Problem Sheet 2b - Probability Mass Distributions

Question 1

1. There are 30 candy covered chocolates in a bag M&M's. There is a .1 probability that that the candy is red. If X is the number of red M&M's in the bag.

i Give the binomial probability mass function for X.

ANSWER:

$$\Pr(k) = \left(\begin{array}{c} 30 \\ k \end{array} \right) (0.1)^k (0.9)^{30-k}, \quad k = 0, 1, 2, ...30,$$

where k is the number of red M & Ms.

ii Find the probability of less than 2 red M&Ms in the bag.

ANSWER:

```
n<-30
k<-1
MandM<-0:n # Number of Games
Pr_Red<-0.1 # Probability of Success
Pr_Reds<-c(dbinom(0:n,n,Pr_Red)) # Binomial Probability
Q1ii<-c(dbinom(0:k,n,Pr_Red))
E_X<- n*Pr_Red # Expected Value
Var_X<-10*Pr_Red*(1-Pr_Red) # Variance
```

$$\Pr(0) = \left(\begin{array}{c} 30 \\ 0 \end{array} \right) (0.1)^0 (0.9)^{30-0} = 0.0423912$$

$$\Pr(1) = \left(\begin{array}{c} 30 \\ 1 \end{array} \right) (0.1)^1 (0.9)^{30-1} = 0.1413039$$

$$Pr(< 2) = Pr(0) + Pr(1) = 0.0423912 + 0.1413039 = 0.183695,$$

As it is a Binomial Distribution we can state that the expected number of Red is

$$E[k] = 30 \times 0.1 = 3,$$

the variance of the distribution is

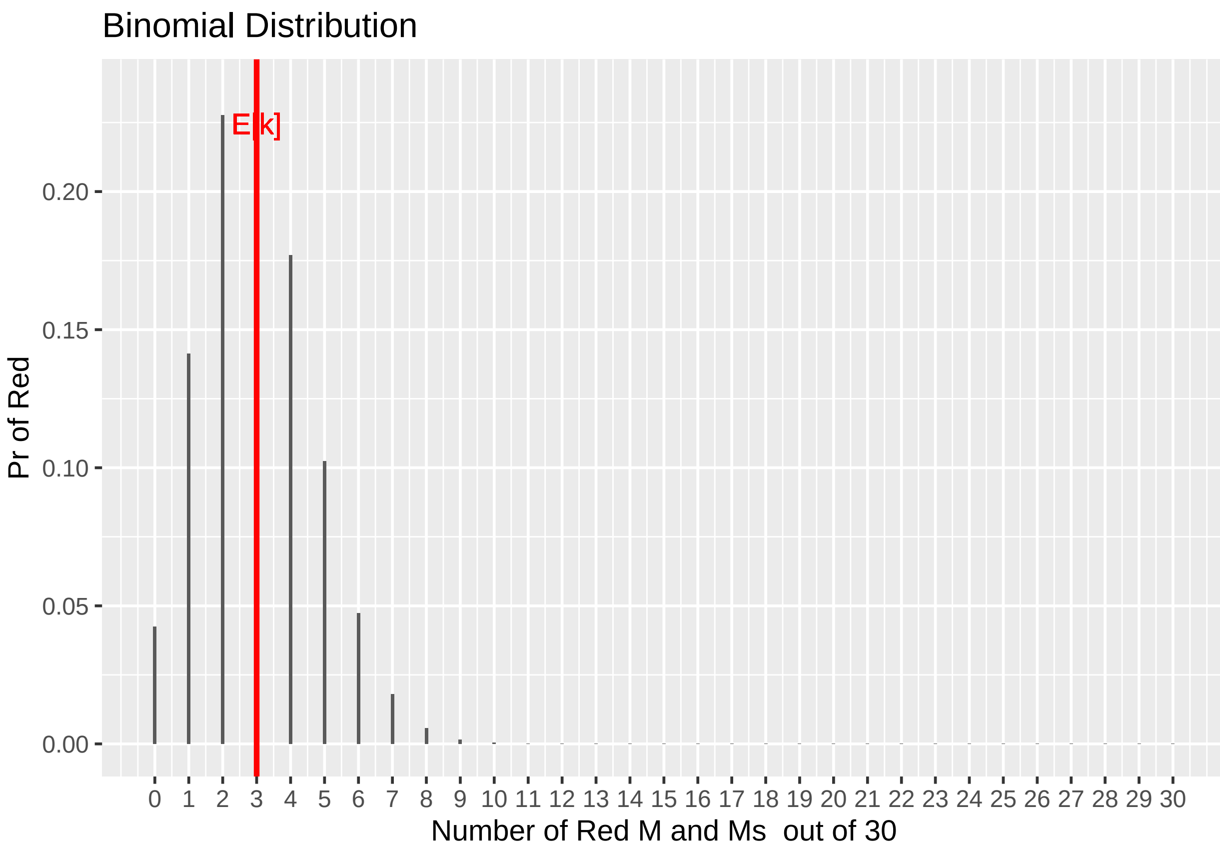
$$Var[k] = 30 \times 0.1 \times (1 - 0.1) = 0.9.$$

