Reduction Properties

1) Since we know that a valid proof to show that a TM(B) is undecidable is to show the A_{TM} can be reduced to B ($A_{TM} \leq_m B$). This means that it suffices to show that if a TM(A) that we know is undecidable, reduces to B then B is undecidable. $A \leq_m B$ & A is undecidable therefore B is undecidable.

2) To show that if $A \leq_m B$ & $B \leq_m C$, then $A \leq_m C$ we need to show that there is a function from A to C. Lets say that the function that reduces A to B is f, & that the function from B to C is g. Therefore if $f(A) = B$ & $g(B) = C$ then $g(f(A)) = C$. Since there is a function from A to C $A \leq_m C$.

3) Let's assume for a contradiction that A is undecidable. Since A is undecidable & $A \leq_m B$ B must be undecidable. However we know B is decidable. (A contradiction!) Therefore A must be decidable.

4) Assume for a contradiction that A_{OTM} is decidable relative to A_{TM}. Let T be a TM that decides A_{OTM}. We will now construct a TM N

- on input M
  - If $\langle M \rangle$ isn't a TM with an oracle for A_{TM} then reject
  - else, run T on $\langle M, M \rangle$ if T accepts then reject, if T rejects accept

However if we run N on itself it will produce outputs opposite to how we defined above. Therefore A_{OTM} must be undecidable relative to A_{TM}

Recursion Theorem

Assume for a contradiction that TM E enumerates L. Now we will construct a TM A that on input x A obtains via the recursion theorem, own description. Run on E untill a machine B appears where B halts on input 0. But A includes the description of B so B doesn't halt on input 0 (A contradiction) Therefore L is not Enumerable

Kolmogorov Complexity

Assume for a contradiction that K(x) is computable. A TM C computes K(x).

The following TM B runs C on strings until it finds one where K(x) > 1Gb.

```
loop_one{
  if w doesn't equal the length of
string x then increment w by 1
  else break
}
loop-two{
  print all possible strings of size w
}
```

Since this program would be less than 1Gb & K(x) is the minimum number of bits to print the string. This creates a contradiction meaning K(x) is not computable.