Impact of Relativity on Doppler Effect

- Some of most important observations involve measurements on the radiation from atoms/nuclei
  (Esp important when don't have direct access to atoms eg space / too hot / to extreme an environment)
- The apparent frequency of emitted radiation depends on relative motion of source & observer

$\hookrightarrow$ Expect Relativistic effects to be important

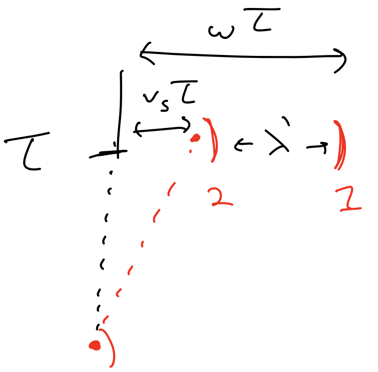
Dependence of frequency on relative motion referred to as "Doppler Effect"

Classical Doppler Effect

Reminder (?) $\qquad$ w - speed in medium

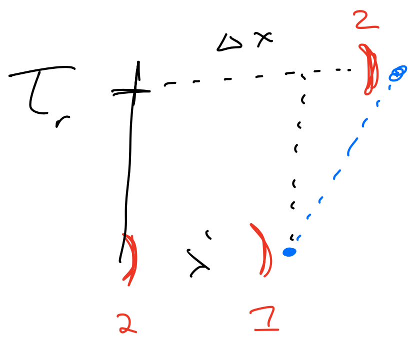
Source emitting pulses w/ $f_0 = \frac{1}{\tau_s}$

moving source



$$\lambda' = w\tau - v_s\tau$$
$$= (w - v_s)\tau$$

moving reciever



$$\tau_r = \frac{(\lambda' + v_r\tau_r)}{w}$$

$$w\tau_r = \lambda' + v_r\tau_r$$

$$\tau_r = \frac{\lambda'}{w - v_r} = \tau\frac{(w - v_s)}{(w - v_r)}$$

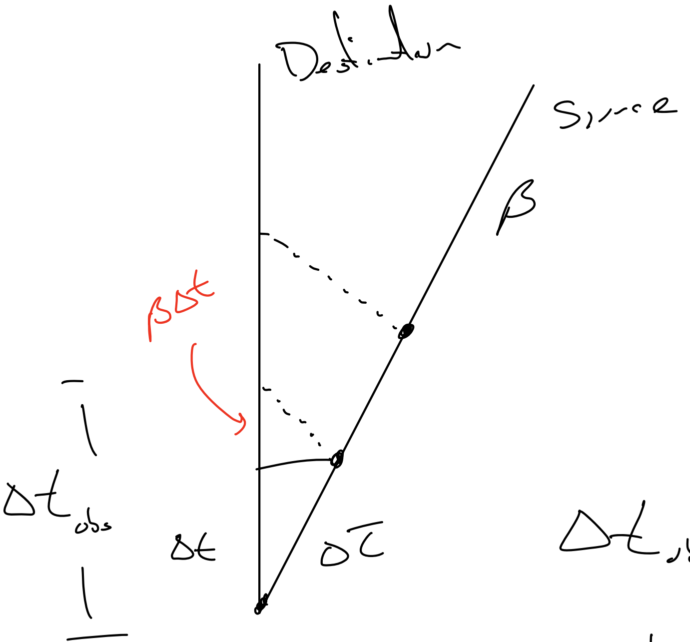
$$= \tau\left(\frac{1 - v_s/w}{1 - v_r/w}\right)$$

So

$$f_{rec} = \frac{1}{\tau_r} = \frac{1}{\tau}\left(\frac{1 - v_r/w}{1 - v_s/w}\right)$$

$$= f_0\left(\frac{1 - v_r/w}{1 - v_s/w}\right)$$

Relativistic Doppler Effect (Receding)



f_0 – frequency in rest frame of source

$$f_0 = \frac{1}{\Delta\tau} \qquad f_{obs} = \frac{1}{\Delta t_{obs}}$$

$$\Delta t_{obs} = \Delta t + \beta\Delta t$$

& $\Delta t = \gamma\,\Delta\tau$

$$\Rightarrow \Delta t_{obs} = \Delta t(1+\beta)$$

$$= \gamma\,\Delta\tau(1+\beta)$$

$$\frac{\Delta t_{obs}}{\Delta\tau} = \gamma(1+\beta) = \sqrt{\frac{(1+\beta)^2}{(1-\beta^2)}} = \sqrt{\frac{(1+\beta)^2}{(1+\beta)(1-\beta)}}$$

$$= \sqrt{\frac{(1+\beta)}{(1-\beta)}}$$

$$= \frac{f_0}{f_{rec}}$$

$$\boxed{f_{rec} = f_0\sqrt{\frac{(1-\beta)}{(1+\beta)}}}$$

Red Shift

Astronomers define "red shift" $z \equiv \dfrac{f_0 - f}{f}$

$z = 0 \Rightarrow$ no red shift

$z > 0$ redshift

$z = \dfrac{f_0}{f} - 1$ or $\dfrac{f_0}{f} = z + 1$

Relativistic Red Shift

$$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}} \Rightarrow \left(\frac{f_0}{f}\right)^2 = \frac{1+\beta}{1-\beta}$$

