Dimensional Analysis and "~"

Put in the right physics to get answers to within *"geometric factors"*
- Dont worry about factors of 2 or π etc
- Use "~" not "="

Examples (Volume of something) ~ (size)^3

$$\text{Cube} = R^3 \sim R^3$$
$$\text{Sphere} = 4/3\pi R^3 = 4.2\, R^3 \sim R^3$$
$$= 1/6\pi(D)^3 = 0.4\, D^3 \sim D^3$$
$$\text{Cylinder} = R\times\pi R^2 = \pi\, R^3 \sim R^3 \quad (\text{if two scales use } r^2R)$$

Kinematic energy = $1/2\, mv^2 \sim mv^2$

Ive been doing this already: "$\Delta p\Delta x \geq h$"

(…it is really $\Delta p\Delta x\geq h/(4\pi)$)

Units

I hate units! All numbers are really unit-less

Always comparing some quantity relative to some standard

We will work in "Natural Units"

<u>Natural Units</u>

- The right way to think about the world

  (How physicists think, what makes them seem smart to other people)

- Very easy. Much easier than Metric/British/cgm/mks …
- Standard is set by basic physical principles

  $\Rightarrow$ numbers have direct physical interpretations

$c \equiv 1$: [Distance]/[Time] $\equiv 1$

- Time and distance have same units
- $E = m$

You are already familiar with this:
"Its about an hour from here"

$h \equiv 1$: [Energy]$\times$[Time] $= 1$ and [Energy]$\times$[Distance] $= 1$

- Energy (or Mass) is inversely related to distance or time.

Write everything in terms of **[Energy]:** use 1 GeV ~ mp as basic unit

Examples

Everything in terms of GeV. Use conversions to get back to human units

Conversions:

$$\mathrm{GeV} = 10^{-27}\ \mathrm{kg}$$

$$\mathrm{GeV}^{-1} = 10^{-16}\ \mathrm{m}$$

$$\mathrm{GeV}^{-1} = 6 \cdot 10^{-25}\ \mathrm{s}$$

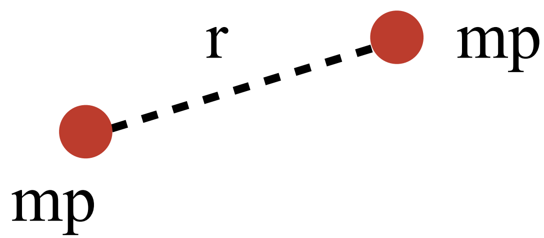
Proton Weight: GeV

Proton Size: GeV^{-1}

My height: $1\mathrm{m} \sim 10^{16}\ \mathrm{GeV}^{-1}$

My weight: $100\ \mathrm{kg} \sim 10^{29}\ \mathrm{GeV}$

EM and Gravitation Interactions



Electromagnetic Energy

$$\underbrace{E}_{\mathrm{GeV}} = -\underbrace{\frac{e^2}{4\pi}}_{\text{Pure number}}\ \underbrace{\frac{1}{r}}_{\mathrm{GeV}}$$

Pure number: α
Its small: 1/137

Gravitational Energy

$$\underbrace{E}_{\mathrm{GeV}} = -\underbrace{G_N}_{\text{Dimensionful number}}\ \underbrace{\frac{m_p^2}{r}}_{\mathrm{GeV}^3}$$

Dimensionful number
$$G_N m_p^2 = 10^{-39}$$

The world with 4 numbers

Claim: ~everything in world combination of these numbers

$$m_p \sim 1\ \mathrm{GeV} \qquad \alpha = \frac{1}{137} \sim 10^{-2}$$

$$m_e \sim 10^{-3}\ \mathrm{GeV} \qquad \alpha_G \equiv G_N m_p^2 = 10^{-39}$$

Will work through some quick examples.

