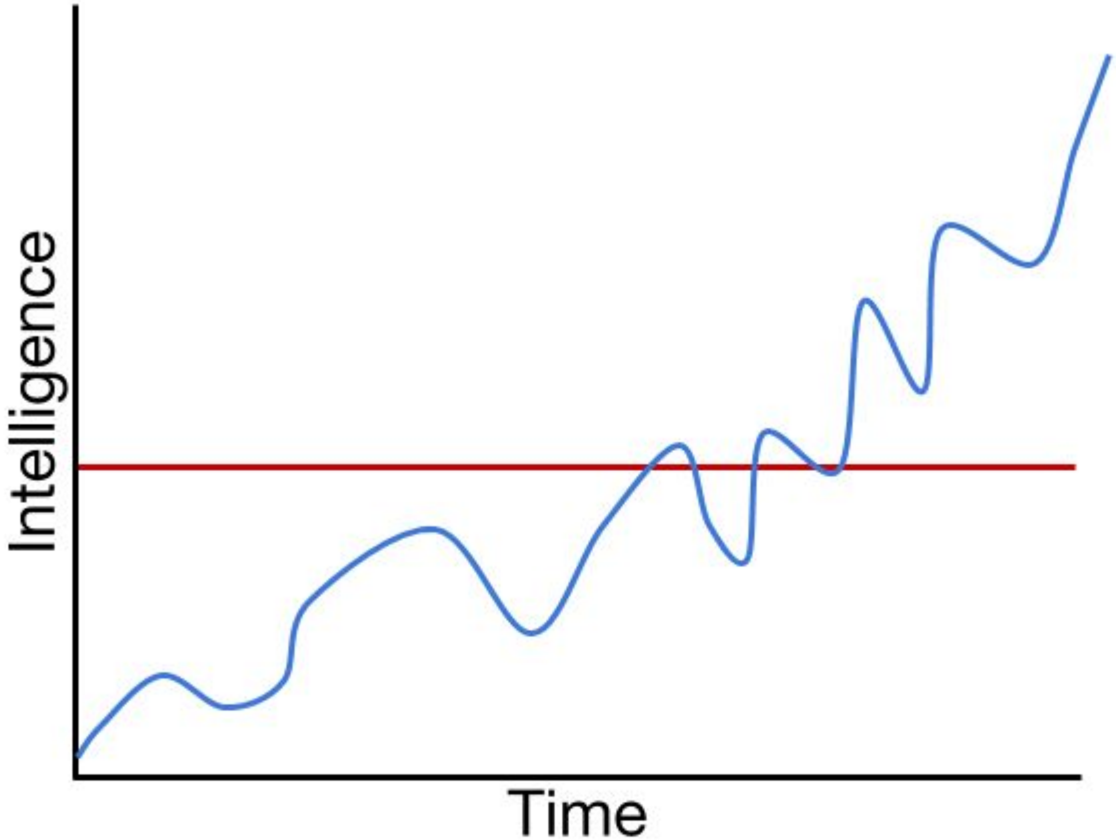
Thermal Radiation

John Lewis

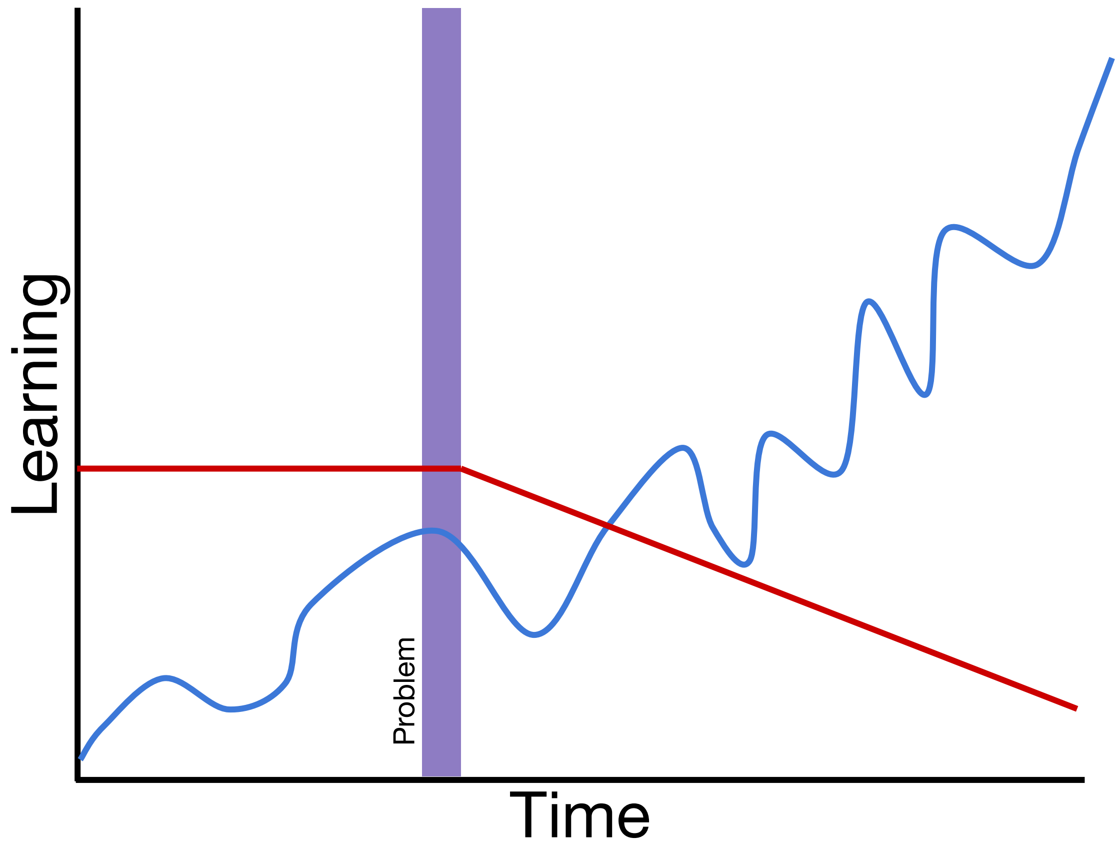
Outline for the Week

- Day 1: Radiation Fundamentals
  - Intensity
- Day 2: Thermal Radiation
  - Blackbody Radiation
- Day 3: Thermal Emitters
  - Useful results using blackbody radiation
- Day 4: Applications
  - Blackbodies in Astronomy

Ask Questions



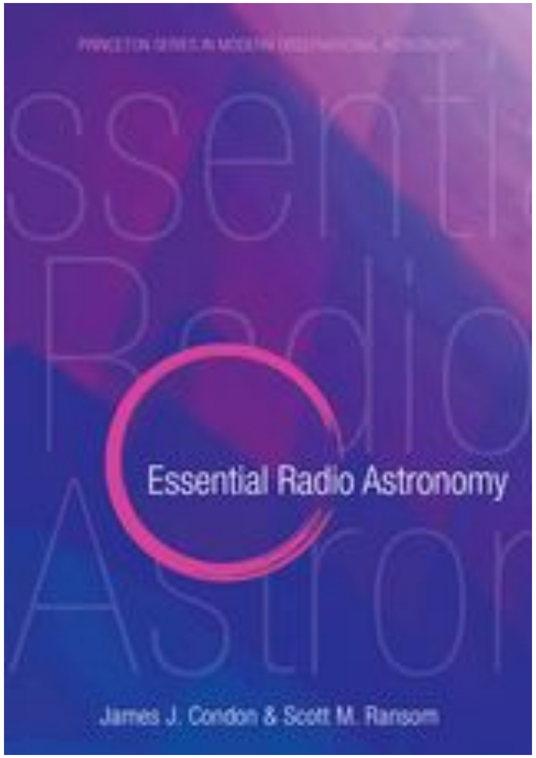
Ask Questions

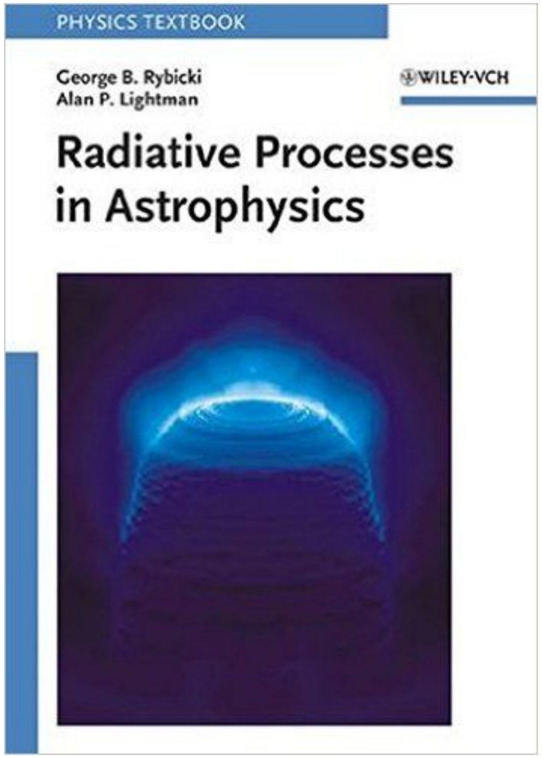


Radiation Fundamentals

Useful resources

- *Essentials of Radio Astronomy*, Ch. 2
  - Free online textbook. http://www.cv.nrao.edu/~sransom/web/Ch2.html
- Rybicki & Lightman - *Radiative Processes in Astrophysics*
  - Available in Wolbach Library...amongst other sundry places



