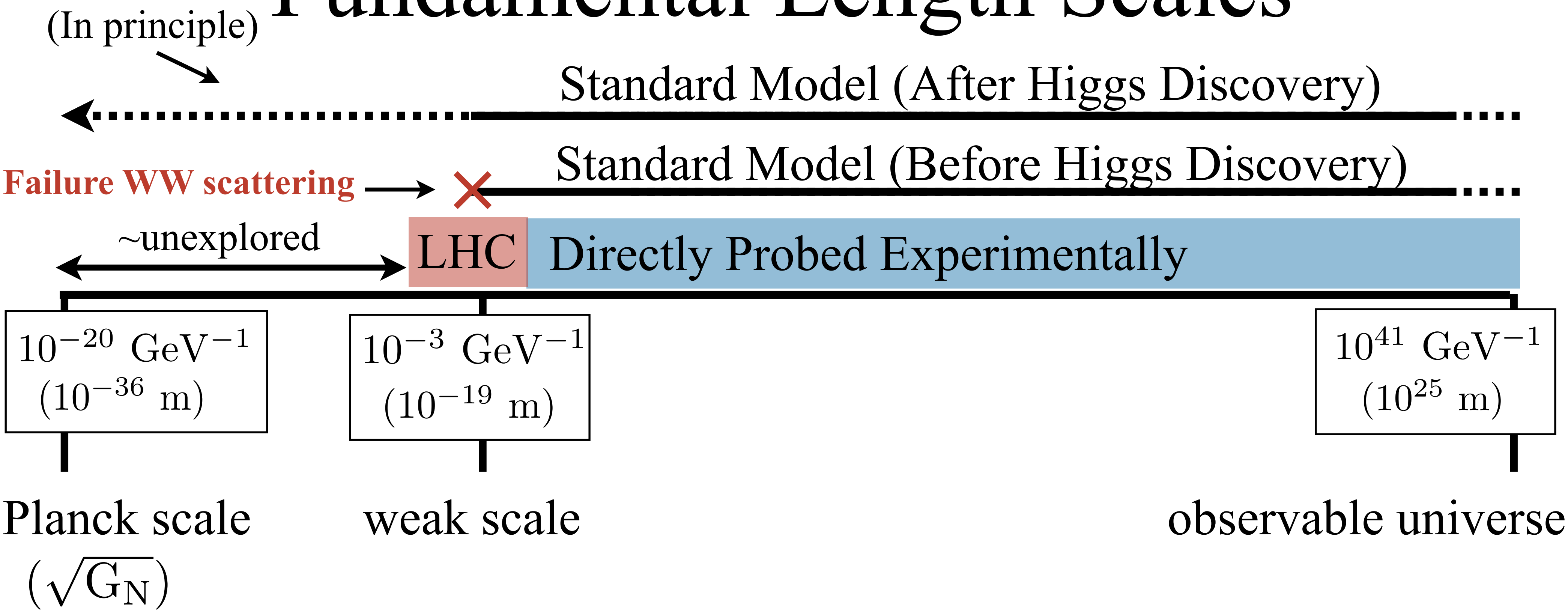
Fundamental Length Scales

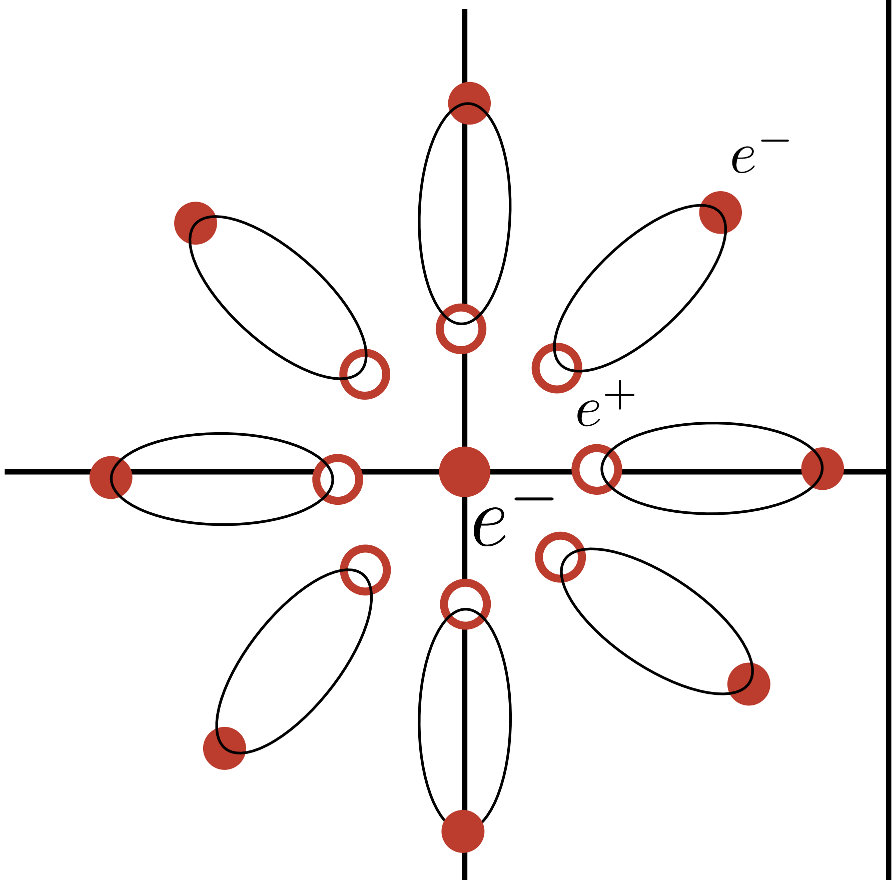


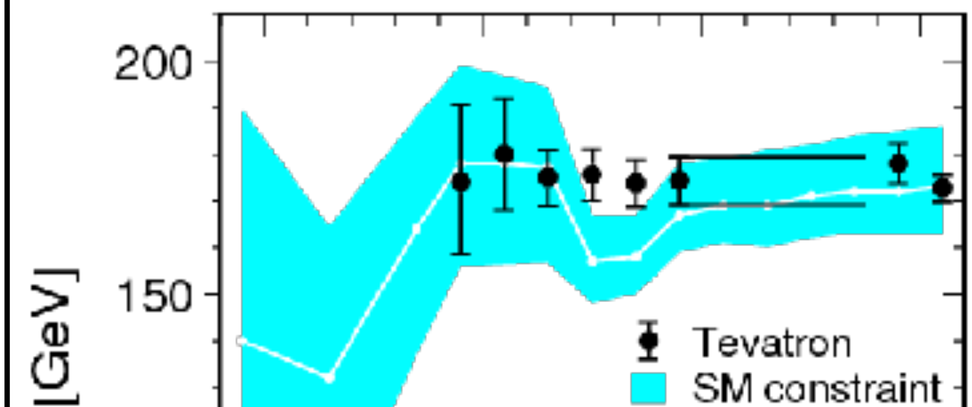
Problems with Weak and Hubble Scales

Problems associated with other two scales close related to one another

- Both come down to vacuum fluctuations

Vacuum Fluctuations *ARE REAL !*

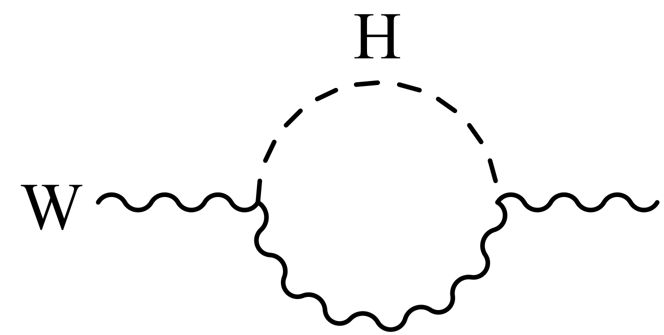




Precisely predict magnetic properties

g/2 = 1.0011596521809(8),

(Agree to better than one part in a trillion.)



m_H

(Predicted)

