Homework #2 — John Walczak, Mth 351

5.e. Rearrange the function to ameliorate loss of significance error.

$$f(x) = \frac{\sqrt{4+x} - 2}{x}$$

when $x$ is near 0 we are subtracting 2 nearly identical numbers which leads to loss-of-significance error.

$$= \frac{\sqrt{4+x} - 2}{x} \frac{(\sqrt{4+x} + 2)}{(\sqrt{4+x} + 2)}$$

$$= \frac{4 + x - 4}{x(\sqrt{4+x} + 2)}$$

$$= \frac{1}{\sqrt{4+x} + 2}$$

2.3.11 let $f(x) = (x-1)(x-2)\cdots(x-n)$ $f(1) = 0$
estimate $f(1+10^{-4})$ w/ $X_T = 1$ for
$n = 2, 3, \ldots 12$

Recall eqn 2.43 which states (from MVT)

$$f(X_A) \approx f(X_T) - f'(X_T)(X_T - X_A)$$

$n=2$ $\quad f(x) = (x-1)(x-2)$

$f'(x) = (x-1) + (x-2)$

$f'(1) = -1$

$f(1+10^{-4}) \approx 0 - 1(1+10^{-4}-1)$

$= -10^{-4}$

$n=3$ $\quad f(x) = (x-1)(x-2)(x-3)$

$f'(x) = (x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)$

$f'(1) = -1(-2) + 0 + 0 = 2$

$f(1+10^{-4}) \approx 0 + 2(10^{-4}) = 2 \times 10^{-4}$

now we can see a pattern that only the first term of the product rule w/o the $(x-1)$ will survive thus

$n=4$ $\quad f'(x) = (x-2)(x-3)(x-4) + \cdots$

$f'(1) = -1(-2)(-3) = -6$

$f(1+10^{-4}) \approx -6(10^{-4}) = -6 \times 10^{-4}$

$n=5$ $\quad f'(1) = (x-2)(x-3)(x-4)(x-5)$

$= -1(-2)(-3)(-4)$

$= 24$

$f(1+10^{-4}) = 24 \times 10^{-4}$

$n=6$ $\quad f'(1) = (x-2)(x-3)(x-4)(x-5)(x-6)$

$= -1(-2)(-3)(-4)(-5)$

$= -120$

$f(1+10^{-4}) = -120 \times 10^{-4}$

$n = 7 \qquad f'(1) = -120(-6) = 720$

$$f(1+10^{-4}) \approx 720 \times 10^{-4}$$

$n = 8 \qquad f'(1) = 720(-7) = -5040$

$$f(1+10^{-4}) = -0.5040$$

$n = 9 \qquad f'(1) = -5040(-8) = 40320$

$$f(1+10^{-4}) = 4.0320$$

$n = 10 \qquad f'(1) = 40320(-9) = -322560$

$$f(1+10^{-4}) = -32.2560$$

$n = 11 \qquad f'(1) = -322560(-10) = 3225600$

$$f(1+10^{-4}) = 322.5600$$

$n = 12 \qquad f'(1) = 3225600(-11) = -35481600$

$$f(1+10^{-4}) = -3548.1600$$

when $n = 8$ we have

$$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)(x-8)$$

when $x$ is an integer from $1, 2, 3, \ldots 8$
$f(x)$ is zero thus $\{1, 2, 3, \ldots 8\}$
are the roots of $f(x)$.

2.4.4

when $n=2$ $\quad S = (a_1 + a_2)$

$$S_2 = fl(a_1 + a_2)$$

now $S - S_2 = a_1 + a_2 - fl(a_1 + a_2)$

$$= (a_1 + a_2) - (1 + \varepsilon_2)(a_1 + a_2)$$

$$= a_1 + a_2 - a_1 - a_2 - \varepsilon_2 a_1 - \varepsilon_2 a_2$$

$$= -a_1 \varepsilon_2 - a_2 \varepsilon_2$$

so eqn 2.50 holds for the $n=2$ case

when $n=3$ $\quad S = a_1 + a_2 + a_3$

$$S_3 = fl(a_1 + S_2)$$

$$= (1+\varepsilon_3)(a_1 + (1+\varepsilon_2)(a_1 + a_2))$$

thus $S - S_3 = a_1 + a_2 + a_3 - (1+\varepsilon_3)(a_1 + (1+\varepsilon_2)(a_1 + a_2))$

* assuming $\varepsilon_i \varepsilon_j \approx 0 \ \forall i,j$ *

$$= a_1 + a_2 + a_3 - a_1 - a_2 - a_3 - \varepsilon_2 a_1 - \varepsilon_2 a_2 - \varepsilon_3 a_3 - \varepsilon_3 a_1 - \varepsilon_3 a_2 + 0 + 0$$

$$= -\varepsilon_2 a_1 - \varepsilon_2 a_2 - \varepsilon_3 a_3 - \varepsilon_3 a_2 - \varepsilon_3 a_1$$

$$= -a_1(\varepsilon_2 + \varepsilon_3) - a_2(\varepsilon_2 + \varepsilon_3) - a_3 \varepsilon_3$$

thus the formula holds for $n = 3$

When $n = 4$

$$S = (a_1 + a_2 + a_3 + a_4)$$

$$S_4 = (a_4 + S_3)$$

$$S - S_4 = a_1 + a_2 + a_3 + a_4 - (1+\varepsilon_4)(a_4 + S_3)$$

$$= a_1 + a_2 + a_3 + a_4 - (1+\varepsilon_4)(a_4 + S_3)$$

from the last part we have

$$S_3 = a_1 + a_2 + a_3 + a_1(\varepsilon_2+\varepsilon_3) + a_2(\varepsilon_2+\varepsilon_3) + a_3\varepsilon_3$$

thus $\quad S_4 = (1+\varepsilon_4)(a_4 + S_3)$ so

$$S_4 = a_1+a_2+a_3+a_4 + a_1(\varepsilon_2+\varepsilon_3+\varepsilon_4) + a_2(\varepsilon_2+\varepsilon_3+\varepsilon_4) + a_3(\varepsilon_3+\varepsilon_4) + a_4\varepsilon_4$$

thus $\quad S - S_4 = -a_1(\varepsilon_2+\varepsilon_3+\varepsilon_4) - a_2(\varepsilon_2+\varepsilon_3+\varepsilon_4) - a_3(\varepsilon_3+\varepsilon_4) - a_4\varepsilon_4.$

which also obeys eqn 2.50.