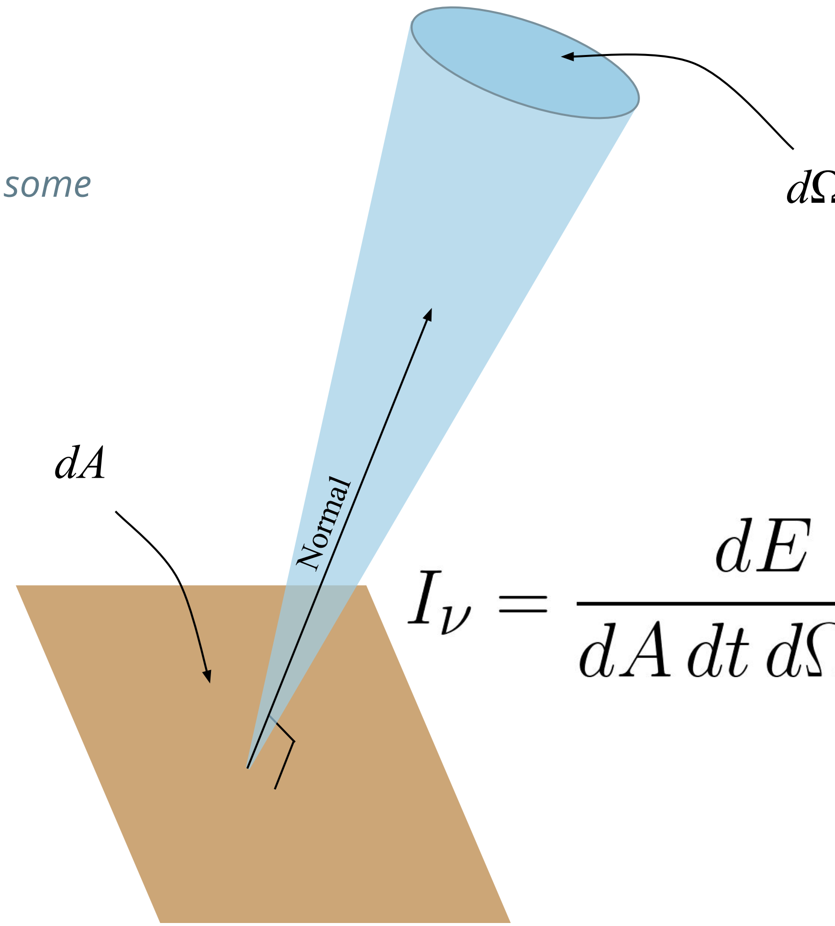


Definitions

Specific Intensity:

The energy in some frequency range passing through some area into some solid angle over some period of time.

$$dE = I_v \, dA \, dt \, d\Omega \, dv$$



$$I_\nu = \frac{dE}{dA\,dt\,d\Omega\,d\nu}$$

Units are:

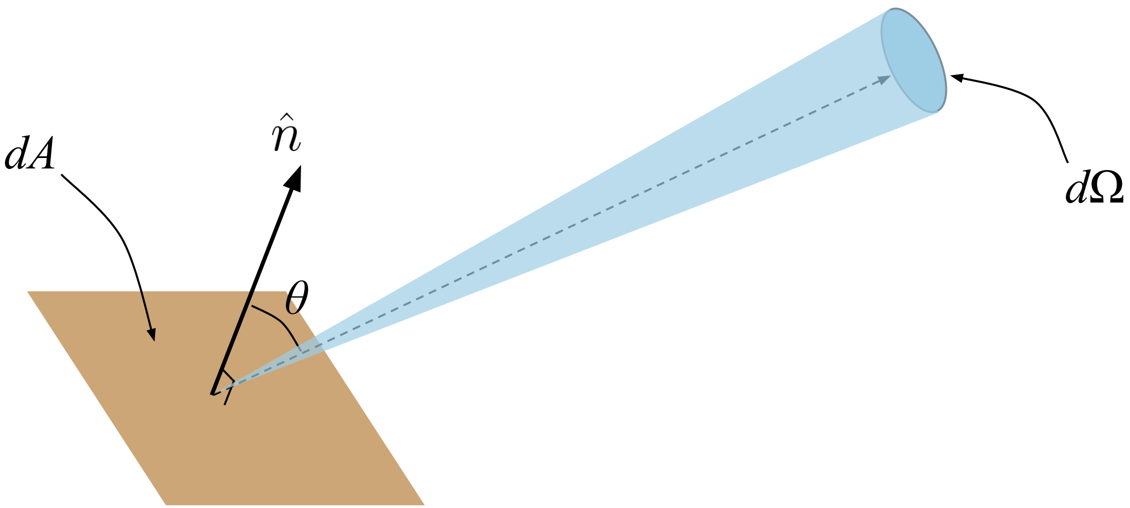
$$[I_v] = \mathrm{ergs\ cm^{-2}\ s^{-1}\ ster^{-1}\ Hz^{-1}}$$

Definition w/ Non-perpendicularity

If the ray is not perpendicular to the target area, then only the projected area matters. So dA → dA cos(θ)

$$dE = I_\nu \, d\Omega \, (\cos\theta \; dA) \, d\nu \, dt$$

Units are still: ergs^{-1} cm^{-2} s^{-1} ster^{-1} Hz^{-1}



Definitions from different perspectives

Intensity is …. *per unit time, per unit frequency*

The energy emitted per unit area of the emitter into some solid angle

The energy received from some solid angle per unit area of the receiver

Question 1: Specific Intensity

I will introduce *thermal emitters* tomorrow; but for now, suffice it to say, the equation of a thermal emitter is

$$B_\nu(T) \equiv \frac{2\nu^2}{c^2} \frac{h\nu}{e^{\left(\frac{h\nu}{kT}\right)} - 1}.$$

I submit to you, that this is a specific intensity. Determine the units of B_ν(T).
Plank's constant: h = 6.623 x 10^{-27} erg s
Boltzmann's constant: k = 1.38 x 10^{-16} erg K^{-1}

What units do you get?

~~Specific~~ Intensity

Specific intensity describes the intensity of electromagnetic waves with frequency between ν ± dν.

We can remove this dependence by integrating over frequency.

$$I = \int_0^\infty I_\nu \, d\nu$$

Quick note on Spherical Coordinates

Coordinate directions

- Radial
- Azimuthal
  - angle from z-axis to x-y plane
- Polar
  - (counter-clockwise) angle around z-axis in x-y plane

Conventions* (radial, azimuthal, polar)

- Mathematics (r,θ,ϕ)
  - Maintains consistency with polar and cylindrical coordinates. θ has same definition.
- Physics and Astronomy (r,ϕ,θ)
  - Maintains consistency with language. The angle we care most about we usually call θ, and since we often have azimuthal symmetry, we make the polar angle θ.

