Local gauge invariance of SU(2)

So we have ... $\phi(x) \to e^{ig\vec{\alpha}(x)\cdot\vec{\sigma}} \phi(x)$ implying

that

$$\phi(x) = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$ ← two component spinor "Weak iso spin"

Requires the addition of 3 gauge fields via

$$\partial_\mu \to D_\mu = \partial_\mu + ig\vec{W}^\mu \cdot \vec{\sigma}$$

$\vec{W} = \{W_1, W_2, W_3\}$

$$\mathcal{L} = i\bar{\Phi}\gamma_\mu \partial^\mu \Phi = i\bar{\nu}_e \gamma_\mu \partial^\mu \nu_e + i\bar{e}\gamma_\mu \partial^\mu e$$

→ 4-component solutions to D.E.

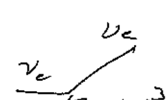
$$\mathcal{L} \to \mathcal{L}' = i\bar{\Phi}\gamma_\mu D^\mu \phi + \begin{pmatrix} \text{kinetic term} & \sim F_{\mu\nu}F^{\mu\nu} \\ \text{for } W\text{s} & \end{pmatrix}$$

$$= i\bar{\Phi}\gamma_\mu \left(\partial_\mu + ig(W_1^\mu\sigma_1 + W_2^\mu\sigma_2 + W_3^\mu\sigma_3\right)\phi + \ldots$$

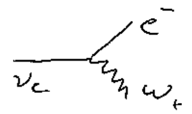
$$\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2) = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{cases} \qquad \vec{W}\cdot\vec{\sigma} = W_1^\mu\sigma_1 + W_2^\mu\sigma_2 + W_3^\mu\sigma_3$$
$$= W_+^\mu\sigma_+ + W_-^\mu\sigma_- + W_3^\mu\sigma_3$$

define $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)$

$$\mathcal{L} \supset i\bar{\Phi}\gamma_\mu^\mu\left(\partial_\mu + ig\left(W_+^\mu\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + W_-^\mu\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + W_3^\mu\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)\right)\phi$$



$$= i\bar{\nu}_e\gamma_\mu\partial^\mu\nu_e + ie\gamma_\mu\partial^\mu e + \underline{i^2 g\gamma_\mu \bar{\nu}_e W_+^\mu e} + \underline{i^2 g\gamma_\mu e W_-^\mu \nu_e} + i^2 g\gamma_\mu^\mu \nu_e W_3^\mu \nu_e$$
$$+ i^2 g\gamma_\mu e W_3^\mu e$$



Problem is weak interaction only talks to left-handed particles

gauge group is really $SU(2)_L$

To deal with this only introduce the left handed particles in isospin doublets

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \quad \begin{pmatrix}u\\ d\end{pmatrix}_L \quad \begin{pmatrix}c\\ s\end{pmatrix}_L \quad \begin{pmatrix}t\\ b\end{pmatrix}_L$$

Treat the R.H. particles as "singlets"

$$e_R \quad \mu_R \quad \tau_\tau \quad u_R \quad c_R \quad t_R \quad d_R \quad s_R \quad b_R$$

$\phi_R \to \phi_R \qquad \phi_L \to e^{ig\alpha\cdot\tau}\phi_L \qquad$ *Weird but true

<u>Now some history</u>

Tempting to identify $W^{1,2}$ w/ $W^{+/-}$ + W^3 w/ Z

turns out that doesn't work. The W^3 only couples to LH particles whereas the Z couples to both (Although not equally).

However at the time ~~a~~ all of this was being sorted out No one had seen $W^{+/-}$ or Z particles only had γ and the fermi X theory. It was believed that this theory needed to be extended to include charged force carriers. Mainly through the observation of "charged currents" $\nu \searrow\!\nearrow e^-$ $\quad n \searrow\!\nearrow p$

No indication of a "neutral current" ν ν e e

So the first response to $SU(2)$ was that it was just wrong. hard to see EM dominates

W^3 did not look like γ which couples to both LH & RH particles. Then GSW added another group

Add $U(1)_Y$ as new gauge symmetry to $SU(2)_L$

"hypercharge

function

$$\phi_L \to e^{ig\vec{\alpha}\cdot\sigma + ig' y(x)}\phi_L \qquad \phi_R \to e^{ig' y(x)}\phi_R$$

$$\partial_\mu \to D_\mu = \partial_\mu + ig\vec{W}_\mu\cdot\sigma + ig' B_\mu$$ ← Need New gauge field

Now it looks like we may have enough freedom except

$m\bar{e}e \sim m e_L e_R$ is no longer gauge invariant

B/c $e_L \to e^{ig\vec{\alpha}\cdot\sigma} e_L$ under $SU(2)_L$ + e_R doesn't

So lets ignore fermion masses for now and try to give the Bosons masses with the Higgs

Saw 2 lectures ago how to use the Higgs Mechanism to generate mass for a photon in U(1) symmetry

Now do the same for $SU(2)_L \times U(1)$ "GSW"

<u>Note</u> In the Higgs mechanism we are just making things up, So there is freedom in what you can do. eg we are just Adding fields to $\mathcal{L}$. We will go through the simplest case, but again we are putting this in by hand, so it could certainly be more complicated ...

