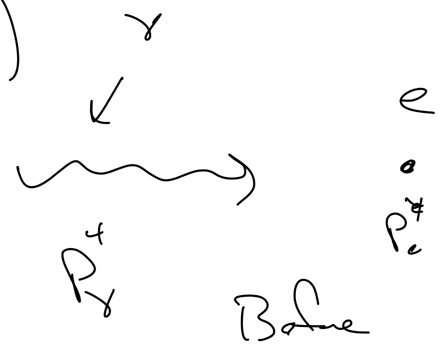
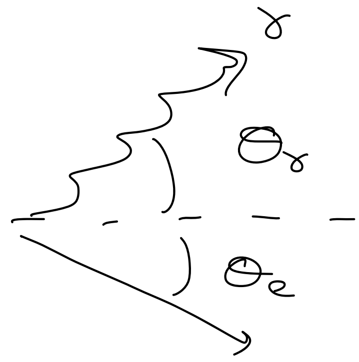
2)



Before



After

$$p^4_\gamma = E_\gamma \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad p^4_e = \begin{pmatrix} m_e \\ \vec{0} \end{pmatrix} \qquad \bar{p}^4_\gamma = \bar{E}_\gamma \begin{pmatrix} 1 \\ \cos\theta_\gamma \\ \sin\theta_\gamma \\ 0 \end{pmatrix} \quad \bar{p}^4_e = \begin{pmatrix} \bar{E}_e \\ E_\gamma - \bar{E}_\gamma \cos\theta_\gamma \\ -\bar{E}_\gamma \sin\theta_\gamma \\ 0 \end{pmatrix}$$

Solve for $\bar{E}_e$ — Conserves $\vec{p}$ Explicitly

$$E^2 - p^2 = m_e^2$$

$$\bar{E}_e^2 - (E_\gamma - \bar{E}_\gamma \cos\theta)^2 - (\bar{E}_\gamma \sin\theta)^2 = m_e^2$$

$$\bar{E}_e^2 - E_\gamma^2 + 2E_\gamma \bar{E}_\gamma \cos\theta - \bar{E}_\gamma^2 \cos^2\theta - \bar{E}_\gamma^2 \sin^2\theta = m_e^2$$

$$\bar{E}_e^2 - E_\gamma^2 + 2E_\gamma \bar{E}_\gamma \cos\theta - \bar{E}_\gamma^2 = m_e^2$$

$$\bar{E}_e^2 = m_e^2 + E_\gamma^2 + \bar{E}_\gamma^2 - 2E_\gamma \bar{E}_\gamma \cos\theta$$

Crossover E

$$E_\gamma + m_e = \bar{E}_\gamma + \bar{E}_e = \bar{E}_\gamma + \sqrt{m_e^2 + E_\gamma^2 + \bar{E}_\gamma^2 - 2E_\gamma \bar{E}_\gamma \cos\theta}$$

$$\underbrace{\left(E_\gamma - \bar{E}_\gamma + m_e\right)^2} = m_e^2 + E_\gamma^2 + \bar{E}_\gamma^2 - 2E_\gamma \bar{E}_\gamma \cos\theta$$

$$(E_\gamma - \bar{E}_\gamma)^2 + 2(E - \bar{E})m_e + m_e^2$$

$$\cancel{E_\gamma^2} - \cancel{2}E_\gamma \bar{E}_\gamma + \cancel{\bar{E}_\gamma^2} + \cancel{2}(E - \bar{E})m_e + \cancel{m_e^2} =$$

$$\cancel{m_e^2} + \cancel{E_\gamma^2} + \cancel{\bar{E}_\gamma^2} - \cancel{2}E_\gamma \bar{E}_\gamma \cos\theta$$

$$(E_\gamma - \bar{E}_\gamma)m_e = E_\gamma \bar{E}_\gamma (1 - \cos\theta_\gamma)$$

