2a) $$\frac{1}{\lambda_H} = R\left(1 - \frac{1}{4}\right) = \frac{3}{4}R$$

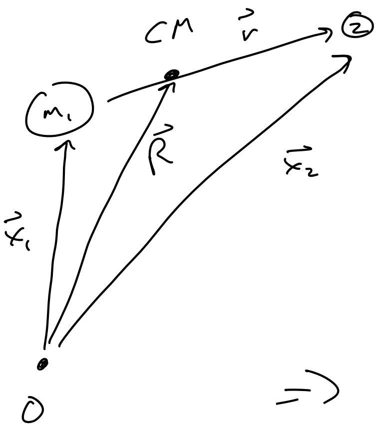
$$R = \frac{mk^2e^4}{4\pi c\hbar^3}$$

$$= 1.097 \times 10^7 \ m^{-1}$$

$$\frac{1}{\lambda_{He}} = Z^2 R \frac{3}{4}$$

$$\frac{\lambda_H}{\lambda_{He}} = Z^2 = 4$$

b)



$$\vec{r} = \vec{x}_2 - \vec{x}_1 \implies \vec{x}_2 = \vec{r} + \vec{x}_1$$

$$\vec{R} = \frac{m_1\vec{x} + m_2\vec{x}_2}{m_1 + m_2}$$

$$\implies (m_1+m_2)\vec{R} = m_1\vec{x}_1 + m_2\vec{x}_2$$

$$= m_1\vec{x}_1 + m_2(\vec{r} + \vec{x}_1)$$

$$= (m_1+m_2)\vec{x}_1 + m_2\vec{r}$$

$$\implies \vec{x}_1 = \frac{(m_1+m_2)\vec{R} - m_2\vec{r}}{m_1+m_2} = \vec{R} - \frac{m_2}{(m_1+m_2)}\vec{r}$$

Same logic

$$\vec{x}_2 = \vec{R} + \left(\frac{m_1}{m_1+m_2}\right)\vec{r}$$

Now the momentum

time derivative

$$\vec{P}_1 = m_1 \dot{\vec{x}}_1 = m_1 \dot{\vec{R}} - \left(\frac{m_1 m_2}{m_1+m_2}\right)\dot{\vec{r}}$$

$$\vec{P}_2 = m_2 \dot{\vec{x}}_2 = \underbrace{m_2 \dot{\vec{R}}}_{\text{Mom of CM}} + \underbrace{\left(\frac{m_1 m_2}{m_1+m_2}\right)\dot{\vec{r}}}_{\text{Relative momentum}}$$

In the center of mass frame

$$\vec{R} = 0 \quad \& \quad \dot{\vec{R}} = 0$$

then

$$\vec{x}_1 = -\frac{m_2}{m_1+m_2}\vec{r} \qquad \vec{x}_2 = \frac{m_1}{m_1+m_2}\vec{r}$$

$$E = \underbrace{\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2}_{KE} + \underbrace{U(\vec{r})}_{PE}$$

$$KE = \frac{1}{2}m_1\left(-\frac{m_2}{m_1+m_2}\dot{\vec{r}}\right)^2 + \frac{1}{2}m_2\left(\frac{m_1}{m_1+m_2}\dot{\vec{r}}\right)^2$$

$$= \frac{1}{2}\frac{m_1 m_2^2}{(m_1+m_2)^2}\dot{r}^2 + \frac{1}{2}\frac{m_2 m_1^2}{(m_1+m_2)^2}\dot{r}^2$$

$$= \frac{1}{2} \frac{m_1 m_2 (m_1 + m_2)}{(m_1 + m_2)^2} \dot{r}^2 = \frac{1}{2} \underbrace{\left(\frac{m_1 m_2}{(m_1 + m_2)} \right)}_{\equiv \mu \text{ "reduced mass"}} \dot{r}^2$$

$$= \frac{1}{2} \mu \dot{r}^2$$

$$E = \frac{1}{2} \mu \dot{r}^2 + U(r)$$

Bohr energies scale as before w/ $m_e \to \mu$.

