The Maths of Predators and Preys

Don’t get Eaten by the Wolf

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frontiers FOR YOUNG MINDS

MATHEMATICS
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THE CIRCLE OF LIFE: THE MATHEMATICS OF PREDATOR-PREY RELATIONSHIPS

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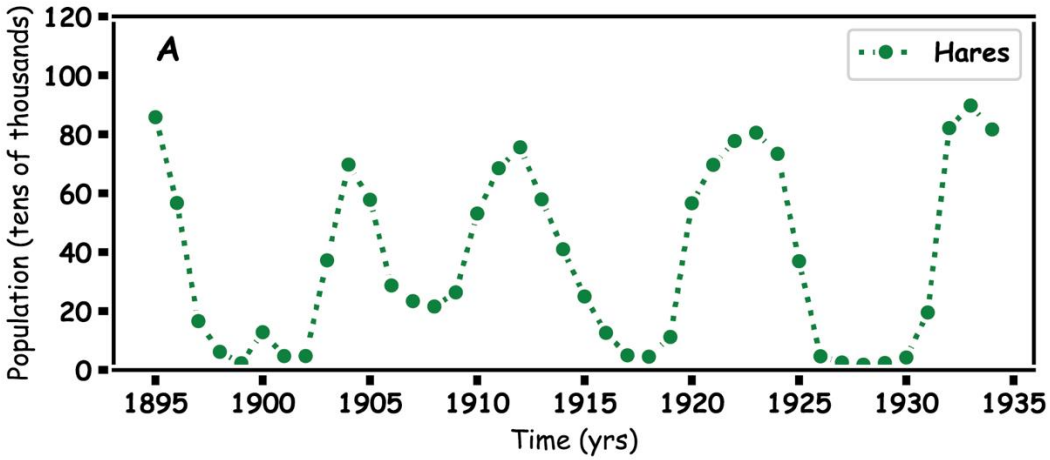
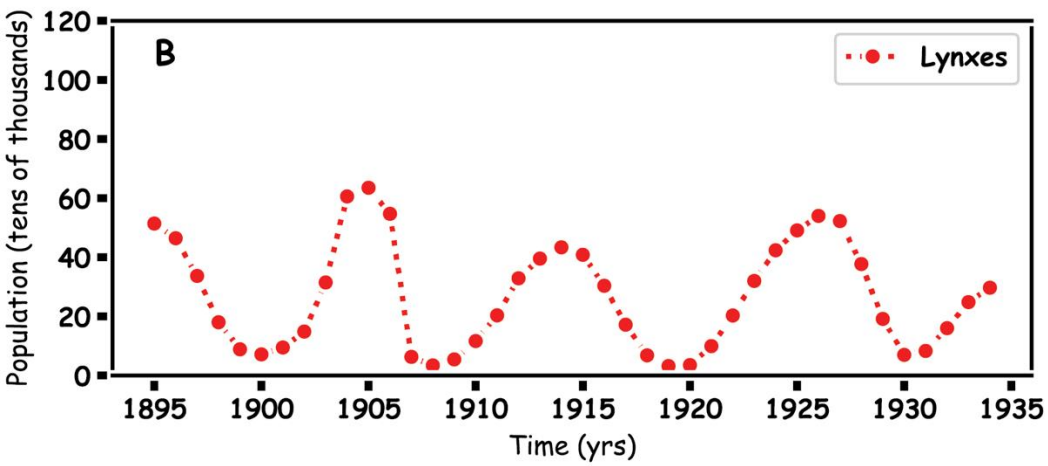
Predator Prey

Hares and Lynx

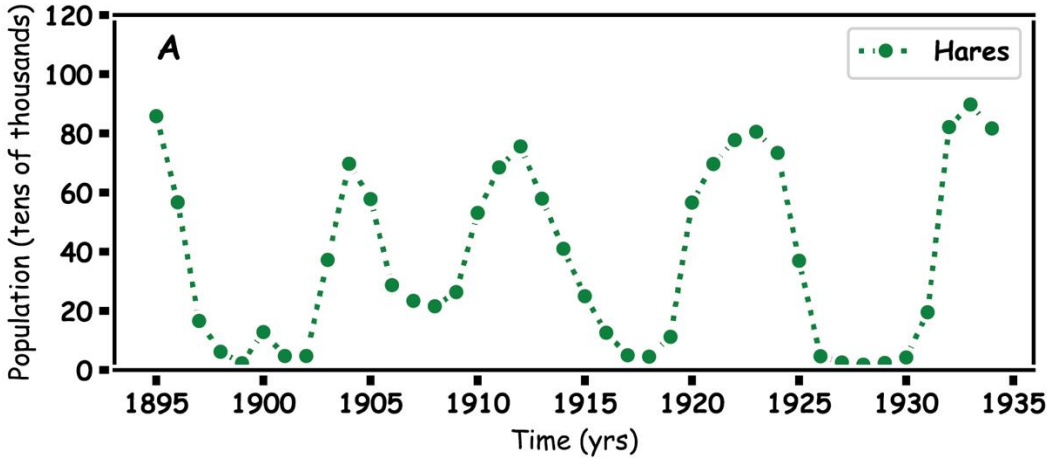
Prey Hares

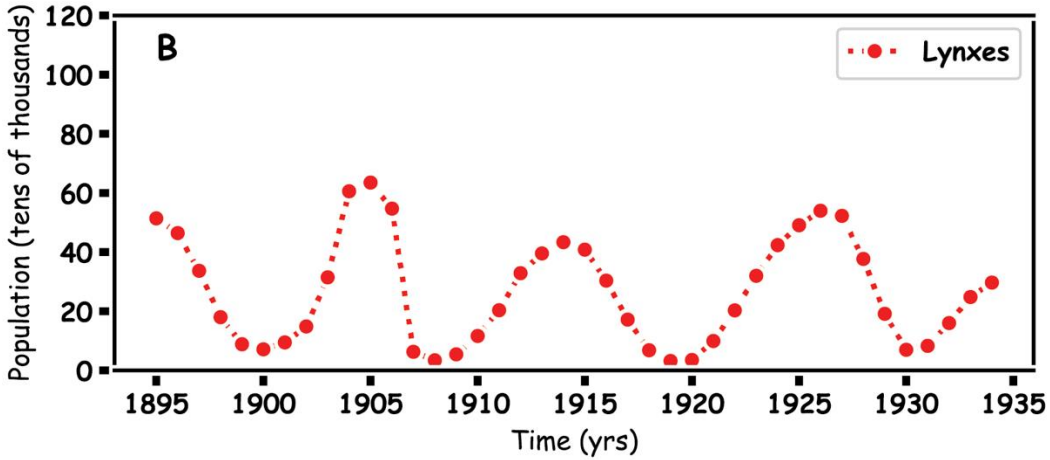

A
Hares
Population (tens of thousands)
120
100
80
60
40
20
0
1895
1900
1905
1910
1915
1920
1925
1930
1935
Time (yrs)

Predator


B
Lynxes
Population (tens of thousands)
120
100
80
60
40
20
0
1895
1900
1905
1910
1915
1920
1925
1930
1935
Time (yrs)

