Read Ch 5

- Will now start bridging historical discussion (where people are just guessing things) to a theory QM w/ firm logical basis.

Saw Before "Waves As Particles"
↳ Slogan Behind | Photons, P.E. B.B.R.

This lecture, "How to make a million dollars; the easy way"

Louis de Broglie PhD thesis

We have two classes of physical entities

Waves - Light, Sound etc

Particles - Atoms, e^-'s, α's

Seen that Waves can behave like particles

Can Particles " " Waves?

(Still just guessing things, Motivated guessing...)

Like Symmetry. Lets try to make that idea work.

Reminder

Waves

$$h(\vec{x},t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) = A\cos(kx-\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Scalar

$$\vec{k} = \frac{2\pi}{\lambda}$$

Vector

$$= A e^{i\omega t - kx}$$

(when we take Real part.

Particles

$$p^4 = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$$

Sure doesn't look like a wave...

(However, do have 1 scalar, 1 vector)

Planck / Einstein

$E = hf = \hbar\omega$

$$p^\mu = \begin{pmatrix} \hbar\omega \\ \vec{p} \end{pmatrix}$$

Maybe this can be written in terms of $\vec{k}$?

De Broglie

How to win a Nobel Prize.

Guess $\vec{p} = \hbar k$ $\qquad p^\mu = \hbar \begin{pmatrix} \omega \\ \vec{k} \end{pmatrix}$

Sanity

$$\boxed{\begin{aligned} &\lambda_c = \frac{h}{m_e c} = \frac{h}{m} \Rightarrow [h] = E \cdot l \\ &k = \frac{2\pi}{\lambda} \Rightarrow [k] = \frac{1}{l} \qquad [\hbar k] = E = [p] \end{aligned}}$$

Lets assume true:

$$\vec{p} = \hbar \vec{k} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \frac{h}{\lambda}$$

$\Rightarrow$ Particle $\vec{p}$ has wavelength $\lambda = \frac{h}{p}$

Isn't this Obviously wrong? Ping Pong Ball 10^{-22} nm

10 eV electron ~ e's in atoms (non-relativistic OK)

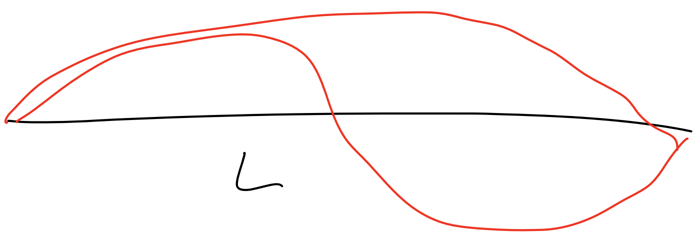
$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}} \sim 0.4 \text{ nm}$$

~ the size of atom (Suggestive ...)

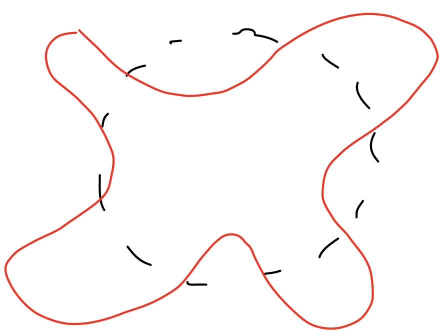
Maybe related to bohrs model ...

Standing waves on a string are quantized



$$n\frac{\lambda}{2} = L$$

Maybe Atoms are like strings w/ e's in standing waves?



need $n\lambda = C$ for this Boundary condition

What does this get us...

- $n\lambda = 2\pi r_e$

$$n\frac{h}{p} = 2\pi r_e \Rightarrow n\frac{h}{mv} = 2\pi r_e$$

or

$$mvr_e = n\hbar$$ (Exactly what Bohr Assumed!)

Here is a <u>much</u> more satisfying alternative history. ("Could have been")

- Atoms don't work (unstable) (No Bohr)
- Wave / Particle Asymmetry,
- Don't like Asymmetry, let particles act like waves
- $\Rightarrow \vec{p} = \hbar\vec{k}$, Notice that $\lambda \sim r_{atom}$
- $\Rightarrow$ Need to treat e^-'s as waves in atoms
- Stable configuration exist $\Rightarrow$ Standing waves
- Predict Bohr's crazy axiom. ... History

De Broglie Relations

Assuming for $m \neq 0$

$$E = h\nu = \hbar\omega$$ (Already saw for photons

$$\vec{p} = \frac{h}{\lambda} = \hbar\vec{k}$$

Question: What is waving?

light: E & B fields

e's ?

Ping Pong Balls ?

Will come back to this...

Being able to talk about waves obviously important.

Let's Begin.

Wave Equation 1D

y - height of wave

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v_p^2} \frac{\partial^2 y}{\partial t^2}$$

Linear partial differential equation

Change in space

"phase velocity"

Change in time

Solution

$$y(x,t) = y_0 e^{i(kx - \omega t)}$$

(try yourself w/ cos)

$$\frac{\partial^2 y}{\partial x^2} = (ik)^2 \underbrace{y_0 e^{i(kx-\omega t)}}_{y(x,t)} = -k^2 y(x,t)$$

$$\frac{\partial^2 y}{\partial t^2} = (-i\omega)^2 \underbrace{y_0 e^{i(kx-\omega t)}}_{y(x,t)} = -\omega^2 y(x,t)$$

Wave-eq

$$-k^2 y(x,t) = -\frac{\omega^2}{v_p^2} y(x,t)$$

Cos w/ spatial frequency k moves to the right

$\Rightarrow$ Solution if $v_p = \frac{\omega}{k}$ $\quad y(x,t) = y_0 e^{i(k(x - vt))}$

These are nice functions to work w/ But they can't describe particles

-) they have infinite extent

-) they are eternal.

