Homework Set #4

Solutions

1) Find the generators of the "Little Group" for Massive particles ***(5 points)***

The little group equation is:

$$W \cdot k = k \Rightarrow \omega \cdot k = 0$$

where

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & a & b & c \\ -a & 0 & A & B \\ -b & -A & 0 & C \\ -c & -B & -C & 0 \end{bmatrix}$$

For a massive particle we can take k to be k = (m,0,0,0).

So $\omega \cdot k = (0, -ma, -mb, -mc)$. So the little group generators for massive particles are

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & A & B \\ 0 & -A & 0 & C \\ 0 & -B & -C & 0 \end{bmatrix}$$

which are the rotation matricies.

2) Heisenberg Equation of Motion ***(5 points)***

a)

$$\begin{aligned} \frac{dA(t)_H}{dt} &= \tfrac{d}{dt}\left(e^{iHt}A_s e^{-iHt}\right) \\ &= iH\left(e^{iHt}A_s e^{-iHt}\right) - i\left(e^{iHt}A_s H e^{-iHt}\right) \\ &= -i\left(\underbrace{e^{iHt}A_s e^{-iHt}}_{A_H(t)} H - H \underbrace{e^{iHt}A_s H e^{-iHt}}_{A_H(t)}\right) \\ &= -i\left[A_H(t), H\right] \end{aligned}$$

b) $\phi_H(x,t) = e^{-iE_p t}\phi_S(x)$

$$\frac{d\phi_H(x,t)}{dt} = -iE_p\phi_H(x,t)$$

and

$$\begin{aligned}
[\phi_H(x,t), H] &= [\int \cancel{dp} e^{ip\cdot x} a^\dagger, \int dp' E_{p'} a^\dagger a] \\
&= \int \cancel{dp} e^{ip\cdot x} \int dp' E_{p'} [a^\dagger, a^\dagger a] \\
&= \int \cancel{dp} e^{ip\cdot x} \int dp' E_{p'} a^\dagger [a^\dagger, a] \\
&= \int \cancel{dp} e^{ip\cdot x} \int dp' E_{p'} a^\dagger \delta^3(\vec{p} - \vec{p'}) \\
&= \int \cancel{dp} e^{ip\cdot x} E_p a^\dagger \\
&= E_p \int \cancel{dp} e^{ip\cdot x} a^\dagger = E_p \phi_H(x,t)
\end{aligned}$$

So

$$\frac{d\phi_H(x,t)}{dt} = -i\left[\phi_H(x,t), H\right]$$

3) Show that $\int \cancel{d^3 p} \equiv \int \frac{d^3\vec{p}}{2E_p}$ is Lorentz invariant. ***(2 points)***

(Hint: $\int d^4p\, \delta(E^2 - (|\vec{p}|^2 + m^2))$ is clearly Lorentz invariant.)

4) Anti-Particles ***(5 points)***

a) Expand $\Phi^{\dagger 2}\Phi^2$ in terms of a, $a^\dagger$, b, and $b^\dagger$ (Ignore the exponentials and integrals)

b) Sketch diagrams of the processes that each term corresponds to.

c) Let the charge (Q) of particle a be q_a and the charge of particle b be q_b. Calculate ΔQ for each process.

d) What happens if you take $q_a = -q_b$?