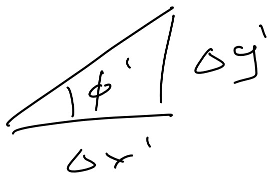
Last time

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

$\beta = \frac{v}{c}$ $\qquad$ $\gamma = \frac{1}{\sqrt{(1-\beta^2)}}$

$[0 - 1]$ $\qquad$ $[1 - \infty)$

Transformation of angles

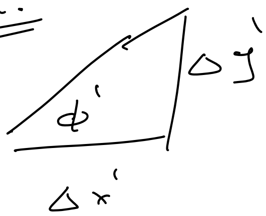


G.T.: $\Delta y' \to \Delta y$

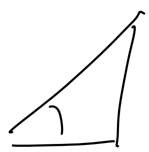
$$\begin{pmatrix} \Delta x' \\ \Delta t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ 0 \end{pmatrix}$$

$$\Delta x' = \Delta x \qquad \phi' \to \phi$$

L.T.

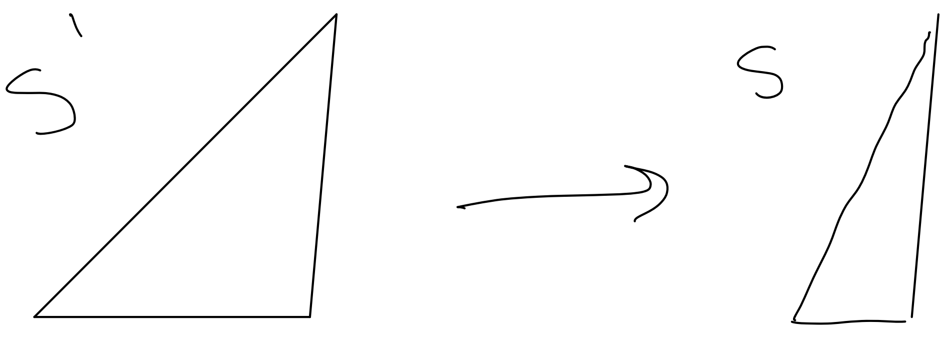


$\longrightarrow$



$\Delta y = \Delta y'$

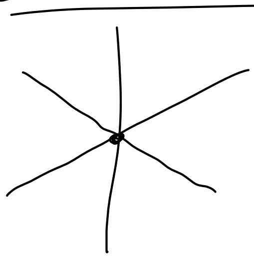
$\Delta x = \frac{1}{\gamma} \Delta x'$ ← Length contraction!



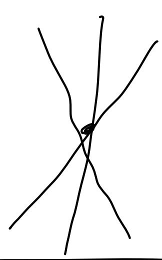
$$\tan\phi' = \frac{\Delta y'}{\Delta x'} \longrightarrow \tan\phi = \gamma \frac{\Delta y'}{\Delta x'}$$

$$= \gamma \tan\phi'$$

E-fields affected



$\longrightarrow$



Reminder

Can Skip in real time

$\Delta y' \to \Delta y$

$$\begin{pmatrix} \Delta x' \\ \Delta t' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \Delta x \\ 0 \end{pmatrix}$$

