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MTH 434 Date: January 20, 2018

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Total time was roughly **35** minutes

1

Define $\gamma(t) = (t, \sin(t))$. Find the curvature function.

$$\begin{aligned}
\gamma'(t) &= v(t) = (1, \cos(t)) \\
\gamma''(t) &= a(t) = (0, \cos(t)) \\
\kappa(t) &\equiv \frac{|\mathbf{t}'(t)|}{|v(t)|} \\
\mathbf{t} &= \frac{v}{|v|} = \Big((1+\cos^2(t))^{-1/2}, \cos(t)(1+\cos^2(t))^{1/2}\Big) \\
\mathbf{t}' &= \Big(\frac{\sin(t)\cos(t)}{(1+\cos^2(t))^{3/2}}, -\frac{\sin(t)}{(1+\cos^2(t))^{3/2}}\Big) \\
|\mathbf{t}'| &= \sqrt{\Big(\frac{\sin(t)\cos(t)}{(1+\cos^2(t))^{3/2}}\Big)^2 + \Big(-\frac{\sin(t)}{(1+\cos^2(t))^{3/2}}\Big)^2} \\
\Rightarrow \kappa(t) &= \frac{\sqrt{\Big(\frac{\sin(t)\cos(t)}{(1+\cos^2(t))^{3/2}}\Big)^2 + \Big(-\frac{\sin(t)}{(1+\cos^2(t))^{3/2}}\Big)^2}}{\sqrt{1+\cos^2(t)}}
\end{aligned}$$

2

Let $\kappa : (a, b) \to \mathbb{R}$ be a function with $\kappa(t) > 0 \ \forall t \in (a, b)$ and let κ be integrable. Show that there exists a regular curve whose curvature is κ.

$$\begin{aligned}
\phi(s) &\equiv \int_{s_0}^{s} \kappa(u)du \\
\alpha(s) &= \Big(\int_{s_0}^{s} \cos(\phi(t))dt, \int_{s_0}^{s} \sin(\phi(t))dt\Big) \\
\alpha'(s) &= (\cos(\phi(s)), \sin(\phi(s))) \quad \text{(by the F.T.C)} \\
|\alpha'(s)| &= 1 \\
&\Rightarrow \alpha(t) \text{ is a regular curve} \\
&\Rightarrow \kappa(t) = |a(t)| \\
|a(t)| &= |\alpha''(t)| \\
\alpha''(t) &= (-\phi'\sin(\phi), \phi'\cos(\phi)) \\
|a(t)| &= \sqrt{\phi'^2} = \phi'(s) \\
&= \frac{d}{ds}\int_{s_0}^{s} \kappa(u)du \\
&= \kappa(s) \quad \text{(by F.T.C.)}
\end{aligned}$$

Therefore, given κ is an integrable function, we have shown there exists a regular curve with curvature κ.