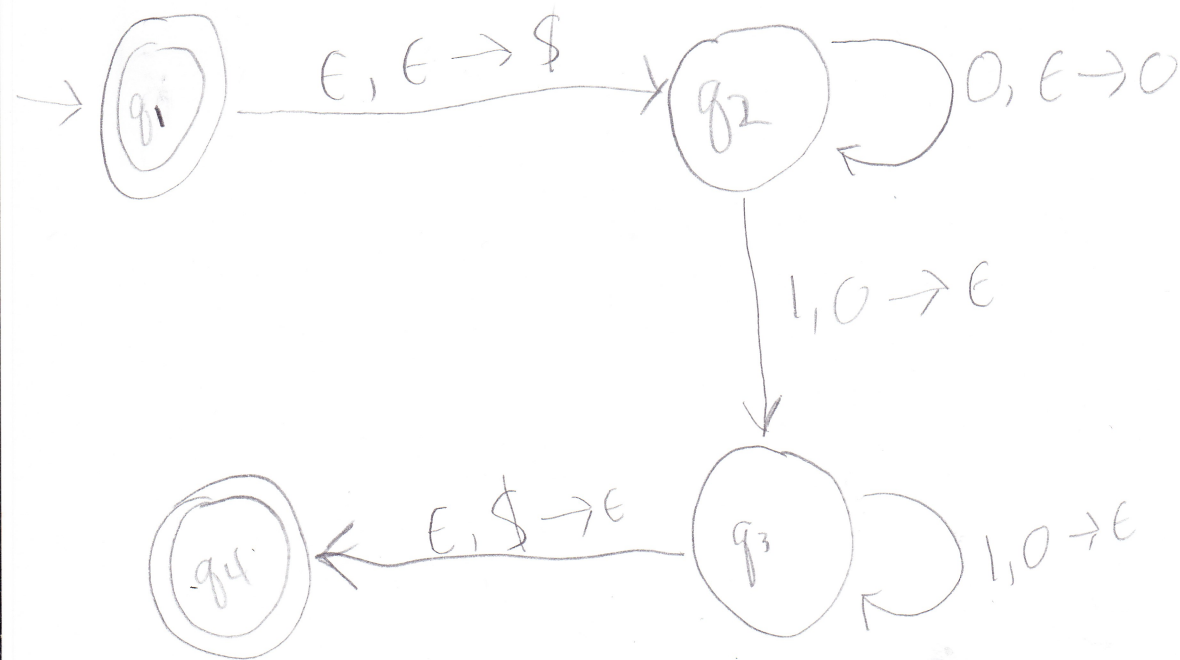# PDA to CFG

1) A_{ij} if $i==j$ the $A_{ij} \Rightarrow \epsilon$

1)
$A_{q_1 q_1} \rightarrow \epsilon$
$A_{q_2 q_2} \rightarrow \epsilon$
$A_{q_3 q_3} \rightarrow \epsilon$
$A_{q_4 q_4} \rightarrow \epsilon$

2) For states p, q, r $A_{pq} \Rightarrow A_{pr} A_{rq}$

3) $A_{pq} \Rightarrow a A_{rs} b$

2) $A_{q_1 q_2} \rightarrow A_{q_1 q_3} A_{q_3 q_2} \mid A_{q_1 q_4} A_{q_4 q_2} \mid A_{q_1 q_1} A_{q_1 q_2} \mid A_{q_1 q_2} A_{q_2 q_2}$

$A_{q_1 q_3} \rightarrow A_{q_1 q_2} A_{q_2 q_3} \mid A_{q_1 q_4} A_{q_4 q_3} \mid A_{q_1 q_1} A_{q_1 q_3} \mid A_{q_1 q_3} A_{q_3 q_3}$

$A_{q_2 q_3} \rightarrow A_{q_2 q_1} A_{q_1 q_3} \mid A_{q_2 q_4} A_{q_4 q_3} \mid A_{q_2 q_2} A_{q_2 q_3} \mid A_{q_2 q_3} A_{q_3 q_3}$

