Midterm

1) Units/Dimensional Analysis: Muonic Atoms *(6 points)*

a) The muon was the first elementary particle discovered that does not appear in ordinary atoms. Negative muons can, however, form muonic atoms by replacing an electron in an ordinary atom. Estimate the size and binding energy of muonic hydrogen in GeV. A muon is 200 times heavier than an electron.

b) How much bigger/smaller would matter be in a world made out of muonic atoms ?

2) Relativity: How many generators does the Lorentz group have ? What transformations do they correspond to ? *(3 points)*

3) Quantum Mechanics: *(4 points)*

How do position eigenstates transform under Translations ?
eg: $T(\vec{a})\,|\vec{x}\rangle$ =?

How do momentum eigenstates transform under Translations ?
eg: $T(\vec{a})\,|\vec{p}\rangle$ =?

4) Why does combining QM and Relativity require the existence of anti-particles? *(5 points)*

5) What is the little group? Why is it useful ? *(2 points)*

6) Lorentz Transforms *(6 points)*

a) How does a **massive** particle transform under a general Lorentz transformation (Λ^{ν}_{μ}) ?
eg: $U(\Lambda]\,|p^{\mu}, \sigma\rangle$ =?

b) How does a **mass-less** particle $|p^{\mu}, h\rangle$ transform under a general Lorentz transformation (Λ^{ν}_{μ})?
eg: $U(\Lambda]\,|p^{\mu}, h\rangle$ =?

7) Relativistic Wave Equations *(4 points)*

a) What modifications where required to the Schrodinger equation to make it relativistic?

b) What modifications to Maxwell's equations where required to make them relativistic?

8) Fields *(3 points)*

In QFT why do we write interactions in terms of fields (Fourier transforms of creation and annihilation operators) instead of the creation and annihilation operators directly?

9) Lagrangians *(4 points)*

Consider the following Lagrangian:

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 + \bar{\psi}[i\gamma_\mu \partial^\mu]\psi + g_1 \phi\psi\psi + g_2 \psi\psi\psi\psi + g_3 \phi\phi\phi\phi$$

a) What is the dimension of g_1 ?

b) What is the dimension of g_2 ?

c) What is the dimension of g_3 ?

10) Feynman Diagrams *(6 points)*

a) What is the S-matrix ?

b) What is a Feynman diagram ? and what is its relationship to the s-matrix?

11) What are <u>two</u> constraints on the interactions of mass-less Spin-1 particles, apart form coming from a Lorentz-invariant Lagrangian? *(6 points)*

12) Can the SM have interactions between fermions and mass-less particles with Spin 2 ? If not, why not. What about interactions between mass-less Spin 3 particles and fermions ? If not, why not. *(4 points)*