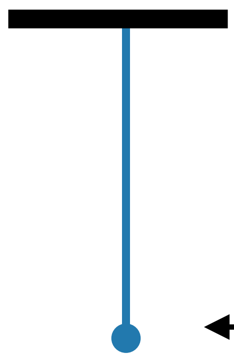
50 —— 150 GeV

115 GeV (Discovered)

Vacuum Has Energy

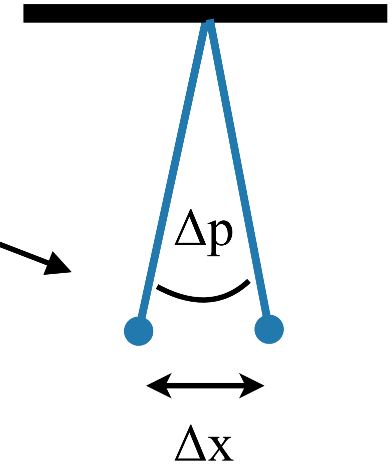
<u>Classically (w/o QM)</u>

<u>Quantum World</u>

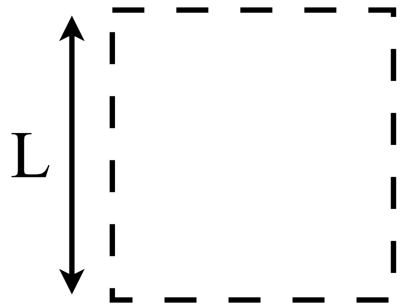


Minimum non-zero energy: E~hω

Lowest possible energy is 0



Estimate energy density in region of empty space: *Dimensional Analysis*

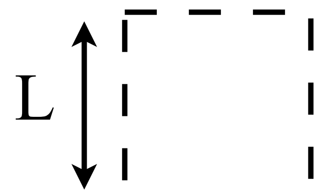


$\Lambda \sim \frac{\mathrm{E}}{\mathrm{V}} \sim \frac{1}{\mathrm{L}^4}$

$(\mathrm{V} \sim \mathrm{L}^3)$

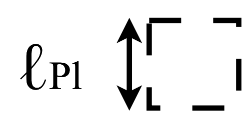
$(\mathrm{E} \sim \frac{1}{\mathrm{L}})$

Smaller Box



Λ much bigger

Reach: Cut-off



$\Lambda \sim \frac{1}{{\ell_{\mathrm{Pl}}}^4}$

...this is a problem

Cosmological Constant Problem

Without gravity constant energies (Λ) can be ignored *(overall offset)*
With gravity, constant energy warps space-time, interacts gravitationally

Uniform matter/energy controls size/expansion of overall Universe

$$t_{\mathrm{Double}} \sim \frac{1}{\sqrt{G_N \Lambda}} \sim \frac{1}{\sqrt{{\ell_{\mathrm{Pl}}}^2 \Lambda}}$$

- Naive cut off at ℓ_{Pl}: $\Rightarrow$ $t_{\mathrm{Double}} \sim 10^{-43}\ \mathrm{s}$

(would be bad for atoms/planets/people…)

- Conservative cut-off at 100 GeV: $\Rightarrow$ $t_{\mathrm{Double}} \sim 10\ \mathrm{ns}$

(would be bad for atoms(?)/planets/people…)

Measured: $t_{\mathrm{Double}} \sim 10^{10}$ years $\Rightarrow$ cut off of 10μm !

Cosmological Constant Problem

How do we deal with this in the current theory ?

from the vacuum fluctuations

Constant.
Input parameter to theory

$$\Lambda = \Lambda_{\mathrm{QM}} + \Lambda_{\mathrm{Classical}}$$

$$= \; 3.\,342\,862\,210 \ldots 554 \ldots \times {\ell_{\mathrm{Pl}}}^{-4} \qquad \boxed{\Lambda_{\mathrm{QM}}}$$

120 digits

$+$

$$-\; 3.\,342\,862\,210 \ldots 541 \ldots \times {\ell_{\mathrm{Pl}}}^{-4} \qquad \boxed{\Lambda_{\mathrm{Classical}}}$$

120 digits

Cosmological Constant Problem

How do we deal with this in the current theory ?

from the vacuum fluctuations

Constant.
Input parameter to theory

Classical

"Fine Tuning"

$10^{-20}\ \mathrm{GeV}^{-1}$
$(10^{-36}\ \mathrm{m})$

Planck scale

$10^{-3}\ \mathrm{GeV}^{-1}$
$(10^{-19}\ \mathrm{m})$

weak scale

$10^{41}\ \mathrm{GeV}^{-1}$
$(10^{25}\ \mathrm{m})$

Hubble scale

?

