Early Quantum theory guesswork motivated by failures of applying classical physics to

- atoms
- Radiation
- Interaction of matter + radiation

Last time we talked about establishment of atomic theory ($\Rightarrow$ quantization matter / charge)

Today will talk about radiation in 19 C

Hot things radiate light
- accelerating charged particles
- thermal radiation
- hotter $\Rightarrow$ more light / change colour

Late 19C Maxwell's Equations in hand

$\Rightarrow$ waves of E & B fields propagate in vacuum w/ $v = c$

2 polarizations of light.

$\boxed{1856}$ - Announcement of unexplained orbit of Mercury
- Origin of Species
- Kirchhoff challenge. explain BBR $\longrightarrow$ Solution in QM

Kirchhoff

Notes that if you have radiation in thermal equilibrium at temperature T

Define $\mathcal{E}(\lambda, T)\,d\lambda$ $\quad\left(\mathcal{E}(\nu, T)\,d\nu\right)$

Energy/Volume of radiation λ

then $\mathcal{E}$ is independent of any property of the enclosure except T.

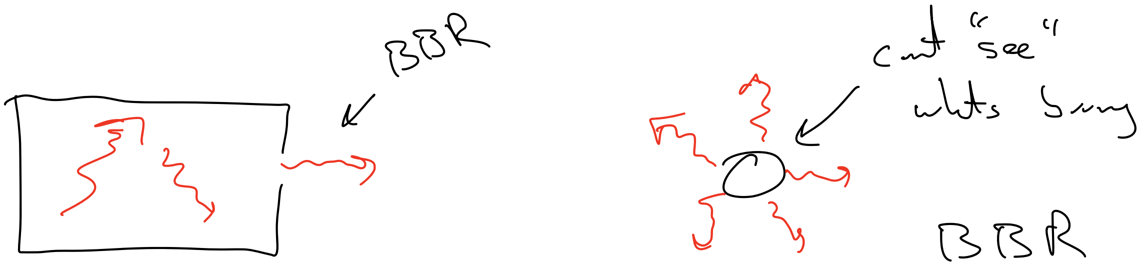
Clue that this quantity has deep physical explanation.

should have

"Black Body Radiation"

"Black" - meaning all radiation falling on object is absorbed

e.g.



"Black" is simple B/c you don't have to worry abt reflection properties which would be material specific

Intensity when Kirchhoff points this out, not much known abt BBR experimentally or theoretically

Launches major research program.

Find Experimentally?

~ Geometry

1st Note $\mathcal{E}(\nu, T) = \frac{4}{c} R(\nu, T)$ $\left(\begin{matrix} \mathcal{E}(\lambda, T) \\ R(\lambda, T) \end{matrix} \right)$

$\frac{\text{energy}}{\text{volume}}$ $\frac{\text{Power}}{\text{Area}}$ easier to measure!

"Stefan - Boltzmann" Law (Empirical)

