2017 Fall - Math 355 - Homework 10

Due: Friday, November 10 *in class.*[1]
Unless specified otherwise, you must always show your work and justify your answers.

(1) Compute the determinant of the matrix below in two different ways: first by expanding along a column of your choice; second by expanding along a row of your choice. Show your work.

$$\begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 3 & 1 & 1 \\ -1 & 0 & 0 & -2 \\ 1 & -1 & 3 & 1 \end{pmatrix}$$

(2) Let $\gamma\colon \mathbb{C} \to \mathbb{C}$ be the map which sends a complex number to its conjugate

$$z \mapsto \gamma(z) = \bar{z}$$

explicitly, if $z = a + ib$, with $a, b \in \mathbb{R}$, we have

$$a + ib \mapsto \gamma(a + ib) = a - ib.$$

(a) Is γ $\mathbb{R}$-linear?
(b) Is γ $\mathbb{C}$-linear?

(3) Consider the set of complex numbers $\mathbb{C}$ as a vector space over $\mathbb{R}$.
(a) Write an $\mathbb{R}$-basis for $\mathbb{C}$.
(b) Show that $\mathbb{C}$ is isomorphic to $\mathbb{R}^2$, as real vector spaces. [In particuar, $\dim_{\mathbb{R}} \mathbb{C} = 2$.]

(4) Consider now $\mathbb{C}$ as a $\mathbb{C}$-vector space.
(a) Write down a $\mathbb{C}$-basis for $\mathbb{C}$ and show that $\dim_{\mathbb{C}} \mathbb{C} = 1$.
(b) From the basis you chose, how do you extract an $\mathbb{R}$-basis for $\mathbb{C}$?

(5) Consider now $\mathbb{C}^2$ as a complex vector space.
(a) Show that $\mathbb{C}^2$ can be seen as a real vector space (by restricting the multiplication by scalars).
(b) Pick a $\mathbb{C}$-basis for $\mathbb{C}^2$ and show that $\dim_{\mathbb{C}} \mathbb{C}^2 = 2$.
(c) How do you extract an $\mathbb{R}$-basis for $\mathbb{C}^2$, from the $\mathbb{C}$-basis you just chose?
(d) Show that $\mathbb{C}^2$ is isomorphic to $\mathbb{R}^4$ as real vector spaces, in particular $\dim_{\mathbb{R}} \mathbb{C}^2 = 4$.

(6) Finally, consider V an arbitrary $\mathbb{C}$-vector space.
(a) Show that V is also an $\mathbb{R}$-vector space, by restricting the scalar multiplication.
(b) If $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a $\mathbb{C}$-basis for V, how can we extract an $\mathbb{R}$-basis for V?
(c) Show that $\dim_{\mathbb{R}} V = 2 \dim_{\mathbb{C}} V$.

[1]This file was last updated at 08:51 on Sunday 5th November, 2017.