Advanced Learning Algorithms

Welcome!

Advanced learning algorithms

Neural Networks 
  inference (prediction)
  training
Practical advice for building machine learning systems 
Decision Trees 

Neural Networks Intuition

Neurons and the brain

Neural networks

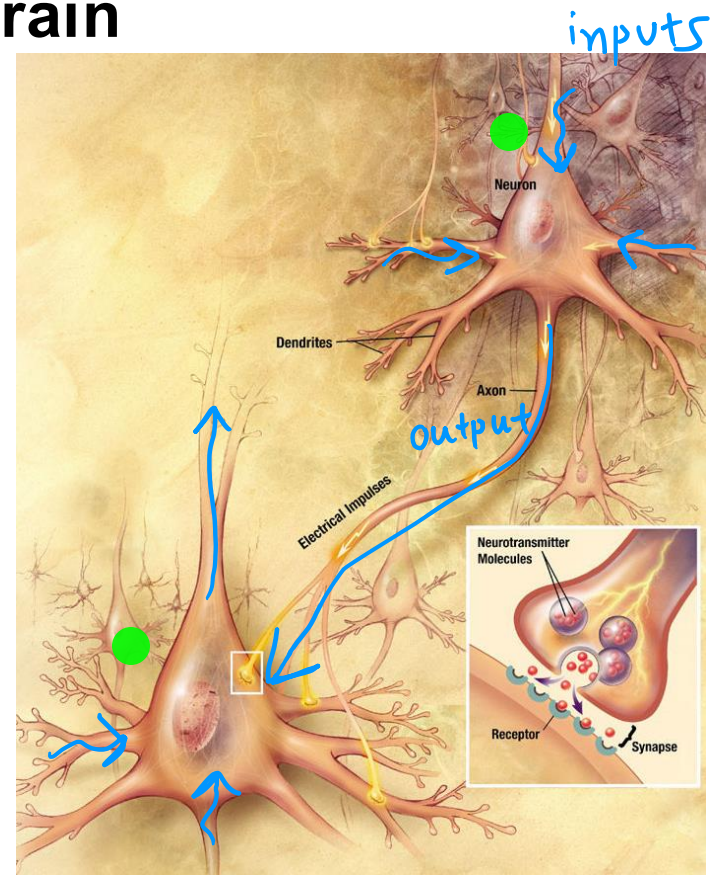
Origins: Algorithms that try to mimic the brain.

Used in the 1980's and early 1990's.
Fell out of favor in the late 1990's.

Resurgence from around 2005.

speech → images → text (NLP) → ...

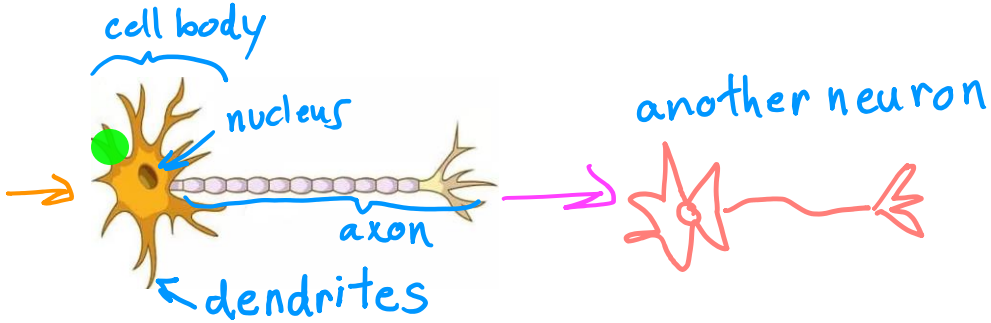
Neurons in the brain



[Credit: US National Institutes of Health, National Institute on Aging]

Biological neuron

inputs outputs



Simplified mathematical model of a neuron

inputs outputs

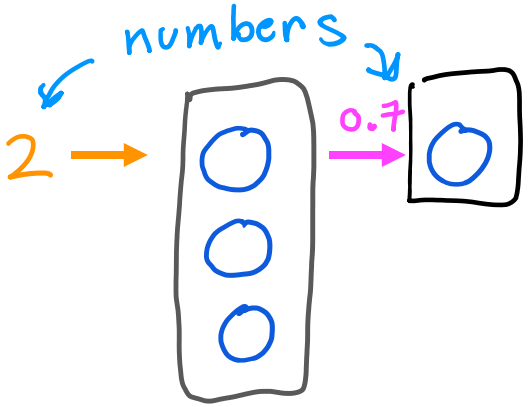
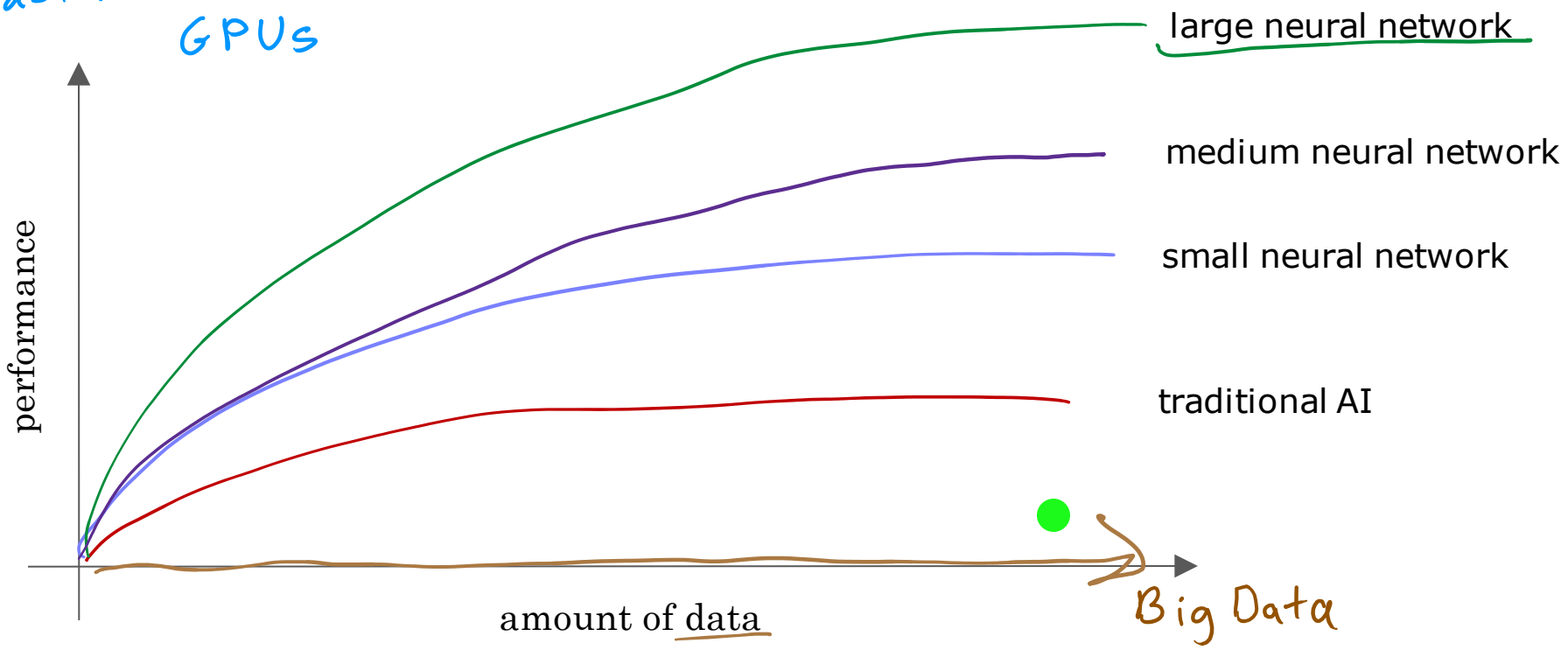


image source: https://biologydictionary.net/sensory-neuron/

Why Now?

