Today 1 lecture on GR!

Really need ~ semester length graduate course to really learn GR.

- Discussion will be historical/qualitative
- Give you an idea about what it's about.

General Relativity

Extends ideas of relativity to non-inertial frames.

Its fundamental theory of gravity

Mathematically MUCH more complicated than S.R.

Turns out need description of "curved Space-time"

Requires Riemannian Geometry / "Differential Geometry"

Graduate-level math.

Main idea: why Curved Space has something to do w/ Gravity

Review: How do objects move in gravitational field?

$$F = m_g\, g = m_i\, a$$

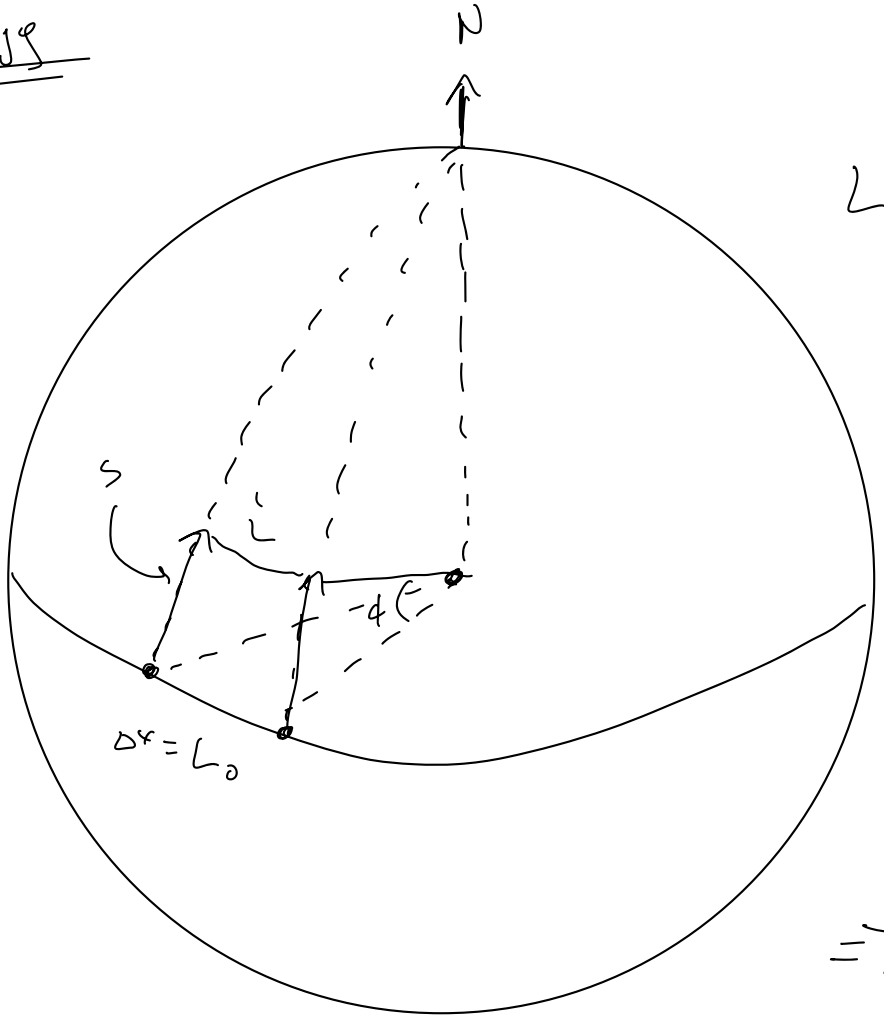
m_g: "Gravitational mass"

m_i: "inertial mass"

$a = \left(\frac{m_g}{m_i}\right) g$ We observe that $m_i = m_g$ to very high accuracy 10^{-12}

$\Rightarrow$ $a = g$ independent of what is falling!

Analogy



Latitude ϕ

$L' = L_0 \cos\phi$

ϕ small

$L' = L_0\left(1 - \frac{\phi^2}{2}\right)$

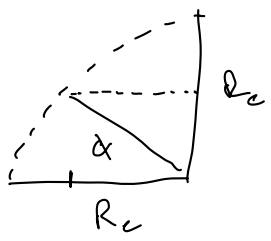
$\phi = \frac{S}{R}$

$\Rightarrow L_0 - L' = L_0 \frac{\phi^2}{2}$

$= L_0\left(\frac{S^2}{2R^2}\right)$

$C(\phi) = 2\pi r(\phi)$

$= 2\pi R_e \cos\phi$



$\frac{\Delta x}{C(\phi)} = \text{const}$

$\Delta x(0) \equiv L_0 = \alpha\, C(0)$

$= \alpha' R_e$

$\Delta x(\phi) = \Delta x(0) \cos\phi$

Note $L' \neq L_0$ B/c the surface of earth is curved.
Not flat

↳ A direct way to measure the geometry of the globe

So travelers who set out on parallel treks are seen (despite all precautions) to be slowly approaching each other!

