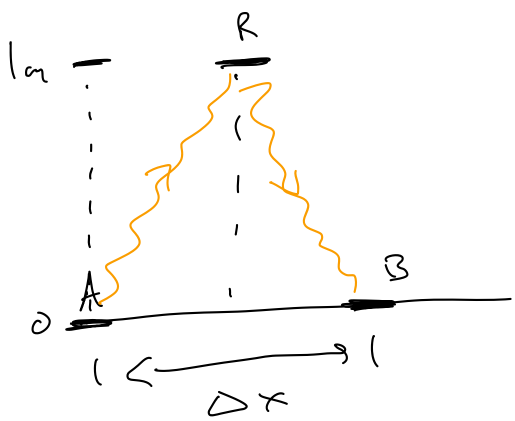
# Spacetime Diagrams

Simple way of looking at the clocks we looked at last time

## Lab frame (S)

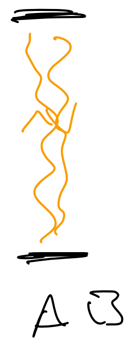



$$x_{emit} = t_e = 0$$

$$x_{reception} = x_r \geq 0$$

$$t_r = \sqrt{2^2 + x_r^2} \geq 2m$$

## Rocket Frame (S')



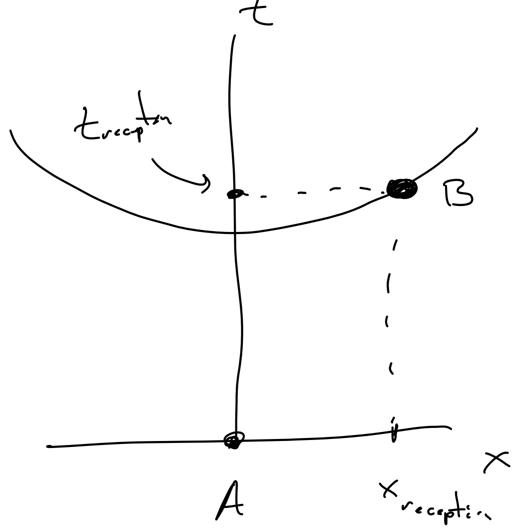

$$\Delta x' = 0$$

$$x'_e = t' = 0$$

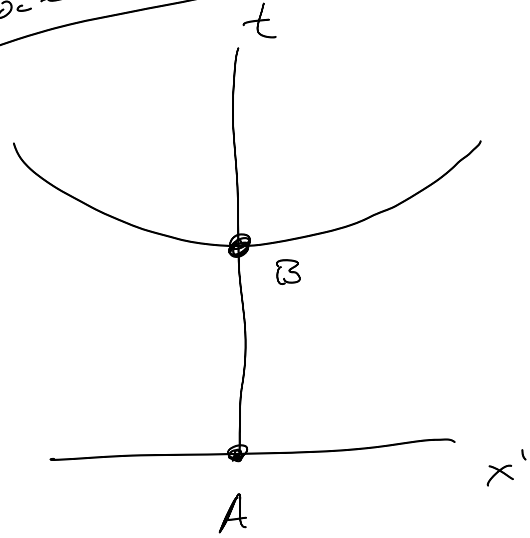
$$x'_r = 0$$

$$t'_r = 2m$$

Lab frame




Rocket frame




↳ Note now not specifying what this is