Why is the universe so big ?

Vacuum Fluctuations: Higgs Particle

Closely related problem

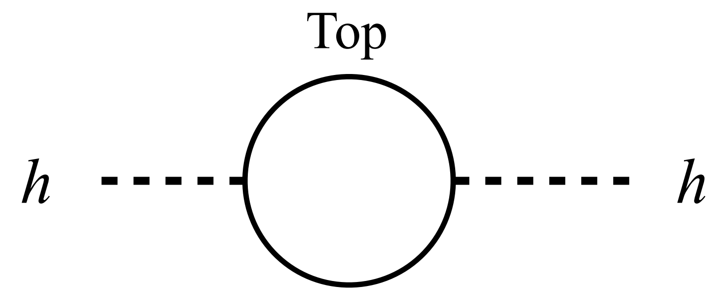
Vacuum fluctuations of Higgs mass (m_H^2)

$$mH^2 = \underbrace{2.569678321 \ldots 554}_{\text{30 digits}} \ldots \times \ell_{\mathrm{Pl}}^{\;2}$$

$$+$$

$$- \underbrace{2.569678321 \ldots 453}_{\text{30 digits}} \ldots \times \ell_{\mathrm{Pl}}^{\;2}$$

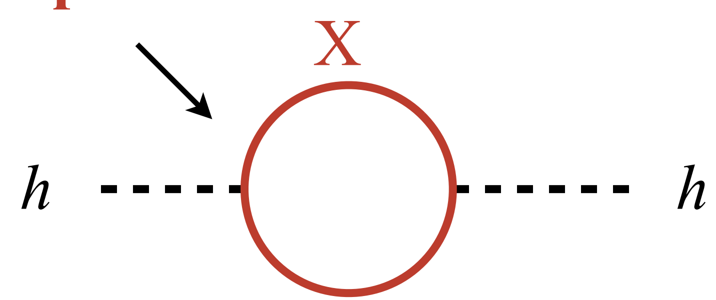
- Estimated mass corrections unreasonably large
- Instability of the Higgs mass



$$\sim \Lambda^2 \;\Rightarrow\; mH \sim 10^{20}\ \mathrm{GeV}$$

new heavy particle

mH ~ mX



Vacuum Fluctuations: Higgs Particle

Closely related problem

Top

Vacuum fluctuations of Higgs mass (m_H^2)

