Supplementary Section

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1 Factors in Debris Dynamics

In our model we treat space debris s, as all those objects in space, which orbit earth, specifically in low orbits, and which do not carry a specific purpose to humanity. The following list provides a detailed description of different processes, which drive the amount of space debris. Further, the list shows how we expect each process to mathematically affect our model.

Explosions and Mission Erosion

To date, the most prominent source of space debris is due to exploding debris objects. Old tanks and rockets that remained in orbit can explode unexpectedly due to remaining elements of fuel or old batteries. Other fragmentation events, such as shedding of coatings, insulation, electrical subsystems, solar panels or fuel leakage are believed to contribute less to the overall number of space debris.
We distinguish between explosions with rate a_{exp} and other more general forms of non-collision fragmentation events with rate a_{frac}.

$$\Delta s = a_{exp} s \quad , \quad \Delta s = a_{frac} s \quad .$$

Anti-Satellite Actions / Deliberate Actions

Deliberate destruction of satellites and rockets can lead to a drastic accumulation of space debris. The number of anti-satellite actions is hard to predict and will not depend on the number of satellites, but purely on political incentives. We model this process therefore as independent from the amount of missions or space debris.

$$\Delta s = a_{asa}$$

Debris Cleanup

Debris Cleanup includes efforts to actively remove debris from space as well as passive removal due to atmospheric drag.
Passive debris cleanup can be expected to scale linearly with space debris in orbit. However, depending on the altitude and orbit of debris objects, rates of passive removal can differ substantially. Active debris cleanup rates will depend on innovations in clean up technology and societal incentives. For the sake of parsimony, we study the basic case of linear debris cleanup, accounting for both passive as well as active clean up procedures.

$$\Delta s = -b_{dc} s$$

Launched Missions

All active missions increase the amount of debris in space eventually, due to fracture of materials, payload erosion, fuel leakage, explosions and other disintegration events.
$M(s)$ is the debris-dependent rate of active missions. A natural assumption could be that new missions scale with the probability of casualties, such that $sM(s) = \text{const.}$, keeping the overall probability of collisions with space debris constant. Economical incentives for exploitation could alternatively also

Parameter	Influence
α	Collisions with Debris
β	Collisions with Missions, Cleanup
γ	Collisions with Missions, Anti-Satellite Actions and New Missions
δ	New Missions

Table 1: This table shows how the aggregated parameters in Eq. 2 are influenced by various processes in the model. The rate of new launches, collisions, fracture and explosions will contribute to aggregate parameters α, β, γ and δ.

render $M(s)$ to be independent of space debris, in other words, relevant agents are ignorant towards space debris populations.

$$\Delta s = a_{new} M(s)$$

Collisions

Collisions can occur among active missions or uncontrolled space debris, leading to vast fragmentation events. Throughout the space era, such events have been rare, but if debris continues to accumulate, the contribution of collisions to the creation of new space debris will become substantial.
We model collisions as interactions within the population of space debris and active missions following assumptions of quadratic dependence often made within particle kinetics:

$$\Delta s = a_{coll} s^2, \quad \Delta s = a_{coll} s M(s) \quad .$$

2 Model Formulation

We collect the above defined processes contributing to space debris in a differential equation describing the dynamics of debris numbers,

$$\dot{s} = a_{coll} s^2 + a_{coll} s M(s) - b_{dc} s + a_{exp} s + a_{frac} s + a_{asa} + a_{new} M(s) \quad , \tag{1}$$

and after aggregation of parameters:

$$\dot{s} = \alpha s^2 + \beta s + \gamma + \delta s^{-1} \quad . \tag{2}$$

Here α controls the amount of collisions, while β captures processes of debris removal and degradation. The last two terms, containing γ and δ, change with the rate of deliberate actions and new missions. Details on the parameter aggregation can be found in table 1.

