Tutorial 4 Solutions

Named Discrete Distributions

Question 1

1. There are 30 candy covered chocolates in a bag M&M's. There is a .1 probability that that the candy is red. If X is the number of red M&M's in the bag.

i. Give the binomial probability mass function for X.

ANSWER:

$$\Pr(k) = \left(\begin{array}{c} 30 \\ k \end{array} \right) (0.1)^k (0.9)^{30-k}, \quad k = 0, 1, 2, ...30,$$

where k is the number of red M & Ms.

ii. Find the probability of less than 2 red M&Ms in the bag.

ANSWER:

$$\Pr(0) = \left(\begin{array}{c} 30 \\ 0 \end{array} \right) (0.1)^0 (0.9)^{30-0} = 0.0423912$$

$$\Pr(1) = \left(\begin{array}{c} 30 \\ 1 \end{array} \right) (0.1)^1 (0.9)^{30-1} = 0.1413039$$

$$Pr(< 2) = Pr(0) + Pr(1) = 0.0423912 + 0.1413039 = 0.183695,$$

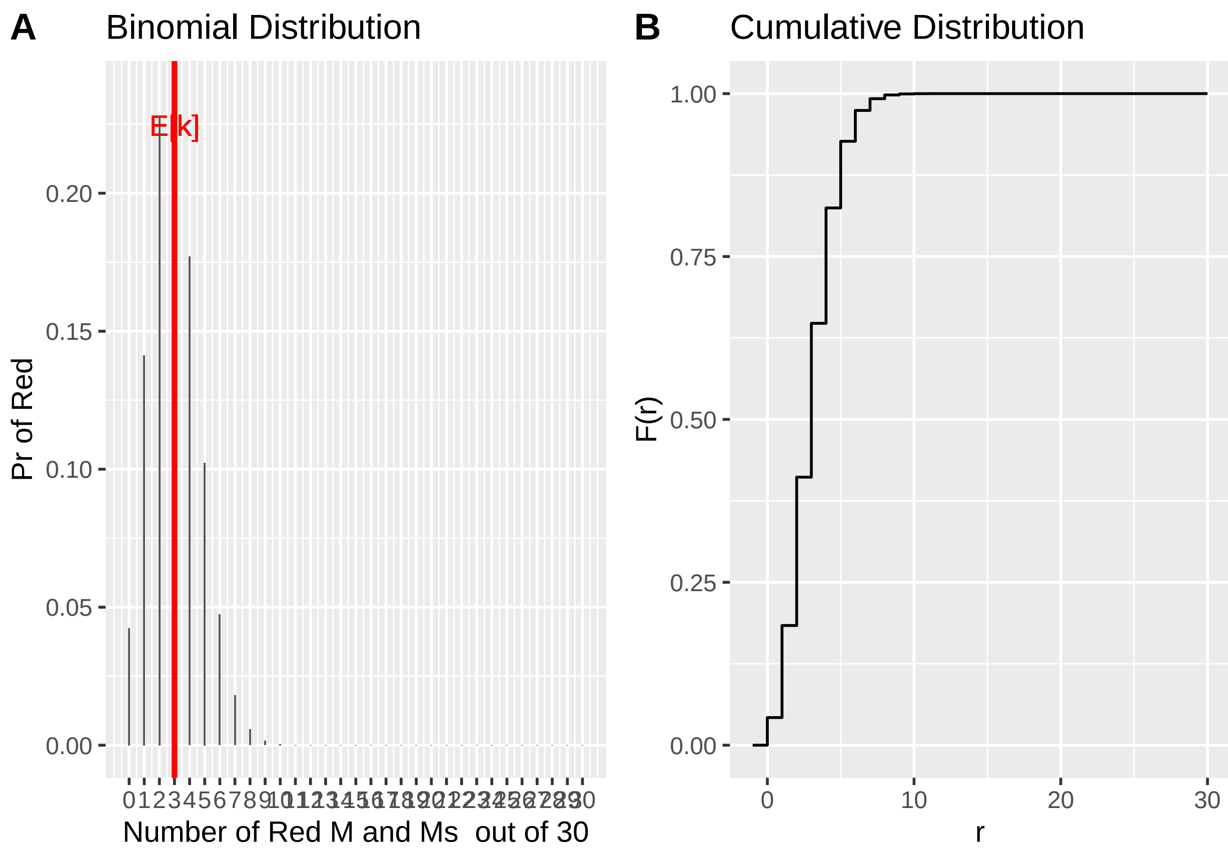
As it is a Binomial Distribution we can state that the expected number of Red is

