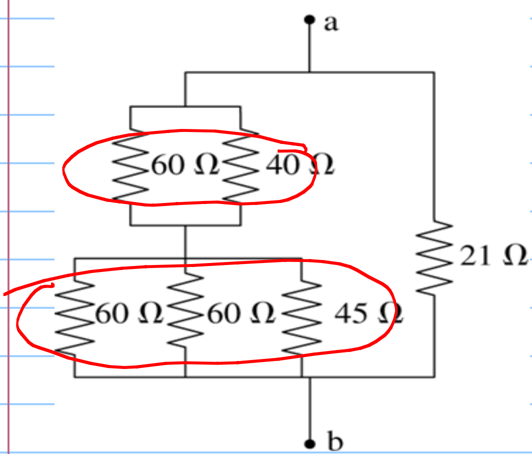
A) 9 V
B) 17 V
C) 1 V
D) 8 V



Question 4: What is the current in the R_4 resistance in the previous circuit?

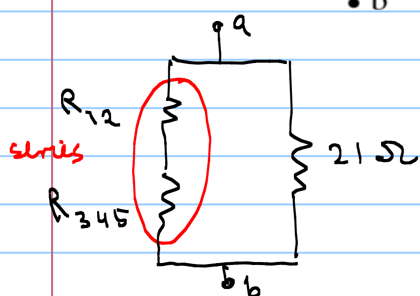
A) 2 A, up
B) 2 A, down
C) 0.2 A, up
D) 0.2 A, down

Question 5: Find the equivalent resistance between points *a* and *b*.



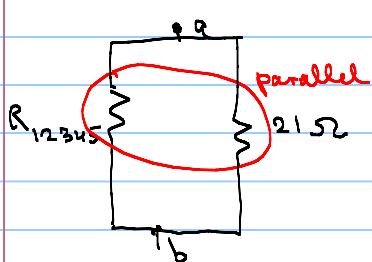
A) 42 ohm
B) 14 ohm
C) 20 ohm

(B) ✓

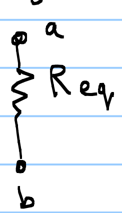


with $R_{12} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{60} + \frac{1}{40}\right)^{-1} = 24\,\Omega$

$R_{345} = \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{60} + \frac{1}{60} + \frac{1}{45}\right)^{-1} = 18\,\Omega$



with $R_{12345} = 24 + 18 = 42\,\Omega$



with $R_{eq} = \left(\frac{1}{R_{12345}} + \frac{1}{R_6}\right)^{-1} = \left(\frac{1}{42} + \frac{1}{21}\right)^{-1} = 14\,\Omega$

Example: Find the equivalent resistance of the circuit. Also find *I* and *V* in every resistor.

Diagram I

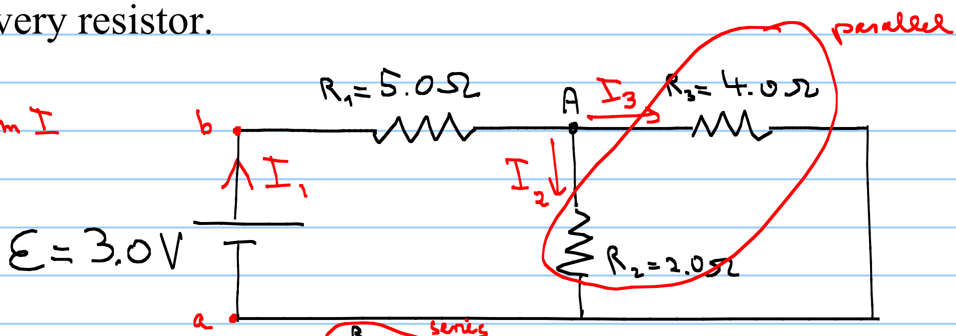
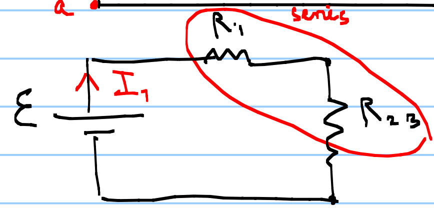
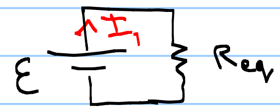


Diagram II



with $R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{4}\right)^{-1} = \left(\frac{3}{4}\right)^{-1} = \frac{4}{3}\,\Omega = 1.33\,\Omega$

Diagram III



with $R_{eq} = R_1 + R_{23} = 5 + \frac{4}{3} = \frac{15+4}{3} = \frac{19}{3} = 6.33\,\Omega$

In Diagram III Ohm's law

$$I_1 = \frac{\Delta V_{Req}}{R_{eq}} = \frac{\mathcal{E}}{R_{eq}} = \frac{3V}{\frac{19}{3}\,\Omega} = \frac{9}{19}\,A = 0.474\,A$$

In Diagram II

$$\Delta V_1 = I_1 R_1 = \frac{9}{19} \times 5 = \frac{45}{19} = 2.37\,V$$

$$\Delta V_{23} = I_1 R_{23} = \frac{9}{19} \times \frac{4}{3} = \frac{12}{19}\,V = 0.632\,V$$

Now $\Delta V_2 = \Delta V_3 = \Delta V_{23} = \frac{12}{19}\,V$ parallel resistors and their equivalent have the same potential difference

In Diagram I

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{12}{19} \times \frac{1}{2} = \frac{6}{19} = 0.32\,A$$

$$I_3 = \frac{\Delta V_3}{R_3} = \frac{12}{19} \times \frac{1}{4} = \frac{3}{19} = 0.16\,A$$

Check: Kirchoff's junction law at A: $I_1 = I_2 + I_3$; $\frac{9}{19}\,A = \frac{6}{19} + \frac{3}{19}$ ✓

Check: " loop law in left branch:

$$\mathcal{E} - I_1 R_1 - I_2 R_2 = 0 \;;\; 3 - \frac{45}{19} - \frac{12}{19} = 0$$ ✓

Homework: Solve the same problem using Kirchoff 's laws.

Very nice online circuit simulator from the university of colorado: the circuit construction icon at:
http://phet.colorado.edu/en/simulations/category/physics/electricity-magnets-and-circuits

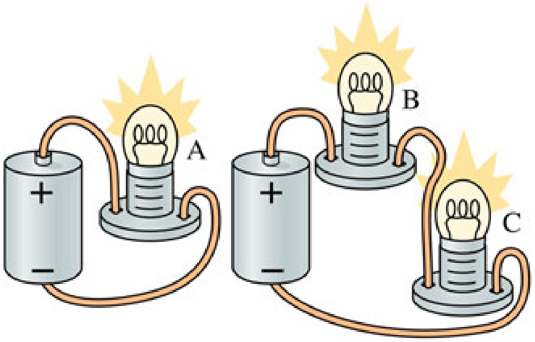
Question 6: The three bulbs are identical and the two batteries are identical. Compare the brightnessess of the bulbs. Hint: Brightness increases with current.

A) A > B > C
B) A > C > B
C) A > B = C
D) A < B = C
E) A = B = C

(C) ✓

$R_{BC} = 2R_B > R_A$

$\rightarrow I_A > I_{BC}$



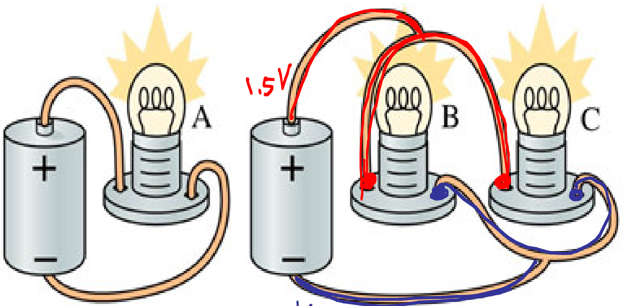
ΔV batteries = 3 V combined

	ΔV on each bulb (V)	I in each bulb (A)	P dissipated in each bulb (W) V × I	P total (W)
number of bulbs				
1	2.6	0.23	0.60	0.60
2 series	1.4	0.17	0.24	0.48
2 parallel	2.4	0.23	0.55	1.1

Question 7: The three bulbs are identical and the two batteries are identical. Compare the brightnessess of the bulbs.

A) A > B > C
B) A > C > B
C) A > B = C
D) A < B = C
E) A = B = C

(E) ✓

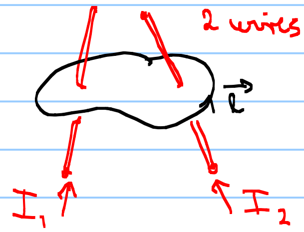


$\Delta V_A = \Delta V_B = \Delta V_c = \mathcal{E}$

$I = \frac{\Delta V}{R}$ ← same on each bulb; R ← same

2 - Ampere's Law, 3 - The Magnetic Force

- For the shown path around loop *l* find the closed line integrals:



$$\oint \vec{E} \cdot d\vec{l} = V_A - V_A = 0 \quad \text{potential}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{through} = \mu_0 (I_1 + I_2)$$

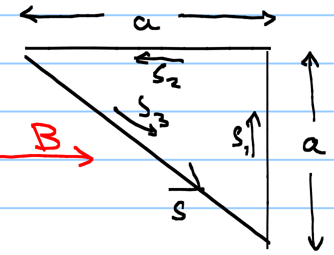
- A charge is moving inside a magnetic field and experiences a magnetic force F_B.
    - Does the charge accelerate inside that magnetic field?

    - Does it change its speed inside a magnetic field?

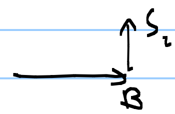
Question: A 2.0 T magnetic field points everywhere along the *x*-direction. Length *a* in the figure = 3.0 m. The line integral of $\vec{B}$ along the path $\vec{s}$ shown is:

A) 12 T.m
B) -12 T.m
C) 8.5 T.m
D) 0

(D) ✓



along path S_1 : $\int \vec{B}\cdot d\vec{s}_1 = 0$

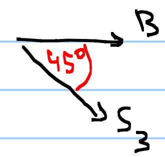


along path S_2 : $\int \vec{B}\cdot d\vec{s}_2 = \int B\, dS_2 \cos\theta$

$$= B\cos\theta \int dS_2 = B S_2 \cos\theta$$
$$= B\ a\ \underbrace{\cos 180^\circ}_{-1} = -Ba$$

along path S_3: $\int \vec{B}\cdot d\vec{s}_3 = B\cos\theta\, S_3$

$$= B\cos(45^\circ)\sqrt{a^2+a^2}$$
$$= B\,\frac{1}{\cancel{\sqrt{2}}}\,a\cancel{\sqrt{2}} = Ba$$



$$\oint \vec{B}\cdot d\vec{s} = \int \vec{B}\cdot d\vec{s}_1 + \int \vec{B}\cdot d\vec{s}_2 + \int \vec{B}\cdot d\vec{s}_3 = 0$$

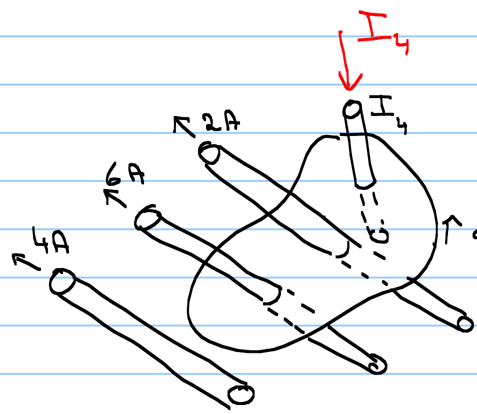
Question: The line integral $\oint \vec{B}\cdot d\vec{s}$ along the path shown is $2\pi \times 10^{-6}$ T m. The current I_4 in the far right wire is:

A) -3 A
B) -7 A
C) +7 A
D) -12 A
[Hint: $\mu_o = 4\pi .10^{-7}$]

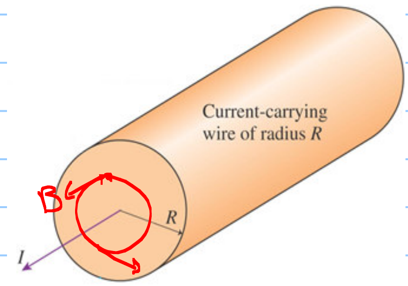
(A) ✓

$$\oint \vec{B}\cdot d\vec{s} = \mu_o I_{through}$$

$$\cancel{2\pi} \times 10^{-6} = \overset{2}{\cancel{4\pi}} \times 10^{-7}(6A + 2A + I_4)$$
$$5 = 8 + I_4$$
$$I_4 = -3A$$



Example- A wire of radius R carries current I.
Find the magnetic field: (a) inside the wire at $r < R$ from the axis.
(b) outside the wire at distance $r > R$.

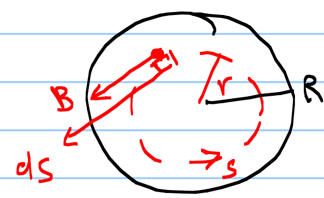


We will use Ampere's law.

a) At $r < R$

1- From symmetry considerations, B of a wire is *constant* along circles around the wire and points tangent to the circles. Hence choose a concentric circle ($r < R$) as your Amperian loop of integration, so B is parallel to *ds*.

2- Along the loop calculate the line integral of B.



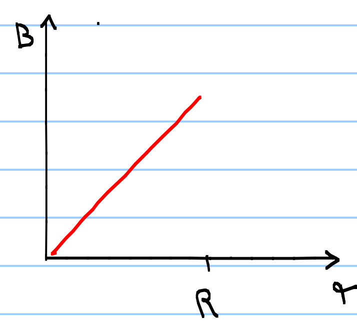
$$\oint \vec{B}\cdot\vec{ds} = \oint B\, ds \cos\theta = \oint B\, ds \underbrace{\cos\theta}_{1} = B\int ds$$
$$= B\,S = B\, 2\pi r$$

3- Inside that Amperian loop find $I_{through}$.

$$\frac{I_{through}}{A_{loop}} = \frac{I_{wire}}{A_{wire}} \quad (= J \leftarrow \text{the current density})$$

$$I_{through} = (I_{wire})\left(\frac{A_{loop}}{A_{wire}}\right) = I\,\frac{\pi r^2}{\pi R^2} = I\,\frac{r^2}{R^2}$$

4- Apply Ampere's law.



$$\oint \vec{B}\cdot\vec{ds} = \mu_0 I_{through}$$

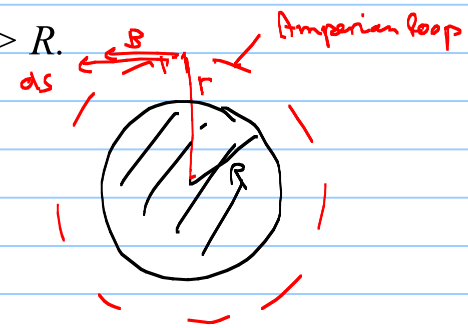
$$B\, 2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{R^2}$$

$$B = \frac{\mu_0 I}{2\pi R^2}\, r.$$

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi R^2}\, r, \ \text{tangent to circles around the wire in ccw direction}\right)$$

b) At $r > R$

Choose an Amperian loop outside the wire of radius $r > R$.



Similarly

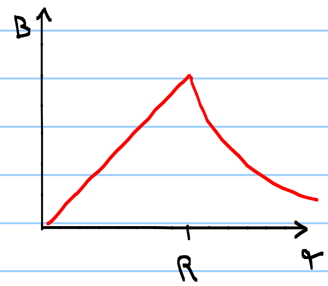
$$\oint \vec{B} \cdot d\vec{l} = B\, 2\pi r$$

$$I_{through} = I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B\, 2\pi r = \mu_0 I$$

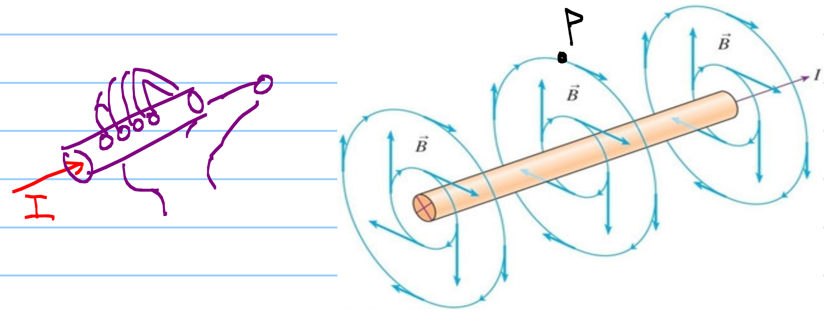
$$B = \frac{\mu_0 I}{2\pi r}$$



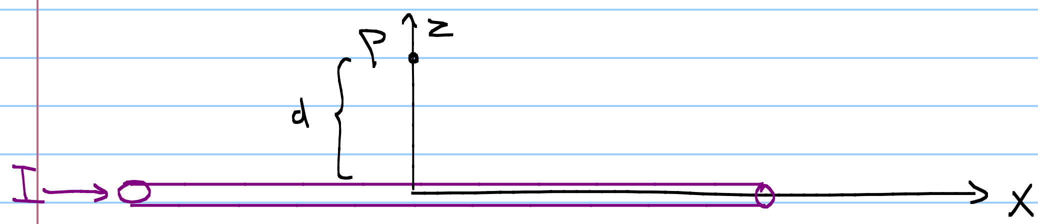
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

B outside a long current-carrying wire

(direction of *B* by right hand rule - tangent to a circle around wire through point of interest).



Example: A long wire carries a 10 A current from left to right. When an electron is 1.0 cm above the wire traveling to the right, its speed is 1.0×10^7 m/s. What are the magnitude and direction of the magnetic force on the electron? (Hint: Using Ampere's law we found that B of a long wire is $\mu_o I / (2\pi d)$).



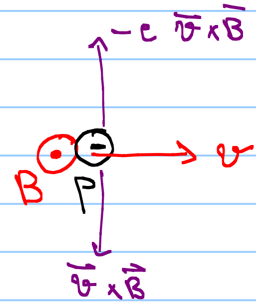
$I = 10\ A$
$d = 1.0 \times 10^{-2}\ m$
$v = 1.0 \times 10^{7}\ m/s$

① Find $\vec{B}$ at point P

$$B = \frac{\mu_o I}{2\pi d} = \frac{(\cancel{4}^{2}\cancel{\pi} \times 10^{-7})(10)}{\cancel{2\pi}(1 \times 10^{-2})} = 2 \times 10^{-4}\ T$$

$\vec{B} = (2 \times 10^{-4}\ T,$ out of the page$)$

② Find $\vec{F}$ on the electron that is moving through point P



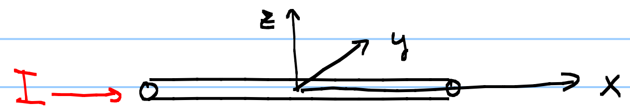
$$\vec{F} = q\, \vec{v} \times \vec{B}$$
$$F = |q|\,|\vec{v} \times \vec{B}| = |-e|\,|vB\sin\theta|$$
$$= e\,v\,B\underbrace{\sin 90^\circ}_{1} = e\,v\,B$$
$$= (1.6 \times 10^{-19})(1 \times 10^{7})(2 \times 10^{-4}) = 3.2 \times 10^{-16}\ N$$

$\vec{F} = (3.2 \times 10^{-16}\ N,$ $\underbrace{\text{up along } +\hat{z}}$$)$

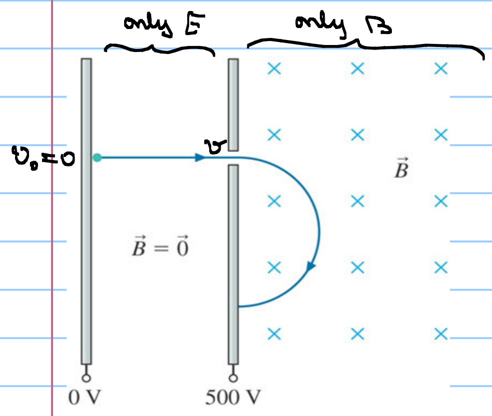
cross product points down
but for electron force points then up.

Question: A 2.0 g, 1.0 cm-long wire and a magnetic field are aligned parallel to the floor. The magnetic field is parallel to the *y*-axis. The 1.4 A current in the wire points along +*x*. What is the magnetic field if the wire floats in air?

