Central Forces Homework 10

Due 6/8/18, 4 pm

Sensemaking: For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

PRACTICE:

1. (McIntyre 8.6) Calculate the probability that the electron is measured to be within one Bohr radius of the nucleus for the $n = 2$ states of hydrogen. Discuss the differences between the results for the $l = 0$ and $l = 1$ states.

2. Consider the initial state $\frac{1}{\sqrt{2}}(|1, 0, 0\rangle + |2, 1, 0\rangle)$. Note, this is **not** an *sp* hybrid orbital such as occurs in chemistry in the study of molecular bonding.

   (a) If you measure the energy of this state, what possible values could you obtain?

   (b) What is this state as a function of time?

   (c) Calculate the expectation value $\langle \hat{L}^2 \rangle$ in this state, as a function of time. Did you expect this answer? Comment.

   (d) Write the time-dependent state in wave function notation.

   (e) Calculate the expectation value $\langle \hat{z} \rangle$ as a function of time. Do you expect this answer?

3. Complete the attached table for the hydrogen atom.

Hydrogen Atom

	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Hamiltonian			
Eigenvalues of Hamiltonian			
Normalized Eigenstates of Hamiltonian			
Coefficient of the energy eigenstate with quantum numbers n, ℓ, m			
Probability of measuring E_n			

Hydrogen Atom

	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Operator for square of the angular momentum			
Eigenvalues of L^2			
Normalized Eigenstates of L^2			
Coefficient of the eigenstates of L^2 with quantum numbers n, ℓ, m			
Probability of measuring $\hbar^2 \ell(\ell+1)$ for the square of the angular momentum			

Hydrogen Atom

	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Operator for z-component of angular momentum			
Eigenvalues of L_z			
Normalized Eigenstates of L_z			
Coefficient of m^{th} eigenstates of L_z			
Probability of measuring $m\hbar$ for z -component of angular momentum			

REQUIRED:

4. McIntyre 8.14

   A hydrogen atom is initially in the superposition state

   $|\psi(0)\rangle = \frac{1}{\sqrt{14}}|2,1,1\rangle - \frac{2}{\sqrt{14}}|3,2,-1\rangle + \frac{3}{\sqrt{14}}|4,2,2\rangle$.

   (a) What are the possible results of a measurement of the energy and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of the energy.

   (b) What are the possible results of a measurement of the angular momentum operator L^2 and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of L^2.

   (c) What are the possible results of a measurement of the angular momentum component operator L_z and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of L_z.

5. (McIntyre 8.7) Calculate the probability that the electron is measured to be in the classically forbidden region for the $n = 2$ states of hydrogen. Discuss the differences between the results for the $l = 0$ and $l = 1$ states.