OR

$$\frac{1}{\bar{E}_\gamma} - \frac{1}{E_\gamma} = \frac{1}{m_e}\left(1 - \cos\theta_\gamma\right)$$

$$\frac{\bar{\lambda}}{hc} - \frac{\lambda}{hc} = \frac{1}{mc^2}\left(1 - \cos\theta_\gamma\right)$$

$$\bar{\lambda} - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta_\gamma\right)$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_e}{\lambda}\left(1 - \cos\theta_\gamma\right)$$

$$\lambda_e \sim 2 \; 10^{-12} \, m = 2 \; 10^{-3} \, nm$$

$$E_\gamma = \frac{hc}{\lambda_e} = \frac{1240 \, eV \, nm}{2 \; 10^{-3} \, nm} = 2480 \; 10^{3} \, eV = 3 \; 10^{6} \, eV$$

$$= 3 \, MeV$$

When scattering on proton all is the same

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_p}{\lambda} \left(1 - \cos\theta_\gamma \right)$$

$$\lambda_p \sim 1 \; 10^{-15} \, m = 10^{-6} \, nm$$

$$E_\gamma = \frac{hc}{\lambda_p} = \frac{1.2 \; 10^{3} \, eV \, nm}{10^{-6} \, nm} = 1.2 \; 10^{9} \, eV$$

$$\sim GeV$$

3a) Large Angle scattering. $\Rightarrow$ atom had small hard core.

3b)

(1 D)

P & E are conserved

$$P_i = m_1 v_1^i + m_2 v_2^i = P_f \quad (i \to f)$$

$$E_i = m_1 v_1^{i\,2} + m_2 v_2^{i\,2} = E_f \quad (i \to f)$$

$$\Rightarrow v_2^f = \frac{m_1 v_1^i + m_2 v_2^i - m_1 v_1^f}{m_2}$$

Assume $v_2^i = 0$ & $m_1\ m_2\ v_1^i$ all known

$$v_2^f = \frac{m_1 v_1^i - m_1 - v_1^f}{m_2} = \frac{m_1}{m_2}\left(v_1^i - v_1^f\right)$$

$$E_i = m_1 v_1^{i\,2} = m_1 v_1^{f\,2} + m_2 v_2^{f\,2}$$

$$= m_1 v_1^{f\,2} + m_2 \underbrace{\left(\frac{m_1}{m_2}\left(v_1^i - v_1^f\right)\right)^2}$$

$$m_1 v_1^{i\,2} = m_1 v_1^{f\,2} + \frac{m_1^2}{m_2}\left(v_1^i - v_1^f\right)^2$$

Divide m_1, expand

$$v_1^{i\,2} = v_1^{f\,2} + \frac{m_1}{m_2}\left(v_1^{i\,2} - 2v_1^i v_1^f + v_1^{f\,2}\right)$$

$$\left(1+\frac{m_1}{m_2}\right) v_1^{f\,2} + \left(-\frac{2m_1}{m_2} v_1^i\right) v_1^f - \left(1-\frac{m_1}{m_2}\right) v_1^{i\,2} = 0$$

$$v_1^f = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\boxed{\begin{aligned} \det &= 4\left(\frac{m_1}{m_2}\right)^2 v_1^{i\,2} + 4\left(1+\frac{m_1}{m_2}\right)\left(1-\frac{m_1}{m_2}\right) v_1^{i\,2} \\ &= \quad '' \quad + 4\left(1-\left(\frac{m_1}{m_2}\right)^2\right) v_1^{i\,2} \\ &= 4 v_1^{i\,2}\left(\left(\frac{m_1}{m_2}\right)^2 + 1 - \left(\frac{m_1}{m_2}\right)^2\right) \\ &= 4 v_1^{i\,2} \end{aligned}}$$

$$= \frac{2\frac{m_1}{m_2} v_1^i \pm 2v_1^i}{2\left(1+\frac{m_1}{m_2}\right)} = \frac{\frac{m_1}{m_2} v_1^i \pm v_1^i}{1+\frac{m_1}{m_2}}$$

