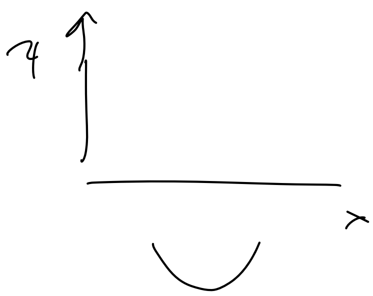
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

Cannot be solved in general.

But for $V(x) = V_0$, easy

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - V)\psi$$

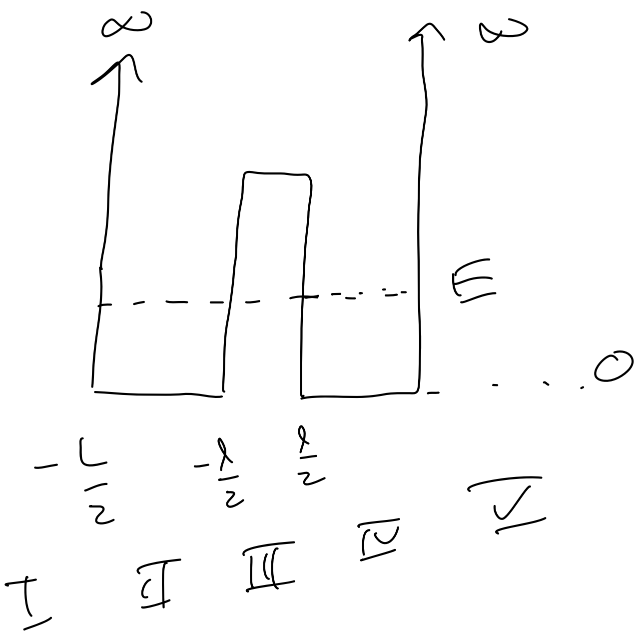
$\underline{\underline{E > V}}$ $$\frac{d^2\psi}{dx^2} = -k^2\psi$$



$\underline{\underline{E < V}}$ $$\frac{d^2\psi}{dx^2} = k^2\psi$$

General Properties

- $V(x) = V(-x)$ $\Rightarrow$ Solutions even or odd
- Ground State has no zeros
- Each higher state has additional 0.



<u>In I & V</u>

$V = \infty$

$\Rightarrow \psi(x) = 0$

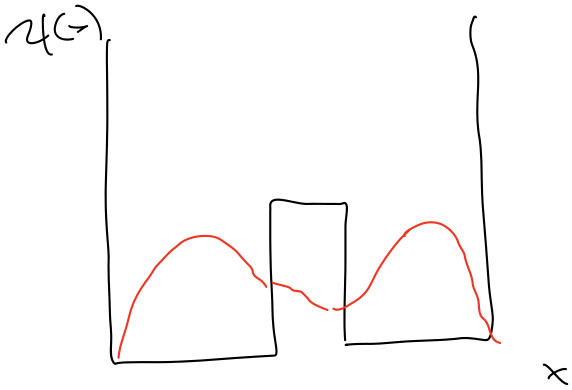
<u>In II & IV</u>

$k \equiv \frac{\sqrt{2mE}}{\hbar}$

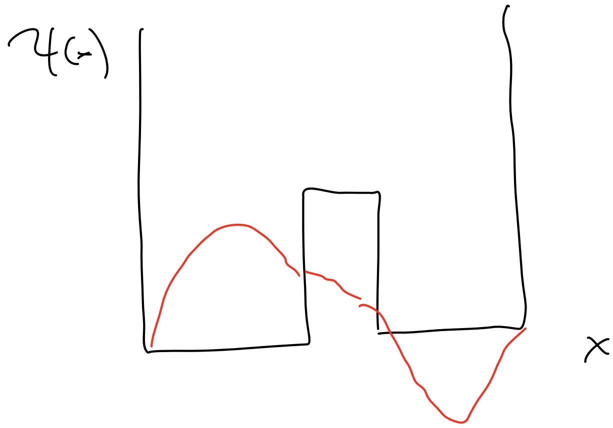
$V = 0 \qquad \psi(x) = A \sin kx + B \cos kx$

$$\psi\left(-\frac{L}{2}\right) = 0$$

$$\psi(x) \begin{cases} 0 & x < -\frac{L}{2} \\ A \sin kx + B \cos kx & -\frac{L}{2} < -\frac{l}{2} \\ C e^{kx} + D e^{-kx} & -\frac{l}{2} < 0 \\ \psi(-x) & > 0 \end{cases}$$



