2) S' β $\quad \beta' = \begin{cases} \beta_x \\ \beta_y \\ \beta_z \end{cases} \Rightarrow \gamma_{\beta'} \quad E' \ \& \ p'$

1st work out how γ transforms

$$\frac{1}{\gamma_\beta^2} = 1 - \left(\left(\frac{\beta'_x + \beta_s}{(1+\beta'_x\beta_s)} \right)^2 + \frac{\beta'^2_y}{\gamma_{\beta_s}^2(1+\beta'_x\beta_s)^2} + \frac{\beta'^2_z}{\gamma_{\beta_s}^2(1+\beta'_x\beta_s)^2} \right)$$

$$= 1 - \frac{1}{(1+\beta'_x\beta_s)^2}\left[(\beta'_x+\beta_s)^2 + \frac{\beta'^2_y + \beta'^2_z}{\gamma_{\beta_s}^2} \right]$$

$$\boxed{\beta'^2 - \beta'^2_x = \beta'^2_y + \beta'^2_z}$$

$$= 1 - \frac{1}{(1+\beta'_x\beta_s)^2}\left[\beta'^2_x + 2\beta'_x\beta_s + \beta_s^2 + (1-\beta_s^2)(\beta'^2 - \beta'^2_x) \right]$$

$$= 1 - \frac{1}{(1+\beta'_x\beta_s)^2}\left[\cancel{\beta'^2_x} + \underline{2\beta'_x\beta_s} + \beta_s^2 + \beta'^2 - \cancel{\beta'^2_x} - \beta_s^2\beta'^2 + \underline{\beta_s^2\beta'^2_x} \right]$$

$$2\beta'_x\beta_s + \beta_s^2\beta'^2_x = (1+\beta'_x\beta_s)^2 - 1$$

$$= 1 - \frac{1}{(1+\beta'_x\beta_s)^2}\left[(1+\beta'_x\beta_s)^2 - 1 + \beta_s^2 + \beta'^2 - \beta_s^2\beta'^2 \right]$$

$$= \frac{1}{(1+\beta'_x\beta_s)^2}\left[\cancel{(1+\beta'_x\beta_s)^2} - \cancel{(1+\beta'_x\beta_s)^2} + 1 - \beta_s^2 - \beta'^2 + \beta_s^2\beta'^2 \right]$$

$$= \frac{1}{(1+\beta'_x\beta_s)^2}\left[(1-\beta_s^2)(1-\beta'^2)\right]$$

$$\gamma_\beta^2 = \frac{(1+\beta'_x\beta_s)^2}{(1-\beta_s^2)(1-\beta'^2)}$$

$$\text{or} \quad \gamma_\beta = \gamma_{\beta_s}\gamma_{\beta'}(1+\beta'_x\beta_s)$$

Now Come to E & P

$$E = m\gamma_\beta = \gamma_{\beta'}\gamma_{\beta_s}(1+\beta_s\beta'_x)m$$

$$= \gamma_{\beta_s}(\underbrace{\gamma_{\beta'}m}_{E'} + \beta_s\underbrace{\gamma_{\beta'}\beta'_x m}_{P'_x})$$

$$= \gamma_{\beta_s}(E' + \beta_s P'_x)$$

$$P_x = m\gamma_\beta\beta_x = m\left(\gamma_{\beta'}\gamma_{\beta_s}\cancel{(1+\beta_s\beta'_x)}\right)\left(\frac{\beta'_x+\beta_s}{\cancel{1+\beta_s\beta'_x}}\right)$$

$$= \gamma_{\beta_s}(m\gamma_{\beta'}(\beta'_x+\beta_s))$$

$$= \gamma_{\beta_s}\left(\beta_s \underbrace{m\gamma_{\beta'}}_{E'} + \underbrace{m\gamma_{\beta'}\beta_x'}_{P_x}\right)$$

$$P_y = m\gamma_\beta \beta_y = m\left(\gamma_{\beta'}\cancel{\gamma_{\beta_s}}(1\cancel{+\beta_s\beta_x'})\right)\left(\frac{\beta_y'}{\cancel{\gamma_{\beta_s}}(1\cancel{+\beta_s\beta_x'})}\right)$$

