Lecture 35

OK left off discussing atmospheric νs

Saw that there are two robust predictions that you can measure:

- muon vs electron ratio and

- top vs down ratio.

Expect the same amount from above and from below.
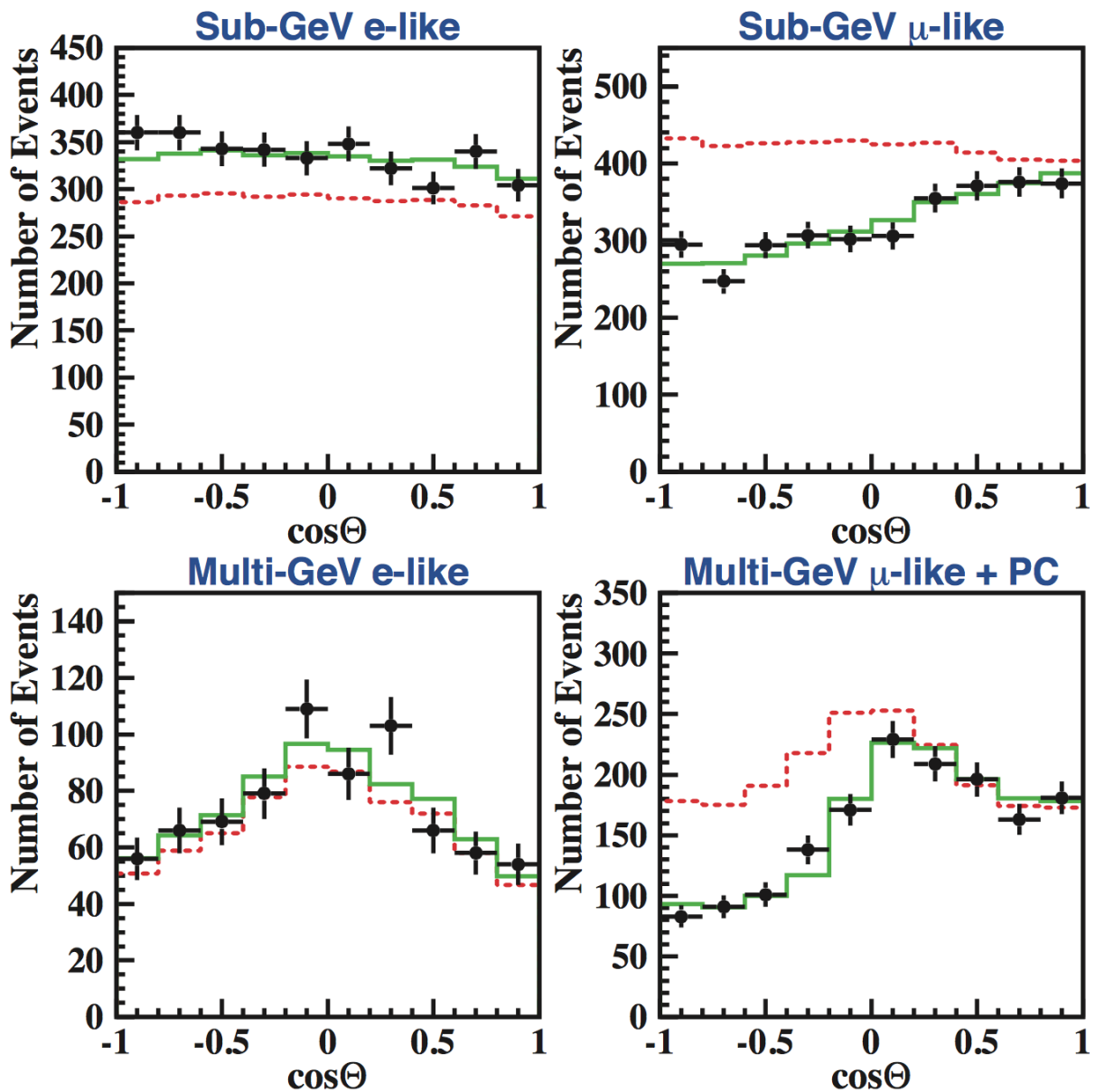
So that's what people know about atmospheric νs.

