

Simplifying the Radial Equation

In the following activity, we will simplify the radial orbit equation so that we can solve for the shape of the orbits.

Currently, the radial equation is

$$\mu\ddot{r} = -\frac{d}{dr}V(r) + \frac{\ell^2}{\mu r^3} \quad (1)$$

1. Simplify $\dot{\phi} \frac{d}{d\phi}$
2. Using this expression, rewrite \dot{r} as a derivative with respect to ϕ .
3. Repeat the process, and re-express \ddot{r} as derivatives with respect to ϕ
4. Perform the variable substitution $r = \frac{1}{u}$ to re-expression \dot{r} and \ddot{r}

5. Using $r = \frac{1}{u}$, find $\frac{d}{dr}$ in terms of u .
6. Use your expressions for \ddot{r} and $\frac{d}{dr}$ to re-express the radial equation in terms of $u(\phi)$
7. Assuming a potential of the form $V(r) = -\frac{k}{r}$, solve your differential equation to find $u(\phi)$
8. Re-express your solution in terms of $r(\phi)$