

W1D1 – John Okon Ansa

Pattern 1: “for all” and “there exists” pattern

Given an array A, if all elements of A are positive, return 1. Else return 0.

Algorithm 1. (Counting method)

```
count = 0;  
for (i = 0; i < n; i++)  
    if (A[i] > 0) count++;  
(count == n)? true : false;
```

Check the condition. If true, increment the counter. If the counter value is n, return true.

Else return false.

Algorithm 2. (Test the opposite condition method)

```
for (i = 0; i < n; i++)  
    if (A[i] <= 0) return false;  
return true;
```

Check the “opposite condition”. If true, return false. Return true outside the loop.

Time Complexity Table:

	Best case	Average case	Worst case
Algorithm 1	O(n)	O(n)	O(n)
Algorithm 2	O(1)	O(n)	O(n)

Lower bound of the problem

What is the minimum information required to conclude the problem statement in the affirmative?

You need to check all elements of A. Hence the lower bound of the problem is $\Omega(n)$.

Since the lower bound of the problem is equal to the worst case time complexity in Solution 1 (Solution 2), Solution 1 (Solution 2) is optimal.

Exercise 1

An array is defined to be a 235 array if the number of elements divisible by 2 plus the number of elements divisible by 3 plus the number of elements divisible by 5 plus the number of elements not divisible by 2, 3, or 5 is equal to the number of elements of the array.

Write a method named is123Array that returns 1 if its array argument is a 235 array, otherwise it returns 0.

Note: A number can be divisible by more than one number. For example, 10 is divisible by both 2 and 5.

Algorithm 3. (Counting method)

```
count = 0;
for (i = 0; i < n; i++) {
    if (A[i] % 2 == 0) count++;
    if (A[i] % 3 == 0) count++;
    if (A[i] % 5 == 0) count++;
    if (A[i] % 2 != 0 && A[i] % 3 != 0 && A[i] % 5 != 0) count++;
}
if (count == n) return 1;
else return 0;
```

Algorithm 4. (Test the opposite condition method)

```
for (i = 0; i < n; i++) {
    if (A[i] % 2 != 0 && A[i] % 3 != 0 && A[i] % 5 != 0 && !(A[i] % 2 != 0 && A[i] % 3 != 0
&& A[i] % 5 != 0))
        return 0;
}
return 1;
```

Time Complexity Table for Exercise 1:

	Best case	Average case	Worst case
Algorithm 3	O(n)	O(n)	O(n)
Algorithm 4	O(1)	O(n)	O(n)

Lower bound of the problem

You need to check all elements of the array to determine if it satisfies the 235 condition. Therefore, the lower bound of the problem is $\Omega(n)$.