$$R = \sigma T^4$$

Absolute temperature

Power radiated per unit area

Constant $5.7 \; 10^{-8} \frac{W}{m^2 K^4}$

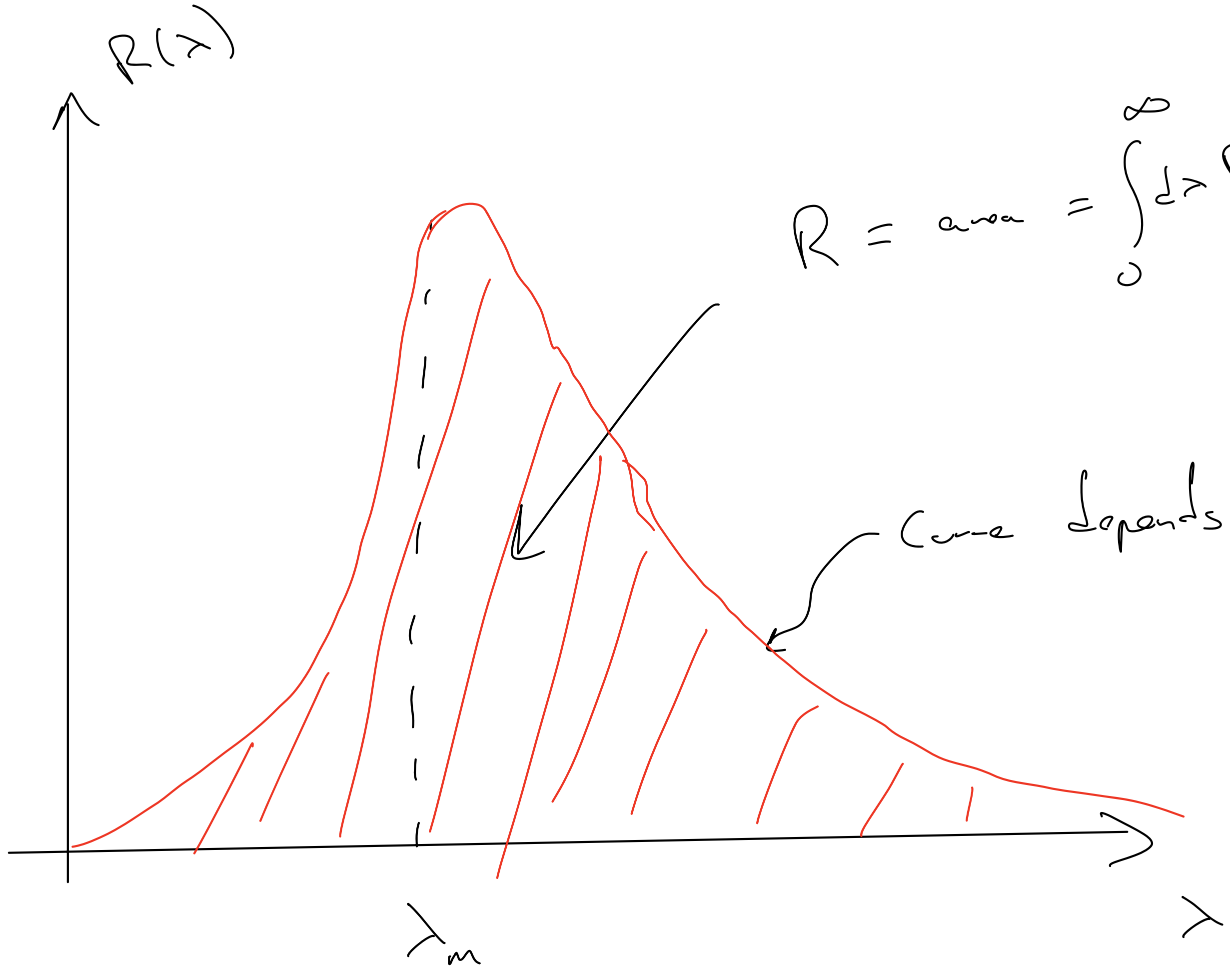
(0.3 in Natural units)

- Note particularly strong temperature dependence.

Bulk property total power/Area

Relatively easy to measure.

Big push to measure differentially in λ, ν



R = area = $\int_0^\infty d\lambda \, R(\lambda) = \sigma T^4$

Curve depends on T

Another Empirical Law "Wien's Displacement"

$\lambda_{max} \propto \frac{1}{T}$

$\lambda_{max} T = 2.9 \; 10^{-3}$ Km

(or $\lambda_{max} T = 1.3$)

