Photons Random Walking Out of a Star

1. Consider a photon that has just been created via a nuclear reaction in the center of the Sun. The photon now starts a long and arduous journey to the Earth to be enjoyed by Ay16 students studying on a nice Spring day.

   (a) The photon does not travel freely from the Sun's center to the surface. Instead it random walks, one collision at a time. Each step of the random walk traverses an average distance l, also known as the mean free path. On average, how many steps does the photon take to travel a distance Δr?

   (*Hint: Be sure to draw a picture. If each step is a vector $\vec{r_i}$, then $\vec{D} = \sum \vec{r_i}$. However, this displacement is zero for a large number of random-walk journeys. Instead, calculate $\vec{D}^2$ and take the square root of the result to be $\Delta r = (\vec{D}^2)^{1/2}$. This is the root–mean–squared displacement, which is a scalar rather than vector quantity. Note that 'multiplying' two vectors isn't as simple as multiplying two scalars. You must take the dot product, which looks like $\vec{A} \cdot \vec{B} = AB\cos\theta$, where θ is the angle between the two vectors.*)

   (b) What is the photon's average velocity over the total displacement after many steps? Call this $\vec{v}_{\text{diff}}$, the diffusion velocity.

   (c) The "mean free path" l is the characteristic (i.e. average) distance between collisions. Consider a photon moving through a cloud of electrons with a number density n. Each electron presents an effective cross-section σ. Give an analytic expression for "mean free path" relating these parameters.

   (d) The mean free path l can also be related to the **mass** density of absorbers ρ, and the "absorption coefficient" κ (cross-sectional area of absorbers per unit mass). How is κ related to σ? Express v_{diff} in terms of κ and ρ using dimensional analysis.

   You have now developed the tools to attack the problem of radiative diffusion.

   (e) What is the diffusion timescale for a photon moving from the center of the sun to the surface? The cross section for electron scattering is $\sigma_T = 7 \times 10^{-25}$ cm^2 and you can assume pure hydrogen for the Sun's interior. Be careful about the mass of material through which the photon travels, not just the things it scatters off of. Assume a constant density, ρ, set equal to the mean Solar density (*N.B.: the subscript T is for Thomson. The scattering of photons by free (i.e. ionized) electrons where both the K.E. of the electron and λ of the photon remain constant—i.e. an elastic collision—is called Thomson scattering, and is a low-energy process appropriate if the electrons aren't moving too fast, which is the case in the Sun.*)