OLLSCOIL TEICNEOLAÍOCHTA
BHAILE ÁTHA CLIATH
DUBLIN
TECHNOLOGICAL
UNIVERSITY DUBLIN

Counting Rules

Dr. John S. Butler

Useful counting Rules

- Consider selecting or sampling r objects from a group of n distinct objects, sampling with replacement.
- The total number of possible ordered samples
- $n \times n \times \cdots \times n = n^r$
- The probability is $\frac{1}{n^r}$

Useful counting Rules

- Consider selecting or sampling r objects from a group of n distinct objects, sample without replacement

- $p_r^n = \frac{n!}{(n-r)!}$
- This equals the number of arrangements (permutations) of any *r* objects selected from a group of *n* distinct objects

Useful counting Rules

- Consider selecting or sampling r objects from a group of n distinct objects, sampling without replacement. The total number of possible non-ordered samples

$$\frac{p_r^n}{r!} = \frac{n!}{((n-r)!\,r!)} = C_r^n = \binom{n}{r}$$

- (Binomial coefficient)

Useful counting Rules

- The number of distinct arrangements of n objects of which n_1 are of one kind n_2 of a second kind, …, n_k are of kth kind is given by multinomial coefficient

$$\frac{n!}{(n_1!n_2!\ldots n_k!)} \text{ where } \sum_{i=1}^{k} n_i = n$$

- Example: What is the probability that in 6 throws of a fair die all faces turn up?

Example

- A bank issues bank cards with PINs consisting of 4 digits, each one {0,1,2,…,9}. How many unique PINs are there if

1. Any 4-digit code can be used
2. The digits must be different

Example

- In a game of 5 card poker what are the number of

1. Different possible hands are there?
2. A hand with a pair
3. A hand with two pair
4. A hand with Three of a kind
5. A hand with a Flush (all the same suit)