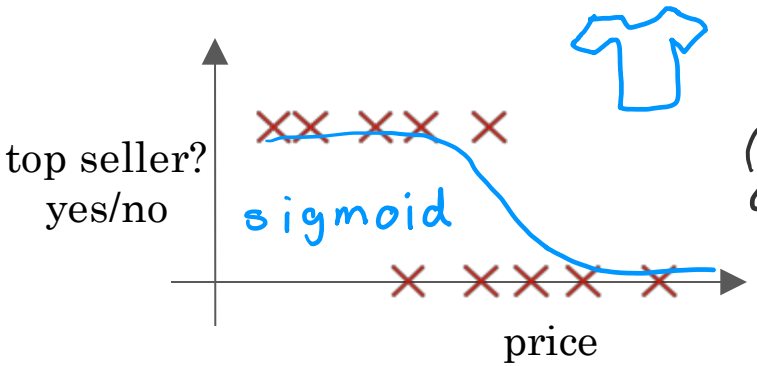
Faster computer processors
GPUs



Neural Network Intuition

Demand Prediction

Demand Prediction



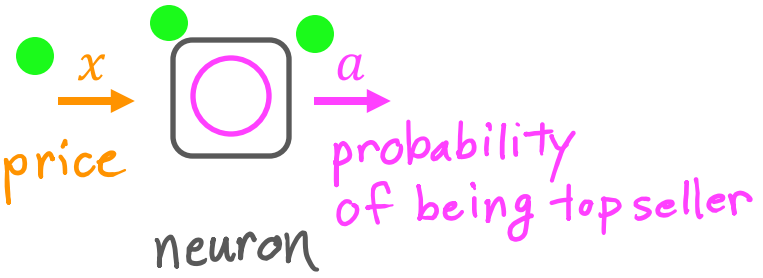
$x = price$

input

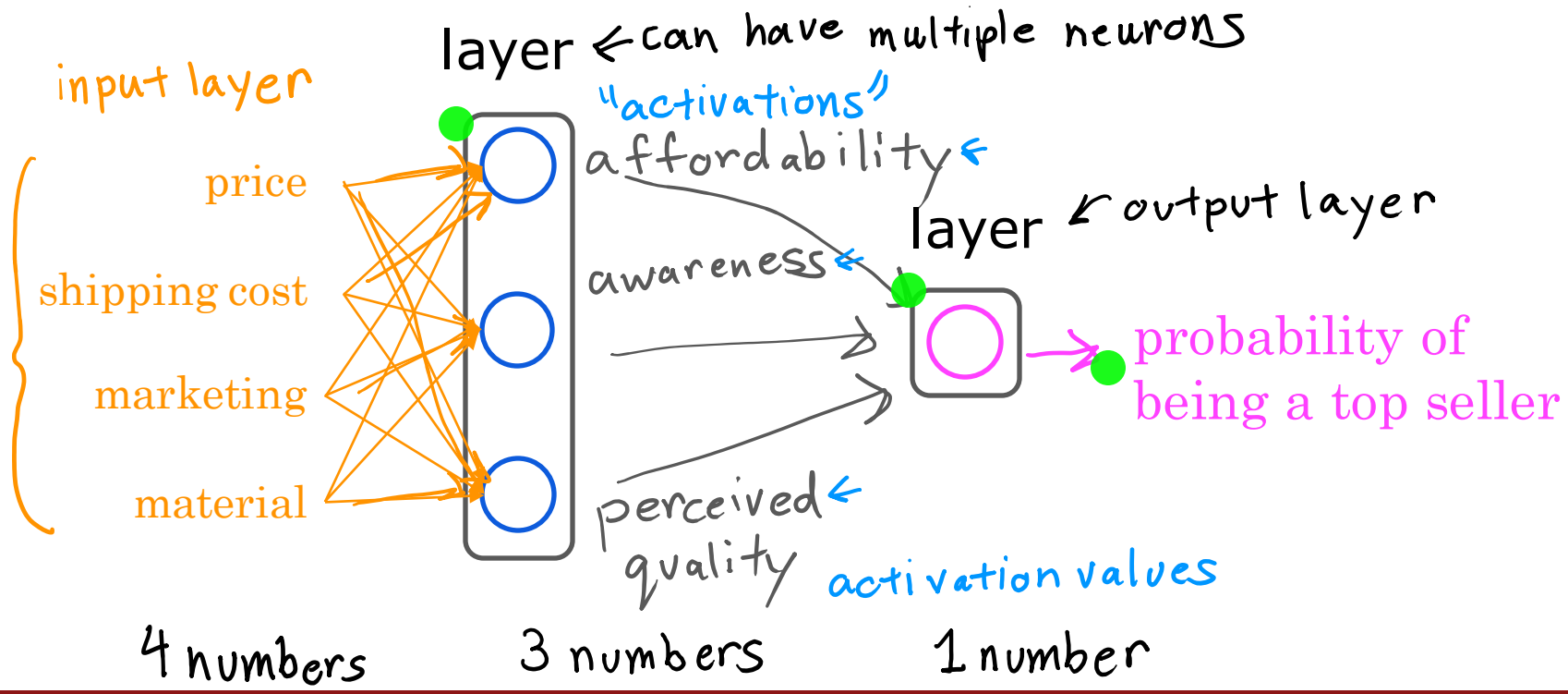
$a = f(x) = \frac{1}{1 + e^{-(wx+b)}}$

output

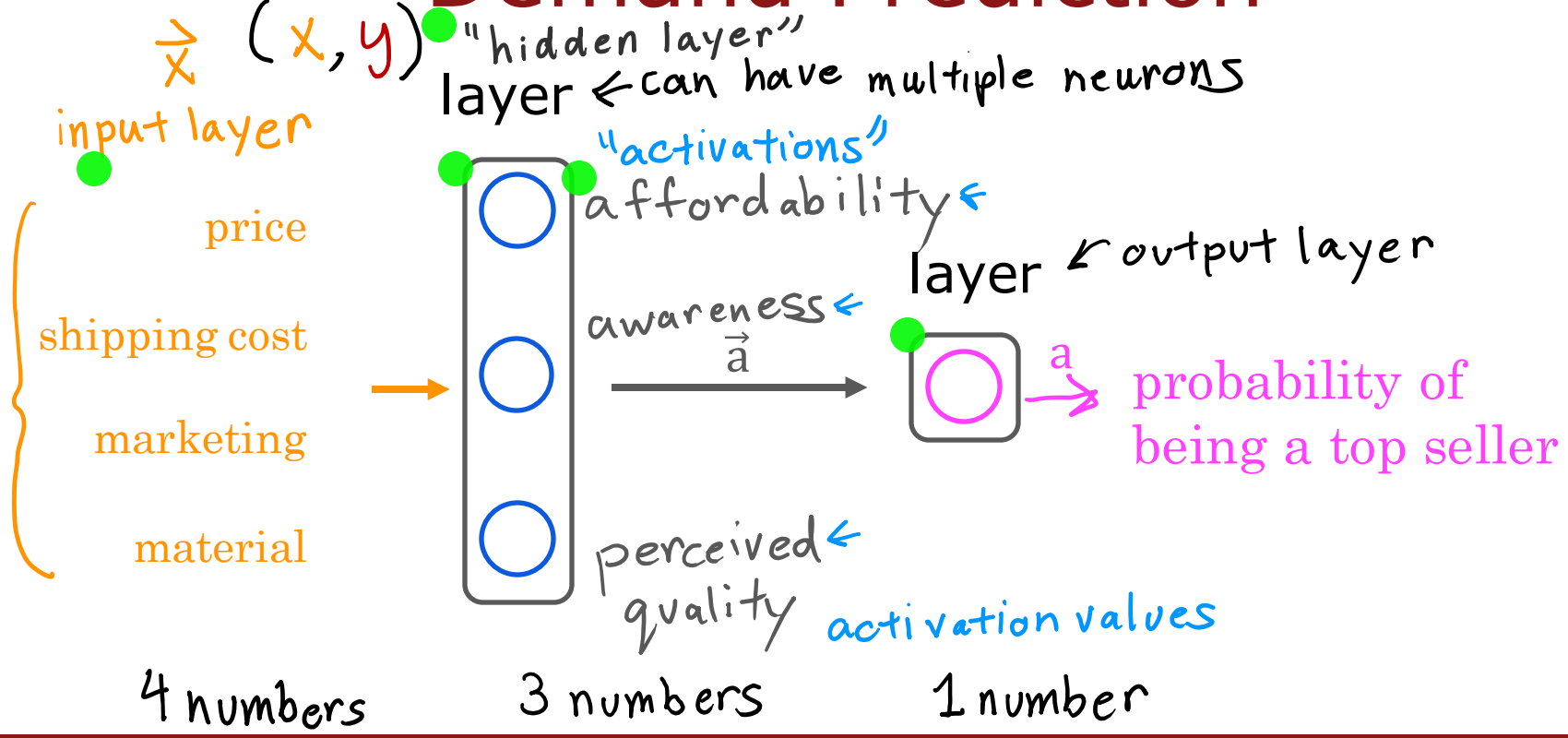
activation



Demand Prediction



Demand Prediction



Demand Prediction

feature engineering

x_1 x_2

$\vec{x}$ (x, y)

input layer

- price
- shipping cost
- marketing
- material

4 numbers

"hidden layer"

layer ← can have multiple neurons

"activations"

affordability ←

awareness ←

perceived quality ←

$\vec{a}$

3 numbers

activation values

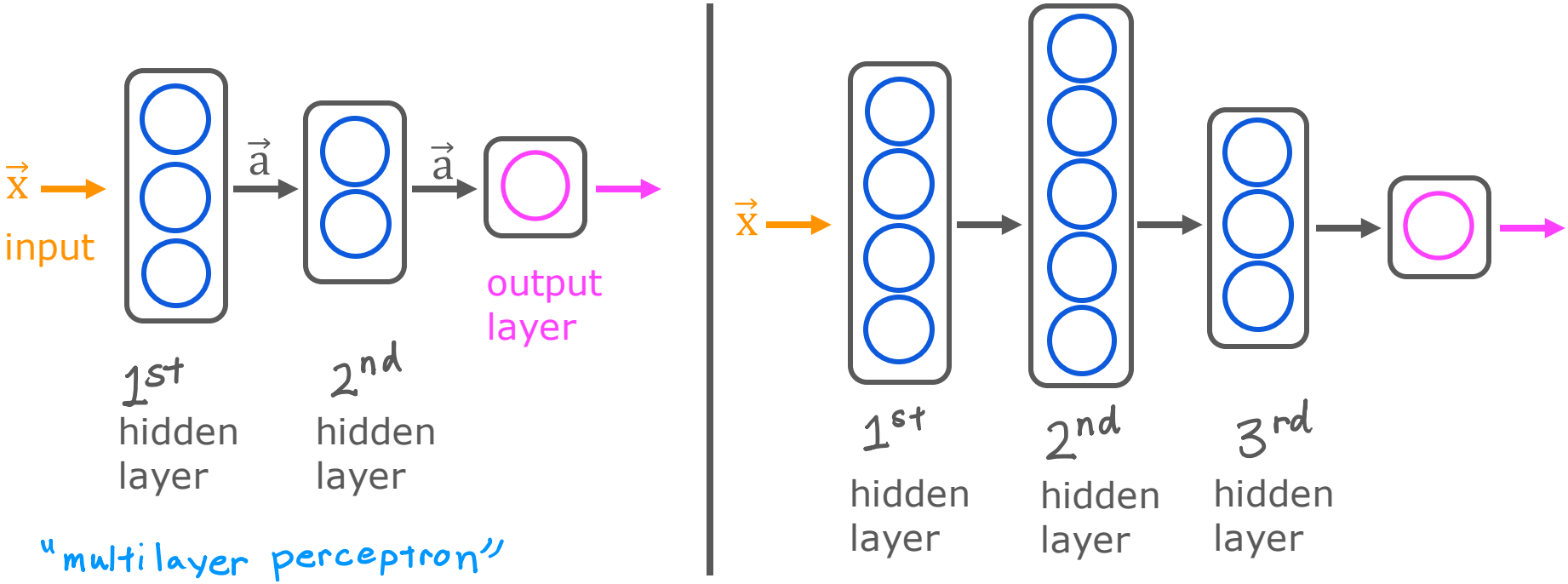
layer ← output layer

a

1 number

probability of being a top seller

Multiple hidden layers

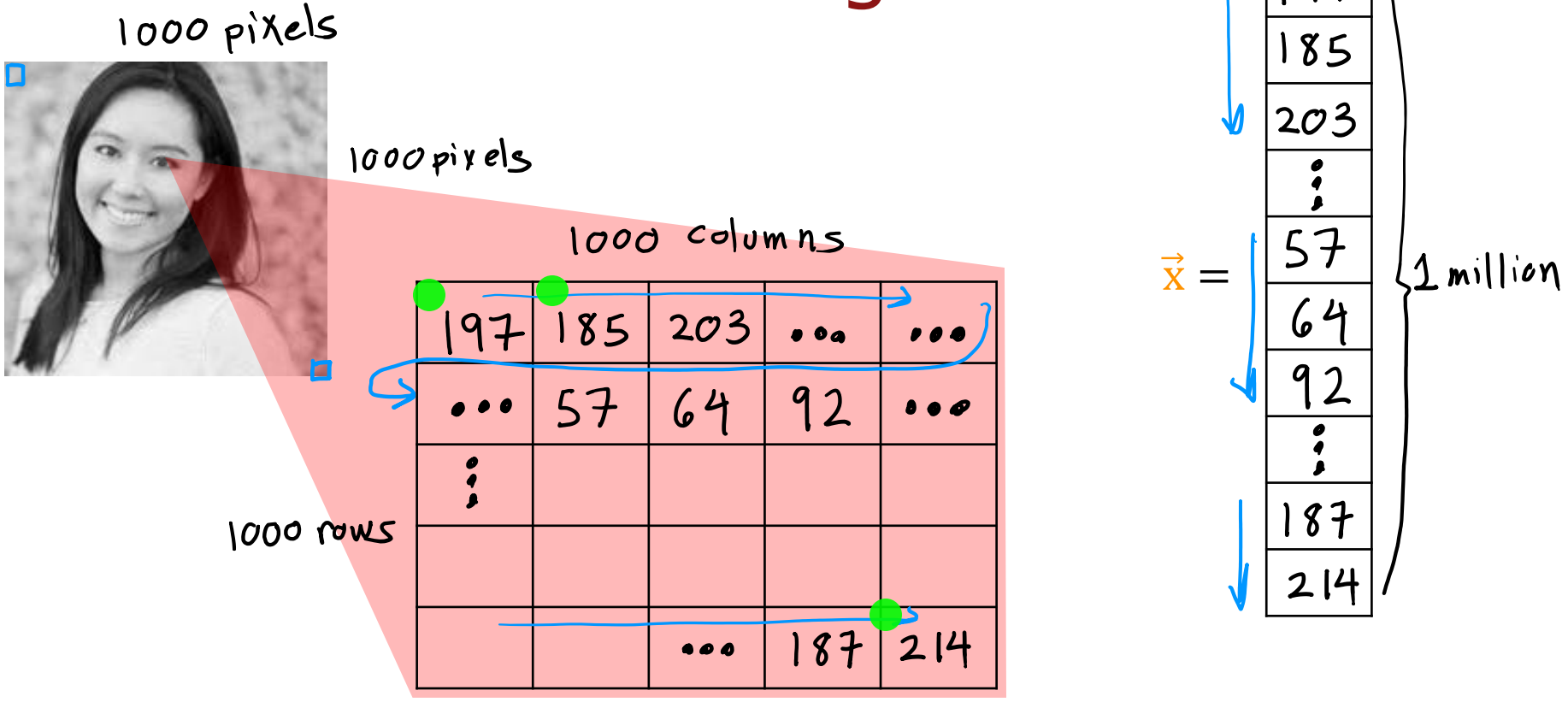


neural network architecture

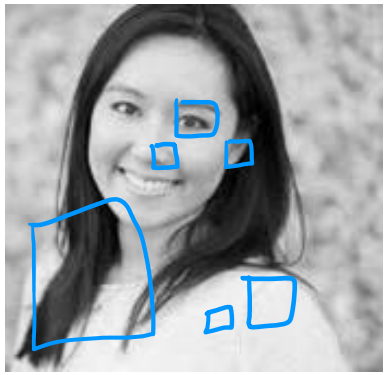
Neural Networks Intuition

Example: Recognizing Images

Face recognition



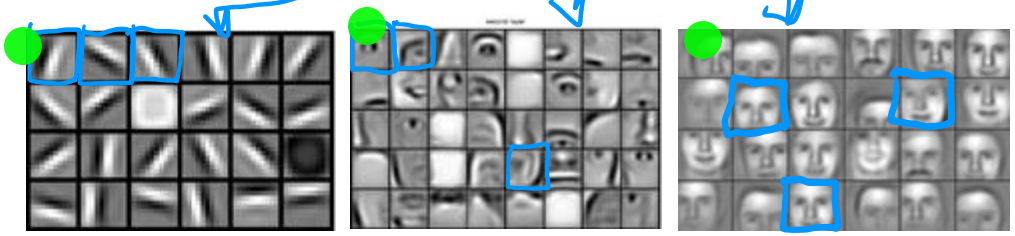
Face recognition



$\vec{x}$
input

output layer

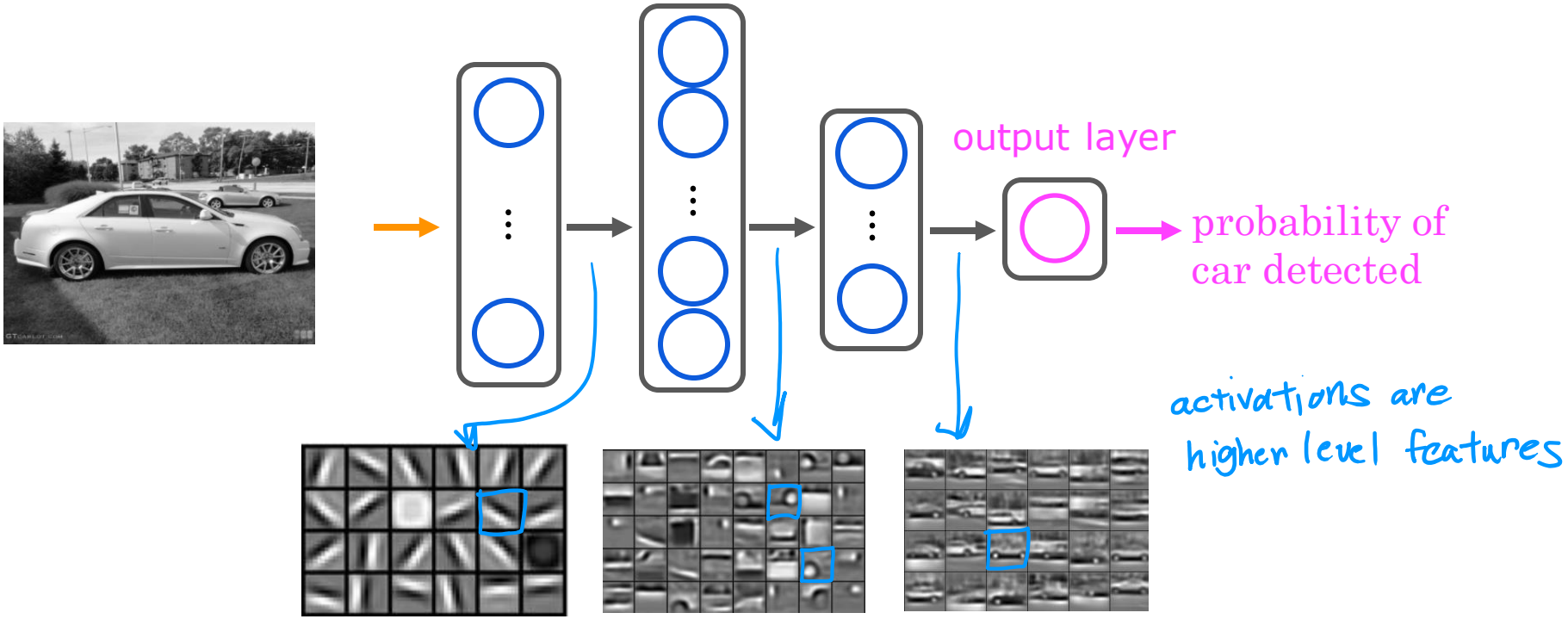
probability of being person 'XYZ'



activations are higher level features

source: Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations by Honglak Lee, Roger Grosse, Ranganath Andrew Y. Ng

