Lecture 30

Recap: $SU(2)_L \times U(1)$

Start with:

$$\underbrace{\phi_1, \quad \phi_2, \quad \phi_3, \quad \phi_4,}_{\text{DoF: } 4\times 1 \text{ (scalars)}} \quad \underbrace{W^1, \quad W^2, \quad W^3, \quad B}_{4\times 2 \text{ (mass-less spin-1)}}$$

So 12 total degrees of freedom.

When $\mu^2 < 0$, left with

$$\underbrace{h}_{\text{DoF: } 1} \quad \underbrace{W^+ \quad W^- \quad Z}_{3\times 3 \text{ (massive spin 1)}} \quad \underbrace{\gamma}_{2}$$

Total degrees of freedom 12, as needed!

$$A_\mu = \frac{1}{\sqrt{g_W^2 + g'^2}}(g' W_\mu^3 + g_W B_\mu) \equiv \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$Z_\mu = \frac{1}{\sqrt{g_W^2 + g'^2}}(g_W W_\mu^3 - g' B_\mu) \equiv -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$

$$\frac{g'}{g} = \tan\theta_W \qquad m_\gamma = 0 \qquad m_Z = \frac{1}{2}\frac{g}{\cos\theta_W}v$$

$$v^2 = \frac{-\mu^2}{\lambda} \simeq 250\text{GeV} \qquad \frac{m_W}{m_Z} = \cos\theta_W \qquad m_H^2 = 2\lambda v^2$$

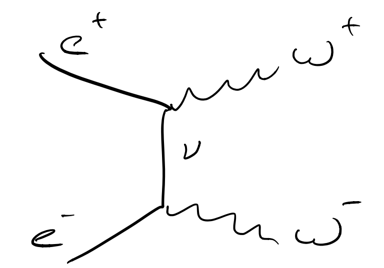
Predictions coming from Electro-weak unification. eg: relationship between W and Z boson masses.

Higgs mechanism on $SU(2)_L \times U(1)$ generates the correct Electro-weak spectra.

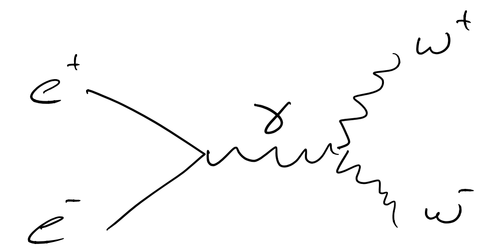
Comments

- Weak force carriers charged, implies relationship between weak and EM force.
- Coupling constants not so different $\frac{1}{137}$ vs $\frac{1}{50}$
- Also strong theoretical arguments that they must be related...Talk about this now

Can produce pairs of W's from e^+e^- collisions



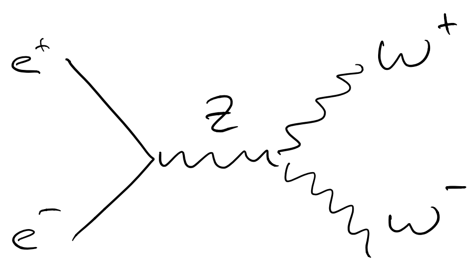
$\sim \alpha_W^2$



$\sim \alpha \times \alpha_W$

with these σ increases with Energy without limit. Eventually probability not conserved. (Calculated WW flux exceeds e^+e^- flux)

Including 3rd diagram resolves problem with negative interference:



Only works because relative couplings are related in very specific way.

Fermion Masses

Remarkably, the stupidest Higgs mechanism can also be used to generate Fermion masses... Lets see how.

Note that a Fermion mass term in the Lagrangian:

$$-m\psi\psi = -m(\psi_R\psi_L + \psi_L\psi_R)$$

does not respect $SU(2)_L$.

$\Rightarrow$ "bare" mass terms cannot be included in $\mathcal{L}_{SM}$.

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad\qquad \psi_R = e_R \qquad\qquad \text{etc...}$$

Now ϕ_{Higgs} is a doublet that transforms under $SU(2)_L$.

So the term $\psi_L\phi_{\text{Higgs}}$ is invariant under $SU(2)_L$ (and U(1)).
$\Rightarrow$ the term $\psi_L\phi_{\text{Higgs}}\psi_R$ is invariant under $SU(2)_L \times U(1)$.

