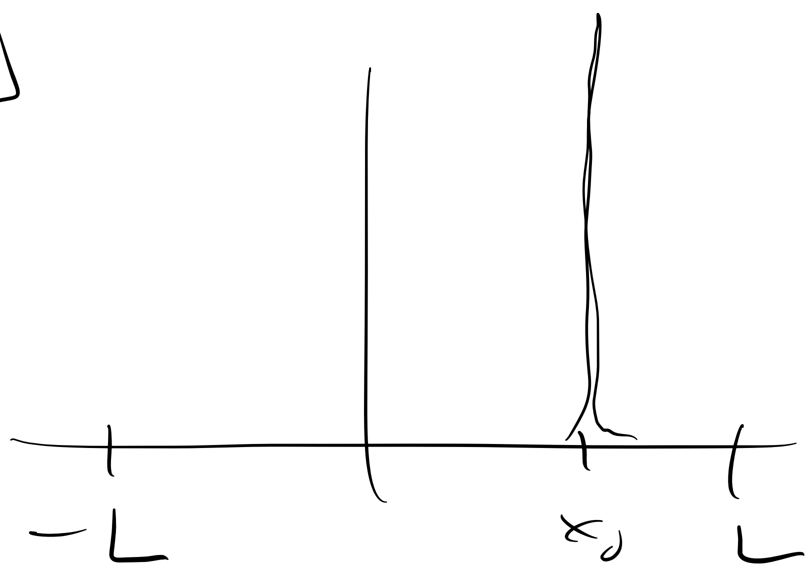
2. a) $\langle x \rangle = 0$ for both

$\langle x^2 \rangle$ bigger for ψ_b

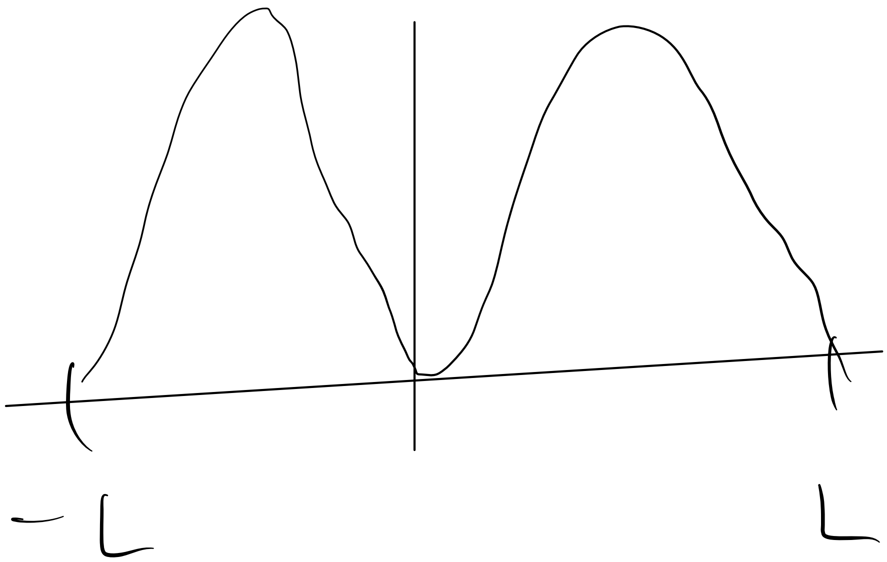
$\langle p \rangle$ bigger for ψ_b, more $\frac{d\psi}{dx}$

2b) first measurement would give value x_0 distributed according to $|\psi_b|^2$, all subsequent measurements would give x_0

eg)



2 c) Now each measurement is distributed according to $|\psi_s|^2$



3)

$$\langle x \rangle = \int x |\psi|^2 \, dx$$

$$\frac{d\langle x \rangle}{dt} = \int x \frac{\partial}{\partial t} |\psi|^2 \, dx$$

$$= \frac{i\hbar}{2m} \int x \frac{\partial}{\partial x} \left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right) dx$$

