Tutorial Sheet 3 Solutions

Probability Mass Distributions

Question 1

1. The probability mass function of a discrete random variable X is given in the following table:

Table 1: Q1: Probability Mass Function.

i	1	2	3	4	5
x	0	1	2	3	4
p(x)	0.1	0.2	0.4	0.2	0.1

i. Find the E[X] and Var [X].

ANSWER The formula for the Expected value, E[X] is:

$$E[X] = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_5p_5 = \sum_{i=1}^{5} x_ip_i$$

Doing the E[X] calculation in tabular form:

Table 2: Q1: Expected Value Calculation

i	1	2	3	4	5
x	0	1	2	3	4
p(x)	0.1	0.2	0.4	0.2	0.1
xp(x)	0.0	0.2	0.8	0.6	0.4

$$E[X] = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) = 2$$

The expected value E[X] is 2.

The formula for the Variance value, VAR[X], is:

$$VAR[X] = (x_1 - E[X])^2p_1 + (x_2 - E[X])^2p_2 +$$

$$(x_3 - E[X])^2p_3 + (x_4 - E[X])^2p_4 + (x_5 - E[X])^2p_5 = \sum_{i=1}^{5}(x_i - E[X])^2p_i$$

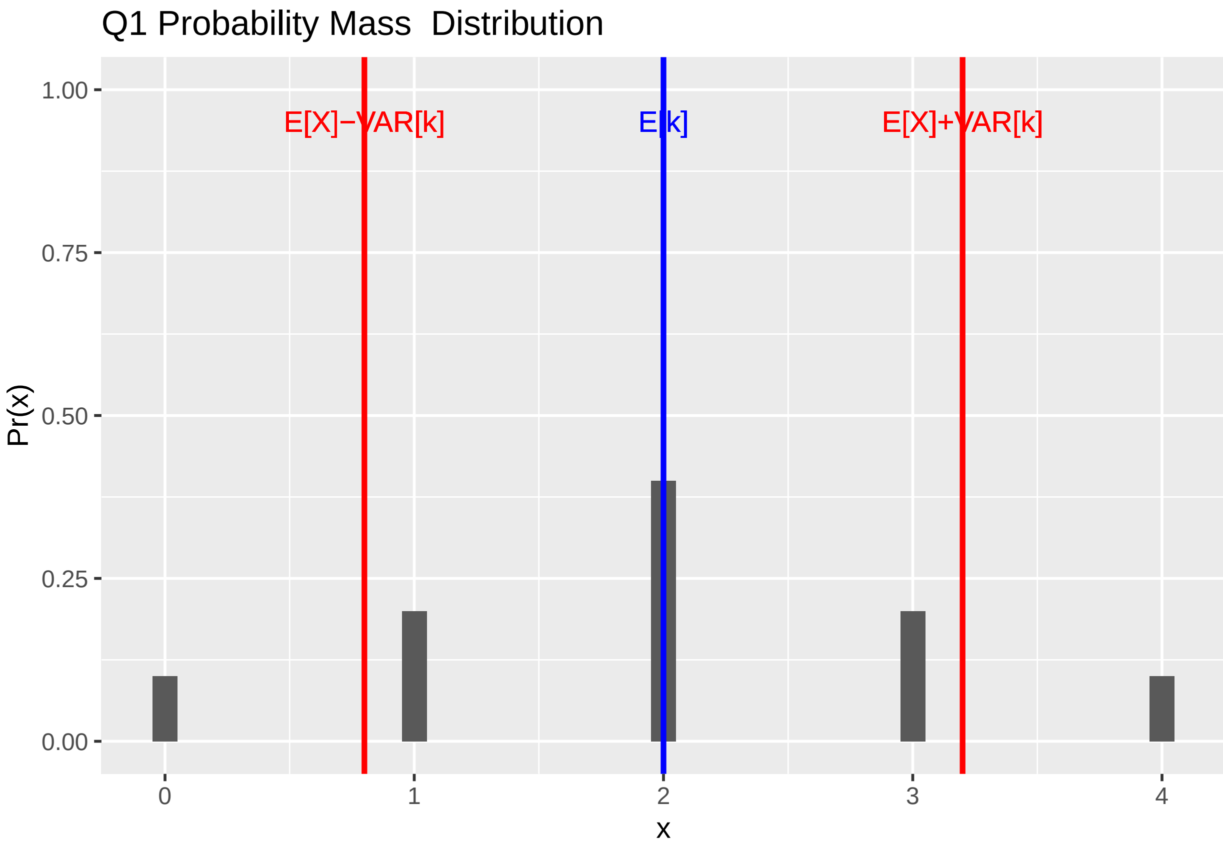
In tabular form:

Table 3: Q1: Variance Calculation

i	1	2	3	4	5
x	0	1	2	3	4
p(x)	0.1	0.2	0.4	0.2	0.1
xp(x)	0.0	0.2	0.8	0.6	0.4
(x-E[x])^2p(x)	0.4	0.2	0.0	0.2	0.4

$$VAR[X] = (0-2)^2 0.1 + (1-2)^2 0.2 +$$
$$(2-2)^2 0.4 + (3-2)^2 0.2 + (4-2)^2 0.1 = 1.2.$$

The Variance value VAR[X] is 1.2.

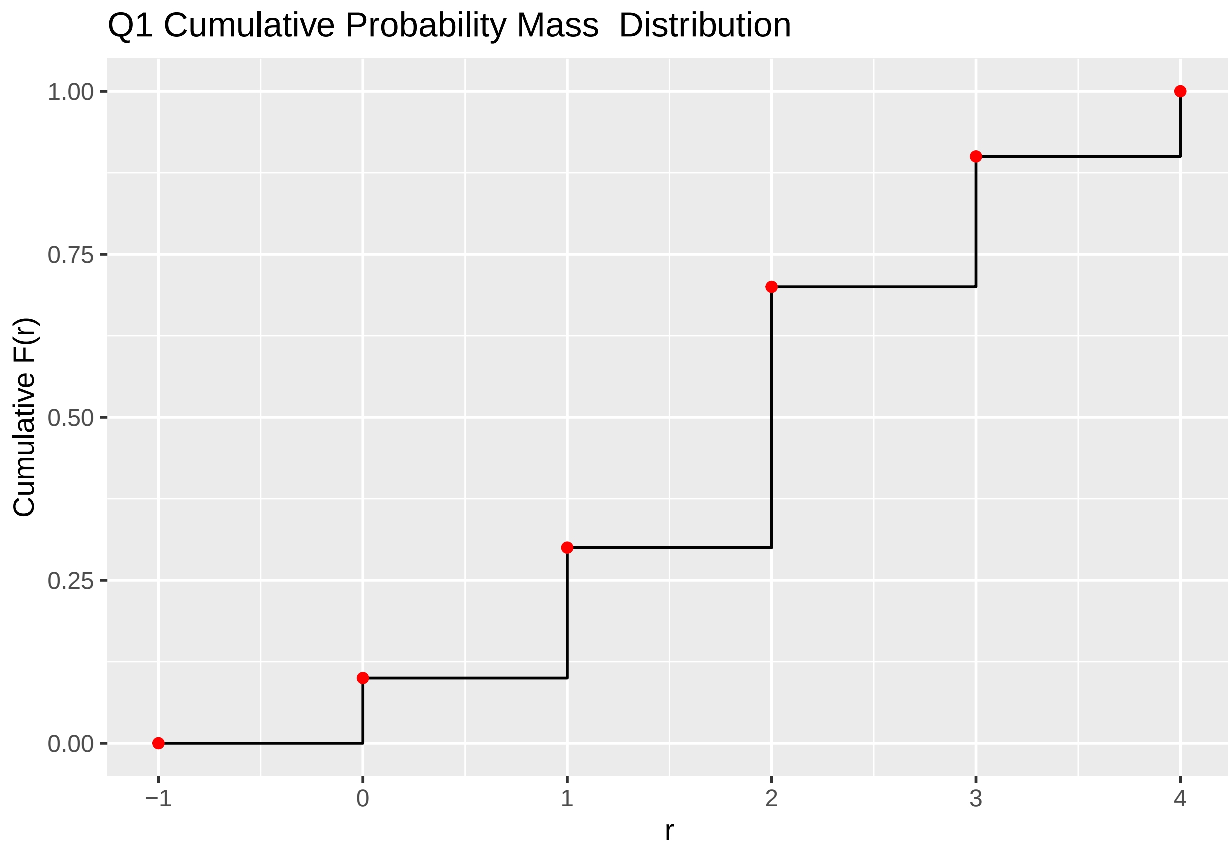


ii. The Cumulative probability mass function

The cumulative probability mass function of a discrete random variable X is given in the following table:

Table 4: Q1: Cumulative Probability Mass Function.

r	<0	$0 \leq x < 1$	$1 \leq x < 2$	$2 \leq x < 3$	$3 \leq x < 4$	x>4
F(r)	0.0	0.1	0.3	0.7	0.9	1.0



Question 2

The probability mass function of a discrete random variable X is given in the following table:

Table 5: Q2: Probability Mass Function.

i	0	1	2	3	4
x	-2	-1	0	1	2
p(x)	0.1	0.3	0.3	0.2	0.1

i. Show that p3 = 0.2

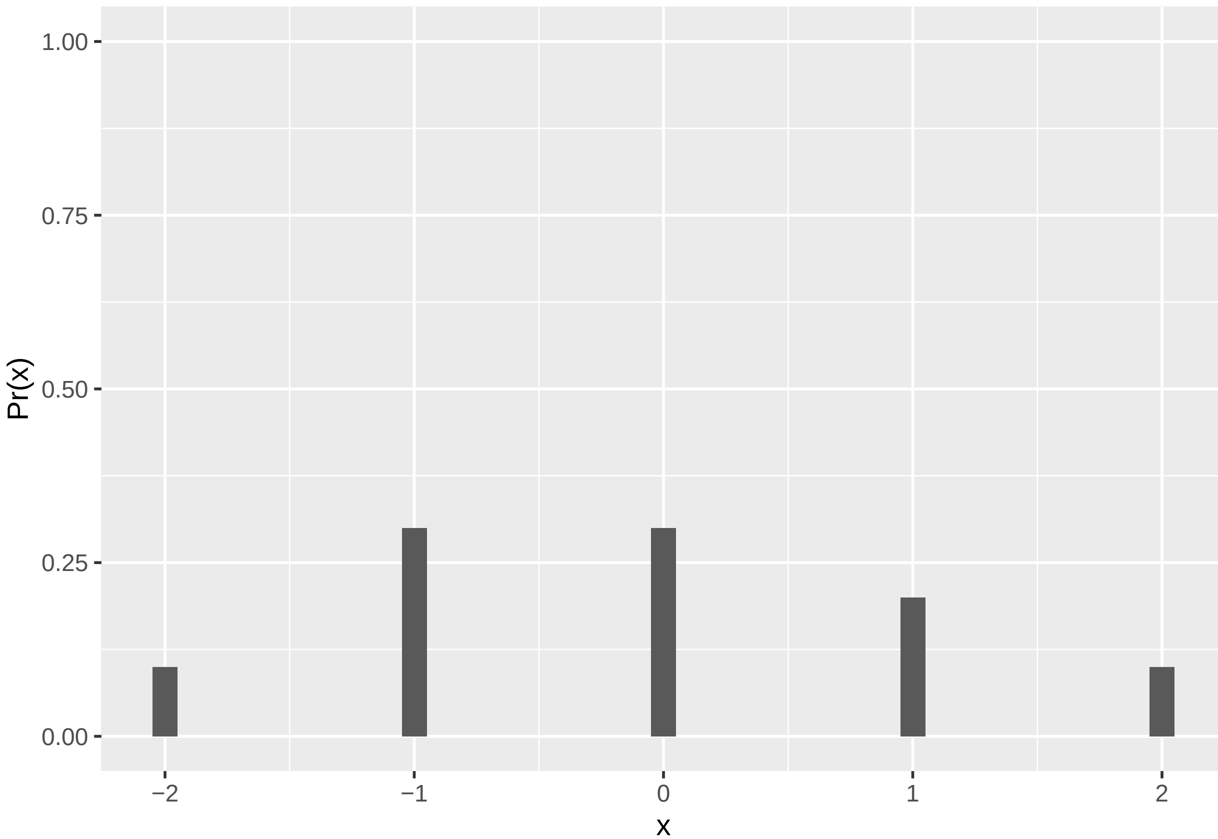
ANSWER

$$0.1 + 0.3 + 0.3 + p_3 + 0.1 = 1$$

re-arranging

$$p_3 = 1 - 0.1 + 0.3 + 0.3 + 0.1$$

$$p_3 = 1 - 0.8 = 0.2$$



ii. Calculate the E[X] and Var[X].