Introduce

complex

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \mathcal{L} = (\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \mu^2\phi^+\phi + \lambda(\phi^+\phi)^2$$

$\mu^2 < 0$ has infinit set of degenerate minima satisfying

$$\phi^+\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

expand about one of these minama just as we did before well pick "ϕ_3 direction"

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix} \longrightarrow \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Pick gauge to be

Lets see what happens ...

Imposing gauge invariance $\quad \phi \to e^{i g \vec{\alpha}\cdot\vec{\sigma} + g' y(x)} \phi \qquad \partial \to D$

$(D_\mu \phi)^\dagger (D^\mu \phi) \qquad D_\mu \phi = (\partial_\mu + i g_w \tau \cdot W_\mu + i g' B_\mu)\phi$ (dropping many factors of 2)

or
$$\begin{pmatrix} \partial_\mu + i g_w W^3_\mu + i g' B_\mu & i g_w (W^1_\mu - i W^2_\mu) \\ i g_w (W^1_\mu + i W^2_\mu) & \partial_\mu - i g_w W^3_\mu + i g' B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$= \begin{pmatrix} i g_w (W^1_\mu - i W^2_\mu)(v+h) \\ (\partial_\mu - i g_w W^3_\mu + i g' B_\mu)(v+h) \end{pmatrix}$$ ← from this can get $(D_\mu \phi)^\dagger$

$$(D_\mu \phi)^\dagger (D^\mu \phi) = (\partial h)^2 + g_w^2 (W^1_\mu + i W^2_\mu)(W^1_\mu - i W^2_\mu)(v+h)^2$$
$$+ (g_w W^3_\mu - g' B_\mu)(g_w W^\mu_3 - g' B^\mu)(v+h)^2$$

Lets look @ terms that go like v^2

$$g_w^2 v^2 (W^1_\mu + i W^2_\mu)(W^1_\mu - i W^2_\mu) = \underbrace{v^2 g_w^2}_{m_w^2} (W^{1\,2} + W^{2\,2})$$

→ W^1 & W^2 are massive with mass $m_w = g_w v$

Same mass

Also a term like

$$v^2 (g_w W^3_\mu - g' B_\mu)(g_w W^{3\mu} - g' B^\mu)$$

$$= v^2 (W^3_\mu \ B_\mu) \underbrace{\begin{pmatrix} g_w^2 & -g_w g' \\ -g_w g' & g'^2 \end{pmatrix}}_{M} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

M - non-diagonal mass matrix

Non diagonal mass matrix is telling you that the W^3_μ & B_μ fields "mix"

So pure W^3_μ or B_μ has probability to turn into the other

those are not eigenstates of the free hamiltonian.

Need to solve for linear combination of W^3 + B which diagonalizes M $\longrightarrow$ those would
1) Not mix
2) Have definite mass
3) Be energy eigenstates

Solve $\det(M - \lambda I) = \emptyset$

$$\Rightarrow \quad (g_w^2 - \lambda)(g'^2 - \lambda) - g_w^2 g'^2 = \emptyset$$

$$\Rightarrow \quad \lambda = 0 \quad \text{or} \quad \lambda = g_w^2 + g'^2$$

In diagonal Basis

$$v^2 (A_\mu \ \ Z_\mu) \begin{pmatrix} 0 & 0 \\ 0 & g_w^2 + g'^2 \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

$$A_\mu = \frac{1}{\sqrt{g_w^2 + g'^2}} \left(g' W^3_\mu + g_w B_\mu \right) \qquad Z_\mu = \frac{1}{\sqrt{g_w^2 + g'^2}} \left(g_w W^3_\mu - g' B_\mu \right)$$

$$m_A = 0 \qquad\qquad M_Z = v^2 \sqrt{g_w^2 + g'^2}$$

There is also a term that goes like h^2

HW

Recap

Start with

$$\underbrace{\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4}_{} \qquad \underbrace{W^1 \quad W^2 \quad W^3 \quad B}_{}$$

DoF : 4×1 Scalars $\qquad$ 4×2 Massless Spin 1 $\qquad = 12$

when $\mu^2 < 0$

left with

$$h \qquad W^+ \quad W^- \quad Z \qquad \gamma$$

DoF $\quad 1$ Scalar massive $\qquad 3 \times 3$ massive Spin 1 $\qquad 2 \qquad = 12$