Homework Set #1

Solutions

2) Solid State Physics *(5 points)*

(a) We worked out in class $r_{\text{atom}} \sim \frac{1}{Z\alpha m_e}$. (using $E \sim -\frac{Z\alpha}{r} + \frac{p^2}{m_e}$ and $p \times r \sim 1$) So a solid has a spacing of r_{atom}.

(b) To probe distances of order r_{atom} need to photons with Energy $\sim \frac{1}{r_{\text{atom}}} \sim Z\alpha m_{\text{electron}}$. Assuming $Z \sim 10$,
Energy $\sim 10 \cdot 10^{-2} \cdot 10^{-3}$ GeV $\sim 10^{-4}$GeV $\sim 10^{2}$ keV

(c) 10^2 keV photons are x-rays.

3) Strength of Gravity on Earth *(5 points)*

(a) From class, $R_{\text{Planet}} \sim \sqrt{\frac{\alpha}{\alpha_G}} \times r_{\text{atom}}$, $\rho_{\text{solid}} \sim \frac{Zm_{\text{proton}}}{r^3_{\text{atom}}}$, and $M_{\text{Planet}} \sim \rho_{\text{Solid}} \times R^3_{\text{Planet}}$

$$g_{\text{local}} \sim G_N \frac{M_{\text{Planet}}}{R^2_{Planet}} \sim \frac{G_N Z m_{\text{proton}} R_{\text{Planet}}}{r^3_{\text{atom}}} \sim \sqrt{\alpha_G \alpha}\frac{Z}{m_{\text{proton}} r^2_{\text{atom}}}$$

(b)

$$\begin{aligned} g_{\text{local}} &\sim (\alpha_G \alpha)^{1/2} \cdot Z \cdot r^{-2}_{\text{atom}} \cdot m^{-1}_{\text{proton}} \\ &\sim (10^{-39} 10^{-2})^{1/2} \cdot 10 \cdot 10^{-8} \text{ GeV}^{-2} \cdot \text{GeV} \\ &\sim 10^{-27.5}\text{GeV} \end{aligned}$$

Need to convert GeV to $\frac{m}{s^2}$ which has units of [distance]×[time]$^{-2}$. c has units of [distance]×[time]$^{-1}$, h has units of [energy]×[time]. So, can convert from [energy] to [distance]×[time]$^{-2}$ by multiplying by c/h. c = 10^8 m/s, h = 10^{-15} eV·s = 10^{-24} GeV· s. So, $c/h = 10^8 \cdot 10^{24} = 10^{32} \frac{m}{\text{GeV}s^2}$

$$\begin{aligned} g_{\text{local}} &\sim 10^{-27.5}\text{GeV} \times (1) \\ &\sim 10^{-27.5}\text{GeV} \times \frac{c}{h} \\ &\sim 10^{-27.5}\text{GeV} \times 10^{32} \frac{m}{\text{GeV}s^2} \\ &\sim 10^4 \frac{m}{s^2} \end{aligned}$$

(c) Not so close to $10\frac{m}{s^2}$, If we has used $r_{\text{atom}} \sim 10^{-10}m$ instead of 10^{-12} which accounts for the screening of multple electrons in the atom. Would have calculated factor of 10^{-4} smaller or g_{local} 1, which is pretty close.

4) Neutron Stars *(5 points)*

(a) Estimate the radius, mass, and speed of sound for neutron stars in terms of α, α_G, m_{proton}, and m_{electron}. (Assume: A neutron star is a solid made of neutrons and $m_{\text{proton}} \sim m_{\text{neutron}}$)

(b) What are you estimated values in mks units ?

(c) Compare your estimates to actual values for Neutron Stars quoted online.

(d) Look up m_{neutron}. How does this compare with the assumption of $m_{\text{proton}} \sim m_{\text{neutron}}$?

5) 2D Rotations *(3 points)*

(a) Show that $R(\Theta) = e^{I\Theta} = cos(\Theta) + I sin(\Theta)$ where, $I = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(b) Show that 2D rotations commute. (ie: $R(\Theta_1)R(\Theta_2) = R(\Theta_2)R(\Theta_1)$)

6) 3D Rotations *(5 points)*

(a) Work out the "algebra" of the generators of 3D rotations J_i.
Where $J_3 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $J_2 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$ $J_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$
(These generators different from the T's derived in class by a factor of i)
Working out the algebra means calculating the commutation relations $[J_i, J_j]$.

(b) Let M be a traceless 2×2 hermitian matrix and U be a 2×2 hermitian matrix with determinant = 1. Show that $M' = U^\dagger M U$ is also traceless and hermitian and that is has the same determinant as M.