**See Wolfram MathWorld for an exhaustive comparison of conventions.*

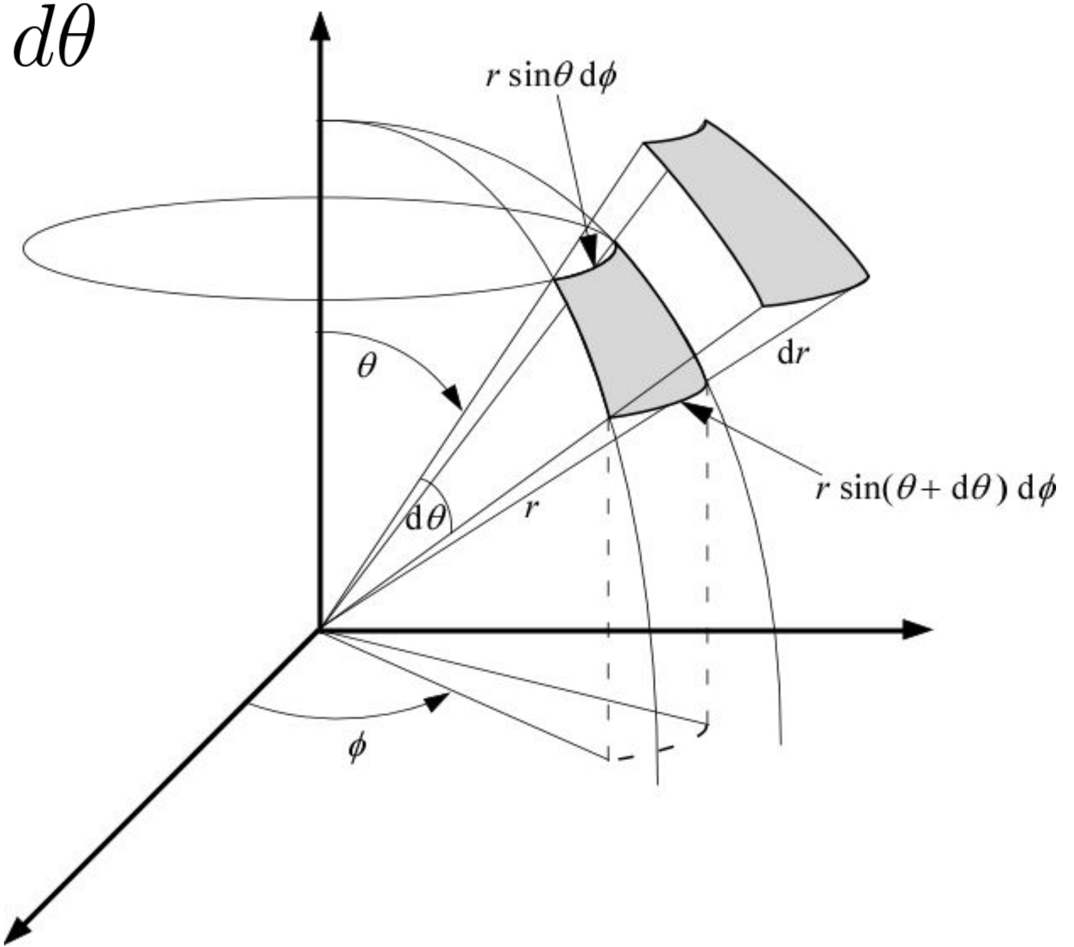
Question 2: Solid Angle

Intensity is measured per "steradian" (pronounces "stair-radian"), which is a unit of solid angle. Solid *what?* A solid angle is a differential element of the surface of a sphere, measured in spherical coordinates. The units are steradians, but they are actually unitless in cgs, like radians.

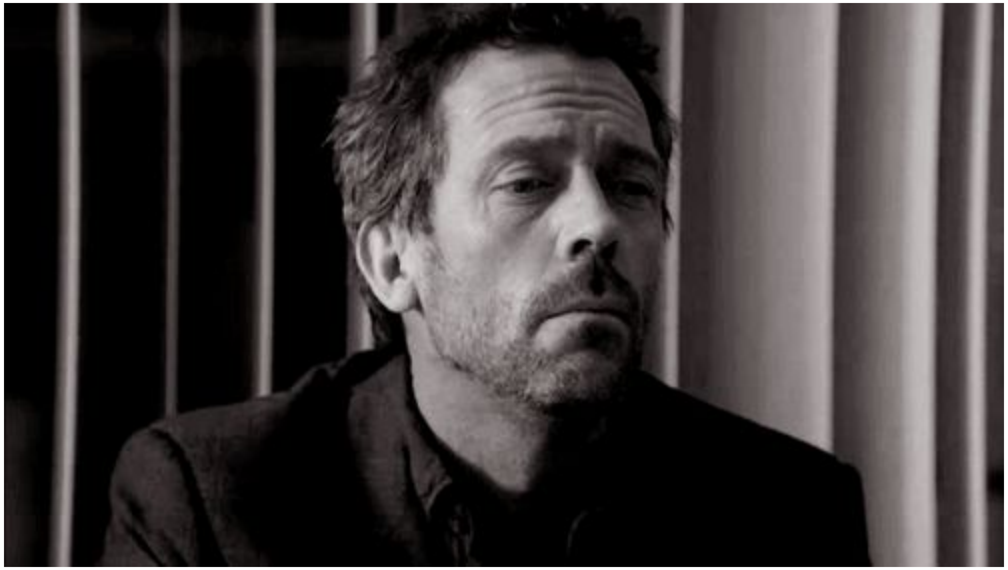
Draw a diagram illustrating this differential element of solid angle, and write down the definition of $d\Omega$.

Blackboard Solution 2: Solid Angle

$$\mathrm{d}\Omega \equiv \frac{\mathrm{d}A}{r^2} = \sin(\theta)\ d\phi\, d\theta$$

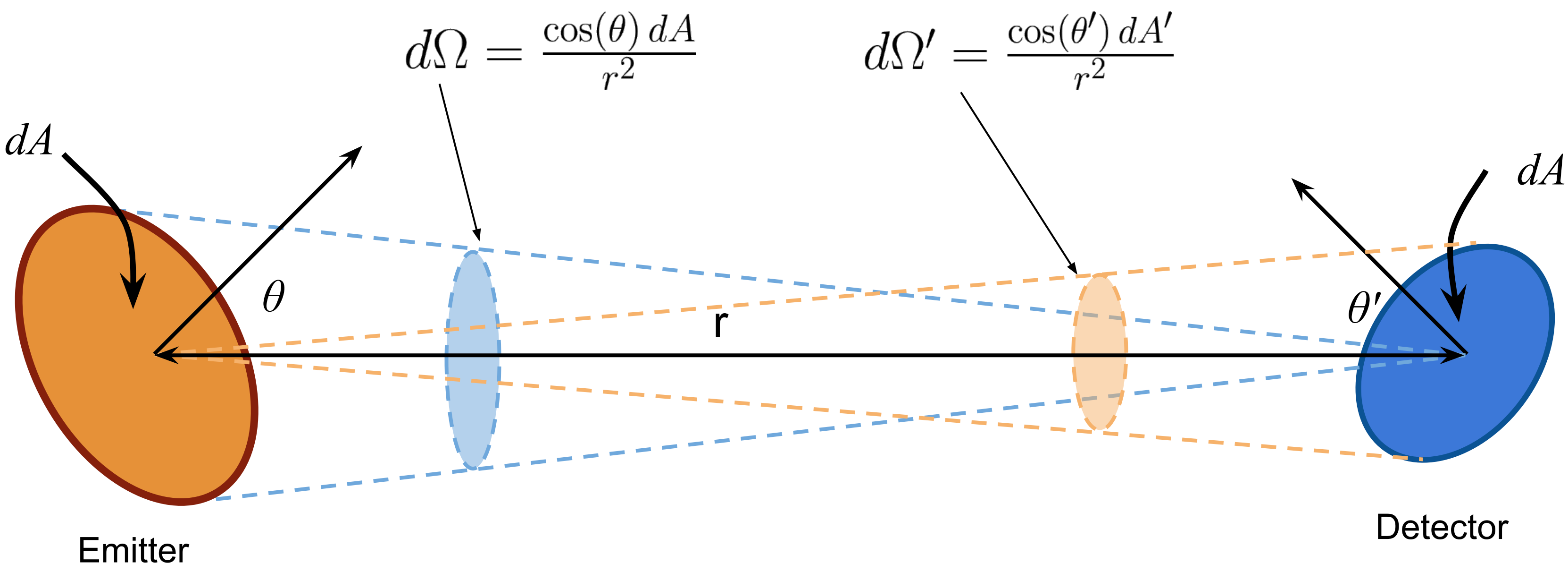


Why is Solid Angle so Important?