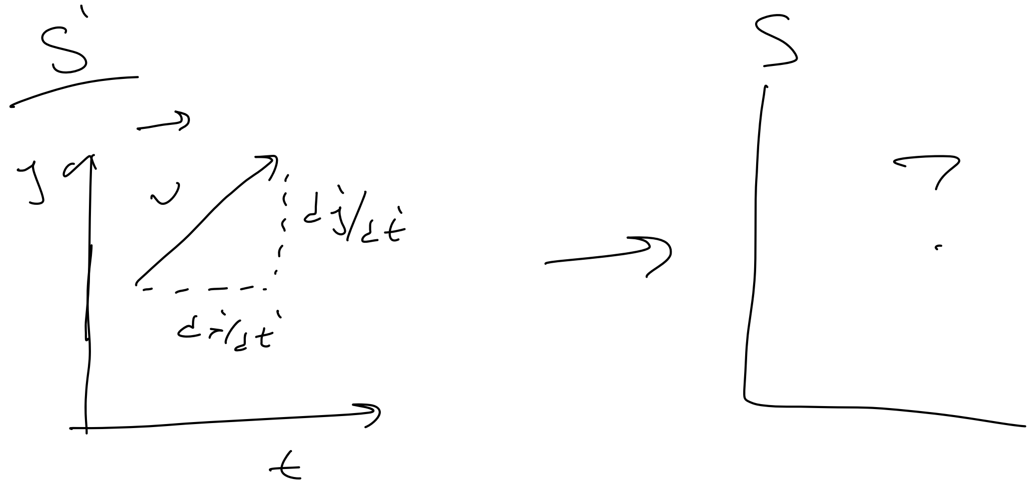
Δx is the difference
At the same time

$$\Delta x' = \gamma \Delta x$$

$$\hookrightarrow \Delta x = \frac{1}{\gamma} \Delta x'$$

(Length contraction we saw before

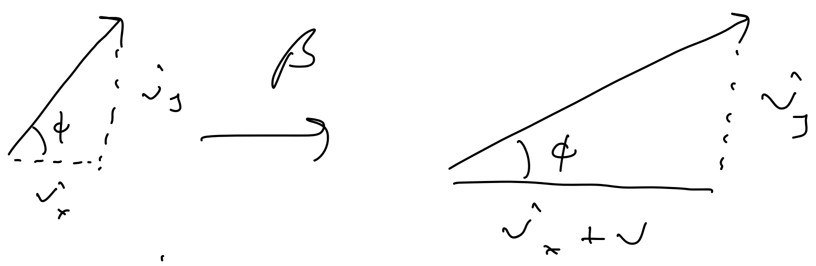
Transformation of Velocities



GT

$dt = dt'$ $\quad dy = dy'$ $\Rightarrow$ $\dfrac{dy}{dt} = \dfrac{dy'}{dt'}$

$$\begin{pmatrix} dx \\ dt \end{pmatrix} = \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} dx' \\ dt' \end{pmatrix} \qquad \frac{dx}{dt} = \frac{dx' + v\,dt}{dt} = \frac{dx'}{dt'} + v$$



$$\tan\phi' = \frac{v'_y}{v'_x} \longrightarrow \tan\phi = \frac{v'_y}{v'_x + v}$$

$$= \tan\phi' \left(\frac{1}{1 + v/v'_x} \right)$$

LT

Start w/ β_y, $\perp$ distances unaffected

But, unlike G.T, time is affected

$dt \to \gamma\, dt'$ (Consider here y-component $dx' = 0$)

$$\beta_y = \frac{dy}{dt} = \frac{1}{\gamma}\frac{dy'}{dt'} = \frac{1}{\gamma}\beta_y'$$

Time dilation hits denominator (numerator unaffected)

Now β_x

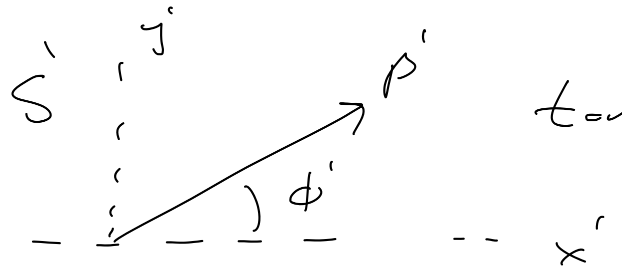
$$\begin{pmatrix} dx \\ dt \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} dx' \\ dt' \end{pmatrix}$$

factor out dt'

$$\frac{dx}{dt} = \frac{\gamma dx' + \beta\gamma dt'}{\beta\gamma dx' + \gamma dt'} = \frac{dx' + \beta dt'}{\beta dx' + dt'} = \frac{\beta_x' + \beta}{1 + \beta_x'\beta}$$

$\boxed{\text{Note: Not "} v + v_s \text{" like GT! Gives "} v + v_s \text{" when } \beta_x'\beta \ll 1}$

