# Measuring Plank's Constant: Lab 1

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## 1 Introduction

Plank's constant is a fundamental physical number that appears all throughout physics, particularly in quantum mechanics. For the purposes of this experiment, this number serves as the proportionality constant for the equation relating the energy of a photon of light to its energy:

$$E = \frac{hc}{\lambda}$$

Here, c is the speed of light which is approximately $3.0 \cdot 10^8$ m/s.[1]

In order to determine this constant, two experiments were performed. The first measured the wavelength of each of the three colors of an LED using a simple interference apparatus. The second test analyzed the LED circuit in order to determine the minimum energy that can produce photons- essentially the energy per photon. The results of these two experiments together allow us determine the value of Plank's constant quantitatively by the relation above.

The first key concept underpinning this lab is the notion of interference. When sent through a number of slits whose width is within a few orders of magnitude of its wavelength, light behaves in a fashion characteristic of the classical wave model. Figure one below shows a diagram of the paths a monochromatic light source takes through a multiple slit diffraction grating.

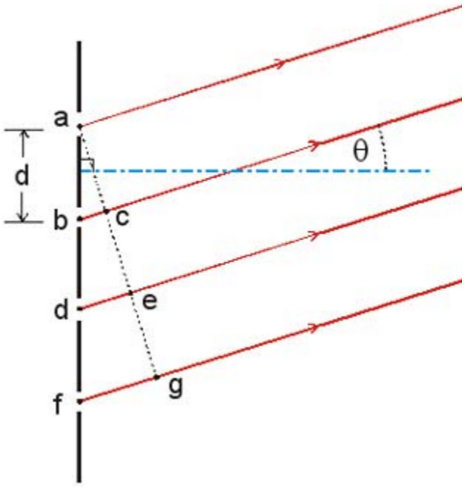



*Figure 1:* Light traveling through a diffraction grating[2]

For each wavelength there exists a special angle *theta* such that the path of light traveling through each slit differs by an integer multiple of $\lambda$. This means that light traveling through the grating experiences constructive interference- appearing brighter as the beam reaches its destination in phase. Likewise for any angle not equal to our special angle $\theta$, differences in path lengths increase until there exists a path of light traveling through another slit that is half a wavelength longer or shorter. This in turn means that light diffracted at these angles reaches its destination out of phase and so deconstructively interferes resulting in no visible light. This special angle applies only if the distance between slits is much smaller than the distance between the grating and the beam's measurement. If this condition is met, the angle can be directly from the triangular geometry shown in figure one and given in the following relation:

$$d \sin \theta = \lambda$$

The second key concept is the idea of the Energy per photon and it's relation to minimum voltage required to turn on an LED. Recall that the energy of a charge carrier is given by:

$$U_{elec} = q \cdot V$$

Where q is the charge of the carrier and V is the potential applied to it. This experiment relies on the idea that an *ideal* LED converts 100 percent of the electrical energy to light. For our purposes $q = e$, the charge of an electron which is approximately $1.6 \cdot 10^{-}19$ Coulombs. The potential also corresponds to the potential difference across the leads of the LED thus at the right current, we can theoretically calculate the energy per photon using this equation.

The following report describes the methods by which the wavelengths and energies were determined as well as how those values are used to determine Plank's constant.

# 2 Procedure

The following sections describe the methods employed in each part of the lab. The goal of the first part was to characterize the wavelength of each color of the RGB LED. The second experiment aimed to find the energy of each photon emitted.

## 2.1 Measuring $\lambda$

A three color RGB LED was used as a variable light source with a potentiometer to control the intensity output. Figure 2 below shows the circuit diagram used to power the LED.

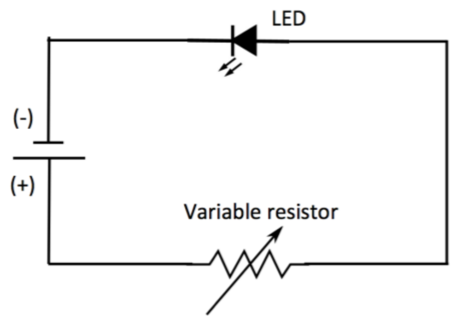



*Figure 2:* LED circuit diagram [3]

The circuit was placed on a large whiteboard. Next to the LED, a large slit was connected to the circuit via a rubber band in order to help collimate the light. A straight line was then drawn with a ruler to denote the path of the light. At its end another line was drawn orthogonal to the first. At the intersection was placed a small diffraction grating.

