Categories for the Curious

A Light Intro to Category Theory

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Table of contents

Preface

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1 Introduction

This site contains the course notes for an informal one-semester seminar on category theory at CSU East Bay.

2 Partially Ordered Sets

2.1 The "Divides" Relation

Let m and n be integers. We say m *divides* n, and we write $m \mid n$ if there exists an integer q such that $mq = n$.

For example, 3 divides 6 since $3 \cdot 2 = 6$, and -5 divides 10 since $-5 \cdot 2 = 10$.

Exercise 2.1. Show that for any integer a we have $a \mid a$.

This is called *reflexivity*, and we say that $\mid$ is *reflexive* because it satisfies this property. For all that follows, we denote by $\mathbb{N}$ the *natural numbers*, consisting of the non-negative integers $0, 1, 2, \ldots$. Take special care to notice that *we are including zero in this set.*[1] It will soon be apparent why we do this.

Another important note about the "divides" relation is that saying "a divides b" is *not* the same as saying "b is divisible by a."

Exercise 2.2. Show that for natural numbers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.[2]

Solution

If $a \mid b$, then there exists a natural number k_1 such that $ak_1 = b$. Similarly, $b \mid a$ implies there exists a natural number k_2 such that $bk_2 = a$. Hence, $b = ak_1 = bk_1k_2$. We may similarly show that $a = k_1k_2a$. The first equation tells us that $b(1 - k_1k_2) = 0$. Then either $b = 0$ or $k_1 = k_2 = 1$.

If $b = 0$, then because $ak_1 = b$ we have either $a = 0$ (and thus $a = b$), or we have $k_1 = 0$. This would imply that $a = k_1k_2a = 0 \cdot k_2a = 0$. So $a = b$.

If $b \neq 0$ (i.e., $k_1 = k_2 = 1$), then certainly $a = 1 \cdot a = k_1 \cdot a = b$. In either case, $a = b$.

Exercise 2.3. Show that if a, b, and c are integers, and $a \mid b$ and $b \mid c$, then $a \mid c$.

[1] This differs from another convention that excludes zero, which you may be used to.

[2] If we allow a and b to be negative, then the most general statement we can make is that the magnitudes are equal.

2.2 Defining Posets

Let $\preccurlyeq$ denote a partial order relation.

References