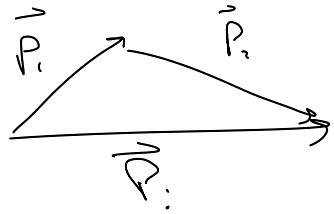
$\gamma \to \gamma\gamma$?

$\vec{p}$ needs to be conserved

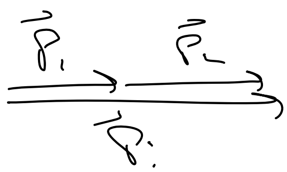
$\vec{p}_i = \vec{p}_1 + \vec{p}_2$



But, $E_i = E_1 + E_2$

& $E = |\vec{p}|$

only works if $\vec{p}_1$ & $\vec{p}_2$ are colinear



Need $\vec{p}_i = \vec{p}_1 + \vec{p}_2$

& $|\vec{p}_i| = |\vec{p}_1| + |\vec{p}_2|$

Turns out $\gamma \to \gamma\gamma$ not possible even in this case for γ's ("γ has no charge") but is possible for gluons ($m_g = 0$) "gluons have strong charge (color)"

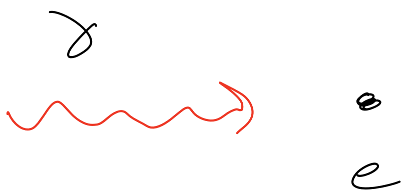
"Compton Scattering"

γ's (x-rays) can be scattered by electrons & have less energy after the scattering than before

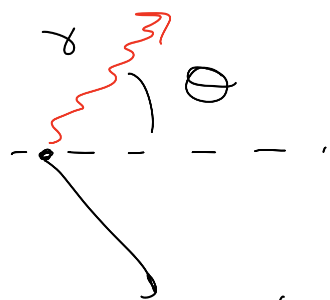
(Most important experimental work in 1920's)

↪ See why later...

Initial



Final



$$P_\gamma^4 = \begin{pmatrix} p_\gamma \\ p_\gamma \\ 0 \\ 0 \end{pmatrix} \qquad \bar{P}_\gamma^4 = \bar{p}_\gamma \begin{pmatrix} 1 \\ \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$P_e^4 = \begin{pmatrix} m_e \\ \vec{0} \end{pmatrix} \qquad \bar{P}_e^4 = \begin{pmatrix} \bar{E}_e \\ p_\gamma - \bar{p}_\gamma \cos\theta \\ -\bar{p}_\gamma \sin\theta \\ 0 \end{pmatrix}$$

$$\bar{E}_e = \sqrt{\bar{p}_e^2 + m_e^2}$$

$$\bar{p}_e^2 = \bar{p}_\gamma^2 \sin^2\theta + (p_\gamma - \bar{p}_\gamma \cos\theta)^2$$

$$= \underbrace{\bar{p}_\gamma^2 \sin^2\theta + \bar{p}_\gamma^2 \cos^2\theta}_{\bar{p}_\gamma^2} + p_\gamma^2 - 2 p_\gamma \bar{p}_\gamma \cos\theta$$

$$= \bar{P}_\gamma^2 + P_\gamma^2 - 2P_\gamma \bar{P}_\gamma \cos\theta$$

Conserve E

$$P_\gamma + m_e = \bar{P}_\gamma + \bar{E}_e = \bar{P}_\gamma + \sqrt{\bar{P}_\gamma^2 + P_\gamma^2 - 2P\bar{P}\cos\theta + m_e^2}$$

