Lecture 28

Bonus

Clarification from last time ...

used “obvious” expansion about $\phi_1 = v$ and $\phi_2 = 0$

$$\phi = (v + \eta + i\epsilon)$$

Put this into L and got

$$L' = \frac{1}{2}(\partial\epsilon)^2 + \frac{1}{2}(\partial\eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A^2 - evA_\mu\partial^\mu\epsilon - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{(interactions)}$$

then we said that this looked like there was unphysical Dof and we starter over with

$$\phi = (v + h)e^{i\theta/v}$$

A (potentially) cleaner way to see whats happening is to notice

$$\frac{1}{2}(\partial\epsilon)^2 + evA_\mu\partial^\mu\epsilon + \frac{1}{2}ev^2A^2 = \frac{1}{2}\left[A_\mu + \frac{1}{ev}\partial_\mu\epsilon\right]^2$$

Now I can pick a gauge where

$$A_\mu \to A_\mu = \frac{1}{ev}\partial_\mu\epsilon$$

$\Rightarrow$ I only have the $\frac{1}{2}e^2v^2A^2$ term.

(of course this is all equivalent as above)

Electroweak Unification

Now that we have seen how we can use the Higgs Mechanism to generate masses for gauge bosons, we will see it works in the SM.

We will take the L_{SM} with 3 massless spin-1 bosons and add a scalar (Higgs) w/the "Mexican hat" potential $\mu^2 < 0$.

So we first need to talk about the initial L_{SM} w/o the Higgs.

This will lead us to talk about the Weak interaction group and "Electroweak" unification. (Major step forward in physics)

QED (EM)

Gauge invariance led us to the L_{QED}

$$\psi_e(x) \to e^{iq\alpha(x)}\psi_e(x)$$

for arbitrary $\alpha(x)$ which is function of spacetime.

We say that this is a "local U(1) phase transformation" (a number is a generator of U(1))

What will it take to make physics invariant under this transformation ?

Couple way to think about this.

Lets see what to the equations of motion (here the dirac equation)

$$(i\gamma^\mu\partial_\mu - m)\psi_e = 0 \to (i\gamma^\mu\partial_\mu - m)e^{iq\alpha(x)}\psi_e = (i\gamma^\mu\partial_\mu - q\gamma^\mu\partial_\mu\alpha - m)e^{iq\alpha(x)}\psi_e = 0$$

or

$$i\gamma^\mu\left[(\partial_\mu + iq\partial_\mu\alpha) - m\right]\psi_e = 0$$

Which is not the Dirac Eq!

$\Rightarrow$ the free particle Dirac Equation does not posess local U(1) invariance. We have shown h that its not possible for a free theory (one with out interactions).

We need something to cancel the $iq\partial_\mu\alpha$ term.

So lets add a term

$$iqA_\mu \to iqA_\mu - iq\partial_\mu\alpha$$

or

$$A_\mu \to A_\mu - \partial_\mu\alpha$$

So the new "Dirac Eq" (really the D.E. with an interaction to a gauge boson) is

$$i\gamma^\mu\left[(\partial_\mu + iqA_\mu) - m\right]\psi_e = 0$$

This is invariant under $\psi \to e^{iq\alpha(x)}\psi$. The derivitive term brings down a piece that is exactly canceled by the A_μ transformation.

This of $(\partial_\mu + iqA_\mu)$ as a "fancy" derivitive $D_\mu = (\partial_\mu + iqA_\mu)$ then goet back the same for as the Dirac Eq with $\partial_\mu \to D_\mu$

$$(i\gamma^\mu D_\mu - m)\psi = 0$$

Similarly the L that gives the Dirac equation is modified by $\partial_\mu \to D_\mu$.

This was all for a U(1) symmetry (phase symmetry).

This describes EM.

In a world with out the weak interaction this would be quantum electrodynamics (QED).

<u>Now to the weak interaction ...</u> (GSW)