John L. Volakis
ElectroScience Lab
Elecrical Engineering Dept.
The Ohio State University
1320 Kinnear Rd.
Columbus, OH 43212
+1 (614) 292-5846 Tel.
+1 (614) 292-7297 (Fax)
volakis.1@osu.edu (email)

David B. Davidson
Dept. E&E Engineering
University of Stellencosch
Stellenbosch 7600, South Africa
(+27) 21 808 4458
(+27) 21 808 4981 (Fax)
davidson@ing.sun.ac.za (e-mail)

Foreword by the Editors

Chiral materials sprang to prominence as a research topic in the 1990s, in particular with the tantalizing promise of adding additional degrees of freedom to permit enhanced absorption, with obvious applications in "stealth." From a theoretical viewpoint, the presence of additional terms in the constitutive parameters required re-visiting a number of classical analytical solutions, and new solutions including the chiral parameters were derived.

In this contribution, the authors review an extension of the classic Mie-series solution to spheres with chiral coatings, and present a *MATLAB* implementation of the theory, which is available for downloading. Since *non-chiral* bodies – such as dielectric and PEC spheres – can also be solved, their code promises to be very useful whenever benchmark results are needed for code validation. Although a large number of workers presumably have their own codes in this regard, surprisingly few, if any, appear to be publicly available, and this contribution is thus especially welcome.

Finally, it should be noted that both the correct characterization of chiral materials, as well as the benefits (or otherwise) of the use of chiral materials for absorbers, have been controversial topics. In this context, readers might find reference [1] of interest.

1. J. H. Cloete, M. Bingle and D. B. Davidson, "The Role of Chirality and Resonance in Synthetic Microwave Absorbers." *Int. J. Electron. Comm. (AEU)*, 55, 4, July/August 2001, pp. 233-239.

A Graphical User Interface (GUI) for Plane-Wave Scattering from a Conducting, Dielectric, or Chiral Sphere

Veysel Demir[1], Atef Elsherbeni[1], Denchai Worasawate[2], and Ercument Arvas[3]

[1] The Center of Applied Electromagnetic Systems Research (CASER), Electrical Engineering Department
The University of Mississippi
University, MS 38677 USA
E-mail: vdemir@olemiss.edu, atef@olemiss.edu

[2]Electrical Engineering Department
Kasetsart University
Bangkok, Thailand
E-mail: fengdcw@ku.ac.th

[3]Electrical Engineering and Computer Science Department
Syracuse University, Syracuse NY 13244 USA
E-mail: earvas@syr.edu

Abstract

Various numerical techniques have been developed for modeling electromagnetic field propagation in various novel complex media. The validity of these techniques is usually verified by comparison to the exact solutions of canonical problems. Recently, research has focused on chiral media, a subclass of materials known as bianisotropic materials, and numerical techniques have been developed in order to calculate the interaction of electromagnetic fields with chiral objects. One canonical problem for these techniques is plane-wave scattering from a chiral sphere. This paper presents a software package that displays and saves the calculated data for the scattering from a chiral, dielectric. or a perfectly conducting sphere using a friendly graphical user interface (GUI).

Keywords: Chiral media; spheres; electromagnetic scattering by anisotropic media; graphical user interfaces

1. Introduction

The interaction of electromagnetic fields with chiral materials has been studied over the years. Chiral media have been used in many applications involving antennas and arrays, antenna radomes, microstrip substrates, and waveguides. A chiral object is, by definition, a body that lacks bilateral symmetry, which means that it cannot be superimposed on its mirror image either by translation or rotation. This is also known as handedness. Objects that have the property of handedness are said to be either right-handed or left-handed. Chiral media are optically active: a property caused by asymmetrical molecular structure that enables a substance to rotate the plane of incident polarized light, where the amount of rotation in the plane of polarization is proportional to the propagation distance through the medium, as well as to the light wavelength [1-5]. A chiral medium therefore has an effect on the rate of attenuation of the right-hand and left-hand circularly polarized waves. Unlike dielectric or conducting cylinders, chiral scatterers produce both co-polarized and cross-polarized scattered fields. Coating with chiral material has therefore been attempted for reducing the radar cross section of targets.