$\Rightarrow$ knowing $R(\lambda)$ gives T

λ_{max} gives T

$\lambda_{max} T = 3 \; 10^{6}$ Knm

Example Black Body Radiation

Blue	450	nm
Green	500	nm
Yellow	600	nm
Red	650	nm

Sun's spectrum peaks at

~ 500 nm

$$\Rightarrow T_{surface} = \frac{3}{5} 10^4 \, K$$

$$= 6000 \, K$$

Measure star w/ longer $\lambda_{max} \sim 1000$nm

$$T^{*}_{surface} = 3000 \, K$$

Looks red

From distance + Luminosity $P_* = 100 P_\odot$

Can solve for size

$$R_* = \frac{P_*}{Area} = \frac{100 P_\odot}{4\pi R_*^2} = \sigma T_*^4$$

$$R_\odot = \frac{P_\odot}{Area} = \frac{P_\odot}{4\pi R_\odot^2} = \sigma T_\odot^4$$

$$\Rightarrow \frac{100 R_\odot^2}{R_*^2} = \left(\frac{T_*}{T_\odot}\right)^4 \qquad R_* = 10 \left(\frac{T_\odot}{T_*}\right)^2 R_\odot$$

$\sim 40 R_\odot$ "Giant"

Now theoretically, can we predict the spectrum?

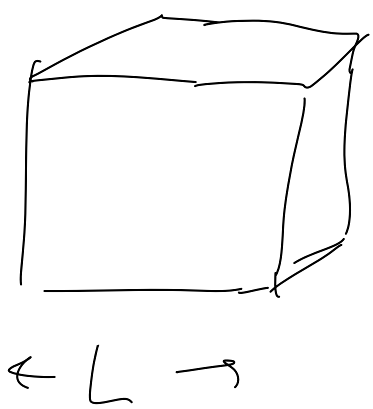
Yes! Only need 19C physics E&M + Stat Mech

E+M ⇒ | - Radiation is EM waves
| - At equilibrium have standing waves
| - 2× modes for each possible wave.

→ This tells us the total nDoF.

Stat Mech ⇒ | - At temp. T have kT for each DoF

So, How many standing waves fit in box?



Assume $L >> \lambda$

$\boxed{\lambda, \; k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad \omega = ck}$

You will show ...

"mode density at k"

$$\frac{N_{modes}}{V} = \frac{k^2}{\pi^2} dk \equiv g_k(k)\, dk$$

What is mode density vs λ

$$K = \frac{2\pi}{\lambda}$$

$$g_\lambda(\lambda)\, d\lambda = g_k(k)\, dk$$

$$g_\lambda(\lambda)\, d\lambda = g_k(k) \left| \frac{dk}{d\lambda} \right|$$

$$= g_k\left(\frac{2\pi}{\lambda}\right) \frac{2\pi}{\lambda^2}$$

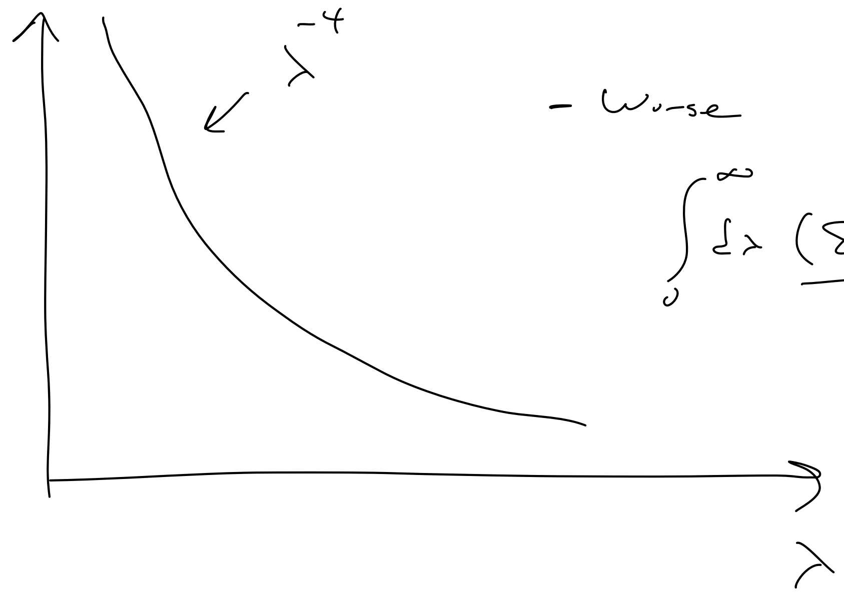
$$= \frac{\left(\frac{2\pi}{\lambda}\right)^2}{\pi^2} \frac{2\pi}{\lambda^2} = \frac{8\pi}{\lambda^4}$$

$$g_\lambda(\lambda)\, d\lambda = \frac{8\pi}{\lambda^4}\, d\lambda \quad (\text{eq } 3\text{-}8)$$

$$\mathcal{E}(\lambda, T) = \frac{8\pi k T}{\lambda^4}$$ "Rayleig - Jeans Law"

E&M + Boltzmann
(Equipartition theorem)

Problem!

