(4b) Talk about all these particle states in a more convenient way.

For every given momenta we have stacks of Hilbert Space
- ∞ many possibilities for Bosons $0 \to N$
- fixed # depending on the Spin for fermions.

Ultimately interested in interactions between particles.
- Defined by some hamiltonian.
- Could specify the ~~that~~ hamiltonian by describing how it acts on all states in the hilbert space.
- Instead for our convience introduce creation + anhilation operators.
  ↳ keeps track of states in a simple way.

vacuum state

$|0\rangle$

These are primary

$|p,\sigma\rangle \equiv a^{\dagger}_{p\sigma}|0\rangle$

$\hookrightarrow$ defines $a^{\dagger}$

$|p_1\sigma_1, p_2\sigma_2\rangle = a^{\dagger}_{p_1\sigma_1} a^{\dagger}_{p_2\sigma_2}|0\rangle$

$\vdots$

$a_{p\sigma}|0\rangle = 0$ $\quad$ a - removes states.

$\vdots$

Can encode boson/fermion statistics in ~~x~~ a & $a^{\dagger}$.

$[a^{\dagger}_{p_1\sigma_1}\ a^{\dagger}_{p_2\sigma_2}] = 0$ $\quad$ Bosons

$\{a^{\dagger}_{p_1\sigma_1}, a^{\dagger}_{p_2\sigma_2}\} = 0$ $\quad$ Fermions

$[a_{p_1\sigma_1}\ a_{p_2\sigma_2}] = 0$

$\{a_{p_1\sigma_1}, a_{p_2\sigma_2}\} = 0$

Normalization

$$\langle p'_i \sigma'_i | p, \sigma \rangle = \delta_{\sigma\sigma'} \, \delta^3(\vec{p} - \vec{p}\,')$$ ← use compressed Notation

$$= \delta_{\sigma\sigma'} \, \delta_{pp'}$$

$\langle p'\sigma' | p\sigma \rangle = \delta\delta$
$\langle 0 | a_{p'\sigma'} a^\dagger_{p\sigma} | 0 \rangle$
$= \langle 0 | a^\dagger a + [a\, a^\dagger] | 0 \rangle$
$= 0 + \langle 0 | [a\, a^\dagger] | 0 \rangle - \delta\delta$

Normalization is our convience.

$$[a_{p\sigma}, a^\dagger_{p'\sigma'}] = \delta_{\sigma\sigma'} \, \delta_{pp'} \quad \text{Bosons}$$

$$\{a_{p\sigma}, a^\dagger_{p'\sigma'}\} = \delta_{\sigma\sigma'} \, \delta_{pp'} \quad \text{Fermions}$$

Note, these are operators that We have defined for our convience. Makes it easier to talk about the states.

Usualy you see it presented as,
- Start w/ fields, quantize, then find those commutation relations

But really the fields are secondary concepts, what comes first is the particles. "Not one doop thing going on here"

What would the free hamiltonian be?

$$H = \sum_{p\sigma} E_p \, a^\dagger_{p\sigma} a_{p\sigma}$$

really an integral.

- Labeling states by 4-momenta, but the energy is constrained ($E^2 = p^2 + m^2$)

Can label by the 3-momentum

$$\langle \vec{p}\,'\sigma' | \vec{p}\,\sigma \rangle = (2\pi)^3 \, 2E_p \, \delta^3(\vec{p} - \vec{p}\,')$$

Note d^3p is not Lorentz invariant, but $\frac{d^3p}{2E_p}$ is Lorentz invariant.

$$\int \frac{d^3p}{(2\pi)^3 \, 2E_p}$$

$$\int d^4p \, \delta(p^2 - m^2) = \int dE \, d^3\vec{p} \, \delta(E^2 - (\vec{p}^{\,2} + m^2))$$

$$= \int \frac{dE^2}{2E} d^3\vec{p} \, \delta(E^2 - (\vec{p}^{\,2} + m^2))$$

Can do $\int dE$

$$= \int \frac{d^3\vec{p}}{2E}$$

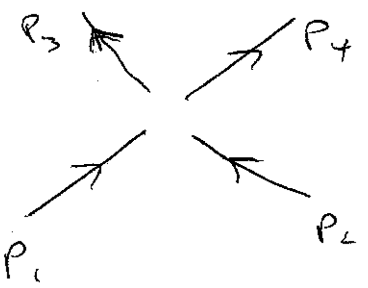
Make my life eisier by defing.

$$\int \bar{d}^3 p = \int \frac{d^3 p}{(2\pi)^3 2E}$$

$$H^{Free} = \sum_{p\sigma} E_p \, a^\dagger_{p\sigma} a_{p\sigma}$$

$$= \int \bar{d}^3 p \, E_p \, a^\dagger_{p\sigma} a_{p\sigma}.$$

Now lets imagine building interactions. Add interaction hamiltonian.



$$+ \int \bar{d}^3 p_1 \, \bar{d}^3 p_2 \, \bar{d}^3 p_3 \, \bar{d}^3 p_4 \, \delta(\vec{p}_1 \cdots \vec{p}_4) \, \delta(E_1 \cdots E_4) \underbrace{a^\dagger_{p_4\sigma_4} a^\dagger_{p_3\sigma_3} a_{p_2\sigma_2} a_{p_1\sigma_1}} \, V^{\sigma_1\sigma_2\sigma_3\sigma_4}(p_1 \, p_2 \, p_3 \, p_4) \Big) + H.C.$$

↳ Acts on the initial state and gives the final state.

