Spontaneous Symmetry Breaking "Hidden Symmetry" ①

Last time ran into a crisis.
Needed both massive boson (force carrier) and gauge invariance. Can't get this from m^2A^2.

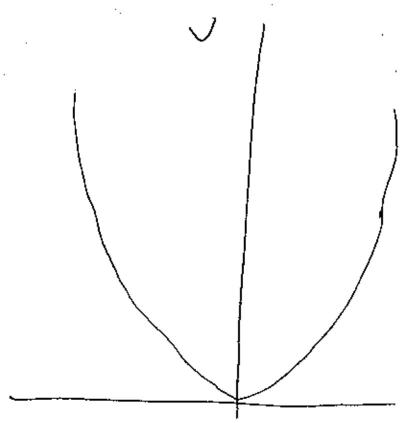
Difficult Subject We will build up to the full picture by going through a few toys.

Toy 1

$$\mathcal{L} = T - V = \frac{1}{2}(\partial\phi)^2 - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right) \qquad \lambda > 0$$

Invariant under $\phi \to -\phi$
(dropped higher order terms)

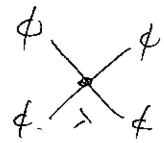
Note the mass sign comes in w/ relative "−" sign



Case a) $\mu^2 > 0$

describes a scalar field w/ $m = \mu$

$\lambda\phi^4$ leads to



interaction

ground state $\phi = 0$

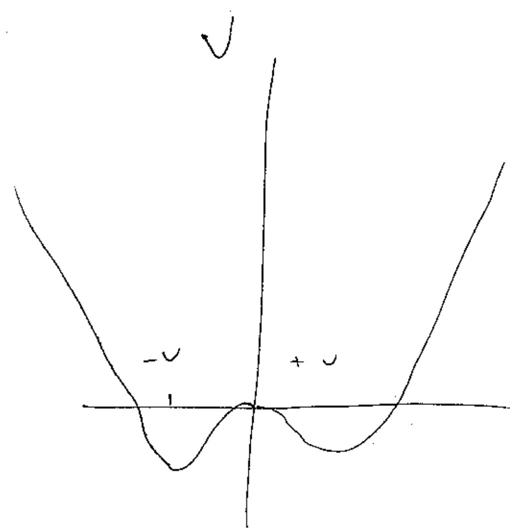
Case b) $\mu^2 < 0$

Now the $\mathcal{L}$ has a mass term w/ the wrong ~~thing~~ sign. (Relative "+" sign w/ Kine term)

$$\frac{\partial V}{\partial\phi} = \phi(\mu^2 + \lambda\phi^2) = 0$$

$$\phi = \pm v \qquad v = \sqrt{-\mu^2\lambda}$$

minimum



Now, $\phi = 0$ does not correspond to a minimum.

Perturbative calculations should involve expansion about minimum either $+v$ or $-v$.

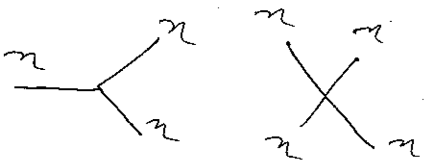
Take $+v$ at random ("Spontaneously")

$$\phi(x) = v + \underbrace{\eta(x)}_{\text{fluctuations about the min.}}$$

Can rewrite $\mathcal{L}$ as $\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 - \underbrace{\lambda v^2 \eta^2}_{\text{mass term w/ correct sign.}} - \lambda v \eta^3 - \frac{1}{2}\lambda \eta^4 + \text{consts}$

$m_\eta = \sqrt{-2\mu^2}$

Other terms give



Mass of η was "generated" ("or "revealed") as "Spontaneous Symmetry Breaking"

$\phi \to -\phi$ Symmetry of $\mathcal{L}$ is broken by the choice of the ground state.

Example seen in condensed matter systems
(eg large ferromagnet. Spins will align, All directions equally likely
Ground State (spins alligned in some direction) breaks rotational symmetry.

- Superconductors

- Buckling of Needle.

