Midterm

1) Units/Dimensional Analysis: Muonic Atoms *(6 points)*

a) The muon was the first elementary particle discovered that does not appear in ordinary atoms. Negative muons can, however, form muonic atoms by replacing an electron in an ordinary atom. Estimate the size and binding energy of muonic hydrogen in GeV. A muon is 200 times heavier than an electron.

$m_\mu << m_p \Rightarrow$ to first order all motion in muon.

$$E \sim \frac{p^2}{m} - \frac{\alpha}{r}$$

QM $\Rightarrow p \sim \frac{1}{r}$

$$E \sim \frac{1}{m_\mu r^2} - \frac{\alpha}{r}$$

Equilibrium when terms are $\sim$ equal.

$$\frac{1}{m_\mu r^2} = \frac{\alpha}{r} \Rightarrow r \sim \frac{1}{\alpha m_\mu} = 10^2\ 10^1\ GeV^{-1} = 10^3\ GeV^{-1}$$

Can get E from plugging this back into either of the two terms.

$$E \sim \frac{\alpha}{r} \sim \alpha^2 m_\mu \sim 10^{-4}\ 10^{-1}\ GeV = 10^{-5}\ GeV$$

For regular hydrogen, same arguments as above with $m_\mu \rightarrow m_e$

$\Rightarrow$

$$\frac{E^\mu}{E^e} = \frac{\alpha^2 m_\mu}{\alpha^2 m_e} = \frac{m_\mu}{m_e}$$

b) How much bigger/smaller would matter be in a world made out of muonic atoms ?

From above,

$$\frac{r_{muonic}}{r_{normal}} \sim \frac{1}{\alpha m_\mu} \times \frac{\alpha m_e}{1} \sim \frac{m_e}{m_\mu} \sim \frac{1}{200}$$

Distances would be 200 times smaller.

2) Relativity: How many generators does the Lorentz group have ? What transformations do they correspond to ? *(3 points)*

6: 3-rotations about x,y,z and 3 boosts along x,y,and z

3) Quantum Mechanics: *(4 points)*

How do position eigenstates transform under Translations ?

$T(\vec{a})\,|\vec{x}\rangle = |\vec{x} + \vec{a}\rangle$

How do momentum eigenstates transform under Translations ?

$T(\vec{a})\,|\vec{p}\rangle = e^{ip\cdot a}\,|\vec{p}\rangle$

4) Why does combining QM and Relativity require the existence of anti-particles? *(5 points)*

Causality or , more precisely, the need for the Hamiltonitan to commute outside the light cone

5) What is the little group? Why is it useful ? *(2 points)*

Subgroup of Lorentz transformations that leave the momentum invariant. They are useful b/c they simplify the problem of finding the other labels (spin and helicity) that the lorentz group acts on.

6) Lorentz Transforms *(6 points)*

a) How does a **massive** particle transform under a general Lorentz transformation (Λ^{ν}_{μ}) ?

$\Lambda^{\nu}_{\mu}\,|p^{\mu}, \sigma\rangle = \sum_{\sigma'} R_{\sigma\sigma'}\,|\Lambda p, \sigma'\rangle$, where R is a rotation matrix.

b) How does a **mass-less** particle $|p^{\mu}, h\rangle$ transform under a general Lorentz transformation (Λ^{ν}_{μ})?

$\Lambda^{\nu}_{\mu}\,|p^{\mu}, \sigma\rangle = e^{ih\theta}\,|\Lambda p, \sigma'\rangle$, where h is the particles helicity.xo

7) Relativistic Wave Equations *(4 points)*

a) What modifications where required to the Schrodinger equation to make it relativistic?

Need to use the same number of space and time derivatives; eg: klien gordon equation or the dirac equation.

b) What modifications to Maxwell's equations where required to make them relativistic?

None. The Maxwell's equations are already relativistic.

8) Fields *(3 points)* In QFT why do we write interactions in terms of fields (Fourier transforms of creation and annihilation operators) instead of the creation and annihilation operators directly?

In order to make the interactions local in space-time.

9) Lagrangians *(4 points)*

Consider the following Lagrangian:

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 + \bar{\psi}[i\gamma_\mu \partial^\mu]\psi + g_1\phi\psi\psi + g_2\psi\psi\psi\psi + g_3\phi\phi\phi\phi$$

a) What is the dimension of g_1 ?

dimension-less

b) What is the dimension of g_2 ?

$[\mathrm{GeV}]^{-2}$

c) What is the dimension of g_3 ?

dimension-less

10) Feynman Diagrams *(6 points)*

a) What is the S-matrix ?

Operator that takes asymptotic states in the far past to asymptotic states in the far future.

b) What is a Feynman diagram ? and what is its relationship to the s-matrix?

A feynman diagram is a term in the expansion of the s-matrix. It (roughly) describes one possible way an interaction could have occured.

11) What are <u>two</u> constraints on the interactions of mass-less Spin-1 particles, apart form coming from a Lorentz-invariant Lagrangian? *(6 points)*

- they have to couple to particles of the same mass

- Charge must be conserved

- Interaction must obey a lie group

12) Can the SM have interactions between fermions and mass-less particles with Spin 2 ? If not, why not. What about interactions between mass-less Spin 3 particles and fermions ? If not, why not. *(4 points)*

- Spin-2: yes, althought the couplingsa are unique.

- Spin-3 No, would violate $p \cdot M = 0$ as required by Lorentz invariance.