Einstein's Principle of Relativity

All laws of physics same in every inertial reference frame

Inertial means non accelerating, not subject to net force.

(See Book for details)

Corollary Both the form of laws and the numerical values of the physical constants.

Idea Choice of particular coordinate system is arbitrary

The laws of physics shouldn't be sensitive to which choice was made.

$\Rightarrow$ The form of all the constants must be the same.

Comment of Principle of Relativity

w/ G.T saw

$$F = \frac{dp}{dt} \quad S$$

$$\parallel \qquad \parallel$$

$$F' = \frac{dp'}{dt'} \quad S'$$

Satisfies the P.o.R., But even more restrictive

PoR only requires:

$$F = \frac{dp}{dt} \quad S$$

& $\neq \qquad \neq$ ⟵ Ok if not equal.

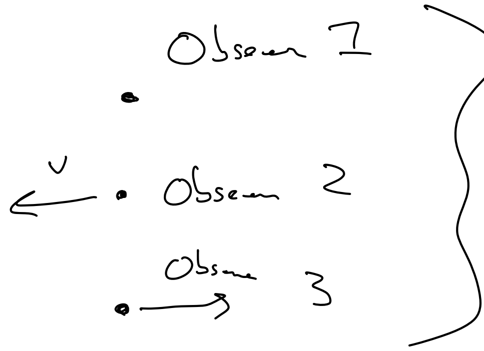
$$F' = \frac{dp'}{dt'} \quad S'$$

This is how E&M can satisfy PoR when $\vec{E} \longleftrightarrow \vec{B}$ in charged Bar example

But one of the constants is a speed! c.

How can that not depend on your reference frame

e.g.)



All measure speed of light c!

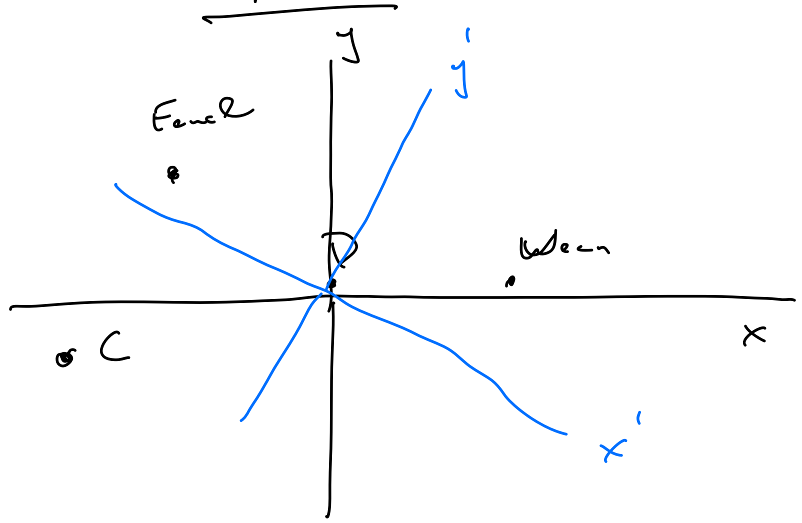
Saw a preview of how this works w/ General Coordinate Transforms & V_x

Will now go through it from different starting point.

What must be true if different observers agree on speed of light?

Warm Up Geometric Analogy

CMU



Two surveying contractors
- Company (Engineering)
- Company' (Chemistry)

Surveyed CMU w/ slightly different orientations

x-coordinate measured in meters

y- " " " miles

	$(x \quad y)$	$(x' \quad y')$
D	0 0	0 0
W	$(x_w \; y_w)$	$(x'_w \; y'_w)$
F	⋮	
C		

Companies detailed accurate measurements, but don't talk to each other

One day, open minded physics student came & studied Both

took daring step to use same dimension for x & y

Conversion factor k

- Then discovered the quantity

$$\sqrt{(x)^2+(ky)^2} = \sqrt{(x')^2+(ky')^2} \equiv \text{Distance}$$

"Discovered principle of invariance of distance"

Relativity

- Points → Events → Space & at time

(See Book discussion about lattice of synchronized clocks)

$$\text{Event} - (x, y, z, t) \xrightarrow{?} (x', y', z', t')$$

↑

Relative motion along x-axis.

Big lesson from geometric analog!