(See notes)

$$\boxed{\begin{aligned} &\textbf{Integration By Parts} \\ &\frac{d}{dx} vw = v \frac{dw}{dx} + w \frac{dv}{dx} \\ &v \frac{dw}{dx} = \frac{d(vw)}{dx} - w \frac{dv}{dx} \\ &\text{or} \\ &\int v \frac{dw}{dx} = \underbrace{\int \frac{d(vw)}{dx}}_{vw\big|_{-\infty}^{\infty}} - \int w \frac{dv}{dx} \end{aligned}}$$

$\psi(\pm\infty) = 0$

$$\frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{2m} \int \underbrace{\left(\frac{\partial x}{\partial x}\right)}_{=1} \left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right)$$

$$+ \frac{i\hbar}{2m} \cancelto{0}{x \left(\psi^* \frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x} \psi \right) \Bigg|_{-\infty}^{+\infty}}$$

$$\frac{d\langle x\rangle}{dt} = -\frac{i\hbar}{2m}\int\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx$$

$$\boxed{\text{But, } \int\frac{\partial\psi^*}{\partial x}\psi\,dx = -\int\psi^*\frac{\partial\psi}{\partial x}\,dx + \cancelto{0}{\psi^*\psi}\Big|_{-\infty}^{\infty}}$$

$$= -\frac{i\hbar}{m}\int\psi^*\frac{\partial\psi}{\partial x}\,dx$$

or

$$\langle p\rangle = -i\hbar\int\psi^*\frac{\partial\psi}{\partial x}\,dx$$

4a) $m \dfrac{d^2\langle x\rangle}{dt^2} = \dfrac{d\langle p\rangle}{dt} = -i\hbar \displaystyle\int \frac{\partial}{\partial t}\left(\Psi^* \frac{\partial \Psi}{\partial x}\right) dx$

$$= -i\hbar \int \left(\frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial x} + \Psi^* \frac{\partial^2 \Psi}{\partial t \partial x} \right) dx$$

$$\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar} V \Psi^*$$

&

$$\int \Psi^* \frac{\partial}{\partial t} \frac{\partial \Psi}{\partial x} dx = \int \Psi^* \frac{\partial}{\partial x} \frac{\partial \Psi}{\partial t} dx$$

$$= -\int \frac{\partial \Psi^*}{\partial x} \frac{\partial \Psi}{\partial t} dx \leftarrow \text{Integration By Parts}$$

and

$$\boxed{\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m} \frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar} V \Psi}$$

$$-\psi^* \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^3}$$

$$2 \times (\text{IBP})$$

$$\int \left(\frac{\partial \psi^*}{\partial t}\frac{\partial \psi}{\partial x} + \psi^* \frac{\partial^2\psi}{\partial t\, \partial x}\right) dx$$

$$= \int \left(\cancel{-\frac{i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2}} + \frac{i}{\hbar} V\psi^*\right)\frac{\partial\psi}{\partial x} -$$

$$\underbrace{\frac{\partial\psi^*}{\partial x}\left(\cancel{\frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}}}_{-\psi^*\frac{i\hbar}{2m}\frac{\partial^3\psi}{\partial x^3}\ (\text{IBP})} - \frac{i}{\hbar}V\psi\right)$$

$$= \frac{i}{\hbar}\int V\cancel{\psi^*\frac{\partial\psi}{\partial x}} + \underbrace{\frac{\partial\psi^*}{\partial x} V\psi}_{= -\psi^*\left(\frac{\partial V}{\partial x}\psi + \cancel{V\frac{\partial\psi}{\partial x}}\right)\ (\text{I. B. P.})}$$

$$= \frac{i}{\hbar}\int -\psi^*\left(\frac{\partial V}{\partial x}\right)\psi\, dx$$