Predator and Prey

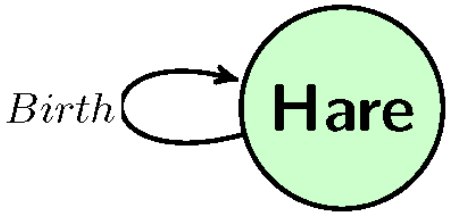


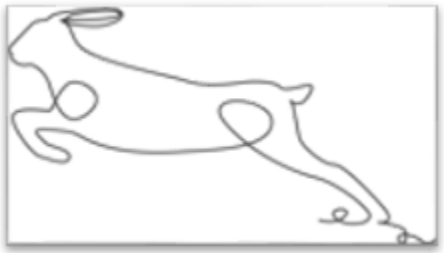


Mathematics

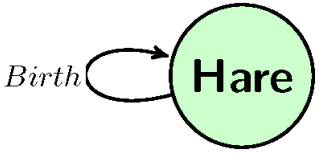
Hares

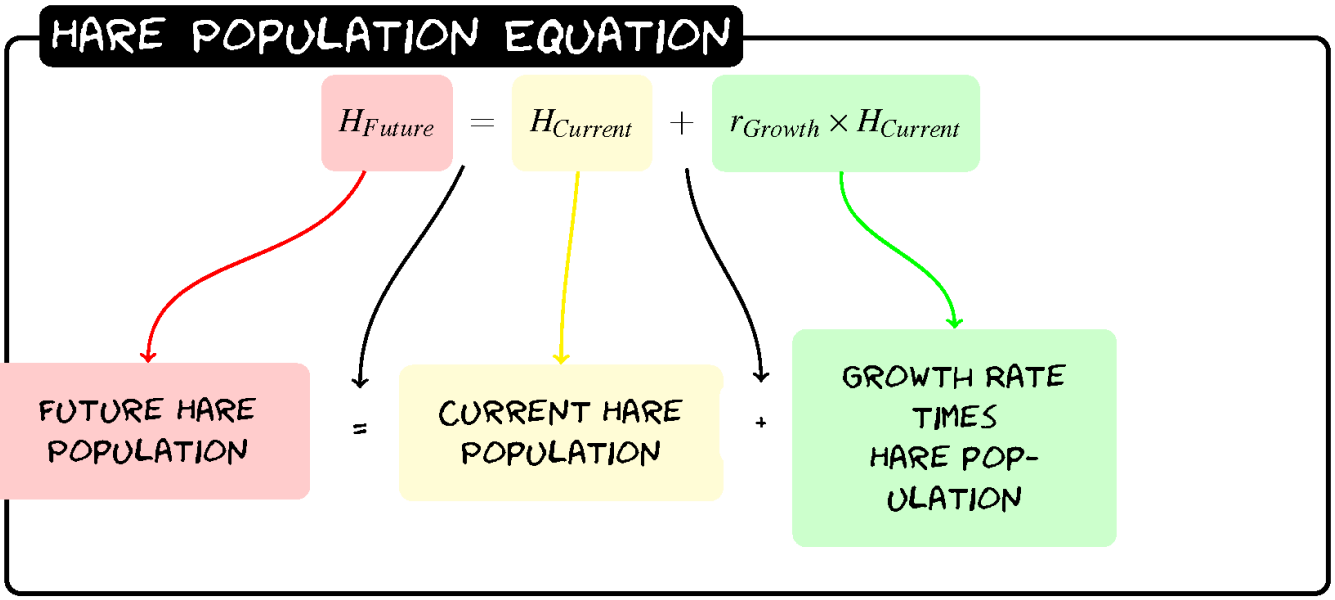
- How to we make a model for a Hare population



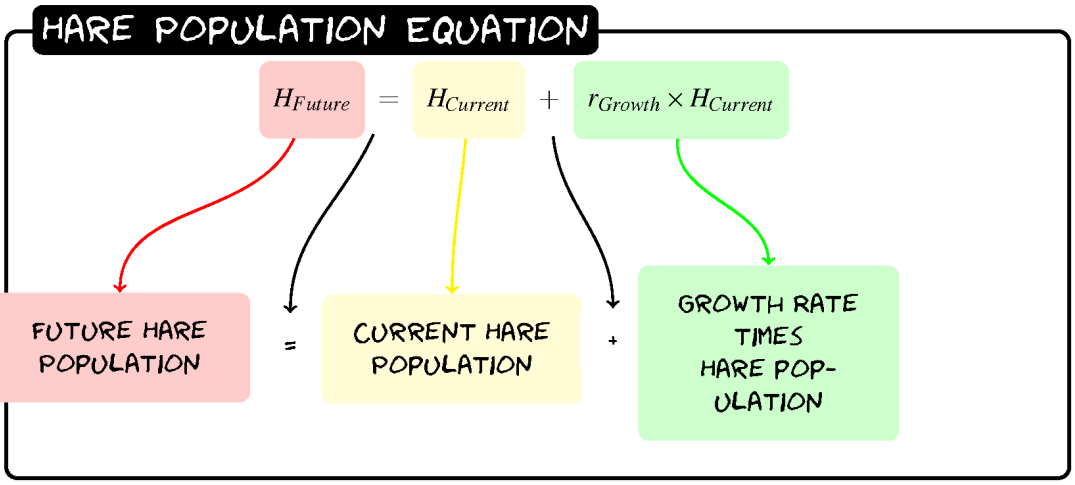


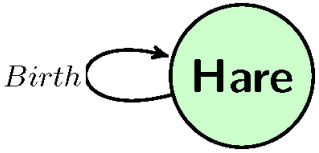
Hares


Birth
Hare


HARE POPULATION EQUATION
$H_{Future} = H_{Current} + r_{Growth} \times H_{Current}$
FUTURE HARE POPULATION
=
CURRENT HARE POPULATION
+
GROWTH RATE TIMES HARE POP-ULATION

