Exam #3

More than one Particle

So far, only talked about systems w/ one DoF
(assumed proton/nuclei stationary)

More complicated atoms, have to deal w/
other electrons. eg

Two new complications

-) $\psi(x) \longrightarrow \psi(x_1, x_2)$

$V(x) \longrightarrow V(x_1, x_2)$

typically Not seperable

$\Rightarrow$ Sch cannot be solved analytically

(Analogs in Classical Physics)

Not much else to say here.

-) Indistinguishability of identical Particles (New to QM)

Indistinguishability

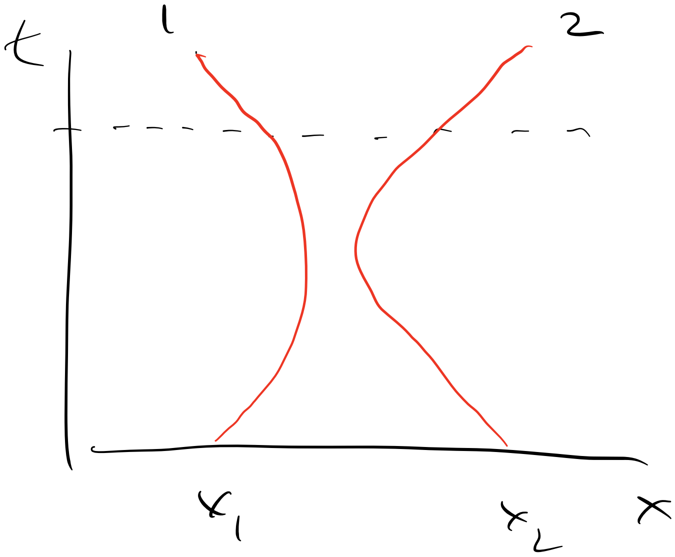
ex: 2 non-interacting particles in 1D infinite well.

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi(x_1, x_2)}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_2^2}\right) + V\psi = E\psi$$

Separable $V = 0$ (usually $V(|x_1 - x_2|)$ + not separable)

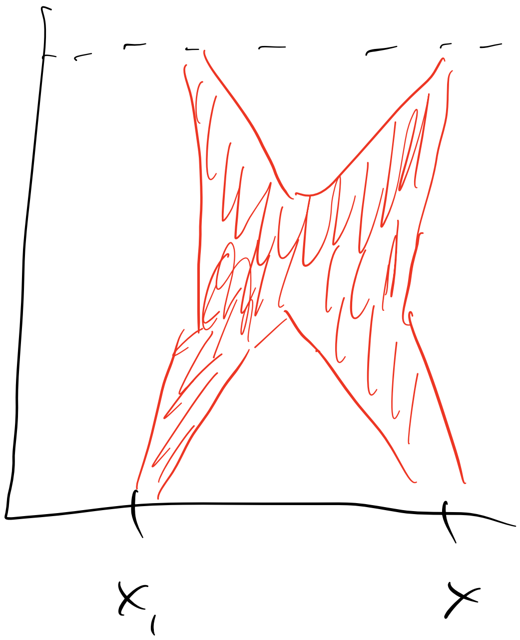
$$\psi_{nm}(x_1, x_2) = \psi_n(x_1)\,\psi_m(x_2)$$ (1D ∞-well solutions)

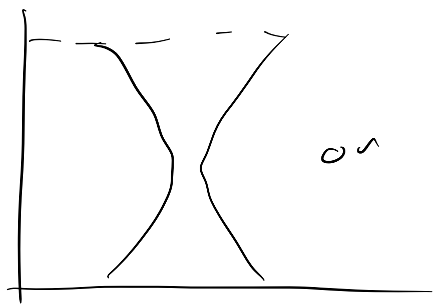
$$= C \sin\frac{n x}{L} \sin\frac{m x}{L}$$



Classically

No way of knowing





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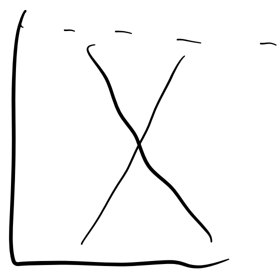

