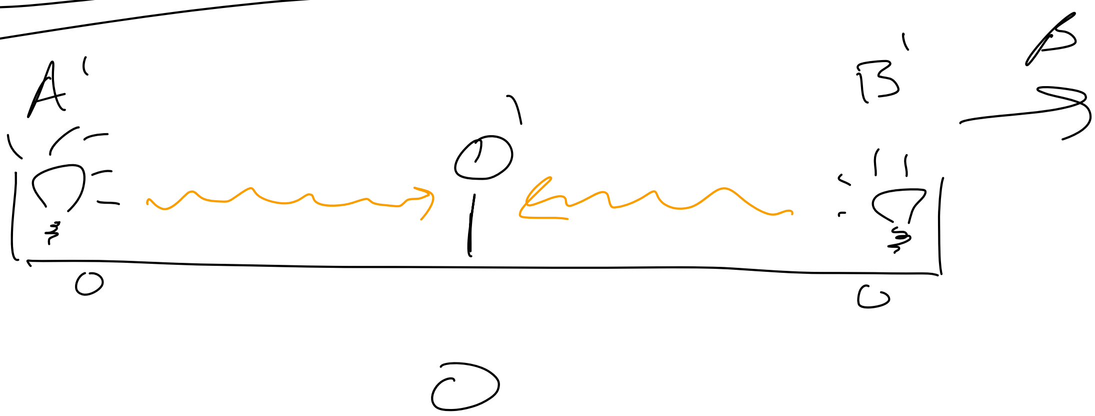
Go through a couple problems in detail
which appear at 1st sight like a paradox.

- Relativity of Simultaneity "Train Paradox"
- Related to time dilation "Train Paradox"
- " " Length contraction "Barn Paradox"

Train Paradox



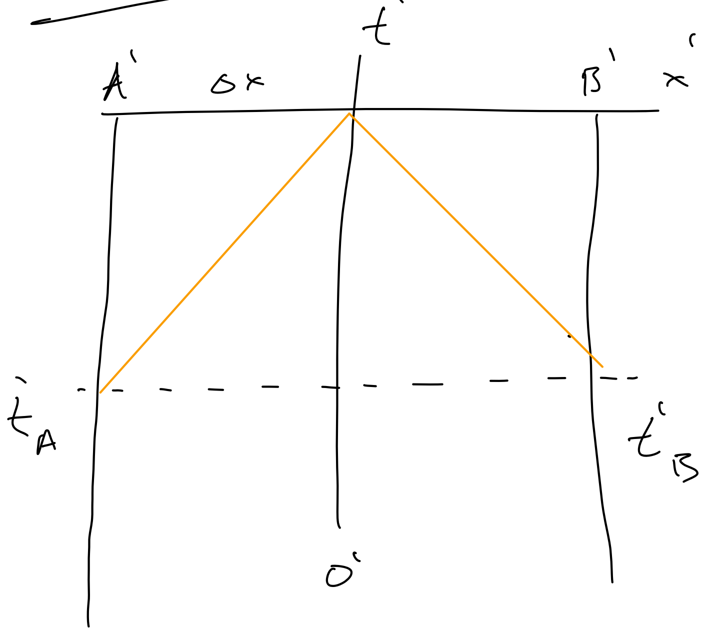
Both light flashes arrive at $O' = O$ at $t = 0$

Who emitted the flash first?