Or

$$\frac{d\langle p\rangle}{dt} = -\int \Psi^* \left(\frac{\partial V}{\partial x}\right) \Psi \, dx$$

$$= -\left\langle \frac{\partial V}{\partial x} \right\rangle$$

4b) if $V \to V + V_0$

claim $\Psi \to e^{-iV_0 t/\hbar}\Psi$

Assume Ψ solves $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$

Define $\Psi' = e^{-iV_0 t/\hbar}\Psi$

then,
$$\frac{\partial \psi'}{\partial t} = e^{-iV_0 t/\hbar} \frac{\partial \psi}{\partial t} - \frac{iV_0}{\hbar} e^{-iV_0 t/\hbar} \psi$$

So
$$i\hbar \frac{\partial \psi'}{\partial t} = \underbrace{i\hbar}\, e^{-iV_0 t/\hbar} \underbrace{\frac{\partial \psi}{\partial t}}_{= -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi} + V_0 \psi'$$

OR,
$$-i\hbar \frac{\partial \psi'}{\partial t} = e^{-iV_0 t/\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \right) + V_0 \psi'$$
$$= -\frac{\hbar^2}{2m} \frac{\partial^2 \psi'}{\partial x^2} + (V + V_0)\psi'$$

Has no impact on dynamics

$$\psi^* \psi = \psi'^* \psi'$$

4 c) $\frac{p^2}{2m} = \frac{3}{2} k_B T$

<u>Solids</u> $d \sim 0.3\text{ nm}$

$\lambda = \frac{h}{p}$ $\qquad p^2 = 3 m k_B T$

$p = \sqrt{3 m k_B T}$

QM important when

$\lambda \gtrsim d$ or

$\frac{h}{\sqrt{3 m k_B T}} \gtrsim d \Rightarrow T \lesssim \frac{h^2}{d^2} \frac{1}{3 m k_B}$

for m_e: $T < 1.3 \cdot 10^5\text{ K}$

m_N: $T < 3\text{ K}$

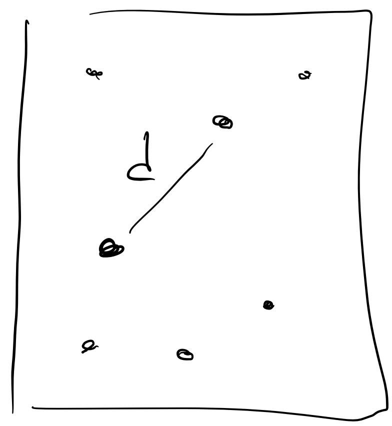
Moral: the free electrons in a solid are always QMical. the nuclei in a solid almost never are.

Gases

$$P_r V = N k_B T$$

$$V = d^3 N$$

$$\Rightarrow \frac{V}{N} \sim d^3$$



$$\frac{V}{N} = \frac{k_B T}{P_r} \qquad d = \left(\frac{k_B T}{P_r}\right)^{1/3}$$

$$T \lesssim \frac{h^2}{d^2} \frac{1}{3 m k_B} = \frac{h^2}{(k_B T)^{2/3}} \frac{P_r^{2/3}}{3 m k_B}$$

or

$$T^{5/3} \le \frac{h^2}{(k_B)^{5/3}} \frac{P_r^{2/3}}{3m}$$

$$T \leq \frac{1}{k_B}\left(\frac{h^2}{3m}\right)^{3/5} \rho^{2/5}$$

$$T = 2.9 \text{ K}$$

for Hydrogen in outter space

$$T < 6 \; 10^{-14} \text{ K}$$

definitly in classical regime!

4d) $$\Psi(x,t) = \Psi(x) e^{-i(E+i\Gamma)t/\hbar}$$

$$= \Psi(x) e^{-iEt} e^{+\Gamma t}$$

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \int |\Psi(x)|^2 e^{2\Gamma t}$$

$$\frac{d}{dt} \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 2\Gamma \int |\Psi(x,t)|^2 dx$$

$$\neq 0 \quad \text{unless } \Gamma = 0$$