Electromagnetic wave propagation in chiral and bi-isotropic media has recently been modeled by various numerical techniques in various studies. In most of these studies, the validity of the developed techniques was verified by comparing the numerical results to the results of one-dimensional and two-dimensional problems that have known, exact solutions. For the techniques for solving three-dimensional problems, plane-wave scattering from a chiral sphere was the benchmark. The exact analytical solution of the scattering by a chiral sphere has been introduced by Bohren [6], and a detailed analysis of the solution was given by Worasawate [7]. This formulation has been used for verification of the scattering from arbitrary shaped three-dimensional chiral objects using a Method of Moments analysis [8] and a Finite-Difference Time-Domain analysis [9].

In this contribution, a software package is developed and presented to calculate plane-wave scattering from a chiral sphere. The package involves a user-friendly GUI, which enables the user to enter the scattering parameters and observe the results in near real time, and to save the calculated data and displayed figures. As will be discussed in the following sections, due to the nature of the chiral constitutive relations, the developed program can be used to calculate scattering from a dielectric or a perfectly conducting sphere, as well. The presented program is based on the exact solution provided in [7], which is summarized here for the reader's convenience.

2. Plane Wave Scattering from a Chiral Sphere

The constitutive relations for a chiral media can be written as

$$\bar{D} = \varepsilon\bar{E} - j\kappa\sqrt{\mu_0\varepsilon_0}\bar{H}\,, \tag{1}$$

$$\bar{B} = \mu\bar{H} + j\kappa\sqrt{\mu_0\varepsilon_0}\bar{E}\,, \tag{2}$$

where κ is the chirality parameter. Equations (1) and (2) can be alternatively written as

$$\bar{D} = \varepsilon\bar{E} - j\xi\bar{H}\,, \tag{3}$$

$$\bar{B} = \mu\bar{H} + j\xi\bar{E}\,, \tag{4}$$

where ξ_r is the relative chirality. The relative chirality is defined as $\xi_r = \dfrac{\xi}{\sqrt{\mu\varepsilon}} = \dfrac{\kappa}{\sqrt{\mu_r\varepsilon_r}}$.

The electromagnetic field in a chiral medium can be decomposed into two parts, the right-handed wave ($\bar{E}_+$, $\bar{H}_+$) and the left-handed wave ($\bar{E}_-$, $\bar{H}_-$). These waves see the chiral medium as equivalent isotropic media characterized by ($\varepsilon_\pm, \mu_\pm$). Electric displacement vectors $\bar{D}_\pm$, magnetic flux densities $\bar{B}_\pm$, and wave impedances $\eta_\pm$ for the equivalent media are defined by

$$\bar{D}_\pm = \varepsilon_\pm\bar{E}_\pm\,, \tag{5}$$

$$\bar{B}_\pm = \mu_\pm\bar{H}_\pm\,, \tag{6}$$

$$\eta_\pm = \sqrt{\frac{\mu_\pm}{\varepsilon_\pm}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}}\,, \tag{7}$$

where $\mu = \mu_o\mu_r$, $\varepsilon = \varepsilon_o\varepsilon_r$, and $\eta_o = \sqrt{\dfrac{\mu_o}{\varepsilon_o}}$ is the free-space wave impedance, and

$$\varepsilon_\pm = \varepsilon \pm \frac{\xi}{\eta}\,, \tag{8}$$

$$\mu_\pm = \mu_\pm \pm \xi\eta\,. \tag{9}$$

The electromagnetic fields ($\bar{E},\bar{H}$) are the sum of the right-handed waves ($\bar{E}_+,\bar{H}_+$) and the left-handed waves ($\bar{E}_-,\bar{H}_-$):

$$\bar{E} = \bar{E}_+ + \bar{E}_-, \tag{10}$$

$$\bar{H} = \bar{H}_+ + \bar{H}_-, \tag{11}$$

where

$$\bar{E}_\pm = \frac{1}{2}\left[\bar{E} \mp j\eta\bar{H}\right] = \mp j\eta H_\pm, \tag{12}$$

$$\bar{H}_\pm = \frac{1}{2}\left[\bar{H} \pm \frac{j}{\eta}\bar{E}\right] = \pm\frac{j\bar{E}_\pm}{\eta}. \tag{13}$$

