

# Simplifying the Radial Equation

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In the following activity, we will simplify the radial orbit equation so that we can solve for the shape of the orbits.

Currently, the radial equation is

$$\mu \ddot{r} = -\frac{d}{dr}V(r) + \frac{\ell^2}{\mu r^3} \quad (1)$$

1. Simplify  $\dot{\phi} \frac{d}{d\phi}$
2. Using this expression, rewrite  $\dot{r}$  as a derivative with respect to  $\phi$ .
3. Repeat the process, and re-express  $\ddot{r}$  as derivatives with respect to  $\phi$
4. Perform the variable substitution  $r = \frac{1}{u}$  to re-expression  $\dot{r}$  and  $\ddot{r}$

5. Using  $r = \frac{1}{u}$ , find  $\frac{d}{dr}$  in terms of  $u$ .
6. Use your expressions for  $\ddot{r}$  and  $\frac{d}{dr}$  to re-express the radial equation in terms of  $u(\phi)$
7. Assuming a potential of the form  $V(r) = -\frac{k}{r}$ , solve your differential equation to find  $u(\phi)$
8. Re-express your solution in terms of  $r(\phi)$