we know $L_A$ is defined by $A \subset O(3)$ thus we have $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ s.t.

$A^T = A^{-1}$ Now we want to take

$\sigma(u,v) = (u, v, uv)$ and so

Now $\cancel{A\sigma(u,v) =} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} u \\ v \\ uv \end{pmatrix}$

$A^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ ~~so~~

$\cancel{A^T A^T =} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$

$=$

For this to work all we should do is apply an orthogonal ~~map~~ $L_A \in O(3)$ to $\sigma(u,v) = (u, v, uv)$ (I think)

then if $A\,\sigma(u,v) = (x, y, z)$

we just need to show that $x \cdot y = z$ to show we've sent $S$ to itself.