From the end last time

- Every particle has a value for its invariant mass ("rest mass")

A system of particles also has an invariant mass that is a function of the E-mom 4-vectors of constituents

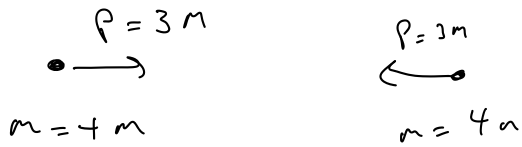
Note Mass of a system is **NOT** $\sum$ mass constituents

e.g.)
$$E_{tot} = \sum_i E_i = \sum_i (m_i + KE_i)$$

In rest frame of total $\vec{p}$

$$M_{tot} = E_{tot} = \sum_i m_i + \sum_i KE_i$$

e.g.)



$$E_i^2 = (3^2 + 4^2)m^2$$
$$E_i = 5m$$
$$E_t = M_t = 10m \neq 8M \,!!!$$

Start today by another (much more algebraic) way of looking at E + P transformations

Relativistic Energy

$m\vec{\beta}$ not conserved in collisions, $m\gamma\vec{\beta}$ is

$\Rightarrow$ $\vec{F} = m\vec{a}$ is not relativistic, implies $m\vec{\beta}$ conserved

$= \frac{d(m\vec{\beta})}{dt}$ $\rightarrow$ Can almost read off what the new expression must be.

$\vec{F} = \frac{d\, m\gamma\vec{\beta}}{dt}$ will conserve the "right" momentum in absence of net force

Can now proceed to define KE as the work needed to accelerate particle from rest to β

Reminder Classically

$$\boxed{KE = \int_{v=0}^{v=v} F \cdot dx = \int_0^v \frac{dp}{dt}\, dx = \int_0^v m \frac{dv}{dt}\, dx = \int_0^v m \underbrace{\frac{dx}{dt}}_{v}\, dv = \frac{1}{2} m v^2}$$

$$KE = \int_0^\beta F \cdot dx = \int \frac{d\, m\gamma\beta}{dt}\, dx = \int \beta\, d(m\gamma\beta)$$

$$d(m\gamma\beta) = m\, d(\gamma\beta) = m \frac{d(\gamma\beta)}{d\beta}\, d\beta$$

$$\frac{d(\gamma\beta)}{d\beta} = \beta \frac{d\gamma}{d\beta} + \gamma \qquad \frac{d\gamma}{d\beta} = -\frac{1}{2}(1-\beta^2)^{-3/2}(-2\beta) = \frac{\beta}{(1-\beta^2)^{3/2}} = \beta\gamma^3$$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}$$

$$\frac{d(\gamma\beta)}{d\beta} = \beta^2\gamma^3 + \gamma = \gamma^3(\beta^2 + \gamma^{-2})$$

$$= \gamma^3(\beta^2 + (1-\beta^2))$$

$$= \gamma^3$$

So ...

$$KE = \int \beta m \gamma^3 \, d\beta = \int_0^\beta \frac{m\beta}{(1-\beta^2)^{3/2}} \, d\beta$$

$$\frac{d\gamma}{d\beta} = \frac{\beta}{(1-\beta^2)^{3/2}} \; !!$$

$$z \equiv \beta^2$$

$$dz = 2\beta \, d\beta$$

$$= m \int_0^\beta \frac{d\gamma}{d\beta} \, d\beta = m \int_{\gamma(0)}^{\gamma(\beta)} d\gamma$$

$$= m\gamma(\beta) - m\gamma(0)$$

$$= m\gamma - m$$

w/ c's

$$KE = m\gamma c^2 - mc^2$$

Makes sense

we had $E = \gamma m c^2 = mc^2 + K.E$

Massless Particles

$$\Delta s^2 = \left|\begin{pmatrix} t \\ \vec{x} \end{pmatrix}\right|^2 \begin{cases} >0 & \text{time-like} \\ =0 & \text{"light-like"} \\ <0 & \text{space like} \end{cases}$$

Are there analogous categories for E-M

$$m^2 = \left|\begin{pmatrix} E \\ \vec{p} \end{pmatrix}\right|^2 \begin{cases} >0 & \text{Already discussed} \\ <0 & \text{Unphysical (violate causality } \beta>1) \\ =0 & \text{"massless" particle} \end{cases}$$

