2) $d_x = v_\perp \times \text{time in } D$

$= (a_\perp \times \text{time in } d) \times \frac{D}{v}$

$= \frac{F}{m} \times \frac{d}{v} \times \frac{D}{v}$

$d_e = \frac{eEdD}{mv^2}$ $\qquad d_B = \frac{e\cancel{v}BdD}{mv^{\cancel{2}}}$

$\frac{d_e}{d_B} = \frac{\frac{\cancel{e}\,E\cancel{dD}}{\cancel{m}\,v^{\cancel{2}}}}{\frac{\cancel{e}B\cancel{dD}}{\cancel{m}\cancel{v}}} = \frac{E}{Bv}$

$\Rightarrow$

$v = \frac{E}{B}\frac{d_B}{d_e}$

meas I: $v = \frac{1.5\ 10^4\ \frac{N}{C}}{5.5\ 10^{-4}\ N/(A\ m)}$

$= \frac{1.5}{5.5}\ 10^8\ \frac{A}{C}\ m \sim 0.3\ 10^8\ \frac{m}{s}$

meas 2: $v = \dfrac{1.8 \; 10^{4}}{5 \; 10^{-4}} \quad 7$

$$= 0.36 \; 10^{8} \; m/s$$

$$\frac{e}{m} = \frac{d_B \, v}{B \, d \, D} = \frac{d_B^2 \, E}{d_e \, B^2 \, d \, D}$$

<u>Units</u>

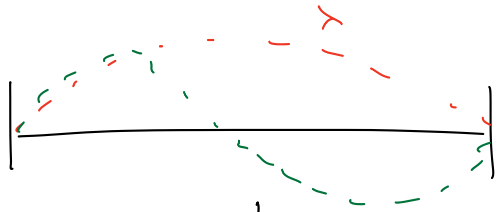
$$\frac{m^2 \; N/C}{m^3 \left(\frac{N}{A\,m}\right)^2} = \frac{A^2 \, m^2}{C \, m \, N} = \frac{C^2 \; m}{s^2 \, C \; N} = \frac{C \; \cancel{m}}{\cancel{s^2} \; kg \, \frac{m}{s^2}}$$

meas I: $\dfrac{8 \, 10^{-2} \; 1.5 \; 10^{4}}{(5.5 \; 10^{-4})^2 \; 5 \, 10^{-2} \; 1.1} = \dfrac{12 \; 10^{2}}{(30 \; 10^{-8}) \; 5 \, 10^{-2}}$

$$= \frac{12}{150} \; 10^{12} = 10^{11} \; \frac{C}{m}$$

meas 2: $\dfrac{6 \, 10^{-2} \quad 1.8 \; 10^{4}}{}$

3 a)



L

Have standing wave when

$$\frac{\lambda}{2} n = L \quad \text{for} \quad n = 1, 2, \dots$$

Allowed λ's $\quad \lambda = \frac{2L}{n}$

$$y = A \sin \frac{2\pi x}{\lambda} \sin 2\pi f t$$

$$y = A \sin kx \sin \omega t$$

$$k = \frac{2\pi}{\lambda} \Rightarrow k_n = \frac{\pi n}{L}$$

3 b)

$$\vec{k} = \frac{\pi}{L}(n_x, n_y, n_z)$$

3 c)

$$|\vec{k}| = \frac{\pi}{L}|\vec{n}| \qquad n = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

$$N_{modes}(|n|) = \frac{4\pi |n|^2 dn}{8}$$ ← n's positive!

$$N_{modes}(|\vec{k}|) = \left(\frac{\pi}{L}\right) \frac{4\pi}{8} \frac{L^2}{\pi^2} k^2 \frac{L}{\pi} dk$$

$$= \frac{L^3}{2\pi^2} k^2 dk$$

But 2 modes for each k!

$$N_{modes}(|\vec{k}|) = \frac{L^3}{\pi^2} k^2 dk$$

$$= \frac{V}{\pi^2}$$

$$\frac{N_{modes}}{V} = \frac{k^2}{\pi^2} dk$$

3d) $k = \frac{2\pi}{\lambda}$

$$g_\lambda(\lambda)\, d\lambda = g_k(k)\, dk$$

$$g_\lambda(\lambda)\, d\lambda = g_k(k) \left| \frac{dk}{d\lambda} \right|$$

$$= g_k\left(\frac{2\pi}{\lambda}\right) \frac{2\pi}{\lambda^2}$$

$$= \frac{\left(\frac{2\pi}{\lambda}\right)^2}{\pi^2} \frac{2\pi}{\lambda^2} = \frac{8\pi}{\lambda^4}$$

$$g_\lambda(\lambda)\, d\lambda = \frac{8\pi}{\lambda^4}\, d\lambda \quad (\text{eq } 3\text{-}8)$$