$$= m\gamma_{\beta'}\beta_y'$$

$$= P_y' \quad (\text{Same for } P_z)$$

So

$$\begin{pmatrix} E \\ P_x \end{pmatrix} = \begin{pmatrix} \gamma_{\beta_s} & \beta_s\gamma_{\beta_s} \\ \beta_s\gamma_{\beta_s} & \gamma_{\beta_s} \end{pmatrix}\begin{pmatrix} E' \\ P_x' \end{pmatrix}$$

$$P_{y,z} = P'_{y,z}$$

3.1 $100\,W \sim 100\,J/s$

$1y \sim \pi \cdot 10^7\,s$

$E = 100\,J \times \pi\, 10^7$

$\sim \pi\, 10^9\,J$

$E = mc^2 \Rightarrow m = \frac{E}{c^2} = \frac{\pi\, 10^9\,J}{9\, 10^{16}\,(m/s)^2} \sim \frac{1}{3}\, 10^{-7}\,kg$

$\sim 3\, 10^{-8}\,kg$

3.2)

$3\, 10^{12}\,kWh = 3\, 10^{15}\,\frac{J}{s} \times \frac{3600\,s}{} = \underbrace{\underbrace{3 \times 3.6}_{10.8}\, 10^{18}\,J}_{10^{19}\,J}$

$m = \frac{E}{c^2} = \frac{10^{19}}{9\, 10^{16}}\,kg \sim \frac{1}{10}\, 10^3\,kg$

$\sim 10^2\,kg$

3.3)

$\text{Power} = \frac{1}{2}\,hp \times 4 = 2hp \sim 1.5\, 10^3\,W = 1.5\, 10^3\,\frac{J}{s}$

$\varepsilon \sim 25\%$

Losing mass at a rate of $\frac{\Delta m}{s} \sim \frac{1.5\, 10^3}{9\, 10^{16} (m/s)^2}\,\frac{J}{s} = \underbrace{\frac{1}{10}\, 10^{-13}\,\frac{kg}{s}}_{10^{-14}\,\frac{kg}{s}}$

$1\,lb \sim \frac{1}{2}\,kg$

$t_{1lb} \sim \frac{1}{2}\,kg \times \frac{1}{10^{-14}\,kg/s} \sim 0.5\, 10^{14}\,s$

or $\frac{1}{6}\, 10^7\,y$

Note: Chemical burning at the gym produces waste products which when eliminated (I guess mostly through Breath) carries away much more mass.

3.4) $1.4\ kW/m^2$ $\qquad r_e = 150 \cdot 10^6\ km$

$$= 1.5\ 10^8\ km$$

$$= 1.5\ 10^{11}\ m$$

fraction of Suns energy in $1\ m^2$ at r_e is

$$f = \frac{1\ m^2}{4\pi r_e^2} = \frac{1}{12}\ \frac{1}{2}\ 10^{-22} = \frac{1}{24}\ 10^{-22}$$

$$= \frac{1}{2}\ 10^{-23}$$

$$P_{sun} = \frac{1.4\ 10^3\ W}{f} = 3\ 10^{26}\ W\ \left(\frac{J}{s}\right)$$

$$\frac{Mass}{s} = \frac{P_{sun}}{c^2} = \frac{3\ 10^{26}}{9\ 10^{16}}\ \frac{kg}{s} = \underbrace{\frac{1}{3}\ 10^{10}\ \frac{kg}{s}}_{3\ 10^{9}\ kg/s}$$

In a year

$$m = \frac{m}{s} \times \pi\, 10^7\, s = \underbrace{9\ 10^{16}\ kg}_{\sim 10^{17}\ kg}$$

Of this the fraction $\dfrac{\pi\, r_{earth}^2}{4\pi\, r_{earth\ orbit}^2}$ reaches earth

