```
In[1]:= A = {{7, 0, 0}, {0, 1, -I}, {0, I, -1}}
```

Out[1]= $\{\{7, 0, 0\}, \{0, 1, -i\}, \{0, i, -1\}\}$

```
In[2]:= vals = Eigenvalues[A]
        vecs = Eigenvectors[A]
```

Out[2]= $\{7, -\sqrt{2}, \sqrt{2}\}$

Out[3]= $\{\{1, 0, 0\}, \{0, i(-1+\sqrt{2}), 1\}, \{0, -i(1+\sqrt{2}), 1\}\}$

```
In[4]:= vecs = {Normalize[vecs[[1]]], Normalize[vecs[[2]]], Normalize[vecs[[3]]]}
```

Out[4]= $$\left\{\{1, 0, 0\}, \left\{0, \frac{i(-1+\sqrt{2})}{\sqrt{1+(-1+\sqrt{2})^2}}, \frac{1}{\sqrt{1+(-1+\sqrt{2})^2}}\right\}, \left\{0, -\frac{i(1+\sqrt{2})}{\sqrt{1+(1+\sqrt{2})^2}}, \frac{1}{\sqrt{1+(1+\sqrt{2})^2}}\right\}\right\}$$

Now we want to show that his orthonormal basis is complete. I.e. that $\Sigma_i\ |\ \phi_n\rangle\langle\phi_n\ | = \mathbf{Id}$

```
In[5]:= A1 = KroneckerProduct[vecs[[1]], vecs[[1]]]
```

Out[5]= $\{\{1, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

```
In[6]:= A2 = KroneckerProduct[vecs[[2]], Conjugate[vecs[[2]]]]
```

Out[6]= $$\left\{\{0, 0, 0\}, \left\{0, \frac{(-1+\sqrt{2})^2}{1+(-1+\sqrt{2})^2}, \frac{i(-1+\sqrt{2})}{1+(-1+\sqrt{2})^2}\right\}, \left\{0, -\frac{i(-1+\sqrt{2})}{1+(-1+\sqrt{2})^2}, \frac{1}{1+(-1+\sqrt{2})^2}\right\}\right\}$$

```
In[7]:= A3 = KroneckerProduct[vecs[[3]], Conjugate[vecs[[3]]]]
```

Out[7]= $$\left\{\{0, 0, 0\}, \left\{0, \frac{(1+\sqrt{2})^2}{1+(1+\sqrt{2})^2}, -\frac{i(1+\sqrt{2})}{1+(1+\sqrt{2})^2}\right\}, \left\{0, \frac{i(1+\sqrt{2})}{1+(1+\sqrt{2})^2}, \frac{1}{1+(1+\sqrt{2})^2}\right\}\right\}$$

In[8]:=
```
MatrixForm[A3]
MatrixForm[A2]
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\left(1+\sqrt{2}\right)^2}{1+\left(1+\sqrt{2}\right)^2} & -\frac{i\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2} \\ 0 & \frac{i\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2} & \frac{1}{1+\left(1+\sqrt{2}\right)^2} \end{pmatrix}$$

Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\left(-1+\sqrt{2}\right)^2}{1+\left(-1+\sqrt{2}\right)^2} & \frac{i\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2} \\ 0 & -\frac{i\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2} & \frac{1}{1+\left(-1+\sqrt{2}\right)^2} \end{pmatrix}$$

In[10]:=
```
FullSimplify[MatrixForm[A1 + A2 + A3]]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[11]:=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

Out[11]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

In[12]:=

In[13]:=