Maxwell's equations in a source-free region for the equivalent media are

$$\nabla\times\bar{E}_\pm = \pm k_\pm\bar{E}_\pm = -j\omega\mu_\pm\bar{H}_\pm, \tag{14}$$

$$\nabla\times\bar{H}_\pm = \pm k_\pm\bar{H}_\pm = +j\omega\varepsilon_\pm\bar{E}_\pm, \tag{15}$$

where $k_\pm$ are the wave numbers for the chiral media, given in terms of the free-space wavenumber, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, as

$$k_\pm = \omega\sqrt{\mu_\pm\varepsilon_\pm} = k_0\sqrt{\mu_r\varepsilon_r}\left(1\pm\xi_r\right). \tag{16}$$

The spherical vector wave functions, $\bar{M}^{(i)}_{\{e,0\}mn}$ and $\bar{N}^{(i)}_{\{e,0\}mn}$, required for the representation of the fields in spherical coordinates, are

$$\bar{M}^{(i)}_{\{e,0\}mn}(k_\pm r) = \hat{a}_\theta\frac{\hat{B}_n(k_\pm r)}{k_\pm r}\frac{mP_n^m(\cos\theta)}{\sin\theta}[-\sin(m\phi),\cos(m\phi)]$$
$$+\hat{a}_\phi\frac{\hat{B}_n(k_\pm r)}{k_\pm r}\sin\theta P_n^{m\prime}(\cos\theta)[\cos(m\phi),\sin(m\phi)], \tag{17}$$

$$\bar{N}^{(i)}_{\{e,0\}mn}(k_\pm r)$$
$$= \hat{a}_r n(n+1)\frac{\hat{B}_n(k_\pm r)}{(k_\pm r)^2}P_n^m(\cos\theta)\left[\cos(m\phi),\sin(m\phi)\right]$$
$$-\hat{a}_\theta\frac{\hat{B}'_n(k_\pm r)}{k_\pm r}\sin\theta P_n^{m\prime}(\cos\theta)\left[\cos(m\phi),\sin(m\phi)\right]$$
$$+\hat{a}_\phi\frac{\hat{B}'_n(k_\pm r)}{k_\pm r}\frac{mP_n^m(\cos\theta)}{\sin\theta}\left[-\sin(m\phi),\cos(m\phi)\right], \tag{18}$$

where

$$\hat{B}_n(z) = zb_n(z), \tag{19}$$

$$\hat{B}'_n(z) = \frac{d}{dz}\left[zb_n(z)\right], \tag{20}$$

$$P_n^{m\prime}(x) = \frac{d}{dx}\left(P_n^m(x)\right). \tag{21}$$

P_n^m is the associated Legendre polynomial of order m and degree n, and the superscript (i) indicates the choice of the spherical Bessel function $b_n(kr)$. Since $b_n(kr)$ is $j_n(kr)$ when $i=1$, $b_n(kr)$ is $y_n(kr)$ when $i=2$, $b_n(kr)$ is $h_n^{(1)}(kr)$ when $i=3$, and $b_n(kr)$ is $h_n^{(2)}(kr)$ when $i=4$. Because the field components should be finite at the origin, only the terms for which $i=1$ are used in the solutions for the fields inside the sphere, and for the scattered field in the region outside the sphere, only terms for which $i=4$ are used in the solutions to satisfy the radiation conditions. The incident plane wave can be represented in terms of the spherical vector wave functions in order to apply the appropriate boundary conditions. Therefore, considering an x-polarized and z-traveling incident plane wave, such that

$$\bar{E}^{inc} = \hat{a}_x E_0 e^{-jk_0 z} = \hat{a}_x E_0 e^{-jk_0 r\cos\theta}, \tag{22}$$

$$\bar{H}^{inc} = \hat{a}_y\frac{E_0}{\eta_0}e^{-jk_0 z} = \hat{a}_y\frac{E_0}{\eta_0}e^{-jk_0 r\cos\theta}, \tag{23}$$

and after some mathematical manipulations, the incident electric and magnetic field vectors can be written in terms of spherical vector wave functions as

$$\bar{E}^{inc} = -E_0\sum_{n=1}^{\infty}\left\{\frac{j^{-n}(2n+1)}{n(n+1)}\left[\bar{M}^{(1)}_{01n}(k_0 r) + j\bar{N}^{(1)}_{e1n}(k_0 r)\right]\right\}, \tag{24}$$

$$\bar{H}^{inc} = \frac{E_0}{\eta_0}\sum_{n=1}^{\infty}\left\{\frac{j^{-n}(2n+1)}{n(n+1)}\left[\bar{M}^{(1)}_{e1n}(k_0 r) - j\bar{N}^{(1)}_{01n}(k_0 r)\right]\right\}. \tag{25}$$