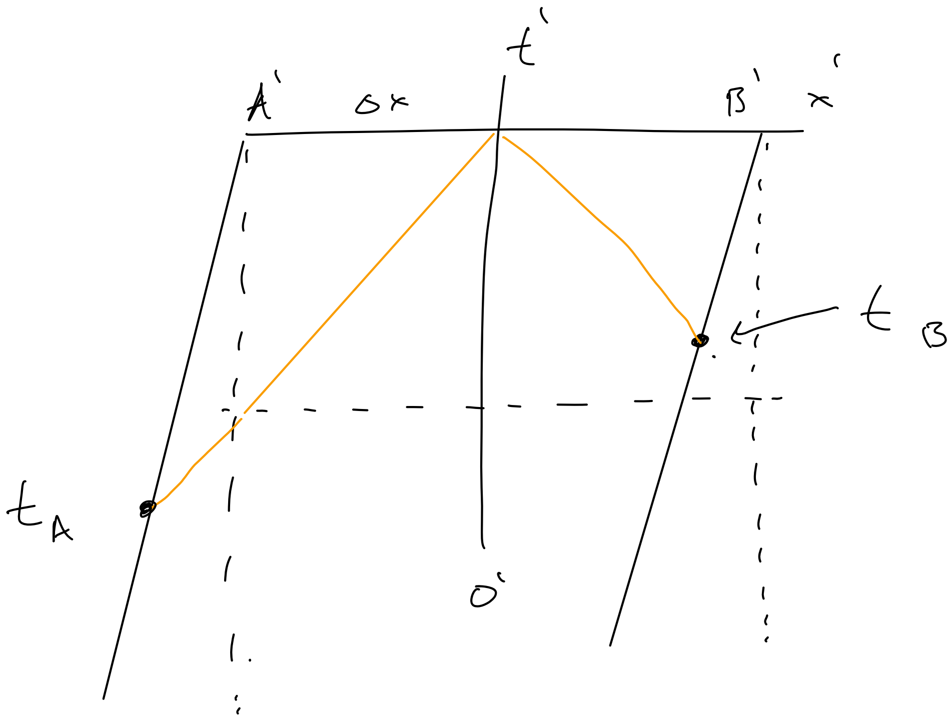
Turns out O & O' give different answers!

Notion of Simultaneity "Two events occur at the same time is relative"

S' frame



S frame



Different observers disagree on the Simultaneity of Events.

No invariant matter of fact about if two things happen at the same time.

"Twin Paradox"

- Start w/ twins (synchronized clocks) Winklevi twins

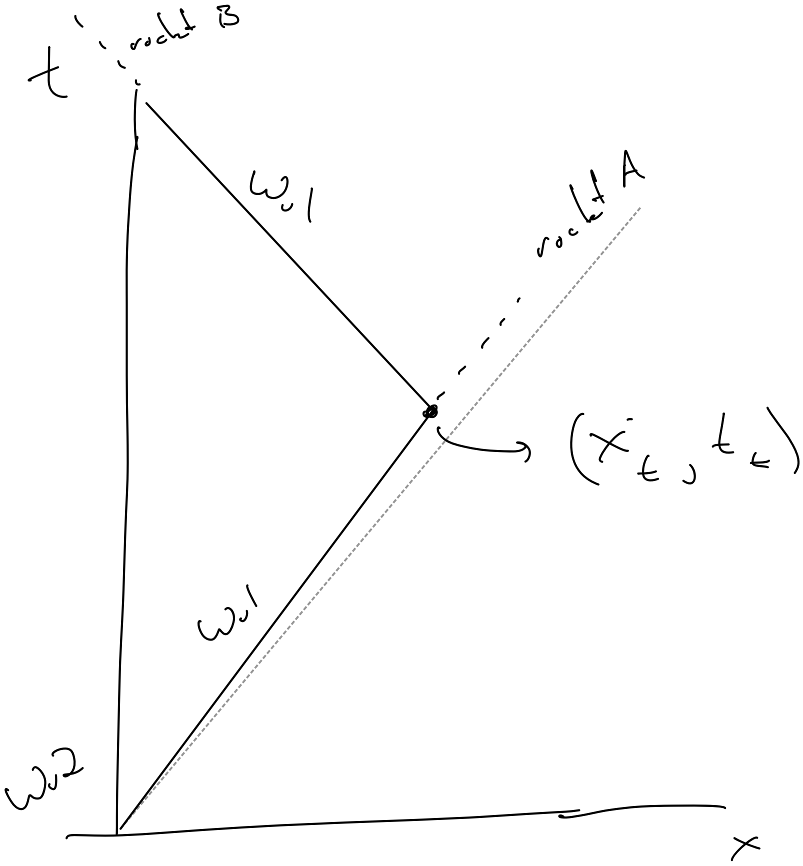
One twin goes on high speed journey (Wu #1)

Travels at β w/ γ for time τ years (his time)

then returns at the same speed τ years

-) Winklevi #1 clearly ages 2τ years

-) Winklevi #2 experiences ...



