Housekeeping

- Anyone not got book? / Not got Canvas?
- Exams 2/10, 3/3, 4/12 + final
- Office hours T: 3:30 – 4:30
  H: 2:30 – 3:30

(Intro discussion about Coordinate Changes to solve polling

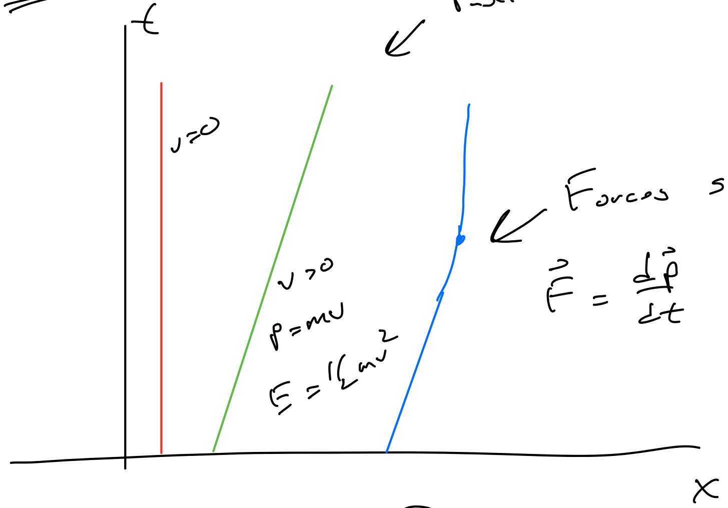
 → Does the ball land in Bucket? Airplane.

Classical Physics "Newtonian" (Reminder)

World described by objects moving in Space + time.
(particles)

eg 1D

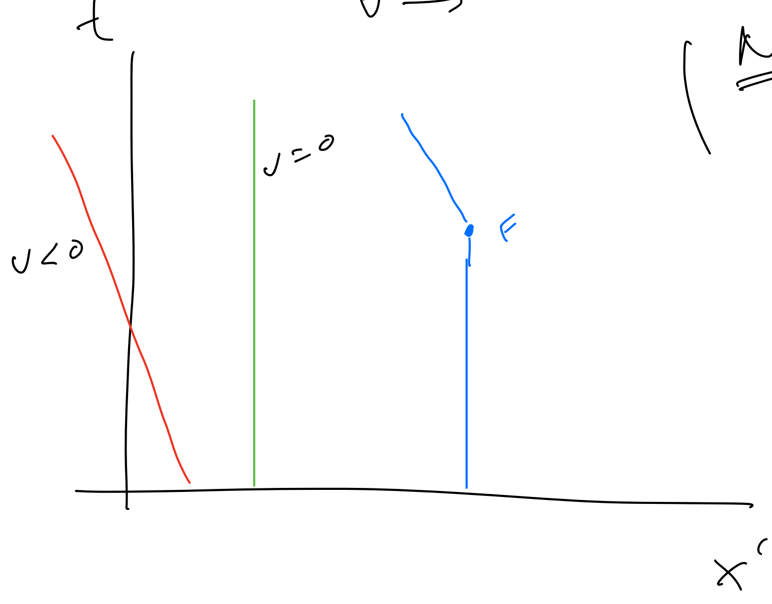
Particles w/ no forces move in straight lines



