So,

$$d\sigma = \frac{V}{T} \frac{1}{|v_1 - v_2|} dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} d\Pi$$

$\rightarrow$ region of final state momenta we are considering

On interval size L

p available are ~~...~~ $p_n = \frac{2\pi n}{L}$

$$N = \int \frac{V}{(2\pi)^3} d^3p$$

$\hookrightarrow N_S = P \times N$

$$d\Pi = \prod_j \frac{V}{(2\pi)^3} d^3p_j$$

$\hookrightarrow$ over final state particles

$\langle f|f\rangle = \langle i|i\rangle \neq 1$ $\leftarrow$ Not L. I

$$\langle p'|p\rangle = (2\pi)^3 2E\, \delta^3(p'-p)$$

$$\langle p|p\rangle = (2\pi)^3 2E_p\, \delta^3(0)$$

$$= 2E_p V$$

"Regulated by V

$$\delta^3(p) = \frac{1}{(2\pi)^3}\int d^3x\, e^{i\vec{p}\cdot\vec{x}}$$

$$\delta^3(0) = \frac{1}{(2\pi)^3}\int d^3x = \frac{V}{(2\pi)^3}$$

$\Rightarrow$

$$\langle i|i\rangle = \langle p_1 p_2|p_1 p_2\rangle = 2E_1 V\, 2E_2 V$$

$$\langle f|f\rangle = \prod_j (2E_j V)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated perturbatively

$$S = \underbrace{\mathbb{1}}_{\text{free theory}} + iT$$

$\rightarrow$ Perturbation "small"

Know that S matrix should vanish if momentum not conserved

$$\langle f|T|i\rangle = (2\pi)^4 \delta^4(\Sigma p)\, \mathcal{M} \leftarrow \text{"Matrix Elements"}$$

$$\hookrightarrow \delta^4\left(\sum_i p_i - \sum_f p_f\right)$$

Might worry that we have to square δ function

$$|\langle f|T|i\rangle|^2 = \delta^4(0)\, \delta^4(\Sigma p)(2\pi)^8 |\langle f|M|i\rangle|^2$$
$$= \delta^4(\Sigma p)\, TV\, (2\pi)^4\, |M|^2$$

So,
$$dP = \frac{\delta^4(\Sigma p)\, TV\, (2\pi)^4}{(2E_1 V)(2E_2 V)} \frac{1}{\prod_j (2E_j V)} |M|^2 \prod_j \frac{V}{(2\pi)^3} d^3 p_j$$

$$= \frac{T}{V} \frac{1}{(2E_1)(2E_2)} |M|^2\, d\Pi_{LIPS}$$

$\hookrightarrow$ LI Phase Space $\equiv (2\pi)^4 \delta^4(\Sigma p) \prod \frac{d^3 p}{(2\pi)^3 2E_p}$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|\vec{v}_1 - \vec{v}_2|} |M|^2\, d\Pi_{LIPS}$$

"Fermis Golden Rule"

$$\boxed{\vec{v} = \frac{\vec{p}}{p_0}}$$

<u>decay rate</u> Probability that a one-particle state turns into a multiparticle state ~~st~~ over the T.

$$p_1 \longrightarrow \{p_i\}$$ really $1 \to N$ scattering.

follow same steps as above

$$d\Gamma = \frac{1}{2E_1} |M|^2 \, d\Pi_{\text{Lips}}$$

"Feynman Diagrams"

Last piece we need is LI matrix element

$$M(1+2 \to 3+4+\cdots j) = \langle \{P_j\}_{out} | P_1 P_2 \rangle_{in}$$

↳ Represents an element of the S-matrix

QFT + $\mathcal{L}$ gives a procedure for calculating M

Very nice interpretation in terms of pictures "Feynman diagrams"

QM Perturbation Theory

$$H = H_0 + V \qquad H_0|\phi\rangle = E|\phi\rangle$$

↑ states in the free theory

After $H|\psi\rangle = E|\psi\rangle$

formally, $$|\psi\rangle = |\phi\rangle + \frac{1}{E_0 - H_0}\overbrace{V|\psi\rangle}^{T|\phi\rangle}$$

$$\Rightarrow T = V + V\frac{1}{E-H_0}T$$

↳ Can solve perturbatively in V.

$$T = V + V\frac{1}{(E-H)}V + V\frac{1}{(E-H)}V\frac{1}{(E-H)}V + \cdots$$

$$\underbrace{\langle \phi_f | T | \phi_i \rangle}_{T_{fi}} = \underbrace{\langle \phi_f | V | \phi_i \rangle}_{V_{fi}} + \sum_j \frac{\langle \phi_f | V | \phi_j \rangle \langle \phi_j | V | \phi_i \rangle}{(E - H_0)} + \cdots$$

etc.

Example Scattering of 2 electrons

$|i\rangle = |\psi_e^1 \psi_e^2\rangle$

$|f\rangle = |\psi_e^3 \psi_e^4\rangle$

$$T_{fi} = V_{fi} + \sum_n V_{fn} \frac{1}{E_i - E_n} V_{ni} + \dots$$

E_i: initial = final energy

E_n: energy of intermediate state

Now $\sum_n$ is over everything in the Hilbert (Fock space) but only certain states will be non-0.

In relativistic theory action-at-a-distance of EM replaced by process where 2 electrons interact via γ which travels @ c. (tells us there should be γ in intermediate state)

$$V = \frac{1}{2} e \int d^3x\, \psi A \psi \quad \text{(Ignoring spin)}$$

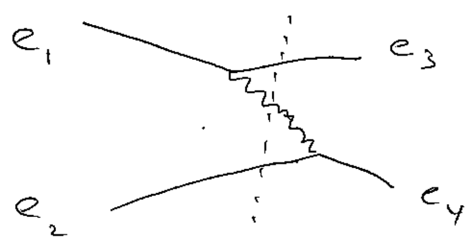
$\longrightarrow$ (terms like $a^\dagger_{\psi_e^3} a^\dagger_\gamma a_{\psi_e^1}$)

B/c all terms involve $a_\gamma^{(+)}$ & $|i\rangle$ $|f\rangle$ have no γ

$$V_{fi} = 0$$

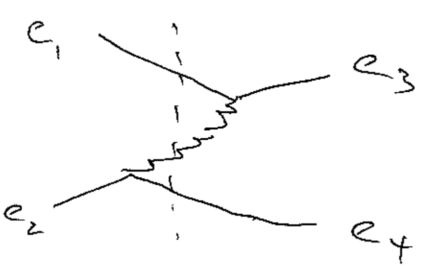
to get non-zero term need $|n\rangle$ with a photon

2 ways   time $\rightarrow$



$|n\rangle = |\psi_e^3 \phi_\gamma \psi_e^2\rangle$

(1)



$|n\rangle = |\psi_e^1 \phi_\gamma \psi_e^4\rangle$

(2)