

In[30]:= **\$Assumptions = {m > 0, R > 0, R > 2 m}**

Out[30]= {m > 0, R > 0, R > 2 m}

Do the integral for outside of the horizon

In[31]:= **Int1 = Integrate** $\left[\frac{-1}{\sqrt{\frac{2m}{r} * \left(1 - \frac{2m}{r}\right)}} + \frac{\sqrt{\frac{2m}{r}}}{\left(1 - \frac{2m}{r}\right)}, \{r, R, 2 m\}\right]$

Out[31]= $-\frac{4 m}{3} + \frac{\sqrt{2} R^{3/2}}{3 \sqrt{m}}$

In[32]:= **FullSimplify** $\left[-\frac{4 m}{3} + \frac{\sqrt{2} R^{3/2}}{3 \sqrt{m}}\right]$

Out[32]= $-\frac{4 m}{3} + \frac{\sqrt{2} R^{3/2}}{3 \sqrt{m}}$

Take the limit as R→0

In[33]:= **Limit** $[-\text{Int1}, R \rightarrow 0]$

Out[33]= $\frac{4 m}{3}$

Do the integral

In[34]:= **Int = Integrate** $\left[\frac{-1}{\sqrt{\frac{2m}{r} - 1}}, \{r, 2 m, 0\}\right]$

Out[34]= $m \pi$

Define the constants

In[35]:= **G = 6.674 * 10 ^ (-11)**

c = 2.88 * 10 ^ (8)

m = 2 * 10 ^ (30)

Out[35]= 6.674×10^{-11}

Out[36]= 2.88×10^8

Out[37]= 2 000 000 000 000 000 000 000 000 000 000 000

Calculate the total time for the upper bound in seconds

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In[38]:= (Int * G) / (c ^ 2)
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Out[38]= 5055.7
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