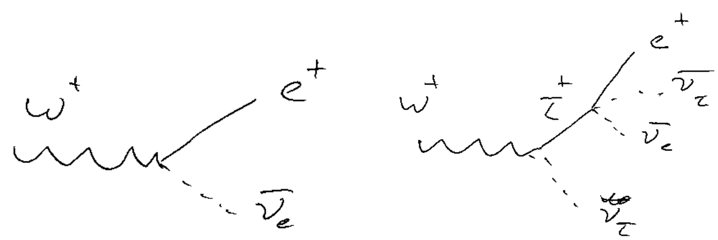
Homework Set #9

4) *W* boson decays to electrons. *(3 points)*

a) The W can decay directly to an electron or to electron by decaying through a τ. Draw the corresponding diagrams.



b)

$$Br(W \to \ell\nu) = \frac{1}{\underbrace{3}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{2 quark generations}}} = \frac{1}{9} = 0.11$$

(Note the top-quark is heavier than the W, so the $W \to t, b$ decay is forbidden.)

Now,

$$Br(\tau \to e\nu\bar{\nu}) = \frac{1}{\underbrace{2}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{1 quark generations}}} = \frac{1}{5} = 0.2$$

Here the τ can decay to two lepton generation (es and μ s), but only has enough mass to decay to one quark generation (u, d).

So,

$$Br(W \to e + X) = \underbrace{\frac{1}{9}}_{W\to e\nu} + \underbrace{\frac{1}{9}}_{W\to\tau\nu} \times \underbrace{\frac{1}{5}}_{\tau\to e\nu} = \frac{1.2}{9} = 0.13$$

4) $H \to WW \to e\mu$ decaus. *(3 points)*

$$Br(W \to \ell\nu) \sim \frac{1}{9}$$

for e or μ can include the decays through τs as in problem 3 to get $\frac{1.2}{9}$

$$Br(WW \to e\mu + X) = 2 \times \left(\frac{1.2}{9}\right)^2 \sim 0.036$$

Factor of two becuase you can get $e^+\mu^-$ or $e^-\mu^+$

2) Tracking Detectors *(10 points)*

a)

$$F = ma \Rightarrow mv^2/r_c = qvB \Rightarrow r_c = p_T/qB$$

Now,

$$r_c^2 = \left(\frac{L}{2}\right)^2 + (r_c - s)^2$$

Or (Ignoring terms

$$\frac{L^2}{4} = 2r_c s - s^2$$

B/c $r_c >> s$, can safely drop s^2 relative to $r_c s$. Thus

$$s = \frac{L^2}{8r_c} = \frac{qBL^2}{8p_T}$$

b)

$$p_T = \frac{qBL^2}{8s}$$

So,

$$\Delta p_T = \frac{qBL^2}{8s^2}\Delta s$$

and

$$\frac{\Delta p_T}{p_T} = \frac{\Delta s}{s} = \frac{8p_T}{qBL^2}\Delta s$$

c) For N = 50, ϵ = 100 μm, L = 1 m, and B = 1 T, $\Delta s \sim 50\mu m = 5010^{-6}m$

Now $T = 2 \times 10^{-16} GeV^2$
$e = 0.3$

$$\Delta p_T = \frac{8(p_T[GeV])^2}{0.3 \times 2 \times 10^{-16}} \frac{50 \times 10^{-6}}{5 \times 10^{15} GeV^{-1}} \sim 3 \times 10^{-3}(p_T[GeV])^2 GeV$$

At 1 GeV the uncertainty is $\sim 10^{-3}$ GeV, At 100 GeV the uncertainty is 10 GeV.

3) Limits of the Tracking System. *(5 points)*

a)

$$r_c \sim 3\frac{p_T[GeV]}{Q[e]B[T]}$$

Particles dont make it to the calorimeter when $r_{calo} \sim 2 \times r_c$

or

$$p_T \sim \frac{qBr_{calo}}{6} = \frac{2 \times 1.1}{6} \sim 400MeV$$

b) Estimate upper limit when $s \sim 17\mu m \sim 20 \times 10^{-6}m$

$$p_T \sim \frac{0.3 \times 2 \times 10^{-16}GeV^2}{8}\frac{0.5}{20 \times 10^{-6}}0.5 \times 5 \times 10^{15}GeV^{-1}p_T \sim 0.5 \times 10^3GeV$$

c) At the limit $\Delta s/s \sim 1 \Rightarrow \Delta p_T/p_T \sim 1$, so $\Delta p_T \sim 500$ GeV

4) Rapidity. *(15 points)*

a) Under a boost along Z

$$E \to E\gamma - \beta\gamma p_z$$
$$p_z \to p_z\gamma - \beta\gamma E$$

So,

$$\begin{aligned} y &\to \frac{1}{2}\log\frac{(E\gamma - \beta\gamma p_z) + (p_z\gamma - \beta\gamma E)}{(E\gamma - \beta\gamma p_z) - (p_z\gamma - \beta\gamma E)} \\ &= \frac{1}{2}\log\frac{\gamma - \beta\gamma}{\gamma + \beta\gamma}\frac{E + p_z}{E - p_z} = \frac{1}{2}\log\frac{E + p_z}{E - p_z} + \frac{1}{2}\log\frac{\gamma - \beta\gamma}{\gamma + \beta\gamma} \\ &= y + \frac{1}{2}\log\frac{\cosh\eta - \sinh\eta}{\cosh + \sinh\eta} = y + \frac{1}{2}\log\frac{e^{-\eta}}{e^{+\eta}} \\ &= y + \frac{1}{2}\log e^{-2\eta} = y - \eta \end{aligned}$$

b. $y = \eta$ for mass-less particles

d. Green are electrons / Red are muons.

e I got:

eta1 = -1 / phi1 = 70 / pt1 = 30

eta2 = 0 / phi2 = 255 / pt2 = 30

eta3 = -0.2 / phi3 = 70 / pt3 = 20

eta4 = 0.5 / phi4 = 200 / pt4 = 25

f. I get: (124.8, -14.2, 9.5, -26.3) GeV (E,vecP)

h. 68. probably Zboson

i. 43 probably off shell z

j. 121 GeV probably a higgs