Homework Set #8

1) Z boson decays: *(5 points)*

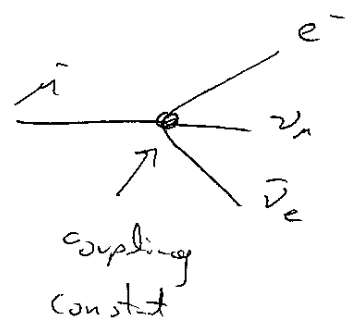
We assumed that the Z-couplings were universal, that the phase space intergrals were the same for all decay products, and that no higher-order diagrams were relevant. (The phase space intergrals will be the same if we can neglect the decay products masses.)

$$Br(Z \to ee) \sim \frac{1}{21} = 0.048 \text{ vs } 0.034 \text{ in PDG}$$

$$Br(Z \to bb) \sim \frac{3}{21} = 0.143 \text{ vs } 0.156 \text{ in PDG}$$

2) Muon decays: *(10 points)*

a)



b) We have 4-bosons (each of dim 3/2) and the coupling constant. The total dimensions have to add up to 4.

$$4 \times \frac{3}{2} + [\text{coupling constant}] = 4$$

$\Rightarrow$

$$[\text{coupling constant}] \sim -2 \text{ or GeV}^{-2}$$

c)

$$\Gamma \sim |M|^2 \sim [\text{coupling constant}]^2 = \text{GeV}^{-4}$$

But we also know that Γ has to come out to have overall dimensions of $\frac{1}{\text{time}}$ or GeV.

$\Rightarrow$

$$\Gamma \sim m_\mu^5$$

d) $m_\mu \sim 0.1\text{GeV}$, $m_\tau \sim 1\text{GeV}$, $\tau_\mu \sim 1\mu s$

Now from c)

$$\Gamma_\tau \sim m_\tau^5$$

and we know

$$\tau_\mu = \Gamma_\mu^{-1}$$
$$\tau_\tau = \Gamma_\tau^{-1}$$

so,

$$\frac{\tau_\tau}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_\tau}$$

$\Rightarrow$

$$\tau_\tau = \tau_\mu \frac{\Gamma_\mu}{\Gamma_\tau} = \tau_\mu \frac{m_\mu^5}{m_\tau^5} = \tau_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 = 1\mu s(10^{-1})^5 = 10^{-6}s \times 10^{-5} = 10^{-11}s$$

e) with a direct three-point $\mu \to e\gamma$ vertex, the only mass scale is m_μ. (b/c the $(\mu e\gamma)$- coupling is dimensionless)

So, $\Gamma_{\mu\to e\gamma} \sim m_\mu$ (to get the dimensions on Γ right)

We know from above that with the four-point interaction in Fermi theory $\Gamma_{SM} \sim m_\mu^5 m_W^{-4}$

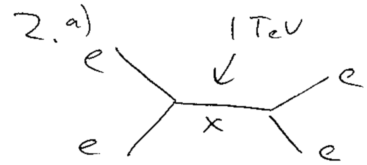
So,

$$\frac{\tau_{new}}{\tau_{SM}} \sim \frac{m_\mu^5 m_W^{-4}}{m_\mu} \sim \left(\frac{m_\mu}{m_W}\right)^4 \sim \left(\frac{0.1\text{ GeV}}{100\text{ GeV}}\right)^4 \sim 10^{12}$$

The direct $\mu \to e\gamma$ would dominate (by a factor 10^{12}!)
The weak interaction is damned weak.

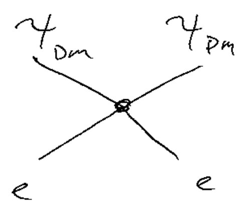
3) A new force. *(5 points)*

a)



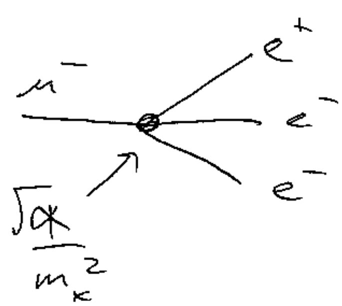
Range $\sim \frac{1}{m_X} \sim \frac{1}{1000\text{GeV}} \sim 10^{-3}\ \text{GeV}^{-1} \sim 10^{-19}\ m$

b)



Four fermion interaction $\Rightarrow$ Units of coupliing $\text{GeV}^{-2} \sim \frac{1}{m_X^2}$

c)



$$\Gamma_{\text{New}} \sim \frac{m_\mu^5}{m_X^4} \qquad \text{(see problem 2 for login on why } m_\mu^5\text{)}$$

$$\Gamma_{\text{SM}} \sim \frac{m_\mu^5}{m_W^4} \qquad \text{(from problem 2)}$$

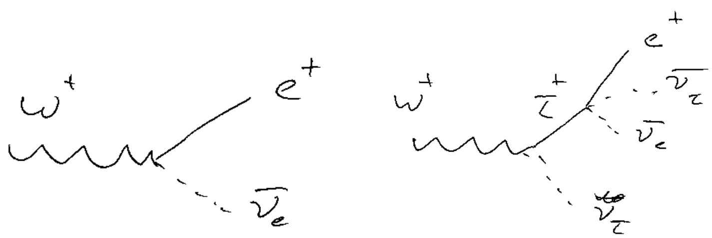
So,

$$\frac{\tau_{\text{New}}}{\tau_{\text{SM}}} \sim \left(\frac{m_X}{m_w}\right)^4 \sim 10^4$$

$\Rightarrow$ SM decays dominate!