A) 1.4 T, along –y
B) 1.4 T, along +y
C) 0.14 T, along –y
D) 0.14 T, along +y



Example: An electron is accelerated from rest through a potential difference of 500 V, then injected into a uniform magnetic field. Once in the magnetic field, it makes half a circuit of radius 8.5 mm. a) What is the magnitude of the magnetic field? b)What is the time the electron spends inside the magnetic field before hitting the plate?



$v_0 = 0$

$\Delta V = 500V$

$r_{cyc} = 8.5 \cdot 10^{-3}$ m

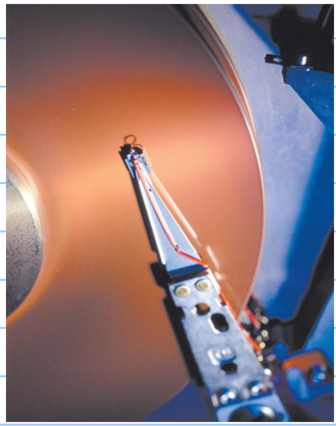
find B

Magnetism: (Chapter 29)

1- The magnetic field $\vec{B}$ - properties of magnets
   - B of moving charge and of current (Biot-Savart law)

2- Ampere's law - B of a wire - B of a solenoid (uniform B)

3- Magnetic force F_B - F_B on a moving charge
   - F_B on a current carrying wire - cyclotron motion



A magnetic hard drive stores digital information as patches of magnetism.

Electromagnet

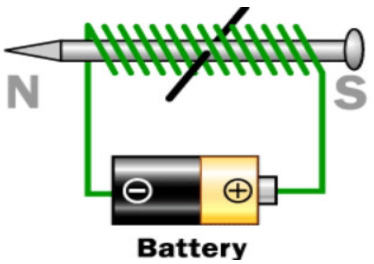


Image from www.themagnetguide.com

1- The magnetic field B:

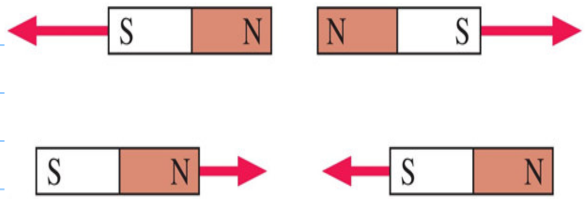
The origins of a magnetic field are moving charges, current, magnets.
In magnets it is the spin of the electron as well as orbiting motion around the nucleus that causes a magnetic field (imagine the electron spinning in adition to orbiting the nucleus much like the earth spins as it orbits the sun).

Units of magnetic field is the Tesla [$1\ T = 1\ N\ A^{-1}\ m^{-1}$].

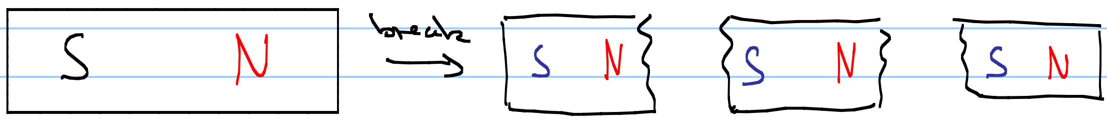
For example, the magnetic field of earth ~ 10^{-5} T.

Some properties of magnets:
Magnetism is very different than electric charges, even if they display similar behavior.

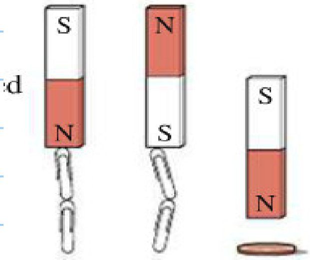
- A magnet has a *north pole and a south pole* (a magnetic dipole).
- *Like poles of magnets repel*, *opposite poles attract*.



- If you cut a magnet into half, you get two magnets. *Monopoles do not exist.*

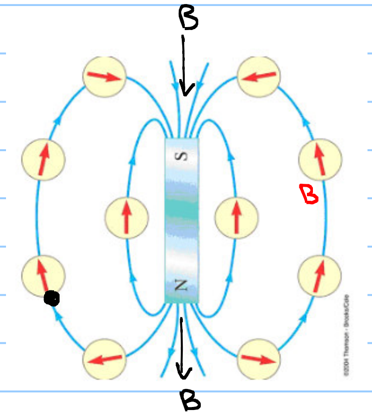


- *Some paper clips are picked up* by a magnet, others not.
  Materials that are attracted to a magnet are called *magnetic materials*.
  They are attracted to *both poles* of a magnet.
  - A penny made of copper (Cu), and objects made of aluminum, or plastic are not picked up by a magnet.
  - A nickel made of Ni and objects containing iron (Fe) or cobald (Co) will be picked up.



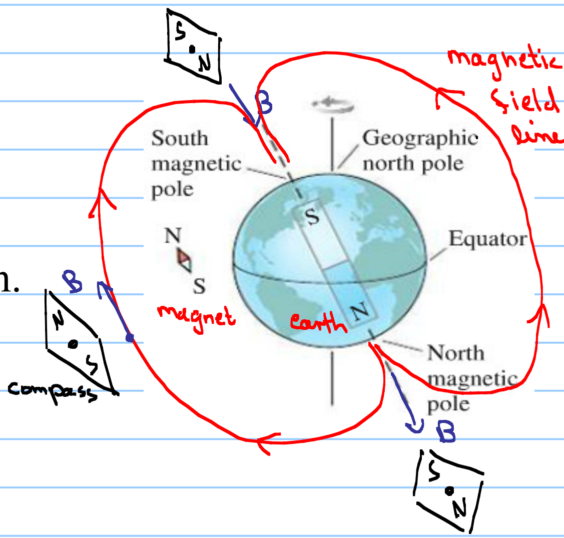
Magnetic field lines

Magnetic field lines always form *closed loops*. The *magnetic field* at any point in space *points along the tangent* to the magnetic field lines.



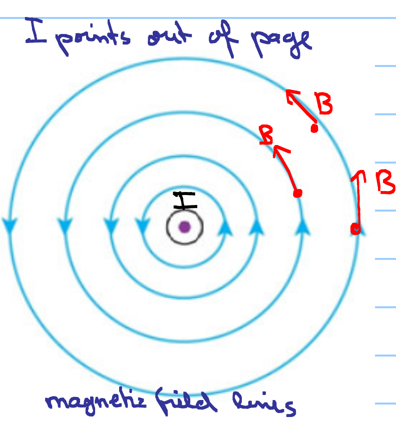
- *For a magnet*:
  Magnetic field lines point outwards from the north pole and inwards at the south pole of a magnet.

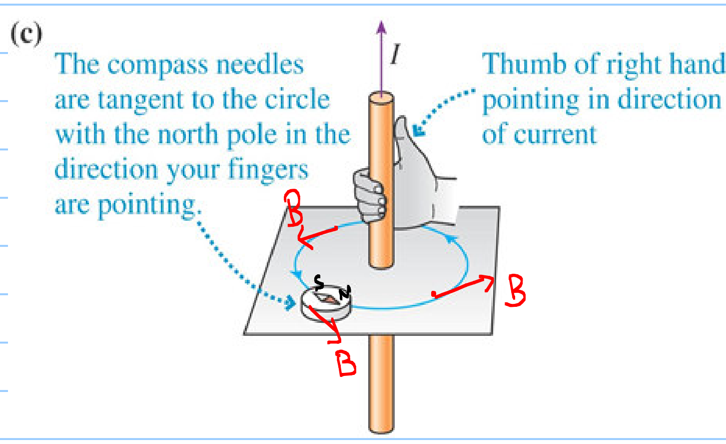
Earth itself is a large magnetic dipole.
The north pole of a compass (magnet) points north (towards geographic north pole, which is the south magnetic pole), the south pole points south, when free to align.



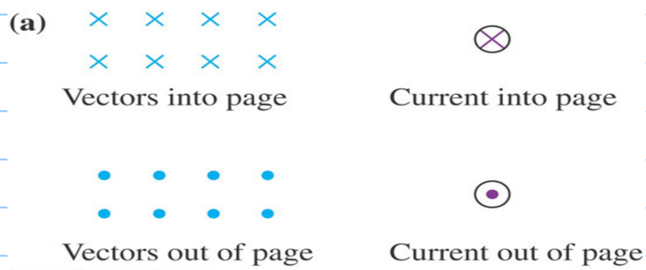
The needle of a compass is a small magnet.

- *For a wire carrying a current*:
  Hans Oersted noticed that the current in a wire caused a compass needle to turn.
  For a straight wire, magnetic field lines are circles around the current, with direction as shown.



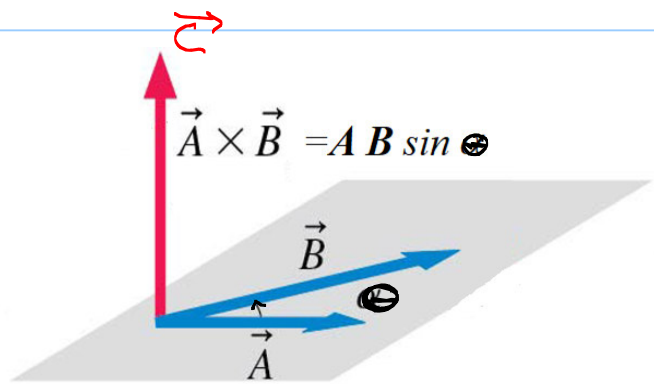


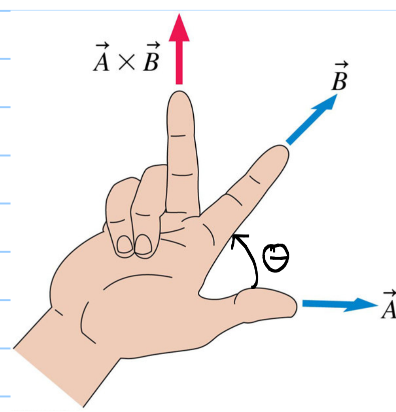
Vectors and cross products: We will deal with three dimensions, so let us introduce the new notation for vectors.



The cross product of two vectors and the right hand rule:
The cross product $\vec{A}$ x $\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a third vector $\vec{C}$ that points *PERPENDICULAR to the plane that contains the two vectors*.
Point your thumb towards $\vec{A}$ and your index finger towards $\vec{B}$, then your middle finger points in the direction of the cross product A x B.

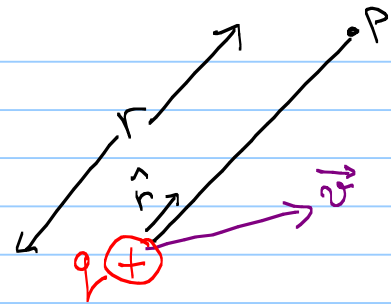
Its magnitude is: $|\vec{A} \times \vec{B}| = |A||B| \sin\theta$
where $\theta \leq 180°$ is the angle between the two vectors





perpendicular to the plane containing both $\vec{A}$ and $\vec{B}$

The magnetic field of moving charges
The Biot-Savart law:
Moving charges are the source of a magnetic field.
The magnetic field at a distance *r* (at point P) away from a charge *q* moving at velocity $\vec{v}$ is:



Unit vector $\hat{r}$ points from q to point P.

Permeability constant:
$\mu_0 = 4\pi \times 10^{-7}$ Tm/A = 1.257×10^{-6} Tm/A

Direction to point where we're measuring the magnetic field.

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} \quad \text{(magnetic field of a point charge)}$$

The Biot-Savart law for a moving charge

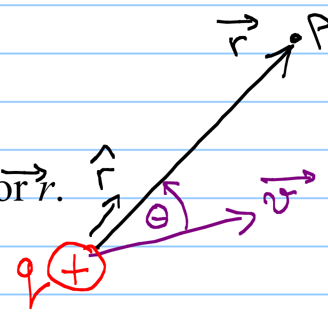
Electric charge of particle.

Velocity of particle.

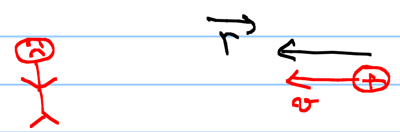
Distance to the point where we're measuring the magnetic field.

$\mu_0 = 4\pi \times 10^{-7}$ T m/A   The permeability of free space

with $|\vec{v} \times \hat{r}| = |v| \cdot 1 \sin\Theta = v \sin\Theta$
where θ is the angle between the velocity vector *v* and vector $\vec{r}$.

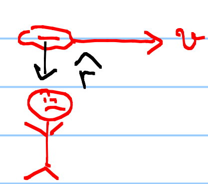


Questions:
- A charged weapon is moving towards you. Do you experience a magnetic field (treat both as particles)? No!



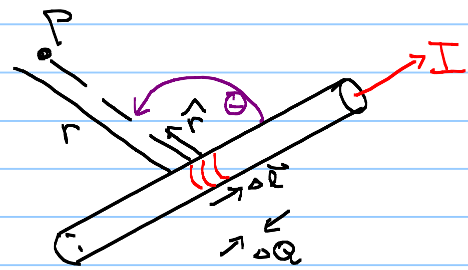
$|\vec{v} \times \hat{r}| = 0 \longrightarrow B = 0$
$\vec{v} \parallel \hat{r}$

- A charged cloud above you moves east.
Do you experience a magnetic field? Yes!



$|\vec{v} \times \hat{r}| = v \cdot 1 \cdot \sin 90° = v$
$\underbrace{q}_{-ve}\ \underbrace{\vec{v} \times \hat{r}}_{\text{into page}} \longrightarrow$ B points out of page.

The magnetic field of a current:
A current inside a wire is made up of many charges moving at velocity *v*.
Let ΔQ be the amount of charge inside a length Δl,
and Δt be the time it takes ΔQ to move through.



Then $I = \frac{\Delta Q}{\Delta t} = \frac{\Delta Q}{\Delta l} v$

$$\Delta Q \vec{v} = I \Delta \vec{l}$$

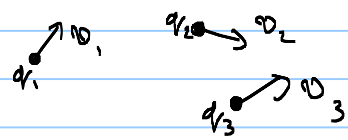
$\vec{B}$ (or call it $\Delta \vec{B}$) a distance *r* away from a current segment Δl is then:

$$\boxed{\underset{(\text{or } \Delta\vec{B})}{\vec{B}} = \frac{\mu_0}{4\pi} I \frac{\Delta \vec{l} \times \hat{r}}{r^2} = \left(\frac{\mu_0}{4\pi} \frac{I \Delta l \sin\theta}{r^2}, \text{ direction by right hand rule} \right)}$$

The Biot-Savart law for a current

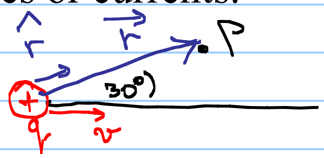
where Δl is the path length taken in the direction of *I*.

Superposition of magnetic field: If there are multiple sources of magnetic field, the resulting B_{net} at a point in space is the superposition of $\vec{B}_i$ at that point due to $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \cdots$



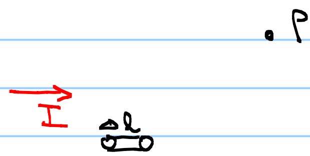
P $\bullet$ $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

Question: Find the direction of the magnetic field at point P for the following charges or currents.

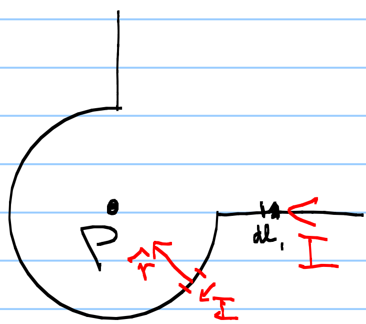


$\vec{v} \times \hat{r}$ points out of page

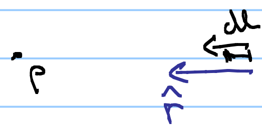
$\vec{B} \propto \underbrace{q}_{+ve}\ \vec{v}\times\hat{r}$ points out of page



$\vec{B}$ points out of page

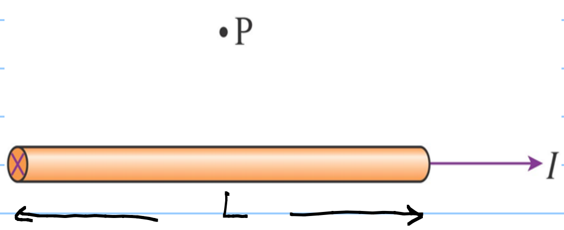


- The two straight segments contribute 0 to $\vec{B}$



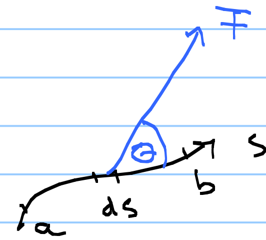
$d\vec{l} \parallel \hat{r}$

- The circular segments contributes a B pointing into the page.



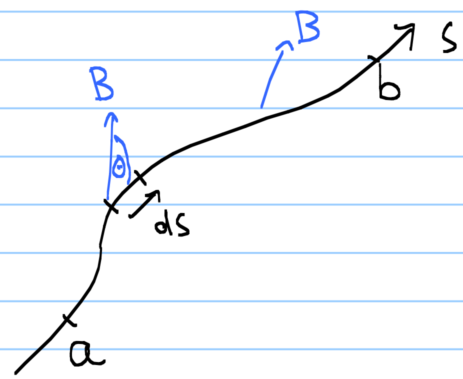
Line Integrals:
Remember: Work done by a force *F* between points *a* and *b* along the line $\vec{s}$ is the line integral of F:

$$\int_a^b \vec{F}\cdot\vec{ds} = \int_a^b F\, ds \cos\theta$$



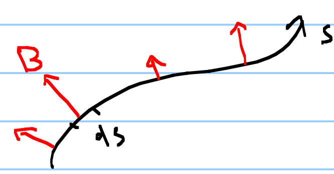
Similarly the line integral of the magnetic field *B* between points *a* and *b* along the line $\vec{s}$ is:

$$\int_a^b \vec{B}\cdot\vec{ds}$$



where we have divided the line *s* into tiny segments of length *ds* .The direction of $\vec{ds}$ is along the line.

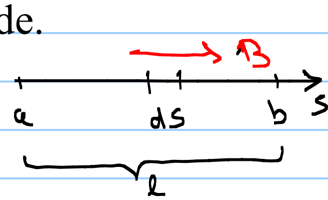
In general this is a difficult integral to carry out.
Two simple cases arises:
- If B is everywhere *perpendicular* to the line



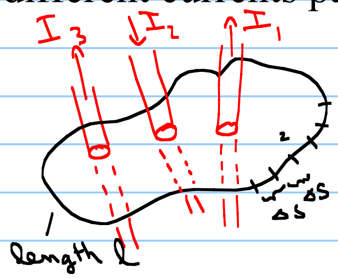
$$\int_a^b \vec{B}\cdot\vec{ds} = 0$$

- If B is always *tangent* to the line and has *constant* magnitude.

$$\int_a^b \vec{B}\cdot\vec{ds} = \int_a^b \underset{\uparrow\,\text{constant}}{B}\, ds\, \underbrace{\cos 0}_{1} = B\int_a^b ds = Bs\Big|_a^b = Bl$$



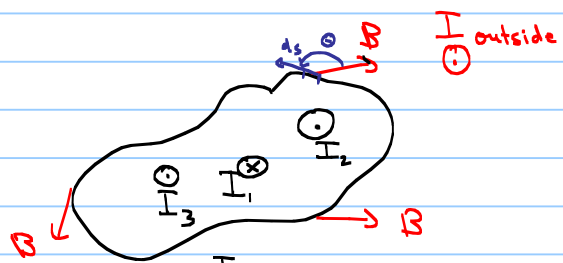
2- Ampere's law:
If symmetry allows, Ampere's law is a useful way to find the magnetic field due to currents. Consider the following area inside the *closed loop* s, with different currents passing through it.