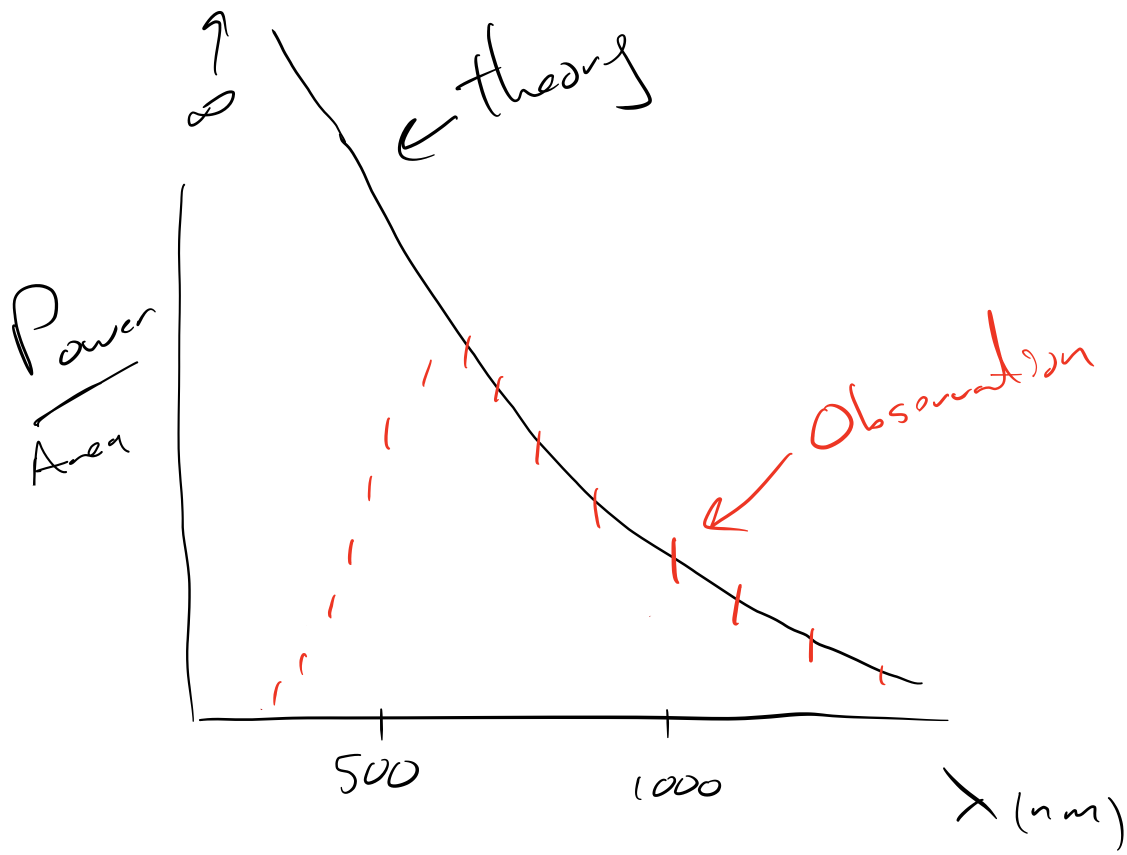


- No characteristic λ_{max}

- Worse

$$\int_0^\infty d\lambda \left(\frac{8\pi kT}{\lambda^4}\right) = \infty!$$

"ultra-violet catastrophe"



