

## Problem Sheet 2a - Probability Mass Distributions

### Question 1

1. The probability mass function of a discrete random variable X is given in the following table:

Table 1: Q1: Probability Mass Function.

x	Pr
0	0.1
1	0.2
2	0.4
3	0.2
4	0.1

Find the  $E[X]$  and  $\text{Var}[X]$ .

### ANSWER

$$E[X] = x_0p_0 + x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 = \sum_{i=0}^4 x_i p_i$$

Table 2: Q1: Expected Value Calculation

i	x	Pr	xPr
0	0	0.1	0.0
1	1	0.2	0.2
2	2	0.4	0.8
3	3	0.2	0.6
4	4	0.1	0.4

$$E[X] = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) = 2$$

The expected value  $E[X]$  is 2.

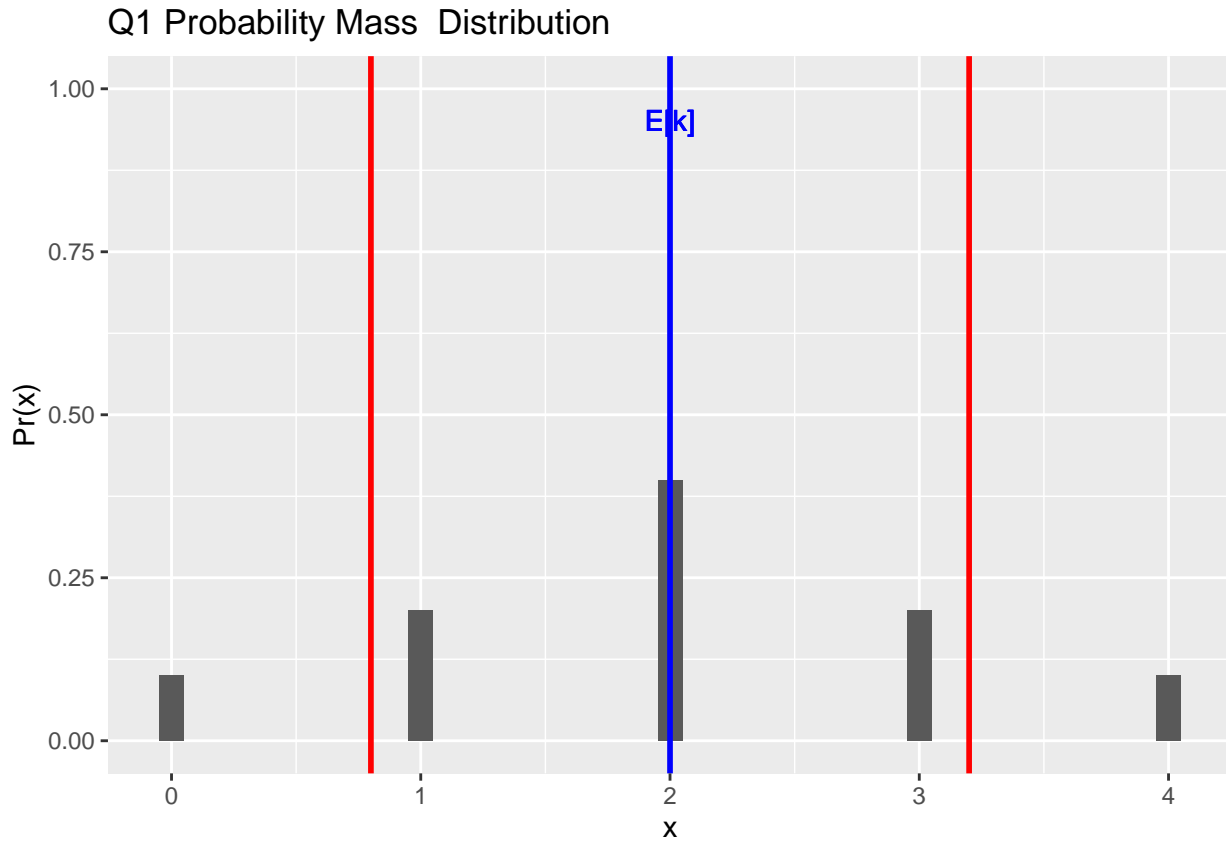
Table 3: Q1: Variance Calculation

i	x	Pr	xPr	var_x
0	0	0.1	0.0	0.4
1	1	0.2	0.2	0.2
2	2	0.4	0.8	0.0
3	3	0.2	0.6	0.2
4	4	0.1	0.4	0.4

