

# Fooling sets

$$1) L_1 = \{w \mid w \in (a, b)^* \text{ and } n_a(w) = 2n_b(w)\}$$

$$F_L = (2a^* + b^*)$$

$$u = a$$

$$v = ab$$

$$x = a$$

$$ux = ab \notin L \quad (F_L \text{ is distinguishable})$$

$$vx = aba \in L$$

Since  $F_L$  is distinguishable & the set is infinite  $L_1$  is not regular

$$2) L_2 = \{w \mid w \in \{a, b\}^* \text{ and } w \text{ is balanced}\}$$

$$= L_2 = \{w \mid w \in \{a, b\}^* \text{ and } n_a(w) = n_b(w)\}$$

$$F_L = (a^* + b^*)$$

$$u = a$$

$$v = ab$$

$$x = a$$

$$ux = ab \in L \quad (F_L \text{ is distinguishable})$$

$$vx = aba \notin L$$

Since  $F_L$  is infinite & distinguishable  $L_2$  is not regular



3  $L_2 = \{w \mid w \in \{a, b\}^* \text{ and } w \text{ is balanced}\}$

1)  $S \rightarrow aSbS \mid bSaS \mid \epsilon$

2) CFG is ambiguous

3)  $S \rightarrow \epsilon \mid XY \mid YX$

$X \rightarrow AS$

$Y \rightarrow BS$

$A \rightarrow a$

$B \rightarrow b$