Problem

Statistical Mechanics $\Rightarrow E/\text{mode} \sim kT$

EM $\Rightarrow$ # of modes $\rightarrow \infty$ as $\lambda \rightarrow 0$

Enter Max Planck

Plank found he could describe the data empirically by making a strange assumption

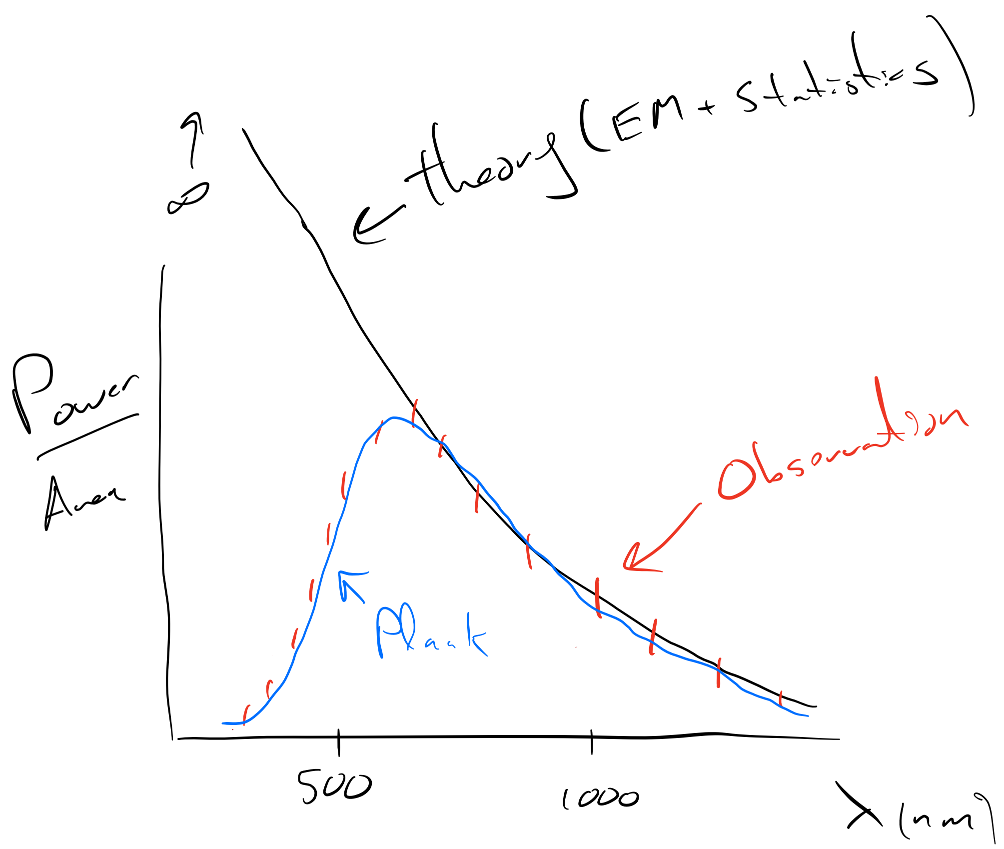
All empirical guess work.

Energy of oscillating charges & photons (γs) could only have discrete values $0, \varepsilon, 2\varepsilon, \ldots$

Or $$E_n = n\varepsilon = nhf \qquad n = 0, 1, 2, \ldots$$

New constant

frequency



What to make of this?

Implies that energy is quantized!

Already seen

Matter / Charge quantized

this much more surprising!

this new assumption $E_n = nhf$

$\Rightarrow$ (HW)

$$\langle E \rangle = \frac{\frac{hc}{\lambda}}{e^{hc/\lambda kT} - 1}$$

So, $u_\lambda(\lambda) = g_\lambda(\lambda)\,\mathcal{E}(\lambda, T)$

$$= \frac{8\pi}{\lambda^4}\left(\frac{\frac{hc}{\lambda}}{e^{hc/\lambda kT} - 1}\right) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

"Planck's formula"

Can fit the data for both h & k

$R = \frac{k}{m_e}$

get N_A