Ampere's law states: When a total current $I_{through}$ passes through an area bounded by a closed curve, the line integral of the magnetic field B around the curve is determined by $I_{through}$:

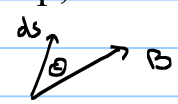
net current passing through loop

$$\boxed{\oint \vec{B}\cdot \vec{ds} = \mu_0 I_{through}}$$



$$I_{through} = \underbrace{I_2 + I_3}_{\text{assumed positive if } \vec{ds} \text{ is chosen ccw}} - I_1$$

where $\oint$ means integration over a closed loop,
ds is an infinitesimal line segment,
the dot product $\vec{B}\cdot\vec{ds} = B\, ds \cos\theta$
and $\mu_o = 4\pi \times 10^{-7}$ T.m/A

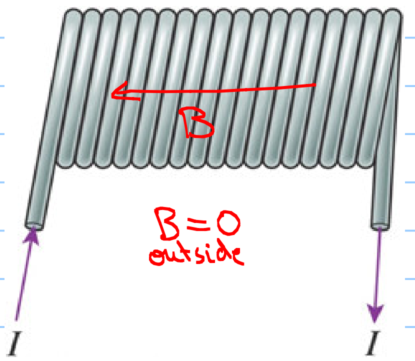


To determine which currents are positive and which are negative, use the right hand rule. If you curl your right fingers around the loop in the direction of $\vec{ds}$, then any currents pointing in the direction of your thumb are positive.

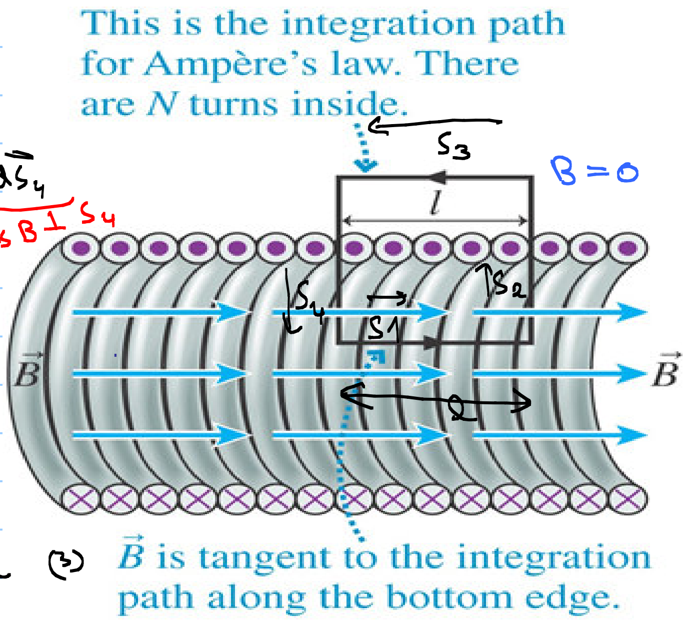
Use Ampere's law if along sections of the Amperian loop:
- B is constant and parallel to the section (then dot product becomes *B l*)
- or B is perpendicular to the section (then dot product = 0).

l = length of section

Solenoids: Very important circuit components!
A wire wrapped into *many narrow turns* is a solenoid.
- *Inside the solenoid* the magnetic field is *CONSTANT* and *PARALLEL* to the axis.
- *Outside the solenoid*, the magnetic field is *ZERO*.



Example: Use Ampere's law to find the uniform magnetic field inside a solenoid that has a linear wire density n (= N turns per L length) and carries a current I.



Apply Ampere's law to the path s.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} \qquad (1)$$

$$\oint \vec{B}\cdot d\vec{s} = \int \vec{B}\cdot d\vec{s}_1 + \underbrace{\int \vec{B}\cdot d\vec{s}_2}_{=0 \text{ as } B \perp s_2} + \underbrace{\int \vec{B}\cdot d\vec{s}_3}_{=0 \text{ as } B=0} + \underbrace{\int \vec{B}\cdot d\vec{s}_4}_{=0 \text{ as } B \perp s_4}$$

$$= \int \underbrace{B}_{\text{constant}}\, ds_1 \underbrace{\cos\theta}_{1} \qquad \xrightarrow{ds_1} B$$

$$= B\int ds_1 = B s_1 = B l \qquad (2)$$

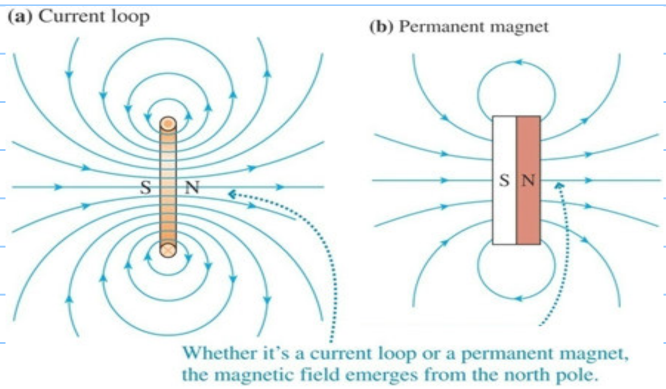
$$I_{\text{through}} = \underbrace{nl}_{\text{number of wires in length } l}\ I \qquad (3)$$

(2) and (3) in (1)

$$B\cancel{l} = \mu_0 n \cancel{l} I$$

$$\boxed{B = \mu_0 n I = \mu_0 \frac{N}{L} I}$$

Similarity between Current loop and permanent magnet:
In our daily use, we often use current loops, let us compare the magnetic field lines of a current loop and that of a permanent magnet.

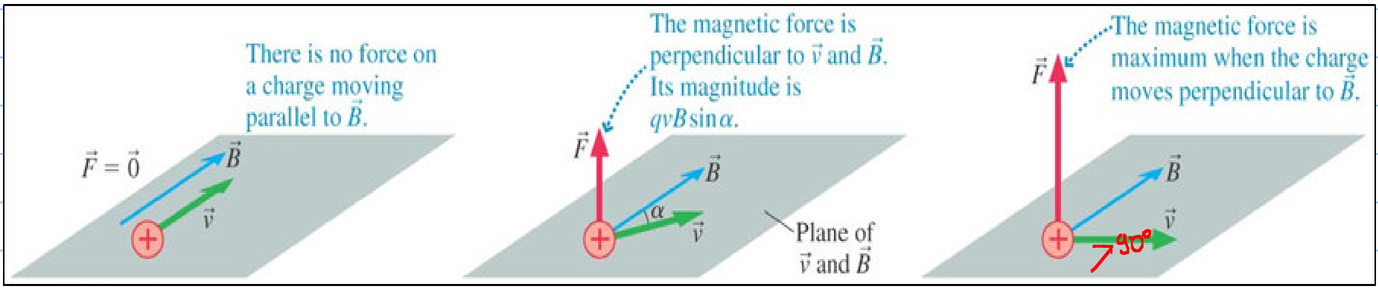


3- The Magnetic Force

a- Magnetic Force on Moving Charges:

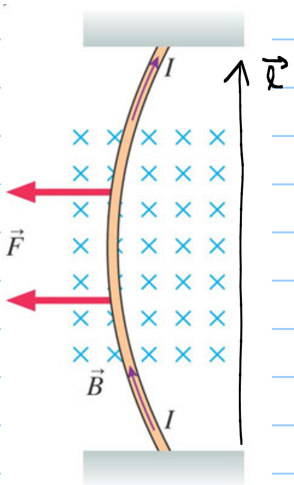
The force $\vec{F}$ on a charge moving at velocity $\vec{v}$ in an external magnetic field $\vec{B}$ is:

$$\boxed{\begin{aligned}\vec{F} &= q\,\vec{v}\times\vec{B}\\ &= (q\,v\,B\sin\theta,\ \text{direction by right-hand rule})\end{aligned}}$$



b- Magnetic Force on a Current-Carrying Wire:

Consider a wire of length l carrying a current I placed inside a magnetic field B. Let q be the amount of charge inside that wire, and Δt be the time it takes this charge to move through.



$$q\,\vec{v} = q\,\frac{\vec{l}}{t} = I\,\vec{l}$$

hence $\boxed{\vec{F}_{\text{on wire}} = I\,\vec{l}\times\vec{B}}$ Magnetic force on wire

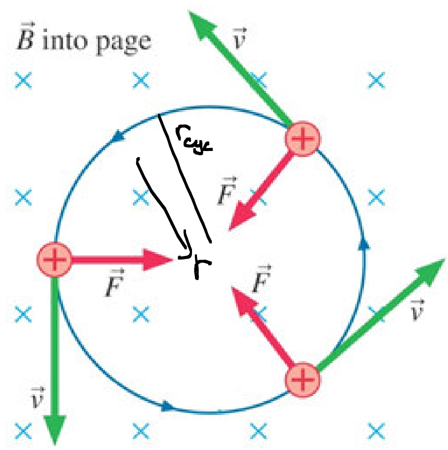
where $\vec{l}$ is a vector of magnitude l and of same direction as I.

c- **Cyclotron Motion**:

- Consider a charged particle that moves in a uniform magnetic field that is *perpendicular* to its velocity.
- Hence, the magnetic force $q\vec{v} \times \vec{B}$ is perpendicular to its velocity (and B).
- Hence the particle undergoes uniform circular motion.

($F \perp v \rightarrow$ centripetal acceleration a_r)

changes direction ↙ ↘ constant speed



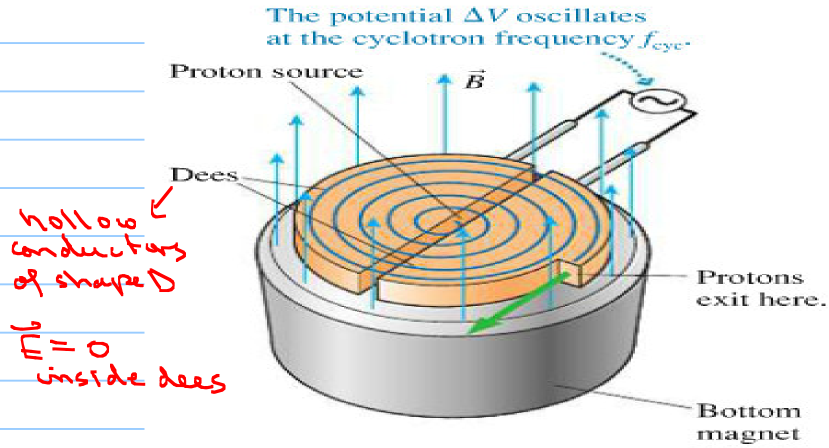
The magnetic force is always perpendicular to $\vec{v}$, causing the particle to move in a circle.

Find the radius of the cyclotron orbit:

Because the speed is constant, the particle makes a full circle in the same time (the period) T and at constant frequency f.

Note: f is independent of v and r, which is practical when designing a cyclotron.

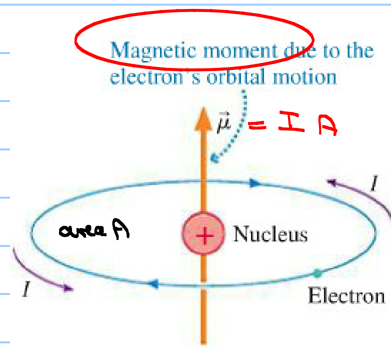
The cyclotron (the first practical particle accelerator)



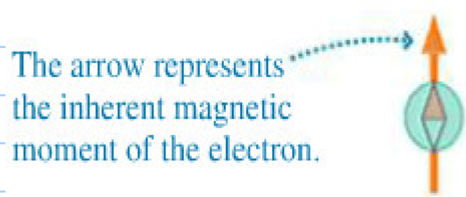
Used for example in creation of radioisotopes for medicine or particle accelerators.

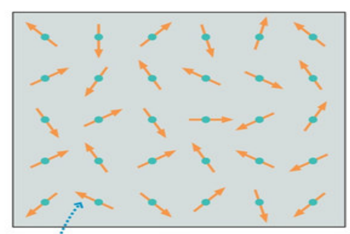
The cyclotron operates by taking advantage of the fact that the cyclotron frequency f_{cyc} of a charged particle is independent of the particle's speed. An *oscillating* potential difference ΔV is connected across the dees and adjusted until its frequency is exactly the cyclotron frequency. There is almost no electric field inside the dees (you learned in Chapter 27 that the electric field inside a hollow conductor is zero), but a strong electric field points from the positive to the negative dee in the gap between them.

Why are some material magnets, while others not?
Electrons orbit the nucleus, see figure.
Hence they act like little current loops.
However an atom has many electrons orbiting in all direction, hence the resulting magnetic field of nearby electrons cancels.



Electron Spin
Electrons have *mass*, so they interact with *gravitational fields*.
They have *charge*, so they interact with *electric fields* and *magnetic fields*.
Electrons have another property, *spin*. You can imagine them as microscopic magnets, with magnetic moments.

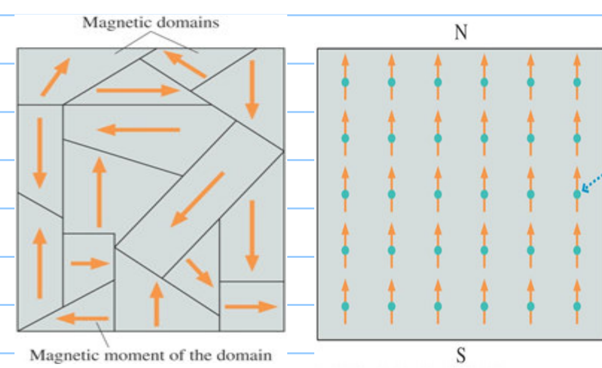




The atomic magnetic moments due to unpaired spins point in random directions. The sample has no net magnetic moment.

In some materials, called *ferromagnetic*, the electron spins lign up in magnetic domains.

If you apply a magnetic field to the ferromagnet, the spins in all domains lign up and you end up with a magnet.



Examples of ferromagnetic materials:
ferrite=iron (Fe), nickel (Ni), cobalt (Co).

Group / Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H 1.008																	2 He 4.0026
2	3 Li 6.94	4 Be 9.0122											5 B 10.81	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.180
3	11 Na 22.990	12 Mg 24.305											13 Al 26.982	14 Si 28.085	15 P 30.974	16 S 32.06	17 Cl 35.45	18 Ar 39.948
4	19 K 39.098	20 Ca 40.078	21 Sc 44.956	22 Ti 47.867	23 V 50.942	24 Cr 51.996	25 Mn 54.938	26 Fe 55.845	27 Co 58.933	28 Ni 58.693	29 Cu 63.546	30 Zn 65.38	31 Ga 69.723	32 Ge 72.63	33 As 74.922	34 Se 78.96	35 Br 79.904	36 Kr 83.798

Magnetism: (Chapter 29)

1- What are sources of a magnetic field?

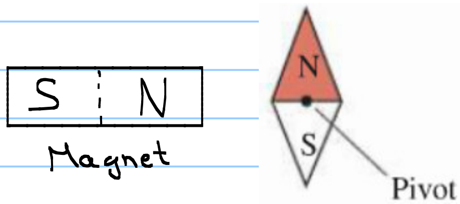
- Moving charges
- Current
- Spin

Question 1: The compass needle rotates

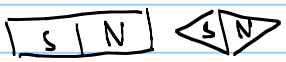
A) clockwise (cw) (A) ✓

B) counterclockwise (ccw)

C) not at all



a compass is a magnet

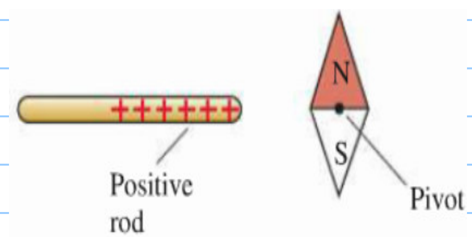


Question 2: Does the compass needle rotate

A) clockwise (cw)

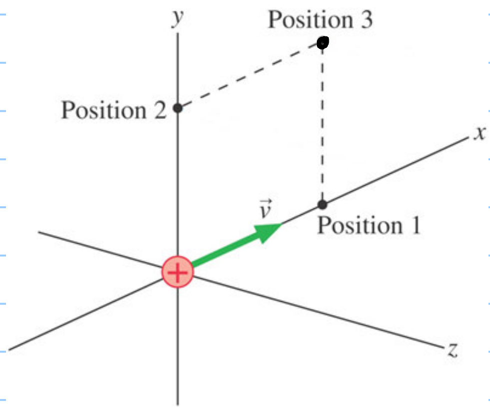
B) counterclockwise (ccw)

C) not at all (C)



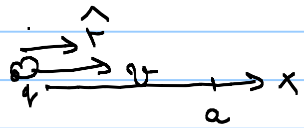
B= 0 at compass

Example: A proton moves along the x axis with velocity $v_x = 1.0 \times 10^7$ m/s. As it passes the origin, what is the magnetic field at the following (x, y, z) positions
a) (1 mm, 0 mm, 0 mm),
b) (1 mm, 1 mm, 0 mm)?



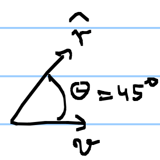
$v_x = 1.0 \times 10^7$ m/s
$q_p = 1.6 \times 10^{-19}$ C
$\mu_0 = 4\pi \times 10^{-7}$ T m.s/C

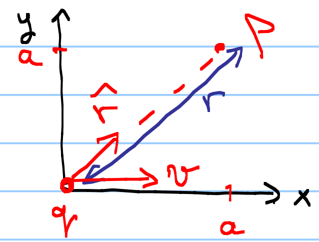
a) $|\vec{v} \times \hat{r}| = 0$ as $\vec{v} \parallel \hat{r}$
$B = 0$



b) $r = \sqrt{a^2 + a^2} = \sqrt{2a^2}$

$|\vec{v} \times \hat{r}| = v \cdot 1 \cdot \sin\Theta = v \sin(45°)$



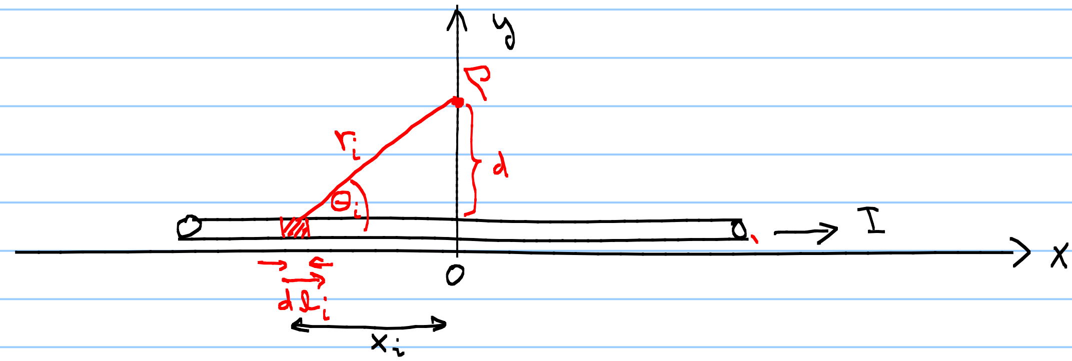


$$B = \frac{\mu_0}{4\pi} \frac{|q \, \vec{v} \times \hat{r}|}{r^2} = \frac{\mu_0}{4\pi} \frac{|q| \, v \sin(45)}{2a^2}$$

$$= \frac{(4\pi \times 10^{-7})}{4\pi} \frac{(1.6 \times 10^{-19})(1 \times 10^7) \sin(45)}{2 \times (1 \times 10^{-3})^2} = 5.7 \times 10^{-14} \text{ T}$$

$\vec{B} = (5.7 \times 10^{-14}$ T, out of page$) = 5.7 \times 10^{-14}$ T $\hat{z}$

Example: Find the magnetic field a distance d from an infinite long wire carrying a current I travelling in the positive x direction.