Car classification

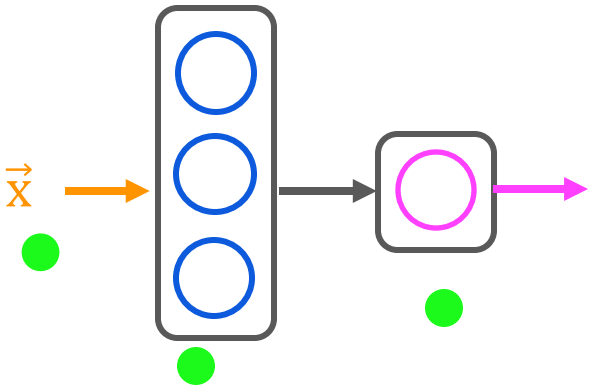


source: Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations by Honglak Lee, Roger Grosse, Ranganath Andrew Y. Ng

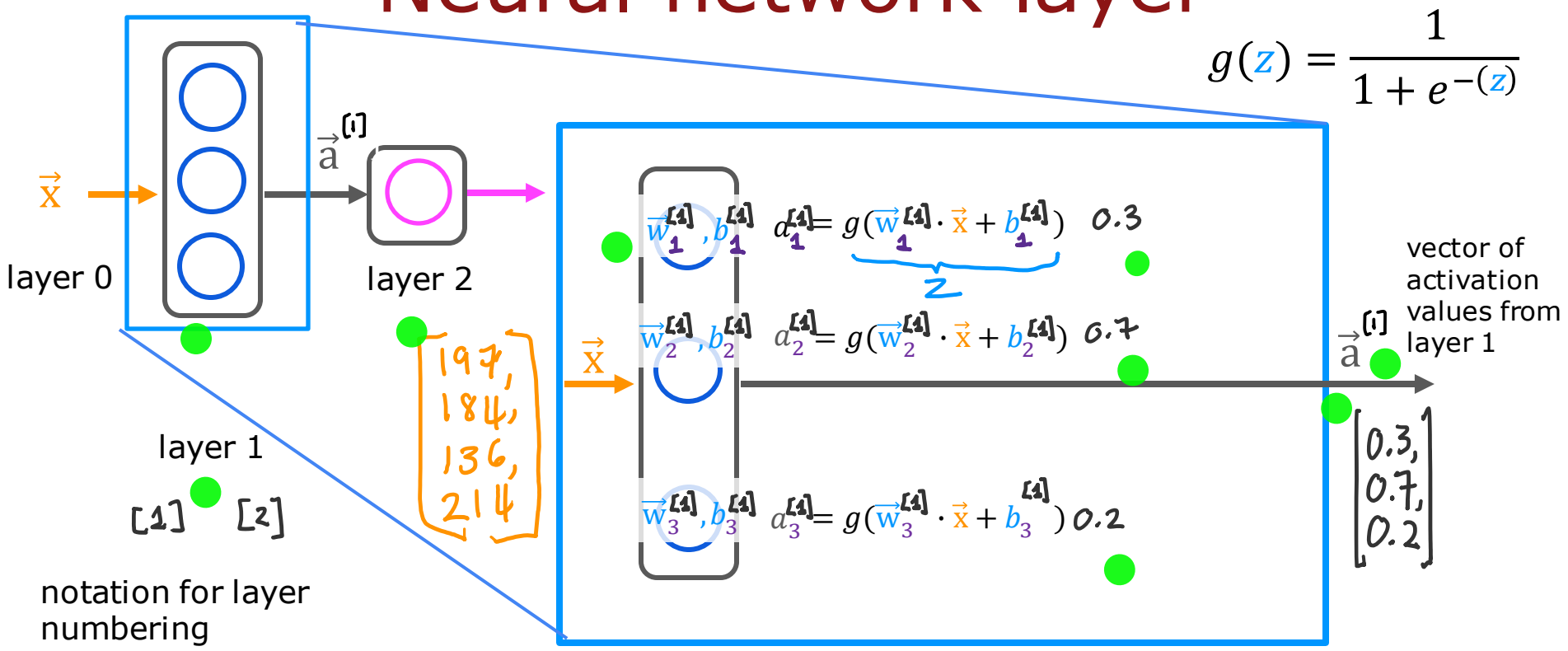
Neural network model

Neural network layer

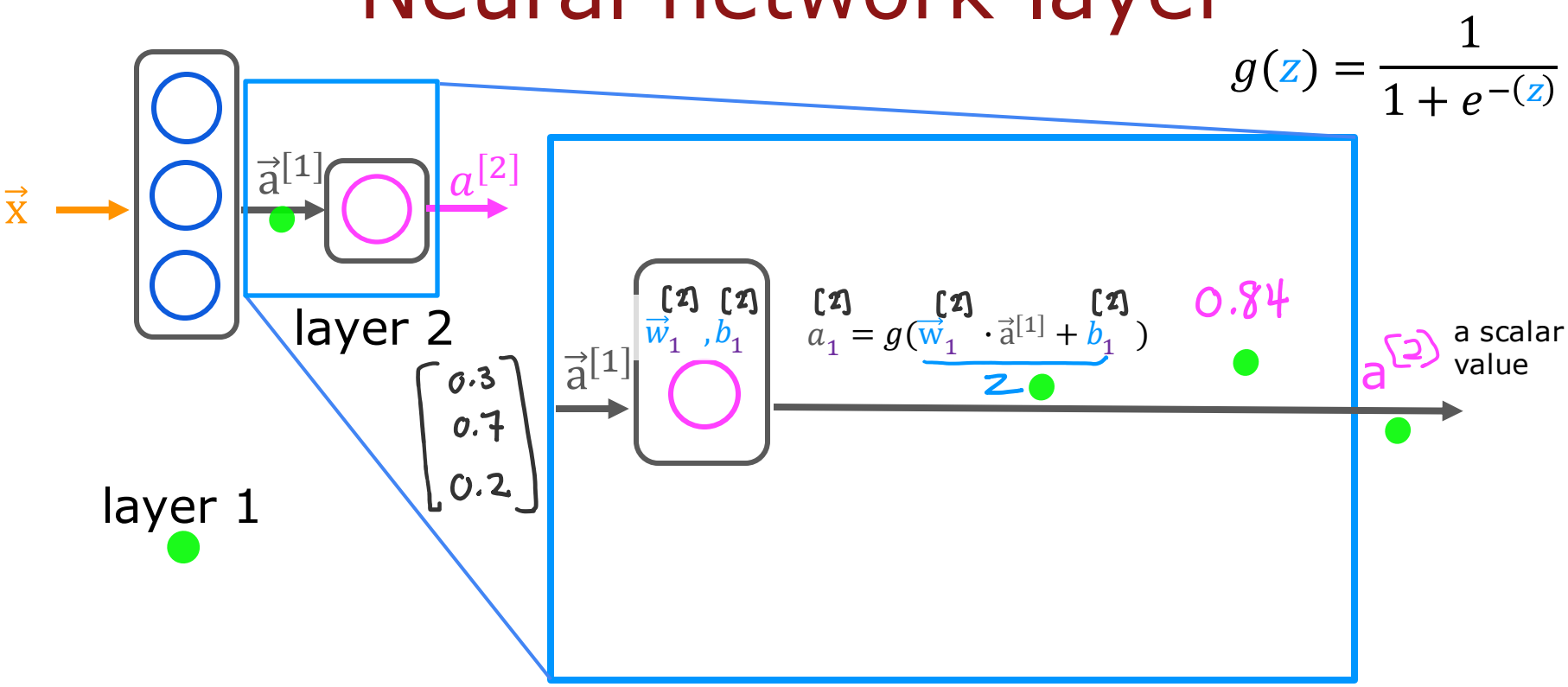
Neural network layer



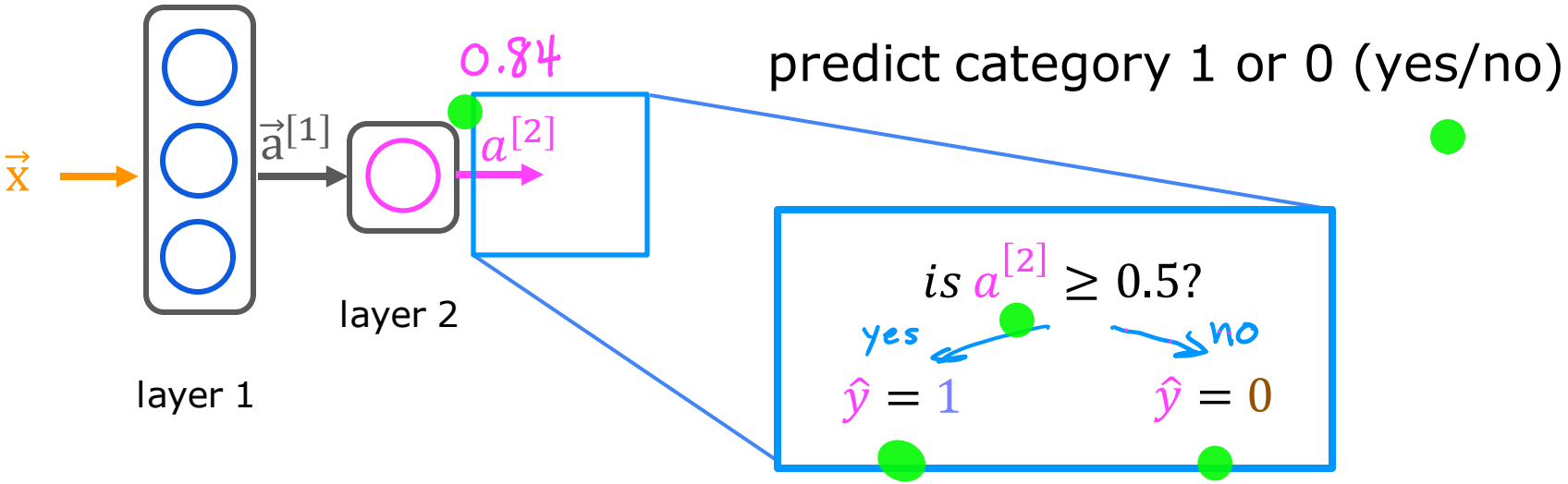
Neural network layer



Neural network layer



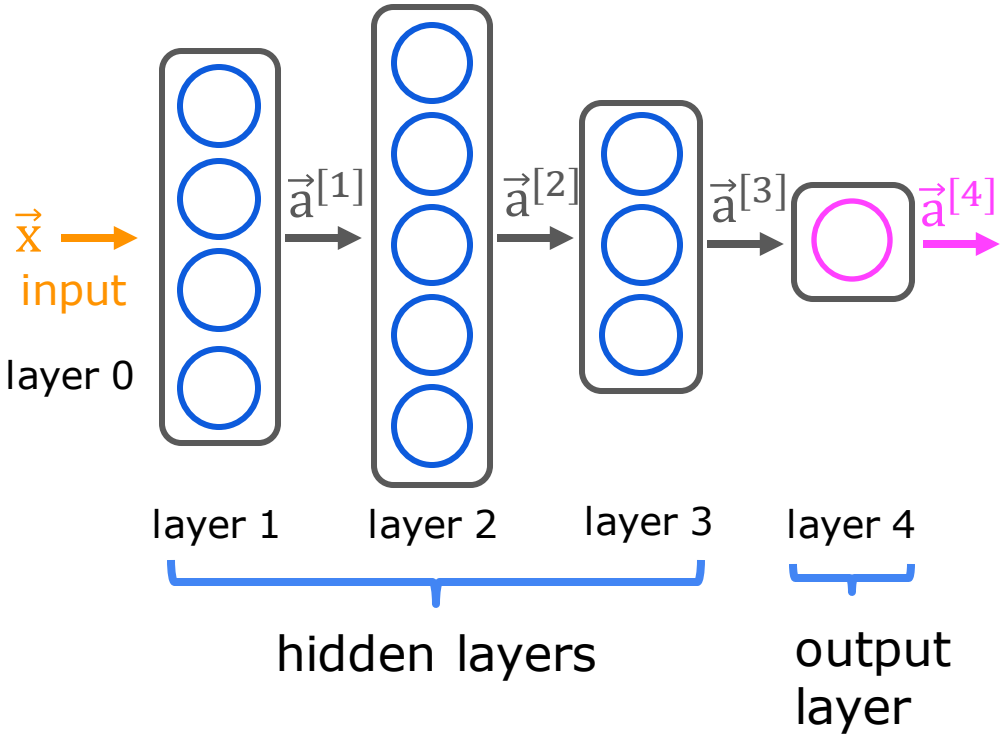
Neural network layer



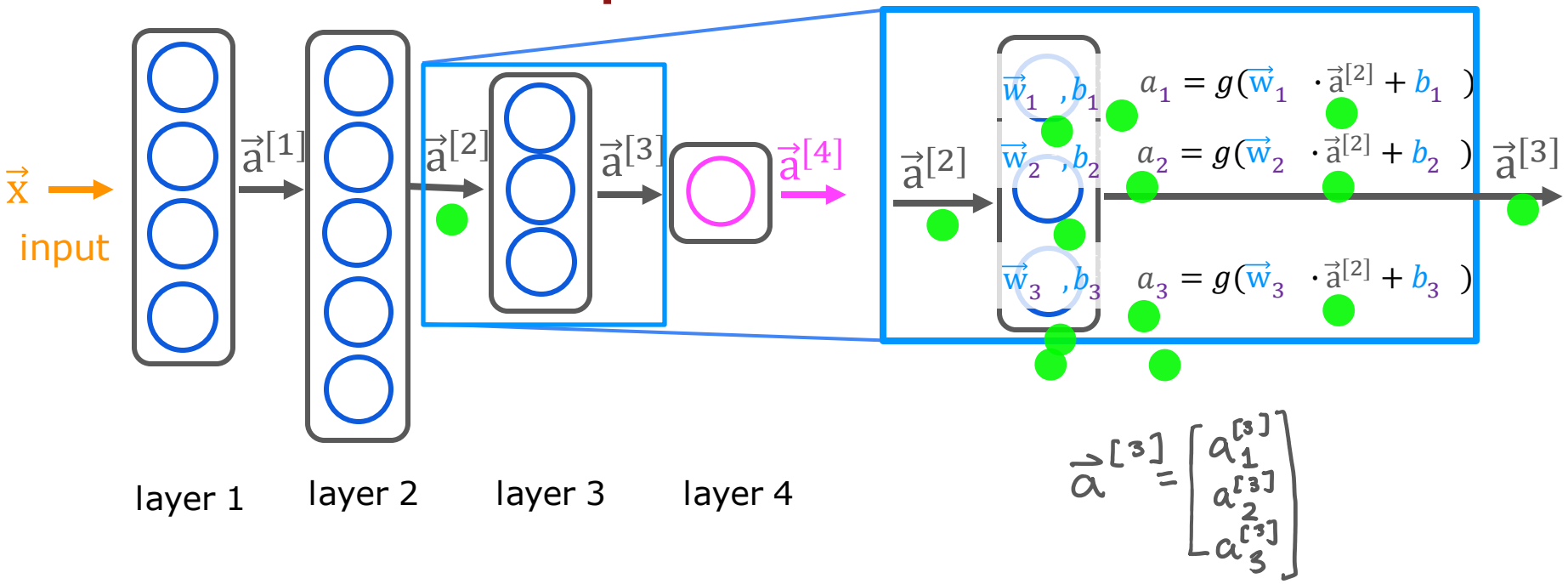
Neural Network Model

More complex neural networks

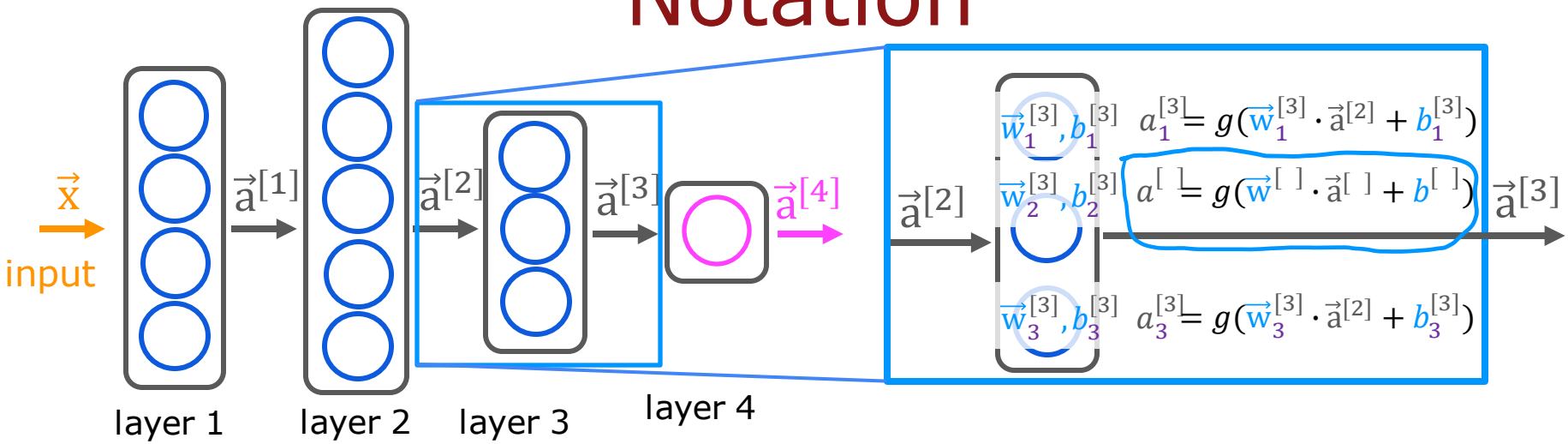
More complex neural network



