Spontaneous Symmetry Breaking "Hidden Symmetry" ①

Last time ran into a crisis.

Needed both massive boson (force carrier) and gauge invariance. Can't get this from $m^2 A^2$.
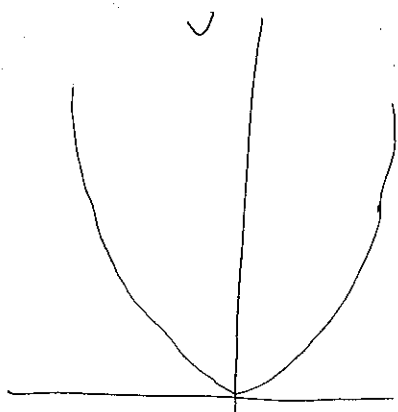
<u>Difficult Subject</u> We will build up to the full picture by going through a few toys.

<u>Toy 1</u>

$$\mathcal{L} = T - V = \tfrac{1}{2}(\partial\phi)^2 - \left(\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4\right)$$

$\lambda > 0$
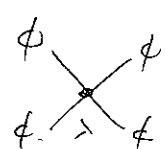
Invariant under $\phi \to -\phi$
(dropped higher order terms)

Note the mass sign comes in w/ relative "$-$" sign

(Case a) $\mu^2 > 0$

describes a scalar field w/ $m = \mu$

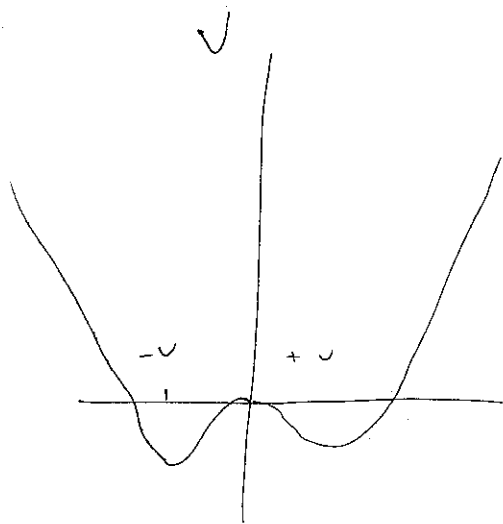$\lambda\phi^4$ leads to interaction

ground state $\phi = 0$

Case b) $\mu^2 < 0$

Now the $\mathcal{L}$ has a mass term w/ the wrong sign. (Relative "+" sign w/ kine term)

$$\frac{\partial V}{\partial\phi} = \phi(\mu^2 + \lambda\phi^2) = 0$$

$\phi = \pm v$ $v = \sqrt{-\mu^2/\lambda}$ $\to$ minimum

-v   +v

Ⓜ

Now, $\phi = 0$ does not correspond to a minimum.
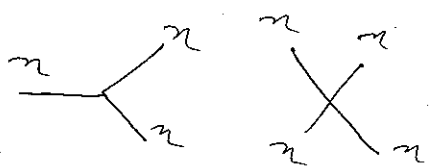Perturbative calculations should involve expansion about
minimum either $+v$ or $-v$.

Take $+v$ at random ("Spontaneously")

$$\phi(x) = v + \eta(x)$$

$\longrightarrow$ fluctuations about the min.

Can rewrite $\mathcal{L}$ as $\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{2}\lambda \eta^4 +$ consts

other terms give

$\longrightarrow$ mass term w/ correct sign.

$$m_\eta = \sqrt{-2\mu^2}$$

Mass of η was "generated" (or "revealed") as "Spontaneous symmetry Breaking"

$\phi \rightarrow -\phi$ Symmetry of $\mathcal{L}$ is broken by the choice of the ground state.

Example seen in condensed matter systems
(eg large ferromagnet. Spins will align, All directions equally likely
Ground State (spins aligned} in some direction)
breaks rotational symmetry.

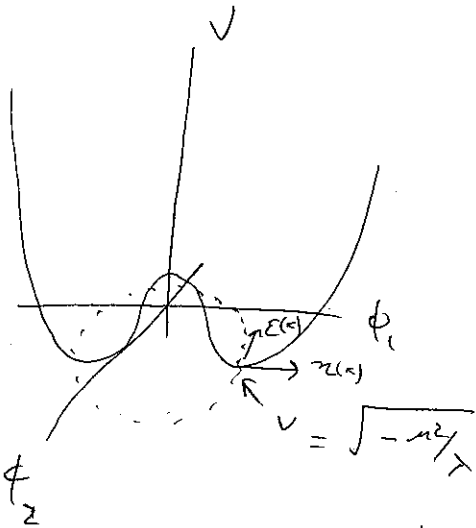- Superconductors

- Buckling of Needle.