Algebra ...

$$\beta = \frac{\left(\frac{f_0}{f}\right)^2 - 1}{\left(\frac{f_0}{f}\right)^2 + 1} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

eg $z = 1 \Rightarrow \beta = \dfrac{2^2 - 1}{2^2 + 1} = \dfrac{3}{5}$

Muons

$\tau \sim 2\ \mu s$ $\qquad N(t) = N(t=0)\, e^{-t/\tau}$

- Created by cosmic rays Altitude $\sim 9\ 10^{3}\ m$
- typical $\beta \sim 0.998$

How many of these make it to ground?

Naive calculation $\quad \Delta t = \dfrac{9\ 10^{3}\ m}{3\ 10^{8}\ m/s} \sim 3\ 10^{-5}\ s$

$\sim 30\ \mu s$

$$\frac{N(30\ \mu s)}{N(0)} = e^{-30/2} = e^{-15} \sim \underline{\underline{3\ 10^{-7}}}$$

almost none!

We see $\sim 10^{4}\ \frac{\mu's}{m^2\ s}$! what gives

1) Time dilation $\qquad \gamma \sim 16$

τ_μ : $2\ \mu s \to 32\ \mu s$ in earth frame

$$\frac{N(30)}{N(0)} = e^{-30/2\gamma} = e^{-30/32} \sim 0.4$$

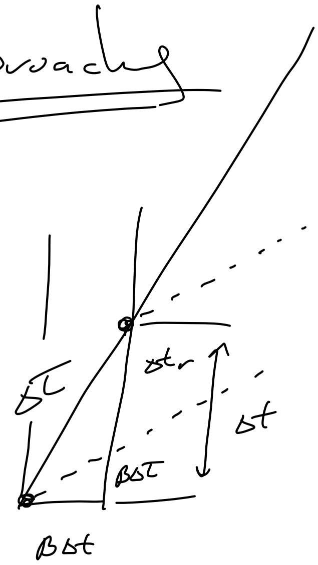
2) Length contraction

$$L' = \frac{L}{\gamma} \qquad \Delta t' = \frac{L}{\gamma c}$$

$\Rightarrow$ fraction down $\quad \dfrac{N(\text{earth})}{N(0)} = e^{\frac{L}{c\gamma\tau}} = 0.4$

Different effects depending on different frames. Overall consistent.

Approach



$$\Delta t = \gamma \Delta\tau$$

$$\Delta t_r = \Delta t - \beta \Delta t$$

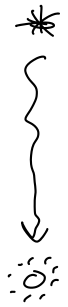
$$= \Delta t (1 - \beta)$$

$$\Delta t_r = \gamma \Delta\tau (1 - \beta)$$

$$\frac{\Delta t_r}{\Delta\tau} = \sqrt{\frac{(1-\beta)^2}{(1+\beta)(1-\beta)}} = \sqrt{\frac{1-\beta}{1+\beta}}$$

44) Aberration of Starlight

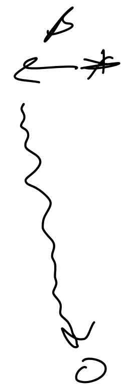
Sun reference frame



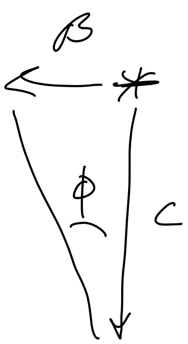
$v^\gamma_y = -c$

$v^\gamma_x = 0$

Earth frame

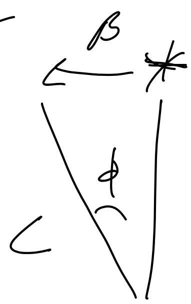


Newton



$\tan\phi = \beta$

w/ Relativity



$\sin\phi = \beta$

for small ϕ difference in

Newton $\beta = \phi + \frac{\phi^3}{3} + \dots$

Relativity $\beta = \phi + \frac{\phi^3}{2\,3}$