Exponential Map **John Waczak**
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Tapp 5.21

Prove Corollary 5.24. If $f : S \to \tilde{S}$ *is an isometry between regular surfaces, and* γ *is a geodesic in* S*, then* $f \circ \gamma$ *is a geodesic in* $\tilde{S}$*.*

Tapp 5.22

Explicitly describe the surface patch for normal polar coordinates when $S = S^2$*,* $p = (0, 0, 1)$*,* $e_1 = (1, 0, 0)$*, and* $e_2 = (0, 1, 0)$*.*

Tapp 5.25

In Fig. 5.4 on page 252, the purple geodesic is asymptotic to the light-blue latitudinal curve.

1. *Prove that a geodesic on a surface of revolution could not be asymptotic to a latitudinal curve unless the latitudinal curve is itself a geodesic.*
2. *Rigorously justify the assertions in the discussion of Fig. 5.4*
3. *Let* $\gamma : \mathbb{R} \to S$ *be a geodesic in the paraboloid*

$$S = \{(x, y, z) \in \mathbb{R}^3 | z = x^2 + y^2\}$$

Prove that the height function, $h(x, y, z) = z$*, composed with* γ *has exactly one critical point* t_0*; it is decreasing on* $(-\infty, t_0)$ *and increasing on* (t_0, ∞)*.*