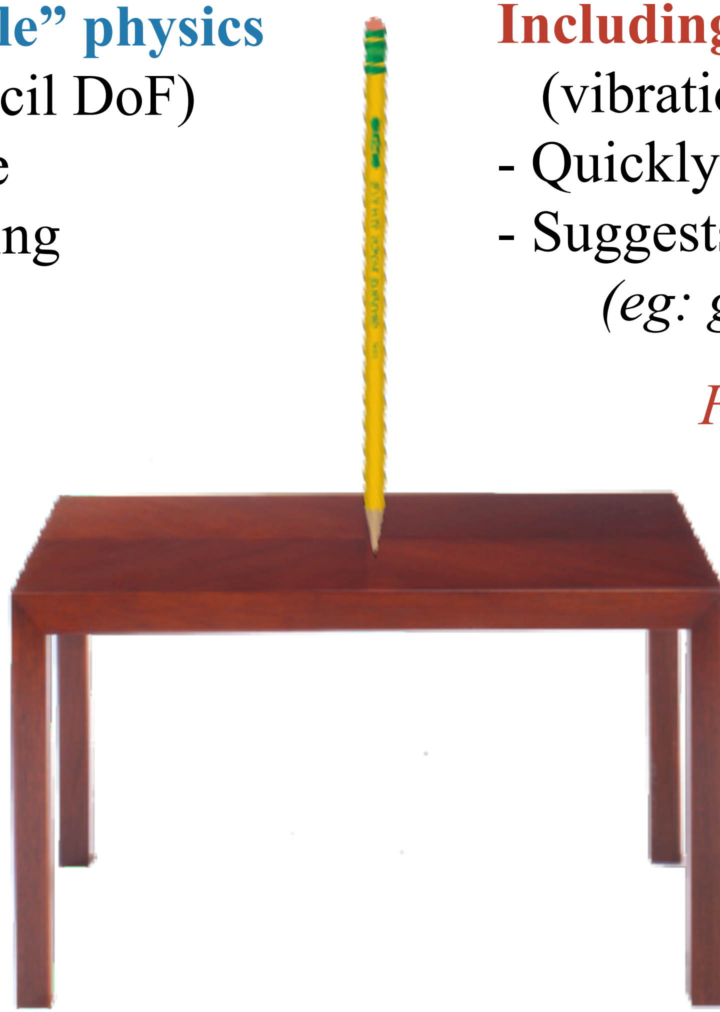
h

Without "small scale" physics

(only gravity + pencil DoF)

- Bizarre, but stable
- Suggests fine tuning

Higgs mass in SM



Including physics at smaller scales

(vibrations/ air molecules / atoms)

- Quickly lead to instability
- Suggests active mechanism

(eg: glue / string)

Higgs mass including new, high mass scale physics

eV

X

-

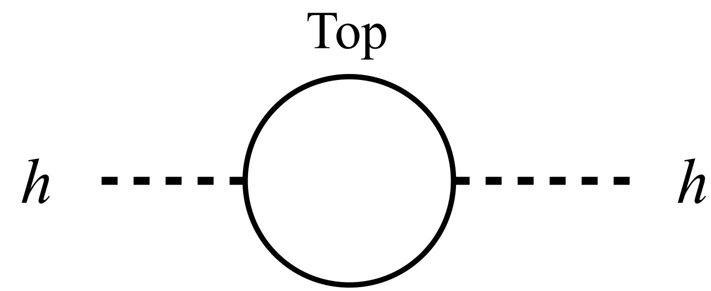
-

h

Vacuum Fluctuations: Higgs Particle

Closely related problem

Vacuum fluctuations of Higgs mass (m_H^2)



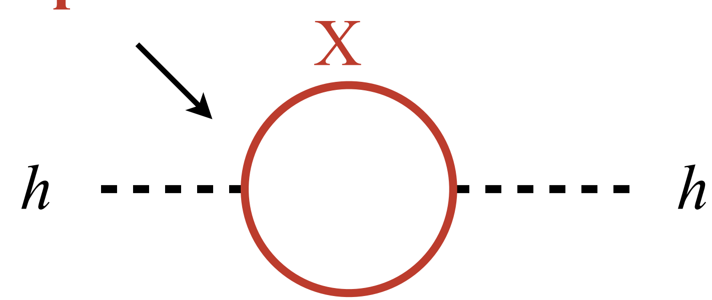
$$\sim \Lambda^2 \Rightarrow \mathrm{mH} \sim 10^{20}\ \mathrm{GeV}$$

$$\mathrm{mH}^2 = \underbrace{2.569678321 \ldots 554}_{\text{30 digits}}\ldots \times \ell_{Pl}^{\ 2}$$

$$+ \quad - \underbrace{2.569678321 \ldots 453}_{\text{30 digits}}\ldots \times \ell_{Pl}^{\ 2}$$

new heavy particle

$\mathrm{mH} \sim \mathrm{mX}$



- Estimated mass corrections unreasonably large
- Instability of the Higgs mass

Particular to Spin-0 particles

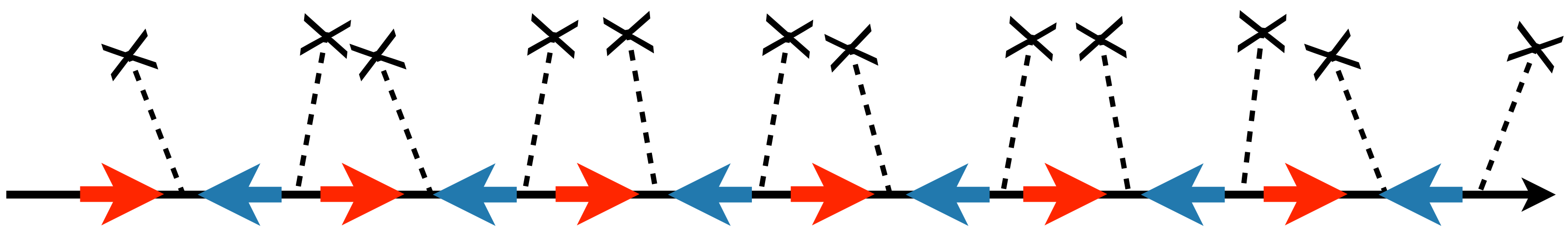
- Spin 1/2 Protected by charge conservation.
  Need interactions with v to get their mass
- Spin 1, 3/2, 2: need needed the extra particles ω/Ω-from

