MTH. 437 4/1/2019

Summary So far

Curvature	Flat	Curved	Flat
Line Element	dx^2+dy^2	$r^2(d\theta^2+\sin^2\theta d\phi^2)$	dx^2-dt^2
Signature	0	0	1

What is the basis for trigonometry?

Circle Trig v.s. ~~Triangle Trig~~

"you measure distance with ds"

① equation of a circle: $x^2+y^2=r^2$

② measure arc length: $ds^2=dx^2+dy^2$

③ define angle: $\theta = s/r$

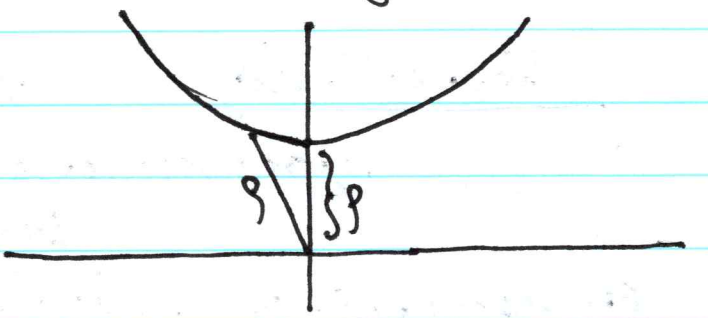
④ trig functions = coordinates on the circle : $(r\cos\theta, r\sin\theta)$

Hardest Part: Integrate for arclength w/o using a trig substitution (because we haven't yet defined the trig!).

Special Relativity: Hyperbola Geometry
note: Not hyperbolic geometry

① define hyperbolic circle e.g. hyperbola

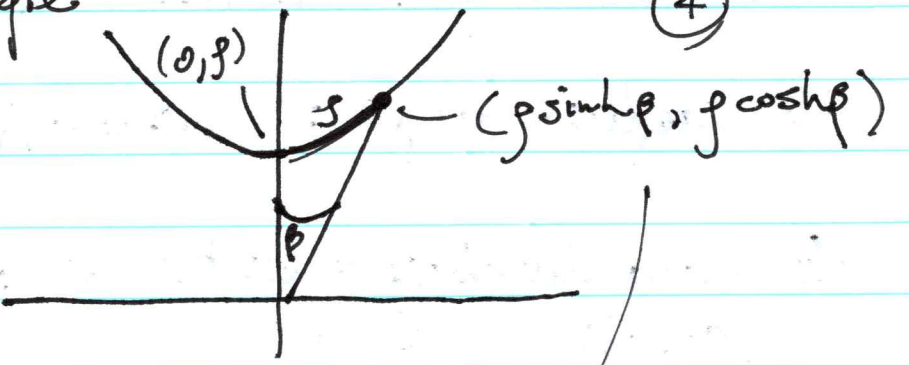
$$t^2 - x^2 = \rho^2$$



② $ds^2 = dx^2 - dt^2$ ⟶ define hyperbolic angle

③ $\beta \equiv s/\rho$

④

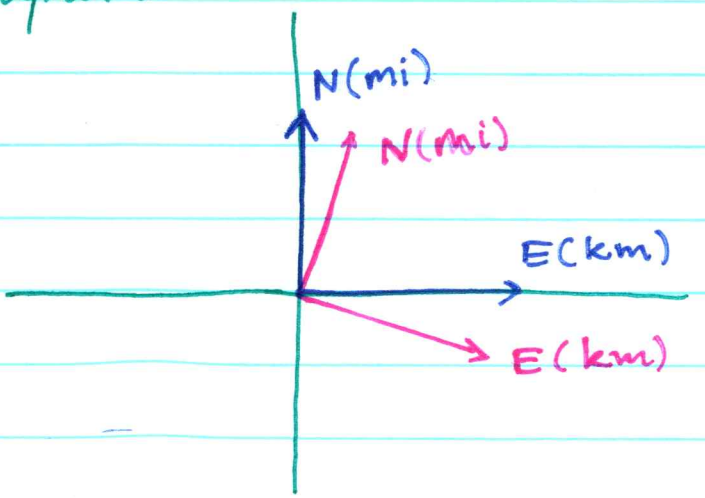


Note: ordering is flipped because β is measured from the y-axis (twice axis).

The Surveyor's Parable

4/3/19

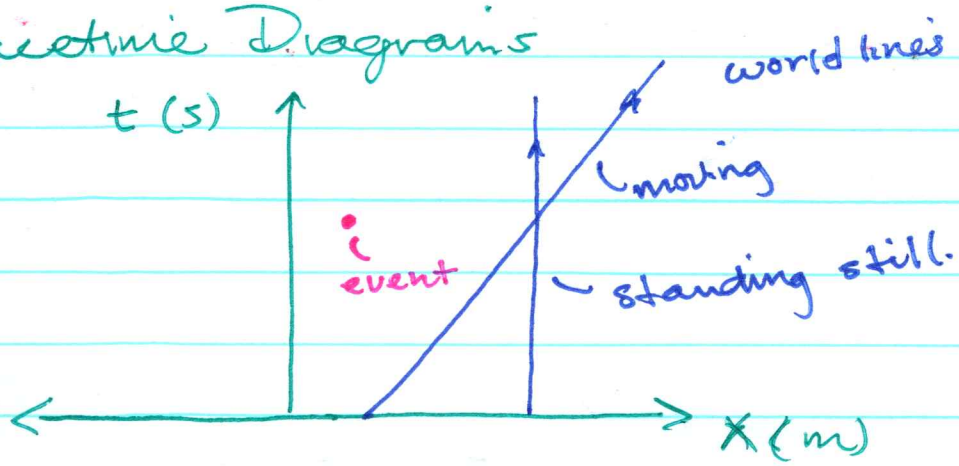
we have two groups of surveyors
One measures north and east relative to sun and other measures north and east by compass



Conclusions:

(1) use the same units (km or mi)
(2) distances are invariant
(3) convert between surveyors using rotations

Spacetime Diagrams



dilema: we need to choose common units in order for "distance" to make sense.

