

Problem 2

In[1]:= **A** = {{0, 4, 0}, {4, 0, 1}, {0, 1, 0}}

Out[1]= {{0, 4, 0}, {4, 0, 1}, {0, 1, 0}}

In[2]:= **Eigenvals** = Eigenvalues[A]

Eigenvecs = Eigenvectors[A]

Out[2]= $\{-\sqrt{17}, \sqrt{17}, 0\}$

Out[3]= $\{\{4, -\sqrt{17}, 1\}, \{4, \sqrt{17}, 1\}, \{-1, 0, 4\}\}$

In[4]:= **NormedVecs** =

{Normalize[Eigenvecs[[1]]], Normalize[Eigenvecs[[2]]], Normalize[Eigenvecs[[3]]]}

Out[4]= $\left\{\left\{2\sqrt{\frac{2}{17}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{34}}\right\}, \left\{2\sqrt{\frac{2}{17}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{34}}\right\}, \left\{-\frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}}\right\}\right\}$

In[5]:= **e0** = {0, 0, 1}

Out[5]= {0, 0, 1}

In[6]:= **Norm**[Dot[NormedVecs[[1]], e0]]^2

Out[6]= $\frac{1}{34}$

In[7]:= **Norm**[Dot[NormedVecs[[2]], e0]]^2

Out[7]= $\frac{1}{34}$

In[8]:= **Norm**[Dot[NormedVecs[[3]], e0]]^2

Out[8]= $\frac{16}{17}$

Problem 5

In[9]:= **A** = {{1, 0, 0}, {0, 0, 1}, {0, 1, 0}}

B = {{0, 0, -1}, {0, 0, i}, {-1, -i, 4}}

C' = {{2, 0, 0}, {0, 1, 3}, {0, 3, 1}}

Out[9]= {{1, 0, 0}, {0, 0, 1}, {0, 1, 0}}

Out[10]= {{0, 0, -1}, {0, 0, i}, {-1, -i, 4}}

Out[11]= {{2, 0, 0}, {0, 1, 3}, {0, 3, 1}}

In[12]:= **MatrixForm[A.B - B.A]**

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & -1 \\ -1 & -2i & 4 \\ 1 & -4 & 2i \end{pmatrix}$$

In[13]:= **MatrixForm[A.C' - C'.A]**

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[14]:= **MatrixForm[B.C' - C'.B]**

Out[14]//MatrixForm=

$$\begin{pmatrix} 0 & -3 & 1 \\ 3 & 6i & -12 \\ -1 & 12 & -6i \end{pmatrix}$$

In[15]:=

In[16]:= **Avals = Eigenvalues[A]**

Bvals = Eigenvalues[B]

Cvals = Eigenvalues[C']

Avecs = Eigenvectors[A]

Bvecs = Eigenvectors[B]

Cvecs = Eigenvectors[C']

Out[16]= $\{-1, 1, 1\}$

Out[17]= $\{2 + \sqrt{6}, 2 - \sqrt{6}, 0\}$

Out[18]= $\{4, -2, 2\}$

Out[19]= $\{\{0, -1, 1\}, \{0, 1, 1\}, \{1, 0, 0\}\}$

Out[20]= $\left\{\left\{-\frac{1}{2 + \sqrt{6}}, \frac{i}{2 + \sqrt{6}}, 1\right\}, \left\{\frac{1}{-2 + \sqrt{6}}, -\frac{i}{-2 + \sqrt{6}}, 1\right\}, \{-i, 1, 0\}\right\}$

Out[21]= $\{\{0, 1, 1\}, \{0, -1, 1\}, \{1, 0, 0\}\}$

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In[22]:= Avecs = {Normalize[Avecs[[1]]], Normalize[Avecs[[2]]], Normalize[Avecs[[3]]]}
Bvecs = {Normalize[Bvecs[[1]]], Normalize[Bvecs[[2]]], Normalize[Bvecs[[3]]]}
Cvecs = {Normalize[Cvecs[[1]]], Normalize[Cvecs[[2]]], Normalize[Cvecs[[3]]]}
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Out[22]= {{0, -1/Sqrt[2], 1/Sqrt[2]}, {0, 1/Sqrt[2], 1/Sqrt[2]}, {1, 0, 0}}
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Out[23]= {{-1/((2+Sqrt[6]) Sqrt[1+2/(2+Sqrt[6])^2]), i/((2+Sqrt[6]) Sqrt[1+2/(2+Sqrt[6])^2]), 1/((2+Sqrt[6]) Sqrt[1+2/(2+Sqrt[6])^2])},
{1/((-2+Sqrt[6]) Sqrt[1+2/(-2+Sqrt[6])^2]), -i/((-2+Sqrt[6]) Sqrt[1+2/(-2+Sqrt[6])^2]), 1/((-2+Sqrt[6]) Sqrt[1+2/(-2+Sqrt[6])^2])},
{-i/Sqrt[2], 1/Sqrt[2], 0}}
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Out[24]= {{0, 1/Sqrt[2], 1/Sqrt[2]}, {0, -1/Sqrt[2], 1/Sqrt[2]}, {1, 0, 0}}
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