Homework Set #9

1) Synchrotron Losses *(10 points)*

a. In class, I gave the wrong formula. The power per turn should be (note the 10^{-6} !!!)

$$P \sim 0.3 \cdot 10^{-6} \left(\frac{E}{m}\right)^4 \left(\frac{km}{R}\right)^2$$

This gives the power lost from synchrotron radiation per proton at the LHC to be about 40 MeV/s. The total number of protons N_p in the LHC ring is the number of bunches times the number of protons per bunch.

$$N_p = 2808 \cdot 1.15 \times 10^{11} \sim 3.2 \times 10^{14}$$

Therefore, the total power radiated away in synchrotron radiation is $P_{tot} \sim 3.2 \times 10^{14} \times 40 MeV/s \sim 1.3 \times 10^{13} GeV/s$. Converting this to J/s, we recall that 1 GeV is 1.6×10^{-10} J. Therefore, the power lost in watts is

$$P_{tot} \sim 1.6 \times 10^{-10} \cdot 1.3 \times 10^{13} W \sim 2 \times 10^3 W$$

That is, the power in synchrotron radiation at the LHC is equivalent to about two microwave ovens. Everyone who attempted this problem with the old formula will get full credit.

b.

$$\frac{P_{LEP}}{P_{LHC}} = \left(\frac{m_p E_{LEP}}{m_e E_{LHC}}\right)^4$$

Now,

$$\frac{m_p E_{LEP}}{m_e E_{LHC}} \sim \frac{1 \times 200}{10^{-3} 6500} \sim 30$$

or

$$P_{LEP} \sim 30^4 P_{LHC} \sim 8^5 \times 40 MeV/s \sim 3 \times 10^4 GeV/s$$

c.

$$\frac{P_{LEP}}{P_{LHC}} = 1 = \left(\frac{m_p E_{LEP}}{m_e E_{LHC}}\right)^4$$

or

$$\frac{E_{LHC}}{E_{LEP}} = \left(\frac{m_p}{m_e}\right) \sim 2000$$

2) Tracking Detectors *(10 points)*

a)
$$F = ma \Rightarrow mv^2/r_c = qvB \Rightarrow r_c = p_T/qB$$

Now,
$$r_c^2 = \left(\frac{L}{2}\right)^2 + (r_c - s)^2$$

Or (Ignoring terms
$$\frac{L^2}{4} = 2r_c s - s^2$$

B/c $r_c >> s$, can safely drop s^2 relative to $r_c s$. Thus
$$s = \frac{L^2}{8r_c} = \frac{qBL^2}{8p_T}$$

b)
$$p_T = \frac{qBL^2}{8s}$$

So,
$$\Delta p_T = \frac{qBL^2}{8s^2}\Delta s$$

and
$$\frac{\Delta p_T}{p_T} = \frac{\Delta s}{s} = \frac{8p_T}{qBL^2}\Delta s$$

c) For N = 50, $\epsilon = 100\ \mu m$, L = 1 m, and B = 1 T, $\Delta s \sim 50\mu m = 5010^{-6}m$

Now $T = 2 \times 10^{-16} GeV^2$
$e = 0.3$
$$\Delta p_T = \frac{8(p_T[GeV])^2}{0.3 \times 2 \times 10^{-16}} \frac{50 \times 10^{-6}}{5 \times 10^{15} GeV^{-1}} \sim 3 \times 10^{-3} (p_T[GeV])^2 GeV$$

At 1 GeV the uncertainty is $\sim 10^{-3}$ GeV, At 100 GeV the uncertainty is 10 GeV.

3) Limits of the Tracking System. *(5 points)*

a)

$$r_c \sim 3\frac{p_T[GeV]}{Q[e]B[T]}$$

Particles dont make it to the calorimeter when $r_{calo} \sim 2 \times r_c$

or

$$p_T \sim \frac{qBr_{calo}}{6} = \frac{2 \times 1.1}{6} \sim 400MeV$$

b) Estimate upper limit when $s \sim 17\mu m \sim 20 \times 10^{-6}m$

$$p_T \sim \frac{0.3 \times 2 \times 10^{-16}GeV^2}{8}\frac{0.5}{20 \times 10^{-6}}0.5 \times 5 \times 10^{15}GeV^{-1}p_T \sim 0.5 \times 10^3 GeV$$

c) At the limit $\Delta s/s \sim 1 \Rightarrow \Delta p_T/p_T \sim 1$, so $\Delta p_T \sim 500$ GeV

4) Rapidity. *(15 points)*

a) Under a boost along Z

$$E \to E\gamma - \beta\gamma p_z$$
$$p_z \to p_z\gamma - \beta\gamma E$$

So,

$$\begin{aligned} y \to \frac{1}{2}\log\frac{(E\gamma - \beta\gamma p_z) + (p_z\gamma - \beta\gamma E)}{(E\gamma - \beta\gamma p_z) - (p_z\gamma - \beta\gamma E)} \\ = \frac{1}{2}\log\frac{\gamma - \beta\gamma}{\gamma + \beta\gamma}\frac{E + p_z}{E - p_z} = \frac{1}{2}\log\frac{E + p_z}{E - p_z} + \frac{1}{2}\log\frac{\gamma - \beta\gamma}{\gamma + \beta\gamma} \\ = y + \frac{1}{2}\log\frac{\cosh\eta - \sinh\eta}{\cosh + \sinh\eta} = y + \frac{1}{2}\log\frac{e^{-\eta}}{e^{+\eta}} \\ = y + \frac{1}{2}\log e^{-2\eta} = y - \eta \end{aligned}$$

b. $y = \eta$ for mass-less particles

c.d. Green are electrons / Red are muons.

c.e I got:

eta1 = -1 / phi1 = 70 / pt1 = 30

eta2 = 0 / phi2 = 255 / pt2 = 30

eta3 = -0.2 / phi3 = 70 / pt3 = 20

eta4 = 0.5 / phi4 = 200 / pt4 = 25

f. I get: (124.8, -14.2, 9.5, -26.3) GeV (E,vecP)

h. 68. probably Zboson

i. 43 probably off shell z

j. 121 GeV probably a higgs