$$E[k] = 30 \times 0.1 = 3,$$

the variance of the distribution is

$$Var[k] = 30 \times 0.1 \times (1 - 0.1) = 2.7.$$

The plot below shows the Binomial Distribution M & Ms:



Question 2

2. A baby wakes on average 0.25 times every hour.

i. If X is the number of times a baby wakes in an hour, give the poisson probability mass function for X.

ANSWER:

The distribution is described by the average, $\lambda = 0.25$,

$$\Pr(k) = \frac{0.25^k e^{-0.25}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of times the baby wakes every hour.

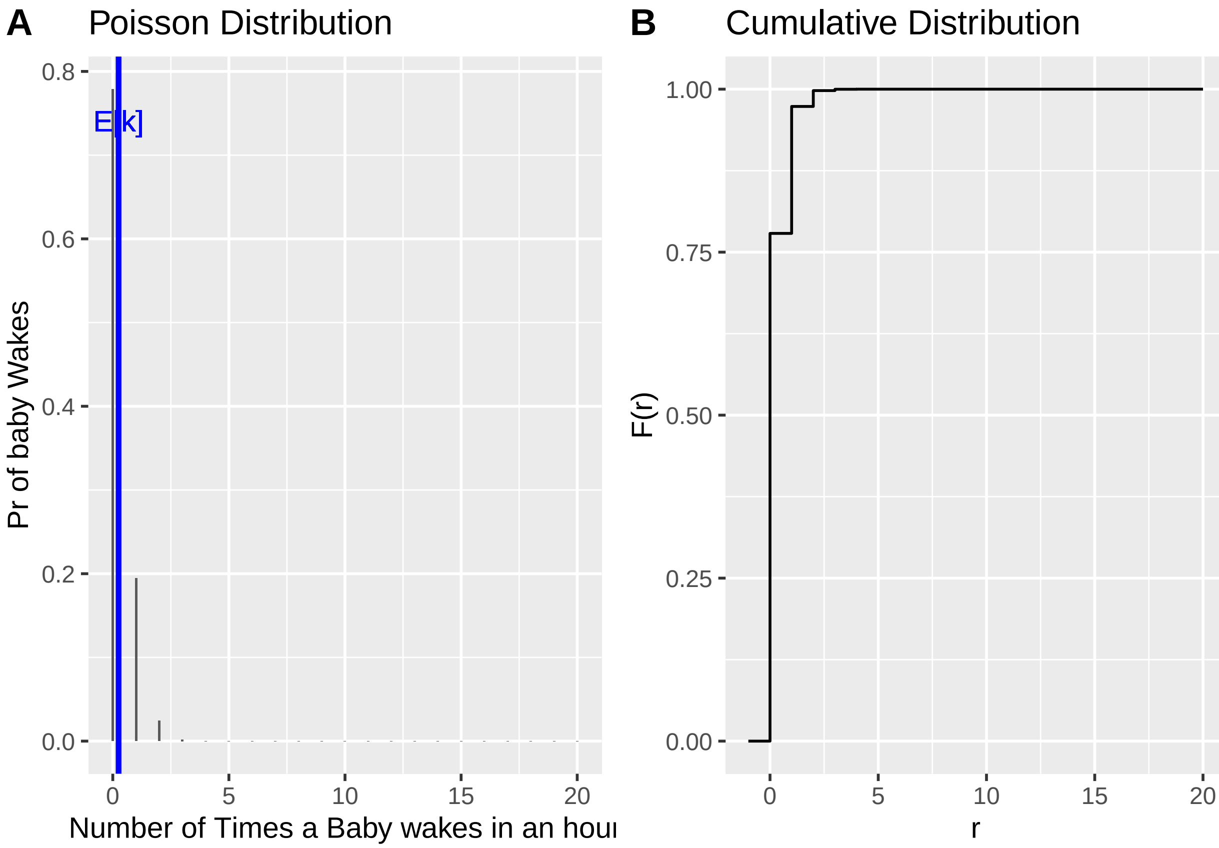
The expected value of the Poisson Distribution is

