

Lecture 9

Particles Interactions

“Quantum Field Theory in a week”

Summary From Last Time

Massive:

$|P, \sigma\rangle$ and $U[\Lambda] |P, \sigma\rangle = \sum_{\sigma'} R_{\sigma, \sigma'} |\Lambda P, \sigma'\rangle$, where the R is a rotation matrix.

Massless:

$|P, h\rangle$ and $U[\Lambda] |P, h\rangle = e^{ih\Theta(W)} |\Lambda P, h\rangle$, where the coefficient is just a phase.

Now talk about all these particles states in a more convenient way.

For every given momenta we have stacks of Hilbert space

- infinitely many possibilities for Bosons $0 \rightarrow N$
- fixed number (depending on the spin) for fermions

Ultimately interested in the interactions between particles.

- Defined by some Hamiltonian
- Could specify the Hamiltonian by describing how it acts on all states in the Hilbert space
- Instead for our convenience introduce creation and annihilation operators These keep track of the states in a simple way...

First define the vacuum state: $|0\rangle$. Then define single particle states as:

$$\underbrace{|P\sigma\rangle}_{\text{These are primary}} \equiv a_{p\sigma}^\dagger |0\rangle$$

This defines $a_{p\sigma}^\dagger$.

$$|P_1\sigma_1, P_2, \sigma_2\rangle = a_{p_1\sigma_1}^\dagger a_{p_2\sigma_2}^\dagger |0\rangle$$

etc. etc. etc.

Similarly, can define annihilation operators

$$\begin{aligned} a_{P,\sigma} |0\rangle &= 0 \\ a_{P,\sigma} |P, \sigma\rangle &= |0\rangle \end{aligned}$$

a - removes states.

Can encode boson/fermion statistics in a and a^\dagger .

Bosons	Fermions
$[a_{P_1,\sigma_1}^\dagger, a_{P_2,\sigma_2}^\dagger] = 0$	$\{a_{P_1,\sigma_1}^\dagger, a_{P_2,\sigma_2}^\dagger\} = 0$
$[a_{P_1,\sigma_1}, a_{P_2,\sigma_2}] = 0$	$\{a_{P_1,\sigma_1}, a_{P_2,\sigma_2}\} = 0$