Interactions made out of Strings of a's and $a^\dagger$'s.

Also easy in this picture to talk about creation and destruction of particles (Not just scattering)

Very convient to use this to map between differet particle states

So far we havnt said the word "feild."

Now comes the challange;
- these are interactions between momentum eigenstates.
- mom eigen states are like big plane waves.

A totally generic coefficient $\left(\delta(\vec{p}_1 \cdots \vec{p}_4) \, \delta(E_1 \ldots E_4) \, V^{\sigma_1 \ldots \sigma_4}(p_1 \ldots p_4) \right)$ is not going to correspond to point-like local interactions.

Would like to come up with some engine to allow us to build interaction Hamiltonians where we can just see explicitly that the interactions are local.

↳ this is where the utility of the field concept comes in.

The states that we defined act very nicely under the translation operator, (just pick up phase) but to get interactions local in space need x to make an appearance.

Build out of these other operators

$$\phi(x) \qquad T: \phi(x) \rightarrow \phi(\vec{x}+\vec{a})$$

Very nice way of doing this, fourier transforms.

<u>Define</u>

$$\phi_+(\vec{x}) = \int đ^3p \; a^\dagger_{\vec{p}} \, e^{i\vec{p}\cdot\vec{x}} \qquad \phi_-(x) = \phi_+^\dagger(x)$$

$$\phi_-(\vec{x}) = \int đ^3p \; a_{\vec{p}} \, e^{-i\vec{p}\cdot\vec{x}}$$

Indeed ϕ's behave as above under translations.

Now can go back to the free hamiltonian and write it very simply using ϕ's

H^{free} <u>non relativistic for the moment.</u> $E_p = \frac{p^2}{2m}$

$$H^{free} = \int d^3x \, \frac{(\vec{\nabla}\phi_+)^\dagger (\nabla\phi_+)}{2m}$$

<u>Same as</u> $H^{free} = \sum_{p\sigma} E_p \, a^\dagger_{p\sigma} a_{p\sigma}$

Now its clear how you could write down interactions that take place locally.

$$H^{free} + \int d^3x \, C\left[\phi_+(x)\,\phi_+(x)\,\phi_-(x)\,\phi_-(x)\right] + \cdots$$

Its totally clear now that this is local in space.

Taking this an expanding out gives something like we just talked about with 2 a's and 2 $a^\dagger$'s. All the rest comes along for the ride.

(Note this is done non-relativistically for the moment to stress that this has nothing to do w/ relativity. This is about making interactions local)

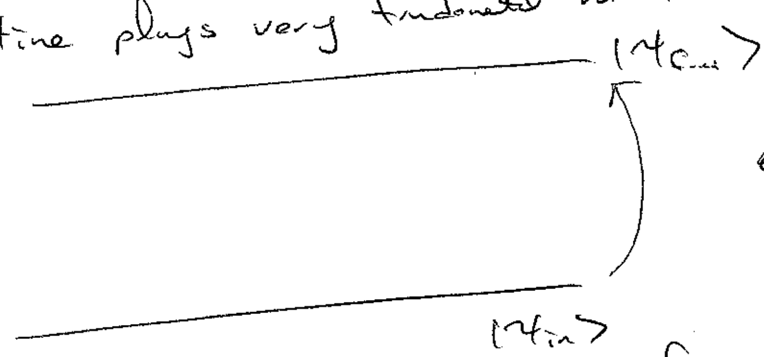
Why we use fields. Makes local interactions of particles manifest.

Hardwired into the description of particles.
locality

Where does relativity come in? What is the dificulty?

Time evolution

QM - time plays very fundamental role.



Different observers see different space-like sheets

Have a hope of Lorentz invariance if we start a $-\infty$ & go to $+\infty$.

Throw particles in from ∞ let them scatter + go back out to ∞. S-matrix

$$t=-\infty \quad |p_1\sigma_1 \cdots p_n\sigma_n\rangle \longrightarrow \overset{t=+\infty}{S|\vec{p}_1\sigma_1 \cdots \vec{p}_n\sigma_n\rangle}$$

$\hookrightarrow$ might be (at least a hope) Lorentz invariant.