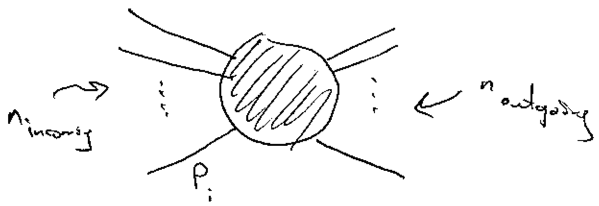
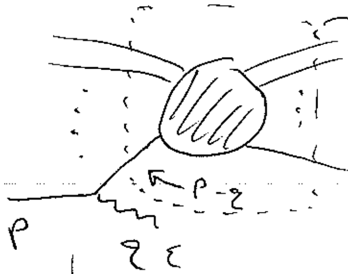
Now do the same thing to a more general interaction



$$\equiv iM_0(p_i)$$

Consider what happens if we attach "photon" to incoming leg

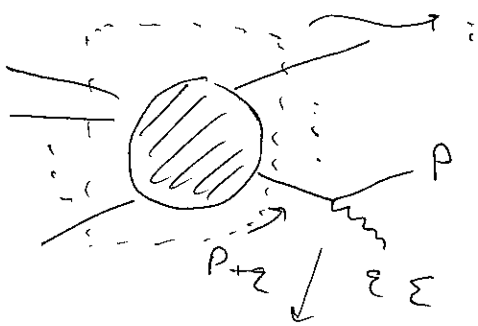


$\to iM_0(p_i - q)$

$$= Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\, iM_0(p_i - q)$$

$$i(iQ)\frac{\left(\varepsilon_\mu\left(p_i^\mu + (p_i^\mu - q^\mu)\right)\right)}{(p_i - q)^2 - m^2}$$

Can also attach photon to outgoing leg



$\to iM_0(p_i + q)$

$$= -Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\, iM(p_i + q)$$

$$(-Q)\,\frac{\varepsilon_\mu\left(p_i^\mu + (p_i^\mu + q^\mu)\right)}{(p+q)^2 - m^2}$$

Total Amplitude

$$M = \sum_{incoming} Q_i\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\, iM_0(p - q) + \sum_{outgoing} -Q_i\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\, iM(p_i + q)$$

Soft limit $\quad M_0(p \mp q) \to M_0(p)$

$$= iM_0\left(\sum_{incoming} Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)} + \sum_{outgoing} -Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\right)$$

now as before $\epsilon_\mu \to \epsilon'_\mu + q_\mu$ means that M must vanish when $\epsilon_\mu \to q_\mu$

OR under LT

$$\epsilon \cdot M \to \epsilon' \cdot M' + \underbrace{iM\left(\sum_{\text{incoming}} Q - \sum_{\text{outgoing}} Q\right)}$$

$\hookrightarrow$ $= 0$ only if $\sum_{\text{incoming}} Q = \sum_{\text{outgoing}} Q$ (charge conserved!)

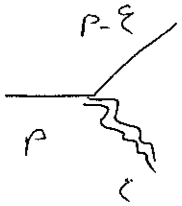
Now same logic for Spin-2 (~~...~~ describes interaction w/ Gravitons)

Same as above except 2-component polarization vector

$$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + \underbrace{\lambda_\mu q_\nu + \lambda_\nu q_\mu + \lambda q_\mu q_\nu}$$

$\nearrow$ under little group

$\hookrightarrow$ effect from all these needs be 0 as before.



$= i(iK)\,\epsilon_{\mu\nu}\dfrac{(2p^\mu p^\nu)}{-p\cdot q}$ (Same idea with outgoing leg)

$$\epsilon_{\mu\nu} \to \epsilon_{\mu\nu} + q_\mu \lambda_\nu$$

$$\epsilon_{\mu\nu} M^{\mu\nu} \to \epsilon'_{\mu\nu} M'^{\mu\nu} + M\left(\sum_{\text{incoming}} K_i \lambda_\nu p^\nu - \sum_{\text{outgoing}} K_o \lambda_\nu p^\nu\right)$$

$$+ M\lambda_\nu\left(\sum_{\text{in}} K_i p^\nu - \sum_{\text{out}} K_{o} p^\nu\right) \Rightarrow$$ $K_i p_i^\nu$ is conserved!

We know that p_i^ν is conserved by E & mom.

Only way can have nontrivial solutions is if

$$k_i = k \quad \text{for all } i$$

All particles interact w/ gravity with the same strength

Gravitational interaction is universal!

"Principle of Equivalence"

Can keep going ...

For massless spin 3 we would need

$$\sum_{\text{incoming}} \beta_i \, p_\nu^i \, p_\mu^i = \sum_{\text{outgoing}} \beta_i \, p_\nu^i \, p_\mu^i$$

eg $\mu\nu = 0$ $\quad \displaystyle\sum_{\text{incoming}} \beta_i E_i^2 = \sum_{\text{outgoing}} \beta_i E_i^2$

way too constraining only way if $\beta_i = 0$

No interacting theories of massless particles of Spin greater than 2