- Choose point P a distance d away from the wire.
- Establish a coordinate system, let us choose the origing as shown.
- Divide the wire into N segments located at coordinate x_i each of length Δx_i.
- At point P, the magnetic field ΔB_i due to any segment i:

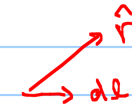
$$\Delta \vec{B}_i = \frac{\mu_0}{4\pi} I \frac{\Delta \vec{l}_i \times \hat{r}_i}{r_i^2}$$

$$\Delta B_i = \frac{\mu_0 I}{4\pi} \frac{\Delta l_i \cdot 1 \cdot \sin\theta_i}{r_i^2} \qquad \text{take } \Delta l_i = \Delta x_i \qquad \sin\theta_i = \frac{d}{r_i} \qquad r_i = \sqrt{d^2 + x_i^2}$$

$$\Delta B_i = \frac{\mu_0 I}{4\pi} \frac{\Delta x_i}{r_i^2} \frac{d}{r_i} = \frac{\mu_0 I d}{4\pi} \frac{\Delta x_i}{\left[(d^2 + x_i^2)^{\frac{1}{2}}\right]^3}$$

$$= \frac{\mu_0 I d}{4\pi} \frac{\Delta x_i}{(d^2 + x_i^2)^{3/2}}$$

- Look at direction of B:



At P, for any segment i, always B points out of the page,
hence we can simply sum all ΔB_i

- Sum over B_i and take the limit when Δx_i goes to zero:

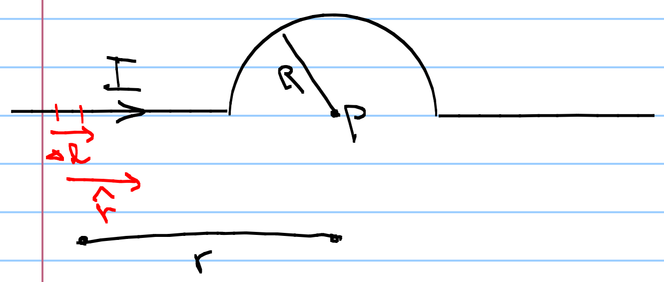
$$B = \sum_{i=1}^{N} \Delta B_i = \frac{\mu_0 I d}{4\pi} \sum_i \frac{\Delta x_i}{(d^2 + x_i^2)^{3/2}}$$

$$= \frac{\mu_0 I d}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(d^2 + x^2)^{3/2}} \qquad \text{integral provided}$$

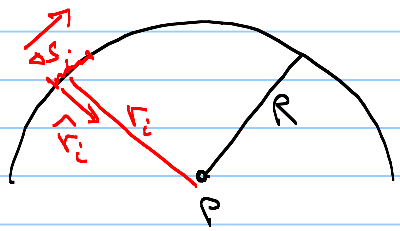
$$= \frac{\mu_0 I d}{4\pi} \left[\frac{1}{d^2} \frac{x}{(d^2 + x^2)^{1/2}} \right]_{-\infty}^{\infty} = \frac{\mu_0 I d}{4\pi d^2} \left[1 - (-1) \right] = \frac{\mu_0 I}{2\pi d}$$

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi d}, \text{ out of page} \right)$$

Example: A wire bent into the shape shown carries current *I*. Find the magnetic field at the center of the semicircle.



For both the straight wires, $\vec{B}_{at\,P} = 0$, as $\vec{\Delta l} \parallel \hat{r}$.



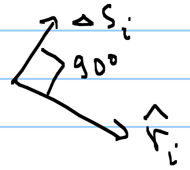
For the half circle of current:

$$\vec{\Delta B_i} = \frac{\mu_0 I}{4\pi} \frac{\vec{\Delta l_i} \times \hat{r}_i}{r_i^2}$$

$\Delta l_i = \Delta s_i \leftarrow$ arc length

$$|\vec{\Delta l_i} \times \hat{r}_i| = |\Delta s_i| \cdot 1 \times \sin 90° = \Delta s_i$$

$$r_i = R$$



$$\Delta B_i = \frac{\mu_0 I}{4\pi} \frac{|\vec{\Delta l_i} \times \hat{r}_i|}{r_i^2} = \frac{\mu_0 I}{4\pi} \frac{\Delta s_i}{R^2}$$

Look for direction of ΔB_i.
For every current segment i, ΔB_i points into the page
$\longrightarrow$ simply add ΔB_i

$$B = \sum_i \Delta B_i = \frac{\mu_0}{4\pi} \frac{I}{R^2} \underbrace{\sum_i \Delta s_i}_{\text{length of half a circumference}} = \frac{\mu_0 I}{4\pi R^2} \frac{\cancel{2}\pi R}{\cancel{2}}$$

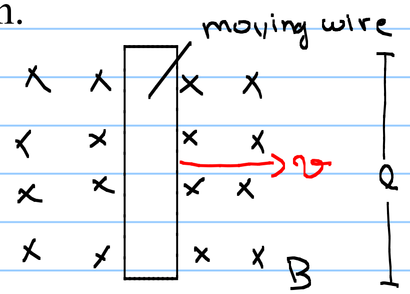
$$\vec{B} = \left(\frac{\mu_0 I}{4R}, \text{ into the page} \right)$$

Electromagnetic Induction (Chapter 30)
- Motional *emf*
- Magnetic flux Φ_B
- Electromagnetic induction and Faraday's law
- Lenz's law (for direction of current and *emf*)
- AC generator
- Inductance
- LC circuits

Question: What is the difference between a motor and a current generator? Can we explain the physics of both?

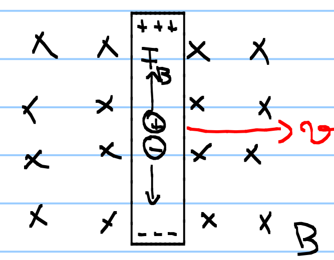
Motional *emf* (ε) and induced current (I_{ind}):
- Consider a conducting wire of length l being pulled to the right at speed v in a stationary magnetic field B pointing into the page.
- The charges within the wire get separated by the magnetic force F_B. Positive charge q moves up, negative charge moves down.

$$F_B = q|v \times B| = qvB$$



- The separated charges produce
1- a potential difference across the wire ΔV
(we call it the motional emf ε)
2- an electric field E inside the wire, pointing down.

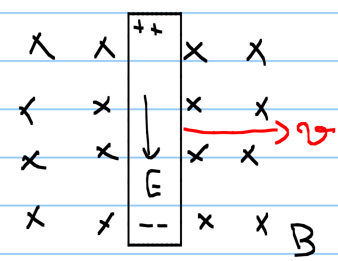
- The electric field keeps growing up to the points when the net forces on each charge balance.



$$F_E = F_{mag}$$
$$qE = qvB$$
$$E = vB$$

The *motional emf* ε (difference in POTENTIAL) is: $\mathcal{E} = \Delta V = El$

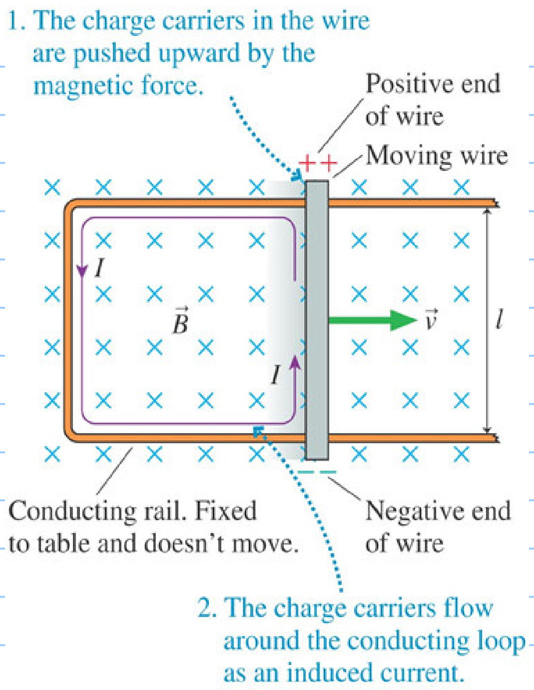
$$\boxed{\mathcal{E} = vBl}$$



Cool: the moving wire acts as a source of emf (kind of a battery!)

If you form a loop, by connecting this moving wire into a closed circuit, an *induced current* will flow in the loop. By Ohm's law:

$$\boxed{I = \frac{\mathcal{E}}{R} = \frac{vBl}{R}}$$

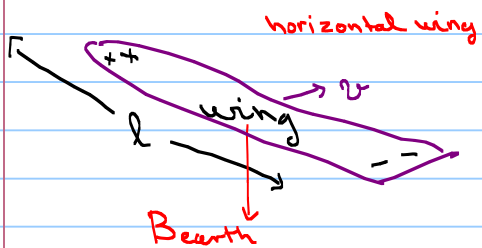


Direction of *I* (in the external circuit) is from positive to negative potential, same as for a battery in a circuit.

We have *turned mechanical energy* (motion) into *electrical energy* (*emf* and current). This concept is used in **generators** *to produce current*. The source of mechanical energy may vary widely from a hand crank to an internal combustion engine. Generators provide power for electric power grids.

This is opposite of what a motor does.
A **motor** turns electrical energy into mechanical energy.

Example: It is known that the earth's magnetic field over northern Canada points straight down. The crew of a Boeing 747 aircraft flying at 260 m/s over northern Canada finds a 0.95 V potential difference between the wing tips. The wing span of a Boeing 747 is 65 m. What is the magnetic field strength there?

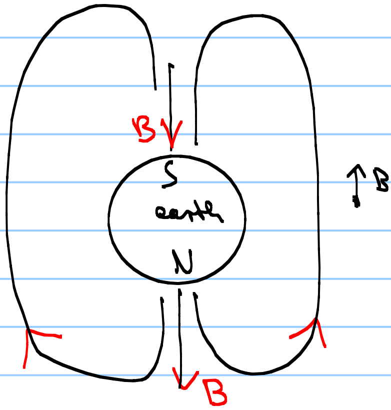


known: $\Delta V = 0.95V$

$v = 260\ m/s$

$l = 65\ m$

find B_{earth}

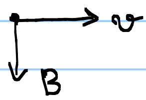


Model the wing as a wire moving $\perp$ to the magnetic field B_{earth}

The motional emf $\mathcal{E}$ is

$$\mathcal{E} = v\, l\, B$$

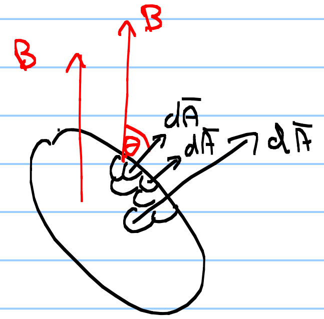
$$B = \frac{\mathcal{E}}{v l} = \frac{0.95}{260 \cdot 65} = 5.6 \cdot 10^{-5}\ T$$



The magnetic flux:
The magnetic flux through an area is defined as

$$\boxed{\Phi_m = \int B_\perp dA = \int \vec{B}\cdot d\vec{A}} = \int B\, dA \cos\theta$$

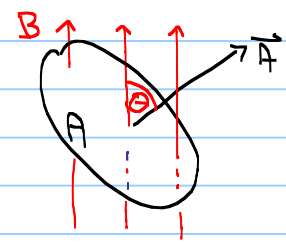
perpendicular to area



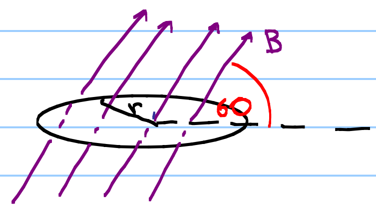
$d\vec{A}$ is the area element vector (its magnitude is dA, its direction is perpendicular to the surface).
Units of Φ_m is the Weber [1 W = 1 T m^2].

For a flat surface A and constant $\vec{B}$:

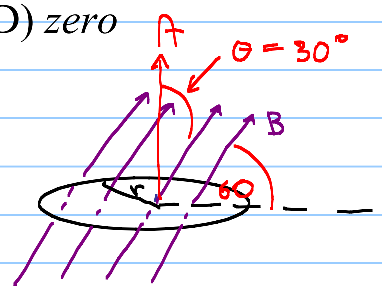
$$\Phi_m = BA\cos\theta$$



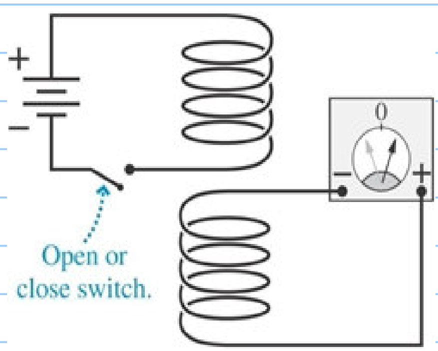
Example: A uniform magnetic field B makes 60° with a plane containing a loop of radius r. What is the magnetic flux through the loop?



A) $B\,\pi r^2 \cos(60^0)$
B) $B\,\pi r^2 \cos(30^0)$ (B) ✓
C) $B\,\pi r^2$
D) *zero*

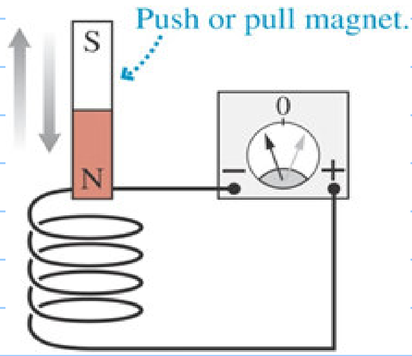


Electromagnetic Induction:
- Oersted discovered that a current creates a magnetic field.
- Does a magnetic field create a current, or more general an electric field?
- Faraday performed three experiments, in each of which *the magnetic flux changes* by different mechanisms. In all three experiments a current was observed.

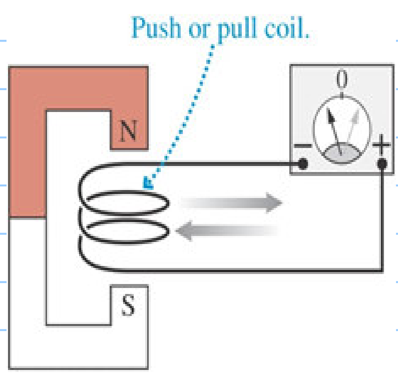


→ momentary current

new physics!



→ momentary current

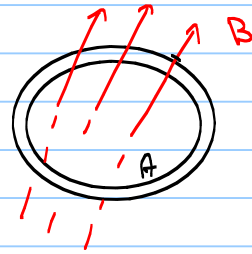


old physics!
motional emf!

→ momentary current

Faraday's Law:

- If the *magnetic flux through a closed loop changes* with time, an *emf ε is induced* around the loop:



opposes change in flux

$$\boxed{\mathcal{E} = -\frac{d\Phi_m}{dt}}$$

Induced *emf* in a closed loop due to a change in magnetic flux

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right|$$

- If the loop has *N* turns, each contributes to *ε*.

$$\boxed{\mathcal{E} = N\left|\frac{d\Phi_{m\,(\text{through 1 turn})}}{dt}\right|}$$

- Accordingly an induced current I_{ind} flows in the loop.

$$\boxed{I_{ind} = \frac{\mathcal{E}}{R}}$$

Induced current

resistance of loop.

The induced electric field

- So back to our question, does **B** create an electric field?

Answer: A time *varying* magnetic field (or more general a *varying* magnetic flux) induces an electric field $\vec{E}$ around it. The *emf* in a closed loop is:

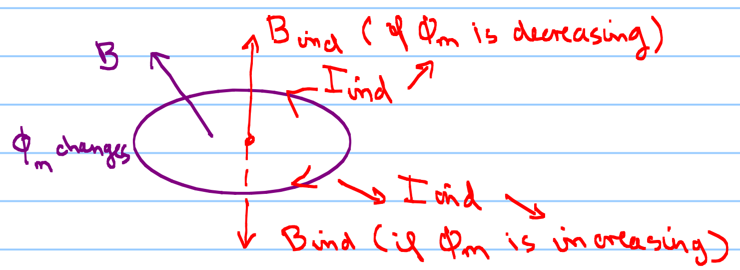
$$\boxed{\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_m}{dt}}$$

This is one of the four Maxwell's equations.

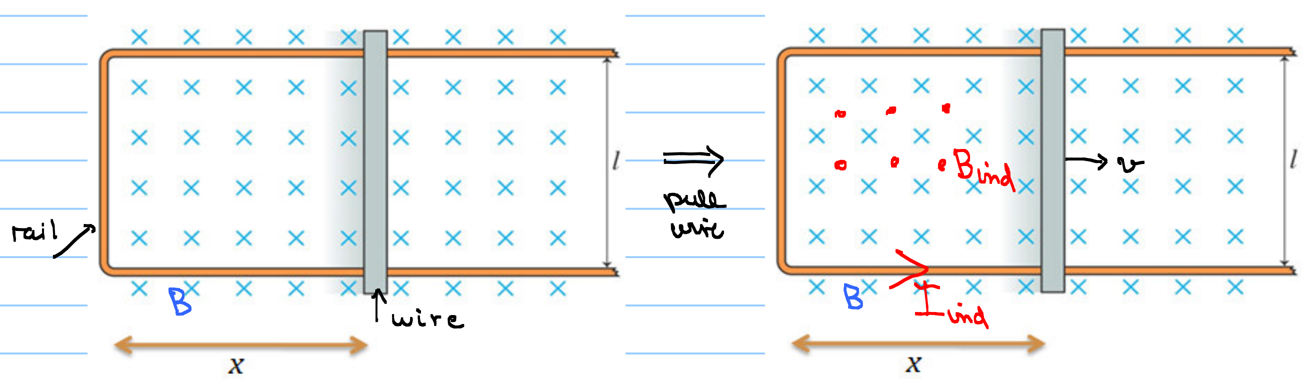
Lenz's Law:
It gives the direction of the induced current and hence induced *emf*.
Lenz's law states:

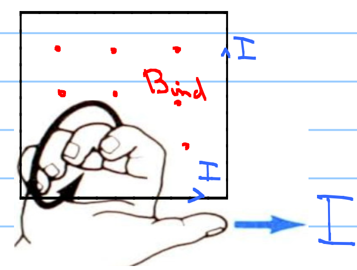
- A *changing magnetic flux* ϕ_m through a closed conducting loop produces an *induced current* I_{ind}.
- The *direction of* I_{ind} is such that the *induced magnetic field* B_{ind} OPPOSES the *change in flux*.

- *i.e.* Determine the loop through which the magnetic flux changes.
  - if ϕ_m is increasing within a loop, then B_{ind} is opposite B.
  - if ϕ_m is decreasing within a loop, then B_{ind} is in same direction as B.
  - choose the induced current direction I_{ind} accordingly.



Question: The wire inside the constant magnetic field is being pulled at speed v as shown in figure. It slides on a conducting rail. The direction of I_{ind} is: A) cw B) ccw.





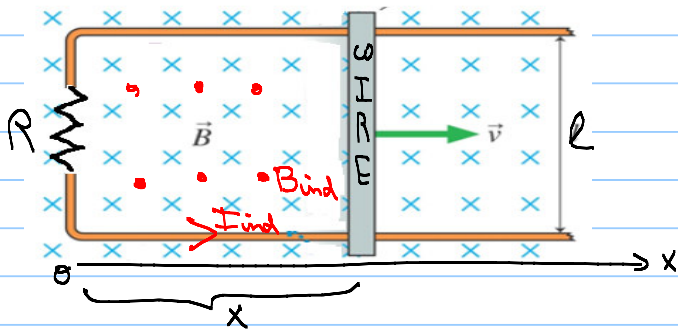
Example: You are pulling the conducting wire of length $l = 1.0$ m to the right at constant speed 5.0 cm/s inside a magnetic field of 0.20 T. The resistanceless wire is connected to a 0.10 ohm resistor through a resistanceless rail.

a) Find the *emf* induced across the wire? Use Faraday's law.

b) Find the induced current in the conducting wire (magnituce & direction)?

c) Find the magnetic force on the wire due to the induced current?

d) Find the force you need to apply to move the wire at constant speed v?



known: $\ell = 1.0$ m

constant $v = 0.05$ m/s

$B = 0.20$ T

$R = 0.10\ \Omega$

find: emf, I, F_m, F_{app}

a) Loop consists of resistor R, rail and wire. Its area increases over time.

From Faraday's law

area of loop; angle between $\vec{B}$ and $\vec{A}$

$$\mathcal{E} = N\left|\frac{d\Phi_m}{dt}\right| \quad \text{with } \Phi_m = \int \vec{B}\cdot d\vec{a} = BA\cos\theta = BA\cos 180° = -BA$$

constant

$$\mathcal{E} = N\left|\frac{d}{dt}(BA)\right| = NB\left|\frac{dA}{dt}\right|$$

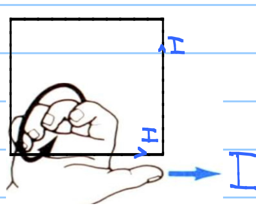
$A = \ell x$; $N=1$; velocity of wire being pulled

$$\mathcal{E} = NB\left|\frac{d}{dt}(\ell x)\right| = NB\ell\left|\frac{dx}{dt}\right| = NB\ell v$$

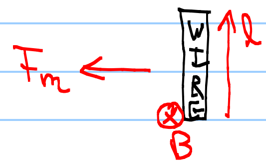
$$= B\ell v = (0.2\,T)(1\,m)(0.05\,m/s) = 0.010\,V$$

b) $$I_{ind} = \frac{\mathcal{E}}{R} = \frac{0.01}{0.1} = 0.10\,A$$

I_{ind} is ccw, according to Lenz's law.



