PH 481: Lab 6 - Aperture Diffraction

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I. Introduction

The experiments in this lab analyzed the different diffraction effects that can be produces by both square and round apertures. We looked in the far field at the Fraunhofer effect as well as in the case of the near field for Fresnel diffraction. Using the theory, we were able to back calculate the aperture sizes.

II. Theory

No source of light in the lab is an ideal point source. We saw this in lab 5 where we had to consider how the single slit diffraction influenced the pattern produced by the double slit interference. In this lab, we take the idealized point sources of square and round apertures and consider now how their geometry leads to unique diffraction patterns.

Previously, the slits we used were long and narrow. This meant that the predominant effect was due to the diffraction in one dimension across the narrow width of the slit. In this lab we considered apertures that had nonzero lengths of the same order in both directions as well as the case for the more natural circular aperture (e.g. telescopes, eyes, etc...). There are two important regimes for diffraction that are characterized by how far away the ”viewing screen” is away from the slit. The first case, called Fraunhofer diffraction, applies in the far field. The general rule of thumb is that Fraunhofer diffraction occurs when either the distance of the source to the aperture or the distance from the aperture to the screen (R) obeys:

$$R > \frac{b^2}{\lambda} \tag{1}$$

where b is the width of the aperture.

For a rectangular aperture in the Fraunhofer regime, there is diffraction in both the vertical and horizontal directions governed by the same conditions as for the single slit:

$$d \sin \theta_m = m\lambda \tag{2}$$

Where θ_m is the angle from the center to the m^{th} fringe. Thus if we want to measure the dimensions of the square width we can analyze the successive fringes in both the y and x directions on the viewing screen (camera).

The case of the round aperture, the intensity no longer behaves as a $sinc^2\beta$ function but instead the round nature leads to Bessel functions of the first kind. Rather than obeying the same exact equation, the fringes instead follow the following relationship:

$$y = m\frac{1.22\lambda L}{2a} \tag{3}$$

Where y is the height of the fringe from the beam axis, L is the distance from the aperture and a is the aperture radius. Thus we can use this equation as well as (2) with a small angle approximation (valid for Fraunhofer regime) to back calculate the slit geometry.

For the case of the Fresnel diffraction, things become a little more tricky as we have to calculate some nasty integrals without approximation. We can however calculate the number of ”Fresnel zones” that fit within the aperture according to the equation:

$$m = \frac{a^2}{x\lambda} \tag{4}$$

Where x is the distance from the slit and a is the aperture radius. This is of course assuming the incoming light is a plane wave which can be accomplished by using a telescope to collimate the beam. When the number Fresnel zones is even, we know from the vibration curve that the intensity is nearly minimum. Thus we can encode this by saying the distance x_n at the n^{th} intensity minimum is given by:

$$x_n = \frac{a^2}{2n\lambda} \tag{5}$$

Thus using this equation we can calculate the aperture radius by fitting the data for the x_n's.

III. Experiment

In the first experiment, we took images of the Fraunhofer diffraction fringes produced by square and round apertures in order to calculate the aperture size. We began by aligning the laser down the optical rail and then added a -25mm diverging lens at the beginning of the optical rail. Fifty millimeters behind this we placed a 75 mm positive lens to collimate the expanded light into a beam. This beam was directed towards the aperture that was fitted into a holder some distance down the rail. Finally we used a 200 mm lens to focus the diffraction pattern into the camera. The following figure illustrates our experimental table geometry.

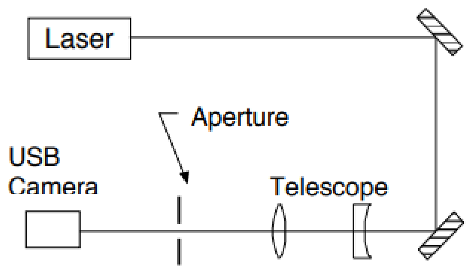

Fig. 1. Optical table geometry. **NOTE** for the Fraunhofer measurements we added a 200 mm lens to the left of the aperture in the figure

Using the camera we recorded images of the Fraunhofer diffraction patterns for each aperture. Then using Ali's Labview program we were able to measure the distance between fringes. This coupled with the distance to the 200 mm lens from the camera and the 632.8 nm wavelength of our He-Ne laser enabled us to calculate the size of the apertures.

