3. Define a matrix $M \in \mathcal{M}_3(\mathrm{R})$ that represents the coefficients of integration for each of our l_i functionals. Then create three column vectors to represent the results of applying this matrix to our basis for $P_2(R)$

In[25]:=
```
M = {{Integrate[1, {x, 0, 1}], Integrate[x, {x, 0 , 1}], Integrate[x^2, {x, 0, 1}]},
   {Integrate[1, {x, 0, 2}], Integrate[x, {x, 0 , 2}], Integrate[x^2, {x, 0, 2}]},
   {Integrate[1, {x, 0, -1}], Integrate[x, {x, 0 , -1}],
    Integrate[x^2, {x, 0, -1}]}};
e1 = {1, 0, 0};
e2 = {0, 1, 0};
e3 = {0, 0, 1} ;
MatrixForm[M]
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 2 & \frac{8}{3} \\ -1 & \frac{1}{2} & -\frac{1}{3} \end{pmatrix}$$

Solve the equation matrix equation $Mb_i = e_i$ for each of our new basis vectors b_i

```
b1 = LinearSolve[M, e1]
```

Out[18]= $\left\{1, 1, -\frac{3}{2}\right\}$

```
b2 = LinearSolve[M, e2]
```

Out[19]= $\left\{-\frac{1}{6}, 0, \frac{1}{2}\right\}$

```
b3 = LinearSolve[M, e3]
```

Out[20]= $\left\{-\frac{1}{3}, 1, -\frac{1}{2}\right\}$

Now we check that these vectors are linearly independent (and therefore form a basis)

In[30]:=
```
Det[{b1, b2, b3}]
```

Out[30]= 1

---

4. Row reduce the R^4 representations of matrices A, B, C, and D to decide if they are linearly independent.

In[21]:=
```
K = {{1, 1, 0, 1}, {1, 0, 1, 3}, {0, 1, 1, 0}, {1, 1, 0, 1}};
MatrixForm[K]
```

Out[22]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

In[24]:=
```
MatrixForm[RowReduce[K]]
```

Out[24]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore, we can see that A, B, C are linearly independent but D=2A-B+C