Homework Set #10

Due Date: Before class **Monday** April 15th

1) Discrete Symmetries *(10 points)*

a. Under Parity:

$$\vec{E} \to -\vec{E}, \vec{\nabla} \to -\vec{\nabla}, \rho \to \rho, \vec{B} \to \vec{B}, \vec{J} \to -\vec{J}, \frac{\partial}{\partial t} \to \frac{\partial}{\partial t}$$

So

$$\vec{\nabla} \cdot \vec{E} = \rho, \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

Are invariant

b. Under Charge Conjugation

$$\vec{E} \to -\vec{E}, \vec{\nabla} \to \vec{\nabla}, \rho \to -\rho, \vec{B} \to -\vec{B}, \vec{J} \to -\vec{J}, \frac{\partial}{\partial t} \to \frac{\partial}{\partial t}$$

So ME are are invariant.

c. Under Time Reversal

$$\vec{E} \to \vec{E}, \vec{\nabla} \to \vec{\nabla}, \rho \to \rho, \vec{B} \to -\vec{B}, \vec{J} \to -\vec{J}, \frac{\partial}{\partial t} \to -\frac{\partial}{\partial t}$$

So ME are are invariant.

d. We assume that the Hamiltonian is time independent. So $\mathcal{T} H = H$

Then

$$i\frac{\partial}{\partial t}\psi = H\psi \to \mathcal{T}(i)\mathcal{T}(\frac{\partial}{\partial t})\psi = \mathcal{T}(H)\psi$$

or

$$\mathcal{T}(i) - (\frac{\partial}{\partial t})\psi = H\psi$$

Which is invariant if $\mathcal{T}(i) = -i$

2) Kinematics of the IceCube Experiment. *(10 points)*

a) Read the wikipedia article on Cherenkov radiation: https://en.wikipedia.org/wiki/Cherenkov_radiation

b) $E = E_\nu, m = m_N$

$$p_\mu = \frac{Em}{m + E(1 - \cos\theta)}(1, 0, sin\theta, cos\theta)$$

$$p_n = \left(E + m - \frac{Em}{m + E(1 - \cos\theta)}, 0, -\frac{Em\sin\theta}{m + E(1 - \cos\theta)}, E - \frac{Em\cos\theta}{m + E(1 - \cos\theta)}\right)$$

c) Figure 2 is a plot from the IceCube experiment that shows the deposited energy into the experiment versus the declination angle of the anti-muon from high-energy neutrino scattering. This plot shows the 37 highest energy events recorded by IceCube. There are three fantastically high energy events observed above 1000 TeV (= 1 Peta-electron Volt) of deposited energy. The highest energy event is affectionately called "Big Bird," while the second and third highest energy events are called "Bert" and "Ernie." (Actually, all of the events on this plot are named after Muppets.)

If the Big Bird anti-muon deposited 2 PeV of energy in IceCube, then what is the corresponding largest and smallest energy that the initial anti-neutrino could have had?

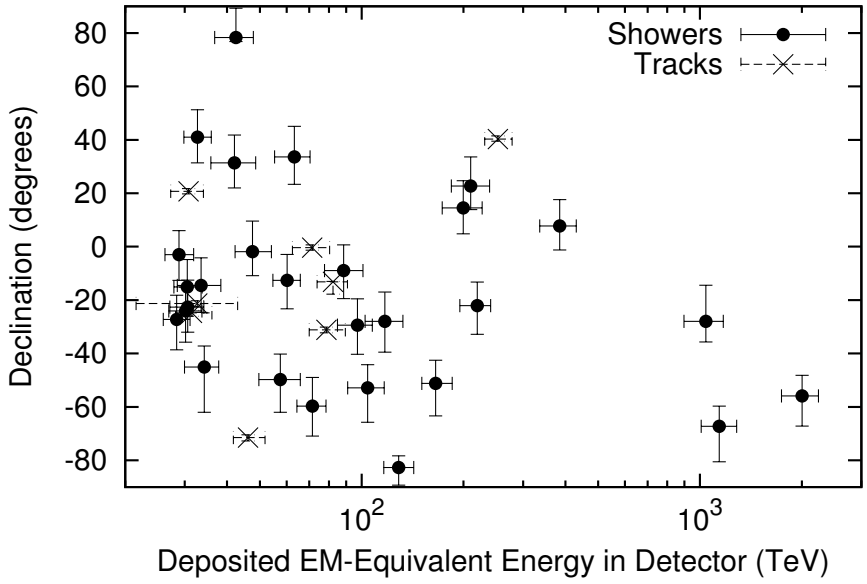

Figure 1

d) What are the corresponding maximum and minimum scattering angles in degrees between the initial anti-neutrino and the measured Big Bird anti-muon? Call the maximum angle $\theta_{\max}$ and the minimum angle $\theta_{\min}$. For simplicity, set the mass of the proton/neutron to $m_N = 1$ GeV. With any possible anti-neutrino energy, is the momentum of the anti-muon close to the direction of the anti-neutrino? You can safely assume that $m_N << E_\mu$, so Taylor expanding would likely help.