For the square aperture, we made measurements in both the vertical and horizontal directions using the program's ”transpose” feature. This let us calculate the aperture width in both directions. The circular aperture produces round Airy discs in the Fraunhofer regime which we used to calculate the aperture's radius. It was slightly difficult to get a good measurement from the first minimum as the intensity curve didn't exactly reach zero instead we used the second minimum, making sure to set $m = 2$ in equation (2) for our calculations.

Finally for the Fresnel diffraction of the circular aperture, we began by finding the first Airy disc in the Fraunhofer regime. We then moved towards the aperture until this disc disappeared and then noted this distance as the $n = 1$ distance. Since λ is known, all we needed to do is slowly decrease the separation between the camera and the aperture and record the position when the center bright fringe turns dark (as this indicates 2n Fresnel zones). Using Dr. McIntyre's χ^2 fitting procedure with Excel's solver feature, we the fit our data to equation (5) in order to determine the aperture radius.

IV. Results

The square aperture produced the following diffraction pattern in the Fraunhofer regime.

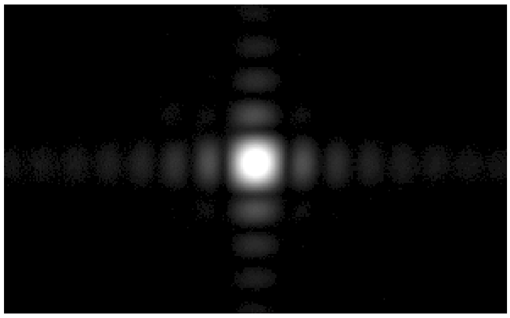

Fig. 2. Square aperture Fraunhofer diffraction pattern

Figure 2 illustrates what I explained in the previous section as we can see that there are diffraction fringes occurring in both the vertical and horizontal directions giving a sort of cross shaped pattern. The following graph shows an example of the Intensity output given by Ali's Labview program:

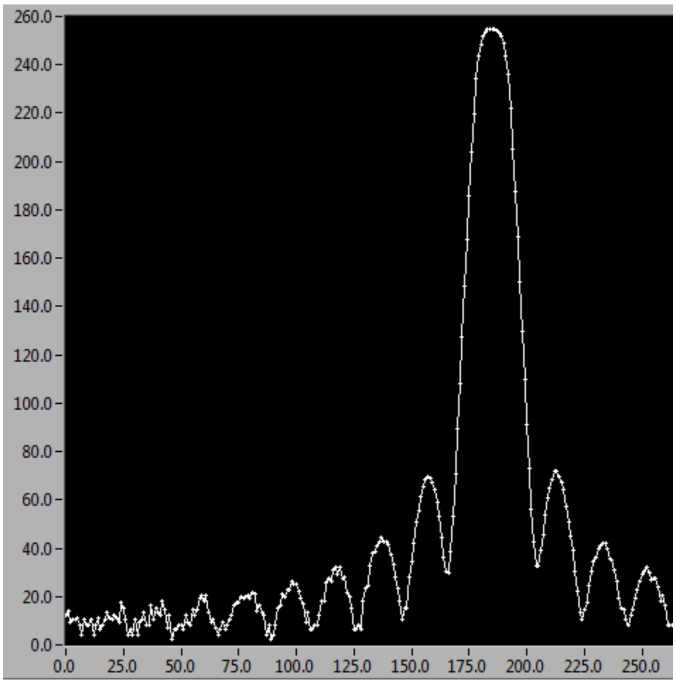

Fig. 3. Intensity profile for one cross-section of the square aperture diffraction pattern

As figure 3 illustrates, we were not able to get the first minimum to be a true minimum with a zero intensity value so instead we used the second ($m = 2$) minimum for our calculations. The following table lists the data and the determined slit widths for the square aperture.