ANSWER

$$E[X] = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_5p_5 = \sum_{i=1}^{5} x_ip_i$$

Table 6: Q2: Expected Value Calculation

i	1	2	3	4	5
x	-2	-1	0	1	2
p(x)	0.1	0.3	0.3	0.2	0.1
xp(x)	-0.2	-0.3	0.0	0.2	0.2

$$E[X] = -2(0.1) + -1(0.3) + 0(0.3) + 1(0.2) + 2(0.1) = -0.1$$

The expected value E[X] is -0.1.

Calculating the Var[X] in tabular form:

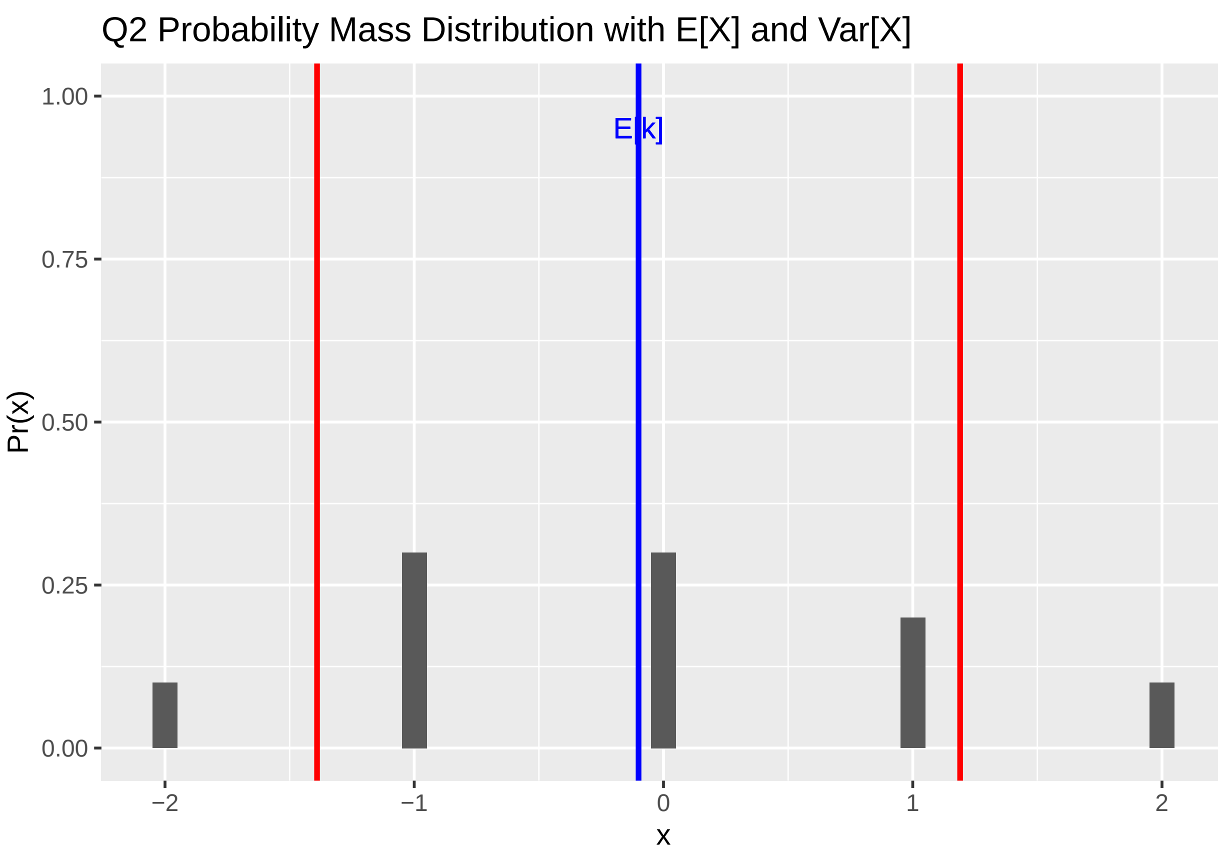
Table 7: Q2: Variance Calculation

i	1	2	3	4	5
x	-2	-1	0	1	2
p(x)	0.1	0.3	0.3	0.2	0.1
xp(x)	-0.2	-0.3	0.0	0.2	0.2
(x-E[x])^2p(x)	0.361	0.243	0.003	0.242	0.441

The formula for the Variance value, VAR[X], is:

$$VAR[X] = \sum_{i=1}^{5}(x_i - E[X])^2 p_i$$

$$VAR[X] = 1.29.$$



Question 3

3. 20% of the Irish population watched Ireland beat France in the Rugby World Cup. A representative from TV3 marketing was sent to Grafton Street to ask passersby their opinion of the match coverage. Let X denote the number of people need to be asked til the marketer successfully finds someone who watched the game.

i Give the Geometric probability mass function for X.

ANSWER

The probability of “success” is

$$p = 0.2,$$

the probability of “failure” is

$$q = 1 - p = 0.8.$$

This gives the general definition of the distribution as:

$$\Pr(k) = 0.8^{(k-1)}0.2, \quad k = 1, 2, ...$$

where k is the number of people asked by the marketer.

ii Find the probability that the marketer had to ask exactly 2 people.

ANSWER

$$\Pr(2) = 0.8^{(2-1)}0.2 = 0.8(0.2) = 0.16$$

iii What is the E [X] and V ar [X] of the distribution.

