Homework Set #6

Due Date: Before class Friday March 21st

1) Reading *(2 points)*

Read chapter 4.

2) Compton Scattering *(15 points)*

Derive Compton's formula for relative shift of the wavelength of a photon scattered off an electron.

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_e}{\lambda}(1 - \cos\theta)$$

where, λ is the wavelength of the incoming photon, $\Delta\lambda$ is the difference between the incoming and outgoing photon wavelengths, θ is the angle between the direction incoming and outgoing photons and $\lambda_e = h/m_e c$ is the Compton wavelength of the electron. What is the energy of a photon with wavelength equal to the Compton wavelength of the electron ?

Repeat for the case of a photon scattered of a proton. What is the numerical value for the "Compton wavelength" of the proton?

What is the energy of a photon with wavelength equal to the Compton wavelength of the proton?

3) Rutherford. *(15 points)*

(a) What was unexpected about Rutherford's alpha-ray scattering experiment? How did it qualitatively change our picture of atoms?

(b) (Rutherford experiment in 1D) Suppose two particles with masses m_α and m_N and initial velocities v_α and $v_N = 0$ along some line collide head on, emerging with velocities v'_α and v'_N along the same line. Use conservation of momentum and energy to find v'_α as a function of m_α, m_N, and v_α. If the alpha particle "bounces back" with significant velocity, eg: $v'_\alpha \sim -|v_\alpha|$, what does that say about the relative size of the masses m_α, m_N ?

(c) What problems arise when applying classical physics to the "solar system" picture of atoms ? eg: what does the theory predict that is not observed?

(d) Estimate an upper limit on the size of a gold nucleus from the distance of closest approach of a 1D head-on collision in which the incoming 8 MeV α particle returns with $\theta = 180^o$.

4) Bohr. *(15 points)*

(a) How are the sharp lines in atomic spectra explained in the Bohr model of the atom?

(b) Show that the Bohr radius and the lowest energy for the hydrogen atom can be written as

$$a_0 = \frac{hc}{\alpha mc^2} = \frac{\lambda_e}{2\pi\alpha} \qquad E_1 = \frac{1}{2}\alpha^2 mc^2$$

where $\lambda_e = \frac{h}{mc}$ is the Compton wavelength of the electron and $\alpha = \frac{ke^2}{\hbar c}$ is the fine-structure constant. Use these expressions to check the numerical values of the constants a_0 and E_1.

(c) If the angular momentum of Earth in its motion around the Sun were quantized like a hydrogen electron according to Bohr's quantization hypothesis, what would Earth's quantum number be? How much energy would be released in a transition to the next lowest level? Would that energy release (presumably as a gravity wave) be detectable? What would be the change in the radius of the earth's orbit ? (The radius of Earth's orbit is 1.50×10^{11} m.)

(d) On average, a hydrogen atom will exist in an excited state for about 10^{-8} sec before making a transition to a lower energy state. About how many revolutions does an electron in the n=2 state make in 10^{-8} sec?