It Pays to use the <u>same units</u> for all coordinates

Instead of t $[t] = s$ use ct $[ct] = m$

Will measure time in meters

"One meter of time" = light meter = t it takes light to go 1m

Deep: c is <u>not</u> some sacred physical constant

Simply a conversion factor $s \to m$

Analogous to k miles $\to m$

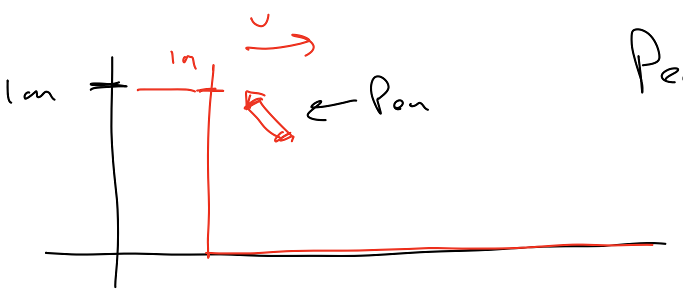
Ok, so what do the co-ordinate transforms have to be

$$(ct, x, y, z) \xrightarrow{v} (ct', x', y', z')$$

to keep c constant?

Start easy $y' = y \quad z' = z$

Direction $\perp$ to relative motion un affected



Pen marks measure Lab y coordinate of the $y' = 1$ Rocket frame

In Lab frame the marks will be on $y = 1$.

If they weren't then Both observers will agree that the rocket passed inside / outside $y = 1$ (By looking at the mark!)

This would be a way to distinguish the two frames. Violates PoR. (Ditto z)

Move on to the more interesting Coordinates

Can now use invariance of $\perp$ distances as a clean way to compare clocks.

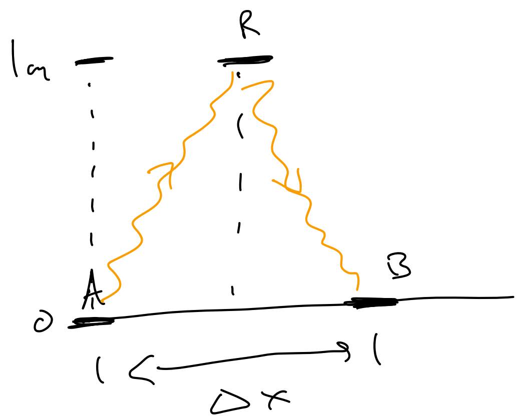
clock "ticks" every time light flash returns to the origin

Now, compare the clocks in the two frames.

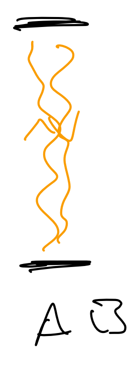
Consider events A - emission of light

B - the 1st "tick"

Lab Frame (S)



Rocket Frame (S')



$\Delta x' = 0$

A happens at $t = t' = 0$ when the coordinate systems coincide

$x_A = 0 \qquad x'_A = 0$

$t_A = 0 \qquad t'_A = 0$

<u>In the rocket frame</u> light goes straight up 1m and " down 1m

$x'_B = 0 \qquad t'_B = 2m \quad (t = 2m/c \ s)$

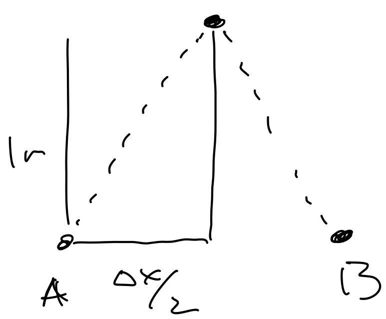
Difference in Coordinates:

$\Delta x' = 0 \qquad \Delta t' = 2m$

<u>In Lab frame</u> there is non-zero Δx Between A & B (the rocket moves wrt Lab!)

Large $v \Rightarrow$ Large Δx

Small $v \Rightarrow$ Small Δx



Light travels total distance $= 2 \times \sqrt{1 + \left(\frac{\Delta x}{2}\right)^2}$

Greater than 2m!

The time difference between A & B is different in the two frames

$\Rightarrow$ Moving Clocks tick at different (slower) rates!

(Before just assumed $t = t'$. Now we see it isn't so.)