This property of the scattering angle from part is extremely important for determining the astrophysical source of the high energy neutrinos observed in IceCube. In 2016, scientists on the Fermi Gamma Ray Space Telescope found evidence that the neutrino that was responsible for the Big Bird muon was created in a blazar, an enormously energetic radiation source believed to be generated by a supermassive black hole at the center of a galaxy.

e) Draw the Feynman diagram for $\bar{\nu}_\mu + p \rightarrow \mu^+ n$ assuming a point-like interaction and treating the proton and neutron as fundamental particles.

f) Use dimensional analysis to estimate the size of the $\bar{\nu}_\mu + p \rightarrow \mu^+ n$ cross section. *Hint: The dependence on E_ν is very weak so express your answer only in terms of m_p and m_W.*

g) IceCube ran for three years to find three events which each deposited more than a PeV of energy into the antarctic ice. Using the cross section above, approximately how many PeV neutrinos passed right through IceCube during that time? The total volume of the IceCube detector is about 1 km^3, and the density of ice is about 1000 $\frac{kg}{m^3}$.

Figure 3 is a display of the photomultiplier tube response in the Big Bird neutrino event. The strings on this figure correspond to the strings of photo-multiplier tubes, while the bubble region represents the detection of Cherenkov radiation. Larger bubbles corresponds to higher energies Cherenkov light detected.

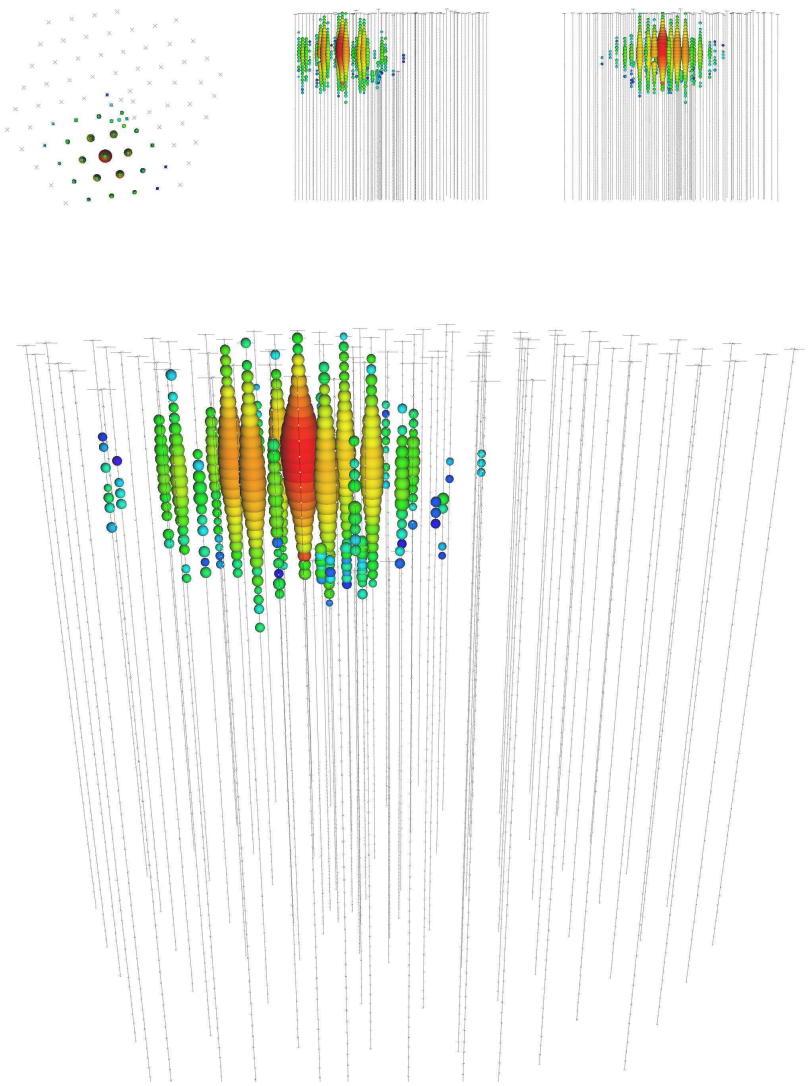

Figure 2