$$E[k] = 0.25,$$

with the variance

$$Var[k] = 0.25.$$

The plot below shows the Poisson Distribution for $\lambda = 0.25$ average number of times a baby wakes in an hour:



ii. If X is the number of times a baby wakes in eight hour, give the poisson probability mass function for X.

ANSWER:

The distribution is described by the average, $\lambda = 0.25 \times 8 = 2$,

$$\Pr(k) = \frac{2^k e^{-2}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of times the baby wakes every 8 hours.

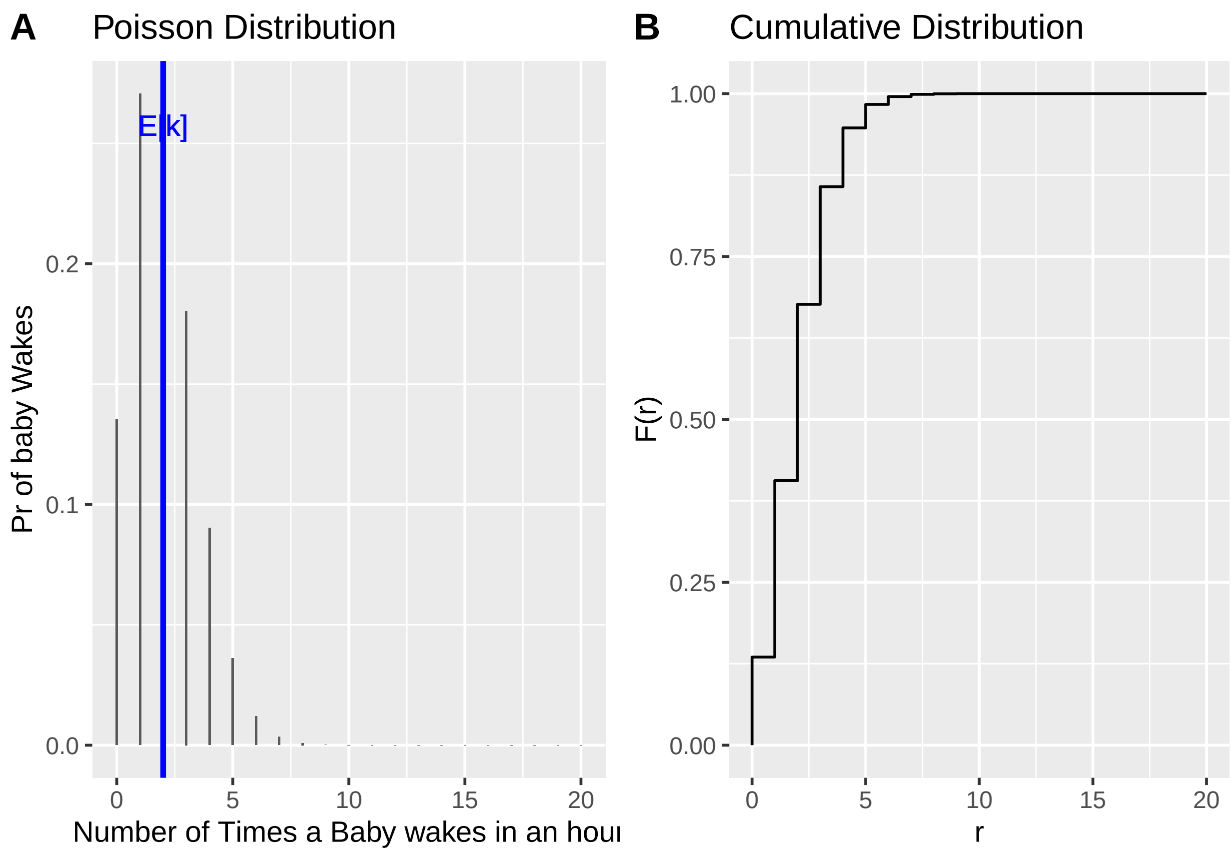
The expected value of the Poisson Distribution is

