2017 Fall - Math 355 - Homework 4

Due: Friday, September 29 *in class.*[1]

(1) Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_4 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \vec{v}_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$.

   (a) Show that $\mathrm{Span}\{\vec{v}_1, \ldots, \vec{v}_5\} = \mathbb{R}^4$.

   (b) Extract two different bases $\mathbb{B}_1, \mathbb{B}_2$ of $\mathbb{R}^4$ from $\vec{v}_1, \ldots, \vec{v}_5$.
   [Note: reordering the terms does *not* count as different.]

   (c) Express $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 5 \\ 1 \end{pmatrix}$ in the bases $\mathbb{B}_1, \mathbb{B}_2$.

(2) Let $W < \mathbb{R}^5$ be a subspace. Assume $\dim W = 5$. Without appealing directly to any of the Facts stated in class on Friday, show that $W = \mathbb{R}^5$.
   [Hint: argue by contradiction]

(3) Consider the homogeneous system of linear equations

$$\begin{cases} x_1 - x_2 + x_3 + 4x_4 - 6x_5 = 0 \\ x_3 + x_5 = 0 \end{cases}$$

   (a) What is the dimension of the solution space Sol?

   (b) Find a basis for Sol.

   (c) Extend that basis to a basis of $\mathbb{R}^5$ in at least two different ways.
   [Not a hint, but: although we haven't covered it in class, you are probably familiar with the dot product of $\mathbb{R}^5$. To complete to a basis, you can always hunt for vectors in $\mathrm{Sol}^{\perp}$, the orhogonal complement. If this makes no sense, ignore it.]

   (d) Using the two bases you found, express the vector $\begin{pmatrix} 0 \\ -1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$.

[1] This file was last updated at 07:02 on Saturday 23$^{\text{rd}}$ September, 2017.