Recap $SU(2)_L \times U(1)$

Start w/ $\underbrace{\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4}_{\text{4-scalar fields}}$ $\underbrace{W^1 W^2 W^3 \; B}_{\text{4 massless Spin-1}}$

DoF 4×1 4×2 $= 12$

After Symmetry Breaking

h $\underbrace{W^+ \quad W^- \quad Z}$ γ

massive scalar massive Spin 1

1 3×3 2 $= 12$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right) \equiv \cos\theta_W B_\mu + \sin\theta_W W^3_\mu$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g W^3 - g' B_\mu\right) \equiv -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu$$

$$\frac{g'}{g} = \tan\theta_W \qquad m_\gamma = 0 \qquad m_Z = \frac{1}{2}\frac{g}{\cos\theta_W} v$$

$$v^2 = \frac{-\mu^2}{\lambda} \simeq 250\text{ GeV} \qquad \frac{m_W}{m_Z} = \cos\theta_W \qquad m_H^2 = 2\lambda v^2$$

~~Fermion Masses~~

Higgs mechanism on $SU(2)_L \times U(1)$ generates the correct Electroweak spectra

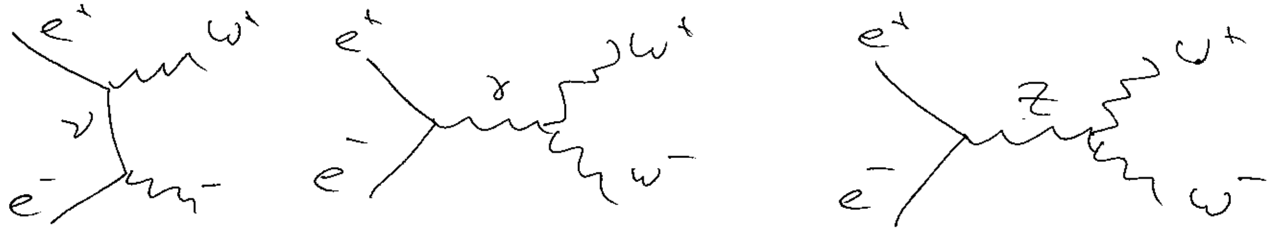
→ Comments

-) Weak force carriers charged, implies relationship

-) Coupling constants not so different $\frac{1}{137}$ vs $\frac{1}{50}$

-) Also strong theoretical arguments that they must be related

Can produce pairs of W's from e^+e^- collisions



$\sim \alpha_w$

$\sim \alpha$

With these σ increases w/ E w/o limit. Eventually probability not conserved. (Calculated WW flux exceeds e^+e^- flux)

Adding 3rd diagram resolves problem w/ negative interference

Only works B/c couplings are related in specific way.

Fermion Masses

Remarkably the stupidest Higgs mechanism can also be used to generate fermion masses... lets see how.

$$-m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$ ← does not respect $SU(2)_L \times U(1)$

⇒ "bare" mass terms cannot be included in $\mathcal{L}_{SM}$

$$\phi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \phi_R = e_R \quad \text{ect.}$$

Now ϕ_{Scalar} is a doublet that transforms under $SU(2)_L$

So the term $\bar{\Phi}_L \phi$ is invariant under $SU(2)_L$ ($+ U(1)$)

⇒ the term $\bar{\Phi}_L \phi e_R$ is invariant under $SU(2)_L \times U(1)$

So we are free to add a term

$$\mathcal{L} \supset -g_e(\bar{\Phi}_L \phi e_R + \bar{e}_R \phi^\dagger \phi_L)$$

or $$-g_e\left[(\bar{\nu}\ e)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + e_R(\phi^{+*}\ \phi^{*})\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L\right]$$

"electron Yukawa" coupling

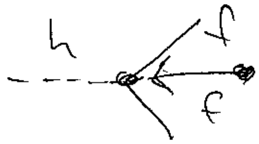
After Electroweak Symmetry Breaking $\phi \to \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

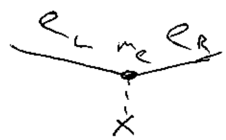
$$\mathcal{L} \supset \underbrace{-\frac{g_e}{\sqrt{2}} v(e_L e_R + e_R e_L)}_{} - \frac{g_e}{\sqrt{2}} h(x)(e_L e_R + e_R e_L)$$

→ exactly whats needed for electron mass term if $g_e = \sqrt{2}\,\frac{m_e}{v}$ (Not predicted by Higgs Mechanism But allowed in gauge invariant way)

$$\mathcal{L}_e = -m_e \bar{e}e - \frac{m_e}{v}\bar{e}eh$$

true for all fermions

$\Rightarrow$  $\frac{m_f}{v}$



So Can construct all fermion masses this way.

$$g_f = \sqrt{2}\,\frac{m_f}{v} \qquad v = 250\ \text{GeV}$$

Interestingly for top $m_t = 173.5$ $\quad \sqrt{2}\,m_t \simeq v \quad g_t \simeq 1 \ (0.997)$

$g_b \sim 0.03$

$g_e \sim 10^{-6} \qquad g_\nu \lesssim 10^{-12}$

$$\boxed{\text{Bonus} \quad \phi_c = i\tau_2\phi^* \xrightarrow{\text{Breaking}} \frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix}}$$

How do we know any of this is correct?

