NE 806: NEUTRONICS
Problem Set No. 4

due: Thursday, Nov. 15, 2018

1. For the following one-speed diffusion problems write: (1) the appropriate form of the diffusion equation (with a sketch showing the problem geometry), (2) the form of the most general solution (including any particular solution), (3) the boundary/source conditions you would use to find values for any arbitrary constants in your general solution, and (5) an explicit expression for the flux density. Assume a vacuum surrounds the media.

   (a) An infinite homogeneous slab of thickness T has a volumetrically distributed source with a strength $S(x) = S_o x^2$ neutrons cm^{-3} s^{-1} where x is measured from the left surface of the slab.

   (b) An infinite homogeneous cylinder of diameter T contains a uniformly distributed source of strength S_o neutrons cm^{-3} s^{-1}.

   (c) A homogeneous sphere of diameter T contains a uniformly distributed source of strength S_o neutrons cm^{-3} s^{-1}.

   (d) Two infinite homogeneous slabs each of thickness T are placed a distance T apart. The left slab has a distributed source $S(x) = S_o \cos \alpha x$ neutrons cm^{-3} s^{-1} where x is measured from the outer surface. The outer surface of the other slab is illuminated uniformly by a perpendicular neutron beam of strength I_o neutrons cm^{-2} s^{-1}.

2. Write a computer program to calculate the steady-state flux density distribution in a homogeneous infinite slab, sphere, and infinite cylinder each of which may contain an arbitrarily distributed volumetric source of neutrons. Assume vacuum boundary conditions.

   (a) Derive the first-order finite-difference form of the appropriate one-speed diffusion equation. Assume equal mesh spacing.

   (b) Write the resulting equations in matrix form $\mathbf{A}\boldsymbol{\phi} = \mathbf{s}$.

   (c) Derive the TDMA algorithm to solve this set of equations and write a subroutine to implement this algorithm.

   (d) Write a program to solve the 1-D finite-difference diffusion equations developed in part (a). Input should include: geometry type, system size, number of mesh points desired, parameters Σ_a and D, and the source distribution $S(r_i)$.

3. Use your program to solve the first three problems of question 1. Data to be used are: $D = 0.600$ cm, $\Sigma_a = 0.005$ cm^{-1}, $T = 45$ cm, and $S_o = 10^8$ neutrons cm^{-2} s^{-1}. Plot both the numerical solution and the analytical flux profile for each problem. Also investigate the effect of mesh size on the accuracy of the solutions.