Central Forces Homework 8

Due 6/1/18, 4 pm

Sensemaking: For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

REQUIRED:

1. Use your favorite tool (*e.g.* Maple, Mathematica, Matlab, pencil) to generate the Legendre polynomial expansion to the function $f(z) = \sin(\pi z)$. How many terms do you need to include in a partial sum to get a "good" approximation to $f(z)$ for $-1 < z < 1$? What do you mean by a "good" approximation? How about the interval $-2 < z < 2$? How good is your approximation? Discuss your answers. Answer the same set of questions for the function $g(z) = \sin(3\pi z)$

2. Show that if a linear combination of ring energy eigenstates is normalized, then the coefficients must satisfy
$$\sum_{m=-\infty}^{\infty} |c_m|^2 = 1$$

3. Answer the following questions for a quantum mechanical particle confined to a ring. You may want to use the Mathematica activity on time dependence of a particle on the ring from the course website (cfqmring.nb) to help you figure out the answers.

   (a) Characterize the states for which the probability density does not depend on time.

   (b) Characterize the states that are right-moving.

   (c) Characterize the states that are standing waves.

   (d) Compare the time dependence of the three states:
$$\begin{aligned}|\Psi_1\rangle &= \frac{1}{\sqrt{2}}\left(|3\rangle + |-3\rangle\right)\\ |\Psi_1\rangle &= \frac{1}{\sqrt{2}}\left(|3\rangle - |-3\rangle\right)\\ |\Psi_1\rangle &= \frac{1}{\sqrt{2}}\left(|3\rangle + i|-3\rangle\right)\end{aligned}$$

4. Consider the following normalized state for the rigid rotor given by:
$$|\psi\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle + \frac{1}{\sqrt{3}}|1,0\rangle + \frac{i}{\sqrt{6}}|0,0\rangle$$

   (a) What is the probability that a measurement of L_z will yield $2\hbar$? $-\hbar$? $0\hbar$?

(b) If you measured the z-component of angular momentum to be $-\hbar$, what would the state of the particle be immediately after the measurement is made? $0\hbar$?

(c) What is the expectation value of L_z in this state?

(d) What is the expectation value of L^2 in this state?

(e) What is the expectation value of the energy in this state?

5. Let $P_l(z)$ be a solution of the Legendre's equation

$$(1-z^2)\frac{d^2y(z)}{dz^2} - 2z\frac{dy(z)}{dz} + l(l+1)y(z) = 0.$$

Show that $(1-z^2)^{m/2}\frac{d^m P_l(z)}{dz^m}$ is a solution of the associated Legendre equation

$$(1-z^2)\frac{d^2y(z)}{dz^2} - 2z\frac{dy(z)}{dz} + \left[l(l+1) - \frac{m^2}{1-z^2}\right]y(z) = 0.$$

Hint: You may want to start by differentiating the Legendre's equation m times with respect to z and apply Leibniz's theorem for the mth derivative of a product

$$\frac{d^m}{dz^m}\left[f(z)g(z)\right] = \sum_{r=0}^{m}\binom{m}{r}\frac{d^r f(z)}{dz^r}\frac{d^{m-r}g(z)}{dz^{m-r}}$$