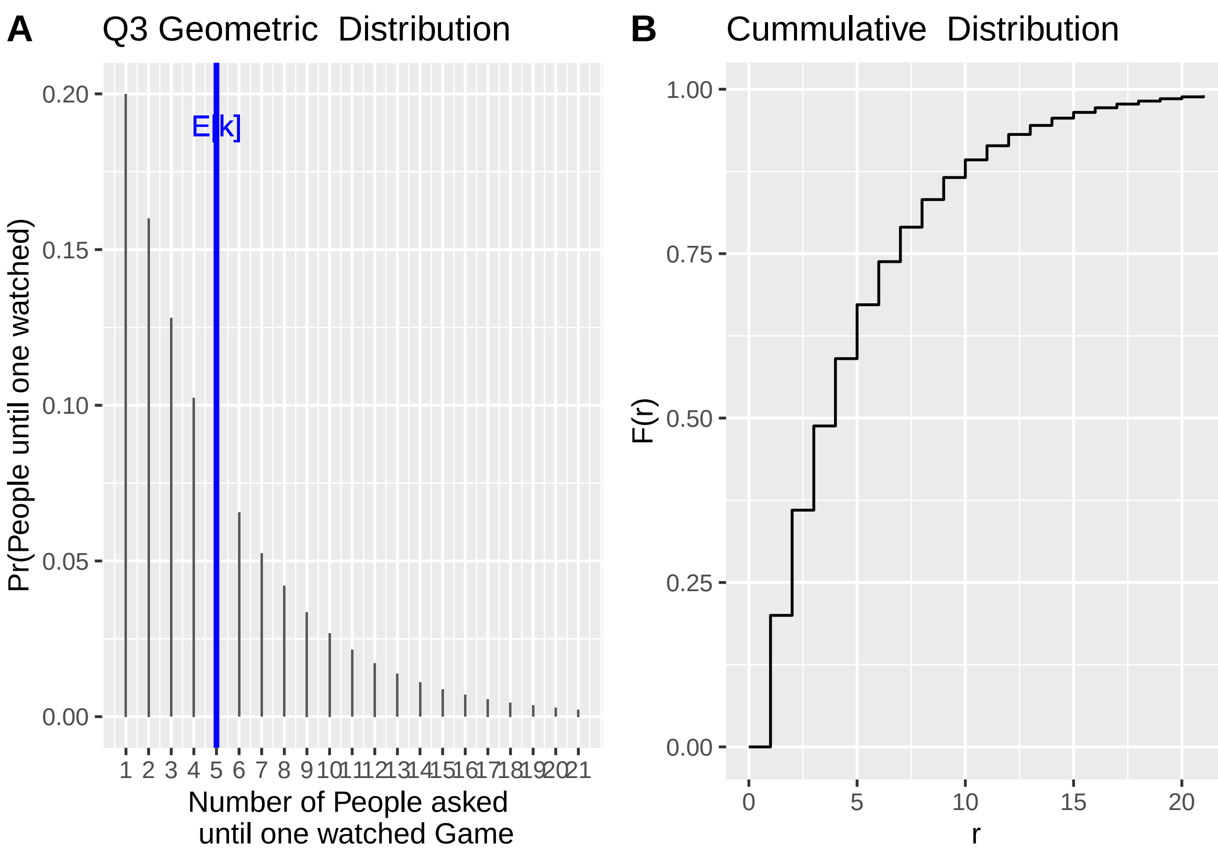
ANSWER

The expected outcome of,

$$E[k] = \frac{1}{p} = \frac{1}{0.2} = 5,$$

and variance of

$$Var[k] = \frac{q}{p^2} = \frac{0.8}{0.2^2} = 20.$$



Very Optional Question 4

4. The probability mass function of a Bernoulli random variable X is given in the following table:

Table 8: Q4: Probability Mass Function.

x	0	1
p(x)	q	p

a. Find the mean E[X] and variance Var[X]. **Answer:**

$$q = 1 - p, p = 1 - q, 1 = p + q.$$

$$E[X] = \Sigma_x xp(x) = 0(q) + 1(p) = p.$$

$$Var[X] = \Sigma_x (x - E[X])^2 p(x) = (0 - p)^2(q) + (1 - q)^2(p) = p^2q + q^2p$$
$$Var[X] == p^2q + q^2p = qp(p + q) = qp$$

MCQ Question 5

Which of the following is true about a probability mass function (PMF) of a discrete random variable?

A) The PMF can take any value greater than or equal to 0.

B) The sum of all values of a PMF is 1.

C) The PMF can take negative values.

D) The PMF is defined for continuous random variables.

Solution

Correct Answer: B) The sum of all values of a PMF is 1.

MCQ Question 6

The probability mass function of a discrete random variable X is given in the following table:

Table 9: Q6: Probability Mass Function.

i	1	2	3	4
x	-4	-2	2	6
p(x)	0.1	0.3	0.4	0.2

What is the probability that X takes a value greater than -2?

A) 0.1

B) 0.2

C) 0.6

D) 0.4

E) 0.7

Solution

Answer: C) 0.6

MCQ Question 7

Which of the following best describes the geometric distribution?

