PH 335 Homework 1

John L. Waczak

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3. Golf Ball: A golf ball is hit from ground level with speed v_0 in a direction that is due east and at an angle θ above the horizontal.

(a) *Assumptions:* To solve this problem I will assume that friction and air resistance are negligible. Thus we can cast the problem from three dimensions into two as there is no initial velocity in the North-South direction nor are there any forces along this axis. This means that we only need to consider West-East and vertical motion which we will thus label using x and z. There is no information given regarding the forces initially so we will only consider the force due to gravity on the ball. Similarly, the initial position of the ball is not specified so we will say $x(0) = y(0) = 0 \quad [m]$.

Newton's second law allows us to write:

$$\sum \mathbf{F} = m\mathbf{a}$$

This is a vector equation which can be brokend down into two equations component wise.

$$\sum F_x = ma_x$$
$$\sum F_z = ma_z$$

Let's first look at the equation for the y component of the forces. Our assumptions suggest that the only force in this direction is that due to gravity. Thus we have:

$$\sum F_z = ma_z = F_g = -mg \tag{1}$$

This tells us that $a_z = g$ which is a first order, separable differential equation in velocity and a second order differential equation in position.

$$\begin{aligned} a_z &= -g \\ \frac{dv_z}{dt} &= -g \\ dv_z &= gdt \\ \int dv_z &= \int -gdt \\ v_z(t) &= -gt + C \end{aligned}$$

We can solve for the integration constant C using the initial condition given for $v_{0,z} = v_0 sin\theta$. Thus we have that:

$$v_z(t) = -gt + v_0 sin\theta \tag{2}$$

To solve for the vertical displacement, we repeat the same process.

$$\frac{dz}{dt} = -gt + v_0 sin\theta$$

$$\int dz = \int (-gt + v_0 sin\theta)dt$$

$$z(t) = -\frac{1}{2}gt^2 + v_0 sin\theta t + C$$

Since y(0) was defined to be 0 we conclude:

$$z(t) = -\frac{1}{2}gt^2 + v_0 sin\theta t \tag{3}$$

Now we will repeat the same process to solve for $x(t)$.

$$\sum F_x = ma_x = 0$$

$$a_x = 0$$

$$\frac{dv_x}{dt} = 0$$

$$\int dv_x = \int 0dt$$

$$v(t) = C$$

$$v(t) = v(0) = v_0 cos\theta$$

$$\frac{dx}{dt} = v_0 cos\theta$$

$$\int dx = \int v_0 cos\theta dt$$

$$x(t) = v_0 cos\theta t + C$$

$$x(0) = C = 0$$

In the final line above we applied the initial condition that the initial position is 0 m. This results in:

$$x(t) = v_0 cos\theta t \tag{4}$$

(b) First we will solve for the time for the golf ball to return to the ground. Then by considering the velocity equations we can determine the peak height. Finally we will use the time of flight to determine the horizontal distance the ball travels.

To solve for the time of flight we can equate eqn. 3 to 0 and solve for t.

$$0 = -\frac{1}{2}gt_{flight}^2 + v_0 sin\theta t_{flight}$$

$$\frac{1}{2}gt_{flight}^2 = v_0 sin\theta t_{flight}$$

$$t_{flight} = \frac{2v_0 sin\theta}{g}$$

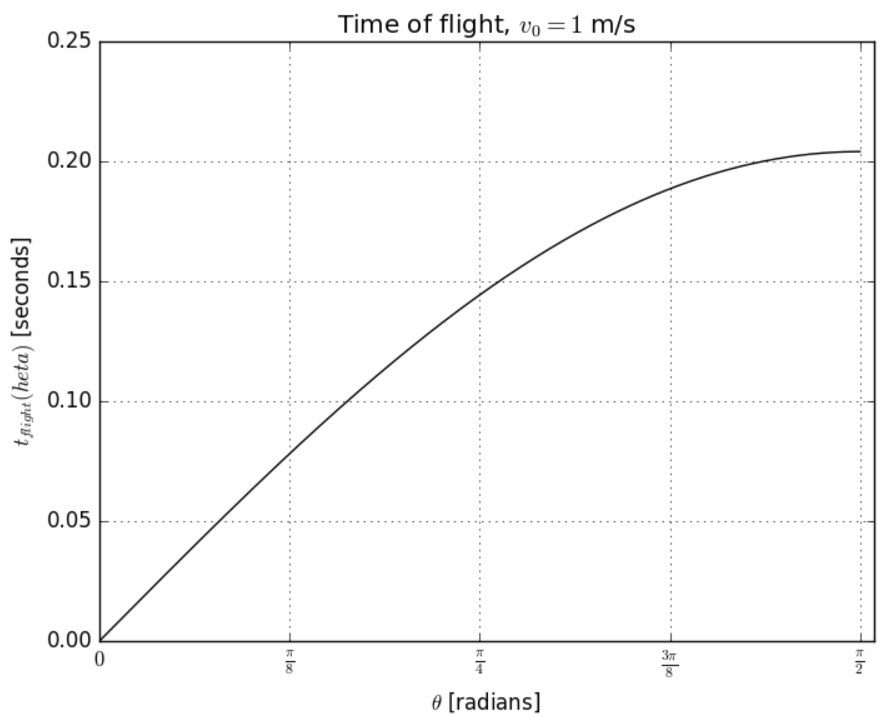
Now, to solve for the maximum height reached we use the fact that the y component of the velocity will be zero.

