PH 481: Lab 5 - Slit Diffraction

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with Darlene Focht and
some data from Attila Varga

I. Introduction

In this set of experiments, we analyzed the diffraction patterns produced by a host of long slit combinations. Single slits, double slits, multiple slits, as well as a diffraction grating and compact-disk (cd) were all examined. Using the principles of Fourier analysis for how the different slits convolve with each other, we back calculated the slit widths and separations for each case.

II. Theory

Consider Huygen's principle which postulates that every point on the incident plane wave of light can be treated as it's own point source radiator that produces spherical waves. Imagine putting N of these oscillators in a line within the width of a single slit. The resulting pattern on a viewing screen some distance away is the result of the interference of the N spherical waves. Figure 1 illustrates this idea.

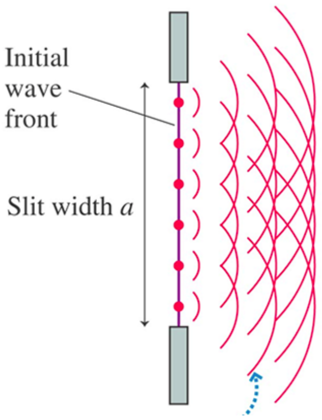

Fig. 1. Huygen principle applied point sources inside of slit

If we now take the limit as the number of points goes to ∞ and the distance between points goes to the infinitesimal dy it can be shown that the resulting intensity distribution takes the following form:

$$I(\theta) = I(0)\Big(\frac{sin(\beta)}{\beta}\Big)^2 \tag{1}$$

$$\beta = k\frac{b}{2}\sin(\theta) \tag{2}$$

This equation is the square of the sinc function and has a minimum when $\beta = m\pi$. Thus the condition for the angle at which the m^{th} dark fringe can be found is given by the following:

$$\begin{aligned} \beta &= m\pi \\ \Rightarrow \frac{kb}{2}\sin(\theta_m) &= m\pi \\ b\sin(\theta_m) &= m\frac{2\pi}{k} \\ b\sin(\theta_m) &= m\lambda \end{aligned} \tag{3}$$

Now recall Young's double slit experiment which found that for two idealizes point sources, the interference pattern produced followed went like $\cos^2(\alpha)$ where $\alpha = \delta/2$ and $\delta = ka\sin(\theta)$. Because the intensity is given by a cosine function, the constructive interference condition occurs when the phase difference $\delta = 2m\pi$. Thus,

$$a\sin(\theta_m) = m\lambda \tag{4}$$

is our second important equation. This tells us where the bright fringes occur as a result of the double slit interference. For the intensity curve we can imagine that the slit apparatus is acting as a Fourier transform and so for the double slit, the resulting intensity is a convolution of our $sinc(\beta)^2$ from the finite slit size with the $\cos^2(\alpha)$ for two point-sources. This wraps the many Youngs double fringes in an sinc function envelope. The net effect is shown below in figure 2.

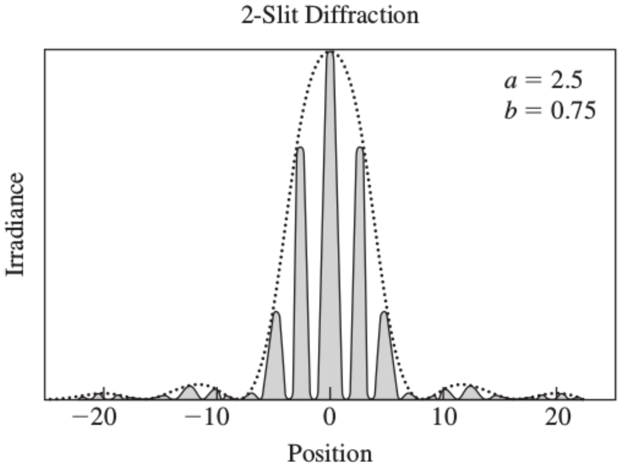

Fig. 2. Double slit intensity profile for given slit width and separation. Notice the sinc function envelope(dotted line)

When we consider multiple slits, we can perform the same treatment: take the convolution of the sinc function from the single slit with the N point source interference from idealized multiple interference. As we found for the case of two source interference, the equation describing the distribution of intensity maxima for multiple sources is given by:

$$a \sin(\theta_m) = m\lambda$$

We now have all of the mathematical tools required to analyze the results of this experiment.

