Can ask, Is it alive in a certain direction. the state is not an eigenstate of that operator. But we could take many copies (N) of the $|S\rangle$ state and ask the same thing. As $N \to \infty$, becomes an eigenstate of "Is alive in a certain direction" w/ eigenvalue that is the probability its moving in that direction.

Why/How the probability postulate of QM can be derived. Tells you, you have to do the experiment ∞ many times.

Exercise (Binomial expansion, Stirling approx)

$$|\Psi_n\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)^N$$

$$P_{\uparrow(\downarrow)} = \left(\frac{N_{\uparrow(\downarrow)}}{N}\right)$$

Probability Operators

$$\lim_{N\to\infty} P_\uparrow|\Psi_n\rangle = |\alpha|^2 |\Psi_n\rangle$$

Issue is identical to

my brain

$$\frac{1}{\sqrt{2}}|\uparrow\rangle|``\uparrow"\rangle|``\text{Its up!}"\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle|``\downarrow"\rangle|``\text{Its down}"\rangle$$

Why do we ~~do~~ feel like we got a definite answer even if we are in some entangled Quantum state.

Same as the is it alive question.

Now **QM + SR** "Quantum Field theory in 45 min"

Particles! Particles! Particles!

Fundamentally everything we are talking about is the interaction of Particles. No such thing as sometimes waves, sometimes Particles, its all particles. No particle-wave duality ← old fashion.

QM Particles.

Sometimes, macroscopic collections of QM particles (when they are bosons) have nice interpretation as classical waves

Macroscopic collections of fermions look like classical particles.

fields are a secondary notion. a convenient way of talking about the interactions of particles.

What particles are: Single particle states are irreducible representations of the Poincaré group.

We have symmetries, good idea to talk about what they can act on. the things they act on can be broken down into these irreducible representations.

Associated w/ any Poincaré transformation, there should be some unitary Matrix, operator that acts on the Hilbert Space of the theory.

Why do particles have something to do with the symmetries?

Momentum eigenstates behave nicely under translations.

In a world that is translationally invariant, useful to talk about momentum eigenstates.

We use the same word for particles moving in different directions even though they are different states, because they are related under rotations.

What are the labels you can have on states for which translations + rotations act nicely?

translations are labeled by $\vec{P}$, Rotations by spin.

The labels are momentum + spin. ↖ Non relativistic

We want to generalize to relativity.

translations, rotations, Boosts the whole Lorentz group

What ~~are~~ are the possible labels?

The answer is going to end up being

-) Massive: Same thing we are used to. $\vec{P}_\mu$, Spin

-) Mass-less: Always $\vec{P}_\mu$ but now labeled by helicity, not spin.

↳ Different # DoF than massive Particles.

Basic, deep + striking feature of relativistic QM. B/c I can't boost

Sketch the argument more formally.

Start w/ translations

label the states w/ $\vec{P}_\mu$

$$|p^\mu, \sigma\rangle$$

↳ what ever else they have call it σ

$x^\mu \to x^\mu + a^\mu$

$$U(T(a^\mu))\,|p^\mu, \sigma\rangle = e^{ip\cdot a}\,|p^\mu, \sigma\rangle$$

Now have to talk about how Lorentz transforms act. where things get more interesting...

for $\Lambda^\mu{}_\nu$, there is some unitary operator $U[\Lambda]$ that acts on the states.

Need to know, $U[\Lambda]\,|p, \sigma\rangle$. what can the action possibly be?

Most Nice answer, ↳ $|\Lambda p, \sigma\rangle$.

this would give action under $\Lambda^\mu{}_\nu$, but not the most general one.

Most general one.

$$U[\Lambda]\,|p, \sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(\Lambda)\,|\Lambda p, \sigma'\rangle$$

↳ Unitary matrix in σ space.

What can the D's possibly be?

