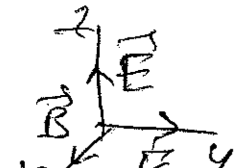
Lecture # 10

Interaction of an atom with an EM wave (cont.)

Last time: $H = H_0 + \underbrace{\frac{e}{mc}\vec{P}\cdot\vec{A}(\vec{r},t)}_{①} + \underbrace{\frac{e}{mc}\vec{S}\cdot\vec{B}(\vec{r},t)}_{②} + \underbrace{\frac{e^2}{2mc^2}\vec{A}^2(\ldots}_{③}$

$V_{①} \gg V_{②} \gg V_{③}$



$\vec{A} = \left(A_0 e^{i(ky-\omega t)} + A_0^* e^{-i(ky-\omega t)}\right)\vec{e}_z$

$\vec{E} = E_0\vec{e}_z\cos(ky-\omega t)$; $\frac{E_0}{B_0} = \frac{\omega}{k} = c$; $i\omega A_0 = \frac{E_0}{2}$

$\vec{B} = B_0\vec{e}_x\cos(ky-\omega t)$; $ikA_0 = \frac{B_0}{2}$

$V_{①} = \frac{e}{mc}\vec{P}\cdot\vec{A}(\vec{r},t) = \underbrace{\frac{e}{mc}P_z\left(A_0 e^{iky}\right)}_{\hat{V}_0} e^{-i\omega t} + c.c.$

$\hat{V}_0 = \underbrace{\hat{V}_{0DE}}_{\text{electric dipole}} + \ldots$

Transition rate $i \to f \Rightarrow$

$P_{i\to f} = \frac{2\pi}{\hbar}\left(\frac{e}{mc}\right)^2|A_0|^2\,|\langle f|e^{iky}P_z|i\rangle|^2$, $E_f = E_i + \hbar\omega$

Electric-dipole approx.:

$e^{iky} \approx 1 \Rightarrow \langle f|e^{iky}P_z|i\rangle \approx \langle f|P_z|i\rangle =$

$= im\omega_{fi}\langle f|z|i\rangle \Rightarrow$ selection rules

$\uparrow$
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$\Delta l = \pm 1$
$\Delta m = 0$ (or ± 1 if x- or y-polar

Now take into account higher-order terms (2) $\Rightarrow$

$$e^{iky} \simeq 1 + \underbrace{iky}_{} + O(ky)^2$$

$$V_{0_{①}} - V_{0\,DE} = \frac{e}{mc} P_z \underbrace{A_0 \cdot (iky)}_{= \frac{B_0}{2}} = \frac{e}{mc}\frac{B_0}{2} P_z y =$$

neglect terms $\sim (ky)^2$

$$= \frac{e}{mc}\frac{B_0}{2}\left[\frac{1}{2}\underbrace{(P_z y - z P_y)}_{= L_x} + \frac{1}{2}(P_z y + z P_y)\right] \qquad (11.1)$$

$$V_{0_{②}} = \frac{e}{mc}\vec{S}\cdot\vec{B} = \frac{e}{mc} S_x \frac{B_0}{2} \qquad (11.2)$$

only time-independent part (factor out $e^{\pm i\omega t}$)

$e^{iky} \approx 1$

$V_{②}(t) = \frac{e}{mc} S_x B_0 \underbrace{\cos\omega t}_{= \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})}$

Combine (11.1) and (11.2) $\Rightarrow$

$$V_0 = V_{0_{DE}} + \underbrace{\frac{e}{mc}\frac{B_0}{2}\cdot\frac{1}{2}(L_x + 2S_x)}_{= V_{0_{DM}}} + \underbrace{\frac{e}{mc}\frac{\overset{= \frac{E_0}{c}}{B_0}}{2}\frac{1}{2}(P_z y + z P_y)}_{V_{0_{QE}}}$$

magnetic dipole transitions

electric quadrupole transitions

the same order!

Selection rules?

