Example: $\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3$

Consider cross-section for $\phi\phi \to \phi\phi$ scattering.

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|} |M|^2 \, d\Pi_{LIPS} \qquad P_1 + P_2 \to P_3 + P_4$$

In com frame, $\vec{P}_1 = -\vec{P}_2 \quad \vec{P}_3 = -\vec{P}_4 \qquad E_1 + E_2 = E_3 + E_4 \equiv E_{cm}$

$$d\Pi_{LIPS} = (2\pi)^4 \delta^4(\Sigma p) \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3 p_4}{(2\pi)^3} \frac{1}{2E_4}$$

Integrating over $\vec{P}_4$

$$= \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3} \frac{1}{E_4} \delta(E_3 + E_4 - E_{cm})$$

$$p_f = |\vec{P}_3| = |\vec{P}_4| \qquad E_3 = \sqrt{m^2 + p_f^2} = E_4 \qquad \int d^3p_3 = \int dp_f \, p_f^2 \, d\Omega$$

Now $p_f \to x = E_3 + E_4 - E_{cm}$

$$dx = \frac{d}{dp}(E_3 + E_4 - E_{cm}) dp_f = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{\overbrace{E_3 + E_4}^{E_{cm}}}{E_3 E_4} p_f \, dp_f$$

$$\Rightarrow \quad \frac{dp_f \, p_f^2}{E_3 E_4} = \frac{dx \, p_f}{E_{cm}}$$

$$d\Pi_{LIPS} = \frac{1}{16\pi^2} d\Omega \int_{m_3+m_4-E_{cm}}^{\infty} dx \frac{p_f}{E_{cm}} \delta(x) = \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{cm}} \begin{cases} \text{if } E_{cm} > m_3 + m_4 \\ 0 \text{ otherwise} \end{cases}$$

$$d\sigma = \frac{1}{(2E_1)(2E_2)|v_1 - v_2|} \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{cm}} |M|^2$$

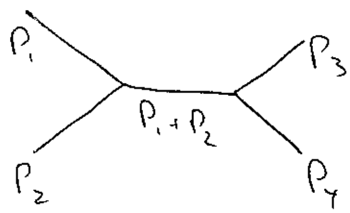
$$|v_1 - v_2| = \left| \frac{|\vec{P}_1|}{E_1} + \frac{|\vec{P}_2|}{E_2} \right| = P_i \frac{E_{cm}}{E_1 E_2} \qquad \boxed{|\vec{P}_i| = |\vec{P}_1| = |\vec{P}_2|}$$

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{64\pi^2 E_{cm}^2} \frac{p_f}{p_i} |M|^2 \qquad \text{if masses are equal } p_f = p_i$$

$$= \frac{1}{64\pi^2 E_{cm}^2} |M|^2$$

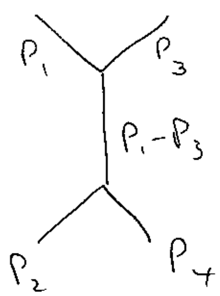
<u>Now to M</u>

"s-channel" diagram



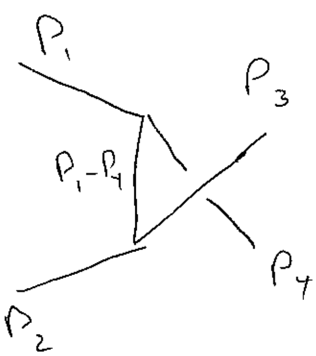
$$= (ig) \frac{i}{\underbrace{(p_1+p_2)^2}_{\equiv s} - m^2} (ig) = \frac{-ig^2}{s-m^2}$$

<u>t-channel</u>



$$= (ig) \frac{i}{\underbrace{(p_1-p_3)^2}_{\equiv t} - m^2} ig = \frac{-ig^2}{t-m^2}$$

<u>u-channel</u>

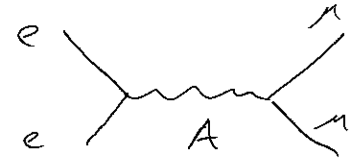


$$= (ig) \frac{i}{(p_1-p_4)^2 - m^2} ig = \frac{-ig^2}{u-m^2}$$

$$\frac{d\sigma}{d\Omega}(\phi\phi \to \phi\phi) = \frac{g^4}{64\pi^2 E_{cm}^2} \left[\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right]^2$$

$s+t+u = \sum m_i^2 \qquad s, t, u$ are L.I.

Example 2a



$\mathcal{M}$ - dimensionless
given by appropriate spin
projections.

~~think~~ focus on projections of initial spins to γ polarizations
then γ polarization to final μ-spins

$$\mathcal{M}(s_1 s_2 \to s_3 s_4) = \sum_{\varepsilon} \underbrace{\langle s_3 s_4 | \varepsilon \rangle}_{\text{spins of outgoing } \mu} \underbrace{\langle \varepsilon | s_1 s_2 \rangle}_{\text{spins of incoming } e}$$

ε → photon polarizations

At high-energies take e & μ massless

$$P_1 = (E, 0, 0, E) \qquad P_2 = (E, 0, 0 -E)$$

In this limit think of electron as having helicity

Linear Basis (you will do circular in HW)

$|s_1 s_2\rangle = |\leftrightarrow\leftrightarrow\rangle$ (along x), $|\updownarrow\updownarrow\rangle$ (along y), $|\updownarrow\leftrightarrow\rangle$ or $|\leftrightarrow\updownarrow\rangle$ Spin 1/2

only $|\leftrightarrow\leftrightarrow\rangle$ & $|\updownarrow\updownarrow\rangle$ can project on to a spin = 1 state

photon polarizations $\quad \varepsilon^1 = (0,1,0,0) \quad$ or $\quad \varepsilon^2 = (0,0,1,0)$

$|\leftrightarrow\leftrightarrow\rangle$ gives ε^1
$|\updownarrow\updownarrow\rangle$ " ε^2

Now μ's are also spin 1/2 (Also have ~~two~~ 4 spin states)

In general, μ not moving along $\quad P_3 = E(1, 0, \sin\theta, \cos\theta)$

$$P_4 = E(1, 0 -\sin\theta, -\cos\theta)$$

Also azimuthal angle ϕ can be set to 0 by cylindrical symmetry.

For muons 2 possible directions of photon polarizations

$$\vec{\varepsilon}^{\,1} = (0, 1, 0, 0) \qquad \bar{\varepsilon}^{2} = (0, 0, \cos\theta, -\sin\theta)$$

(Can check they are $\perp$ to P_3 & P_4)

In general hard to measure spins. Sum over all μ spins.

Must sum over all possible combinations of $\begin{cases}\text{initial} \\ \text{final}\end{cases}$ polarization

For us only $M_1 = M(|\leftrightarrow\leftrightarrow\rangle \to |\bar{\varepsilon}_1\rangle) = \varepsilon^1 \bar{\varepsilon}_1 = -1$

& $M_2 = M(|\updownarrow\updownarrow\rangle \to |\bar{\varepsilon}_2\rangle) = \varepsilon^2 \bar{\varepsilon}_2 = -\cos\theta$

are non-zero.

If our initial beams are unpolarized, sum over initial spins

$$|M|^2 = |M_1|^2 + |M_2|^2 = 1 + \cos^2\theta$$

$$\alpha = \frac{e^2}{4\pi} \qquad \alpha^2 = \frac{e^4}{16\pi^2}$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 E_{cm}^2}(1 + \cos^2\theta)$$

$$= \frac{\alpha^2}{4 E_{cm}^2}(1 + \cos^2\theta)$$