Problem Sheet 1 - Solutions

Question 0 *

Give an example of each of the following data types:

i. Categorical

SOLUTION:

Dog or Cat

ii. Interval

SOLUTION:

Height

iii. Ordinal

SOLUTION:

What did you think of the first class:

Hated it, Neutral, Loved it

Measures of Location

Question 1

Describe each of the following measures of location including their the pros and cons

i. Mean

The mean is the point at which the sum of the deviations is 0:

Pros:

- Easy to calculate;
- Uses all the data.

Con:

- Sensitive to extreme values.

ii. Median

The median is the middle value of the ordered set of values:

Pros:

- Median gives the center of the data;
- Not sensitive to extreme values.

Con:

- Does not use all the data.

iii. Variance

The variance is the spread around the mean:

Pro:

- Takes all data into account.

Cons:

- Hard to interpret;
- Can be influenced by extreme values.

iv. Skewness

Skewness is a measure of symmetry of a distribution: Pro

- Takes all data into account;

Con

- Not intuitive.

Counting Questions

Question 2

In a given card game hand each player gets four cards

i. How many different combinations of four cards can be made for a 52 card deck, when **order does** matter.

SOLUTION:

$$P_4^{52} = \frac{52!}{(52-4)!} = (52 \times 51 \times 50 \times 49) = 6,497,400$$

ii. How many different combinations of four cards can be made for a 52 card deck, when order **does not** matter.

SOLUTION:

Order **does not** matter

$$C_4^{52} = \binom{52}{4} = \frac{52!}{(52-4)!4!} = \frac{(52 \times 51 \times 50 \times 49)}{(4 \times 3 \times 2 \times 1)} = 2.70725 \times 10^5$$

iii. What is the probability of being dealt a hand with the 2 of clubs, the 3 of clubs, the 4 of clubs and the 5 of clubs, **in exact order**?

SOLUTION:

$$\frac{1}{P_4^{52}} = \frac{1}{\frac{52!}{(52-4)!}} = \frac{1}{(52 \times 51 \times 50 \times 49)} = 1.5390772 \times 10^{-7}$$

iv. What is the probability of being dealt a hand with the 2 of clubs, the 3 of clubs, the 4 of clubs and the 5 of clubs, **in any order**?

SOLUTION:

$$\frac{1}{C_4^{52}} = \frac{1}{\frac{52!}{(52-4)!4!}} = \frac{4 \times 3 \times 2 \times 1}{(52 \times 51 \times 50 \times 49)} = 3.6937852 \times 10^{-6}$$

v. What is the probability of being dealt a hand with the 2 of diamonds, the 7 of spades, the 9 of clubs and the Queen of diamonds, in any order?

SOLUTION:

$$\frac{1}{C_4^{52}} = \frac{1}{\frac{52!}{(52-4)!4!}} = \frac{4 \times 3 \times 2 \times 1}{(52 \times 51 \times 50 \times 49)} = 3.6937852 \times 10^{-6}$$

vi. What is the probability of being dealt a hand with at least one King, in any order?

SOLUTION:

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Number of hands with at least one king in any order: Hands with exactly One King

$$\frac{(4 \times 48 \times 47 \times 46)}{(1 \times 3 \times 2 \times 1)} = 6.9184 \times 10^4$$

Hands with exactly Two Kings

$$\frac{(4 \times 3 \times 48 \times 47)}{(2 \times 1 \times 2 \times 1)} = 6768$$

Hands with exactly Three Kings

$$\frac{(4 \times 3 \times 2 \times 48)}{(3 \times 2 \times 1 \times 1)} = 192$$

Hands with exactly Four Kings

$$\frac{(4 \times 3 \times 2 \times 1)}{(4 \times 3 \times 2 \times 1 \times 1)} = 1$$

Hands with exactly Zero Kings

$$\frac{(48 \times 47 \times 46 \times 45)}{(4 \times 3 \times 2 \times 1)} = 1.9458 \times 10^5$$

All possible hands 2.70725×10^5 2.70725×10^5

At least one King is all possible hands minus no kings 7.6145×10^4

$$\frac{\frac{(4 \times 51 \times 50 \times 49)}{(1 \times 3 \times 2 \times 1)}}{\frac{52!}{(52-4)!4!}} = \frac{4 \times 51 \times 50 \times 49}{(52 \times 51 \times 50 \times 49)} = 0.2812633$$

Question 3

A bank issues bank cards with PINs consisting of 4 digits, each one {0,1,2,. . . ,9}.