Upon using Equations (17) and (18) with $i=4$, the scattered-field vectors, $\bar{E}^s$ and $\bar{H}^s$, are given by

$$\bar{E}^s = -E_0\sum_{m,n}\frac{j^{-n}(2n+1)}{n(n+1)}\left\{\left[ja_{mn}\bar{N}^{(4)}_{emn}(k_0 r) + jb_{mn}\bar{N}^{(4)}_{omn}(k_0 r)\right.\right.$$
$$\left.\left.+c_{mn}\bar{M}^{(4)}_{emn}(k_0 r) + d_{mn}\bar{M}^{(4)}_{0mn}(k_0 r)\right]\right\}, \tag{26}$$

$$\bar{H}^s = \frac{E_0}{\eta_0}\sum_{m,n}\frac{j^{-n}(2n+1)}{n(n+1)}\left\{\left[-jc_{mn}\bar{N}^{(4)}_{emn}(k_0 r) - jd_{mn}\bar{N}^{(4)}_{omn}(k_0 r)\right.\right.$$
$$\left.\left.+a_{mn}\bar{M}^{(4)}_{emn}(k_0 r) + b_{mn}\bar{M}^{(4)}_{0mn}(k_0 r)\right]\right\}, \tag{27}$$

while upon using Equations (17) and (18) with $i=1$, the fields inside the chiral sphere, $\bar{E}^{chiral}$ and $\bar{H}^{chiral}$, are given by

$$\bar{E}^{chiral} = -E_0\sum_{m,n}\frac{j^{-n}(2n+1)}{n(n+1)}\left\{jg_{mn}\left[\bar{N}^{(1)}_{emn}(k_+ r) + \bar{M}^{(1)}_{emn}(k_+ r)\right]\right.$$
$$+jh_{mn}\left[\bar{N}^{(1)}_{0mn}(k_+ r) + \bar{M}^{(1)}_{0mn}(k_+ r)\right]$$
$$+u_{mn}\left[\bar{N}^{(1)}_{emn}(k_- r) - \bar{M}^{(1)}_{emn}(k_- r)\right]$$
$$\left.+w_{mn}\left[\bar{N}^{(1)}_{0mn}(k_- r) - \bar{M}^{(1)}_{0mn}(k_- r)\right]\right\}, \tag{28}$$

