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I took roughly **30 min** to finish and type up this homework

1.38

Compute the curvature for the helix directly from definition and then by reparametrizing by arc length.

a)

$$
\begin{aligned}
\gamma(t) &= (\cos(t), \sin(t), t) \\
\gamma'(t) &= v(t) = (-\sin(t), \cos(t), 1) \\
\gamma''(t) &= a(t) = (-\cos(t), -\sin(t), 0) \\
a(t)^{\parallel} &= \mathrm{proj}_a v = \frac{\langle a, v\rangle}{|v|^2} v \\
|\gamma'(t)| &= \sqrt{2} \\
\langle a, v\rangle &= \sin(t)\cos(t) - \sin(t)\cos(t) = 0 \\
\Rightarrow a^{\perp} &= a \\
|a| &= \sqrt{1} = 1 \\
|v| &= \sqrt{2} \\
\kappa(t) &= \frac{|a^{\perp}|}{|v|^2} = \frac{1}{2}
\end{aligned}
$$

b)

$$
\begin{aligned}
|\gamma'(t)| &= \sqrt{2} \\
s &= \int_0^t \sqrt{2} dt' \\
&= \sqrt{2} t \\
\Rightarrow t &= \frac{1}{\sqrt{2}} s \\
\gamma(s) &= (\cos(\frac{1}{\sqrt{2}} s), \sin(\frac{1}{\sqrt{2}} s, \frac{1}{\sqrt{2}} s) \\
\gamma'(s) &= \frac{1}{\sqrt{2}}(-\sin(\frac{1}{\sqrt{2}} s), \cos(\frac{1}{\sqrt{2}} s), 1) \\
\gamma''(s) &= \frac{1}{\sqrt{2}}\Big(-\frac{1}{\sqrt{2}}\cos(\frac{1}{\sqrt{2}} s), -\frac{1}{\sqrt{2}}\sin(\frac{1}{\sqrt{2}} s), 0\Big) \\
\kappa(s) &= |a(s)| = |\gamma''(s)| = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{2}} \\
&= \frac{1}{2}
\end{aligned}
$$

Thus we have shown that both definitions serve to calculate the curvature.