Thermal Physics - PH441 **John Waczak**
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More On Density Of States

Let's derive the Fermi-energy. Last time we had

$$N = \frac{V}{2\pi^2}\Big(\frac{2m}{\hbar^2}\Big)^{3/2}\frac{2}{3}\varepsilon_f^{3/2}$$

Using this we can solve for the Fermi energy to find:

$$\varepsilon_f = \frac{\hbar^2}{2m}\Big(\frac{N2\pi^2}{V}\Big)^{2/3}$$

and so we can simplify our density of states to arrive at

$$D(\varepsilon) = \frac{3}{2}N\frac{\varepsilon^{1/2}}{\varepsilon_f^{3/2}}$$

Now we can use ε_f to define a whole bunch of *Fermi* things.

$$\begin{aligned}
k_f &= \Big(\frac{N}{V}2\pi^2\Big)^{1/3} \\
p_f &= \hbar k_f \\
v_f &= \frac{p_f}{m} \\
T_f &= \frac{\varepsilon_f}{k_B}
\end{aligned}$$

Example

Solve for U of a Fermi-gas at zero-Temperature:

$$U = \int_0^\infty D(\varepsilon)\varepsilon f(\epsilon) d\varepsilon$$

Where $D(\varepsilon)$ tells us how many orbitals there are at an energy ε per unit energy and $f(\varepsilon)$ is the Fermi-Dirac distribution function that tells us what fraction of orbitals are occupied at a given ε. At zero temperature $f(\varepsilon)$ becomes a step function that turns off at ε_f. Thus we have

$$\begin{aligned}
U_{T=0} &= \int_0^{\varepsilon_f} D(\varepsilon)\varepsilon d\varepsilon \\
&= \frac{3}{2}N\int_0^{\varepsilon_f}\frac{\varepsilon^{3/2}}{\varepsilon_f^{3/2}}d\varepsilon \\
&= \frac{3}{5}\varepsilon_f
\end{aligned}$$

Example

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How would you find C_v?

$$C_v = \left(\frac{\partial U}{\partial T}\right)_{V,N}$$
$$U = \int_0^\infty D(\varepsilon) f(\varepsilon) \varepsilon d\varepsilon$$

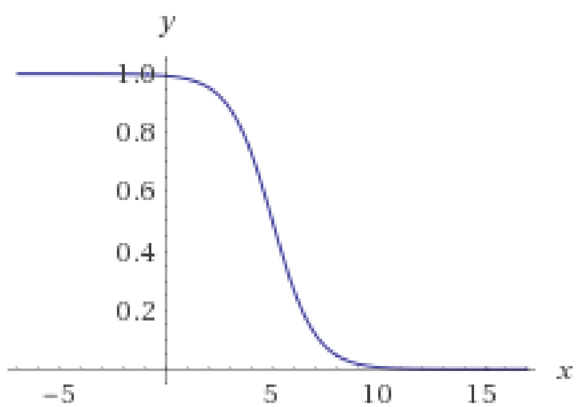

Figure 1: The shape of the Fermi-Dirac distribution