

Homework Set #9

1) Synchrotron Losses

(10 points)

- a. In class, I gave the wrong formula. The power per turn should be (note the 10^{-6} !!!)

$$P \sim 0.3 \cdot 10^{-6} \left(\frac{E}{m} \right)^4 \left(\frac{km}{R} \right)^2$$

This gives the power lost from synchrotron radiation per proton at the LHC to be about 40 MeV/s. The total number of protons N_p in the LHC ring is the number of bunches times the number of protons per bunch.

$$N_p = 2808 \cdot 1.15 \times 10^{11} \sim 3.2 \times 10^{14}$$

Therefore, the total power radiated away in synchrotron radiation is $P_{tot} \sim 3.2 \times 10^{14} \times 40 \text{ MeV/s} \sim 1.3 \times 10^{13} \text{ GeV/s}$. Converting this to J/s, we recall that 1 GeV is 1.6×10^{-10} J. Therefore, the power lost in watts is

$$P_{tot} \sim 1.6 \times 10^{-10} \cdot 1.3 \times 10^{13} \text{ W} \sim 2 \times 10^3 \text{ W}$$

That is, the power in synchrotron radiation at the LHC is equivalent to about two microwave ovens. Everyone who attempted this problem with the old formula will get full credit.

- b.

$$\frac{P_{LEP}}{P_{LHC}} = \left(\frac{m_p E_{LEP}}{m_e E_{LHC}} \right)^4$$

Now,

$$\frac{m_p E_{LEP}}{m_e E_{LHC}} \sim \frac{1 \times 200}{10^{-3} 6500} \sim 30$$

or

$$P_{LEP} \sim 30^4 P_{LHC} \sim 8^5 \times 40 \text{ MeV/s} \sim 3 \times 10^4 \text{ GeV/s}$$

- c.

$$\frac{P_{LEP}}{P_{LHC}} = 1 = \left(\frac{m_p E_{LEP}}{m_e E_{LHC}} \right)^4$$

or

$$\frac{E_{LHC}}{E_{LEP}} = \left(\frac{m_p}{m_e} \right) \sim 2000$$

2) Tracking Detectors

(10 points)

a)

$$F = ma \Rightarrow mv^2/r_c = qvB \Rightarrow r_c = p_T/qB$$

Now,

$$r_c^2 = \left(\frac{L}{2}\right)^2 + (r_c - s)^2$$

Or (Ignoring terms

$$\frac{L^2}{4} = 2r_c s - s^2$$

B/c $r_c \gg s$, can safely drop s^2 relative to $r_c s$. Thus

$$s = \frac{L^2}{8r_c} = \frac{qBL^2}{8p_T}$$

b)

$$p_T = \frac{qBL^2}{8s}$$

So,

$$\Delta p_T = \frac{qBL^2}{8s^2} \Delta s$$

and

$$\frac{\Delta p_T}{p_T} = \frac{\Delta s}{s} = \frac{8p_T}{qBL^2} \Delta s$$

c) For $N = 50$, $\epsilon = 100 \mu m$, $L = 1 \text{ m}$, and $B = 1 \text{ T}$, $\Delta s \sim 50 \mu m = 50 \times 10^{-6} \text{ m}$

Now $T = 2 \times 10^{-16} \text{ GeV}^2$

$e = 0.3$

$$\Delta p_T = \frac{8(p_T[\text{GeV}])^2}{0.3 \times 2 \times 10^{-16}} \frac{50 \times 10^{-6}}{5 \times 10^{15} \text{ GeV}^{-1}} \sim 3 \times 10^{-3} (p_T[\text{GeV}])^2 \text{ GeV}$$

At 1 GeV the uncertainty is $\sim 10^{-3} \text{ GeV}$, At 100 GeV the uncertainty is 10 GeV.

3) Limits of the Tracking System.

(5 points)

a)

$$r_c \sim 3 \frac{p_T [GeV]}{Q[e]B[T]}$$

Particles dont make it to the calorimeter when $r_{calo} \sim 2 \times r_c$

or

$$p_T \sim \frac{qBr_{calo}}{6} = \frac{2 \times 1.1}{6} \sim 400 MeV$$

b) Estimate upper limit when $s \sim 17\mu m \sim 20 \times 10^{-6}m$

$$p_T \sim \frac{0.3 \times 2 \times 10^{-16} GeV^2}{8} \frac{0.5}{20 \times 10^{-6}} 0.5 \times 5 \times 10^{15} GeV^{-1} p_T \sim 0.5 \times 10^3 GeV$$

c) At the limit $\Delta s/s \sim 1 \Rightarrow \Delta p_T/p_T \sim 1$, so $\Delta p_T \sim 500 GeV$

4) Rapidity.

(15 points)

a) Under a boost along Z

$$E \rightarrow E\gamma - \beta\gamma p_z$$

$$p_z \rightarrow p_z\gamma - \beta\gamma E$$

So,

$$\begin{aligned} y &\rightarrow \frac{1}{2} \log \frac{(E\gamma - \beta\gamma p_z) + (p_z\gamma - \beta\gamma E)}{(E\gamma - \beta\gamma p_z) - (p_z\gamma - \beta\gamma E)} \\ &= \frac{1}{2} \log \frac{\gamma - \beta\gamma}{\gamma + \beta\gamma} \frac{E + p_z}{E - p_z} = \frac{1}{2} \log \frac{E + p_z}{E - p_z} + \frac{1}{2} \log \frac{\gamma - \beta\gamma}{\gamma + \beta\gamma} \\ &= y + \frac{1}{2} \log \frac{\cosh \eta - \sinh \eta}{\cosh \eta + \sinh \eta} = y + \frac{1}{2} \log \frac{e^{-\eta}}{e^{+\eta}} \\ &= y + \frac{1}{2} \log e^{-2\eta} = y - \eta \end{aligned}$$

b. $y = \eta$ for mass-less particles

c.d. Green are electrons / Red are muons.

c.e I got:

$$\eta_1 = -1 / \phi_1 = 70 / p_{t1} = 30$$

$$\eta_2 = 0 / \phi_2 = 255 / p_{t2} = 30$$

$$\eta_3 = -0.2 / \phi_3 = 70 / p_{t3} = 20$$

$$\eta_4 = 0.5 / \phi_4 = 200 / p_{t4} = 25$$

f. I get: (124.8, -14.2, 9.5, -26.3) GeV (E,vecP)

h. 68. probably Zboson

i. 43 probably off shell z

j. 121 GeV probably a higgs