Homework Set Branching Ratios

0) Chapter 4 *(3 points)*

The cross section is used to characterize the probability of particles interacting. The cross section has dimensions of area or of GeV^{-2}. The cross section scales as the matrix element squared. ie: $\sigma \sim |M|^2$.

1) Z boson decays: *(5 points)*

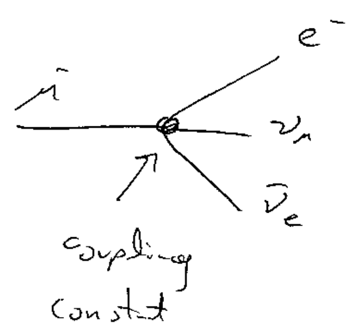
We assumed that the Z-couplings were universal, that the constants of proportionality (called the phase space integrals) were the same for all decay products, and that no higher-order diagrams were relevant. (The phase space intergrals will be the same if we can neglect the decay products masses.)

$$Br(Z \to ee) \sim \frac{1}{21} = 0.048 \text{ vs } 0.034 \text{ in PDG}$$

$$Br(Z \to bb) \sim \frac{3}{21} = 0.143 \text{ vs } 0.156 \text{ in PDG}$$

2) Muon decays: *(10 points)*

a)



b) We have 4-bosons (each of dim 3/2) and the coupling constant. The total dimensions have to add up to 4.

$$4 \times \frac{3}{2} + [\text{coupling constant}] = 4$$

$\Rightarrow$

$$[\text{coupling constant}] \sim -2 \text{ or GeV}^{-2}$$

c)

$$\Gamma \sim |M|^2 \sim [\text{coupling constant}]^2 = \text{GeV}^{-4}$$

But we also know that Γ has to come out to have overall dimensions of $\frac{1}{\text{time}}$ or GeV.

$\Rightarrow$

$$\Gamma \sim m_\mu^5$$

d) $m_\mu \sim 0.1\text{GeV}$, $m_\tau \sim 1\text{GeV}$, $\tau_\mu \sim 1\mu s$

Now from c)

$$\Gamma_\tau \sim m_\tau^5$$

and we know

$$\tau_\mu = \Gamma_\mu^{-1}$$
$$\tau_\tau = \Gamma_\tau^{-1}$$

so,

$$\frac{\tau_\tau}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_\tau}$$

$\Rightarrow$

$$\tau_\tau = \tau_\mu \frac{\Gamma_\mu}{\Gamma_\tau} = \tau_\mu \frac{m_\mu^5}{m_\tau^5} = \tau_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 = 1\mu s (10^{-1})^5 = 10^{-6} s \times 10^{-5} = 10^{-11} s$$

e) with a direct three-point $\mu \to e\gamma$ vertex, the only mass scale is m_μ. (b/c the $(\mu e\gamma)$- coupling is dimensionless)

So, $\Gamma_{\mu \to e\gamma} \sim m_\mu$ (to get the dimensions on Γ right)

We know from above that with the four-point interaction in Fermi theory $\Gamma_{SM} \sim m_\mu^5 m_W^{-4}$

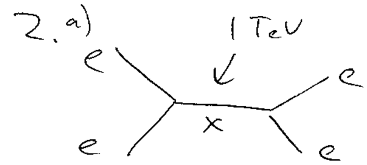
So,

$$\frac{\tau_{new}}{\tau_{SM}} \sim \frac{m_\mu^5 m_W^{-4}}{m_\mu} \sim \left(\frac{m_\mu}{m_W}\right)^4 \sim \left(\frac{0.1 \text{ GeV}}{100 \text{ GeV}}\right)^4 \sim 10^{12}$$

The direct $\mu \to e\gamma$ would dominate (by a factor 10^{12}!)
The weak interaction is damned weak.

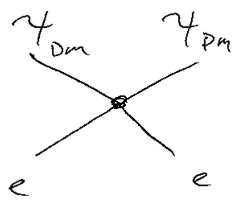
3) A new force. *(5 points)*

a)



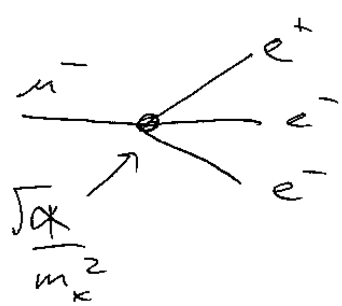
Range $\sim \frac{1}{m_X} \sim \frac{1}{1000\text{GeV}} \sim 10^{-3}\ \text{GeV}^{-1} \sim 10^{-19}\ m$

b)



Four fermion interaction $\Rightarrow$ Units of coupliing $\text{GeV}^{-2} \sim \frac{1}{m_X^2}$

c)



$$\Gamma_{\text{New}} \sim \frac{m_\mu^5}{m_X^4} \qquad \text{(see problem 2 for login on why } m_\mu^5\text{)}$$

$$\Gamma_{\text{SM}} \sim \frac{m_\mu^5}{m_W^4} \qquad \text{(from problem 2)}$$

So,

$$\frac{\tau_{\text{New}}}{\tau_{\text{SM}}} \sim \left(\frac{m_X}{m_w}\right)^4 \sim 10^4$$

$\Rightarrow$ SM decays dominate!