$\psi_s \, , E_s$



$\psi_A \, , E_A \qquad E_A > E_S$

$\left(\text{more } \frac{d\psi}{dx} \right)$

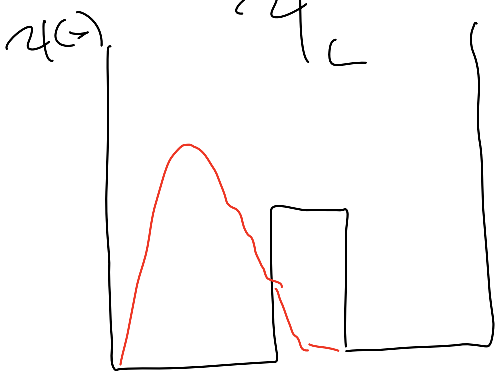
Now Consider Dynamics of "Putty particle 'in one side'"

Normalized if ψ_A & ψ_S are

$$\psi_L = \frac{1}{\sqrt{2}}\left(\psi_s + \psi_A\right)$$

$$\psi_R = \frac{1}{\sqrt{2}}\left(\psi_s - \psi_A\right)$$

ψ_L



ψ_R

Note ψ_L & ψ_R Do Not have definite Energies

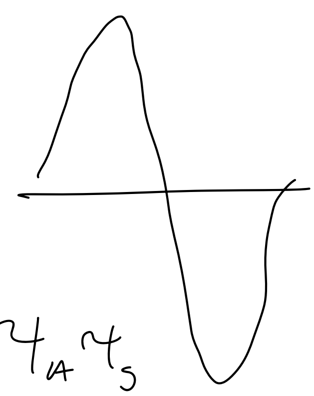
Not Stationary States

Lets look at time dependence

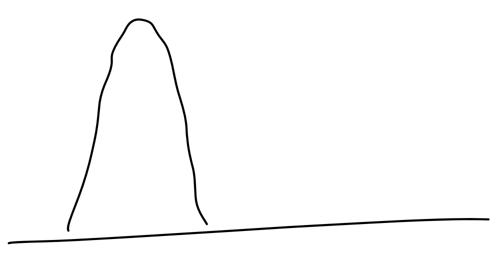
Say we start w/ particle on left

$$\Psi(x,0) = \psi_L(x) = \frac{1}{\sqrt{2}}\left(\psi_A(x) + \psi_S(x)\right)$$

$$P(x,0) = |\Psi(x,0)|^2 = \frac{1}{2}\left(\psi_A^2(x) + \psi_S^2(x) + 2\psi_A\psi_S\right)$$

 +  + 2 

ψ_A^2 $\qquad$ ψ_S^2 $\qquad$ $\psi_A\psi_S$

$P(x,0)$  ✓

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left(\Psi_A(x,t) + \Psi_S(x,t)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\psi_A\, e^{iE_A t} + \psi_S\, e^{iE_S t}\right)$$

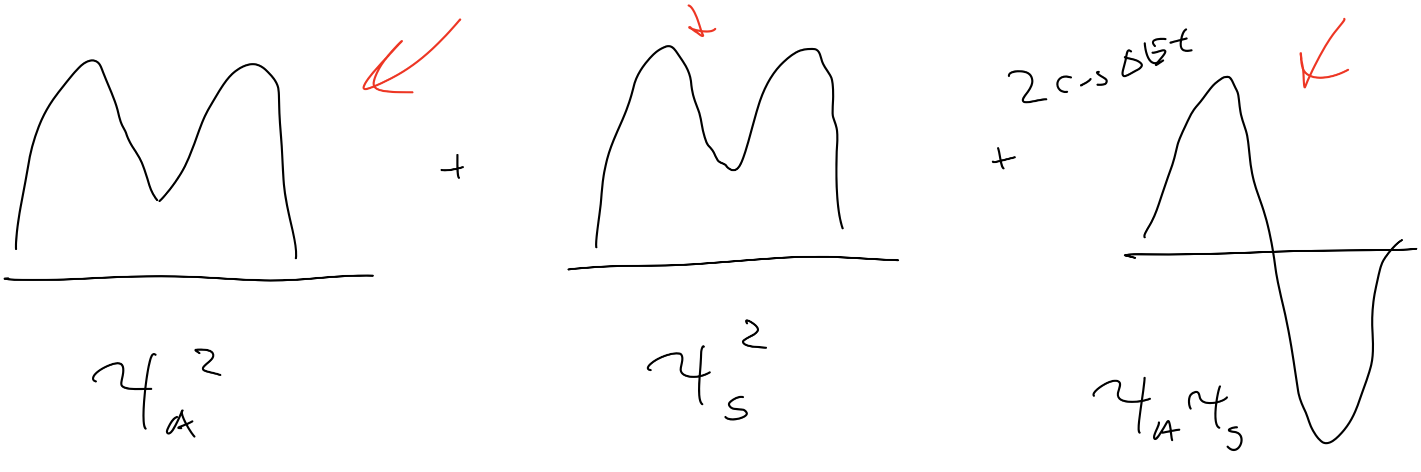
$$P(x,t) = |\Psi|^2 = \frac{1}{2}\Big(\psi_A^2 + \psi_S^2 + \psi_A\psi_S\left(e^{-iE_A t}e^{iE_S t} + e^{iE_A t}e^{-iE_S t}\right)\Big)$$