TABLE I
SQUARE APERTURE RESULTS

orientation	m	L	y	d
vertical	2	0.187 m	0.0002155 m	0.0010985 m
horizontal	2	0.187m	0.00021 m	0.00112 m

Of note is the last column which shows our calculated aperture width and height. Both are near 1 millimeter.

Figure 4 shows the diffraction pattern for the circular aperture in the Fraunhofer regime. Clearly visible are the characteristic rings with the bright Airy disc at the center.There are some noticeable aberrations we attribute to imperfections in the aperture.

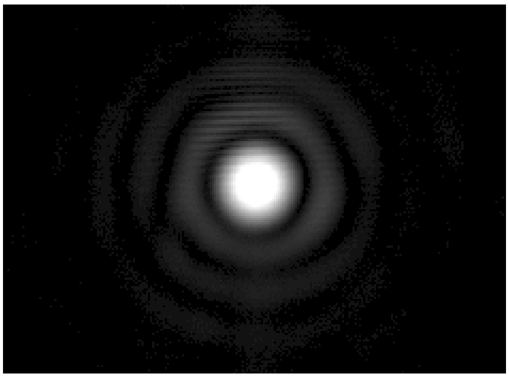

Fig. 4. Circular aperture Fraunhofer diffraction pattern

As we mentioned in the theory section, the circular aperture displays Bessel function behavior in it's radial intensity profile. This leads to the zeros in intensity occurring at weird positions defined by equation (3). The following figure shows the output of the labview program for the intensity of the circular aperture:

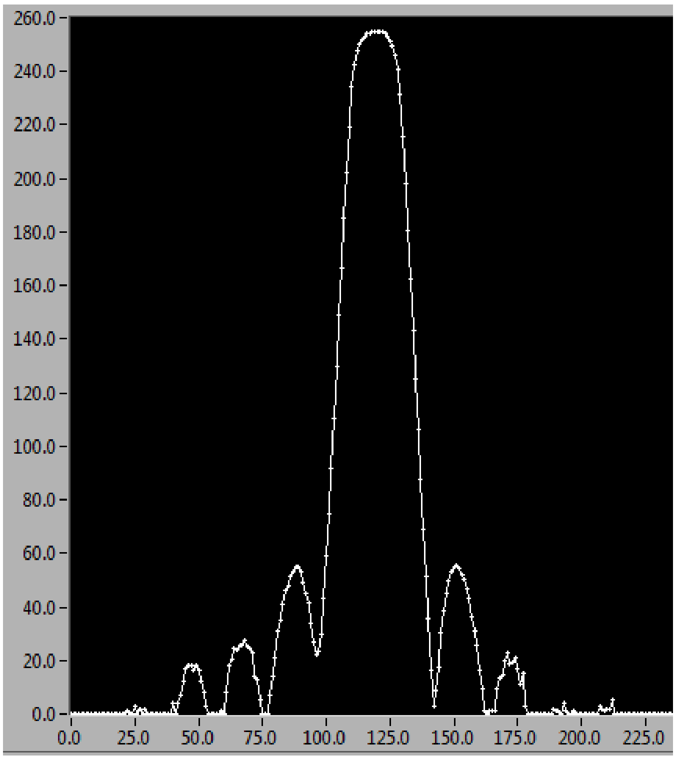

Fig. 5. Intensity profile for circular aperture in Fraunhofer regime

From figure 5 we measured the distance between the central maximum and the first minimum to be $y = 0.00012$ m at a distance of 0.209 m from the aperture. Using equation (3) this gives a radius of

$a = 0.00065$ mm which is ≈ 0.5 mm.

