Homework Set #4

Solutions

1) Find the generators of the "Little Group" for Massive particles   *(5 points)*

The little group equation is:

$$W \cdot k = k \Rightarrow \omega \cdot k = 0$$

where

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & a & b & c \\ -a & 0 & A & B \\ -b & -A & 0 & C \\ -c & -B & -C & 0 \end{bmatrix}$$

For a massive particle we can take k to be k = (m,0,0,0).

So $\omega \cdot k = (0, -ma, -mb, -mc)$. So the little group generators for massive particles are

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & A & B \\ 0 & -A & 0 & C \\ 0 & -B & -C & 0 \end{bmatrix}$$

which are the rotation matrices.

2) Find the generators of the "Little Group" for Massless particles   *(5 points)*

Same as above but now, for a massless particle, we can take k to be k = (E,E,0,0).

So $\omega \cdot k = E(a, -a, -b - A, -c - B)$. So the little group generators for massless particles must have a=0, b=A, and c=B

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ -A & -A & 0 & C \\ -B & -B & -C & 0 \end{bmatrix}$$

which are the results we had in class.

3) Show that $\int d^3\!\!\!/p \equiv \int \frac{d^3\vec{p}}{2E_p}$ **is Lorentz invariant.**   *(2 points)* We will start with a manifestly Lorentz invariant integral and show that it is the same as $\int d^3\!\!\!/p$.

$$\int d^4p \, \delta(E^2 - (|\vec{p}|^2 + m^2)) = \int dE d^3p \, \delta(E^2 - (|\vec{p}|^2 + m^2))$$

Now, $dE^2 = 2EdE$ or $dE = dE^2/2E$, so

$$= \int \frac{dE^2}{2E} d^3 p \, \delta(E^2 - (|\vec{p}|^2 + m^2))$$

Can now do the integral over dE^2

$$= \int \frac{d^3 p}{2 \sqrt{|\vec{p}|^2 + m^2}} = \int \widetilde{d^3 p}$$

4) Anti-Particles *(5 points)*

a) Expand $\Phi^{\dagger^2}\Phi^2$ in terms of a, $a^\dagger$, b, and $b^\dagger$ (Ignore the exponentials and integrals)

$$\Phi^{\dagger^2}\Phi^2 = \left(\phi_-^a + \phi_+^b\right)^2 \left(\phi_+^a + \phi_-^b\right)^2 \sim \left(a + b^\dagger\right)^2 \left(a^\dagger + b\right)^2$$

$$= \left(aa + 2ab^\dagger + b^\dagger b^\dagger\right)\left(a^\dagger a^\dagger + 2a^\dagger b + bb\right)$$

$$= \left(aaa^\dagger a^\dagger + 2aaa^\dagger b + aabb + 2ab^\dagger a^\dagger a^\dagger + 4ab^\dagger a^\dagger b + 2ab^\dagger bb + b^\dagger b^\dagger a^\dagger a^\dagger + 2b^\dagger b^\dagger a^\dagger b + b^\dagger b^\dagger bb\right)$$

b) See figure.

c) Let the charge (Q) of particle a be q_a and the charge of particle b be q_b. Calculate ΔQ for each process.

See figure each term goes like $(q_a + q_b)$

d) What happens if you take $q_a = -q_b$? When $q_a = -q_b$, all processes conserve charge.

$a\ a\ a^\dagger\ a^\dagger$

$a \qquad a$

$a \qquad a$

$$\Delta Q = 0$$

$a\ a\ a^\dagger\ b$

a

$a \quad a \quad b$

$$\Delta Q = -z_a - z_b$$
$$= -(z_a + z_b)$$

$a\ a\ b\ b$

$a \quad a \quad b \quad b$

$$\Delta Q = -2z_a - 2z_B$$
$$= -2(z_a + z_B)$$

$a\ b^\dagger\ a^\dagger\ a^\dagger$

$b \quad a \quad a$

a

$$\Delta Q = (z_a + z_B)$$

$a\ b^\dagger\ a^\dagger\ b$

$a \qquad b$

$a \qquad b$

$$\Delta Q = 0$$

$a\ b^\dagger\ b\ b$

b

$b \quad b \quad a$

$$\Delta Q = -(z_a + z_B)$$

$b^\dagger\ b^\dagger\ a^\dagger\ a^\dagger$

$b \quad b \quad a \quad a$

$$\Delta Q = 2(z_a + z_B)$$

$b^\dagger\ b^\dagger\ a^\dagger\ b$

$b \quad b \quad a$

b

$$\Delta Q = (z_a + z_B)$$

$b^\dagger\ b^\dagger\ b\ b$

$b \qquad b$

$b \qquad b$

$$\Delta Q = 0$$