$$= \psi_A^2 + \psi_S^2 + \psi_A\psi_S\underbrace{\left(e^{-i(E_A - E_S)t} + e^{i(E_A-E_S)t}\right)}_{2\cos((E_A - E_S)t)}$$

$$(E_A - E_S) \neq 0$$

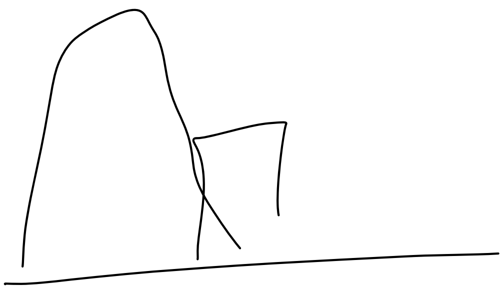
So

$$P(x,t) = \psi_A^2 + \psi_S^2 + 2\cos\Delta E t\,(\psi_A\psi_S)$$

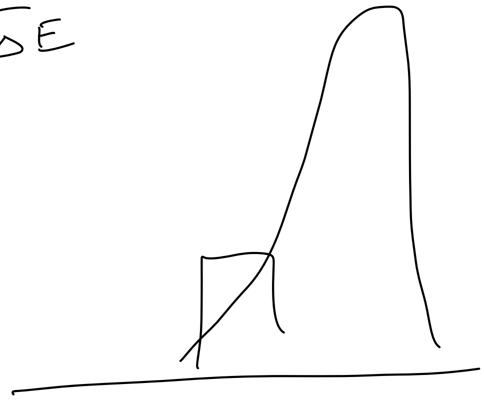


Now

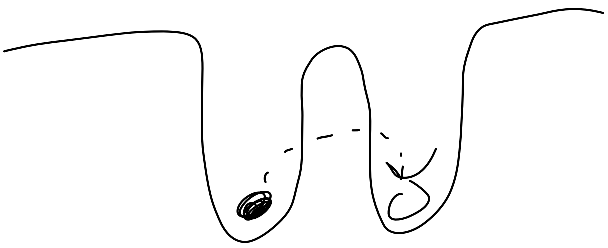
$t = 0$



$t = \frac{\pi}{\Delta E}$



"Quantum tunnelling"



