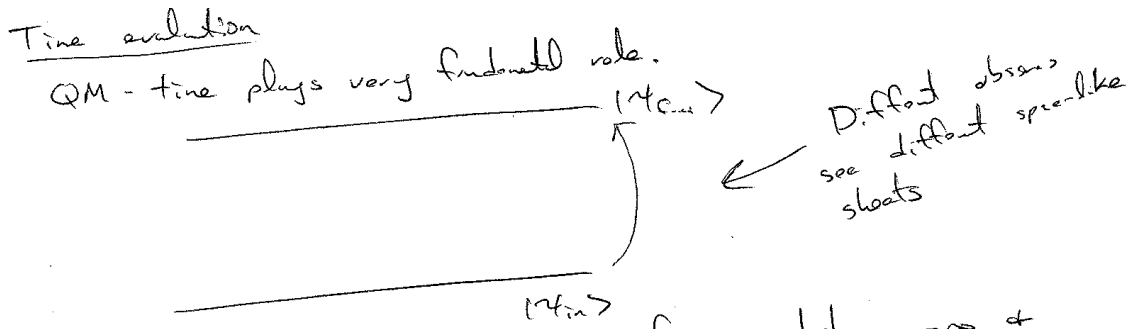


Lecture 10

QFT Continued...

Summary From Last Time



Only have a hope of Lorentz invariance if we start at $-\infty$ and go to $+\infty$.

Throw particles in from ∞ let them scatter & go back out to ∞ .

Define S-Matrix

$$\underbrace{|p_1\sigma_1, \dots, p_n\sigma_n\rangle}_{t=-\infty} \rightarrow \underbrace{\mathcal{S} |p_1\sigma_1, \dots, p_n\sigma_n\rangle}_{t=+\infty}$$

\mathcal{S} might be (at least a hope) Lorentz Invariant.

Big Picture: The plan is to Figure out what \mathcal{S} is in a totally generic theory, then see what it would take to make it Lorentz Invariant.

Sure doesn't look like it will be L.I. \mathcal{S} is the only object that you could even have a hope to make L.I.

We will see that for very special choices of the interaction it will barely be possible for it to be Lorentz Invariant. These choices force on us anti-particle and the connection between spin and statistics.

Something annoying that we should get rid of right away. Free evolution, just evolves w/phase. Totally irrelevant part.

Standard way of removing the free evolution “Interaction Representation”.

$$H = H_{\text{free}} + H_{\text{Int}}$$

$$i \frac{d}{dt} |\psi\rangle = (H_{\text{free}} + H_{\text{Int}}) |\psi\rangle$$

For $H_{\text{Int}} = 0$

$$|\psi\rangle = e^{-iH_f t} |\psi_{\text{in}}\rangle$$

Now, we don't have a free theory, but if the interaction is small going to be pretty close to evolving like this.

$$|\psi\rangle = e^{-iH_f t} \underbrace{|\psi_I\rangle}_{\text{definition}} \quad (1)$$

If $H_{\text{Int}} = 0$, $|\psi_{\text{Int}}\rangle$ doesn't evolve at all. Bc there is H_{Int} , $|\psi_{\text{Int}}\rangle$ will evolve.

$$\begin{aligned} i \frac{d}{dt} |\psi\rangle &= H_f |\psi\rangle + e^{-iH_f t} i \frac{d}{dt} |\psi_I\rangle \\ &= (H_f + H_{\text{int}}) e^{-iH_f t} |\psi_I\rangle \end{aligned}$$

Note: first line from derivative of 1, the second from the Schrodinger Equation.

The RHSs imply,

$$i \frac{d}{dt} |\psi_I\rangle = \underbrace{e^{-iH_f t} H_{\text{Int}} e^{-iH_f t}}_{\text{Interaction Hamiltonian in the interaction representation}} |\psi_I\rangle$$

So,

$$i \frac{d}{dt} |\psi_I\rangle = H_I |\psi_I\rangle$$

where H_I can be time dependent.

Lets formally solve this

Just integrating gives,

$$|\psi_I(t_2)\rangle = |\psi_I(t_1)\rangle - i \int_{t_1}^{t_2} dt H_I |\psi_I(t)\rangle$$

Now we can keep iterating,

$$= |\psi_I(t_1)\rangle - i \int_{t_1}^{t_2} dt H_I(t) \left(|\psi_I(t_1)\rangle - i \int_{t_1}^t dt' H_I(t') |\psi_I(t')\rangle \right)$$

or

$$= |\psi_I(t_1)\rangle - i \int_{t_1}^{t_2} dt H_I |\psi_I(t_1)\rangle + (-i)^2 \int_{t_1}^{t_2} dt \int_{t_1}^t dt' H_I(t) H_I(t') |\psi_I(t')\rangle$$

Pattern is clear, can keep going...

$$\begin{aligned} |\psi_I(t_2)\rangle = & [\quad 1 \quad +(-i) \int_{t_1}^{t_2} dt H_I(t) \\ & + \quad (-i)^2 \int_{t_1}^{t_2} dt \int_{t_1}^t dt' H_I(t) H_I(t') \\ & + \quad (-i)^3 \int_{t_1}^{t_2} dt \int_{t_1}^t dt' \int_{t_1}^{t'} dt'' H_I(t) H_I(t') H_I(t'') \\ & + \quad \dots] |\psi_I(t_1)\rangle \end{aligned}$$