Lecture 13

Noether's Theorem

Lagrangian may be invariant under some type of transformation (variation)

eg: $\phi \to \phi + \delta$

This transformation is a symmetry of the Lagrangian

Say ϕ is complex: 2 DoF ϕ and ϕ^*.

And you have a Lagrangian given by

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi \phi^*$$

symmetry

$$\phi \to e^{-i\alpha}\phi \qquad\qquad \phi^* \to e^{i\alpha}\phi^*$$

Whenever we have a continuous symmetry (meaning there is a continuous limit)

$$\begin{aligned}
\frac{\delta \mathcal{L}}{\delta \alpha} = 0 \;&=\; \sum_n \left[\frac{\partial \mathcal{L}}{\partial \phi_n}\frac{\delta \phi_n}{\delta \alpha} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)}\frac{\delta(\partial_\mu \phi_n)}{\delta \alpha}\right] \\
&=\; \sum_n \left[\underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi_n} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)}\right]}_{=0 \text{ Euler Lagrange}}\frac{\delta \phi_n}{\delta \alpha} + \partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)}\frac{\delta \phi_n}{\delta \alpha}\right]\right]
\end{aligned}$$

$$\phi_n = \{\phi, \phi^*\}$$

$\Rightarrow$

$$\partial_\mu J^\mu = 0$$

with

$$J^\mu = \sum_n \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right]$$

J^μ is a conserved current. “Noether’s Current”

total “charge”

$$Q \equiv \int d^3x\, J_0$$

$$\partial_t Q = \int d^3x\, \partial_t J_0 = \underbrace{\int d^3x\, \vec{\nabla} \cdot \vec{J}}_{\text{Vanishes on the Boundary}} = 0$$

Q does not change with time!

Very general and important theorem “Noether’s Theorem”

If $\mathcal{L}$ has (“enjoys”) a continuous symmetry, there exists an associated current that is conserved.

$$\phi(x) \to \phi(x + \epsilon) = \phi(x) + \epsilon^\mu \partial_\mu \phi(x)$$

This leaves $\mathcal{L}$ and $\mathcal{S}$ invariant and gives a four vector of noether currents.

$\Rightarrow$ Noether’s theorem tells us why energy and momentum are conserved.

Cross Sections And Decay Rates

20th century witnessed the development of collider physics. Effective means to determine which particles exist and their properties and interactions.

- Rutherford discovery of the nucleus using α 1911
- Andersen's discovery of anti-electrons 1932

These were made with "Natural accelerators" α's or cosmic rays

Around 1930 man made collisions started winning.

eg: 1 MeV

Now 13 TeV at the LHC.

Collisions map free fixed momenta initial states $\rightarrow$ final fixed momentum states.

QFT predicts probability for projections to occur.

Probabilities typically dependant on parameters (angles, momenta, etc)

$$P(v_1, ...v_n) \text{ - differential probabilities}$$

Given by

$$|\langle\psi_{final}, +\infty|\psi_{initial}, -\infty\rangle|^2$$

$$\langle f|S|i\rangle \qquad \text{S-matrix}$$

QFT tell us how to calculate S given some Lagrangian (next week)

S-matrix elements are the primary object of interest for particle physics.

In this lecture we will relate S-matrix elements to scattering $\left|\begin{array}{l}\text{cross sections}\\ \text{decay rates}\end{array}\right.$ which we can measure experimentally

Cross Sections

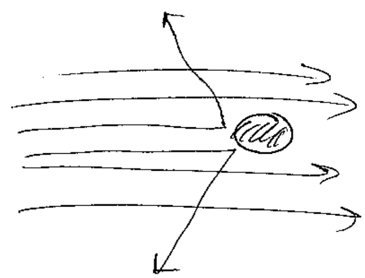
> Aside:
> Probabilities are dimensionless and [0-1] absolute ⇒ extremely subtle
> to calculate P need to know all possible outcomes a priori!
> QM⇒ need complete basis.
> Usually impossible in QCD | # final states ∞
> final states not fully known
> (Particle decays is an exception)

Natural quantity to measure.

eg: Rutherford was interested in the size of the nucleus.

By colliding α-particles w/gold foil and measuring how many partilces are scattered, can determine $\sigma = \pi r^2$

Single nucleus



$$\sigma = \frac{\text{\#- scattered}}{\text{time} \times (\text{Number density in beam}) \times \text{velocity of beam}} = \frac{1}{T}\frac{1}{\Phi}N$$

Real Experiment other factors: | number density of nuclei in foil
cross sectional area of the beam (if smaller than foil)

This stuff and T and Φ depend on details of the experiment.

In contrast, σ is a property of particles being scattered.

In QM generalize notion of cross sectional area to "cross section" | -units of area
-abstract measure of interaction strength

eg: Classically the α will either scatter or not.

Quantum Mechanically, there is some probability for scattering.

$$d\sigma = \frac{1}{T} \underbrace{\frac{1}{\Phi}}_{\substack{\text{Normalized}\\ \text{to one particle}}} \underbrace{dP}_{\text{QM probability of scattering}}$$

$\left.\begin{array}{l} \mathrm{d}\sigma \\ \mathrm{dP} \end{array}\right.$ - differential in kinematic variables θ's and Ps

$$\underbrace{dN}_{\text{\#of scatters}} = \underbrace{L}_{\text{``integrated luminosity}} \times d\sigma$$

(take eq as definition of L)

So number of observed events is direct measurement of cross section (See in presentation and papers)

<u>Relate to S-matrix</u>

practically impossible to collide more than two particles at a time.

$|i\rangle$ will always be a 2 particle state

$$P_1 + P_2 \to \{P_j\}$$

<u>Rest frame</u> of one partilces $\Phi = \frac{|\vec{v}|}{V}$

<u>Center of mass frame</u> $\Phi = \frac{|\vec{v_1} - \vec{v_2}|}{V}$

So,

$$d\sigma = \frac{V}{T} \frac{1}{|\vec{v_1} - \vec{v_2}|} dP$$

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} \underbrace{d\Pi}_{\substack{\text{Region of final state momenta}\\ \text{we are considering}}}$$

On interval of size L, the momenta of available states are $P_n = \frac{2\pi n}{L}$ (from particle in a box).

$\Rightarrow$ throughout a volume V

$$N = \int \frac{V}{(2\pi)^3} d^3 p$$

$$d\Pi = \prod_j \frac{V}{(2\pi)^3} d^3 p_j$$

where j runs over final state partilces.

OK, lets deal with the normalization factors.

Note, $\langle f|f\rangle$ and $\langle i|i\rangle \neq 1$ (The inner products are not Lorentz invariant...)

$$\begin{aligned}
\langle p'|p\rangle &= (2\pi)^3\, 2E\, \delta^3(p'-p) \\
\langle p|p\rangle &= (2\pi)^3\, 2E_p\, \delta^3(0) \\
&= 2E_p V
\end{aligned}$$

we say the $\delta^3(0)$ is "regulated by V".

$$\delta^3(p) = \frac{1}{(2\pi)^3} \int d^3x\, e^{ipx}$$

$$\delta^3(0) = \frac{1}{(2\pi)^3} \int d^3x = \frac{V}{(2\pi)^3}$$

$\Rightarrow$

$$\langle i|i\rangle = \langle p_1 p_2|p_1 p_2\rangle = 2E_1 V\, 2E_2 V$$

$$\langle f|f\rangle = \prod_j (2E_j V)$$

Now have to deal with $\langle f|S|i\rangle$

S elements always calculated pertubatively

$$S = \underbrace{1}_{\text{free theory}} + \underbrace{iT}_{\text{perturbatively small}}$$