3. Solution:

(**a**) We wish to calculate the necessary velocity changes Δv and $\Delta v'$ in order to achieve the orbit shown in the above figure. Recall from class that the total energy of an orbiting body is related to the length of the semi-major axis by

$$E = -\frac{k}{2a} \tag{1}$$

The total energy E is also defined as the sum of the kinetic (T) and potential ($U(r)$) energies so that we must have

$$-\frac{k}{2a} = E = \frac{1}{2}mv_1^2 - \frac{k}{R} \tag{2}$$

Solving this equation for v_1, yields the total velocity of the satellite for the smaller circular orbit. That is,

$$v_1 = \sqrt{\frac{k}{mR}} \tag{3}$$

The transfer orbit is an ellipse whose semi-major axis is determined from figure 1 to be

$$2a_t = R + R' \tag{4}$$

We can now use this to solve for the speed of the satellite in the elliptical transfer orbit at the point where it changes from the green circular orbit. That is,

$$E_t = -\frac{k}{R+R'} = \frac{1}{2}mv_t^2 - \frac{k}{R} \tag{5}$$

Note that here we are using R as distance from the sun in agreement with the green orbit. This results in a speed

$$v_{t1} = \sqrt{\frac{2k}{mR}\left(\frac{R'}{R+R'}\right)} \tag{6}$$

Therefore, the necessary change in speed for the satellite to leave the circular orbit and enter the yellow elliptical transfer orbit is just

$$\Delta v = v_{1t} - v_1 \tag{7}$$

Similarly, we can solve for $\Delta v'$ by finding the speed of the two orbits at the point where they overlap. They are

$$v_2 = \sqrt{\frac{k}{mR'}} v_{t2} = \sqrt{\frac{2k}{mR'}\left(\frac{R}{R+R'}\right)} \tag{8}$$

The necessary change in velocity is therefore,

$$\Delta v' = v_2 - v_{t2} \tag{9}$$

(**b**) The total time to make the transfer T_t is a half-period of the transfer orbit. Using Kepler's third law, the time is found to be

$$T_t = \pi\sqrt{\frac{k}{m}} a_t^{3/2} = \pi\sqrt{\frac{k}{m}}\left(\frac{R+R'}{2}\right)^{3/2} \tag{10}$$