3e)

$$\mathcal{E}(\lambda, T) = kT \times \#\text{ modes}$$

$$= \frac{8\pi kT}{\lambda^4}$$

3f) $E_n = nh\nu \qquad n = 0, 1, 2, \ldots$

$$f_n = A e^{-E_n/kT} = A^{-nh\nu/kT}$$

<u>find A</u> $\quad \sum_n f_n = 1$

or $\quad \sum_0^\infty A e^{-nC} = 1 \qquad C = \frac{h\nu}{kT}$

$$\boxed{\sum_0^\infty x^n = \frac{1}{1-x} \qquad x < 1}$$

$$\Rightarrow \frac{1}{A} = \sum_0^\infty \left(e^{-C}\right)^N = \frac{1}{1-e^{-C}}$$

or $\quad A = \left(1 - e^{-C}\right) = \left(1 - e^{-h\nu/kT}\right)$

$$\mathcal{E}(f,T) = \sum_{0}^{\infty} E_n f_n = \sum_{0}^{\infty} E_n A e^{-E_n/kT}$$

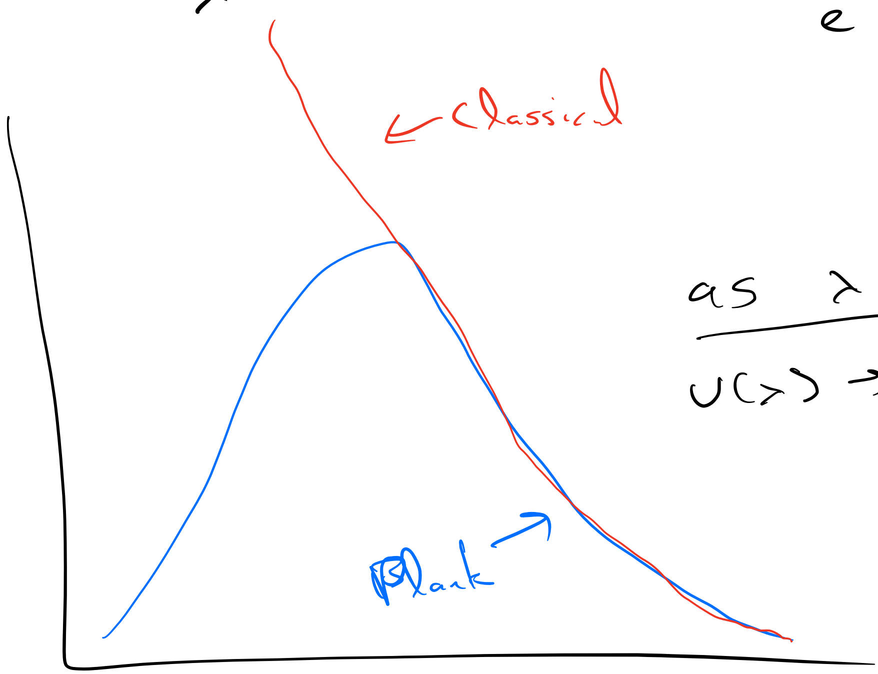
$$= A \sum_{0}^{\infty} n h\nu \, e^{-nC}$$

$$= A h\nu \sum_{0}^{\infty} n e^{-nC}$$

$$\boxed{\sum_{0}^{\infty} n e^{-nC} = -\frac{d}{dC} \sum_{0}^{\infty} e^{-nC} = -\frac{d}{dC} (1-e^{-C})^{-1} = + \frac{e^{-C}}{(1-e^{-C})^2}}$$

$$\bar{E} = Ahf \frac{e^{-C}}{(1-e^{-C})^2} = \frac{hf \, e^{-C}}{(1-e^{-C})} = \frac{hf}{e^{+C}-1} = \frac{hf}{e^{hf/kT}-1} \qquad (3\text{-}17)$$

$$U(\lambda) = \frac{8\pi}{\lambda} \times \frac{hc}{\lambda(e^{hf/kT}-1)} = \frac{8\pi hc \lambda^{-5}}{e^{hc/\lambda kT}-1}$$



as $\lambda \to 0$

$$U(\lambda) \to \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda kT}}$$

$$\to 0$$

λ

as $\lambda \to \infty$ $\quad e^{\frac{hf}{\lambda kT}} \simeq 1 + \frac{hc}{\lambda kT}$