Forces specify how the lines curve

$\vec{F} = \frac{d\vec{p}}{dt}$

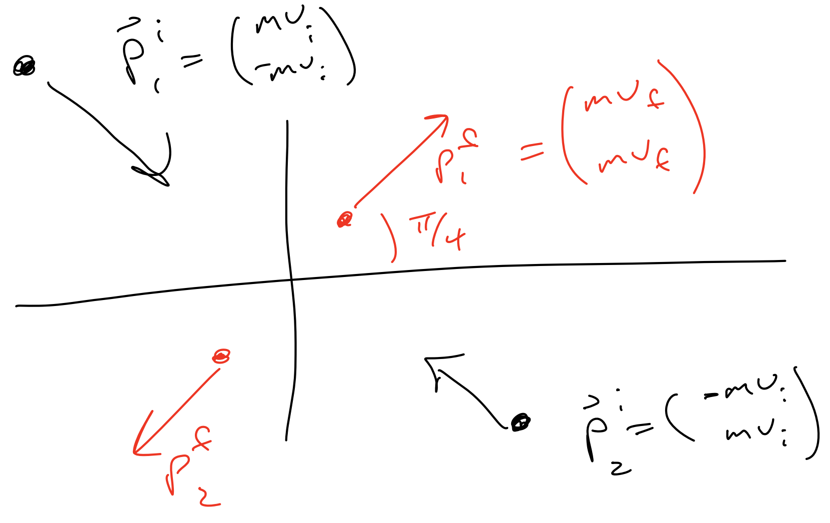
Different Reference frame



Nomenclature:

(Note $\vec{p}$ & E are not invariant)

masses are invariant



By symmetry

$$|\vec{p}_1^{\,f}| = |\vec{p}_2^{\,f}|$$

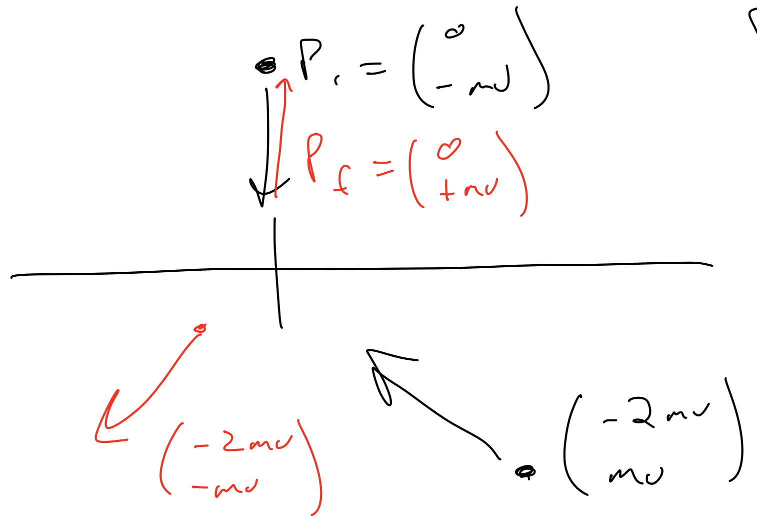
By E conservation

$$v_i = v_f$$

$$\vec{p}^{\,i}_{tot} = 0 = \vec{p}^{\,f}_{tot}$$

$$v^i_{tot} = \sqrt{2}\, v_i \qquad E = 2 \times \tfrac{1}{2} m\, 2 v_i^2$$
$$= 2 \ \tfrac{1}{2} m v_i^2 = m v_i^2$$

Go to frame F' w/ v →



$$|v_i| = \sqrt{5}\, v$$

$$P^i_{tot} = \begin{pmatrix} -2mv \\ 0 \end{pmatrix} \qquad E^i = \tfrac{1}{2} m v^2 + \tfrac{1}{2} m (5v^2) = 3mv^2$$

$$P^f_{tot} = \begin{pmatrix} -2mv \\ 0 \end{pmatrix} \qquad E^f = \tfrac{1}{2} m v^2 + \tfrac{1}{2} m (5v^2)$$

In those examples we needed a coordinate system to describe the motion $\left(x(t) , \frac{dx(t)}{dt} \right)$

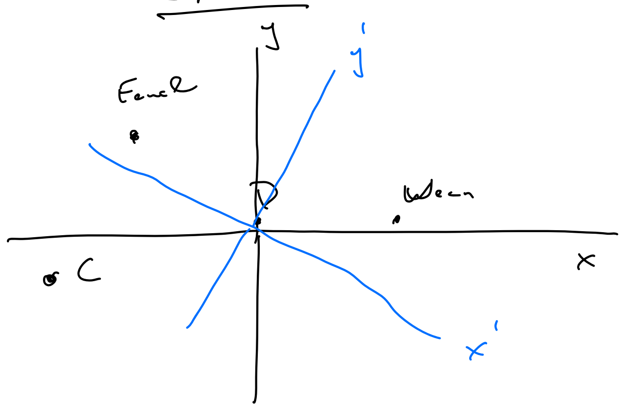
Our choice of coordinates is arbitrary!

