

Lecture 9

Particles Interactions

“Quantum Field Theory in a week”

Summary From Last Time

Massive:

$|P, \sigma\rangle$ and $U[\Lambda] |P, \sigma\rangle = \sum_{\sigma'} R_{\sigma, \sigma'} |\Lambda P, \sigma'\rangle$, where the R is a rotation matrix.

Mass-less:

$|P, h\rangle$ and $U[\Lambda] |P, h\rangle = e^{ih\Theta(W)} |\Lambda P, h\rangle$, where the coefficient is just a phase.

Now talk about all these particles states in a more convenient way.

For every given momenta we have stacks of Hilbert space

- infinitely many possibilities for Bosons $0 \rightarrow N$
- fixed number (depending on the spin) for fermions

Ultimately interested in the interactions between particles.

- Defined by some Hamiltonian
- Could specify the Hamiltonian by describing how it acts on all states in the Hilbert space
- Instead for our convenience introduce creation and annihilation operators.

These keep track of the states in a simple way...

First define the vacuum state: $|0\rangle$. Then define single particle states as:

$$\underbrace{|P\sigma\rangle}_{\text{These are primary}} \equiv a_{p\sigma}^\dagger |0\rangle$$

This defines $a_{p\sigma}^\dagger$.

$$|P_1\sigma_1, P_2, \sigma_2\rangle = a_{p_1\sigma_1}^\dagger a_{p_2\sigma_2}^\dagger |0\rangle$$

etc. etc. etc.

Similarly, can define annihilation operators

$$\begin{aligned} a_{P,\sigma} |0\rangle &= 0 \\ a_{P,\sigma} |P, \sigma\rangle &= |0\rangle \end{aligned}$$

a - removes states.

Can encode boson/fermion statistics in a and a^\dagger .

Bosons	Fermions
$[a_{P_1,\sigma_1}^\dagger, a_{P_2,\sigma_2}^\dagger] = 0$	$\{a_{P_1,\sigma_1}^\dagger, a_{P_2,\sigma_2}^\dagger\} = 0$
$[a_{P_1,\sigma_1}, a_{P_2,\sigma_2}] = 0$	$\{a_{P_1,\sigma_1}, a_{P_2,\sigma_2}\} = 0$

Normalization

$$\langle p', \sigma' | p, \sigma \rangle = \delta_{\sigma', \sigma} \delta^3(\vec{p} - \vec{p}')$$

will use compressed notation

$$\langle p', \sigma' | p, \sigma \rangle = \delta_{\sigma', \sigma} \delta_{p, p'}$$

This implies

For Bosons,

$$[a_{p,\sigma}, a_{p',\sigma'}^\dagger] = \delta_{\sigma', \sigma} \delta_{p, p'}$$

For Fermions,

$$\{a_{p,\sigma}, a_{p',\sigma'}^\dagger\} = \delta_{\sigma', \sigma} \delta_{p, p'}$$

Note, these are operators that We have defined four our convenience. Makes it easier to talk about the states. (Of course what really matters is the states)

Usually you see it presented as:

- Start with fields
- Quantize them
- then find these commutation relation

But really the fields are secondary concepts, what comes first is the particles.
“Not one deep thing going on here”

OK, What would be the free Hamiltonian ?

$$H = \sum_{p,\sigma} E_p a_{p,\sigma}^\dagger a_{p,\sigma}$$

We're labeling states by 4-momenta, but the energy is constrained ($E^2 = p^2 + m^2$).

Can label by the 3-momentum

$$\langle \vec{p}', \sigma' | \vec{p}, \sigma \rangle = (2\pi)^3 2E_p \delta^3(\vec{p} - \vec{p}')$$

Note $d^3 p$ is not Lorentz invariant, but $d^3 p / 2E_p$ is. (HW).

To make my life easier by defining

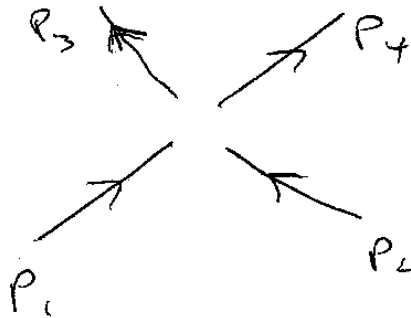
$$\int d^3 p = \int \frac{d^3 p}{2E}$$

So can write the free Hamiltonian as:

$$\begin{aligned} H &= \sum_{p,\sigma} E_p a_{p,\sigma}^\dagger a_{p,\sigma} \\ &= \int d^3 p E_p a_{p,\sigma}^\dagger a_{p,\sigma} \end{aligned}$$

Now let's imagine building interactions. Add interaction hamiltonian.

eg: an interaction where two particles come in and two particles go out:



This would correspond to adding an interaction term to the Hamiltonian of the form:

$$\int d^3 p_1 d^3 p_2 d^3 p_3 d^3 p_4 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta(E_1 + E_2 - E_3 - E_4) \underbrace{a_{p_4, \sigma_4}^\dagger a_{p_3, \sigma_3}^\dagger a_{p_2, \sigma_2} a_{p_1, \sigma_1}}_{\text{Acts on the initial state and gives the final state}} V(p's, \sigma's) + \text{Hermitian Conjugate}$$

Interactions are made out of strings of a 's and a^\dagger 's. Also easy in this picture to talk about the creation and destruction of particles (Not just scattering)

Very convenient to use this to map between particle states.

So far we have not said the word "Field".