More complex neural network

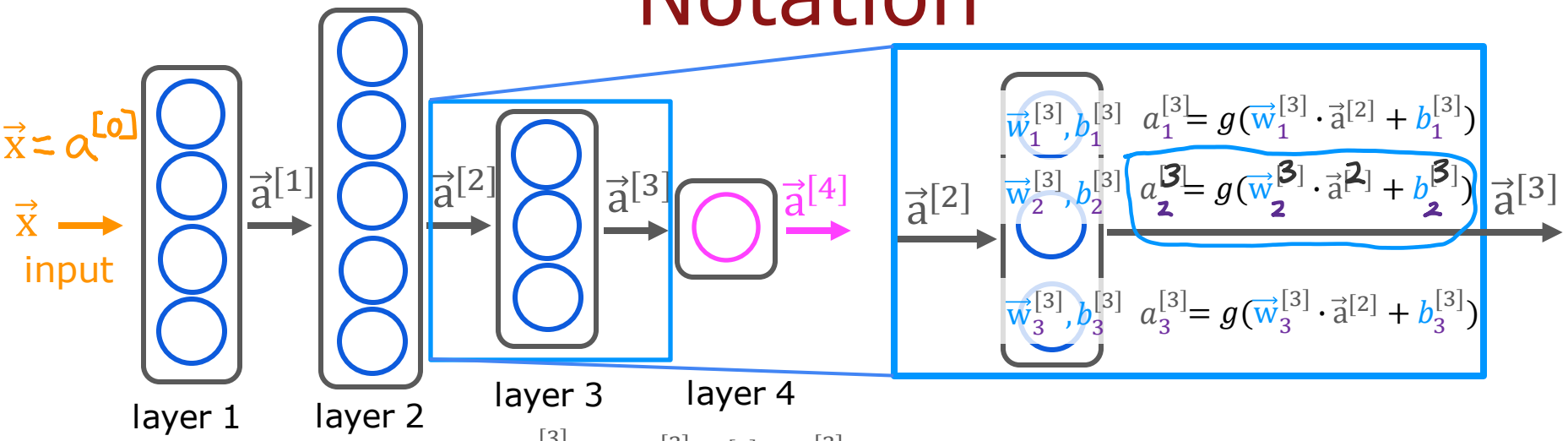


Notation



Question:
Can you fill in the superscripts and subscripts for the second neuron?

Notation



$$a_2^{[3]} = g(\vec{w}_2^{[3]} \cdot \vec{a}^{[2]} + b_2^{[3]})$$

Activation value of layer l, unit(neuron) j

output of layer $l-1$ (previous layer)

$$a_j^{[l]} = g(\vec{w}_j^{[l]} \cdot \vec{a}^{[l-1]} + b_j^{[l]})$$

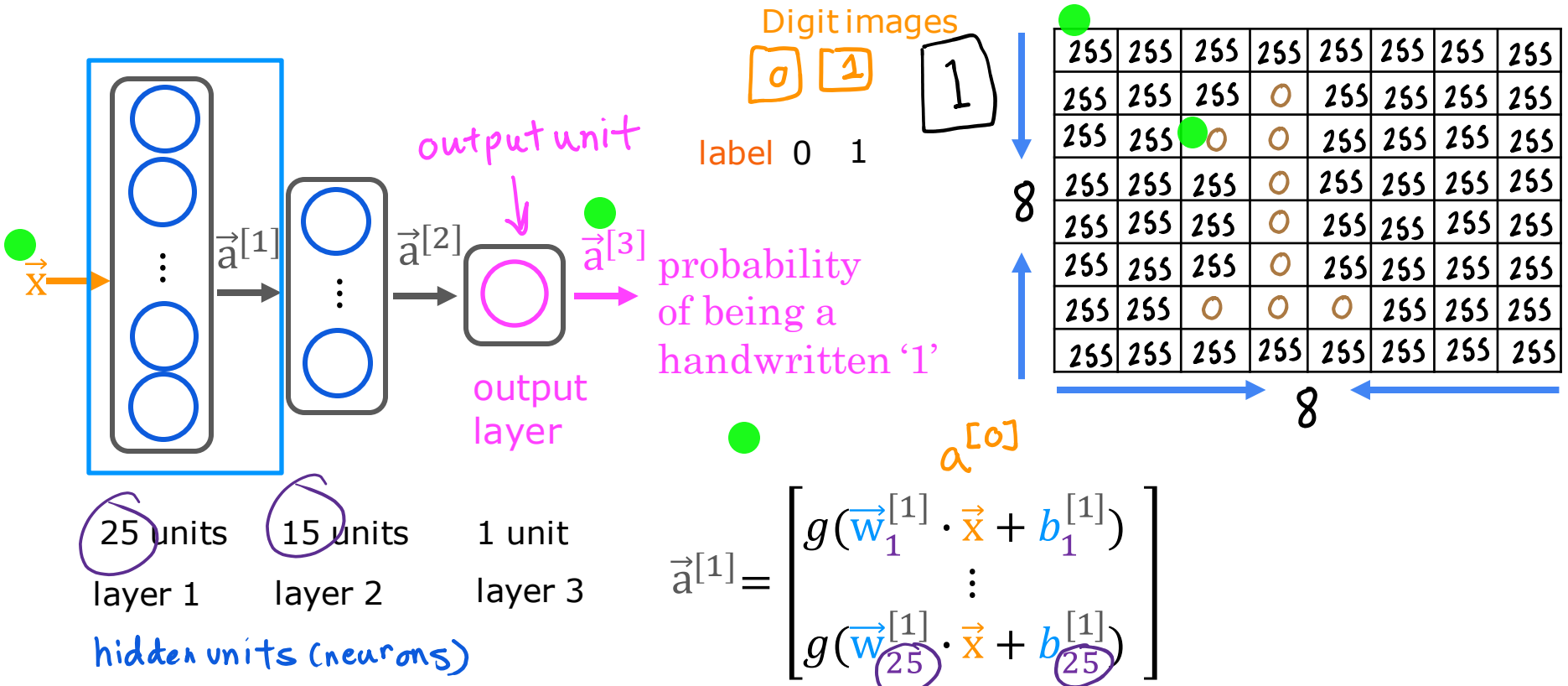
sigmoid "activation function"

Parameters w & b of layer l, unit j

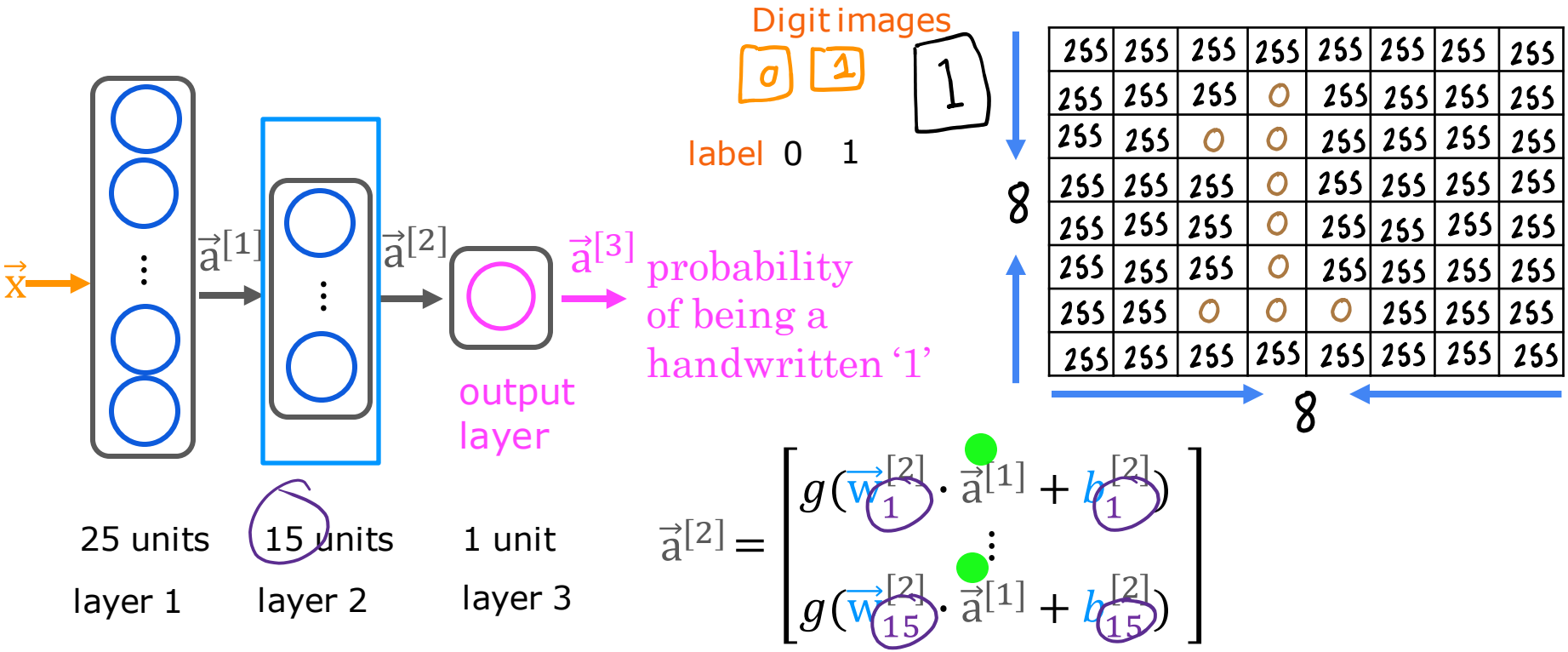
Neural Network Model

Inference: making predictions (forward propagation)

