PH 427 Homework 5

5.1 Select a research article

Find a partner for your journal club presentation, and agree on a research article on which to present.

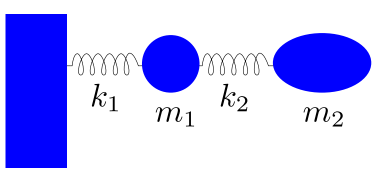
Submit a copy of the first page of your chosen research article (title, authors and abstract), along with the name of your partner and a short statement/agreement about how your will share the workload of preparing and giving the presentation.

Furthermore, please submit a preference for when (which week) you will be scheduled to present your talk.

I will let you know if there is a problem with the article you selected, or with your work plan.

5.2 Two coupled masses

Two masses m_1 and m_2 are connected to each other by a spring of constant k_2, and mass m_1 is connected to a fixed support by another spring with spring constant k_1. Assume motion is allowed in a horizontal straight line from or toward the support, with no friction as shown in the figure.



Let x_1 and x_2 represent the position of m_1 and m_2, respectively, relative to the equilibrium positions of the masses (i.e. when the springs are not stretched).

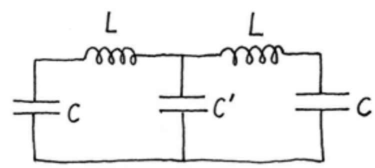
(a) Derive an expression for the frequencies of characteristic small-amplitude oscillation in terms of the above parameters.

(b) Your formula for the frequency at this stage will probably look a bit confusing. Check that it is correct in at least one limiting case where you can easily determine the solution.

(c) Let $k_1 = k_2 = k$ and $m_1 = m_2 = m$. What are the characteristic frequencies now?

5.3 Coupled electronic oscillators

(a) Find a pair of coupled differential equations that describe the time dependent behavior of the circuit in the figure. Solve these coupled differential equations to find the natural frequencies of the system.



(b) One of the natural frequencies is independent of C'—discuss the physical significance of this (what does the pattern of charge motion look like?).

(c) If $C' \gg C$, and the correct initial conditions are chosen, charge in the right-side of the circuit will oscillate while gradually transferring energy to the left-side of the circuit. What initial conditions would generate this behavior?

Suggested approach for part (a): First, solve the simplest system with one capacitor and one inductor. This will remind you how to describe circuit dynamics in terms of charge on a capacitor plate and the sum of the voltages in a loop. Second, analyze the figure in terms of charge on leftmost capacitor, and charge on the rightmost capacitor. The charge in the center capacitor can be determined from those two charges.

5.4 Complex numbers

Find all the values of the following complex functions:

(a) $\ln(-1)$

(b) $\sin(2i)$

(c) $(-8)^{1/3}$

5.5 Unitary matrix and eigenvectors

Consider a (generic) 3×3 Hermitian matrix M with three orthonormal basis vectors $|u\rangle$, $|v\rangle$, $|w\rangle$. Suppose that you know M and its basis vectors in a particular representation, so that M is a particular matrix and the basis vectors are (known) columns. Build the matrix U out of the columns that represent the basis vectors:

$$U = \begin{pmatrix} |u\rangle & |v\rangle & |w\rangle \end{pmatrix} \tag{1}$$

(a) Show that U is unitary.

(b) Show that $U^\dagger M U$ is a diagonal matrix and that the diagonal entries are the eigenvalues of M. (You will show taht any matrix is diagonal in its own eigenstates.)