c) $m \to \mu = \dfrac{m M_N}{m + M_N} = \dfrac{m}{1 + m/M_N}$

$$R \to R\left(\frac{1}{1 + m/m_N}\right)$$

$$\frac{\lambda_H}{\lambda_{He}} = \frac{Z^2 R_{He}}{R_H} = 4\left(\frac{\left(\frac{1}{1 + m_e/m_{He}}\right)}{\left(\frac{1}{1 + \frac{m_e}{m_H}}\right)}\right)$$

$$= 4\left(\frac{1 + m_e/m_H}{1 + m_e/m_{He}}\right)$$

$m_e = 0.5$ MeV $= 0.5\, 10^{-3}$ GeV

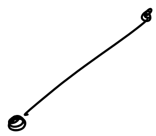
$m_H = 0.9$ GeV

$m_{He} = 3.7$ GeV

$$\frac{1 + \frac{0.5\, 10^{-3}}{0.9}}{1 + \frac{0.5\, 10^{-3}}{3.7}} = \frac{1 + 0.5\, 10^{-3}}{1 + 0.14\, 10^{-3}}$$

$$\frac{m_{He} + \frac{m_{He}}{m_H} m_e}{m_{He} +}$$

d) Muonic Hydrogen



μ

p

$m_e \to m_\mu$ But we should probably use the reduced mass here

$$\mu = \frac{m_\mu m_p}{m_\mu + m_p} = \frac{m_\mu}{1 + \frac{m_\mu}{m_p}}$$

$m_\mu \sim 0.1$ GeV

$m_e \sim 0.5 \; 10^{-3}$ GeV

$m_p \sim 1$ GeV

$$\frac{m_\mu}{m_e} \sim 200$$

$$= \frac{200 \, m_e}{1 + 0.1} \sim 180 \, m_e$$

Bohr radius

$$r_1 = \frac{\hbar^2}{\mu \alpha} = \frac{\hbar^2}{m_e \alpha} \frac{1}{180} \sim 5 \, 10^{-3} \, a_0$$

$$E_1 = \frac{\mu \alpha^2}{2} = 180 \, \frac{m_e \alpha^2}{2} = 180 \, (13.6 \text{ eV})$$

e) Positronium

Now $\mu = \frac{m_e^2}{m_e + m_e} = \frac{1}{2} m_e$

$r \to r = 2 a_0$

$E_1 \to 2 E_0$

3a) $KE = \frac{3}{2} kT$

λ_N @ 300 K $\quad KE \sim 3 \cdot 10^{-2}$ eV

$\lambda = \frac{h}{p}$ $\qquad KE = \frac{p^2}{2m}$

$p = \sqrt{2 m KE}$

$= (2 \cdot 14 \, m_p \, KE)^{1/2}$

$= (28 \text{ GeV } 3 \, 10^{-11} \text{ GeV})^{1/2}$

$= 3 \cdot 10^{-5}$ GeV

$$\lambda = \frac{4 \cdot 10^{-15} \text{ eV} \cdot s}{3 \cdot 10^{-5} \text{ GeV}} \sim \frac{4}{3} \frac{10^{-24} \text{ GeV} \cdot s}{10^{-5} \text{ GeV}} \cdot c$$

$$= \frac{4}{3} 10^{-19} \, s \times 3 \, 10^{8} \frac{m}{s} = 4 \cdot 10^{-11} \, m$$

$= 0.04$ nm

3b) $KE \sim 0.02\ eV$

$$KE = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2 KE}} = \frac{1240\ eV\,nm}{\sqrt{2 \cdot 10^9 \cdot 2\,10^{-2}\ eV^2}}$$

$$= \frac{1240\ eV\,nm}{2 \cdot 3\ 10^3\ eV} \sim \frac{2\,10^2}{10^3}\ nm$$

$$\sim 2 \cdot 10^{-1}\ nm = 0.2\ nm$$

3a) $\sigma_x \sigma_p \sim \hbar$

if $\sigma_x \sim \lambda = \frac{h}{p}$

then

$$\sigma_p \sim \frac{\hbar}{\sigma_x} \sim p$$

$\Rightarrow \sim 100\%$ uncertainty in p

3d) $\sigma_x \sim 5\ 10^{-12}\ m$

if $\lambda_\gamma = 5\ 10^{-12}\ m$ $\qquad\qquad \lambda\nu = c$

$$E = h\nu = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{5\ 10^{-3}\ nm}$$

$$\sim \frac{2.5\ 10^{2}}{10^{-3}} = 2\ 10^{5}\ eV \sim 0.2\ MeV$$

$$p = \frac{E}{c} = 8\ 10^{-7}\ eV\ \frac{s}{m}$$

$$\sigma_p = \frac{h}{\sigma_x} = \frac{4\ 10^{-15}\ eV\cdot s}{5\ 10^{-12}\ m} = 8\ 10^{-4}\ \frac{eVs}{m}$$

Moral Δp for electron is 1000 times larger than for a γ

4) $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Assume $y_n(x,t)$ are solutions.

$$y(x,t) = \sum_n c_n y_n(x,t)$$

$$\frac{\partial^2 y}{\partial x^2} = \sum_n c_n \left(\frac{\partial^2 y_n}{\partial x^2}\right) = \sum_n c_n \left(\frac{1}{v^2}\frac{\partial^2 y_n}{\partial t^2}\right)$$

$$= \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\left(\sum_n c_n y_n\right) = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$