1)

$$\Psi(x,t) = c_1 \Psi_1(x,t) + c_2 \Psi_2(x,t)$$

$$\Psi(x,t) = c_1 \Psi_1(x) e^{-iE_1 t/\hbar} + c_2 \Psi_2(x) e^{-iE_2 t/\hbar}$$

$$P(x,t) = |\Psi(x,t)|^2$$

$$= \left(c_1 \Psi_1(x) e^{-iE_1 t/\hbar} + c_2 \Psi_2(x) e^{-iE_2 t/\hbar}\right)$$

$$\times\left(c_1 \Psi_1(x) e^{+iE_1 t/\hbar} + c_2 \Psi_2(x) e^{+iE_2 t/\hbar}\right)$$

$$= c_1^2 \Psi_1^2 + c_1 c_2 \Psi_1 \Psi_2 e^{-i(E_1 - E_2)t/\hbar} + c_1 c_2 \Psi_1 \Psi_2 e^{+i(E_1 - E_2)t/\hbar}$$

$$+ c_2^2 \Psi^2$$

$$= c_1^2 \Psi_1^2 + c_2^2 \Psi_2^2 + 2 c_1 c_2 \Psi_1 \Psi_2 \cos\frac{(E_2 - E_1)t}{\hbar}$$

P - oscillates w/ simple harmonic motion

w/ frequency $\frac{E_2 - E_1}{\hbar}$

Not stationary State.

2a) $L = 10\,fm = 10^{-14}\,m$

$nm = 10^{-9}$
$fm = 10^{-15}$

1) m_e

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2 m L^2}$$

$eV\,nm$
$= 10^{-6} MeV\, 10^{6} fm$

$$E_1 = \frac{(hc)^2}{8(mc^2)(10\,fm)^2} = \frac{(1240\,MeV\,fm)^2}{8(0.5\,MeV)(10)^2}$$

$$= 3760\,MeV$$

b) m_p

$$E_1 = \frac{(1240\,MeV\,fm)^2}{8(1000\,MeV)\,10^2} = 2\,MeV$$

c)

$$\Delta E = E_{n_2} - E_{n_1} = (n_2^2 - n_1^2)\frac{\pi^2\hbar^2}{2mL^2}$$

$$= (n_2^2 - n_1^2)\,E_1$$

for $2 \to 1$ for electron

$$\Delta E = 3 \times 3760\,\text{MeV}$$

$$\sim 10{,}000\,\text{MeV}$$ too big!

for proton

$$\Delta E \sim 6\,\text{MeV}$$ about right.

2b) $\lambda = 694.3\,\text{nm}$

Assume from $2 \to 1$

$$E_\gamma = \frac{hc}{\lambda} \qquad E_{2\to 1} = 3E_1 = \frac{3\pi^2\hbar^2}{2mL^2}$$

$$\frac{hc}{\lambda} = \frac{3(hc)^2}{8(mc^2)L^2} \Rightarrow L^2 = \frac{3\,(hc)\,\lambda}{8\,(mc^2)}$$

$$L^2 = \frac{3}{8} \frac{1240 \text{ eV nm} \quad 694 \text{ nm}}{(0.5\ 10^6 \text{ eV})}$$

$$= 0.6 \text{ nm}^2$$

$$\Rightarrow L = 0.8 \text{ nm}$$

2c) $\langle p^2 \rangle = \int \psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi \, dx$

$$= \int \psi^* \underbrace{\left(-\hbar^2 \frac{\partial^2}{\partial x^2}\right)}_{= 2m[E - V(x)]} \psi \, dx$$

By t-ind. Schr Eq

$$= \int \psi^* \left(2m[E - V(x)]\right) \psi \, dx$$

$$= \langle 2m[E - V(x)] \rangle$$

$\Rightarrow \langle p^2 \rangle$ (for ∞-well)

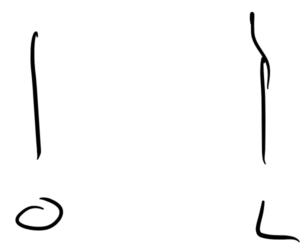
$$= \langle 2mE \rangle = 2m\left(\frac{n^2\pi^2\hbar^2}{2mL^2}\right)$$

$$= \frac{n^2\pi^2\hbar^2}{L^2}$$

Or $\frac{\pi^2\hbar^2}{L^2}$ for ground state

2d) for σ_x need $\langle x \rangle$ + $\langle x^2 \rangle$

$\langle x \rangle = \frac{L}{2}$ by symmetry



$\langle p \rangle = 0$

$\langle p^2 \rangle = \frac{\pi^2\hbar^2}{L^2}$ (see above)

