

In[14]:= \$Assumptions = $a_0 > 0$ and $a_0 \in \text{Reals}$ and $\tau > 0$

Out[14]= $a_0 > 0 \in \text{and } \mathbb{R} \tau > 0$

Calculate integral for matrix element

$$\begin{aligned} \text{In[15]:= } \psi_{210}[r_-, \theta_-, \phi_-] &:= \frac{1}{2\sqrt{\pi}} \left(\frac{1}{2a_0} \right)^{3/2} * \frac{r}{a_0} * \text{Exp}[-r/(2a_0)] * \text{Cos}[\theta] \\ \psi_{100}[r_-, \theta_-, \phi_-] &:= \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{3/2} * \text{Exp}[-r/a_0] \end{aligned}$$

$$\begin{aligned} \text{In[17]:= } &\sqrt{\frac{4\pi}{3}} * \text{Integrate}[\psi_{210}[r, \theta, \phi] * r * \text{SphericalHarmonicY}[1, 0, \theta, \phi] * \\ &\psi_{100}[r, \theta, \phi] * r^2 * \text{Sin}[\theta], \{r, 0, \infty\}, \{\theta, 0, \pi\}, \{\phi, 0, 2\pi\}] \end{aligned}$$

$$\text{Out[17]= ConditionalExpression}\left[\frac{128\sqrt{2}a_0}{243}, \text{Re}[a_0] > 0\right]$$

Integrate for the probability

$$\text{In[18]:= Nasty_Int} = \text{Integrate}[\text{Exp}[-t^2/\tau^2] * \text{Exp}[i*\omega*t], \{t, -\infty, \infty\}]$$

$$\text{Out[18]= ConditionalExpression}\left[\frac{e^{-\frac{1}{4}\tau^2\omega^2}\sqrt{\pi}}{\sqrt{\frac{1}{\tau^2}}}, \text{Re}[\tau^2] > 0\right]$$

$$\text{In[23]:= } \frac{q^2 * \epsilon^2}{h^2} * \left(\frac{128\sqrt{2} * a_0}{243} \right)^2 * \left(\frac{e^{-\frac{1}{4}\tau^2\omega^2}\sqrt{\pi}}{\sqrt{\frac{1}{\tau^2}}} \right)^2$$

$$\text{Out[23]= } \frac{32768 e^{-\frac{1}{2}\tau^2\omega^2} \pi q^2 \epsilon^2 \tau^2 a_0^2}{59049 h^2}$$

$$\text{In[24]:= } \frac{2^{15} e^{-\frac{1}{2}\tau^2\omega^2} \pi q^2 \epsilon^2 \tau^2 a_0^2}{3^{10} h^2}$$

$$\text{Out[24]= } \frac{32768 e^{-\frac{1}{2}\tau^2\omega^2} \pi q^2 \epsilon^2 \tau^2 a_0^2}{59049 h^2}$$