Kerr Geometry

Initialization

Run the following code to initialize $\LaTeX$ output and load (Tevian's frontend to) the differential forms package, either by clicking on "Evaluate" or by typing Shift+Enter.

```
import urllib
url="http://oregonstate.edu/~drayt/MTH437/handouts/einstein.txt"
exec(eval(urllib.urlopen(url).read()))
Parallelism().set(nproc=8)
```

Evaluate

```
Initialization loaded
```

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Code

Now enter a line element in the box below, adapting the given code as needed. First declare any parameters or functions, then provide a list of coordinates using the `MakeC` command as shown below (note the double parentheses). Finally, set the (covariant) metric to the (inverse of the) matrix relating the coordinate and orthonormal 1-form bases. The result should be the line element in tensor notation.

```
m,w=var('m,omega')
MakeC(('t','x','y','z'))
Q=M.automorphism_field()
#Qinv=matrix([[sqrt(Delta)/rho,0,0,sqrt(Delta)/rho*a*sin(theta)^2],[0,rho/sqrt(Delta),0,0],[0,0,rho,0],[a*sin(theta)/rho,0,0,sin(theta
Qinv = matrix([[(1/(sqrt(2)*w)),0,(1/(sqrt(2)*w))*exp(x),0],[0, (1/(sqrt(2)*w)),0,0],[0,0,(1/(2*w))*exp(x),0],[0,0,0,(1/(sqrt(2)*w))]]
Q[:]=Qinv.inverse()
e=XX.frame().new_frame(Q,'e')
M.set_default_frame(e)
g=M.metric('g',M._dim-2)
g[1,1],g[2,2],g[3,3],g[4,4]=-1,1,1,1
g.display(XX.frame())
```

Evaluate

$$g = -\frac{1}{2\,\omega^2}\mathrm{d}t\otimes\mathrm{d}t - \frac{e^x}{2\,\omega^2}\mathrm{d}t\otimes\mathrm{d}y + \frac{1}{2\,\omega^2}\mathrm{d}x\otimes\mathrm{d}x - \frac{e^x}{2\,\omega^2}\mathrm{d}y\otimes\mathrm{d}t - \frac{e^{(2\,x)}}{4\,\omega^2}\mathrm{d}y\otimes\mathrm{d}y + \frac{1}{2\,\omega^2}\mathrm{d}z\otimes\mathrm{d}z$$

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List the nonzero components of the (covariant) metric tensor (in a coordinate basis!).

```
g.display_comp(XX.frame())
```

Evaluate

$$
\begin{aligned}
g_{t\,t} &= -\frac{1}{2\,\omega^2} \\
g_{t\,y} &= -\frac{e^x}{2\,\omega^2} \\
g_{x\,x} &= \frac{1}{2\,\omega^2} \\
g_{y\,t} &= -\frac{e^x}{2\,\omega^2} \\
g_{y\,y} &= -\frac{e^{(2\,x)}}{4\,\omega^2} \\
g_{z\,z} &= \frac{1}{2\,\omega^2}
\end{aligned}
$$

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List the nonzero Christoffel symbols (in an orthonormal frame).

```
nab=g.connection()
nab.display()
```

Evaluate

$$
\begin{aligned}
\Gamma^{1}{}_{2\,3} &= \omega \\
\Gamma^{1}{}_{3\,2} &= -\omega \\
\Gamma^{2}{}_{1\,3} &= \omega \\
\Gamma^{2}{}_{3\,1} &= \omega \\
\Gamma^{2}{}_{3\,3} &= -\sqrt{2}\omega \\
\Gamma^{3}{}_{1\,2} &= -\omega \\
\Gamma^{3}{}_{2\,1} &= -\omega \\
\Gamma^{3}{}_{2\,3} &= \sqrt{2}\omega
\end{aligned}
$$

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Compute and display the components of the Ricci tensor R_{ij}. The Kerr geometry is a vacuum solution of Einstein's equation!

```
ric=g.ricci()
ric[:]
```

Evaluate

$$
\begin{pmatrix} 2\,\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

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Enter any further code you wish below.

```

```

Evaluate