Lecture 34

OK left off discussing atmospheric νs

Saw that there are two robust predictions that you can measure: muon vs electron ratio and top vs down.

Expect the same amount from above and from below.

So thats what people know about atmosphereic νs.

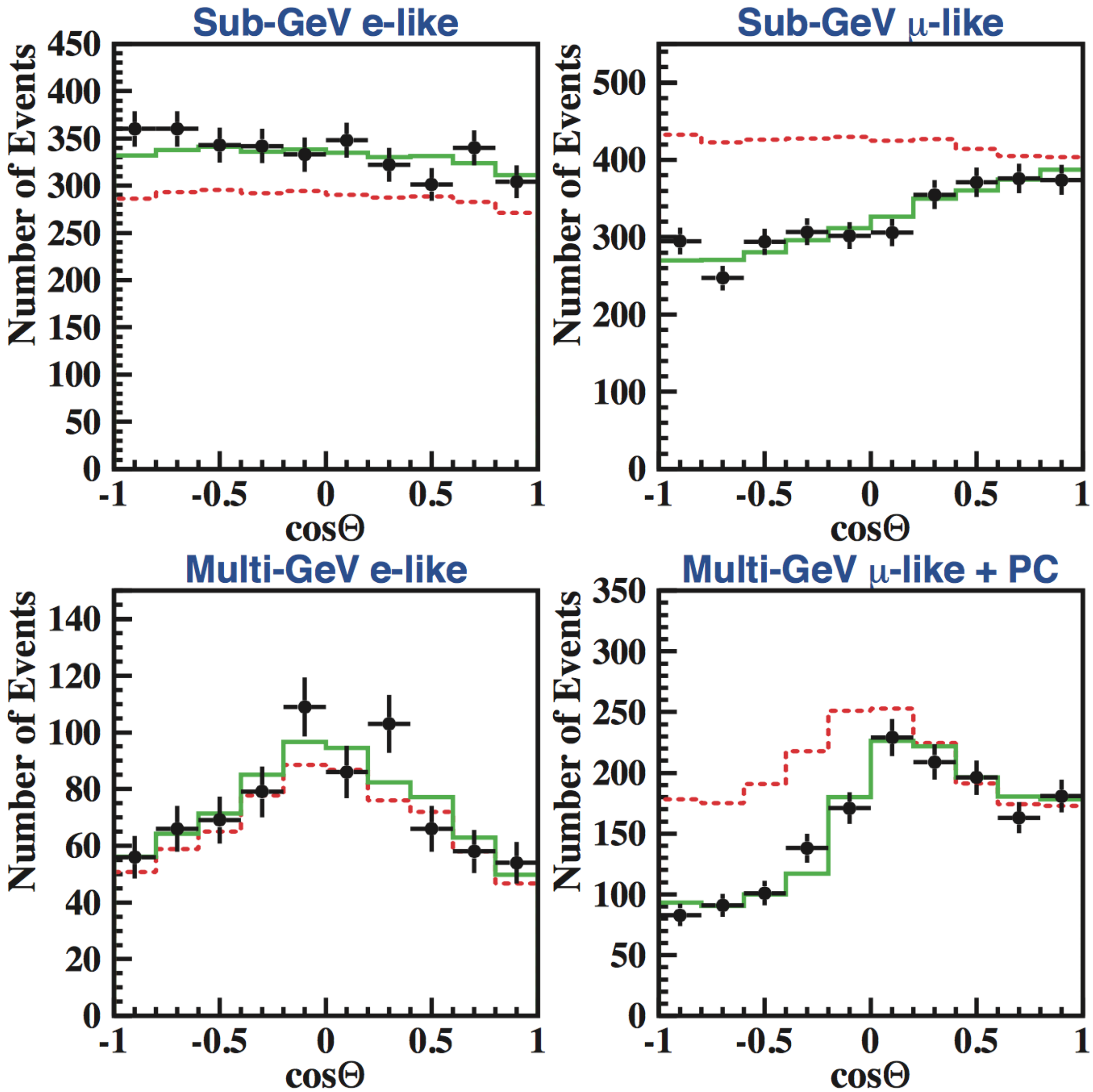
There were some experiments that were dedicated to measure atomsphereic νs.

