So far have been applying conservation of p to elastic collisions (K.E. conserved)

-) Once to derive definition of $\vec{p} = m\gamma\vec{v}$

  Saw we nearly had 4-vector, added "p_+"

-) Again in mass free to show $p_+ = E$

-) Today we will apply p conservation for 3rd time to inelastic collision

  ↳ Conserve momentum, but Not K.E.

  eg balls of putty

Inelastic Collisions

Classically: $\vec{p}$ - Conserved, KE - Not Conserved

Some of the initial KE lost to heat
rotation
binding of chemical bonds

How can the 4-vector (Only knows about $E + \vec{p}$) give an adequate description of these complications?!

Seems like we are going to hit a wall.

Just saw that we can't require mom conservation w/o also Energy conservation!! ...

Example

Initial

Final

Let's try: $\vec{p}_f = \vec{p}_i = p_1$

$$E_f = E_i = E_1 + E_2$$

$$= E_1 + m_2$$

Then for the final mass we get

$$m_f^2 = E_f^2 - p_f^2$$

$$= (E_1 + m_2)^2 - p_1^2$$

$$= E_1^2 + 2E_1 m_2 + m_2^2 - p_1^2$$

$$= m_1^2 + m_2^2 + 2E_1 m_2$$

$$\hookrightarrow (m_1 + KE_1)$$

$$= (m_1^2 + 2m_1 m_2 + m_2^2) + 2(KE)_1 m_2$$

$$= (m_1 + m_2)^2 + (K.E)_1 (2m_2)$$

Note

- $m_f > (m_1 + m_2)$!
- the size of the discrepancy depends on the initial K.E. of the impact

* Momentum & Energy conserved also in inelastic collisions

Answer: the rest mass of final system is greater than the sum of the original rest masses.

New feature of Space-time physics.

The increase in mass precisely accounts for the lost KE that has gone into heat/rotation/other internal excitations of system!

In this example initial KE converted to rest mass

$$A + B \rightarrow C$$

Also have example: $A + B \rightarrow C + D + E + F$

$$e^- + e^+ \rightarrow e^+ + e^- + e^+ + e^-$$

Can also convert rest mass (internal energy) into KE

$$A \rightarrow B + C$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Note everyday release of internal energies
- Burning of coal / gas
- explosion of TNT

} Seem large, when work in mass units

Only $\sim 10^{-7}$ rest mass gets converted into K.E.

Described by the same formalism.

Need to turn to world of
Nuclear + Particle Physics
to properly test the relativistic conservation laws.
We will study many examples.

In center of momentum frame

$$P_i = P_f = 0 \qquad E_f = m_f = E_i$$

$$E_i = E_1 + E_2$$

$$m_f^2 = E_i^2 = (E_1 + E_2)^2$$

$$= ((m_1 + KE_1) + (m_2 + KE_2))^2$$

$$= ((m_1 + m_2) + (\underbrace{KE_1 + KE_2}_{\equiv KE_{tot}}))^2$$

$$= ((m_1 + m_2)^2 + 2(m_1 + m_2) KE_{tot} + KE_{tot}^2)$$

"Rest mass can be transformed into Energy &
Energy can be turned into Rest Mass"

More Accurate

-) the total momentum-Energy 4-vector is unchanged
-) the invariant magnitude of the mom-E 4-vector is equal to the rest mass

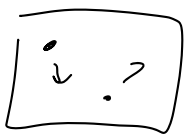
Potential Pits of Confusion

- Mass is invariant, but can change in collisions.

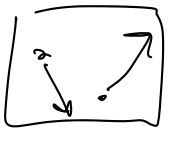
- $E_{tot} = \sum_i E_i = \sum_i (m_i + KE_i)$

In rest frame of total P. $M_{tot} = E_{tot} = \sum_i m_i + \sum_i KE_i$

- Rest mass $\neq$ physical

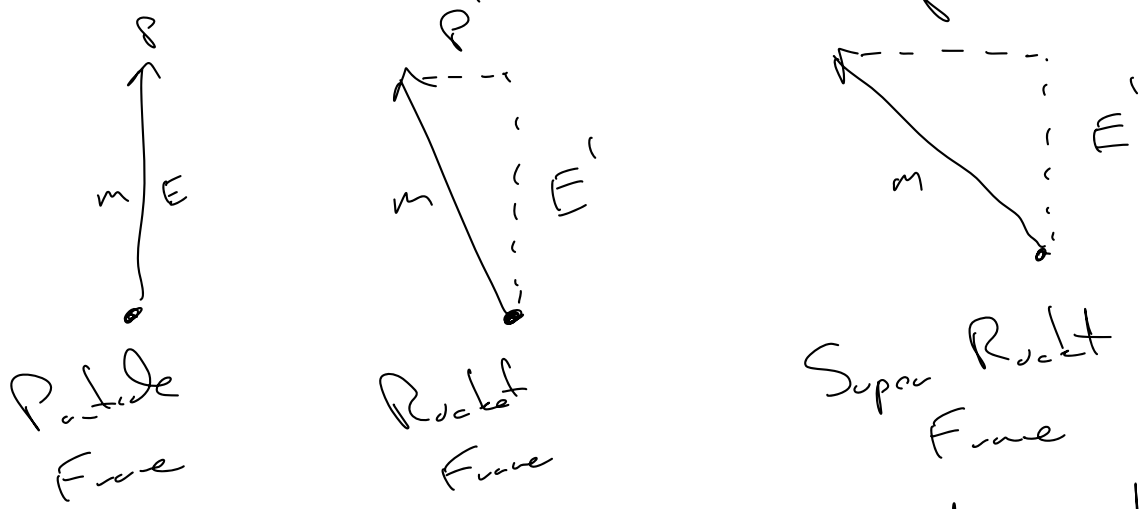


more $\vec{a}$ than for F



In fact most of the known rest mass of the Universe comes from exactly this form

Mass vs Energy



But remember, we are in Lorentz geometry

$$m^2 = E^2 - p^2$$