Lecture 7

Review Quantum Mechanics (Dynamics)

$$|\alpha, t_0\rangle \to |\alpha, t\rangle$$

This is what we mean by time evolution.

In QM, then there has to be an operator associated with taking the first state to the second.

Time Evolution Operator: $U(t, t_0)$

$$|\alpha, t\rangle = U(t, t_0)\,|\alpha, t_0\rangle$$

$U(t, t_0)$ Properties

1. $U^\dagger(t, t_0)U(t, t_0) = 1$ Unitary See this from $\langle \alpha t_0 | \alpha t_0\rangle = \langle \alpha t | \alpha t\rangle$
2. $U(t_2, t_0) = U(t_2, t_1)U(t_1, t_0)$ Composition Rule
3. $U(t_0, t_0) = 1$

How can we possibly deterine what the time operator is ???? (Should be getting old by now...)

Start with infintesimal time evolution

$$U(t + \epsilon, t) = 1 - i\Omega\epsilon$$

where Ω is a Hermitian operator (b/c) U is unitary

So,

$$|\alpha, t + \epsilon\rangle = (1 - i\epsilon\Omega)\,|\alpha, t\rangle$$

OR,

$$\Omega\,|\alpha, t\rangle = i\frac{|\alpha, t + \epsilon\rangle - |\alpha, t\rangle}{\epsilon} = i\frac{\partial}{\partial t}\,|\alpha, t\rangle$$

in limit $\epsilon \to 0$

Physical Meaning of Ω:

As before to get the generate for the finite movement you have to exponetiate

$$U(t) = e^{-i\Omega t}$$

Note that Ω has units 1/[time]. Just like energy.

Identify $\Omega = \frac{1}{\hbar}H$ where H is the Hamiltonian operator.

$$i\frac{\partial}{\partial t}|\psi\rangle = \Omega|\psi\rangle = \frac{E}{\hbar}|\psi\rangle$$

Schrodinger Equation

$$i\frac{\partial}{\partial t}|\psi\rangle = H(t)|\psi\rangle$$

Non-relativistically: $H \sim \frac{p^2}{2m} = -\frac{1}{2m}\frac{\partial^2}{\partial^2 x}$

Time Evolution

$$|\alpha, t\rangle = U(t, t_0)|\alpha, t_0\rangle$$

Case I: H is time independent

$$U(t, t_0) = \lim_{N\to\infty}\prod_{i}^{N} e^{-\frac{i}{\hbar}H\Delta t} = e^{\frac{-iH(t-t_0)}{\hbar}}$$