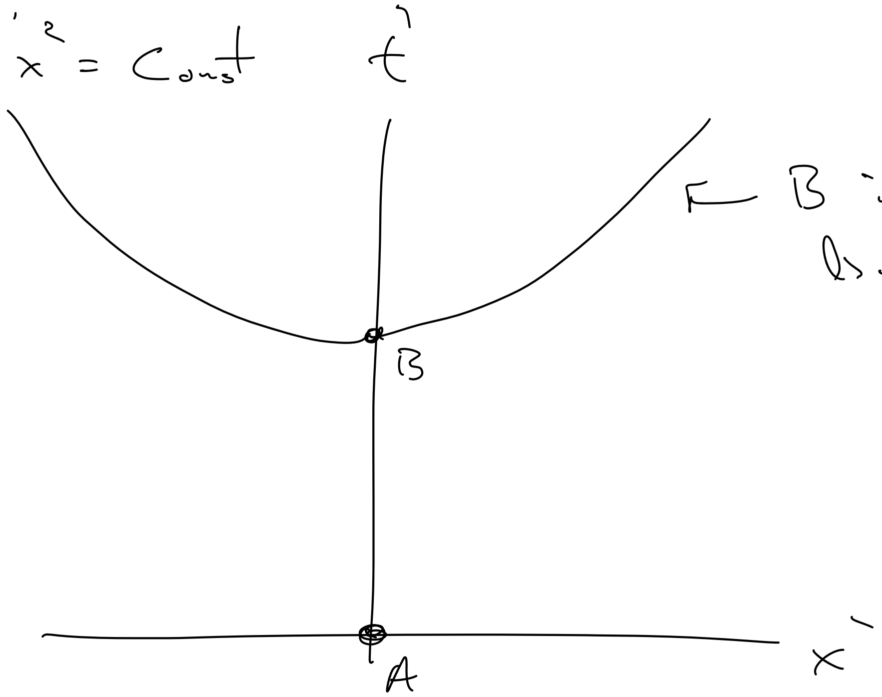
But $\geq 0$

A & B are the <u>same events</u> w/ different coordinates

All satisfy $\Delta t^2 - \Delta x^2 \equiv (\text{Interval})^2 = \text{constant}$

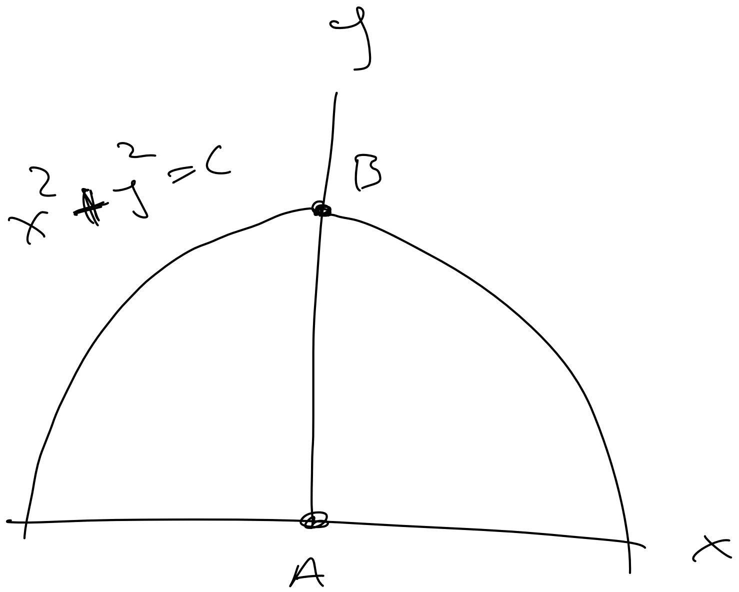
↳ defines hyperbola

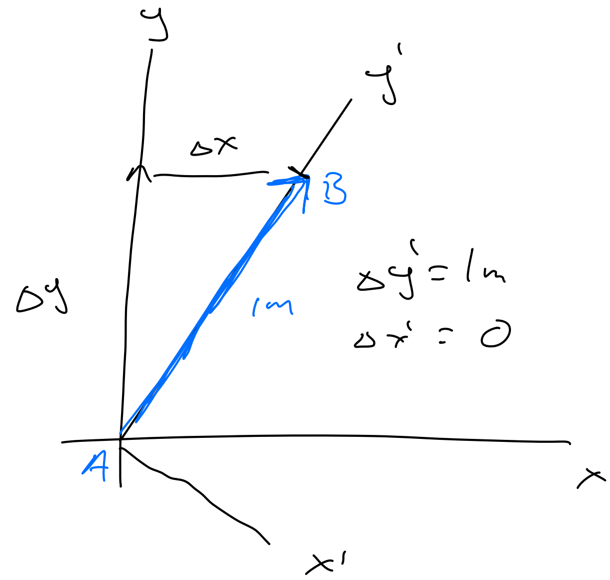
$t'^2 - x'^2 = \text{Const}$




← B in all coordinate frames lives on this curve

# Geometric Analogy




This is the fundamental difference between
Euclidean geometry & "Lorentz" Geometry of Spacetime

