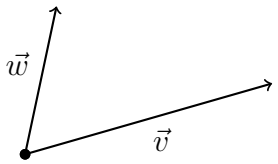
2017 Fall - Math 355 - Homework 2

Due: Friday, September 16 *in class.*

(1) Let $\vec{v}$ and $\vec{w}$ be the vectors pictured below. Sketch the following vectors (one different sketch for each part).



(a) $\vec{v} + \vec{w}$
(b) $3\vec{v}$
(c) $-\vec{w}$
(d) $\vec{v} - \vec{w}$
(e) $\vec{w} - \vec{v}$

(2) Let $a, b \in \mathbb{R}$ be two fixed real numbers. Consider the system

$$\begin{cases} x + y + 2z = 0 \\ 2x + z + 6w = 1 \end{cases}$$

(a) Which variables are leading? Which variables are free?
(b) Describe the solution set. Does the system have no solutions, a unique solution or infinitely many solutions?
(c) Describe the solution set as $\text{Sol} = \{\vec{p} + c_1\vec{v}_1 + \cdots + c_k\vec{v}_k \mid c_i \in \mathbb{R}\}$, where $\vec{p}$ is a particular solution and the $\vec{v}_i$ are solutions to the associated homogeneous system.
(d) What is the associated homogenous system?

(3) Let V be an abstract vector space.[1] Suppose $\vec{v}_0 \in V$ has the property that, for any $\vec{w} \in V$ we have

$$\vec{v}_0 + \vec{w} = \vec{w}.$$

Show that $\vec{v}_0 = \vec{0}$. [You are only allowed to use the axioms from the definition: use them very *carefully*!]

(4) Let V be an abstract vector space. Let $\vec{v} \in V$. Suppose $\vec{u}, \vec{w}$ are such that

$$\vec{u} + \vec{v} = \vec{0}$$

and

$$\vec{v} + \vec{w} = \vec{0}$$

Show that $\vec{u} = \vec{w}$. [Once again: you are only allowed to use the axioms from the definition!]

[1]The full definition may be found on page 78 of Hefferon.