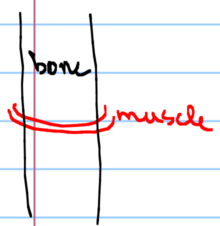
c) $$\vec{F}_{m\text{ on wire}} = I\vec{\ell}\times\vec{B}$$
$$= (I\ell B\sin(90°),\ \text{left})$$
$$= (0.020\,N,\ \text{left})$$

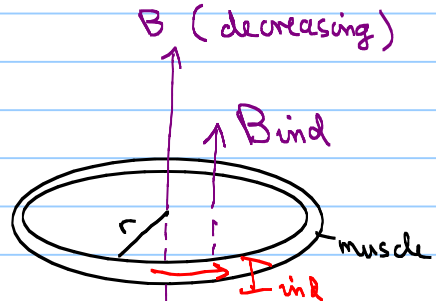


d) Because v is constant, $\rightarrow$ $\vec{F}_{net}$ on wire $= 0$

$$\vec{F}_{applied} = -\vec{F}_m$$
$$= (0.020\,N,\ \text{right})$$

Example: Muscle in an MRI machine:
Consider a muscle (a conductor) that circles the bone of your arm, forming an 8.0 cm diameter ring. The muscle's resistance is 4800 Ω. Suppose the magnetic field along the axis of the loop drops homogeneously in 0.30 s from 1.6 T to 0 T.
a) What current is induced in your muscle?
b) How much power is dissipated in your muscle?





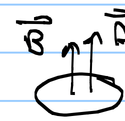
given:

$r = \frac{0.08\text{ m}}{2} = 0.04\text{ m}$ (radius)

$R = 4800\ \Omega$ (resistance)

$\frac{\Delta B}{\Delta t} = \frac{B_f - B_i}{\Delta t} = -\frac{1.6}{0.3}\frac{T}{s}$

a) Use Faraday's law. The loop we consider is the muscle.



$\Phi_m = BA\cos\theta \leftarrow \theta = 0 \quad = BA$

area surrounded by muscle

homogeneous decrease.

$\mathcal{E}_{loop} = N\left|\frac{d\Phi_m}{dt}\right| = 1\left|\frac{d}{dt}(BA)\right| = A\left|\frac{dB}{dt}\right| \stackrel{\downarrow}{=} A\left|\frac{\Delta B}{\Delta t}\right|$

$= (\pi r^2)\left|\frac{\Delta B}{\Delta t}\right| = 0.0268\text{ V}$

By Ohm's Law

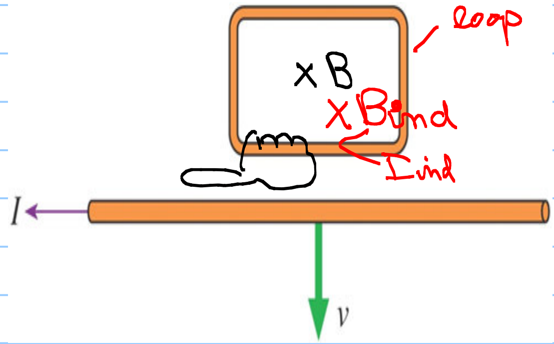
$I_{ind} = \frac{\mathcal{E}}{R} = \frac{0.0268}{4800} = 5.6 \times 10^{-6}\text{ A}$ (ccw)

b) $P = I\mathcal{E}$ $\quad (I_{ind}^2 R)$

$= 1.5 \times 10^{-7}\text{ W}$

Question: A current carrying wire is pulled away from a conducting loop in the direction shown.

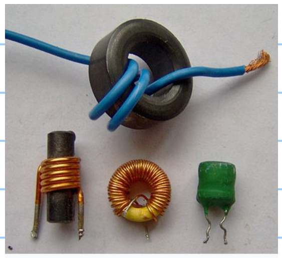
The direction of B_{wire} inside the loop is:
A) cw (clockwise)
B) ccw (counter-clockwise)
C) into the page (C) ✓
D) out of the page
E) there is no magnetic field



The direction of the current induced (I_{ind}) in the loop is:
A) cw (A) ✓
B) ccw
C) into the page
D) out of the page
E) there is no current.

wire gets further away from loop
Φ_m through loop decreases.
B_{ind} must be in same direction as B

The Inductance of an inductor:

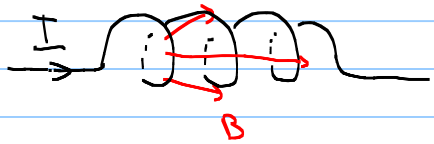

A selection of low-value inductors

wikipidea: inductors

- An *inductor* is a coil of wire that stores energy U_L in the magnetic field, if a current flows through it.

- The inductance L is the ratio of the magnetic flux through the inductor to the current through it.

$$\boxed{L = \frac{\Phi_m}{I}}$$ Inductance



Units of L is the Henry [1 H = 1 T m^2 / A].

- *L depends on inductor's properties,* for example, its shape, number of turns of the coil and the material inside it, but not the current.

- The energy stored inside the inductor is:

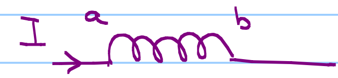
$$\boxed{U_L = \frac{1}{2} L I^2}$$

Inductors in circuits:

Potential difference across an inductor:

If an inductor is placed inside a circuit that has *a changing current,* then

- B and hence ϕ_m inside each turn of the inductor changes,



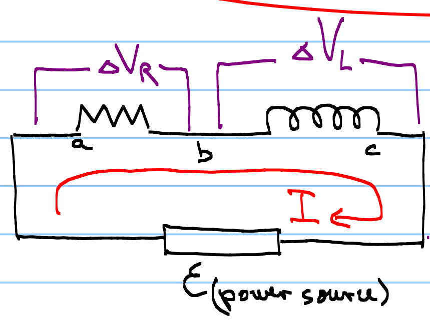
- an *emf* ε_L is induced in the circuit.

$$\varepsilon_L = -N\frac{d\Phi_m \text{ per turn}}{dt} = -\frac{d\Phi_m (\text{of inductor})}{dt} = -\frac{d}{dt}(\overset{\text{constant}}{L} I)$$

$$\varepsilon_L = \boxed{\Delta V_L = -L\frac{dI}{dt}}$$

- This formula (with the minus sign in front) gives the potential difference ΔV_L across the inductor measured *ALONG the direction of the current*

along I:
$\Delta V_R = V_b - V_a = -IR$



along I:
$\Delta V_L = V_c - V_b = -L\frac{dI}{dt}$

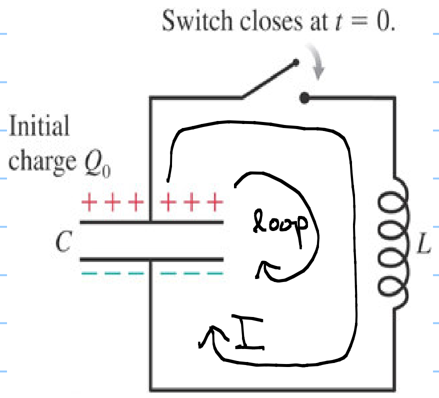
Note: ΔV_L across the inductor is:

- *positive or negative or zero*, depending on the change in I.
- zero for a *constant current* and assuming the inductor is ideal (zero resistance).

LC Circuits (oscillating circuits)

A *charged* capacitor of capacitance C is connected through a switch to an inductor of inductance L. Let Q_0 be the initial charge on the capacitor at the starting time when the switch is closed (at $t = 0$).

Kirchoff's loop law states:



$$\Delta V_c + \Delta V_L = 0$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} - L\frac{d}{dt}\left(-\frac{dQ}{dt}\right)$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = \frac{-Q}{LC}$$ Differential equation.

This is a 2nd order differential equation describing simple harmonic motion with boundary conditions at $t = 0$: $Q = Q_o$ and $I = 0$. Its solution is:

$$\boxed{Q = Q_0\cos(\omega t)}$$ Q on capacitor (1)

Find ω by plugging Q into the differential equation:

$$\frac{dQ}{dt} = Q_0\,\omega\left(-\sin(\omega t)\right) \quad (2)$$

$$\frac{d^2Q}{dt^2} = -Q_0\omega\,\omega\cos(\omega t) = -Q_0\omega^2\cos(\omega t) \quad (3)$$

$$-\cancel{Q_0}\omega^2\cancel{\cos(\omega t)} = -\frac{1}{LC}\cancel{Q_0}\cancel{\cos(\omega t)}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$ Natural (resonance) angular frequency ω of an LC circuit

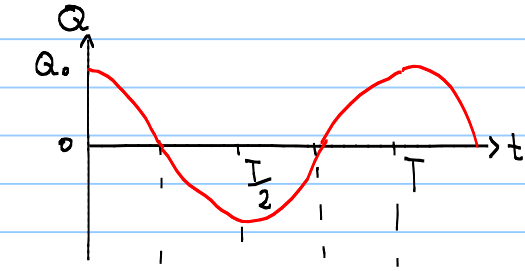
Remember also:

$$(\underbrace{\omega}_{\text{angular frequency}} = 2\pi f)$$ ← frequency

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$ Natural frequency f of oscillation and Period T (time of a complete oscillation)

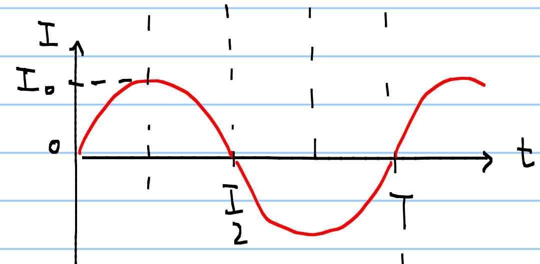
The current is given by:

$$I = -\frac{dQ}{dt} \overset{\text{from (2)}}{=} \underbrace{Q_0\,\omega}_{I_0} \sin(\omega t)$$

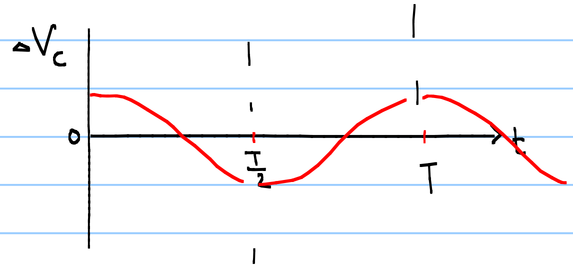


$Q = Q_0 \cos(\omega t)$

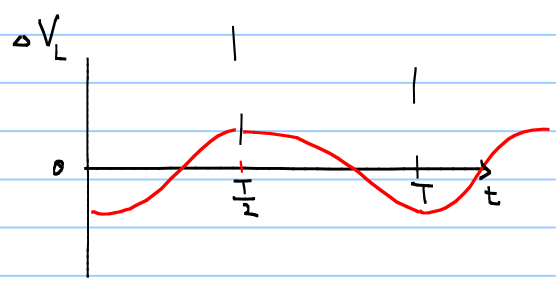
where $\omega = 2\pi f = \frac{2\pi}{T}$

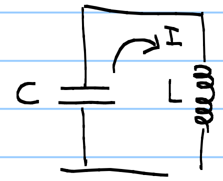


$I = I_0 \sin(\omega t)$
$= Q_0 \omega \sin(\omega t)$



$\Delta V_c = \frac{Q}{C} = \frac{Q_0}{C} \cos(\omega t)$





by Kirchoff's loop law.

$\Delta V_L + \Delta V_c = 0$

$\rightarrow \Delta V_L = -\Delta V_c$

The energy in an LC circuit is constant, *NO energy* is dissipated.
It oscillates back and forth between the capacitor and the inductor.

Tuning :

- LC circuits are used either for generating signals at a particular frequency, or picking out a signal at a particular frequency from a more complex signal.

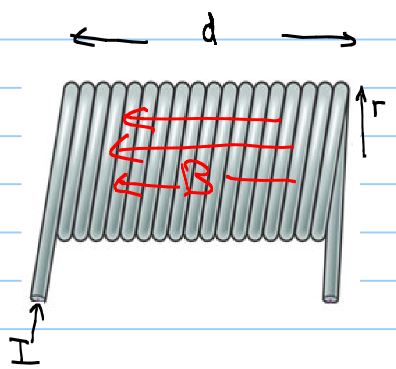
- An LC circuit wants to *respond* only at its *natural oscillation frequency* $\omega = 1 / \mathrm{sqrt}(LC)$. A strong response at that natural frequency is called resonance, and resonance is the basis for all telecommunications.

- The input circuits in radios, televisions, and cell phones is an LC circuit, driven by the signal picked up by the antenna (for example, imagine a secondary coil that carries the broadcast signal wrapped around the inductor, the broadcast signal will cause an induced current in the LC circuit - Farady's law. That current has the same frequency as the signal. The strength of that induced current depends in part on that frequency and is maximum at the natural frequency). So, although many different signals are picked up by the antenna, the *LC* circuit responds mainly to that signal that matches its natural frequency.

Example:

a) Find the inductance L of a solenoid of length 1.0 cm, diameter 0.60 cm and having 30.0 turns.

b) If the current passing through the solenoid decreases uniformly from 1.4 to 0.80 A in 0.30 ms, what is the potential difference ΔV_L.

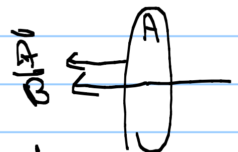


known $d = 0.01\ m$

$r = \frac{0.6}{2}\ cm = 0.003\ m$

$N = 30$

find L



$\Phi_m(1\ turn) = BA\cos\theta = BA$

a) $L = \frac{\Phi_m(solenoid)}{I} = \frac{N\Phi_m(1\ turn)}{I}$

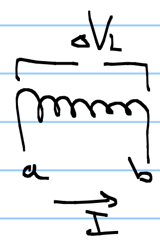
$= \frac{N}{I} BA \underbrace{(\cos\theta)}_{1}$

see lecture on Ampere's law

$= \frac{N}{I}(\mu_0 n I) A$ ← # turns per length $= \frac{N}{I}\left(\mu_0 \frac{N}{d} I\right)(\pi r^2)$

$= \mu_0 \frac{N^2 \pi r^2}{d} = 3.2 \times 10^{-6}\ H$

b)



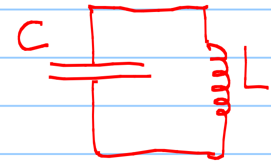
$\frac{\Delta I}{\Delta t} = \frac{0.8 - 1.4}{0.3 \times 10^{-3}}$

$\Delta V_L = -L\frac{dI}{dt}$ uniform change

$= -L\frac{\Delta I}{\Delta t} = 6.4 \times 10^{-3}\ V$

$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$

Example: You have a 1.0 mH inductor. You choose to make an oscillator with a frequency of 910 kHz, which is near the center of the AM radio band.



- What is the angular frequency of the oscillator?

A) 5.8×10^{6} rad/s (A) ✓

B) 1.4×10^{5} rad/s

C) 1.1×10^{-6} rad/s

$f = 910$ kHz

$\omega = 2\pi f = 2\pi \times 910 \times 10^{3}$ rad/s

- What is the capacitor you choose?

A) 1.0×10^{-11} F

B) 3.0×10^{-11} F (B) ✓

C) 1.7×10^{-4} F

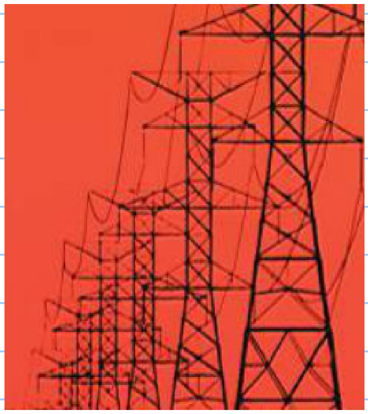
$\omega = \frac{1}{\sqrt{LC}}$

$\omega^2 = \frac{1}{LC}$

$C = \frac{1}{L\omega^2}$

AC Circuits (Chapter 32)
The wires that transport electricity across the country use *alternating current* called AC. This is especially useful as:
- transformers allow an AC voltage to be "stepped up" to a higher voltage, hence lower current, (*power* is the same).
- a lower current dosn't overheat the wires, reducing energy loss.

Hence you can step up the voltage at the point of transmission, then at the end of the transmission line bring the voltage down again.



AC circuit:
- AC generator
- Phasors
- Resistor circuits
- Capacitor circuits
- Inductor circuits
- Series RLC circuits

An AC Generator (alternator):

- A generator is a device that *turns motion, i.e. mechanical energy (for example: falling water, wind, hamster spinning a wheel, rotating car wheels)* into a current (electric energy).
- It is the opposite of a motor, which turns a current into motion.

- Consider a coil with N turns and area A inside a magnetic field of strength B. You rotate the coil at a frequency f.

An external mechanical force rotates the loop at frequency f

$$\mathcal{E} = -N\frac{d\Phi}{dt}$$

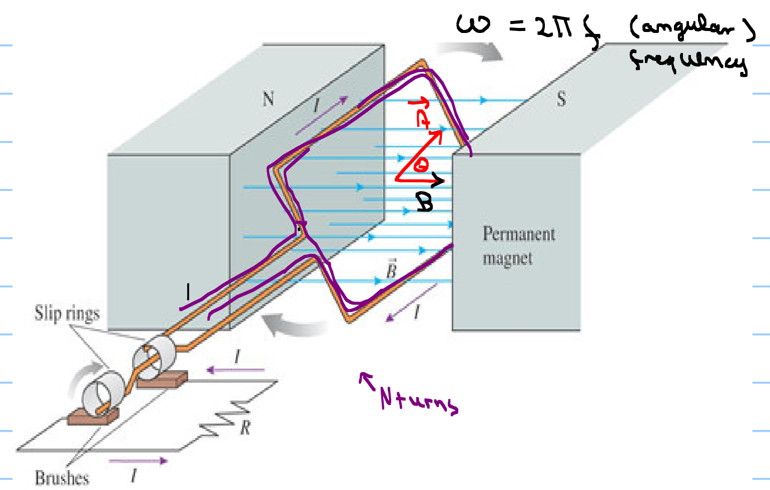
$$= -N\frac{d}{dt}\left[BA\cos(\underbrace{\omega t}_{\theta})\right]$$

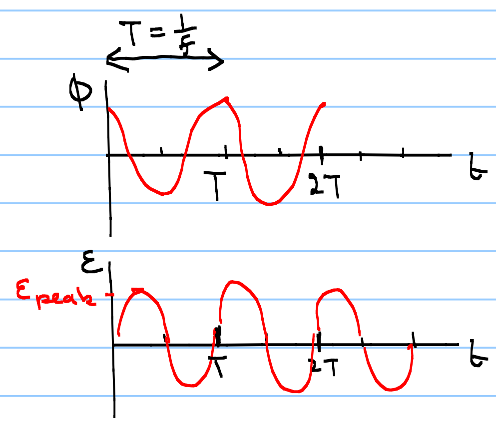
with $\omega = 2\pi f$

$$= -NBA\left[-\omega\sin(\omega t)\right]$$

$$= N\omega BA\sin(\omega t)$$

$$I = \frac{\mathcal{E}}{R}$$





Phasors: Before analyzing AC circuits such as resistor, capacitor, RC filter, inductor and RLC circuits, we will introduce phasors.
Phasors are an alternative way to describe the oscillation with time.