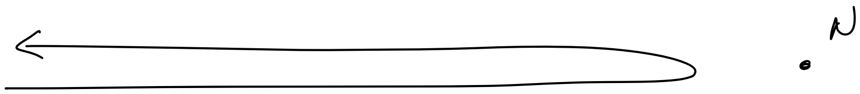
$$= \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{m_1}{m_2} \pm 1\right) v_1^i = \left(\frac{m_1 \pm m_2}{m_1+m_2}\right) v_1^i$$

If $v_1^f \sim -v_1^i$, then $m_2 \gg m_1$

3c) Classical Physics applied to the "solar system" atomic model Predicts

-) Continuous Spectrum of emitted γ's

-) That atoms are unstable.

3d) 

$E_\alpha = 8\,\mathrm{MeV}$

At closest approach

$$E_\alpha = PE = \frac{k(2e^+)(79e^+)}{r_{DCA}} = 160\,\frac{ke^2}{r_{DCA}} = 160\,\frac{2.3\ 10^{-28}\ \mathrm{J\,m}}{r_{DCA}}$$

$$r_{DCA} = \frac{160\ \ 2.3\ 10^{-28}}{8\ \ 1.6\ 10^{-13}\ \mathrm{J}}\ \mathrm{J\,m} = 20\ \ 10^{-15}\ \mathrm{m} \sim 10^{-14}\ \mathrm{m}$$

Natural units

$8 \; 10^{-3}\ \text{GeV} = 160 \; \frac{\alpha}{r} \sim \frac{1}{r}$

$200\ \text{MeV} = \text{fm}^{-1}$

r

$r = \frac{1}{8}\, 10^{3}\ \text{GeV}^{-1}$

$0.2\ \text{GeV} = \text{fm}^{-1}$

$= 0.1 \; 10^{3}\, (0.2\,\text{fm})$

$\text{GeV}^{-1} = 0.2\ \text{fm}$

$= 20\ \text{fm} \sim 10^{-14}\ \text{m}$

4a) Sharp lines from transitions between allowed orbits.

Only certian fixed orbits

$\Rightarrow$ only certian allowed $E = h\nu$

4b)

$$a_0 = \frac{\hbar^2}{mke^2} \qquad \lambda_c = \frac{h}{mc} \Rightarrow m_e = \frac{h}{\lambda_c c}$$

$$a_0 = \frac{\hbar^2}{\alpha \hbar c m} = \frac{\hbar c}{\alpha m c^2} \qquad \alpha = \frac{ke^2}{\hbar c} \Rightarrow ke^2 = \alpha \hbar c$$

So

$$a_0 = \frac{\hbar^2}{\left(\frac{h}{\lambda_c c}\right)(\alpha \hbar c)} = \frac{\lambda_c}{2\pi \alpha}$$

$$E_n = -\frac{ke^2}{2r_n} \Rightarrow E_1 = -\frac{ke^2 \, mke^2}{2\hbar^2} = -\frac{\alpha^2 mc^2}{2}$$

$$\& \; r_n = \frac{n^2 \hbar^2}{mke^2} \qquad \boxed{E_1 = \frac{1}{2}\alpha^2 mc^2}$$

Note Chemistry at scales

$\sim (10^{-2})^2$ MeV $\sim$ 10's eV

$$a_0 = \frac{2.4 \; 10^{-12} \text{ m}}{2\pi \; 10^{-2}} \sim 10^{-10} \text{ m} \quad \text{which is } r_{atom}$$