$u(\lambda) \to 8\pi kT \lambda^{-4}$ = Classical

g) Can be explained high λ behavior

Cannot low λ

Stephen - Boltzmann

Wiens Law.

h) Total Energy Density given by

$$\mathcal{E}(T) = \int_0^\infty \mathcal{E}(\lambda, T)\, d\lambda = \int_0^\infty \frac{8\pi hc \lambda^{-5}}{e^{hc/\lambda kT} - 1}\, d\lambda$$

$$\boxed{x \equiv \frac{hc}{\lambda kT} \quad dx = -\frac{hc}{kT\lambda^2} d\lambda \Rightarrow d\lambda = -\lambda^2 \left(\frac{kT}{hc}\right)}$$

$$\mathcal{E}(T) = -\int_\infty^0 \frac{8\pi hc \lambda^{-5}}{e^x - 1} \lambda^2 \left(\frac{kT}{hc}\right) dx = 8\pi hc \left(\frac{kT}{hc}\right) \int_0^\infty \frac{\lambda^{-3}}{e^x - 1} dx$$

$$= 8\pi kT \int_0^\infty \frac{\left(\frac{kTx}{hc}\right)^3}{e^x - 1} dx = \frac{8\pi (kT)^4}{(hc)^3} \underbrace{\int_0^\infty \frac{x^3}{e^x - 1} dx}_{\text{finite}}$$

$$\mathcal{E}(T) \sim T^4$$

i)

$$\mathcal{E}(\lambda, T) = \frac{8\pi hc\,\lambda^{-5}}{e^{hc/\lambda kT} - 1} \equiv C\,\lambda^{-5}\left(e^{a/\lambda} - 1\right)^{-1}$$

$$C = 8\pi hc \qquad a = \frac{hc}{kT}$$

$$\frac{d\mathcal{E}}{d\lambda} = C\,\lambda^{-5}\left(-\left(e^{a/\lambda} - 1\right)^{-2} e^{a/\lambda}\left(-\frac{a}{\lambda^2}\right)\right) + C\left(e^{a/\lambda} - 1\right)^{-1}\left(-5\lambda^{-6}\right)$$

$$\frac{d\mathcal{E}}{dx} = 0 \Rightarrow$$

$$0 = \cancel{C} a\lambda^{-7} e^{a/\lambda}\left(e^{a/\lambda} - 1\right)^{-2} - 5\cancel{C}\lambda^{-6}\left(e^{a/\lambda} - 1\right)^{-1}$$

Mult by $\lambda^7\left(e^{a/\lambda} - 1\right)^2$

$$0 = a e^{a/\lambda} - 5\lambda\left(e^{a/\lambda} - 1\right)$$

$$\Rightarrow 5\lambda\left(e^{a/\lambda} - 1\right) = a e^{a/\lambda}$$

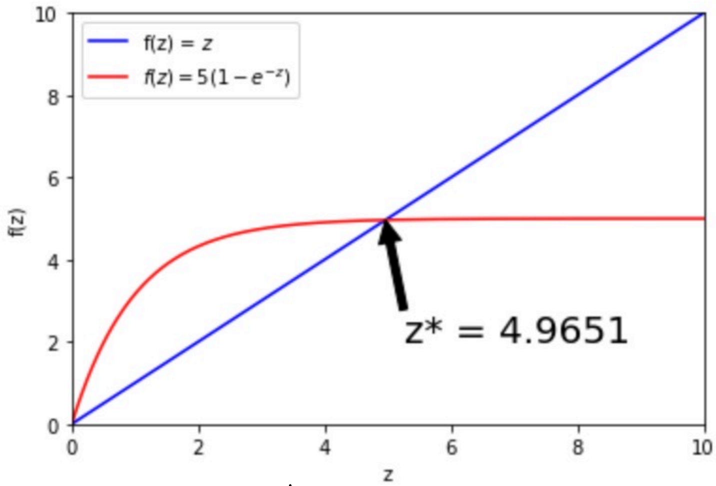
Mult by $e^{-a/\lambda}$ $\Rightarrow$ $a = 5\lambda\left(1 - e^{-a/\lambda}\right)$

or $\dfrac{a}{\lambda} = 5\left(1 - e^{-a/\lambda}\right)$

$$z = 5\left(1 - e^{-z}\right)$$

Solution $z^* = 4.965$ See Notebook

https://gitlab.cern.ch/johnda/notebooks/-/blob/master/QM-225/ModernEssentials_HW5.ipynb