3 Model With Lethal Non-Trackable Debris

The model formulated above does not make a destinction between trackable and non-trackable debris. Debris objects, which are too small to be tracked, however pose a serious risk and can - despite their size - collide with payloads. Since lethal non-trackable debris is not directly observed, their actual numbers can only be estimated based on detailed simulations of the space environment, which is beyond this manuscript. Here, we present an extension of our basic model, including an additional distinct population of non-trackable debris with a priori unknown size z. The extended model includes debris creation from collisions with non-trackable debris occuring at rate ω, extending Eq. 2 to become

$$\dot{s} = \alpha s^2 + \omega\, s z + \beta s + \gamma + \delta s^{-1} \quad . \tag{3}$$

This can be rewritten as

$$\dot{s} = \left(\alpha + \omega \frac{z}{s}\right) s^2 + \beta s + \gamma + \delta s^{-1} \quad . \tag{4}$$

Small debris enhances the collision rate α to become a debris dependent rate $\alpha'(z/s) = \alpha + \omega z/s$ compared with the baseline model (Eq. 2). If the relative frequency of z to s stays constant at all times α' becomes a constant and it is sufficient to simply use a higher level of α in Eq. 2 to account for non-trackable debris. Such an assumption is supported by phenomenological size distributions of debris often taking a fixed power-law form.

In the main text, such an approximation, $z/s = \text{const.}$, is studied by including small impactor events into the calibration procedure of α. Alternatively z and s could grow at different rates, resulting in a potentially growing effective collision rate α'. The model analysed in the main text therefore only presents a best-case scenario with respect to small debris.

4 Model Behavior

$\delta = 0$, New Missions do not adapt

If missions do not adapt, but stay constant then we can set $\delta = 0$ and solving Eq. 2 for $\dot{s} = 0$ yields the solution for the fixed points:

$$s_1^* = -\frac{\beta}{2\alpha}\left(1 + \sqrt{1 - \frac{4\gamma\alpha}{\beta^2}}\right) \tag{5}$$

$$s_2^* = -\frac{\beta}{2\alpha}\left(1 - \sqrt{1 - \frac{4\gamma\alpha}{\beta^2}}\right) \tag{6}$$

Either β or γ have to be negative in order for the dynamics to yield stability for any $s \geq 0$. A condition for stable solutions can be derived by requiring the discriminant to be greater than 0,

$$1 - \frac{4\gamma\alpha}{\beta^2} \geq 0 \quad ,$$

resulting in i

$$\beta \geq \sqrt{4\gamma\alpha} \quad \text{and} \quad \beta \leq -\sqrt{4\gamma\alpha} \tag{7}$$

Only the second inequality provides physical solutions with $s > 0$.

Further inspection of Eq. 5 allows a stability analysis of a scenario with no further launches, $\gamma = 0$. The critical value s_1^*, beyond which a population of debris s becomes instable, even in the absence of further space exploitation, then equals

$$s^*_{1(\gamma=0)} = -\frac{\beta}{\alpha} \quad . \tag{8}$$

$\delta > 0$, More Debris, Fewer Missions

If $\delta > 0$, indicating a debris-compensating strategy, equation 2 becomes of third order and analytical solutions are harder to derive. We utilized numerical methods instead.

