PH 481: Lab 4 - Fabry-Perot Interferometer

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I. Introduction

In this lab we created an optical device known as a Fabry-Perot (FP) interferometer. Different from last week's Michelson interferometer, the FP uses a set of mirrors set a fixed distance reflect an incident source of light multiple times resulting in sharper interference rings. Using the table top laser we were able to make a crude measurement of the finesse and then used the device to confirm the wavelengths of doublet created by the light from a sodium lamp.

II. Theory

As was the case for the Michelson interferometer, the interference fringes produced by the FP obey the equation:

$$2d\cos\theta_m = m\lambda \tag{1}$$

where d denotes the inter-mirror spacing and m denotes the m^{th} interference fringe at an angle of θ_m from the central beam axis. Figure 1 illustrates the beam geometry of the FP interferometer. The key difference is that the multiple reflections inside of the cavity lead to multiple-beam interference on the viewing screen.

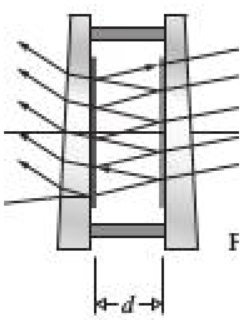

Fig. 1. Optical cavity similar to that of FP interferometer

Due to this multiple beam interference, the radial distribution of intensity does not follow the same cosine squared relationship as for the Michelson interferometer but rather has a central bright ring followed by decreasing intensity as determined by the following equation.

$$I(\delta) = \frac{I_0}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\frac{\delta}{2}} \tag{2}$$

Here $\delta = 2kd\cos(\theta)$ is the phase shift and F denotes the "finesse" which is the ratio of the fringe spacing to the fringe width. Finesse is an important measure as it relates th how sharp the interference fringes will appear. The ratio is dimensionless and so can be determined by a number of different measures (wavelength, frequency, etc...). For this lab we define the finesse in the frequency domain by the free-spectral-range (FSR), i.e. the spacing between adjacent fringes, and the Δf as measured roughly halfway up the peak. Thus,

$$F = \frac{\text{FSR}}{\Delta f} \tag{3}$$

It is difficult do determine the wavelengths of the sodium lamp just from one image of the interference fringes. Furthermore, for certain distances d, the phase difference is just right so that the intensities of the two wavelengths appear on top of one another. The literature values for the wavelengths are $\lambda_1 = 589.00$ nm and $\lambda_2 = 589.59$ nm respectively. If we define λ to be the average of the two wavelengths, then when we set the distance to make the wavelengths overlap we can count the number of fringes that pass when we move the mirrors a distance Δd to see the two wavelengths overlap again. Using equation 1 for the $\theta = 0$ center fringe this becomes:

$$\lambda = \frac{2\Delta d}{N} \tag{4}$$

Once λ has been determined, we can massage these equations to give us an approximate expression for $\Delta\lambda$, the difference between the wavelengths of the doublet. If there are N fringes for λ_1 between locations of overlap then there will be $N+1$ fringes for λ_2 Plugging this in via equation 1 allows us to derive the following:

$$\lambda_1 = \frac{2\Delta d}{N} \tag{5}$$

$$\lambda_2 = \frac{2\Delta d}{N+1} \tag{6}$$

$$\Rightarrow \Delta\lambda = \lambda_1 - \lambda_2 \tag{7}$$

$$= 2\Delta d\Big(\frac{1}{N} - \frac{1}{N+1}\Big) \tag{8}$$

$$= \frac{2\Delta d}{N(N+1)} \tag{9}$$

It is much easier to to measure the distance between when we see successive doublets and relate this to our average λ in order to reduce potential human error in counting. To do this we make the following argument:

$$\Delta\lambda = \frac{\lambda_1}{N} \text{ (from 8)} \tag{10}$$

$$\Rightarrow N^2 = \Big(\frac{\lambda_2}{\Delta\lambda}\Big)^2 \tag{11}$$

$$N(N+1) \approx N^2 \tag{12}$$

$$\Rightarrow \Delta\lambda = 2\Delta d \frac{1}{\frac{\lambda_2^2}{\Delta\lambda^2}} \tag{13}$$

$$\Rightarrow \Delta\lambda = \frac{\lambda_2^2}{2\Delta d} \tag{14}$$

$$\Delta\lambda \approx \frac{\lambda^2}{2\Delta d} \tag{15}$$

Thus using equations (9) and (15) we can experimentally determine λ and $\Delta\lambda$. For reference the true values are $\lambda_T = 589.295$ nm and $\Delta\lambda_T = 0.59$

III. Experiment

As per usual, we first aligned the laser down the optical axis using by adjusting the two mirrors so that the beam passed through two irises on the optical rail. Figure 2 illustrates the experimental design for the FP interferometer. Whereas the Michelson had two arms that were somewhat troublesome to align, this design simply puts two mirrors along the optical axis with the second being manipulated by a translation stage to allow us to vary the separation distance d.