To take data, first the color of the LED was selected by connecting power to the appropriate leg of the LED. Next, we looked through the diffraction back towards the LED. Sweeping from the LED out to one side (left or right), we looked until a small visible dot of reflection could be found. This location was marked on the whiteboard so that a line could be drawn to the intersection of the two lines previously marked on the board. Then, this third line was connected to that coming from the LED in order to form a right triangle.

The value of the height and length of this triangle were recorded so that theta could be determined using the trigonometric identity:

$$\theta = \arctan \frac{h}{d}$$

Where d is the length and h is the height. The lines were then redrawn and the measurement was made once more so that 5 data points were obtained. After, the color of the LED was changed and the process was repeated. Careful efforts were taken to insure that the device was off between trials to protect the LED and insure battery life.

## 2.2 Determining $E_{photon}$

For the second experiment, the same circuit was built however a voltmeter was connected in parallel to the LED and the ammeter was placed in series as shown below in figure 3.

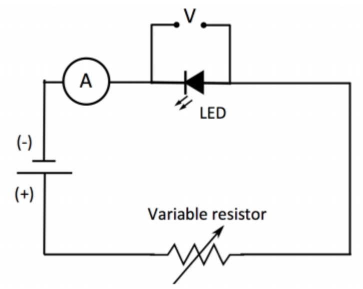



*Figure 3:* LED circuit diagram [4]

Using this design, the potential difference across the LED was measured for each color at the currents: 2mA, 3mA, 5mA, and 9mA. These results were then plotted in a Current vs Voltage graph and fitted with a linear trend line so that the minimum voltage (the x intercept of the graph) could be determined. This voltge was then used with the equation shown in the introduction, $E = e\cdot$ in order to determine the photon energy for each wavelength.

# 3 Data

Table one below shows the interference data used to determine the wavelength of each color of LED light. The units for the length measurements are in centimeters and the angles are measured in degress as opposed to radians.

| Blue | | | Green | | | Red | | |
|---|---|---|---|---|---|---|---|---|
| *h* | *d* | *theta* | *h* | *d* | *theta* | *h* | *d* | *theta* |
| 15.5 | 29.5 | 27.71850163 | 13.5 | 23.5 | 29.87599269 | 12.5 | 15 | 39.80557109 |
| 16 | 24.5 | 33.14699583 | 12 | 19 | 32.27564431 | 13 | 15 | 40.91438322 |
| 11 | 20 | 28.81079374 | 12.5 | 20.5 | 31.37300514 | 13 | 15.5 | 39.98688624 |
| 11 | 21 | 27.64597536 | 12.2 | 20 | 31.38319106 | 18 | 20 | 41.9872125 |
| 10 | 18.5 | 28.39301942 | 13.5 | 22.5 | 30.96375653 | 20 | 25.5 | 38.10757688 |
| | **Average** | **29.1430572** | | **Average** | **31.17431795** | | **Average** | **40.16032599** |
| | **STDEV** | **2.289928648** | | **STDEV** | **0.869771367** | | **STDEV** | **1.001433021** |

*Table 1:* Part 1 raw measurements

As mentioned in the previous section, data for the height and length of the triangle formed by the reflected LED light were recorded and the angle $\theta$ was calculated using the *arctan* function. Now that the angles corresponding to each color of light have been determined, calculating the wavelength of each color is accomplished using the second equation from the introduction. The following tables list the determined wavelengths in meters and nanometers. below I have also included a low bound and high bound by using the standard deviation with angle. I feel like this is a good representation for the potential spread of data as one standard deviation covers almost 70 percent of a distribution. The slit distance used for the calculation was determined from the fact that the diffraction grating was listed as 1000 slits per mm giving a slit width of $1.0 \cdot 10^{-}6$ m.

| | Blue | | Green | | Red | |
|---|---|---|---|---|---|---|
| | (m) | (nm) | (m) | (nm) | (m) | (nm) |
| *Lambda* | 4.87E-07 | 487.0 | 5.18E-07 | 517.6 | 6.45E-07 | 644.9 |
| *Upper bound* | 5.22E-07 | 521.5 | 5.31E-07 | 530.6 | 6.58E-07 | 658.2 |
| *Lower bound* | 4.52E-07 | 451.7 | 5.05E-07 | 504.6 | 6.31E-07 | 631.5 |

*Table 2:* Part 1 wavelength data

For part 2 of the lab, data was collected for the voltage across the leads of the LED for various currents ranging from 2mA to 9mA. Table 3 below lists the data collected.