Proof that Intensity is Conserved

Suppose we have a universe with a single emitter and a single detector, then, by conservation of energy, the power emitted into the solid angle of the detector equals the power received from the solid angle of the emitter

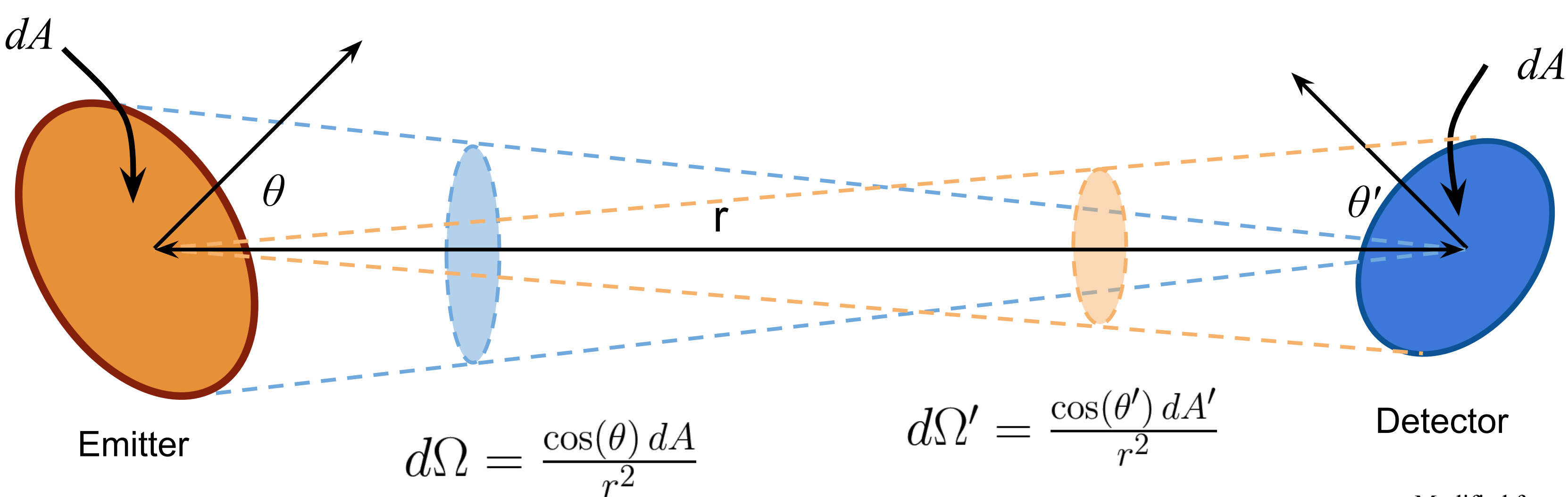


Modified from AstroBaki

Proof that Intensity is Conserved

Suppose we have a universe with a single emitter and a single detector, then, by conservation of energy, the power emitted into the solid angle of the detector equals the power received from the solid angle of the emitter

$$P_e = I_e \cos(\theta)\ dA\, d\Omega'$$
$$P_r = I_r \cos(\theta')\ dA'\, d\Omega$$



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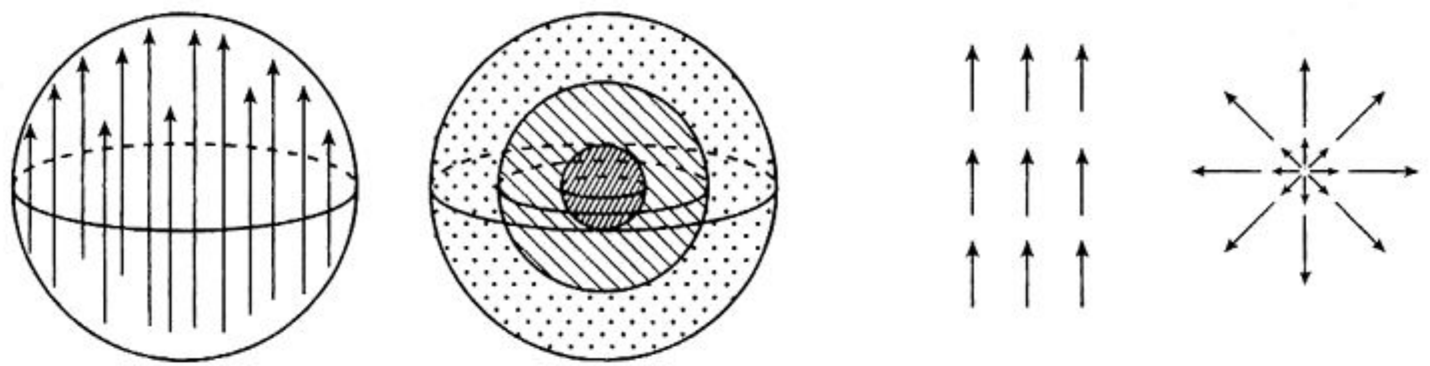
Isotropic point sources



Homogeneous
Not isotropic

Isotropic
Not homogeneous

Homogeneous
Not isotropic

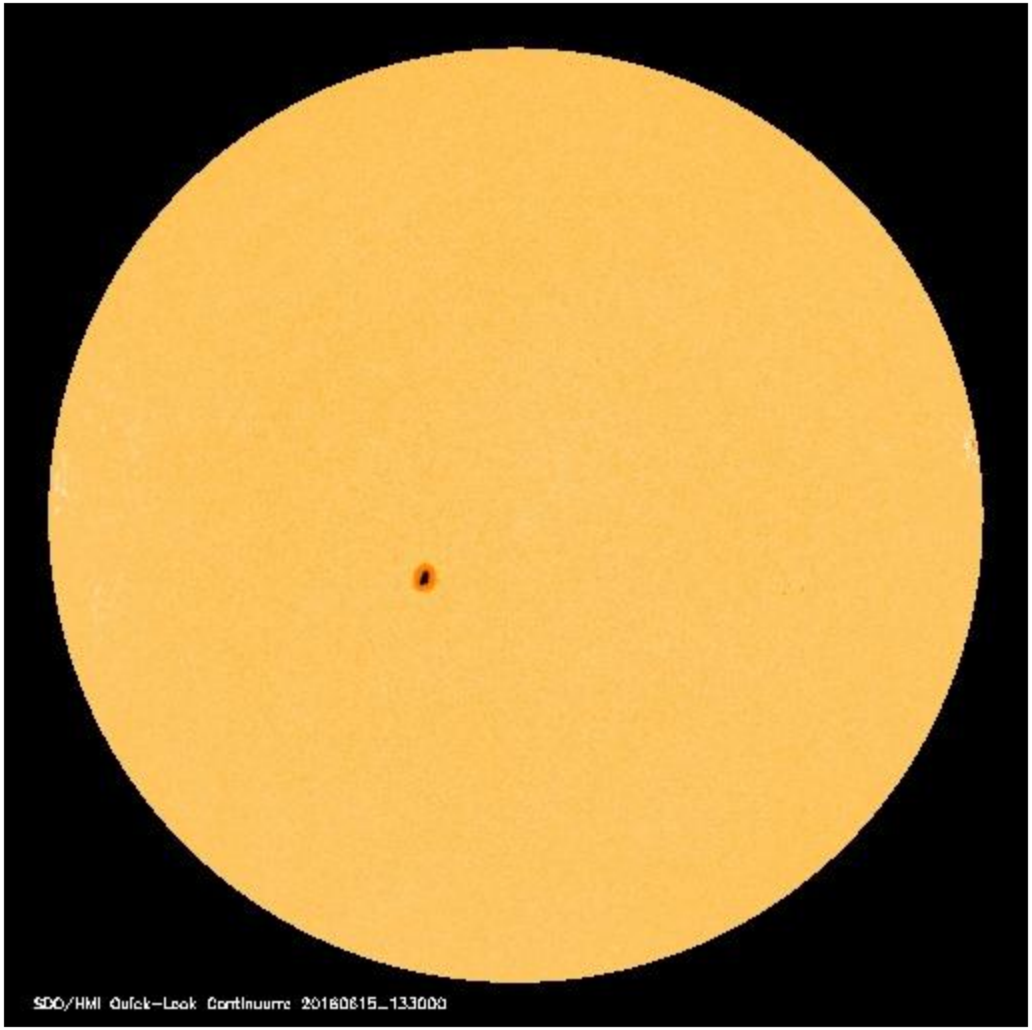
Isotropic
Not homogeneous

Isotropic sources, even non-point sources, have the same appearance from any angle.

Isotropic sources

An isotropic source looks the same from any angle. This means all rays leaving the source have the same intensity. In physics, an isotropic radiator is a point radiation source. Spherical sources with uniform brightness are approximately isotropic radiators at a distance.

As we will discuss later, blackbodies are uniform emitters



Moments of Intensity

1st Moment - Specific Flux

Specific flux, in radiation, is a quantity with units of

energy per *time* per *frequency* per *area*

We must remember that intensity is a directional quantity, but it's emitting area is not. While we can describe intensity from the point of view of either the emitter or the detector, we must always include in the description the relative angle between the EM-wave ($\hat{k}$) and the detector or emitting direction ($\hat{\Omega}$).