$$VAR[X] = (x_0 - E[X])^2 p_0 + (x_1 - E[X])^2 p_1 + (x_2 - E[X])^2 p_2 + (x_3 - E[X])^2 p_3 + (x_4 - E[X])^2 p_4 = \sum_{i=0}^4 (x_i - E[X])^2 p_i$$

$$VAR[X] = (0 - 2)^2 0.1 + (1 - 2)^2 0.2 + (2 - 2)^2 0.4 + (3 - 2)^2 0.2 + (4 - 2)^2 0.1 = 1.2.$$

The Variance value  $VAR[X]$  is 1.2.



## Question 2

The probability mass function of a discrete random variable X is given in the following table:

Show that  $p_3 = 0.2$

**ANSWER**

$$0.1 + 0.3 + 0.3 + p_3 + 0.1 = 1$$

re-arranging

$$p_3 = 1 - 0.1 + 0.3 + 0.3 + 0.1$$

$$p_3 = 1 - 0.8 = 0.2$$

Table 4: Q2: Probability Mass Function.

x	Pr
-2	0.1
-1	0.3
0	0.3
1	0.2
2	0.1

and calculate the  $E[X]$ .

**ANSWER**

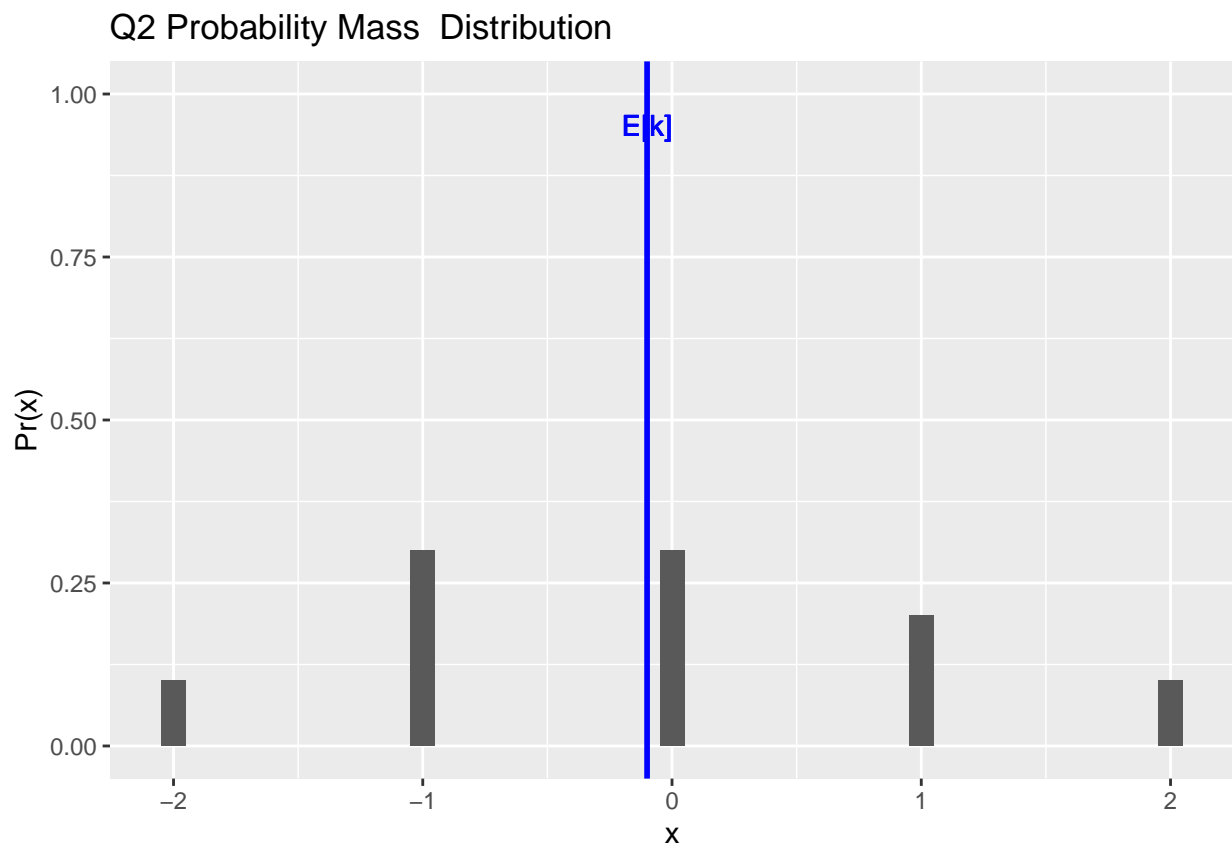
$$E[X] = x_0p_0 + x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 = \sum_{i=0}^4 x_i p_i$$