The experiments that measured atmospheric νs where giant water tanks.

Kamiokande - Name Kamioka place / nde - "nucleon decay experiment. "

Built to look for proton decay, but atmospheric νs where a background one you cant get rid of.

When you measure the flux from above and below turns out the answer is not one: 1/2.

They could actually do better. Tell muons from electrons. These plots show log/high energy electrons and muons separately.

Sub-GeV e-like — **Sub-GeV μ-like** — **Multi-GeV e-like** — **Multi-GeV μ-like + PC**

Number of Events vs cosΘ

Very interesting results. Points are the observations, dashed line is the prediction. Notice a few things:

- For the electrons everything works more or less OK,

- For the muons it doesn't work well at all.

- νs coming from above work OK,

- νs coming from below don't work well. (Missing about half of them.)

You have an effect that says you don't understand what the ν_μs are doing and that effect depends on the energy and how far they are propagating.

Very exciting result. Because you're sure the measurements are correct. Observable is robust. Implies that the νs are doing something. And whatever they are doing depends on the energy and the baseline (how much they have traveled)

Why is this important ? need a hypothesis for what is going on. Maybe the νs are being absorbed ? νs that go through the earth are getting absorbed by the earth. Know Cant be true, cross section would have to be too high.

What else could be going on? νs decaying or changing flavour. Only looking for muon and electron νs, so if they were converting into ν_τs, this could explain all this data.

Side-note:
Only massive particles know about time.
What ever they are doing If νs can tell time ==> νs have mass.
OR Lorentz invariance is wrong.

So that's where we were. Bottom line νschange flavour as a function of E and distance.

Mass-induced flavour oscillations.

What happens if the νs have mass ?

$$\nu_1 \text{ with } m_1$$
$$\nu_2 \text{ with } m_2$$
$$\nu_3 \text{ with } m_3$$

If you raise the hypothesis that νs have mass, then there are νs states that you can label with different masses.

We also have ν_e, ν_μ and ν_τ these are the interaction eigenstates the νs you produce when you have a weak interaction.

The question is then which one of these is the ν_e, ν_μ and ν_τ?

The answer is it doesn't have to be any of them. What we know for sure is that the ν_e is a linear combination of νs with a well-defined bass

$$\nu_e = U_{ei}\nu_i$$

same for mus and taus.

$$\nu_\mu = U_{\mu i}\nu_i$$
$$\nu_\tau = U_{\tau i}\nu_i$$

also know that the ν_e, ν_μ and ν_τ or orthogonal to one another (ie: they are different states)

These can then be organised into a unitary matrix.

$$\nu_\alpha = U_{\alpha i}\nu_i$$

where, $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$.

$U_{\alpha i}$ is Unitary mixing matrix. We will see, just by raising this hypothesis you can explain all the data.

Parallel in Quark Sector

Actually similar thing happens in the quark sector, we haven't talked about this yet, but its true.

Have do identify who are the "real particles". Meaning what are the eigenstates of the free Hamiltonian.

In the quark section that's the

$$u\ c\ t$$
$$d\ s\ b$$

In the lepton sector we choose that to be

$$e\ \mu\ \tau$$
$$\nu_1\ \nu_2\ \nu_3$$

There is no such thing as an ν_e, doesn't exist.

but the weak interactions couple to linear combinations of these real particles.

$W \rightarrow t + b, s, b$ w/coupling $V_{t,b}^{CKM}$

the same thing happens in the lepton sector:

$W \rightarrow e + \nu_1, \nu_2, \nu_3$ w/coupling $U_{e,i}$

physics is the same, the consequences turn out to be very different. Main reason is that the ν masses are very small. Bc the masses are small (as well talk about in a second) you have a phenomena of ν oscillations. Doesn't happen for quarks.

