Lecture 2

The right way to think about the world

Talked last time about what our world consists of Matter/Forces etc. This class we will start by talking about the "right way" to view all of this.

Something that is not often taught (even in gradschool). However it is easy and extremely powerful....

Compton lectures that I gave. Got into trouble. Diagrams like: Feature of Life -> Evolution -> DNA -> molecules -> chemistry -> Atoms -> Electrons-> Quantum Mechanics -> Standard Model "Newton's Dream"

Go through a few examples of this kind of reasoning: Teeth behind these statements

Idea that can describe world around us using a few basic physical parameters. Powerful (Fun!) way of estimating ~anything to order of magnitude.

Dimensional Analysis and "$\sim$"

Put in the right physics to get answers to within "geometric factors":

- Wont worry about factors of 2 or π etc
- Use "$\sim$" not "="

Examples

(Volume of something) $\sim$ (size)^3 Cube = $R^3 \sim R^3$
Sphere = $4/3\pi R^3$ = 4.2 R3 $\sim R^3$
Sphere = $1/6\pi D^3 \sim D^3$
Cylinder = R$\times\pi R^2 \sim R^3$ (if two scales use $r^2 R$)

Kinematic energy = 1/2 m$v^2 \sim$ mv^2
Ive been doing this already: "Δp$\Delta X \geq$ h" (...it is really Δp$\Delta X \geq$ h/(4π))

Natural Units

Units...

I hate units! All numbers are really unit-less. Always comparing some quantity relative to some standard. We will work in "Natural Units".

Very easy. (Much easier than Metric/British/cgm/mks ...)
- Standard is set by basic physical principles.
- Numbers have direct physical interpretations.

$c \equiv 1$: [Distance]/[Time] $\equiv 1$
- Time and distance have same units
- Already familiar with this: "Its about an hour from here"
- $E = m$

$h \equiv 1$: [Energy]$\times$[Time] $= 1$ and [Energy]$\times$[Distance] $= 1$
- Energy (or Mass) is inversely related to distance or time.

Write everything in terms of [Energy]: Will often use 1 GeV $\sim m_{\text{proton}}$ as basic unit.

Will now do everything in terms of GeV. Can use conversions to get back to human units

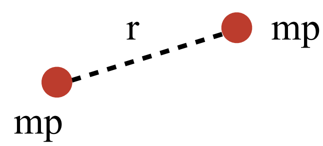
Conversions:
GeV = $10^{-}27$ kg
GeV^{-1}= 10^{-16} m
GeV^{-1}= 6×10^{-25} s

Examples:
Proton Weight: GeV
Proton Size: GeV^{-1}
My height: 1m $\sim 10^{16}$ GeV^{-1}*(I am as tall as* 10^{16} *protons stacked on top of each other)*
My weight: 100kg $\sim 10^{29}$ GeV *(I am made of* 10^{29} *protons)*

The world with 4 numbers.
Claim: ~everything in world combination of these numbers

- m_{proton}= 1 GeV
- $\alpha = 1/137$
- m_{electron}= 10^{-3} GeV
- $\alpha_G \equiv G_N m_{\text{proton}}^2 = 10^{-39}$

EM and Gravitation Interactions



Electromagnetic Energy

$$E = -\underbrace{\frac{e^2}{4\pi}}\frac{1}{r}$$

GeV GeV

Pure number: α
Its small: 1/137

Gravitational Energy

$$E = -\underbrace{G_N}\frac{m_p^2}{r}$$

GeV GeV^3

Dimensionful number
$G_N m_p^2 = 10^{-39}$

Will work through some quick examples.

