Thermal Physics - PH441 **John Waczak**
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Things to address from homework 1

On problem 1 we had an expression like $S = k\beta U + k\ln Z$ and then we zapped with d and took the differential. The biggest issue was not considering the fact that β was changing. k_b is definitely constant and also V implicitly. Since $\beta = \frac{1}{k_b T}$ we have to allow β to vary in our differential.

We want to show we can extend proof from number two to n systems. let:

$$\begin{aligned} S_{AB} &= S_A + S_B \quad \text{from first part} \quad \checkmark \\ \text{define } S_N &= S_1 + S_{N-1} \\ &. \\ &. \\ &. \\ S_N &= NS_1 \end{aligned}$$

New stuff... deriving Boltzmann Factor

Last time we established the Micro-canonical definition of temperature, which was:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V \tag{1}$$

Furthermore:

$$\begin{aligned} S(E) &= k_b \ln(E) \\ g_{AB}(E_{AB}) &= \sum_{E_A} g_A(E) g_b(E_{AB} - E_A) \\ P(E_A) &= \frac{g_A(E_A) g_B(E_{AB})}{\sum\limits_{E'_A} g_A(E'_A) g_B(E_{AB} - E'_A)} \end{aligned}$$

Assume we have two systems A,B with $A << B$. Now this will allow us to use a power series expansion

$$
\begin{aligned}
S_B(E_B) &= k_B \ln(g_B(E_B)) \\
S_B(E_{AB}-E_A) &\approx S_B(E_{AB}) - \frac{1}{T}E_A \quad \text{from taking derivative in expansion in } E_A \\
\to \frac{S_B(E_{AB})}{k} - \frac{E_A}{kT} &= \ln g_B(E_{AB}-E_A) \\
\to g_B(E_{AB}-E_A) &\approx e^{\frac{S_B(E_{AB})}{k} - \frac{E_A}{\beta}} \\
\text{thus} \quad P(E_A) &\approx \frac{g_A(E_A)e^{\frac{S_B(E_{AB})}{k} - \frac{E_A}{\beta}}}{\sum\limits_{E'_A} g_A(E'_A)e^{\frac{S_B(E_{AB})}{k} - \frac{E'_A}{\beta}}} \\
&= \frac{g_A(E_A)e^{-\frac{E_A}{\beta}}}{\sum\limits_{E'_A} g_A(E'_A)e^{-\frac{E'_A}{\beta}}} \\
&= \frac{g_A(E_A)e^{-\frac{E_A}{\beta}}}{\sum\limits_{\mu'} e^{-\frac{E_{\mu'}}{\beta}}} \\
&= \frac{g_A(E_A)e^{-\frac{E_A}{\beta}}}{Z} \\
&= \frac{e^{-\beta E_\mu}}{Z}
\end{aligned}
$$

The *Boltzmann factor* is taken as a ratio of two probabilities (aka Boltzmann ratio).

Internal Energy U

$$
U = \sum_\mu E_\mu P_\mu = \sum_\mu \frac{E_\mu e^{-\beta E_\mu}}{Z}
$$

Note that: $\displaystyle \frac{\partial}{\partial \beta} \sum_\mu e^{-\beta E_\mu} = -\sum_\mu E_\mu e^{-\beta E_\mu}$

$$
\text{so...}\ U = \frac{-\frac{\partial Z}{\partial \beta}}{Z} = -\frac{\partial \ln Z}{\partial \beta} \quad \text{This is a trick... NEVER START HERE you wont remember it}
$$