Now look at a few more applications of
Relativity

Mass Units

$$[\text{mass}] = [\text{momentum}] = [\text{Energy}]$$

$$[m] = [m\beta\gamma] = [m\gamma]$$

Need to pick a good unit to talk about the world

Going to use "GeV" (One of gods units)

$$\text{GeV} = 10^{9}\ \text{eV}$$

eV = energy gained by an electron when accelerated by potential difference 1V.

$$1\ \text{eV} = 1.6\ 10^{-19}\ \text{C} \cdot 1\text{V} = 1.6\ 10^{-19}\ \text{J}$$

translation of eV to "mks" Energy

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To get units of "mks" momentum, divide by c

$$[E]_{mks} = kg\,\frac{m^2}{s^2} = [\text{momentum}][\text{velocity}]$$

$$1\,eV/c = \frac{1.6\;10^{-19}\;J}{3\;10^{8}\;m/s} = 0.5\;10^{-27}\;kg\,\frac{m}{s}$$

To get units of "mks" mass, divide by c^2

$$[E]_{mks} = kg\,\frac{m^2}{s^2} = [\text{mass}][\text{velocity}]^2$$

$$\frac{1\,eV}{c^2} = \frac{1.6\;10^{-19}\;J}{9\;10^{+16}\;m^2/s^2} = 0.17\;10^{-35}\;kg$$

$$\sim 10^{-36}\;kg$$

$$\frac{GeV}{c^2} = 10^{-27}\;kg$$

$$GeV = 10^{-27}\;kg$$

Why would we do this?!

$m_{\text{proton}} = 0.938 \text{ GeV} \sim 1 \text{ GeV}$

$m_{\text{Neutron}} \sim 1 \text{ GeV}$

$(m_N - m_p) \sim 1 \text{ MeV}$

$m_e \sim 0.5 \text{ MeV} \sim 10^{-3} \text{ GeV}$

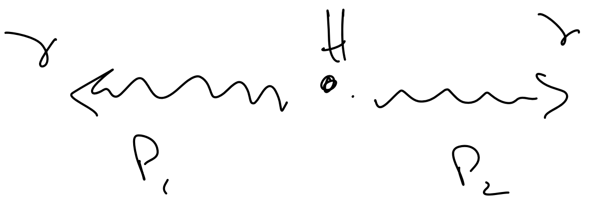
Example Cosmic μ's

$m_\mu \approx 100 \text{ MeV}$ typical μ $E \sim 1.5 \text{ GeV}$

$\Rightarrow \quad \gamma = 15$

e.g. Higgs Boson

$H \to \gamma\gamma$



$P_1 = -60$ GeV $\qquad P_2 + 65$ GeV

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 = 1 \text{ GeV}$$

$$E_{tot} = E_1 + E_2 = 125 \text{ GeV}$$

$$M_{tot} = \sqrt{125^2 - 1^2} = 124.9 \text{ GeV}$$

$$P_1^4 + P^4 = P_+^4$$

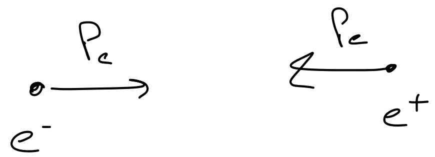
$$\begin{pmatrix} E \\ \vec{p} \end{pmatrix} \underbrace{\begin{pmatrix} 60 \text{ GeV} \\ -60 \text{ GeV} \\ 0 \\ 0 \end{pmatrix}}_{m=0} + \underbrace{\begin{pmatrix} 65 \text{ GeV} \\ +65 \text{ GeV} \\ 0 \\ 0 \end{pmatrix}}_{m=0} = \underbrace{\begin{pmatrix} 125 \text{ GeV} \\ 1 \text{ GeV} \\ 0 \\ 0 \end{pmatrix}}_{m \neq 0}$$

m_H

Anti-Particles can Annihilate Matter & turn into pure radiation

$$e^- + \underset{\text{"positron"}}{e^+} \to \text{light}$$

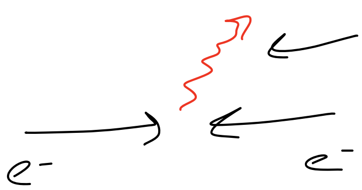
Question: Can the light be just one photon?



$$\underbrace{\begin{pmatrix} \sqrt{m_e^2+p_e^2} \\ +p_e \\ 0 \\ 0 \end{pmatrix}}_{m_e \checkmark}^{p^4_{e^-}} \quad \underbrace{\begin{pmatrix} \sqrt{m_e^2+p_e^2} \\ -p_e \\ 0 \\ 0 \end{pmatrix}}_{m_e \checkmark}^{p^4_{e^+}} \quad \overset{?}{=} \quad \underbrace{\begin{pmatrix} 2\sqrt{m_e^2+p_e^2} \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{m \neq 0}^{p^4_{\gamma}}$$

Not possible!

Another way of seeing it



γ cannot be at rest

cannot have $\vec{p}_+ = 0$

Two Photons?