The plot below shows the Binomial Distribution M & Ms:

```
df <- data.frame(MandM,Pr_Reds)
## Plot
binomial.p_dist<-ggplot(df, aes(x=MandM,y=Pr_Reds)) + geom_col(width = 0.1)+
  xlab("Number of Red M and Ms  out of 30")+
  ylab("Pr of Red")+ggtitle("Binomial Distribution")+
```

```
  scale_x_continuous(breaks=0:n)+
  geom_vline(xintercept = E_X, color = "Red", size=1)+
  geom_text(aes(x=E_X, label="E[k]", y=max(Pr_Reds)),vjust=2,
            colour="red",  text=element_text(size=18))

binomial.p_dist
```




```
ggsave("Q1.png",dpi=300, width = 4, height = 2)
```

Question 2

2. A baby wakes on average 0.25 times every hour.

i If X is the number of times a baby wakes in an hour, give the poisson probability mass function for X. **ANSWER:**

The distribution is described by the average, $\lambda = 0.25$,

$$\Pr(k) = \frac{0.25^k e^{-0.25}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of times the baby wakes every hour.

The expected value of the Poisson Distribution is

$$E[k] = 0.25,$$

with the variance

$$Var[k] = 0.25.$$

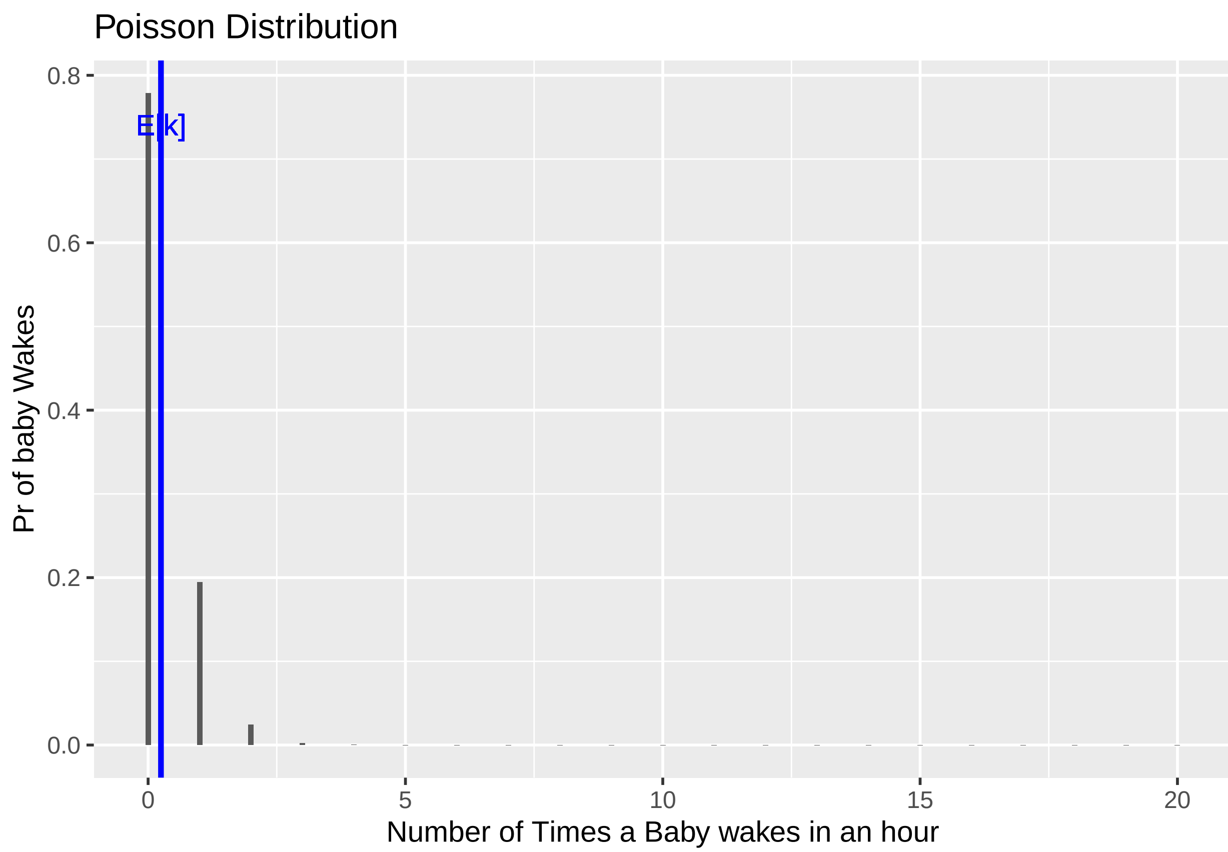