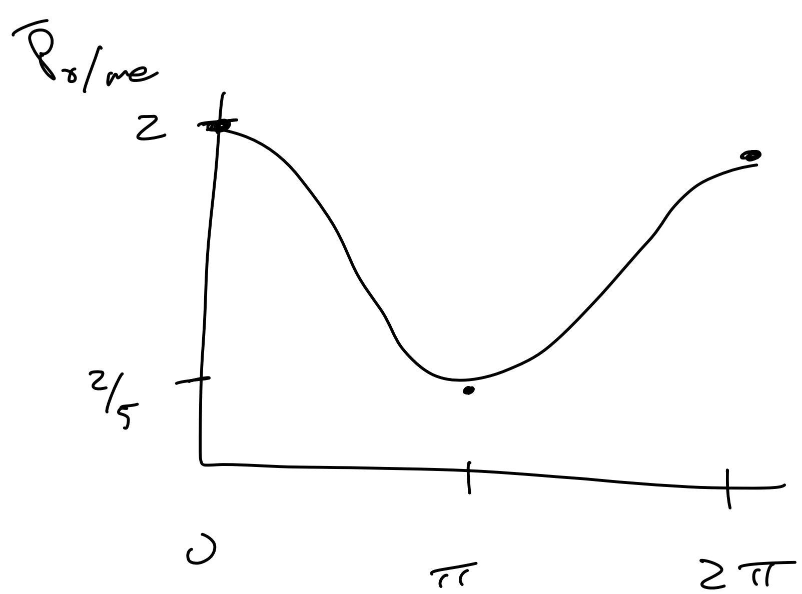
$$\underbrace{(P_\gamma - \bar{P}_\gamma + m_e)^2}_{(P-\bar{P})^2 + 2(P-\bar{P})m_e + m_e^2} = \bar{P}_\gamma^2 + P_\gamma^2 - 2\bar{P}_\gamma P_\gamma \cos\theta + m_e^2$$

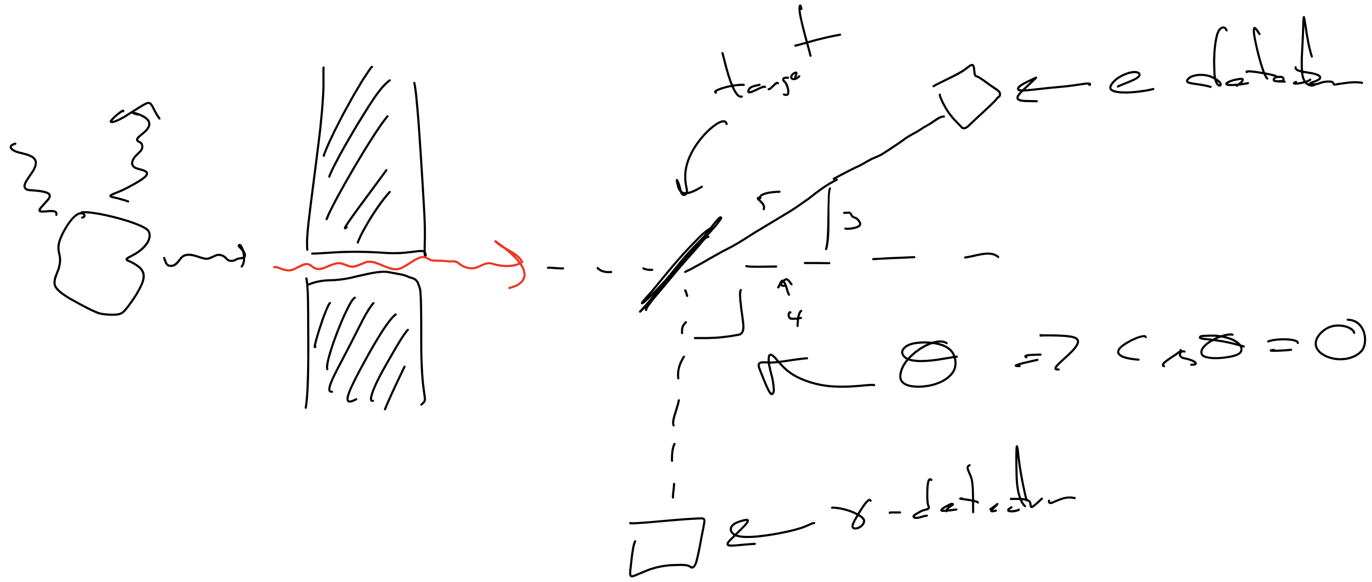
$$\cancel{P^2} - 2P\bar{P} + \cancel{\bar{P}^2} + 2(P-\bar{P})m_e + \cancel{m_e^2} = \cancel{\bar{P}_\gamma^2} + \cancel{P_\gamma^2} - 2P\bar{P}\cos\theta + \cancel{m_e^2}$$

$$(P_\gamma - \bar{P}_\gamma)m_e = P_\gamma \bar{P}_\gamma (1 - \cos\theta)$$

$$\Rightarrow \quad \bar{P}_\gamma = \frac{P_\gamma}{1 + \left(\frac{P_\gamma}{m_e}\right)(1 - \cos\theta)}$$