And then there are a differnet kind of detectors. Giant water tanks Kameiokande Kamiokande - Name Kamioka place / nde - “nuclean decay experiment. “ Built ot look for proton decay.

Atmosheric νswhere a background one you cant get rid of.

WHen you measure the flux from above and below turns out the answer is not one: 1/2. This is a really robust prediction.

They could actually do better. Tell muons from electrons. These plots show log/high energy electrons and muons separately.


Sub-GeV e-like
Sub-GeV μ-like
Multi-GeV e-like
Multi-GeV μ-like + PC
Number of Events
cosΘ
450
400
350
300
250
200
150
100
50
0
500
140
120
80
60
40
20
-1
-0.5
0
0.5
1

Very intersting result. Points are the observations, solid line is the prediction. Notice a few think; For teh electrons everything works more or less OK, however for the muons it doesn work well at all. You have an effect that says you dont understand what the muon νsare doing and that effect dependson the energy and how far they are popogating. νscoming from above, νscoming from below dont work well. Missing about half of them.

Very exciting result. B/c your sure the measurements are correct. Observable is robust. Implies that the νsare doign someing. And whatever they are doing depends on the energy and the baseline (how much they have traveled)

Why is thsi important ? need a hypothesis for what is going on. Maybe the νsare being absorbed ? νsthat go through the earth are gettins absorbed by the earth. Know Cant be true, cross section would bave to be too high.

What esle could be going on? νsdecaying /changing flavour Only looking for muon and electorn νs, so if they were converting into τs, this could explain all this data. Only massive particles know about time. What ever they are doing If νscan tell time ==> νshave mass. OR Lorentz invariance is wrong.

So thats where we were. Bottom line νschange flavour as a function of E and distance.

Mass-induced flavour oscillations.

What happens if the νshave mass ?

$$\nu_1 \text{ with } m_1$$

$$\nu_2 \text{ with } m_2$$

$$\nu_3 \text{ with } m_3$$

If you raise the hypothesis that νshave mass, then there are νsstates that you can label with different masses.

We also have $\nu_e\nu_\mu$and $\nu_\tau$these are the interaction eigenstatss the νsyou produce when you have a weak interaction. The quesitno is then which one of these is the $\nu_e\nu_\mu$and ν_τ?

The answer is it doesnt have to be any of them, know for sureis that the $\nu_e$is a linear combination of νswith a well-defined bass

$$\nu_e = U_e i \nu_i$$

same for mus and taus.

$$\nu_\mu = U_m u i \nu_i$$

$$\nu_\tau = U_t a u i \nu_i$$

also know that the ν_e, $\nu_\mu$and $\nu_\tau$or orthognal to one another (ie: they are different states)

These Us can be organised into a unitary matrix.

$$\nu_\alpha = U_{\alpha\ i} \nu_i$$

where, $\alpha = e, \mu, \tau$

i = 1,2,3

$U_{\alpha i}$ is Unitary mixing matrix We will see, just by raising this hypothesis you can explain all the data.

Actually similar thing happens in the quark sector, we havent talked about this yet, but its true.

Have do identify who are the "real particles". Meaning what are the eigenstates of the free hamiltoninan.

In the quark section thats the

u c t d s b

In the lepton sector we choose that to be

e mu tau nu1 nu2 nu3

There is no such thing as an ν_e, doesnt exist.

but the weak interactions

$W \to t + b, s, b$ w/coupling V_{CKM} U_t, α

the same thing happens in the lepton sectior:

$W \to e + \nu_1, \nu_2, \nu_3$ w/coupling U_e, i

physics is the same, the consequesnces turn out to be very differnt. Main reaon is that the ν masses are very small. Bc the masses are small (as well talk about in a second) you have a phenomina of ν oscillations. Doesnt happen for quarks.

OK lets set this up.

What does it mean to be a particle with a well defined mass? QM POV, eigen state of free particle hamiltoninaon.

$$|\nu_1\rangle = e^{-iE_1 t} |\nu_1\rangle$$

this is what ti means to be a ν with a well-defined mass.

