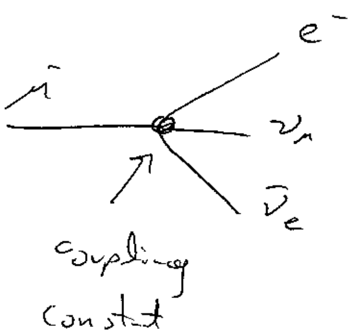
1. a)



b) $4 \times \frac{3}{2} + [\text{coupling constant}] = 4$

$\Rightarrow \quad [\text{coupling constant}] = -2$

OR $\quad GeV^{-2}$

c) $\Gamma \propto |M|^2 \sim [\text{coupling const}]^2 = GeV^{-4}$

But Γ has to have dimension $\frac{1}{\text{time}}$ or Mass

$\Rightarrow \quad \Gamma \propto m_\mu^{\,5}$

d) $m_\mu \sim 0.1\ GeV$ $\qquad m_\tau \sim 1\ GeV$ $\qquad$ from c) $\Gamma_\tau \propto m_\tau^{\,5}$

$\tau_\mu \sim 1\ \mu s$

$\tau_\mu = \Gamma_\mu^{-1}$ $\qquad \frac{\tau_\tau}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_\tau} \Rightarrow \tau_\tau = \tau_\mu \frac{\Gamma_\mu}{\Gamma_\tau}$

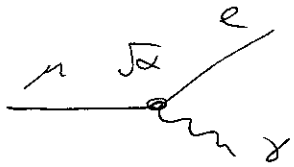
$\tau_\tau = \Gamma_\tau^{-1}$

$$\tau_\tau = (1\ \mu s)\left(\frac{m_\mu}{m_\tau}\right)^5 = 1\ \mu s\left(10^{-1}\right)^5$$

$$= 1 \cdot 10^{-6}\ s\ 10^{-5}$$

$$= 10^{-11}\ s$$

(e)



$$\Gamma_{\mu \to e\gamma} \sim \cancel{\alpha}\, m_\mu$$

$$\Gamma_{SM} \propto m_W^{-4} m_\mu^5$$

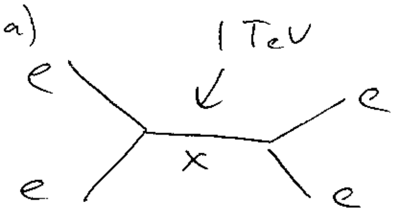
$$\frac{\tau_{new}}{\tau_{SM}} = \frac{m_W^{-4} m_\mu^5}{\cancel{\alpha}\, m_\mu} \sim \left(\frac{m_\mu}{m_W}\right)^4 \sim \left(\frac{0.1\ \text{GeV}}{100\ \text{GeV}}\right)^4$$

$$\sim 10^{12}$$

the 

decay would dominate (by factor 10^{12}!)

2. a)



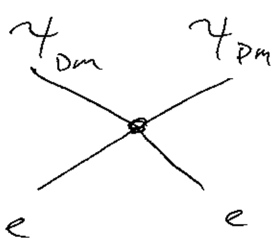
$$\text{range} \sim \frac{1}{m_X} \sim \frac{1}{1000\ \text{GeV}}$$

$$\boxed{\text{GeV}^{-1} = 10^{-16}\ \text{m}}$$

$$\sim 10^{-3}\ \text{GeV}^{-1}$$

$$\sim 10^{-19}\ \text{m}$$

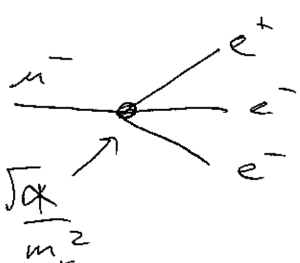
b)



Units of coupling constant GeV^{-1}

$\Rightarrow$ coupling constant $\sim \dfrac{\sqrt{\cancel{4}\alpha}}{m_X^2}$

c)



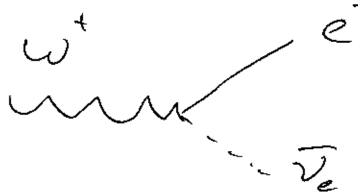
$$\Gamma_{New} \sim \frac{\alpha}{m_X^4} m_\mu^5$$

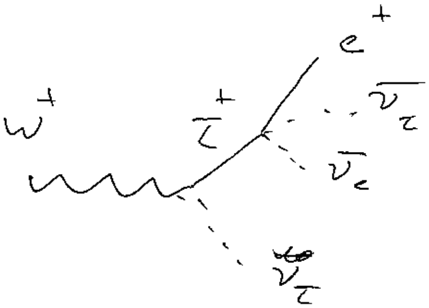
$$\Gamma_{SM} \sim \frac{\alpha}{m_W^4} m_\mu^5$$

$$\frac{\tau_{New}}{\tau_{SM}} = \left(\frac{m_X}{m_W}\right)^4 \sim 10^4$$

SM decays dominate!