So we are free to add terms like:

$$\mathcal{L} \supset -g_F\left(\bar{\psi}_L\phi_{\text{Higgs}}\psi_R + \bar{\psi}_R\phi^\dagger_{\text{Higgs}}\psi_L\right)$$

eg:

$$\mathcal{L} \supset -g_e\left(\begin{pmatrix} \nu_e & e \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + e_R \begin{pmatrix} \phi^+ & \phi^{0*} \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L\right)$$

g_e - refereed to as the "electron Yukawa" coupling. (Coupling constant. not a dimensional mass parameter)
Note dimensions on these terms $\Rightarrow$ g_e -dimensionless.

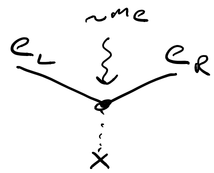
Now, after Electro-weak Symmetry Breaking, $\phi_H \rightarrow \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

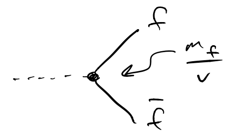
$$\mathcal{L} \rightarrow \mathcal{L}' \supset \quad \underbrace{\frac{-g_e}{\sqrt{2}} v\left(e_L e_R + e_R e_L\right)}_{\text{Exactly whats needed for mass term!}} \quad - \frac{g_e}{\sqrt{2}} h(x)\left(e_L e_R + e_R e_L\right)$$

require $g_e = \sqrt{2}\frac{m_e}{v}$

Note, not predicted by the Higgs Mechanism, but allowed in gauge invariant way.

$$\mathcal{L}_e = \underbrace{-m_e e_L e_R} - \underbrace{\frac{m_e}{v} e_R e_L h}$$





Can construct all massive fermions this way.

$$g_F = \sqrt{2}\frac{m_f}{v} \qquad\qquad v = 250\text{GeV}$$

Interestingly for the top quark: $m_t = 173.5$ $\sqrt{2}m_t \sim v\ g_t \sim 1(0.997)$

Other numbers small:

$$g_b \sim 0.03 \qquad\qquad g_e \sim 10^{-6} \qquad\qquad g_\nu \sim 10^{-12}$$

Bonus:

$$\phi_c \equiv i\sigma_2\phi^2 \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix}$$

Now, how do we know any of this is correct ?

- Predicted neutral currents. Then found.
- Predicted value of m_W and m_Z. Later discovered where predicted.

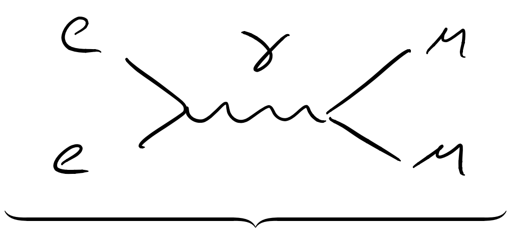
Other precise predictions of the electro-weak model was confronted with equally precise measurements of $W^\pm$, Z properties at LEP (Large Electron Positron Collider) and Tevatron.

Will say a few things about these tests...

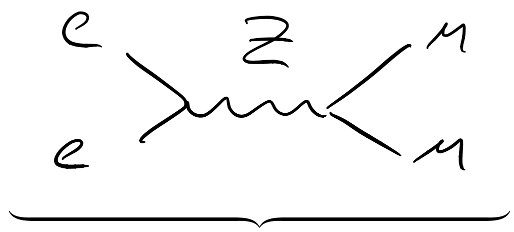
LET produced large quantities of e^+e^- collisions.

Many "on the Z resonance"

Any process with γ can be replaced with Z eg:



$M_\gamma \sim \frac{e^2}{q^2}$



$M_Z \sim \frac{g_Z^2}{q^2 - M_Z^2}$

In these "s-channel" diagrams the 4-momentum of the internal line is equal to E_{CM}.

B/c $m_Z^2 = (90 \text{ GeV})^2$

- For $E_{CM}^2 << m_Z^2$: $M_\gamma >> M_Z$ (EM dominates)
- For $E_{CM}^2 >> m_Z^2$: $M_\gamma >> M_Z$ (Both are important $\alpha \sim \alpha_W$)
- For $E_{CM}^2 \sim m_Z^2$: $M_Z >> M_\gamma$ (Weak Interaction (Z-boson production) dominates)

In fact, when $E_{CM} = m_Z$ is naively infinite ... B/c doesn't account for Z being an unstable particles.

Number of way to account for this.

Think of the Z-boson wave-function $\psi \sim e^{imt}$ (in the Z rest frame $E \sim m$).

