Bonus

Clarification from last time ...

used "obvious" expansion about $\phi_1 = v$ $\phi_2 = 0$

$$\phi = (v + \eta + i\xi)$$

Put this in $\mathcal{L}$ and got $\mathcal{L}' = \ldots + \frac{1}{2}(\partial\xi)^2 + ev A_\mu \partial^\mu \xi + \frac{1}{2}e^2v^2A^2$

then we said this ~~was~~ looked like there was an unphysical DoF and we started over w/ $\phi = (v+h)e^{i\xi/v}$

A (potentially) cleaner way to see whats happening is to notice

$$\frac{1}{2}(\partial\xi)^2 + ev A_\mu \partial^\mu \xi + \frac{1}{2}ev^2A^2 = \frac{1}{2}e^2v^2\left[A_\mu + \frac{1}{ev}\partial_\mu\xi\right]^2$$

Now I can pick a gauge where

$$A_\mu \rightarrow A_\mu - \frac{1}{ev}\partial_\mu \xi$$

$\Rightarrow$ I only have the $\frac{1}{2}e^2v^2A^2$ term.

(Of course this is equivalent as above)

Electroweak Unification

Now that we see how we can use the Higg's Mechanism to generate masses for gauge bosons, we will see how it works in the SM.

We will take the $\mathcal{L}^{SM}$ with massless bosons $(W, Z\, \gamma)$ and add a scalar w/ the "mexican hat" potential $\mu^2 < 0$.

So we first need to talk about the initial $\mathcal{L}^{SM}$ w/o the higgs

This will lead us to talk about the Weak interaction group

& "Electroweak" unification.

$\hookrightarrow$ Major Step forward in physics

QED (EM)

gauge invariance led us to the $\mathcal{L}^{QED}$

$\psi_e(x) \to e^{iq\alpha(x)}\psi_e(x)$ for arbitrary function $\alpha(x)$

electron

"local U(1) phase transformation"

$\hookrightarrow$ a number is the generator of U(1)

What will it take to make physics invariant under this transformation?

Couple ways to think about this.

Lets see what happens to the equations of motion (Dirac Eq)

$$(i\gamma^\mu\partial_\mu - m)\psi_e = 0 \to (i\gamma^\mu\partial_\mu - m)e^{iq\alpha(x)}\psi_e = (i\gamma^\mu\partial_\mu \underbrace{}_{} -q\gamma^\mu A_\mu - m)e^{iq\alpha}\psi_e = 0$$

or $i\gamma^\mu[(\partial_\mu + iq\partial_\mu\alpha) - m]\psi = 0$ which is not the Dirac Eq!

$\Rightarrow$ the free particle D.E. does not possess the local $U(1)$ invariance. Turns out not possible for a free theory.
↳ one w/o interactions

~~However if we add~~ We need something to cancel the $iq\partial_\mu\alpha$ term.

So lets add a term $iqA_\mu \rightarrow iqA_\mu - iq\partial_\mu\alpha$

or $A_\mu \rightarrow A_\mu - \partial_\mu\alpha$

So the new "Dirac Eq" (Really the D.E. w/ an interaction to a gauge Boson ...) is

$$i\gamma^\mu\left[(\partial_\mu + iqA_\mu) - m\right]\psi = 0$$

$\hookrightarrow$ this is invariant under $\psi_e \rightarrow e^{i\alpha}\psi_e$
the derivitive term brings down a piece that is exactly canceled by A_μ transformation

Think of $(\partial_\mu + iqA_\mu)$ as 'fancy' derivitive $D_\mu = (\partial_\mu + iqA_\mu)$
$\hookrightarrow$ "Covariant"
then get back the same form as the Dirac equation. w/ $\partial_\mu \rightarrow D_\mu$

$$(i\gamma^\mu D_\mu - m)\psi = 0$$

Similarly the $\mathcal{L}$ that gives the D.E. is modified by $\partial_\mu \rightarrow D_\mu$

This was all for a $U(1)$ symmetry (phase symmetry)

This describes E.M. In a world w/o the weak interaction this would be Q.E.D.

Now to the weak interaction ... (GSW)

Would like to follow QED as closely as possible

So lets do the same thing but pick a different group.

SU(2) is a group we looked at before.

Can use it to describe rotations in 3D w/ complex spinors.

Generators of SU(2) are the pauli matricies...

So now postulate

$$\phi_L \rightarrow e^{i(\text{generators})} \phi_L \quad \text{for } SU(2), \quad \phi \rightarrow e^{ig\vec{\alpha}\cdot\vec{\sigma}} \phi$$

finite transformation

Lets unpack this

Consider infintesimal 1st

$$\phi \rightarrow e^{i\varepsilon\vec{\alpha}\cdot\vec{\sigma}} \phi = (1 + \varepsilon(\alpha_i \sigma_i)) \phi$$

$$= \left(1 + \varepsilon\left(a_1(x)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + a_2(x)\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + a_3(x)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)\right)\phi$$

ϕ is not a number its a vector

$$\phi(x) = \begin{pmatrix} \nu_e(x) \\ e(x) \end{pmatrix}$$

2×2 unit matrix

Dirac Equation would be $\left(i\gamma^\mu \partial_\mu \mathbb{1} - m\mathbb{1}\right)\phi = 0$

Now when we do $\phi \rightarrow e^{ig\vec{\alpha}\cdot\sigma}\phi$ the derivative brings

down 3 factors $\sim \partial_\mu \alpha_1(x) \quad \sim \partial_\mu \alpha_2(x) \quad \partial_\mu \alpha_3(x)$

Under local SU(2) transformation. $e^{ig(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)}$

$$\left(i\gamma^\mu\partial_\mu \mathbb{1} - m\mathbb{1}\right)\phi = 0 \rightarrow \left(i\gamma^\mu\partial_\mu \cancel{\mathbb{1}} - m\right)e^{ig\vec{\alpha}\cdot\vec{\sigma}}\phi = 0$$

$$\rightarrow \Big[\underbrace{i\gamma^\mu\partial_\mu}_{\mathbb{1}} + ig\underbrace{(\partial_\mu\alpha_1(x))\sigma_1}_{2\times2} + ig\underbrace{(\partial_\mu\alpha_2(x))\sigma_2}_{2\times2} + ig\underbrace{(\partial_\mu\alpha_3(x))\sigma_3}_{2\times2} - \underbrace{m}_{\mathbb{1}}\Big]\phi = 0$$

Clear that the only way to make this invarint under the local transformation is to add 3 new particles.

$$W^1_\mu \rightarrow W^1_\mu - \partial_\mu\alpha_1 \qquad \partial_\mu \rightarrow D_\mu = \underbrace{\partial_\mu}_{\mathbb{1}} + ig\vec{W}_\mu\cdot\vec{\sigma}$$

$$W^2_\mu \rightarrow W^2 - \partial_\mu\alpha_2$$

$$W^3_\mu \rightarrow W^3 - \partial_\mu\alpha_3$$

Then the "D.E." $(i\gamma_\mu \underbrace{D^\mu}_{\mathbb{1}} - m)\phi = 0$ is invarint.

Questions? ...? B/c here is where it is going to get wierd...

$\boxed{\text{Stress the direct connection between the number of generators \& the number of gauge bosons.}}$

The local gauge symmetry is dictating the particle content.