Circles: $\Delta y^2 + \Delta x^2 = (\text{Distance})^2$ Distance $\geq 0$

hyperbolas: $\Delta t^2 - \Delta x^2 = (\text{Interval})^2$

Interval $\begin{cases} \leq 0 & \text{"time-like"} \\ = 0 & \text{"light-like"} \\ \geq 0 & \text{"space-like"} \end{cases}$

Note the sign of the Interval
(just like the interval itself)
is <u>invariant</u>

Physical property of difference
Between events

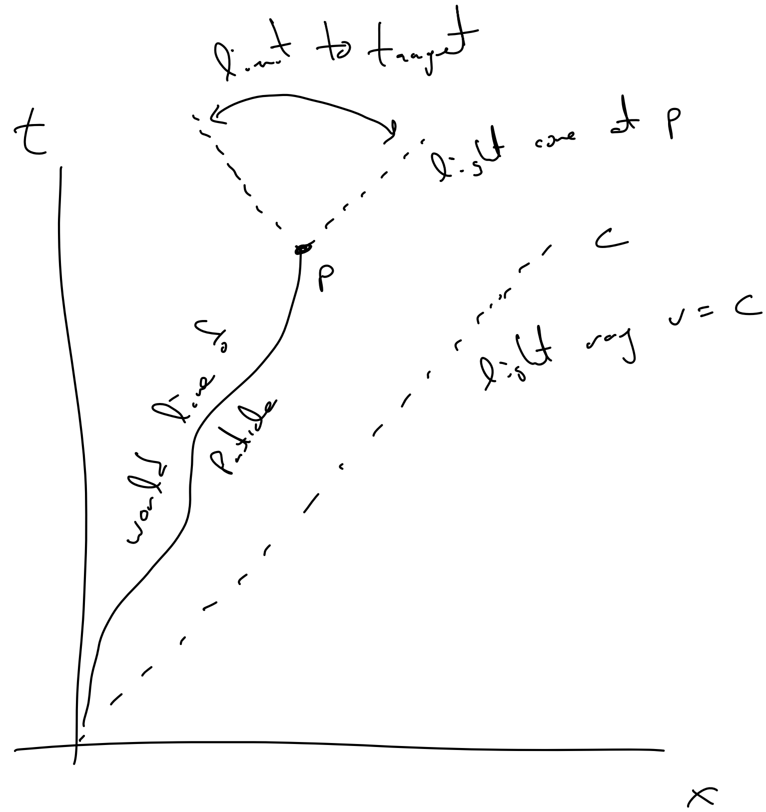
The time-like difference between events when $\Delta x = 0$ (4)

Called "proper time" $\Delta \tau$

$$\Delta \tau = \sqrt{(\Delta t)^2 - (\Delta x)^2}$$

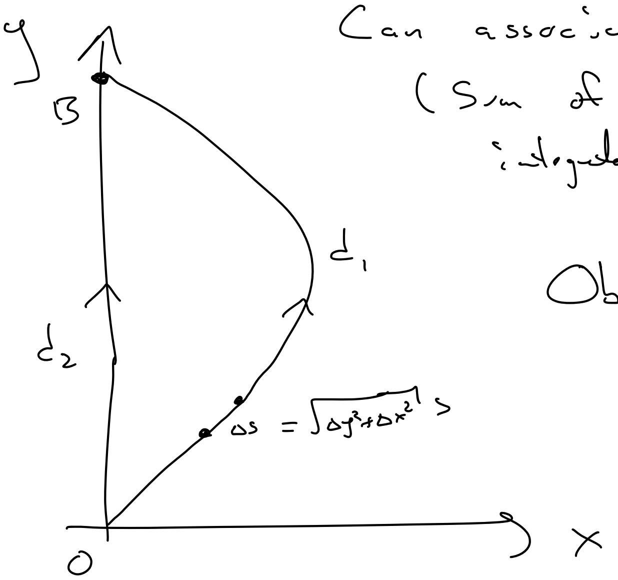
The space-like separation between simultaneous events $\Delta t = 0$

