Midterm

1) Why are energy and momentum conserved in the Standard Model ? What would it mean if we found evidence for non-conservation of Energy ? What about non-conservation of Momentum? *(3 points)*

2) What are three major consequences of combining QM and Relativity? *(3 points)*

3) Muonic Atoms *(7 points)*

a) The muon was the first elementary particle discovered that does not appear in ordinary atoms. Negative muons can, however, form muonic atoms by replacing an electron in an ordinary atom. Estimate the size and binding energy of muonic hydrogen in GeV.

b) What is the ratio of the muonic hydrogen binding energy to that of regular hydrogen?

4) Why do we label particle states by momentum? *(2 points)*

5) What are two restrictions to particle interactions that are a consequence gauge invariance ? *(2 points)*

6) Lorentz Transforms *(4 points)*

a) How does a **massive** particle $|p^\mu, \sigma\rangle$ transform under a little group transformation (W_μ^ν) ?

b) How does a **massless** particle $|p^\mu, \sigma\rangle$ transform under a little group transformation (W_μ^ν) ?

c) How does a **massive** particle $|p^\mu, \sigma\rangle$ transform under a general Lorentz transformation (Λ_μ^ν) ?

d) How does a **massless** particle $|p^\mu, \sigma\rangle$ transform under a general Lorentz transformation (Λ_μ^ν)?

7) Can the SM have interactions between fermions and massless particles with Spin 2 ? If not, why not. What about interactions between mass-less Spin 3 particles and fermions ? If not, why not. *(2 points)*

8) Yukawa's Theory. *(2 points)*

In the 1930s, Hideki Yukawa predicted the existence of a new particle, now called the pion, which is responsible for binding protons and neutrons together in atomic nuclei. Estimate the mass of the pion from the assumed range of the force. (10^{-15} meters). Express the mass in GeV.

9) How many generators does the Lorentz group have ? *(3 points)*

What transformations do they correspond to ?

10) Fields *(2 points)*

In QFT why do we write interactions in terms of fields (Fourier transforms of creation and annihilation operators) instead of the creation and annihilation operators directly?

11) Why is the weak interaction so much weaker than then electromagnetic interactions at low energies? *(2 points)*

12) Lagrangians *(3 points)*

Consider the following Lagrangian:

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 + \bar{\psi}[i\gamma_\mu \partial^\mu]\psi + g_1 \phi\psi\psi + g_2 \psi\psi\psi\psi + g_3 \phi\phi\phi\phi$$

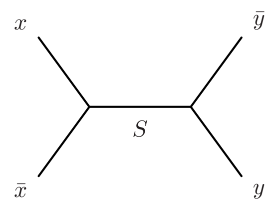
a) What is the dimension of g_1 ?

b) What is the dimension of g_2 ?

c) What is the dimension of g_3 ?

13) Feynman Diagrams *(15 points)*

Fermions of type x scatter into fermions of type y through the diagram shown below, where S is a massive scalar. At low energies ($P_x P_y << m_S$) the cross section for this process is given by σ_0. Assume m_x and m_y are both negligible.



a) How does this cross section change if the "S-charge" of the x particle is doubled ? (S-charge being the $x\bar{x} \to S$ coupling)

b) How does the cross section change if the mass of the S-particle is doubled ?

c) At low energies, how does the cross section depend on the center of mass energy of the scattering E_{CM} ? *(Hint: At low energies you can think of the $xx \to yy$ scattering as described by an effective 4-point xxyy interaction, as Fermi did for neutrino scattering.)*