Only need $\langle x^2 \rangle$

$$\langle x^2 \rangle = \int_0^L \psi_1^* \, x^2 \, \psi_1 \, dx$$

$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

$$\langle x^2 \rangle = \int_0^L \frac{2}{L} x^2 \sin^2 \frac{\pi x}{L} \, dx$$

$$z = \frac{\pi x}{L} \qquad x = \frac{Lz}{\pi} \qquad dx = \frac{L}{\pi} dz$$

$$\langle x^2 \rangle = \left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)^3 \underbrace{\int_0^{\pi} z^2 \sin^2 z \, dz}_{\frac{1}{12}\pi\left(2\pi^2 - 3\right)}$$

wolfram alpha

$$\langle x^2 \rangle = \frac{2L^2}{\pi^3}\left(\frac{\pi^3}{6} - \frac{\pi}{4}\right) = \frac{L^2}{\pi^2}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$$

$$= \frac{L^2}{3} - \frac{L^2}{2\pi^2}$$

So $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$$= \left(\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \frac{L^2}{4} \right)^{1/2}$$

$$= L \sqrt{\frac{1}{12} - \frac{1}{2\pi^2}} = 0.181\ L$$

$$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{\pi^2 \hbar^2}{L^2}} = \frac{\pi \hbar}{L}$$

$$\sigma_x \sigma_p = 0.181\ \pi \hbar$$

$$= 0.568\ \hbar$$

3 - Distinguishable Particles

$$\psi_{ground}(x_1, x_2) = A \sin\left(\frac{\pi x_1}{L}\right) \sin\left(\frac{\pi x_2}{L}\right)$$

$$E_{ground} = \underbrace{\frac{\pi^2 \hbar^2}{2mL^2}}_{E_1} (1 + 1) = 2E_1$$

- Identical Bosons

$$\psi_{ground}(x_1, x_2) = \psi_{ground}(x_2, x_1)$$

$$\Rightarrow \psi(x_1, x_2) = A \sin\left(\frac{\pi x_1}{L}\right) \sin\left(\frac{\pi x_2}{L}\right)$$

$$E_{ground} = 2E_1$$

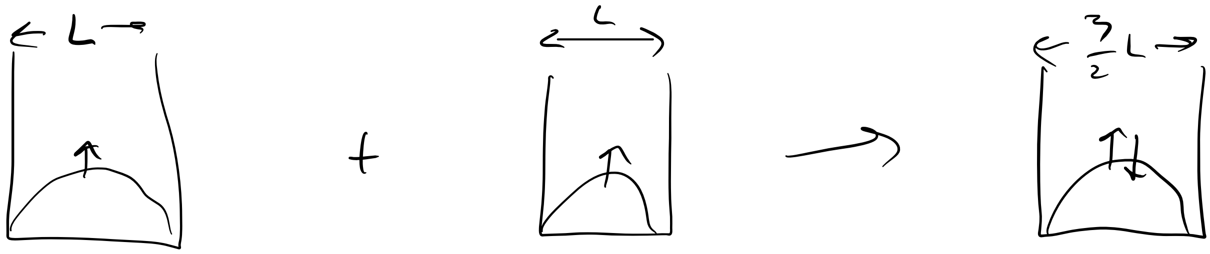
- Identical Fermions $\quad \psi_{ground}(x_1, x_2) = -\psi_{ground}(x_2, x_1)$

$$\Rightarrow \psi(x_1, x_2) = A \left[\sin\frac{\pi x_1}{L} \sin\frac{2\pi x_2}{L} - \sin\frac{2\pi x_1}{L} \sin\frac{\pi x_2}{L} \right]$$

$$E = E_1 (1^2 + 2^2) = 5E_1$$

4)

$$H + H \rightarrow H_2$$



$$E = \frac{\pi^2\hbar^2}{2mL^2} \qquad \frac{\pi^2\hbar^2}{2mL^2} \qquad 2 \times \frac{\pi^2\hbar^2}{2m(\frac{3}{2}L)^2}$$

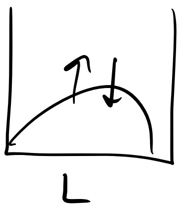
$$\equiv E_1$$

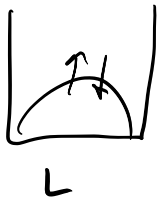
$$2E_1 \quad \text{vs} \quad \frac{8}{9}E_1$$

Energy Lower when combined

$\Rightarrow$ Expect Binding

$He + He \longrightarrow He_2$





Here need 2 e^-'s in $n=1$
2 e^-'s in $n=2$

$$\underbrace{2E_1 + 2E_1}_{4E_1} \quad \text{vs} \quad 2\frac{\pi^2\hbar^2}{2m(\frac{3}{2}L)^2} + 2\frac{\pi^2\hbar^2 2^2}{2m(\frac{3}{2}L)^2}$$

$$\frac{8}{9}E_1 + \frac{32}{9}E_1$$

$$\frac{40}{9}E_1 = \frac{36+4}{9}E_1$$

$$= 4E_1 + \frac{4}{9}E_1$$

Energy higher when combined

$\Rightarrow$ Expect <u>No</u> Binding