

Lecture 16

Example:

$$L = -\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi)^2 - \frac{1}{2}m^2 \phi^2 + \frac{g}{3!}\phi^3$$

Consider cross-section for $\phi\phi \rightarrow \phi\phi$ scattering.

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|} |M|^2 d\Pi_{LIPS}$$

$$p_1 + p_2 \rightarrow p_3 + p_4$$

In COM frame, $\vec{p}_1 = -\vec{p}_2$ and $\vec{p}_3 = -\vec{p}_4$

Also, $E_1 + E_2 = E_3 + E_4 = E_{CM}$

$$d\Pi_{LIPS} = (2\pi)^4 \delta^4(\sum p) \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3 p_4}{(2\pi)^3} \frac{1}{2E_4}$$

(integrating over \vec{p}_4)

$$= \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3} \frac{1}{E_4} \delta(E_3 + E_4 - E_{CM})$$

where $p_f = |\vec{p}_3| = |\vec{p}_4|$, $E_3 = \sqrt{m^2 + p_f^2}$, and $\int d^3 p_f = \int dp_f p_f^2 d\Omega$

Now change variables,

$$p_f \rightarrow x = E_3 + E_4 - E_{CM}$$

$$dx = \frac{d}{dp_f}(E_3 + E_4 - E_{CM}) dp_f = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{E_3 + E_4}{E_3 E_4} p_f dp_f$$

\Rightarrow

$$\frac{dp_f p_f^2}{E_3 E_4} = \frac{dx p_f}{E_{CM}}$$

$$\begin{aligned}
d\Pi_{LIPS} &= \frac{1}{16\pi^2} d\Omega \int_{m_3+m_4-E_{CM}}^{\infty} dx \frac{p_f}{E_{CM}} \delta(x) \\
&= \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{CM}} \text{if } E_{CM} > m_3 + m_4 \text{ else } 0
\end{aligned}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|} \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{CM}} |M|^2$$

$$|v_1 - v_2| = \left| \frac{|\vec{p}_1|}{E_1} + \frac{|\vec{p}_2|}{E_2} \right| = p_i \frac{E_{CM}}{E_1 E_2}$$

\Rightarrow

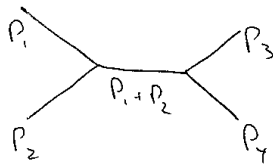
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2} \frac{p_f}{p_i} |M|^2$$

(if masses are equal $p_f = p_i$)

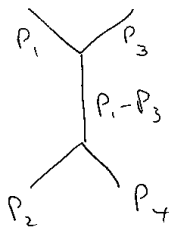
Now to M

Now to M

"s-channel" diagram

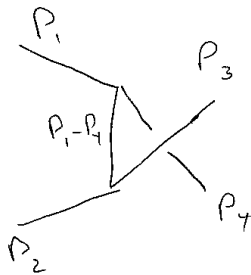


$$= (ig) \frac{i}{\underbrace{(p_1 + p_2)^2 - m^2}_{\equiv s}} (ig) = \frac{-ig^2}{s - m^2}$$



t-channel

$$= (ig) \frac{i}{\underbrace{(p_1 - p_3)^2 - m^2}_{\equiv t}} ig = \frac{-ig^2}{t - m^2}$$



u-channel

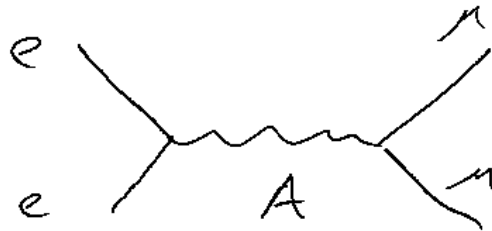
$$= (ig) \frac{i}{\underbrace{(p_1 - p_4)^2 - m^2}_{\equiv u}} ig = \frac{-ig^2}{u - m^2}$$

$$\frac{d\sigma}{d\Omega}(\phi\phi \rightarrow \phi\phi) = \frac{g^4}{64\pi^2 E_{CM}^2} \left[\frac{1}{s - m^2} + \frac{1}{t - m^2} + \frac{1}{u - m^2} \right]^2$$

$$s + t + u = \sum m_j^2 \text{ (s,t,u are L. I.)}$$

Example 2

Electron-positron to muons scattering.



Note from:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2} \frac{p_f}{p_i} |M|^2$$

that M is dimensionless.

It is given by combination of dimensionless couplings and appropriate spin projections.

Focus on the projections on

- initial spins to the γ polarization
- γ polarization to final spins

$$M(s_1 s_2 \rightarrow s_3 s_4) = \sum_{\substack{\epsilon \\ \text{photon polarizations}}} \langle \underbrace{s_3 s_4}_{\text{Spin of } \mu s} | \epsilon \rangle \langle \epsilon | \underbrace{s_1 s_2}_{\text{Spin of } e s} \rangle$$

At high-energies take e and μ to be massless.

$$P_1 = (E, 0, 0, E)$$

$$P_2 = (E, 0, 0, -E)$$

In this limit think of the electron as having helicity.

We will solve this now in linear basis (you will do circular in HW)

$$|s_1 s_2\rangle = \underbrace{|\leftrightarrow\leftrightarrow\rangle}_{\text{along-x}}, \underbrace{|\uparrow\uparrow\rangle}_{\text{along-y}}, |\leftrightarrow\uparrow\rangle, |\uparrow\leftrightarrow\rangle$$

where the spins are 1/2.

However, only two combinations can project onto a spin-1 photon

$$|\leftrightarrow\leftrightarrow\rangle, |\uparrow\uparrow\rangle,$$

photon polarizations

$$\epsilon^1 = (0, 1, 0, 0)$$

$$\epsilon^1 = (0, 0, 1, 0)$$

$|\leftrightarrow\leftrightarrow\rangle$ gives ϵ^1

$|\uparrow\uparrow\rangle$ gives ϵ^2

Now, the μ 's are also spin 1/2. (Also have 4 spin states)

In general, μ not moving along same direction as the incoming electrons.

Can parameterize with θ , (symmetric under ϕ can rotate such that $\phi = 0$)

$$P_3 = E(1, 0, \sin \theta, \cos \theta)$$

$$P_4 = E(1, 0, -\sin \theta, -\cos \theta)$$

for muons, 2 possible directions of photon polarizations

$$\bar{\epsilon}^1 = (0, 1, 0, 0)$$

$$\bar{\epsilon}^1 = (0, 0, \cos \theta, -\sin \theta)$$

(Can check that these are perpendicular to P_3 and P_4)

In general, hard to measure spins. Sum over all μ spins.

Must sum over all possible $\left| \begin{array}{c} \text{initial} \\ \text{final} \end{array} \right|$ polarizations

For us only

$$- M_1 = M(|\leftrightarrow\leftrightarrow\rangle \rightarrow |\bar{\epsilon}^1\rangle) = \epsilon^1 \bar{\epsilon}^1 = -1$$

$$- M_2 = M(|\uparrow\uparrow\rangle \rightarrow |\bar{\epsilon}^2\rangle) = \epsilon^2 \bar{\epsilon}^2 = -\cos \theta$$

are non-zero.

If our initial beams are unpolarized, sum over initial spins.

$$|M|^2 = |M_1|^2 + |M_2|^2 = 1 + \cos^2 \theta$$

$$\frac{d\sigma}{d\Omega} \sim \frac{\alpha^2}{E_{CM}^2} (1 + \cos^2 \theta)$$