

Assignment27

John Bute

2024-11-08

R Markdown

1. As $\lambda \rightarrow \infty$, will \hat{g}_1 or \hat{g}_2 have the smaller training RSS?

The regularization term will dominate as λ reaches infinity, thus the penalty for non-smoothness becomes increasingly large. Therefore, the third derivative $[g^{(3)}(x)]$, will change the curvature of the function $g(x)$ and make it so that the function approaches a quadratic curve, while penalizing the fourth derivative makes the function approach a cubic curve. (the idea is that when you take, say, the third derivative of a quadratic function, you end up with 0, therefore as λ approaches infinity, we force g_1 , in this case, to become a quadratic function, as the solution that minimalizes the entire function will be quadratic)

Thus, we have a smoother curve (quadratic function) and a less smoother curve (cubic function). Since cubic functions tend to fit data better than quadratic functions, it must have a smaller training RSS, and therefore we will see g_2 having a smaller training RSS.

2. As $\lambda \rightarrow \infty$, will \hat{g}_1 or \hat{g}_2 have the smaller test RSS?

g_1 will have a smaller test RSS than g_2 , as we said before, g_1 will converge to a quadratic function, while g_2 will converge to a cubic function. Therefore, a quadratic function tends to overfit less the data rather than a cubic, and therefore has the lower test RSS.

3. For $\lambda = 0$, will \hat{g}_1 or \hat{g}_2 have the smaller training and test RSS?

They will both have the same training RSS, as if you remove the regularization term, we allow g_1 and g_2 to fit the data as closely as possible, leading to overfitting and a OLS regression model. However, since both models are unregularized, we will have the same but way higher test RSS.