Lecture 10

QFT Continued...

Summary From Last Time

$$\phi_+(\vec{x}) = \int \not{d}^3 p \; e^{-i\vec{p}\vec{x}} \; a^\dagger_{\vec{p}}$$

(use scalars for the moment)

Need to build H_I out of $\phi_{+/-}$ in the interaction representation.

Note:

$$\phi^I_+(x,t) = e^{+iH_f t}\phi(x)e^{-iH_f t} \to e^{iE_p t}\phi(x)$$

The e^{iHt} will go to e^{-iEt}, then $\phi(x)$ will create a new particle with energy E_p, and then the e^{+iHt} will give $e^{+i(E+E_p)t}$

$$\begin{aligned}
\phi^I_+(x,t) &= e^{+iH_f t}\phi(x)e^{-iH_f t} \\
&= \int \not{d}^3 p \; e^{-i\vec{p}\vec{x}} \; e^{+iE_p t} \; a^\dagger_{\vec{p}} \\
&= \int \not{d}^3 p \; e^{ip^\mu x_\mu} \; a^\dagger_{\vec{p}}
\end{aligned}$$

this behaves nicely under Lorentz Transforms $\phi(\Lambda x) = \phi(x)$

We seem to be in awesome shape, lets write down an interaction.

$$H_I = \int d^3 x \; k \left[\phi^I_+(x)...\phi^I_-(x)\right]$$

and now we know how to turn this into the $\mathcal{S}$-matrix

$$\mathsf{T}\left(e^{(-i)\int_{-\infty}^{+\infty} dt \; d^3x \; k\left[\phi_+(x)...\phi_-(x)\right]}\right)$$

$$\mathsf{T}\left(e^{(-i)\int_{-\infty}^{+\infty} d^4x \; k\left[\phi_+(x)...\phi_-(x)\right]}\right)$$

All of this is Lorentz Invariant! Seems like we are done.

What is the problem?

Problem is the time ordering. Only thing that is not necessarily L.I.

This would be LI if $\top$ was LI. But space-like separated objects are not time-ordered in a LI way.

Diagram: (see lecture for space time diagram)

Only way to be LI if $H_I(x)$ and $H_I(x')$ commute when x and x' are space-like separated.

Lorentz Invariant $\Rightarrow [H_I(x), H_I(x')] = 0$ if $x - x' < 0$ (space-like).

Now can ask if the $\phi^\dagger$'s or the ϕ's commute outside the light cone. <u>They do Not</u>.

However can find a new combination which does.

Now just state some results...

<u>Turns out</u>

$$[\phi_+(x), \phi_+^\dagger(x')] \neq 0 \text{ for } x - x' < 0$$

But we can define

$$\Phi(x) = \phi_+(x) + \phi_-(x) = \int \not{d}^3 p (a^\dagger e^{-ipx} + a e^{ipx})$$

Can show that,

$$[\Phi(x), \Phi(x')] = 0 \text{ for } x - x' < 0$$

Only if $[a, a^\dagger] = 0$, not if $\{a, a^\dagger\} = 0$

Tells us that Scalars have to be Bosons!

Tells us that we need to build H_I out of Φ (not ϕ_+ and ϕ_- separately)

eg:

$$
\begin{aligned}
H_I &= \lambda\Phi^4(x) \\
&= \lambda(\phi_+(x) + \phi_-(x))^4 \\
&= \lambda\left[\underbrace{\overset{\text{2 go in 2 come out}}{\phi_+^2\phi_-^2}}_{\text{term saw before}} + \underbrace{\phi_+^4 + \phi_+\phi_-^3 \ldots}_{\text{New terms}}\right]
\end{aligned}
$$

No charges associated with this scalar.

Can create it or destroy it at will (eg: $\phi_+\phi_-^3$)

How do we talk about particles with conserved charge? This will not work ! Only have one choice, introduce another operator. a's and b's.

$$
\begin{aligned}
\phi^a &= \int a_p^\dagger\, e^{-ipx} \\
\phi^b &= \int b_p^\dagger\, e^{-ipx}
\end{aligned}
$$

And construct Φ as

$$
\begin{aligned}
\Phi &= \phi_+^a + \phi_-^b \\
&= \int \not{d}^3 p(ae^{ipx} + b^\dagger e^{-ipx})
\end{aligned}
$$

Again we have,

$$[\Phi(x), \Phi(x')] = 0 \text{ for } x - x' < 0$$

(Note: Φ is <u>not</u> Hermitian.) But

$$\int d^3x\, \Phi^\dagger(x)^2\Phi(x)^2$$

is Hermitian

Expand this out and find that everything would conserve change provided that particles of type b have opposite charge to a.

So, if you want to talk about particles that carry some well defined charge, you must have anti-particles.

If you put this together with s=1/2, you find that they have to { } = 0 for the Hamiltonian to vanish outside the light cone.