III. Experiment

As always, we began by aligning the laser along the optical axis. We then expanding the beam by creating a Keplerian telescope from a -25mm diverging lens coupled with a 100 mm positive lens. This gave us a wide enough beam to be able to hit the entirety of the slit plate. We then added a 75 mm lens to focus the resulting pattern to a smaller point that was directed into a camera. To make sure the intensity wasn't too much for the camera, we added two polarizes in between the mirrors on the optical axis.

Our first task was to determine width of a single slit. We aligned the slit plate and then focused the camera to obtain the following image of the diffraction pattern:

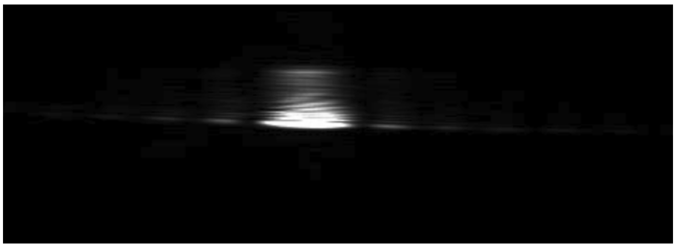

Fig. 3. Focused image of the single slit interference pattern. The intensity was reduced using polarizers to protect the camera.

Using this image and Ali's labview program we were able to obtain a graph of the intensity distribution. From this we determined the slit width.

Next we determined the slit width and separation for one of the double slits on the slit plate. Again we used the camera to take the following focused raw image of the interference pattern.

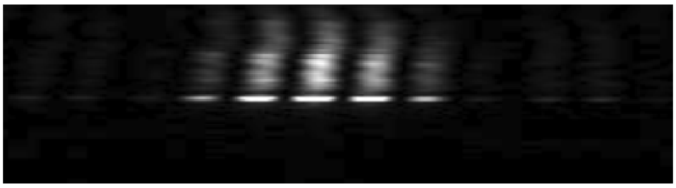

Fig. 4. Double slit interference pattern

Using this image, we again determined the slit width as well as the separation. This same process was then repeated for 3, 4, and 10 slit patterns. Finally, for fun, we performed measurements to deduce the separation width of a diffraction grating and a the indentation separation for the grooves of a CD.

IV. Results

Unfortunately, I misplaced my binder this Monday and haven't had any luck trying to find it. The following data is courtesy of Attila Varga and Katy Chase. The following figures are from our old data though.

For the single slit we found that the length required to make a focused image was $\ell = 7$cm. The following figure shows the output of the intensity distribution:

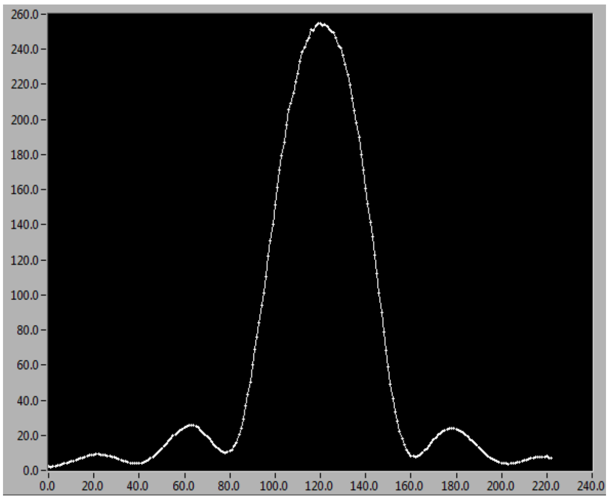

Fig. 5. Intensity distribution from single slit

From this graph we measured the position difference between the central maximum and the first dark fringe to be $y = 233\mu m$. Our He-Ne laser has a wavelength of 632.8 nm and so using this data with a small angle approximation in equation 3 yields a width of:

$$\begin{aligned} b &= \frac{\lambda \ell}{y} \\ &= \frac{632.8 \cdot 10^{-9} \cdot 0.07}{233 \cdot 10^{-6}} \\ b &= 0.1901 \text{ mm} \end{aligned} \tag{5}$$

The reported value for this slit was 0.2 mm.

