ψ - complex spinor Use $\bar{\psi}$ for complex conjugate

$$S[\bar{\psi},\psi] = \int d^4x \, \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi$$

Varying action w.r.t. $\bar{\psi}$ gives dirac eq.

Now Spin 1

EM first LI theory.

Maxwells Equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times B - \frac{\partial E}{\partial t} = \vec{J}$$

$$\nabla \cdot B = 0 \qquad \vec{\nabla}\times E + \frac{\partial B}{\partial t} = 0$$

Not manifestly L.I.

Can define $J^\mu = (\rho, \vec{J})$ E & B 6 - components

$F_{\mu\nu} = -F_{\nu\mu}$ has $16 - 4 - 6 = 6$ components

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & -B_z & B_y \\ " & & 0 & B_x \\ & & & 0 \end{pmatrix} \qquad F_{\mu\nu} \to \Lambda_\rho{}^\mu \Lambda_\sigma{}^\nu F_{\rho\sigma}$$

$$\partial_\mu F^{\mu\nu} = J^\nu$$ L.I. "eq of motion" Maxwells eq's

Can also express E & B in terms of potentials

$$\vec{E} = \vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

Can use potentials defined up to "gauge transformation"

$$V \to V + \frac{\partial \lambda}{\partial t} \qquad \vec{A} \to \vec{A} + \vec{\nabla}\lambda$$ give same physics

Introduce

$$A_\mu = (V, \vec{A}) \qquad A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \left\{ \begin{array}{l} \text{Note that } F_{\mu\nu} \text{ is invariant} \\ \text{to gauge transformations.} \end{array} \right.$$

Gauge transformations <u>not</u> deep. More of a redundancy not transformation

Really saying that

$$A_\mu \quad \& \quad A_\mu + \partial_\mu \lambda$$

Represent the same state.

2nd Requirement
-1) L.I.
-2) "Gauge Invariance"

$$\mathcal{L}_{E\&M} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu$$

<u>Photon equation of motion</u>

$\boxed{\text{HW: Check gauge Invariance}}$

Almost always most useful to think in terms of A_μ directly.

A_μ "photon field" directly describes photon D.o.F.
boson

Assume no sources $J_\mu = 0$

$$\partial_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad \partial^2 A_\mu - \partial_\mu \partial \cdot A = (\eta_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) A^\nu = 0$$

Ansatz

$$A_\mu = \varepsilon_\mu(p) e^{-ip\cdot x} \qquad \varepsilon(p) \text{ - polarization vector}$$

eq motion $\Rightarrow$ $(\eta_{\mu\nu} \cancel{p^2}^{\,0} - p_\mu p_\nu) \varepsilon^\nu = 0$

or $\quad p \cdot \varepsilon = 0$

$P = (E, 0, 0, E)$ <u>WLOG</u>

$\varepsilon(P) = (a, b, c, a)$ ← Most general form

Convient to express b & c as

$\varepsilon_R(p) = \frac{1}{\sqrt{2}}(0, 1, i, 0)$ $\quad\quad$ $\varepsilon_L(p) = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$

$\varepsilon_R \cdot \varepsilon_L = -1$ $\quad\quad$ $\varepsilon_R^2 = \varepsilon_L^2 = 0$

What about a? (Note: only expect 2 DoF from Little group)

Gauge invariance $\Rightarrow$ a is non-physical.

$A_\mu = 0$ is <u>the same</u> as $A_\mu = \partial_\mu \lambda$ ← Any λ

↳ direction along p_μ

<u>Lagrangian Terms</u> (dont usually talk about K.E. P.E.)

<u>Kinetic Terms</u> (bilinear - exactly two fields)

$\partial_\mu \phi \partial^\mu \phi$, $\bar{\psi} \not{\partial} \psi$, $\frac{1}{4} F_{\mu\nu}^2$ $\quad \partial_\mu \phi_1 \partial^\mu \phi_2$, ...

<u>Interactions</u> (3 - or more fields)

$\lambda \phi^3$ $\quad g \bar{\psi} \gamma_\mu A^\mu \psi$ $\quad g \partial_\mu \phi A^\mu \phi$ $\quad y^2 A_\mu^2 A_\nu^2$ ...

4b) Start off doing a little dimensional analysis that will turn out to have shockingly large ramifications.

- Do the dim. analysis
- Draw very nice conclusions

} Skips ~~over~~ 40 years of massive confusion in the field
late 30's — late 60's

The infinities that emerge when you actually do calculations, clouded these results, turns out to all be a red hering.

Correct way of thinking about it is "Wilsonian Way" EFTs

There is something deep underneath that legitimizes the nice answer, but it turns out the simple answer is correct.

One of the most important things thats happened in physics in the last 50 years is understanding this deeper way of thinking about QFT.

"Could tell you the words, but wont appriciate until you suffer the normal bad way of talking about it"

What are the units of the feilds we just talked about?