Called "Proper distance" $\Delta \sigma$




Events along the world line always time-like (within the light cone)

# Path Lenghts in Euclidean Geom



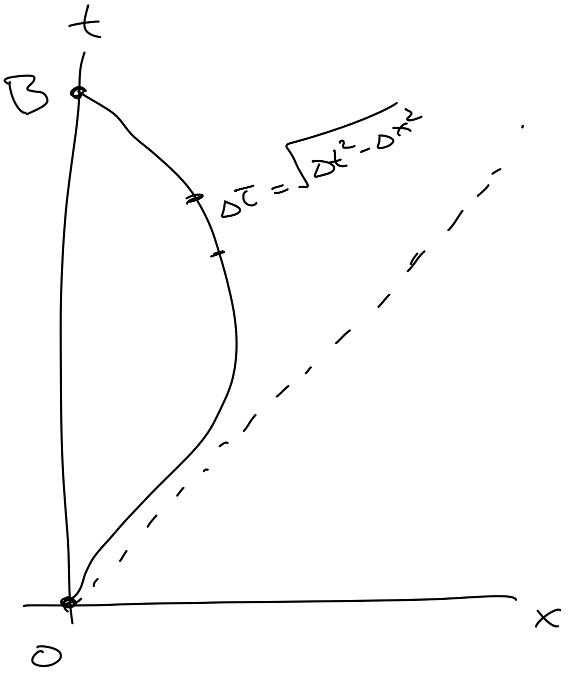

Can associate an invariant distance (Sum of distances between points on path integrated over path)

Obvious that different paths have different lengths

Curved paths are longer

# Same Logic Applies to

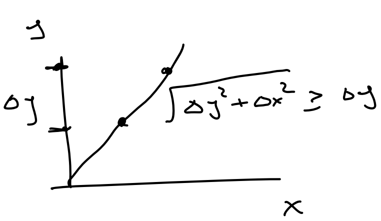
# Proper time along world lines in Lorentz Geometry

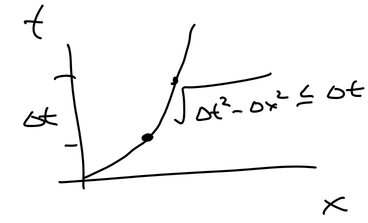


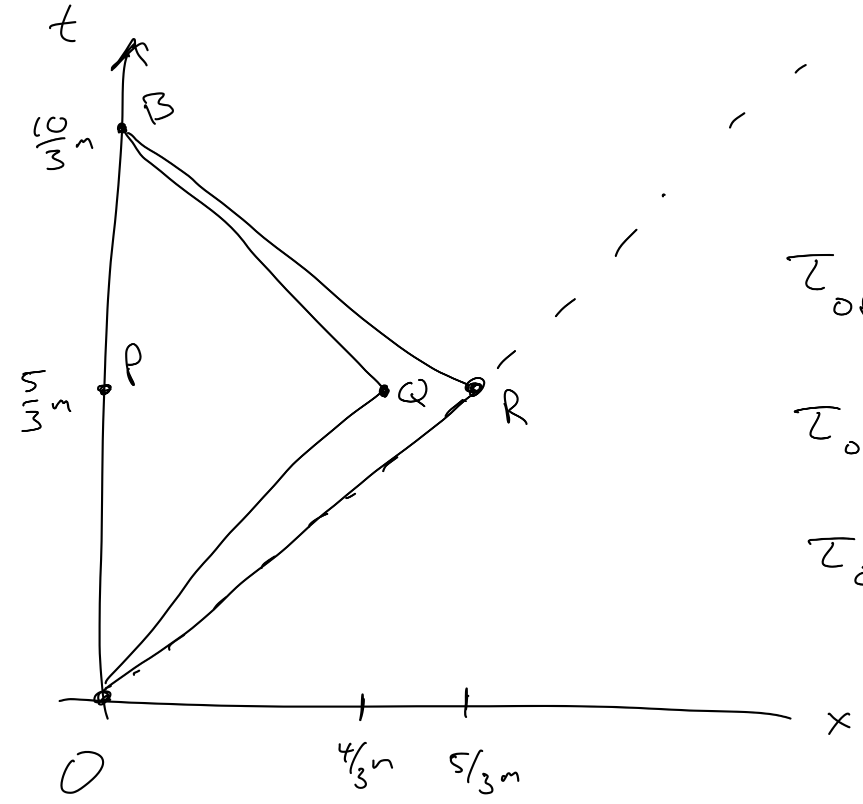

Proper time is an invariant associated with each world line

Different world lines between events have different proper times.

