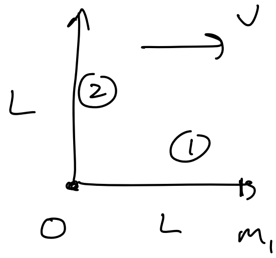
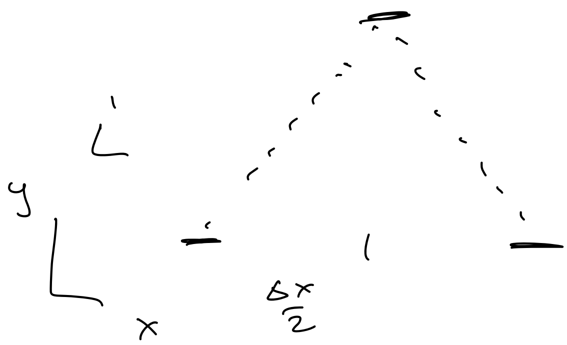
1) Reading

2)



Leg 2 first



$$\Delta x = \beta \Delta t$$

$$\Delta t = 2\sqrt{L'^2 + \frac{\beta^2 \Delta t^2}{4}}$$

or

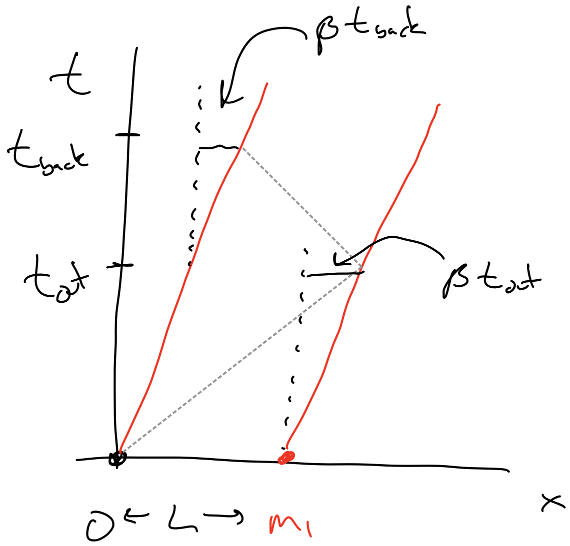
$$\Delta t^2 = 4L'^2 + \beta^2 \Delta t^2$$

$$\Delta t = \frac{2L'}{\sqrt{1-\beta^2}} \equiv \Delta t_2$$

$$= 2L'\gamma$$

Leg 1

Now t-x diagram



$$\Delta t_1 = t_{out} + t_{back}$$

$$t_{out} = L + \beta t_{out}$$

$$\Rightarrow t_{out} = \frac{L}{1-\beta}$$

$$t_{back} = L - \beta t_{back}$$

$$\Rightarrow t_{back} = \frac{L}{1+\beta}$$

$$\Delta t_1 = L\left(\frac{1}{1-\beta} + \frac{1}{1+\beta}\right)$$

$$= \frac{2L}{1-\beta^2} = 2L\gamma^2$$

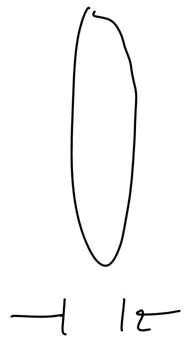
Not $2L\gamma$!

But we forgot length contraction

$$L = \frac{L'}{\gamma} \Rightarrow \Delta t_1 = 2L'\gamma = \Delta t_2$$

3)

b



$\frac{L_0}{\gamma}$

4) $d = 10 \text{ ly}$ $\beta = 0.5$

__Way one__   According to earth frame

$$t = \frac{10 \text{ ly}}{1/2 \, c} = 20 \text{ years}$$

But moving clocks run slow by factor of γ

$$t = \gamma \tau \qquad \gamma = \frac{1}{\sqrt{1 - \frac{1}{4}}} = \frac{1}{\sqrt{3/4}} = \frac{2}{\sqrt{3}}$$

~ 1.155

$$\tau = \frac{1}{\gamma} 20 \text{ years} \times \frac{365 \text{ meals}}{y}$$

$$= \frac{20 \cdot 365}{\gamma} \text{ meals} = 6320$$

(would be 7300
w/o time dilation

way 2 distance is contracted by factor $\frac{1}{\gamma}$

$$t = \frac{L}{\beta} = \frac{L/\gamma}{1/2} = \frac{20\ ly}{\gamma}$$

$\Rightarrow$ Same answer ✓

5) $\beta_p^2 = 0.9$ $\quad S'' \equiv S^2 \quad S' \equiv S^1 \quad S \equiv S^0$