- Predicted neutral currents, then found
- " the value of m_W & m_Z, later discovered.

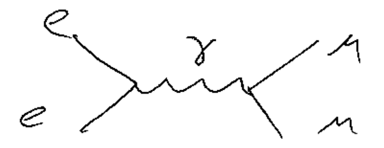
Other precise predictions of electroweak model were confronted w/ equally precise measurements of $W^\pm$, Z properties @ LEP & Tevatron. Say a few things about those tests.

LEP produced large quantities of e^+e^- collisions.

Many "on the Z resonance"

Any process w/ γ can be replaced w/ Z

eg)



$$\mathcal{M}_\gamma \propto \frac{e^2}{q} \text{ from propagator} \qquad \mathcal{M}_Z \propto \frac{g_Z^2}{q^2 - M_Z^2}$$

In these "s-channel" diagrams the 4-momentum of internal line is equal to E_{cm}. B/c of $m_Z^2 \approx 90$ GeV^2

$$\mathcal{M}_\gamma >> \mathcal{M}_Z \text{ for } E_{cm}^2 << m_Z^2$$

for $E_{cm}^2 >> m_Z^2$ Both are important $\alpha \sim \alpha_w$

However for $E_{cm} \sim m_Z$ the Z process dominates

(In fact, naively infinite... B/c doesn't account for Z being unstable particle)

Number of ways to account for this. Think of the Z boson wave function $\psi \propto e^{imt}$ for unstable particle

In Z rest frame $E = m$

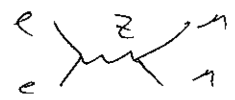
$\psi \to \psi \propto e^{imt} e^{-\Gamma t/2}$ to account for the decay rate

Implies $\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau}$

$\Rightarrow m \to m - i\frac{\Gamma}{2}$ for unstable particles.

$$m_Z^2 \to \left(m_Z^2 - i\frac{\Gamma_Z}{2}\right)^2 = m_Z^2 - i m_Z \Gamma_Z - \frac{1}{4}\Gamma_Z^2 \qquad \text{for } Z,\ \Gamma_Z \ll m_Z$$

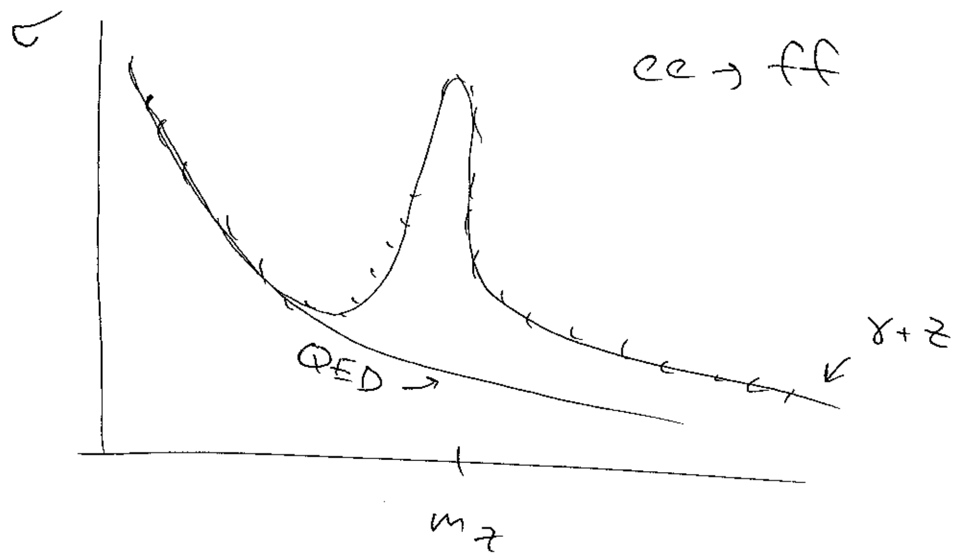
$$\mathcal{M}_Z \propto \frac{g_Z^2}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$$



$$\sigma \propto |\mathcal{M}|^2 \propto \left| \frac{1}{E_{cm}^2 - m_Z^2 + i m_Z \Gamma_Z} \right|^2 = \frac{1}{(E_{cm}^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

$\Rightarrow$ $ee \to Z$ cross section sharply peaked @ $E_{cm} = m_Z$

Dependence on mass referred to as Breit-Wigner



m_Z was measured to 0.002% precision

- Required correcting for distortions of earth due to moon
- " " " electrical currents induced by french train system.

total width was also measured

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

Remember $\quad \Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{hadrons} + N_\nu \Gamma_\nu$

Only 3 generations observed know they come in doublets

Maybe 4th generation heavy... $\begin{pmatrix} \nu_x \\ x \end{pmatrix}$ Should get $Z \to \nu_x \bar{\nu}_x$

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{hadrons})}{\Gamma_{\nu\nu}^{SM}}$$

$\Gamma_{\ell\ell}$ from $ee \to Z \to \mu\mu$

$\Gamma_{hadrons}$ from $ee \to Z \to$ jets $\qquad N_\nu = 2.9840 \pm 0.0082$

$\Rightarrow$ exactly 3 generations of light ν's

Probably only 3 generations