

# Homework Set #1

**Due Date:** Before class Friday January 27th

## 1) You

(2 points)

- (a) What is your major(s)/minor(s) ?
- (b) Have you studied Matrices ? Relativity ? or QM before?
- (c) What do you most want to get out of this course ?

## 2) Reading

(2 points)

- Read sections 1-1,1-2,1-3,1-4

## 3) Matrix Multiplication

(2 points)

$$\text{Let, } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

- (a) What is  $A \times C$  ?
- (b) What is  $A \times B$  ?
- (b) What is  $n \times A$  ? where  $n$  is a number.
- (c) Watch this: The adult way to think about matrices.

## 3) Galilean Transformations

(10 points)

Consider an isolated system of  $N$  particles. The total momentum is given by  $P_{\text{total}} = \sum_i P_i$ , where the momentum of the  $i$ th particle is  $m_i v_i$  and the total energy by  $E_{\text{total}} = \sum_i E_i$ , where the energy of the  $i$ th particle is  $1/2 m_i v_i^2$

- (a) Show that if the total momentum is conserved in one reference frame (S) it is also conserved in a reference frame (S') moving with velocity  $v$  with respect to the first frame.
- (b) Show that if energy is conserved in reference frame S it is also conserved in reference frame (S')

#### 4) General Linear Coordinate Transformations

(35 points)

Fill in the details for argument for the general coordinate transformation sketched in class. Write down the arbitrary linear transformation we assumed in class. The unknowns are arbitrary functions of  $v$ , but not (by assumption  $x, x', t$ , or  $t'$ ).

- (a) Impose the constraint  $x=vt$  when  $x'=0$ . What is the most general form now?
- (b) Impose the constraint  $x'=-vt'$  when  $x=0$ . What is the most general form now?
- (c) Require that the combination of two transforms ( $v_1$ ) and ( $v_2$ ) yields another transform. (Hint: What relation do the diagonal elements have to have to form a valid transformation consistent with b)? ) (Hint #2 This should yield a free parameter given by the separation of variables constant)
- (d) You should now have one free parameter (the separation constant) and one unknown arbitrary function of  $v$ . Solve for the arbitrary function by requiring the two transforms ( $v$ ) and ( $-v$ ) to give the identity (ie:  $x'=x$  and  $t'=t$ )
- (e) You should now have the most general linear coordinate transformation. Write the separation constant in terms of a velocity  $v_*$  as we did in class. Show that if something is moving with  $v_*$  in one frame it move with the speed in the other frame. (ie:  $x' = v_* t' \implies x = v_* t$ )
- (f) Show that  $v_*^2 t^2 - x^2$  is invariant under coordinate transformations.
- (h) How does the most general coordinate transformation relate to the Galilean transformations in classical physics? eg: What choice does Newton make for  $v_*$  ? A pithy way to summarize the difference between Newtonian physics and Relativistic physics is simply a different choice for the free parameter in the general coordinate transformation.

#### 5) Proton Collisions in Newtonian Mechanics

(10 points)

A proton moving with speed  $v_i$  strikes a proton at rest in the lab frame. After the collision the protons are observed to scatter symmetrically about the x-axis. (ie: one proton makes an angle  $+\theta$  with respect to the x-axis, the other makes an angle  $-\theta$ . ) What does Newtonian physics predict the angle between the protons should be in the lab frame ?

#### 6) How long does it take light to travel a foot ?

(2 points)

**7) What is an inertial frame ?**

*(6 points)*

- Are you in an inertial frame when riding on a smooth high speed train ? Why or Why not?
- Are you in an inertial frame when riding on a merry-go-round ? Why or Why not?

**8) Two runners**

*(5 points)*

Two runners (A and B) are racing along a straight track of length  $L$ . B travels at a constant speed  $v$ , A goes half of the way with speed  $2v$  and half of the way with speed  $v/2$ . Who wins?

**9) State the postulate(s) of Special Relativity.**

*(2 points)*

**10) Different kinds of clocks.**

*(5 points)*

There are many different kinds of clocks: sand clocks, electric clocks, mechanical clocks, light clocks and biological clocks. We have shown that light clocks in motion slow down, does this necessarily imply that all clocks will be affected equally?