Practice Assignment 6

Due: Thursday, March 10th at 2PM

To get credit, each student must submit their own solutions (which need not be typeset) to Canvas by the due date above – no late submissions are allowed. Note that while you must each submit solutions individually, you are free to work through the problem sets in groups. Solutions will be posted shortly after class and may be discussed in class. These will be graded on the basis of *completion* alone, not *correctness*.

1. Duckwheat is produced in Kansas and Mexico and consumed in New York and California. Kansas produces 15 shnupells of duckwheat and Mexico 8. Meanwhile, New York consumes 10 shnupells and California 13. The transportation costs per shnupell are \$4 from Mexico to New York, \$1 from Mexico to California, \$2 from Kansas to New York, and \$3 and from Kansas to California. Write a linear program that decides the amounts of duckwheat (in shnupells and fractions of a shnupell) to be transported from each producer to each consumer, so as to minimize the overall transportation cost.

   Let $x_{i,j}$ be the number of shnupells of duckwheat produced in city i and consumed in city j.

$$\begin{array}{ll} \min & 4 \cdot x_{M,N} + 1 \cdot x_{M,C} + 2 \cdot x_{K,N} + 3x_{K,C} \\ s.t. & x_{K,N} + x_{K,C} \leq 15 \\ & x_{M,N} + x_{M,C} \leq 8 \\ & x_{K,N} + x_{M,N} \leq 10 \\ & x_{K,C} + x_{M,C} \leq 13 \\ & x_{K,N}, x_{M,N}, x_{M,N}, x_{M,C} \geq 0 \end{array}$$

2. A film producer is seeking actors and investors for his new movie. There are n available actors; actor i charges s_i dollars. For funding, there are m available investors. Investor j will provide p_j dollars, but only on the condition that certain actors $Lj \subseteq \{1, 2, ..., n\}$ are included in the cast (all of these actors L_j must be chosen in order to receive funding from investor j).

   The producer's profit is the sum of the payments from investors minus the payments to actors. The goal is to maximize this profit.

   Question: Express this problem as an integer linear program in which the variables take on values $\{0, 1\}$.

   Let $x_j = 1$ mean that investor j is investing in the movie, 0 otherwise.

   Let $y_i = 1$ mean that actor i is acting in the movie, 0 otherwise.

$$\begin{array}{ll} \max & \sum_{j=1}^{m} x_j p_j - \sum_{i=1}^{n} y_i s_i \\ s.t. & x_j |L_j| \leq \sum_{i \in L_j} y_i \\ & x_j, y_i \in \{0, 1\} \end{array}$$

3. (a) I have a problem X that I can solve by using 3SAT as a black box, with an additional polynomial amount of time. (That is, problem X poly-time reduces to 3SAT.) Are the following statements true or false/unknown?

      i. X is in NP.

      ii. X is NP-hard.

   (b) I have two problems, X and Y. 3SAT reduces to problem X and problem X reduces to problem Y. (These reductions are poly-time reductions.) Is the following statement true or false/unknown?

      i. Y is NP-hard.

(c) Problem X and problem Y are both NP-complete. Is the following statement true or false/unknown?

i. Problem X reduces to problem Y and vice versa.

For solutions, work through `https://courses.ecampus.oregonstate.edu/cs325/np-hard/story.html`

4. We are feeling experimental and want to create a new dish. There are various ingredients we can choose from and we'd like to use as many of them as possible, but some ingredients don't go well with others. If there are n possible ingredients (numbered 1 to n), we write down an $n \times n$ binary matrix where the (i, j) entry is 1 if i and j can go together and 0 otherwise. Notice that this matrix is necessarily symmetric; and that the diagonal entries are always 0.

We wish to solve the following problem:

EXPERIMENTAL CUISINE :

input: n, the number of ingredients to choose from; B, the $n \times n$ binary matrix that encodes which items go well together

output: the maximum number of ingredients which can be selected together

Show that if EXPERIMENTAL CUISINE can be solved in polynomial time, then P=NP.

By reduction from INDEPENDENT SET: we can use an algorithm that solves EXPERIMENTAL CUISINE to solve INDEPENDENT SET.

For INDEPENDENT SET, we wish to know if a graph G has a set of k vertices such that no two vertices are adjacent. To solve this, we create an instance of the EXPERIMENTAL CUISINE problem. For each vertex of the graph, we create an ingredient. The matrix B is the complement of the adjacency matrix for G: $B(i, j)$ is 0 if there is an edge between vertices i and j in G (in this case we would not be allowed to select these ingredients). If the algorithm for EXPERIMENTAL CUISINE finds an answer $\geq k$, then the solution to INDEPENDENT SET is true and otherwise false.

So, if we can solve EXPERIMENTAL CUISINE in poly-time, we can solve INDEPENDENT SET in poly-time.

Since INDEPENDENT SET is NP-complete, solving INDEPENDENT SET in poly-time would imply that P=NP.