They blame this on the existence of some mysterious "Gravitation Force" which deflects their paths.

What to do? Study the nature of this force.

Use direct means of trial | fast cars
bikes trucks ...

Always find the same shortening of separation!
↳ accelerate w/ ϕ

they know

$$F = ma$$

Conclude that the force of "Gravity" is directly proportional to mass of vehicle.

$$F_g = m_g g$$

$$a = \left(\frac{m_g}{m_i}\right) g$$

"Look we have discovered something great $m_g = m_i$!"

Of course we know, m_g vs m_i obscures the truth.
gravitational force

Curvature alone is enough to explain the increasing rate at which the travelers approach each other.

Summarize GR

Einstein did the same thing for Space-time

Curvature of Space-time all that is required to explain "gravitational" attraction of 2 bodies.

Our simple example was simple (constant curvature) 2D surface

Need to do this for arbitrary curvature in 4D Space-time.

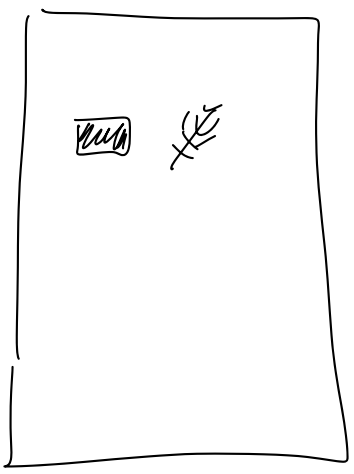
Einsteins Insight

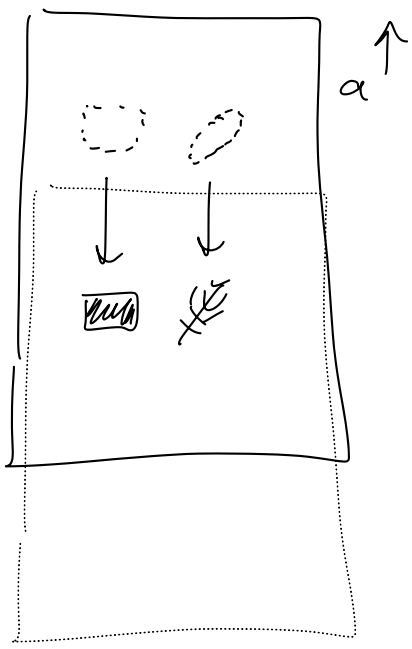
- If gravity due to curvature then gravitational acceleration always the same. Can't tell acceleration vs gravitational field.

"Equivalence Principle"

Homogenous Grav field | indistinguishable to uniformly accelerated frame!

eg)





