Lecture 29

Continuing with Electroweak Unification...

So we have ... Local gauge invariance of SU(2)

$$\phi(x) \rightarrow e^{ig\vec{\alpha}(x)\cdot\sigma}\phi(x)$$

implying that

$$\phi(x) = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

two compoent spinor: “weak iso spin”

Requires the addition of 3 gauge feilds via

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig\vec{W}_\mu \cdot \vec{\sigma}$$

$\vec{W}_\mu = \{W_\mu^1, W_\mu^2, W_\mu^3\}$

Before adding the gauge invariance.

$$\mathcal{L} = i\bar{\phi}\gamma_\mu\partial^\mu\phi = i\nu_e\gamma_\mu\partial^\mu\nu_e + ie\gamma_\mu\partial^\mu e$$

where e and ν_e are 4-component solutions to the Dirac Equation.

With the gauge invariance.

$$\begin{aligned} \mathcal{L} \rightarrow \mathcal{L}' &= i\bar{\phi}\gamma_\mu D^\mu\phi + (\text{kinetic term for Ws} \sim F_{\mu\nu}F^{\mu\nu}\ ) \\ &= i\bar{\phi}\gamma_\mu\left(\partial_\mu + ig(W_1^\mu\sigma_1 + W_2^\mu\sigma_2 + W_3^\mu\sigma_3)\right)\phi + ... \end{aligned}$$

Now define

$$\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2) = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{cases}$$

So,

$$\begin{aligned}\vec{W}\cdot\vec{\sigma} &= W_1^\mu\sigma_1 + W_2^\mu\sigma_2 + W_3^\mu\sigma_3 \\ &= W_+^\mu\sigma_+ + W_-^\mu\sigma_- + W_3^\mu\sigma_3\end{aligned}$$

define $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)$

So,

$$\mathcal{L} \supset= i\bar{\phi}\gamma_\mu\left(\partial_\mu + ig\left(W_+^\mu\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} + W_-^\mu\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix} + W_3^\mu\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right)\right)\phi$$