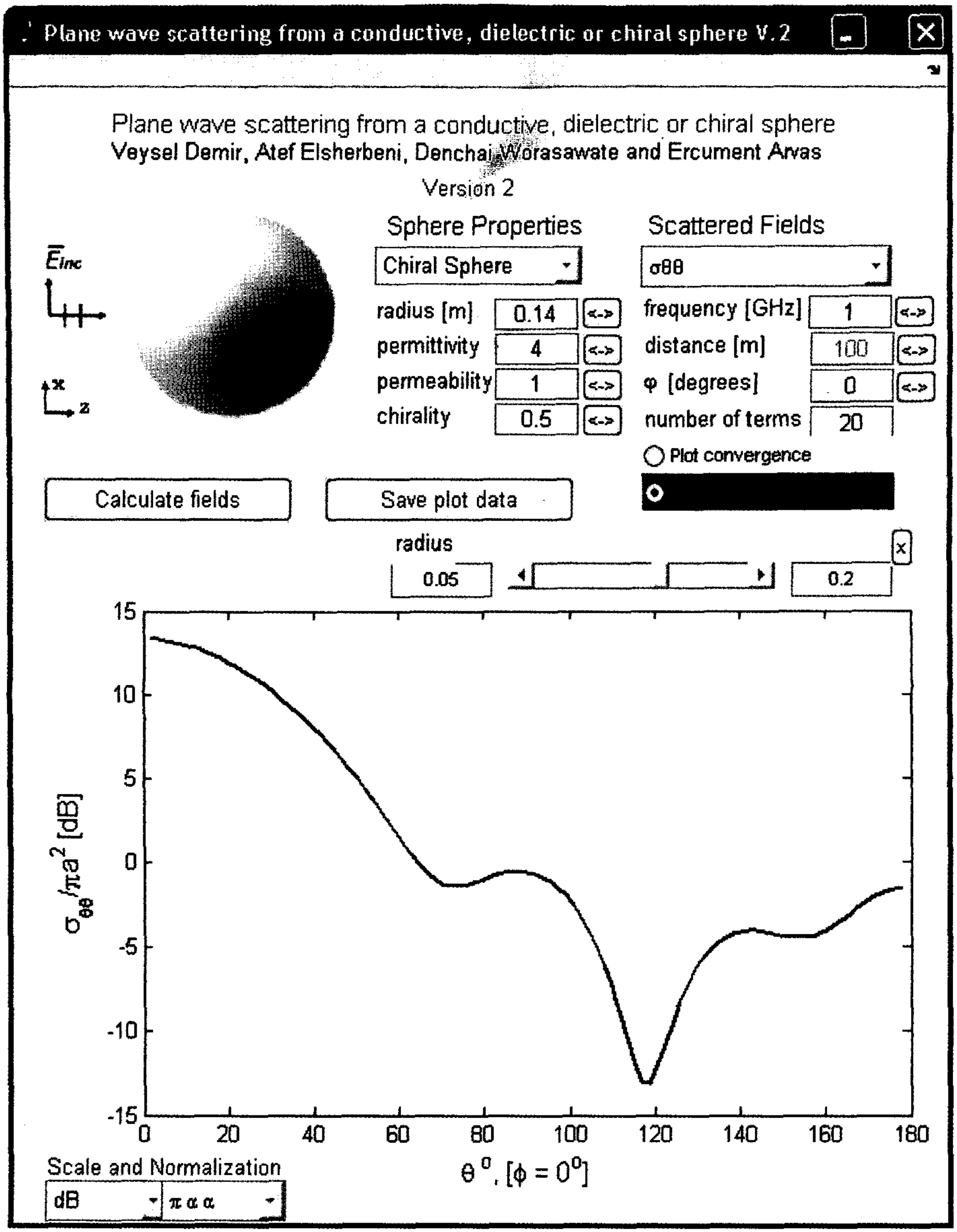

Figure 1. A GUI for plane-wave scattering from a chiral, dielectric, or PEC sphere.

$$\bar{H}^{chiral} = \frac{E_0}{\eta_0} \sum_{m,n} \frac{j^{-n}(2n+1)}{n(n+1)} \left\{ g_{mn} \left[\bar{N}_{emn}^{(1)}(k_+ r) + \bar{M}_{emn}^{(1)}(k_+ r) \right] \right.$$
$$+ h_{mn} \left[\bar{N}_{0mn}^{(1)}(k_+ r) + \bar{M}_{0mn}^{(1)}(k_+ r) \right]$$
$$+ j u_{mn} \left[\bar{N}_{emn}^{(1)}(k_- r) - \bar{M}_{emn}^{(1)}(k_- r) \right]$$
$$\left. + j w_{mn} \left[\bar{N}_{0mn}^{(1)}(k_- r) - \bar{M}_{0mn}^{(1)}(k_- r) \right] \right\}. \quad (29)$$

The scattered electromagnetic field in the presence of a chiral sphere of radius $r = a$ can be obtained using Equations (24)-(29). These equations are used to construct a set of simultaneous equations to solve for the unknown coefficients a_{mn}, b_{mn}, c_{mn}, d_{mn}, g_{mn}, h_{mn}, u_{mn}, and w_{mn}. The incident-field excitations contain only the terms for which $m = 1$. Therefore, only the $m = 1$ terms are included in the solutions for the scattered field and the electromagnetic field inside the chiral sphere. Thus, by applying the boundary conditions that require the tangential components of the electric and magnetic fields be continuous at $r = a$, and after some manipulations the unknowns, a_{1n}, b_{1n}, c_{1n}, and d_{1n} are found as

