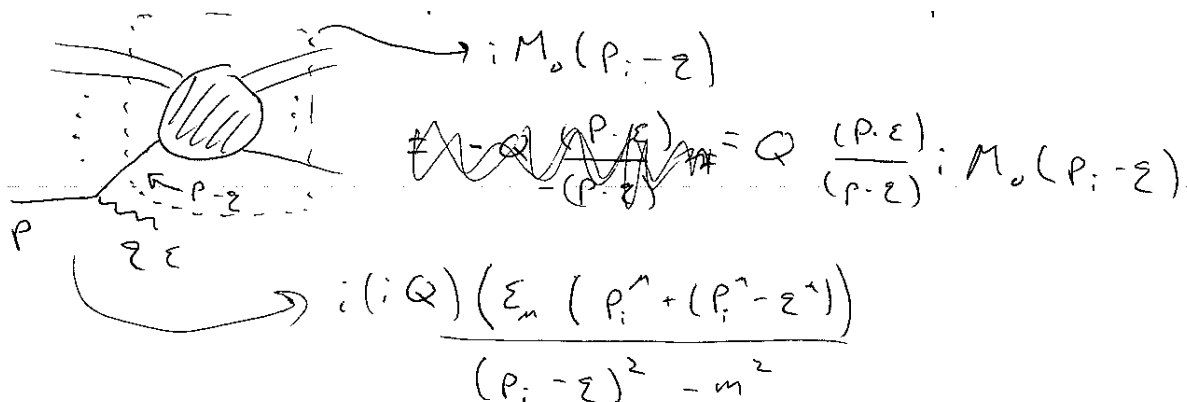


Lecture 18

Now do the same thing to a more general interaction



Consider what happens if we attach a "photon" to an incoming leg



Can also attach photon to outgoing leg

$$iM_0(p+q) = -Q \frac{(p \cdot \epsilon)}{(p \cdot q)} iM_0(p+q)$$

$$\frac{(-Q) \epsilon_\mu (p^\mu + (p^\mu + q^\mu))}{(p+q)^2 - m^2}$$

Total Amplitude is then given by

$$M = \sum_{\text{incoming}} Q_i \frac{(p \cdot \epsilon)}{(p \cdot q)} iM_0(p - q) + \sum_{\text{outgoing}} -Q_i \frac{(p \cdot \epsilon)}{(p \cdot q)} iM_0(p + q)$$

Take soft limit: $M_0(p \pm q) \rightarrow M_0(p)$

$$M = iM_0 \left(\sum_{\text{incoming}} Q_i \frac{(p \cdot \epsilon)}{(p \cdot q)} + \sum_{\text{outgoing}} -Q_i \frac{(p \cdot \epsilon)}{(p \cdot q)} \right)$$

Now as before $\epsilon_\mu \rightarrow \epsilon'_\mu + q_\mu$ means that M must vanish when $\epsilon_\mu \rightarrow q_\mu$.

OR under a Lorentz Transform

$$\epsilon_\mu \cdot M \rightarrow \epsilon'_\mu \cdot M' + iM_0 \underbrace{\left(\sum_{\text{incoming}} Q_i + \sum_{\text{outgoing}} -Q_i \right)}_{\substack{=0 \text{ only if} \\ \sum_{\text{incoming}} Q_i = \sum_{\text{outgoing}} Q_i}}$$

Charge has to be conserved!

Now same logic for Spin-2 (describes interaction w/Gravitons)

Same as above except 2-component polarization vector.

$$\epsilon_{\mu\nu} \xrightarrow{\text{under little group}} \epsilon_{\mu\nu} + \underbrace{A_\mu q_\nu + B_\mu q_\mu + C q_\mu q_\nu}_{\text{effect from all of these need to be 0 as before}}$$

where A, B C's are non-zero and depend on the particular little group transformation done.



$$= i(iK_i)\epsilon_{\mu\nu} \frac{(2p^\mu p^\nu)}{-p \cdot q}$$

(Same idea with the outgoing leg)

Now, (lets focus on piece that goes like $\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + q_\mu B_\nu$)

$$\begin{aligned} \epsilon_{\mu\nu} \rightarrow \epsilon'_{\mu\nu} M'^{\mu\nu} + M \left(\sum_{\text{incoming}} K_i B_\nu p^\nu - \sum_{\text{outgoing}} K_i B_\nu p^\nu \right) \\ + MB_\nu \left(\sum_{\text{incoming}} K_i p^\nu - \sum_{\text{outgoing}} K_i p^\nu \right) \end{aligned}$$

$\Rightarrow K_i p_i^\nu$ is conserved

We know that p_i^ν is conserved by E and momentum conservation.

Only way can have nontrivial solutions is if $k_i = k$ for all i

All particles interact with gravity with the same strength.

Gravitational interaction is Universal !

Discovered the “Principle of Equivalence” that is the starting point of General Relativity!

Can keep going...

For a massless spin-3 particle we would do the same exercise.

We would find we need

$$\sum_{\text{incoming}} \beta_i p_i^\mu p_i^\nu = \sum_{\text{outgoing}} \beta_i p_i^\mu p_i^\nu$$

eg: $\mu\nu = 0$

$$\sum_{\text{incoming}} \beta_i E_i^2 = \sum_{\text{outgoing}} \beta_i E_i^2$$

Way too constraining.

Only way if $\beta_i = 0$

There can be no interacting theories of massless particles of Spin greater than 2 !
