Lecture 16

Example:

$$L = -\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3$$

Consider cross-section for $\phi\phi \to \phi\phi$ scattering.

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|}|M|^2 d\Pi_{LIPS}$$

$p_1 + p_2 \to p_3 + p_4$

In COM frame, $\vec{p}_1 = -\vec{p}_2$ and $\vec{p}_3 = -\vec{p}_4$
Also, $E_1 + E_2 = E_3 + E_4 = E_{CM}$

$$d\Pi_{LIPS} = (2\pi)^4 \delta^4(\sum p)\frac{d^3 p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3 p_4}{(2\pi)^3}\frac{1}{2E_4}$$

(integrating over $\vec{p}_4$)

$$= \frac{1}{16\pi^2}d\Omega \int dp_f \frac{p_f^2}{E_3}\frac{1}{E_4}\delta(E_3 + E_4 - E_{CM})$$

where $p_f = |\vec{p}_3| = |\vec{p}_4|$, $E_3 = \sqrt{m^2 + p_f^2}$, and $\int d^3 p_f = \int dp_f p_f^2 d\Omega$

Now change variables,

$p_f \to x = E_3 + E_4 - E_{CM}$

$$dx = \frac{d}{dp_f}(E_3 + E_4 - E_{CM})dp_f = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{E_3 + E_4}{E_3 E_4}p_f dp_f$$

$\Rightarrow$

$$\frac{dp_f p_f^2}{E_3 E_4} = \frac{dx p_f}{E_{CM}}$$

$$
\begin{aligned}
d\Pi_{LIPS} &= \frac{1}{16\pi^2} d\Omega \int_{m_3+m_4-E_{CM}}^{\infty} dx \frac{p_f}{E_{CM}} \delta(x) \\
&= \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{CM}} \text{if } E_{CM} > m_3 + m_4 \text{ else } 0
\end{aligned}
$$

$$
d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|} \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{CM}} |M|^2
$$

$$
|v_1 - v_2| = \left| \frac{|\vec{p}_1|}{E_1} + \frac{|\vec{p}_2|}{E_2} \right| = p_i \frac{E_{CM}}{E_1 E_2}
$$

$\Rightarrow$

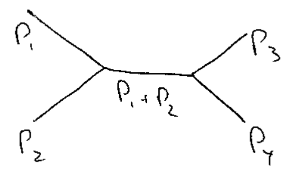
$$
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2} \frac{p_f}{p_i} |M|^2
$$

(if masses are equal $p_f = p_i$)

Now to M

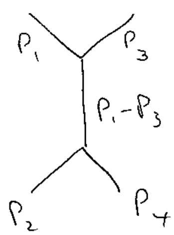
Now to M

"s-channel" diagram



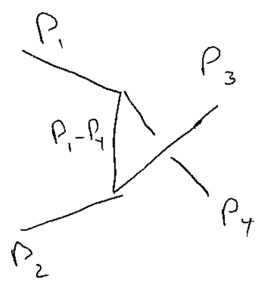
$$= (ig)\,\frac{i}{\underbrace{(p_1+p_2)^2}_{\equiv s} - m^2}\,(ig) = \frac{-ig^2}{s-m^2}$$

t-channel



$$= (ig)\,\frac{i}{\underbrace{(p_1-p_3)^2}_{\equiv t} - m^2}\,ig = \frac{-ig^2}{t-m^2}$$

u-channel



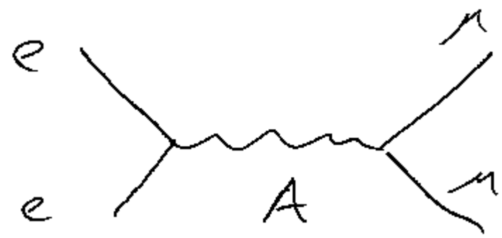
$$= (ig)\,\frac{i}{(p_1-p_4)^2 - m^2}\,ig = \frac{-ig^2}{u-m^2}$$

$$\frac{d\sigma}{d\Omega}(\phi\phi \to \phi\phi) = \frac{g^4}{64\pi^2 E_{CM}^2}\left[\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right]^2$$

$s + t + u = \sum m_j^2$ (s,t,u are L. I)

Example 2

Electron-positron to muons scattering.



Note from:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2}\frac{p_f}{p_i}|M|^2$$

that M is dimensionless.

It is given by combination of dimensionless couplings and appropriate spin projections.

Focus on the projections on

- initial spins to the γ polarization
- γ polarization to final spins

$$M(s_1 s_2 \to s_3 s_4) = \underbrace{\sum_{\epsilon}}_{\text{photon polarizations}} \langle \underbrace{s_3 s_4}_{\text{Spin of } \mu\text{s}} |\epsilon\rangle\langle\epsilon| \underbrace{s_1 s_2}_{\text{Spin of } e\text{s}} \rangle$$

At high-energies take e and μ to be massless.

$$P_1 = (E, 0, 0, E) \qquad\qquad P_1 = (E, 0, 0, -E)$$

In this limit think of the electron as having helicity.

We will solve this now in linear basis (you will do circular in HW)

$$|s_1 s_2\rangle = \underbrace{|\leftrightarrow\leftrightarrow\rangle}_{\text{along-x}}, \underbrace{|\updownarrow\updownarrow\rangle}_{\text{along-y}}, |\leftrightarrow\updownarrow\rangle, |\updownarrow\leftrightarrow\rangle$$

where the spins are 1/2.

However, only two combinations can poject onto a spin-1 photon

$$|\leftrightarrow\leftrightarrow\rangle, |\updownarrow\updownarrow\rangle,$$

<u>photon polarizations</u>

$$\epsilon^1 = (0, 1, 0, 0) \qquad\qquad \epsilon^1 = (0, 0, 1, 0)$$

$|\leftrightarrow\leftrightarrow\rangle$ gives ϵ^1
$|\updownarrow\updownarrow\rangle$ gives ϵ^2

Now, the μ's are also spin 1/2. (Also have 4 spin states)

In general, μ not moving along same direction as the incoming electrons.

Can paramaterize with θ, (symmetric under ϕ can rotate such that $\phi = 0$)

$$P_3 = E(1, 0, \sin\theta, \cos\theta) \qquad\qquad P_4 = E(1, 0, -\sin\theta, -\cos\theta)$$

for muons, 2 possible directions of photon polarizations

$$\bar{\epsilon}^1 = (0, 1, 0, 0) \qquad\qquad \bar{\epsilon}^1 = (0, 0, \cos\theta, -\sin\theta)$$

(Can check that these are perpindicular to P_3 and P_4)

In general, hard to measure spins. Sum over all μ spins.

Must sum over all possible $\left|\begin{array}{l}\text{initial}\\ \text{final}\end{array}\right.$ polarizations

For us only

- $M_1 = M(|\leftrightarrow\leftrightarrow\rangle \to |\bar{\epsilon^1}\rangle = \epsilon^1\bar{\epsilon^1} = -1$
- $M_2 = M(|\updownarrow\updownarrow\rangle \to |\bar{\epsilon^2}\rangle = \epsilon^2\bar{\epsilon^2} = -cos\theta$

are non-zero.

If our initial beams are unpolarized, sum over initial spins.

$$|M|^2 = |M_1|^2 + |M_2|^2 = 1 + \cos^2\theta$$

$$\frac{d\sigma}{d\Omega} \sim \frac{\alpha^2}{E_{CM}^2}(1 + \cos^2\theta)$$