An example of the diffraction pattern produced by the circular aperture in the Fresnel regime is shown in the following figure:

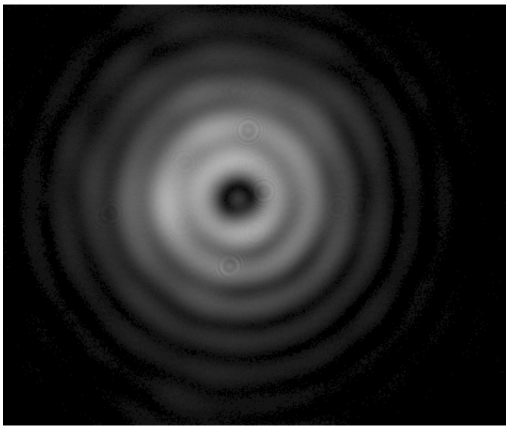

Fig. 6. Diffraction pattern produced by circular aperture in Fresnel regime

Of note is that in figure 6 we can see the bright Airy disc has disappeared. We know this should happen when m is even (i.e. $m = 2n$) since this corresponds to anti-parallel vectors on the vibration curve. The following figure shows a graph of the x_ns we measured as a function of n. Plotted against this curve is the theoretical best fit line found using Dr. McIntyre's χ^2 Excel method.

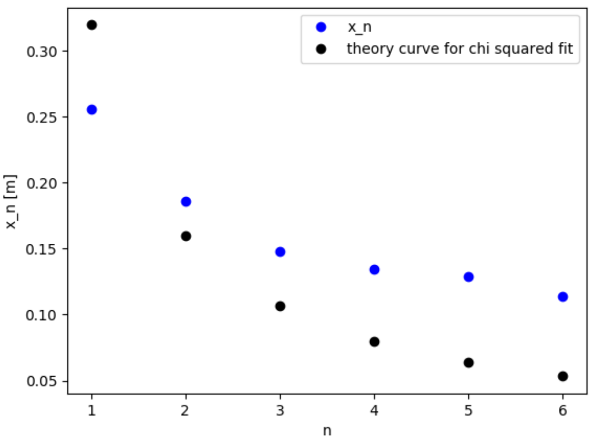

Fig. 7. Figure showing our data for the x_ns as a function of n versus the best fit line for a radius of 0.00064 m

The best fit procedure for the data in Figure 7 gave us an aperture radius of 0.64 mm.

V. Discussion

The measurements for the Fraunhofer diffraction went very well as the labview program gave us the intensity curves in real time next to the image. This enabled us to move the camera to try and focus the image as well as manipulate the sample so that we could try and get the clearest possible figure possible. By manipulating the exposure we could make the contrast between the bright and dark regions clearer and this led us to very reasonable measurements of 1.09, 1.12, and 0.65 millimeters for the two square aperture widths and the round aperture radius.

The largest source of error for the Fraunhofer measurements was in the distance measurements between the aperture and the camera. It was difficult to read off a good value using the markings on the optical rail due to the room's darkness as well as the fact that the camera's sensor lies recessed a few millimeters behind the front face of the camera.

As for the Fresnel measurements, it was very difficult to make sure we weren't skipping interfering zones as equation (5) has an inverse relationship between x_n and n. This meant that as we moved the camera closer to the aperture, the frequency of these interfering positions increased. We found that after about 5 we had difficulty telling if we skipped a region. Regardless, our χ^2 fit gave us a radius of 0.64 millimeters which agrees with our previous conclusion from the Fraunhofer measurement of 6.5 millimeters quite well (1.5 %).

VI. Conclusions

By taking measurements of the diffraction patterns produced in the Fraunhofer and Fresnel regimes we were able to determine the width and height for a square aperture to be ≈ 1 mm. Our measurements for the radius of the circular aperture were ≈ 0.65 with a 1% discrepancy.

VII. References

[1] *Optics*, Eugene Hecht

[2] Darlene Focht

[3]http://physics.oregonstate.edu/~mcintyre/
COURSES/ph481/LABS/Lab5.pdf