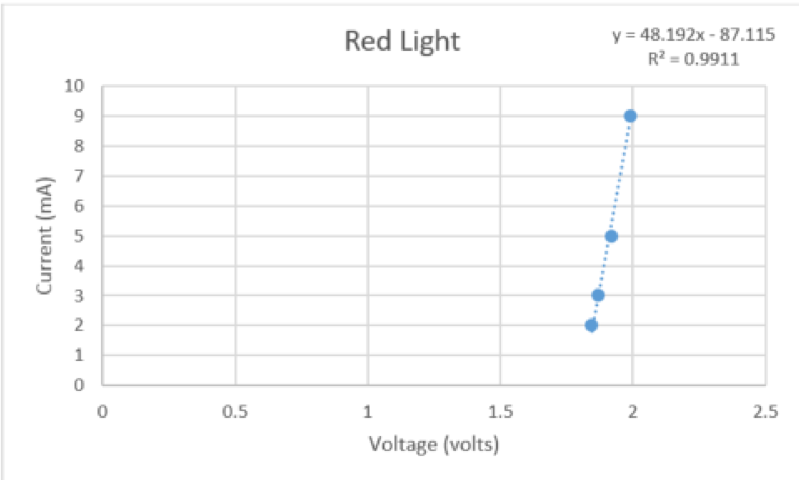
| ***Red*** | |
|---|---|
| *V* | *I (mA)* |
| 1.845 | 2 |
| 1.87 | 3 |
| 1.92 | 5 |
| 1.99 | 9 |

| ***Blue*** | |
|---|---|
| *V* | *I (mA)* |
| 2.68 | 2 |
| 2.72 | 3 |
| 2.8 | 5 |
| 2.91 | 9 |

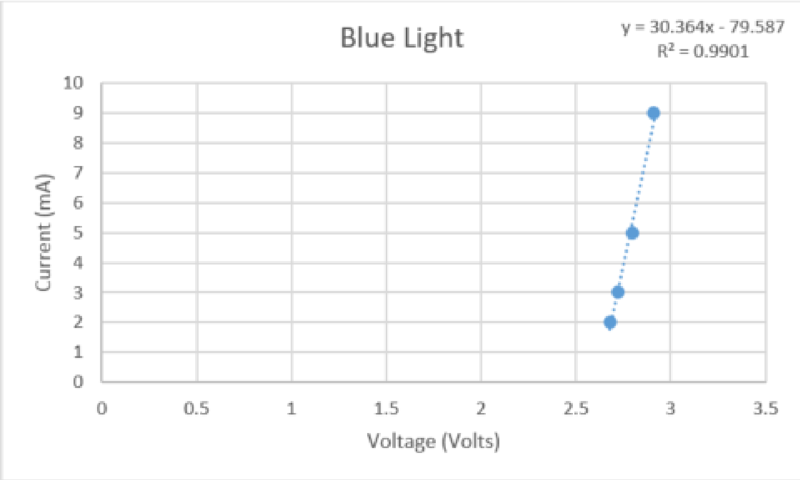
| ***Green*** | |
|---|---|
| *V* | *I (mA)* |
| 2.42 | 2 |
| 2.48 | 3 |
| 2.57 | 5 |
| 2.71 | 9 |

*Table 3:* Part 2 Voltage data

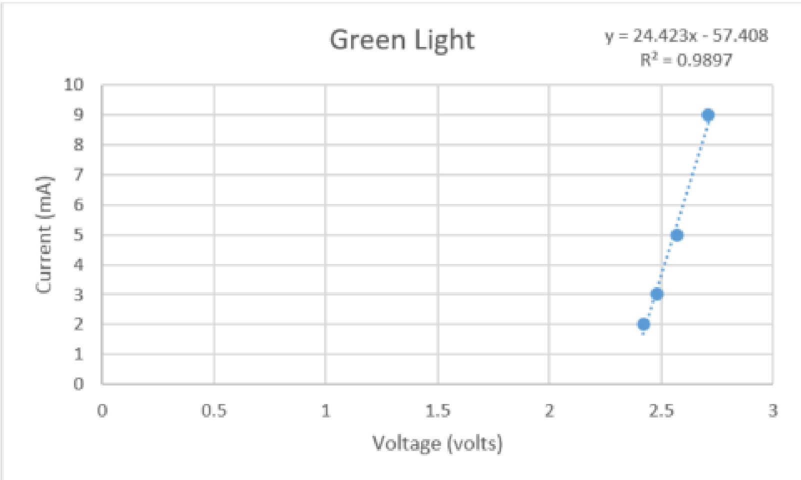
Graphs of the data are shown below. Each was fitted with a linear trend line in order to solve for the x-intercept minimum voltage. $R^2$ values have been included to give a sense of how good the fit is.