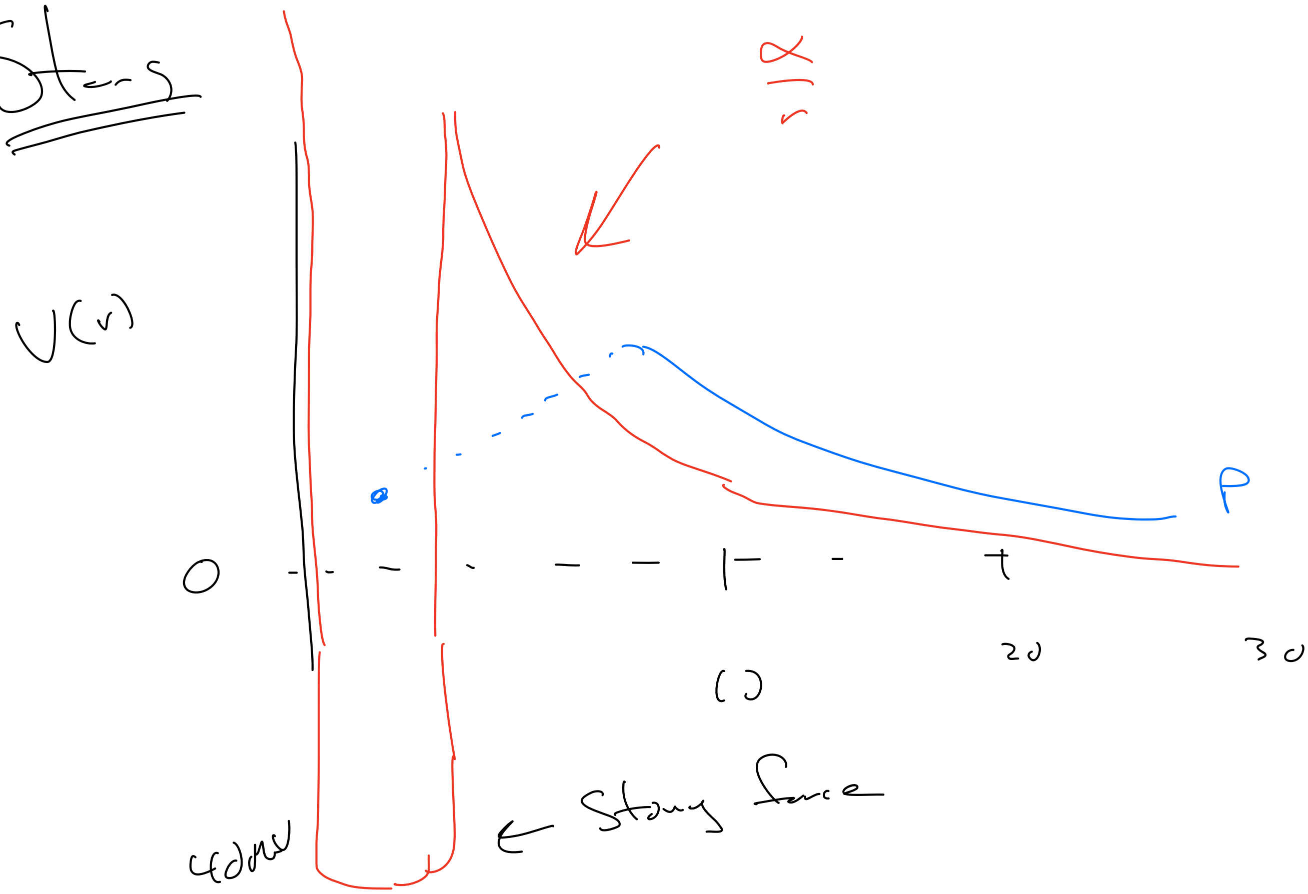
turns out ΔE very sensitive to l.
(exponentially sensitive)

Scanning tunnelling microscope

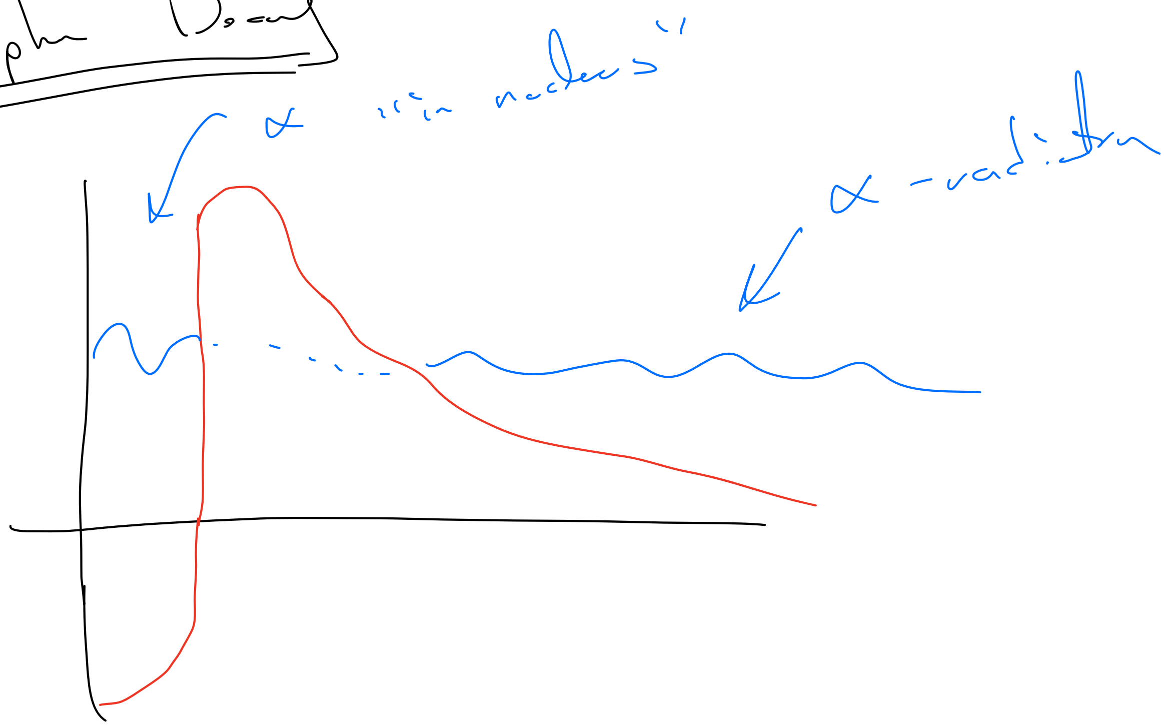
Measure current $\Rightarrow$ Δx from needle to sample.

