Homework Set Branching Ratios

Due Date: Friday March 25th

0) Read Chapter 4, Sections 4.1 and 4.2 *(3 points)*
What physicical quantity is used to characterize the probability of particles interacting? What is the dimension of this quantity ? How does this quantity scale with the Matrix element calculated from Feynment Diagrams?

1) Z boson decays: *(5 points)*
In class we estimated the branching ratio of $Z \to ee$. Repeat this calculation. Calculate the branching ratio of $Z \to bb$ decays. What approximations where made ? Compare this rough estimate to the experimentally measured values using the Particle Data Group's (PDG) Review of Particle Physics. You can find it at http://pdg.lbl.gov. All measured properties of all known particles are recorded here.

2) Muon decays: *(10 points)*

a) The muon decays via the weak interaction, At low energy ($E << m_W$), this can be approximated as a point-like interaction. Draw the diagram describing muon decay to an electron assuming a point-like weak interaction.

b) What are the dimensions of the coupling constant, associated to this diagram ?

c) How does the decay rate Γ (decays/unit time) depend on the muon mass ?

d) The muon has a mass of ~0.1 GeV and a lifetime of $\sim 1\mu s$. The tau lepton has a mass of ~1 GeV. Estimate the lifetime of the tau lepton in μs.

e) Suppose that the photon could couple at the same vertex to the muon and the electron. Then the muon could decay as $\mu \to e\gamma$. Estimate the ratio of the μ lifetime in this world to that in our world without this interaction.

3) A new force. *(7 points)*
Assume there is another force of nature felt by electrons associated with the exchange of a new X boson of mass 1 TeV (= 1000 GeV).

a) Estimate an upper limit on the range of this new force in meters.

b) Assume that this new X boson could also decay to spin-1/2 dark matter particles ψ_{DM}. At low energies (« 1 TeV) the X interaction can be described by a point-like interaction. Estimate the coupling constant associated to dark matter scattering $e\psi_{DM} \to e\psi_{DM}$. (Assume the X coupling at high energies is the same as for EM)

c) Assume there is also a direct $X \to e\mu$ interaction. This would allow the muon to decay via $\mu^- \to e^-e^+e^-$. Draw the corresponding diagram and estimate the impact of the muon lifetime from this process. How does it compare to the lifetime in the standard model?