$r_{earth} \sim 6\ 10^6\ m$

$$f_{to\ earth} \sim \frac{1}{4}\left(\frac{6\ 10^6\ m}{1.5\ 10^{11}\ m}\right)^2 \sim \frac{1}{4}\left(4\ 10^{-5}\right)^2$$

$$\sim 4\ 10^{-10}$$

$$\frac{m_{to\ earth}}{y} \sim f_{to\ earth} \times \frac{m}{y}$$

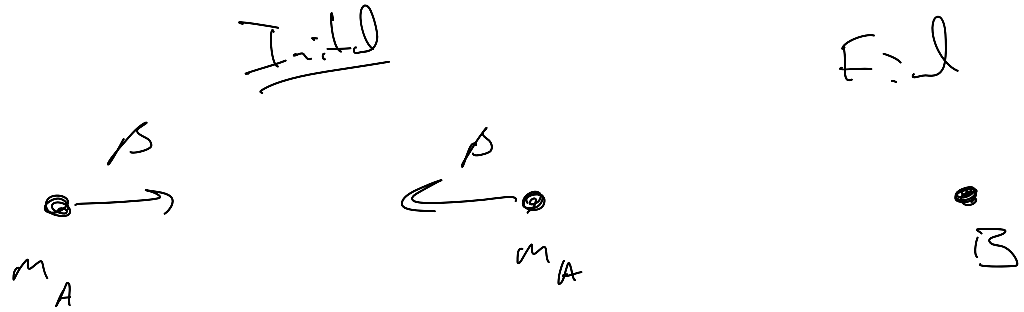
$$\sim 4\ 10^{-10} \times 10^{17}\ kg \sim 3\ 10^7\ kg$$

3.5) $v = \dfrac{100\ mi}{h} \times \dfrac{1.6\ 10^3\ m}{1\ mi} \times \dfrac{1\ h}{3600\ s} \sim \dfrac{1.6}{3.6}\ \dfrac{10^5}{10^3} \sim \dfrac{1}{2}\ 10^2\ \dfrac{m}{s}$

$\beta = \dfrac{\frac{1}{2}\ 10^2}{3\ 10^8} \sim \dfrac{1}{6}\ 10^{-6} \sim 10^{-7} \Rightarrow$ Newton OK

$KE = 2 \times \frac{1}{2} m \beta^2 = 10^8\ kg\ (10^{-7})^2 \sim 10^{-6}\ kg$

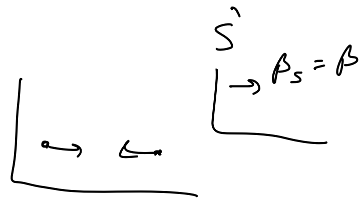
4) Inelastic Collisions



$$E_i = 2\, m_A \gamma_\beta \qquad E_F = M_B$$

$$M_B = 2 m_A \gamma_\beta$$

In S'



Initial

(1) $\beta_1 = \dfrac{\beta_1' + \beta}{1 + \beta_1' \beta} \qquad \beta_1' = 0$

$= \beta$ ✓

(2) $\beta_2 = \dfrac{\beta_2' + \beta}{1 + \beta_2' \beta} = -\beta \Rightarrow -\beta - \beta_2' \beta^2 = \beta_2' + \beta$

$$-2\beta = (1 + \beta^2)\beta_2'$$

$$\Rightarrow \beta_2' = \frac{-2\beta}{1 + \beta^2}$$

$$E_i = E_1 + E_2$$

$$= m_A + m_A \underbrace{\gamma_{\beta_2'}}$$

$$1 - \beta_2'^2 = \frac{(1+\beta^2)^2}{(1+\beta^2)^2} - \frac{4\beta^2}{(1+\beta^2)^2}$$

$$= \frac{1}{(1+\beta^2)^2}\left[1 + 2\beta^2 + \beta^4 - 4\beta^2\right]$$

$$= \frac{1 - 2\beta^2 + \beta^4}{(1+\beta^2)^2} = \frac{(1-\beta^2)^2}{(1+\beta^2)^2}$$

So: $\gamma_{\beta_2'} = \dfrac{1+\beta^2}{1-\beta^2}$

$$E_i = m_A\left(1 + \frac{1+\beta^2}{1-\beta^2}\right) = m_A\left(\frac{1-\beta^2 + 1 + \beta^2}{1-\beta^2}\right) = 2 m_A \gamma_\beta^2$$

from Above!

