Homework Set #8

Due Date: Friday March 27th

1) Z boson decays: *(5 points)*
In class we estimated the branching ratio of $Z \to ee$ decays and $Z \to bb$ decays. Repeat these calculations. What approximations where made ? Compare this rough estimate to the experimentally measured values using the Particle Data Group's (PDG) Review of Particle Physics. You can find it at http://pdg.lbl.gov. All measured properties of all known particles are recorded here.

2) Muon decays: *(10 points)*

a) The muon decays via the weak interaction, At low energy ($E << m_W$), this can be approximated as a point-like interaction. Draw the diagram describing muon decay to an electron assuming a point-like weak interaction.

b) What are the dimensions of the coupling constant, associated to this diagram ?

c) How does the decay rate Γ (decays/unit time) depend on the muon mass ?

d) The muon has a mass of ~0.1 GeV and a lifetime of ~ $1\mu s$. The tau lepton has a mass of ~1 GeV. Estimate the lifetime of the tau lepton in μs.

e) Suppose that the photon could couple at the same vertex to the muon and the electron. Then the muon could decay as $\mu \to e\gamma$. Estimate the ratio of the μ lifetime in this world to that in our world without this interaction.

3) A new force. *(5 points)*
Assume there is another force of nature felt by electrons associated with the exchange of a new X boson of mass 1 TeV (= 1000 GeV).

a) Estimate an upper limit on the range of this new force in meters.

b) Assume that this new X boson could also decay to spin-1/2 dark matter particles ψ_{DM}. At low energies (« 1 TeV) the X interaction can be described by a point-like interaction. Estimate the coupling constant associated to dark matter scattering $e\psi_{DM} \to e\psi_{DM}$. (Assume the X coupling at high energies is the same as for EM)

c) Assume there was a direct $X \to e\mu$ interaction. This would allow the muon to decay via $\mu^- \to e^- e^+ e^-$. Draw the corresponding diagram and estimate the impact of the muon lifetime from this process. How does it compare to the lifetime in the standard model?

4) W boson decays to electrons. *(3 points)*

a) The W can decay directly to an electron or to electron by decaying through a τ. Draw the corresponding diagrams.

a) Estimate how often the W decays to an electron.

5) Higgs Boson Decays. *(3 points)*

The Higgs boson decays at a high rate to pairs of W-bosons. A clean way to observe this signal is in the $e\mu$ + Missing transverse energy final state. How often do $H \to WW$ events lead to a $e\mu$ final state?

6) Galactic Collisions. *(10 points)*

The Milky Way and Andromeda (M31) galaxies will collide in about 4 billion years. Both galaxies are spiral-type and contain about 100 billion stars each, which is comparable to the number of protons in a bunch at the LHC.

a) The two galaxies are similar in size and disk-shaped, with a diameter of about 10^5 light-years and a thickness of about 10^3 light-years. If the relative velocity of the Milky Way and Andromeda galaxies is about 10^5 m/s, estimate the flux factor (luminosity) at collision. How does this compare to the luminosity of proton collisions at the LHC?

b) What is the average distance between two stars in these galaxies? Assuming that an average star is about the size of our Sun (radius 7×10^8 m), what is the ratio of the average distance to star radius? Correspondingly, what is the ratio between the average distance between protons in a bunch at the LHC to the proton radius?

c) If the process of collision of the Milky Way and Andromeda galaxies takes about a billion years, approximately how many individual stars will collide? Is this similar to the number of proton collisions per bunch crossing at the LHC?