# QM Linear Algebra in a complex vector space.

State of a system is a vector (ray) in the Complex Vector Space

$|\alpha\rangle$

state vector

**Linear Superpositions**

$$|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$$

complex #'s

is another vector.

## Dual Space!

for every vector $|\alpha\rangle$ (ket) there is another vector $\langle\alpha|$ (bra) in a "dual" space.

Mirror image of the ket space.

$|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$ then $\langle\psi| = c_1^*\langle\alpha_1| + c_2^*\langle\alpha_2|$

Complex Conj.

**Inner Product** - given 2 vectors can get a "c #"

$\langle\alpha|\beta\rangle$

**Properties**

1. $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$
2. $\langle\alpha|\alpha\rangle \geq 0$
3. $\langle\beta|(c_1|\alpha_1\rangle + c_2|\alpha_2\rangle) = c_1\langle\beta|\alpha_1\rangle + c_2\langle\beta|\alpha_2\rangle$

$|\alpha\rangle$ + $|\beta\rangle$ are orthogonal if $\langle\alpha|\beta\rangle = 0$

States can be normalized $\langle\alpha|\alpha\rangle = 1$

Operators $X|\alpha\rangle = |\alpha'\rangle$ In general, not commutative $XY \neq YX$

Product $(YX)|\alpha\rangle = Y(X|\alpha\rangle)$

But they are associative. $\langle\alpha'| = \langle\alpha|X^\dagger$

$X^\dagger \neq X$ in general
Hermitian if so

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System characterized by single observable $A$
eg: Position / Momentum / Energy

Measurement of $A$ gives possible values

$a_1, a_2, \ldots$

$|a\rangle$ = State for which $A$ has value $a$

$$\sum_a |a\rangle\langle a| = 1 \qquad \langle a|a'\rangle \, \delta_{aa'}$$

Any physical observable corresponds to an operator like

$$A = \sum_{a'} a'|a'\rangle\langle a'|$$

$$\boxed{\begin{aligned} A|a\rangle &= \Big(\sum a'|a'\rangle\langle a'|\Big)|a\rangle \\ &= a|a\rangle \end{aligned}}$$

A is Hermitian

$$A^\dagger = \sum_a a^*|a\rangle\langle a| = \sum a|a\rangle\langle a| = A$$

B/c $a$ is real

Physical Observables are Hermitian Operators

## Probabilities

Consider a filter $M(a) = |a\rangle\langle a|$ on general states $|s\rangle$

$$M(a)|s\rangle = |a\rangle\langle a|s\rangle$$
$$= \langle a|s\rangle|a\rangle$$

$\hookrightarrow$ c # tells you something about what fraction of the time you get through

$\langle a|s\rangle$ related to pass fraction

* But, a) not real b) not normalized

However, $|\langle a|s\rangle|^2 = \langle a|s\rangle\langle s|a\rangle = \langle s|a\rangle\langle a|s\rangle$

1. real
2. normalized $\sum_a \langle s|a\rangle\langle a|s\rangle = \langle s|s\rangle = 1$

## Interpretation

$|\langle a|s\rangle|^2$ = Probability that a system prepared in state $|s\rangle$ will be found in state $|a\rangle$ with value $a$ for an observable $A$ after measurement.

~~Example~~

Comments on the measurement Problem