Homework Set #11

Due Date: Before class Friday April 26th

1) Spontaneous Symmetry Breaking *(10 points)*

a. Let V(ϕ) be

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

where ϕ is a real scalar feild and μ and λ are constants. If $\mu^2 > 0$, where are the minima ?

b. If $\mu^2 < 0$, where are the minima?

c. Now let ϕ be a complex field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$. And let,

$$\mathcal{L} = (\partial_\mu\phi^*)(\partial^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

with $\mu^2 < 0$ and $\lambda > 0$. Expand the Lagrangian about the minimum as we did in class with $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\epsilon(x))$. Write out all the terms.

d. Do the same thing with the U(1) Lagrangian. Expand about the minimum and write the Lagrangian including all interaction terms.

2) Higgs Self-Interaction *(10 points)*

a. Let V(ϕ) be

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

with $\mu^2 < 0$ and $\lambda > 0$. where ϕ is a real scalar field and μ and λ are constants, with $\mu^2 < 0$ and $\lambda > 0$. What is the coupling constant associated to the h^3 interaction in terms of m_h and the position of the minimum ?

b. Assume the potential is:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^6$$

with $\mu^2 < 0$ and $\lambda > 0$. What is the position of the minimum ? What is the coupling constant associated to the h^3 interaction in terms of m_h and the position of the minimum ?

3) LEP: Z and the Higgs *(15 points)*

a Estimate the branching fraction of Z to νs ?

b Estimate the ratio of the ee→Z to ee→H cross sections.

c LEP collected about 17 million Z bosons, for a discovery and study of the Higgs in direct e+e- collisions how many events would LEP have to collect ?

d If LEP collided electrons at 40MHz like the LHC, how long would it take to collect this data ?

e You can also produce a higgs by "radiation it off of a virtual Z boson. Draw this diagram.

f Estimate the cross section for this process.