Midterm

X) What are three major consequences of combining QM and Relativity?   *(2 points)*

X) Neutrino masses. The seesaw mechanism is a proposed source for neutrino masses. In this scenario the neutrino gets a mass from the following diagram.   *(2 points)*
HHνν What is the dimension of the coupling constant associated with this diagram ?

X) List or draw a diagram of the particles in the Standard model.   *(2 points)*
What is the spin of each particle ?

X) How many generators does the Lorentz group have ?   *(2 points)*
What transformations to they correspond to ?

X) Why is the weak interaction so much weaker than EM and low energies?   *(2 points)*

X) Can the SM have interactions between fermions and massive particles with Spin 3 ? If not, why not. What about interactions between mass-less Spin 3 particles and fermions ? If not, why not.   *(2 points)*

X) Lorentz Transforms   *(5 points)*

a) How does a massive particle $|p^\mu, \sigma\rangle$ transform under a general Lorentz transformation (Λ^ν_μ) ?

b) How does a massive particle $|p^\mu, \sigma\rangle$ transform under a little group transformation (W^ν_μ) ?

c) How does a mass-less particle $|p^\mu, \sigma\rangle$ transform under a general Lorentz transformation (Λ^ν_μ) ?

d) How does a mass-less particle $|p^\mu, \sigma\rangle$ transform under a general a little group transformation (W^ν_μ) ?

e) How does a massive scalar particle $|p^\mu, \sigma\rangle$ transform under a general Lorentz transformations (Λ^ν_μ) ?

X) GZK cutoff energy   *(5 points)*
High-energy cosmic rays (high energy protons), lose energy by interacting with CMB photons by producing a neutral pion:

$$p + \gamma_{\text{CMB}} \rightarrow p + \pi_0.$$

The proton energy at which this process can occur is called the GZK cutoff. Estimate the GZK cutoff energy. The energy of CMB photons is 10^{-13} GeV (which follows from the measurement of CMB temperature of about 2.7 K). Use 0.1 GeV for the mass of the pion and 1 GeV for the mass of the proton

X) *ee → ττ* **scattering** *(2 points)*

a) At high energy ($E_{CM} >> m_\tau$), what is the dependence of the $ee \to \tau\tau$ cross section on E_{CM} ?

b) At high energy ($E_{CM} >> m_\tau$), derive the dependence of the cross section on scattering angle θ.

c) Sketch a graph of $\frac{\sigma(ee \to \tau\tau)}{\sigma(ee \to \mu\mu)}$ as a function of E_{CM} from $2 \times m_\mu$ to 1 TeV.

X) Muon decays: *(5 points)*

a) The muon decays via the weak interaction, At low energy ($E << m_W$), this can be approximated as a point-like interaction. Draw the diagram describing muon decay to an electron assuming a point-like weak interaction.

b) What are the dimensions of the coupling constant, associated to this diagram ?

c) How does the decay rate Γ (decays/unit time) depend on the muon mass ?

d) The muon has a mass of ~0.1 GeV and a lifetime of ~ $1\mu s$. The tau lepton has a mass of ~1 GeV. Estimate the lifetime of the tau lepton in μs.

e) Suppose that the photon could couple at the same vertex to the muon and the electron. Then the muon could decay as $\mu \to e\gamma$. Estimate the ratio of $\mu \to e\gamma$ lifetime to the corresponding weak decay.

X) Yukawa's Theory. *(2 points)*
In the 1930s, Hideki Yukawa predicted the existence of a new particle, now called the pion, which is responsible for binding protons and neutrons together in atomic nuclei. Estimate the mass of the pion from the assumed range of the force. (10^{-15} meters). Express the mass in GeV.

X) A new force. *(5 points)*
Assume there is another force of nature felt by electrons associated with the exchange of a new X boson of mass 1 TeV.

a) Estimate an upper limit on the range of this new force in meters.

b) Assume muons also interact with this new boson. How could you determine the spin of X ?

c) Assume that this new X boson could also decay to spin-1/2 dark matter particles ψ_{DM}. At low energies (« 1 TeV) the X interaction can be described by a point-like interaction. Estimate the coupling constant associated to dark matter scattering $e\psi_{DM} \to e\psi_{DM}$. (Assume the X coupling at high energies is the same as for EM)

3) Neutrino interactions *(2 points)*

Assuming lepton universality, would you expect a difference between the way ν_e and ν_μ interact with matter ? Justify your answer by drawing the leading order diagrams for $\nu_e + e \rightarrow \nu_e + e$ and $\nu_\mu + e \rightarrow \nu_\mu + e$.

3) Exotic Atoms *(5 points)*

a) The muon was the first elementary particle discovered that does not appear in ordinary atoms. Negative muons can, however, form muonic atoms by replacing an electron in ordinary atoms. Estimate the size of muonic hydrogen in GeV.

b) Positronium is a system consisting of an electron and its anti-particle, a positron, bound together into an exotic atom. Estimate the size of positronium in GeV.

c) Calculate the ratios of muonic hydrogen to hydrogen and positronium to hydrogen.