From the PoV of the Box looks like all objects falling "down" at same $\vec{a}$

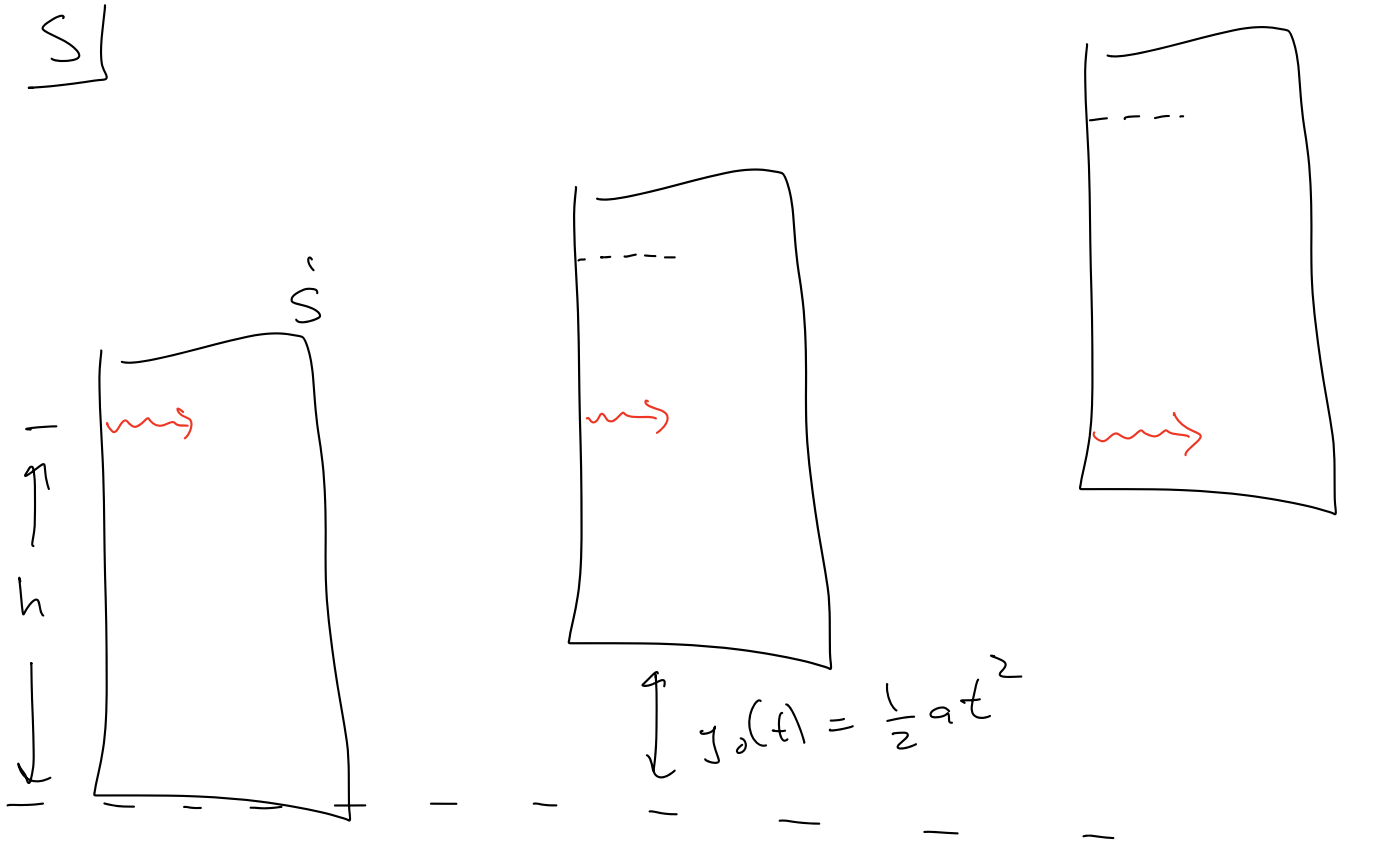
Notion that $m_i = m_g$ is baked in to this description.

Have any new content? Or just reformulation?

Consider how light ($m=0$!) behaves

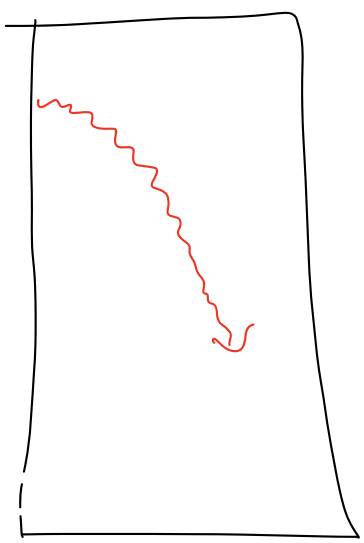
Newton says $F_g = mg = 0$

S



$x'(t) = t$

$y'(t) = h - y_0(t) = h - \frac{1}{2}at$



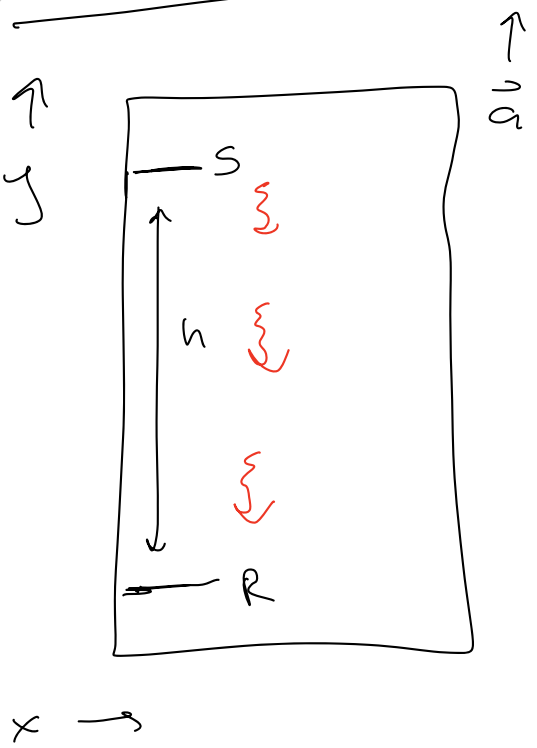
$\Rightarrow$ Light must also fall in "gravitational field"

Can calculate that the apparent positions of stars shift wrt background when close in sky to Sun

(Need eclipse to see stars!)

This is the thing that really made Einstein the famous public figure.

