

Lecture 13

Noether's Theorem

Lagrangian may be invariant under some type of transformation (variation)

eg: $\phi \rightarrow \phi + \delta$

This transformation is a symmetry of the Lagrangian

Say ϕ is complex: 2 DoF ϕ and ϕ^* .

And you have a Lagrangian given by

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi \phi^*$$

symmetry

$$\phi \rightarrow e^{-i\alpha} \phi \qquad \phi^* \rightarrow e^{i\alpha} \phi^*$$

Whenever we have a continuous symmetry (meaning there is a continuous limit)

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta \alpha} = 0 &= \sum_n \left[\frac{\partial \mathcal{L}}{\partial \phi_n} \frac{\delta \phi_n}{\delta \alpha} + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \frac{\delta (\partial_\mu \phi_n)}{\delta \alpha} \right] \\ &= \sum_n \left[\underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi_n} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \right]}_{=0 \text{ Euler Lagrange}} \frac{\delta \phi_n}{\delta \alpha} + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right] \right] \end{aligned}$$

$$\phi_n = \{\phi, \phi^*\}$$

\Rightarrow

$$\partial_\mu J^\mu = 0$$

with

$$J^\mu = \sum_n \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right]$$

J^μ is a conserved current. “Noethers Current”

total “charge”

$$Q \equiv \int d^3x J_0$$

$$\partial_t Q = \int d^3x \partial_t J_0 = \underbrace{\int d^3x \vec{\nabla} \cdot \vec{J}}_{\text{Vanishes on the Boundary}} = 0$$

Q does not change with time!

Very general and important theorem “Noethers Theorem”

If \mathcal{L} has (“enjoys”) a continuous symmetry, there exists an associated current that is conserved.

$$\phi(x) \rightarrow \phi(x + \epsilon) = \phi(x) + \epsilon^\mu \partial_\mu \phi(x)$$

This leaves \mathcal{L} and \mathcal{S} invariant and gives a four vector of noether currents.

\Rightarrow Noether’s theorem tells us why energy and momentum are conserved.

Cross Sections And Decay Rates

20th century witnessed the development of collider physics. Effective means to determine which particles exist and their properties and interactions.

- Rutherford discovery of the nucleus using α 1911
- Anderson's discovery of anti-electrons 1932

These were made with “Natural accelerators” α 's or cosmic rays

Around 1930 man made collisions started winning.

eg: 1 MeV

Now 13 TeV at the LHC.

Collisions map free fixed momenta initial states \rightarrow final fixed momentum states.

QFT predicts probability for projections to occur.

Probabilities typically dependant on parameters (angles, momenta, etc)

$P(v_1, \dots, v_n)$ - differential probabilities

Given by

$$|\langle \psi_{final}, +\infty | \psi_{initial}, -\infty \rangle|^2$$

$\langle f | S | i \rangle$ S-matrix

QFT tell us how to calculate S given some Lagrangian (next week)

S-matrix elements are the primary object of interest for particle physics.

In this lecture we will relate S-matrix elements to scattering $\left| \begin{array}{l} \text{cross sections} \\ \text{decay rates} \end{array} \right.$ which
we can measure experimentally

Cross Sections

Aside:

Probabilities are dimensionless and [0-1] absolute \Rightarrow extremely subtle to calculate P need to know all possible outcomes a priori!

QM \Rightarrow need complete basis.

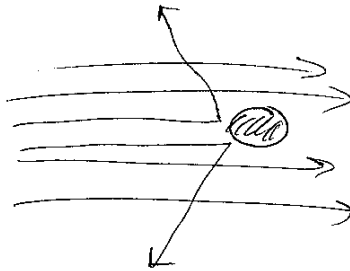
| | |
|-----------------------------------|------------------------------|
| Usually impossible in QCD | # final states ∞ |
| (Particle decays is an exception) | final states not fully known |

Natural quantity to measure.

eg: Rutherford was interested in the size of the nucleus.

By colliding α -particles w/gold foil and measuring how many particles are scattered, can determine $\sigma = \pi r^2$

Single nucleus



$$\sigma = \frac{\text{\#- scattered}}{\text{time} \times (\text{Number density in beam}) \times \text{velocity of beam}} = \frac{1}{T} \frac{1}{\Phi} N$$

| | |
|---------------------------------------|---------------------------------------------------------|
| <u>Real Experiment</u> other factors: | number density of nuclei in foil |
| | cross sectional area of the beam (if smaller than foil) |

This stuff and T and Φ depend on details of the experiment.

In contrast, σ is a property of particles being scattered.

| | |
|---------------------------------------------------------------------------|-------------------------------------------|
| <u>In QM</u> generalize notion of cross sectional area to “cross section” | -units of area |
| | -abstract measure of interaction strength |

eg: Classically the α will either scatter or not.

Quantum Mechanically, there is some probability for scattering.

$$d\sigma = \frac{1}{T} \underbrace{\frac{1}{\Phi}}_{\text{Normalized to one particle}} \underbrace{dP}_{\text{QM probability of scattering}}$$

$\left| \frac{d\sigma}{dP} \right|$ - differential in kinematic variables θ 's and P 's

$$\underbrace{dN}_{\text{\# of scatters}} = \underbrace{L}_{\text{"integrated luminosity"}} \times d\sigma$$

(take eq as definition of L)

So number of observed events is direct measurement of cross section (See in presentation and papers)

Relate to S-matrix

practically impossible to collide more than two particles at a time.

$|i\rangle$ will always be a 2 particle state

$$P_1 + P_2 \rightarrow \{P_j\}$$

Rest frame of one particle $\Phi = \frac{|\vec{v}|}{V}$

Center of mass frame $\Phi = \frac{|\vec{v}_1 - \vec{v}_2|}{V}$

So,

$$d\sigma = \frac{V}{T} \frac{1}{|\vec{v}_1 - \vec{v}_2|} dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} \underbrace{d\Pi}_{\text{Region of final state momenta we are considering}}$$