(e) Calculate the expectation value ⟨z⟩ as a function of time. Do you expect this answer?

To solve this, first we will set up the definition for the time dependent wavefunction and then will use this to evaluate the weighted inner product to find ⟨z⟩ .

In[1]:=
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$Assumptions = {r, n, a0, t, h, ϵ, μ} ∈ Reals
Z = 1
```

Out[1]= (r | n | a0 | t | h | ϵ | μ) ∈ Reals

Out[2]= 1

In[3]:=
```
ρ[n_] := Z r / (n a0);
R[n_, l_, r_] := -Sqrt[(2 Z / (n a0)) ^3 (n - l - 1) ! / (2 n ((n + l) !)) ^3]
  Exp[-ρ[n]] (2 * ρ[n]) ^l LaguerreL[n + l, 2 l + 1, 2 ρ[n]]
Y[l_, m_, θ_, ϕ_] := SphericalHarmonicY[l, m, θ, ϕ]
En[n_] := -1 / (2 * n^2) * (Z * Exp[2] / (4 * π * ϵ)) ^2 * μ / h^2
T[n_, t_] := Exp[-I * En[n] / h * t]

Ψket[n_, l_, m_] := R[n, l, r] * Y[l, m, θ, ϕ] * T[n, t]
ψ[r_, θ_, ϕ_, t_] := (1 / Sqrt[2]) * (Ψket[2, 0, 0] + Ψket[2, 1, 0])
ψ[r, θ, ϕ, t]
```

Out[10]=
$$\frac{-\frac{\sqrt{\frac{1}{a0^3}}\, e^{-\frac{r}{2\, a0}+\frac{i\, e^4\, t\, \mu}{128\, h^3\, \pi^2\, \epsilon^2}}\left(6\, a0^2-6\, a0\, r+r^2\right)}{64\, a0^2\sqrt{2\,\pi}}-\frac{\left(\frac{1}{a0^3}\right)^{3/2} e^{-\frac{r}{2\, a0}+\frac{i\, e^4\, t\, \mu}{128\, h^3\, \pi^2\, \epsilon^2}}\, r\left(120\, a0^3-90\, a0^2\, r+18\, a0\, r^2-r^3\right)\mathrm{Cos}[\theta]}{576\, a0\sqrt{2\,\pi}}}{\sqrt{2}}$$

In[11]:=
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Integrate[Conjugate[ψ[r, θ, ϕ, t]] * ψ[r, θ, ϕ, t] * r^3 * Cos[θ] * Sin[θ],
 {r, 0, Infinity}, {θ, 0, π}, {ϕ, 0, 2 π}]
```

Out[11]= $\mathrm{ConditionalExpression}\left[-\frac{5}{64}\, a0\, \mathrm{Sign}[a0]^3,\ a0>0\right]$

Which we can see is not time dependent (as expected). This makes sense as both of the basis states have the same n value and therefore the same energy. Since the time dependence depends exclusively on the energy of the state, when normalized, the norm squared of the n-energy turns to 1, removing any time dependence.