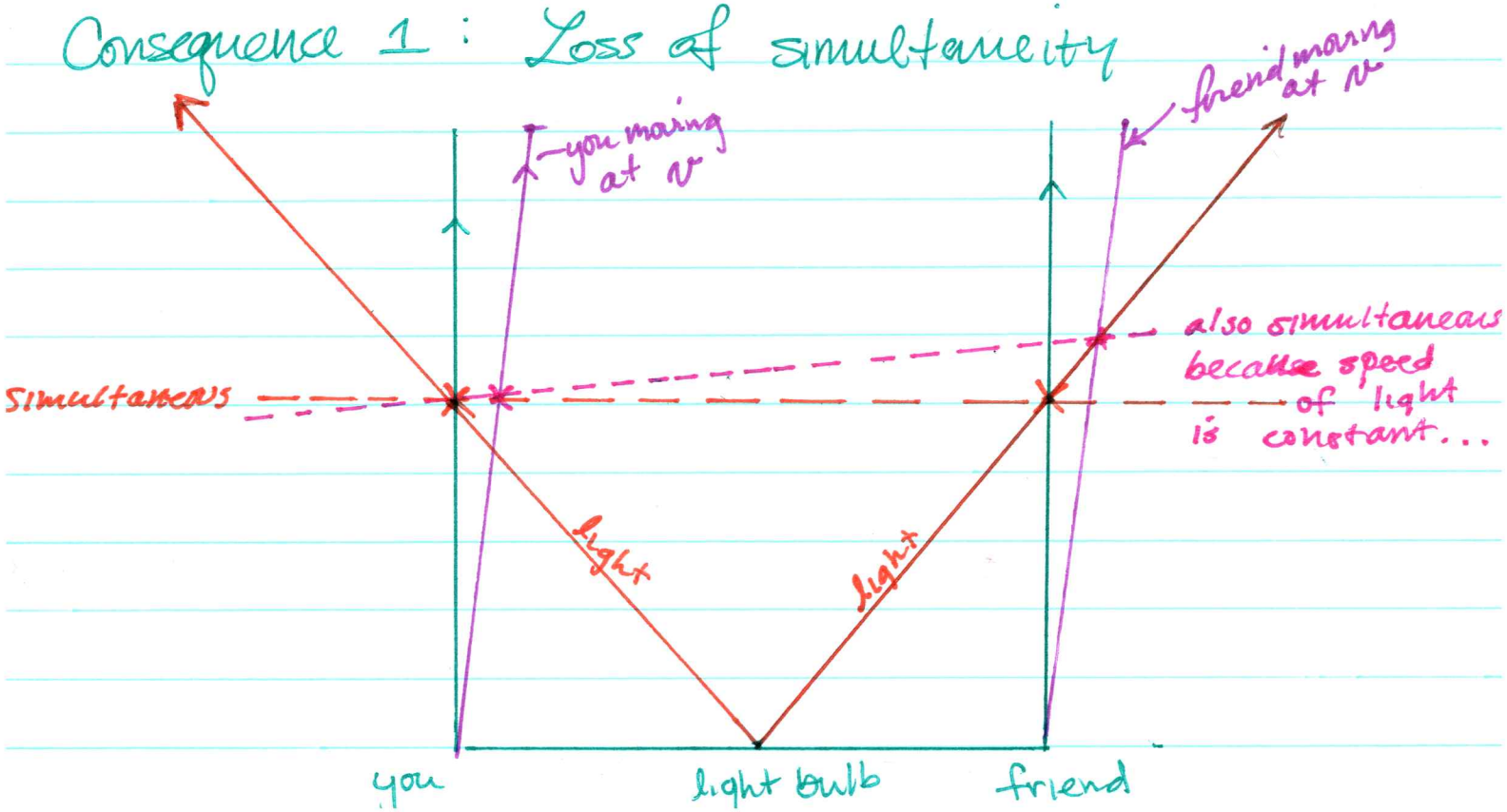
Standard fix: $t \rightarrow ct$ (meters)

"measure time as a distance"

$\boxed{c = 3.0 \times 10^{10} \text{ cm/s}}$ ⇝ speed of light

Postulate: It is an observational fact that the speed of light is the same in all inertial reference frames.

Consequence 1: Loss of simultaneity



2. Inner products in $\mathbb{M}^2$

metric: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

$\longrightarrow \quad \hat{t}\cdot\hat{t} = -1 \quad , \quad \hat{x}\cdot\hat{x} = 1 \quad , \ldots$

Terminology

Let $\vec{u}, \vec{v} \in \mathbb{M}^2$, then

$\vec{u}\cdot\vec{u} > 0 \iff$ spacelike

$\vec{u}\cdot\vec{u} < 0 \iff$ timelike

$\vec{u}\cdot\vec{u} = 0 \iff$ light like (null) assuming $\vec{u} \neq \vec{0}$.