John Walczak

(1)

Tappi 3.80

(1) prove that if f is a rigid motion and $S \subset \mathbb{R}^3$ is a regular surface then $f(S)$ is a regular surface.

Since S is a regular surface we have

$$\sigma : U \subset \mathbb{R}^2 \to V \subset \mathbb{R}^3 \text{ is a smooth bijection}$$

and U is an open set in $\mathbb{R}^2$.

Now we want to show $f(S)$ is a regular surface. Because f is a rigid motion we can say $f = T_q \circ L_A$ where T_q is a translation by q and L_A is an orthogonal matrix. Now we already have that U is open in $\mathbb{R}^2$. Observe the following diagram for the map $f \circ \sigma$

$$f \circ \sigma : U \subset \mathbb{R}^2 \longrightarrow f(V) \subset \mathbb{R}^3$$

this is equivalent to

$$U \subset \mathbb{R}^2 \xrightarrow{\sigma} V \subset \mathbb{R}^3 \xrightarrow{f} f(V) \subset \mathbb{R}^3.$$

Thus all we need to show is that because σ is a diffeomorphism, $f \circ \sigma$ must also be a diffeomorphism