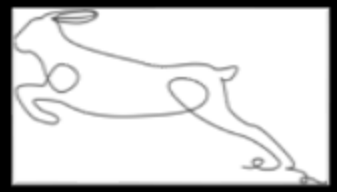
Hares



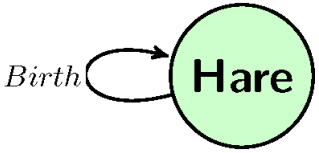


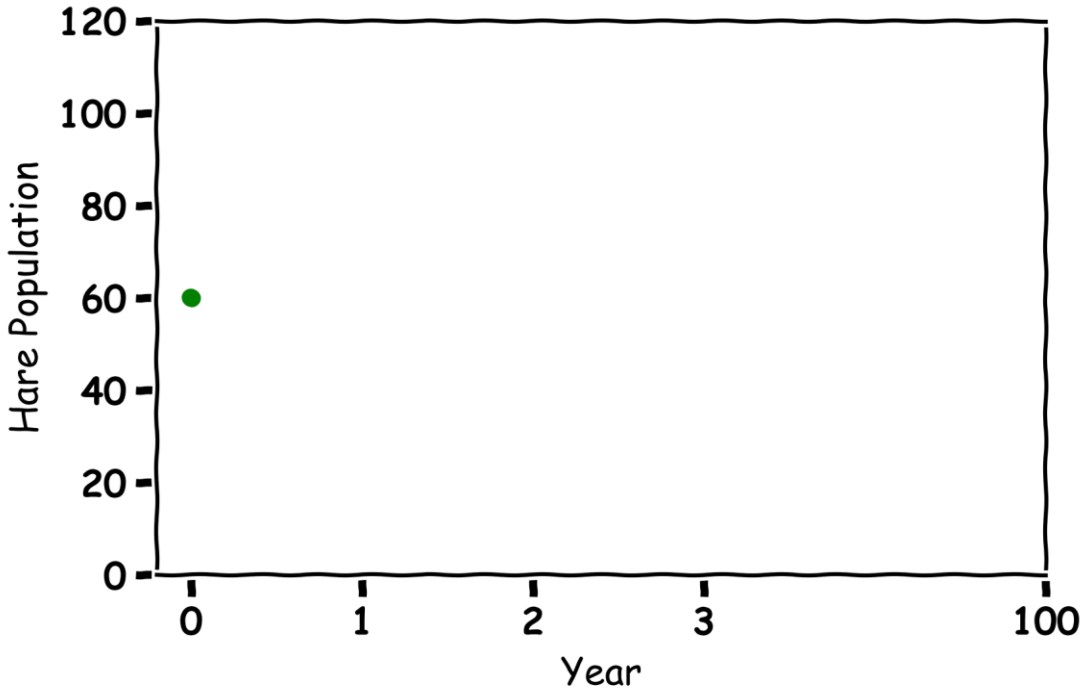
Year	Hare Population
0	60
1	
2	
3	

Hares

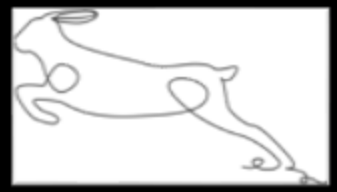


Year	Hare Population
0	60
1	60+0.2(60)=72
2	
3	

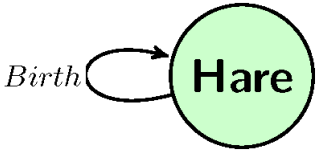


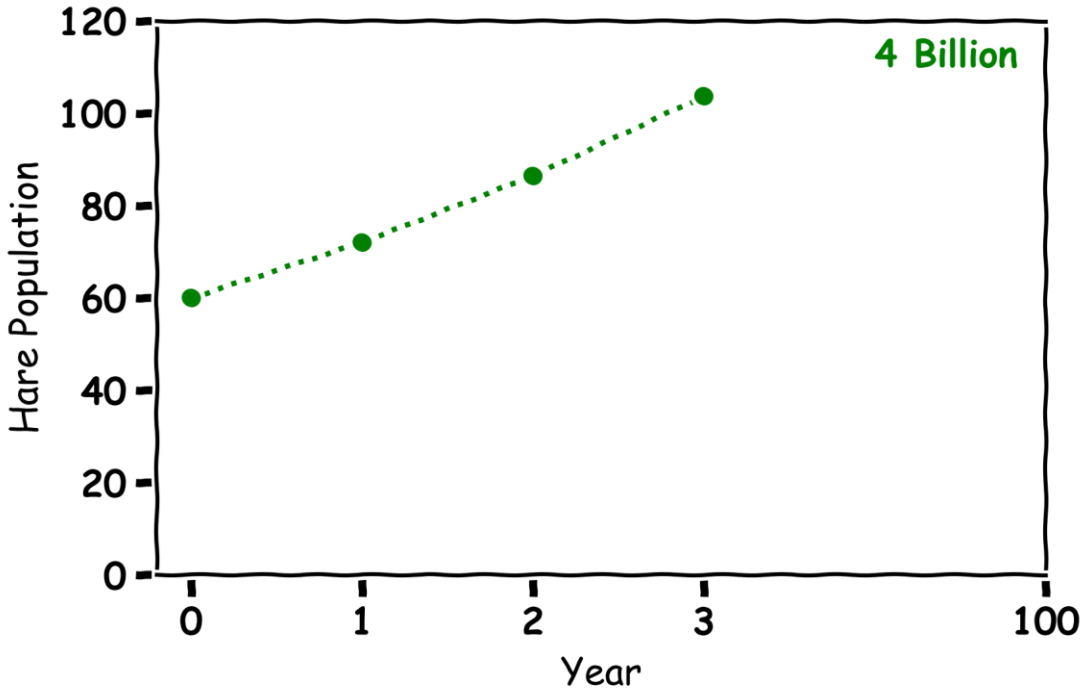


Hares



Year	Hare Population
0	60
1	72
2	86
3	103

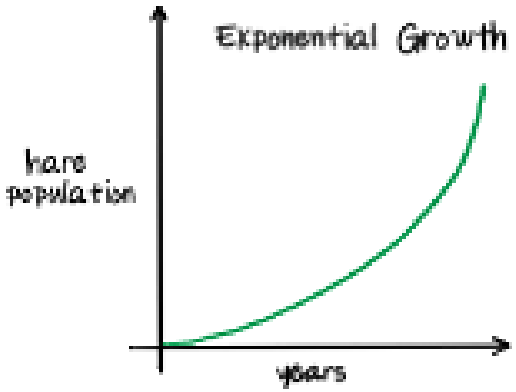




Hare Population

Ⓐ Hare Population

H_c : current hare population H_F : Future hare population



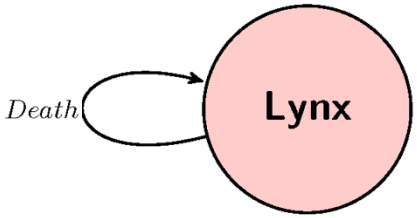
r_{growth} = Births - Deaths

Change in hare population = $r_{growth} \times H_c$

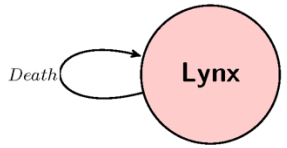
$H_F = H_c$ + Change in hare population

