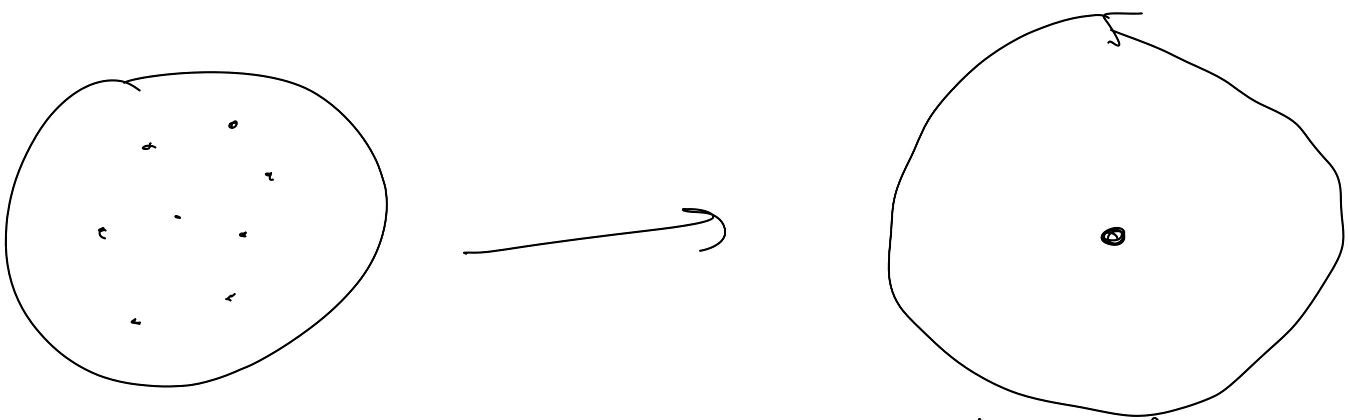
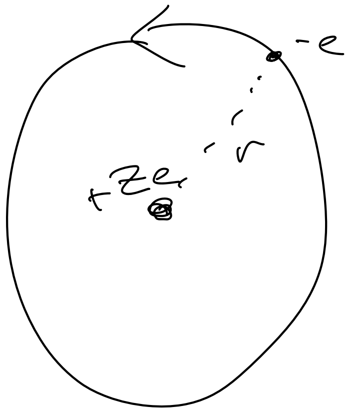
Saw last time that the obvious atomic model not viable



α - scattering forces "Solar System" like picture

explains large ⊖ scattering, But is Problematic . . .



$$F = \frac{kZe^2}{r^2} = ma = m\frac{v^2}{r}$$

$$\Rightarrow \quad mv^2 = \frac{kZe^2}{r}$$

$$v = \sqrt{\frac{kZe^2}{mr}}$$

$$E_e = KE + PE$$

$$= \underbrace{\frac{1}{2}mv^2}_{+\frac{1}{2}\frac{kZe^2}{r}} - \frac{kZe^2}{r} = -\frac{1}{2}\frac{kZe^2}{r}$$

Now

. . .

Problem E&M $\Rightarrow$ accelerating charge radiates

$$\nu = \text{frequency of Periodic motion} = \frac{v}{2\pi r} \sim \frac{1}{r^{3/2}}$$

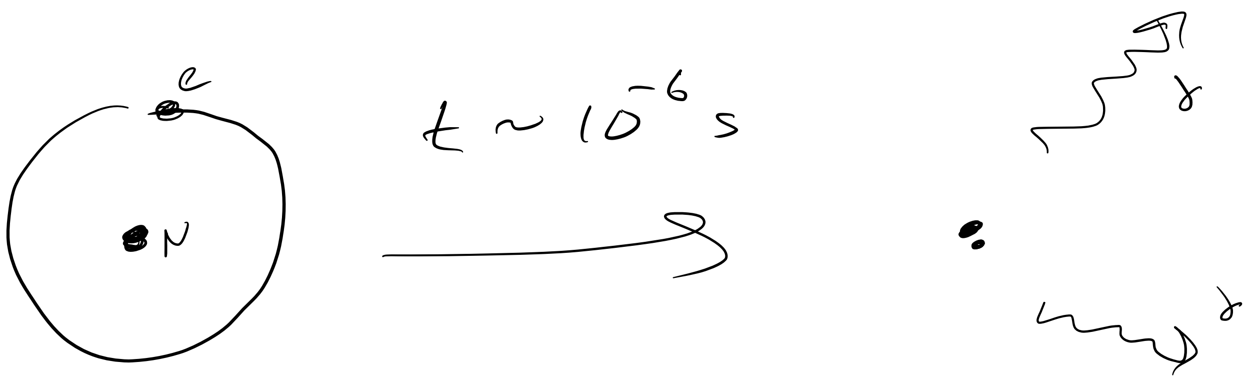
frequency radiation

$$F = ma \Rightarrow v \sim \sqrt{\frac{1}{r}}$$

above

Physics (Newton + E&M) Predicts run away effect

- Electron in Circle $\left(E \sim -\frac{1}{r}\right)$
- Radiates $\Rightarrow$ loses energy $\Rightarrow \frac{1}{r}$ bigger $\Rightarrow r$ smaller
- Smaller r means more radiation $E \sim f \sim \frac{1}{r^{3/2}}$