Massless Particles make No Sense Classically

they transfer No momentum $= m\beta = 0$

& No energy $= \frac{1}{2} m \beta^2 = 0$

Is $m=0$ permitted in Relativity

Seems No $\quad p = m\gamma\vec{\beta} \qquad E = m\gamma$

However, if take $\beta \to 1$ as $m \to 0$

$$m\gamma \to 0 \times \infty \sim \text{finite!}$$

<u>Possible if $\beta = 1$</u> $\quad E = |\vec{p}|$

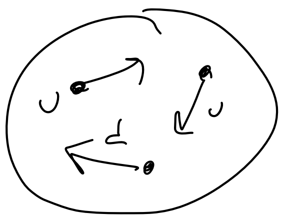
Known Examples of Massless Particles

1) Photon γ — "Particle of light"

(Makes sense that this moves w/ c!)

Will hear all about this in Part II

2) Gluons "Strong Interaction"



responsible for keeping quarks in proton/neutron

3) Graviton (Not yet confirmed)

force carrier of gravity.

4) ν's massless, chargeless e's

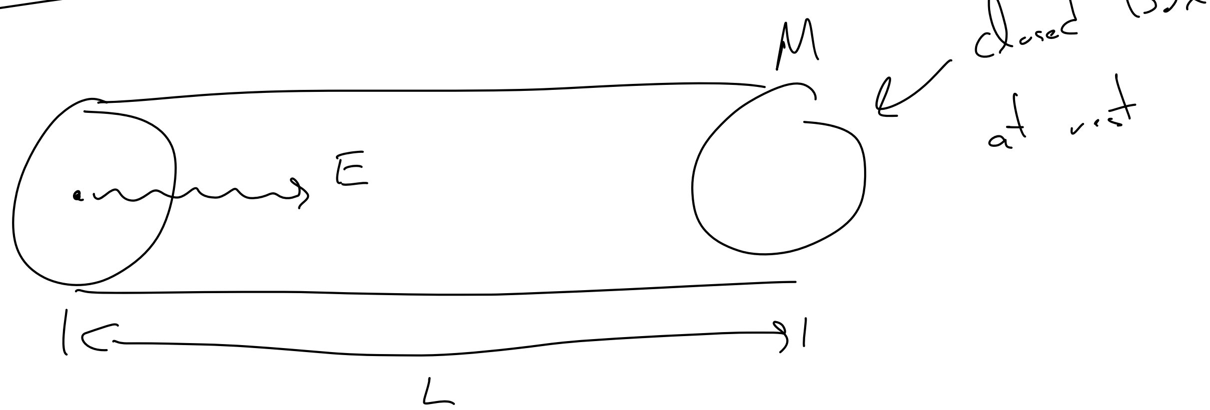
Now known to have a mass

$\nu_e \leftrightarrow \nu_\mu$ only if $m^2 \neq 0$

$m_\nu < 10^{-6} m_e$ m_e already small

Story of Sun

Alternative derivation of $E = m$



- Box at rest $P_i = 0$
- Burst of light (EM) energy / photons emitted from left wall, travels length of Box absorbed at right wall.

Know radiation carries P & E

Radiation exerts pressure on LHS during emission

Box pushed left w/ mom $-p$

$P_i = 0 \Rightarrow$ radiation $+p$ to right

Consider the center of mass of the box.

C.O.M. not moving, Box transports Mass during light travel

$\Rightarrow$ light must carry mass

$P = E$ (BTW can also derive this from Maxwell eq's)

During transit Box moving slowly $\Rightarrow$ $m\beta = p$

$$M\beta = -p = -E$$

or $\beta = -\dfrac{E}{M}$

$t \sim L$ Box moves $\Delta x = \beta t = -\dfrac{EL}{m}$

If radiation carried no mass,

the Δx would be a Net displacement

But (Assume) an isolated system cannot set itself into motion

$\Rightarrow$ must be some countervailing displacement of mass to the right (Feature of the radiation)

m_{rad} distance moved by radiation L

$$M\Delta x + m_{rad} L = 0$$

$$m_{rad} = -\frac{M\Delta x}{L} = -\frac{M}{L}\left(-\frac{EL}{m}\right) = E \quad !$$

Conclude transport of Energy E is equivalent to the transport of Mass E from one end of the box to the other.

Mass equivalence of radiant Energy implies the mass equivalence of thermal energy

Whatever the details of the mechanisms by which the light was emitted & absorbed the Net Effect is the transfer of heat energy from one end to the other.

$\Rightarrow$ Mass moves when thermal energy changes location