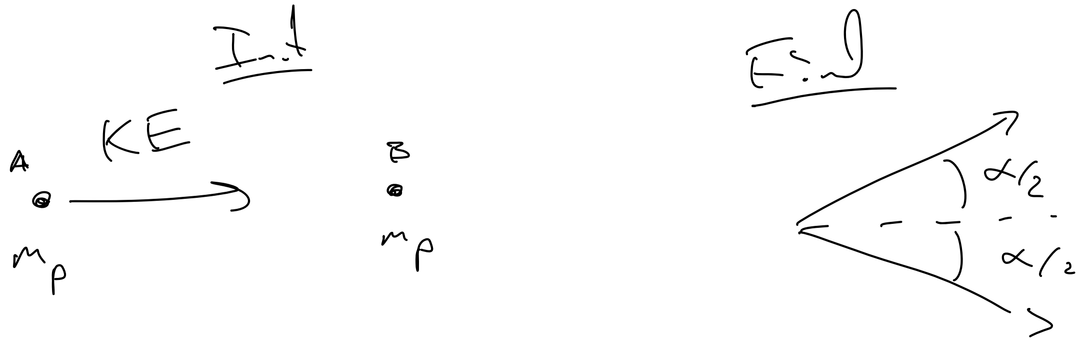
$$E_F = \gamma_\beta M_B = \gamma_\beta (2 m_A \gamma_\beta) = 2 m_A \gamma_\beta^2 = E_i$$

Now directly via L.T.

$$E_F^S = M_B = 2 m_A \gamma_\beta \qquad P_{xF}^S = 0$$

$$E_F^{S'} = \gamma E_F^S - \cancel{\beta\gamma P_{xF}^S} = \gamma E_F^S = 2 m_A \gamma_\beta^2 \checkmark$$

5) Symmetric Collisions



$$E_i = 2m_p + KE \qquad E_f = 2\sqrt{m^2 + p_f^2}$$

$$E_i^2 = 4m_p^2 + 4m_p KE + KE^2 \qquad E_f^2 = 4(m_p^2 + p_f^2)$$

$$E_f^2 = E_i^2 \Rightarrow p_f^2 = \frac{1}{4}\left(4m_p^2 + 4m\,KE + KE^2\right) - m_p^2$$

$$= \cancel{m_p^2} + m\,KE + \frac{1}{4}KE^2 - \cancel{m_p}$$

$$|p_f| = \sqrt{KE\left(m + \frac{1}{4}KE\right)}$$

$$p_x^f = 2|p_f| \cos\frac{\alpha}{2}$$

$$p_i^x = m_A \gamma_A \beta_A$$

$$\frac{1}{\gamma^2} = 1 - \beta^2$$

$$\gamma_A = \frac{E_A}{m}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\gamma\beta = \sqrt{\gamma^2 - 1} \Rightarrow p_i^x = m\sqrt{\frac{E^2}{m} - 1} = \sqrt{E^2 - m^2}$$

$$E_A = m + KE$$

$$E^2 - m^2 = m^2 + 2m\,KE + KE^2 - m^2 = KE(2m + KE)$$

$$P_i^{x^2} = P_f^{x^2} =$$

$$KE\cancel{2}\,(2m + KE) = 4\left(KE\left(m + \tfrac{1}{4}KE\right)\right)\cos^2\frac{\alpha}{2}$$

$$= 4\,K\cancel{E}\left(m + \tfrac{1}{4}KE\right)\cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{2m + KE}{4m + KE}$$

$$\boxed{\cos^2\frac{\alpha}{2} = \frac{\cos\alpha + 1}{2}}$$

$$\frac{\cos\alpha + 1}{2} = \frac{2m + KE}{4m + KE} \quad \text{or} \quad \cos\alpha = 2\left(\frac{2m + KE}{4m + KE}\right) - 1$$

