Lecture 8

Particles

Now, QM + SR. ("Quantum Feild Theory in a week")

Particles! Particles! Particles!

Fundamentally everything we are talking about is the interaction of particles. No such thing as sometimes waves, sometimes particles, its all particles. No particle-wave duality (old fashion).

QM Particles Sometimes macroscopic collictions of QM particles (when they are bosons) have a nice interpretation as classical waves . Macroscopic collictions of fermions look like classical particles.

(By the way do people know what I'm talking about when I say bosons vs fermions? QM, identical particles are fundamentally indestinguishable.

$$|\psi(p_1, p_2)|^2 = |\psi(p_2, p_1)|^2 \Rightarrow \psi(p_1, p_2) = \pm\psi(p_2, p_1)$$

Catagorizes all particles in nature into two classes (NR QM effect)
$\psi(p_1, p_2) = +\psi(p_2, p_1)$ (for bosons)
$\psi(p_1, p_2) = +\psi(p_2, p_1)$ (for fermions))

Feilds are also a secondary notion. They are a convient way of talking about the interactions of particles.

What particles are: single particle states are irreducible representations of the Poincare group. Poincare group = (translation + Lorentz transformations.)

We have symmetries, (handed down to us from the 1st half o the twenty century) its a good idea to talk about what they can act on. The things they act on can be broken down into the irreducible representations. Associated with every Poincare transformation, there should be some unitary operator that acts on the Hilbert space of the theory.

Why do particles have something to do with symmetries? Momentum eigenstates behave nicely under translations. In a world that is translationally invariant, useful to talk about momentum eigenstates. We use the same word for particles moving in different directions even thought they are differnt states, because they are related under rotations.

So, what are the labels you can have on states for which translations and rotations act nicely? Non-relativistically, translations labeled by $\vec{P}$, Rotations by spin.

We want to generalize this to relativity. Translations, Rotations and Boosts, the whole Lorentz group.

What are the possible labeles?

The answer is going to end up being

-) Massive: Same thing we are used to $\vec{P_\mu}$ and spin

-) Massless: Always $\vec{P_\mu}$, but now labeled by helecity, not spin

Different number of DoF than massive particles. **Basic, deep and striking feature of relativitic QM** B/c I cant boost to a frame where the massless particle is at rest.

Sketch the argument more formally

Start with translations

$$|P^\mu, \sigma\rangle$$

Label the states with P^μ and whatever else they are (which we will come to shortly) call σ.

Under $x^\mu \to x^\mu + a^\mu$,

$$U(T(a^\mu))\,|P^\mu, \sigma\rangle = e^{iP_\mu a^\mu}\,|P^\mu, \sigma\rangle$$

Now have to talk about how Lorentz transformations act. where things get interesting...

For $\Lambda^\mu{}_\nu$ there is some unitary operator $U[\Lambda]$ that acts on the states. Need to know, $U[\Lambda]\,|P, \sigma\rangle$. What can this action possible be?

Most niave answer: $U[\Lambda]\,|P, \sigma\rangle = |\Lambda P, \sigma\rangle$.

This would give action under $\Lambda^\mu{}_\nu$, but not the most general one.

Most general one,

$$U[\Lambda]\,|P,\sigma\rangle = \sum_{\sigma'} D_{\sigma,\sigma'}(\Lambda)\,|\Lambda P,\sigma'\rangle$$