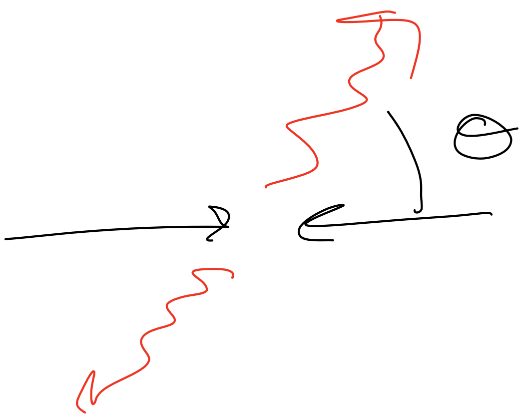
$$p^4_{e^+e^-} \qquad\qquad p^4_{\gamma_1} \qquad\qquad p^4_{\gamma_1}$$

$$\begin{pmatrix} 2E \\ \vec{0} \end{pmatrix} \stackrel{?}{=} E^{\gamma}_1 \underbrace{\begin{pmatrix} 1 \\ \hat{n}_1 \end{pmatrix}}_{m=0} + E^{\gamma}_2 \underbrace{\begin{pmatrix} 1 \\ \hat{n}_2 \end{pmatrix}}_{m=0}$$

$\hat{n}$ - unit vectors

Only works if $E^{\gamma}_1 = E^{\gamma}_2$ & $\hat{n}_1 = -\hat{n}_2$

Photons have equal energy & come out "Back-to-Back"



How PET scans work

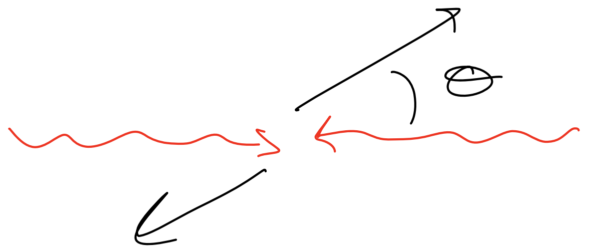
$N^A_Z \to e^+ \nu_e N^A_{Z-1}$

$$^{18}F \xrightarrow{20\text{ min}} {}^{18}O$$

Can Also Create Matter / Antimatter from radiation

Can clearly do it w/ 2 photons:

$$p^4_{\gamma_1} + p^4_{\gamma_2} \to p^4_{e^+} + p^4_{e^-} \quad \text{(from above!)}$$



Can we make $e^+ e^-$ from 1 photon?

$$\underset{m=0}{\overset{p^4_\gamma}{\begin{pmatrix} E_\gamma \\ E_\gamma \\ 0 \\ 0 \end{pmatrix}}} \to \underset{m=e}{\overset{p^4_{e^-}}{\begin{pmatrix} E_\gamma/2 \\ \frac{E_\gamma}{2}\sqrt{1+\left(\frac{2m_e}{E_\gamma}\right)^2} \\ 0 \\ 0 \end{pmatrix}}} + \underset{m=e}{\overset{p^4_{e^+}}{\begin{pmatrix} E_\gamma/2 \\ \frac{E_\gamma}{2}\sqrt{1+\left(\frac{2m}{E_\gamma}\right)^2} \\ 0 \\ 0 \end{pmatrix}}}$$

Don't quite work!

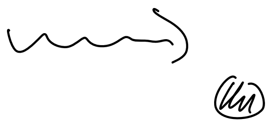
$$\left(\frac{E_\gamma}{2}\right)^2 - p_e^2 = m_e^2$$

$$p_e = \sqrt{\left(\frac{E_\gamma}{2}\right)^2 + m_e^2}$$

$$= \frac{E_\gamma}{2}\sqrt{1+\left(\frac{2m_e}{E_\gamma}\right)^2} > \frac{E_\gamma}{2}$$

Can't Conserve <u>Both</u> E & P

However lets add Nucleus that barely moves



N

$$\underbrace{\begin{pmatrix}E_\gamma\\ E_\gamma\\ 0\\ 0\end{pmatrix}}_{m=0} + \underbrace{\begin{pmatrix}M_N\\ \vec{0}\end{pmatrix}}_{M_N} = \underbrace{\begin{pmatrix}E'_\gamma/2\\ \frac{E'_\gamma}{2}\sqrt{1+\frac{2m}{E_i}}\\ \vdots\end{pmatrix}}_{m_e} + \begin{pmatrix}E'_\gamma/2\\ \vdots\end{pmatrix} + \underbrace{\begin{pmatrix}M_N\sqrt{1+\left(\frac{\mathcal{E}}{M_N}\right)^2}\\ \vec{\mathcal{E}}\end{pmatrix}}_{M_N}$$

$$E_\gamma + M_N = E'_\gamma + \underbrace{M_N\sqrt{1+(\mathcal{E}/M_N)^2}}_{M_N\left(1+\frac{1}{2}\frac{\mathcal{E}^2}{M_N^2}\right) = M_N + \frac{\mathcal{E}^2}{2}}$$

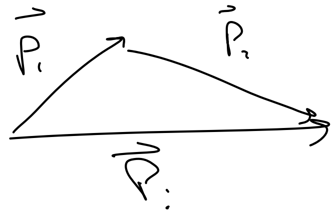
$$E'_\gamma = E_\gamma - \mathcal{E}^2/2$$

This is how we "x-ray"

our LHC detectors

$\gamma \to \gamma\gamma$?

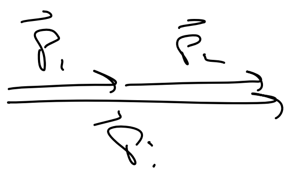
$\vec{p}$ needs to be conserved $\qquad \vec{p}_i = \vec{p}_1 + \vec{p}_2$



But, $E_i = E_1 + E_2$

& $E = |\vec{p}|$

only works if $\vec{p}_1$ & $\vec{p}_2$ are collinear



Need $\vec{p}_i = \vec{p}_1 + \vec{p}_2$

& $|\vec{p}_i| = |\vec{p}_1| + |\vec{p}_2|$

Turns out $\gamma \to \gamma\gamma$ not possible even in this case for γ's ("γ has no charge") but is possible for gluons ($m_g = 0$) "gluons have strong charge (color)"