Lecture 13

From Last time...

$$d\sigma = \frac{V}{T}\frac{1}{|\vec{v_1} - \vec{v_2}|}dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \underbrace{d\Pi}_{\text{Region of final state momenta we are considering}}$$

On interval of size L, the momenta of available states are $P_n = \frac{2\pi n}{L}$ (from particle in a box).

$\Rightarrow$ throughout a volume V

$$N = \int \frac{V}{(2\pi)^3}d^3p$$

$$d\Pi = \prod_j \frac{V}{(2\pi)^3}d^3p_j$$

where j runs over final state partilces.

OK, lets deal with the normalization factors.

Note, $\langle f|f\rangle$ and $\langle i|i\rangle \neq 1$ (The inner products are not Lorentz invariant...)

$$\begin{aligned}\langle p'|p\rangle &= (2\pi)^3\, 2E\, \delta^3(p'-p)\\ \langle p|p\rangle &= (2\pi)^3\, 2E_p\, \delta^3(0)\\ &= 2E_pV\end{aligned}$$

$\Rightarrow$

$$\langle i|i\rangle = \langle p_1p_2|p_1p_2\rangle = 2E_1V\, 2E_2V$$

$$\langle f|f\rangle = \prod_j(2E_jV)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated pertubatively

$$S = \underbrace{1}_{\text{free theory}} + \underbrace{iT}_{\text{perturbatively small}}$$

Know that S matrix should vanish if momentum not conserved

$$\langle f|T|i\rangle = (2\pi)^4\delta^4(\sum p) \underbrace{M}_{\text{“}MatrixElement''}$$

Now, might worry that we have to square the δ function

$$\begin{aligned} |\langle f|T|i\rangle|^2 &= (2\pi)^8\, \delta^4\left(\sum p\right)\, \delta^4(0)\, |M|^2 \\ &= (2\pi)^4\, \delta^4\left(\sum p\right)\, TV|M|^2 \end{aligned}$$

So,

$$\begin{aligned} dP &= \frac{(2\pi)^4\, \delta^4\,(\sum p)\;\, TV}{(2E_1V)(2E_2V)} \frac{1}{\prod_j (2E_jV)} |M|^2 \prod_j \frac{V}{(2\pi)^3} d^3p_j \\ &= \frac{T}{V}\frac{1}{(2E_1)(2E_2)}|M|^2 \qquad \underbrace{d\Pi_{\text{LIPS}}}_{\substack{\text{L.I. Phase space} \\ =(2\pi)^4\ \delta^4(\sum p)\ \prod_j \frac{d^3p}{(2\pi)^3 2E_p}}} \end{aligned}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|\vec{v_1} - \vec{v_2}|}|M|^2 d\Pi_{\text{LIPS}}$$

where $\vec{v} = \vec{p}/p_0$

known as “Fermis Golden Rule”

Decay rate probability that a one-particle state turns into a multi-particle state over time T.

$$p_1 \to \{P_j\}$$

thing of it as $1 \to N$ scattering.

follow same steps as above

$$d\Gamma = \frac{1}{2E_1}|M|^2 d\Pi_{\text{LIPS}}$$