The plot below shows the Poisson Distribution for $\lambda = 0.25$ average number of times a baby wakes in an hour:

```r
Wake<-0:20 # Number of Games
Baby_wakes<-0.25 # Lambda
Pr_wakes<-c(dpois(Wake,Baby_wakes)) # Poisson Probabilities
E_X<-Baby_wakes # Expected Outcome

df <- data.frame(Wake,Pr_wakes)

## Plot
Poisson.p_dist<-ggplot(df, aes(x=Wake,y=Pr_wakes)) +
  geom_col(width = 0.1)+xlab("Number of Times a Baby wakes in an hour")+
  ylab("Pr of baby Wakes")+ggtitle("Poisson Distribution")+
  geom_vline(xintercept = E_X,  color = "blue", size=1)+
  geom_text(aes(x=E_X, label="E[k]", y=max(Pr_wakes)),vjust=2,
            colour="blue",text=element_text(size=18))

Poisson.p_dist
```




```r
ggsave("Q2i.png",dpi=300, width = 4, height = 2)
```

ii If X is the number of times a baby wakes in eight hour, give the poisson probability mass function for X.

ANSWER:

The distribution is described by the average, $\lambda = 0.25 \times 8 = 2$,

$$\Pr(k) = \frac{2^k e^{-2}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of times the baby wakes every 8 hours.

