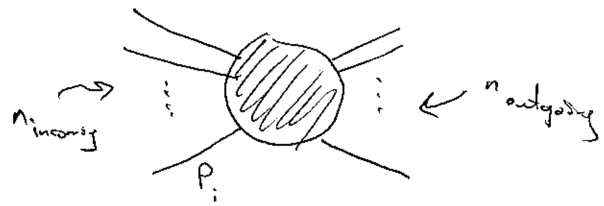
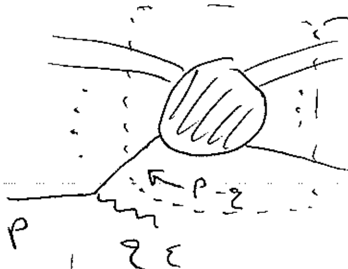
Now do the same thing to a more general interaction



$$\equiv iM_0(p_i)$$

Consider what happens if we attach "photon" to incoming leg

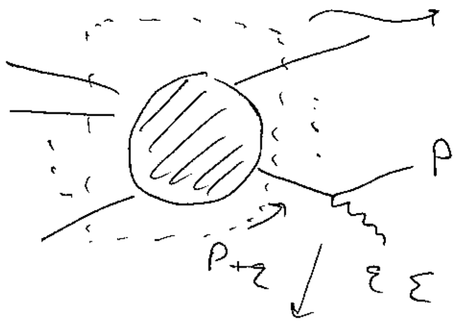


$$\to iM_0(p_i - q)$$

$$= Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\,iM_0(p_i - q)$$

$$i(iQ)\,\frac{\left(\varepsilon_\mu\left(p_i^\mu + (p_i^\mu - q^\mu)\right)\right)}{(p_i - q)^2 - m^2}$$

Can also attach photon to outgoing leg



$$\to iM_0(p_i + q)$$

$$= -Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\,iM(p_i + q)$$

$$(-Q)\,\frac{\varepsilon_\mu\left(p_i^\mu + (p_i^\mu + q^\mu)\right)}{(p+q)^2 - m^2}$$

Total Amplitude

$$M = \sum_{\text{incoming}} Q_i\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\,iM_0(p - q) + \sum_{\text{outgoing}} -Q_i\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\,iM(p_i + q)$$

Soft limit $M_0(p\mp q) \to M_0(p)$

$$= iM_0\left(\sum_{\text{incoming}} Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)} + \sum_{\text{outgoing}} -Q\,\frac{(p\cdot\varepsilon)}{(p\cdot q)}\right)$$

now as before $\varepsilon_\mu \to \varepsilon'_\mu + q_\mu$ means that M must vanish when $\varepsilon_\mu \to q_\mu$

OR under LT

$$\varepsilon \cdot M \to \varepsilon' \cdot M' + \underbrace{iM\left(\sum_{\text{incoming}} Q - \sum_{\text{outgoing}} Q\right)}$$

$\hookrightarrow = 0$ only if
$\sum_{\text{incoming}} Q = \sum_{\text{outgoing}} Q$ (charge conserved!)

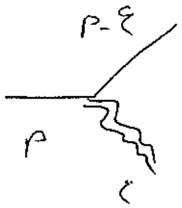
Now same logic for Spin-2 (describes interaction w/ Gravitons)

Same as above except 2-component polarization vector

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + \underbrace{\Lambda_\mu q_\nu + \Lambda_\nu q_\mu + \Lambda q_\mu q_\nu}$$

under little group

$\hookrightarrow$ effect from all these needs be 0 as before.



$$= i(iK)\,\varepsilon_{\mu\nu}\frac{(2p^\mu p^\nu)}{-p\cdot q}$$ (Same idea with outgoing leg)

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + q_\mu \Lambda_\nu$$

$$\varepsilon_{\mu\nu} M^{\mu\nu} \to \varepsilon'_{\mu\nu} M'^{\mu\nu} + M\left(\sum_{\text{incoming}} K_i \Lambda_\nu p^\nu - \sum_{\text{outgoing}} K_o \Lambda_\nu p^\nu\right)$$

$$+ M\Lambda_\nu\left(\sum_{\text{in}} K_i p^\nu - \sum_{\text{out}} K_{o} p^\nu\right) \Rightarrow$$ $K_i p_i^\nu$ is conserved!

We know that p_i^ν is conserved by E & mom.

Only way can have nontrivial solutions is if

$$k_i = k \text{ for all } i$$

All particles interact w/ gravity with the same strength

Gravitational interaction is universal!

"Principle of Equivalence"

Can keep going ...

For massless spin 3 we would need

$$\sum_{\text{incoming}} \beta_i p_\nu^i p_\mu^i = \sum_{\text{outgoing}} \beta_i p_\nu^i p_\mu^i$$

eg $\mu\nu = 0$ $$\sum_{\text{incoming}} \beta_i E_i^2 = \sum_{\text{outgoing}} \beta_i E_i^2$$

way too constraining only way if $\beta_i = 0$

No interacting theories of massless particles of Spin greater than 2