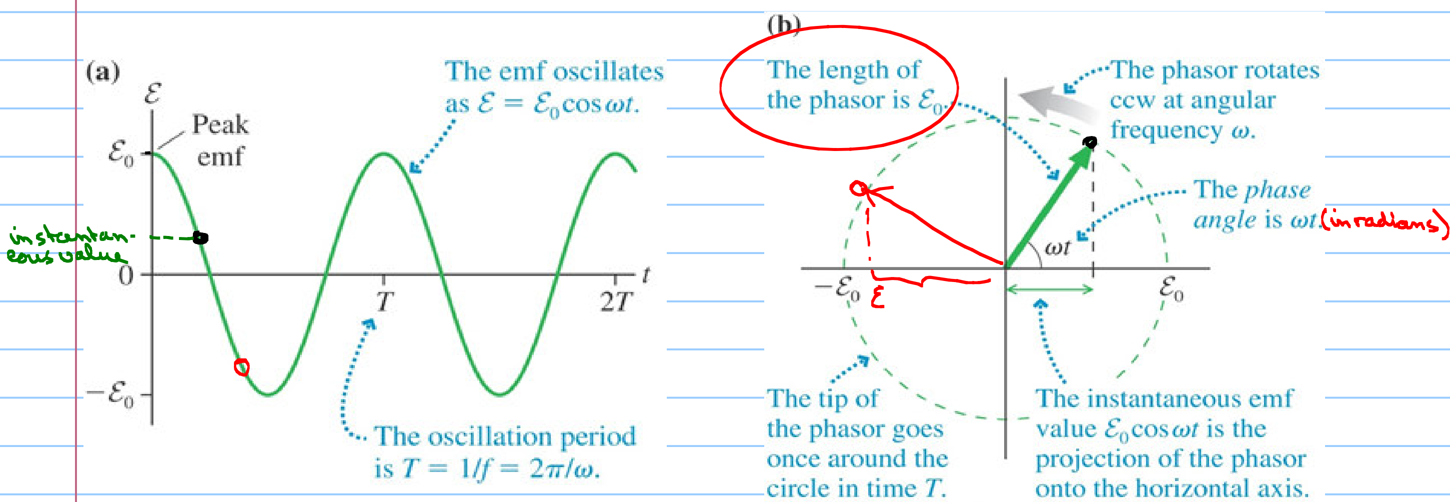
The *emf* of an AC source can be described by a sinusoidal time dependence:

$$\underbrace{\mathcal{E}}_{\text{instantaneous emf}} = \underbrace{\mathcal{E}_0}_{\text{peak emf}} \cos(\overbrace{\omega t}^{\text{phase angle}})$$

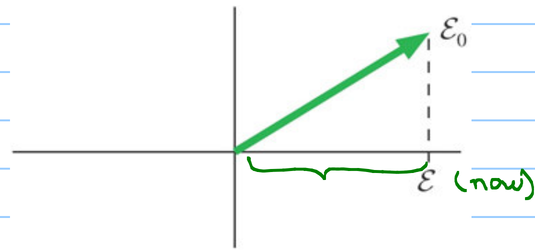
t: time

ω: angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$ (f: frequency, T: period)

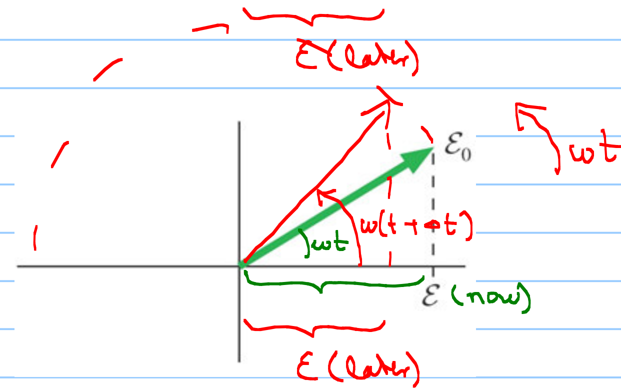
The dot in diagram (a) is represented by the phasor show in (b).



Question: The magnitude of the instantaneous value of the *emf* represented by this phasor is:



A) Increasing
B) Decreasing (B) ✓
C) Constant
D) Not possible to tell



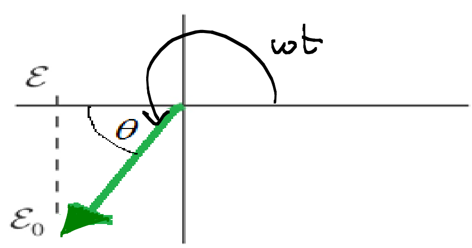
Question: If the instantaneous *emf* represented by this phasor is -5.0 V and θ is 60°, then the peak *emf* $\mathcal{E}_0$ is:

A) 10 V (A) ✓
B) 2.5 V
C) 5.8 V
D) 4.3 V

$$\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$$

$$\mathcal{E}_0 = \frac{\mathcal{E}}{\cos(\omega t)} = \frac{-5V}{\cos(240^\circ)}$$

$$= 10V$$

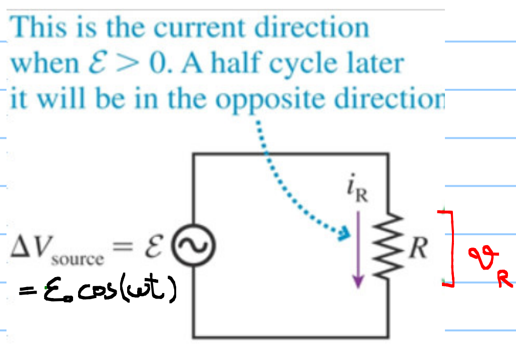


Phase shifts between i and v:
Instantaneous and Peak values:
Let us for ac applications agree on the following notation:
- small letters *v, i* and ε for instantaneous voltages, current and *emf*.
- capital letter *V* and *I* and letter ε_0 for peak values.

Let's establish the phase relation between the current *i* in the circuit and the voltage *v* across a resistor, a capacitor and an inductor.
$v_R = iR$, $v_C = q/C$, $v_L = L\, di/dt$.

Resistor circuit: Consider a resistor whose voltage varies sinusoidally as $v_R = V_R \cos(wt)$ (for example connecting it to an ac source $\varepsilon_0 \cos(wt)$).
instantaneous peak

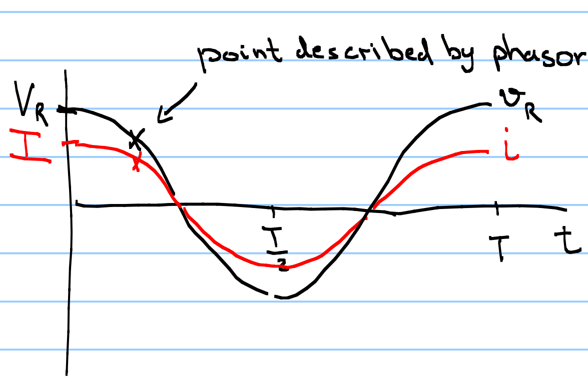
From Ohm's law:
The *instantaneous current i* through the resistor is:

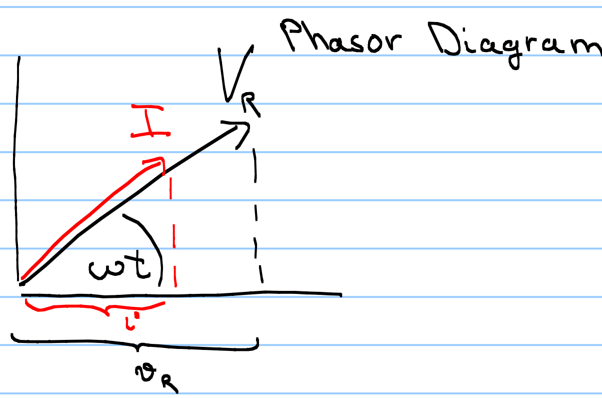


$$\boxed{i = \frac{v_R}{R} = \underbrace{\frac{V_R}{R}}_{I} \cos(\omega t)}$$

Kirchoffs loop law: $\mathcal{E} + \Delta V_R = 0$
$\mathcal{E} - \underbrace{iR}_{} = 0$
$\mathcal{E}_0 \cos \omega t - v_R = 0$

Summary: Comparing the phase angle of *i* and v_R (both are ωt) we conclude that:
The current *i* and v_R are in phase.
(*i.e.* they reach +ve maxima at same time, and reach zeros at same time)

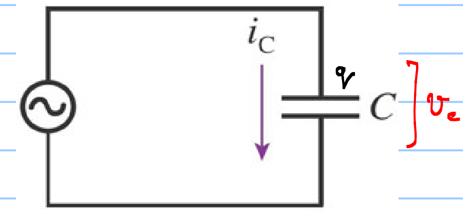




The *current peak* value *I* is:

$$\boxed{I = \frac{V_R}{R} \quad \text{and} \quad R = \frac{V_R}{I}}$$

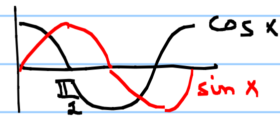
Capacitor Ciruits: Consider a capacitor whose voltage varies sinusoidally as $v_C = V_C \cos(wt)$ (by for example connecting it to an ac source).
The *instantaneous current* in the circuit can be found:



$$i = \frac{dq}{dt} = \frac{d}{dt}\left[C v_c\right] = C \frac{d}{dt}\left[V_c \cos(\omega t)\right]$$

$$= C V_c \left[-\omega \sin(\omega t)\right] = -C V_c \omega \sin(\omega t)$$

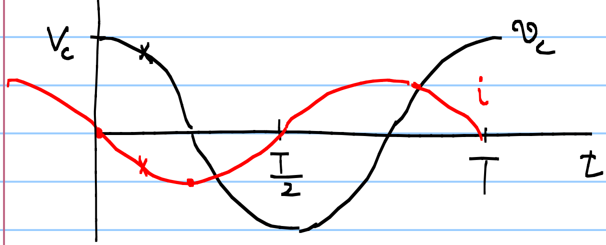
and by using $\sin(\omega t) = -\cos(\omega t + \frac{\pi}{2})$

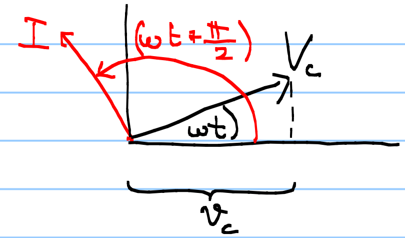


$$\boxed{\begin{aligned} i &= \underbrace{\omega C V_c}\cos(\omega t + \tfrac{\pi}{2}) \\ &= I \cos(\omega t + \tfrac{\pi}{2}) \end{aligned}}$$

[i leads v_c by $\frac{\pi}{2}$]

Summary: By comparing i and v_C, we see that:
The current i leads the capacitor voltage v_c by $\pi/2$.





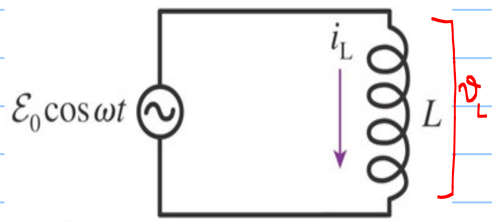
Capacitive Reactance:
The *current peak* value is $I = \omega C V_c$

$$\boxed{I = \frac{V_c}{X_c} \quad \text{with} \quad X_c = \frac{1}{\omega C}}$$

capacitive reactance

X_c is the *capacitive reactance* of the circuit [in units of ohm].
X_c decreases as ω increases.

Inductor circuits: Consider an inductor whose voltage varies sinusoidally according to $v_L = V_L \cos(wt)$ (by for example connecting it to an ac source).



$$v_L = L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v_L}{L} = \frac{V_L}{L}\cos(\omega t)$$

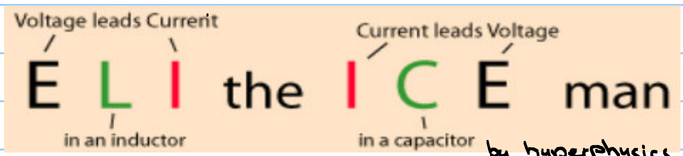
$$\int di = \int \frac{V_L}{L}\cos(\omega t)\, dt$$

$$i = \frac{V_L}{L}\frac{\sin(\omega t)}{\omega}$$

and using $\sin(\omega t) = \cos\left(\omega t - \frac{\pi}{2}\right)$

$$\boxed{i = \underbrace{\frac{V_L}{L\omega}}_{I}\cos\left(\omega t - \frac{\pi}{2}\right)}$$

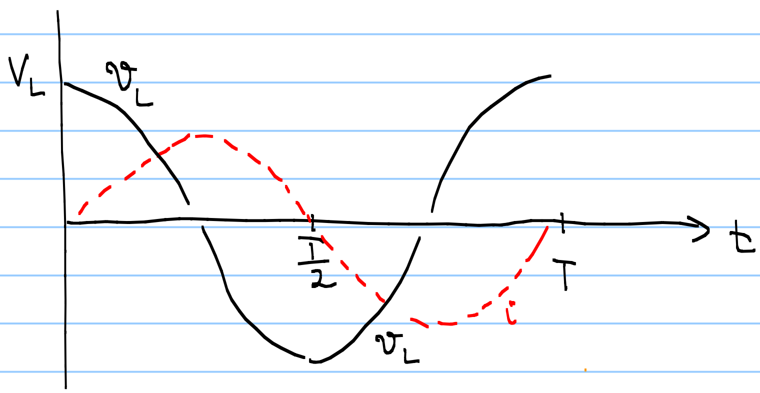
[i lags v_L by $\frac{\pi}{2}$]

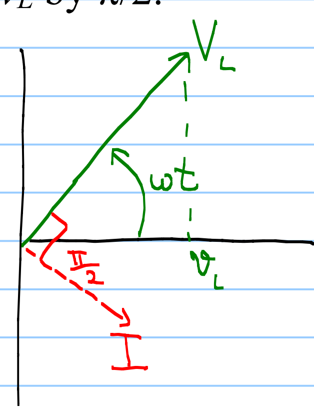


by hyperphysics

Summary: By comparing i and v_L, we conclude that:

The current i lags the inductance voltage v_L by $\pi/2$.





Inductive Reactance:

The *current peak* value is $I = \frac{V_L}{L\omega}$

$$\boxed{I = \frac{V_L}{X_L} \quad \text{with} \quad X_L = \omega L}$$

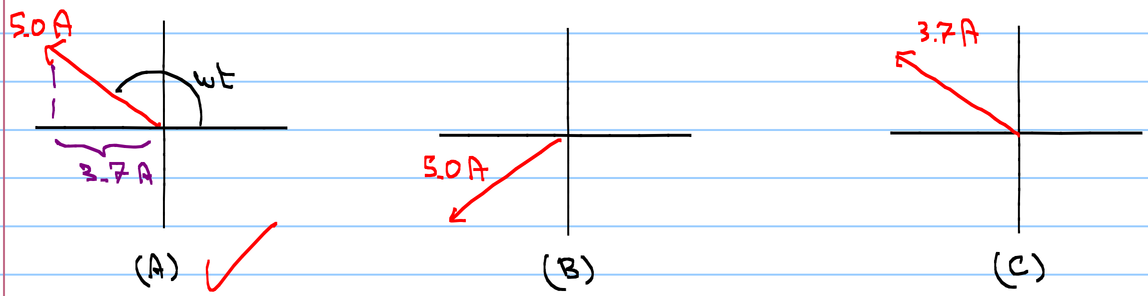
inductive reactance

X_L is the *inductive reactance* of the circuit [in units of ohm].

X_L increases as ω increases.

$\omega = 4.0$ rad/s $\qquad (i = I\cos(\omega t))$

Question: The current phasor at $t = 0.60$ s describing the current relation $\underbrace{i = 5.0}_{\text{peak current } I}\cos(4.0\,t)$ where t is in seconds and I is in A is:



$\omega t = 4t = 4 \times 0.6 = 2.4$ rad

$= 2.4\,\cancel{\text{rad}} \times \dfrac{180^\circ}{\pi\,\cancel{\text{rad}}} = 138^\circ$

Example: What is the capacitive reactance of a 0.10 μF capacitor at
(a) an 100 Hz audio frequency
(b) an 100 MHz FM-radio frequency?

$C = 0.10\ \mu F$

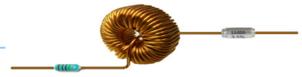
a) find X_C at $f_1 = 100$ Hz

b) find X_C at $f_2 = 100 \times 10^6$ Hz

a) $X_C = \dfrac{1}{\omega_1 C} = \dfrac{1}{(2\pi f_1)C} = \dfrac{1}{2\pi(100)\,0.1\times10^{-6}} = 16000\ \Omega$

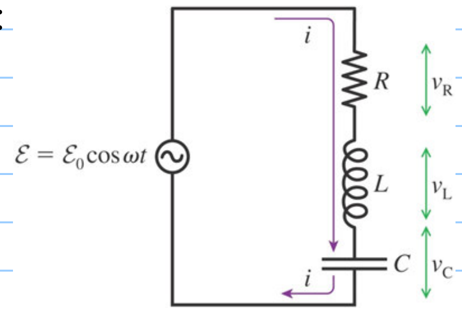
b) $X_C = \dfrac{1}{\omega_2 C} = \dfrac{1}{(2\pi f_2)C} = 0.016\ \Omega$

It is clear, a capacitor resists low frequency signals.



wiki. RLC circuit

RLC circuits:
Series RLC circuits have a resistor, capacitor and inductor connected in series to an ac source. $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$
The instantaneous voltages satisfy Kirchoff's loop law:



$$\mathcal{E} = -v_R \mp v_C \mp v_L$$

$$\mathcal{E} = iR + \frac{q}{C} + L\frac{di}{dt}$$

Let ϕ be phase angle between i and $\mathcal{E}$.

$$i = I\cos(\omega t - \phi)$$

$$\mathcal{E}_0 \cos(\omega t) = V_R \cos(\omega t - \phi) + V_C \cos\left(\omega t - \phi - \frac{\pi}{2}\right) + V_L \cos\left(\omega t - \phi + \frac{\pi}{2}\right)$$

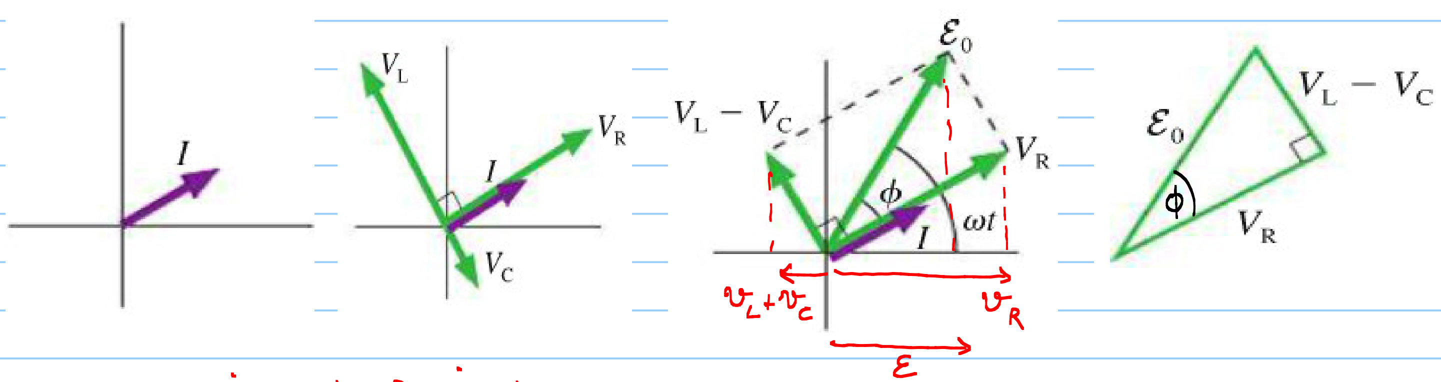
v_R is in phase with i v_C lags i by $\frac{\pi}{2}$ v_L leads i by $\frac{\pi}{2}$

The peak voltages are related to the peak current *I* by:

$$\boxed{V_R = IR \qquad V_L = IX_L, \qquad V_C = IX_C}$$

with $X_L = \omega L$ and $X_C = 1/(\omega C)$ and ω is the angular frequency of the power supply.

We can show the relations of the instantaneous voltages and current by a phasor diagram. Start by placing the current phasor at any angle.



i and v_R in phase
i lags v_L by $\frac{\pi}{2}$
i leads v_C by $\frac{\pi}{2}$

Hence the peak voltages are related to the peak *emf* by:

$$\boxed{\mathcal{E}_0 = \sqrt{V_R^2 + (V_L - V_C)^2}} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

The instantaneous potentials satisfy Kirchoff's loop law, but the peak values DO NOT.

The Peak current and the Impedence *Z*:

An RLC circuit resists the current with an *impedence Z*:

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{I = \frac{\mathcal{E}_0}{Z}}$$

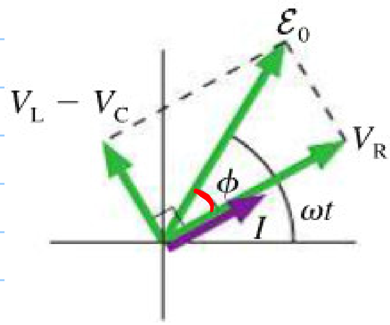
where $\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$ Impedance of an RLC circuit [in units of Ω]

The phase angle ϕ:

The current i lags the *emf* by some angle which we call the phase angle ϕ, ϕ extend from $-\pi/2$ to $+\pi/2$.

