MATH355 2017-09-13 BONUS

As usual, V denotes an abstract vector space.

Fact 1. If $W < V$ is a subspace, then $\operatorname{Span} W = W$.

Why is that? Well, we always have $W \subset \operatorname{Span} W$. Indeed, if $\vec{w} \in W$, then $1\vec{w}$ is a linear combination! So we just need to show that $\operatorname{Span} W \subset W$. But W is closed under linear combinations! More precisely, any element of $\operatorname{Span} W$ of the form $\vec{v} = a_1\vec{w}_1 + \cdots + a_k\vec{w}_k$ with $a_i \in \mathbf{R}$, $\vec{w}_i \in W$ for all i. But clearly $\vec{v} \in W$. So, $\operatorname{Span} W \subset W$.

Suppose $\vec{v} \in V$.

Fact 2. $\operatorname{Span} \vec{v} = \{a\vec{v} \mid a \in \mathbf{R}\}$.

We saw in class an example of this: if you write any linear combination we'll have

$$a_1\vec{v} + a_2\vec{v} + \cdots + a_k\vec{v}_k = (a_1 + \cdots + a_k)\vec{v} = a\vec{v}$$

where $a := a_1 + \cdots + a_k$.

In particular,

Fact 3. $\operatorname{Span}\{\vec{0}\} = \{\vec{0}\}$.

Date: John Calabrese, September 14, 2017.