In[∘]:=
```
A = {{7, 0, 0}, {0, 1, -i}, {0, i, -1}}
```
Out[∘]= $\{\{7, 0, 0\}, \{0, 1, -i\}, \{0, i, -1\}\}$

In[∘]:=
```
vals = Eigenvalues[A]
vecs = Eigenvectors[A]
```
Out[∘]= $\{7, -\sqrt{2}, \sqrt{2}\}$

Out[∘]= $\{\{1, 0, 0\}, \{0, i\left(-1+\sqrt{2}\right), 1\}, \{0, -i\left(1+\sqrt{2}\right), 1\}\}$

In[∘]:=
```
vecs = {Normalize[vecs[[1]]], Normalize[vecs[[2]]], Normalize[vecs[[3]]]}
```
Out[∘]= $\left\{\{1, 0, 0\}, \left\{0, \frac{i\left(-1+\sqrt{2}\right)}{\sqrt{1+\left(-1+\sqrt{2}\right)^2}}, \frac{1}{\sqrt{1+\left(-1+\sqrt{2}\right)^2}}\right\}, \left\{0, -\frac{i\left(1+\sqrt{2}\right)}{\sqrt{1+\left(1+\sqrt{2}\right)^2}}, \frac{1}{\sqrt{1+\left(1+\sqrt{2}\right)^2}}\right\}\right\}$

Now we want to show that his orthonormal basis is complete. I.e. that $\Sigma_i \mid \phi_n\rangle\langle\phi_n \mid = \mathbf{Id}$

In[∘]:=
```
A1 = KroneckerProduct[vecs[[1]], vecs[[1]]]
```
Out[∘]= $\{\{1, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

In[∘]:=
```
A2 = KroneckerProduct[vecs[[2]], Conjugate[vecs[[2]]]]
```
Out[∘]= $\left\{\{0, 0, 0\}, \left\{0, \frac{\left(-1+\sqrt{2}\right)^2}{1+\left(-1+\sqrt{2}\right)^2}, \frac{i\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2}\right\}, \left\{0, -\frac{i\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2}, \frac{1}{1+\left(-1+\sqrt{2}\right)^2}\right\}\right\}$

In[∘]:=
```
A3 = KroneckerProduct[vecs[[3]], Conjugate[vecs[[3]]]]
```
Out[∘]= $\left\{\{0, 0, 0\}, \left\{0, \frac{\left(1+\sqrt{2}\right)^2}{1+\left(1+\sqrt{2}\right)^2}, -\frac{i\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2}\right\}, \left\{0, \frac{i\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2}, \frac{1}{1+\left(1+\sqrt{2}\right)^2}\right\}\right\}$

In[•]:=
```
MatrixForm[A3]
MatrixForm[A2]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\left(1+\sqrt{2}\right)^2}{1+\left(1+\sqrt{2}\right)^2} & -\frac{\mathrm{i}\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2} \\ 0 & \frac{\mathrm{i}\left(1+\sqrt{2}\right)}{1+\left(1+\sqrt{2}\right)^2} & \frac{1}{1+\left(1+\sqrt{2}\right)^2} \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\left(-1+\sqrt{2}\right)^2}{1+\left(-1+\sqrt{2}\right)^2} & \frac{\mathrm{i}\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2} \\ 0 & -\frac{\mathrm{i}\left(-1+\sqrt{2}\right)}{1+\left(-1+\sqrt{2}\right)^2} & \frac{1}{1+\left(-1+\sqrt{2}\right)^2} \end{pmatrix}$$

In[•]:=
```
FullSimplify[MatrixForm[A1 + A2 + A3]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[•]:=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

Out[•]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

In[•]:=

In[•]:=