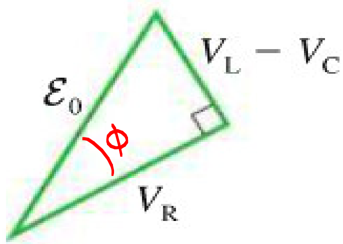
In an RLC circuit, if the *emf* is: $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$

the instantaneous current i is: $i = I \cos(\omega t - \phi)$



where $\tan\phi = \dfrac{V_L - V_C}{V_R}$

$$\boxed{\begin{aligned}\phi &= \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)\\ &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\end{aligned}}$$



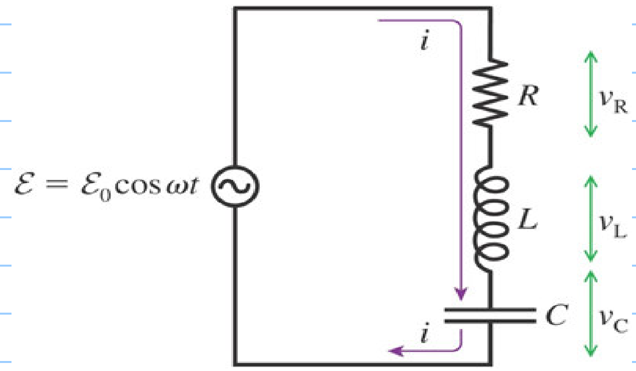
Let's put all together:
An RLC circuit with resistance R, capacitance C and inductance L in series with an ac source: $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$

has $i = I \cos(\omega t - \phi)$ phase angle ϕ

with $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

and $X_L = \omega L$ (inductive reactance)

$X_C = \frac{1}{\omega C}$ (capacitive reactance)



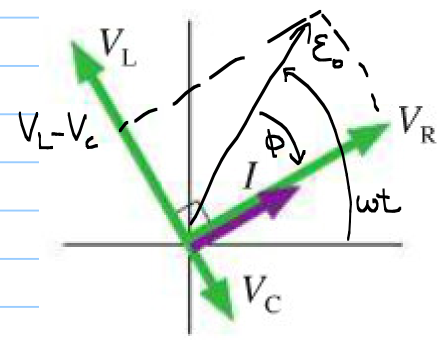
The peak values are:

$I = \frac{\mathcal{E}_0}{Z}$ ← impedance

with $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$V_R = I R$, $V_L = I X_L$, $V_C = I X_C$

$\mathcal{E}_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$



For the instantaneous values

$\mathcal{E} = v_R + v_L + v_C$ (Kirchoff's law)

$v_R = V_R \cos(\omega t - \phi)$ v_R has same phase as i

$v_L = V_L \cos(\omega t - \phi + \frac{\pi}{2})$ v_L leads i by $\frac{\pi}{2}$

$v_C = V_C \cos(\omega t - \phi - \frac{\pi}{2})$ v_C lags i by $\frac{\pi}{2}$

Root mean square values:

In ac applications, instead of stating the peak value, we can use some average value, the root mean square (*rms*).

For example, the root mean square value of the ac current I_{rms} is :

peak

$$I_{rms} = \frac{I}{\sqrt{2}} \sim 0.71\, I$$

Similarly $V_{R(rms)}$::

$$V_{R(rms)} = \frac{V_R}{\sqrt{2}} \quad \text{and} \quad \mathcal{E}_{(rms)} = \frac{\mathcal{E}_0}{\sqrt{2}}$$

Be aware how we describe a source *emf:*

$$\mathcal{E} = \mathcal{E}_0 \cos(\underbrace{2\pi \times \underset{Hz}{60}}_{\omega}\; t) \qquad \text{or} \qquad \mathcal{E} = 120V \,/\, 60\text{ Hz}$$

peak value

f

$\mathcal{E}_{rms}$

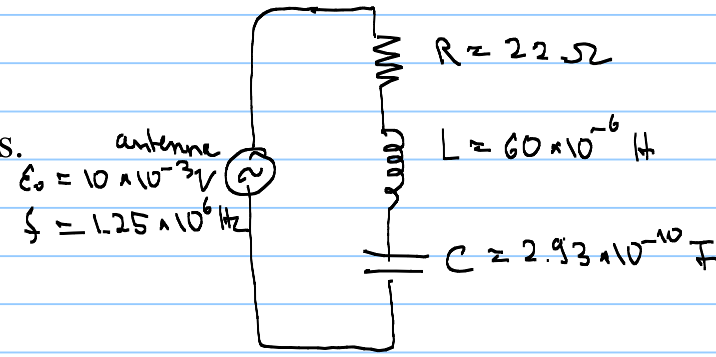
f

$$\mathcal{E}_{peak} = \mathcal{E}_{rms}\sqrt{2}$$

Example: The antenna of a series *RLC* circuit receives a 1.25 MHz signal with a peak voltage of 10.0 mV. The circuit consists of a 60.0 μH inductor, 2.93 x 10^{-10} F capacitor and 22.0 Ω resistance. Find

a) The angular frequency.

$\omega = 2\pi f = 7.854 \times 10^{6}$ rad/s



b) The inductive and capacitive reactances.

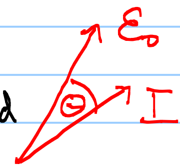
$X_L = \omega L = 471.2\ \Omega$

$X_C = \frac{1}{\omega C} = 434.6\ \Omega$

c) The phase angle ϕ in rad.

$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = 58.99° = 59.0°$

calculator on rad $= 1.03$ rad



d) The impedance .

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 42.70 = 42.7\ \Omega$

e) The peak current .

$I = \frac{\mathcal{E}_0}{Z} = \frac{10 \times 10^{-3}}{42.7} = 2.342 \times 10^{-4}$ A

f) The 3 peak potentials.

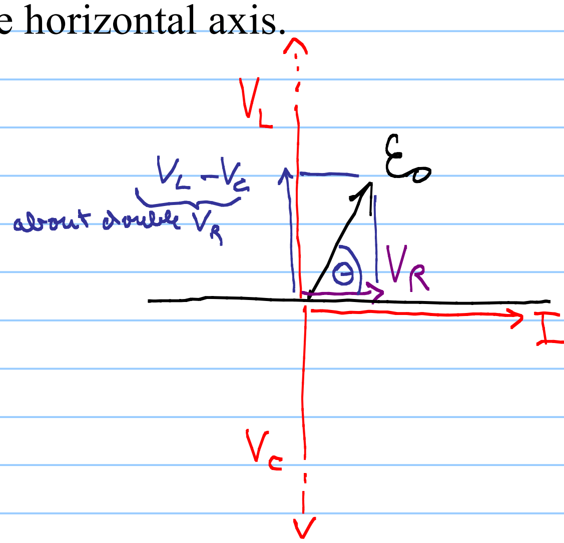
$V_R = I R = 51.52 \times 10^{-4}$ V

$V_C = I X_C = 1018 \times 10^{-4}$ V

$V_L = I X_L = 1104 \times 10^{-4}$ V

$\mathcal{E}_0 = 100 \times 10^{-4}$ V

g) Plot the phasor diagram, showing all 4 potential phasors and the current phasor. Place the current phasor on the positive horizontal axis.



h) Find the resonance frequency of the circuit? This would be the frequency
that corresponds to the minimum impedance.

$$Z = \sqrt{R^2 + \underbrace{(X_L - X_C)^2}}$$

at resonance : $X_L - X_C = 0$ and Z is minimum $Z = R$

$$X_L - X_C = 0$$

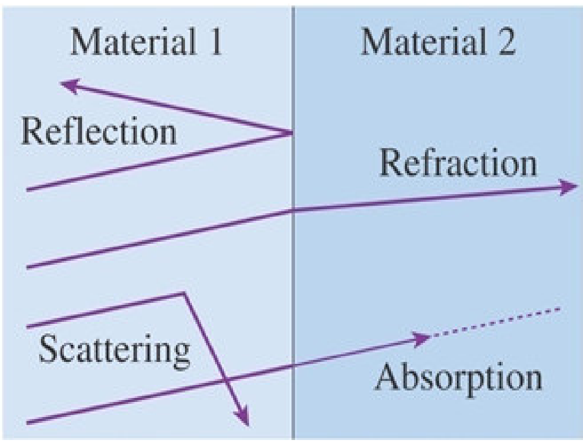
$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 1.20 \times 10^6 \text{ Hz}$$

Ray Optics (chapter 34):
- Reflection
- Refraction
- Total internal reflection
- Thin lenses

The Ray Model of Light:
Light travels in a straight line.
When it crosses a border its direction changes.



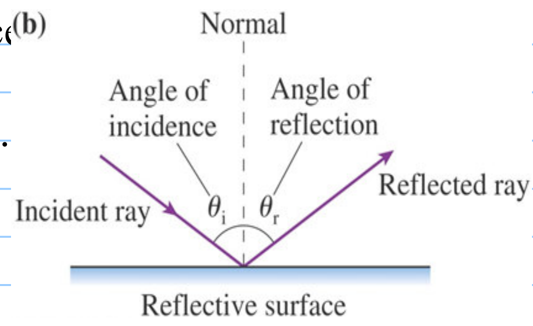
Reflection:

- The incident ray and the reflected ray are in the same plane normal to the surface. (called *plane of incidence*). *The plane of incidence* is the plane containing both the incident and reflected rays. *It is normal to the surface*.

- The angle of reflection equals the angle of incidence

$$\boxed{\theta_r = \theta_i}$$

with both angles defined with respect to the normal.

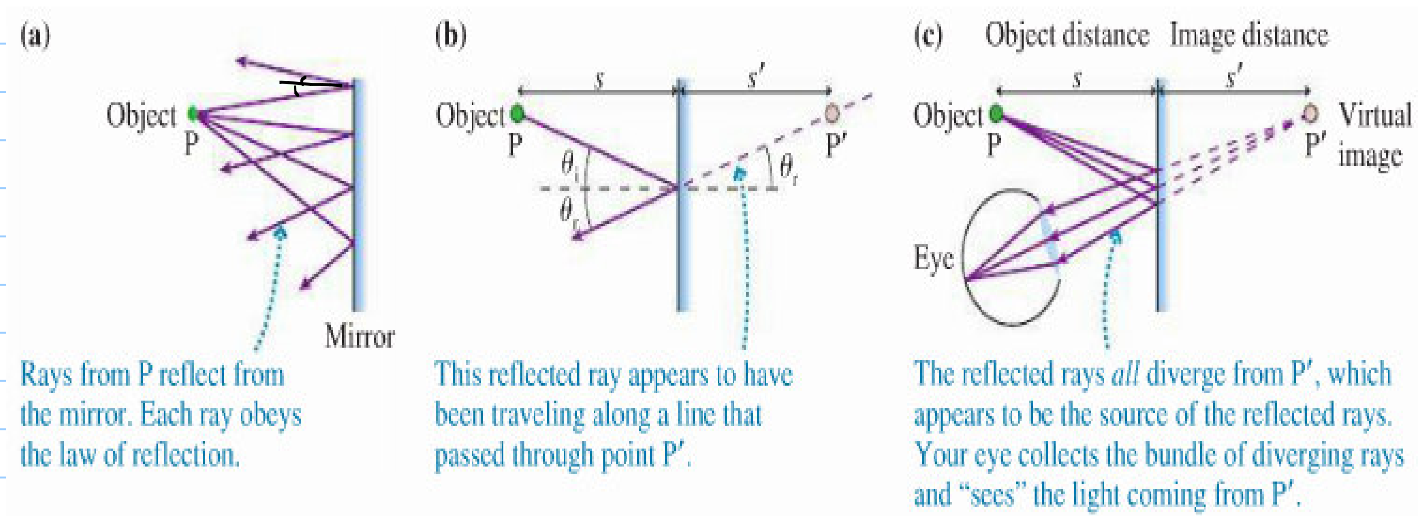


The Plane Mirror:

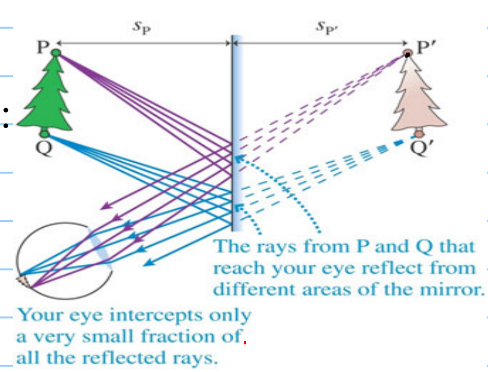
Consider a point source at point P. The eye sees the image as coming from P' behind the mirror.

For a plane mirror: $\boxed{s' = s}$

image distance (image to mirror) object distance (object to mirror)

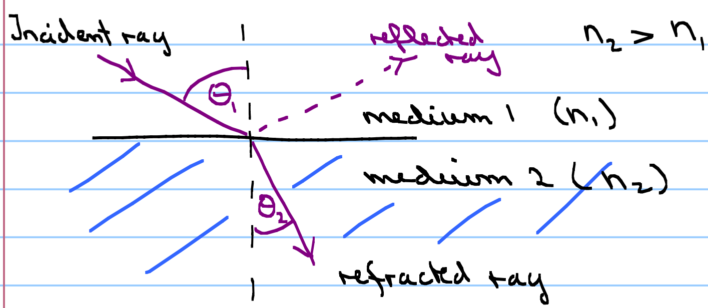


For an extended body, each point acts as a point source:



Refraction:
When light is incident on a smooth boundary between two transparent materials
- part of the light is *reflected*.
- part of the light is *refracted*, *i.e.* it is transmitted into the other material at a different angle.

Consider a ray incident at an angle θ_1 from medium 1 of refractive index n_1 onto medium 2 of refractive index n_2.

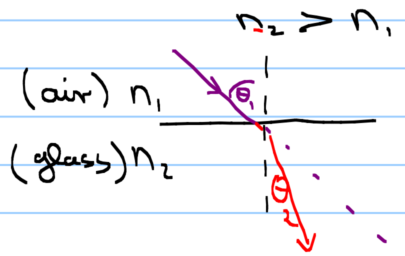


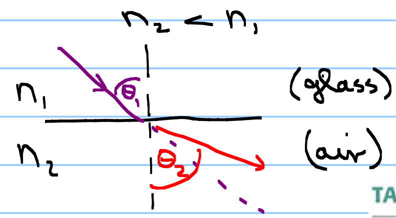
Snell's law relates the angle of refraction to the angle of incidence.

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Snell's law of refraction

So, if $n_2 > n_1$, the ray bends towards the normal and *vice versa*.





The index of refraction is related to the speed of light:

$$\boxed{n = \frac{c}{v_{medium}}}$$

where c, the speed of light is 3.0×10^8 m/s

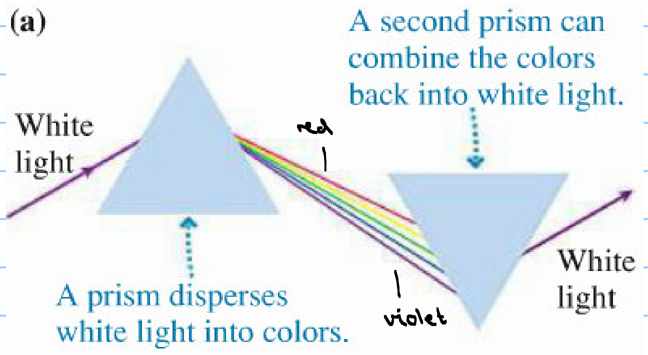
TABLE 23.1 Indices of refraction

Medium	n
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

Color:

White light consists of several color, each has a different wavelength.

- In air, all colors travel at the same speed $c = 3x10^8$ m/s. ($n_{air} = 1$)
- In glass, violet travels slightly slower than red.
- $n_{glass\text{-}violet}$ is slightly greater than $n_{glass\text{-}red}$ ($n = c / v$).
- A prism disperses white colors into the different colors, violet is deflected stronger than red from its original path when emerging from the prism after being refracted twice.

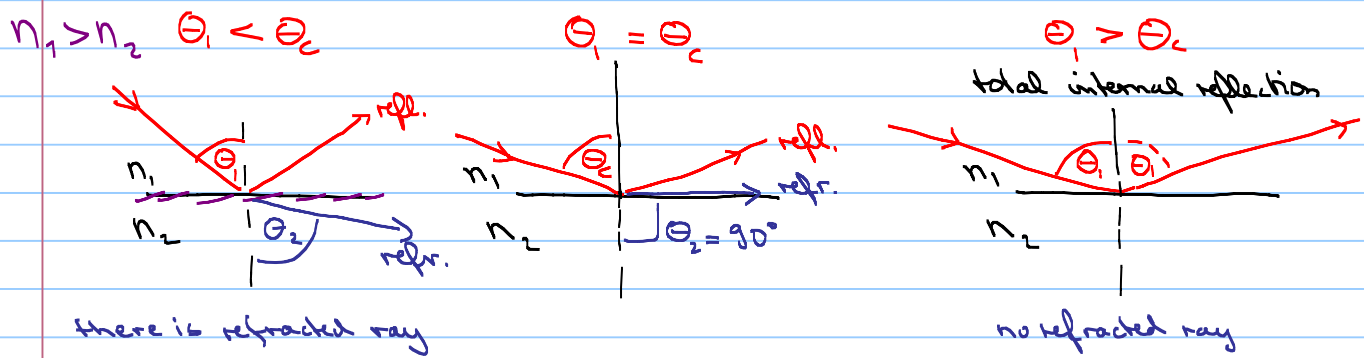


Total internal reflection:

Consider a ray incident from medium n_1 into medium n_2, where $n_1 > n_2$.

Hence the refracted angle θ_2 is larger than the incident angle θ_1. $\theta_2 > \theta_1$.

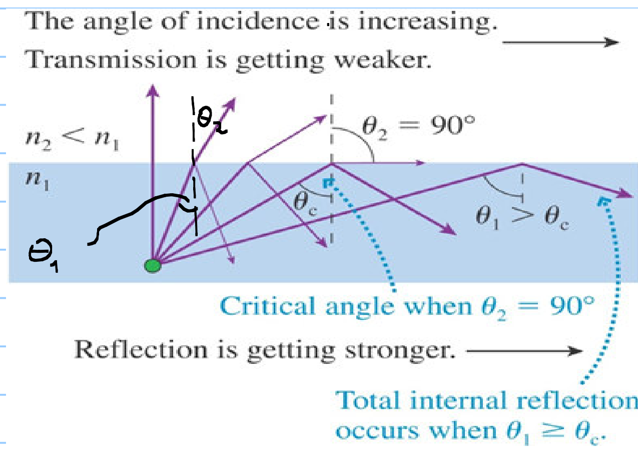
As θ_1 is increased, θ_2 will eventually reach 90º, beyond which refraction ceases.



This specific angle of incidence is called the critical angle θ_c:

When $\theta_i > \theta_c$:

- no ray will be transmitted.
- the ray undergoes *total internal reflection*.



From Snell's law:

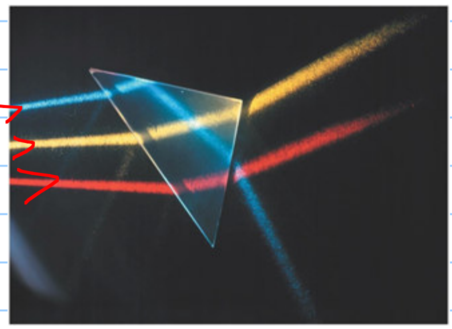
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

At the critical angle ($\theta_1 = \theta_c$), $\theta_2 = 90^0$.

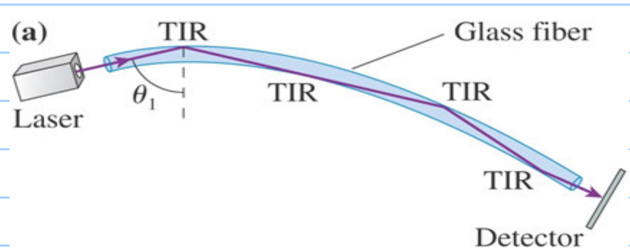
$$n_1 \sin\theta_c = n_2 \underbrace{\sin 90^\circ}_{1}$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$ critical angle

For $\theta_1 > \theta_c$ we get total internal reflection

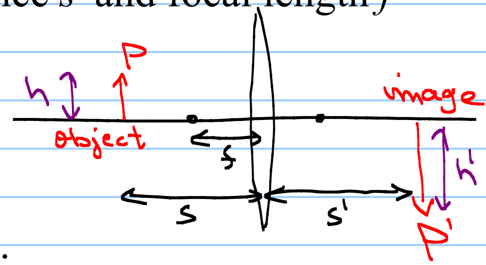


Total internal reflection is used for example to transmit light through *fibre optics cables*.



