

Homework Set #11

Due Date: Before class Friday April 26th

1) Spontaneous Symmetry Breaking

(10 points)

- a. Let $V(\phi)$ be

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

where ϕ is a real scalar field and μ and λ are constants. If $\mu^2 > 0$, where are the minima?

- b. If $\mu^2 < 0$, where are the minima?

- c. Now let ϕ be a complex field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$. And let,

$$\mathcal{L} = (\partial_\mu\phi^*)(\partial^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

with $\mu^2 < 0$ and $\lambda > 0$. Expand the Lagrangian about the minimum as we did in class with $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\epsilon(x))$. Write out all the terms.

- d. Do the same thing with the U(1) Lagrangian. Expand about the minimum and write the Lagrangian including all interaction terms.

2) Higgs Self-Interaction

(10 points)

- a. Let $V(\phi)$ be

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

with $\mu^2 < 0$ and $\lambda > 0$. where ϕ is a real scalar field and μ and λ are constants, with $\mu^2 < 0$ and $\lambda > 0$. What is the coupling constant associated to the h^3 interaction in terms of m_h and the position of the minimum?

- b. Assume the potential is:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^6$$

with $\mu^2 < 0$ and $\lambda > 0$. What is the position of the minimum? What is the coupling constant associated to the h^3 interaction in terms of m_h and the position of the minimum?

3) LEP: Z and the Higgs

(15 points)

- a Estimate the branching fraction of Z to $\nu\bar{\nu}$?
- b Estimate the ratio of the $ee \rightarrow Z$ to $ee \rightarrow H$ cross sections.
- c LEP collected about 17 million Z bosons, for a discovery and study of the Higgs in direct e^+e^- collisions how many events would LEP have to collect ?
- d If LEP collided electrons at 40MHz like the LHC, how long would it take to collect this data ?
- e You can also produce a higgs by “radiation it off of a virtual Z boson. Draw this diagram.
- f Estimate the cross section for this process.