Homework 2 **John Waczak**
MTH 443 Date: October 8, 2018
Dr. Schmidt

---

(1). (a) Clearly state under what conditions the range and null space of a linear transformation T are the same set.

Consider a general linear transformation $T : V \to W$ where V and W are $\mathbb{F}$-vector spaces. In order for the range of T to be equal to its null space, we must have the following

1. $\text{rank}(T) = \text{nullity}(T)$
2. $\dim(V)$ is even
3. W = V

(b) Prove your assertion

Proof. Recall from class that the Dimension Theorem (2.3) asserts

$$\text{rank}(T) + \text{nullity}(T) = \dim(V)$$

□

(c) Give an example

Example: Consider the linear transformation $T : \mathbb{F}^2 \to \mathbb{F}^2$ given by

$$T(x, y) = (0, x)$$

Acting this transformation on the canonical basis vectors e_1, e_2 generates the matrix representation for T denoted $A_T \in \mathcal{M}_{2\times 2}$.

$$A_T = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Then solutions to the equation $Ax = 0$ are easily shown by row operations to be of the form

$$\begin{pmatrix} 0 \\ \lambda \end{pmatrix}, \quad \forall \lambda \in \mathbb{F}$$

Therefore the null space of this linear transformation is

$$\ker(T) = \{(x, y) \in \mathbb{F}^2 | T(x, y) = (0, 0)\} = \text{span}(e_2)$$