Critical Mental Model

	$(x \;\; y)$	$(x' \;\; y')$
D	$0 \;\; 0$	$0 \;\; 0$
W	$(x_w \; y_w)$	$(x'_w \; y'_w)$
F	$\vdots$	
C		

$$x^2 + y^2 = x'^2 + y'^2$$

CMU



Puzzle

How can coordinates be both Required + Arbitrary?!

Answer Coordinates are more than just numbers!

Numbers + | Rules / Prescription | to | change the / transform

"Equivalence class" of groups of numbers related by transformations

How to transform between coordinates?

F $\qquad$ F' - moves w/ v $\qquad$ $F(x_i - x_j)$

$x(t)$ $\qquad$ $x'(t)$

transformation : $x(t) \longrightarrow x'(t')$

"time is time" $\Rightarrow$ $t = t'$

for x, need to account relative motion of S'

$$x'(t') = x(t) - v_s t \qquad \begin{pmatrix} y' = y \\ z' = z \end{pmatrix}$$

$$t' = t$$

Matrix Multiplication / Crash Course

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_{11} c_1 + a_{12} c_2 \\ a_{21} c_1 + a_{22} c_2 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{pmatrix}$$

Probably all we will need.

$$x \to x' = x + vt$$

$$t \to t$$

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} x + vt \\ t \end{pmatrix}$$

<u>Note 2 transformations</u> $\quad x \xrightarrow{v_1} x' \xrightarrow{v_2} x''$

$$v_c = v_1 + v_2$$

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = \begin{pmatrix} 1 & v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & v_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$= \begin{pmatrix} 1 & v_1 + v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$G(v_2)G(v_1) = G(v_c)$$

$$v_c = v_1 + v_2$$

"Galilean Transforms
form a group"

<u>Assumption</u> Physics shouldn't depend on coordinate choice
(Assertion)

$$x' = x(t) - v_s t \quad \Rightarrow \quad F(x_1' - x_2') = F(x_1 - x_2)$$

$$\boxed{\text{what is } v'(t)?} = \frac{dx'(t')}{dt'} = \frac{dx'(t)}{dt} = \frac{d(x(t) - v_s t)}{dt}$$

$$= \frac{dx}{dt} - v_s \frac{dt}{dt} = v(t) - v_s$$

$$v' = v(t) - v_s \quad \Rightarrow \quad a' = \frac{dv'}{dt} = \frac{dv}{dt} = a$$

If $F = ma$ holds in one coordinate system
holds in all.

⑦

$\boxed{HW}$

Notes if $\frac{d}{dt}\sum_i P_i' = \sum_i m_i \frac{d(v(t) - v_s)}{dt}$

$= \sum_i m_i \frac{dv}{dt} = \frac{d}{dt}\sum P_i$

$\boxed{HW}$

E conserved if P "

$$\frac{d}{dt}\sum_i E_i' = \sum_i \frac{d}{dt}\frac{m_i}{2}v_i'^2 = \sum_i \frac{m_i}{2}\frac{d}{dt}(v - v_s)^2$$

$$= \sum_i \frac{m_i}{2}\, 2(v_i - v_s)\frac{dv_i}{dt}$$

$0 = \frac{dP_{tot}}{dt}$

$$= \underbrace{\sum_i \left(\frac{m_i}{2}\, 2v_i \frac{dv_i}{dt}\right)}_{\frac{d}{dt}E_{tot}} - \cancel{\sum_i v_s m_i \frac{dv_i}{dt}}$$

These transformations seem <u>very straight forward</u>

But (as we will see) they are also wrong...

Took A.E. to figure out why.

<u>1st hint</u> Maxwell's equations <u>do Not</u> Satisfy the G.T.

- Speed of light not constant
- $\vec{E} \longleftrightarrow \vec{B}$ under G.T