$A_{q_4 q_2} \rightarrow A_{q_4 q_1} A_{q_1 q_2} \mid A_{q_4 q_3} A_{q_3 q_2} \mid A_{q_4 q_2} A_{q_2 q_2} \mid A_{q_4 q_4} A_{q_4 q_2}$
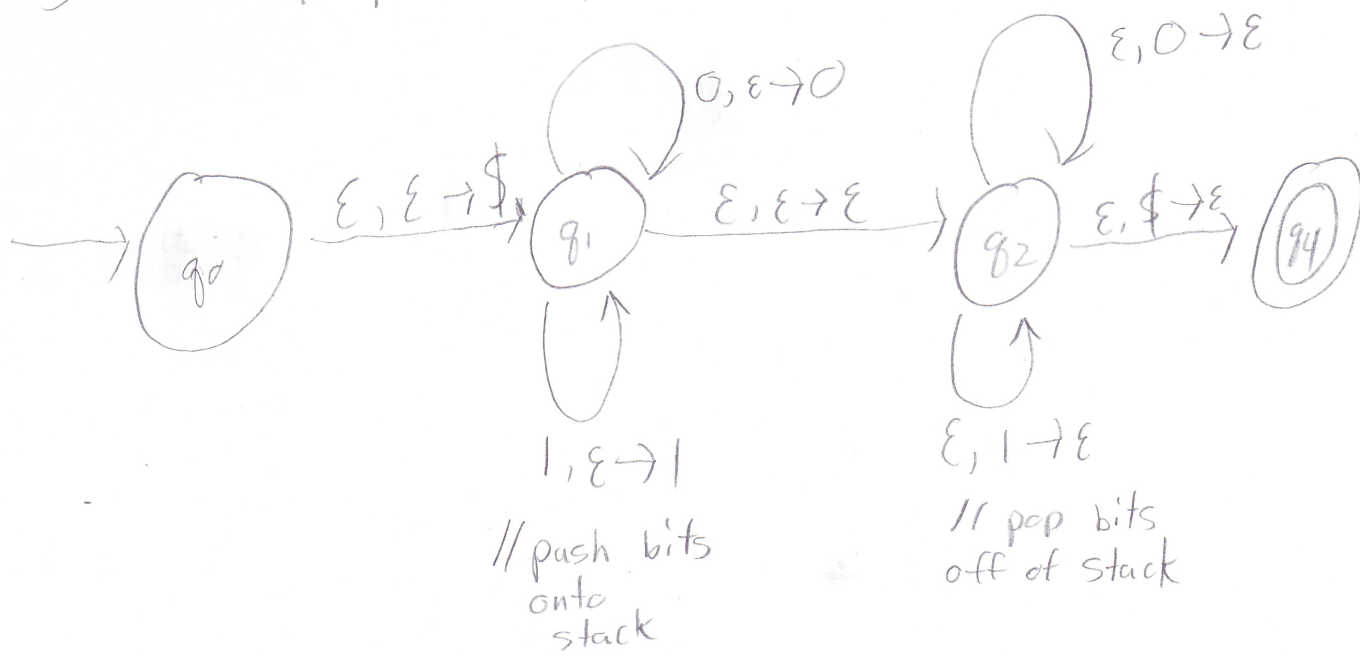
$A_{q_4 q_3} \rightarrow A_{q_4 q_1} A_{q_1 q_3} \mid A_{q_4 q_2} A_{q_2 q_3} \mid A_{q_4 q_4} A_{q_4 q_3} \mid A_{q_4 q_3} A_{q_3 q_3}$

3) $A_{q_1 q_4} \rightarrow \epsilon A_{q_2 q_3} \epsilon$

$S \rightarrow A_{q_1 q_4}$

CFG to PDA

$$S \to 0 \mid 1 \mid 0S0 \mid 1S1 \mid \varepsilon$$

$0, \varepsilon \to 0$

$\varepsilon, \varepsilon \to \$$

$\varepsilon, \varepsilon \to \varepsilon$

$\varepsilon, \$ \to \varepsilon$

$\varepsilon, 0 \to \varepsilon$

$1, \varepsilon \to 1$

// push bits onto stack

$\varepsilon, 1 \to \varepsilon$

// pop bits off of stack

3) pumping lemma

$$S = 0^P 1^{2P} 0^P \in L$$

L is context free

S = string

P = pumping length

By Pumping lemma

$S = uvxyz$, $|vy| > 0$, $|vxy| \leq P$

$i \geq 0$, $uv^i x y^i z \in A$

$S = 0^n 1^{2n} 0^n$ $n = 2$

$S = 0^2 1^4 0^2$

$S = 0011110 0$

$u = 00$
$v = 11$ $uv^2 x y^2 z^2$ $00 111111 10 100 = S'$
$x = 1$
$y = 10$ S' is not a palindrome
$z = 0$
$\qquad$ So by pumping lemma B is not
$\qquad$ context free

3) 2) Pumping Lemma

$C = \{ w \in \{0,1,2,3,4\}^* \mid \#$ of (1s & 2s) are the same & # of (3s & 4s) are the same $\}$

$S = 1^P 3^P 2^P 4^P$

$p =$ pumping length

1) $u v^i x y^i z \in C, \forall i \geq 0$

2) $vy > 0$

3) $vxy \leq p$

$i = 2$

If vxy contains a 1, there cannot be a 2 & vice versa.

If vxy contains a 3, there cannot be a 4 & vice versa.

Therefore from 2) it violates the pumping lemma & C is not context free.