Homework #12

1) $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$

$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$

$t \sim L$

$$|\nu_e(t)\rangle = \cos\theta\, e^{-i\frac{m_1^2}{2E}L}|\nu_1\rangle + \sin\theta\, e^{-i\frac{m_2^2}{2E}L}|\nu_2\rangle$$

$$\langle\nu_\mu|\nu_e(L)\rangle = -\sin\theta\cos\theta\, e^{-i\frac{m_1^2}{2E}L} + \sin\theta\cos\theta\, e^{-i\frac{m_2^2}{2E}L}$$

$$= e^{-i\frac{m_1^2}{2E}L}\left(-\frac{1}{2}\sin(2\theta) + \frac{1}{2}\sin 2\theta\, e^{-i\frac{(m_2^2-m_1^2)}{2E}L}\right)$$

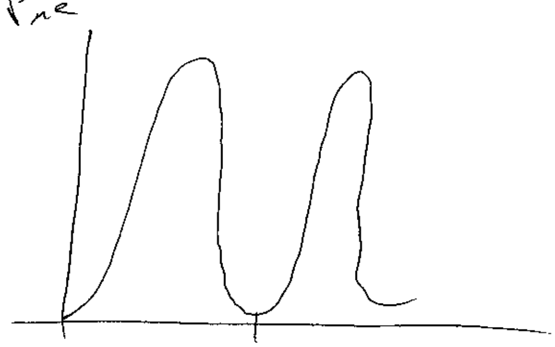
$$= \frac{1}{2}e^{-i\frac{m_1^2}{2E}L}\sin(2\theta)\left(-1 + e^{-i\frac{\Delta m^2}{2E}L}\right)$$

$$|\langle\nu_\mu|\nu_e(L)\rangle|^2 = \frac{1}{4}\sin^2(2\theta)\left(1 + 1 - \underbrace{\left(e^{-i\frac{\Delta m^2}{2E}L} + e^{-i\frac{\Delta m^2}{2E}L}\right)}_{2\cos\frac{\Delta m^2}{2E}L}\right)$$

$$= \frac{1}{2}\sin^2(2\theta)\left(1 - \cos\frac{\Delta m^2}{2E}L\right)$$

$$= \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right)$$

$$\boxed{\begin{array}{l}\text{Note} \\ P_{ee} = 1 - \sin^2 2\theta\, \sin^2\left(\frac{\Delta m^2}{4E}L\right) \\ \Rightarrow\ P_{ee} + P_{\mu e} = 1 \quad \checkmark\end{array}}$$



2 Cosmic Rays are

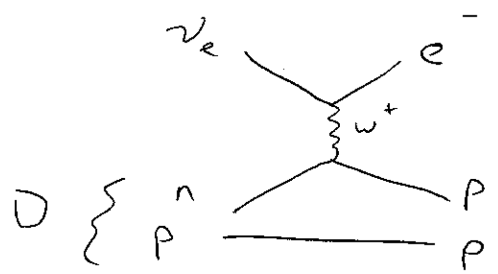
$p \to \pi^\pm \to \mu^+ \bar{\nu}_\mu \to e^+ \nu_e \bar{\nu}_\mu$

$\searrow \mu^- \bar{\nu}_\mu \to e^- \bar{\nu}_e \nu_\mu$

$\Rightarrow$ for every ν_e there will be 2 ν_μ's

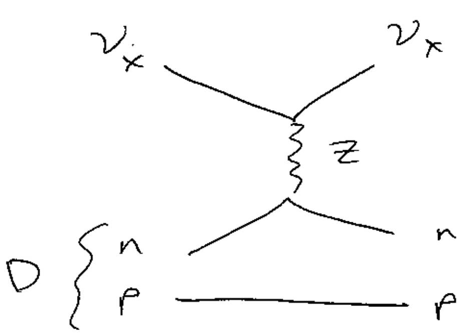
At higher E the μ's will not decay, so you will only get ν_μ's

3 "charged current"



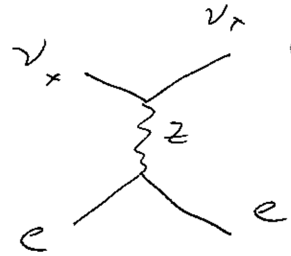
only possible $\leftarrow$ for ν_e's

"neutral current"

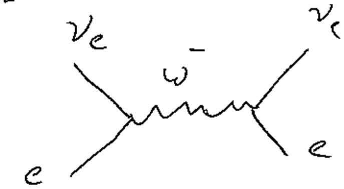


Same for all flavors

"Electron Scattering"



Same for All X



Sum is Different for ν_e than ν_τ, ν_μ

only for ν_e

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a) Just like cosmic-rays, smash protons into something

b)

$$p \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

ν_e's + ν_μ's

c) Protons will give more π^+ which will give more ν's

d) You could enhance the ν fraction by removing negative particles eg: only "focus" positive π^+s

e) Same idea but only focus π^-'s