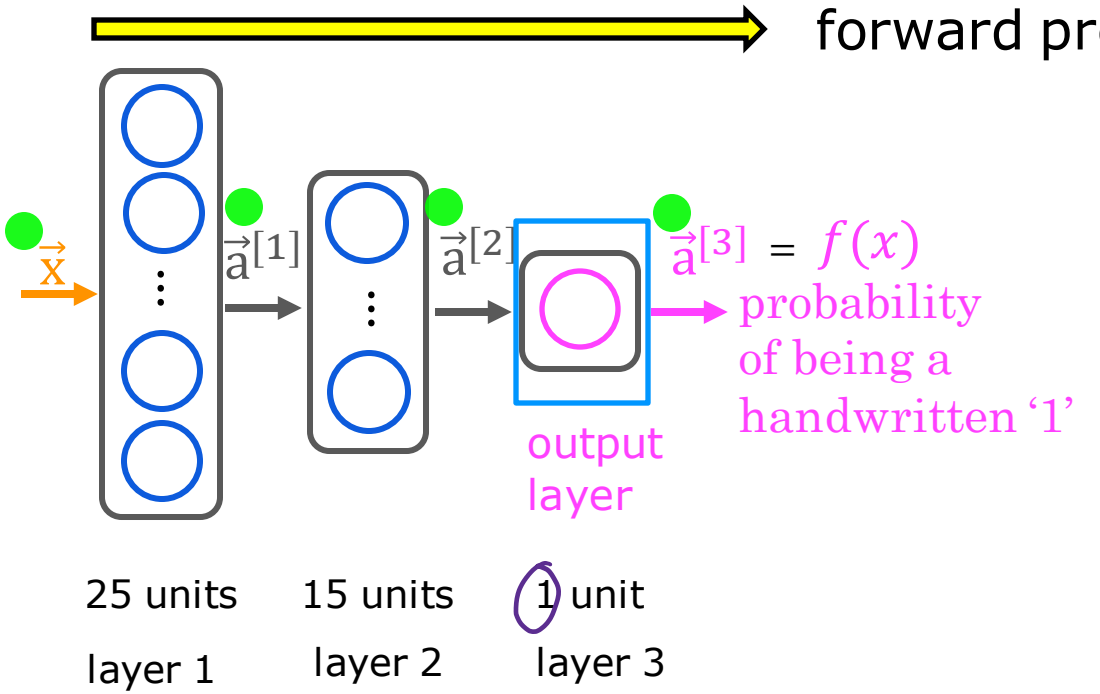
Handwritten digit recognition



Handwritten digit recognition



Handwritten digit recognition



$$\vec{a}^{[3]} = \left[g\left(\vec{w}_1^{[3]} \cdot \vec{a}^{[2]} + b_1^{[3]} \right) \right]$$

$is\ a_1^{[3]} \geq 0.5?$

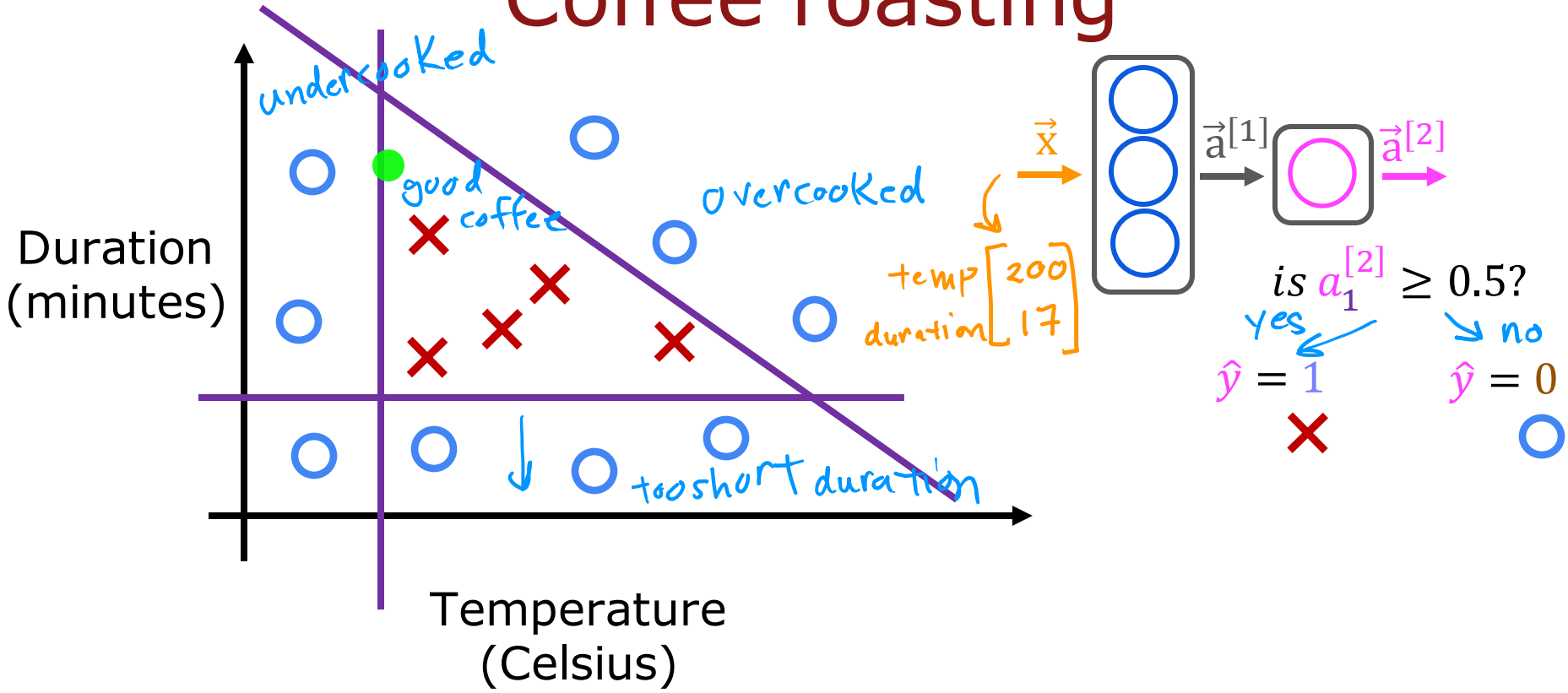
yes → $\hat{y} = 1$ image is digit 1

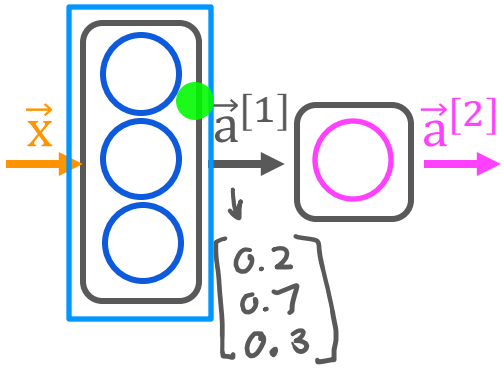
no → $\hat{y} = 0$ image isn't digit 1

TensorFlow implementation

Inference in Code

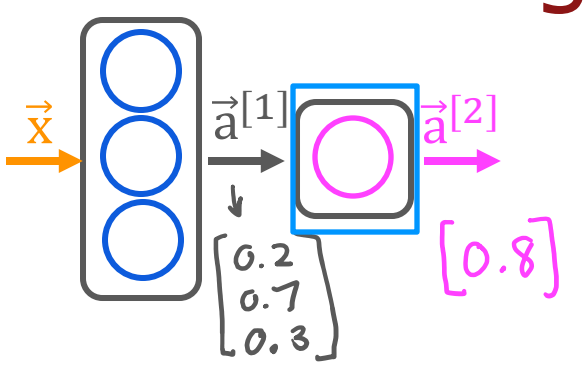
Coffee roasting






```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

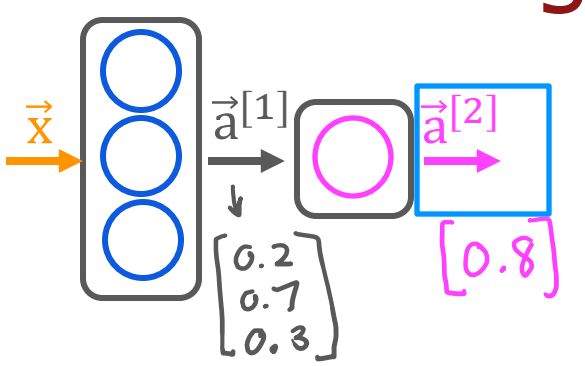
Build the model using TensorFlow

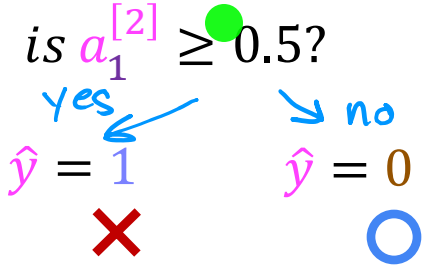



```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

Build the model using TensorFlow



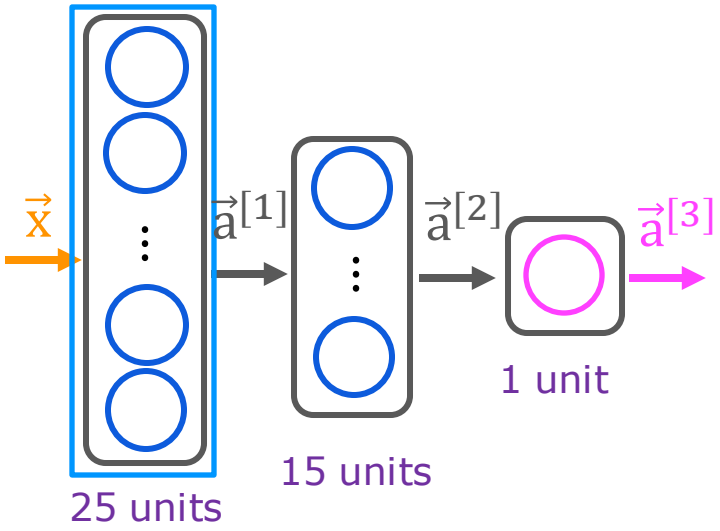



```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

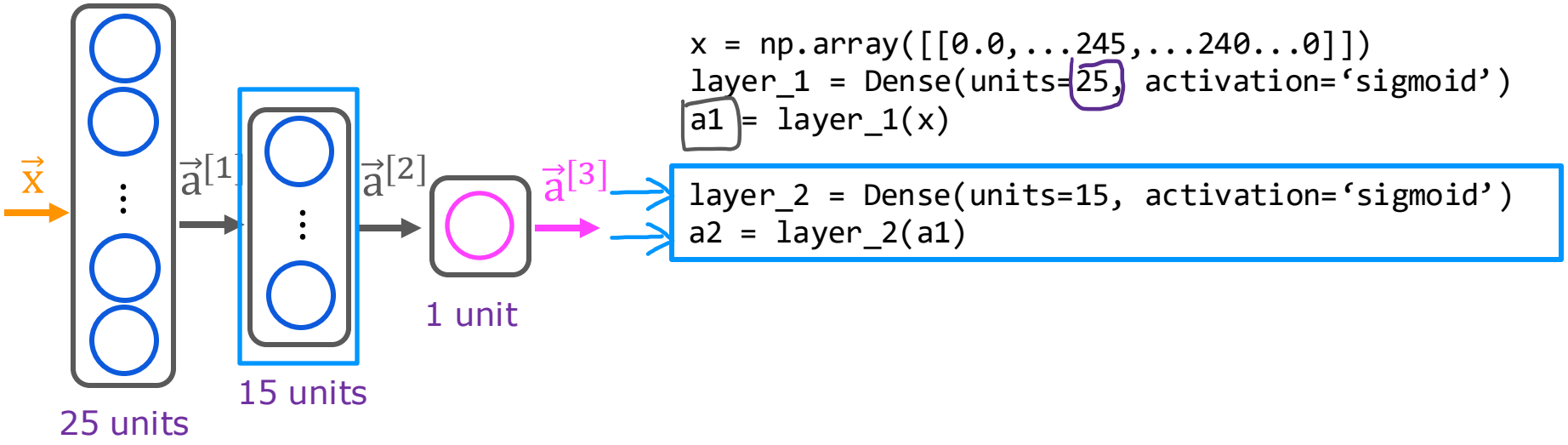
```
if a2 >= 0.5:
  yhat = 1
else:
  yhat = 0
```

Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)
```