In reality waves/particles often localised in space/time

Are there localised solutions to wave eq.

Yes! Superpositions of simple harmonic waves

"Fourier Analysis"

Example w/ Basic idea

$$y(x,t) = y_0 e^{i(k_1 x - \omega_1 t)} + y_0 e^{i(k_2 x - \omega_2 t)} \qquad e^{ix} = \cos x + i \sin x$$

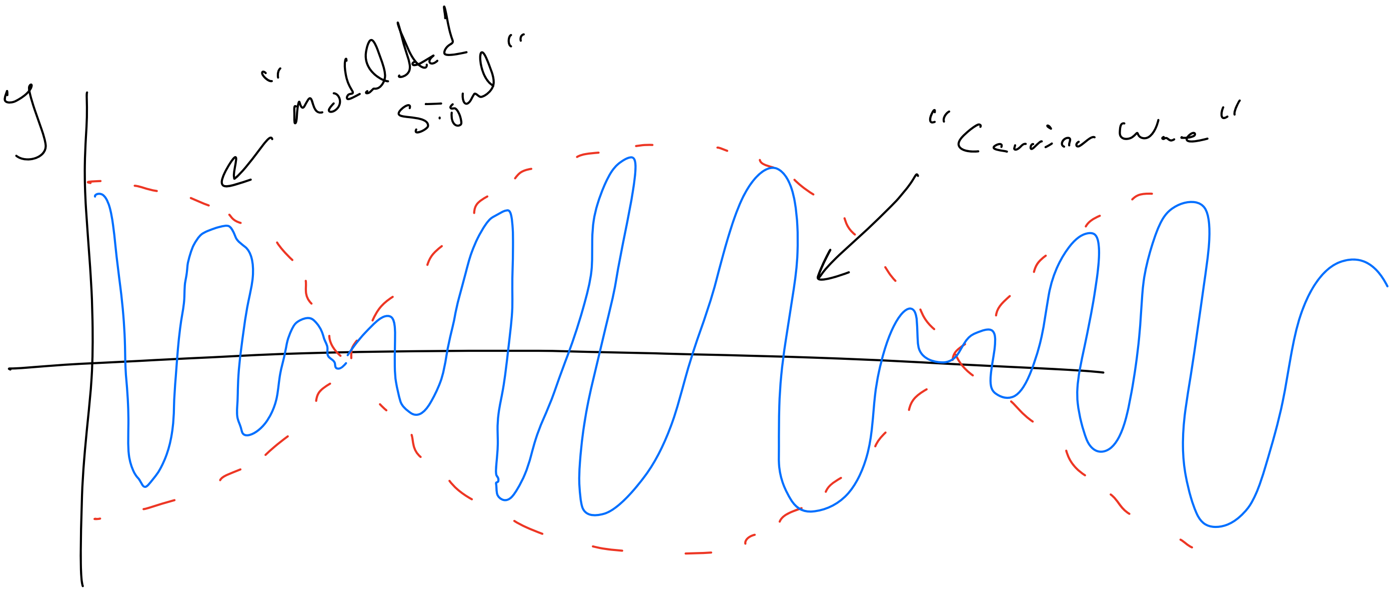
$$\boxed{\begin{aligned} e^{ik_1x} + e^{ik_2x} &= e^{i\frac{k_1}{2}x}\left(e^{i\frac{k_1}{2}x} + e^{i(k_2 - \frac{k_1}{2})x}\right) \\ &= e^{i\frac{k_1}{2}x} e^{i\frac{k_2}{2}x}\left(e^{i\frac{k_1-k_2}{2}x} + e^{i\left(\frac{k_2-k_1}{2}\right)x}\right) \\ &= e^{i\bar{k}x}\left(2\cos\frac{\Delta k}{2}x\right) \end{aligned}}$$

$$y(x,t) = 2y_0 \cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos\left(\bar{k}x - \bar{\omega}t\right)$$

Consider the case $k_1 \sim k_2 \sim \bar{k}$, $\omega_1 \sim \omega_2 \sim \bar{\omega}$

And $\frac{\Delta k}{2} \ll \bar{k}$ $\frac{\Delta \omega}{2} \ll \bar{\omega}$

We then get a "modulated" carrier wave w/ $(\bar{k}, \bar{\omega})$ that has a much slower modulation $(\Delta k, \Delta \omega)$ on top



Carrier Waves move w/ phase velocity $\frac{\bar{\omega}}{\bar{k}}$

Modulation moves w/ "group velocity" $\frac{\Delta \omega}{\Delta k}$

Adding more terms suppresses other modulation peaks

Figure in Book

$V_p = \frac{\omega}{k}$ group velocity $\rightarrow$ $V_g = \frac{d\omega}{dk}$