Lecture 15

From Last time...

Example of scattering of 2 electrons

$$|i\rangle = |e_1, e_2\rangle \qquad\qquad |f\rangle = |e_3, e_4\rangle$$

$$T_{fi} = V_{fi} + \sum_n V_{fn} \frac{1}{E_i - E_n} V_{ni} + ...$$

- E_i- is the initial (= final) energy
- E_n - is the energy of the intermediate state

Now, the $\sum_n$ runs over energy thing in the Hilbert (Fock) space, however only certain states will be non-0.

In relativistic theory, the action-at-a-distance of EM is replaced by a process where 2 electrons interact with a γ which travels at c. (This tells us there should be γ in the intermediate state)

$$V \sim e \int d^3 x \psi \phi \psi \qquad \text{(Ignoring spin)}$$

this operator will have terms that go like ($\sim a^\dagger_{e_3}\, a^\dagger_\gamma\, a_{e_1}$)

However, here all terms involve $a^\dagger_\gamma$.
Because $|i\rangle$ and $|f\rangle$ do not contain a γ, $V_{fi} = 0$

to get a non-zero term, we need $|n\rangle$ with a photon.

$$T_{fi} = \frac{\langle e_3 e_4 | V | e_3 \gamma e_2 \rangle \langle e_3 \gamma e_2 | V | e_1 e_2 \rangle}{(E_1 + E_2) - (E_3 + E_4 + E_\gamma)} + \text{(2nd term)}$$

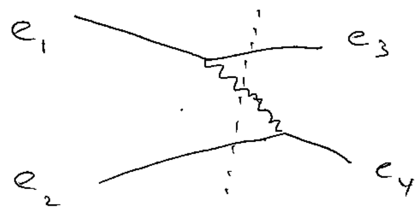
Note: $E_n \neq E_i$ which is allowed by uncertainty principle.

Look at

$$\langle e_3 \gamma e_2 | V | e_1 e_2 \rangle = \langle \gamma e_3 | V | e_1 \rangle$$

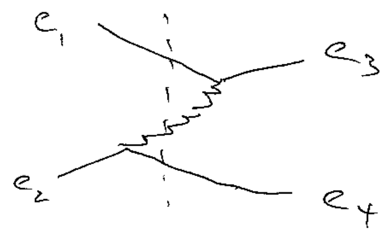
(up to overall normalization from $\langle e_2 | e_2 \rangle$.

2 ways time $\rightarrow$



$|n\rangle = |\psi_e^3 \phi_\gamma^\gamma \psi_e^2\rangle$

(1)



$|n\rangle = |\psi_e^1 \phi_\gamma \psi_e^4\rangle$

(2)

$$\boxed{\langle\gamma|\phi_\gamma(x)|0\rangle = e^{-ip_\gamma\cdot x}}$$

$$\begin{aligned}
\langle\gamma e_3|V|e_1\rangle &= e\int d^3x\,\langle\gamma e_3|\psi_e\phi_\gamma\psi_e|e_1\rangle \\
&= e\int d^3x e^{-i(p_3+p_\gamma-p_1)x} = e(2\pi)^3\delta(p_3+p_\gamma-p_1)
\end{aligned}$$

Other product

$$\langle e_4|V|\gamma e_2\rangle = e(2\pi)^3\delta(p_4-p_\gamma-p_2)$$

Combining this gives

$$T_{fi}^1 \sim \int d^3p_\gamma\,\delta\delta\frac{e^2}{E_i-E_n}$$

where $E_i = (E_1+E_2)$ and $E_n = (E_3+E_2+E_\gamma)$.

$$T_{fi}^1 \sim \frac{e^2}{(E_1+E_2)-(E_3+E_2+E_\gamma)} = \frac{e^2}{(E_1-E_3)-E_\gamma}$$

Same logic for 2nd term leads to

$$T_{fi}^2 \sim \frac{e^2}{(E_2-E_4)-E_\gamma}$$

End of the day, need to add the two processes.

Note:

$$E_1 + E_2 = E_3 + E_4$$

$$E_1 - E_3 = E_4 - E_2 \equiv \Delta E$$

$$T^1 + T^2 = \frac{e^2}{\Delta E - E_\gamma} + \frac{e^2}{-\Delta E - E_\gamma} = \frac{2e^2 E_\gamma}{(\Delta E)^2 - E_\gamma^2}$$

define $k^\mu \equiv p_3^\mu - p_1^\mu = (\Delta E, \vec{p_\gamma})$

Note k^μ is not the photon momentum! $k^2 \neq 0 (= (\Delta E)^2 - E_\gamma^2)$

$$T_{fi} = \underbrace{2E_\gamma}_{\text{Related to normalization}} \frac{e^2}{k^2}$$

Summary Standard "old-fashion" perturbation theory

- All states are physical (on-shell)
- Matrix element V_{ij} vanishes unless 3-momentum conserved
- Energy not conserved at each vertex
- Add all time orderings

Modern way to interpret same thing "Feynam rules"

Summary Feynman Rules

- Draw diagrams ignoring time ordering
- Vertices come from interactions in Lagrangian: factor of i times coupling constant
- Internal lines get "propagators" = $\frac{i}{p^2 - m^2}$
- Lines connected to external points do not get propagators (scalars $\times$ 1 / spinors $\times$ u or v / spin-1 $\times \epsilon$)
- Four momenta is conserved at each vertex
- Integrate over all undetermined 4-momenta

Other Spins

Scalar:

$$\Phi(\vec{x}) = \int d\!\!\!{}^-{}^3p \; a^\dagger e^{-ipx} + be^{+ipx}$$

Spin-1:

$$A^\mu(\vec{x}) = \int d\!\!\!{}^-{}^3p \; a^\dagger_\gamma \epsilon^\mu e^{-ipx} + a_\gamma \epsilon^\mu e^{+ipx}$$

where, ϵ^μ is the photon polarization vector, the solution for the Spin-1 relativistic equation of motion (ie: Maxwell's equations). Here, we are taking the spin-1 particles to be thier own anti-particles (as is true for photons).

Spin-1/2:

$$\Psi^\mu(\vec{x}) = \int d\!\!\!{}^-{}^3p \; a^\dagger v(p) e^{-ipx} + bu(p) e^{+ipx}$$

where, $u(p)$ and $v(p)$ are the dirac spinors, the solution for the Spin-1/2 relativistic equation of motion (ie: the dirac equation).