Controlled gates

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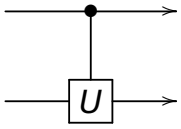
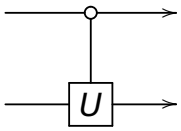
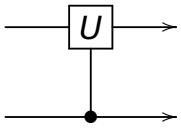
I 4x4



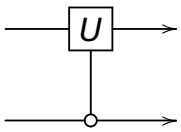
$$= \quad |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{U}} \\ 0 & 0 & & \end{pmatrix}$$



$$= \quad |0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes I \quad = \quad \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{U}} & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$= \quad I \otimes |0\rangle\langle 0| + U \otimes |1\rangle\langle 1| \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & U_{11} & 0 & U_{12} \\ 0 & 0 & 1 & 0 \\ 0 & U_{21} & 0 & U_{22} \end{pmatrix}$$



$$= \quad U \otimes |0\rangle\langle 0| + I \otimes |1\rangle\langle 1| \quad = \quad \begin{pmatrix} U_{11} & 0 & U_{12} & 0 \\ 0 & 1 & 0 & 0 \\ U_{21} & 0 & U_{22} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$