

Homework Set #6

Due Date: Before class Friday March 24th

1) Reading

(2 points)

Read chapter 4.

2) Compton Scattering

(15 points)

Derive Compton's formula for relative shift of the wavelength of a photon scattered off an electron.

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_e}{\lambda}(1 - \cos\theta)$$

where, λ is the wavelength of the incoming photon, $\Delta\lambda$ is the difference between the incoming and outgoing photon wavelengths, θ is the angle between the direction incoming and outgoing photons and $\lambda_e = h/m_e c$ is the Compton wavelength of the electron. What is the energy of a photon with wavelength equal to the Compton wavelength of the electron ?

Repeat for the case of a photon scattered of a proton. What is the numerical value for the "Compton wavelength" of the proton?

What is the energy of a photon with wavelength equal to the Compton wavelength of the proton?

3) Rutherford.

(15 points)

- (a) What was unexpected about Rutherford's alpha-ray scattering experiment? How did it qualitatively change our picture of atoms?
- (b) (Rutherford experiment in 1D) Suppose two particles with masses m_α and m_N and initial velocities v_α and $v_N = 0$ along some line collide head on, emerging with velocities v'_α and v'_N along the same line. Use conservation of momentum and energy to find v'_α as a function of m_α, m_N , and v_α . If the alpha particle "bounces back" with significant velocity, eg: $v'_\alpha \sim -|v_\alpha|$, what does that say about the relative size of the masses m_α, m_N ?
- (c) What problems arise when applying classical physics to the "solar system" picture of atoms ? eg: what does the theory predict that is not observed?
- (d) Estimate an upper limit on the size of a gold nucleus from the distance of closest approach of a 1D head-on collision in which the incoming 8 MeV α particle returns with $\theta = 180^\circ$.

4) Bohr.*(15 points)*

- (a) How are the sharp lines in atomic spectra explained in the Bohr model of the atom?
- (b) Show that the Bohr radius and the lowest energy for the hydrogen atom can be written as

$$a_0 = \frac{hc}{\alpha mc^2} = \frac{\lambda_e}{2\pi\alpha} \quad E_1 = \frac{1}{2}\alpha^2 mc^2$$

where $\lambda_e = \frac{h}{mc}$ is the Compton wavelength of the electron and $\alpha = \frac{ke^2}{\hbar c}$ is the fine-structure constant. Use these expressions to check the numerical values of the constants a_0 and E_1 .

- (c) If the angular momentum of Earth in its motion around the Sun were quantized like a hydrogen electron according to Bohr's quantization hypothesis, what would Earth's quantum number be? How much energy would be released in a transition to the next lowest level? Would that energy release (presumably as a gravity wave) be detectable? What would be the change in the radius of the earth's orbit ? (The radius of Earth's orbit is 1.50×10^{11} m.)
- (d) On average, a hydrogen atom will exist in an excited state for about 10^{-8} sec before making a transition to a lower energy state. About how many revolutions does an electron in the $n=2$ state make in 10^{-8} sec?