Assignment13

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Assignment 13

To prove that the R^2 statistic is equal to the square of the correlation r between X and Y, we start by noting the definitions:

1. The R^2 statistic is defined as:

$$R^2 = \frac{\text{Explained Sum of Squares (ESS)}}{\text{Total Sum of Squares (TSS)}} = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2}$$

   Since

$$\bar{y} = \bar{x} = 0$$

   , we can rewrite the expression as:

$$R^2 = \frac{\text{Explained Sum of Squares (ESS)}}{\text{Total Sum of Squares (TSS)}} = \frac{\sum(\hat{y}_i)^2}{\sum(y_i)^2}$$

2. In simple linear regression, $Y = \beta_0 + \beta_1 X$ with $\bar{x} = \bar{y} = 0$, so we can safely say the the intercept of the regression function will be 0.

$$\hat{y}_i = \beta_1 x_i$$

3. The total sum of squares (TSS) is:

$$\text{TSS} = \sum(y_i - \bar{y})^2 = \sum y_i^2$$

   due to the assumption earlier

4. The explained sum of squares (ESS) is:

$$\text{ESS} = \sum(\hat{y}_i)^2 = \sum(\beta_1 x_i)^2 = \beta_1^2 \sum x_i^2$$

5. We know that the formula for β_1 is:

$$\beta_1 = \frac{\sum x_i y_i}{\sum x_i^2}$$

6. Substituting β_1 into the ESS, we get:

$$\text{ESS} = \frac{(\sum x_i y_i)^2}{(\sum x_i^2)^2} * \sum x_i^2 = \frac{(\sum x_i y_i)^2}{\sum x_i^2}$$

7. The R^2 statistic can now be expressed as:

$$R^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{\frac{(\sum x_i y_i)^2}{\sum x_i^2}}{\sum y_i^2} = \frac{(\sum x_i y_i)^2}{\sum x_i^2 * \sum y_i^2}$$

8. The correlation r between X and Y is defined as:

$$r = \frac{\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2}}$$

9. Squaring r, we get:

$$r^2 = \left(\frac{\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2}} \right)^2 = \frac{(\sum x_i y_i)^2}{\sum x_i^2 \sum y_i^2}$$

Therefore, we have shown that R^2 = r ^2