Wage Constant Derivation

The conditional forecast exercise requires formal assumptions about both the immediate path and steady state value of V/U; both of these assumptions are described in detail in the paper. Because of the assumed steady state for V/U, however, we must make an adjustment to the constant term of the wage equation for the simulation. The following text details how to derive the adjustment term, β_{20} in the wage equation below.

Begin with the empirical wage and price equations. Recall that for any period t, the empirical wage equation is

$$\begin{aligned} gw_t = &\beta_1 gw_{t-1} + \beta_2 gw_{t-2} + \beta_3 gw_{t-3} + \beta_4 gw_{t-4} \\ &+ \beta_5 cf1_{t-1} + \beta_6 cf1_{t-2} + \beta_7 cf1_{t-3} + \beta_8 cf1_{t-4} \\ &+ \beta_9 magpty_{t-1} + \beta_{10} vu_{t-1} + \beta_{11} vu_{t-2} + \beta_{12} vu_{t-3} + \beta_{13} vu_{t-4} \\ &+ \beta_{14} catchup_{t-1} + \beta_{15} catchup_{t-2} + \beta_{16} catchup_{t-3} + \beta_{17} catchup_{t-4} \\ &+ \beta_{18} dummy_{q2} + \beta_{19} dummy_{q3} \\ &+ \beta_{20}, \end{aligned}$$

where $dummy_{q_i} = 1$ if and only if quarter $q_i \in \{\text{Q2 2020}, \text{Q3 2020}\}$, subject to homogeneity constraint

$$\sum_{i=1}^{8} \beta_i = 1.$$

Likewise, the empirical price equation is

$$\begin{aligned} gcpi_t = &\gamma_1 magpty_t + \gamma_2 gcpi_{t-1} + \gamma_3 gcpi_{t-2} + \gamma_4 gcpi_{t-3} + \gamma_4 gcpi_{t-4} + \\ &+ \gamma_6 gw_t + \gamma_7 gw_{t-1} + \gamma_8 gw_{t-2} + \gamma_9 gw_{t-3} + \gamma_{10} gw_{t-4} \\ &+ \gamma_{11} grpe_t + \gamma_{12} grpe_{t-1} + \gamma_{13} grpe_{t-2} + \gamma_{14} grpe_{t-3} + \gamma_{15} grpe_{t-4} \\ &+ \gamma_{16} grpf_t + \gamma_{17} grpf_{t-1} + \gamma_{18} grpf_{t-2} + \gamma_{19} grpf_{t-3} + \gamma_{20} grpf_{t-4} \\ &+ \gamma_{21} shortage_t + \gamma_{22} shortage_{t-1} + \gamma_{23} shortage_{t-2} + \gamma_{24} shortage_{t-3} + \gamma_{25} shortage_{t-4} \\ &+ \gamma_{26} \end{aligned}$$

subject to

$$\sum_{i=2}^{10} \gamma_i = 1.$$

Also recall that catchup is assumed to be specified as follows:

$$catchup_t = \frac{1}{4}\left(gcpi_t + gcpi_{t-1} + gcpi_{t-2} + gcpi_{t-3}\right) - cf1_t.$$

To evaluate the steady state value of the wage price constant β_{20}, set all variables equal to their long-run equilibrium values. In other words, we have

$$gw^* = \left(\sum_{i=1}^{4} \beta_i\right) gw^* + \left(\sum_{i=1}^{8} \beta_i\right) cf1^* + \beta_9 magpty^* + \left(\sum_{i=10}^{14} \beta_i\right) vu^* + \left(\sum_{i=14}^{17} \beta_i\right) catchup^* + \beta_{20}$$

and

$$\begin{aligned} gcpi^* = \gamma_1 magpty^* + \left(\sum_{i=2}^{6} \gamma_i\right) gcpi^* + \left(\sum_{i=6}^{10} \gamma_i\right) gw^* + ... \\ ... + \left(\sum_{i=11}^{15} \gamma_i\right) grpe^* + \left(\sum_{i=16}^{20} \gamma_i\right) grpf^* + \left(\sum_{i=21}^{25} \gamma_i\right) shortage^* + \gamma_{26}. \end{aligned}$$

Note that long run catch up is zero, yielding the fact that long-run expectations are equal to inflation, or $cf1^* = gcpi^*$. Furthermore, dummy variables are assumed zero in the long run. Using the equations above, it is simple to derive the following result:

$$\begin{aligned} \beta_{20}^* = -\left(\sum_{i=10}^{13} \beta_i\right) vu^* - \left(1 - \sum_{i=1}^{4} \beta_i\right) \Bigg[\left(\frac{\gamma_1}{1 - \sum_{j=2}^{5} \gamma_j} + \frac{\beta_9}{1 - \sum_{i=1}^{4}}\right) magpty^* + ... \\ ... + \frac{\sum_{k=21}^{25}}{1 - \sum_{j=2}^{5}} shortage^* + \frac{\gamma_{26}}{1 - \sum_{j=2}^{5} \gamma_j}\Bigg] \end{aligned}$$

Note that if any changes are made to the specification of the estimated wage or price equations, the above calculation of β_{20}^* must change as well.