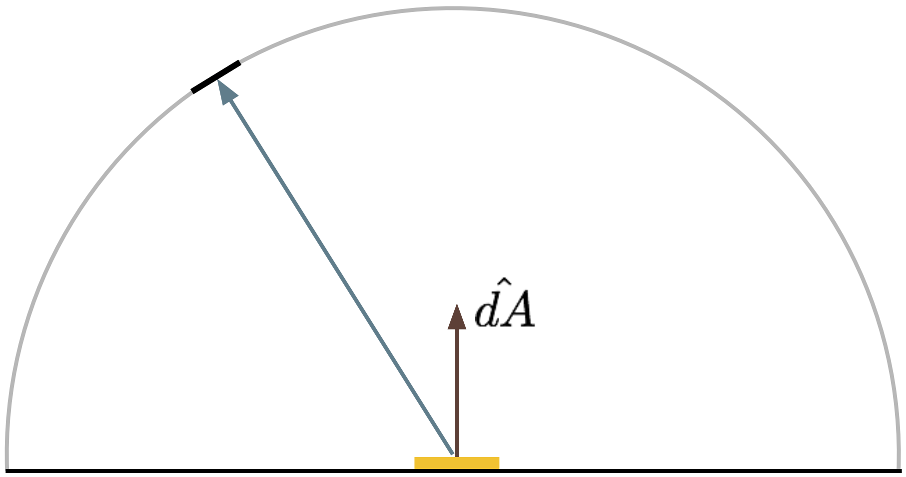
$$dE = F_\nu \, dA \, d\nu \, dt$$

$$F_\nu = \int I_\nu(\theta, \phi) \ \cos\theta \ d\Omega$$

Question 3: Flux from Patch

There is a uniformly, isotropically emitting patch. This means in every direction the intensity is constant, B.

What is the flux, F_ν , emitted into space?



Question 3: Flux from Patch

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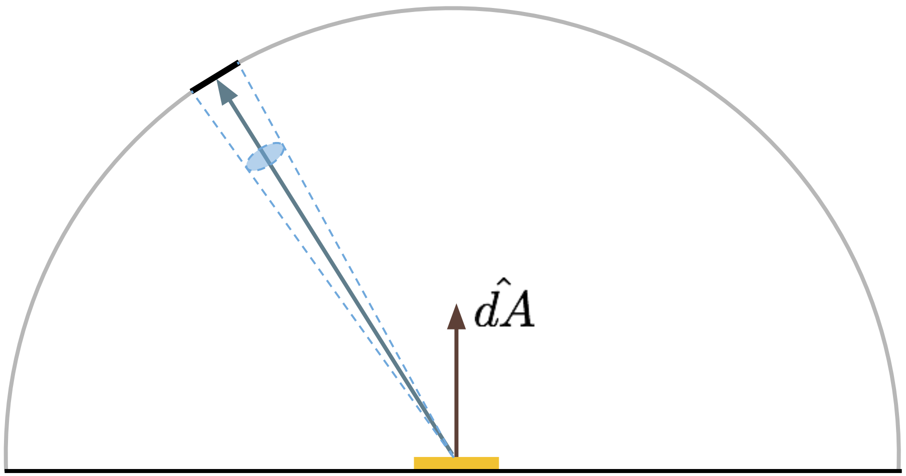
What is the flux, F_ν , emitted into space?

Things to think about:

- Units of flux: erg s^{-1} cm^{-2} Hz^{-1}
- What area does a small patch of sky see?

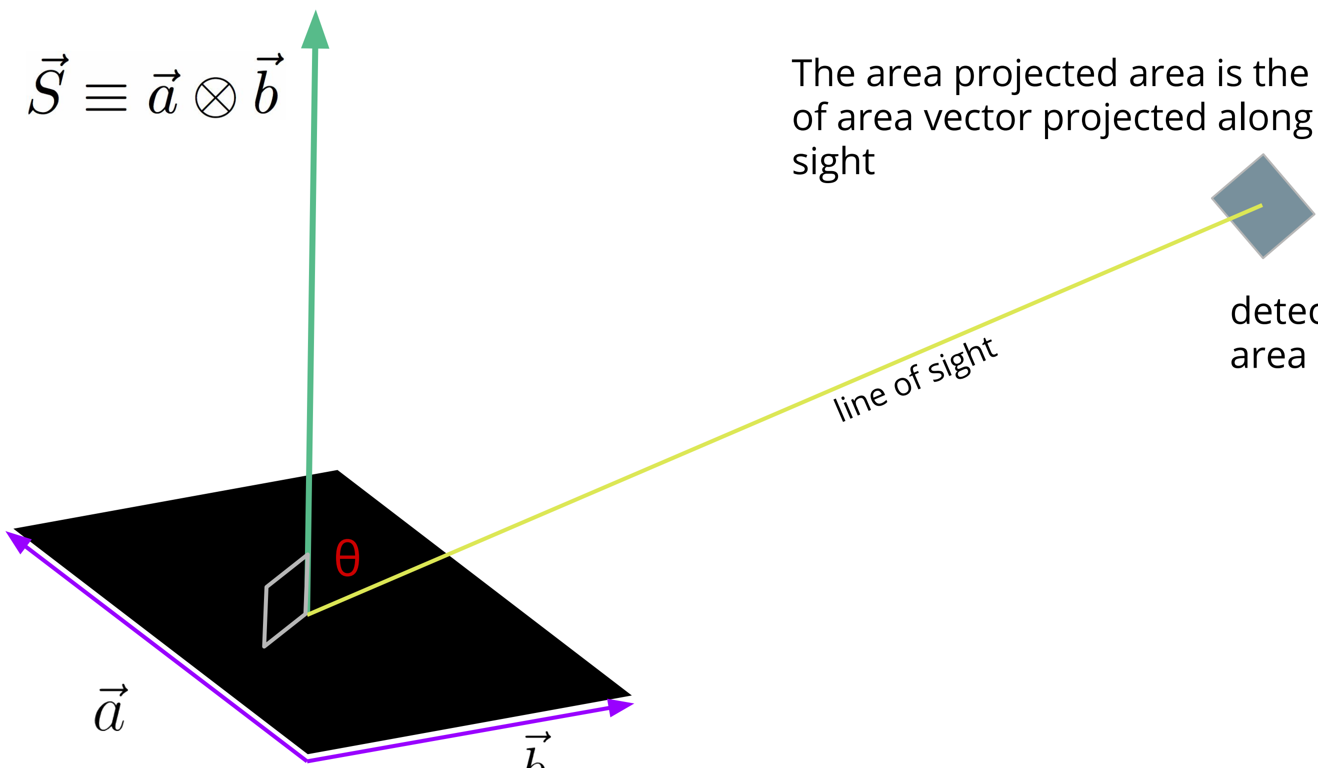


Projected area of a patch

The magnitude of an area vector is the area. We often work with normal vectors, which are normalized area vectors

$$\hat{n} \equiv \frac{\vec{a} \otimes \vec{b}}{||\vec{a} \otimes \vec{b}||}$$

The area projected area is the magnitude of area vector projected along the line of sight



detector sees
area = $|\boldsymbol{S}| \cos(\theta)$

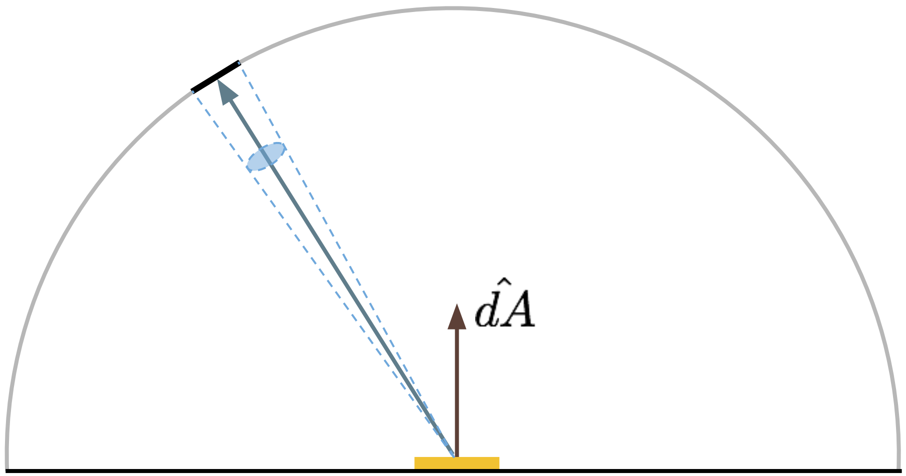
Flux from Patch

"detector" sees

$$I_\nu \; \hat{dA} \cdot \hat{d\Omega} = I_\nu \cos(\theta) \; dA \, d\Omega$$

being emitted from the patch. So the intensity it sees is: $I_\nu \cos(\theta)$

Integrated this over the entire sky.



