

Comparison of Bessel and Gaussian beams

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Received July 20, 1987; accepted October 26, 1987

A comparison of beam divergence and power-transport efficiency is made between Gaussian and Bessel beams when both beams have the same initial total power and the same initial full width at half-maximum.

Recently we reported some experimental observations¹ that verified the theoretical predictions by Durnin² on the depth of field of Bessel modes. Some of the features of these modes are strikingly different from corresponding features of the well-known Gaussian modes of the paraxial wave equation. For example, in their ideal form these modes represent perfectly diffraction-free waves with a sharply defined central maximum. Here diffraction-free means that the wave amplitude $\psi(x, y, z, t)$ has the property

$$\psi(x, y, z, t) = \exp[-i(\omega t - \beta z)]\psi(x, y, z = 0, t = 0), \quad (1)$$

so that the time-averaged wave intensity satisfies

$$I(x, y, z) = |\psi(x, y, z, t)|^2 = |\psi(x, y, z = 0, t = 0)|^2 \quad (2)$$

That is, according to Eq. (1) the wave travels in the z direction, and according to Eq. (2) the transverse intensity distribution is independent of the propagation distance z .

The simplest diffraction-free beam solution is²

$$\psi(x, y, z = 0, t = 0) = J_0(\alpha\rho), \quad (3)$$

where J_0 is the zero-order Bessel function of the first kind, $\rho = \sqrt{(x^2 + y^2)}$, α and β are real constants that satisfy $\alpha^2 + \beta^2 = (\omega/c)^2$, and c is the speed of light. The effective width of the beam is proportional to $1/\alpha$ and can be extremely small, of the order of one wavelength (FWHM). We have shown¹ that finite aperture approximations to a Bessel beam are easily realizable in the laboratory by using only simple optical elements.

Although Bessel beams are diffraction-free in the sense defined above, they are not so well localized as Gaussian beams. Each lobe (i.e., the area between two successive zeros) of a Bessel beam carries approximately the same energy as the central spot, and thus a Bessel beam with 20 lobes, for example, contains only about 5% of the total energy within the central spot. This property of Bessel beams appears strongly disadvantageous when questions of power transport are considered. In comparison with Gaussian beams,

which contain 50% of their total energy within their FWHM, it appears that a Bessel beam wastes most of its energy off axis.

The purpose of this Letter is to explain that this conclusion, although apparently obvious, is not at all correct. We present order-of-magnitude calculations that compare the properties of Bessel and Gaussian beams with respect to both beam divergence and power transport efficiency. The comparisons are based on observations made at a plane $z = d$ given the following two initial constraints on field distribution in the plane $z = 0$: same initial total power and same initial FWHM.

Consider an aperture of radius R at $z = 0$ and an observation plane at $z = d$ having a circle of radius a upon it, as shown in Fig. 1. We shall assume that the field distributions for the Gaussian and Bessel beams in the plane $z = 0$ are of the form

$$E(x, y, z = 0) = \xi_G \exp(-\rho^2/2\sigma^2) \quad \text{Gaussian}, \quad (4)$$

$$E(x, y, z = 0) = \xi_B J_0(\alpha\rho) \quad \text{Bessel} \quad (5)$$

for all $\rho < R$, and zero for all $\rho > R$. The total initial power contained in each beam is given by

$$P_T = \left(\frac{c}{2\pi}\right) \frac{\pi}{2} \xi_G^2 \sigma^2 \quad \text{Gaussian}, \quad (6)$$

$$P_T = \left(\frac{c}{2\pi}\right) \xi_B^2 \frac{R}{\alpha} \quad \text{Bessel}, \quad (7)$$

assuming that $R \gg \sigma$, $1/\alpha$ (i.e., the aperture radius R is large compared with the central spot sizes of the two beams).

We now pose the following question: Given the wavelength λ , the total initial power P_T , and a propagation distance d , what is the maximum possible fraction F of total initial power P_T that can be transported to the disk of radius a ? The case of greatest practical interest, and that which we shall consider, is when $d \gg a > \lambda$. The parameters to be optimized are ξ_G and σ for the Gaussian beam and ξ_B and R for the Bessel beam.