Non relativistically, if the particle had spin D would be nothing other than the rotation acting on the spin. (But we don't know that yet)

In order to figure out what the D's are.

Pick some reference momentum k^μ eg: massive partcl $k^\mu = (m, 000)$

Any other momentum Λk

$$P^\mu = L^\mu{}_\nu(P)\, k^\nu$$

even the L's are my choice, Often more than one lorentz transfm you can do to get the same state. (Not a unique $L(P)$)

eg: to get from ~~one~~ k to P. I can boost to P and then do any rotation I like, ~~so~~ we just pick canonical L.

Define what I mean by $|P,\sigma\rangle \underset{\text{def}}{\equiv} \mathcal{U}(L(P))|k,\sigma\rangle$

Now defined every state in the thing.

$$\mathcal{U}(\Lambda)|P,\sigma\rangle \; ?$$

$$= \mathcal{U}(\Lambda)\,\mathcal{U}(L(P))|k,\sigma\rangle$$

(Know the ultimate momentum is ΛP. tempted to write as $\mathcal{U}(L(\Lambda p))|k,\sigma\rangle$)

$$= \left(\mathcal{U}[L(\Lambda p)]\,\mathcal{U}[L^{-1}(\Lambda p)]\right)\mathcal{U}[\Lambda]\,\mathcal{U}[L(P)]|k,\sigma\rangle$$

$\mathcal{U}$'s form a group, so $U(g_1)U(g_2) = U(g_1 g_2)$

$$= \mathcal{U}[L(\Lambda p)]\,\mathcal{U}[\underbrace{L^{-1}(\Lambda p)\,\Lambda\, L(p)}_{W(\Lambda,p)\text{ (for wigner)}}]|k,\sigma\rangle$$

What does $W(\Lambda,p)$ do to k?

Takes $k \to P \to \Lambda p \to k$.

$$\boxed{W^\mu{}_\nu(\Lambda p)\, k^\nu = k^\mu}$$

$$\mathcal{U}(W)|k,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}|k,\sigma'\rangle$$

Now have a simpler problem.

$$\mathcal{U}(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda,p))\,|\Lambda p,\sigma'\rangle$$

Nice and interesting equation. Tells us about what acts on the σ's alone. the things that act on them alone are not general lorentz transformations. The matrices D have to furnish the representations of a different group. the W's are lorentz transformations that leave k invariant

"Little Group" $W \cdot k = k$

the indices σ furnish representation of the little group.

Much simpler problem.

For massive particles, a nice choice for $k = (m, 000)$

what is the little group? Rotations: leave k invariant.

σ indices have to furnish representation of rotation group.

Massive particles are labeled by spin. Part of their label. Don't need to think of Deeper origin. Particles are just the representations.

Now lets do the more interesting

Massless case

Mass-less Case

What is the little group in this case? Harder to visualize

One obvious one, if the particle is moving along the z-dir and we rotate around the z-axis, leaves momentum invariant

Two other generators, there should be 3.

- 3 that left massive particles invariant.
- This number cannot change discontinuously.

Literal analog between Galilean relativity + SR.

- The Galilean group has just as many generators as Lorentz group
- Galilean group is deformation of Lorentz group in limit $c \to \infty$

What might this be?

When a particle is massive its rotations, which is like taking a sphere and rotating it in any way.

What contraction might such a symmetry have? $R \to \infty$

full rotations $\to$ rotations about z, $\underbrace{\text{translations } x + y}_{\text{commute, full rotations do not.}}$

Lets find the generators

$$\omega^\mu{}_\nu = \delta^\mu{}_\nu + \varepsilon \omega^\mu{}_\nu$$

want $\omega^\mu{}_\nu k^\nu = k^\mu \Rightarrow \omega^\mu{}_\nu k^\nu = 0$

most general $\omega^\mu{}_\nu$ of Lorentz transformations

$$\omega^\mu{}_\nu = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & a & b & c \\ 1 & a & 0 & A & B \\ 2 & b & -A & 0 & C \\ 3 & c & -B & -C & 0 \end{array}$$

want special ones that annihilate k

$$k = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\omega^\mu{}_\nu = \begin{bmatrix} 0 & 0 & A & B \\ 0 & 0 & A & B \\ A & -A & 0 & C \\ B & -B & -C & 0 \end{bmatrix}$$

most general ω for massless particles.

