Exam #2

1) Chased by a comet *(5 points)*

A comet is chasing a spaceship. Let β, P and E be the speed, momentum, and energy of the comet as seen by the astronaut when it hits the spaceship. In what way would the increasing the spaceship's speed alter the astronaut's perceived values of β, P and E?

a) β, P and E will all be constant not change at all.

b) β, P and E will all decrease.

c) β, P will get smaller, E will not change.

d) β, E will get smaller, P will not change.

e) P and E will get smaller, β will not change.

2) Chased by a photon *(5 points)*

A photon is chasing a spaceship. Let β, P and E be the speed, momentum, and energy of the photon as seen by the astronaut when it hits the spaceship. In what way would the increasing the spaceship's speed alter the astronaut's perceived values of β, P and E?

a) β, P and E will all be constant not change at all.

b) β, P and E will all decrease.

c) β, P will get smaller, E will not change.

d) β, E will get smaller, P will not change.

e) P and E will get smaller, β will not change.

3) Strong Box *(3 points)*

Suppose an atomic bomb was exploded in a box that was strong enough to contain all the energy released by the bomb. After teh explosion the box would weigh:

a) more than before the explosion

b) less than before

c) the same as before

4) General Relativity *(5 points)*

Which of the following statements are TRUE about general relativity

a) Clocks higher up in a gravitaional potential run faster than clocks lower down.

b) Measurements made in a uniform gravitaional feild are indistinguishable from measurements made is a uniformly accelerating reference frame.

c) The bending of light in a gravitational field is an illusion: viewed from far away the light path is a straight line.

d) In general relativity, the interial and gravitational mass of an object are not exactly the same in a gravitational potential well.

e) general relativity implies a direct connection between space-time curvature and the electro-magnetic force

5) Mass *(12 points)*

Consider three particles A, B, C. Particle A has 10 GeV of total energy and is moving at $\beta = \frac{3}{5}$. Particle B has 8 GeV of total energy and 2 GeV of momentum. Particle C has 12 GeV of total energy and 3 GeV of kinetic energy. Which particle is the most massive ? Which is the least massive?

$E_A = 10 \quad \beta = \frac{3}{5} \Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{9}{25}}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{5}{4}$

$E_A = m_A \gamma \Rightarrow m_A = \frac{E_A}{\gamma} = 8 \text{ GeV}$

$E_B = 8 \quad P_B = 2 \qquad m_B^2 = E_B^2 - P_B^2 = 64 - 4 = \sqrt{60}$

$\sim 7.X$

$E_C = 12 \quad KE_C = 3$

$E_C = m_C + 3 \Rightarrow m_C = 9$

$m_B < m_A < m_C$

6) Elastic Collisions *(25 points)*

A particle of mass m in incident with kinetic energy KE on an identical particle at rest relative to an inertial system S. The collision is elastic and such that the outgoing scattered particles make the same angle wrt to the x-axis.

a) Find the angle between the direction of motion of the two particles according to non-relativistic (Newtonian) physics

I

F

$P_i = m v_i \qquad P_f = 2 m v_f \cos\alpha$

$P_i = P_f \Rightarrow v_i = 2 v_f \cos\alpha$

$v_i^2 = 4 v_f^2 \cos^2\alpha$

$KE = \frac{1}{2} m v_i^2$

$\Rightarrow v_i = \sqrt{\frac{2KE}{m}}$

from E

$E_i = \frac{1}{2} m v_i^2 \qquad E_f = m v_f^2 \Rightarrow \frac{v_i^2}{v_f^2} = 2$

$\frac{v_i^2}{v_f^2} = 4\cos^2\alpha$

$\cos^2\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{4}$

$\cos\alpha = \frac{1}{\sqrt{2}}$

b) Find the angle between the direction of motion of the two particles (or the cosine of the angle) assuming relativisitic physics. (Note: $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$)