- Easiest way of talking about this is using the Lagrangian formalism, but B/C we didnt talk about that its more complicated.

$$H = \int \frac{d^3p}{(2\pi)^3} E_p a_p^+ a_p$$

round-off units of a

a - units E^{-2}

$$\phi_+ = \int d^3p \; a_p^+ e^{ipx}$$

ϕ_+ - has mass dimension 1

The mass dimension of ϕ is telling you about ~~its~~ the size of its fluctuations.

For positive mass dimension the fluctuations in ϕ get smaller & smaller @ larger distances, Conversely they get larger & larger @ smaller distances.

$$\int d^4x \; \lambda \Phi^4(x)$$

$\hookrightarrow$ units of λ - dimensionless

$$+ \mu \Phi^3(x)$$

$\hookrightarrow$ mass dimension 1

$$\underbrace{\frac{1}{M^2}\Phi^6(x) + \frac{1}{M^4}\Phi^8(x) + \frac{1}{M^2}\Phi^2 \partial\phi \, \partial\phi + \cdots}$$

tons of things you could write down, but the vast majority of interactions have a strength that is an inverse power of mass.

Only a few interactions are dimensionless, ~~or~~ or have positive mass dimension.

dimensionless - marginal

positive mass dim - relevant

negative mass dim - IRREL

Gravity is example of Irrel operator, No invariant sense of weather strong or weak

- At low energy scales so weak that unimportant
- At high energy scales so important that we need to change theory.

~~ever~~ Any irrelevant theory comes equipped w/ scale at which it breaks down.

In any given theory, if you want to describe physics around some fixed energy scale, All you need to know are
-) What are the particles around. (fields associated w/ them)
-) What are the possible marginal or relevant interactions between them? Write them all down

Only a few parameters, what ever is going on @ these energy scales can be accurately described by those few parameters and nothing else.

If you want to predict things very accurately then maybe you need to include the next terms.

Relevant terms only important at very low energies.
Always like to think of approximation, where the particles are moving fast compared to thier masses, energies are high
In that approximation the only things that matter are the marginal interactions.

In all cases in physics there is some physical scale M in which your description of the physics is wrong. (often called UV cut-off)
eg: Plank scale the whole picture is wrong.
But if we are at energies much lower than M_p we dont even

Whats happening at the higher scales is encoded in the higher dim operators.

The way to organize your thinking is scale-by-scale.
All the thinking goes into determining the interactions.
Garanteed to be finite # of parameters at any given scale to describe the physics accurately.

Any theory that we write down is an effective theory that is only accurate to order powers of $\frac{E}{M_{New}}$

(There is something always to E/M_p that we don't know even if we understand 1 TeV, 10 – 100 ... etc.)

Tower of effective field theories each accompanied by its own cutoff.

This is the bottomline to Wilson's way of thinking.

Massive restriction to the kinds of interactions that we can have and that are relevant.
↳ incredibly different from non-relativistic QM!
- Any crappy potential that I want ($\frac{1}{r}$ ... $\frac{1}{r^{2.3}}$ random linear combinations etc)

- framework itself is incredibly unconstrained

The second we put in SR + QM,
- dramatic kinematic things (Anti-particles / Spin states)
- the number of interactions that we are talking about are numbers we can count on our hands.

In SM ~ 19 marginal interactions
↳ Not 1, but not continuous ∞, which is what you get in a NR picture of the world.

Now for a little bit more ooo

think about limit in which can neglect the mass of all particles, imagining high energy processes relative to mass of particles.

(Not Explained, but not so important)

All this comes out of same logic but for fermions.

$$\int d^4x \; \tfrac{1}{2}(\partial\phi)^2 \qquad + \; \bar{\psi}\, \partial\, \psi$$

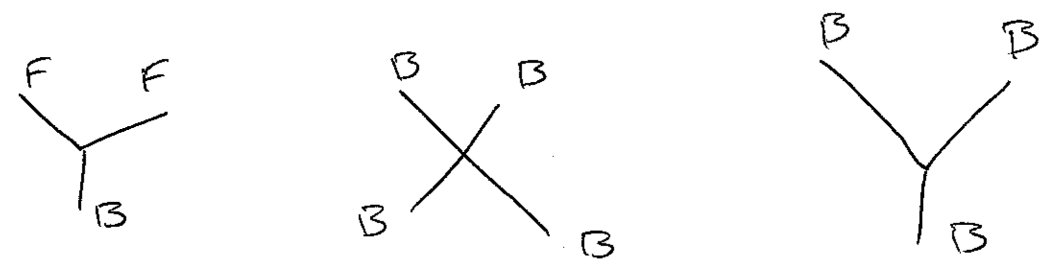
free theory of scalars — free theory of fermions

mass dim(Bosons) = 1 — mass dim(fermions) = $3/2$

Can now write down possible interactions in 4-dim.

ϕ^4 $\quad \phi^2 \partial\phi \quad$ $\psi\psi\phi$ $\quad$ thats it. for the marginal operators.

Basic interactions in nature



At sufficiently low energies compared to M_{Pl}, SR + QM say we need a theory of Fermions & Bosons, and these are the only interactions that are important

Dimension Counting tells us there can only be a few types of interactions,

Now last bit of logic for why the world is such a damn constrained place ...