Flux from Patch

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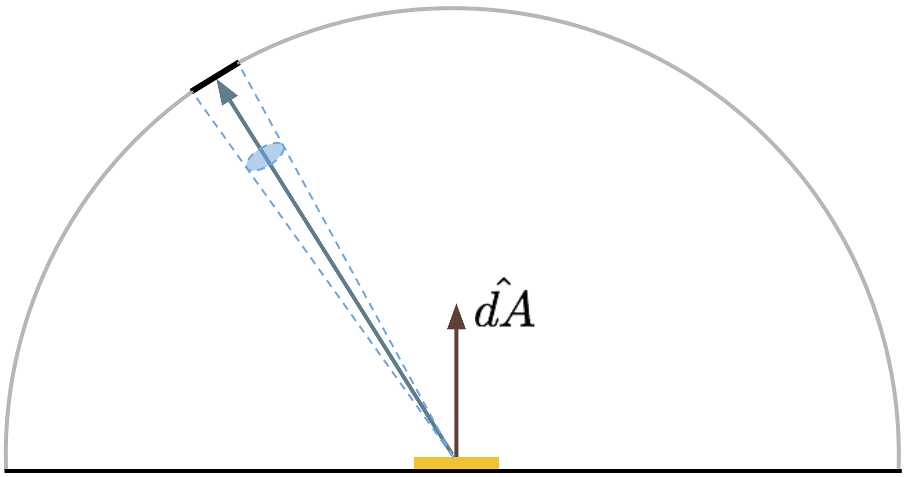
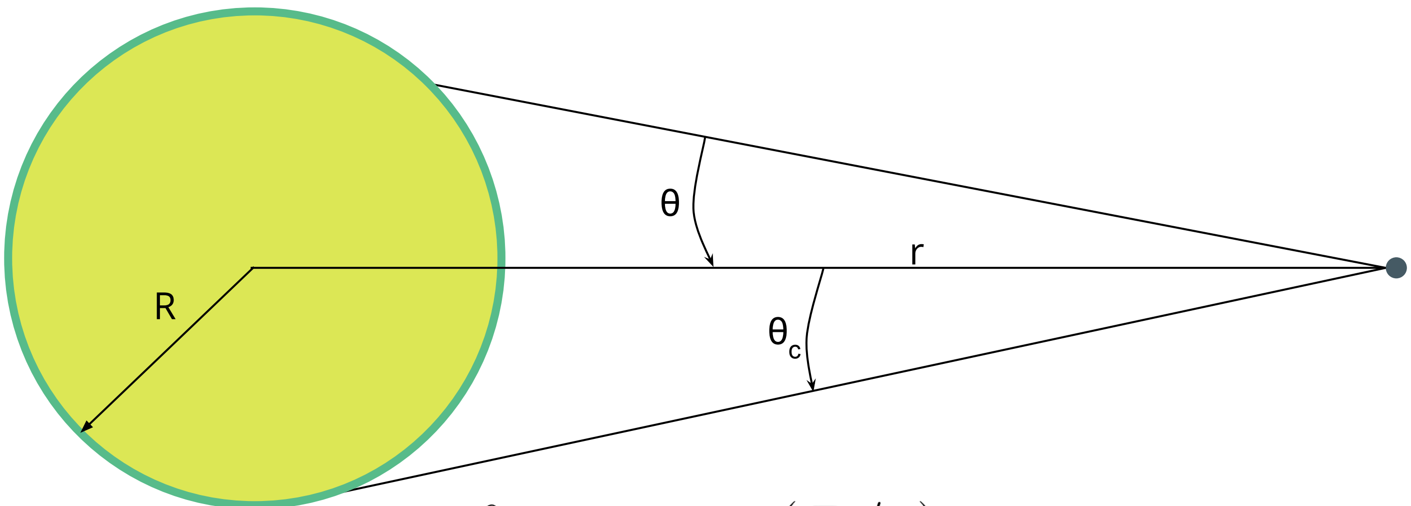


$$\begin{aligned} F &= \int I \cos\theta \; d\Omega \\ &= I \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta \;\; \cos\theta \;\; d\theta \\ &= \pi I \end{aligned}$$

Switching perspectives…

Question 4: Flux from Sphere

Calculate the flux received from the solid angle of a sphere. The sphere has uniform brightness I and as such is isotropic (all rays leaving the sphere have the same brightness).

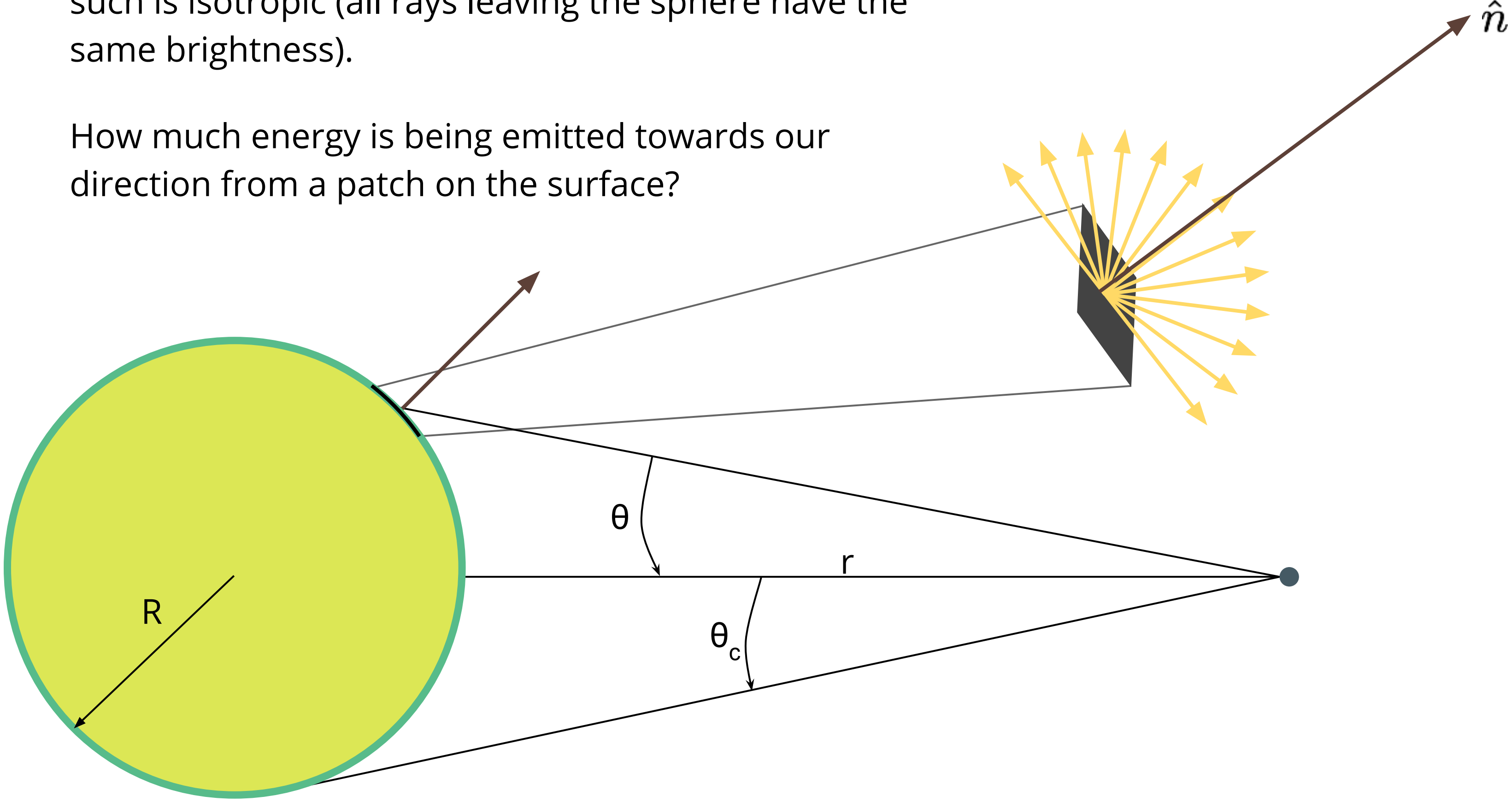


$$\theta_c = \arcsin(R/r)$$

Question 4: Flux from Sphere

Calculate the flux received from the solid angle of a sphere. The sphere has uniform brightness I and as such is isotropic (all rays leaving the sphere have the same brightness).