The expected value of the Poisson Distribution is

$$E[k] = 2,$$

with the variance

$$Var[k] = 2.$$

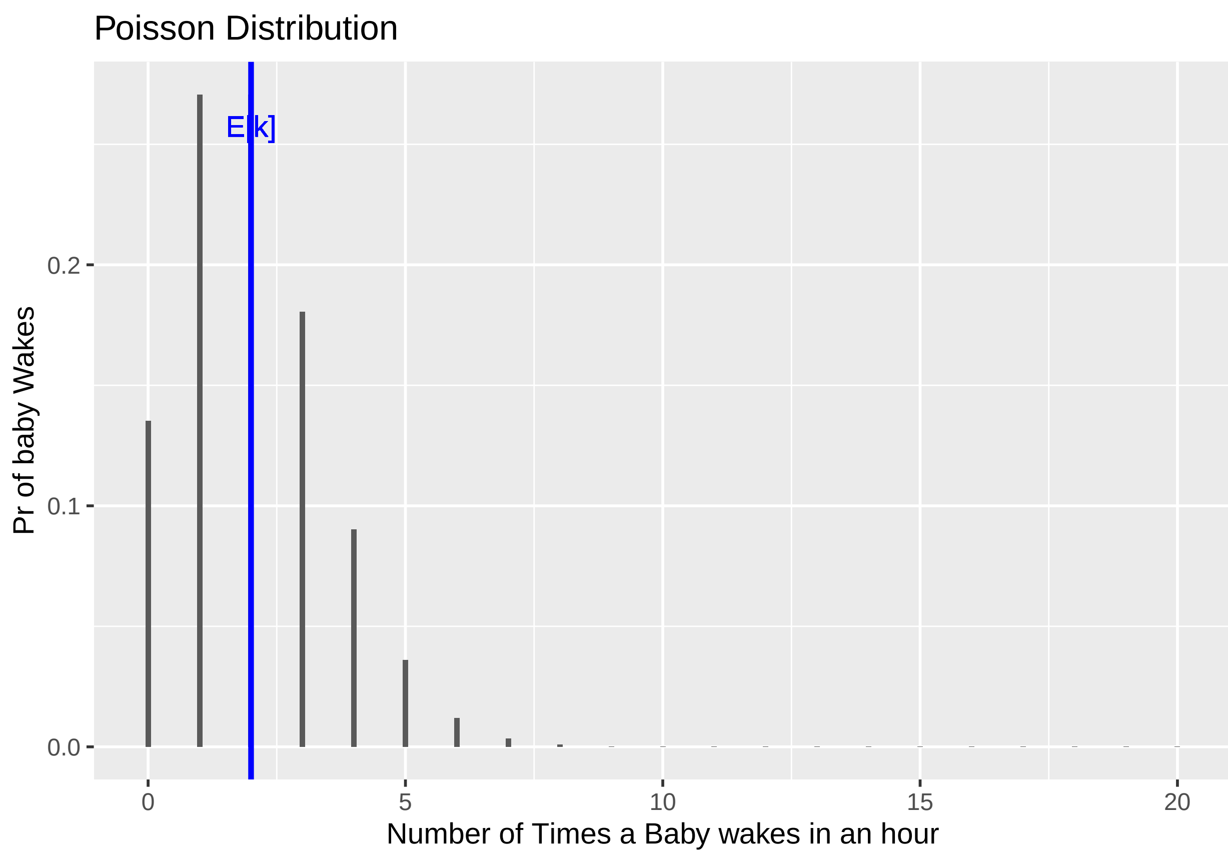
The plot below shows the Poisson Distribution for $\lambda = 2$ average number of times a baby wakes in eight hours:

```
Baby_wakes<-0.25*8 # Lambda
Pr_wakes<-c(dpois(Wake,Baby_wakes)) # Poisson Probabilities
E_X<-Baby_wakes # Expected Outcome

df <- data.frame(Wake,Pr_wakes)

## Plot
Poisson.p_dist<-ggplot(df, aes(x=Wake,y=Pr_wakes)) +
  geom_col(width = 0.1)+xlab("Number of Times a Baby wakes in an hour")+
  ylab("Pr of baby Wakes")+ggtitle("Poisson Distribution")+
  geom_vline(xintercept = E_X,  color = "blue", size=1)+
  geom_text(aes(x=E_X, label="E[k]", y=max(Pr_wakes)),vjust=2,
            colour="blue",text=element_text(size=18))

Poisson.p_dist
```



```
ggsave("Q2i.png",dpi=300, width = 4, height = 2)
```

iii What is the probability that the baby does not wake during the 8 hours.

ANSWER:

```
Baby_wakes<-0.25*8 # Lambda
Pr_wakes_zero<-dpois(0,Baby_wakes) #
```

$$\Pr(0) = \frac{0.25^0 e^{-0.25}}{0!} = 0.1353353.$$

Question 3

3. Give the features of a

i Geometric Experiment.

