Heat capacity for Fermi-gas

Recall that the heat capacity is given by $C_V = \left(\frac{\partial U}{\partial T}\right)_V$. Thus we can find this using our density of states

$$
\begin{aligned}
U &= \int D(\varepsilon)\varepsilon f(\varepsilon) d\varepsilon \\
\Rightarrow C_V &= \int D(\varepsilon)\varepsilon\Big(\frac{\partial f(\varepsilon)}{\partial T}\Big)_\varepsilon d\varepsilon \\
f(\varepsilon) &= \frac{1}{e^(\varepsilon - \mu)/kT + 1} \\
\frac{\partial f}{\partial T} &= \frac{-e^{\beta(\varepsilon-\mu)}}{(e^{\beta(\varepsilon-\mu)}+1)^2}\Big[\frac{(\varepsilon-\mu)}{kT^2} - \beta\frac{\partial\mu}{\partial T}\Big]
\end{aligned}
$$

where the $\partial_T\mu$ term goes to zero if we assume that μ isn't changing much with temperature. Thus we have $\mu = \varepsilon_f$ as well. So,

$$
\partial_T f(\varepsilon) = \frac{1}{\Big(e^{\beta(\varepsilon-\varepsilon_f)}+1\Big)\Big(e^{-\beta(\varepsilon-\varepsilon_f)}+1\Big)}\frac{\varepsilon-\varepsilon_f}{kT^2}
$$