Toy 2 Repeat above w/ complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

$\hookrightarrow$ invariant under $\phi \to e^{i\alpha}\phi$ for some # α

Global U(1) gauge symmetry $\lambda > 0, \mu^2 < 0$

$$\mathcal{L} = \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Again, need to expand about min.

<u>WOLG</u> expand around $\phi_1 = v$ $\phi_2 = 0$
(like picking +v before)

$$\phi(x) = \sqrt{\frac{1}{2}}\left(v + \eta(x) + i\,\varepsilon(x)\right)$$

$v = \sqrt{-\mu^2/\lambda}$

Get new $\mathcal{L}' = \frac{1}{2}(\partial\varepsilon)^2 + \frac{1}{2}(\partial\eta)^2 + \mu^2\eta^2 + \mathcal{O}(\eta^3) + \mathcal{O}(\eta^4)$
$$+ \mathcal{O}(\varepsilon^3) + \mathcal{O}(\varepsilon^4)$$

$\mu^2\eta^2 = -\frac{1}{2}m_\eta^2\eta$ mass term for η just as before.

However now have ε which is a <u>massless</u> scalar.
                                              "Goldstone Boson"

Problem, tried to give mass to a gauge boson
       and created a massless boson as well.

Intuitively, direction along ε is flat, $\Rightarrow$ no resistance to
       excitations along that direction. <u>Crisis</u>

Have not observed these massless gauge bosons

Lets try a local gauge theory (A miracle is about to happen ...)

Higgs Mechanism   U(1) simplest example (SM. does this in

$SU(2)_L$ getting closer)

$$\phi \to e^{i\alpha(x)}\phi \quad \text{for arbitrary function } \alpha(x)$$

As for EM $D_\mu = \partial_\mu - ieA_\mu$ ← gives coupling of ϕ to A

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\mathcal{L} = (D_\mu \phi)^*(D^\mu \phi) - \mu^2 \phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(if $\mu^2 > 0$ QED w/ massive charged scalar of mass μ w/ $\phi^\dagger X^\dagger_\phi$ term

Now for $\mu^2 < 0$, Again we do $\phi \to \frac{1}{\sqrt{2}}[v + \eta(x) + i\,\epsilon(x)]$

$$\mathcal{L}' = \frac{1}{2}(\partial\epsilon)^2 + \frac{1}{2}(\partial\eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A^2 - evA_\mu\partial^\mu\epsilon - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+ (\text{interactions})$$

3 particles (Apparently ...)

$$m_\epsilon = 0 \qquad m_\eta = \sqrt{2\lambda v^2} \qquad m_A = ev$$

↗   ↖ dynamically generated mass for gauge field

Still problem

This however can't be the spectrum B/c there are only

$~~$4 DoF before the expansion, but now seems like

$\underbrace{(2 \text{ scalar} + 2 \text{ Massless Spin 1})}$ 5 $2 + \underbrace{3}_{\to \text{ massive spin 1}}$

So some of these apparent DoF are unphysical.
(Similar to picking a gauge in QED)

Try different expansion

$$\phi \rightarrow \frac{1}{\sqrt{2}}(v + h(x)) e^{i\theta(x)/v}$$

Particular choice of gauge
← that gives h - real

$$A_\mu \rightarrow A_\mu + \frac{1}{ev}\partial\theta$$

$$\mathcal{L}'' = \frac{1}{2}(\partial h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\bullet^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2 A^2 h^2$$
$$+ v e^2 A^2 h - \frac{1}{4}F^2$$

↱ Now Goldstone boson does not
appear in the theory!

The apparent extra DoF was spurious
(only corresponds to gauge transformation)

Spectrum (Physical)

2 particles interacting massive

$$A_\mu \quad - 3 \text{ DoF massive Spin 1}$$

$$h \quad - 1 \text{ DoF massive Spin 0}$$

↳ "higgs boson"

Massless Goldstone Boson "eaten" by the A_μ to become
the extra DoF for the longitudinal polarization.
                needed
                    "Higgs Mechanism"