Reductions

1) $L = \{\langle M \rangle \mid$ M accepts ww whenever M accepts w$\}$
is undecidable because we can reduce
$A_{TM} = \{\langle M, w \rangle$ M is a TM and M accepts w$\}$
A_{TM} accepts whenever M accepts w.
We now reduce A_{TM} so it accepts ww whenever M accepts w
$A_{TM} = \{\langle M, w \rangle$ M is a TM and accepts ww whenever M accepts w$\}$
Since we can reduce A_{TM} to a langage equivalent to L. L must be undecidable

2) CaDo is undecidable if we can reduce A_{TM} to CaDo.

To reduce A_{TM} to CaDo we map w in A_{TM} to strings containing CAT as a substring

$w \rightarrow f(w) \rightarrow$ substrings containg CAT

Since a Language that decides whether a TM will accept on an input containing CAT is undecidable the CaDo must be undecidable

3) We will represent the problem as the language.

$U_{TM} = \{\langle M, q \rangle \; M$ is a TM & q is a useless state in $M\}$

$E_{TM} = \{\langle M \rangle \mid M$ is a TM and $L = \emptyset\}$

E_{TM} is not decidable

q_{accept} will be a useless state if $L = \emptyset$

We will now reduce E_{TM} to U_{TM}.

$E_{TM} = \{\langle M, q_{accept} \rangle \mid M$ is a TM and q_{accept} is a useless state$\}$

Since we can reduce E_{TM} to U_{TM}, U_{TM} must be undecidable

4) Assume (for a contradiction) that BB is computable. We now will make a TM R that computes BB.

- R is a TM that, on an input of n 1s halts on BB(n) 1s

We now will make a doubler machine D that starts with n 1s, on the tape and ends with 2n 1s on the tape.

A machine like this can be made with a constant (x) amount of states.

Since a machine that writes n 1s to a tape will take at most n states. Therefore the max states in a TM T that writes n 1s & doubles it can be expressed as $(n+x) \rightarrow BB(n+x)$.

If we run R on T we get BB(2n).

This implies $2n \leq n+x$ for all n. Since $2n > n+x$ when $n > x$, BB must not be computable.

PCP

1010⊔ ↑ = head

1) Input = 1010

$\delta\{q_0, 0\} \Rightarrow (q_2, 0, R)$ 0010⊔

$\delta\{q_2, 0\} \Rightarrow (q_3, 1, L)$ 0110⊔

$\delta\{q_3, 0\} \Rightarrow (q_3, 0, R)$ 0110⊔

$\delta\{q_3, 1\} \Rightarrow (q_3, 1, R)$ 0110⊔

$\delta\{q_3, 1\} \Rightarrow (q_3, 1, R)$ 0110⊔

$\delta\{q_3, 0\} \Rightarrow (q_3, 0, R)$ 0110⊔

$\delta\{q_3, \sqcup\} \Rightarrow (q_a, \sqcup, L)$ ACCEPT

Set of Dominoes = $\{\delta\{q_0,0\} \Rightarrow (q_2,0,R)$, $\delta\{q_2,0\} \Rightarrow (q_3,1,L)$, $\delta\{q_3,0\} \Rightarrow (q_3,0,R)$, $\delta\{q_3,1\} \Rightarrow (q_3,1,R)$, $\delta\{q_3,\sqcup\} \Rightarrow (q_a,\sqcup,L)\}$

Since the TM M enters an accept state the set of dominoes contains a match

2) Input = 1111   1111⊔   ↑ = head

$\delta\{q_0, 1\} \Rightarrow (q_2, 0, R)$ 0111⊔

$\delta\{q_2, 1\} \Rightarrow (q_2, 1, R)$ 0111⊔

$\delta\{q_2, 1\} \Rightarrow (q_2, 1, R)$ 0111⊔

$\delta\{q_2, 1\} \Rightarrow (q_2, 1, R)$ 0111⊔

$\delta\{q_2, ⊔\} \Rightarrow (q_r, ⊔, L)$ REJECT

Since the TM M enters a reject state the set of dominoes does not contain a match.