ANSWER:

- Bernoulli Trial • Goes until you "win" • E[X]=1/p, VAR[X]=q/p^2

ii Binomial Experiment.

ANSWER:

- Bernoulli Trial • Play a specific number of times • E[x]=np, VAR[X]=npq

iii Poisson Experiment.

ANSWER:

- Well known mean • Number of "wins" • E[X]=lambda, VAR[X]=lambda

Question 4

4. Every day a production line makes 100 computers of which 10% are defective. If X is the number of defective computers in a day.

a) Give the binomial probability mass function for X.

ANSWER:

$$\Pr(k) = \left(\begin{array}{c} 100 \\ k \end{array}\right) (0.1)^k (0.9)^{30-k}, \quad k = 0, 1, 2, ...100,$$

where k is the number of defective computers.

b) Find the probability that there is more than 2 computers defective in a day.

ANSWER:

```
n<-100
k<-2
Computers<-0:n # Number of Games
Pr_Def<-0.1 # Probability of Success
Pr_Defs<-c(dbinom(0:n,n,Pr_Def)) # Binomial Probability
Q4ii<-c(dbinom(0:k,n,Pr_Def))
E_X<- n*Pr_Def # Expected Value
Var_X<-10*Pr_Def*(1-Pr_Def) # Variance
```

$$\mathrm{Pr}(0) = \binom{100}{0}(0.1)^0(0.9)^{100-0} = 2.6561399 \times 10^{-5},$$

$$\mathrm{Pr}(1) = \binom{100}{1}(0.1)^1(0.9)^{100-1} = 2.9512665 \times 10^{-4},$$

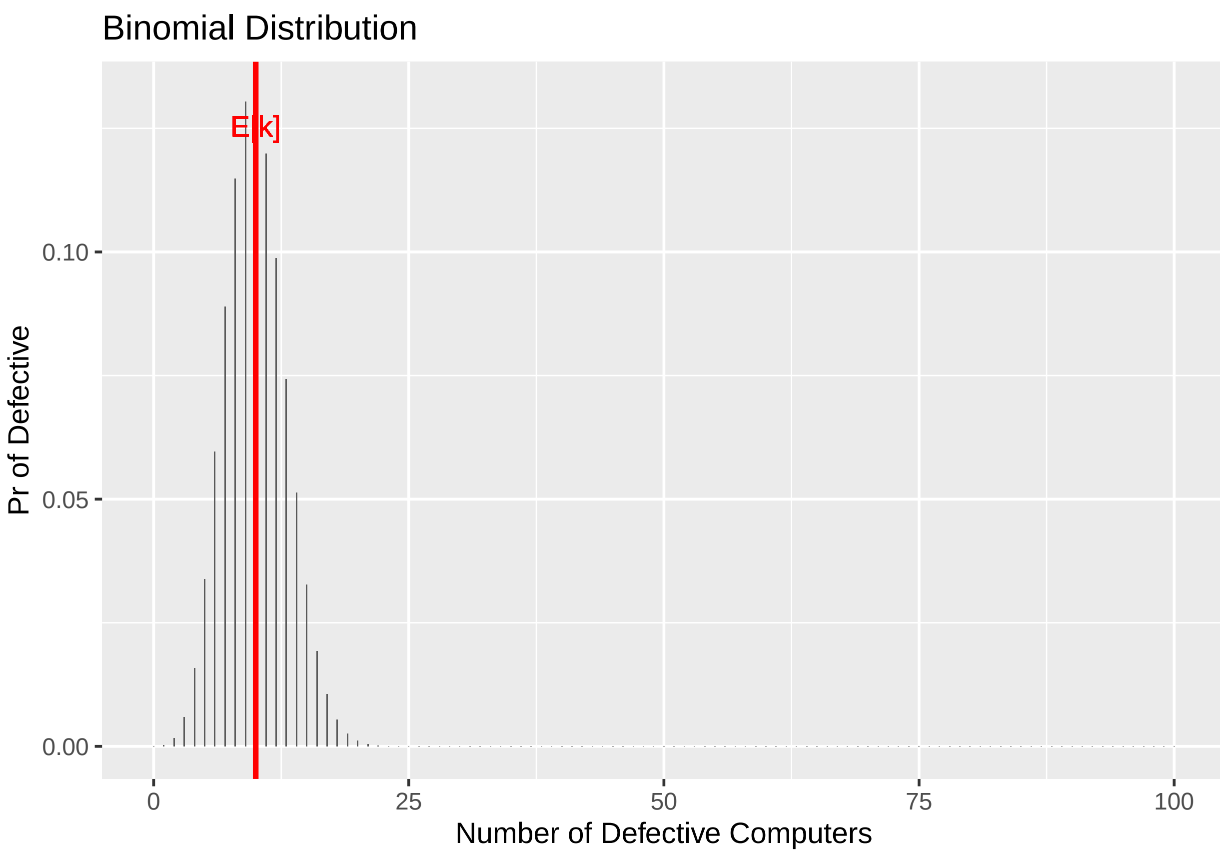
$$Pr(2) = \binom{100}{2}(0.1)^2(0.9)^{100-2} = 0.0016232.$$