ν Oscillations

OK lets set up this physics behind this idea..

What does it mean to be a particle with a well defined mass? QM point-of-view, eigenstates of free particle Hamiltonian.

$$|\nu_1(t)\rangle = e^{-iE_1 t}|\nu_1\rangle$$

this is what it means to be a ν with a well-defined mass.

Now, what happens if you don't have one of these, but a linear superposition ?

Lets say you have the ν_e and, to make life easy, lets pretend that we only have 2 νs.

Now ν_e will be a linear combination of ν_1 and ν_2.

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

For completeness, can also write ν_μ, which is also a linear combination of ν_1 and ν_2, but it is a orthogonal to ν_e.

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Some obvious things, we know $\cos\theta$ and $\sin\theta$ are some coefficients that we don't know, but we know the sum of squares is 1. That's why we write it like an angle.

(Can work this out from $\langle v_e | v_e \rangle = 1$ and $\langle v_e | v_\mu \rangle = 0$) That's why are allowed to paramaterize things this way

How do these states evolve as a function of time?

$$|v_e(t)\rangle = \cos\theta e^{-iE_1 t}|v_1\rangle + \sin\theta e^{-iE_2 t}|v_2\rangle$$

This is the heart of what v oscillations are all about. Because these phases are different, what you get is no longer proportional to the v_e state. Or in fancier words, v_e is not an eigenstate of the free Hamiltonian.

Now this is very very simple physics. Literally a 2 level system that you learned in undergraduate QM.

Remember everything is going to be relativistic. vs are going to be propagating plane waves.

Relativistic version

$$|v_e(\vec{x}, t)\rangle = \cos\theta e^{-ip_1^\mu x_\mu}|v_1\rangle + \sin\theta e^{-ip_2^\mu x_\mu}|v_2\rangle$$

where x is the $(t, \vec{x})$ four vector.

Ultra relativistic approx.

$$t \sim L$$

Phase factors very close to being zero, depend on difference between energy and momentum $(E_1 - p_1)$

Now let me calculate this difference in the following way,

$$(E_1 - p_1)(E_1 + p_1) = m_1^2 \Rightarrow (E_1 - p_1) = \frac{m_1^2}{2E}$$

in the ultra relativistic approx $E \sim P$

$$|v_e(L)\rangle = \cos\theta e^{-i\frac{m_1^2}{2E}L}|v_1\rangle + \sin\theta e^{-i\frac{m_2^2}{2E}L}|v_2\rangle$$

OK, now lets calculate something...

Lets calculate the probability that this object here, when it hits something produces an electron.

Which is just given by

$$\langle \nu_e | \nu_e(L) \rangle = \cos^2 \theta e^{-i\frac{m_1^2}{2E}L} + \sin^2 \theta e^{-i\frac{m_2^2}{2E}L}$$

is the amplitude for having an ν_e be born somewhere, propagate some distance L and then be detected as an ν_e. So the probability is this thing squared:

$$
\begin{aligned}
|\langle \nu_e | \nu_e(L) \rangle|^2 &= \left| \cos^2 \theta e^{-i\frac{m_1^2}{2E}L} + \sin^2 \theta e^{-i\frac{m_2^2}{2E}L} \right|^2 \\
&= \left| \cos^2 \theta + \sin^2 \theta e^{-i\frac{m_2^2 - m_1^2}{2E}L} \right|^2 \\
&= \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^2 \theta \, 2 \cos\left(\frac{\Delta m^2 L}{2E} \right)
\end{aligned}
$$

where $\Delta m^2 = m_2^2 - m_1^2$. Can simply this...

$$
\begin{aligned}
&= \left(\cos^2 \theta + \sin^2 \theta \right)^2 - 2\cos^2 \theta \sin^2 \theta + 2\cos^2 \theta \sin^2 \theta \cos\left(\frac{\Delta m^2 L}{2E} \right) \\
&= 1 - 2\cos^2 \theta \sin^2 \theta \left(1 - \cos\left(\frac{\Delta m^2 L}{2E} \right) \right)
\end{aligned}
$$

