Lecture 13

Cross Sections And Decay Rates

20th century witnessed the development of collider physics. Effective means to determine which particles exist and their properties and interactions.

- Rutherford discovery of the nucleus using α 1911
- Andersen's discovery of anti-electrons 1932

These were made with "Natural accelerators" α's or cosmic rays

Around 1930 man made collisions started winning.

eg: 1 MeV

Now 13 TeV at the LHC.

Collisions map free fixed momenta initial states → final fixed momentum states.

QFT predicts probability for projections to occur.

Probabilities typically dependant on parameters (angles, momenta, etc)

$$P(v_1, ...v_n) \text{ - differential probabilities}$$

Given by

$$|\langle\psi_{final}, +\infty|\psi_{initial}, -\infty\rangle|^2$$

$$\langle f|S|i\rangle \qquad \text{S-matrix}$$

QFT tell us how to calculate S given some Lagrangian (next week)

S-matrix elements are the primary object of interest for particle physics.

S-matrix elements always calculated pertubatively

$$S = \underbrace{1}_{\text{free theory}} + \underbrace{iT}_{\text{perturbatively small}}$$

QFT + Lagrangian gives a procedure (recipe) for calculating S

Very nice interpretation in terms of picture "Feynman diagrams"

QM Perturbation Theory

$H = H_0 + V$

Remember we are interested in how some free state at early times $(-\infty)$ evolve to some (potentially) other free state at late times.

At early times have a state with a given energy E, which is an eigenstate of H_0

$$H_0 |\phi\rangle = E |\phi\rangle$$

Including the interaction piece, will also be eigenstate of the full Hamiltonian with the same energy.

$$H |\psi\rangle = E |\psi\rangle$$

Now we can formally write,

$$|\psi\rangle = |\phi\rangle + \frac{1}{E - H_0} V |\psi\rangle$$

which can be verified by multiplying through by $(E - H_0)$.

Whats happening here is that the interaction at intermediate times is inducing transitions among the states $|\phi\rangle$, which are non-interacting at early (and late) times.

So the full state $|\psi\rangle$ is given by the free state $|\phi\rangle$ plus a scattering term.

Really want to express the full state $|\psi\rangle$ entirely in terms of $|\phi\rangle$.

We do this by defining operator T: $V |\psi\rangle = T |\phi\rangle$.

This gives us

$$|\psi\rangle = |\phi\rangle + \frac{1}{E - H_0} T |\phi\rangle$$

or

$$\begin{aligned} V |\psi\rangle &= V |\phi\rangle + V \frac{1}{E - H_0} T |\phi\rangle \\ &= T |\phi\rangle \end{aligned}$$

So we get a nice iterate equation for T

$$T = V + V\frac{1}{E - H_0}T$$

which we can solve perturbatively in V.

eg

$$T = V + V\frac{1}{E - H_0}V + V\frac{1}{E - H_0}V\frac{1}{E - H_0}V + ...$$

Of course, we are interested in inner products of these with the initial/final states

$$\underbrace{\langle\phi_f|T|\phi_i\rangle}_{T_{fi}} = \underbrace{\langle\phi_f|V|\phi_i\rangle}_{V_{fi}} + \sum_j \frac{\langle\phi_f|V|\phi_j\rangle\langle\phi_j|V|\phi_i\rangle}{E - H_0} + ...$$

<u>Example of scattering of 2 electrons</u>

$$|i\rangle = |e_1, e_2\rangle \qquad\qquad |f\rangle = |e_3, e_4\rangle$$

$$T_{fi} = V_{fi} + \sum_n V_{fn}\frac{1}{E_i - E_n}V_{ni} + ...$$

- E_i- is the initial (= final) energy
- E_f - is the energy of the intermediate state

Now, the $\sum_n$ runs over energy thing in the Hilbert (Fock) space, however only certain states will be non-0.

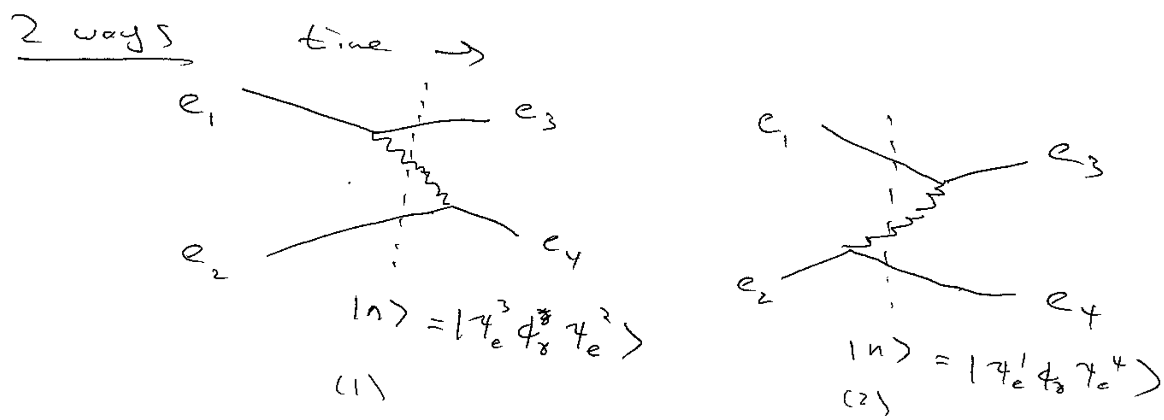
In relativistic theory, the action-at-a-distance of EM is replaced by a process where 2 electrons interact with a γ which travels at c. (This tells us there should be γ in the intermediate state)

$$V \sim e\int d^3x\psi\phi\psi \qquad \text{(Ignoring spin)}$$

this operator will have terms that go like ($\sim a^\dagger_{e_3}\, a^\dagger_\gamma\, a_{e_1}$)

However, here all terms involve $a^\dagger_\gamma$.
Because $|i\rangle$ and $|f\rangle$ do not contain a γ, $V_{fi} = 0$

to get a non-zero term, we need $|n\rangle$ with a photon.



$$T_{fi} = \frac{\langle e_3 e_4|V|e_3\gamma e_2\rangle\,\langle e_3\gamma e_2|V|e_1 e_2\rangle}{(E_1+E_2)-(E_3+E_4+E_\gamma)} + \text{(2nd term)}$$

Note: $E_n \neq E_i$ which is allowed by uncertainty principle.

Look at

$$\langle e_3\gamma e_2|V|e_1 e_2\rangle = \langle \gamma e_3|V|e_1\rangle$$

(up to overall normalization from $\langle e_2|e_2\rangle$.