

Homework Set #4

Due Date: Before class Friday February 15th

1) Find the generators of the “Little Group” for Massive particles *(5 points)*

2) Heisenberg Equation of Motion *(5 points)*

- a) Show that $\frac{dA(t)_H}{dt} = -i[A(t)_H, H]$. Where $A(t)_H$ is an operator in the Heisenberg representation and H is the Hamiltonian. This equation is referred to as the Heisenberg equation of motion.
- b) Show that $\phi_H(x, t) = e^{-iE_p t} \phi_S(x)$ satisfies the Heisenberg equation of motion. Where $\phi_S(x) = \int d\vec{p} e^{i\vec{p} \cdot \vec{x}} a^\dagger$ is the operator in the Schrodinger representation and $H = \int dp E_p a^\dagger a$

3) Show that $\int d^3\vec{p} \equiv \int \frac{d^3\vec{p}}{2E_p}$ is Lorentz invariant. *(2 points)*

(Hint: $\int d^4p \delta(E^2 - (|\vec{p}|^2 + m^2))$ is clearly Lorentz invariant.)

4) Anti-Particles *(5 points)*

- a) Expand $\Phi^{\dagger 2} \Phi^2$ in terms of a, a^\dagger, b , and b^\dagger (Ignore the exponentials and integrals)
- b) Sketch diagrams of the processes that each term corresponds to.
- c) Let the charge (Q) of particle a be q_a and the charge of particle b be q_b . Calculate ΔQ for each process.
- d) What happens if you take $q_a = -q_b$?