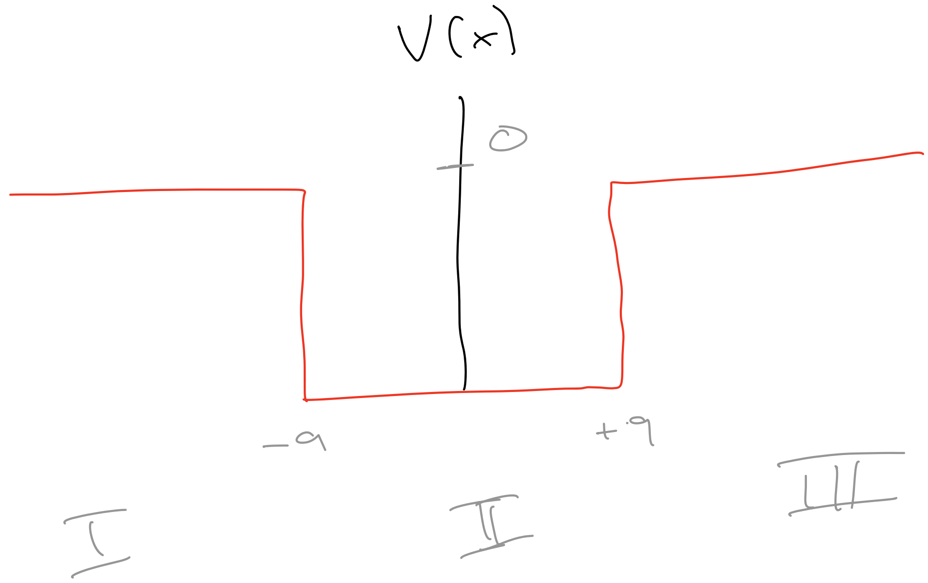
Finite Square Well

(More realistic Particle in Box)

Assume $E<0$ $\Rightarrow$ Bound states



Strategy: Solve Schr. eq separately in 3 regions

Impose Continuity (ψ & $\frac{d\psi}{dx}$) at Boundaries

Region I

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \quad \text{or} \quad \frac{d^2\psi}{dx^2} = K^2\psi$$

$$K = \frac{\sqrt{-2mE}}{\hbar}$$

Note $E<0 \Rightarrow K$ real & positive

Solution $\psi(x) = Ae^{-kx} + Be^{+kx}$

$|\psi|^2 = 1 \Rightarrow A = 0 \qquad \psi(x) = Be^{kx}$

Region II

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi \quad \text{or} \quad \frac{d^2\psi}{dx^2} = -l^2\psi$$

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

(real & positive)

$$\psi(x) = C\sin(lx) + D\cos(lx)$$

Region III

Same logic as Region I

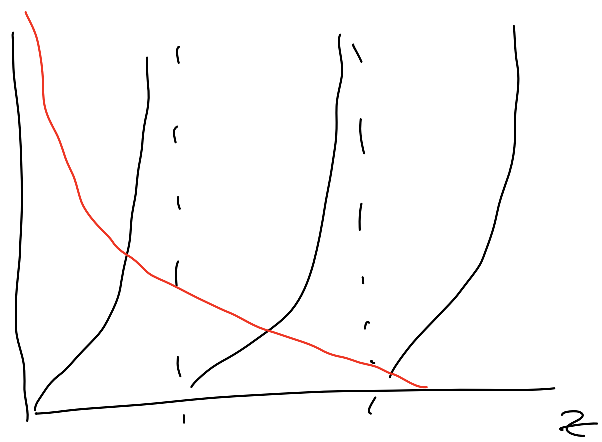
$$\psi(x) = Fe^{-kx}$$

Fix constants w/ Boundary Conditions

ψ & $\frac{d\psi}{dx}$ continuous @ a & $-a$

Get transcendental eq.