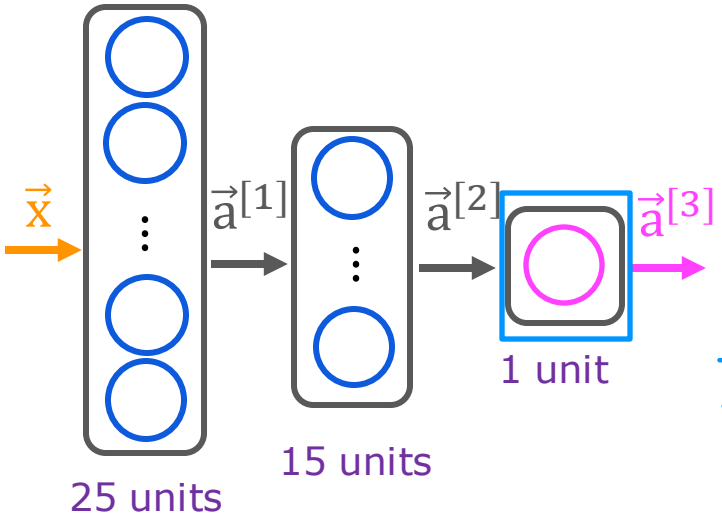
Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)
```

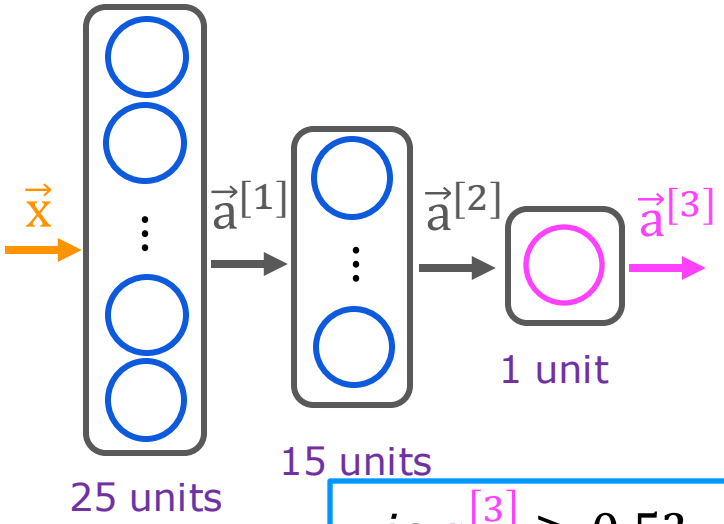
```
layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)
```

Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)

layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)

layer_3 = Dense(units=1, activation='sigmoid')
a3 = layer_3(a2)
```

Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)

layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)

layer_3 = Dense(units=1, activation='sigmoid')
a3 = layer_3(a2)
```

$is\ a_1^{[3]} \geq 0.5?$

$\hat{y} = 1$ ✗ $\hat{y} = 0$ ○

```
if a3 >= 0.5:
  yhat = 1
else:
  yhat = 0
```

TensorFlow implementation

Data in TensorFlow

Feature vectors

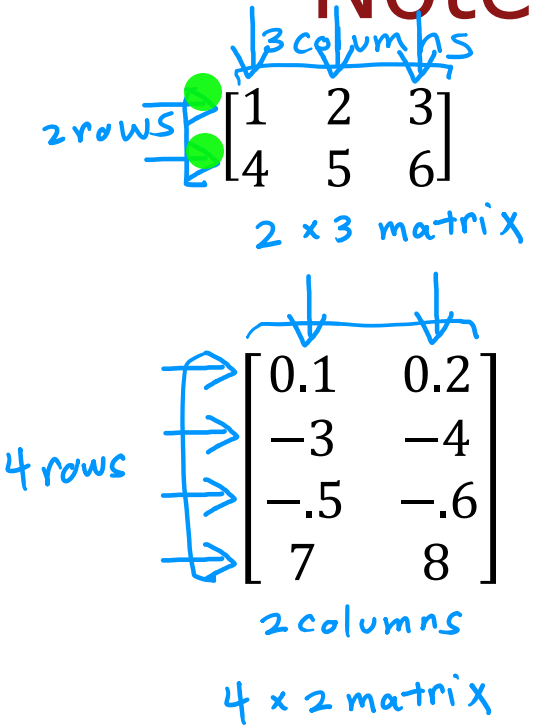
temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
...	...	...

```
x = np.array([[200.0, 17.0]])
```

```
[[200.0, 17.0]]
```

Why?

Note about numpy arrays




```
x = np.array([[1, 2, 3],
              [4, 5, 6]])
```

```
[[1, 2, 3],
 [4, 5, 6]]
```

```
x = np.array([[0.1, 0.2],
              [-3.0, -4.0,],
              [-0.5, -0.6,],
              [7.0, 8.0,]])
```

```
[[0.1, 0.2],
 [-3.0, -4.0,],
 [-0.5, -0.6,],
 [7.0, 8.0,]]
```

2D array

2 x 3

4 x 2

1 x 2

2 x 1

Note about numpy arrays

```
x = np.array([[200, 17]])
```

$\begin{bmatrix} 200 & 17 \end{bmatrix}$

1 x 2

```
x = np.array([[200],
              [17]])
```

$\begin{bmatrix} 200 \\ 17 \end{bmatrix}$

2 x 1

```
x = np.array([200,17])
```

1D
"Vector"

Feature vectors

temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
...	...	...

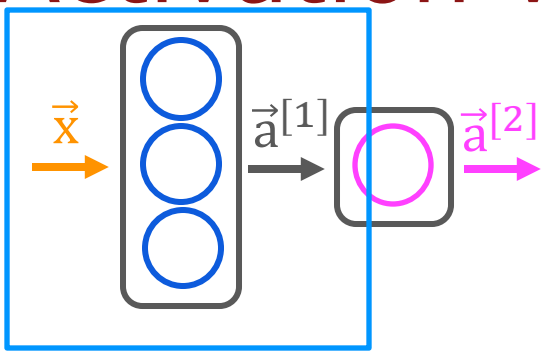
```
x = np.array([[200.0, 17.0]])
```

```
[[200.0, 17.0]]
```

1 x 2

$[200.0 \quad 17.0]$

Activation vector




```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

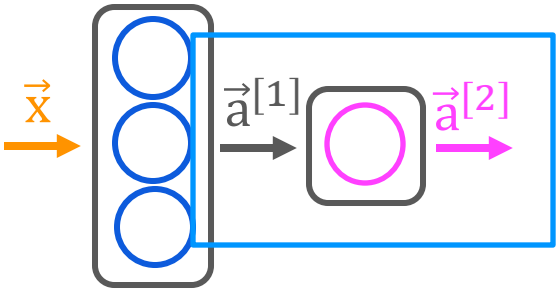
[[0.2, 0.7, 0.3]] 1 x 3 matrix

```
tf.Tensor([[0.2 0.7 0.3]], shape=(1, 3), dtype=float32)
```

```
a1.numpy()
```

```
array([[1.4661001, 1.125196 , 3.2159438]], dtype=float32)
```

Activation vector




```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

[[0.8]]

1 x 1

```
tf.Tensor([[0.8]], shape=(1, 1), dtype=float32)
```

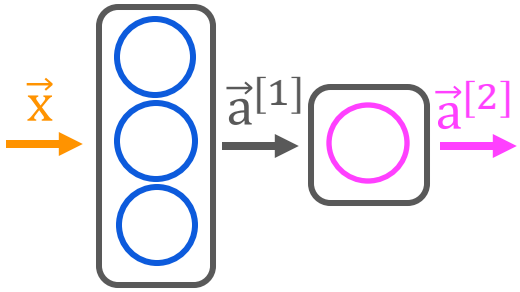
```
a2.numpy()
```

```
array([[0.8]], dtype=float32)
```

TensorFlow implementation

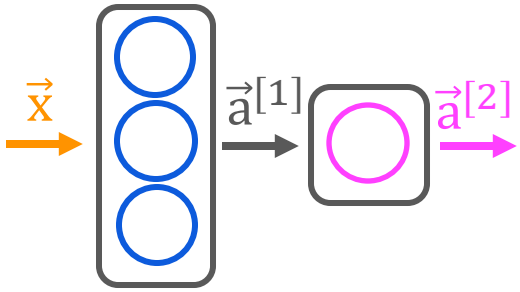
Building a neural network

What you saw earlier




```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation="sigmoid")
a1 = layer_1(x)

layer_2 = Dense(units=1, activation="sigmoid")
a2 = layer_2(a1)
```

Building a neural network architecture



		y
200	17	1
120	5	0
425	20	0
212	18	1

layer1

```
layer_1 = Dense(units=3, activation="sigmoid")
layer_2 = Dense(units=1, activation="sigmoid")
```

layer2

```
model = Sequential([layer_1, layer_2])

x = np.array([[200.0, 17.0],
              [120.0, 5.0],
              [425.0, 20.0],
              [212.0, 18.0]])
```

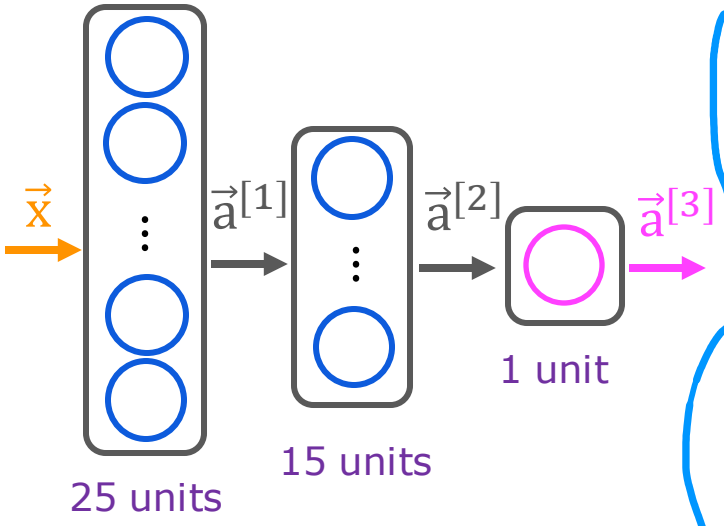
4 x 2

targets

```
y = np.array([1,0,0,1])

model.compile(...)
model.fit(x,y)

model.predict(x_new)
```

more about this next week!

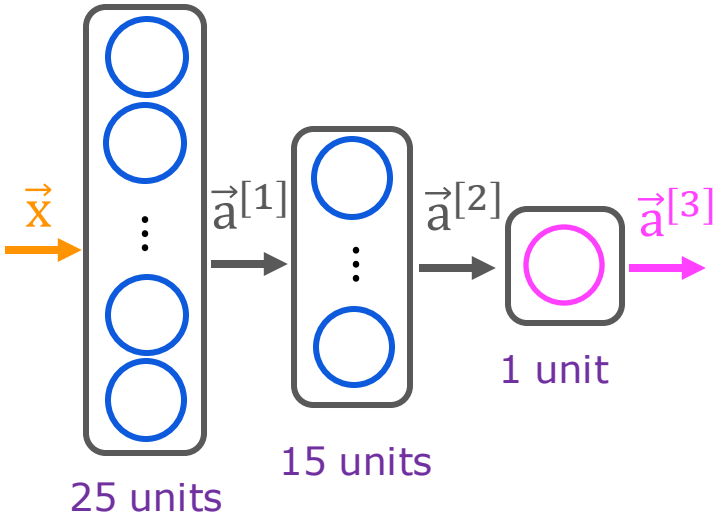
Digit classification model




```
layer_1 = Dense(units=25, activation="sigmoid")
layer_2 = Dense(units=15, activation="sigmoid")
layer_3 = Dense(units=1, activation="sigmoid")
model = Sequential([layer_1, layer_2, layer_3])
model.compile(...)
x = np.array([[0..., 245, ..., 17],
              [0..., 200, ..., 184]])
y = np.array([1,0])
model.fit(x,y)
model.predict(x_new)
```

more about this next week!

Digit classification model

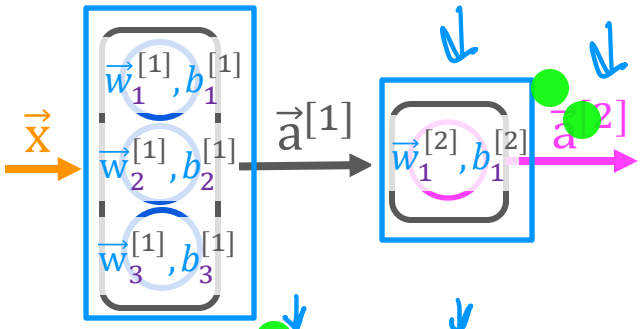



```
model = Sequential([
    Dense(units=25, activation="sigmoid"),
    Dense(units=15, activation="sigmoid"),
    Dense(units=1, activation="sigmoid")])

model.compile(...)

x = np.array([[0..., 245, ..., 17],
              [0..., 200, ..., 184])
y = np.array([1,0])

model.fit(x,y)

model.predict(x_new)
```

Neural network implementation in Python

Forward prop in a single layer

forward prop (coffee roasting model)



$a_1^{[2]} = g(\vec{w}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]})$