Now use some trig IDs...

$$
\begin{aligned}
&= 1 - 4\sin^2 \theta \cos^2 \theta \sin^2\left(\frac{\Delta m^2 L}{2E} \right) \\
&= 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{2E} \right)
\end{aligned}
$$

So what we learn at the end of the day is, if your born as ν_e and you propagate a certain distance L, and you're detected as an ν_e, the probability is given by,

$$P_{ee}(L) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{2E} \right)$$

Possible to be born as ν_e and detected as ν_e with less than 100% probability!

So we have derived this very simple two flavour oscillation formula. And we can actually get a lot of mileage out of this.

Turns out that is what we need to explain almost all the ν data we have seen so far. Trivial expression, but very important one.

Oscillates as a function of L/E. (We knew from that beginning that whatever we got had to depend on L/E (Lorentz invariance).)

As a function of L for fixed E looks like a sine wave. Two important behaviours of this function

1) L is very small

2) Oscillations are very fast.

In these two limits life is very simple:

L small $\Rightarrow 1 - \sin^2 2\theta \left(\Delta m^2 L/4E\right)^2$ (behaviour that turns on slowly as L^2)

L large $\Rightarrow 1 - \frac{1}{2}\sin^2 2\theta$

Two physics parameters that you can fit: θ and Δm^2. This tell what sorts of experiments you need to do. θ some number between 0 and 1. Δm^2 tells you what values of L and E you need to pick in order to get a significant result.

For scale, with a 1 GeV ν and $\Delta m^2 \sim eV^2$ oscillation length order km.

"Explains" why we didn't see this in the first types of ν experiments that we built.

Turns out that $\Delta m^2 \sim 10^{-3} - 10^{-4} eV^2$ So for a 1 GeV ν to see large effect, need baselines L 100-10,000 km. This leads to ridiculous experiments, Have to be very far away from your ν source. On the other hand, nice that the earth has roughly the right size that you can see this with atmospheric νs.

Talk about this now.

Explain how this fits some of the data that we looked at before.

Lets look back at the Super K data.

Look at the electrons first. Dashed lines are predicted flux. Data agrees, means that the electrons νs aren't doing anything. (Don't know how to predict the overall normalization accurately.)

The muons νs don't work out well. So we will use oscillations to explain that. Lets look at the high energy ν_μ's The νs coming from above look OK. The νs from the other side of the earth don't look good. The way to make this work is pretty simple. Pick Δm^2 such that for νs coming from above, $\sim$ 50 km, the oscillation events are small.

Now go to the other side of the earth, and we need a large effect: missing 50% of the νs. Pick Δm^2 in such a way that the oscillations are very fast, compared to with the resolutions of your detector. Pick a Δm^2 such that the oscillations average out. To get factor of 2, need $\sin^2 2\theta \sim 1$ which is as big as it can be. Turns out you need $\Delta m^2 \sim 10^{-3} eV^2$

Whats exciting is once you've don't that you've exhausted all your physics parameters. All the other panels have to be fit by the same choices you made. That's what the solid lines shows. It works really well.

Very high-energy νs there is not a big effect from the oscillation. An incredible successful story that measures these parameters really well.

To convince you, we have repeated this experiment in beam experiments (Japan and the US) Once you know the Δm^2 you know the kind of distances you need.

We say earlier that the ν_e's weren't doing anything. That means that whatever oscillation was going on it is interpreted to be between ν_μs and ν_τs.

(If that's whats happening, how do we convince our self of that: Also doing experiments where you try to see the ν_τ interactions. Seeing ν_τs is very hard. Can use neutral currents: in SuperK. Expect an excess of NC events b/c of tau decays. Hard but $\sim 3\sigma$, probably wont claim ν_τ appearance based on this alone.

So there is OPERA detector CERN. Whole point it is to see the tau track. Have convincing evidence that this is what is indeed going on.)