John Beverley
Lunchtime Talk 10/24/2014

# A Taste of Curry's Paradox

**I.** Logical Consequence and Motivation

- Logical consequence in formal systems, one constituent: T-scheme
  - **T-Scheme**: $\varphi \leftrightarrow Tr[\varphi]$
  - Virtue: Captures intuitions about truth, i.e. what 'true' means

**II.** Ingredients of **C-Curry**

- T-Scheme coupled with **Diagonalization Sentence**: $\varphi \leftrightarrow (Tr[\varphi] \rightarrow \psi)$…

| | | |
|---|---|---|
| 1. | $\quad\varphi$ | Assume for Conditional Proof |
| 2. | $\quad\varphi \rightarrow \psi$ | 1, Substitution via Diagonalization Sentence |
| 3. | $\quad\psi$ | 1,2 MP |
| 4. | $\varphi \rightarrow \psi$ | 1-3 Discharge Conditional Assumption |
| 5. | $\varphi$ | 4, Substitution via Diagonalization Sentence |
| 6. | $\psi$ | 4,5, MP |

- …*trivializes* the formal system.
  - Options: Removing Conditional Proof or Removing Contraction
    - **Conditional Proof**: If $A \vdash B$ then $\vdash (A \rightarrow B)$
    - **Contraction**: If $A, A \vdash B$ then $A \vdash B$

**III.** Ingredients of **V-Curry**

- Logical consequence in formal systems, another constituent: V-Scheme
  - **V-Scheme**: $\alpha \vdash \beta \leftrightarrow Val[\alpha, \beta]$
  - Virtue: Captures intuitions about validity, i.e. what 'follows from' means
    - **VI**: If $\alpha \vdash \beta$ then $\vdash Val[\alpha, \beta]$ (Validity Introduction 'Rule')
    - **VE**: $\alpha, Val[\alpha, \beta] \vdash \beta$ (Validity Elimination 'Rule')

- VI, VE, coupled with the analogous Diagonalization Sentence: $\alpha \leftrightarrow Val[\alpha, \beta]$…

| | | |
|---|---|---|
| 1. | $\quad\alpha$ | Assume for VI |
| 2. | $\quad Val[\alpha , \beta]$ | 1, Substitution via Diagonalization Sentence |
| 3. | $\quad\beta$ | 1,2 VE |
| 4. | $\alpha \rightarrow \beta$ | 1-3 Discharge VI Assumption |
| 5. | $\alpha$ | 4, Substitution via Diagonalization Sentence |
| 6. | $\beta$ | 4,5, VE |

- …*trivializes* the formal system.
  - Options Considered (Beall/Murzi): Removing VI or Removing Contraction
    - But, VI is part of the V-Schema (it *is* validity), reject Contraction!

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**IV.** The Error

- V-Scheme parallels T-Scheme, is intuitive, and captures aspect of Logical Consequence
- However, V-Scheme is *not equivalent* to VI and VE (pace Beall/Murzi), recall -
    - V-Scheme: $\alpha \vdash \beta \leftrightarrow \mathrm{Val}[\alpha, \beta]$
    - VI: If $\alpha \vdash \beta$ then $\vdash \mathrm{Val}[\alpha, \beta]$
    - VE: $\alpha, \mathrm{Val}[\alpha, \beta] \vdash \beta$
- However, for *equivalence*, we must change VI, VE, and V-Scheme, perhaps to…
    - V-Scheme*: $\alpha \vdash \beta \leftrightarrow \boldsymbol{\alpha}, \mathrm{Val}[\alpha, \beta]$
    - VI*: If $\alpha \vdash \beta$ then $\vdash \boldsymbol{\alpha}, \mathrm{Val}[\alpha, \beta]$
    - VE*: **If** $\alpha, \mathrm{Val}[\alpha, \beta]$ **then** $\boldsymbol{\alpha \vdash \beta}$
- …or change VE to...
    - VE#: **If** $\mathrm{Val}[\alpha, \beta]$ **then** $\boldsymbol{\alpha \vdash \beta}$
        - However, either option precludes V-Curry (for the same reason, no way to discharge assumptions, only substitutions!)

**V.** Conclusion

- Beall/Murzi attempt to motivate rejecting Contraction by showing the high cost of rejecting the alternative (VI corresponding to Conditional Proof)
    - Yet, they mischaracterize the V-Scheme
    - Thus, the motivation is no motivation at all
- C-Curry is still on the menu, but V-Curry has been removed as logically unappetizing

*****VI.** Supplement

- Despite the above, there are good reasons to reject Contraction
    - Avoids paradoxes other than C-Curry
    - Formal systems with Contraction Conditions (C1-C4) are susceptible to paradox

**C1**: $A \rightarrow B \vdash A \phi B$
**C2**: $A, A \phi B \vdash B$
**C3**: $A \phi (A \phi B) \vdash A \phi B$
**C4**: If $A \vdash B$ then $\vdash A \phi B$

| | | |
|---|---|---|
| 1. | $A$ | Assumption for C4 |
| 2. | $A \phi B$ | 1, Substitution with Diagonalization Sentence: $A \leftrightarrow (A \phi B)$ |
| 3. | $B$ | 1, C2 |
| 4. | $A \phi B$ | 1-3 Discharge C4 |
| 5. | $A$ | 4, Substitution of Diagonalization Sentence: $A \leftrightarrow (A \phi B)$ |
| 6. | $B$ | 4,5 C2 |

- Despite the above, there may be good reasons *not* to reject Contraction
    - We may also have to abandon **Cantor's Theorem**…