Central Forces Homework 3

Due 5/16/18, 4 pm

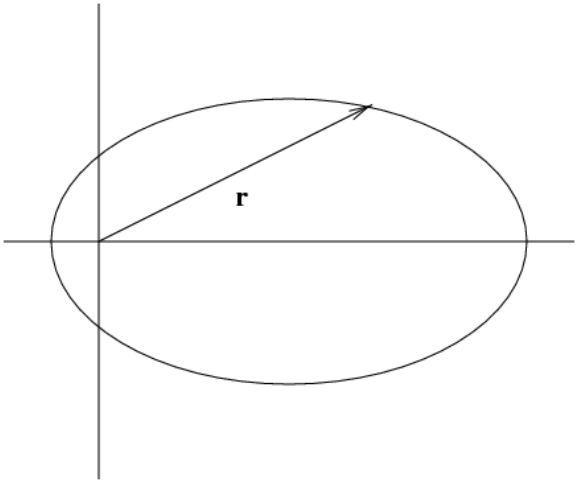
Sensemaking: For every problem, before you start the problem, make a brief statement of the form that a correct solution should have, clearly indicating what quantities you need to solve for. This statement will be graded.

PRACTICE:

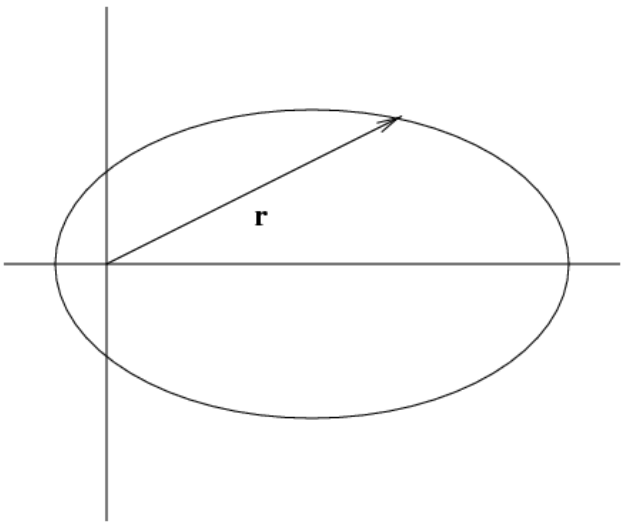
1. Show that the plane polar coordinates we have chosen are equivalent to spherical coordinates if we make the choices:
   (a) The direction of z in spherical coordinates is the same as the direction of $\vec{L}$.
   (b) The θ of spherical coordinates is chosen to be $\pi/2$, so that the orbit is in the equatorial plane of spherical coordinates.
2. Show that the plane of the orbit is perpendicular to the angular momentum vector $\vec{L}$.

REQUIRED:

3. The figure below shows the position vector $\mathbf{r}$ and the orbit of a "fictitious" reduced mass.
   (a) Assuming that $m_2 = m_1$, draw on the figure the position vectors for m_1 and m_2 corresponding to $\mathbf{r}$. Also draw the orbits for m_1 and m_2. Describe a common physics example of central force motion for which $m_1 = m_2$.



   (b) Repeat the previous problem for $m_2 = 3m_1$.



4. Consider a system of two particles.

   (a) Show that the total kinetic energy of the system is the same as that of two "fictitious" particles: one of mass $M = m_1 + m_2$ moving with the speed of the CM (center of mass) and one of mass μ (the reduced mass) moving with the speed of the relative position $\vec{r} = \vec{r}_2 - \vec{r}_1$.

   (b) Show that the total angular momentum of the system can be similarly decomposed into the angular momenta of these two fictitious particles.

5. The general equation for a straight line in polar coordinates is given by:

$$r(\phi) = \frac{r_0}{\cos(\phi - \delta)}$$

   Find the polar equation for the following straight lines:

   (a) $y = 3$

   (b) $x = 3$

   (c) $y = -3x + 2$