Physics 315: Homework 4 Solutions

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1. Preparing for the term paper

Propose a topic for your term paper. Suggest one graph and one original calculation that would satisfy the assignment guidelines.

I'd like to analyze the contributions of CO2 increases in atmosphere to the climate versus those of methane specifically taking note of:

- trapping efficiency
- atmospheric lifetime
- human / human related production

Calculations: The total energy reflected back to the Earth's surface by 1 kilogram of CO2 in its airborne lifetime vs. the same for methane. Graph: Pie charts of distribution of green house gas emissions as well as a graph of the total intensity reflected as a function of mass of the green house gas (starting with one kg and ending at the end of the element's lifetime). I want to know regardless of how much we currently emit what

2. Heat conductivity

Using technically accurate physics language, describe why a wood block at 15 deg C feels warmer to the human touch than a metal block at 15 deg C. Note that the human body temperature is 37 deg C.

Object	thermal conductivity
Aluminum	205.0
Wood	0.12 – 0.04

Data from hyperphysics.com

Because each block is colder than your body, in both cases heat transfers from you to the block. The two feel different though because the rate at which they allow heat to transfer (thermal conductivity) is different.

Aluminum metal for example has a conductivity that is on the order of 2000 times larger than wood's so the temperature of your hand and that of the metal equilibrate *much* faster than your hand and the wood. Given enough time your hand and each block would all reach the same temperature. It just happens much faster for the aluminum so you perceive the block as colder.

3. The ideal gas law

Use the equipartition theorem to derive the ideal gas law.

From the posted video we can conclude the following about a system of N particles confined to a box of volume V:

$$\begin{aligned}
\Delta p_{particle} &= 2 \cdot m \cdot v_z \\
N_{collisions} &= \frac{1}{2}\frac{N}{V} \cdot v_z \cdot \Delta t A \\
F_{net} &= \frac{N_{collisions} \cdot \Delta p_{particle}}{\Delta t} \\
&= \frac{\frac{1}{2}\frac{N}{V} \cdot v_z \cdot \Delta t A \cdot 2m \cdot v_z}{\Delta t} \\
P &= \frac{F_{net}}{A} \\
P &= \frac{N}{V} \cdot m v_z^2 \\
PV &= N m v_z^2
\end{aligned}$$

Applying the equipartition theorem for the velocity degree of freedom gives the final solution:

$$\begin{aligned}
\frac{1}{2} m v_z^2 &= \frac{1}{2} K_B T \\
PV &= 2 \cdot N \frac{1}{2} K_B T \\
&= N K_B T
\end{aligned}$$

4. Changes in pressure and volume

(a) a monatomic ideal gas with initial pressure of 80 kPa is confined to a cylinder with a volume of 600 cm3. What is the pressure now?

Consider the ideal gas law: $PV = NK_B T$. A simple rearrangement gives $\frac{PV}{T} = NK_B =$ constant. Thus because the equation is constant for a given system, we can conclude with the ideal gas law:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Because Temperature remains constant during the process, we can drop the T's in the equation giving:

$$\begin{aligned} P_1 V_1 &= P_2 T_2 \\ 80 \cdot 600 &= P_2 \cdot 450 \\ P_2 &= \frac{80 \cdot 600}{450} \\ &\approx 107 \quad kPa \end{aligned}$$

(b) A diatomic ideal gas with an initial pressure of 60 kPa is confined to a cylinder with a volume of 600 cm3. We then compress the gas adiabatically until its volume has decreased to 450 cm3. What is it's pressure now?

Because the gas is ideal, its adiabatic expansion is governed by the law $PV^\gamma = constant$ where γ depends on the degrees of freedom and is given by $\gamma = \frac{f+2}{f} \approx 1.4$. Thus we have:

$$\begin{aligned} PV^\gamma &= constant \\ P_1 V_1^\gamma &= P_2 V_2^\gamma \\ P_2 &= \frac{P_1 V_1^\gamma}{V_2^\gamma} \\ P_2 &= \frac{60 \cdot 600^{1.4}}{450^{1.4}} \\ P_2 &= 89.8 \quad kPa \end{aligned}$$

(c) An ideal gas is constrained to follow the three step cyclic process shown in the graph. Specify the sign of Q, W, and ΔU for each step of the process. What is the net work flowing into or our of the gas for the entire cyclic process?

	Q	W	ΔU
A-B	$+$	$-$	$+$
B-C	$+$	0	$+$
C-A	$-$	$+$	0

The values for the sign of the work in the above table were determined in the following section by integrating the curves shown in the graph. Then using the rule for an ideal gas that $\Delta U = \frac{f}{2} N K_B \Delta T$, I reasoned out the sign of the Internal energy based on what was happening during each process. From A-B Pressure is constant so by the combined gas law, if V increases then T must also increase. Similarly, for B to C, since pressure increases at constant V, T must also increase. For C to A however we Have that the decrease in V is proportional to the decrease in P so we would not expect a Temperature change. Once I had figured out the second two columns, I simply deduced the required sign of Q for each process.

To solve for the work done on the system, we use the relation that $W = -\int pdV$ and so because the system is represented by a simple triangle, we can use geometry to calculate the work done during each step then sum them up.