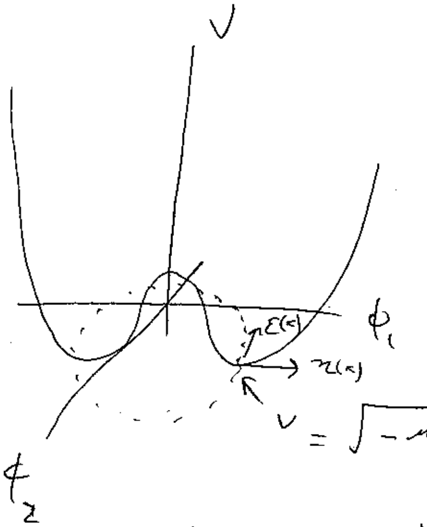
Toy 2 Repeat above w/ complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$\hookrightarrow$ invariant under $\phi \to e^{i\alpha}\phi$ for some # α

Global U(1) gauge symmetry $\lambda > 0$, $\mu^2 < 0$

$$\mathcal{L} = \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$



$v = \sqrt{-\mu^2/\lambda}$

Agian, need to expand about min.

<u>WOLG</u> expand around $\phi_1 = v$ $\phi_2 = 0$

(like picking $+v$ before)

$$\phi(x) = \sqrt{\tfrac{1}{2}}\left(v + \eta(x) + i\,\xi(x)\right)$$

Get now $\mathcal{L}' = \frac{1}{2}(\partial\xi)^2 + \frac{1}{2}(\partial\eta)^2 + \mu^2\eta^2 + \mathcal{O}(\eta^3) + \mathcal{O}(\eta^4) + \mathcal{O}(\xi^3) + \mathcal{O}(\xi^4)$

$\mu^2\eta^2 = -\frac{1}{2}m_\eta^2\eta$ mass term for η just as before.

However now have ξ which is a <u>massless</u> scalar.

"Goldstone Boson"

Problem, tried to give mass to a gauge boson and created a massless boson as well.

Intuitively, direction along ξ is flat, $\Rightarrow$ no resistance to excitations along that direction. <u>Crisis</u>

Have not observed these massless gauge bosons

Lets try a local gauge theory (A miracle is about to happen ...)

<u>Higgs Mechanism</u> U(1) simplest example (SM. does this in $SU(2)_L$ getting closer)

$$\phi \to e^{i\alpha(x)}\phi \quad \text{for arbitrary function } \alpha(x)$$

As for EM $\quad D_\mu = \partial_\mu - ieA_\mu$ ← gives coupling of ϕ to A

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(if $\mu^2 > 0$ QED w/ massive charged Scalar of mass μ w/ $\phi\phi\phi\phi$ term

Now for $\mu^2 < 0$, Again we do $\phi \to \frac{1}{\sqrt{2}}[v + \eta(x) + i\xi(x)]$

$$\mathcal{L}' = \frac{1}{2}(\partial\xi)^2 + \frac{1}{2}(\partial\eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A^2 - evA_\mu\partial^\mu\xi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

+(interactions)

<u>3 particles</u> (Apparently ...)

$m_\xi = 0 \qquad m_\eta = \sqrt{2\lambda v^2} \qquad m_A = ev$ ← dynamically generated mass for gauge field

Still problem

This however can't be the spectrum B/c there are only 4 DoF before the expansion, but now seems like 5 = 2 + 3

(2 scalar + 2 Massless Spin 1) $\qquad$ 3 → massive Spin 1

So some of these apparent DoF are unphysical. 5

(Similar to picking a gauge in QED)

Try different expansion

$$\phi \to \frac{1}{\sqrt{2}}(v + h(x))e^{i\theta(x)/v}$$ Particular choice of gauge

$$A_\mu \to A_\mu + \frac{1}{ev}\partial\theta$$ that gives h - real

$$\mathcal{L}'' = \frac{1}{2}(\partial h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2v^2A^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2A^2h^2 + ve^2A^2h - \frac{1}{4}F^2$$

Now Goldstone boson does not appear in the theory!

The apparent extra DoF was spurious (only corresponds to gauge transformation)

<u>Spectrum (Physical)</u>

2 particles interacting massive

A_μ - 3 DoF massive Spin 1

h - 1 DoF massive Spin 0

→ "higgs boson"

Massless Goldstone Boson "eaten" by the A_μ to become the extra DoF needed for the longitudinal polarization.

"Higgs Mechanism"