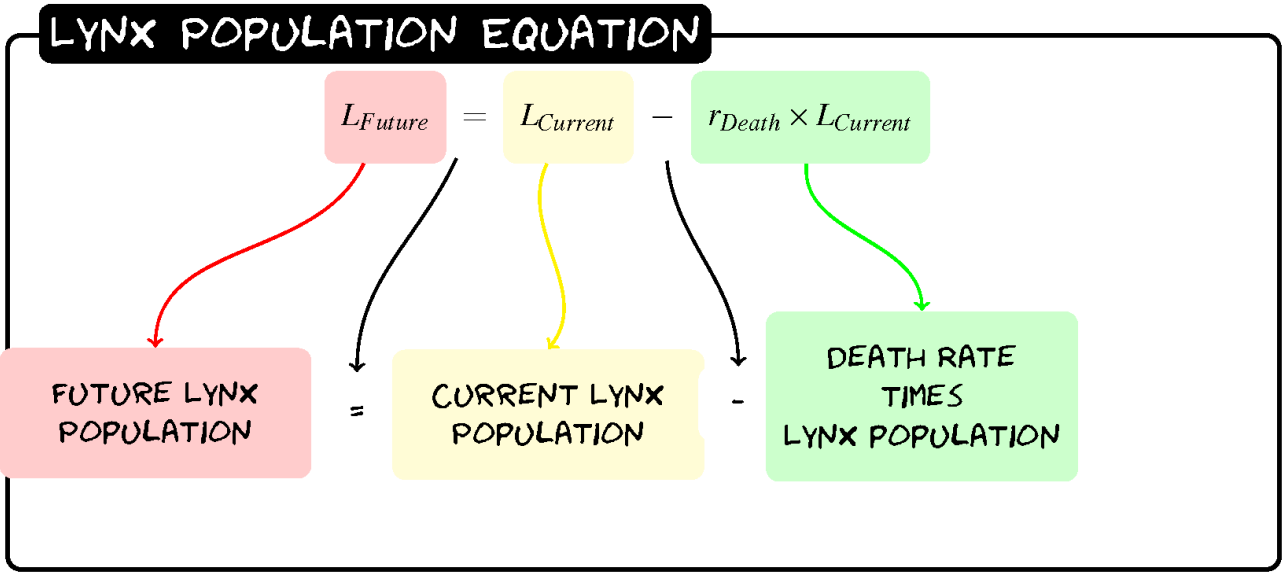
Lynx

- How to we make a model for a Lynx population

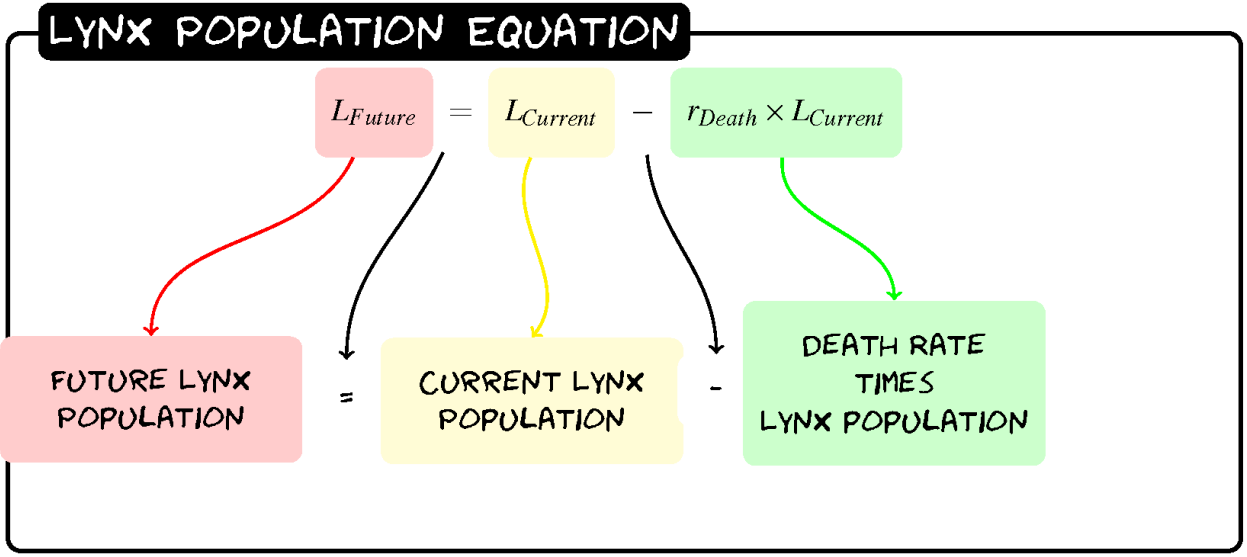


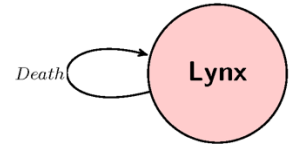
Lynx


Death
Lynx


LYNX POPULATION EQUATION
$L_{Future} = L_{Current} - r_{Death} \times L_{Current}$
FUTURE LYNX POPULATION
=
CURRENT LYNX POPULATION
-
DEATH RATE TIMES LYNX POPULATION

Lynx

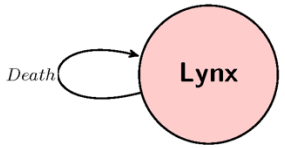


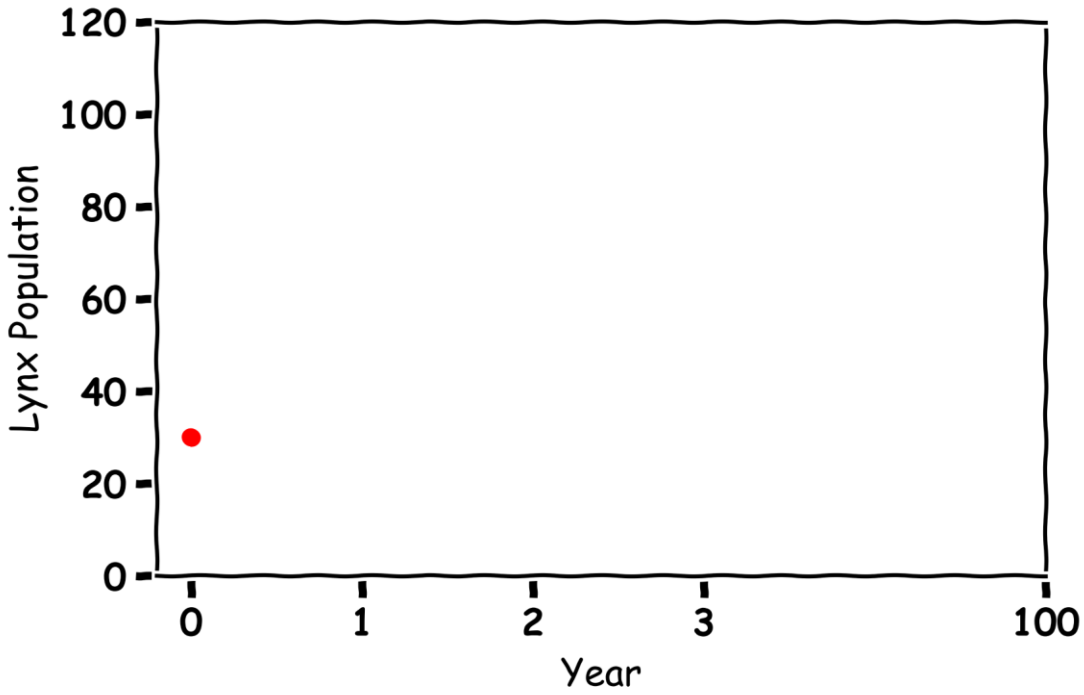


Year	Lynx Population
0	30
1	
2	
3	

Lynx

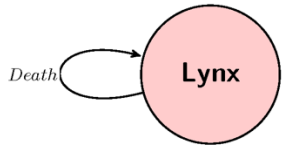
Year	Lynx Population
0	30
1	
2	
3	

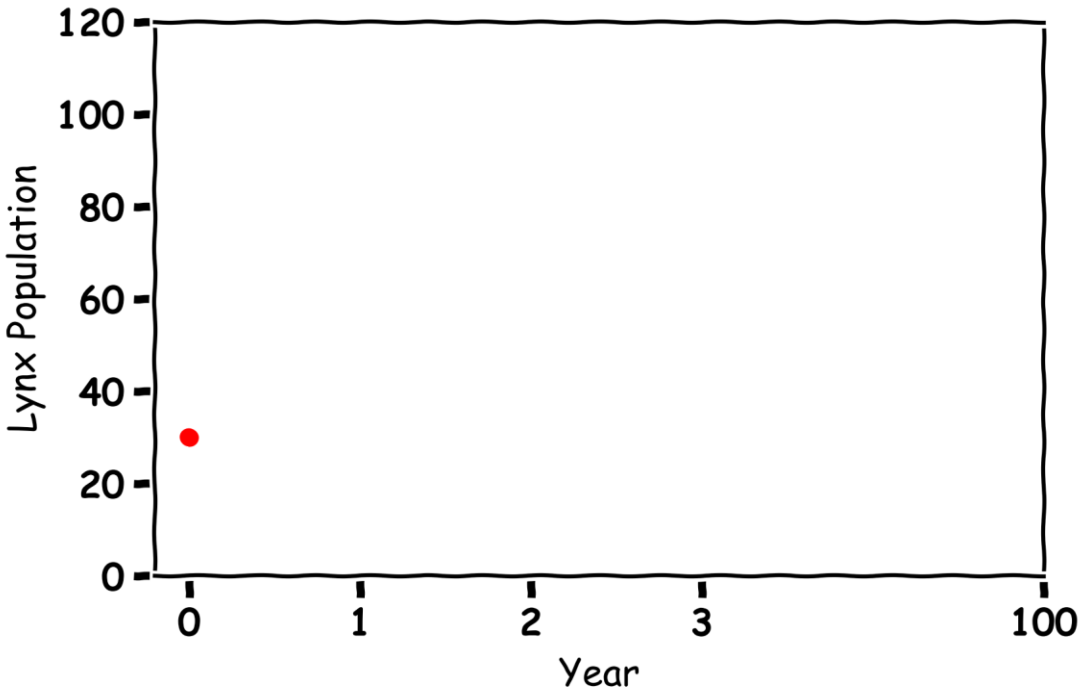




Lynx

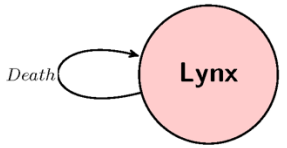
Year	Lynx Population
0	30
1	30-0.1*30=27
2	
3	

