$$(1-z^2)\,P''(z) - 2z\,P'(z) + \ell(\ell+1)\,P(z)=0$$

$$\boxed{\begin{array}{l} z = \cos\theta \\ dz = -\sin\theta\, d\theta \end{array}} \; \Big\rangle \text{ plug in}$$

$$(1-\cos^2(\theta))\,\frac{d}{-\sin\theta\, d\theta}\left(\frac{d\,P(\theta)}{-\sin\theta\, d\theta}\right) - 2\cos\theta\,\frac{dP(\theta)}{-\sin\theta\, d\theta} + \ell(\ell+1)\,P(\theta)=0$$

$$1-\cos^2(\theta)=\sin^2\theta$$

$$\sin^2\theta\,\frac{d}{\sin\theta\, d\theta}\left(\frac{1}{\sin\theta}\,\frac{dP(\theta)}{d\theta}\right) + 2\cot(\theta)\,\frac{dP(\theta)}{d(\theta)} + \ell(\ell+1)\,P(\theta)=0$$

$$\sin^2\theta\,\frac{1}{\sin\theta}\underbrace{\left[\frac{-\cos\theta}{\sin^2\theta}\,\frac{dP(\theta)}{d\theta} + \frac{1}{\sin\theta}\,\frac{d^2P(\theta)}{d\theta^2}\right]}_{\text{product rule}} + 2\cot(\theta)\,\frac{dP(\theta)}{d\theta} + \ell(\ell+1)\,P(\theta)=0$$

$$-\frac{\cos\theta}{\sin\theta}\,P'(\theta) + P''(\theta) + 2\cot(\theta)\,P'(\theta) + \ell(\ell+1)\,P(\theta)=0$$

$$-\cot(\theta)\,p'(\theta) + p''(\theta) + 2\cot(\theta)\,p'(\theta) + \ell(\ell+1)\,p(\theta)=0$$

$$\Longrightarrow \quad \boxed{p''(\theta) + \cot(\theta)\,p'(\theta) + \ell(\ell+1)\,p(\theta)=0}$$

Later we will see this equation and will use the reverse to convert to Legendre's equation in order to use power-series methods.