Table 5: Q2: Expected Value Calculation

i	x	Pr	xPr
0	-2	0.1	-0.2
1	-1	0.3	-0.3
2	0	0.3	0.0
3	1	0.2	0.2
4	2	0.1	0.2

$$E[X] = -2(0.1) + -1(0.3) + 0(0.3) + 1(0.2) + 2(0.1) = -0.1$$

The expected value  $E[X]$  is -0.1.



## Geometric Distribution

A Geometric distribution is used to describe the probability distribution if you do an experiment until you succeed, the experiment has two possible outcomes “success” or “failure”. The probability of “success” is  $p$ , the probability of “failure” is  $q = 1 - p$ . This gives the general definition of the distribution as:

$$\Pr(k) = q^{(k-1)}p, \quad k = 1, 2, \dots$$

with the expected outcome of,

$$E[k] = \frac{1}{p},$$

and variance of

$$\text{Var}[k] = \frac{q}{p^2}.$$

### Question 3

3. 20% of the Irish population watched Ireland beat France in the Rugby World Cup. A representative from TV3 marketing was sent to Grafton Street to ask passerbys their opinion of the match coverage. Let  $X$  denote the number of people need to be asked til the marketer successfully finds some who watched the game.

i Give the Geometric probability mass function for  $X$ .

#### ANSWER

The probability of “success” is

$$p = 0.2,$$

the probability of “failure” is

$$q = 1 - p = 0.8.$$

This gives the general definition of the distribution as:

$$\Pr(k) = 0.8^{(k-1)}0.2, \quad k = 1, 2, \dots$$

where  $k$  is the number of people asked by the marketer.

ii Find the probability that the marketer had to ask exactly 2 people.

#### ANSWER

$$\Pr(2) = 0.8^{(2-1)}0.2 = 0.8(0.2) = 0.16$$

iii What is the  $E[X]$  and  $\text{Var}[X]$  of the distribution.

#### ANSWER

The expected outcome of,

$$E[k] = \frac{1}{p} = \frac{1}{0.2} = 5,$$

and variance of

$$\text{Var}[k] = \frac{q}{p^2} = \frac{0.8}{0.2^2} = 20.$$

Table 6: Q3

People	Pr_win
1	0.2000000
2	0.1600000
3	0.1280000
4	0.1024000
5	0.0819200
6	0.0655360
7	0.0524288
8	0.0419430
9	0.0335544
10	0.0268435
11	0.0214748
12	0.0171799
13	0.0137439
14	0.0109951
15	0.0087961
16	0.0070369
17	0.0056295
18	0.0045036
19	0.0036029
20	0.0028823
21	0.0023058