Atoms

$$\mathrm{p} \times \mathrm{r} \sim 1$$

$$\mathrm{E} \sim -\frac{\mathrm{Z}\alpha}{\mathrm{r}} + \frac{\mathrm{p}^2}{\mathrm{m_e}} \qquad \mathrm{E} \sim -\frac{\mathrm{Z}\alpha}{\mathrm{r}} + \frac{1}{\mathrm{m_e r^2}}$$

$$\mathrm{r_{atom}} \sim \frac{1}{\mathrm{Z}\alpha \mathrm{m_e}}$$

Z	Prediction	Actual Value
1	$\sim 10^{-11}\mathrm{m}$	$2.5 \cdot 10^{-11}\mathrm{m}$
10	$\sim 10^{-12}\mathrm{m}$	$4.0 \cdot 10^{-11}\mathrm{m}$
>10	$\sim 10^{-12}\mathrm{m}$	$\sim 10^{-10}\mathrm{m}$

Details of electron screening needed for high Z

(Will use 10^-10 when Z >10)

Atoms

$$\mathrm{p \times r \sim 1}$$

$$\mathrm{E \sim -\frac{Z\alpha}{r} + \frac{p^2}{m_e}} \qquad \mathrm{E \sim -\frac{Z\alpha}{r} + \frac{1}{m_e r^2}}$$

$$\mathrm{r_{atom} \sim \frac{1}{Z\alpha m_e}} \qquad \mathrm{r_{nucleus} \sim \frac{Z^{1/3}}{m_p}}$$

$$\mathrm{\frac{r_{nucleus}}{r_{atom}} \sim \frac{\alpha m_e}{Z^{2/3} m_p} \sim 10^{-5}}$$

Atoms

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$$r_{atom} \sim \frac{1}{Z\alpha m_e} \qquad r_{nucleus} \sim \frac{Z^{1/3}}{m_p}$$

$$\frac{r_{nucleus}}{r_{atom}} \sim \frac{\alpha m_e}{Z^{2/3} m_p} \sim 10$$

Number of different atoms ~ $1/\alpha$

$$p_e \sim \frac{1}{r_{atom}} \sim m_e(Z\alpha) \qquad v_e \sim (Z\alpha)$$

- Why we could do QM first with out relativity: ($v << 1$ for $Z \sim 1$)
- Why electricity more stronger everyday than magnetism.

Atoms

$$\mathrm{p} \times \mathrm{r} \sim 1$$

$$\mathrm{E} \sim -\frac{Z\alpha}{\mathrm{r}} + \frac{\mathrm{p}^2}{\mathrm{m_e}} \qquad \mathrm{E} \sim -\frac{Z\alpha}{\mathrm{r}} + \frac{1}{\mathrm{m_e r^2}}$$

$$\mathrm{r_{atom}} \sim \frac{1}{Z\alpha \mathrm{m_e}} \qquad \mathrm{r_{nucleus}} \sim \frac{Z^{1/3}}{\mathrm{m_p}}$$

$$\frac{\mathrm{r_{nucleus}}}{\mathrm{r_{atom}}} \sim \frac{\alpha \mathrm{m_e}}{Z^{2/3}\mathrm{m_p}} \sim 10^{-5}$$

$$\mathrm{p_e} \sim \frac{1}{\mathrm{r_{atom}}} \sim \mathrm{m_e}(Z\alpha) \qquad \mathrm{v_e} \sim (Z\alpha)$$

$$\mathrm{E_{atom}} \sim \frac{Z\alpha}{\mathrm{r_{atom}}} \sim Z^2\alpha^2\mathrm{m_e}$$

For Hydrogen

10^{-4} 0.5 MeV $\sim$ 50 eV

(Actually is 13.6 eV)

Solids

(To within our ~) Solids just atoms stacked next to each other

Mass Density: Mass/Volume

$$\rho_{\rm solid} \sim \frac{\rm Zm_p}{(\rm r_{atom})^3} \sim \rm Z^4\alpha^3 m_p m_e^3$$

Pressure of Solid: Force/Area or Energy/Volume

$$\rm P_{solid} \sim \frac{Z^2\alpha^2 m_e}{(r_{atom})^3} \sim Z^5\alpha^5 m_e^4$$

(Ratio of two give the speed of sounds)

$$\rm v_{sound} \sim \sqrt{\frac{P_{solid}}{\rho_{solid}}} \sim \sqrt{\frac{\alpha}{m_p r_{atom}}}$$

