Dimensional Analysis and "~"

Put in the right physics to get answers to within *"geometric factors"*

- Dont worry about factors of 2 or π etc
- Use "~" not "="

Examples (Volume of something) ~ (size)^3

Cube $= R^3$ $\sim R^3$

Sphere $= 4/3\pi R^3 = 4.2\ R^3 \sim R^3$

$= 1/6\pi(D)^3 = 0.4\ D^3 \sim D^3$

Cylinder $= R \times \pi R^2 = \pi\ R^3 \sim R^3$ (if two scales use r^2R)

Kinematic energy $= 1/2\ mv^2 \sim mv^2$

Ive been doing this already: "$\Delta p \Delta x \geq h$"

(…it is really $\Delta p \Delta x \geq h/(4\pi)$)

Units

I hate units! All numbers are really unit-less
Always comparing some quantity relative to some standard
We will work in "Natural Units"

Natural Units

- The right way to think about the world
  (How physicists think, what makes them seem smart to other people)
- Very easy. Much easier than Metric/British/cgm/mks …
- Standard is set by basic physical principles
  ⇒ numbers have direct physical interpretations

c ≡ 1: [Distance]/[Time] ≡ 1
- Time and distance have same units
- E = m

You are already familiar with this:
"Its about an hour from here"

h ≡ 1: [Energy]×[Time] = 1 and [Energy]×[Distance] = 1
- Energy (or Mass) is inversely related to distance or time.

Write everything in terms of **[Energy]:** use 1 GeV ~ mp as basic unit

Examples

Everything in terms of GeV. Use conversions to get back to human units

Conversions:

$$\mathrm{GeV} = 10^{-27}\ \mathrm{kg}$$

$$\mathrm{GeV}^{-1} = 10^{-16}\ \mathrm{m}$$

$$\mathrm{GeV}^{-1} = 6 \cdot 10^{-25}\ \mathrm{s}$$

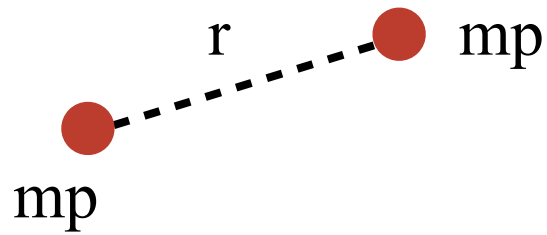
Proton Weight: GeV

Proton Size: GeV^{-1}

My height: $1\mathrm{m} \sim 10^{16}\ \mathrm{GeV}^{-1}$

My weight: $100\ \mathrm{kg} \sim 10^{29}\ \mathrm{GeV}$

EM and Gravitation Interactions



Electromagnetic Energy

$$E = -\underbrace{\frac{e^2}{4\pi}}\frac{1}{r}$$

GeV GeV

Pure number: α
Its small: 1/137

Gravitational Energy

$$E = -\underbrace{G_N}\frac{m_p^2}{r}$$

GeV GeV^3

Dimensionful number
$G_N m_p^2 = 10^{-39}$

The world with 4 numbers

Claim: ~everything in world combination of these numbers

$$m_p \sim 1 \text{ GeV} \qquad \alpha = \frac{1}{137} \sim 10^{-2}$$

$$m_e \sim 10^{-3} \text{ GeV} \qquad \alpha_G \equiv G_N m_p^2 = 10^{-39}$$

Will work through some quick examples.

Atoms

$$E \sim -\frac{Z\alpha}{r} + \frac{p^2}{m_e}$$

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Z	Prediction	Actual Value
1	$\sim 10^{-11}$m	$2.5 \cdot 10^{-11}$m
10	$\sim 10^{-12}$m	$4.0 \cdot 10^{-11}$m
>10	$\sim 10^{-12}$m	$\sim 10^{-10}$m

Details of electron screening needed for high Z
(Will use 10^-10 when Z >10)

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Number of different atoms ~ 1/α

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- Why we could do QM first with out relativity: (v << 1 for Z~1)
- Why electricity more stronger everyday than magnetism.

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For Hydrogen

10^{-4} 0.5 MeV $\sim$ 50 eV

(Actually is 13.6 eV)

Atoms

For Atoms Electron mass is king!
(mp doesn't make an appearance)

$$E \sim -\frac{Z\alpha}{r} + \frac{p^2}{m_e}$$

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Mass Density: Mass/Volume

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(Ratio of two give the speed of sounds)

$$v_{\text{sound}} \sim \sqrt{\frac{P_{\text{solid}}}{\rho_{\text{solid}}}} \sim \sqrt{\frac{\alpha}{m_p r_{\text{atom}}}}$$

Predict: ~25,000 m/s

Beryllium	12,890 m/s
Diamond	12,000 m/s
Steel	6000 m/s

Planets

Solids where gravitational pressure balanced by solid pressure

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$$R_{Planet} \sim \sqrt{\frac{1}{G_N m_p^2 Z^3 \alpha m_e^2}} \sim \sqrt{\frac{\alpha}{\alpha_G}} \times r_{atom}$$

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Planets/atoms relative size direct result of EM vs gravity strength

r_{atom}^0 r_{atom}^1

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Prediction: $r_e \sim 10^7 m$ $M_p \sim 10^{25} kg$

Actual: $6.4 \cdot 10^6 m$ $5.9 \cdot 10^{24} kg$

α / G $\times r_{atom}$

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Life

Estimate limit on size of life: Require dont break bones when fall

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Break bones along cross sectional areas

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$$L_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{1}{4}} \times r_{atom} \qquad M_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{4}} \times Z m_p$$

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$$E_{Fall} \sim E_{Bi}$$

$L_A \sim 10$ cm / $M_A \sim 100$ kg

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