(a) $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$ $\qquad \mu^2 > 0$

$\lambda > 0$

$$\frac{\partial V}{\partial \phi} = \mu^2\phi + \lambda\phi^3 = \phi(\mu^2 + \lambda\phi^2)$$

only minima @ $\phi = 0$

b) if $\mu^2 < 0$ have extrema @ $\phi = 0$

$$\& \quad (\lambda\phi^2 - |\mu|^2) = 0 \quad \Rightarrow \quad \phi_{min} = \pm\sqrt{\frac{+|\mu|^2}{\lambda}} = \pm\sqrt{\frac{-\mu^2}{\lambda}}$$

to check max/min can check $\frac{\partial^2 V}{\partial \phi^2} = \mu^2 + 3\lambda\phi^2$

@ $\phi = 0$ $\quad \frac{\partial^2 V}{\partial \phi^2} < 0 = \mu^2 \quad \Rightarrow \quad \phi = 0$ is maximum

$$\left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi_{min}} = \mu^2 + 3\lambda\left(\frac{-\mu^2}{\lambda}\right) = -2\mu^2 > 0$$

ϕ_{min} is indeed minima

c) $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \qquad \mathcal{L} = (\partial_\mu\phi^*)(\partial^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$

$$\phi(x) \rightarrow \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\xi(x)\right)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta - i\partial_\mu\xi)(\partial^\mu\eta + i\partial^\mu\xi) - \frac{\mu^2}{2}\Big[\underbrace{(v+\eta)^2 + \xi^2}_{v^2 + 2v\eta + \eta^2 + \xi^2}\Big] - \frac{\lambda}{4}\Big[\underbrace{((v+\eta)^2 + \xi^2)^2}\Big]$$

$$= \frac{1}{2}(\partial\eta)^2 + \frac{1}{2}(\partial\xi)^2 - \underbrace{\frac{\mu^2 v^2}{2}}_{\text{const}} - \mu^2 v\eta - \frac{\mu^2}{2}\eta^2 - \frac{\mu^2}{2}\xi^2 - \frac{\lambda}{4}\big(\eta^4 + 4\eta^3 v + 6\eta^2 v^2 + 2\xi^2\eta^2 + 4\eta v^3 + 4\xi^2\eta v + \underbrace{v^4} + 2\xi^2 v^2 + \xi^4\big)$$

Now $v = \sqrt{-\frac{\mu^2}{\lambda}}$ $\quad v^2 = -\frac{\mu^2}{\lambda} \Rightarrow -\mu^2 = +\lambda v^2$

$$\mathcal{L} = \frac{1}{2}(\partial\eta)^2 + \frac{1}{2}(\partial\xi)^2 + \cancel{\lambda v^3 \eta} + \underline{\lambda v^2 \eta^2} + \underline{\cancel{\frac{\lambda v^2}{2}\xi^2}}$$

$$-\frac{\lambda}{4}\eta^4 - \lambda v \eta^3 - \underline{\frac{3}{2}\lambda v^2 \eta^2} - \frac{1}{2}\xi^2\eta^2 - \cancel{\lambda \eta v^3}$$

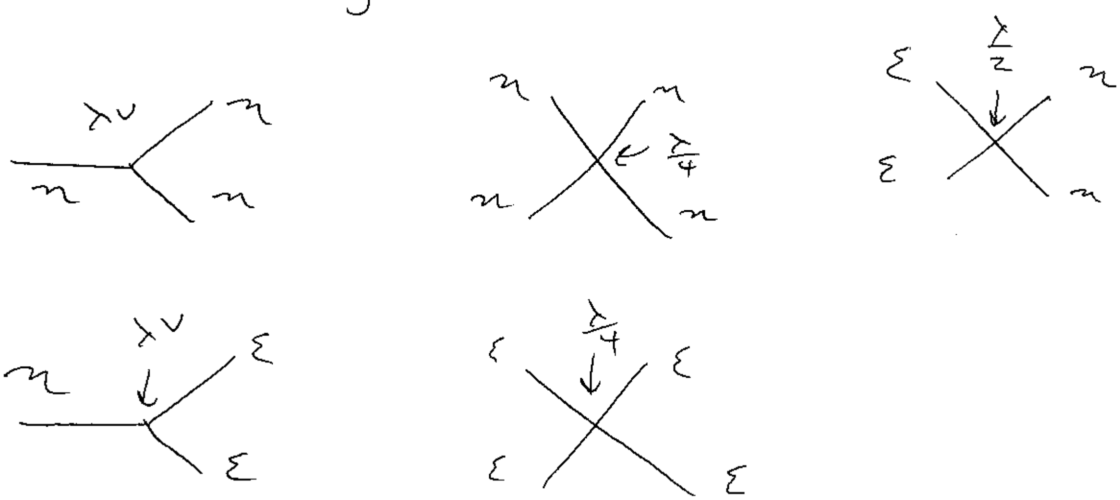
$$-\xi^2 \eta v \lambda - \underline{\cancel{\frac{1}{2}\lambda v^2 \xi^2}} - \frac{\lambda}{4}\xi^4$$