Predict: ~25,000 m/s

Beryllium	12,890 m/s
Diamond	12,000 m/s
Steel	6000 m/s

Planets

Solids where gravitational pressure balanced by solid pressure

$$\mathrm{E_{Gravity}} \sim \frac{\mathrm{G_N M_p^2}}{\mathrm{R_p}} \qquad \mathrm{P_{Gravity}} \sim \frac{\mathrm{E_{Gravity}}}{\mathrm{V_{Planet}}} \sim \frac{\mathrm{G_N M_p^2}}{\mathrm{R_p^4}}$$

$$\mathrm{M_{Planet}} \sim \rho_{\mathrm{solid}} \times \mathrm{R_P^3} \sim \frac{\mathrm{Z m_p R_P^3}}{\mathrm{r_{atom}^3}}$$

$$\mathrm{P_{Gravity}} \sim \frac{\mathrm{G_N Z^2 m_p^2 R_P^2}}{\mathrm{r_{atom}^6}}$$

$$\mathrm{P_{Gravity}} \sim \mathrm{P_{solid}} \qquad \frac{\mathrm{G_N Z^2 m_p^2 R_P^2}}{\mathrm{r_{atom}^6}} \sim \frac{\mathrm{Z}\alpha}{\mathrm{r_{atom}^4}}$$

$$\mathrm{R_{Planet}} \sim \sqrt{\frac{1}{\mathrm{G_N m_p^2 Z^3 \alpha m_e^2}}} \sim \sqrt{\frac{\alpha}{\alpha_{\mathrm{G}}}} \times \mathrm{r_{atom}}$$

Planets

Solids where gravitational pressure balanced by solid pressure

$$E_{\mathrm{Gravity}} \sim \frac{G_N M_p^2}{R_p} \qquad P_{\mathrm{Gravity}} \sim \frac{E_{\mathrm{Gravity}}}{V_{\mathrm{Planet}}} \sim \frac{G_N M_p^2}{R_p^4}$$

$$M_{\mathrm{Planet}} \sim \rho_{\mathrm{solid}} \times R_P^3 \sim \frac{Z m_p R_P^3}{r_{\mathrm{atom}}^3}$$

$$P_{\mathrm{Gravity}} \sim \frac{G_N Z^2 m_p^2 R_P^2}{r_{\mathrm{atom}}^6}$$

$$P_{\mathrm{Gravity}} \sim P_{\mathrm{solid}}$$

Planets/atoms relative size direct result of EM vs gravity strength

r_{atom} r_{atom}

α G $\times\, r_{\mathrm{atom}}$

Prediction:	$r_e \sim 10^7\mathrm{m}$	$M_p \sim 10^{25}\mathrm{kg}$
Actual:	$6.4 \cdot 10^6\mathrm{m}$	$5.9 \cdot 10^{24}\mathrm{kg}$

This is why things are big, despite being governed by microscopic laws

Life

Estimate limit on size of life: Require dont break bones when fall

$$E_{\rm fall} \sim M_A g_{\rm local} L_A$$

$$g_{\rm local} \sim G_N \frac{M_P}{R_P^2} \sim \sqrt{\alpha_G \alpha} \frac{1}{m_p r_{\rm atom}^2}$$

Prediction:	~5 m/s/s
Actual:	9.8 m/s/s

Break bones along cross sectional areas

$$E_{\rm Break\ Bones} \sim N_{\rm atoms\ cross-section} \times E_{\rm atom}$$
$$\sim \left(\frac{L_A}{r_{\rm atom}}\right)^2 \times \frac{Z\alpha}{r_{\rm atom}}$$

$$E_{\rm Fall} \sim E_{\rm Break\ Bones}$$

$$L_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{1}{4}} \times r_{\rm atom} \qquad M_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{4}} \times Z m_p$$

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$E_{\rm Fall} \sim E_{\rm B}$ $\boxed{L_A \sim 10\ \text{cm} \ / \ M_A \sim 100\ \text{kg}}$

$$L_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{1}{4}} \times r_{\rm atom} \qquad M_A \sim \left(\frac{\alpha}{\alpha_G}\right)^{\frac{3}{4}} \times Z m_p$$