Because of the finite aperture radius R , the Bessel beam will have a propagation range given by²

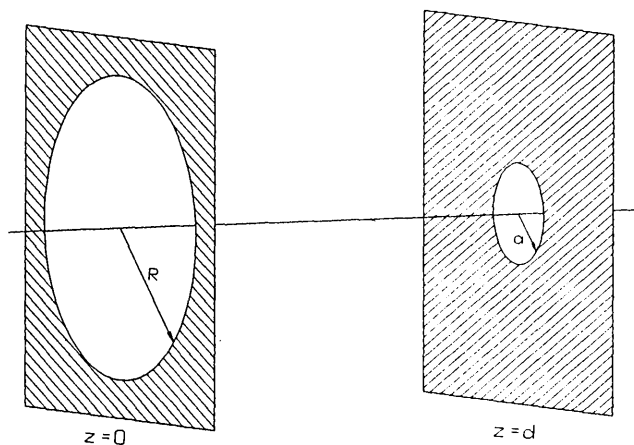


Fig. 1. Aperture geometry. Both beams are assumed to be confined to a finite aperture of radius R in the plane $z = 0$. A disk of radius a located in the plane $z = d$ defines the area over which the beam intensity is to be integrated.

$$Z_{\max} = R[(\kappa/\alpha)^2 - 1]^{1/2}, \quad (8)$$

provided that $\kappa > \alpha > 2\pi/R$, where $\kappa = 2\pi/\lambda$ is the wave number. Over that distance Z_{\max} the central spot radius of the Bessel beam will remain fixed, while the peak intensity oscillates slightly about its initial value. We shall now choose the central spot radius of the Bessel beam to equal the disk radius a so that the fraction of total power contained within that disk at the plane $z = d$ is approximately the same as in the initial plane, namely,

$$F \approx aR \approx 1/(1 + 4N/3), \quad (9)$$

where N is the number of rings in the Bessel function within the aperture radius R . In terms of a , the maximum propagation range [Eq. (8)] of the Bessel function is $Z_{\max} \approx \pi Ra/\lambda$, and we therefore find that if we choose the aperture radius R to be

$$R = \lambda d/\pi a, \quad (10)$$

the fraction F of total initial power P_T that is contained within the disk of radius a becomes

$$F \approx \pi a^2/\lambda d. \quad (11)$$

This also determines the Bessel beam amplitude to be given by

$$\xi_B^2 = (4\pi/c)(P_T/Ra). \quad (12)$$

In the case of the Gaussian beam, one can easily show that both the peak intensity at $z = d$ and the fractional power contained within a disk of radius a located at $z = d$ will be maximized by choosing

$$\sigma^2 = d/\kappa, \quad (13)$$

$$\xi_G^2 = (8\pi/c)(P_T/\lambda d), \quad (14)$$

regardless of the value of a . The maximum fraction of total initial power contained within the disk of radius a at $z = d$ is then found to be

$$F = 1 - \exp(-\pi a^2/\lambda d) \approx \pi(a^2/\lambda d). \quad (15)$$

Thus, just as in the case of the Bessel beam, we find that fractional power incident upon the disk is approximately π times the Fresnel number of the disk, provided that the Fresnel number is much less than 1. (When the Fresnel number is comparable with or greater than 1 the effects of diffraction are sufficiently negligible that a plane wave of radius a suffices for power transport.) With regard to beam divergence, the Gaussian beam spreads by a factor of $\sqrt{2}$, while the central maximum of the Bessel beam does not spread at all.

Let us now consider the case in which the Bessel and Gaussian beams are assumed to have the same FWHM in the plane $z = 0$, i.e., $\sigma^2 \approx a^2$. If we define the range of the Gaussian beam as the distance z at which its peak intensity falls by a factor of 2, we find that

$$\text{Bessel range} \approx N \times \text{Gaussian range}, \quad (16)$$

where N is again the total number of rings in the Bessel function in the plane $z = 0$. However, since the total integrated power contained in each ring is approximately the same as that contained in the central maximum of the Bessel beam, we also find that

$$\text{total Bessel power} \approx N \times \text{total Gaussian power}. \quad (17)$$

Increased depth of field is therefore obtained at the expense of power.

In conclusion, we have shown that (1) comparable efficiencies in power transport are obtained with optimized Gaussian and Bessel beams in practical situations (i.e., when the Fresnel number of the target is much smaller than unity) and (2) the depth of field of a Bessel beam can be made arbitrarily larger than that of a Gaussian beam having the same spot size (FWHM), but at the expense of power. The apparent advantage of greater localization in Gaussian beams, relative to that found in Bessel beams, is therefore nonexistent.

We acknowledge support from the National Science Foundation and from Research Corporation. J. H. Eberly is also with the Institute of Optics, University of Rochester.

References

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