Following particle paths violates the uncertainty principle

$\Rightarrow$ Physics invariant under $x_1 \leftrightarrow x_2$

$$P(x_1\, x_2) = P(x_2, x_1)$$

$$\Rightarrow \quad |\psi(x_1, x_2)|^2 = |\psi(x_2\, x_1)|^2$$

$$\Rightarrow \psi(x_2, x_1) = e^{iC}\psi(x_1, x_2)$$

However, $\psi(x_2, x_1) = e^{iC}\underbrace{\psi(x_1, x_2)}_{e^{iC}\psi(x_2,x_1)}$

$$= e^{2iC}\psi(x_2, x_1)$$

$$\Rightarrow e^{2iC} = 1$$

or $e^{iC} = 1$ or -1

Particles w/

$\psi(x_2, x_1) = +\psi(x_1, x_2)$ "Bosons"

$\psi(x_2, x_1) = -\psi(x_1, x_2)$ "Fermions"

Turns out... Intrinsic Spin tells you type

$s = (n + 1/2)$ "Fermions"

$s = n$ "Bosons"

Electons $S = 1/2 \Rightarrow$ Fermions

Back to our example:

$$\psi_n(x_1)\psi_m(x_2) \neq -\psi_n(x_2)\psi_m(x_1)$$

$\Rightarrow$ this $\psi(x_1, x_2)$ cannot the electons

However given a solution $\psi(x_1, x_2)$ can always construct symmetric or anti-symmetric ψ's

$$\psi_S = C\left[\psi(x_1,x_2) + \psi(x_2,x_1)\right]$$

$$\psi_A = C\left[\psi(x_1,x_2) - \psi(x_2,x_1)\right]$$

$\Rightarrow$ In our example

$$\psi_{nm}(x_1, x_2) = C\left[\psi_n(x_1)\psi_m(x_2) - \psi_n(x_2)\psi_m(x_1)\right]$$

__Note__ for fermions $\psi_{nm} = 0$ if $n = m$

Example "Pauli Exclusion Principle"

Cannot have 2 identical fermions in same Quantum State (w/ same quantum #'s)

If so, $\psi = 0$ By $x_1 \leftrightarrow x_2$

General, Major Implications

Major Simplification of the allowed States for systems of fermions

Spin One more piece to full Hydrogen wave function. Electron Spin.

Fundamental property of electrons
- Intrinsic angular momentum
- Deep origins.
- Intuitively: Think of electron as sphere of charge, then spin measures how much electron is rotating or spinning (Not quite right...)

$$S^2 = s(s+1)\hbar^2$$

where S is a Quantum Number associated to different particles (fundamental label)

Expect $2s+1$ possible values for the z-component $(-s, -s+1, \dots s)$

Aside

Space-time + QM implies can only have

$$S = \left(\underbrace{0 \qquad \tfrac{1}{2}}_{\text{Unconstrained}} \qquad 1 \qquad \tfrac{3}{2} \qquad 2 \right)$$

1: As many as we want. But need to Interact via group theory

3/2: ≤ 8 "Supergravity"

2: Unique

Observe

1 $\qquad$ 45 $\qquad$ 24 $\qquad$ 0 $\qquad$ 0^*

ST + QM

$\Rightarrow$ Spin $\frac{2n+1}{2}$ Fermions

Spin n Bosons

Wave-Particle Duality

electrons - known to be particles,
now seen to also have wave-like behavior

photons - make up light, clearly behaves like wave
also known to interact w/ atoms & e^-s
like particles

Quantum Mechanically all phenomena have both classical
wave & particle properties

Classical Particles - localized, scattered, deposits
energy suddenly in one spot.
Conserves E & p. No interference / diffraction

Classical Waves: Interfer and diffract
E spread out across space/time
Not quantized

Matter and radiation have aspects of both.
When interacting (emission/absorption) particle aspects
When propogating through space wave aspects

Observations characterized by particle-like properties

Predictions are " " wave-like properties

We will get more quantitative next