2017 Fall - Math 355 - Homework 12

Due: Friday, Dec 1 *in class.*[1]
Unless specified otherwise, you must always show your work and justify your answers.

(1) Let $f\colon V \to V$ be a linear map. Let $\vec{v}$ be a λ-eigenvector, let $\vec{w}$ be a μ-eigenvector. Is the following true or false:
 $\vec{v}$ and $\vec{w}$ are linearly independent if and only if $\lambda \neq \mu$.
 Why, or why not?

(2) Let A, B be two matrices.
 (a) If A is similar to B, is it true that A^2 is similar to B^2? Why, or why not?
 (b) If A^2 is similar to B^2, is it true that A is similar to B? Why, or why not?

(3) Consider the polynomial

$$(x-5)^3(x+1)$$

- If such polynomial is the characteristic polynomial of a linear map, what are the possible geometric multiplicities of 5 and -1?
- For each possibility above, find a corresponding matrix A in Jordan form.

(4) For the two matrices above, find the Jordan form and find a basis $\mathbb{B}$ for which $\mathrm{Rep}_{\mathbb{B}}$ is in Jordan form. [For this problem you need to explain what you are doing, but you do not need to show all the arithmetic that goes into it. In particular, I strongly strongly encourage you to use Wolfram Alpha (or similar) to compute the eigenvalues.]

$$A = \begin{pmatrix} 1 & -1 & -1 & 0 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 1 & 0 & 1 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & -2 & 1 & -2 \\ 0 & 3 & 0 & 0 \\ -1 & 2 & 2 & 1 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

(5) Let A be a 4×4 matrix. Suppose that the only eigenvalue of A is 64, with algebraic multiplicity 4, and geometric multiplicity 2. Is this information enough to deduce the Jordan form of A? Why, why not?

[1] This file was last updated at 10:21 on Friday 24th November, 2017.