Transformation of Velocity Directions



$$\tan\phi' = \frac{dy'}{dx'}$$

$$\beta'_x = \beta\cos\phi' = \frac{dx'}{dt'} \qquad \tan\phi' = \frac{\beta'_y}{\beta'_x}$$

$$\beta'_y = \beta\sin\phi' = \frac{dy'}{dt'}$$

LAB frame

$$dt = \beta\gamma \underbrace{dx'}_{\beta'_x dt'} + \gamma dt' = \gamma(1+\beta\beta'_x)dt'$$

$$\frac{dy}{dt} = \frac{dy'}{\gamma(1+\beta\beta'_x)dt'} = \frac{1}{\gamma(1+\beta\beta'_x)}\beta'_y \leftarrow \boxed{\begin{array}{l}\underline{\underline{\text{Note}}}\ \text{Not } \frac{1}{\gamma}\beta'_y \\ \text{B/c } dx' \neq 0\end{array}}$$

$$\frac{dx}{dt} = \frac{\beta'_x+\beta}{1+\beta\beta'_x}$$

$$\tan\phi = \frac{dy}{dx} = \frac{\frac{dy}{dt}dt}{\frac{dx}{dt}dt} = \frac{\frac{\beta'_y}{\cancel{\gamma}\cancel{(1+\beta\beta'_x)}}\ \cancel{\gamma}\cancel{(1+\beta\beta'_x)}dt'}{\frac{\beta'_x+\beta}{\cancel{1+\beta\beta'_x}}\ \gamma\cancel{(1+\beta\beta'_x)}dt'} = \frac{\beta'_y}{\gamma(\beta'_x+\beta)}$$

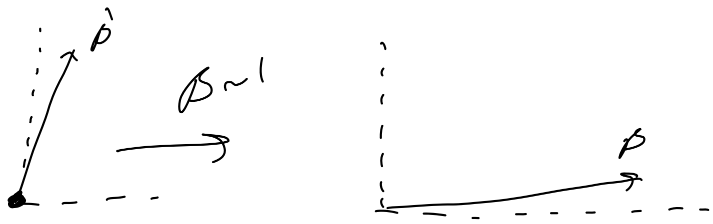
$$= \tan\phi' \frac{1}{\gamma}\left(\frac{1}{1+\frac{\beta}{\beta'_x}}\right) = \tan\phi' \frac{1}{\gamma}\left(\frac{\beta'_x}{\beta'_x+\beta}\right)$$

Note the difference between transformation of angles between lengths + velocities

