TAPP 3.77 --- John Waczak

What is the surface area of the Mobius Strip defined on page 156?

The Mobius strip defined on page 156 has the surface patch:

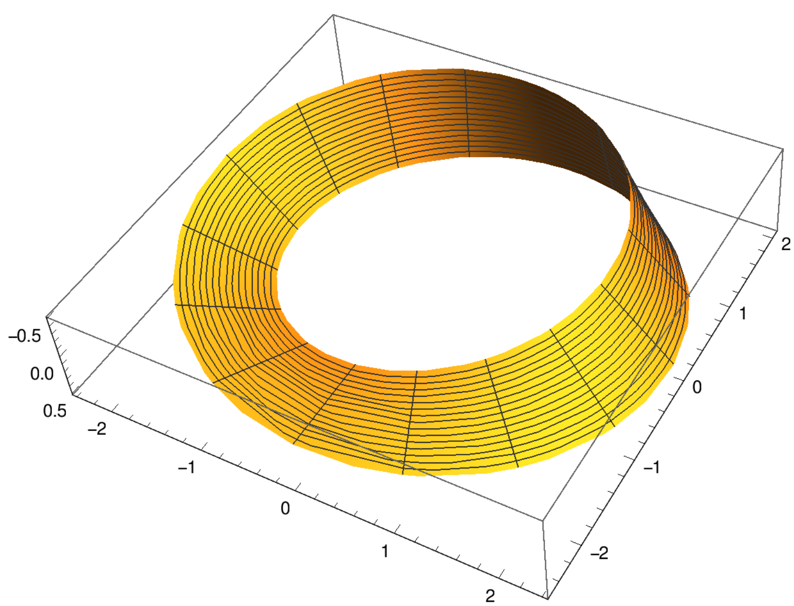
$$\boldsymbol{\sigma(u,v) = (\cos u(2 + v\sin(u/2)), \sin u(2 + v\sin(u/2)), v\cos(u/2))}$$

where **u ∈ [0, 2π)** and **v ∈ (-1/2, 1/2)**

In[9]:=
```
σ[u_, v_] := {Cos[u] * (2 + v * Sin[u / 2]), Sin[u] * (2 + v * Sin[u / 2]), v * Cos[u / 2] }
```

In[10]:=
```
ParametricPlot3D[{σ[u, v][[1]], σ[u, v][[2]], σ[u, v][[3]]},
 {u, 0, 2 π}, {v, -1 / 2, 1 / 2}]
```

Out[10]=



That looks like a Mobius strip to me! Now we need to find the partials of our surface patch.

In[16]:=
```
σ1[u_, v_] := {D[σ[u, v][[1]], u], D[σ[u, v][[2]], u], D[σ[u, v][[3]], u]}
```

In[17]:=
```
σ1[u, v]
```

Out[17]=
$$\left\{\frac{1}{2} v \operatorname{Cos}\left[\frac{u}{2}\right] \operatorname{Cos}[u] - \left(2 + v \operatorname{Sin}\left[\frac{u}{2}\right]\right) \operatorname{Sin}[u],\ \operatorname{Cos}[u] \left(2 + v \operatorname{Sin}\left[\frac{u}{2}\right]\right) + \frac{1}{2} v \operatorname{Cos}\left[\frac{u}{2}\right] \operatorname{Sin}[u],\ -\frac{1}{2} v \operatorname{Sin}\left[\frac{u}{2}\right]\right\}$$

In[19]:=
```
σ2[u_, v_] := {D[σ[u, v][[1]], v], D[σ[u, v][[2]], v], D[σ[u, v][[3]], v]}
```

In[20]:=
```
σ2[u, v]
```

Out[20]= $\left\{\text{Cos}[u]\ \text{Sin}\left[\frac{u}{2}\right],\ \text{Sin}\left[\frac{u}{2}\right]\ \text{Sin}[u],\ \text{Cos}\left[\frac{u}{2}\right]\right\}$

Now that we have the partials, we can find the cross product and take its norm to get the area distortion.

In[40]:=
```
cross[u_, v_] := σ1[u, v] × σ2[u, v]
```

In[41]:=
```
cross[u, v]
```

Out[41]= $$\left\{2\,\text{Cos}\left[\frac{u}{2}\right]\text{Cos}[u] + v\,\text{Cos}\left[\frac{u}{2}\right]\text{Cos}[u]\,\text{Sin}\left[\frac{u}{2}\right] + \frac{1}{2}v\,\text{Cos}\left[\frac{u}{2}\right]^2\text{Sin}[u] + \frac{1}{2}v\,\text{Sin}\left[\frac{u}{2}\right]^2\text{Sin}[u],\right.$$
$$-\frac{1}{2}v\,\text{Cos}\left[\frac{u}{2}\right]^2\text{Cos}[u] - \frac{1}{2}v\,\text{Cos}[u]\,\text{Sin}\left[\frac{u}{2}\right]^2 + 2\,\text{Cos}\left[\frac{u}{2}\right]\text{Sin}[u] + v\,\text{Cos}\left[\frac{u}{2}\right]\text{Sin}\left[\frac{u}{2}\right]\text{Sin}[u],$$
$$\left.-2\,\text{Cos}[u]^2\,\text{Sin}\left[\frac{u}{2}\right] - v\,\text{Cos}[u]^2\,\text{Sin}\left[\frac{u}{2}\right]^2 - 2\,\text{Sin}\left[\frac{u}{2}\right]\text{Sin}[u]^2 - v\,\text{Sin}\left[\frac{u}{2}\right]^2\text{Sin}[u]^2\right\}$$

In[42]:=
```
dσ[u_, v_] := Norm[cross[u, v]]
```

In[44]:=
```
FullSimplify[dσ[u, v]]
```

Out[44]= $$\frac{1}{2}\sqrt{\left(4\,\text{Abs}\left[\text{Sin}\left[\frac{u}{2}\right]\left(2 + v\,\text{Sin}\left[\frac{u}{2}\right]\right)\right]^2 + \text{Abs}\left[v\,\text{Cos}[u] - \left(v + 2\,\text{Csc}\left[\frac{u}{2}\right]\right)\text{Sin}[u]^2\right]^2 + \text{Abs}\left[\text{Cos}\left[\frac{u}{2}\right]\left(4\,\text{Cos}[u] + v\left(\text{Sin}\left[\frac{u}{2}\right] + \text{Sin}\left[\frac{3u}{2}\right]\right)\right)\right]^2\right)}$$

Now we have the area distortion ||**d*σ***||. To find the surface area we now integrate this expression over the domains of u and v.

In[48]:=
```
S = NIntegrate[dσ[u, v], {u, 0, 2π}, {v, -1/2, 1/2}]
```

Out[48]= 12.5996

This shows the calculated result of the numerical integration is **Area = 12.5996**.