Lab

$\Delta x = \Delta x$

$\Delta t = 2\sqrt{1 + \left(\frac{\Delta x}{2}\right)^2}$

Rocket

$\Delta x' = 0$

$\Delta t' = 2$

the coordinate differences differ between the two frames

(Just like $\Delta x \neq \Delta x'$ & $\Delta y \neq \Delta y'$ in geometry analogy.)

Note

$$\Delta t^2 - \Delta x^2 = 4\left(1 + \frac{\Delta x^2}{4}\right) - \Delta x^2$$

$$= 4 + \cancel{\Delta x^2} - \cancel{\Delta x^2}$$

$$= \Delta t'^2 - \Delta x'^2 \equiv (\text{Interval})^2$$

This quantity "The Interval" is invariant.

Coordinate independent separation between events.

(Analogous to distance in the geometry analogy)

Deep! Invariance of the interval implies that time cannot be separated from Space.

Space & time part of single entity

Spacetime

The geometry of Spacetime is truly 4D (Not 3+1 as in Newtonian Physics)

"The direction of the "time axis" depends on the ~~state~~ of motion of the observer.

Just like the direction of surveyors y-axis depends on thier orientation.

HW: What about a frame S'' that is moving faster than S'?

Comments

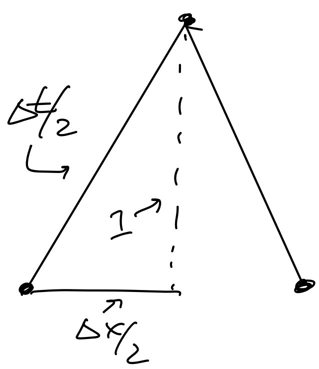
-) that fact that Δx is different is no surprise!

-) $\Delta t > \Delta t'$ "Moving clocks run slow"

$$\frac{\Delta t}{\Delta t'} = \sqrt{1+\left(\frac{\Delta x}{2}\right)^2} \to 1 \quad \text{when } \Delta x \to 0$$

when v small.

-)



$$\left(\frac{\Delta x}{2}\right)^2 + 1^2 = \left(\frac{\Delta t}{2}\right)^2$$

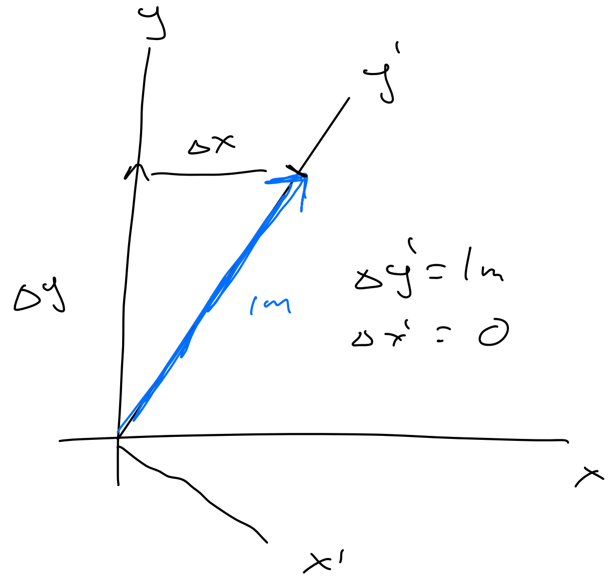
$$\Delta x^2 + 2^2 = \Delta t^2$$

-) PoR used in two ways

-) $\perp$ distances the same.

-) c is the same in both frames

Compare



Simplier frame?	C'	S'
why?	$\Delta x' = 0$	$\Delta x' = 0$
Simplification	1 meter stick need	1 clock needed
Alternative	2 sticks needed x & y	2 clocks needed one at O & X_r
Complicated frame necessarise	Δy (not full interval!)	Δt (Not full interval!)
	$\Delta y = \sqrt{(AB)^2 - (\Delta x)^2} \leq (AB)$	$\Delta t = \sqrt{(AB)^2 + (\Delta x)^2} \geq (AB)$

Inconsistent?

Identical meter sticks give different Δy's?

No 2 meter sticks in one frame 1 in the other

No C meter stick can be said to disagree w/ C' meter stick

Identical clocks different Δts?

No 2 clocks vs 1 clock

No lab clock can be said to disagree w/ rocket clock

Summary

No paradox about different y components. Not fault of m-sticks. "Discrepancy" from structure of (Difference) Euclidean Geom.

No paradox about different time lapses. Not fault of clocks "Discrepancy" from structure of (Difference) Space time.