*Graph 1:* Voltage vs Current - Red Light




*Graph 2:* Voltage vs Current - Blue Light




*Graph 3:* Voltage vs Current - Green Light

# 4 Analyses and Results

From the best fit lines calculated for the graphs in the previous section, the minimum voltage can be determined by solving for the x intercept. The following is a sample calculation showing how the min voltage was calculated for the red light data.

$$\begin{aligned} f(x) &= 48.2x - 87.1 \\ 0 &= 48.2x - 87.1 \\ x &= \frac{87.1}{48.2} \approx 1.08 \quad volts \end{aligned}$$

The min voltages for the blue and green lights were 2.61 and 2.35 volts respectively. The uncertainty in these voltages is $\pm 0.01$ volts as the Voltmeter was analog and this is the smallest unit it reported to. Multiplying these voltages by the charge of an electron $e^-$ gives the energy per photon. Here we use the value $1.6 \cdot 10^{-19}$ Coulombs for charge giving: $1.73E - 19$, $4.18E - 19$, and $3.77E - 19$ Joules. Combining these values with the wavelengths determined for red, blue, and green light determined in part one of the lab we can determine Plank's constant using the first equation listed in the introduction:

$$E_{photon} = \frac{hc}{\lambda}$$

where c is the speed of light – roughly $3.0 \cdot 10^8$ m/s. Below is an example calculation for the constant using red wavelength.

$$\begin{aligned} E_{photon} &= \frac{hc}{\lambda_r} \\ h &= \frac{\lambda_r \cdot E_{photon}}{c} \\ h &= \frac{6.45 \cdot 10^{-7} \cdot 1.73 \cdot 10^{-19}}{3.0 \cdot 10^8} \\ h &\approx 3.72 \cdot 10^{-34} \quad Js \end{aligned}$$

Here, the order of magnitude is correct but the value is off by a factor of approximately 2. Following the same process for blue and green light resulted in values of $6.78 \cdot 10^{-34} \quad Js$, and $6.51 \cdot 10^{-34} \quad Js$.

Sources of error in this experiment are attributable to the difficulty of accurately measuring the diffraction peak because the experimental setup required that we look through the small grating and then mark on the white board using a dry erase pen. We tried to mitigate this error by keeping the distance from the LED to the grating large. This should help make measurements more accurate as the longer distance to the LED makes for a better small angle approximation. One problem that arose from this as seen by the value obtained using red light was that this distance dramatically reduced the intensity of light visible through the grating.

# 5 Discussion

Using the values measured for wavelength and photon energy Plank's constant was determined to the correct order of magnitude. The three values were $6.78 \cdot 10^{-34} \quad Js$ for blue light, $6.51 \cdot 10^{-34} \quad Js$ for green light, and $3.72 \cdot 10^{-34} \quad Js$ for red. The first two values are quite close to the reported value of $6.626 \cdot 10^{-34}$ while the red is off by a factor of two that we attributed in the previous section to poor intensity. Some other difficulty comes in the fact that the current voltage relationship for an LED is non-ohmic meaning that for larger currents, the trend is definitely *not* linear. Because the trend fit was for all of the data points, the x intercept (interpreted as the photon energy) could have been slightly shifted.

If this experiment were to be repeated, it would be interesting try some new wavelengths of light. I am not sure whether this could be accomplished by using three potentiometers and the same LED (this would just be a super position of red green and blue) or if a separate type of LED would be needed altogether. It would also be helpful to build a track or enclosure for the LED so that that more light makes it to the grating. Securing the grating in some kind of base might also make taking data simpler because there was some difficulty in holding the grating steady and leaving room for an observer to look through and take measurements.

Overall it is extremely exciting that Plank's constant could be determined to the correct order with simple tools like an LED and a whiteboard.

# 6 References


1. https://www.britannica.com/science/Plancks-constant

2. https://sites.ualberta.ca/~pogosyan/teaching/PHYS_130/FALL_2010/lect

3. http://physics.oregonstate.edu/~minote/COURSES/ph315 /lib/exe/fetch.php?media=part_1_of_lab.pdf

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