3. $\frac{d}{d\xi}\left(\xi \frac{du}{d\xi}\right) + \left(\frac{1}{2}E\xi + \alpha - \frac{m^2}{4\xi} - \frac{1}{4}F\xi^2\right)u = 0$

a). observe that $\frac{d}{d\xi}\left(\xi \frac{du}{d\xi}\right) = u' + \xi u''$ so that we can say

$$u' + \xi u'' + \left(\frac{1}{2}E\xi + \alpha - \frac{m^2}{4\xi} - \frac{1}{4}F\xi^2\right)u = 0$$

$$\xi^2 u'' + \xi u' + \left(\frac{1}{2}E\xi^2 + \alpha\xi - \frac{m^2}{4} - \frac{1}{4}F\xi^4\right)u = 0$$

we will look for series solutions w/

$$u = \sum_{n=0}^{\infty} a_n \xi^{n+r} \qquad u' = \sum_{n=0}^{\infty}(n+r)a_n\xi^{n+r-1} \qquad u'' = \sum_{n=0}^{\infty}(n+r)(n+r-1)a_n\xi^{n+r-2}$$

plugging this in yields:

$$\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n\xi^{n+r} + \sum_{n=0}^{\infty}(n+r)a_n\xi^{n+r} + \left(\frac{1}{2}E\xi^2 + \alpha\xi - \frac{m^2}{4} - \frac{1}{4}F\xi^3\right)\sum_{n=0}^{\infty}a_n\xi^{n+r} = 0$$

$$0 = \sum_{n=0}^{\infty}(n+r)(n+r-1)a_n\xi^{n+r} + \sum_{n=0}^{\infty}(n+r)a_n\xi^{n+r} + \sum_{n=0}^{\infty}\frac{1}{2}Ea_n\xi^{n+r+2} + \sum_{n=0}^{\infty}\alpha a_n\xi^{n+r+1} - \sum_{n=0}^{\infty}\frac{m^2}{4}a_n x^{n+r} - \sum_{n=0}^{\infty}\frac{1}{4}Fa_n\xi^{n+r+3}$$

$$0 = \sum_{n=0}^{\infty}\left[(n+r)(n+r-1) + (n+r) - \frac{m^2}{4}\right]a_n\xi^{n+r} + \sum_{n=0}^{\infty}\alpha a_n\xi^{n+r+1} + \sum_{n=0}^{\infty}\frac{1}{2}Ea_n\xi^{n+r+2} - \sum_{n=0}^{\infty}\frac{1}{4}Fa_n\xi^{n+r+3}$$

Now we reindex in order to obtain powers of ξ^{n+r}

$$0 = \sum_{n=0}^{\infty}\left[(n+r)(n+r-1) + (n+r) - \frac{m^2}{4}\right]a_n\xi^{n+r} + \sum_{n=1}^{\infty}\alpha a_{n-1}\xi^{n+r} + \sum_{n=2}^{\infty}\frac{1}{2}Ea_{n-2}\xi^{n+r} - \sum_{n=3}^{\infty}\frac{1}{4}Fa_{n-3}\xi^{n+r}$$

and thus

$$0 = \left(r^2 - \frac{m^2}{4}\right)a_0\xi^r + \left((1+r)^2 - \frac{m^2}{4}\right)a_1\xi^{r+1} + \alpha a_0\xi^{r+1} + \left((2+r)^2 - \frac{m^2}{4}\right)a_2\xi^{r+2}$$
$$+ \alpha a_1 x^{r+2} + \frac{1}{2}Ea_0 x^{r+2} +$$
$$+ \sum_{n=3}^{\infty}\left[\left((n+r)(n+r-1) + (n+r) - \frac{m^2}{4}\right)a_n + \alpha a_{n-1} + \frac{1}{2}Ea_{n-2} - \frac{1}{4}Fa_{n-3}\right]\xi^{n+r}$$

So our indicial equation is $r^2 - \frac{m^2}{4} = 0 \Rightarrow \boxed{r = \pm\frac{m}{2}}$

the ξ^{r+1} gives $a_1 = \dfrac{-\alpha}{\left((1+r)^2 - \frac{m^2}{4}\right)}a_0 = \left(\dfrac{\alpha}{\frac{m^2}{4} - (1+r)^2}\right)a_0$