

In[40]:= \$Assumptions = Element[R, Reals] && Element[M, Reals] && Element[h, Reals] && R > 0

$$\rho = \frac{M}{\frac{1}{3} \pi R^2 h}$$

Out[40]= $R \in \text{Reals} \ \&\& \ M \in \text{Reals} \ \&\& \ h \in \text{Reals} \ \&\& \ R > 0$

Out[41]= $\frac{3 M}{h \pi R^2}$

In[45]:=
$$I_{11} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (y^2 + z^2) \, dz \, dy \, dx$$

Out[45]= $\frac{3}{20} M (4 h^2 + R^2)$

In[46]:=
$$I_{22} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (x^2 + z^2) \, dz \, dy \, dx$$

Out[46]= $\frac{3}{20} M (4 h^2 + R^2)$

In[47]:=
$$I_{33} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (x^2 + y^2) \, dz \, dy \, dx$$

Out[47]= $\frac{3 M R^2}{10}$

In[48]:=
$$I_{12} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (-x * y) \, dz \, dy \, dx$$

Out[48]= 0

In[49]:=
$$I_{13} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (-x * z) \, dz \, dy \, dx$$

Out[49]= 0

In[51]:=
$$I_{23} = \rho * \int_{-R}^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{\frac{h*\sqrt{x^2+y^2}}{R}}^h (-y * z) \, dz \, dy \, dx$$

Out[51]= 0