If $P_\gamma = 2m_e$ $\quad \bar{P}_\gamma = \dfrac{2m_e}{1 + 2(1-\cos\phi)} \quad P_\gamma(\phi = \pi) = \dfrac{2m_e}{1 + 2\cdot 2} =$





$$\bar{P}^4_\gamma = (\bar{P}_\gamma, 0, -\bar{P}_\gamma, 0)$$

$$\bar{P}^4_e = (E_e, P_\gamma, \bar{P}_\gamma, 0)$$

from Above

$$(P_\gamma - \bar{P}_\gamma) m_e = P_\gamma \bar{P}_\gamma$$

$$\bar{P}_\gamma = \frac{P m_e}{P + m_e} = \frac{P}{1 + P/m_e}$$

& know that

$$\frac{3}{4} = \frac{\bar{P}_\gamma}{P_\gamma} \Rightarrow \frac{3}{4} = \frac{1}{1 + P/m_e}$$

$$1 + \frac{P}{m_e} = \frac{4}{3}$$

$$P_\gamma = \frac{m_e}{3}$$

Discovering Particles w/ Colliders

Creation of short-lived Particles by protons / electrons

For lower masses, can use naturally occurring cosmic rays

Idea: Convert KE of high-E particles from an accelerator into rest mass of some new heavy particle.

1st example of this was Anti-protons 50's by colliding protons

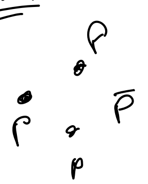
For various reasons (eg: Charge conservation) Anti protons created w/ protons

eg: $p + p \to p + p + \bar{p} + p$

Question What KE does the initial proton have to have to discover $\bar{p}$?

1st Try

Final



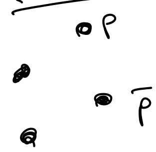
$E^f = 4m_p \Rightarrow$ Need KE of $2m_p$.

Why is this wrong?!

Need to Conserve Momentum!

Initial

Final



Claim that this is the min Energy?

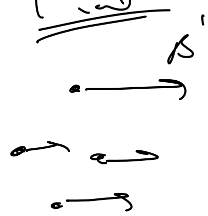
$E_i = 2m\gamma_{\beta'} \Rightarrow \gamma_{\beta'} = 2$

$E_f' = 4m$

Back in the Lab

Initial

Final



$$E = 4m\gamma_{\beta'} = 8m$$

$\Rightarrow$ Need KE of $6m$

Not $2m$!

Only $2m$ of the initial KE is converted to mass

the other $4m$

Colliding Beams much more effective.

Always have to accelerate particles in the LAB frame

If

Initial then Final is OK

For anti-protons lab frame each proton need $KE = m_p$

$KE = 1\, m_p$ vs $6\, m_p$ Need $6\times$ less accelerator

Now take ultra relativistic limit

LAB frame

$E_{cm} = 2E^c$

$\Rightarrow \gamma = \frac{E^c}{m} = \frac{E_{cm}}{2m}$

If "fixed target"

$$E = E_{cm}\,\gamma = \frac{E_{cm}^2}{2m}$$

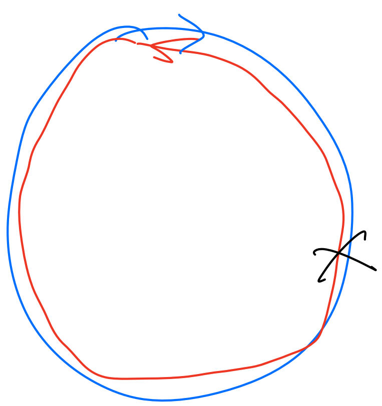
$$\Rightarrow \quad E^{FT} \sim \frac{E_{cm}^2}{2m} = \left(\frac{E_{cm}}{2}\right)\left(\frac{E_{cm}}{m}\right) = E^{c}\left(\frac{E^{cm}}{m}\right)$$

fixed targets need an extra factor $\dfrac{E_{cm}}{m}$!

eg LHC $E_{cm} \sim 10$ TeV $\sim 10^4$ GeV

$E_p^c \sim 5$ TeV $\qquad E_p^{F.T} = 5\text{ TeV}\left(\dfrac{10^4\text{ GeV}}{m_p}\right)$

$= 50{,}000$ TeV !



Interaction point

Advantage that particles that miss come back around.