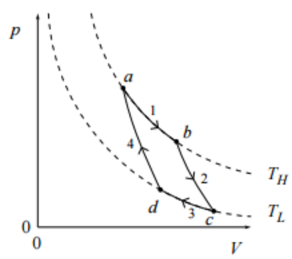
$$\begin{aligned} W_{A-B} &= -20 \cdot 2 = -40 \quad kJ \\ W_{B-C} &= 0kJ \\ W_{C-A} &= \frac{1}{2}20 \cdot 2 + 20 \cdot 2 = 60 \quad kJ \\ \sum W &= -40 + 0 + 60 = 20 \quad kJ \end{aligned}$$

Because the work is done ON the gas, a positive sum means work is flowing into the system. This makes sense as From A - C the gas expands and pressurizes, doing work, but then more work is required to bring the system back to state A.

5. Carnot Efficiency

In class we calculated the coefficient of performance of a heat pump that utilizes the carnot cycle. Do a similar analysis to find the efficiency of an engine that utilizes the Carnot cycle. Express the efficiency in terms of Th and Tc

Recall that the Carnot cycle, the most efficient heat engine, is a cycle consisting of two isotherms and two adiabatic curves.



The P-V diagram for a Carnot Engine

We will need 3 important concepts to solve this problem. The first is that for an isothermal process $\Delta T = 0$ and so $\Delta U = 0$. Thus from the second law, $Q = -W$. The next important idea is that for an adiabatic process $\Delta Q = 0$. Finally, we will need the equation for efficiency which is given as: $\eta = 1 - \frac{Q_c}{Q_h}$.

From the PV diagram, we know then that we only need to consider the two isotherms a-b and c-d as the adiabatic curves do not contribute to any heat exchange.

$$\begin{aligned}
Q_{a-b} &= -W_{a-b} = \int pdV \\
&= \int_{V_a}^{V_b} \frac{NKT_h}{V} dV \\
&= NKT_h \cdot ln(\frac{V_a}{V_b}) \\
Q_{c-d} &= -W_{c-d} = \int pdV \\
&= \int_{V_c}^{V_d} \frac{NKT_c}{V} dV \\
&= -\int_{V_c}^{V_d} \frac{NKT_c}{V} dV \\
&= -NKT_c \cdot ln(\frac{V_c}{V_d}) \\
\eta &= 1 - \frac{NKT_c \cdot ln(\frac{V_c}{V_d})}{NKT_h \cdot ln(\frac{V_a}{V_b})}
\end{aligned}$$

In class we said: $\frac{V_a}{V_b} = \frac{V_c}{V_d}$ so:

$$\eta = 1 - \frac{T_c}{T_h} = \frac{T_h - T_c}{T_h} = \frac{\Delta T}{T_h}$$

6. Formula 1 racing innovation

The latest Prius hybrids have a 40 percent efficient combustion engine (better than typical older cars). Formula 1 race cars are pushing the combustion engine efficiency even further. The current record (2016) for the Mercedes F1 race car is 47 percent. Assuming that the air in the cylinder of the engine is initially at 20 C, estimate the maximum temperature reached by the air.

Assuming that the process for the Prius's engine is reversible, we can apply the Carnot Efficiency which is in terms of the Temperatures:

$$\begin{aligned}
\eta &= 1 - \frac{T_c}{T_h} \\
0.47 &= 1 - \frac{20}{T_h} \\
T_h &= \frac{-293}{0.47 - 1} = 552.8K \approx 279.8 \quad C
\end{aligned}$$

7. Electricity generation and heat pumps

Electrical energy (and heat) is output from a coal-burning power station. The electricity is made by heating/cooling a working gas using $T_h = 200$ C and $T_c = 5$ C. Te electricity is sent to nearby buildings where it is used to run high-performance heat pumps. The outside air temp is 5 c. The temp inside the buildings is 20C. Assume that both the power station and the heat pump are operating at the thermodynamic efficiency limit.

(a) calculate the ratio of heat energy pumped into the buildings to the chemical energy consumed by the power plant

(b) if our goal is to minimize co2 emissions, is it better to burn the coal in our houses, or burn the coal in power stations and use an ideal heat pump?

(a). Recall that the thermodynamic efficiency limit is given by: $\eta = 1 - \frac{T_c}{T_h}$. and is the ratio of $\frac{Work}{E_{coal}}$ where E_{coal} is chemical energy consumed by the power plant. Similarly for a heat pump, a useful quantity dubbed the coefficient of performance (COF) is given by: $COF = \frac{T_h}{T_h - T_c}$ and is the ratio of heat energy pumped out to the work required by the pump: $\frac{Q_p}{Work}$ Thus the desired ratio $\frac{Q_p}{E_{coal}}$ is given by: (making sure to convert to Kelvin)

$$
\begin{aligned}
\frac{Q_p}{E_{coal}} &= \eta_{coal} \cdot CoF_{pumps} \\
&= (1 - \frac{T_c}{T_h})(\frac{T_h}{T_h - T_c}) \\
&= (1 - \frac{278}{473})(\frac{293}{15}) \\
&\approx 8.05
\end{aligned}
$$

(b) it appears that it would be better to pump energy to our houses because this ratio is positive. This means that burning coal all at once and then using that energy in homes to cool buildings gains you a factor of 8. I would imagine then that burning the coal and using that energy to directly cool houses would be more inefficient.