Gravitational Red Shift

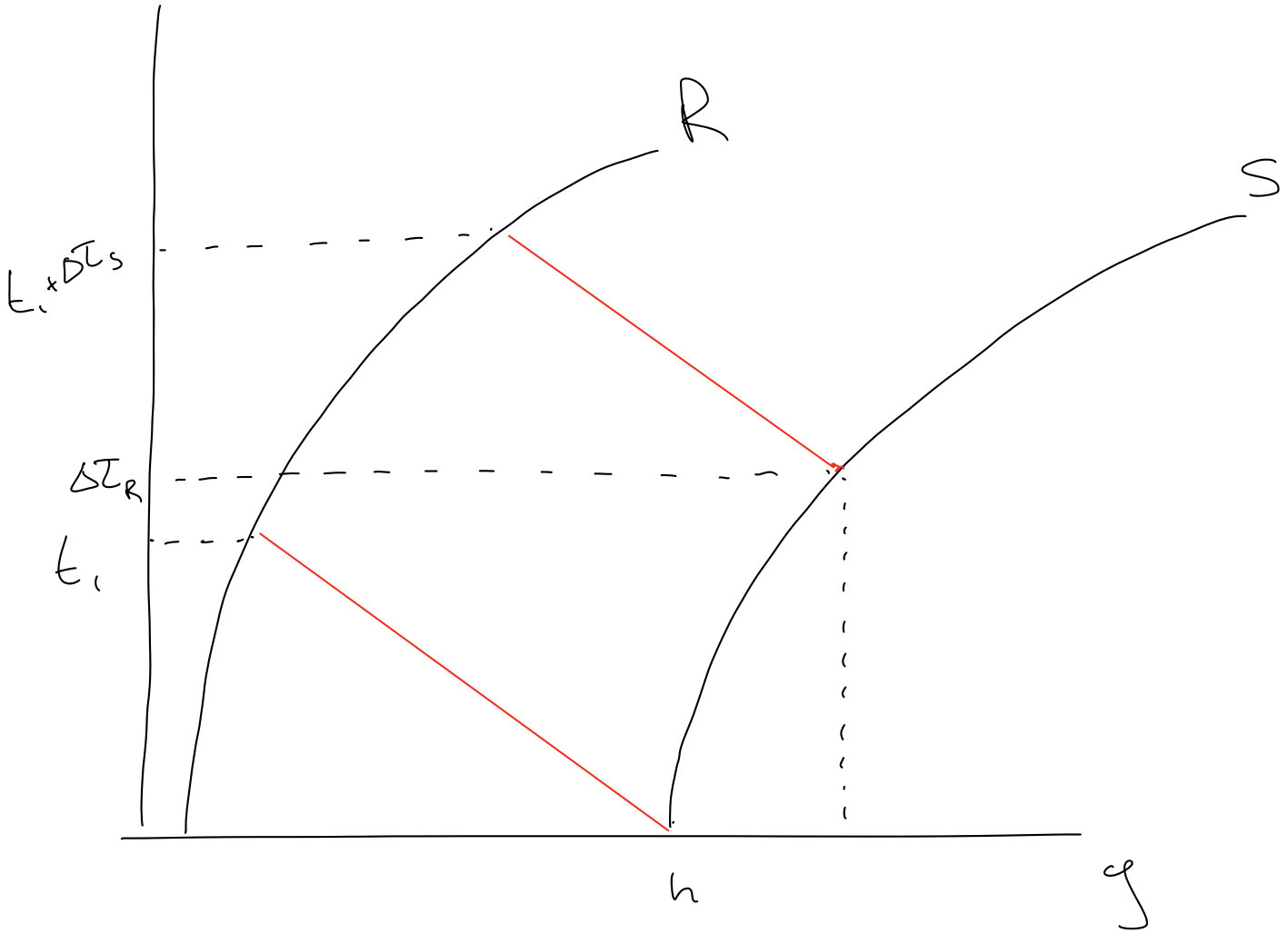


$$y_R = h + \frac{1}{2} g t^2$$

$$y_S = \frac{1}{2} g t^2$$

S emits $t = 0$, $\Delta\tau_S$

R receives $t = t_1$, $t_1 + \Delta\tau_R$



Wont go though Algebra!

$$\Delta T_R (1 + gh) = \Delta T_S$$

$$f_R = (1 + gh) f_S$$

$$= (1 + \Delta\phi_g) f_S$$

g-me potential

$$\phi_S - \phi_R = gh$$

$$f_{obs} = \left(1 - \frac{GM}{R}\right) f_{source}$$

← for star

$\frac{GM}{R} \ll 1$

Need this to get GPS to work!

In general

non-trivial

$$f_{obs} = \left(1 - \frac{2GM}{R}\right)^{1/2} f_{source}$$

Other Impacts of GR

- Gravitational waves!
- Mercury's Orbit (Precession)
- Black Holes
- Global Curvature to universe
  - "Big Bang"
  - "Dark Energy"

Overall Energy Scale Matters