Assignment36

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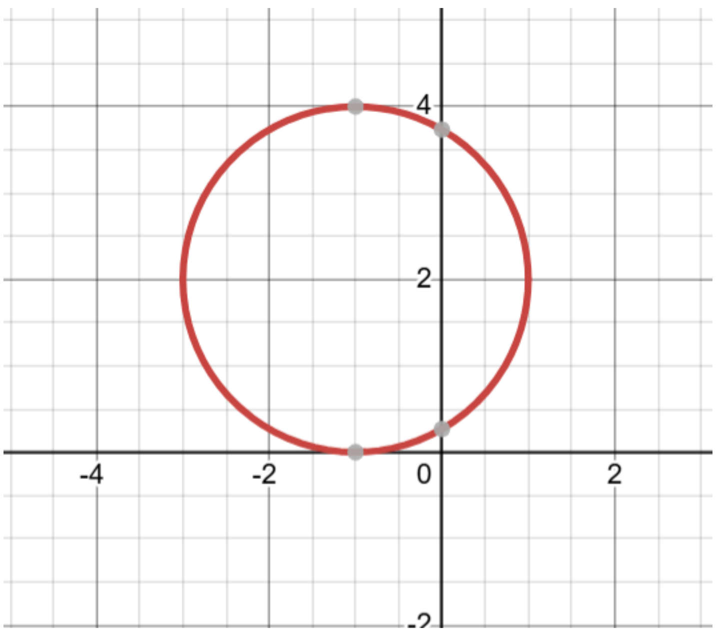
We have seen that in p = 2 dimensions, a linear decision boundary takes the form β0 + β1X1 + β2X2 = 0. We now investigate a non-linear decision boundary. 1. Sketch the curve (1 + X1)^2 + (2 −X2)^2 = 4



As we see, the curve is in fact a circle with a radius of 2. Centered at (-1, -2).

2. On your sketch, indicate the set of points for which (1 + X1)^2 + (2 −X2)^2 > 4, as well as the set of points for which (1 + X1)2 + (2 −X2)2 ≤4.

Any coordinates that satisfy this requirement (1 + X1)^2 + (2 −X2)^2 > 4 are outside the circle.

Take (0, 0) : (1 + 0)^2 + (2 - 0) ^ 2 = 5 > 4.

Meanwhile, the opposity will be inside the circle, or on the circle's perimeter.

3. Suppose that a classifier assigns an observation to the blue class if (1 + X1)^2 + (2 −X2)^2 > 4, and to the red class otherwise. To what class is the observation (0,0) classified? (−1,1)? (2,2)? (3,8)

   As we see, (0,0), (2,2), and (3,8) are outside the circle, while (-1, 1) is inside our circle. Thus we have that

Class(0,0), Class(2,2), Class(3,8) = Blue

Class(-1,1) = Red.

4. Argue that while the decision boundary in 3. is not linear in terms of X1 and X2, it is linear in terms of X1, X1 ^ 2 , X2, and X2 ^ 2

If we expand our equation:

$(1 + X1)^2 + (2 - X2)^2 = 4$ ->

$1 + 2X1 + X1^2 + 4 - 4X2 + X2^2 = 4$ ->

$2X1 + X1^2 - 4X2 + X2^2 + 1 = 0$, which contains quadratic terms and therefore not linear in terms of X1 and X2.

If we transformed this equation into a 4 dimensional space, where:

$Z1 = X1$, $Z2 = X2$, $Z3 = X1^2$, $Z4 = X2^2$, we get the equation

$1 + 2Z1 + Z3 - 4Z2 + Z4 = 0$, which is linear in terms of Z1, Z2, Z3, Z4 (it becomes a hyper plane in a higher dimensional space)