$$\begin{pmatrix} x_t \\ t_t \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$t_t = \gamma \tau$$

$$\Delta t = 2\tau \times \underbrace{\gamma}_{>1}$$

$\boxed{\text{eg:}}$ $\tau = 5$ years

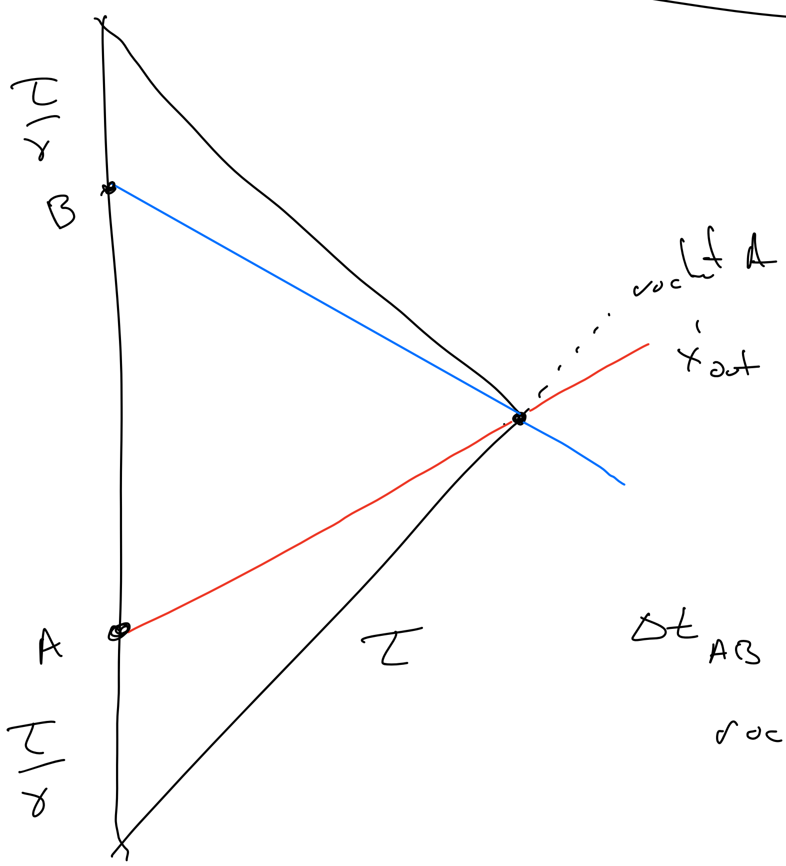
$\gamma = 10$

Wu 1 ages 10 y

Wu 2 ages 100 y

Future oriented time-travel!

Can change return time by tuning τ & γ



Δt_{AB} from the difference in clocks

rocket A $\Rightarrow$ rocket B

Rocket A at $t' = 0$

$$\begin{pmatrix} x' \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \Rightarrow \quad x' = \gamma x - \beta\gamma t$$

$$0 = -\beta\gamma x + \gamma t \Rightarrow x = \frac{t}{\beta}$$

$$x' = \frac{\gamma t}{\beta} - \beta\gamma t = \gamma\left(\frac{1}{\beta} - \beta\right)t$$

$$= \gamma\left(\frac{1 - \beta^2}{\beta}\right)t$$

$$= \gamma \frac{1}{\beta\gamma^2} t = \frac{1}{\beta\gamma} \Rightarrow t = \beta\gamma x'$$

$$t_{now} = \beta\gamma - L$$

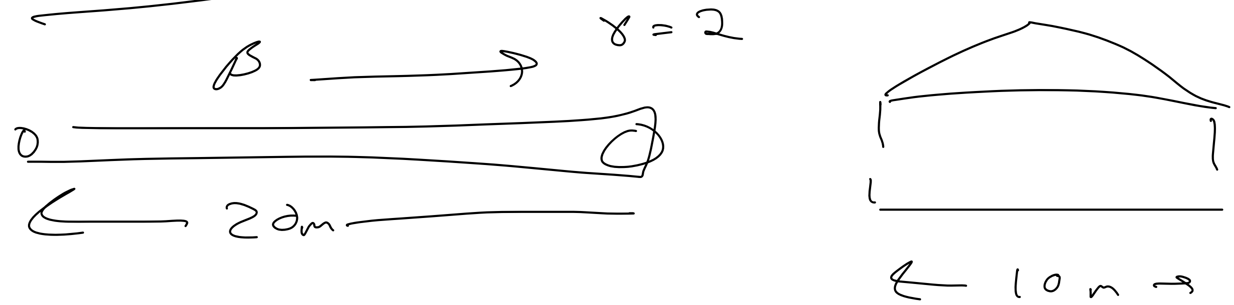
whereas in rocket B

$$t_{now} = -\beta\gamma - L \qquad \Delta t_{now} = 2\beta\gamma L$$

$$L = \tau\beta$$

$$\Delta t_{tt} = 2\left(\frac{\tau}{\gamma}\right) + \Delta t_{turn} = 2\frac{\tau}{\gamma} + 2\beta^2\gamma L = 2\tau\gamma \checkmark$$

"Barn + Pole" Paradox



Can the pole fit in the Barn?