Vacuum Fluctuations: Higgs Field

Another way of talking about same problem

Can perform similar estimate for scale of interaction with condensate v

Same logic $\Rightarrow$ *Scale should be set by the cut-off in the theory*



$\sim \frac{1}{\Lambda}$

Naively, $\Lambda \sim \ell_{\rm Pl}$:

$$\sim \frac{1}{\ell_{\rm Pl}} \sim 10^{-20}\ \mathrm{GeV}^{-1} \sim 10^{-36}\ \mathrm{m}$$

$$\boxed{\text{Measured scale of:}\quad \sim 10^{-3}\ \mathrm{GeV}^{-1} \sim 10^{-19}\ \mathrm{m}}$$

$\Lambda \sim \ell_{\rm Pl}$ would be bad for atoms/planets/people… all blackholes

$$\frac{\mathrm{F_G}}{\mathrm{F_{EM}}} \sim ({\ell_{\rm Pl}}^2\Lambda^2)$$

Expect: ~ 1

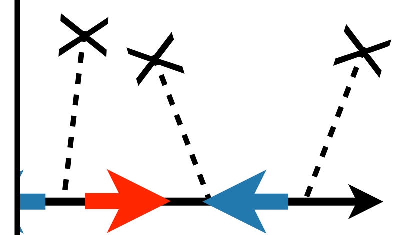
Observe: $\sim 10^{-34}$

Vacuum Fluctuations: Higgs Field

Another way of talking about same problem

C ... n with condensate v

... n the theory



... $\text{GeV}^{-1} \sim 10^{-36}$ m

... $\text{eV}^{-1} \sim 10^{-19}$ m

Weakness of gravity directly responsible
~ all structure around us

$$R_{\text{Planet}} \sim \sqrt{\frac{\alpha}{\alpha_G}} \times r_{\text{atom}}$$

$$R_{\text{Animal}} \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{1}{4}} \times r_{\text{atom}}$$

(Stars ...)

$\Lambda \sim \ell_{\text{Pl}}$ would be bad for atoms/planets/ ... all blackholes

Why is gravity so weak ?

$$\frac{F_G}{F_{EM}} \sim \left(\ell_{\text{Pl}}{}^2 \Lambda^2\right)$$

Expect: ~ 1

Observe: $\sim 10^{-34}$

Vacuum Fluctuations: Higgs Field

Another way of talking about same problem

C … n with condensate v

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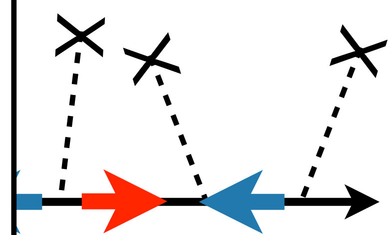
Weakness of gravity directly responsible
~ all structure around us

$$\mathrm{R}_{\mathrm{Planet}} \sim \sqrt{\frac{\alpha}{\alpha_G}} \times \mathrm{r}_{\mathrm{atom}}$$



"Hierarchy Problem"