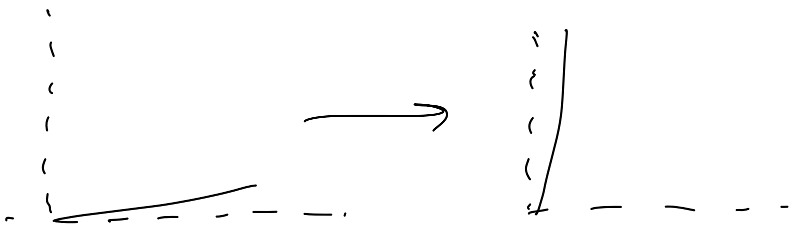
velocity

$\beta \to 1 \quad \gamma \to \infty \qquad dx \sim 1\,dt$

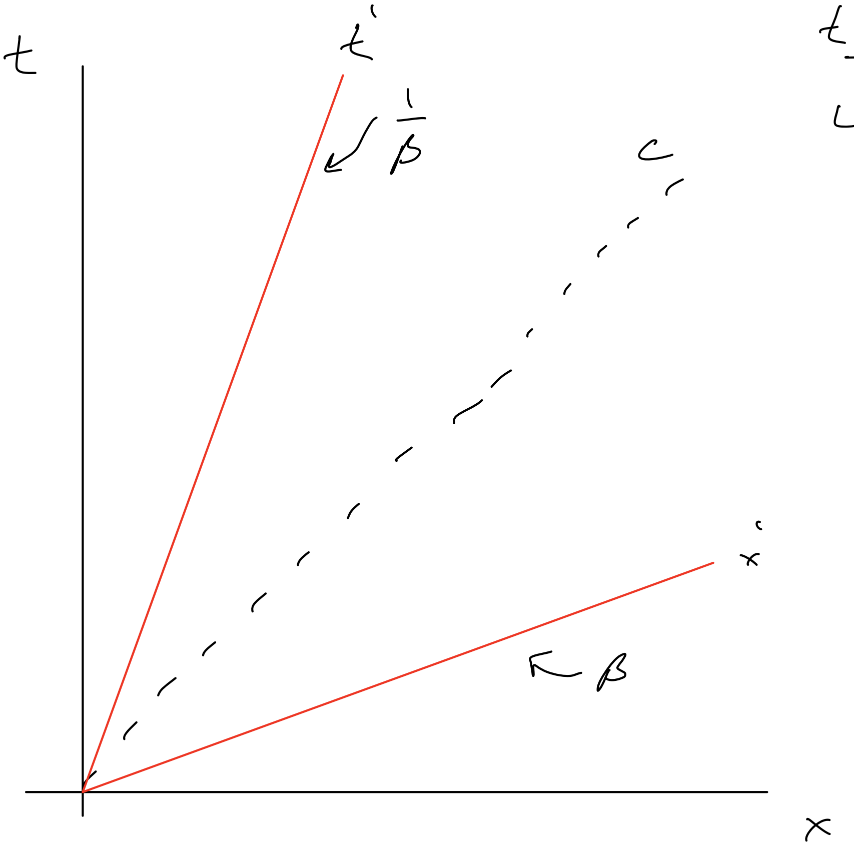
$dy \sim 1$



lengths



Back to Space-time diagrams



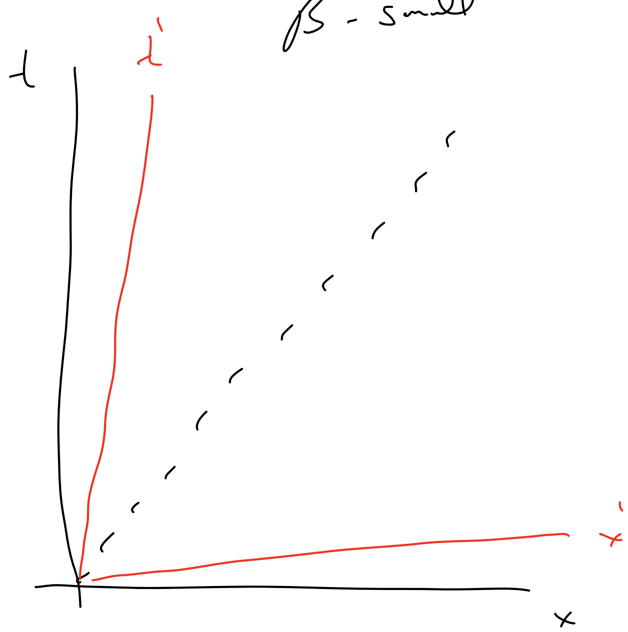
t'-axis when x' = 0

LT ⇒ $x = \beta\gamma t'$