$$a_{1n} = \frac{(ERB - FA)(CH - RGD) + (CER - GA)(HB - RFD)}{\Delta_1}, \quad (30)$$

$$b_{1n} = \frac{R(CF - BG)}{(k_0 a)^2 \Delta_1}, \quad (31)$$

$$c_{1n} = -b_{1n}, \quad (32)$$

$$d_{1n} = \frac{(ARG - CE)(FD - RBH) + (ARF - BE)(GD - RCH)}{\Delta_1}, \quad (33)$$

where

$$\Delta_1 = (CH - RGD)(FD - RBH) + (GD - RCH)(HB - RFD)$$

$$A = \frac{\hat{J}_n(k_0 a)}{k_0 a},$$

$$B = \frac{\hat{J}_n(k_+ a)}{k_+ a},$$

$$C = \frac{\hat{J}_n(k_- a)}{k_- a},$$

$$D = \frac{\hat{H}_n^{(2)}(k_0 a)}{k_0 a},$$

$$E = \frac{\hat{J}_n'(k_0 a)}{k_0 a},$$

$$F = \frac{\hat{J}_n'(k_+ a)}{k_+ a},$$

$$G = \frac{\hat{J}_n'(k_- a)}{k_- a},$$

$$H = \frac{\hat{H}_n^{(2)\prime}(k_0 a)}{k_0 a},$$

$$R = \frac{\eta_0}{\eta},$$

while $\hat{B}_n(z) = \sqrt{\frac{\pi z}{2}} B_{n+\frac{1}{2}}$, and $B_{n+\frac{1}{2}}$ is a cylindrical Bessel function. The co-polarized bistatic radar cross section, $\sigma_{\theta\theta}$, and the cross-polarized bistatic radar cross section, $\sigma_{\phi\theta}$, can then be defined as

$$\sigma_{\theta\theta} = \lim_{r \to \infty} 4\pi r^2 \frac{\left|E_\theta^s\right|^2}{\left|E_\theta^{inc}\right|^2}, \quad (34)$$

$$\sigma_{\phi\theta} = \lim_{r \to \infty} 4\pi r^2 \frac{\left|E_\phi^s\right|^2}{\left|E_\theta^{inc}\right|^2}. \quad (35)$$

With the assumption that the plane of interest is defined by $\phi = 0°$, one can obtain

$$\sigma_{\theta\theta} = \frac{\lambda_0^2}{\pi} \left| \sum_{n=1}^{\infty} \left[\frac{2n+1}{n(n+1)} \left(a_{1n}\tau_n + d_{1n}\pi_n \right) \right] \right|^2 \quad (36)$$

$$\sigma_{\phi\theta} = \frac{\lambda_0^2}{\pi} \left| \sum_{n=1}^{\infty} \left\{ \frac{2n+1}{n(n+1)} \left(b_{1n}\pi_n - c_{1n}\tau_n \right) \right\} \right|^2, \quad (37)$$

where

$$\pi_n = \frac{P_n^1(\cos\theta)}{\sin\theta}, \quad (38)$$

$$\tau_n = -\sin\theta P_n^{1\prime}(\cos\theta). \quad (39)$$

3. Software Description

A program was developed to calculate the scattered fields from a chiral sphere due to an incident x-polarized and z-traveling plane wave. If the chirality vanishes – that is, $\kappa = 0$ – the constitutive relations given in Equations (1) and (2) reduce to those of a dielectric medium. Therefore, this program can be used to calculate the scattering from a dielectric sphere, as well. Furthermore, if a very large value of the dielectric constant is used, the medium behaves like a highly conductive medium. Thus, this program also can calculate the scattering from a highly conductive or a PEC sphere.