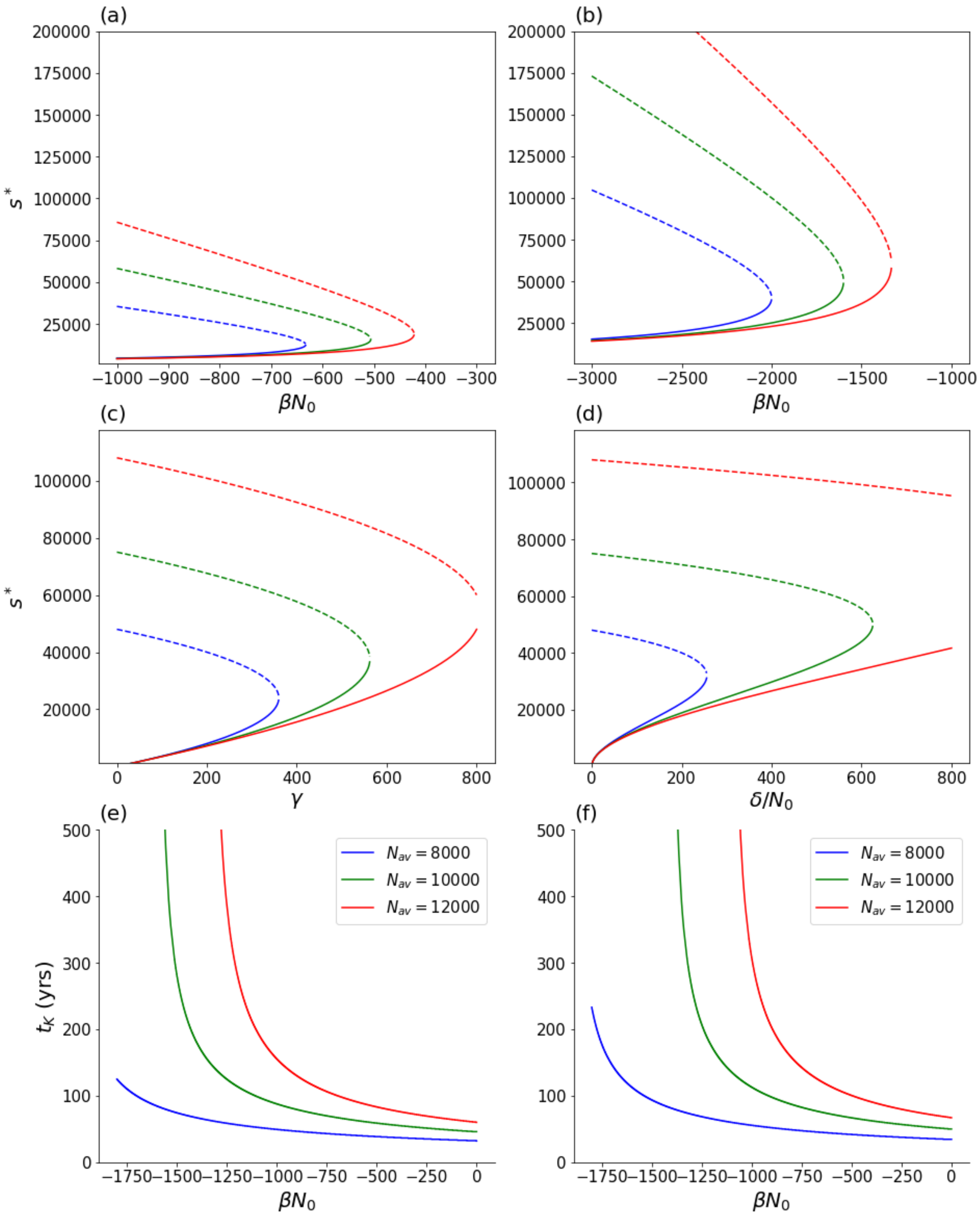

Figure 1: **The influence of different values of effective debris N in the calibration procedure**: For comparison, all panels correspond to **Fig. 3-5** presented in the main text, but show how they change with different values of N_{av}. All green lines show the results for $N_{av} = 10,000$, presented in the main text (blue - 8000, red - 12000). Increasing N_{av}, will decrease the effective values of α as resulting from the calibration procedure. Doubling the value of N_{av}, decreases α four times. **(a-d)**: This results in higher values of s^* for both unstable and stable fixed points, allowing stabilization of space debris at higher overall densities of debris. **(e-f)**: If rates of collision α decrease with N_{av}, space congestion is reached at later times t_k. On the other hand, estimates of α increase with N_{av}, and space congestion results earlier. Parameter Choices, **(a)** [$\gamma = 100$, $\delta = 0$, $\alpha N_{av}^2 = 40$], **(b)** [$\gamma = 1000$, $\delta = 0$, $\alpha N_{av}^2 = 40$], **(c)** [$\delta = 0$, $\beta = -1200 N_0$, $\alpha N_{av}^2 = 40$], **(d)** [$\gamma = 0$, $\beta = -1200 N_0$, $\alpha N_{av}^2 = 40$], **(e)** [$\delta = 0$, $\gamma = 1000$, $\alpha N_{av}^2 = 40$], **(f)** [$\delta / N_0 = 1000$, $\gamma = 0$, $\alpha N_{av}^2 = 40$]