A) It models the number of successes before the first failure in a series of independent Bernoulli trials.

B) It models the number of trials until the first success in a series of independent Bernoulli trials.

C) It models the number of failures before the first success in a series of dependent Bernoulli trials.

D) It models the number of trials until the first success in a series of dependent Bernoulli trials.

Solution ANSWER: B

MCQ Question 8

Spot three reasons why the following table cannot be a probability mass function of a discrete random variable X:

Table 10: Q8: Probability Mass Function.

i	1	2	3	4
x	4	-2	2	6
p(x)	0.1	-0.3	0.4	0.2

Solution

1. Negative probability
2. Probabilities do not sum to 1
3. In wrong order (this is a bit of a weak one)

MCQ Question 9

What is the variance of a Bernoulli random variable with success probability $p = 0.6$?

A) 0.24

B) 0.36

C) 0.4

D) 0.6

Answer: **A**
$\text{Var}[X] = p(1-p) = 0.6 \cdot 0.4 = 0.24$

MCQ Question 10

Which statement best describes a cumulative distribution function (CDF)?

A) It gives the probability of a specific value.

B) It gives the probability that a variable is greater than a value.

C) It gives the probability that a variable is less than or equal to a value.

D) It gives the expected value of a distribution.

Answer: **C**

Question 11

Write your own question for a geometric distribution.