

Lecture 14

From Last time...

$$d\sigma = \frac{V}{T} \frac{1}{|\vec{v}_1 - \vec{v}_2|} dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} \underbrace{d\Pi}_{\text{Region of final state momenta we are considering}}$$

On interval of size L, the momenta of available states are $P_n = \frac{2\pi n}{L}$ (from particle in a box).

\Rightarrow throughout a volume V

$$N = \int \frac{V}{(2\pi)^3} d^3 p$$

$$d\Pi = \prod_j \frac{V}{(2\pi)^3} d^3 p_j$$

where j runs over final state particles.

OK, lets deal with the normalization factors.

Note, $\langle f|f\rangle$ and $\langle i|i\rangle \neq 1$ (The inner products are not Lorentz invariant...)

$$\begin{aligned} \langle p'|p\rangle &= (2\pi)^3 2E \delta^3(p' - p) \\ \langle p|p\rangle &= (2\pi)^3 2E_p \delta^3(0) \\ &= 2E_p V \end{aligned}$$

\Rightarrow

$$\langle i|i\rangle = \langle p_1 p_2 | p_1 p_2 \rangle = 2E_1 V 2E_2 V$$

$$\langle f|f\rangle = \prod_j (2E_j V)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated perturbatively

$$S = \underbrace{1}_{\text{free theory}} + \underbrace{iT}_{\text{perturbatively small}}$$

Know that S matrix should vanish if momentum not conserved

$$\langle f|T|i\rangle = (2\pi)^4 \delta^4(\sum p) \underbrace{M}_{\text{"MatrixElement"}}$$

Now, might worry that we have to square the δ function

$$\begin{aligned} |\langle f|T|i\rangle|^2 &= (2\pi)^8 \delta^4(\sum p) \delta^4(0) |M|^2 \\ &= (2\pi)^4 \delta^4(\sum p) TV |M|^2 \end{aligned}$$

So,

$$\begin{aligned} dP &= \frac{(2\pi)^4 \delta^4(\sum p) TV}{(2E_1 V)(2E_2 V)} \frac{1}{\prod_j (2E_j V)} |M|^2 \prod_j \frac{V}{(2\pi)^3} d^3 p_j \\ &= \frac{T}{V} \frac{1}{(2E_1)(2E_2)} |M|^2 \underbrace{d\Pi_{\text{LIPS}}}_{\text{L.I. Phase space}} \\ &\quad = (2\pi)^4 \delta^4(\sum p) \prod_j \frac{d^3 p}{(2\pi)^3 2E_p} \end{aligned}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|\vec{v}_1 - \vec{v}_2|} |M|^2 d\Pi_{\text{LIPS}}$$

where $\vec{v} = \vec{p}/p_0$

known as “Fermis Golden Rule”

Decay rate probability that a one-particle state turns into a multi-particle state over time T .

$$p_1 \rightarrow \{P_j\}$$

think of it as $1 \rightarrow N$ scattering.

follow same steps as above

$$d\Gamma = \frac{1}{2E_1} |M|^2 d\Pi_{\text{LIPS}}$$

“Feynman Diagrams”

Last piece we need is the LI matrix element

$$M(1 + 2 \rightarrow 3 + 4 + \dots n) = \langle \{p_j\}_{\text{out}} | p_1 p_2 \rangle_{\text{in}}$$

This represents an element of the S-matrix.

QFT + Lagrangian gives a procedure (recipe) for calculating M

Very nice interpretation in terms of picture “Feynman diagrams”

QM Perturbation Theory

$$H = H_0 + V$$

Remember we are interested in how some free state at early times ($-\infty$) evolve to some (potentially) other free state at late times.

At early times have a state with a given energy E, which is an eigenstate of H_0

$$H_0 |\phi\rangle = E |\phi\rangle$$

Including the interaction piece, will also be eigenstate of the full Hamiltonian with the same energy.

$$H |\psi\rangle = E |\psi\rangle$$

Now we can formally write,

$$|\psi\rangle = |\phi\rangle + \frac{1}{E - H_0} V |\psi\rangle$$

which can be verified by multiplying through by $(E - H_0)$.

Whats happening here is that the interaction at intermediate times is inducing transitions among the states $|\phi\rangle$, which are non-interacting at early (and late) times.

So the full state $|\psi\rangle$ is given by the free state $|\phi\rangle$ plus a scattering term.

Really want to express the full state $|\psi\rangle$ entirely in terms of $|\phi\rangle$.

We do this by defining operator T: $V |\psi\rangle = T |\phi\rangle$.

This gives us

$$|\psi\rangle = |\phi\rangle + \frac{1}{E - H_0} T |\phi\rangle$$

or

$$\begin{aligned} V |\psi\rangle &= V |\phi\rangle + V \frac{1}{E - H_0} T |\phi\rangle \\ &= T |\phi\rangle \end{aligned}$$

So we get a nice iterate equation for T

$$T = V + V \frac{1}{E - H_0} T$$

which we can solve perturbatively in V.

eg

$$T = V + V \frac{1}{E - H_0} V + V \frac{1}{E - H_0} V \frac{1}{E - H_0} V + \dots$$

Of course, we are interested in inner products of these with the initial/final states

$$\underbrace{\langle \phi_f | T | \phi_i \rangle}_{T_{fi}} = \underbrace{\langle \phi_f | V | \phi_i \rangle}_{V_{fi}} + \sum_j \frac{\langle \phi_f | V | \phi_j \rangle \langle \phi_j | V | \phi_i \rangle}{E - H_0} + \dots$$

Example of scattering of 2 electrons

$$|i\rangle = |e_1, e_2\rangle \quad |f\rangle = |e_3, e_4\rangle$$

$$T_{fi} = V_{fi} + \sum_n V_{fn} \frac{1}{E_i - E_n} V_{ni} + \dots$$

- E_i - is the initial (= final) energy
- E_f - is the energy of the intermediate state

Now, the \sum_n runs over energy thing in the Hilbert (Fock) space, however only certain states will be non-0.

In relativistic theory, the action-at-a-distance of EM is replaced by a process where 2 electrons interact with a γ which travels at c . (This tells us there should be γ in the intermediate state)

$$V \sim e \int d^3x \psi \phi \psi \quad (\text{Ignoring spin})$$

this operator will have terms that go like $(\sim a_{e_3}^\dagger a_\gamma^\dagger a_{e_1})$

However, here all terms involve a_γ^\dagger .

Because $|i\rangle$ and $|f\rangle$ do not contain a γ , $V_{fi} = 0$

to get a non-zero term, we need $|n\rangle$ with a photon.

