$$\mathcal{D}_z^{1/2}(\phi) = \lim_{n\to\infty}\left[1 - i\frac{S_z}{\hbar}\frac{\phi}{n}\right]^n$$

$$= e^{-iS_z\phi/\hbar}$$

$$\begin{aligned} S_x &= \frac{\hbar}{2}\Big\{|+\rangle\langle-| + |-\rangle\langle+|\Big\} \\ S_y &= \frac{\hbar i}{2}\Big\{-|+\rangle\langle-| + |-\rangle\langle+|\Big\} \\ S_z &= \frac{\hbar}{2}\Big\{|+\rangle\langle+| - |-\rangle\langle-|\Big\} \end{aligned} \tag{1}$$

$$A\,|a\rangle = a\,|a\rangle \Rightarrow f(A)\,|a\rangle = f(a)\,|a\rangle$$

$$\begin{aligned} \langle S_x\rangle_{\text{new}} &= \cos\phi\,\langle S_x\rangle - \sin\phi\,\langle S_y\rangle \\ \langle S_y\rangle_{\text{new}} &= \cos\phi\,\langle S_y\rangle + \sin\phi\,\langle S_x\rangle \end{aligned}$$

$$\begin{pmatrix}\langle S_x\rangle_{\text{new}} \\ \langle S_y\rangle_{\text{new}}\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\langle S_x\rangle \\ \langle S_y\rangle\end{pmatrix} \tag{2}$$

$$\mathcal{D}_z^{1/2}(\phi)\,|\alpha\rangle = \mathcal{D}_z^{1/2}(\phi)\Big(\langle+|\alpha\rangle\,|+\rangle + \langle-|\alpha\rangle\,|-\rangle\Big) \tag{3}$$

$$= \langle+|\alpha\rangle\,e^{-iS_z\phi/\hbar}\,|+\rangle + \langle-|\alpha\rangle\,e^{-iS_z\phi/\hbar}\,|-\rangle \tag{4}$$

$$= e^{-i\phi/2}\,\langle+|\alpha\rangle + e^{i\phi/2}\,\langle-|\alpha\rangle \tag{5}$$

$$\mathcal{D}_z^{1/2}(2\pi)\,|\alpha\rangle = -\,|\alpha\rangle \tag{6}$$

$$\mathcal{U}(t,0) = e^{-iHt/\hbar} \tag{7}$$

$$H = -\left(\frac{e}{m_e c}\right)\mathbf{S}\cdot\mathbf{B} \equiv \omega S_z \tag{8}$$

$$\mathcal{U}(t,0) = e^{-iS_z\omega t/\hbar} \tag{9}$$