In Lorentz Geometry a curved world line between two events is shorter than the direct path.






$\tau_{OPB} = \frac{10}{3}$ m

$\tau_{ORB} = 2 \times 0$ m

$\tau_{OQB} = 2\sqrt{(5/3)^2 - (4/3)^2}$

$= 2$ m

Counter intuitive: Opposite of Euclidean Geometry
Due to the minus sign in the interval.

Might think there is "10/3 m" of time between the events
But if you move fast there is less "time" ($\leq \frac{10}{3}$ m)

A & B are Not separated by certain amount of time, but by a certain amount of Spacetime!

If you sit here, there is just time between the events
But if you get on a rocket & put significant space between the events there will be less time.

# Regions of Spacetime

Note in general $\text{distance}^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$

So $A = (t, x, y, z)$ $B = (t, x+\Delta x, y+\Delta y, z+\Delta z)$

$$\text{Interval}^2 = (\text{time})^2 - (\text{distance})^2$$

$$= (\Delta t)^2 - (\Delta x)^2 - (\Delta y^2) - (\Delta z)^2$$

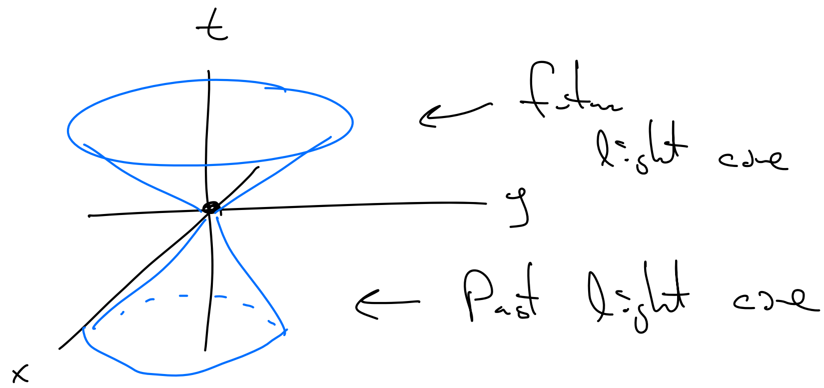
Relative sign is difference between space + time.

Saw above, Interval vanishes when moving at c

$$\Delta t = \pm (\text{distance})$$

The interval between two events is 0 if can be connected by a light ray

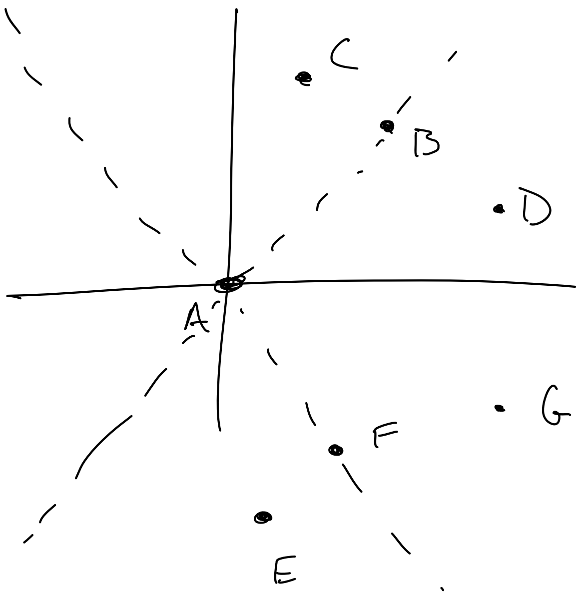
Location of all events w/ 0 interval (can be connected by light ray) called "light cone"




Note Light cones are invariant

Light Cones unique features of L. G.

Imposes Casal structure to physical world.




The causal connection between events is also invariant.

Discussion