Problem Sheet 1

1. Describe each of the following measures of location including their the pros and cons

i Mean

The mean is the point at which the sum of the deviations is 0:

Pros:

- Easy to calculate;
- Uses all the data.

Con:

- Sensitive to extreme values.

ii Median

The median is the middle value of the ordered set of values:

Pros:

- Median gives the center of the data;
- Not sensitive to extreme values.

Con:

- Does not use all the data.

iii) Variance

The variance is the spread around the mean:

Pro:

- Takes all data into account.

Cons:

- Hard to interpret;
- Can be influenced by extreme values.

iv Skewness

Skewness is a measure of symmetry of a distribution: Pro

- Takes all data into account;

Con

- Not intuitive.

Counting

2. How many different combinations of 4 cards can be made for a 52 card deck.

ANSWER:

Order **does not** matter

$$C_4^{52} = \binom{52}{4} = \frac{52!}{(52-4)!4!} = \frac{(52 \times 51 \times 50 \times 49)}{(4 \times 3 \times 2 \times 1)} = 270,725$$

```
52*51*50*49/(4*3*2*1)
```

```
## [1] 270725
```

```
# OR
choose(52,4)
```

```
## [1] 270725
```

Order **does** matter

ANSWER:

$$P_4^{52} = \frac{52!}{(52-4)!} = (52 \times 51 \times 50 \times 49) = 6,497,400$$

```
52*51*50*49
```

```
## [1] 6497400
```

```
# OR
factorial(52)/factorial(48)
```

```
## [1] 6497400
```

3. A bank issues bank cards with PINs consisting of 4 digits, each one {0,1,2,. . . ,9}. How many unique PINs are there if

i. Any 4-digit code can be used.

ANSWER:

$$10^4 = 10 \times 10 \times 10 \times 10 \times 10 = 10,000$$

```
10*10*10*10
```

```
## [1] 10000
```

ii. The digits must be different.

ANSWER:

Order **does** matter

$$P_4^{10} = \frac{10!}{(10-4)!} = (10 \times 9 \times 8 \times 7) = 5,040$$

```
10*9*8*7
```

```
## [1] 5040
```

4. In a lottery, each ticket has 5 one-digit numbers 0-9 which is not repeated on it. i You win if your ticket has the digits in any order. What are the total number of possible combinations?

ANSWER:

Order **does not** matter

$$C_5^{10} = \binom{10}{5} = \frac{10!}{(10-5)!5!} = \frac{(10 \times 9 \times 8 \times 7 \times 6)}{(5 \times 4 \times 3 \times 2 \times 1)} = 252$$

```
choose(10,5)
```

```
## [1] 252
```

```
10*9*8*7*6/(5*4*3*2*1)
```

```
## [1] 252
```

ii You would win only if your ticket has the digits in the required order. What are the total number of combinations?

ANSWER:

Order **does** matter

$$P_5^{10} = \frac{10!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

```
10*9*8*7*6
```

```
## [1] 30240
```

5. How many different combinations of 6 cards can be made for a 52 card deck if

i) order matters.

ANSWER:

Order **does** matter

$$P_6^{52} = \frac{52!}{(52-6)!} = 52 \times 51 \times 50 \times 49 \times 48 \times 47 = 14,658,134,400$$

```
52*51*50*49*48*47
```

```
## [1] 14658134400
```

```
factorial(52)/factorial(46)
```

```
## [1] 14658134400
```

ii) order does not matter

ANSWER:

Order **does not** matter

$$C_6^{52} = \binom{52}{6} = \frac{52!}{(52-6)!6!} = \frac{(52 \times 51 \times 50 \times 49 \times 48 \times 47)}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} = 20,358,520$$

```
choose(52,6)
```

```
## [1] 20358520
```

```
52*51*50*49*48*47/(6*5*4*3*2*1)
```

```
## [1] 20358520
```

6. A poker hand consists of 7 cards:

i) How many different hands are possible, if order does not matter,

ANSWER:

Order **does not** matter

$$C_7^{52} = \binom{52}{7} = \frac{52!}{(52-7)!7!} = \frac{(52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46)}{(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)} = 133,784,560$$

```
choose(52,7)
```

```
## [1] 133784560
```

```
52*51*50*49*48*47*46/(7*6*5*4*3*2*1)
```

```
## [1] 133784560
```

ii) How many hands can be made with at least one king and one queen.

ANSWER:

$$\frac{4 \times 4 \times (50 \times 49 \times 48 \times 47 \times 46)}{(5 \times 4 \times 3 \times 2 \times 1)} = 33,900,160$$

```
4*4*50*49*48*47*46/(5*4*3*2*1)
```

```
## [1] 33900160
```

7. In a game of 5 card poker what are the number of different possible hands are there?

i) A hand with a pair

ANSWER:

$$\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 47 \times 46)}{(3 \times 2 \times 1)} = 1,349,088$$

```
(52*3)/(2*1)*48*47*46/(3*2*1)
```

```
## [1] 1349088
```

aa)A hand with a only pair

ANSWER:

$$\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 44 \times 40)}{(3 \times 2 \times 1)} = 1,098,240$$

```
(52*3)/(2*1)*48*44*40/(3*2*1)
```

```
## [1] 1098240
```

b)A hand with two pair

ANSWER:

$$\frac{\frac{52 \times 3}{2 \times 1} \times \frac{48 \times 3}{2 \times 1}}{2 \times 1} \times \frac{44}{1} = 123,552$$

```
((52*3)/(2*1)*48*3/(2*1))/2*44/1
```

```
## [1] 123552
```

c)A hand with Three of a kind

ANSWER:

$$\frac{52 \times 3 \times 2}{3 \times 2 \times 1} \times \frac{48 \times 3}{2 \times 1} = 3,744$$

```
((52*3*2)/(3*2*1)*48*3/(2*1))
```

```
## [1] 3744
```

d)A hand with a Flush (all the same suit)

ANSWER:

$$\frac{52 \times 12 \times 11 \times 10 \times 9}{5 \times 4 \times 3 \times 2 \times 1} = 5,148$$

```
((52*12*11*10*9)/(5*4*3*2*1))
```

```
## [1] 5148
```