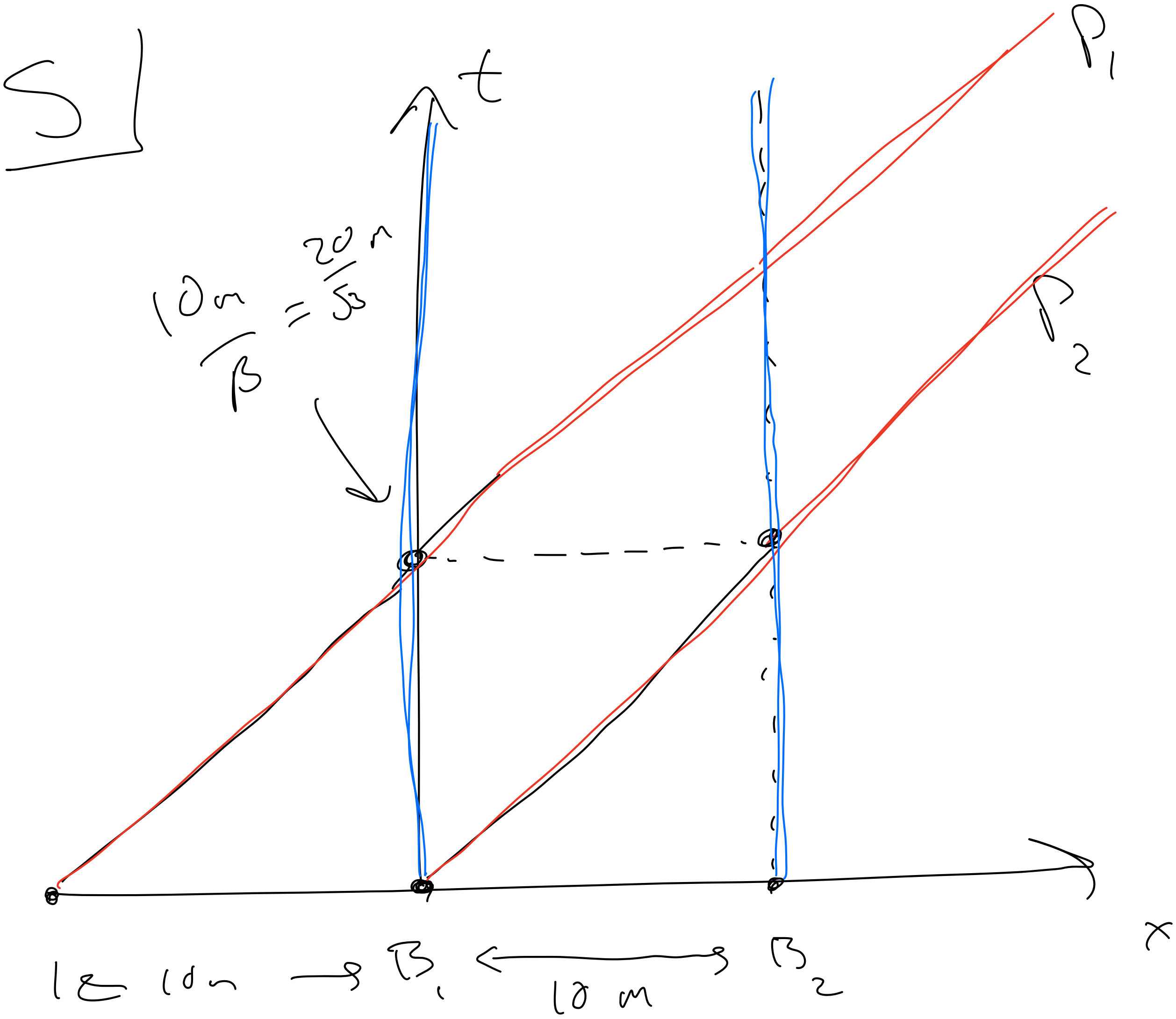
S' observer No!