For the double slit, we captured the following intensity distribution:

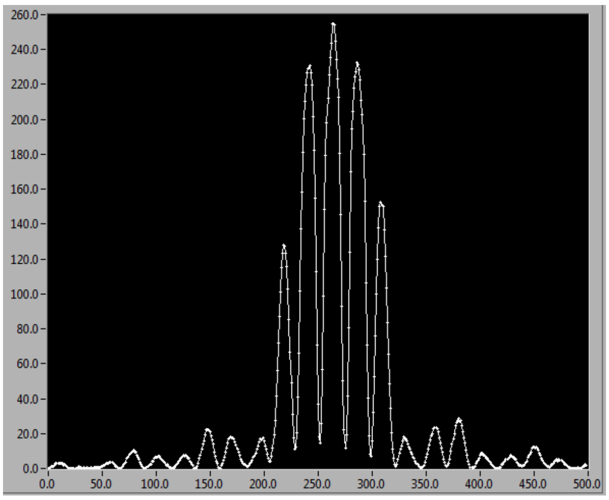

Fig. 6. Intensity distribution for double slit

We had some trouble determining which minimum to use for the slit width determination. To figure this out we looked at the successive peak to peak lengths and found the place where that value drastically changed. This focused image was taken at a distance $\ell = 6.75$ cm away from the slit. The max to min distance for the sinc envelop was measured to be $y_1 = 440.236\mu m$. The peak to peak distance was measured at $y_2 = 72.5094\mu m$. By the same process as for the single slit, this leads to the values:

$$a = 0.589 \text{ mm} \tag{6}$$

$$b = 0.0970 \text{ mm} \tag{7}$$

The reported values for our chosen slits were 0.01 mm wide and 0.5 mm. The ratio of a to b is $a/b = 6.072$. I suspect an easier way to find this ratio would be to take the ratio of our two y values i.e. $y_1/y_2 = 440.236/72.5094 = 6.071$. This makes sense as the two lengths come from similar triangles in the geometry.

The following three images show the intensity distributions for the 3, 4, and 10 slit spots on the slit plate.

The following data tables summarize the results for the multiple slit measurements:

TABLE I
MULTIPLE SLIT MEASUREMENTS

num slits	ℓ	y_1	y_2
3	7.25 cm	323.185 μm	571.789 μm
4	7 cm	340.276 μm	512.2745 μm
10	17 cm	865.97 μm	1373.02 μm

Of note is that for the final row, we switched out the 75 mm focusing lens 200 mm focusing lens to get a clearer picture. To easily determine the number of slits from the intensity graph we cant count the number of local maxima between maxima corresponding to the sinc function. For example in figure IV a we can count 1 small maximum in between the two large. This gives 3 slits. Similarly for 10 we see that there are 8 small maxima between the adjacent large maxima corresponding to the sinc envelope. Taking our data from table I we can deduce the slit width and separations in the same manner as we did for the double and single slits. This is shown in the following table:

TABLE II
SLIT WIDTHS AND SEPARATIONS

num slits	a	b	a/b
3	0.1419 mm	0.0802 mm	1.769
4	0.1302 mm	0.0774 mm	1.682
10	0.1242 mm	0.0783 mm	1.586

Lastly, we used a line of string to measure the distance between the central and first diffraction maxima at a distance of $\ell = 307cm$. This distance ended up as $y = 112cm$. From this we calculated the following:

$$\theta = \tan^{-1}(y/\ell) = 20.04^\circ$$
$$a = \frac{632.8nm}{sin(20.04)} = 1.846\mu m \tag{8}$$

I lost my data for the "for fun" CD diffraction measurement and Attila didn't take data for that so I used my 532 nm green laser pointer and a video game DVD at home to roughly do the same thing. I found that at a distance of $\ell = 32.5in$ the distance between the central maximum and the $m = 1$ maximum was $y = 1in$. This gives a "slit width" of $a = 17\mu m$

