CS 452 - Design and Analysis of Algorithms

Homework 8

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This lab adheres to the JMU honor code,
a copy of which can be accessed here:
http://www.jmu.edu/honor/code.html

Question 1: Interstate Billboards

You just took on a consulting job for Harrisonburg Advertising Ltd. (HAL). The company wants to place advertising billboards along I-81 going from south to north for M miles. The possible sites for billboards are given by positive integers x1, x2, ..., xn, each in the interval [0,M] (specifying their position along the interstate, measured in miles from the southern end. If you place a billboard at location xi, you receive a revenue of ri > 0. However, the Virginia Department of Transportation requires that no two of the billboards be within less than or equal to 5 miles within each other. You need to place billboards at a subset of the sites while maximizing the revenue.

Example:
Suppose M = 20, n=4, {x1, x2, x3, x4} = {6, 7, 12, 14}, and {r1, r 2, r 3, r 4} = {5, 6, 5, 1}. Then the optimal solution would be to place billboards at x1 and x3 for a total revenue of 11.

a) *Set up a recurrence for the revenue from the optimal subsets of sites x1, ..., xj.*
   Hint: Let n(j) be a function/table that returns the northernmost site xi that is more than 5 miles from xj. The values for this function/table can be precomputed in O(n) time. In part (d) you will need to explain how if you want to use this.

Now for each mile in high-way, we need to check whether this mile has option for any bill-board, if not then max-i-mum rev-enue gen-er-ated till that mile would be same as max-i-mum rev-enue gen-er-ated till one mile before.
But if that mile has option for bill-board then we have 2 options

Either we will place the bill-board (ignore the bill-boards in pre-vi-ous 5 miles) and add the rev-enue of bill-board placed, or we will ignore the billboard.
So we will choose the option which will generate the maximum revenue.

MR(i) = Max{MR(i-6) + Revenue[i], MR[i-1]}

Note: Two bill boards has to be more than 5 miles away so actu-ally we add MR(i-6) with revenue

b) *Describe an efficient dynamic programming algorithm to solve this problem.*

This algorithm will use a loop to find the maximum revenue possibility.

```
j = 0;   // j is five miles behind the current position
         // the last valid location for a billboard, if one placed at P[k]
for k := 1 to n
    while (P[ j ] < P[ k ] – 5)
        j := j + 1;
    j := j – 1;
    Opt[k] =Max(Opt[k-1],V[k]+Opt[j]);
```

c) *Describe an algorithm to create the actual set of optimal sites.*

It would be the same algorithm as in part b. And the actual set would be inside the values of the array.

d) *Analyze the time complexities of your algorithms.*

O(M), where M is distance of total Highway. This has been a linear operation, since it is just a comparison of values going through one loop.