PH 481: Lab 7 - Polarization

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I. Introduction

In this set of experiments we considered the properties of polarized light and polarizers. First we verified Malus's law for the intensity transmitted through a system of two polarizers separated by some angle. Next, we reexamined the thin film interference for a microscope slide and a cover slip. Finally, we used a spectrometer to measure the absorption spectrum of an Antimony Oxide thin-film sample.

II. Theory

In our first experiment we examined a two polarizer system. The first polarizer prepared the incoming light into a particular polarization angle. With the observation that placing the analyzer 90 degrees relative to the polarization of the prepared beam leads to 0 intensity transmission it can be shown that the analyzer acts to project the polarization onto the new angle.//

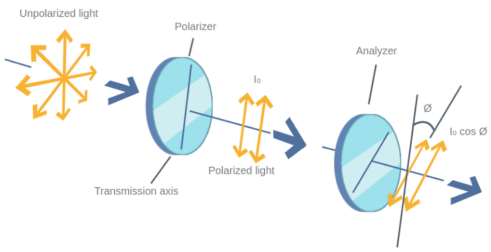

Fig. 1. Malus's law asserts that the analyzing polarizer projects the incident polarization into the new polarization state

Figure 1 illustrates how this projection works. Since intensity is proportional to the square of the Electric field, this relationship is summarized mathematically by

$$I(\theta) = I_0 \cos^2 \theta \tag{1}$$

where θ is the difference in angle between the two polarizers.

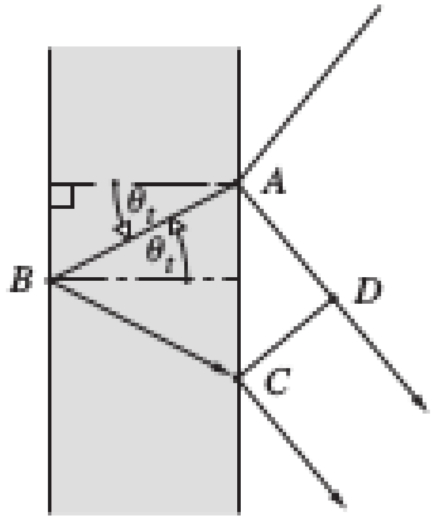

Fig. 2. Illustration of light rays in thin film interference.

Figure 2 shows how light enters and exits a thin film. Part of the beam incident to point A is immediately reflected while the rest is transmitted to then reflect and retransmit out of the material at point C. While in truth there are multiple beams that can interfere with the reflected beam, their impact is minimal compared to that of the first transmitted beam.

In this situation interference is determined by the overall optical path length difference (OPL) between the first reflected beam and that pases through point C. This difference is given by $OPL = n_2(AB + BC) - n_1(AD)$. This OPL leads to the following equation for interference fringes (here taken to be minima of intensity):

$$d \cos \theta_t = m\frac{\lambda}{2} \tag{2}$$

where $\lambda = \lambda_0/n$ is the wavelength inside of the thin film. For the third experiment, the light entered at normal incidence so that $\theta_t = 0$ removing the cosine term from equation (2).

In our third and final experiment, we measured the absorption spectrum of a sample of Antimony Oxide from Dr. Tate's lab. The absorption spectrum as a function of wavelength is given by Beer's law which states:

$$T = (1 - R)e^{-\alpha d} \tag{3}$$

where T and R are the transmission and reflection coefficients, α is the absorption, and d is the width of the thin film. Naturally, the T and R depend on wavelength so if we solve for the absorption coefficient we find:

$$\alpha(\lambda) = \frac{1}{d} \ln\left(\frac{1 - R(\lambda)}{T(\lambda)}\right) \tag{4}$$

Thus we have all of the theoretical framework required to describe the results of the following three experiments.

III. Experiment

First we aligned the table top laser and then proceeded to add a fixed polarizer and then a variable angle polarizer along the optical axis. This enabled us to change the angle of the analyzer polarizer.

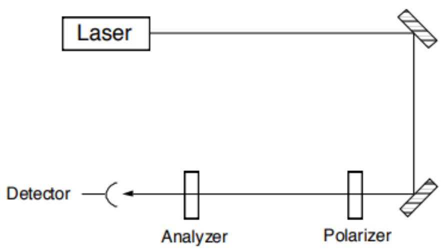

Fig. 3. Table design for polarization measurements

Figure 3 illustrates the table-top setup for the two polarizer measurements. For the first experiment we were charged with verifying the Law of Malus (equation 1). To do this we place a photo-diode at the end of the optical rail connected to the oscilloscope in order to measure the transmitted intensity. We then found the brightest angle possible and defined that to be 0 radians difference between the polarizer and analyzer. Subsequently we varied the analyzer angle and measured the transmitted intensity.