$$Pr(>2) = 1 - Pr(\leq 2) = 1 - (Pr(0) + Pr(1) + Pr(2)) = 1 - 2.6561399 \times 10^{-5} + 2.9512665 \times 10^{-4} + 0.0016232 = 0.9980551$$

The plot below shows the Binomial Distribution for defective computers:

```
df <- data.frame(Computers,Pr_Defs)
## Plot
binomial.p_dist<-ggplot(df, aes(x=Computers,y=Pr_Defs)) + geom_col(width = 0.1)+
  xlab("Number of Defective Computers")+
  ylab("Pr of Defective")+ggtitle("Binomial Distribution")+
  geom_vline(xintercept = E_X, color = "Red", size=1)+
  geom_text(aes(x=E_X, label="E[k]", y=max(Pr_Defs)),vjust=2,
            colour="red",  text=element_text(size=18))

 binomial.p_dist
```



```
ggsave("Q4.png",dpi=300, width = 4, height = 2)
```

Question 5

5. A phone center receives 15 calls every 30 minutes.

a) If X is the number of phone calls in 30 minutes, give the Poisson probability mass function for X.

ANSWER:

The distribution is described by the average, $\lambda = 15$,

$$\Pr(k) = \frac{15^k e^{-15}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of phone calls per half hour.

