2017 Fall - Math 355 - Homework 7

Due: Friday, October 20 *in class.*[1]
Unless specified otherwise, you must always show your work and justify your answers.

(1) Write $\mathbb{R}^5$ as a direct sum of two (non-zero) subspaces.
(2) Write $\mathbb{R}^5$ as a sum of three (non-trivial[2]) subspaces.
(3) Let $U, W < \mathbb{R}^5$ be subspaces, suppose $\dim U = 3$, $\dim W = 3$. What are the possibilities for $\dim U \cap W$, $\dim U + W$? For each, exhibit an example.
(4) Let $\vec{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \vec{b}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ be a basis of $\mathbb{R}^3$. Let $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the unique linear map such that $f(\vec{b}_1) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $f(\vec{b}_2) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $f(\vec{b}_3) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
What is $f(\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix})$? What is $f(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix})$? What is $f(\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix})$?
(5) Let V be a vector space. Let $n = \dim V$. Show that V is the direct sum of n subspaces, all of dimension 1.
(6) Suppose $V = W_1 + W_2 + W_3$. Suppose $\dim W_1 + \dim W_2 + \dim W_3 = \dim V$. Suppose W_i is non-trivial for all i. What can you say about $W_1 \cap W_2$? What can you say about $W_2 \cap W_3$? What can you say about $W_1 \cap W_2 \cap W_3$? Discuss.
(7) Suppose $\dim V = 3$ and suppose $V = W_1 + W_2 + W_3$. Suppose $\dim W_1 + \dim W_2 + \dim W_3 = 4$. Suppose W_i is non-trivial for all i. What can you say about $W_1 \cap W_2 \cap W_3$? Discuss.

[1]This file was last updated at 18:01 on Friday 13$^{\text{th}}$ October, 2017.
[2]A subspace $U < V$ is *non-trivial* if $U \neq \{\vec{0}\}$ but also $U \neq V$.