$$\cos\alpha = \frac{4\cancel{m} + 2KE - (\cancel{4m} + KE)}{4m + KE}$$

$$= \frac{KE}{KE + 4m}$$

Newton $\Rightarrow$ $\frac{KE}{m} \ll 1$ $\qquad \cos\alpha = \frac{KE}{m\left(4 + \frac{KE}{m}\right)} \sim \frac{1}{4}\frac{KE}{m}$

$$\sim 0$$

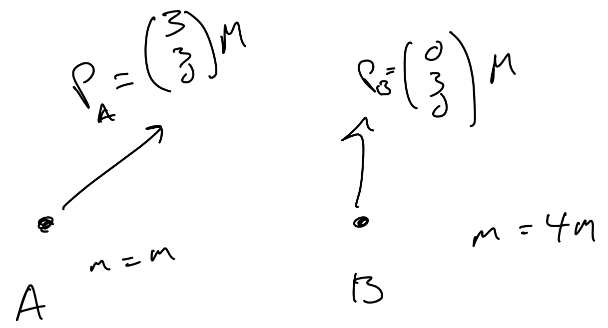
$$\Rightarrow \alpha = \frac{\pi}{2}$$

Relativistic $\frac{m}{KE} \ll 1$

$$\cos\alpha = \frac{1}{1 + 4\frac{m}{KE}} \sim 1 \Rightarrow \alpha = 0$$

6) Asymmetric Collisions

①



$E_A^2 = M^2 + P_A^2$

$= (1 + 18) M^2$

$E_A = \sqrt{19}\, M$

$E_B^2 = 16M^2 + 9M^2$

$= 25 M^2$

$|P|_B = 3M$

$E_B = 5M$

$E_A = M + KE$

$KE_A = (\sqrt{19} - 1) M$

$KE_B = M$

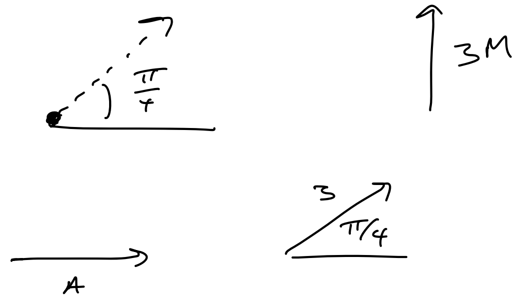
$\beta_A = \dfrac{P_A}{E_A} = \sqrt{\dfrac{18}{19}}$

$\beta_B = \dfrac{3}{5}$

② In the A rest frame

$E_A' = M_A$ rotate by $\frac{\pi}{4}$

$\gamma_A = \dfrac{E_A}{m_A} = \sqrt{19}$



$$E_B' = \gamma_A E_B - \beta_A \gamma_A P_{xB}$$

$$= \sqrt{19}\, E_B - \sqrt{\tfrac{18}{19}}\sqrt{19}\, P_{xB} = \sqrt{19}\,(5M) - \sqrt{18}\, 3 \cos\tfrac{\pi}{4}\, M$$

$$= \left(5\sqrt{19} - 3\sqrt{\tfrac{18}{2}}\right) M = \left(5\sqrt{19} - 9\right) M$$

(3) In the B rest frame $E_B = M_B = 4M$

Rotate

$$\gamma_B = \frac{E_B}{m_B} = \frac{5}{4}$$

$$\beta_B = \frac{3}{5}$$

$$E_A' = \gamma_B E_A - \beta_B \gamma_B P_x^A$$

$$= \frac{5}{4}\left(\sqrt{19}\, M\right) - \frac{3}{4}\sqrt{18}\cos\tfrac{\pi}{4})$$

$$= \left(\frac{5}{4}\sqrt{19} - \frac{9}{4}\right) M$$

4) $$\vec{P}_c = \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} M \qquad |P_c| = \sqrt{9 + 36}\; M = \sqrt{45}\, M$$

