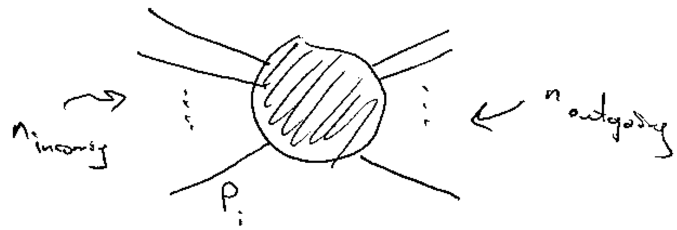
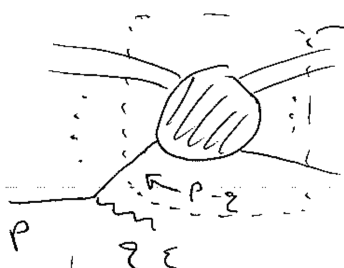
Lecture 18

Now do the same thing to a more general interaction



$$\equiv i\mathcal{M}_0(p_i)$$

Consider what happens if we attach a "photon" to an incoming leg

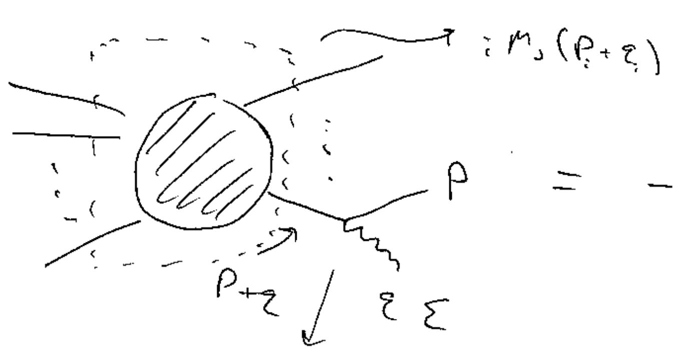


$$\to i\mathcal{M}_0(p_i - q)$$

$$= Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\, i\mathcal{M}_0(p_i - q)$$

$$\to i(iQ)\,\frac{\left(\varepsilon_\mu\left(p_i^\mu + (p_i^\mu - q^\mu)\right)\right)}{(p_i - q)^2 - m^2}$$

Can also attach photon to outgoing leg



$$= -Q \frac{(p\cdot\epsilon)}{(p\cdot q)}\, i M(p_i+q)$$

$$(-Q)\, \frac{\epsilon_\mu \left(p_i^\mu + (p_i^\mu + q^\mu)\right)}{(p+q)^2 - m^2}$$

Total Amplitude is then given by

$$M = \sum_{\text{incoming}} Q_i \frac{(p\cdot\epsilon)}{(p\cdot q)}\, iM_0(p-q) + \sum_{\text{outgoing}} -Q_i \frac{(p\cdot\epsilon)}{(p\cdot q)}\, iM_0(p+q)$$

Take soft limit: $M_0(p \pm q) \to M_0(p)$

$$M = iM_0 \left(\sum_{\text{incoming}} Q_i \frac{(p\cdot\epsilon)}{(p\cdot q)} + \sum_{\text{outgoing}} -Q_i \frac{(p\cdot\epsilon)}{(p\cdot q)} \right)$$

Now as before $\epsilon_\mu \to \epsilon'_\mu + q_\mu$ means that M must vanish when $\epsilon_\mu \to q_\mu$.

OR under a Lorentz Transform

$$\epsilon_\mu \cdot M \to \epsilon'_\mu \cdot M' + iM_0 \underbrace{\left(\sum_{\text{incoming}} Q_i + \sum_{\text{outgoing}} -Q_i \right)}_{\substack{=0 \text{ only if} \\ \sum_{\text{incoming}} Q_i = \sum_{\text{outgoing}} Q_i}}$$

Charge has to be conserved!

Now same logic for Spin-2 (describes interaction w/Gravitons)

Same as above except 2-component polarization vector.

$$\epsilon_{\mu\nu} \underbrace{\rightarrow}_{\text{under little group}} \epsilon_{\mu\nu} + \underbrace{A_\mu q_\nu + B_\mu q_\mu + C q_\mu q_\nu}_{\text{effect from all of these need to be 0 as before}}$$

where A, B C's are non-zero and depend on the particular little group transformation done.

$$= i(iK_i)\epsilon_{\mu\nu}\frac{(p^\mu p^\nu)}{-p\cdot q}$$

(Same idea with the outgoing leg)

Now, (lets focus on piece that goes like $\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + q_\mu B_\nu$

$$\epsilon_{\mu\nu} \rightarrow \epsilon'_{\mu\nu} M'^{\mu\nu} \;+\; M\left(\sum_{\text{incoming}} K_i B_\nu p^\nu - \sum_{\text{outgoing}} K_i B_\nu p^\nu\right)$$
$$+\; MB_\nu\left(\sum_{\text{incoming}} K_i p^\nu - \sum_{\text{outgoing}} K_i p^\nu\right)$$

$\Rightarrow K_i p_i^\nu$ is $\underline{\underline{\text{conserved}}}$

We know that p_i^ν is conserved by E and momentum conservation.

Only way can have nontrivial solutions is if $k_i = k$ for all i

All particles interact with gravity with the same strength.

Gravitational interaction is Universal !

Discovered the "Principle of Equivalence" that is the starting point of General Relativity!

Can keep going...

For a massless spin-3 particle we would do the same exercise.

We would find we need

$$\sum_{\text{incoming}} \beta_i p_i^\mu p_i^\nu = \sum_{\text{outgoing}} \beta_i p_i^\mu p_i^\nu$$

eg: $\mu\nu = 0$

$$\sum_{\text{incoming}} \beta_i E_i^2 = \sum_{\text{outgoing}} \beta_i E_i^2$$

Way too constraining.

Only way if $\beta_i = 0$

There can be no interacting theories of massless particles of Spin greater than 2 !