For unstable particle $\psi \to \psi \sim e^{imt}e^{-\Gamma t/2}$ (to account for the decay rate)

Implies $\psi^*\psi \sim e^{-\Gamma t} = e^{-\frac{t}{\tau}}$

$\Rightarrow$ unstable particles can describe by $m \to m - i\Gamma/2$

$$m_Z^2 \to \left(m_Z^2 - \frac{i\Gamma_Z}{2}\right)^2 = m_Z^2 - im_Z\Gamma_Z - \frac{1}{4}\Gamma_Z^2$$

For well-defined particles $\Gamma_Z << m_Z \Rightarrow$ drop terms $\mathcal{O}(\Gamma_Z)$.

So,

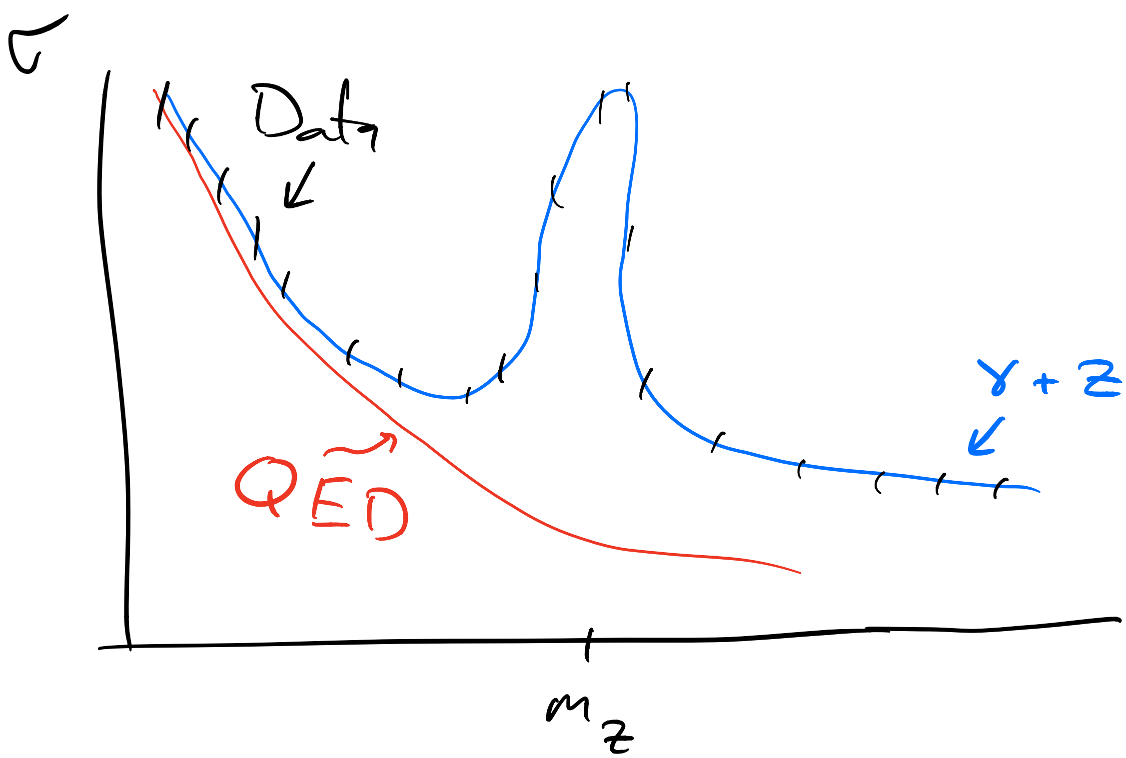
$$M_Z \sim \frac{g_Z^2}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$$

$$\sigma \sim |M_Z|^2 \sim \left|\frac{1}{E_{EM}^2 - m_Z^2 + i m_Z \Gamma_Z}\right|^2 = \frac{1}{\left(E_{EM}^2 - m_Z^2\right)^2 + m_Z^2 \Gamma_Z^2}$$