5) $$M_s^2 = E_s^2 - P_s^2$$

$$= (E_1 + E_2)^2 - 45 M^2$$

$$= \left(\sqrt{19}\, M + 5M\right)^2 - 45 M^2$$

$$= \left[\left(19 + 10\sqrt{19} + 25 \right) - 45 \right] M^2$$

$$= \left(10\sqrt{19} - 1 \right) M^2 \approx 42.5\ M^2$$

vs

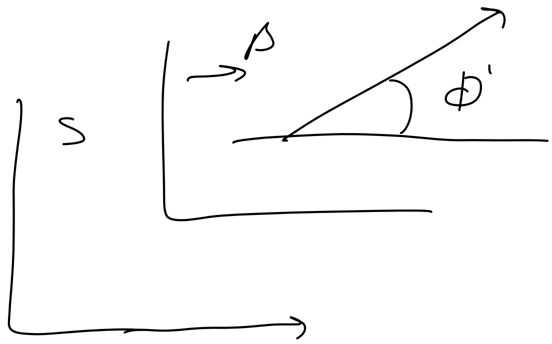
$$(m_A + m_B)^2 = 25\ M^2$$

More mass in combied System B/c some of the initial KE in Mass in the combined system.

6)

$$\beta = \frac{|P_s|}{E_s} = \frac{\sqrt{45}\ M}{(5 + \sqrt{19})\ M} = \left(\frac{\sqrt{45}}{5 + \sqrt{19}} \right)$$

7 In S'



E_γ $\quad p'_x = p'\cos\phi$

$p'_y = p'\sin\phi$

$p' = E'$

In S

$$p_y = p'_y$$

$$p_x = \gamma p'_x + \beta\gamma E'$$

$$= \gamma(E'\cos\phi + \beta E') = E'\gamma(\cos\phi + \beta)$$

$$E = \gamma E' + \beta\gamma p'_x = \gamma E'(1 + \beta\cos\phi')$$

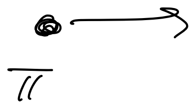
$$p_x = E\cos\phi \Rightarrow \cos\phi = \frac{p_x}{E} = \frac{\gamma E'(\cos\phi' + \beta)}{\gamma E'(1 + \beta\cos\phi')}$$

$$\boxed{\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'}}$$

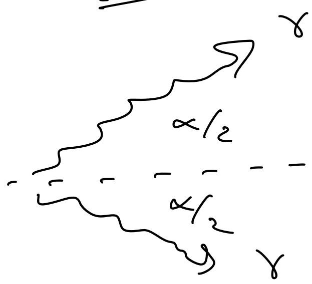
8

Initial

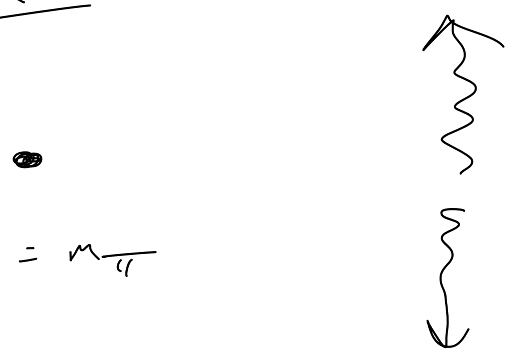
$E_\pi = 2 m_\pi$



Final



Rest frame



$E' = m_\pi/2$

$E' = m_\pi$

$\cos\phi' = 0$

Need β for the π rest frame

$$E_\pi = m_\pi + KE = 2 m_\pi$$

$$\Rightarrow \gamma = \frac{E_\pi}{m_\pi} = 2 \qquad p = \sqrt{E^2 - m^2} = \sqrt{4-1}\; m_\pi = \sqrt{3}\, m_\pi$$

$$\beta = \frac{\sqrt{3}}{2}$$

from #7

$$\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'} = \beta = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6} \text{ or } 30^\circ$$

$$E = \gamma E'(1 + \beta\cos\phi') = \gamma E' = 2\,\frac{m_\pi}{2} = m_\pi$$