1) All about you.

2) Reading

3) See video!!!

a) $$A \times C = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = c_1 \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} + c_2 \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} c_1 a_{11} + c_2 a_{12} \\ c_1 a_{21} + c_2 a_{22} \end{bmatrix}$$

b) $$A \times B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$$

b) $$nA = \begin{bmatrix} n a_{11} & n a_{12} \\ n a_{21} & n a_{22} \end{bmatrix}$$

c) Video.

3) a)

$$\frac{d}{dt}\sum_i P'_i = \sum_i m_i \frac{d(v(t) - v_s)}{dt}$$

$$= \sum_i m_i \frac{dv}{dt} = \frac{d}{dt}\sum P$$

$\Rightarrow$ P'_{tot} conserved if P_{tot} conserved.

b)

$$\frac{d}{dt}\sum_i E'_i = \sum_i \frac{d}{dt}\frac{m_i v'^2_i}{2} = \sum_i \frac{m_i}{2}\frac{d(v - v_s)^2}{dt}$$

$$= \sum_i \frac{m_i}{2} 2(v_i - v_s)\frac{dv_i}{dt} \qquad 0 = \frac{dP_{tot}}{dt}\cdot v$$

$$= \sum_i \underbrace{\left(\frac{m_i}{2} 2v_i \frac{dv_i}{dt}\right)}_{\frac{d}{dt}E_{tot}} - \sum_i \cancel{v_s m_i \frac{dv_i}{dt}}$$

$$= \frac{d}{dt}\sum_i E_i$$

$\Rightarrow$ E'_{tot} conserved if E_{tot} conserved.

4) Assume transform is linear Between reference frame w/ relative velocity v

$$x = ax' + bt' \qquad t = ex' + ft' \qquad \text{or} \quad \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} a & b \\ e & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

w/ (a b e f) unknown functions of v

a) when $x' = 0$ $\quad x = vt$ ← Assumption of velocity v

$$\Rightarrow \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} a & b \\ e & f \end{pmatrix} \begin{pmatrix} 0 \\ t' \end{pmatrix} = \begin{pmatrix} bt' \\ ft' \end{pmatrix}$$

$$\frac{x}{t} = \frac{bt'}{ft'} = v \qquad \text{or} \quad \begin{pmatrix} x \\ t \end{pmatrix} = f \begin{pmatrix} \frac{a}{f} & v \\ \frac{e}{f} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

b) When $x = 0$ $\quad x' = -vt'$ (view from other frame)

$$\begin{pmatrix} 0 \\ t \end{pmatrix} = f \begin{pmatrix} \frac{a}{f} & v \\ e/f & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} ax' + fvt' \\ ex' + ft' \end{pmatrix}$$

$$= \begin{pmatrix} ax' - fx' \\ ex' - \frac{f}{v} x' \end{pmatrix}$$

$$\Rightarrow a = f$$

So General C.T.

$$\begin{pmatrix} x \\ t \end{pmatrix} = a \begin{pmatrix} 1 & v \\ \frac{e}{a} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

c) Now impose C.T. form a group

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} \xrightarrow[v_1]{} \begin{pmatrix} x' \\ t' \end{pmatrix} \xrightarrow[v_2]{} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$\begin{pmatrix} x \\ t \end{pmatrix} = a(v_2) \begin{pmatrix} 1 & v_2 \\ \frac{e}{a}(v_2) & 1 \end{pmatrix} a(v_1) \begin{pmatrix} 1 & v_1 \\ \frac{e}{a}(v_1) & 1 \end{pmatrix} \begin{pmatrix} x'' \\ t'' \end{pmatrix}$$

$$= a(v_2)\, a(v_1) \begin{pmatrix} 1 + \frac{e}{a}(v_1)\cdot v_2 & v_1 + v_2 \\ \frac{e}{a}(v_1) + \frac{e}{a}(v_2) & 1 + \frac{e}{a}(v_2)\cdot v_1 \end{pmatrix} \begin{pmatrix} x'' \\ t'' \end{pmatrix}$$

Now action of 2 C.T's must be described by the action of single combined C.T.

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} \xrightarrow[v_c]{} \begin{pmatrix} x \\ t \end{pmatrix}$$

For this to be the case, the diagonal elements must be equal

$$1 + \frac{e}{a}(v_1)\cdot v_2 = 1 + \frac{e}{a}(v_2)\, v_1$$

$$\Rightarrow \frac{1}{v_1} \frac{e}{a}(v_1) = \frac{1}{v_2} \frac{e}{a}(v_2)$$

B/c we have separation of variables, both sides must be equal to a constant (See this by applying $\frac{\partial}{\partial v_1}$ & $\frac{\partial}{\partial v_2}$)

$$\Rightarrow \frac{1}{v} \frac{e}{a}(v) = g \quad \Rightarrow \frac{e}{a}(v) = g v$$

free constant (independent of v)

So, General C.T.

$$\begin{pmatrix} x \\ t \end{pmatrix} = a \begin{pmatrix} 1 & v \\ g v & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

$\boxed{\begin{array}{l} \underline{\text{Note}} \\ [t] = [g][v][x] \\ \text{or } [g] = \dfrac{[t]}{[x][v]} = \dfrac{1}{[v]^2} \\ g \to \dfrac{1}{v_*^2} \end{array}}$

d) find a, by imposing $\begin{pmatrix} x \\ t \end{pmatrix} \xrightarrow[v]{} \begin{pmatrix} x' \\ t' \end{pmatrix} \xrightarrow[-v]{} \begin{pmatrix} x \\ t \end{pmatrix}$

$$\begin{pmatrix} x \\ t \end{pmatrix} = a(-v) \begin{pmatrix} 1 & -v \\ -\frac{v}{v_*^2} & 1 \end{pmatrix} a(v) \begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$= a(-v)\, a(v) \underbrace{\begin{pmatrix} 1 - \frac{v^2}{v_*^2} & 0 \\ 0 & 1 - \frac{v^2}{v_*^2} \end{pmatrix}}_{\text{must } = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$\Rightarrow \quad a(-v)\, a(v) = \frac{1}{1 - \frac{v^2}{v_*^2}}$$