$\Rightarrow$ max when $\frac{a}{\lambda} = 4.965$

or $\frac{hc}{kT\lambda_m} = 4.965$

or $\lambda_m T = \frac{hc}{k(4.965)} = \frac{1240 \text{ eV nm}}{(4.965)\,k}$

$k = 8.617\ 10^{-5}$ eV/K

$\lambda_{max} T = \frac{1240\ 10^{-9} \text{ m K}}{(4.965)\ 8.617\ 10^{-5}}$

$= 2.898\ 10^{-3}$ m K ✓

4a)

The higher the intensity the more energy in the light. the more energy the more electrons.

4b)

Classical Physics Able to predict

Increase of current w/ intensity

Classical Physics un-able to predict

- Current $\neq 0$ when intensity low
- dependence of light frequency
- lack of time lag (see 2a)

4c) Assume source w/ $1\,W = 1\,J/s$

Intensity at 1 m $\sim \dfrac{10^{19}\,eV}{s}$

$$\frac{P}{4\pi R^2} \sim \frac{10^{19}\,eV}{10\,m^2 s} \sim 10^{18}\,\frac{eV}{m^2 s}$$

$r_{atom} \sim 10^{-10}\,m \Rightarrow$ Power on atom $\sim 10^{18} \cdot 10^{-20}\,\frac{eV}{s}$

$10^{-2}\,eV/s$

Work function $\sim eV$

↪ this is the energy needed to eject an electron.

time to eject $\sim \dfrac{E \text{ needed}}{\text{Power Applied}} = \dfrac{eV}{10^{-2}\,eV}\,s$

$\sim 10^{2}\,s$

$\sim$ minute

this is another problem w/ classical physics. Expect delay of ejected electrons of ~ minute. However experimentally we see current immediately.

5a) 380 nm − 750 nm

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda}$$

$$\boxed{\text{Range} = 3.3\ \text{eV} - 1.7\ \text{eV}}$$

FM at 100 MHz $= 10^2\, 10^6$ Hz

$= 10^8$ Hz

$\Rightarrow \lambda = \frac{c}{\nu} = \frac{3\, 10^8\ \text{m/s}}{10^8\ \frac{1}{s}}$

$= 3$ m

$= 3\, 10^9$ nm

$$E = \frac{1240\ \text{eV}\cdot\text{nm}}{3\, 10^9\ \text{nm}}$$

$$= \frac{12\, 10^2}{3\, 10^9}\ \text{eV}$$

$$\boxed{= 4\, 10^{-7}\ \text{eV}}$$

5b) $E_{Bond} \sim 4.26\,eV$

Need γ w/at least $4.26\,eV$

$$E = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{\lambda} \Rightarrow \lambda = \frac{1240\,eV\,nm}{4.26\,eV}$$

or $\lambda = 291\,nm$

$$\Rightarrow \nu = \frac{c}{\lambda} = \frac{3\,10^{8}\,m/s}{291\,10^{-9}\,m} = \frac{3\,10^{8}}{3\,10^{-7}}\,\frac{1}{s}$$

$$= 10^{15}\,Hz$$

UV photon

5c) 40 w bulb

$T = 3300\ K$

max when $\lambda_m T \sim 3 \cdot 10^{-3}\ mK$

$\Rightarrow \lambda_m = \frac{3e{-3}\ m}{3300} = 9 \cdot 10^{-7}\ m$

$c = \lambda f$

$\sim 900\ nm$

$f = \frac{c}{\lambda} \sim 3 \cdot 10^{14}\ Hz$

Assume All photons w/ λ + f

$E/_{photon} = hf = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{900\ nm}$

$= 1.38\ eV$

$40\ W = 40\ J/s$

$eV = 1.6\ 10^{-19}\ J$

$\sim 40 \cdot 10^{19}\ eV/s \Rightarrow \sim 4 \cdot 10^{20}\ \gamma/s$

Area of eye $\sim \pi (2.5\,mm)^2$

fractions of γ's at 5 m is

$$\frac{\pi (0.0025)^2}{4\pi (5)^2} \sim \frac{1}{4}\left(\frac{0.0025}{5}\right)^2$$

or

$$\frac{1}{\cancel{4}}\left(\frac{0.0025}{5}\right)^2 \cancel{4}\, 10^{20} =$$

$$2.5 \cdot 10^{-7} \qquad \sim 10^{13} \frac{\gamma\text{'s}}{s}$$