$$= \frac{1}{2}(\partial\eta)^2 + \frac{1}{2}(\partial\xi)^2 - \frac{1}{2}\lambda v^2\eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$

$$-\frac{1}{2}\lambda\xi^2\eta^2 - \lambda v \xi^2 \eta - \frac{\lambda}{4}\xi^4$$

One massive Boson η, one massless boson ξ

& the following interactions



1d) $\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{2} F_{\mu\nu}F^{\mu\nu}$

If $\mu^2<0$ $\quad \phi \to \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

$D_\mu = \partial_\mu - ieA_\mu$

$(D_\mu\phi) = \frac{1}{\sqrt{2}}\left(\partial\eta + i\partial\xi - ieA_\mu(v+\eta) + eA\xi\right)$

$(D_\mu\phi)^+ = \frac{1}{\sqrt{2}}\left(\partial\eta \mp i\partial\xi + ieA_\mu(v+\eta) + eA\xi\right)$

$$\mathcal{L}' = \frac{1}{2}(\partial\eta)^2 + \frac{1}{2}(\partial\xi)^2 + (\partial\eta)eA\xi - eA_\mu(v+\eta)\partial^\mu\xi + \frac{1}{2}e^2A^2(v+\eta)^2 + e^2A^2\xi^2$$
$$-\frac{1}{2}\lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 - \frac{1}{2}\lambda\xi^2\eta^2 - \lambda v\xi^2\eta - \frac{\lambda}{4}\xi^4 - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$$

2a) $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$

Know the minimum will be @ $v = \sqrt{\frac{-\mu^2}{\lambda}}$

Letting $\phi = v + h$ $\qquad \Rightarrow -\mu^2 = \lambda v^2$

$$\Rightarrow V = \frac{1}{2}\mu^2(v+h)^2 + \frac{1}{4}\lambda(v+h)^4$$

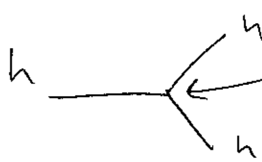
$$= \underbrace{\frac{1}{2}\mu^2 v^2}_{\text{const}} + \mu^2 vh + \frac{1}{2}\mu^2h^2 + \frac{1}{4}\lambda\left[h^4 + 4h^3v + 6h^2v^2 + 4hv^3 + \underbrace{v^4}_{\text{constant}}\right]$$

$$= -\cancel{\lambda v^3 h} - \underline{\frac{1}{2}\lambda v^2h^2} + \frac{1}{4}\lambda h^4 + \lambda v h^3 + \underline{\frac{3}{2}\lambda v^2 h^2} + \cancel{\lambda v^3 h}$$

$$= \lambda v^2 h^2 + \lambda v h^3 + \frac{1}{4}\lambda h^4$$

$$\boxed{\frac{m_h^2}{2} = \lambda v^2 \Rightarrow m_h = \sqrt{2\lambda}\, v} \Rightarrow \lambda = \frac{1}{2}\left(\frac{m_h}{v}\right)^2$$

OR $\quad V = \frac{m_h^2}{2}h^2 + \underline{\frac{1}{2}\frac{m_h^2}{v}h^3} + \frac{1}{8}\left(\frac{m_h}{v}\right)^2 h^4$