3a)





b) $Br(W^+ \to l^+ \nu)$ is $\dfrac{1}{\underbrace{3}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{quarks } \binom{u}{d}\binom{s}{c}}} = \dfrac{1}{9} \sim 0.11$

Now, $Br(\tau \to e + \nu\bar{\nu}) = \dfrac{1}{\underbrace{2}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{1}_{\text{quarks } \binom{u}{d}}} = \dfrac{1}{5} \sim 0.2$

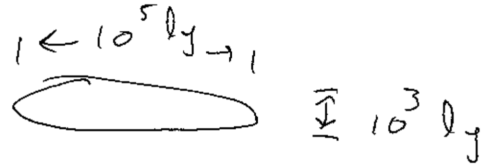
So $Br(W^+ \to e^+ + X) = \underbrace{\dfrac{1}{9}}_{\text{direct } W \to e\nu} + \underbrace{\dfrac{1}{9}}_{W \to \tau\nu} \times \underbrace{\dfrac{1}{5}}_{\tau \to e\nu\nu} = \dfrac{1.2}{9} \simeq 0.13$

4) $Br(W \to l\nu) \sim \dfrac{1}{9}$ for e or μ $Br(W \to l\nu + x) = \dfrac{1.2}{9}$

Including τ decays

$Br(WW \to e\mu + X) = \underbrace{2}_{e^+\mu^- \text{ or } \mu^+ e^-} \times \left(\dfrac{1.02}{9}\right)^2 \sim 0.036$

5) $N_{star} = 10^{11}$

a) 

relative velocity 10^5 m/s

$$\mathcal{L}_{galaxy} = \frac{N_A N_B |v_A - v_B|}{Vol} \sim \frac{10^{22}\ 10^5\ m/s}{Vol}$$

$$Vol = \pi\left(\frac{10^5}{2} ly\right)^2 \times 10^3 ly = \frac{\pi}{4} 10^{13} (ly)^3 \sim 10^{13}\left(3 \cdot 10^8\ m/s \cdot \pi \cdot 10^7 s\right)^3$$

$$\sim 10^{13}(10^{16} m)^3 \sim 10^{13}\ 10^{48}\ m^3 \sim 10^{61}\ m^3$$

$$\mathcal{L}_{galaxy} \sim \frac{10^{27}\ m/s}{10^{61}\ m^3} \sim 10^{-34} \frac{1}{m^2}\frac{1}{s}$$

$$\sim 10^{-38} \frac{1}{cm^2}\frac{1}{s}$$

Now
$1\ m^2 = 10^4\ cm^2$
$\frac{1}{m^2} = \frac{10^{-4}}{cm^2}$

$$\mathcal{L}_{LHC} \sim 10^{34} \frac{1}{cm^2}\frac{1}{s}$$

so ~ 72 orders of mag smaller!

b) $Vol/star \sim \frac{10^{61}\ m^3}{10^{11}} \sim 10^{50}\ m^3$

$$\frac{distance}{star} \sim 10^{50/3}\ m$$

$$\frac{\langle \text{distance to star} \rangle}{\langle R_* \rangle} \sim \frac{10^{50/3}\ m}{7 \times 10^8\ m}$$

$$\sim 10^{-1}\ 10^{26/3} \sim 10^{25/3}\ m$$

$$\sim 10^8\ \cancel{m}$$

@ LHC

w/ squeeze

$$\frac{\text{Volume}}{\text{Proton}} \sim \frac{10^{-10}\ m^2}{10^{11}}\ m \sim 10^{-21}\ m^3$$

$$\text{distance to proton} \sim 10^{-7}\ m \qquad \frac{\langle \text{distance to proton} \rangle}{\langle \text{radius proton} \rangle} \sim \frac{10^{-7}\ m}{10^{-15}\ m} \sim 10^{8}$$

Which is quite close!

c) $N_{collision} = \underbrace{\int dt\, \mathcal{L}} \times \sigma$

$\rightarrow \pi r^2 \sim \pi \times (7 \cdot 10^{8}\ m)^2$

$$10^{9}\ y \times \pi \cdot 10^{7} \frac{s}{y} \times 10^{-34}\ m^{-2}\ s^{-1} \times \pi\ 49 \times 10^{16}\ m^2$$

$$49\ \pi^2\ 10^{9}\ 10^{7}\ 10^{-34}\ 10^{16} = 10^{18}\ 10^{-18} \sim 1 \text{ collision}$$

The LHC has about 10^1 collision / event.