$E_i = 2m + KE \qquad E_f = 2\sqrt{m^2 + P_f^2}$

$4m^2 + 4mKE + KE^2 \qquad E_f^2 = 4(m^2 + P_f^2)$

$E_i^2 = E_f^2$

$\cancel{4m^2} + 4mKE + KE^2 = \cancel{4m^2} + 4P_f^2 \Rightarrow P_f = \sqrt{KE(m + \frac{1}{4}KE)}$

$P_f = 2|P|_f \cos\frac{\alpha}{2}$

$P_i = m\gamma\beta \qquad \gamma\beta = \frac{P_A}{m}$

c) What is the angle in the ultra-relativistic ($KE >> m$) limit ? $\longrightarrow$ 0

$P_A^2 = E^2 - m^2 = \cancel{m^2} + 2mKE + KE^2 - \cancel{m^2} = KE(2m + KE) = (\beta\gamma)$

$P_i^2 = KE(2m + KE) = P_f^2 = 4(KE(m + \frac{1}{4}KE))\cos^2\alpha$

$$\frac{\cancel{KE}(2m+KE)}{4(\cancel{KE}(m+\frac{1}{2}KE)} = \cos^2\frac{\alpha}{2} = \frac{2m+KE}{4m+KE}$$

$$\cos\alpha = \frac{KE}{KE+4m}$$

7) Nuclear Physics *(15 points)*

An unstable nucleus A (with mass M_1 = 50 GeV and proper lifetime = 5s) decays into a photon and another unstable nucleus B (with mass M_2 = 40 GeV and proper lifetime = 1s). What is the energy of the photon and the lifetime of nucleus B as observed from the rest frame of nucleus A?

$E_i = M_1$

$E_F = E_2 + P_\gamma$

$|P_B| = |P_\gamma|$ $\qquad E_2^2 = M_2^2 + |P_\gamma|^2$

$=$

$M_1 = \sqrt{M_2^2 + P_\gamma^2} + P_\gamma$

$(M_1 - P_\gamma)^2 = M_2^2 + P_\gamma^2$

$M_1^2 - 2M_1P_\gamma + \cancel{P_\gamma^2} = M_2^2 + \cancel{P_\gamma^2}$

$P_\gamma = \frac{M_1^2 - M_2^2}{2M_1} = \frac{900}{2\cdot 50} = 9$

$\gamma_B = \frac{E}{m} = \frac{41}{40}$

$\Rightarrow E_2 = \sqrt{40^2 + 9^2} = 41$

$t = \frac{41}{40}$ s

8) Testing the Standard Model *(20 points)*

The Standard Model of particle physics predicts a neutral particle, the Z boson, which has a mass of 100 GeV. The Z can be produced in electron-positron collisions in the reaction $e^+ + e^- \rightarrow Z$.

a) In a "fixed target experiment" the electron is at rest in the lab frame. How much kinetic energy in the lab frame does the positron need to in order to produce a Z ?

I $\quad e^+ \quad e^-$ $\qquad$ S' $\quad \beta'$ $\qquad$ F $\quad Z$

$E = 2 m_e \gamma_{\beta'}$ $\qquad$ $E = M_Z$

$\Rightarrow \gamma_{\beta'} = \frac{1}{2}\frac{M_Z}{m_e}$

Boost to frame when e^- at rest

β $\quad e^+ \quad e^-$ $\qquad$ β' $\quad M_Z$

$E = \gamma M_Z = \frac{1}{2}\frac{M_Z^2}{m_e}$

$KE + 2m = \frac{1}{2}\frac{M_Z^2}{m_e} \simeq KE$

b) In a "collider" the particles move with the same speed in opposite directions in the lab frame. How much kinetic in the lab frame do the electron and positron need in order to produce a Z in a collider?

I $\quad \beta \quad \beta$ $\qquad$ F $\quad M_Z$

$E = 2(m_e + KE)$ $\qquad$ $E = M_Z$

$KE = \frac{M_Z}{2} - m_e \sim \frac{M_Z}{2}$