$t = \gamma t'$ ⇒ $t = \frac{1}{\beta} x$

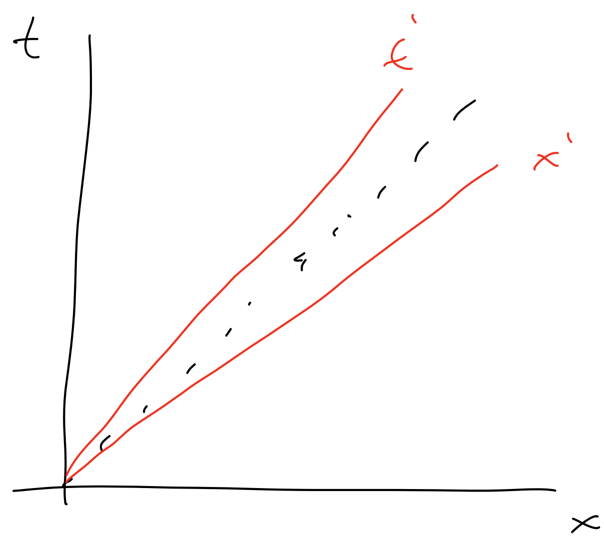
x'-axis when t' = 0

$x = \gamma x'$ ⇒ $t = \beta x$
$t = \beta\gamma x'$

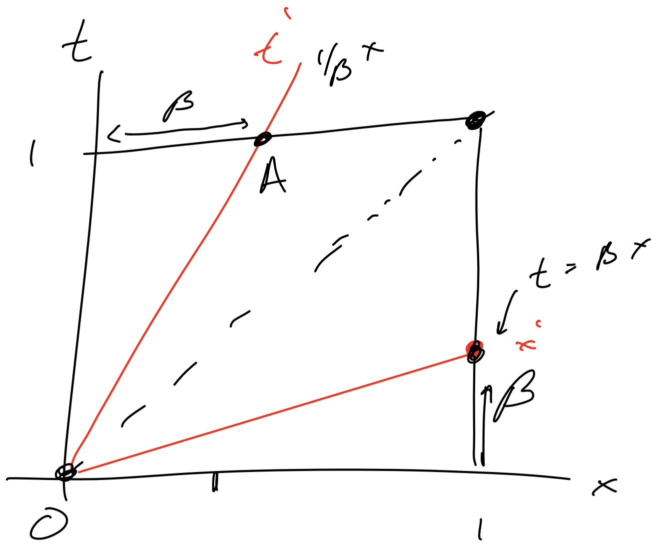
β - small



β → c

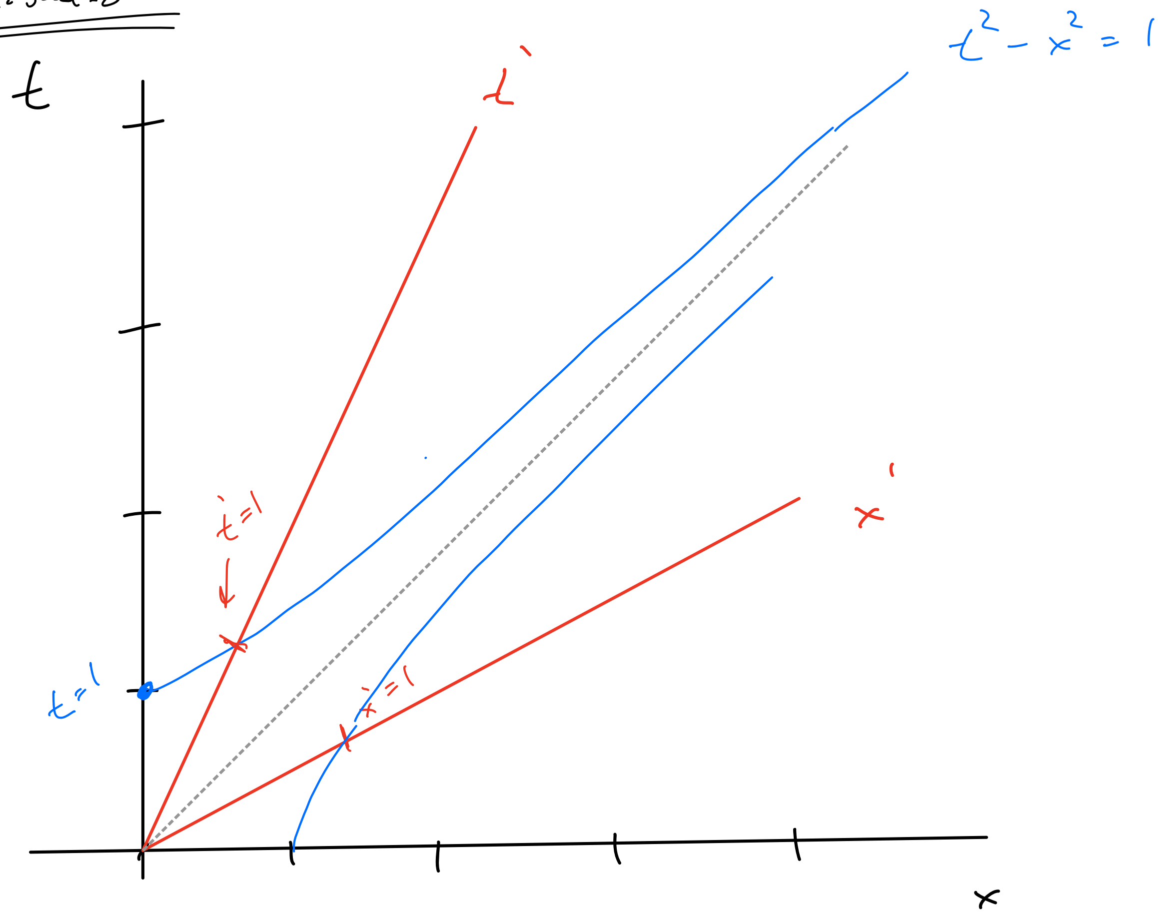


Another way of saying the $x'=0$ axis



