Homework Set #1

Due Date: Before class Friday February 4th

1) You *(2 points)*

(a) What is your major/minor ?

(b) When do you plan on graduating?

(c) What do you want to do after graduation ? (eg: grad school ? if so, what subject ? if not, what industry?)

(d) What do you most want to get out of this course ?

2) Radius of Planets *(5 points)*

(a) Calculate r_{planet} in terms of α, α_G, m_{proton}, and m_{electron}.

(b) Express your answer in terms of r_{atom}.

(c) How does this estimate compare with the radius of the earth ?

3) Solid State Physics *(5 points)*

(a) Assume a solid is composed of closely packed atoms. What is the spacing between atoms? Express your result in terms of α, α_G, m_{proton}, and m_{electron}.

(b) If you wanted to study the crystal structure of a solid material with $Z \sim 10$ using light, what wavelength of photons would you need ? Express your result in terms of α, α_G, m_{proton}, and m_{electron}.

(c) Where in the spectrum of EM radiation do these photons lie?

4) 2D Rotations *(5 points)*

(a) Show that $R(\Theta) = e^{I\Theta} = cos(\Theta) + I sin(\Theta)$ where, $I = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(b) Show that 2D rotations commute and that the multiplciation law is given by $R(\Theta_1)R(\Theta_2) = R(\Theta_1 + \Theta_2)$

(c) **Show that SO(2) $\simeq$ U(1)**. Consider the complex plane and independent variables z and z^* where $z = x + iy$ and z^* is the complex conjugate. What is zz^* in terms of x and y ? Consider the action of the operation: $z \rightarrow e^{i\theta}z, z^* \rightarrow e^{-i\theta}z^*$ Show that this action commutes and satisfies same multiplication law as we found for SO(2). The group of transformations $e^{i\theta}$ is referred to as $U(1)$, for Unitary and 1 dimensional.

5) 3D Rotations *(5 points)*

(a) Work the "algebra" of the generators of 3D rotations J_i.
Where $J_{12} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $J_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ $J_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$
Working out the algebra means calculating the commutation relations $[J_i, J_j]$.

(b) **Show that SO(3) $\simeq$ SU(2)** Let M be a traceless 2×2 hermitian matrix and U be a 2×2 unitary matrix. Write down the most general form of this matrix. Show that it can be written in the form $\vec{\sigma} \cdot \vec{r}$, where $\vec{\sigma}$ is a vector of the 2x2 pauli matrices ie $(\sigma_x, \sigma_y, \sigma_z)$. What is the determinant of M ? Now consider performing a unitary transformation $M' = U^\dagger M U$, where U is a unitrary 2x2 matrix. Show that M' is also traceless and hermitian and therefore can be also written as $\vec{\sigma} \cdot \vec{r'}$ What is the determinant of M'? Comment.

6) Lorentz Transformations *(5 points)*

In class we showed the generator for boosts (in 1D is given by) $I_{\mathrm{B}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Show that $B(\eta) = e^{\eta I_{\mathrm{B}}} = \cosh(\eta) + I_{\mathrm{B}} \sinh(\eta)$

(b) This implies

$$\begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$

Derive the relationship between $\cosh(\eta)$ and $\sinh(\eta)$ to $\beta = v$ and $\gamma = \frac{1}{\sqrt{1-v^2}}$
(Hint: consider the primed reference frame moving at velocity v with respect to the unprimed reference frame)