Lecture 27

Spontaneous Symmetry Breaking "Hidden Symmetry"

Last time we ran into a crisis. Needed both massive boson (force carrier) and gauge invariance. Cant get this from m^2A^2.

Difficult Subject We will build up to the full picture by going through a few toys.

Toy 1

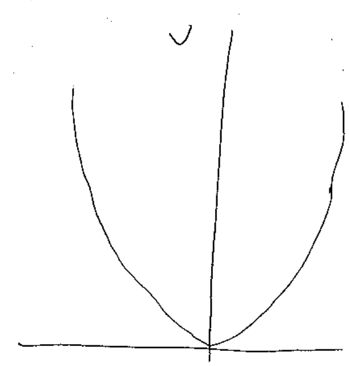
$$L = T - V = \frac{1}{2}(\partial\phi)^2 - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

Note:

- L invariant under $\phi \to -\phi$ (dropped higher order terms)
- we will require $\lambda > 0$
- note the mass sign come in with a relative "-" sign

Case a) $\mu^2 > 0$
Describes a scalar field with mass = μ.

V looks like:

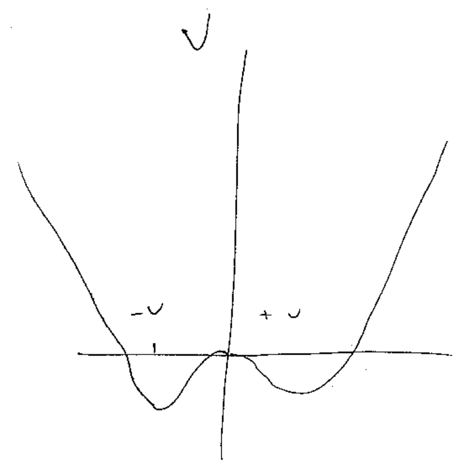


the $\lambda\phi^4$ term leads to a direct four-point ϕ interaction.

Case b) $\mu^2 < 0$

Now the L has a mass term with the wrong sign. (Relative "+" sign of the mass wrt to the kinetic term)

V now looks like:



Minimum at:

$$\frac{\partial V}{\partial \phi} = \phi(\mu^2 + \lambda\phi^2) = 0$$

$$\phi = \pm v \qquad\qquad v = \sqrt{-\mu^2 \lambda}$$

Now, $\phi = 0$ does not correspond to a minimum.

Perturbative calculations should involve the expansion about minimum. (either +v or -v)

Take +v at random ("Spontaneously")

$$\phi(x) = v + \eta(x)$$

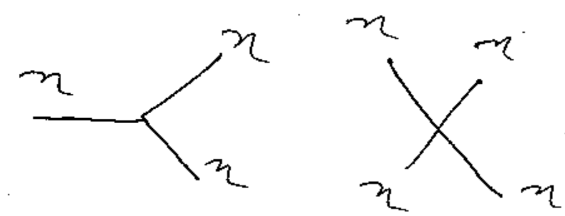
where here $\eta(x)$ describes fluctuations about the minimum.

Can then rewrite L as

$$L' = \frac{1}{2}(\partial\eta)^2 - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{2}\lambda\eta^4 + \text{constants}$$

Note that now the η^2 mass term has the correct sign with $m_\eta = \sqrt{-2\mu^2}$.

Other terms give 3-point and 4-point η interactions:



To summarize,

- Mass of η was "generated" or "revealed" by "Spontaneous Symmetry Breaking"
- The $\phi \to -\phi$ symmetry of L is broken by the choice of the ground state.

This is an example that is seen in (very interesting) condensed matter systems.

(eg: large ferromagnetic. Spins will align. All directions equally likely. Ground stat will be spins aligned in some direction This "choice" of which way the spins point in the ground state breaks the rotational symmetry of the problem)

Superconductors and the buckling of a Needle are other examples of Spontaneous Symmetry Breaking.

