

Homework Set #8

Due Date: Friday March 27th

1) Z boson decays:

(5 points)

We assumed that the Z-couplings were universal, that the phase space integrals were the same for all decay products, and that no higher-order diagrams were relevant. (The phase space integrals will be the same if we can neglect the decay products masses.)

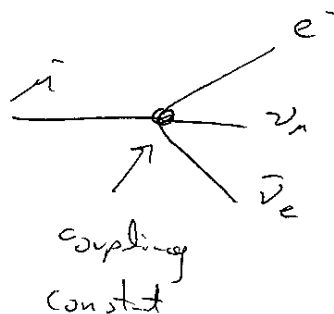
$$Br(Z \rightarrow ee) \sim \frac{1}{21} = 0.048 \text{ vs } 0.034 \text{ in PDG}$$

$$Br(Z \rightarrow bb) \sim \frac{3}{21} = 0.143 \text{ vs } 0.156 \text{ in PDG}$$

2) Muon decays:

(10 points)

a)



b) We have 4-bosons (each of dim $3/2$) and the coupling constant. The total dimensions have to add up to 4.

$$4 \times \frac{3}{2} + [\text{coupling constant}] = 4$$

\Rightarrow

$$[\text{coupling constant}] \sim -2 \text{ or } \text{GeV}^{-2}$$

c)

$$\Gamma \sim |M|^2 \sim [\text{coupling constant}]^2 = \text{GeV}^{-4}$$

But we also know that Γ has to come out to have overall dimensions of $\frac{1}{\text{time}}$ or GeV .

\Rightarrow

$$\Gamma \sim m_\mu^5$$

d) $m_\mu \sim 0.1\text{GeV}$, $m_\tau \sim 1\text{GeV}$, $\tau_\mu \sim 1\mu s$

Now from c)

$$\Gamma_\tau \sim m_\tau^5$$

and we know

$$\begin{aligned}\tau_\mu &= \Gamma_\mu^{-1} \\ \tau_\tau &= \Gamma_\tau^{-1}\end{aligned}$$

so,

$$\frac{\tau_\tau}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_\tau}$$

\Rightarrow

$$\tau_\tau = \tau_\mu \frac{\Gamma_\mu}{\Gamma_\tau} = \tau_\mu \frac{m_\mu^5}{m_\tau^5} = \tau_\mu \left(\frac{m_\mu}{m_\tau} \right)^5 = 1\mu s (10^{-1})^5 = 10^{-6}s \times 10^{-5} = 10^{-11}s$$

e) with a direct three-point $\mu \rightarrow e\gamma$ vertex, the only mass scale is m_μ . (b/c the $(\mu e\gamma)$ - coupling is dimensionless)

So, $\Gamma_{\mu \rightarrow e\gamma} \sim m_\mu$ (to get the dimensions on Γ right)

We know from above that with the four-point interaction in Fermi theory $\Gamma_{SM} \sim m_\mu^5 m_W^{-4}$

So,

$$\frac{\tau_{new}}{\tau_{SM}} \sim \frac{m_\mu^5 m_W^{-4}}{m_\mu} \sim \left(\frac{m_\mu}{m_W} \right)^4 \sim \left(\frac{0.1 \text{ GeV}}{100 \text{ GeV}} \right)^4 \sim 10^{12}$$

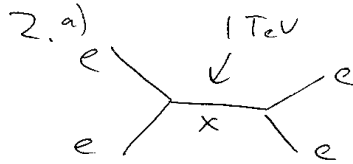
The direct $\mu \rightarrow e\gamma$ would dominate (by a factor 10^{12} !)

The weak interaction is damned weak.

3) A new force.

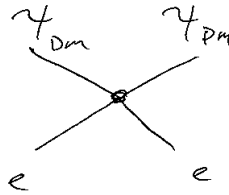
(5 points)

a)



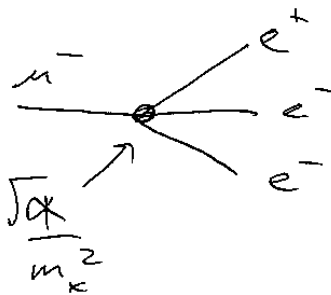
$$\text{Range} \sim \frac{1}{m_X} \sim \frac{1}{1000 \text{ GeV}} \sim 10^{-3} \text{ GeV}^{-1} \sim 10^{-19} \text{ m}$$

b)



Four fermion interaction \Rightarrow Units of coupling $\text{GeV}^{-2} \sim \frac{1}{m_X^2}$

c)



$$\Gamma_{\text{New}} \sim \frac{m_\mu^5}{m_X^4} \quad (\text{see problem 2 for log on why } m_\mu^5)$$

$$\Gamma_{\text{SM}} \sim \frac{m_\mu^5}{m_W^4} \quad (\text{from problem 2})$$

So,

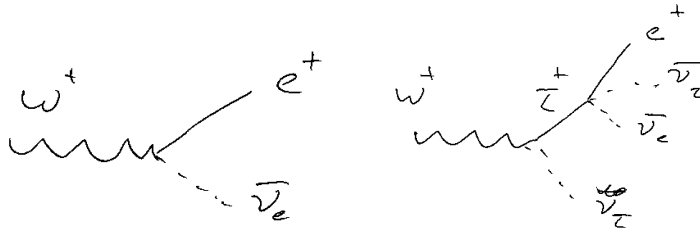
$$\frac{\tau_{\text{New}}}{\tau_{\text{SM}}} \sim \left(\frac{m_X}{m_w}\right)^4 \sim 10^4$$

\Rightarrow SM decays dominate!

