4. continued

John Wacz

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Recall that $\forall$ step function $f: [a,b] \to \mathbb{R}$

$$\int_a^b f(x)\,dx = \sum_{i=1}^{N} c_i\,(x_i - x_{i-1})$$ where c_i's are heights of each ste

Thus we can say

$$\int_a^b f_2(x)\,dx - \int_a^b f_1(x)\,dx = \sum_{i=1}^{N} f(x_i)(x_i - x_{i-1}) - \sum_{i=1}^{N} f(x_{i-1})(x_i - x_{$$

$$= \sum_{i=1}^{N} \left(f(x_i) - f(x_{i-1})\right)(x_i - x_{i-1})$$

Now if we ~~also~~ know that $x_i - x_{i-1} \leq \delta$ the width so

$$< \delta \sum_{i=1}^{N} f(x_i) - f(x_{i-1}) = \delta\left(f(x_a) - f(x_b)\right)$$

strictly because δ is can be bigger than $x_i - x_{i-1}$

Thus if $\delta = \dfrac{\varepsilon}{f(x_a) - f(x_b)}$ then

$$\int_a^b f_2(x) - f_1(x)\,dx < \varepsilon$$

∎