Homework Set #4

Due Date: Before 5pm Friday February 14th

1) Find the generators of the "Little Group" for Massive particles *(5 points)*

2) Heisenberg Equation of Motion *(5 points)*

a) Show that $\frac{dA(t)_H}{dt} = -i[A(t)_H, H]$. Where $A(t)_H$ is an operator in the Heisenberg representation and H is the Hamiltonian. This equation is referred to as the Heisenberg equation of motion.

b) Show that $\phi_H(x,t) = e^{-iE_p t}\phi_S(x)$ satisfies the Heisenberg equation of motion. Where $\phi_S(x) = \int \cancel{d}\cancel{p} e^{i\vec{p}\cdot\vec{x}} a^\dagger$ is the operator in the Schrodinger representation and $H = \int d^3 p E_p a^\dagger a$

3) Show that $\int \cancel{d^3}\cancel{p} \equiv \int \frac{d^3\vec{p}}{2E_p}$ is Lorentz invariant. *(2 points)*

(Hint: $\int d^4p\, \delta(E^2 - (|\vec{p}|^2 + m^2))$ is clearly Lorentz invariant.)

4) Anti-Particles *(5 points)*

a) Expand $\Phi^{\dagger 2}\Phi^2$ in terms of a, $a^\dagger$, b, and $b^\dagger$ (Ignore the exponentials and integrals)

b) Sketch diagrams of the processes that each term corresponds to.

c) Let the charge (Q) of particle a be q_a and the charge of particle b be q_b. Calculate ΔQ for each process.

d) What happens if you take $q_a = -q_b$?