To create this geometry, we first placed the fixed mirror and then adjusted the pitch and tilt

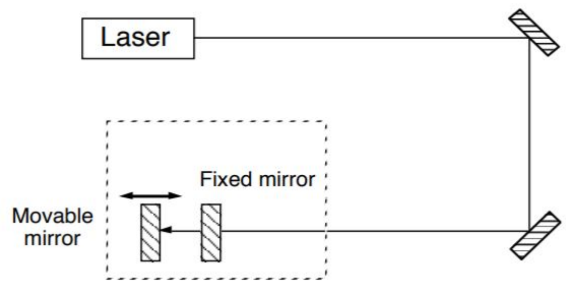

Fig. 2. Table geometry for FP interferometer

so that the reflected beam traveled back towards the laser. Then we added the second mirror to the translation stage being sure to place the bolts in reasonable locations so as to allow both forward and backwards translation of the mirror without colliding with the fixed mirror. Once we approached a suitable distance and were happy with how tight the bolts were, we adjusted the second mirror so that its reflection also returned to the laser.

At this point we were able to see a set of dots on the viewing screen (a piece of card-stock about a foot away). We made a few small adjustments to the fixed mirror so that these dots collapsed into one giving a crude set of interference fringes.

In order to enhance these fringes into the expected rings we introduced a -25mm diverging lens before the fixed mirror to create the effect of a point source of light. Once we observed this pattern we made a few small adjustments to the fixed mirror in order to align the center of the fringes with the optical axis. After this point we were ready to measure the finesse.

We added a single iris behind the translation stage in order to constrict the intensity to the first ring. We then hooked up a photo-diode behind the iris and connected it to the oscilloscope through a resistor which gave us the ability to measure the light's intensity (proportional to voltage from diode).

To measure the finesse, we force the translation stage to "scan" back and forth through a range of distances. This resulted in a waveform on the oscilloscope which we captured and used to determine F. Subsequently, we spent roughly an hour trying to adjust the mirrors to produce the widest and narrowest peaks possible in order to maximize the value of F.

For the second experiment, we turned off the laser and inserted a sodium lamp in its place. We used some electrical tape to restrict the output port of the lamp to a pinhole which we then sent through our FP interferometer. In place of the photo-diode, we inserted a camera which allowed us to capture the image of the interference fringes. We found that removing the -25mm lens led the the sharpest possible image after adjusting the fixed mirror to find the peak finesse.

First we determined the average wavelength. To do this we found a distance where the doublet appeared to overlap resulting in single rings. We then scanned the translation stage and counted each time a new fringe appeared in the center, stopping at 100. We then recorded the change in distance from translation stage software for use with equation (9).

Finally, we moved the stage back to a location so we could clearly see the doublet in the fringes and then measured the distance required to see the clear doublet again. This enables us to use (15) with our result from the first experiment to determine $\Delta\lambda$.

IV. Results

The following figure shows the oscilloscope capture of the best finesse we were able to obtain.

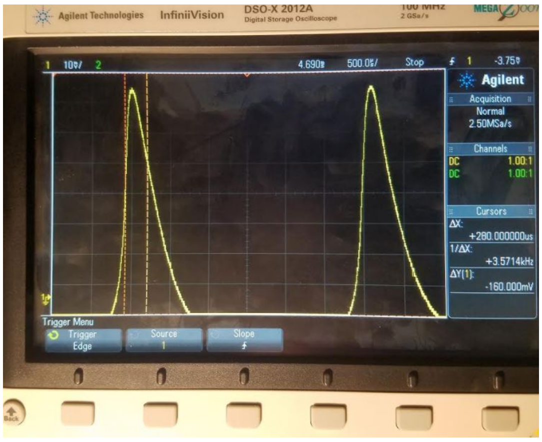

Fig. 3. Oscilloscope output when scanning the translation stage

The FSR was measured to be $3.07 \cdot 10^{-3}$ (s) and the Δf was measured to be $280 \cdot 10^{-6}$ (s) so that our finesse is:

$$F = \frac{3.07 \cdot 10^{-3}}{280 \cdot 10^{-6}} \approx 10.96 \tag{16}$$

For the sodium lamp, we were luck to get a pretty clear image right away as shown in the following two figure.