Predicts: No stable atoms (& continuous spectrum of radiation)

<u>Disaster</u>

Enter Niels Bohr

Comes up w/ "Solution"

Scare quotes B/c, as we will see, this is just making things up.

-) Assumes (Axiom) Special orbits that don't radiate (Solves the problem by fiat) "Stationary States"

Determined by: $L = mvr = n\hbar$ w/ $n = 1, 2, \ldots$

$\frac{h}{2\pi}$ (3^{rd} coming of h)

-) Atoms only radiate when transition between "Allowed" orbits. In which case

$$E_\gamma = h\nu = E_i - E_f$$

-) for large E recover Classical Physics

Looks totally crazy! Physically it is.

Turns out this is actually close to being right.

(Note: Causality Gone!)

Implications

$$r = \frac{n\hbar}{mv} \qquad \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{kZe^2}{mr}}$$

$$r^2 = \frac{n^2\hbar^2}{m^2v^2} = \frac{n^2\hbar^2}{m^2}\left(\frac{mr}{kZe^2}\right)$$

$$\Rightarrow r_n = \frac{n^2\hbar^2}{kZe^2 m} = n^2 \underbrace{\left(\frac{\hbar^2}{kme^2}\right)}_{a_0} \frac{1}{Z} = \frac{n^2 a_0}{Z}$$

a_0 – "Bohr" Radius

$0.5 \; 10^{-10}$ m

$$E_n = -\frac{kZe^2}{2r_n} = -\frac{kZe^2}{2}\left(\frac{kZe^2 m}{n^2\hbar^2}\right)$$

$$= -E_0 \frac{Z^2}{n^2} = -\frac{E_0}{n^2} \quad \text{(for Hydrogen)}$$

E_0: 13.6 eV (An important #)

Stationary Orbits have quantized radii + E.

Transitions

During transition γ emitted w/ energy

$$E_\gamma = E_n - E_m = -E_0 Z^2\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$$

Or

$$\frac{1}{\lambda_{n,m}} = \underbrace{\frac{E_0}{hc}}_{} Z^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$

$= 1.097\ 10^{-7}\ m^{-1}$

Exactly Rydbergs Constant!

Big indication that on the right track

Objection: Explains stuff already known!

Can apply Bohr's model to other systems He^+ ← $Z=2$ one electron

$$\frac{R_{He}}{R_H} = 2^2 = 4 \quad \left(\begin{array}{c}\text{Observed} \\ 4.00168 \\ \uparrow \text{ Due to nuclear motion} \\ \text{HW}\end{array}\right)$$

Sanity Check

$$\boxed{r_n = \frac{n^2 \hbar^2}{k Z e^2 m}}$$

$$m v_n^2 = \frac{k Z e^2}{r_n} =$$

$$= \frac{k^2 Z^2 e^4 m}{n^2 \hbar^2}$$

or

$$v_n = \left(\frac{k e^2}{\hbar} \right) \frac{Z}{n}$$

So

$$\frac{v_n}{c} = \underbrace{\left(\frac{k e^2}{\hbar c} \right)}_{} \frac{Z}{n}$$

$\longrightarrow$ Pure # $= \dfrac{1}{137}$

Ridiculously Important # in physics
(Characterizes strength of E&M)

Intuition that it is small : 10^{-2}

Note: for H
$Z = 1$

$$\frac{v_n}{c} \sim 10^{-2}$$

Means we can figure out QM w/o worry about relativistic effects

relativistic corrections

$$\gamma \sim \left(\frac{v}{c} \right)^2 \sim 10^{-4}$$

Note: Bohr Model applied to H atom

Turns out much more complicated when other $\bar{e}$'s around.

However, Analysis also applies in some other situations

-) Ionized Atoms w/ one e (Already saw this)

-) Inner electrons of heavier atoms (tomorrow)

-) Outer electrons of heavy atom where effects of other $\bar{e}$'s approximated by modifying the effective charge. ($Z_{eff} \to Z$)