For our second experiment, we removed the polarizers and added a rotational stage at the end of the optical rail (in the path of the expanded beam). We then fixed the angle and set up a camera behind a 75 mm positive lens to capture the image of the reflected interference pattern. Using Ali's LabView program, we located one bright fringe (m) and then translated the stage until the m+1 bright fringe walked to where the initial fringe was located. Noting that change in angle we then used equation (2) to determine the sample width by combining the equations for the two different angles.

For the final experiment we used a separate workbench that was already set up with the SbO3 sample and a spectrometer connect to fiber optic cables. In order to measure the absorption spectrum we first needed to measure the transmission and reflection spectra. This was accomplished by first taking a background measurement and then capturing the spectra and subtracting this background away. For the case of the reflection spectrum, we had to multiply our results by a reference spectrum because we used a mirror to determine the background spectrum.

This mirror itself has a wavelength-dependent reflection coefficient and so we first had to interpolate a set of Thor-labs data for the reflectivity as a function of wavelength for the mirror to ensure the number of data points matched our spectrum. This was accomplished using Python's NumPy package.

From the reflection spectrum we then used equation (2) at normal incidence to determine the width of the sample as follows:

$$d = m\frac{\lambda_1}{2} = m\frac{\lambda_{1,0}}{2n}$$
$$d = m'\frac{\lambda_2}{2} = m'\frac{\lambda_{2,0}}{2n}$$

Where λ_1 and λ_2 are the wavelengths inside the sample corresponding to adjacent interference fringes. Because these fringes are next to each other, we can write:

$$
\begin{aligned}
m' &= (m+1) \\
\Rightarrow (m+1)\frac{\lambda_{2,0}}{2n} &= m\frac{\lambda_{1,0}}{2n} \\
m\Delta\lambda_0 &= -\lambda_{2,0} \\
\Rightarrow d &= \frac{\lambda_{2,0}\lambda_{1,0}}{2\Delta\lambda_0 n}
\end{aligned} \tag{5}
$$

Note that here I have gotten rid of the negative sign because we require that d is a positive number. Once we had calculated d, we were able to determine the absorption spectrum from Beer's law.

Finally, we examined the spectra for two different colored-films in order to qualitatively analyze how the spectra are similar and different.

IV. RESULTS

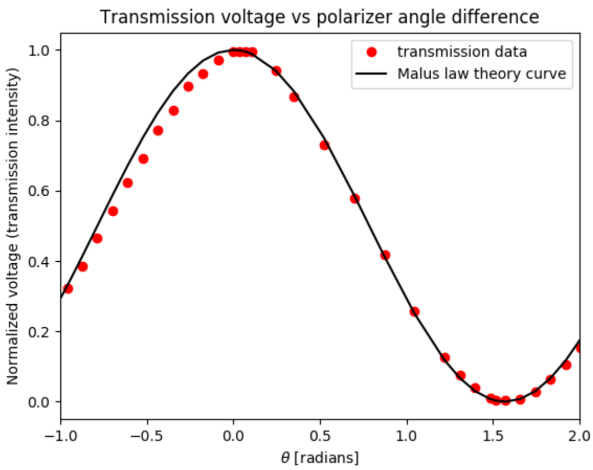

Fig. 4. Transmission data as a function of angular difference between polarizers

Figure 4 shows the data for the transmission amplitude as a function of the difference in angle between the two polarizers. This is graphed against Malus's law which is a cosine squared relationship.

TABLE I
THIN FILM INTERFERENCE RESULTS

sample	θ_t (m)	θ_t (m+1)	d [mm]	d actual
microscope	0.1689	0.1661	0.44016	0.5
cover slip	0.2662	0.2707	0.1743	0.2

The table I table shows the results for the thin film interference measurements performed on the glass microscope slide and the thin cover slip. To make sure our data was in the correct form we had to be careful to use Snell's law and the index of refraction for glass $n_g = 1.5$ in order to turn the incident angle into the transmitted angle from the thin film.

The following figure shows the absorption spectra for the Transmission and Reflection curves as functions of wavelength. Notice the multiple maxima and minima that correspond to wavelengths that are almost entirely transmitted or reflected.

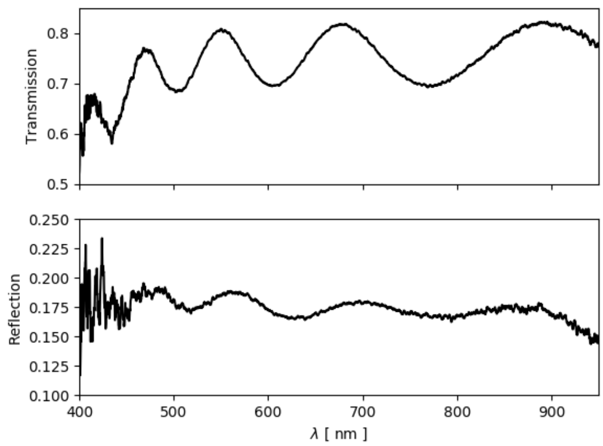

Fig. 5. Transmission and Reflection spectra

In order to create the reflection function in figure 5 we had to interpolate the Thor labs reflectance data to match the resolution of the spectrometer as mentioned in the previous section. After finding these curves we read off $\lambda_1 = 561$ nm and $\lambda_2 = 696$ nm from the Reflection data. This enabled us to find $d = 0.689\ \mu m$ as the sample width (from equation 5). Finally, with these three pieces of information we were able to determine the absorption function via equation (4). This is summarized in the following figure:

Notice how the maxima in the absorption graph appear to coincide with the maxima of the Transmission curve and the minima appear to coincide with the reflection curve. There is a small amount of noise present near low wavelengths in the reflection curve which I suspect led to the

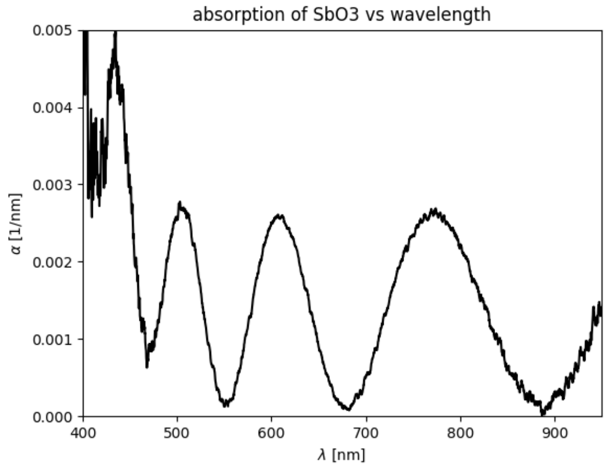

Fig. 6. Absorption curve for the SbO3 sample.

somewhat noisier nature of our absorption spectrum.

The final two figures show the Transmission spectra for two different colored-samples:one blue and one red.

Fig. 7. Transmission curve for blue filter

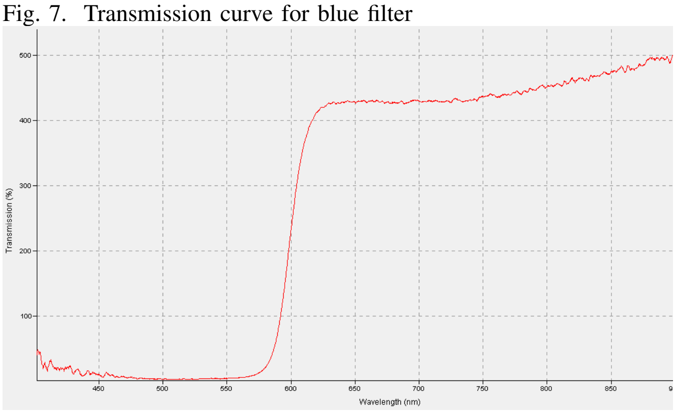

Fig. 8. Transmission curve for red filter

Here we can see that the red filter has nearly zero transmission until wavelengths above roughly 500 nm. Wavelengths in this range are in the long end of the visible spectrum and so appear red. Similarly for the blue transmission spectrum we have nonzero transmission intensity in the short wavelength range of the visible spectrum corresponding to blue light.

V. Discussion

The results of our first experiment appear to agree with Malus's law quite well Nearly all of the data points lie on the theoretical curve or within a very small neighborhood of it. This provides a good verification for Malus's law. I think given more time I would like to have added a second polarizer and then tested intensities over a square grid of analyzer values.

For the interference measurements we were able to determine the width of each sample to a reasonable level of accuracy – that is to the extent at which we can trust caliper measurements to verify our results.

In our third experiment, we found the absorption curve for the Antimony Oxide sample from Dr. Tate's lab. Although I am not entirely confident I performed the interpolation (see attached code) the data appears to make sense as qualitatively, the sample was not opaque and so we do not expect there to be a lot of absorption in the visual range. It is also interesting to not that the local extrema of the absorption curve appear to coincide with those of the transmission (maxima) and reflection (minima) curves.

Finally our transmission spectra for the red and blue samples appear to agree with the colors we see. The transmission curve for the red sample showed little light below 500 nm with most of the intensity in the red portion of the visible range. Similarly, the intensity of the blue sample was mostly in the low wavelength portion of the visible range which we know appears to be blue.

VI. Conclusion

We were able to verify Malus's law by using a polarizer-analyzer system with a high degree of confidence. Using thin film interference, we were able to determine the thickness of two samples: a microscope slide (~ 0.5mm) and a thin cover slip

(~ 0.2 mm). Finally, we computed the absorption spectrum by calculating the width a the SbO3 sample and measuring the transmission and reflection spectra.

VII. References

[1] *Optics*, Eugene Hecht

[2] Darlene Focht

[3]http://physics.oregonstate.edu/~mcintyre/COURSES/ph481/LABS/Lab5.pdf