John Waczak

9) suppose $f:[a,b]\to\mathbb{R}$, $g:[a,b]\to\mathbb{R}$ continuous

prove $\int_a^b |f+g| \leq \int_a^b |f| + \int_a^b |g|$

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observe that because f, g continuous on $[a,b]$ $\exists \int_a^b f, \int_a^b g$. By linearity of integration we have that $\exists \int_a^b f+g$.

Now observe that $\forall \alpha, \beta \in \mathbb{R}$ $\alpha \neq \beta$ by triangle inequality we know

$$|\alpha+\beta| \leq |\alpha| + |\beta|$$

Since f and g are real valued we can say $|f+g| \leq |f| + |g|$ $\forall x \in [a,b]$

all the absolute value does is make the functions strictly positive so $|f+g|, |f|, |g|$ are continuous and therefore int'able on $[a,b]$.

$\therefore$ by Corollary 1 (page 117) we have that

$|f+g|$ and $|f|+|g|$ are int'able on $[a,b]$

and $|f+g| \leq |f|+|g|$ so

$$\int_a^b |f+g| \leq \int_a^b |f|+|g| = \int_a^b |f| + \int_a^b |g|$$

∎