Atoms:

$E \sim -\frac{Z\alpha}{r} + \frac{p^2}{m_e}$

$p \times r \sim 1 \Rightarrow E \sim -\frac{Z\alpha}{r} + \frac{1}{m_e r^2}$ solving... $r_{\text{atom}} \sim \frac{1}{Z\alpha m_e}$

Nucleus is protons + neutrons pacted in 3D volume. $r_{\text{nucleus}} \sim Z^{1/3} r_{\text{proton}} \sim \frac{Z^{1/3}}{m_{\text{proton}}}$

Relative scale: $\frac{r_{\text{nucleus}}}{r_{\text{atom}}} \sim \frac{\alpha m_{\text{electron}}}{Z^{2/3} m_{\text{proton}}} \sim 10^{-5}$ (Justifies claim that most of volume due to electron)

Back to the electron... $p_e \sim \frac{1}{r_{\text{atom}}} \sim m_{\text{electron}}(Z\alpha) \Rightarrow v_{\text{electron}} \sim (Z\alpha)$
Interesting physics content: v cannot be bigger than one ($\equiv$ c). (when ~1 electron relativistic, can get unstable to pair produciton, eg: excite electrons-positions from vacuum.)
$\Rightarrow$ there cannot be more than $\sim \frac{1}{\alpha}$ stable elements!

Also note $v_{\text{electron}} \sim (Z\alpha) << 1$ for low Z. This is why:

- we could do QM first with out relativity!
- electricity tends to be stronger than magnetism in everyday applications/conditions.

$E_{\text{atom}} \sim \frac{Z\alpha}{r_{\text{atom}}} \sim Z^2\alpha^2 m_{\text{electron}}$ (For hydrogen = $10^{-4}\times$ 0.5 MeV ~ 50 eV / Actually 13.6 eV)

For atoms, electron mass is king... dominates.

Solids:

(To within our ~) Solids just atoms stacked next to each other

Mass Density: Mass/Volume $\rho_{\text{solid}} \sim \frac{Z m_{\text{proton}}}{r_{\text{atom}}^3}$

Pressure of Solid: Force/Area or Energy/Volume

$P_{\text{solid}} \sim \frac{Z^2\alpha^2 m_{\text{electron}}}{r_{\text{atom}}^3} \sim Z^5\alpha^5 m_{\text{electron}}^4$

(Ratio of two gqive the speed of sound)

$v_{\text{sound}} \sim \sqrt{\frac{P_{\text{solid}}}{\rho_{\text{solid}}}} \sim \sqrt{\alpha} m_{\text{proton}} r_{\text{atom}}$

Predict ~25,000 m/s
Beryllium 12,890 m/s
Diamond 12,000 m/s
Steel 6000 m/s

Planets:

Solids where gravitational pressure balanced by solid pressure

$E_{\text{Gravity}} \sim \frac{G_N M_{\text{Planet}}^2}{R_{\text{Planet}}}$

$P_{\text{Gravity}} \sim \frac{E_{\text{Gravity}}}{V_{\text{Planet}}} \sim \frac{G_N M_{\text{Planet}}^2}{R_{\text{Planet}}^4}$

$M_{\text{Planet}} \sim \rho_{\text{solid}} \times R_{\text{Planet}}^3 \sim \frac{Z m_{\text{proton}} R_{\text{Planet}}^3}{r_{\text{atom}}^3}$

$P_{\text{Gravity}} \sim \frac{G_N Z^2 m_{\text{proton}}^2 R_{\text{Planet}}^2}{r_{\text{atom}}^6}$

Have a planet when $P_{\text{Gravity}} \sim P_{\text{Solid}}$ or $\frac{G_N Z^2 m_{\text{proton}}^2 R_{\text{Planet}}^2}{r_{\text{atom}}^6} \sim \frac{Z\alpha}{r_{\text{atom}}^4}$

$\Rightarrow R_{\text{Planet}} \sim \sqrt{\frac{1}{G_N m_{\text{proton}}^2 Z^3 \alpha m_{\text{electron}}^2}} \sim \sqrt{\frac{\alpha}{\alpha_G}} \times r_{\text{atom}}$

Planets/atoms relative size direct result or realtive EM vs Gravity strength.

This is why things are big, despite being governed by microscopic laws.

	R_{Earth}	M_{Earth}
Prediction:	10^7 m	10^{25} kg
Actual:	6.4×10^6 m	5.9×10^{24} kg

Life etc.: