Noether's Theorem

Lagrangian may be invariant under some type of variation eg $\phi \to \phi + \delta$

ex $\cancel{\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \quad \phi \to e^{-i\alpha}\phi \quad \forall \alpha}$

transformation is a symmetry of $\mathcal{L}$

ϕ - complex 2 dof ϕ & ϕ^*

$$\mathcal{L} = (\partial_\mu \phi)(\partial_\mu \phi^*) - m^2 \phi \phi^*$$

Symmetry

$$\phi \to e^{-i\alpha}\phi \quad \phi^* \to e^{i\alpha}\phi^*$$

Whenever have a continuous symmetry (there is an infinitesimal limit)

$$\frac{\delta \mathcal{L}}{\delta \alpha} = 0 = \sum_n \left[\frac{\partial \mathcal{L}}{\partial \phi_n} \frac{\delta \phi_n}{\delta \alpha} + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \frac{\delta (\partial_\mu \phi_n)}{\delta \alpha} \right]$$

$$\phi_n = \{\phi, \phi^*\} \qquad = \sum_n \left\{ \underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi_n} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \right]}_{= 0 \ \text{E/L}} \frac{\delta \phi_n}{\delta \alpha} + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right] \right.$$

$$\Rightarrow \quad \partial_\mu J_\mu = 0 \qquad J_\mu = \sum_n \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \quad \text{"Noether Current"}$$

J_μ - "conserved current"

H.W. $\left(J_\mu = -i(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) \right)$

total charge $\equiv Q = \int d^3x \, J_0 \qquad \partial_t Q = \int d^3x \, \partial_t J_0 = \int d^3x \, \vec{\nabla} \cdot J = 0$

J vanishes on boundary

Q does not change with time!

Very General + Important theorem "Noethers Theorem"

If $\mathcal{L}$ has "enjoys" a continuous symmetry, there exists an associated current that is conserved.

$$\phi(x) \to \phi(x+\xi) = \phi(x) + \xi^{\nu}\partial_{\nu}\phi(x)$$

this leaves $\mathcal{L}$ invariant gives
S

energy-momentum tensor Gives a 4-vector of noethers currents

$\Rightarrow$ noethers theorem tells us why energy + mom are conserved.

Cross Sections & Decay Rates

20th century witnessed development of collider physics
effective means to determine which | particles exist
| their properties
| interactions

Rutherford discovery of nucleus using α 1911
Andersons discovery of anti-electrons 1932 } Nature

around 1930's manmade collisions started winning.
1 MeV

Now ~~7~~ 13 TeV @ LHC

Collisions map free fixed momenta initial state
$\longrightarrow$ final fixed momenta state

QM predicts probabilities for projections to occur.
Probabilities typically depend on parameters (angles, momenta)

$P(v_1 \ldots v_n)$ ~ differential probabilities

Given by $|\langle \Psi_{final}\ +\infty | \Psi_{Initial}\ -\infty\rangle|^2$

$\langle f | S | i \rangle$ S-matrix

QFT will tell us how to calculate S
given some Lagrangian. (next week)

S-matrix elements are the primary objects of interest for particle physics.

In this lecture we will relate S-matrix elements to scattering cross sections | decay rates which we can directly measure experimentally.

Aside

Probabilities dimensionless [0–1], absolute => extremely subtle
to calculate P need all possible outcomes a priori!
QM => need complete basis. Impossible in QFT (usually) | # final states ∞ | final states not fully known
↳ not for decays

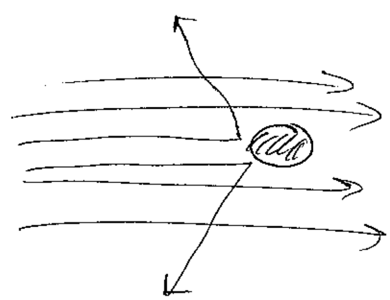
Cross Sections

Natural quantity to measure.

eg Rutherford was interested in size of nucleus (r_n)

By colliding α-particles w/ gold foil and measuring how many particles are scattered, can determine $\sigma = \pi r^2$

Single nucleus



$$\sigma = \frac{\#\text{ - scattered}}{\text{time} \times \left(\begin{matrix}\text{Number density}\\ \text{in beam}\end{matrix}\right) \times \text{velocity beam}} = \underbrace{\frac{1}{T}}_{\text{time}} \, \underbrace{\frac{1}{\Phi}}_{\text{flux}} \, \overbrace{N}^{\#\text{ scatters}}$$

Real experiment

other factors: # density of nuclei in foil
cross sectional area of the beam (if smaller than foil)

This stuff + T & Φ depend on details of experiment

In contrast σ - property of particle being scattered

<u>In QM</u> generalize notion of cross sectional area
to "cross section" → | - units of area
| - abstract measure of interaction strength

eg: Classically ~~th~~ the α will either scatter or not.
QM'ly there is some probability for scattering.

$$d\sigma = \frac{1}{T}\frac{1}{\Phi}\, dP \rightarrow \text{QM probability of scattering}$$

Φ ↪ normalized to one particle

$\left.\begin{matrix} d\sigma \\ dP \end{matrix}\right\}$ differential in kine vars θ's P's

# of scatters

$$dN = L \times d\sigma$$

↪ "integrated luminosity" (take eq as definition L)

So number of observed events is ~~...~~ direct measurement of cross section. (See in presentations & papers)

<u>Relate to S-matrix</u>

practically impossible to collide more than two particles @ a time.

$|i\rangle$ will always be 2 particle state

$$P_1 + P_2 \longrightarrow \{P_j\}$$

<u>Rest frame</u> of one particle

$$\Phi = \frac{|\vec{v}|}{V}$$

<u>CoM frame</u>

$$\Phi = \frac{|v_1 - v_2|}{V}$$

So,

$$d\sigma = \frac{V}{T}\frac{1}{|v_1 - v_2|}\, dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle}\, d\Pi$$

→ region of final state momenta we are considering

On interval size L

p available are ~~...~~ $p_n = \frac{2\pi n}{L}$

$$N = \int \frac{V}{(2\pi)^3} d^3p \qquad d\Pi = \prod_j \frac{V}{(2\pi)^3} d^3p_j$$

$\hookrightarrow N_S = P \times N$ $\qquad$ ↪ over final state particles

$\langle f|f\rangle = \langle i|i\rangle \neq \mathbb{1}$ ← Not L. I

$$\langle p'|p\rangle = (2\pi)^3\, 2E\, \delta^3(p'-p)$$

$$\langle p|p\rangle = (2\pi)^3\, 2E_p\, \delta^3(0)$$

$$= 2E_pV$$

$$\boxed{\begin{gathered}\text{"Regulated by } V \\ \delta^3(p) = \frac{1}{(2\pi)^3}\int d^3x\, e^{i\vec{p}\cdot\vec{x}} \\ \delta^3(0) = \frac{1}{(2\pi)^3}\int d^3x = \frac{V}{(2\pi)^3}\end{gathered}}$$

$\Rightarrow$

$$\langle i|i\rangle = \langle p_1p_2|p_1p_2\rangle = 2E_1V\,2E_2V$$

$$\langle f|f\rangle = \prod_j (2E_jV)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated perturbatively

$$S = \underbrace{\mathbb{1}}_{\text{free theory}} + i\underbrace{T}_{\text{Perturbation "small"}}$$

Know that S matrix should vanish if momentum not conserved

$$\langle f | T | i \rangle = (2\pi)^4 \delta^4(\Sigma p) \mathcal{M} \leftarrow \text{"Matrix Elements"}$$

$$\hookrightarrow \delta^4\left(\sum_i p_i - \sum_f p_f\right)$$

Might worry that we have to square δ function

$$|\langle f | T | i \rangle|^2 = \delta^4(0)\, \delta^4(\Sigma p)\, (2\pi)^8\, |\langle f | \mathcal{M} | i \rangle|^2$$

$$= \delta^4(\Sigma p)\, T V\, (2\pi)^4\, |\mathcal{M}|^2$$

So,

$$dP = \frac{\delta^4(\Sigma p)\, T V\, (2\pi)^4}{(2E_1 V)(2E_2 V)} \frac{1}{\prod_j (2E_j V)} |\mathcal{M}|^2 \prod_j \frac{V}{(2\pi)^3} d^3 p_j$$

$$= \frac{T}{V} \frac{1}{(2E_1)(2E_2)} |\mathcal{M}|^2\, d\Pi_{LIPS}$$

$\hookrightarrow$ LI Phase Space

$$\equiv (2\pi)^4 \delta^4(\Sigma p) \prod \frac{d^3 p}{(2\pi)^3 2E_p}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)\, |\vec{v}_1 - \vec{v}_2|} |\mathcal{M}|^2\, d\Pi_{LIPS}$$

"Fermis Golden Rule"

$$\boxed{\vec{v} = \frac{\vec{p}}{p_0}}$$

<u>decay rate</u> Probability that a one-particle state turns into a multiparticle state ~~ ~~ over the T.

$$P_1 \longrightarrow \{P_i\}$$ really $1 \to N$ scattering.

follow same steps as above

$$d\Gamma = \frac{1}{2E_1} |M|^2 \, d\Pi_{Lips}$$

<u>Example</u> $2 \to 2$ scattering $\quad P_1 + P_2 \to P_3 + P_4$

CoM $\quad P_1 = -P_2 \quad E_1 + E_2 = E_3 + E_4 = E_{cm}$

$P_3 = -P_4$

$$d\Pi_{Lips} = (2\pi)^4 \delta^4(\Sigma p) \frac{d^3 P_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3 P_4}{(2\pi)^3} \frac{1}{2E_4}$$

Integrate over d^3P_4 using δ

$$\oint \; = \frac{1}{16\pi^2} d\Omega \int dp \, P_f^2 \frac{1}{E_3 E_4} \delta(E_3 + E_4 - E_{cm})$$

$\leftarrow P_f = |\vec{P}_3| = |\vec{P}_4|$