A. Given that any 4-digit code can be used. i. How many unique PINs can be generated?

SOLUTION:

$$10^4 = 10 \times 10 \times 10 \times 10 \times 10 = 10^4$$

ii. What is the probability of guessing someone's PIN?

SOLUTION:

$$\frac{1}{10^4} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = 10^{-4}$$

iii. What is the probability of guessing someone's PIN given you know the first number?

SOLUTION:

$$\frac{10}{10^4} = \frac{10}{10 \times 10 \times 10 \times 10 \times 10} = 0.001$$

B. The digits must be different.

i. How many unique PINs can be generated?

SOLUTION:

Order **does** matter

$$P_4^{10} = \frac{10!}{(10-4)!} = (10 \times 9 \times 8 \times 7) = 5,040$$

ii. What is the probability of guessing someone's PIN?

SOLUTION:

$$\frac{1}{P_4^{10}} = \frac{1}{\frac{10!}{(10-4)!}} = \frac{1}{(10 \times 9 \times 8 \times 7)} = 1.984127 \times 10^{-4}$$

iii. What is the probability of guessing someone's PIN give you know the first number?

SOLUTION:

$$\frac{10}{P_4^{10}} = \frac{10}{\frac{10!}{(10-4)!}} = \frac{10}{(10 \times 9 \times 8 \times 7)} = 0.0019841$$

Question 4

In a lottery, each ticket has 5 one-digit numbers 0-9 which is not repeated on it.

A. You win if your ticket has the digits in any order.

i. What are the total number of possible combinations?

SOLUTION:

Order **does not** matter

$$C_5^{10} = \binom{10}{5} = \frac{10!}{(10-5)!5!} = \frac{(10 \times 9 \times 8 \times 7 \times 6)}{(5 \times 4 \times 3 \times 2 \times 1)} = 252$$

ii. What is the probability of winning?

SOLUTION:

Order **does not** matter

$$\frac{1}{\binom{10}{5}} = \frac{1}{\frac{10!}{(10-5)!5!}} = \frac{(5 \times 4 \times 3 \times 2 \times 1)}{(10 \times 9 \times 8 \times 7 \times 6)} = 0.0039683$$

B. You would win only if your ticket has the digits in the required order.

i. What are the total number of possible combinations?

SOLUTION:

Order **does** matter

$$P_5^{10} = \frac{10!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

ii. What is the probability of winning?

$$\frac{1}{P_5^{10}} = \frac{1}{\frac{10!}{5!}} = \frac{1}{10 \times 9 \times 8 \times 7 \times 6} = 3.3068783 \times 10^{-5}$$

Question 5

How many different combinations of 6 cards can be made for a 52 card deck if

i. order matters.

SOLUTION:

Order **does** matter

$$P_6^{52} = \frac{52!}{(52-6)!} = 52 \times 51 \times 50 \times 49 \times 48 \times 47 = 14,658,134,400$$

ii. order does not matter

SOLUTION:

Order **does not** matter

$$C_6^{52} = \binom{52}{6} = \frac{52!}{(52-6)!6!} = \frac{(52 \times 51 \times 50 \times 49 \times 48 \times 47)}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} = 20,358,520$$

Question 6

In a game of 5 card poker what are the number of different possible hands are there?

i. What is the probability of being dealt a hand with a pair?