$\left(\frac{1}{2}\frac{m_h^2}{v^2}\right) v h^3$

2b) $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^6$

$$\frac{\partial V}{\partial \phi} = \mu^2\phi + \frac{3}{2}\lambda\phi^5 = \phi\left(\mu^2 + \frac{3}{2}\lambda\phi^4\right)$$

$$\phi_{min} = \left(\frac{-2\mu^2}{3\lambda}\right)^{1/4}$$

λ - has mass dimension -2

$\lambda \equiv \frac{1}{\rho^2}$

$$= \left(-\frac{2}{3}\mu^2\rho^2\right)^{1/4} \equiv v$$

$\Rightarrow v^4 = -\frac{2}{3}\mu^2\rho^2 \Rightarrow \mu^2 = -\frac{3}{2}\frac{v^4}{\rho^2}$

or $\frac{1}{\rho^2} = -\frac{2}{3}\frac{\mu^2}{v^4}$

Now expand about the vev $\phi \to (v+h)$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\frac{1}{\rho^2}\phi^6$$

$$= \frac{1}{2}\mu^2\phi^2 - \frac{1}{6}\frac{\mu^2}{v^4}\phi^6$$

$$= \frac{1}{2}\mu^2(\underbrace{v^2}_{const} + 2vh + h^2) - \frac{1}{6}\frac{\mu^2}{v^4}(\underbrace{v^6}_{const} + 6hv^5 + 15h^2v^4 + 20h^3v^3 + HoT)$$

$$= \underline{\frac{1}{2}\mu^2h^2} + \cancel{\mu^2vh} - \cancel{\mu^2hv} - \underline{\frac{15}{6}\mu^2h^2} - \frac{20}{6}\frac{\mu^2}{v}h^3 + HoT$$

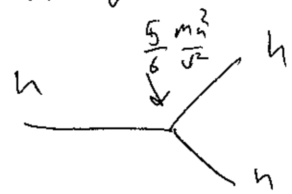
$$= -\frac{12}{6}\mu^2h^2 - \frac{10}{3}\frac{\mu^2}{v^2}vh^3 + HoT$$

$$\Rightarrow \frac{1}{2}m_h^2 = -2\mu^2 \Rightarrow -\mu^2 = \frac{1}{4}m_h^2$$

$$m_h = 2\sqrt{-\mu^2}$$

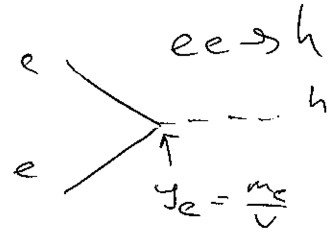
$$V(h) = \frac{1}{2}m_h^2h^2 + \frac{10}{3}\frac{1}{4}\frac{m_h^2}{v^2}vh^3$$

$$+ \frac{10}{12}\frac{m_h^2}{v^2}vh^3$$



3a) Z $\qquad \dfrac{\Gamma(Z \to \nu\nu)}{\Gamma(\text{All})} = \dfrac{3\,(\nu\text{'s})}{3(\ell\text{'s}) + 3(\nu\text{s}) + 3 \times 5(q\text{'s})} = \dfrac{3}{21}$

b) $ee \to Z$



$\sigma_Z \sim \dfrac{\alpha}{\cancel{m_Z^2}}$

$\sigma_h \sim \dfrac{m_e^2}{v^2}\cancel{m_h^2}$

$$\frac{\sigma_Z}{\sigma_h} = \frac{\alpha\, v^2}{m_e^2} = \frac{10^{-2}}{(10^{-6})^2} = 10^{10}$$

$\Rightarrow \sigma_h$ is factor of 10^{10} smaller than σ_Z

c) to study the higgs @ the same level that LEP studied the Z, LEP would have to collect

$$\underbrace{10^{10}}_{} \times \underbrace{10^{7}}_{\#\, Z\text{'s}} = 10^{17} \text{ events total}$$

d) 10^{17} event $\times \dfrac{1\text{ s}}{40\text{e}6} = \dfrac{1}{4}\, 10^{10}\text{ s} \times \dfrac{1 \text{ year}}{\pi\, 10^{7}\text{ s}}$

$= \dfrac{1}{12}\, 10^{3}$ years ~ 100 years

e) 

f) $\sigma_{Zh} \sim \dfrac{\alpha}{(m_Z + m_h)^2}$