Thin-lens equation:
For thin lenses: The object distance s and image distance s' and focal length f are related:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$ (thin-lens equation)



s' is positive for real image, negative for virtual image.
f is positive for converging lens, negative for diverging lens.

The magnification is the ratio of the image height h' to the object height h:

$$\boxed{h' = |m|\, h}$$

The magnification m is given by:
m is positive for upright image, negative for inverted image.

$$\boxed{m = -\frac{s'}{s}}$$

TABLE 23.4 Sign convention for thin lenses

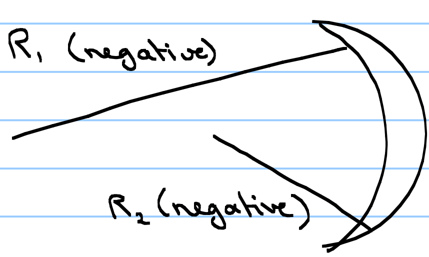
	Positive	Negative
R_1, R_2	Convex toward the object	Concave toward the object
f	Converging lens, thicker in center	Diverging lens, thinner in center
s'	Real image, opposite side from object	Virtual image, same side as object
m	Upright image	Inverted image

Remember!!!

The focal length is related to the radii R of the lens surfaces:

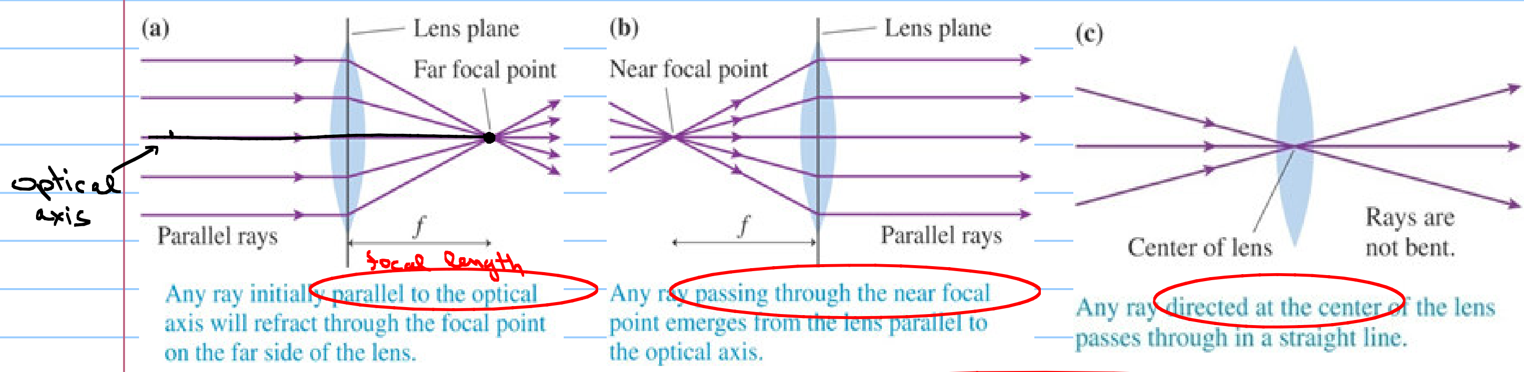
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ (lens maker's equation)

object placed at this side



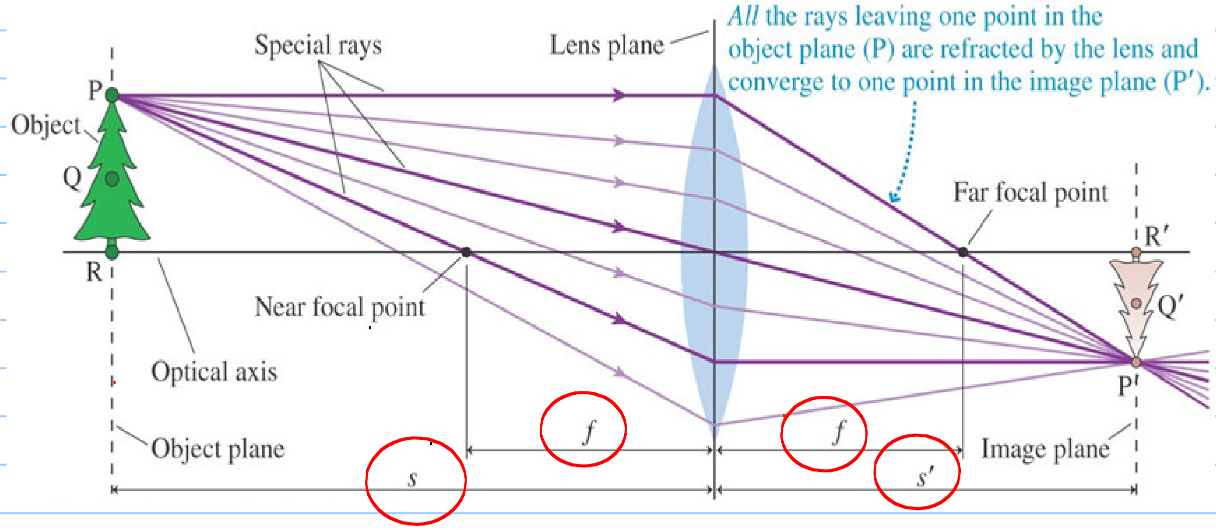
(converging lens)

Converging lenses:
Consider a thin converging lens of focal length f (f is positive).
Notice the three important types of light rays.



a converging lens has positive focal length f

Case $s > f$:
For an object (at P) at a distance s from the lens, with $s > f$ (the near focal point):
- Draw three rays leaving the object (point P): (two are sufficient)
    1- parallel to the optical axis
    2- through the center of the lens
    3- through the near focal point
- The image is where the three rays meet.



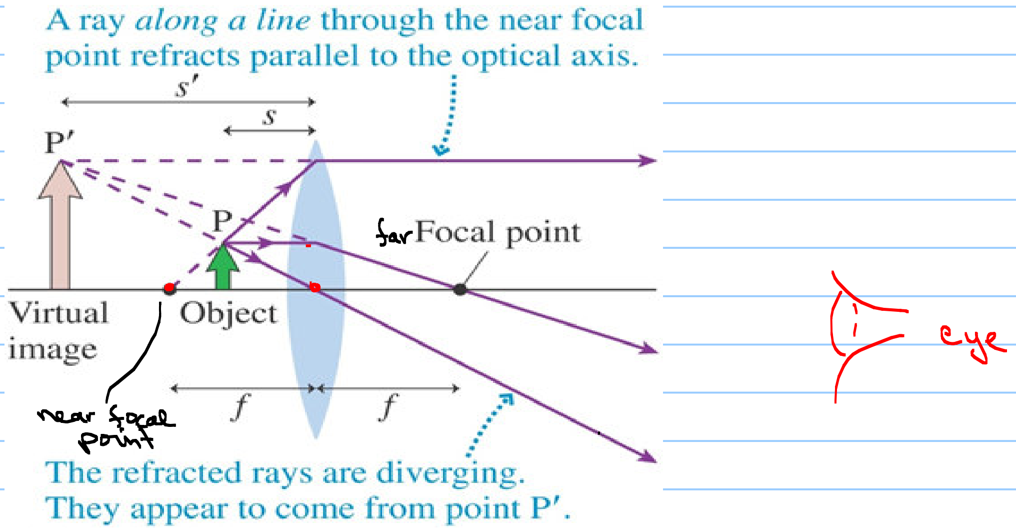
For object distance $s > f$ — f is +ve for converging lens
- a *real image* is formed on the *opposite side* from the lens.
- the image is *inverted*.
- the image can be larger, smaller or equal in size.
- the image distance s' is defined to be positive.

real

Case $s < f$:

Now place the object at a distance $s < f$.

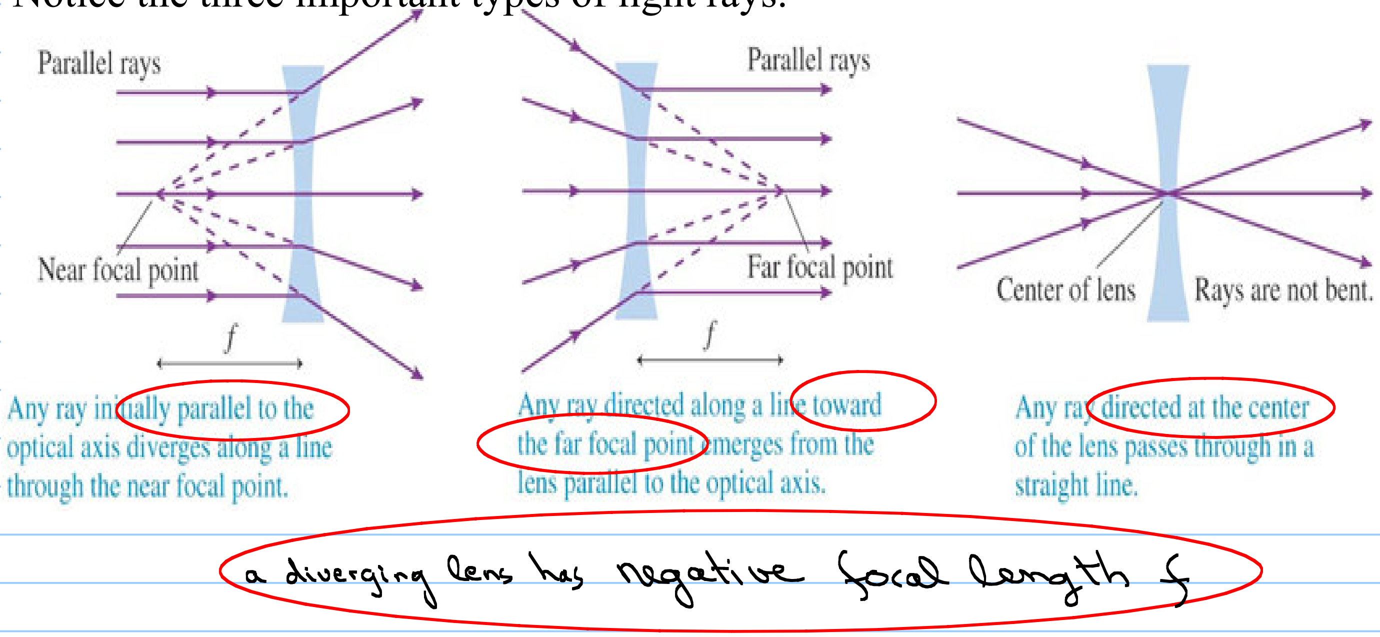
- Draw three rays leaving the object (point P):
  1- parallel to the optical axis
  2- through the center of the lens
  3- as if coming from the near focal point
- The three rays will not meet, but it seems as if they are emitted from same point P'.



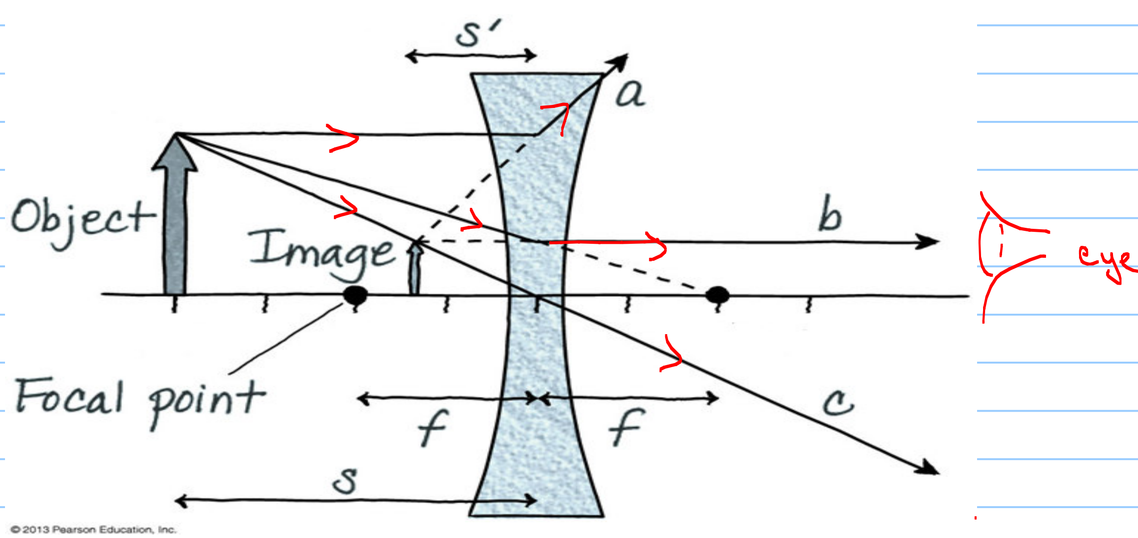
For object distance $s < f$

- a *virtual image* is formed on the *same side* of the object.
- the image is *upright*.
- the image is *magnified*.
- the image distance s' is defined to be negative.

Diverging lenses:
Consider a thin converging lens of focal length *f* (*f* is negative).
Notice the three important types of light rays.

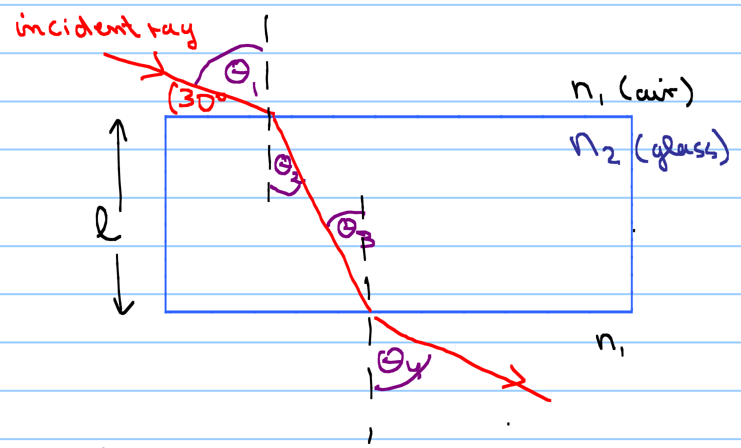


- Draw three rays leaving the object (point P):
  1- parallel to the optical axis
  2- through the center of the lens
  3- directed along a line toward the far focal point
- The three rays will not meet, but it seems as if they are emitted from same point P'.



- A *virtual image* is formed on the *same side* of the object.
- the image is *upright*.
- the image is *smaller*.

Example: A laser beam is aimed at a 1.0 cm thick sheet of glass at an angle 30.0° with the horizontal. The index of refraction *n* of air = 1 and *n* of glass = 1.5. Find the angle with the vertical at which the beam is transmitted.



$\ell = 0.01\ m$
$\alpha = 30°$

$n_1 = n_{air} = 1.0$
$n_2 = n_{glass} = 1.5$

Snell's law at the upper surface

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ with $\theta_1 = 90 - \alpha = 60°$

$\sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin(\theta_1)\right] = \sin^{-1}\left[\frac{1}{1.5}\sin(60°)\right] \simeq 35.26°$$

At the bottom surface.

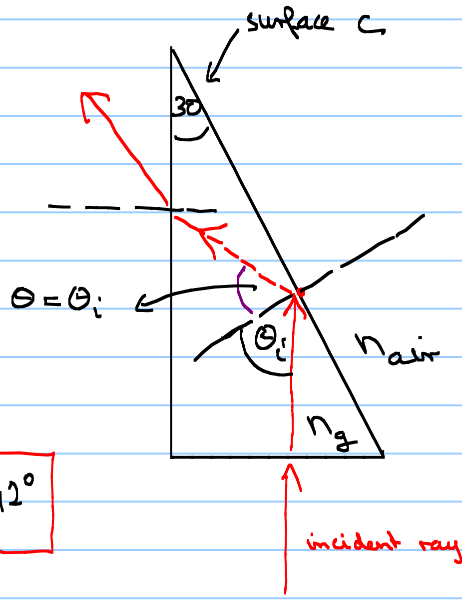
$n_2 \sin\theta_3 = n_4 \sin\theta_4$ with $\theta_3 = \theta_2$

$$\theta_4 = \sin^{-1}\left[\frac{n_2}{n_1}\sin(\theta_2)\right] = 60°$$ with the normal.

Example: A laser beam enters - starting from air (n = 1.0) - a glass prism (n = 1.5), as shown. Find the critical angle at the glass/air interface (surface C) and show the path of the ray until it exits the prism.

At surface C

$\Theta_i = 60°$



Snell's law:

$$\underbrace{n_g \sin \Theta_g}_{\text{incident ray}} = \underbrace{n_{air} \sin \Theta_{air}}_{\text{refracted}}$$

At critical angle

$$n_g \sin \Theta_c = n_{air} \underbrace{\sin 90°}_{1}$$

$$\boxed{\Theta_c = \sin^{-1}\left(\frac{n_{air}}{n_g}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 42°}$$

For our pb.

$\Theta_i > \Theta_c \longrightarrow$ total internal reflection.

i.e. no refracted ray

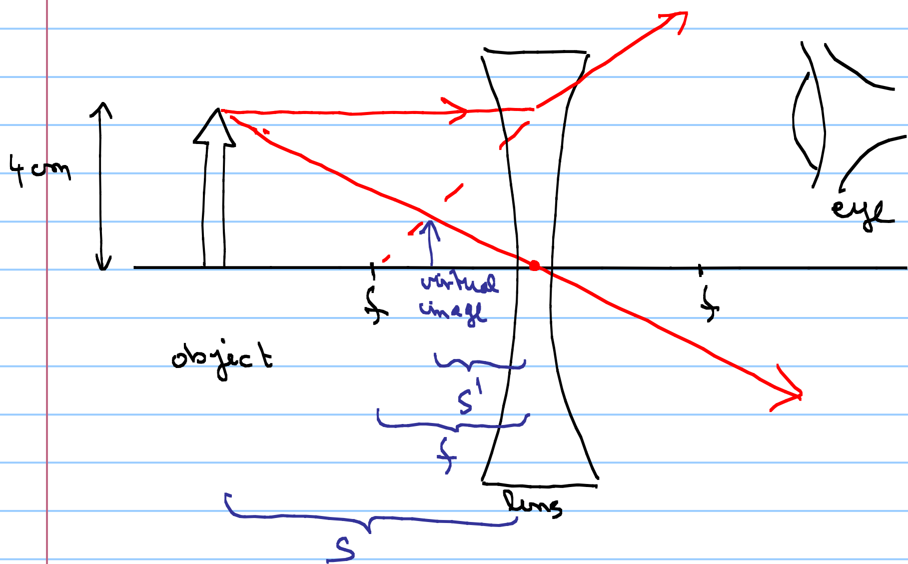
at bottom surface

$$n_{air} \underbrace{\sin \Theta_{air}}_{\sin 0 = 0} = n_g \sin \Theta_g$$

$\rightarrow \sin \Theta_g = 0$

$\Theta_g = 0$

Questions: A 4.00 cm long object is placed 100. cm in front of a diverging lens as shown in figure. The magnitude of the focal length is 50.0 cm.
Find the location of the image s', magnification m, and height h'.
trace the light rays starting at the object and forming the real or virtual image.



from sketch

$s = 100$ cm

$f = -50$ cm

↑ diverging

$h = 4$ cm

find s', m, h'

The image distance s' (including sign) is:
a) 100 cm
b) -100 cm
c) 33.3 cm
d) -33.3 cm (d) ✓

↑ virtual, on same side as object

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-50} - \frac{1}{100} = \frac{-3}{100}$$

$$s' = -33.3 \text{ cm}$$

The magnification is:
a) 0.33 (a) ✓
b) - 0.33
c) 3.0
d) -3.0

+ve → upright

$$m = \frac{-s'}{s} = \frac{-(-33.3)}{100}$$

always +ve

The height is:
a) 0.33 cm
b) - 0.33 cm
c) 1.3 cm (c) ✓
d) -1.3 cm

$$h' = |m|\, h$$

$$= 0.33 \times 4 \text{ cm}$$