Once again there are 3 generators.

- C is rotations about the 1 direction
- A - Boost in y followed by rotation, bring it back

same for B.

J_{23} $\quad T_2 \quad T_3$

$$\begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&-1 \\ 0&0&1&0 \end{bmatrix}$$

exercise, work out the commutation relations between these guys.

what you'll find:

$$[T_2, T_3] = 0$$
$$[T_2, J_{23}] = T_3$$
$$[T_3, J_{23}] = -T_2$$

Exactly what you expect for trans + rotation in a plane

$\boxed{E(2)}$

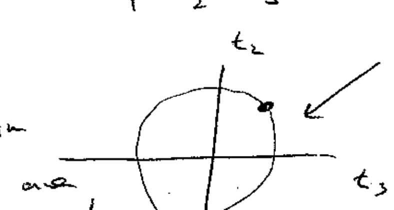
How does this group act on particles?

Need to come up w/ representations. $T_2 + T_3$ commute so we could label states by their eigenvalues

$$|t_2, t_3\rangle$$

Small problem if that is a state and I act on it with J

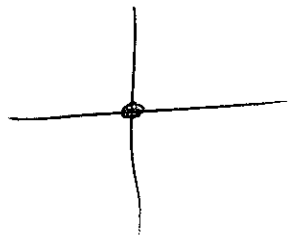
$$e^{i\theta J_{23}} |t_2, t_3\rangle = |t_2' \, t_3'\rangle$$



Can get a continuous ∞ of states $t_2 + t_3$

In other words, if one state in ... there are ... of massless particals

Declare that we are only interested in states w $t_2, t_3 = 0$



what do I label those states by?

Only thing left is eigenvalue J_{23}.

Massless particles only labeled by the spin in the direction of motion.

Only labeled by helicity.

Summary

Massive:

rotation

$|p,\sigma\rangle \qquad U(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} R_{\sigma\sigma'} |\Lambda p, \sigma'\rangle$

Massless

one #

Just a phase

$|p,h\rangle \qquad U(\Lambda)|p,h\rangle = e^{ih\Theta(w)} |\Lambda p\, h\rangle$

Introduced the cast of characters.

(4b) Talk about all these particle states in a more convenient way.

For every given momenta we have stacks of Hilbert Space

- ∞ many possibilities for Bosons $0 \to \infty$
- fixed # depending on the Spin for fermions.

Ultimately interested in interactions between particles.

- Defined by some hamiltonian.
- Could specify the ~~hat~~ hamiltonian by describing how it acts on all states in the hilbert space.
- Instead for our convenience introduce creation + annihilation operators.
  - ↳ keeps track of states in a simple way.

vacuum state

$|0\rangle$

Those are primary

$|p,\sigma\rangle \equiv a^{+}_{p\sigma} |0\rangle$

⟶ defines a^{+}

$|p_1\sigma_1, p_2\sigma_2\rangle = a^{+}_{p_1\sigma_1} a^{+}_{p_2\sigma_2} |0\rangle$

⋮

$a_{p\sigma}|0\rangle = 0$

a – removes states.

Can encode boson/fermion statistics in ~~a~~ a + a^{+}.

$[a^{+}_{p_1\sigma_1}\ a^{+}_{p_2\sigma_2}] = 0$ Bosons

$\{a^{+}_{p_1\sigma_1}, a^{+}_{p_2\sigma_2}\} = 0$ Fermions

$[a \quad a] = 0 \qquad \{a \quad a\} = 0$