Toy 2 Repeat above with complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

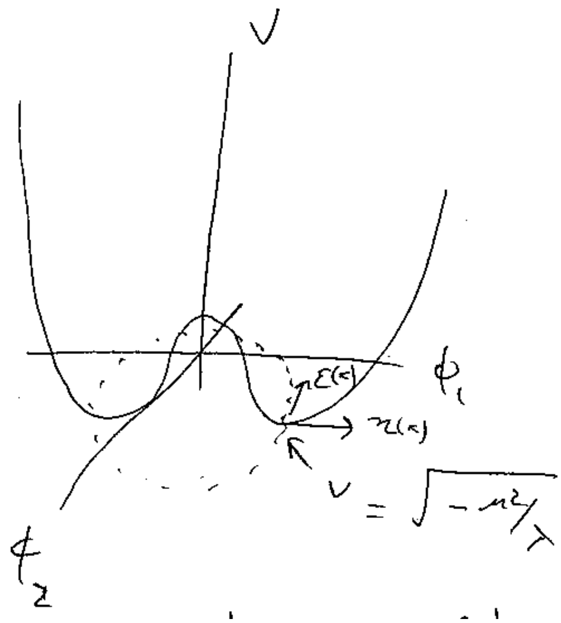
Will now consider the Lagrangian:

$$L = \partial_\mu\phi^*\partial^\mu\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

which is invariant under $\phi \to e^{i\alpha}\phi$ for arbitrary number α. (Global U(1) gauge symmetry.)

Now will take $\lambda > 0$ and $\mu^2 < 0$:

$$L = \frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$



Again, need to expand about min.
With out loss of generality expand around $\phi_1 = v$ and $\phi_2 = 0$.
(like picking +v before)

$$\phi(x) = \sqrt{\frac{1}{2}}(v + \eta(x) + i\epsilon(x))$$

Now get a new Lagrangian

$$L' = \frac{1}{2}(\partial\epsilon)^2 + \frac{1}{2}(\partial\eta)^2 + \mu^2\eta^2 + O(\eta^3) + O(\eta^4) + O(\epsilon^3) + O(\epsilon^4)$$

with $\mu^2\eta^2 = -\frac{1}{2}m_\mu^2\eta^2$ mass term for η just as before.

However now have ϵ which is a mass-less scalar "Goldstone Boson".

Problem, tried to give mass to a gauge boson and created a mass-less boson as well.

Intuitively, direction along ϵ is flat $\Rightarrow$ no resistance to excitations along that direction.
Crisis

Have not observed these mass-less gauge bosons.

Lets try a local gauge theory (A miracle is about to happen...)

Higgs Mechanism U(1) simplest example
(SM does this in $SU(2)_L$, getting closer...)

$$\phi \to e^{i\alpha(x)}\phi \qquad \text{for arbitrary function } \alpha(x)$$

As for EM

$$D_\mu = \partial_\mu - \underbrace{ieA_\mu}_{\text{gives coupling of } \phi \text{ to A}}$$

$$A_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$L = (D_\mu \phi)^*(D^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(if $\mu^2 > 0$ QED w/massive charged scalar of mass μ and a 4-ϕ interaction term)

Now for $\mu^2 < 0$, Again we do $\phi(x) \to \sqrt{\frac{1}{2}}\,(v + \eta(x) + i\epsilon(x))$

$$L' = \frac{1}{2}(\partial\epsilon)^2 + \frac{1}{2}(\partial\eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A^2 - evA_\mu\partial^\mu\epsilon - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{(interactions)}$$

3 particles (Apparently...)

$$\begin{aligned} m_\epsilon &= 0 \text{ (still problem)} \\ m_\eta &= \sqrt{2\lambda v^2} \\ m_A &= ev \text{ dynamically generated mass for gauge field} \end{aligned}$$

This however cant be the spectrum because there are only 4 DoF before the expansion (2 scalar + 2 mass-less spin-1), but now seems like 5 = 2 (scalars) + 3 (massive spin-1).

So some of these apparent DoF are nonphysical.

Can see this explicitly by noticing that:

$$\frac{1}{2}(\partial\epsilon)^2 + evA_\mu\partial^\mu\epsilon + \frac{1}{2}e^2v^2A^2 = \frac{e^2v^2}{2}\left[A_\mu + \frac{1}{ev}\partial_\mu\epsilon\right]^2$$

Now I can pick a gauge where

$$A_\mu \rightarrow A_\mu - \frac{1}{ev}\partial_\mu\epsilon$$

$$L'' = \frac{1}{2}(\partial\eta)^2 - \lambda v^2\eta^2 + \frac{1}{2}e^2v^2A^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 + \frac{1}{2}e^2A^2\eta^2 + ve^2A^2\eta - \frac{1}{4}F^2$$

Now Goldstone boson does not appear in the theory!

The apparent extra DoF was spurious (only corresponds to a gauge transformation)

Spectra (Physical)

2 massive interacting particles

A_μ - 3 DoF massive Spin-1

η - 1 DoF massive Spin-0

η-"Higgs boson"

Mass-less Goldstone Boson "eaten" by the A_μ to become the extra DoF needed for the longitudinal polarization.

This is called the "Higgs Mechanism".