$$E[k] = 2,$$

with the variance

$$Var[k] = 2.$$

The plot below shows the Poisson Distribution for $\lambda = 2$ average number of times a baby wakes in eight hours:



iii. What is the probability that the baby does not wake during the 8 hours.

ANSWER:

```
Baby_wakes<-0.25*8 # Lambda
Pr_wakes_zero<-dpois(0,Baby_wakes) #
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$$\Pr(0) = \frac{2^0 e^{-2}}{0!} = 0.1353353.$$

Question 3

3. Give the features of a

i. Geometric Experiment.

ANSWER:

- The experiment consists of a series of repeated Bernoulli trials
- There are two possible outcomes arbitrary called success and failure
- A success occurs with probability p and a failure occurs with probability q=1-p
- The Random Variable is ordered as 1 if success and 0 if failure
- The random variable is the number of trials performed to yield one success
- $E[X] = 1/p$, $VAR[X] = q/p^2$

ii. Binomial Experiment.

ANSWER:

- The experiment consists of n repeated Bernoulli trials
- The trials are independent
- The probability of success in each trial is constant
- E[x]=np, VAR[X]=npq

iii. Poisson Experiment.

ANSWER:

- The experiment consists of a number of events happening randomly over time or space
- The events occur independently of each other
- The rate of occurrence of the events is a well defined average per unit/space
- The Random Variable is the number of events occurring in a given interval
- E[X]=lambda, VAR[X]=lambda

iv. Negative Binomial Experiment.

ANSWER:

- The trials are independent
- The number of trials to be performed is not known at the start of the experiment
- The probability of success in each trial is constant
- The random variable is the number of trials performed to yield r successes
- $E[X] = r/p$, $VAR[X] = r\frac{q}{p^2}$

Question 4