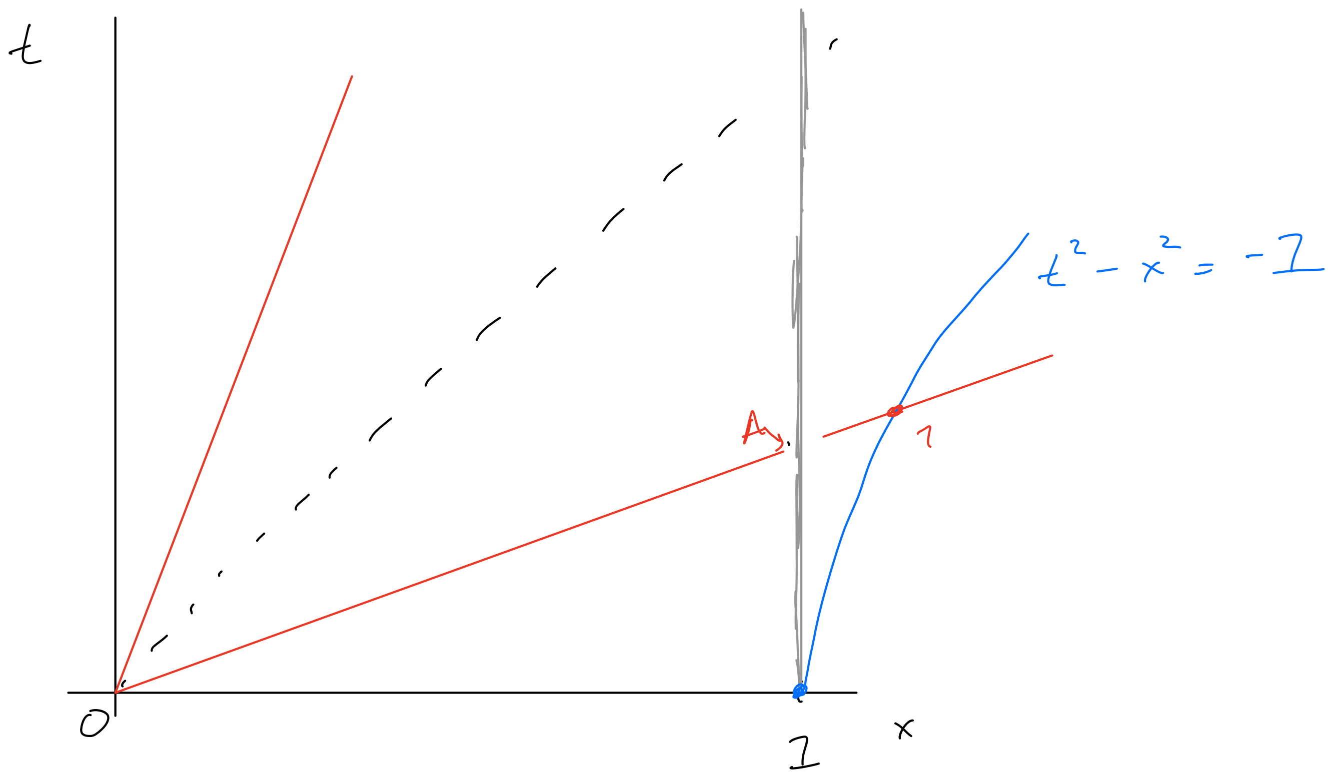
If $c = 1$ in both frames, then
$|OA| = |OB|$
$\Rightarrow$ the triangles must be similar.

Calibration



Geometric view of Length Contraction

meter stick at rest in LAB frame



OA is meter stick at $\Delta t' = 0$ in rocket frame

$\underline{|OA| < 1}$

Meter Stick in Rocket Frame