In[12]:= $u = \left(\frac{s}{R}\right)^2 - 2*\left(\frac{s}{R}\right)*\text{Cos}[\phi - \phi_0]$

Out[12]= $\frac{s^2}{R^2} - \frac{2\,s\,\text{Cos}[\phi - \phi_0]}{R}$

Find terms in expansion up to u^4

In[13]:= $p = -(1/2);$

In[14]:= $\text{Expand}\left[\frac{p}{1}*u\right]$

$\text{Expand}\left[\frac{p\,(p-1)}{2!}*u\wedge 2\right]$

$\text{Expand}\left[\frac{p\,(p-1)\,(p-2)}{3!}*u\wedge 3\right]$

$\text{Expand}\left[\frac{p\,(p-1)\,(p-2)\,(p-3)}{4!}*u\wedge 4\right]$

Out[14]= $-\frac{s^2}{2\,R^2} + \frac{s\,\text{Cos}[\phi - \phi_0]}{R}$

Out[15]= $\frac{3\,s^4}{8\,R^4} - \frac{3\,s^3\,\text{Cos}[\phi - \phi_0]}{2\,R^3} + \frac{3\,s^2\,\text{Cos}[\phi - \phi_0]^2}{2\,R^2}$

Out[16]= $-\frac{5\,s^6}{16\,R^6} + \frac{15\,s^5\,\text{Cos}[\phi - \phi_0]}{8\,R^5} - \frac{15\,s^4\,\text{Cos}[\phi - \phi_0]^2}{4\,R^4} + \frac{5\,s^3\,\text{Cos}[\phi - \phi_0]^3}{2\,R^3}$

Out[17]= $\frac{35\,s^8}{128\,R^8} - \frac{35\,s^7\,\text{Cos}[\phi - \phi_0]}{16\,R^7} + \frac{105\,s^6\,\text{Cos}[\phi - \phi_0]^2}{16\,R^6} - \frac{35\,s^5\,\text{Cos}[\phi - \phi_0]^3}{4\,R^5} + \frac{35\,s^4\,\text{Cos}[\phi - \phi_0]^4}{8\,R^4}$

Collect like terms of $\left(\frac{s}{R}\right)^k$

In[18]:=
$$\text{Integrand1} = \frac{s \operatorname{Cos}[\phi - \phi_0]}{R}$$
$$\text{Integrand2} = -\frac{s^2}{2 R^2} + \frac{3 s^2 \operatorname{Cos}[\phi - \phi_0]^2}{2 R^2}$$
$$\text{Intergrand3} = -\frac{3 s^3 \operatorname{Cos}[\phi - \phi_0]}{2 R^3} + \frac{5 s^3 \operatorname{Cos}[\phi - \phi_0]^3}{2 R^3}$$
$$\text{Integrand4} = \frac{3 s^4}{8 R^4} - \frac{15 s^4 \operatorname{Cos}[\phi - \phi_0]^2}{4 R^4} + \frac{35 s^4 \operatorname{Cos}[\phi - \phi_0]^4}{8 R^4}$$

Out[18]=
$$\frac{s \operatorname{Cos}[\phi - \phi_0]}{R}$$

Out[19]=
$$-\frac{s^2}{2 R^2} + \frac{3 s^2 \operatorname{Cos}[\phi - \phi_0]^2}{2 R^2}$$

Out[20]=
$$-\frac{3 s^3 \operatorname{Cos}[\phi - \phi_0]}{2 R^3} + \frac{5 s^3 \operatorname{Cos}[\phi - \phi_0]^3}{2 R^3}$$

Out[21]=
$$\frac{3 s^4}{8 R^4} - \frac{15 s^4 \operatorname{Cos}[\phi - \phi_0]^2}{4 R^4} + \frac{35 s^4 \operatorname{Cos}[\phi - \phi_0]^4}{8 R^4}$$

Calculate the nasty integral

In[22]:=
$$\text{Int} = \text{Expand}\Big[\int_0^{2\pi} \Big(1 + \operatorname{Cos}[\phi - \phi_0] * \left(\frac{s}{R}\right) + \left(\frac{3}{2}\operatorname{Cos}[\phi - \phi_0]^2 - \frac{1}{2}\right) * \left(\frac{s}{R}\right)^2 + \left(\frac{5}{2}\operatorname{Cos}[\phi - \phi_0]^3 - \frac{3}{2}\operatorname{Cos}[\phi - \phi_0]\right) * \left(\frac{s}{R}\right)^3 + \left(\frac{3}{8} - \frac{15}{4}\operatorname{Cos}[\phi - \phi_0]^2 + \frac{35}{8}\operatorname{Cos}[\phi - \phi_0]^4\right) * \left(\frac{s}{R}\right)^4\Big) \, d\phi_0\Big]$$

Out[22]=
$$2\pi + \frac{\pi s^2}{2 R^2} + \frac{9 \pi s^4}{32 R^4}$$

Multiply by all of the constants to get the potential --> V= $\frac{Q}{4\pi\epsilon_0}$*$\frac{1}{2\pi}$*Integral

In[23]:=

In[24]:=
$$V = \frac{Q}{4\pi\epsilon_0} * \text{Expand}\left[\frac{1}{2\pi} * \text{Int}\right]$$

Out[24]=
$$\frac{Q\left(1 + \frac{s^2}{4 R^2} + \frac{9 s^4}{64 R^4}\right)}{4 \pi \epsilon_0}$$