SOLUTION:

Number of hands with a pair (or more):

$$\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 47 \times 46)}{(3 \times 2 \times 1)} = 1,349,088$$

Number of hands all possible hands

$$\frac{(52 \times 51 \times 50 \times 49 \times 48)}{(5 \times 4 \times 3 \times 2 \times 1)} = 2.59896 \times 10^6$$

Number of all possible hands:

$$p(pair) = \frac{\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 47 \times 46)}{(3 \times 2 \times 1)}}{\frac{(52 \times 51 \times 50 \times 49 \times 48)}{(5 \times 4 \times 3 \times 2 \times 1)}} = 0.5190876$$

(Alternative) A hand with a only pair

SOLUTION:

$$\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 44 \times 40)}{(3 \times 2 \times 1)} = 1.09824 \times 10^6$$

$$p(pair) = \frac{\frac{52 \times 3}{2 \times 1} \times \frac{(48 \times 44 \times 40)}{(3 \times 2 \times 1)}}{\frac{(52 \times 51 \times 50 \times 49 \times 48)}{(5 \times 4 \times 3 \times 2 \times 1)}} = 0.422569$$

ii. What is the probability of being dealt a hand with two pair?

SOLUTION:

Number of two pair hands:

$$\frac{\frac{52\times3}{2\times1}\times\frac{48\times3}{2\times1}}{2\times1}\times\frac{44}{1}=1.23552\times10^5$$

$$p(\text{two pair})=\frac{\frac{\frac{52\times3}{2\times1}\times\frac{48\times3}{2\times1}}{2\times1}\times\frac{44}{1}}{\frac{(52\times51\times50\times49\times48)}{(5\times4\times3\times2\times1)}}=0.047539$$

iii. What is the probability of being dealt a hand with three of a kind and two of kind (Full House)

SOLUTION:

Numer of Full House hands:

$$\frac{52\times3\times2}{3\times2\times1}\times\frac{48\times3}{2\times1}=3,744$$

$$p(\text{Full House})=\frac{\frac{52\times3\times2}{3\times2\times1}\times\frac{48\times3}{2\times1}}{\frac{(52\times51\times50\times49\times48)}{(5\times4\times3\times2\times1)}}=0.0014406$$

iv. What is the probability of being dealt a hand with a Flush (all the same suit)

SOLUTION:

Number of possible flush hands

$$\frac{52\times12\times11\times10\times9}{5\times4\times3\times2\times1}=5148$$

$$p(Flush)=\frac{\frac{52\times12\times11\times10\times9}{5\times4\times3\times2\times1}}{\frac{(52\times51\times50\times49\times48)}{(5\times4\times3\times2\times1)}}=0.0019808$$

Multiple-Choice Questions

MCQ Question 7

Which of the following is an example of an ordinal data type?

A. Temperature in Celsius

B. Eye color

C. Customer satisfaction rating (e.g., Poor, Fair, Good, Excellent)

D. Bank account number

Answer: C. Customer satisfaction rating

MCQ Question 8

Which measure of location is least affected by extreme values?

A. Mean

B. Median

C. Mode

D. Variance

Answer: B. Median

MCQ Question 9

How many different ways can the letters of the word "STATISTICS" be arranged?

A) 3,628,800

B) 50,400

C) 63,504

D) 1,260

ANSWER: B) 50,400

Explanation: This is a permutation problem with repeated elements. The total number of arrangements is given by 10!/(3!3!2!)

MCQ Question 10

You have 10 different players and need to form a lineup of 4 players, where the order in which they are arranged does not matter. What rule should you use to calculate how many different lineups can be made?

A) Combination

B) Permutation

C) Neither

ANSWER: A)

Explanation: This is a combination problem where order does not matter.

MCQ Question 11

What rule would you use to calculate how many ways can you arrange the letters in the word "BALLOON"

A) Combination

B) Permutation

C) Neither

ANSWER: C)

Explanation: As there are two L's and O's you have to take that into account

Question 12

Write your own question or multiple choice question with solution on counting.