```
w2_1 = np.array([-7, 8])
b2_1 = np.array([3])
z2_1 = np.dot(w2_1,a1)+b2_1
a2_1 = sigmoid(z1_1)
```

$w_1^{[2]}$ w2_1

```
x = np.array([200, 17])
```

1D arrays

$a_1^{[1]} = g(\vec{w}_1^{[1]} \cdot \vec{x} + b_1^{[1]})$ $a_2^{[1]} = g(\vec{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]})$ $a_3^{[1]} = g(\vec{w}_3^{[1]} \cdot \vec{x} + b_3^{[1]})$

```
w1_1 = np.array([1, 2])
b1_1 = np.array([-1])
z1_1 = np.dot(w1_1,x)+ b
a1_1 = sigmoid(z1_1)
```

```
w1_2 = np.array([-3, 4])
b1_2 = np.array([1])
z1_2 = np.dot(w1_2,x)+b
a1_2 = sigmoid(z1_2)
```

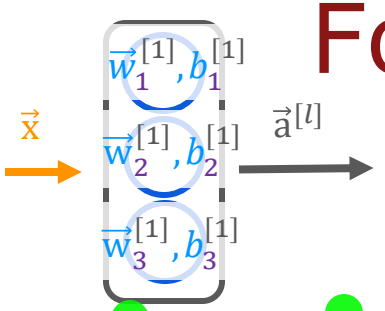
```
w1_3 = np.array([5, -6])
b1_3 = np.array([2])
z1_3 = np.dot(w1_3,x)+b
a1_3 = sigmoid(z1_3)
```

```
a1 = np.array([a1_1, a1_2, a1_3])
```

Neural network implemenation in Python

General implementation of forward propagation

Forward prop in NumPy



$\vec{w}_1^{[1]} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{w}_2^{[1]} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $\vec{w}_3^{[1]} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

```
W = np.array([
    [1, -3, 5]
    [2, 4, -6]]) 2 by 3
```

$b_1^{[l]} = -1 \quad b_2^{[l]} = 1 \quad b_3^{[l]} = 2$

```
b = np.array([-1, 1, 2])
```

$\vec{a}^{[0]} = \vec{x}$

```
a_in = np.array([-2, 4])
```

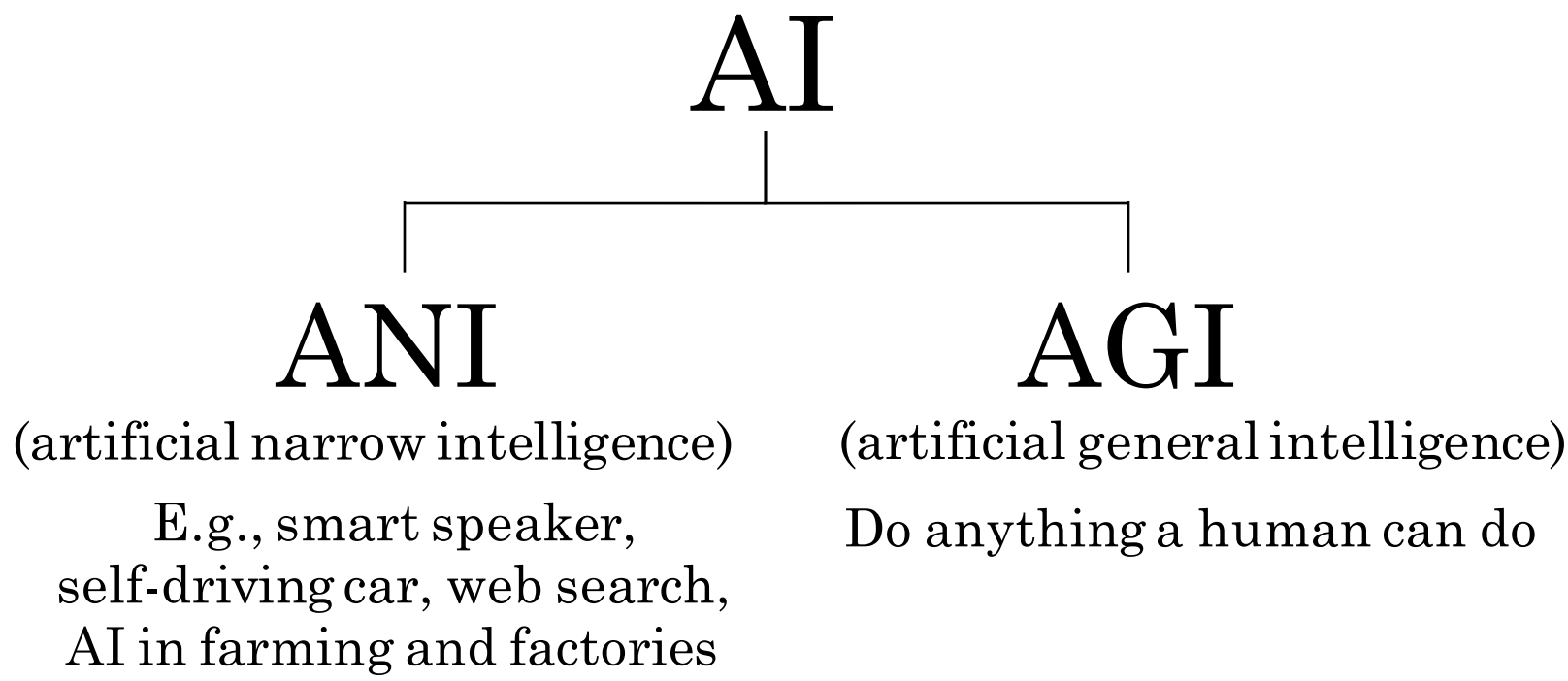
```
def dense(a_in,W,b, g):
    units = W.shape[1]      [0,0,0]
    a_out = np.zeros(units)
    for j in range(units):  0,1,2
        w = W[:,j]
        z = np.dot(w,a_in) + b[j]
        a_out[j] = g(z)
    return a_out
```

```
def sequential(x):
    a1 = dense(x,W1,b1,g)
    a2 = dense(a1,W2,b2,g)
    a3 = dense(a2,W3,b3,g)
    a4 = dense(a3,W4,b4,g)
    f_x = a4
    return f_x
```

capital W refers to a matrix

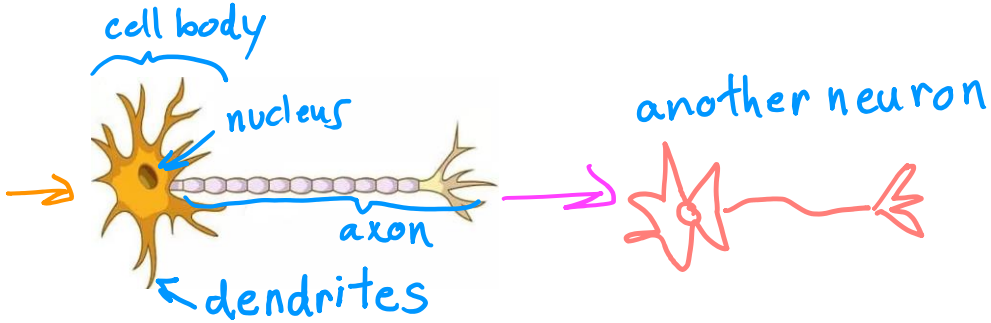
Speculations on artificial general intelligence (AGI)

Is there a path to AGI?


AI
ANI
(artificial narrow intelligence)
E.g., smart speaker,
self-driving car, web search,
AI in farming and factories
AGI
(artificial general intelligence)
Do anything a human can do

Biological neuron

inputs outputs



Simplified mathematical model of a neuron

inputs outputs

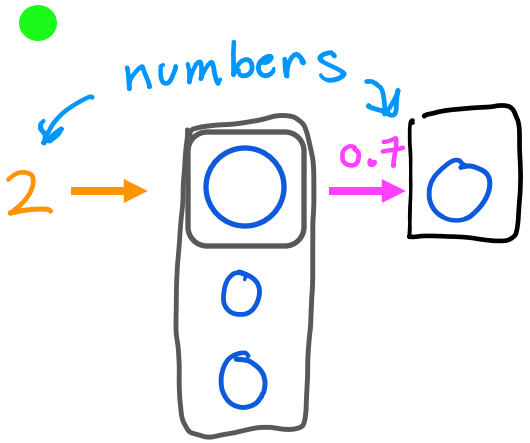
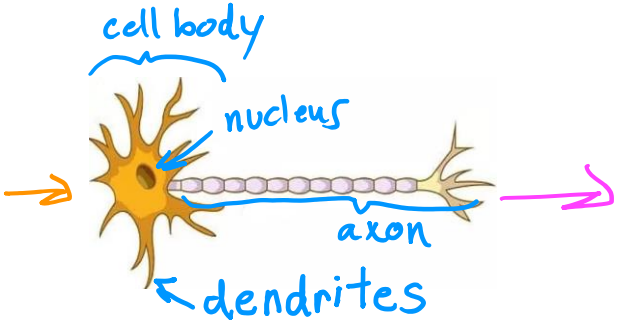


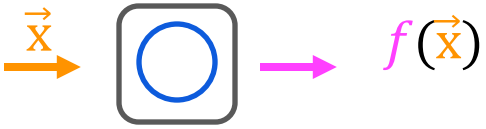
image source: https://biologydictionary.net/sensory-neuron/

Neural network and the brain

Can we mimic the human brain?

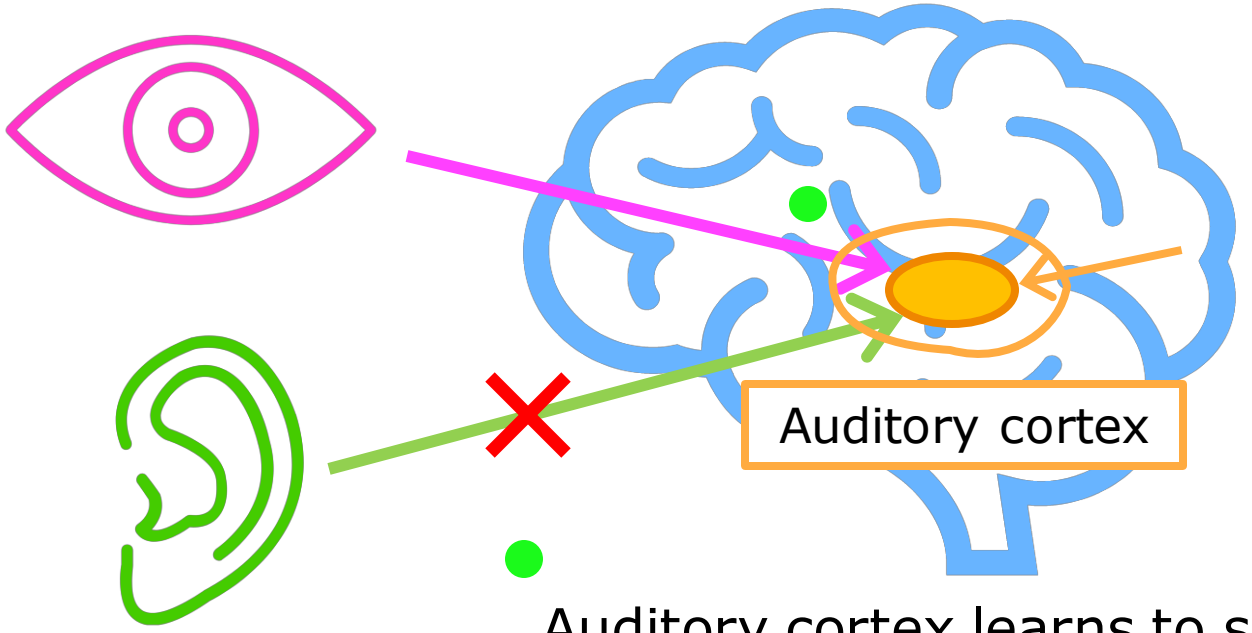


vs



We have (almost) no idea how the brain works

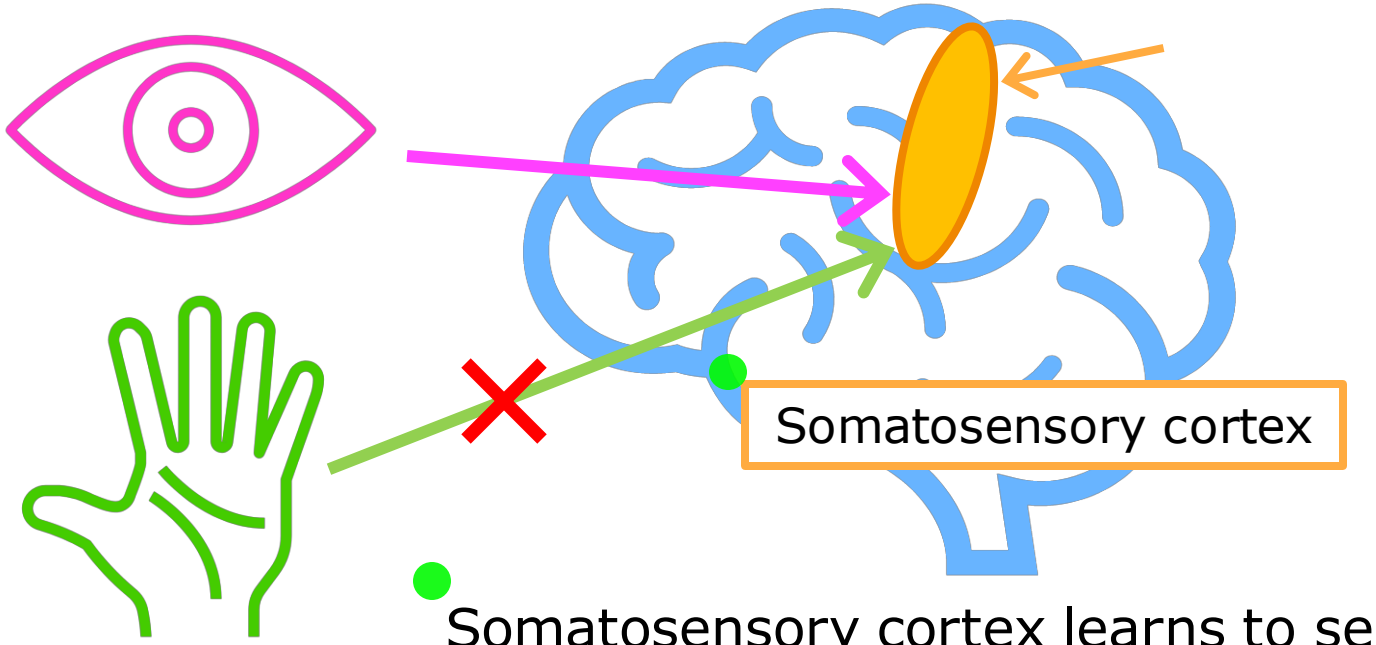
The ”one learning algorithm” hypothesis



Auditory cortex learns to see

[Roe et al., 1992]

The ”one learning algorithm” hypothesis



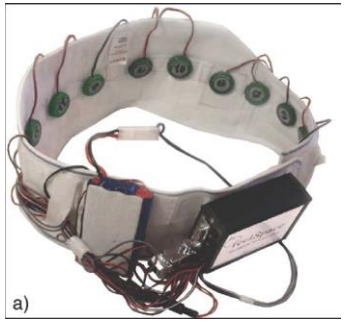
Somatosensory cortex learns to see

[Metin & Frost, 1989]

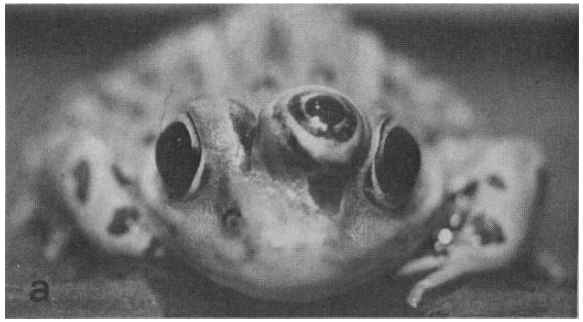
Sensor representations in the brain

Seeing with your tongue

Human echolocation (sonar)



Haptic belt: Direction sense



Implanting a 3rd eye

[BrainPort; Welsh & Blasch, 1997; Nagel et al., 2005; Constantine-Paton & Law, 2009]

Vectorization (optional)

How neural networks are implemented efficiently

For loops vs. vectorization

