① 5B

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Tapp 6.12.

Soccer ball constructed from 20 hexagons and 12 pentagons. Determine V, E, F and calculate χ.

Observe that every vertex is shared by 3 faces, every edge is shared by two faces. Thus:

$$V = \frac{(12\cdot5)+(20\cdot6)}{3} = \frac{180}{3} = 60$$

$$E = \frac{(12\cdot5)+(20\cdot6)}{2} = \frac{180}{2} = 90$$

$$F = 12 + 20 = 32.$$

$$\chi(S) = V - E + F = 60 - 90 + 32 = 2 \quad \checkmark$$

Tapp. 6.13

Torus w/ recctangular triangulation. Explain why $F = V = \frac{E}{2}$

From picture every vertex is shared by 4 faces. Thus there are $\frac{4F}{4} = F$ vertices. Every edge is shared by 2 faces. There are 4 edges per faces w/ each being shared by 2 faces. Thus $\frac{4F}{2} = E \Rightarrow \frac{E}{2} = V$

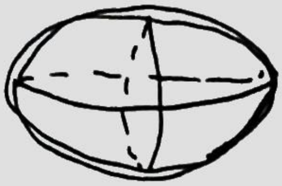
therefore

$$F = V = \frac{E}{2}.$$

Now from this:

$$\chi = V - E + F = F - 2F + F = \emptyset.$$

2) a) Find Gaussian curvature total for Ellipsoid



by 6.13 every connected surface has:

$$\iint K\,dA = 2\pi\,\chi(S)$$

this appears to be homeomorphic to the sphere, thus:

$$\iint K\,dA = 2\pi(2) = 4\pi$$

b)

I am concerned about conditions required to make this claim..

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Connected?

3) $K > 0 \Rightarrow S$ is diffeomorphic to the sphere.

Assume ~~k~~ S surface w/ question's hypotheses. then assume for contradiction that S is not diffeo to S^2.

Because No diffeo, $6.16 \Rightarrow \chi(S) \neq 2$. furthermore we have that if $\chi(S) \neq 2$ $\Rightarrow \chi(S) \leq 0$. Thus

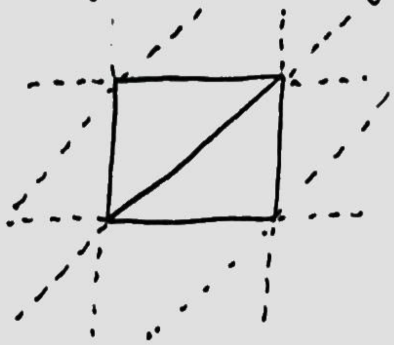
$$\iint K \, dA = 2\pi \chi(S) \leq 0$$

which contradicts hypothesis.

4) ?

5.

a) given rectangles, cut diagonally into triangles.

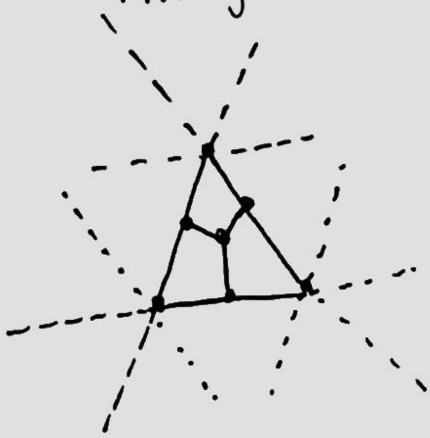


this makes

$$V \mapsto V$$
$$F \mapsto 2F$$
$$E \mapsto E + F$$

thus $\chi^{new}(s) = V' - E' + F' = V - (E+F) - 2F = V - E + F$ ✓

b) triangles into 3 rectangles



this transformation sends

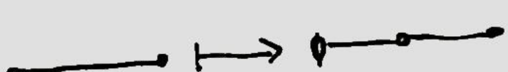
$$F \mapsto 3F$$
$$E \mapsto 2E + 3F$$
$$V \mapsto V + E + F$$

and so $\chi' = V' - E' + F' = V + E + F - (2E + 3F) + 3F$
$= V - E + F$ ✓

c) N- polygon → triangles

imagine discrete function $f(i)$, $i \in \{1, \dots N\}$
such that ~~each step only~~ $f: T' \to T^2$
the triangulations. Only Possible moves each turn are:

add vertex to edge



$(V+1) - (E+1) + F = V - E + F$ ✓

connect two vertices

$V - (E+1) + (F+1)$ ✓

Add interior vertices and construct new ~~polygon~~