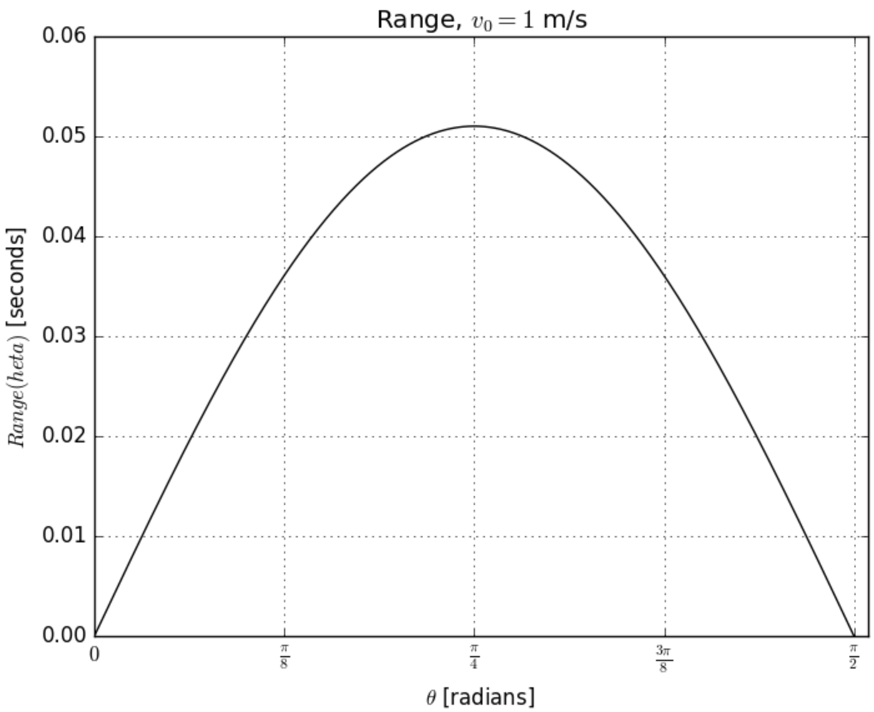
$$0 = -gt_{max} + v_0 sin\theta$$
$$t_{max} = \frac{v_0 sin\theta}{g}$$
$$height_{max} = z(t_{max}) = -\frac{1}{2}(\frac{v_0 sin\theta}{g})^2 + v_0 sin\theta(\frac{v_0 sin\theta}{g})$$

This answer makes sense as the trajectory is parabolic so by symmetry we expect the time for a maximum height to be half of the total fight time since the ball starts from the ground. Finally, to solve for the range of the ball, we will use the t_{flight} and the kinematic equation for the eastward displacement.

$$Range = x(t_{max}) = v_0 cos\theta(\frac{v_0 sin\theta}{g})$$

(c) The following graphs illustrate the time of flight as a function of initial angle and as well as the range as a function of initial angle.





(d) The angle that corresponds to the maximum time of flight is $\pi/2$. In the graph I chose to limit the domain because for angles beyond this point the ball simply flies in the other direction (West). This value makes sense for the maximum time as the initial velocity is completely in the vertical direction the ball must spend the most time in the sky.

(e) The initial angle corresponding to the maximum range is $\pi/4$ (45 degrees). This value makes sense conceptually as this value achieves a large time of flight while still having a large horizontal velocity component.