The expected value of the Poisson Distribution is

$$E[k] = 15,$$

with the variance

$$Var[k] = 15.$$

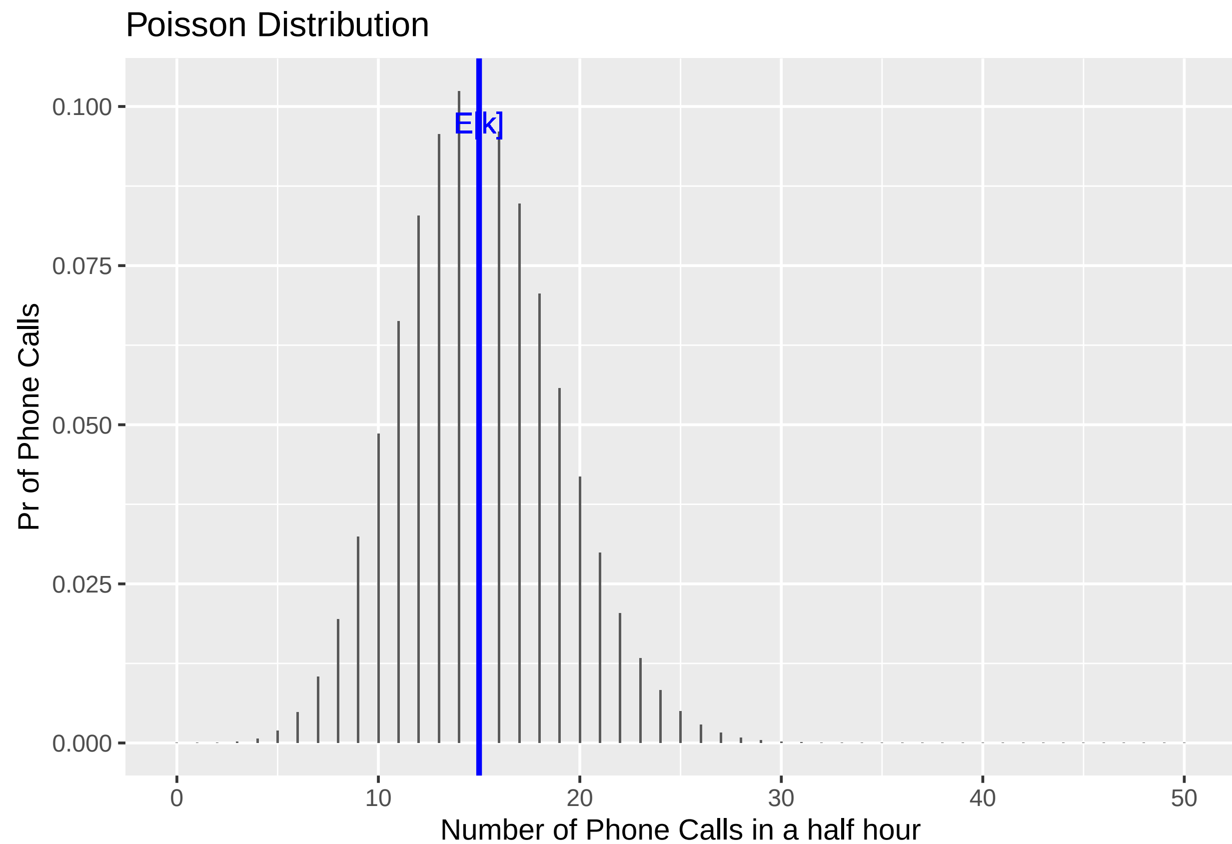
The plot below shows the Poisson Distribution for $\lambda = 15$ average number of calls per half-hour:

```
Calls<-0:50 # Number of Games
No_of_Calls<-15 # Lambda
Pr_Calls<-c(dpois(Calls,No_of_Calls)) # Poisson Probabilities
E_X<-No_of_Calls # Expected Outcome

df <- data.frame(Calls,Pr_Calls)

## Plot
Poisson.p_dist<-ggplot(df, aes(x=Calls,y=Pr_Calls)) +
  geom_col(width = 0.1)+xlab("Number of Phone Calls in a half hour")+
  ylab("Pr of Phone Calls")+ggtitle("Poisson Distribution")+
  geom_vline(xintercept = E_X,  color = "blue", size=1)+
  geom_text(aes(x=E_X, label="E[k]", y=max(Pr_Calls)),vjust=2,
            colour="blue",text=element_text(size=18))

Poisson.p_dist
```




```
ggsave("Q5i.png",dpi=300, width = 4, height = 2)
```

b) What is the probability that there will be exactly 10 phone calls in the first 30 minutes and exactly 20 phone calls in the second 30 minutes.

ANSWER:

```
No_of_Calls<-15 # Lambda
Pr_Calls_10<-dpois(10,No_of_Calls) #  10 Calls Poisson Probability
Pr_Calls_20<-dpois(20,No_of_Calls) #  20 Calls Poisson Probability
Pr_Calls_10_then_20 <- Pr_Calls_10*Pr_Calls_20
```

Probability of 10 calls in the first half-hour:

$$\Pr(10) = \frac{15^{1}0e^{-15}}{10!} = 0.0486108.$$

Probability of 20 calls in the first half-hour:

$$\Pr(20) = \frac{15^{2}0e^{-15}}{20!} = 0.0418103.$$

The combination of both:

$$\Pr(10) \times \Pr(20) = 0.0020324.$$