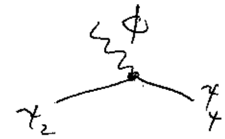
$$T_{fi} = \frac{\langle \psi^3 \psi^4 | V | \psi^3 \phi_\gamma \psi^2 \rangle \langle \psi^3 \phi_\gamma \psi^2 | V | \psi^1 \psi^2 \rangle}{(E_1 + E_2) - (E_3 + E_4 + E_\gamma)} + 2^{nd} \text{ term}$$

$E_n \neq E_i$ allowed by uncertainty principle QM.

$$\langle \psi^3 \psi^4 | V | \psi^3 \phi \psi^2 \rangle = \langle \psi^4 | V | \phi \psi^2 \rangle$$



$$\boxed{\langle \phi^\gamma | \phi(x) | 0 \rangle = e^{-i \vec{p}_\gamma \cdot \vec{x}}}$$

$$= \frac{e}{2} \int d^3x \, \langle \psi^2 \phi | \psi_e(x) \phi(x) \psi_e(x) | \psi^4 \rangle$$

$$= e \int d^3x \, e^{i(p_4 - p_2 - p_\gamma)x} = e(2\pi)^3 \delta(p_4 - p_2 + p_\gamma)$$

Other product

$$\langle \psi^3 \phi | V | \psi^1 \rangle = \cancel{e \int d^3x} \; e(2\pi)^3 \delta(\vec{p}_1 - \vec{p}_3 - p_\gamma)$$

$$T_{fi}^{(1)} \sim \int d^3 p_\gamma \, \delta \, \delta \, \frac{e^2}{E_i - E_n} \qquad E_n = E_3 + E_2 + E_\gamma$$

$E_i \hookrightarrow E_1 + E_2$

$$T_{fi}^{(1)} \approx \frac{e^2}{(E_1 + E_2) - (E_3 + E_2 + E_\gamma)} = \frac{e^2}{(E_1 - E_3) - E_\gamma}$$

Same logic

$$T^{(2)} \sim \frac{e^2}{(E_2 - E_4) - E_\gamma}$$

End of day, need to add processes

$$E_1 + E_2 = E_3 + E_4$$

$$E_1 - E_3 = E_4 - E_2 = \Delta E$$

$$T^{(1)} + T^{(2)} = \frac{e^2}{\Delta E - E_\gamma} + \frac{e^2}{-\Delta E - E_\gamma} = \frac{2 e^2 E_\gamma}{(\Delta E)^2 - E_\gamma^2}$$

define $k^\mu \equiv p_3^\mu - p_1^\mu = (\Delta E, \vec{P}_\gamma)$

← Not the photon momentum!

$k^2 \neq 0$

$= (\Delta E)^2 - E_\gamma^2$

$$T_{fi} = \underline{\underline{2E_\gamma}}\left(\frac{e^2}{k^2}\right)$$

Related to normalization

Summary OFPT

- All states are physical (on-shell)
- Matrix Element V_{ij} 0 unless 3-momentum conserved
- Energy Not conserved @ each vertex

Feynman Rules

- Internal lines get "propagators" $\dfrac{i}{p^2 - m^2 + i\varepsilon}$
- Vertices come from interactions in the Lagrangian. They get factors of the coupling constant times i
- Lines connected to external points do not get propagators (Scalars get $\times 1$ / Spinor by u + v / ~~Sp~~ Spin 1 by ε ε^*)
- 4-momenta is conserved @ each vertex
- Integrate over all undetermined 4-mom.
- Sum over all possible diagrams.

Example: $\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3$

Consider cross-section for $\phi\phi \to \phi\phi$ scattering.

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|}|M|^2\, d\Pi_{LIPS} \qquad P_1 + P_2 \to P_3 + P_4$$

In COM frame, $\vec{P}_1 = -\vec{P}_2 \quad \vec{P}_3 = -\vec{P}_4 \qquad E_1 + E_2 = E_3 + E_4 \equiv E_{cm}$

$$d\Pi_{LIPS} = (2\pi)^4\delta^4(\Sigma p)\frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2E_4}$$

Integrating over $\vec{P}_4$

$$= \frac{1}{16\pi^2}d\Omega\int dp_f\,\frac{p_f^2}{E_3}\frac{1}{E_4}\delta(E_3 + E_4 - E_{cm})$$

$P_f = |\vec{P}_3| = |\vec{P}_4| \qquad E_3 = \sqrt{m_3^2 + p_f^2} \ne E_4 \qquad \int d^3p_3 = \int dp_f\, p_f^2\, d\Omega$

Now $\quad p_f \to x = E_3 + E_4 - E_{cm}$

$$dx = \frac{d}{dp}(E_3 + E_4 - E_{cm})\,dp_f = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{\overbrace{E_3 + E_4}^{E_{cm}}}{E_3E_4}\,p_f\, dp_f$$

$$\Rightarrow \quad \frac{dp_f\, p_f^2}{E_3E_4} = \frac{dx\, p_f}{E_{cm}}$$

$$d\Pi_{LIPS} = \frac{1}{16\pi^2}d\Omega\int_{m_3+m_4-E_{cm}}^{\infty}dx\,\frac{p_f}{E_{cm}}\delta(x) = \frac{1}{16\pi^2}d\Omega\,\frac{p_f}{E_{cm}}\begin{cases}\text{if } E_{cm} > m_3 + m_4 \\ 0 \text{ otherwise}\end{cases}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|}\frac{1}{16\pi^2}d\Omega\,\frac{p_f}{E_{cm}}|M|^2$$

$$|v_1 - v_2| = \left|\frac{|\vec{P}_1|}{E_1} + \frac{|\vec{P}_2|}{E_2}\right| = P_i\frac{E_{cm}}{E_1E_2} \qquad \boxed{|\vec{P}_i| = |\vec{P}_1| = |\vec{P}_2|}$$