Symmetry of Space

$\Rightarrow a(v) = a(|v|)$

or $a(-v)\, a(v) = (a(v))^2$

$$\Rightarrow \quad a(v) = \frac{1}{\sqrt{1 - \frac{v^2}{v_*^2}}}$$

General C.T. parametrized by v_* given by

$$\begin{pmatrix} x \\ t \end{pmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{v_*^2}}} \begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

e) What is v_*? Assume something has velocity v_* in primed coordinates

$$x' = v_* t'$$

In unprimed coordinates,

$$\begin{pmatrix} x \\ t \end{pmatrix} = \gamma_{v_*} \begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} v_* t' \\ t' \end{pmatrix} = \gamma_{v_*} \begin{pmatrix} v_* t' + v t' \\ \frac{v}{v_*} t' + t' \end{pmatrix}$$

$$\frac{x}{t} = \frac{v_* + v}{\frac{v}{v_*} + 1} = \frac{v_* + v}{\frac{v + v_*}{v_*}} = v_*$$

v_* | maximum speed !
| coordinate invariant

f)

$$v_*^2 t^2 - x^2 = v_*^2 \left(\frac{\frac{v}{v_*^2} x' + t'}{\sqrt{1 - \left(\frac{v}{v_*}\right)^2}} \right)^2 - \left(\frac{x' + v t'}{\sqrt{1 - \left(\frac{v}{v_*}\right)^2}} \right)^2$$

$$= \frac{\left(\frac{v}{v_*} x' + v_* t' \right)^2 - (x' + v t')^2}{1 - \left(\frac{v}{v_*}\right)^2}$$

$$= \frac{1}{1 - \left(\frac{v}{v_*}\right)^2} \left(\frac{v^2}{v_*^2} x'^2 + \cancel{2 v x' t'} + v_*^2 t'^2 - x'^2 - \cancel{2 v x' t'} - v^2 t'^2 \right)$$

$$= \frac{1}{1 - \left(\frac{v}{v_*}\right)^2} \left[(v_*^2 - v^2) t'^2 + \left(\frac{v^2}{v_*^2} - 1 \right) x'^2 \right]$$

$$= \frac{1}{1-\left(\frac{v}{v_*}\right)^2}\left[\left(1-\left(\frac{v}{v_*}\right)^2\right) v_*^2 t'^2 - \left(1-\left(\frac{v}{v_*}\right)^2\right) x'^2\right]$$

$$= v_*^2 t'^2 - x'^2$$

g) when $v_* \to \infty$ we recover Galilean Transfrms

$$\begin{pmatrix} x \\ t \end{pmatrix} = \frac{1}{\sqrt{1-\frac{v^2}{v_*^2}}}\begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix}$$

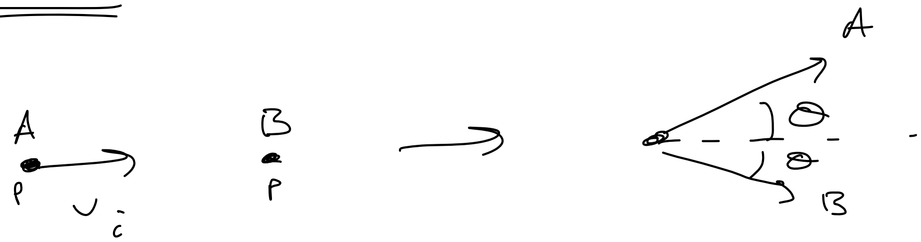
$$\xrightarrow[v_* \to \infty]{} = 1 \begin{pmatrix} 1 & v \\ \emptyset & 1 \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix}$$

or

$$x = x' + vt'$$

$$t = t'$$

5) Collisions



Determine θ

$$\vec{P}_I = m_p v = \vec{P}_F$$

x-direction

$$2 v_f \cos\theta = v_i$$

$$\frac{v_i}{v_f} = 2\cos\theta$$

or

y-direction

$$m v_A^f \sin\theta = m v_B^f \sin\theta \Rightarrow v_A^f = v_B^f$$

$$E_I = m v_i^2 = 2 m v_f^2$$

$$\left(\frac{v_i}{v_f}\right)^2 = 2$$

$$4\cos^2\theta = 2$$

$$\cos^2\theta = \frac{1}{2} \qquad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

6)

1 ns

(Remember this number!)

7) A non-accelerating reference frame with

- Smooth train: technically not, on earth.
- No merry go round is accelerating.

8)

time for B

$$t_B = \frac{L}{v}$$

$$t_A = \frac{\frac{L}{2}}{2v} + \frac{\frac{L}{2}}{v/2}$$

$$= \frac{1}{4}\frac{L}{v} + \frac{L}{v}$$

$$t_A > t_B$$

a) Laws of physics (and all physical constants appearing in them) are the same in all inertial reference frames

(Corollary: the speed of light (one of the physical constants) is the same in all reference frames.

(d) Say two different clocks were adjusted to run at the same speed ("calibrated") at rest and then set in motion, if the motion affects the clocks differently, someone riding along with them could see the clocks running & thus know they were in a moving reference frame.

This is a violation of the Principle of Relativity

$\Rightarrow$ the clocks must be affected in exactly the same way.