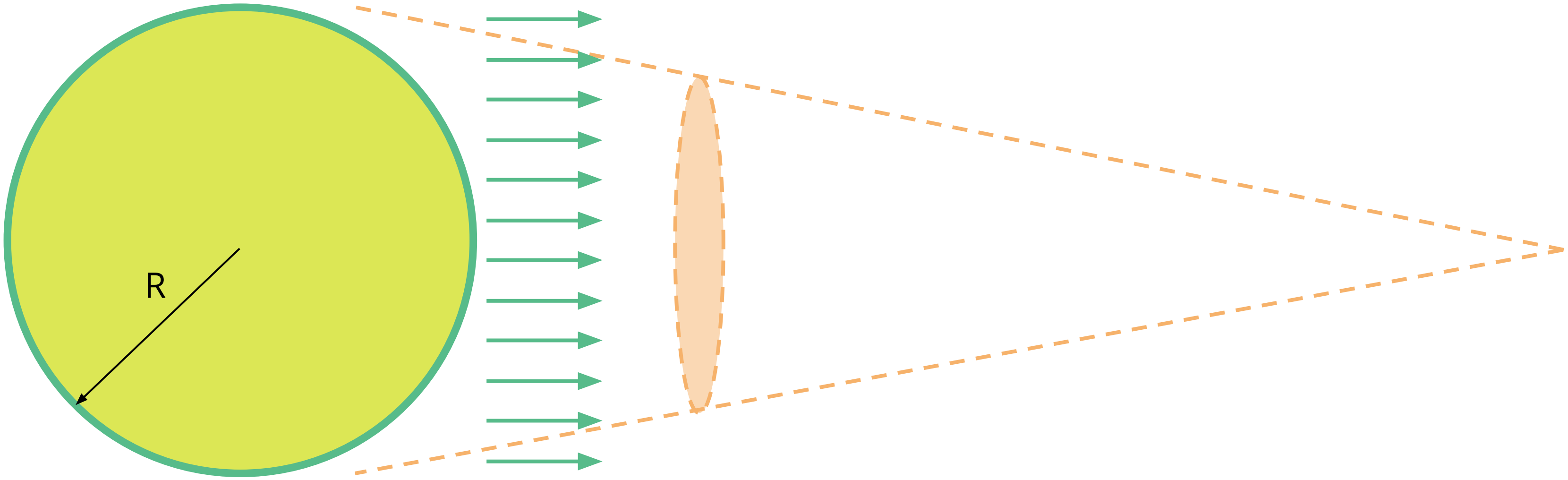
How much energy is being emitted towards our direction from a patch on the surface?



So what is the solid angle of a Sphere?

This must be done far away from the sphere so that we can consider parallel rays

The question boils down to, what is the projected solid angle of a sphere?



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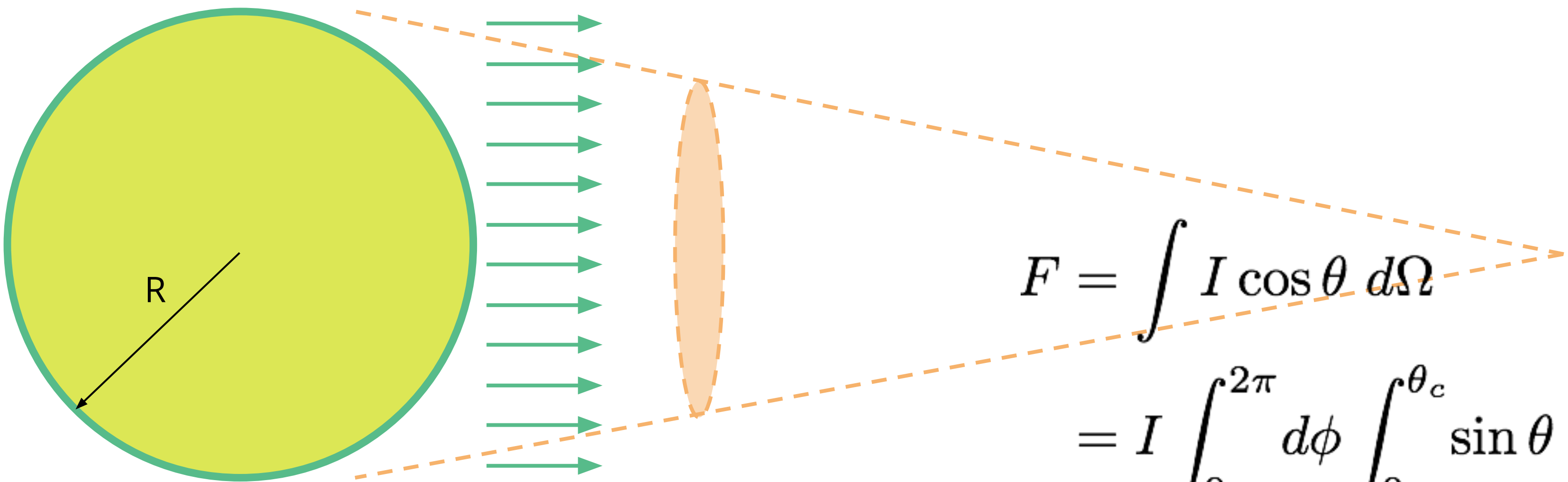


$$
\begin{aligned}
F &= \int I \cos\theta \; d\Omega \\
&= I \int_0^{2\pi} d\phi \int_0^{\theta_c} \sin\theta \;\; \cos\theta \;\; d\theta \\
&= \pi I \sin^2 \theta_c
\end{aligned}
$$

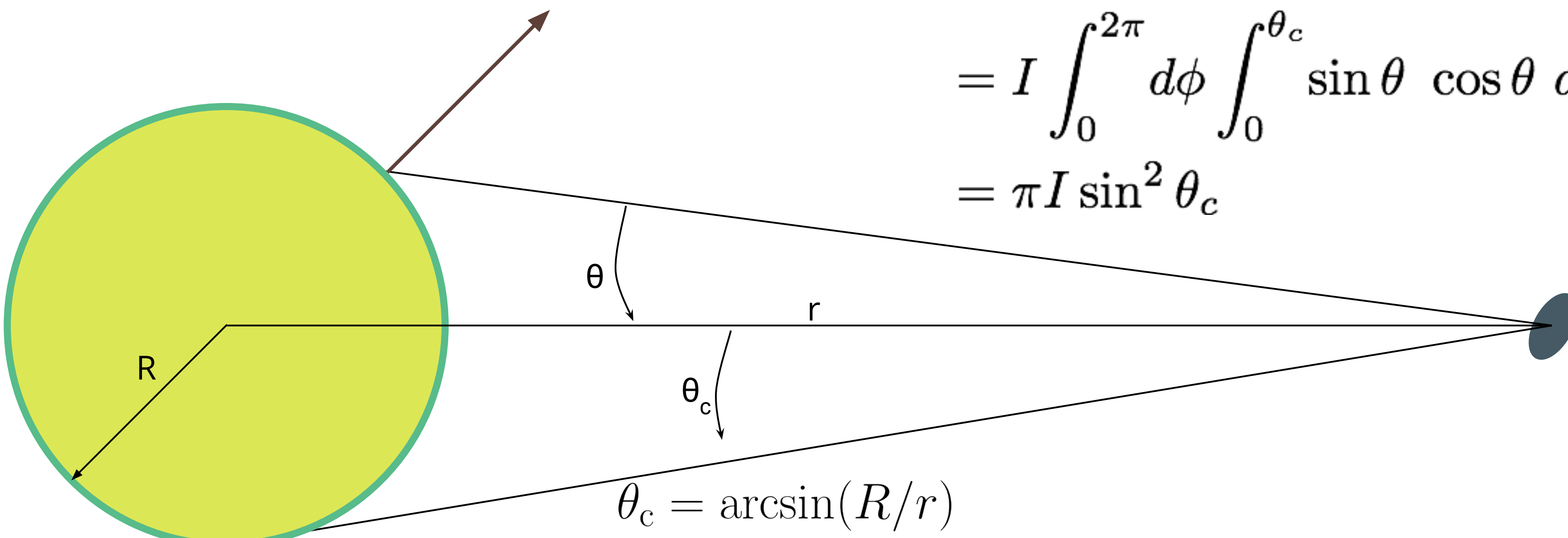
Question 3: Flux

Flux is the amount of energy passing through a surface. Calculate the flux from a sphere of uniform brightness B. Such a source is isotropic (all rays leaving the sphere have the same brightness).