4) W boson decays to electrons.

(3 points)

- a) The W can decay directly to an electron or to electron by decaying through a τ . Draw the corresponding diagrams.



b)

$$Br(W \rightarrow \ell \nu) = \frac{1}{\underbrace{3}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{2 quark generations}}} = \frac{1}{9} = 0.11$$

(Note the top-quark is heavier than the W, so the $W \rightarrow t, b$ decay is forbidden.)

Now,

$$Br(\tau \rightarrow e \nu \bar{\nu}) = \frac{1}{\underbrace{2}_{\text{leptons}} + \underbrace{3}_{\text{color}} \times \underbrace{2}_{\text{1 quark generations}}} = \frac{1}{5} = 0.2$$

Here the τ can decay to two lepton generation (es and μ s), but only has enough mass to decay to one quark generation (u, d).

So,

$$Br(W \rightarrow e + X) = \underbrace{\frac{1}{9}}_{W \rightarrow e \nu} + \underbrace{\frac{1}{9}}_{W \rightarrow \tau \nu} \times \underbrace{\frac{1}{5}}_{\tau \rightarrow e \nu} = \frac{1.2}{9} = 0.13$$

4) W boson decays to electrons.

(3 points)

$$Br(W \rightarrow \ell \nu) \sim \frac{1}{9}$$

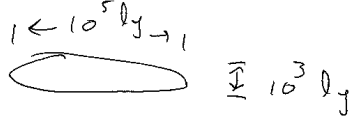
for e or μ can include the decays through τ s as in problem 3 to get $\frac{1.2}{9}$

$$Br(WW \rightarrow e\mu + X) = 2 \times \left(\frac{1.2}{9}\right)^2 \sim 0.036$$

Factor of two because you can get $e^+\mu^-$ or $e^-\mu^+$

6) Galactic Collisions.

(10 points)



a) $N_{star} \sim 10^{11}$

$$v_{rel} \sim 10^5 m/s$$

$$\mathcal{L}_{galaxy} = \frac{N_A N_B |v_A - v_B|}{volume} \sim \frac{10^{22} 10^5 m/s}{volume}$$

$$volume = \pi(10^5/2 ly)^2 \times 10^3 ly = \frac{\pi}{4} 10^{13} (ly)^3 \sim 10^{13} (3.8 m/s \pi 10^7 s)^3 = 10^{61} m^3$$

So,

$$\mathcal{L}_{galaxy} \sim \frac{10^{27} m/s}{10^{61} m^3} \sim 10^{-34} \frac{1}{m^2} \frac{1}{s} = 10^{-38} \frac{1}{cm^2} \frac{1}{s}$$

$$\mathcal{L}_{LHC} \sim 10^{34} \frac{1}{cm^2} \frac{1}{s}$$

So ~ 72 orders of magnitude smaller !

b) In a galaxy,

$$\frac{volume}{star} \sim \frac{10^{61} m^3}{10^{11}} \sim 10^{50} m^3$$

$$\frac{distance}{star} \sim \left(\frac{volume}{star}\right)^{\frac{1}{3}} \sim 10^{50/3} m$$

So,

$$\frac{\langle distance \text{ to star } \rangle}{\langle R_* \rangle} \sim \frac{10^{50/3} m}{7 \cdot 10^8 m} \sim 10^8$$

In a proton bunch (with focusing magnets) at the LHC,

$$\frac{volume}{proton} \sim \frac{10^{-10} m^3}{10^{11}} \sim 10^{-21} m^3$$

So

$$\frac{\text{distance}}{\text{proton}} \sim \left(\frac{\text{volume}}{\text{proton}} \right)^{\frac{1}{3}} \sim 10^{-7} \text{ m}$$

$$\frac{\langle \text{distance to proton} \rangle}{\langle R_p \rangle} \sim \frac{10^{-21} \text{ m}}{10^{-15}} \sim 10^8$$

Which is quite close !

c) For the galaxy,

$$N_{\text{collisions}} = \int dt \mathcal{L} \times \sigma$$

Now,

$$\int dt = 10^9 \text{ y} \times (\pi 10^7) \text{ y/s}$$

$$\mathcal{L} = 10^{-34} \text{ m}^{-2} \text{ s}^{-1}$$

$$\sigma = \pi (7 \cdot 10^8 \text{ m})^2$$

\Rightarrow N-collisions ~ 1 .

The LHC has about ~ 10 proton collisions per bunch crossing.

(Not so different!)