$$\tan z = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}$$



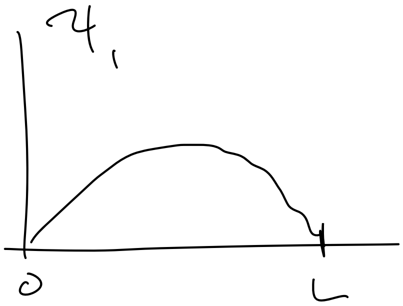
Results

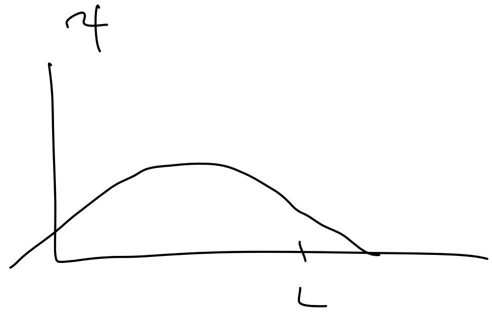
∞-well

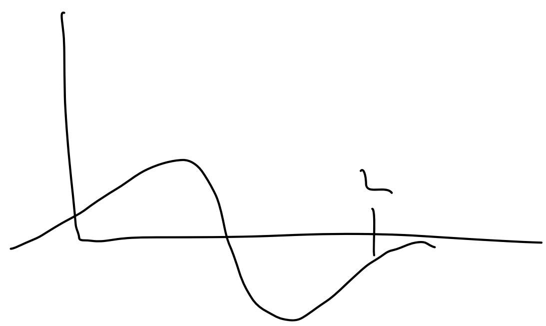
∞ # Solutions

finite well

finite # Solutions







Differences

- λ longer for finite well

  $\Rightarrow \dfrac{\partial \psi}{\partial x}$ smaller, E smaller

- $\psi \neq 0$ outside of the well!

$\psi \sim e^{-\kappa x}$ Completely different from Classical Physics.

The Simple Harmonic Oscillator

$$V(x) = \frac{1}{2} m \omega^2 x^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

* Approximates any potential around a stable equilibrium. → Used as approximate model to describe many physical systems.

Solving the Sch Eq. can be done

- long, tedious (not hard)
- not particularly enlightening.
- We will skip to punch line.

Results very similar to what we've seen

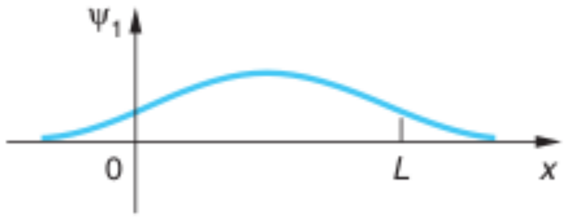
quantum number n characterizes ψ_n, E_n

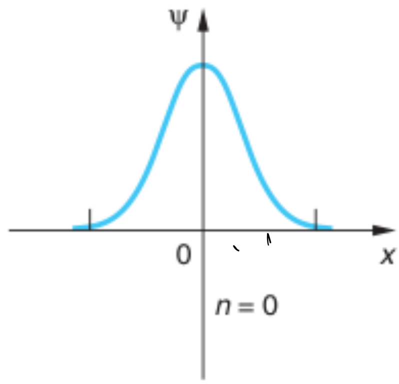
ψ_n even or odd (infinite family)

ψ for finite square well

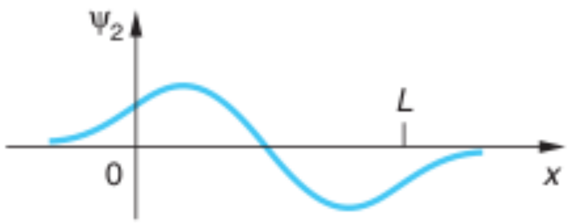
Simple Harmonic Oscillator

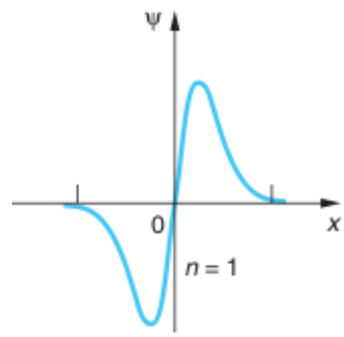
$n=0$



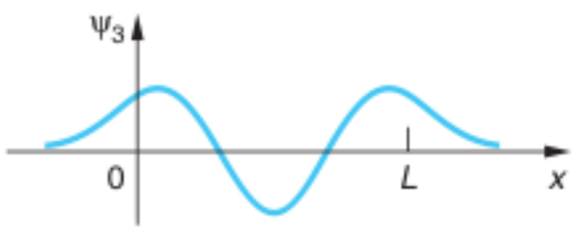


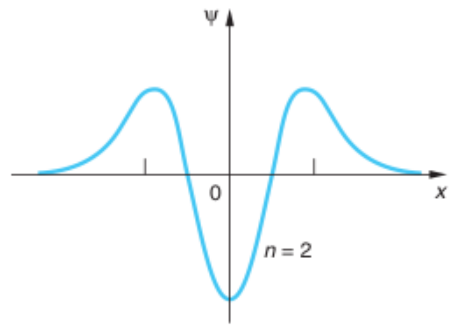
$n=1$





$n=2$





- Wave functions similar
- SHO more peaked at center.