

## Problem Sheet 1b

### Question 1

1. The events A and B are defined on a sample space. The following probabilities are known:  $\Pr(A) = 2/3$ ;  $\Pr(B) = 1/6$  and  $\Pr(A \cap B) = 1/6$

- a) Are A and B mutually exclusive and independent?
- b) Find the following additional probabilities:
- c)  $\Pr(\text{not}A)$
- ii)  $\Pr(A \cup B)$
- iii)  $\Pr(A|B)$

**ANSWER:**

i)

$$\Pr(\text{not}A) = 1 - \Pr(A)$$

1-1/3

## [1] 0.6666667

ii)

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - (\Pr(A \cap B))$$

2/3+1/6-1/6

## [1] 0.6666667

iii)

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

(1/6)/(1/6)

## [1] 1

### Question 2

2. The events A and B are defined on a sample space. The following probabilities are known:  $\Pr(A) = 1/2$ ;  $\Pr(B) = \frac{1}{2}$  and  $\Pr(A \cup B) = \frac{5}{6}$ . Find the following additional probabilities: i  $\Pr(A) = \Pr(\text{not}A)$  ii  $\Pr(A \cap B)$  iii  $\Pr(A|B)$

**ANSWER:**

i)

$$\Pr(\text{not}A) = 1 - \Pr(A)$$

1-1/2

## [1] 0.5

ii)

$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - (\Pr(A \cup B))$$

1/2+1/2-5/6

## [1] 0.1666667

iii)

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

(1/6)/(5/6)

## [1] 0.2

## Question 4

4. A standard green and a standard red die are thrown at the same time. i Define the sample space for all the events. ii What is probability that the sum of the scores on the two dice is greater than 6. iii What is the probability that the green die is less than 5. iv That is the probability that the sum of the two die is greater than six given that the green die is less than 5.

**ANSWER:**

- i) Define the sample space for all the events.

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##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] "1,1" "2,1" "3,1" "4,1" "5,1" "6,1"
## [2,] "1,2" "2,2" "3,2" "4,2" "5,2" "6,2"
## [3,] "1,3" "2,3" "3,3" "4,3" "5,3" "6,3"
## [4,] "1,4" "2,4" "3,4" "4,4" "5,4" "6,4"
## [5,] "1,5" "2,5" "3,5" "4,5" "5,5" "6,5"
## [6,] "1,6" "2,6" "3,6" "4,6" "5,6" "6,6"
```

- ii What is probability that the sum of the scores on the two dice is greater than 6. **ANSWERS:**

$$Pr(\text{Sum} > 6) = \frac{21}{36}$$

- iii What is the probability that the green die is less than 5.

**ANSWERS:**

$$Pr(\text{Green} < 5) = \frac{24}{36}$$

- iv What is the probability that the sum of the two die is greater than six given that the green die is less than 5.

**ANSWERS:**

$$Pr(\text{Green} < 5 \cap \text{Sum} > 6) = \frac{10}{36}$$
$$Pr(\text{Sum} > 6 | \text{Green} < 5) = \frac{\frac{10}{36}}{\frac{24}{36}} = \frac{10}{24}$$

## Conditional Probability

### Question 5

5. 20% of the population owns a smart-phone and a tablet, and 60% of the population own a smart-phone. What is the probability that a someone owns a tablet given that they own a smart-phone?

**ANSWER:**

$$Pr(Tablet|Smart - Phone) = \frac{Pr(Smart - Phone \cap Tablet)}{Pr(Smart - Phone)} = \frac{0.2}{0.6}$$

(.2)/(.6)

## [1] 0.3333333

6. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

**ANSWER:**

$$Pr(Absent|Friday) = \frac{Pr(Absent \cap Friday)}{Pr(Friday)} = \frac{0.03}{0.2}$$

(.03)/(.2)

## [1] 0.15

## Bayes Probability

### Question 7

7. In 2015, of the 200 million people registered to vote in America 4% say they will vote for Donald Trump. Of these voters 50% are climate change skeptics, while only 8.3% of the other voters are climate change skeptics.
- a) Find the probability that a voter selected is a climate change skeptic.

**ANSWER:**

$$Pr(Skeptic) = Pr(Skeptic \cap Trump)Pr(Trump) + Pr(Skeptic \cap NotTrump)Pr(NotTrump)$$

.04\*0.5+0.96\*0.083

## [1] 0.09968

- b) If a voter is selected and they are a climate change skeptic, determine the probability that they will vote for Trump.

**ANSWER:**

$$Pr(Trump|Skeptic) = \frac{Pr(Skeptic \cap Trump)Pr(Trump)}{Pr(Skeptic \cap Trump)Pr(Trump) + Pr(Skeptic \cap NotTrump)Pr(NotTrump)}$$

.04\*0.5/(.04\*0.5+0.96\*0.083)

## [1] 0.2006421

8. In 2015, of the 200 million people registered to vote in America 38% say they will vote for Donald Trump. Of these voters 20% are birthers (people who questioned that President Obama was born in the US), while only 1% of the other voters are birthers.
- a) Find the probability that a voter selected is a birther.

**ANSWER:**

$$Pr(birther) = Pr(birther \cap Trump)Pr(Trump) + Pr(birther \cap NotTrump)Pr(NotTrump)$$

$$0.38*0.2+0.62*0.01$$

## [1] 0.0822

b) If a voter is selected and they are a birther, determine the probability that they will vote for Trump.

**ANSWER:**

$$Pr(Trump|birther) = \frac{Pr(birther \cap Trump)Pr(Trump)}{Pr(birther \cap Trump)Pr(Trump) + Pr(birther \cap NotTrump)Pr(NotTrump)}$$

$$(0.38*0.2)/(0.38*0.2+0.62*0.01)$$

## [1] 0.9245742

9. In a school 30% of the students are male. Of these males 20% have long hair, while 90% of the female students have long hair. i Find the probability that a student selected has long hair.

**ANSWER:**

$$Pr(Long - Hair) = Pr(Long - Hair \cap Male)Pr(Male) + Pr(Long - Hair \cap Female)Pr(Female)$$

$$.3*.2+.7*.9$$

## [1] 0.69

ii If a student is selected and they have long hair, determine the probability that the student is male.

**ANSWER:**

$$Pr(Male|Long - Hair) = \frac{Pr(Long - Hair \cap Male)Pr(Male)}{Pr(Long - Hair \cap Male)Pr(Male) + Pr(Long - Hair \cap Female)Pr(Female)}$$

$$(.3*.2)/(.3*.2+.7*.9)$$

## [1] 0.08695652