4) *W* boson decays to electrons. *(3 points)*

a) The W can decay directly to an electron or to electron by decaying through a τ. Draw the corresponding diagrams.



b)

$$Br(W \to \ell\nu) = \frac{1}{\underbrace{3}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{2 quark generations}}} = \frac{1}{9} = 0.11$$

(Note the top-quark is heavier than the W, so the $W \to t, b$ decay is forbidden.)

Now,

$$Br(\tau \to e\nu\bar{\nu}) = \frac{1}{\underbrace{2}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{1 quark generations}}} = \frac{1}{5} = 0.2$$

Here the τ can decay to two lepton generation (es and μ s), but only has enough mass to decay to one quark generation (u, d).

So,

$$Br(W \to e + X) = \underbrace{\frac{1}{9}}_{W\to e\nu} + \underbrace{\frac{1}{9}}_{W\to\tau\nu} \times \underbrace{\frac{1}{5}}_{\tau\to e\nu} = \frac{1.2}{9} = 0.13$$

4) *W* boson decays to electrons. *(3 points)*

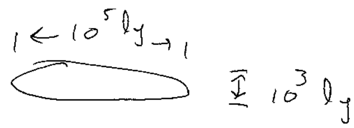
$$Br(W \to \ell\nu) \sim \frac{1}{9}$$

for e or μ can include the decays through τs as in problem 3 to get $\frac{1.2}{9}$

$$Br(WW \to e\mu + X) = 2 \times \left(\frac{1.2}{9}\right)^2 \sim 0.036$$

Factor of two becuase you can get $e^+\mu^-$ or $e^-\mu^+$

6) Galactic Collisions. *(10 points)*



a) $N_{star} \sim 10^{11}$ $\qquad v_{rel} \sim 10^5 m/s$

$$\mathcal{L}_{galaxy} = \frac{N_A N_B |v_A - v_B|}{volume} \sim \frac{10^{22}\ 10^5\ m/s}{volume}$$

$$volume = \pi(10^5/2\ ly)^2 \times 10^3\ ly = \frac{\pi}{4} 10^{13} (ly)^3 \sim 10^{13} (3.8\ m/s\ \pi 10^7\ s)^3 = 10^{61}\ m^3$$

So,

$$\mathcal{L}_{galaxy} \sim \frac{10^{27}\ m/s}{10^{61}\ m^3} \sim 10^{-34} \frac{1}{m^2} \frac{1}{s} = 10^{-38} \frac{1}{cm^2} \frac{1}{s}$$

$$\mathcal{L}_{LHC} \sim 10^{34} \frac{1}{cm^2} \frac{1}{s}$$

So ~ 72 orders of magnitude smaller !

b) In a galaxy,

$$\frac{volume}{star} \sim \frac{10^{61}\ m^3}{10^{11}} \sim 10^{50}\ m^3$$

$$\frac{distance}{star} \sim \left(\frac{volume}{star}\right)^{\frac{1}{3}} \sim 10^{50/3}\ m$$

So,

$$\frac{\text{<distance to star>}}{< R_* >} \sim \frac{10^{50/3}\ m}{7 \cdot 10^8\ m} \sim 10^8$$

In a proton bunch (with focusing magnets) at the LHC,

$$\frac{volume}{proton} \sim \frac{10^{-10}\ m^3}{10^{11}} \sim 10^{-21}\ m^3$$

So

$$\frac{distance}{proton} \sim \left(\frac{volume}{proton}\right)^{\frac{1}{3}} \sim 10^{-7}\ m$$

$$\frac{\text{<distance to proton>}}{< R_p >} \sim \frac{10^{-21}\ m}{10^{-15}} \sim 10^{8}$$

Which is quite close !

c) For the galaxy,

$$N_{\text{collisions}} = \int dt \mathcal{L} \times \sigma$$

Now,

$$\int dt = 10^{9}\ y \times (\pi 10^{7})\ y/s$$
$$\mathcal{L} = 10^{-34}\ m^{-2} s^{-1}$$
$$\sigma = \pi (7 \cdot 10^{8}\ m)^{2}$$

$\Rightarrow$ N-collisions $\sim$ 1.

The LHC has baout $\sim$ 10 proton collisions per bunch crossing.

(Not so different!)