2) a) $B \sim 0.07 \longrightarrow \Delta x \leq \frac{1}{0.14} = 7.14\,m$

I chose $N_x = 2000$, or $\Delta x = \frac{L}{N_x} = 0.5\,m$

b) $\frac{\partial^2 u}{\partial t^2} - c^2(x)\frac{\partial^2 u}{\partial x^2} = 0 \qquad x \in [0, L] \quad t \in [0; 5]$

$u(t, 0) = 0 \qquad \frac{\partial u}{\partial x}(t, L) = 0$

$u(0, x) = f(x) \quad , \quad \frac{\partial u}{\partial t}(0, x) = g(x)$

$L = 1\,km \qquad f(x) = g(x) = \begin{cases} 0 & x \notin (600, 800) \\ \left(1 - \left(\frac{x - 700}{10}\right)^2\right) e^{-\frac{1}{2}\left(\frac{x-700}{10}\right)^2} & x \in (600, 800]\,m \end{cases}$

$c(x) = \begin{cases} 3\,km/s & x \in [0, 500]\,m \\ 1\,km/s & x \in [500, 1000]\,m \end{cases}$

$u^0 = f, \quad u^1 = \frac{1}{2} B u^0 + \Delta t\, g$

$u^{i+1}_{j=0} = 0 \qquad x = 0$

$u^{i+1} = -u^{i-1} + B u^i \quad , \quad x \in (0, L)$

$u^{i+1}_{j=N_x} = -u^{i-1}_{N_x} + 2\sigma(x)^2 u^i_{N_x} + 2(1-\sigma(x)^2) u^i_{N_x-1} \quad x = L, \ j = N_x$

$u^{i+1}_{N_x} = -u^{i-1}_{N_x} + 2\sigma(x)^2 u^i_{N_x} + 2\left(1 - \sigma(x)^2\right) u^i_{N_x - 1}$

$$B = \begin{bmatrix} 2(1-\sigma(x)^2) & \sigma(x)^2 & 0 \\ \sigma(x)^2 & 2(1-\sigma(x)^2) & \sigma(x)^2 & \ddots \\ 0 & \sigma(x)^2 & \ddots & \ddots \\ & & \ddots & \ddots \end{bmatrix}$$

$\sigma(x) = c(x)\frac{\Delta t}{\Delta x}$

c) $N_x = 2000, \quad \boxed{\Delta x = \frac{L}{N_x} = 0.5\,m.}$

$N_t = 50000 \quad , \quad \Delta t \leq \cancel{c(x)\max(c(x))} \frac{\Delta x}{\max(c(x))} = 0.0001\bar{6}\,s$

$\Rightarrow \boxed{\Delta t = 0.0001\,s = 0.1\,ms}$

Scanned with CamScanner

1)

$$\rho \ddot{u}_x = \frac{\partial}{\partial x}\sigma_{xx} + \frac{\partial}{\partial y}\sigma_{xy} + \frac{\partial}{\partial z}\sigma_{xz} + f_x$$

$$= \frac{\partial}{\partial x}\left[(\lambda+2\mu)\varepsilon_{xx} + \lambda\varepsilon_{yy} + \lambda\varepsilon_{zz}\right] + \frac{\partial}{\partial y}(\mu\varepsilon_{xy}) + \frac{\partial}{\partial z}(\mu\varepsilon_{xz}) + f_x$$

$$= \frac{\partial}{\partial x}\left((\lambda)(\Delta)\right) + \frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial x}\right) + f_x$$

$$= \frac{\partial}{\partial x}(\lambda\Delta) + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial x}\right) + f_x$$

$$\boxed{\rho\frac{\partial^2}{\partial t^2}\vec{u} = \vec{\nabla}(\lambda\Delta) + \vec{\nabla}\cdot\left(\mu(\vec{\nabla}\vec{u}) + \mu(\vec{\nabla}\vec{u})^T\right) + \vec{f}}$$

$$\vec{\nabla}\vec{u} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix}$$

um over
repeated indices

$$\boxed{\rho\frac{\partial^2}{\partial t^2}u_j = \partial_j(\lambda\Delta) + \partial_i(\mu\,\partial_i u_j + \mu\,\partial_j u_i) + f_j}$$

If ρ, λ, μ all constant

$$\rho\frac{\partial^2}{\partial t^2}\vec{u} = \lambda\vec{\nabla}\Delta + \mu\nabla^2\vec{u} + \mu\vec{\nabla}\Delta + \vec{f}$$

$$\rho\frac{\partial^2}{\partial t^2}\vec{u} = (\lambda+\mu)\vec{\nabla}\Delta + \mu\nabla^2\vec{u} + \vec{f}$$

$$\Rightarrow \rho\frac{\partial^2}{\partial t^2}(\vec{\nabla}\cdot\vec{u}) = \rho\frac{\partial^2}{\partial t^2}\Delta = (\lambda+\mu)\nabla^2\Delta + \mu\nabla^2\Delta + \vec{\nabla}\cdot\vec{f}$$

$$\rho\frac{\partial^2}{\partial t^2}\Delta - (\lambda+2\mu)\nabla^2\Delta = \vec{\nabla}\cdot\vec{f}$$