$\Downarrow$

$\langle f | V_{0_{DM}} | i \rangle \sim \langle f | L_x + 2S_x | i \rangle =$

$= \langle n_f\, l_f\, m_f\, m_{s_f} | \underset{\uparrow \atop \frac{L_+ + L_-}{2}}{L_x} + 2 \underset{\uparrow \atop \frac{S_+ + S_-}{2}}{S_x} | n_i\, l_i\, m_i\, m_{s_i} \rangle \neq 0$ if

$m_f = m_i \pm 1$

$m_{s_f} = m_{s_i} \pm 1$

If $\vec{B} \parallel Oz \Rightarrow \langle f | L_z + 2S_z | i \rangle \neq 0$ if

$m_f = m_i$

$m_{s_f} = m_{s_f}$

Since V_{DM} doesn't act on l

$\Downarrow$ $\Delta l = l_f - l_i = 0$

So, selection rules for the magnetic dipole transition are $\Delta l = 0$; $\Delta m = \pm 1$ or 0

$\Delta m_s = \pm 1$ or 0

What about $\langle f | V_{0_{QE}} | i \rangle$? $\Rightarrow$

$\langle f | P_z y + z P_y | i \rangle \underset{\uparrow}{=} \langle f | \frac{im}{\hbar}[H_0, z] y + \frac{im}{\hbar}[H_0, y] z | i \rangle$

$= \langle f | \frac{im}{\hbar}[H_0, zy] | i \rangle \circeq$ $\quad P_{z \atop y} = \frac{im}{\hbar}[H_0, z_{\atop y}]$

$\circeq \frac{im}{\cancel{\hbar}} \omega_{fi} \cancel{\hbar} \langle f | \underbrace{zy}_{} | i \rangle$ $\leftarrow$ component of quadrupole moment

$yz \sim r^2 (A Y_2^1 + B Y_2^{-1}) \Rightarrow$

$\langle f | yz | i \rangle \sim \int Y_{\ell_f}^{m_f *} Y_2^{\pm 1} Y_{\ell_i}^{m_i} d\Omega \Rightarrow 0$ unless

also take into account parity of Y_ℓ^m $\longrightarrow$ $\Delta \ell = 0, \pm 2$

$\Delta m = 0, \pm 1, \pm 2$

take into account all $\langle f | xy | i \rangle, \langle f | xz | i \rangle, \ldots$

Further expansion: electric octupole, magnetic quadrupole, ...

Analysis
- Because of selection rules Γ_{DM} and Γ_{QE} never compete with Γ_{DE}
- Γ_{DM} & Γ_{QE} can be separated by observing $\Delta \ell = \pm 2$ transitions $\Rightarrow$ e.g. 557,7 nm line of atomic oxygen

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Back to electric dipole approximation and absorption, $\Rightarrow$

define an absorption cross-section σ_{abs} $\Rightarrow$

$$\sigma_{abs} = \frac{\text{Energy per unit time, absorbed by the atom}}{\text{Energy flux of the radiation field}} =$$

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energy per area per unit time

$$= \frac{\hbar\omega\, P_{i\to f}}{cU} = \frac{\hbar\omega \left(\frac{2\pi}{\hbar}\right)\left(\frac{e}{mc}\right)^2 |A_0|^2 m^2 \omega_{fi}^2 |\langle f|z|i\rangle|^2}{\frac{1}{2\pi}\frac{\omega^2}{c}|A_0|^2} =$$

$cU = \frac{1}{2\pi}\frac{\omega^2}{c}|A_0|^2$

energy density

energy absorbed by atom, $\underline{E_f = E_i + \hbar\omega}$

assume $\delta(E_f - E_i - \hbar\omega)$

$\omega_{fi} \approx \omega$

$$= 4\pi^2 \underbrace{\frac{e^2}{\hbar c}}_{=\alpha} \omega_{fi} |\langle f|z|i\rangle|^2 \delta(\omega_{fi} - \omega)$$

α ← fine-structure constant

Define <u>oscillator strength</u> $f_{fi} = \frac{2m\omega_{fi}}{\hbar}|\langle f|z|i\rangle|^2$

determines the strength of the transition

$$\sum_f f_{fi} = 1$$

all possible final states

Thomas - Reiche - Kuhn sum rule

show! (HW)