```
x = np.array([200, 17])

W = np.array([[1, -3, 5],
              [-2, 4, -6]])
b = np.array([-1, 1, 2])

def dense(a_in,W,b):
    a_out = np.zeros(units)
    for j in range(units):
      w = W[:,j]
      z = np.dot(w,x) + b[j]
      a[j] = g(z)
  return a
```

Vectorized

```
X = np.array([[200, 17]])

W = np.array([[1, -3, 5],
              [-2, 4, -6]])
B = np.array([[-1, 1, 2]])

def dense(A_in,W,B):
    Z = np.matmul(A_in,W) + B
    A_out = g(Z)
    return A_out

[[1,0,1]]
```

2D array

same

1x3 2D array

all 2D arrays

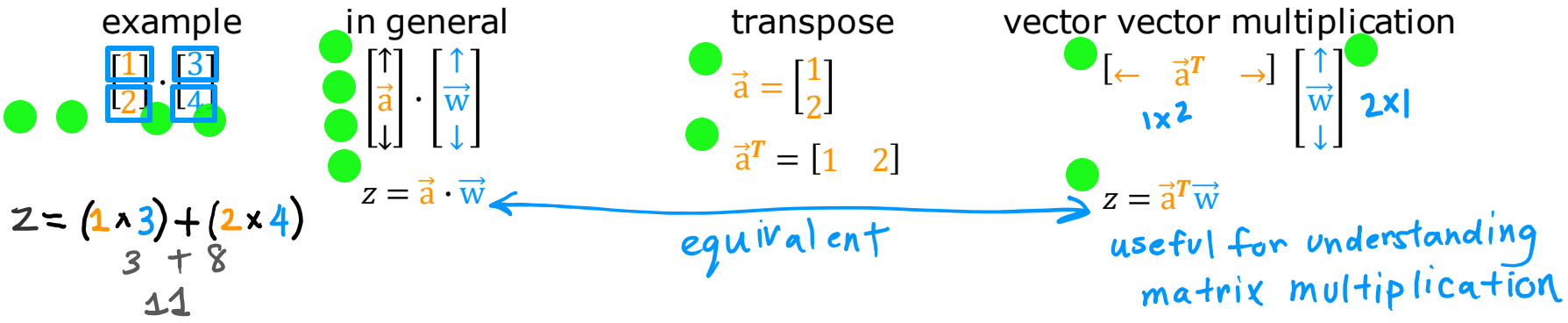
matrix multiplication

```
[1,0,1]
```

Vectorization (optional)

Matrix multiplication

Dot products



Vector matrix multiplication

$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$a = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$= a \begin{bmatrix} \uparrow & \uparrow \\ w_1 & w_2 \\ \downarrow & \downarrow \end{bmatrix}$

1 by 2

$= [\, a w_1 \quad a w_2 \,]$

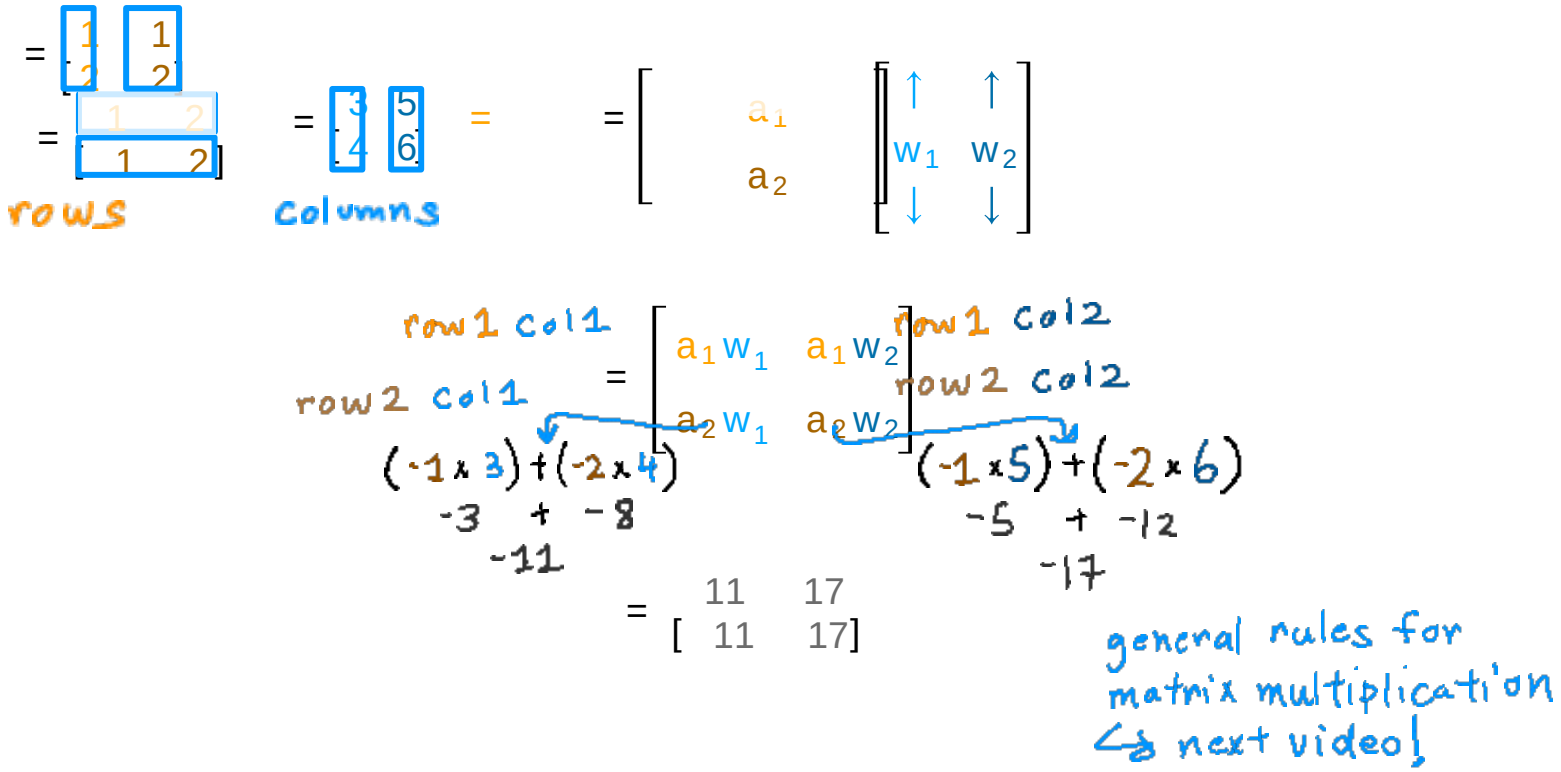
$(1 \quad 3) + (2 \quad 4) \qquad (1 \quad 5) + (2 \quad 6)$

$3 + 8 \qquad 5 + 12$

$11 \qquad 17$

$= [11 \quad 17]$

matrix matrix multiplication



Vectorization (optional)

Matrix multiplication rules

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$$

$\vec{a}_1^T$ $\vec{a}_2^T$ $\vec{a}_3^T$

$\vec{w}_1$ $\vec{w}_2$ $\vec{w}_3$ $\vec{w}_4$

$$Z = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$$

$$Z = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

3 by 4 matrix

$$\vec{a}_1^T \vec{w}_1 = (1 \times 3) + (2 \times 4) = 11$$

row 3 column 2

$$\vec{a}_3^T \vec{w}_2 = (0.1 \times 5) + (0.2 \times 6) = 1.7$$

$0.5 + 1.2$

row 2 column 3?

$$\vec{a}_2^T \vec{w}_3 = (-1 \times 7) + (-2 + 8) = -23$$

$-7 + -16$

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$$

$$Z = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

3 by 4 matrix

$\vec{a}_1^T \vec{w}_1 = (1 \times 3) + (2 \times 4) = 11$

row 3 column 2

$\vec{a}_3^T \vec{w}_2 = (0.1 \times 5) + (0.2 \times 6) = 1.7$

$0.5 + 1.2$

row 2 column 3?

$\vec{a}_2^T \vec{w}_3 = (-1 \times 7) + (-2 + 8) = -23$

$-7 + -16$

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

3 x 2 2 x 4

can only take dot products of vectors that are same length

[0.1 0.2] length 2

$\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ length 2

3 by 4 matrix

↳ same # rows as A^T

same # columns as W

Vectorization (optional)

Matrix multiplication code

Matrix multiplication in NumPy

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad \mathbf{Z} = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

```
A=np.array([1,-1,0.1],
           [2,-2,0.2]])

AT=np.array([1,2],
            [-1,-2],
            [0.1,0.2])

AT=A.T
```

transpose

```
W=np.array([3,5,7,9],
           [4,6,8,0])
```

```
Z = np.matmul(AT,W)
```

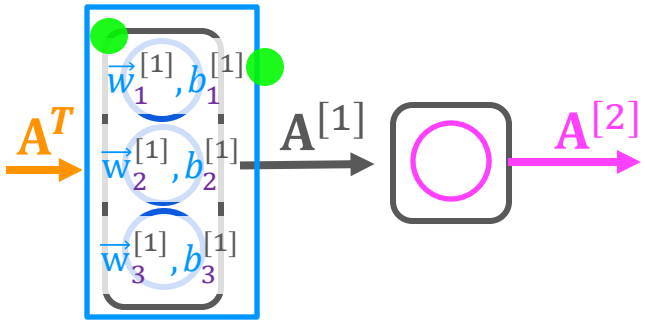
or

```
Z = AT @ W
```

result

```
[[11,17,23,9],
 [-11,-17,-23,-9],
 [1.1,1.7,2.3,0.9]
]
```

Dense layer vectorized



$$\mathbf{A}^T = [200 \quad 17] \qquad 1 \times 2$$

$$\mathbf{W} = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 4 & -6 \end{bmatrix} \qquad 2 \times 3$$

$$\vec{\mathbf{b}} = [-1 \quad 1 \quad 2] \qquad 1 \times 3$$

$$\mathbf{Z} = \mathbf{A}^T\mathbf{W} + \mathbf{B}$$

$$[165 \quad -531 \quad 900]$$

$$z_1^{[1]} \quad z_2^{[1]} \quad z_3^{[1]}$$

$$\mathbf{A} = g(\mathbf{Z})$$

$$[1 \quad 0 \quad 1]$$

A

```
AT = np.array([[200, 17]])
W = np.array([[1, -3, 5],
              [-2, 4, -6]])
b = np.array([[-1, 1, 2]])
```

a_in

```
def dense(AT,W,b,g):
  z = np.matmul(AT,W) + b
  a_out = g(z)
  return a_out
```

a_in

```
[[1,0,1]]
```