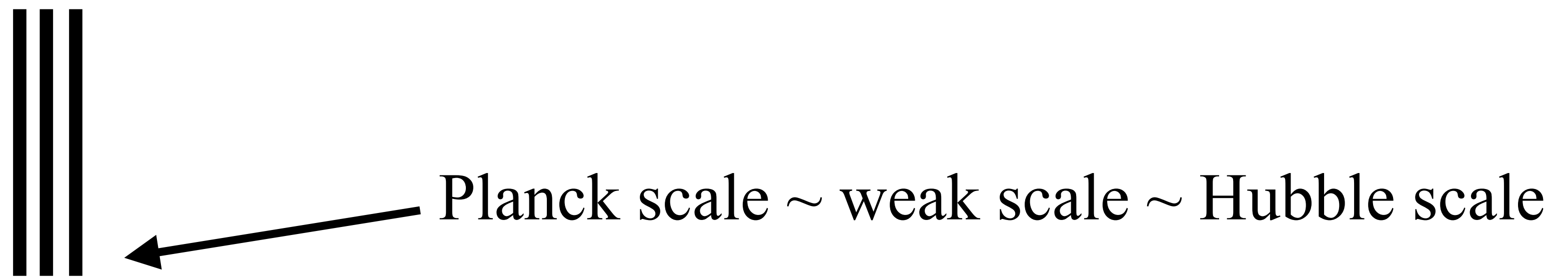
?

Why is gravity so weak ?

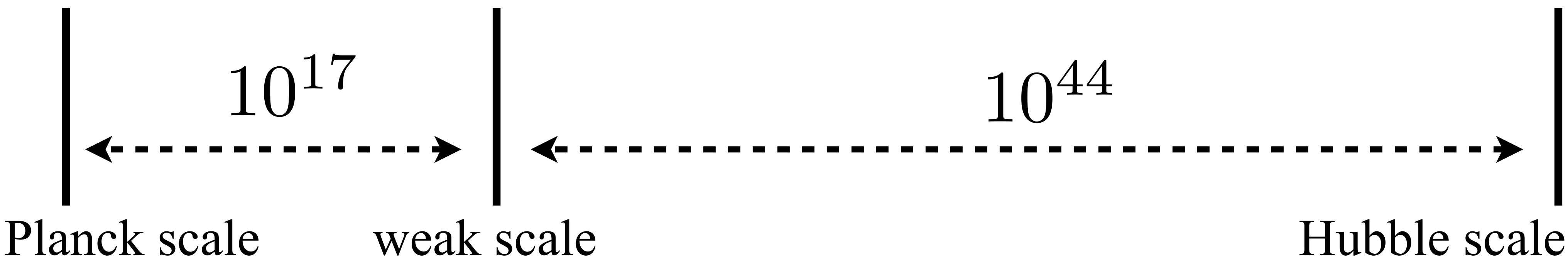
EM … Observe: ~ 10

Length Scales

Quantum Mechanics + Space-time leads us to expect:



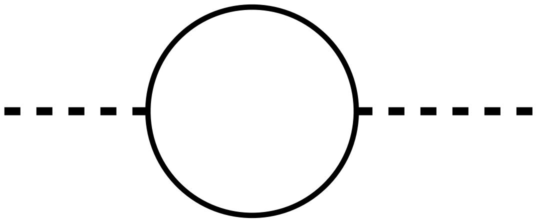
We observe:



Current theory accounts for huge difference w/implausible cancellation

Need modifications QM or Space-time to avoid fine tuning

What scale do we need Modification?

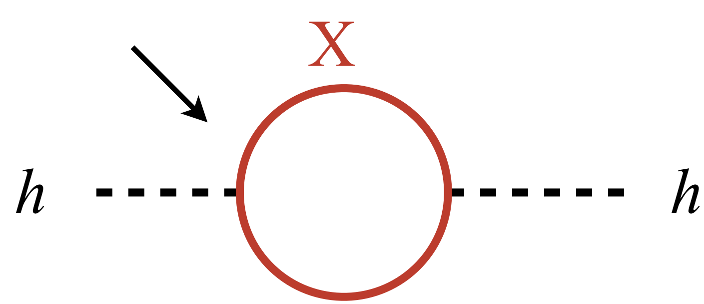
mH $=$ $+$ 

~ weak-scale mHClassical $\sim \Lambda^2$

Can avoid need for fine tuning only if Λ ~ weak-scale.

new particle mX ~ 1000 GeV

Need changes to stop vacuum fluctuations below: 10^{-3} GeV^{-1} $(10^{-19}$ m$)$



Dark Matter

Most natural explanation requires new physics at $10^{-3}\ \mathrm{GeV}^{-1}$ $(10^{-19}\ \mathrm{m})$