Practice Assignment 3

Graph G w/ n vertices and m edges

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initialize T = ∅
 for each v ∈ G          ] Sets blank tree
     Key[v] = ∞
     Parent[v] = null
Key[r] = 0                 r is root or starting point
Q = V
while Q != ∅
    e = min(Q)     e is smallest value in Q
    Q = Q - e
    if Parent(e) != null
       T = T + (e, Parent(e))
    for each v ∈ Adjacent(e)
        if v ∈ Q and weight(e,v) < Key[v]
            Parent[v] = e
            Key[v] = weight(e,v)
Return T
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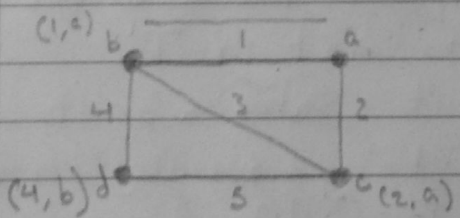


$T = \{\}$
$Q = \{a, b, c, d\}$

$T = \{\ \}$
$Q = \{b, c, d\}$ $\Rightarrow$ $T = \{(a,b)\}$
$Q = \{c, d\}$

$T = \{(a,b)\ (a,c)\}$
$Q = \{d\}$ $\Rightarrow$ $T = \{(a,b), (a,c), (b,d)\}$
$Q = \emptyset$

I believe the complexity of this is $\boxed{O(n^2)}$
but I heard it can be as fast as $O(m \log n)$

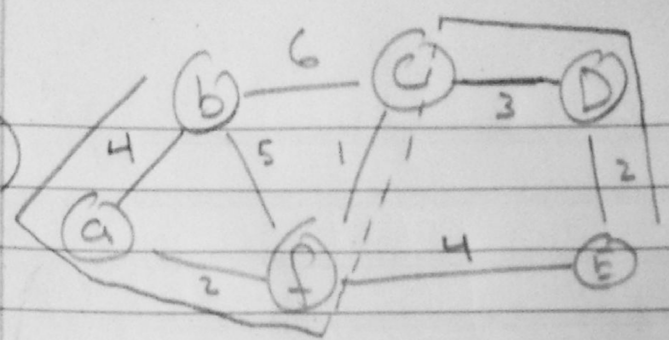
2) G has n vertices and m edges
$T = \emptyset$
sort (edges least to greatest)
For each $(v_1, v_2) \in E$: ← E is sorted
  if v_1 and $v_2 \notin T$
    $T = T \cup \{(v_1, v_2)\}$
    Parent$[v_1] = v_2$
Return T

This runtime will seem to be $O(m \log n)$, as it orders the edges then goes through the list adding edges not in T to T.

3) a) If the weights of edges of G are unique then there exists a unique MST in G
Take T to be a MST of G and Let T' be an MST of G also, but $T \neq T'$, and T, T' have n-1 edges. So there exists some edge E in T that is not in T'. If we remove E from T, it creates a split in G which then cancels T being a spanning tree. Since T is a MST, then E will be the "light" edge, so E exists in all MST's, but $E \notin T'$, which makes T' not a MST which contradicts our original statement

Another Case) Consider T and T' overlapping. Since they overlap they create a cycle with n edges, and T, T' have n-1 edges. Let E be the heaviest in the cycle. Without loss of generality E is in T, but because all edge weights are unique a MST will ignore the heaviest (E) which is in T, therefore T can't be a MST, contradicting our original statement

G



b) $w(a,b) = w(f,e)$

but G has a unique MST

4) Let P be "∃ 1 dry surviver in every odd water gun fight"

$P(x) = 2x - 1$, so look at $P(1)$

$P(1) = 2(1) - 1 = 1$ Base Case is true

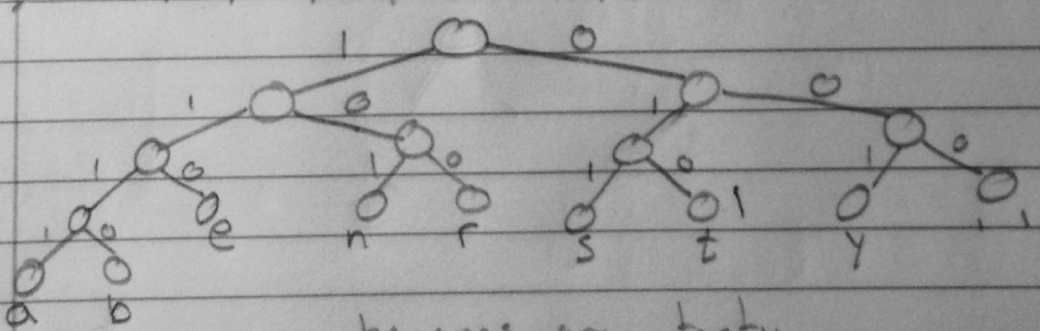
Assume $P(K)$ is true so $P(K) = 2K - 1$ is true

look at $P(K+1) = 2(K+1) - 1 = (2K+2) - 1 = (2K-1) + 2$

$\Rightarrow P(K+1) = (2K-1) + 2 = P(K) + 2$

Since we assume $P(K)$ to be true that at least one surviver is dry adding 2 doesn't make it false. So by induction for all odd numbered water gun fights there will be at least one surviver with $2n-1$ children.

5) a, b, e, n, r, s, t, y, ' ' |9|



bananas are tasty

a = 1111 = 4b $\underbrace{1110}_{b}, \underbrace{1111}_{a}, \underbrace{101}_{n}, \underbrace{1111}_{a}, \underbrace{101}_{n}, \underbrace{1111}_{a}, \underbrace{011}_{s}, \underbrace{000}_{' '}, \underbrace{1111}_{a}, \underbrace{100}_{r}, \underbrace{110}_{e},$

b = 1110 = 4b

e = 110 = 3b $\underbrace{000}_{' '}, \underbrace{010}_{t}, \underbrace{1111}_{a}, \underbrace{011}_{s}, \underbrace{010}_{t}, \underbrace{001}_{y},$

n = 101 = 3b

r = 100 = 3b $1(b) + 5(a) + 1(e) + 2(n) + 1(r) + 2(s) + 2(t) + 1(y) + 2(' ')$

s = 011 = 3b $4 + 20 + 3 + 6 + 3 + 6 + 6 + 3 + 6 = 57$ bits < 58.

t = 010 = 3b

y = 001 = 3b

' ' = 000 = 3b