

1) 1) $H = \{ \langle M, x \rangle \mid M \text{ is a TM that halts on } x \}$

H is Turing recognizable because it is recognized by the following TM:

Turing machine T takes in inputs $\langle M, x \rangle$
where M is a TM and x is its input string, we then run M on x , if M halts then we accept
else reject

2) $H = \{ \langle M, x \rangle \mid M \text{ is a TM that halts on } x \}$

$E =$ Enumerator that enumerates H

$M_i = i^{\text{th}}$ output of E

$S_1, S_2, S_3, S_4, S_5, \dots, S_K$ be input strings for M

We will define a Turing machine T as:

$W = S_i$ ($w = \text{input}$)

Enumerate M on E

Run M_i on W

If M_i accepts then reject

If M_i doesn't accept then accept

Since if we run T on itself we get results opposite to how we defined T , H is not decidable.

- 3) Languages are decidable if and only if it and its complement are recognizable. Since H is recognizable & H is not decidable the \overline{H} is not recognizable.

2) 1) $H_0 = \{ \langle M \rangle \mid M \text{ is a TM that halts on } 0 \}$

H_0 is Turing recognizable because it can be recognized by the following TM:

Turing machine T takes in input M where M is a TM,

If M halts on input 0 then
accept
else,
reject

2) $E =$ Enumerator that enumerates H_0
 $M_i = i^{\text{th}}$ output of E

input string s_i

if $0 \in S_i$ then reject

Define Turing machine T as

$w = S_i$

Enumerate M on E

Run M_i on w

If M_i accepts then reject

If M_i doesn't accept then accept

If we run $T \langle T \rangle$ we will get results opposite to how we defined T above, H_0 must not be decidable.

3) Since a language is decidable if both it & its complement are recognizable since H_0 is recognizable & not decidable then $\overline{H_0}$ must be not recognizable.