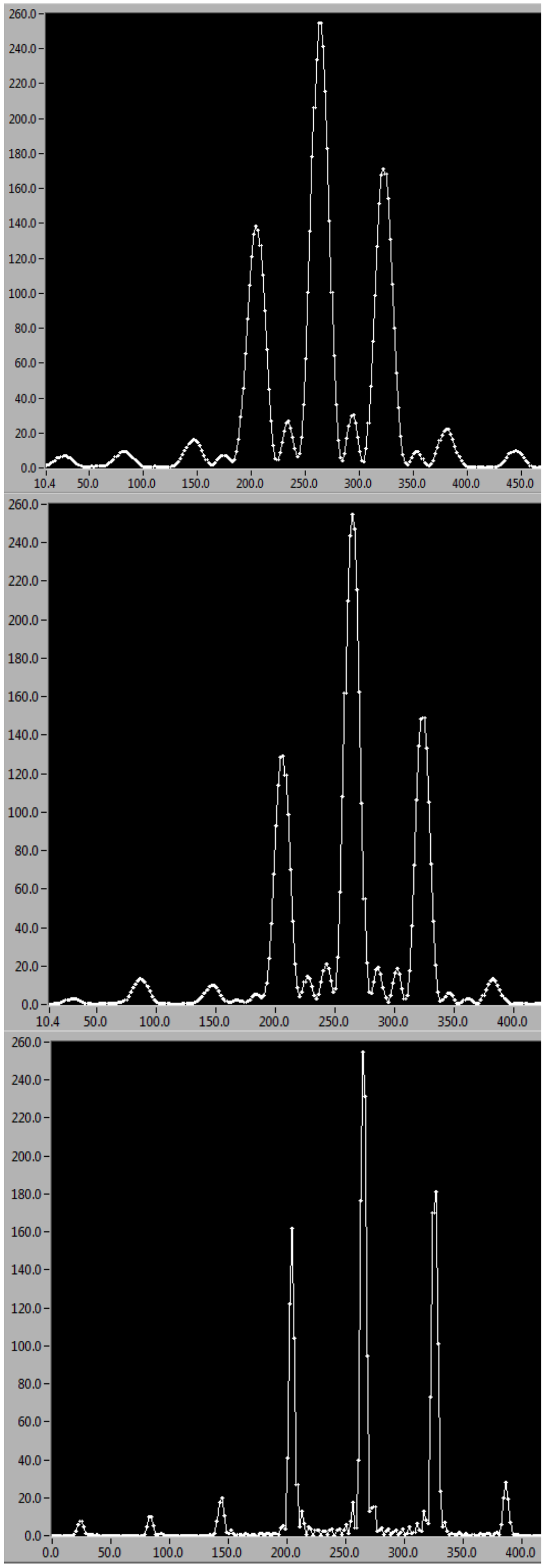

Fig. 7. Multiple slit interference intensities

V. Discussion

Our results from the single and double slits were pretty good, particularly for the slit width as the actual value for the double slit width was 0.1 mm and we measured 0.97 mm (%3 percent error). We had more trouble with the multiple slit diffraction where not only were our values slightly off but the ratio of a to b was decreasing as we increased the number of slits. This was unexpected because the reported values for the multiple slits did not change. The slit width was supposed to be 0.05 mm and the separation distance was supposed to be 0.132 mm.

As far as uncertainty is concerned the part of the problem was determining where the actual minima were. This was pretty challenging for the higher number of slits because the peaks dropped drastically for a pretty wide space. We also had some trouble with keeping the telescope aligned which made it hard take images of the diffraction patterns.

My only lingering question from this lab regards the compact disk. The lab manual says it is an example of an interesting diffraction grating however the CD is not transparent. I wonder how diffraction comes into play for the small bumps on the surface of the CD that encode information. I imagined it more like multiple beam interference and so really I guess I'm confused as to what the "width" corresponds to— depth of the bump or it's actual width.

VI. Conclusion

By observing the interference pattern produced by different variations of slits we were able to determine both the slit widths and separation distances. We then applied the same method to analyze a piece of diffraction grating as well as a compact disk.

VII. References

[1] *Optics*, Eugene Hecht

[2] Darlene Focht

[3]http://physics.oregonstate.edu/~mcintyre/COURSES/ph481/LABS/Lab4.pdf

[4] Attila Varga