$\beta_{21} = 0.9$

$\beta_{10} = 0.9$

1^{st} need $\beta_p^1 = \dfrac{\beta_p'' + \beta_{12}}{1 + \beta_p''\beta_{12}} = \dfrac{2 \cdot 0.9}{1 + (0.9)^2} = \dfrac{1.8}{1.81}$

velocity addition formula

~ 0.9945

then get $\beta_p^0 = \dfrac{\beta_p^1 + \beta_{01}}{1 + \beta_p^1 \beta_{01}} = \dfrac{0.9945 + 0.9}{1 + 0.9(0.9945)}$

$= \dfrac{1.8945}{1.89505} = 0.9997$

Not $3 \times 0.9 \sim 2.7c$!

6)

$$t = \gamma \tau$$

$$= \frac{1}{\sqrt{1-\beta^2}}\tau \sim \left(1 + \tfrac{1}{2}\beta^2\right)\tau$$

$$\boxed{(1-x)^{-1/2} \sim 1 + \tfrac{1}{2}x}$$

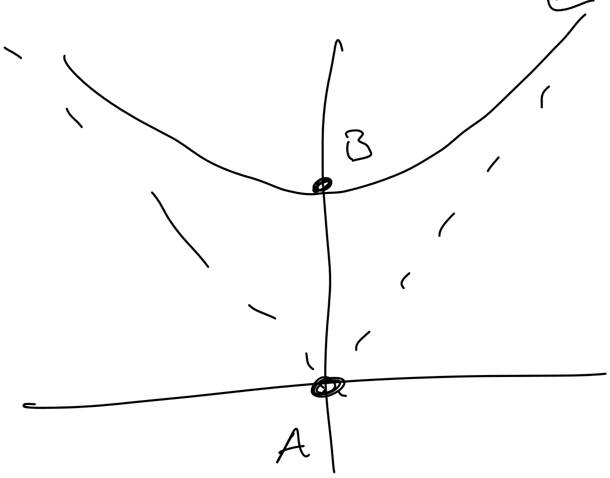
$$\frac{t-\tau}{t} = \frac{t\left(\cancel{1} - \left(\cancel{1} + \frac{1}{2}\beta^2\right)\right)}{t} = \frac{1}{2}\beta^2$$

if $\frac{t-\tau}{t} = 0.01 \Rightarrow \frac{1}{2}\beta^2 = \frac{1}{100}$

$$\beta^2 = \frac{1}{50} \qquad \boxed{\beta \sim \frac{1}{7}}$$

Good rule of thumb.

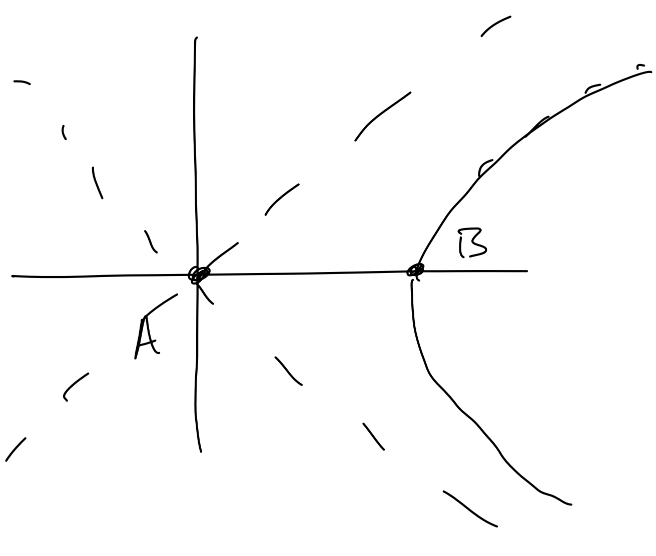
7)



All observers agree B happens on this invariant
$\Rightarrow$ All agree $t_B > t_A$

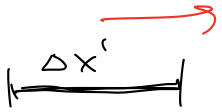
Relative motion cannot change the sign of Δx or δt if separation light-like.

If spatially separated observers will agree on this invariant. where some observers see B as occurring Before A.