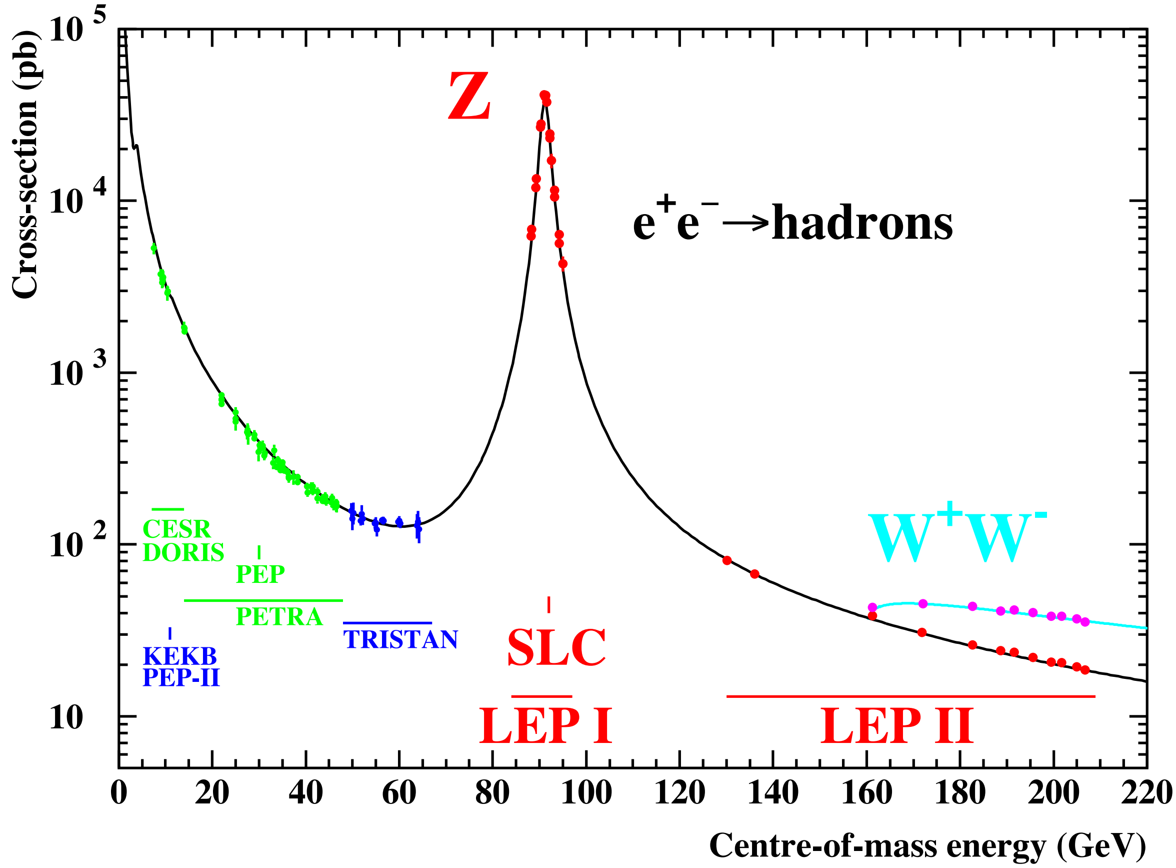
$\Rightarrow ee \to Z$ cross section sharply peaked at $E_{CM} = m_Z$.

This dependence on mass referred to as "Breit-Wigner" distribution.

Cartoon of the behaviour:



Actual data compared with Electro-weak theory:



m_Z was measured to 0.002% precision

- Required correcting for distortions of the earth due to the Moon
- Required correcting for electrical currents induced by French train system

Total width measured to be

$$\Gamma_Z = 2.4952 \pm 0.0023\text{GeV}$$

We can do something cool with this...

Remember

$$\Gamma_Z = 3\Gamma_{\ell^+\ell^-} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\bar{\nu}}$$

Only generations observed. Know they come in doublets.

Maybe there is a 4th generation which has been too heavy to be seen... $\begin{pmatrix} \nu_X \\ X \end{pmatrix}$

Would lead to $Z \to \nu_X \bar{\nu}_X$.

Could turn the above around to get an equation for the number of neutrinos

$$N_\nu = \frac{\Gamma_Z - 3\Gamma_{\ell^+\ell^-} - \Gamma_{\text{hadrons}}}{\Gamma_{\nu\bar{\nu}}}$$

- Measure $\Gamma_{\ell^+\ell^-}$ from $ee \to Z \to \mu\mu$
- Measure Γ_{hadrons} from $ee \to Z \to$ jets

$$\Rightarrow N_\nu = 2.9840 \pm 0.0082$$

Exactly 3 generation of light neutrinos ($m_\nu < m_Z/2$)!

$\Rightarrow$ Probably only 3 generations!