

## Final

1) What are three major consequences of combining QM and Relativity? *(3 points)*

2) Lorentz Transforms *(5 points)*

a) How does a massive particle  $|p^\mu, \sigma\rangle$  transform under a little group transformation ( $W_\mu^\nu$ ) ?

d) How does a mass-less particle  $|p^\mu, \sigma\rangle$  transform under a little group transformation ( $W_\mu^\nu$ ) ?

c) How does a massive particle  $|p^\mu, \sigma\rangle$  transform under a general Lorentz transformation ( $\Lambda_\mu^\nu$ ) ?

d) How does a mass-less particle  $|p^\mu, \sigma\rangle$  transform under a general Lorentz transformation ( $\Lambda_\mu^\nu$ ) ?

**3) List or draw a diagram of the particles in the Standard model.**

*(3 points)*

What is the spin of each particle ?

**4) Why is the weak interaction so much weaker than the electromagnetic interaction at low energies?**

*(2 points)*

**5) Accelerators:**

*(4 points)*

a) What limits the energy of circular proton accelerators ?

b) What limits the energy of circular electron accelerators ?

**6) Muon decays:**

*(10 points)*

- a) The muon decays via the weak interaction. At low energy ( $E \ll m_W$ ), this can be approximated as a point-like interaction. Draw the diagram describing muon decay to an electron assuming a point-like weak interaction.
  
  
  
  
  
  
  
  
  
  
- b) What are the dimensions of the coupling constant, associated to this diagram ?
  
  
  
  
  
  
  
  
  
  
- c) How does the decay rate  $\Gamma$  (decays/unit time) depend on the muon mass ?
  
  
  
  
  
  
  
  
  
  
- d) The muon has a mass of  $\sim 0.1$  GeV and a lifetime of  $\sim 1\mu s$ . The tau lepton has a mass of  $\sim 1$  GeV. Estimate the lifetime of the tau lepton in  $\mu s$ .

- e) Suppose that the photon could couple at the same vertex to the muon and the electron. Then the muon could decay as  $\mu \rightarrow e\gamma$ . Estimate the ratio of the  $\mu$  lifetime in this world to that in our world without this interaction.

**7) Electron-positron Collisions**

*(10 points)*

- a) Consider electron-positron collisions with a center-of-mass energy of 40 GeV. Estimate the ratio of hadron production to di-muon production.
- b) Sketch a graph of the total cross section of  $ee \rightarrow \mu\mu$  as a function of  $E_{CM}$  from 40 GeV to 200. Also sketch the component of the cross section due to the electro-magnetic interaction.

**8) Branching ratios:**

*(9 points)*

- a) How often does a  $\tau$  decay to a charged lepton ?
  
  
  
  
  
  
  
  
  
  
- b) How often does a Z-boson decay to charged leptons ?
  
  
  
  
  
  
  
  
  
  
- c) How often does a W-boson decay to a charged lepton ?

**9) Collider Detectors**

*(6 points)*

- a) In what ways do the detector signatures of electrons and muons look a-like, in what ways are they different ?
  
  
  
  
  
  
  
  
  
  
- b) In what ways do the detector signatures of electrons and photons look a-like, in what ways are they different ?

**10) What are some reasons that hadronic showers are more challenging to measure than electro-magnetic showers ?** *(5 points)*

**11) If you want to improve momentum resolution, is it better to have a tracking detector that is twice as big or that has twice the magnetic field ? Justify your answer.** *(5 points)*

**12) For a new particle  $X$  with mass  $\sim 2$  TeV, would you expect to measure the  $X$  mass more precisely from  $X \rightarrow ee$  or  $X \rightarrow \mu\mu$  ? Justify your answer.** *(5 points)*

**13) How are  $\nu$ s detected at the LHC ?**

*(3 points)*

**14) Spontaneous Symmetry Breaking**

*(9 points)*

- a) What is the particle spectra (ie: for each particle, is it massive or mass-less and what is the spin) from the Lagrangian:

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - V(\phi), \text{ where } \phi = \phi_1 + i\phi_2, V(\phi) = \mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2 \text{ and } \lambda, \mu^2 > 0$$

- b) What is the particle spectra from the setup in a) but with  $\mu^2 < 0$  ?

- c) What is the particle spectra from the Lagrangian:

$$\mathcal{L} = (D_\mu \phi^*)(D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where  $\phi$  and  $V(\phi)$  are as before,  $D_\mu = \partial_\mu - ieA_\mu$ ,  $\lambda > 0$ , and  $\mu^2 < 0$  ?

**15) Higgs Physics** (7 points)

- a) Estimate the ratio of  $\text{Br}(H \rightarrow \mu\mu)/\text{Br}(H \rightarrow \tau\tau)$
  
  
  
  
  
  
  
  
  
  
- b) Draw one possible Feynman diagram for production and decay of the Higgs boson at the LHC.
  
  
  
  
  
  
  
  
  
  
- c) Draw one possible Feynman diagram for the production and decay of the Higgs boson at an electron-positron Collider.

**16) What are some experimental constraints on a fourth generation of leptons ?** (3 points)



## 17) Neutrino Physics

(17 points)

- a) How was the distinction between  $\nu_\mu$  and  $\nu_e$  discovered ?
  
  
  
  
  
  
  
  
  
  
- b) Why was this expected ?
  
  
  
  
  
  
  
  
  
  
- c) What was the main reason to study  $\nu$ s in the 60s and 70s, before we knew they had mass?
  
  
  
  
  
  
  
  
  
  
- d) What are dominant kind(s) (indicate particle or anti-particle and flavor) of  $\nu$ s that are produced (ignore oscillations) from:
  - i) The sun ?
  
  
  
  
  
  
  
  
  
  
  - ii) Nuclear reactors ?
  
  
  
  
  
  
  
  
  
  
  - ii) Cosmic-rays ?
  
  
  
  
  
  
  
  
  
  
  - iv)  $\nu$ -beams ?

**18) In a two  $\nu$  model, what combination of  $\Delta m^2$ , E, and L do the transition probabilities depend on?** *(3 points)*

**19) What problems might Super-Symmetry solve ?** *(3 points)*