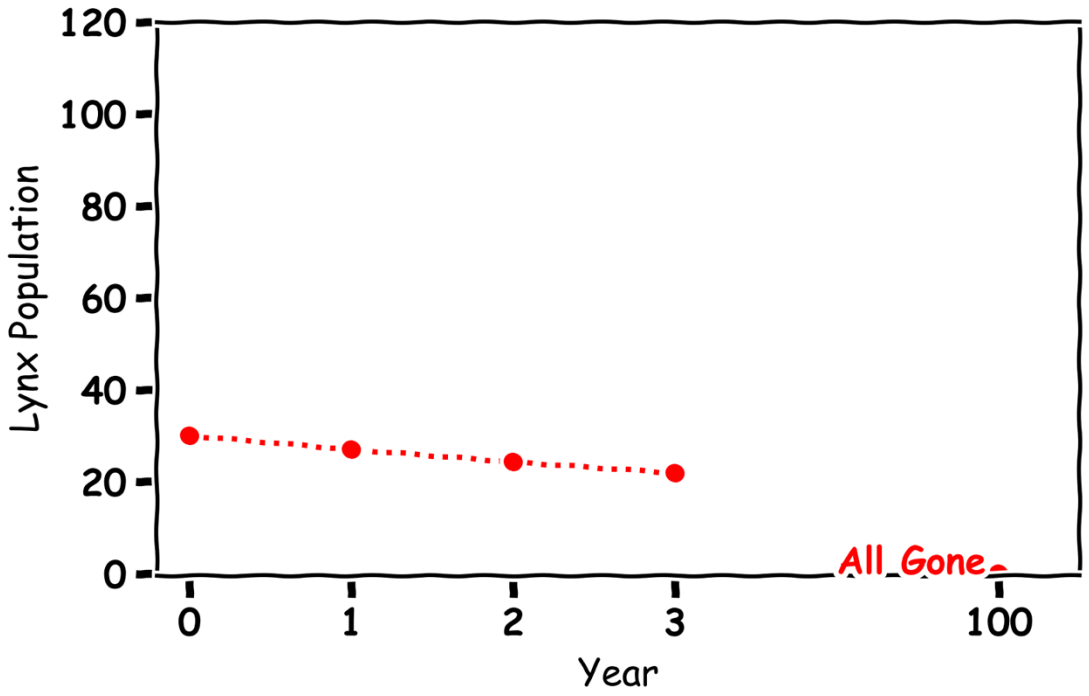




Lynx

Year	Lynx Population
0	30
1	27
2	24
3	22





Lynx

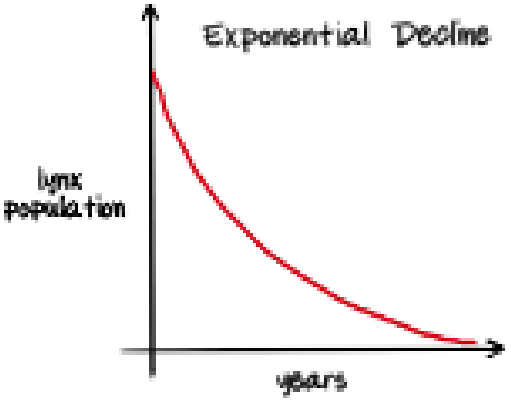
(B) Lynx Population

L_c : current lynx population L_F : Future lynx population

r_{death} = Deaths - Births

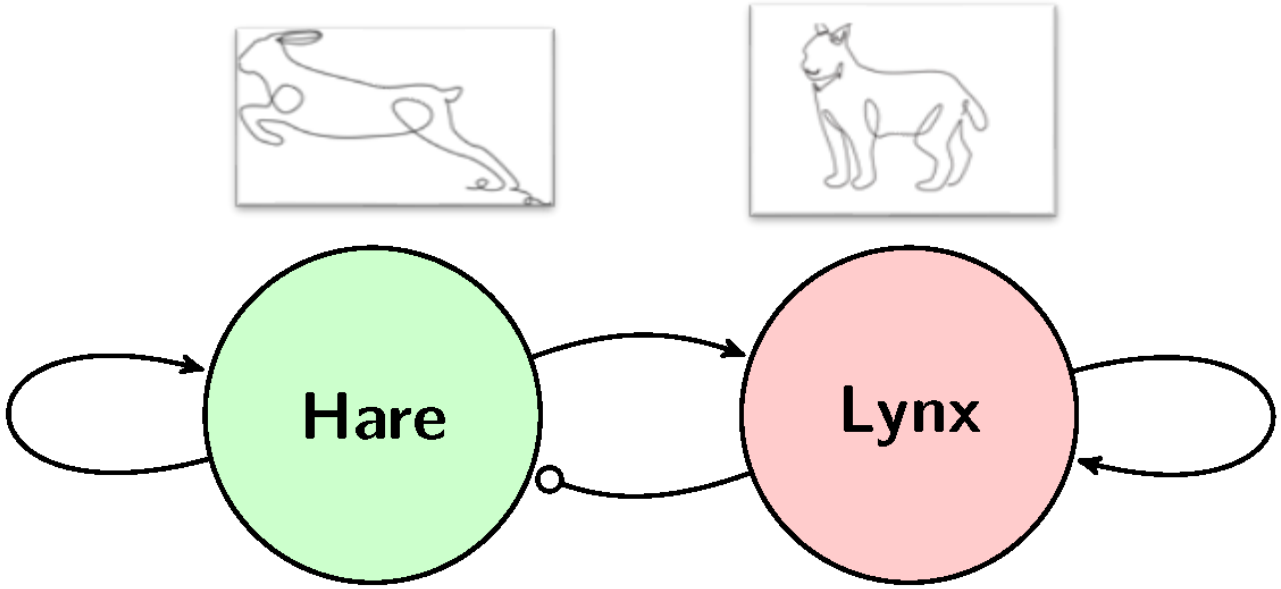
change in lynx population = $r_{death} \times L_c$

$L_F = L_c$ - change in lynx population



Predator and Prey

Predator and Prey



Predator and Prey

HARE POPULATION EQUATION

$$H_{Future} = H_{Current} + r_{Growth} \times H_{Current} - r_{Eat} \times H_{Current} \times L_{Current}$$

FUTURE HARE POPULATION = CURRENT HARE POPULATION + GROWTH RATE TIMES HARE POPULATION - EAT RATE OF HARE POPULATION BY LYNX POPULATION

LYNX POPULATION EQUATION

$$L_{Future} = L_{Current} - r_{Death} \times L_{Current} + r_{Food} \times L_{Current} \times H_{Current}$$

FUTURE LYNX POPULATION = CURRENT LYNX POPULATION - DEATH RATE TIMES LYNX POPULATION + FOOD RATE OF LYNX POPULATION BY HARE POPULATION

Predator and Prey

$$H_{Future} = H + 0.2(H) - 0.005(H)(L)$$

$$L_{Future} = L - 0.1(L) + 0.002(L)(H)$$

Predator and Prey

$H_{Future}=H+0.2(H)-0.005(H)(L)$

$L_{Future}= L-0.1(L)+0.002(L)(H)$

Year	Hare Population	Lynx Population
0	60	30
1		
2		
3		

Predator and Prey

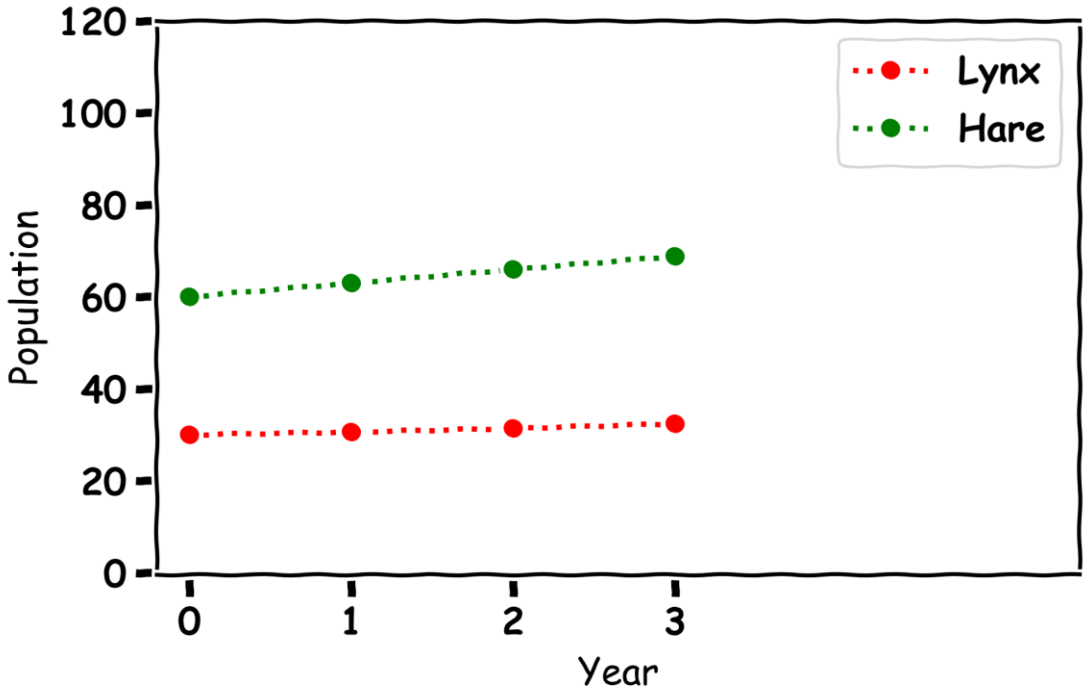
$H_{Future}=H+0.2(H)-0.005(H)(L)$

$L_{Future}= L-0.1(L)+0.002(L)(H)$

Year	Hare Population	Lynx Population
0	60	30
1	60+0.2(60)-0.005(60)(30)=**63**	30-0.1(30)+0.002(30)(60)=**30.6**
2	63+0.2(63)-0.005(63)(30.6)=**65.9**	30.6-0.1(30.6)+0.002(30.6)(63)=**31.3**
3	65.9+0.2(65.9)-0.005(65.9)(31.3)=**68.8**	31.3-0.1(31.3)+0.002(31.3)(65.9)=**32.4**

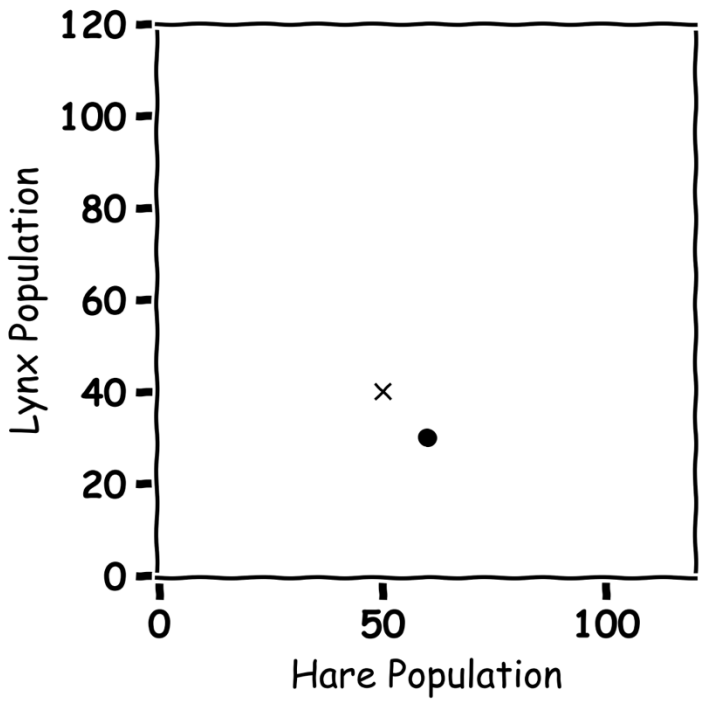
Predator and Prey

Year	Hare Population	Lynx Population
0	60	30
1	63	30.6
2	65.9	31.3
3	68.8	32.4



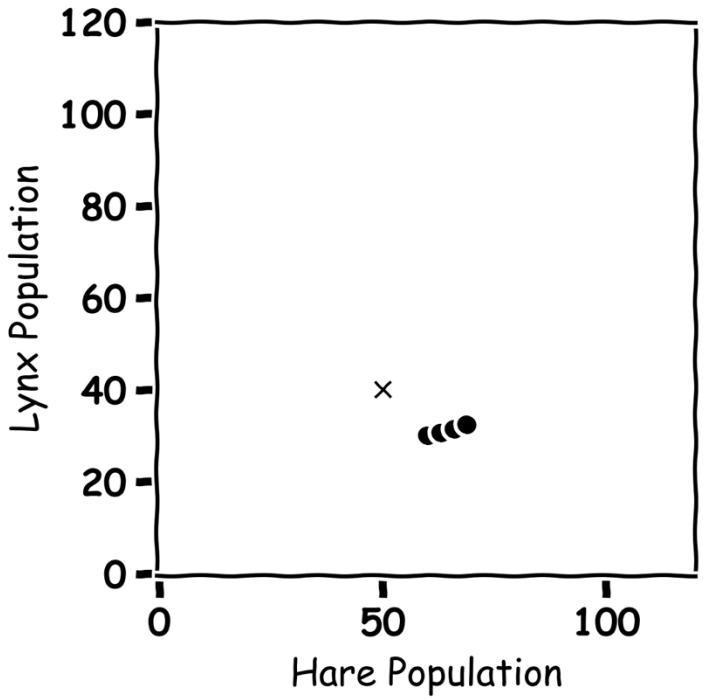
Predator and Prey

Year	Hare Population	Lynx Population
0	60	30
1		
2		
3		

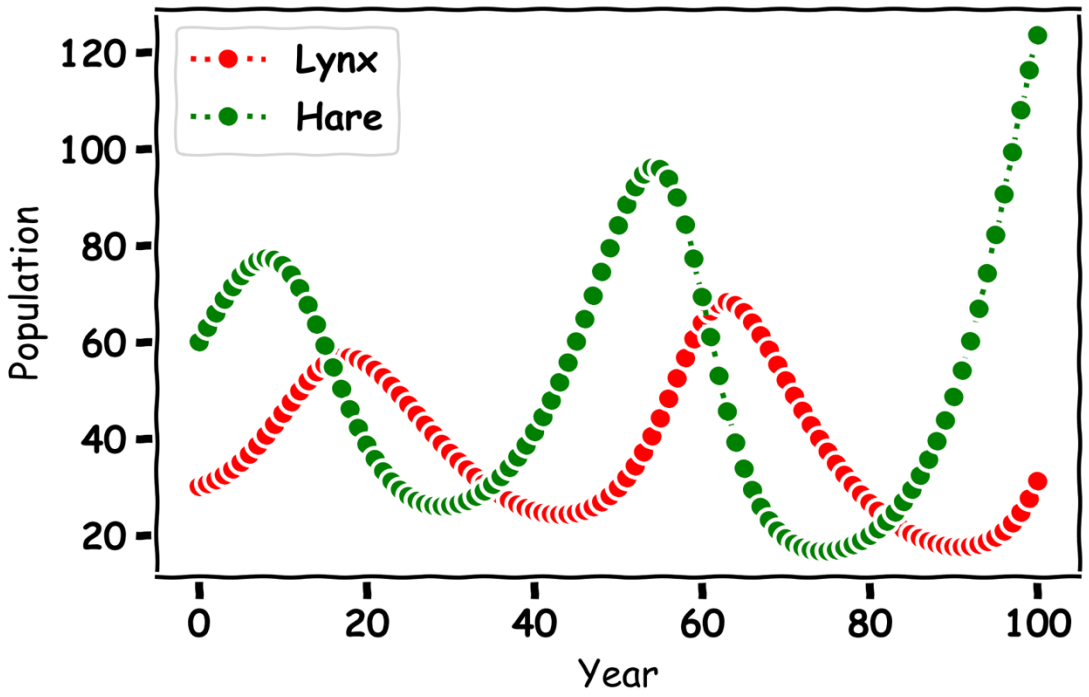


Predator and Prey

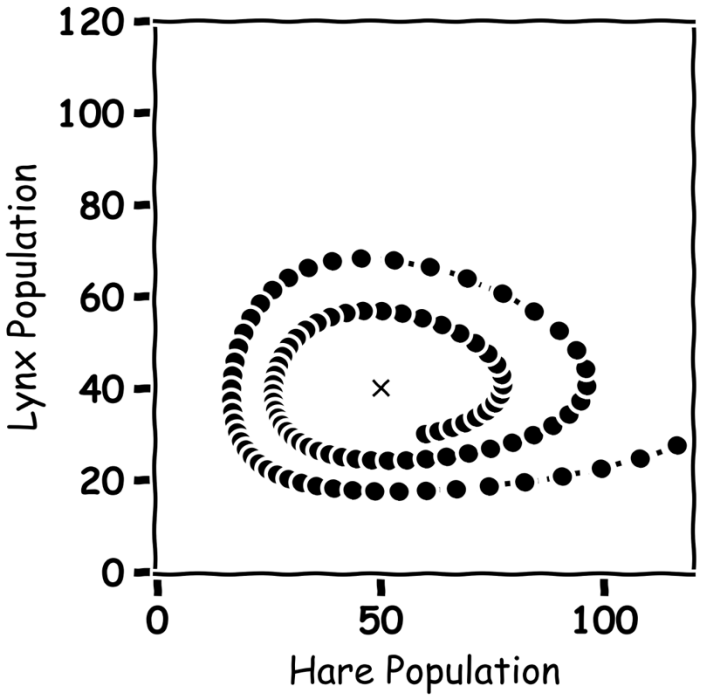
Year	Hare Population	Lynx Population
0	60	30
1	63	30.6
2	65.9	31.3
3	68.8	32.4



Predator and Prey



Predator and Prey



Predator and Prey

© Hare - Lynx Interaction

Hare Equation

r_{eaten} = hares lost to hunting

$\text{hares eaten} = r_{eaten} \times H_0 \times L_0$

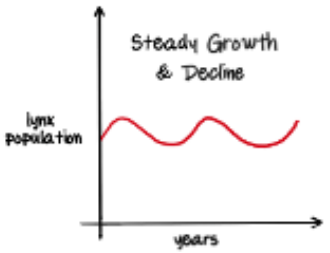
$H_F = H_0 + \text{Change in hare population} - \text{hares eaten}$

Lynx Equation

r_{food} = successful lynx hunts

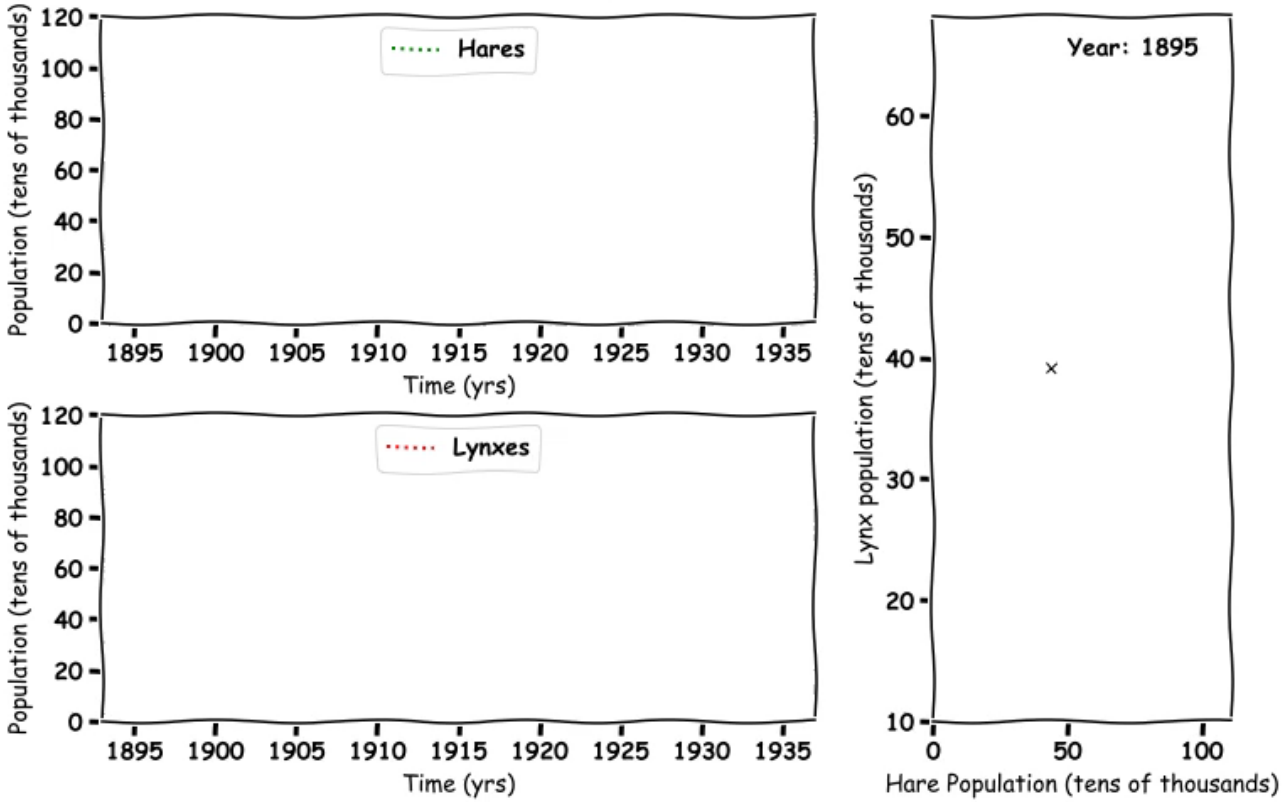
$\text{lynxes fed} = r_{food} \times H_0 \times L_0$

$L_F = L_0 - \text{change in lynx population} + \text{lynxes fed}$



Predator and Prey

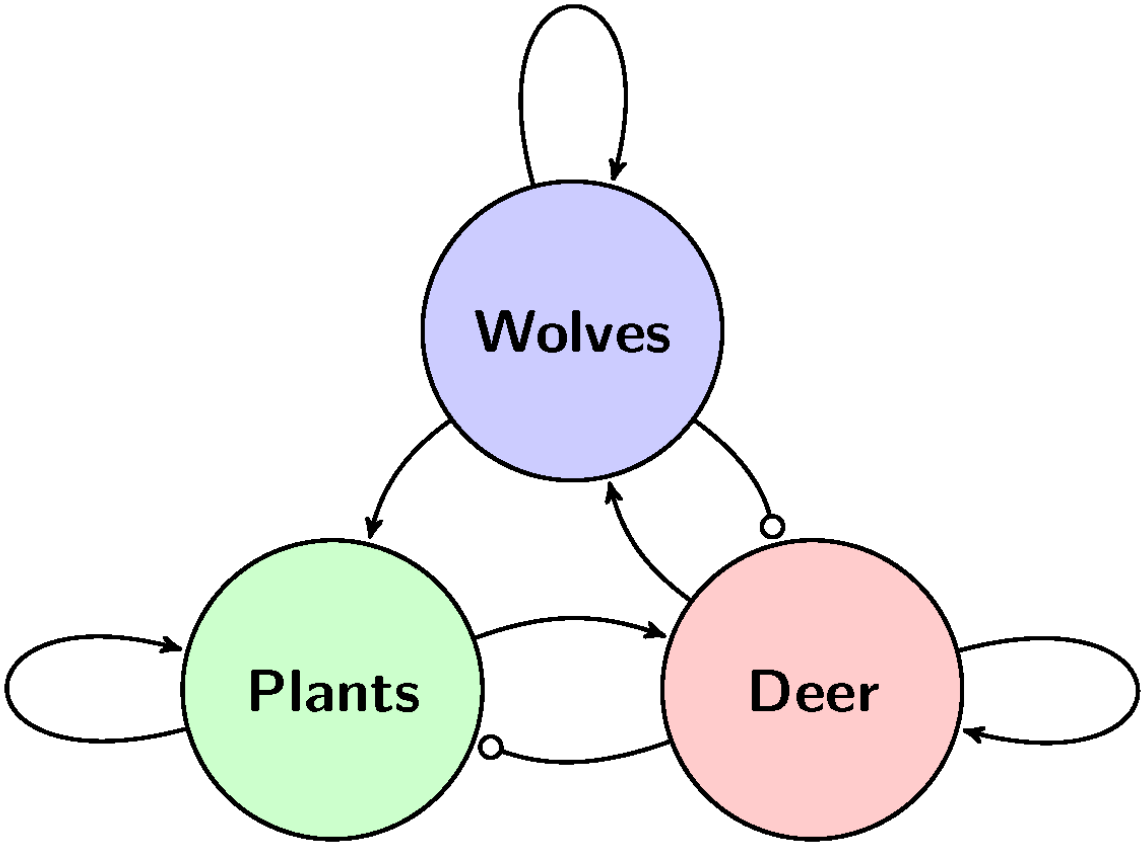
The figure below plots the numerical approximation of the Hare population (green) and the Lynx poulation (red) as a function of time in years between 1895 and 1935.



