Thermal Physics - PH441 **John Waczak**

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Fermion-Statistics

Recall from last class, we discussed how we could take the many bodied wave function $\Psi(\vec{r}_1, \vec{r}_2, ...\vec{r}_n) =$ slater determinant that ensures that we have the correct anti-symmetric combination of orbitals. These orbitals $\phi_i(\vec{r}_j)$ are the energy eigenstates of the single particle system. The *trick* we are going to be using is to take each of the orbitals and treat them as if they were a separate systems... weird.

Now think of our $\phi_i(\vec{r})$ as having an occupancy of 1 or 0.

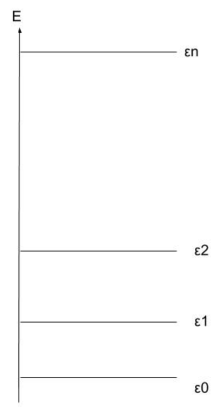

Figure 1: general idea of energy levels four our single particle

We have that our Gibbs sum for a ϕ_i From Puali's exclusion principle, we can only have one particle :

$$
\begin{aligned}
\mathbb{Z} &= \sum_i e^{-\beta(E_i - \mu N_i)} \\
&= 1 + e^{-\beta(\epsilon-\mu)} \\
\Rightarrow \langle E \rangle &= \epsilon \frac{e^{-\beta(\epsilon-\mu)}}{1 + e^{-\beta(\epsilon-\mu)}} \\
&= \epsilon \frac{1}{1 + e^{\beta(\epsilon-\mu)}} \\
\langle N \rangle &= \frac{e^{-\beta(\epsilon-\mu)}}{1 + e^{-\beta(\epsilon-\mu)}} \\
&= \frac{1}{1 + e^{\beta(\epsilon-\mu)}} = \text{Fermi-Dirac function}
\end{aligned}
$$

Definitions The *Fermi-Level* is synonymous with the chemical potential μ. The *Fermi-energy* is $\mu(T = 0)$.

Boson-statistics

We do not have the anti-symmetry requirement and so we can have many combinations of orbitals to make a combined state. Now our orbitals ϕ_i with energy ϵ and occupancy $N = 0, 1, 2, 3, ...\infty$.

$$\begin{aligned}
\mathbb{Z} &= 1 + \sum_{j=1}^{\infty} \left(e^{-\beta(\epsilon j - \mu j)} \right) \\
&= 1 + \sum_{j=1}^{\infty} \left(e^{-\beta(\epsilon - \mu)} \right)^j \\
&= \frac{1}{1 - e^{-\beta(\epsilon-\mu)}} \\
\langle N \rangle &= \sum_j \frac{j e^{-\beta(\epsilon-\mu)j}}{\mathbb{Z}} \\
&= \frac{1}{\mathbb{Z}} \frac{\partial \mathbb{Z}}{\partial(\mu\beta)} \\
&= \frac{e^{-\beta(\epsilon-\mu)}}{1 - e^{-\beta(\epsilon-\mu)}} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1} = f_{BE}(\epsilon)
\end{aligned}$$