Practice Assignment 4

Due: Thursday, February 18 at 2PM

To get credit, each student must submit their own solutions (which need not be typeset but need to be easily legible) to Canvas by the due date above – no late submissions are allowed. Note that while you must each submit solutions individually, you are free to work through the problem sets in groups. Solutions will be posted shortly after class and may be discussed in class. These will be graded on the basis of *completion* alone, not *correctness*.

1. Try to solve the following recurrence relations and give a Θ bound for each of them:
   (a) $T(n) = 2T(n/3) + 1$
   (b) $T(n) = 5T(n/4) + n$
   (c) $T(n) = 7T(n/7) + n$
   (d) $T(n) = 9T(n/3) + n^2$
   (e) $T(n) = 8T(n/2) + n^3$
   (f) $T(n) = T(n-1) + 2$
   (g) $T(n) = T(n-1) + c^n$, where $c > 1$ is some constant
   (h) $T(n) = 2T(n-1) + 1$
   The following ones are a bit more challenging. Try your best to solve them, but they will not be counted toward completeness.
   (i) $T(n) = 49T(n/25) + n^{3/2} \log n$
   (j) $T(n) = T(n-1) + n^c$, where $c \geq 1$ is a constant
   (k) $T(n) = T(\sqrt{n}) + 1$

2. The well-known mathematician George Polya posed the following false "proof" showing through mathematical induction that actually, all horses are of the same color. Identify what is wrong with this proof.
   Base case: If there's only one horse, there's only one color, so of course its the same color as itself.
   Inductive case: Suppose within any set of n horses, there is only one color. Now look at any set of $n+1$ horses. Number them: $1, 2, 3, \ldots, n, n+1$. Consider the sets $\{1, 2, 3, ..., n\}$ and $\{2, 3, 4, ..., n+1\}$. Each is a set of only n horses, therefore within each there is only one color. But the two sets overlap, so there must be only one color among all $n+1$ horses.

3. Given two sorted arrays $a[1, ..., n]$ and $b[1, ..., n]$, given an $O(\log n)$ algorithm to find the median of their combined $2n$ elements. (Hint: use divide and conquer).

4. (a) Explain why the following algorithm sorts its input.

   STOOGESORT($A[0 \ldots n-1]$)
       if $n = 2$ and $A[0] > A[1]$
           swap $A[0]$ and $A[1]$
       else if $n > 2$
           $k = \lceil 2n/3 \rceil$
           STOOGESORT($A[0 \ldots k-1]$)
           STOOGESORT($A[n-k \ldots n-1]$)
           STOOGESORT($A[0 \ldots k-1]$)

   (b) Would STOOGESORT still sort correctly if we replaced $k = \lceil 2n/3 \rceil$ with $m = \lfloor 2n/3 \rfloor$? *(Hint: what happens when $n = 4$?)*
   (c) State a recurrence for the number of comparisons executed by STOOGESORT.
   (d) Solve the recurrence. Simplify your answer.