4. Every day a production line makes 100 computers of which 10% are defective. If X is the number of defective computers in a day.

i. Give the binomial probability mass function for X.

ANSWER:

$$\Pr(k) = \binom{100}{k}(0.1)^k(0.9)^{100-k}, \quad k = 0, 1, 2, ...100,$$

where k is the number of defective computers.

ii. Find the probability that there is more than 2 computers defective in a day.

ANSWER:

$$\Pr(0) = \binom{100}{0}(0.1)^0(0.9)^{100-0} = 2.6561399 \times 10^{-5},$$

$$\Pr(1) = \binom{100}{1}(0.1)^1(0.9)^{100-1} = 2.9512665 \times 10^{-4},$$

$$Pr(2) = \binom{100}{2}(0.1)^2(0.9)^{100-2} = 0.0016232.$$

$Pr(> 2) = 1 - Pr(\leq 2) = 1 - (Pr(0) + Pr(1) + Pr(2)) = 1 - 2.6561399 \times 10^{-5} + 2.9512665 \times 10^{-4} + 0.0016232$

$$Pr(>2) = 0.9980551$$

iii. What is the E[X] and Var[X] of the distribution?

ANSWER:

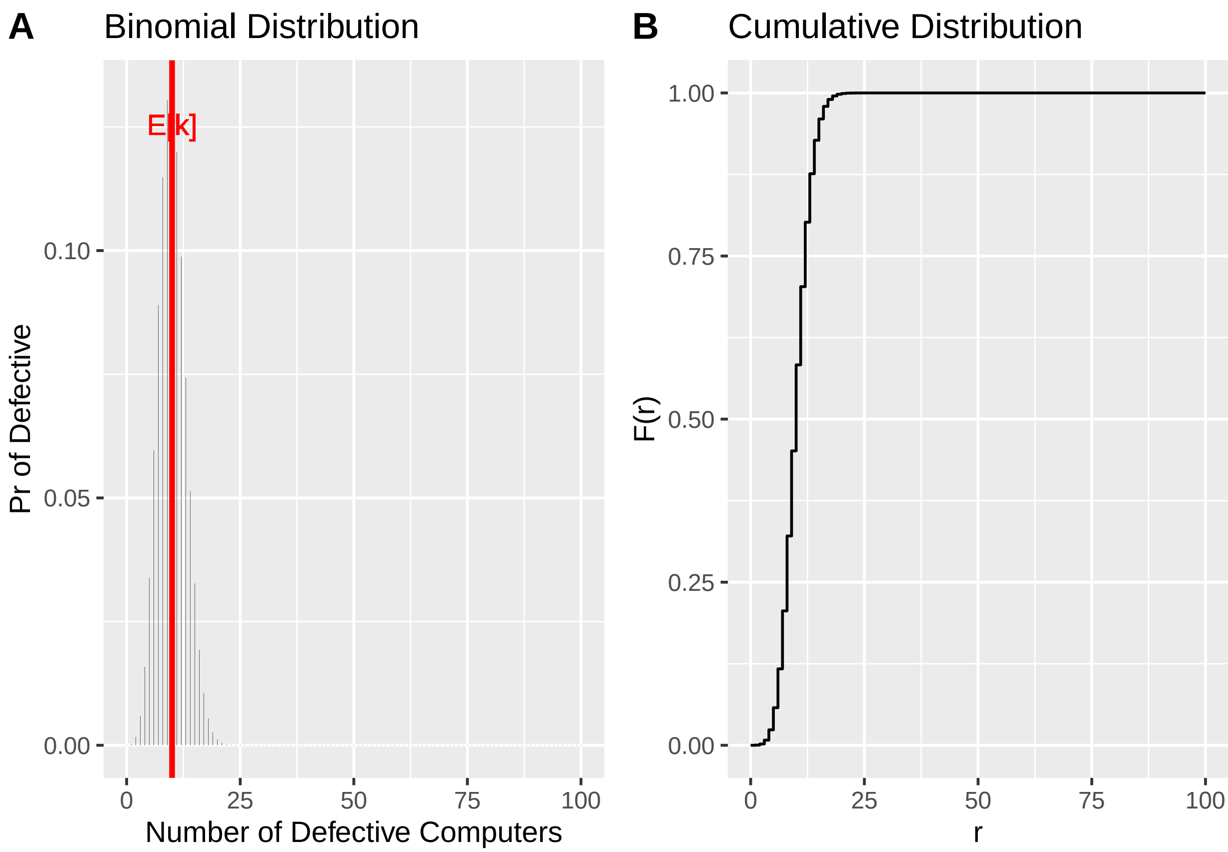
As it is a Binomial Distribution we can state that the expected number is

$$E[k] = 10,$$

the variance of the distribution is

$$Var[k] = 9.$$

The plot below shows the Binomial Distribution and the Cumulative for defective computers:



Question 5

5. A phone center receives 15 calls every 30 minutes.

i. If X is the number of phone calls in 30 minutes, give the Poisson probability mass function for X.

ANSWER:

The distribution is described by the average, $\lambda = 15$,

$$\Pr(k) = \frac{15^k e^{-15}}{k!}, \quad k = 0, 1, 2, ...$$

where k is the number of phone calls per half hour.

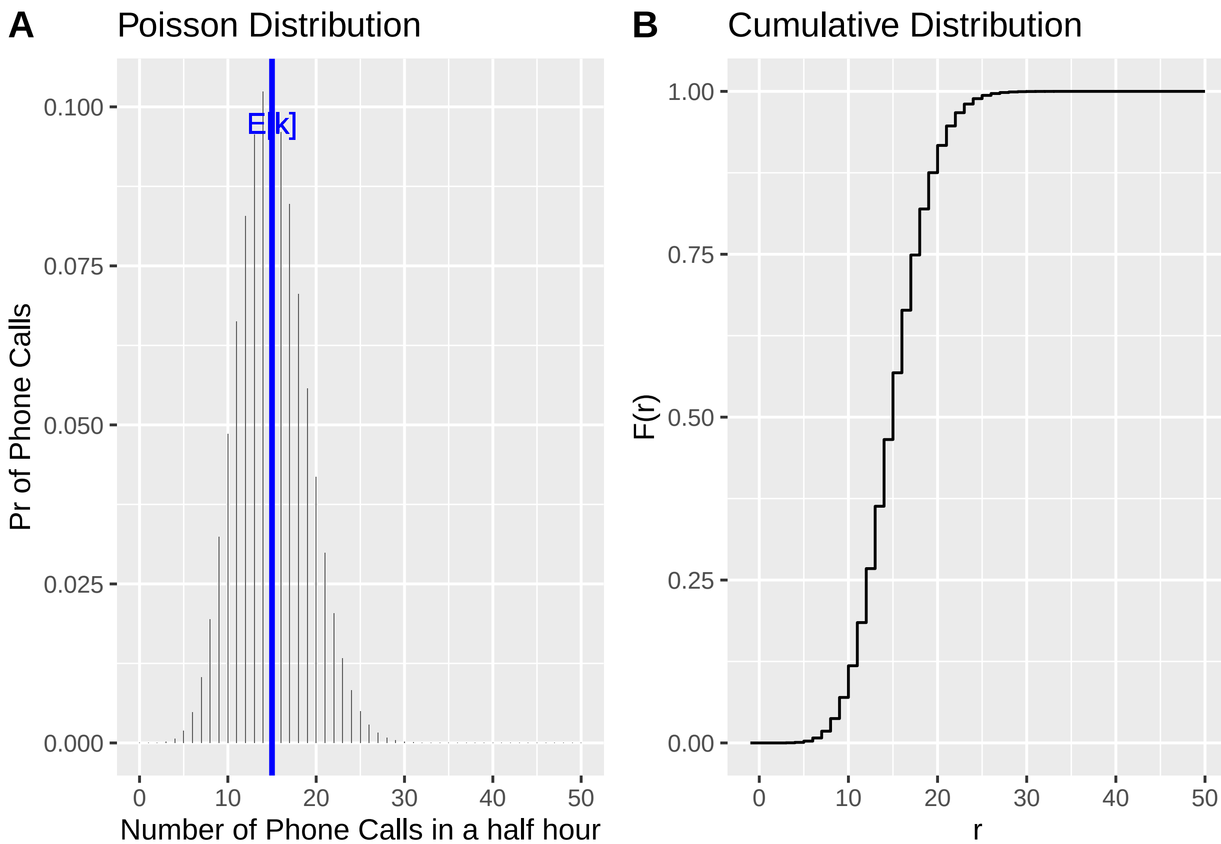
The expected value of the Poisson Distribution is

$$E[k] = 15,$$

with the variance

$$Var[k] = 15.$$

The plot below shows the Poisson Distribution and the Cumulative Distribution for $\lambda = 15$ average number of calls per half-hour:



ii. What is the probability that there will be exactly 10 phone calls in the first 30 minutes and exactly 20 phone calls in the second 30 minutes.

ANSWER:

Probability of 10 calls in the first half-hour:

$$\Pr(10) = \frac{15^{10}e^{-15}}{10!} = 0.0486108.$$

Probability of 20 calls in the first half-hour:

$$\Pr(20) = \frac{15^{20}e^{-15}}{20!} = 0.0418103.$$

The combination of both:

$$\Pr(10) \times \Pr(20) = 0.0020324.$$

iii. What is the E[X] and Var[X] of the distribution.

ANSWER:

$$E[X] = \lambda = 15$$
$$Var[X] = \lambda = 15$$

A basketball player has a 0.3 chance of making a free throw. They keep shooing until they get a basket.

i. Give the appropriate distribution for this situation. **ANSWER:**

This scenario describes repeated independent trials until the first success, which is a **Geometric Distribution**

The probability that the first success occurs on the k-th trial in a geometric distribution is:

$$P(X = k) = (0.7)^{k-1} \cdot 0.3$$

where k=1,2,3,. . . , is the number of free throws until a success

ii. What is the probability they make their first shot on the third attempt? **ANSWER:**

$$P(X = 3) = (0.7)^2 \cdot 0.3 = 0.49 \cdot 0.3 = 0.147$$

iii. What is the E[X] and Var[X] of the distribution.

ANSWER:

For a geometric distribution:

- **Expected Value**:
$$E[X] = \frac{1}{p} = \frac{1}{0.3} = \boxed{3.\overline{3}}$$

- **Variance**:
$$\text{Var}[X] = \frac{1-p}{p^2} = \frac{0.7}{0.09} \approx \boxed{7.78}$$

Question 7

A city installs a sensor at a traffic light that records the number of cars passing through every minute. On average, 3.3 cars pass through per minute.

i. Give the appropriate distribution for this situation.

ANSWER:

This scenario involves counting the number of events (cars passing) in a fixed interval (1 minute), with events occurring independently which is a **Poisson Distribution**