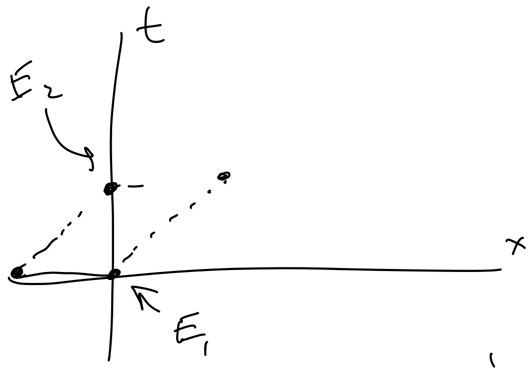


8) Lorentz Contraction

$\Delta x' = 1$ in rocket frame



$E_1 = (0, 0)$ $\quad$ $E_2 = (\Delta t, 0)$



In the rocket frame:

$E_1' = (0, 0)$ $\quad$ $E_2' = \left(\frac{\Delta x'}{\beta}, -\Delta x' \right)$

$$\Delta s' = \sqrt{\frac{\Delta x'^2}{\beta^2} - \Delta x'^2} = \Delta x' \sqrt{\frac{1-\beta^2}{\beta^2}} = \frac{\Delta x'}{\beta} \sqrt{1-\beta^2}$$

$$= \Delta t$$

$$\Rightarrow \quad \Delta x_{\text{inferred}} \equiv \beta \Delta t = \sqrt{1-\beta^2}\, \Delta x' = \frac{\Delta x'}{\gamma} \qquad \gamma \geq 1$$

9) Time Dilation

In rocket frame $\Delta x' = 0$

If lab measures Δx & divides by β

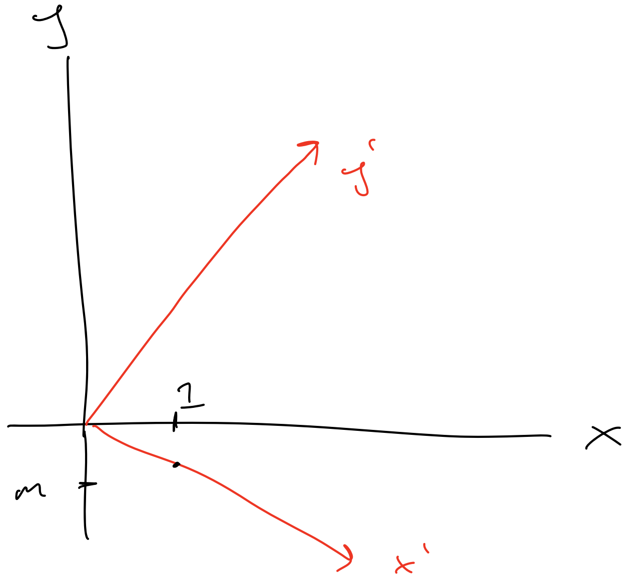
$$\Delta s' = \Delta t' = \sqrt{\frac{\Delta x^2}{\beta^2} - \Delta x^2}$$

$$\Rightarrow \Delta t' = \Delta x \sqrt{\frac{1-\beta^2}{\beta^2}}$$

$$\Delta t_{meas} = \frac{\Delta x}{\beta} = \frac{1}{\beta} \frac{\beta}{\sqrt{1-\beta^2}} \Delta t' = \frac{\Delta t'}{\sqrt{1-\beta^2}}$$

$$= \gamma \Delta t'$$

10) Euclidean "Slope" Transformations



Characterize orientation by m_x slope of x'-axis in x-y frame

$$x'\big|_s = mx$$

$$\hat{x}'\big|_s \propto \begin{pmatrix} 1 \\ m \end{pmatrix} = \frac{1}{\sqrt{1+m^2}} \begin{pmatrix} 1 \\ m \end{pmatrix} \equiv G \begin{pmatrix} 1 \\ B \end{pmatrix}$$

$$\hat{y}'\big|_s \propto \begin{pmatrix} -m \\ 1 \end{pmatrix} = \frac{1}{\sqrt{1+m^2}} \begin{pmatrix} -m \\ 1 \end{pmatrix} = G \begin{pmatrix} -B \\ 1 \end{pmatrix}$$

$$\hat{x}' \cdot \hat{y}' = 0$$

So a vector w/ coordinates $\begin{pmatrix} a \\ b \end{pmatrix}$ in S' frame

Appears as $a\,\hat{x}'\big|_s + b\,\hat{y}'_s$ in S frame

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} G & -BG \\ BG & G \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad G = \frac{1}{\sqrt{1+B^2}}$$

Compare

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$