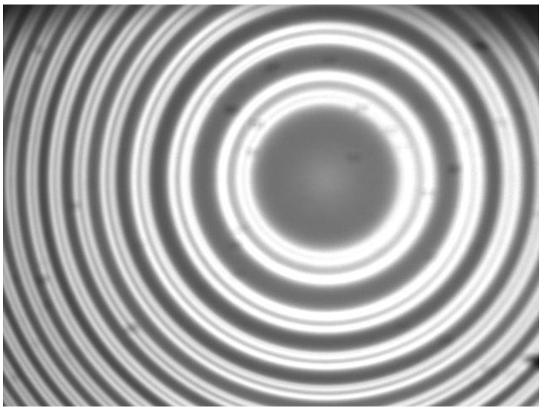

Fig. 4. Sodium interference fringes showing doublet

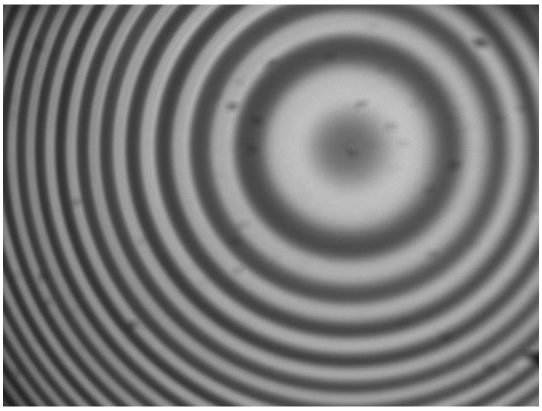

Fig. 5. Sodium interference fringes showing overlap between wavelengths

In figure 4 we can clearly see that there is a series of sets of two rings right next to each other reflecting the presence two different emission wavelengths for sodium. In figure 5 we can see what happens as the distance was changed and the resulting image has the previously separate rings overlapping.

We measured that starting at an initial distance of 5.3353 mm led to a final distance of 5.3055 mm for 100 fringes to pass. This results in a average wavelength of:

$$\lambda = \frac{2(5.3353 - 5.3055) \cdot 10^{-3}}{100 \cdot 101} = 596.00 \text{ nm} \quad (17)$$

For the determination of $\Delta\lambda$ we measured an initial distance of 5.5270 mm and a final distance of 5.2883 mm. This leads to a spread of:

$$\Delta\lambda = \frac{(596 \cdot 10^{-9})^2}{2(5.5270 - 5.2883) \cdot 10^{-3}} = 0.7441 \text{ nm} \quad (18)$$

V. Discussion

We had an incredibly difficult time measuring the finesse. It took us most of the lab period to get our laser aligned to the point where we had a good interference pattern that we could capture on the oscilloscope. After nearly an hour of tweaking the mirror to find the best finesse possible, we settled on our 10.96 value obtained from figure 3. This difficult was due in part to noise affecting the table as well as the fact that our mirrors were not perfectly aligned and so there was a small amount of clipping on the edge that made adjustments difficult.

The reported value for the finesse of our FP is 60 however Dr. McIntyre instructed that the best we could expect to measure would be in the 20 to 60 range. I suspect that if we had better mirror alignment from the beginning (which could have been accomplished by better planning for where to put the translation stage) that we could have approached the 20 mark. There were a couple of other lab groups that came pretty close to this value.

For the sodium wavelength measurements we were extremely fortunate to get a clear picture of the interference fringes right when we first turned on the camera without much adjustment. By this point we were short on time so we were only able to take one measurement of the 100 fringe count for the average wavelength. Darlene stayed late after lab to finish collecting data to determine the wavelength difference $\Delta\lambda$.

Our value for the average wavelength came out to 596.00 nm. This give us a relative error of:

$$Rel(596) = \frac{596 - 589.295}{589.295} = 0.011 \approx 1\% \quad (19)$$

This relative error is in agreement with the error estimation given in the lab manual which reports that for going through 10 overlaps, the error should be roughly 1%.

For the wavelength difference we measured 0.7441 nm. The value based off of the literature sodium wavelengths was determined earlier to be 0.59 nm. This gives us a relative error of:

$$Rel(0.7441) = \frac{0.7441 - 0.59}{0.59} = 0.2611 \approx 25\% \quad (20)$$

This error is much higher than for the average wavelength and illustrates the difficulty in getting the change in distance between successive doublets. Very small translations were required to make it possible to actually track the fringes as they moved. If we take our $\Delta\lambda$ and try and derive the values of λ_1, λ_2 we can mitigate this error a bit more.

VI. Conclusions

We were able to crudely determine the finesse of the FP interferometer to be 10.96. This value has the correct order of magnitude but is roughly half of what we expected to be able to measure. We also were able to determine the average wavelength for the sodium lamp correctly to within 1% error. Our $\Delta\lambda$ was worse but given more time it is my opinion that we could produce a better result. This set of experiments illustrates how such interferometer designs can be used for spectroscopy as the doublets we were able to observe for the sodium lamp were clear and indicative of the element that was radiating light.

VII. References

[1] *Optics*, Eugene Hecht

[2] Darlene Focht

[3]http://physics.oregonstate.edu/~mcintyre/COURSES/ph481/LABS/Lab4.pdf