$$P(X = k) = \frac{e^{-3.3}3.3^k}{k!}$$

where k=0,1,2,. . . is the number of cars in a minute

ii. What is the probability that fewer than 2 cars pass through in a minute? **ANSWER:**

$$P(X < 2) = P(X = 0) + P(X = 1)$$

- $P(X = 0) = \frac{e^{-3.3} \cdot 3.3^0}{0!} = e^{-3.3} = 0.0365$
- $P(X = 1) = \frac{e^{-3.3} \cdot 3.3^1}{1!} = 3.3 \cdot e^{-3.3} = 0.1205$

$$P(X < 2) = 0.0365 + 0.1205 = 0.157$$

iii. What is the expected number of cars in a 10-minute interval?

ANSWER:

$$E[X_{10}] = \lambda \cdot 10 = 3.3 \cdot 10 = 33.$$

Multiple-Choice Questions

MCQ Question 8

A multiple-choice quiz consists of ten questions each with five possible answers of which only one is correct. What is the appropriate probability distribution.

i. Geometric Distribution;

ii. Binomial Distribution;

iii. Poisson Distribution;

iv. Negative Binomial Distribution;

v. Gaussian Distribution.

ANSWER:

Binomial Distribution.

MCQ Question 9

When a person fishing catches a fish, it is too small with a probability of 0.42 and it is returned to the water. On the other hand if it is bigger the person stops fishing.

i. Geometric Distribution;

ii. Binomial Distribution;

iii. Poisson Distribution;

iv. Negative Binomial Distribution;

v. Gaussian Distribution.

ANSWER:

Geometric Distribution

MCQ Question 10

The Poisson distribution is often used to model which of the following scenarios?

A) The number of events occurring within a fixed interval of time or space.

B) The number of successes in a fixed number of independent trials.

C) The number of trials required to get the first success.

D) The number of failures before the first success in a series of Bernoulli trials.

ANSWER:

A

Question 11

Write your own question with a named discrete distribution.