Quantum Tunneling very General feature QM

Strong



Alpha Decay



3D Ch 7

Generalization to 3D straight forward

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$ H - total energy "Hamiltonian"

3D

$$H \rightarrow \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + V(\vec{x})$$

$$p_x \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad p_y \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial y} \qquad p_z \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial z}$$

$$\vec{p} \rightarrow \frac{\hbar}{i}\vec{\nabla}$$

Sch Eq

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Note ψ V now functions of $\vec{x}$ & t

Just as before, we will consider solutions

$$\Psi(\vec{x},t) = \psi(\vec{x})\, e^{-iE_n t/\hbar}$$

where $\psi(x)$ satisfies

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

Questions

Infinite Cube

$$V(x,y,z) = \begin{cases} 0 & \text{if } x,y,z \text{ in } 0-a \\ \infty & \text{otherwise} \end{cases}$$

Outside of cube $\psi(x,y,z) = 0$
(Same logic as in 1D)

Inside the cube

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = E\psi$$

Assume separation of variables

$$\psi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

then (substituting & dividing by ψ)

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{f(x)} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{f(y)} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{f(z)} = \underbrace{-\frac{2m}{\hbar}E}_{\text{Constant}}$$

Only possible if the different $f()$'s are equal to constants (k_x^2, k_y^2, k_z^2)

So, $\dfrac{d^2X}{dx^2} = -k_x^2$, $\dfrac{d^2Y}{dy^2} = -k_y^2$, $\dfrac{d^2Z}{dz^2} = -k_z^2$

And $$E = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right)$$

$\left(p = \hbar k,\ E = \dfrac{p_x^2 + p_y^2 + p_z^2}{2m}\right)$

As expected...

Now easy to solve the separated equ's

$$X(x) = A_x \sin k_x x + \cancel{B_x \cos k_x x}$$

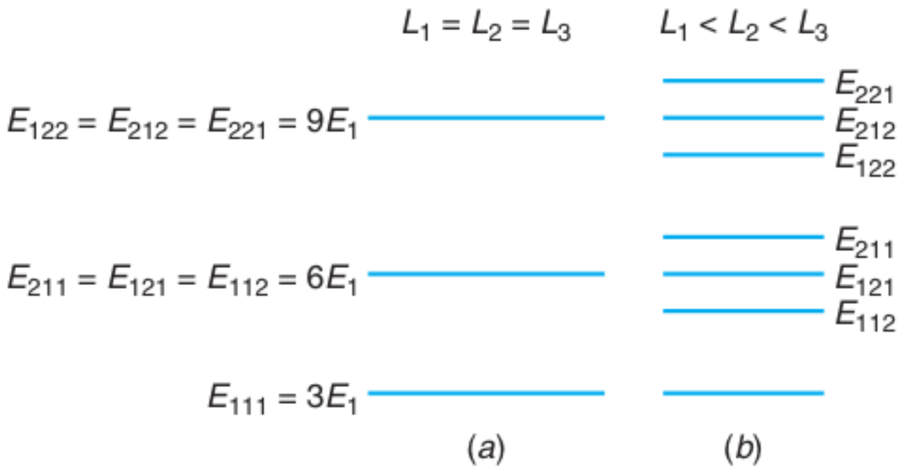
ψ - cont at ϕ

$k_x = \dfrac{n_x \pi}{a}$ (from ψ - cont at a)

Ditto $Y(y)$, $Z(z)$

$$\psi(x,y,z) = A \sin\left(\frac{n_x\pi}{a}\right) \sin\left(\frac{n_y\pi}{a}\right) \sin\left(\frac{n_z\pi}{a}\right)$$

So $\psi_{n_x,n_y,n_z}(x,y,z)$ $\quad E_{n_x,n_y n_z} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2+n_y^2+n_z^2\right)$



First case we see different states with same Energy "degenerate"

Break degeneracy by moving to a rectangular box.

More on this later