A graphical user interface was developed using *MATLAB*, in order to provide a user-friendly environment for the calculation and visualization of the results. A snapshot of this user interface is shown in Figure 1. The user can choose to calculate scattering from a chiral, dielectric, or conductive sphere from a drop-down menu. The radius of the sphere, relative permittivity, relative permeability, and chirality parameters can be entered through entry boxes. The unnecessary parameters are not used during calculations. Thus, chirality will not be used if a dielectric sphere is selected, and permittivity, permeability, and chirality are not used if a conductive sphere is selected. The frequency of the incident field is another parameter that should be supplied by the user. One of four types of results can be viewed in the plot window: co-polarized bistatic radar cross-section, $\sigma_{\theta\theta}$; cross-polarized bistatic radar cross-section, $\sigma_{\phi\theta}$; magnitude of E_θ; or magnitude of E_ϕ. The field components E_θ and E_ϕ are calculated at a specified distance from the center of the sphere, which should be entered by the user. Therefore, the near- as well as far-field components, E_θ and E_ϕ, can be calculated and displayed. The radar cross-section values $\sigma_{\theta\theta}$ and $\sigma_{\phi\theta}$ are computed from far-field components, regardless of the distance value entered by the user. These fields are calculated on an arc defined by $0° \le \theta \le 180°$, $\phi = \phi_0$, in spherical coordinates. The angle ϕ_0 is another parameter that should be entered by the user.

As can be seen in Equation (26), the solution of the scattered fields is the sum of an infinite series. The user can enter the number of terms to be used for calculation; however, the calculation will converge to exact results only with a large enough number of terms. The program provides the user the option to display the solution convergence in terms of the number of terms by a radio button. This feature allows the user to examine the convergence of such complicated summations.

The parameter entry boxes have sliding bars in order to provide a range for each input parameter to the program. Once the parameter is changed with the slider, the corresponding calculations are performed, and displayed in the result window. Therefore, the variation of results with respect to parameters can be viewed in a near-real-time manner.

Any data displayed in the result window can be saved to a *MATLAB* script file, together with the *MATLAB* plotting commands. Once a plot is saved, it can be displayed again later by running the saved script file in *MATLAB*. Numerical data can be extracted from this file if the user wishes to use it in another package.

This package has been developed and tested using *MATLAB* 7, Release 14, and a p-coded version of this package is available for free download from the Applied Computational Electromagnetic Society (ACES) Web site: (http://aces.ee.olemiss.edu).

4. Conclusions

An interactive software package has been developed to calculate and display the scattered fields and radar cross sections of a chiral, dielectric, or perfectly conducting sphere due to an incident plane wave.

5. References

1. M. A. Al-Kanhal and E. Arvas, "Electromagnetic Scattering from a Chiral Cylinder of Arbitrary Cross Section," *IEEE Transactions on Antennas and Propagation*, **AP-44**, 7, July 1996, pp. 1041-1048.

2. N. Engheta and D. L. Jaggard, "Electromagnetic Chirality and its Applications," *IEEE Antennas Propagation Society Newsletter*, **30**, 5, 1988, pp. 6-12.

3. S. F. Mahmoud, "Mode Characteristics in Chirowaveguides with Constant Impedance Walls," *Journal of Electromagnetic Waves and Applications*, **6**, 5/6, 1992, pp. 625-640.

4. S. F. Mahmoud, "Characteristics of a Chiral Coated Slotted Cylindrical Antenna," *IEEE Transactions on Antennas and Propagation*, **AP-44**, 6, June 1996, pp. 814-821.

5. R. D. Graglia, P. L. E. Uslenghi, and C. L. Yu, "Electromagnetic Oblique Scattering by a Cylinder Coated with Chiral Layers and Anisotropic Jump-Admittance Sheets," *Journal of Electromagnetic Waves and Applications*, **6**, no. 5/6, 1992, pp. 625-719.

6. C. F. Bohren, "Light Scattering by an Optically Active Sphere," *Chem. Phys. Lett.*, **29**, December 1974, pp. 458-462.

7. D. Worasawate, *Electromagnetic Scattering from an Arbitrarily Shaped Three-Dimensional Chiral Body*, PhD dissertation, Syracuse University, Syracuse, NY, 2002.

8. D. Worasawate, J. R. Mautz, and E. Arvas, "Electromagnetic Scattering from an Arbitrarily Shaped Three-Dimensional Homogeneous Chiral Body," *IEEE Transactions on Antennas and Propagation*, **AP-51**, 5, May 2003, pp. 1077-1084.

9. Veysel Demir, Atef. Z. Elsherbeni, and Ercument Arvas, "FDTD Formulation for Scattering from Three Dimensional Chiral Objects," 20th Annual Review of Progress in Applied Computational Electromagnetics, ACES'04, Syracuse, NY, April 2004.