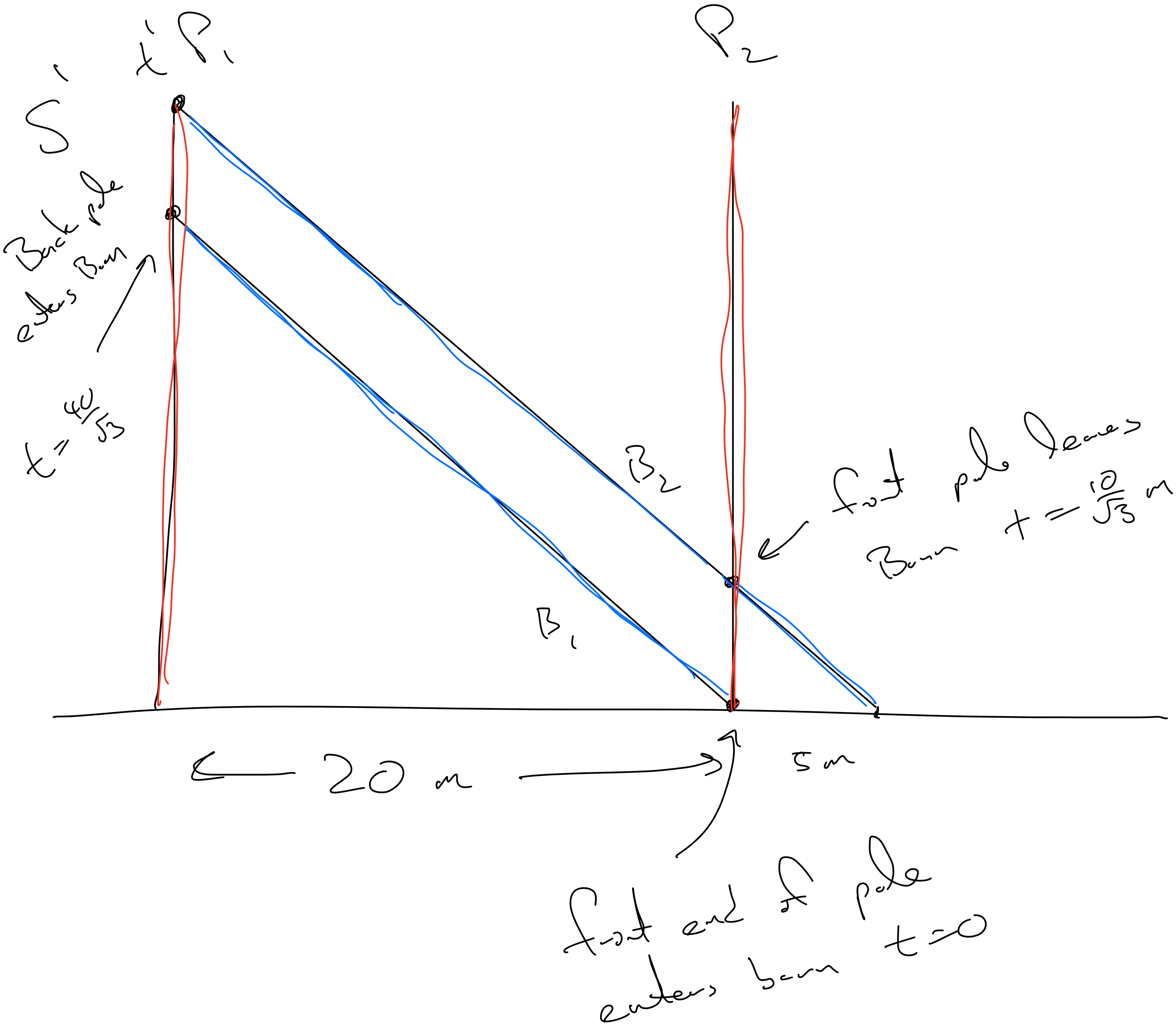
$\Delta x_{pole} = 20m \qquad \Delta x_{Barn} = \frac{10}{\gamma} = 5m$!

S observe Yes!

$\Delta x_{pole} = \frac{20m}{\gamma} = 10m \qquad \Delta x_{Barn} = 10m$

$\gamma = 2 \Rightarrow \beta = \frac{\sqrt{3}}{2}$


S
t
x
P1
P2
10m / β = 20m / √3
← 10m → B1
B1 ← 10 m → B2
B2



S'
t' P₁
P₂
Back pole enters Barn
$t = \frac{40}{\sqrt{3}}$
B_2
B_1
front pole leaves Barn $t = \frac{10}{\sqrt{3}}$ m
← 20 m →
5 m
front end of pole enters barn t = 0

No objective matter of fact about if one moving object can fit in another!

Ultimately relies on the notion of simultaneity.