$$F = \int I \cos\theta \; d\Omega$$

$$= I \int_0^{2\pi} d\phi \int_0^{\theta_c} \sin\theta \;\; \cos\theta \; d\theta$$

$$= \pi I \sin^2\theta_c$$



$$\theta_c = \arcsin(R/r)$$

Optical depth

$$\tau \equiv \int_{s_{in}}^{s_{out}} \alpha_\nu \, ds$$

$$\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu$$

Blackbody Radiation

$$B_\nu(T) \equiv \frac{2\nu^2}{c^2} \frac{h\nu}{e^{\left(\frac{h\nu}{kT}\right)} - 1}$$

stopped here

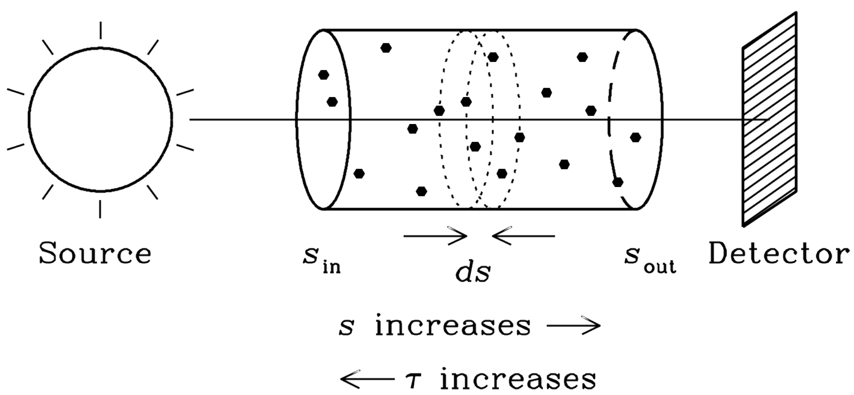
Radiative Transfer

Radiative Transfer

In free space, intensity is conserved along a ray

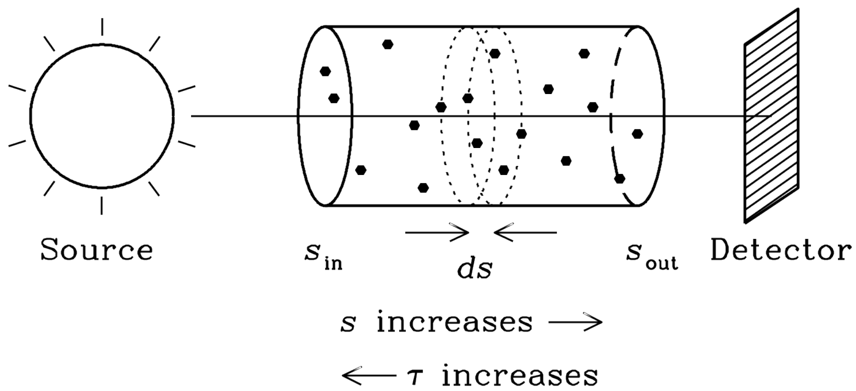
$$\frac{d\,I_\nu}{d\,s} = 0$$

But what if there is stuff in-between us and the source...



Absorption

If the material has some linear absorption coefficient κ (cm^{-1}), the probability of being absorber per unit length, then we add a *sink* term



$$\frac{d\ I_\nu}{d\ s} = -\alpha_\nu I_\nu$$

Question 4: 1D Radiative Transfer with Absorption

Solve the 1D radiative transfer equation for the propogation of a ray through a medium with

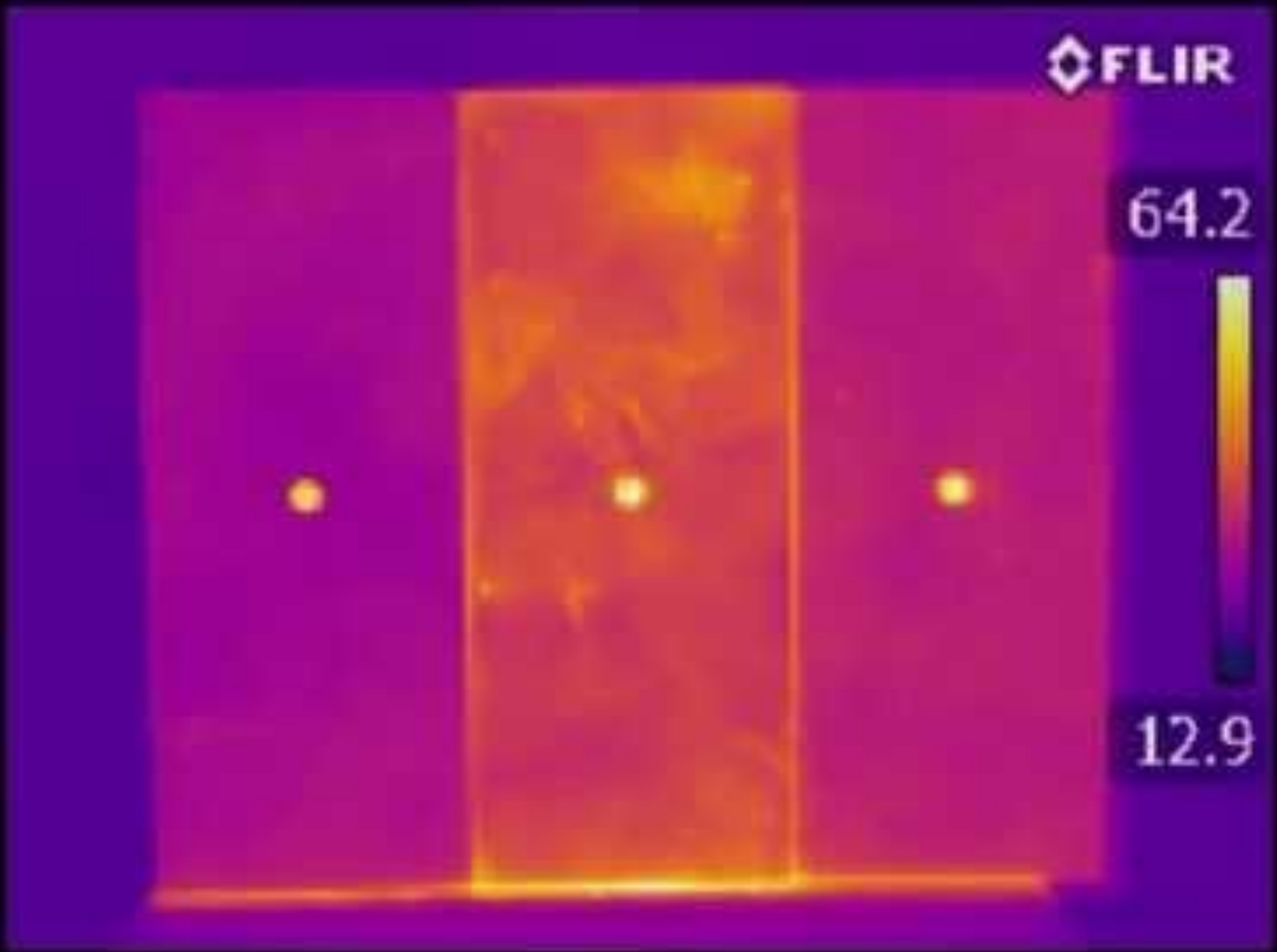
$$\frac{d\ I_\nu}{d\ s} = -\alpha_\nu I_\nu$$

Thermal Radiation I

Thermal radiation is electromagnetic radiation generated by the thermal motion of charged particles in matter. The energy distribution follows the Planck distribution, also known as the blackbody distribution.

$$B_\nu(T) \equiv \frac{2\nu^2}{c^2}\frac{h\nu}{e^{\left(\frac{h\nu}{kT}\right)} - 1}$$

An object is a blackbody if it is opaque to its own radiation. They are also defined as perfect emitters and perfect absorbers. It has previously been known as cavity radiation.


FLIR
64.2
12.9

Specific Intensity

We can think of intensity from two points of view

- Emission
    - The energy emitted from a small patch into some solid an
- Detection
    - The energy received on a small patch from some solid ang