4c) Assume gravity quantized w/

$$L = mvr = n\hbar \qquad n = 1, 2, \ldots$$

$$r_{orbit} = 1.5 \; 10^{11} \, m$$

$$v_e = \frac{2\pi \, r_{orbit}}{year} \qquad year = \pi \cdot 10^7 \, s$$

$$v_e = \frac{2 \cdot 1.5 \; 10^{11} \, m}{10^7 \, s} \sim 3 \cdot 10^4 \, \frac{m}{s}$$

$$m_e = 6 \; 10^{24} \, kg$$

$$L_e = 6 \cdot 10^{24} \, kg \cdot 3 \; 10^4 \, \frac{m}{s} \cdot 1.5 \cdot 10^{11} \, m$$

$$= 27 \cdot 10^{39} \, \frac{kg \, m^2}{s} \sim 3 \; 10^{40} \, \frac{kg \, m^2}{s}$$

$$\hbar = 10^{-34} \, \underbrace{J \cdot s}_{kg \frac{m^2}{s^2} s \; \checkmark}$$

$$\boxed{n_e = \frac{L_e}{\hbar} = \frac{3 \; 10^{40}}{10^{-34}} \sim 3 \; 10^{74}}$$

$$E = \frac{p^2}{2m} - G_N \frac{m_e M_s}{r_e} \qquad \& \qquad F = ma$$

$$\Rightarrow G_N \frac{m_e M_s}{r^2} = \frac{m v^2}{r}$$

$$v^2 = G_N \frac{M_s}{r}$$

$$m v r = n \hbar \implies m^2 v^2 r^2 = n^2 \hbar^2$$

$$\text{or} \quad G_N m^2 M_s r = n^2 \hbar^2$$

$$\implies r_n = \frac{n^2 \hbar^2}{G_N m^2 M_s}$$

$$E = \frac{1}{2} m v^2 - G_N \frac{m M_s}{r}$$

$$= \frac{1}{2} G_N \frac{m M_s}{r} - G_N \frac{m M_s}{r} = -\frac{1}{2} G_N \frac{m M_s}{r}$$

$$E_n = -\frac{1}{2} G_N \frac{m M_s}{r_n} = -\frac{1}{2} \frac{G_N^2 m^3 M_s^2}{n^2 \hbar^2}$$

$$= -\frac{E_0^{GrU}}{n^2} = \frac{4\,10^{182}}{n^2} \text{ J}$$

$$\Delta E = E_0 \left(\frac{1}{(n_e-1)^2} - \frac{1}{n_e^2} \right) = E_0 \left(\frac{n_e^2 - (n_e-1)^2}{n_e^2 (n_e-1)^2} \right)$$

$$= E_0 \left(\frac{n_e^2 - (n_e^2 - 2n_e + 1)}{n_e^2 (n_e-1)^2} \right) = E_0 \left(\frac{2n_e + 1}{n_e^2 (n_e-1)^2} \right)$$

$$\sim E_0 \left(\frac{2n_e}{n_e^4} \right) = \frac{E_0\, 2}{n_e^3} = \frac{8\, 10^{182}}{10^{222}} \sim 10^{-40} \text{ J}$$

Would not be detectible!

$$r_n = \frac{n^2 \hbar^2}{G_N m^2 M_s}$$

$$\Delta r = \frac{\hbar^2}{G_N m_e^2 M_s} \underbrace{\left(n_e^2 - (n_e - 1)^2 \right)}_{\sim 2n_e}$$

$$\Delta r = \frac{\hbar^2}{G_N m_e^2 M_s} 2 N_e$$

$$= 10^{-63} \, m$$

$r \sim$ unchanged.

Moral for big systems well-described by classical physics its hard to know (not obvious) if they are described by some underlying quantum theory or not.

In fact, for gravity we still don't know

4d) $\Delta t_{excited} \sim 10^{-8}$ s

In $n=2$ $\quad mvr = 2\hbar \Rightarrow v = \frac{2\hbar}{mr}$

$$r = 2^2 a_0 \sim 4\ 10^{-10}\ m$$

Number of revolutions $= \frac{rev}{s}\ \Delta t$

$$\frac{rev}{s} = \frac{v}{2\pi r} = \frac{2\hbar}{2\pi m r^2} = \frac{\hbar}{\pi m r^2}$$

$N_{revolutions} = \sim 2\ 10^6$!