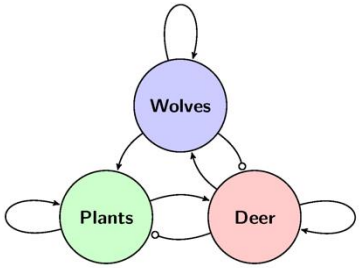
Predator Prey

Deer, Plants and Wolves

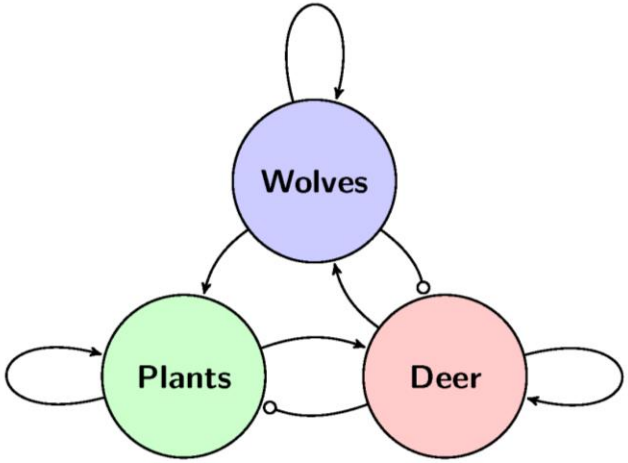
Three Animals

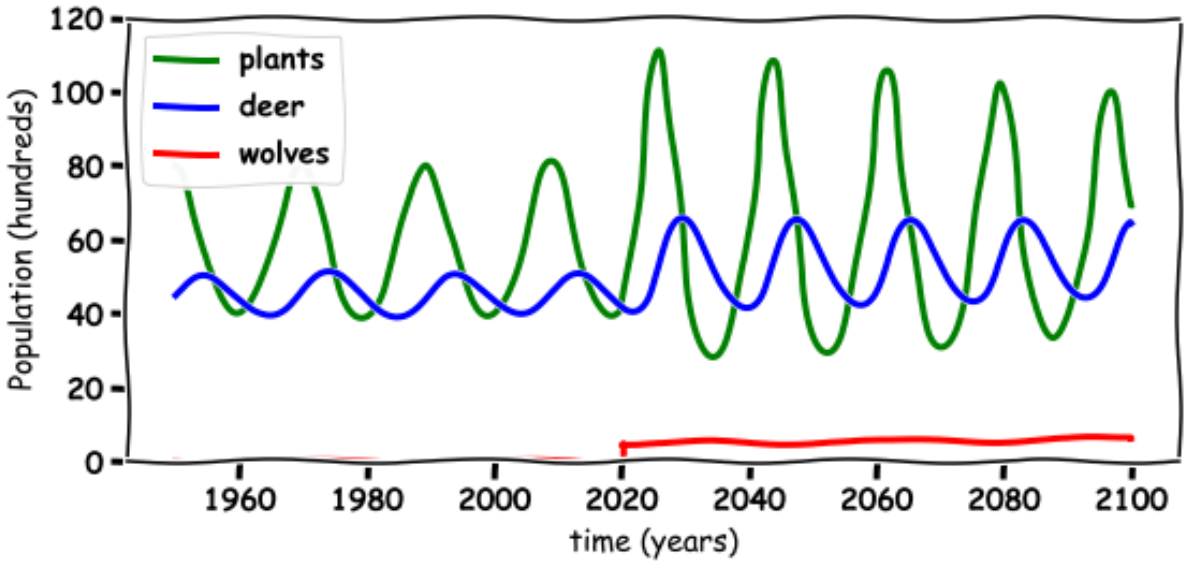


Three Animals

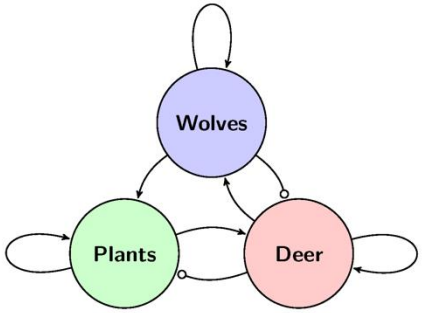


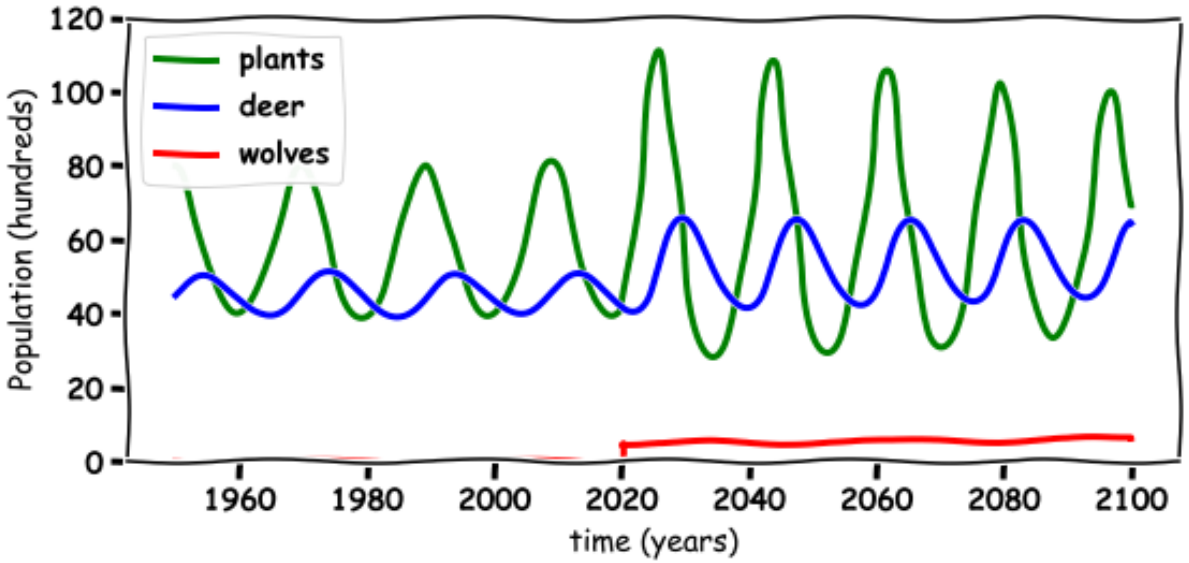
Three Animals

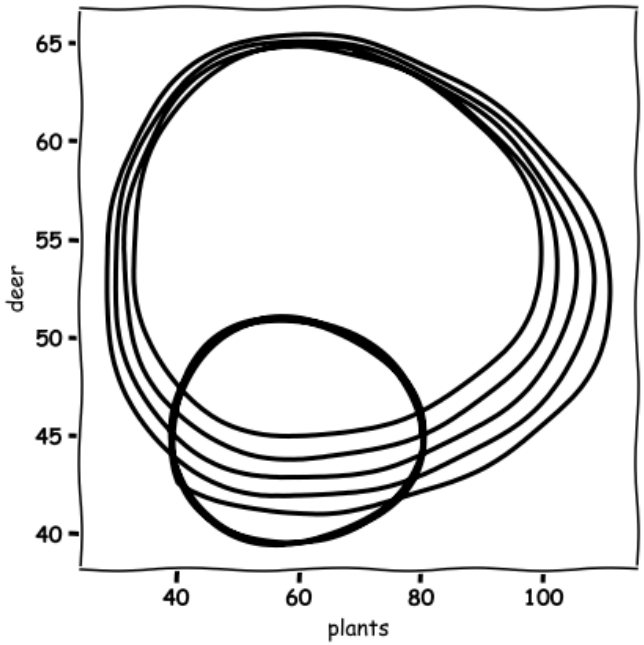




Three Animals







Thank you for Listening