MATH355 2017-11-20

JORDAN

If A is a square matrix, its *characteristic polynomial* is

$$\det(A - xI)$$

where I is the identity matrix. The roots $\lambda_1, \ldots, \lambda_k$ of this polynomial are precisely the eigenvalues of A (why?).

Given a root λ, its multiplicity (as a root) is called the *algebraic multiplicity*. On the other hand, we have the vector space

$$\{\vec{v} \mid A\vec{v} = \lambda\vec{v}\}$$

and its dimension is called the *geometric multiplicity* of λ.

Fact 1. The matrix A is diagonalizable if, for each root λ its geometric multiplicity is the same as its algebraic multiplicity.

Remark 2. Strictly speaking, we should distinguish whether we are working with real or complex vector spaces. If we are working over $\mathbf{C}$, then the Fact above is true as stated (because every polynomial factors as a product of polynomials of degree 1, this is the *fundamental theorem of algebra*). If we are working over $\mathbf{R}$, then the Fact above is ture, provided all the roots are real.

Sometimes a matrix is simply not diagonalizable. So the next best thing is Jordan form.

Have a look at these notes:

`http://math.rice.edu/~friedl/math355_fall04/Jordan.pdf`

where you can find the algorithm we outlined in class. The only difference is that the 1s in our Jordan form are below the diagonal. But the two are equivalent, it's just a matter of switching the order of your basis vectors.

Date: John Calabrese, November 24, 2017.