$\sigma$ is a smooth bijection and so $\sigma^{-1}$ exists and is smooth. Now

$$f \circ \sigma = T_q \circ L_A \circ \sigma.$$

Now the inverse of $f$, is simply

$$f^{-1} = L_A^{-1} \circ T_{-q} \text{ and so}$$

$$(f \circ \sigma)^{-1} = \sigma^{-1} \circ f^{-1} = \sigma^{-1} \circ L_A^{-1} \circ T_{-q}$$

since ~~$L_A$~~ $A \subset O(3)$ we know it is invertible and so $L_A^{-1}$ is defined. Thus we have a map and its inverse and so all we must show is that they are smooth.

The derivatives of $T_q, T_{-q}$ are the identity and so those functions are smooth.

Similarly because $L_A$ is a rotation or reflection (we showed this in exercise 1.1.73) then at most ~~[illegible]~~ $L_A$ multiplies the components of the vector in $\mathbb{R}^3$ by some combination of sine, cosine and ~~or~~ constant.

we also know $dL_A = L_A$ so it is infinitely differentiable

These three functions are smooth so $T_q \circ L_A \circ \sigma$ and $\sigma^{-1} \circ L^{-1} \circ T_{-q}$ are smooth. Thus we have an open set $U \subset \mathbb{R}^2$, a neighborhood $f(V)$ of $p \in f(S)$ and a diffeomorphism $T_q \circ L_A \circ \sigma$. Therefore, $f(S)$ is a regular surface.