Noether's Theorem

Lagrangian may be invariant under some type of variation eg $\phi \to \phi + \delta$

ex $\cancel{\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 \quad \phi \to e^{-i\alpha}\phi \quad \forall \alpha}$

transformation is a symmetry of $\mathcal{L}$

ϕ - complex 2 dof ϕ & ϕ^*

$$\mathcal{L} = (\partial_\mu \phi)(\partial_\mu \phi^*) - m^2 \phi \phi^*$$

Symmetry

$$\phi \to e^{-i\alpha}\phi \quad \phi^* \to e^{i\alpha}\phi^*$$

Whenever have a continuous symmetry (there is an infinitesimal limit)

$$\frac{\delta \mathcal{L}}{\delta \alpha} = 0 = \sum_n \left[\frac{\partial \mathcal{L}}{\partial \phi_n} \frac{\delta \phi_n}{\delta \alpha} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta(\partial_\mu \phi_n)}{\delta \alpha} \right]$$

$\phi_n = \{\phi, \phi^*\}$

$$= \sum_n \left\{ \underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi_n} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \right]}_{=0 \ \text{E/L}} \frac{\delta \phi_n}{\delta \alpha} + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right] \right\}$$

$\Rightarrow \quad \partial_\mu J_\mu = 0 \qquad J_\mu = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha}$ "Noether Current"

J_μ - "conserved current"

H.W. $\left(J_\mu = -i(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) \right)$

total charge $\equiv Q = \int d^3x \, J_0 \qquad \partial_t Q = \int d^3x \, \partial_t J_0 = \int d^3x \, \vec{\nabla} \cdot J = 0$

J vanishes on boundary

Q does not change with time!

Very General + Important theorem "Noethers Theorem"

If $\mathcal{L}$ has "enjoys" a continuous symmetry, there exists an associated current that is conserved.

$$\phi(x) \to \phi(x+\xi) = \phi(x) + \xi^{\nu} \partial_{\nu} \phi(x)$$

↖ this leaves $\mathcal{L}$ invariant gives S

energy-momentum tensor Gives a 4-vector of noethers currents

$\Rightarrow$ noethers theorem tells us why energy + mom are conserved.

Cross Sections & Decay Rates

20th century witnessed development of collider physics
effective means to determine which | particles exist
| their properties
| interactions

Rutherford discovery of nucleus using α 1911
Andersons discovery of anti-electrons 1932 } Nature

around 1930's manmade collisions started wing.
1 MeV

Now 13 TeV @ LHC

Collisions map free fixed momenta initial state
$\longrightarrow$ final fixed momenta states

QM predicts probabilities for projections to occur.
Probabilities typically depend on parameters (angles, momenta)

$P(v_1 \ldots v_n)$ ~ differential probabilities

Given by $|\langle \psi_{final}\ +\infty | \psi_{Initial}\ -\infty \rangle|^2$

$\langle f | S | i \rangle$ S-matrix

QFT will tell us how to calculate S
given some Lagrangian. (next week)

S-matrix elements are the primary objects of interest for particle physics.

In this lecture we will relate S-matrix elements to scattering | cross sections / decay rates which we can directly measure experimentally.

Aside

Probabilities dimensionless [0-1], absolute => extremely subtle

to calculate P need all possible outcomes a priori!

QM => need complete basis. Impossible in QFT | # final states ∞ | final states not fully known

usually

↳ not for decays

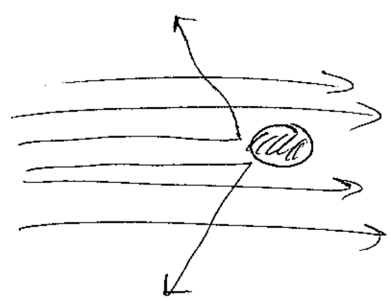
Cross Sections

Natural quantity to measure.

eg Rutherford was interested in size of nucleus (r_n)

By colliding α-particles w/ gold foil and measuring how many particles are scattered, can determine $\sigma = \pi r^2$

Single nucleus



$$\sigma = \frac{\#\text{-scattered}}{\text{time} \times \left(\begin{smallmatrix}\text{Number density}\\ \text{in beam}\end{smallmatrix}\right) \times \text{velocity beam}} = \frac{1}{T}\,\frac{1}{\Phi}\,N$$

T: time, Φ: flux, N: # scatters

Real experiment

other factors: # density of nuclei in foil

cross sectional area of the beam (if smaller than foil)

This stuff + T & Φ depend on details of experiment

In contrast σ - property of particle being scattered

In QM generalize notion of cross section area to "cross section" — units of area
- abstract measure of interaction strength

eg: Classically ~~the~~ the α will either scatter or not.
QM l_g there is some probability for scattering.

$$d\sigma = \frac{1}{T}\frac{1}{\Phi}\, dP \rightarrow \text{QM probability of scattering}$$

Φ → normalized to one particle

$d\sigma$, dP } differential in kine vars θ's P's

# of scatters

$$dN = L \times d\sigma$$

→ "integrated luminosity" (take eq as definition L)

So number of observed events is ~~a~~ direct measurement of cross section. (See in presentations & papers)

Relate to S-matrix

practically impossible to collide more than two particles @ a time.

$|i\rangle$ will always be 2 particle state

$$P_1 + P_2 \rightarrow \{P_j\}$$

Rest frame of one particle

$$\Phi = \frac{|\vec{v}|}{V}$$

CoM frame

$$\Phi = \frac{|v_1 - v_2|}{V}$$

So,

$$d\sigma = \frac{V}{T}\frac{1}{|v_1 - v_2|}\, dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle}\, d\Pi$$

$d\Pi \rightarrow$ region of final state momenta we are considering

On interval size L

p available are ~~...~~ $p_n = \frac{2\pi n}{L}$

$$N = \int \frac{V}{(2\pi)^3} d^3p \qquad d\Pi = \prod_j \frac{V}{(2\pi)^3} d^3p_j$$

$\hookrightarrow N_S = P \times N$

$\hookrightarrow$ over final state particles

$\langle f|f\rangle = \langle i|i\rangle \neq \mathbb{1} \leftarrow$ Not L. I

$$\langle p'|p\rangle = (2\pi)^3\, 2E\, \delta^3(p'-p)$$

$$\langle p|p\rangle = (2\pi)^3\, 2E_p\, \delta^3(0)$$

$$= 2E_pV$$

"Regulated by V

$\delta^3(p) = \frac{1}{(2\pi)^3}\int d^3x\, e^{i\vec{p}\cdot\vec{x}}$

$\delta^3(0) = \frac{1}{(2\pi)^3}\int d^3x = \frac{V}{(2\pi)^3}$

$\Rightarrow$

$$\langle i|i\rangle = \langle p_1 p_2|p_1 p_2\rangle = 2E_1V\, 2E_2V$$

$$\langle f|f\rangle = \prod_j (2E_jV)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated perturbatively

$$S = \underbrace{\mathbb{1}}_{\text{free theory}} + i\underbrace{T}_{\text{Perturbations "small"}}$$