Now What happens if you dont have one of these things, but a linear superposition of these.

Lets say you have the ν_e and to make life easy, lets pretend that we only have 2 νs.

Now ν_e will be a linear combination of ν_1 and ν_2.

$$|\nu_e\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

For completeness, can also write ν_μ, which is also a linear combination of ν_1 and ν_2, but it is a orthognal to ν_e.

$$|\nu_\mu\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

Some obvious things, we know $\cos\theta$ and $\sin\theta$ are some coefficients that we dont know, but we know the sum of squares is 1. (Thats why we write it like an angle)

Can work this out from $\langle\nu_e|\nu_e\rangle = 1$ and $\langle\nu_e|\nu_\mu\rangle = 0$

thats why are allowed to paramaterize things in terms of an angle.

How do these states evolve as a fucntion of time?

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle$$

this is the heart of what ν oscillations are all about. Because these phaces are different, what you get is no longer proportinal to the ν_e state. Or in fancier words, ν_e is not an eigenstate of the free hamiltonian.

NOw this is very very simple physics. Literally a 2 level system that you learned in undergraduate QM.

Rember everything is going to be relativistic. νsare going to be propogating plane waves.

Relativistic version

$$|\nu_e(\vec{x},t)\rangle = \cos\theta e^{-ip_1^\mu x_\mu}|\nu_1\rangle + \sin\theta e^{-ip_2^\mu x_\mu}|\nu_2\rangle$$

where x is the $(t,\vec{x})$ four vector.

Ultra relativistic approx.

$$t \sim L$$

Phase factors very close to being zero, depend on difference between energy and momentum $(E_1 - p_1)$

Now let me calculate this differnce in the following way,

$$(E_1 - p_1)(E_1 + p_1) = m_1^2 \Rightarrow (E_1 - p_1) = \frac{m_1^2}{2E}$$

in the ultra relativistic approx $E \sim P$

$$|\nu_e(L)\rangle = \cos\theta e^{-i\frac{m_1^2}{2E}L}|\nu_1\rangle + \sin\theta e^{-i\frac{m_2^2}{2E}L}|\nu_2\rangle$$

OK, now lets calculate something...

Lets calculate the probablity that this obect here, when it hits something produces an electron.

Which is just given by

$$\langle \nu_e | \nu_e(L) \rangle = \cos^2\theta e^{-i\frac{m_1^2}{2E}L} + \sin^2\theta e^{-i\frac{m_2^2}{2E}L}$$

Amplitude for having an ν_e be born somewhere, propagate some distance L and then be detected as an ν_e.

So the proabablity is this thing squared

$$\begin{aligned} |\langle \nu_e | \nu_e(L) \rangle|^2 &= \left| \cos^2\theta e^{-i\frac{m_1^2}{2E}L} + \sin^2\theta e^{-i\frac{m_2^2}{2E}L} \right|^2 \\ &= \left| \cos^2\theta + \sin^2\theta e^{-i\frac{m_2^2-m_1^2}{2E}L} \right|^2 \\ &= \cos^4\theta + \sin^4\theta + \cos^2\theta \sin^2\theta 2\cos\left(\frac{\Delta m^2 L}{2E}\right) \end{aligned}$$

where $\Delta m^2 = m_2^2 - m_1^2$

Can simply this...

$$\begin{aligned} &= \left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + 2\cos^2\theta\sin^2\theta\cos\left(\frac{\Delta m^2 L}{2E}\right) \\ &= 1 - 2\cos^2\theta\sin^2\theta\left(1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right) \end{aligned}$$

Now use a trig ID for $1 - \cos\theta$...

$$\begin{aligned} &= 1 - 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2 L}{2E}\right) \\ &= 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{2E}\right) \end{aligned}$$

So what we learn at the end of the day is, if your born as $\nu_e$and you propogate a certian distance L, and you're detectred as an ν_e, the probablity is given by,

$$P_e e(L) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{2E} \right)$$

Possible to be born as $\nu_e$and detected as $\nu_e$with less than 100% probability.