2017 Fall - Math 355 - Homework 3

Due: Friday, September 22 *in class.*

(1) Let $M_{2\times 2}$ be the space of two-by-two matrices. Consider the set

$$W := \left\{ \begin{pmatrix} a & 1 \\ c & d \end{pmatrix} \mid a, c, d \in \mathbb{R} \right\}$$

is $W \subset M_{2\times 2}$ a subspace? Why, or why not?

(2) Consider the subset

$$U := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{R}, a + d \geq 0 \right\}$$

Is $U \subset M_{2\times 2}$ a subspace? Why, why not?

(3) If $A \in M_{2\times 2}$ is the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

we call

$$A^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

the *transpose* matrix.

(a) What is the transpose of

$$\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}?$$

(b) Consider the set

$$V := \left\{ A \in M_{2\times 2} \mid A + A^t = 0 \right\}$$

Is $V \subset M_{2\times 2}$ a subspace? Why, or why not?

(4) Let $\alpha \in \mathbb{R}$ be a real number (think of α as some parameter). Determine all the values of α for which

$$\begin{pmatrix} 2+\alpha \\ 3+\alpha \\ 4+2\alpha \end{pmatrix} \in \operatorname{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

(5) Are the vectors $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ linearly dependent or independent? Why?

(6) Are the vectors $\begin{pmatrix} -7 \\ -6 \\ -13 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ linearly dependent or independent? Why?

(7) We know $\vec{e}_1, \vec{e}_2, \vec{e}_3$ is a basis of $\mathbb{R}^3$. Write down a different one. Then write down another one.

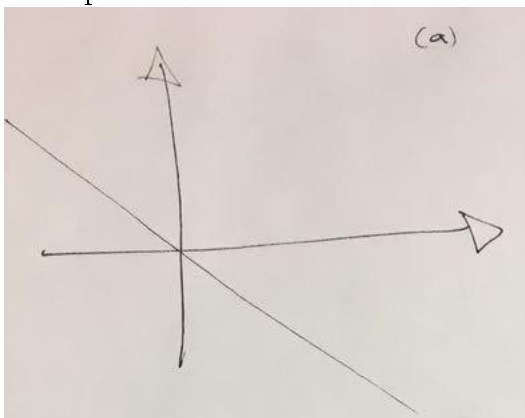
(8) Let V be a vector space. Let $S \subset V, T \subset V$ be two subsets.

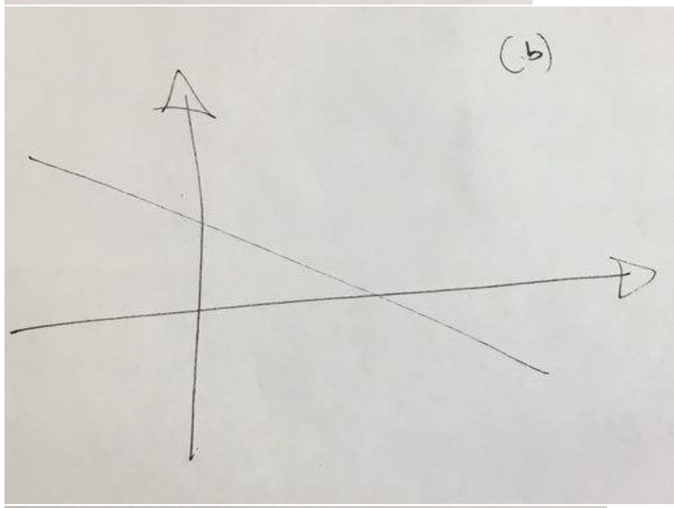
(a) If $S \subset T$, is $\operatorname{Span} S \subset \operatorname{Span} T$? If so, provide a proof. If not, exhibit a counterexample.

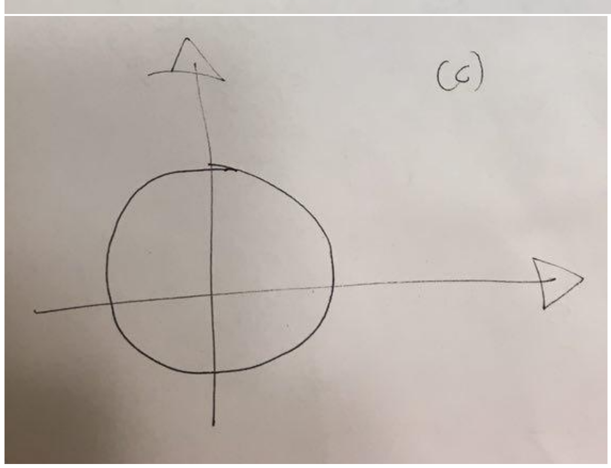
(b) Is $\operatorname{Span}(S \cup T) = \operatorname{Span}(S) \cup \operatorname{Span}(T)$? If so, provide a proof. If not, exhibit a counterexample.

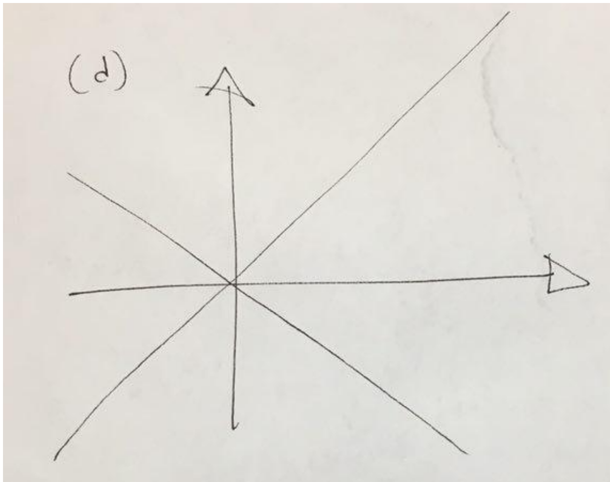
(c) Is $\operatorname{Span}(S \cap T) = \operatorname{Span}(S) \cap \operatorname{Span}(T)$? If so, provide a proof. If not, exhibit a counterexample.

(9) Here are some (incredibly accurate) sketches of subsets of $\mathbb{R}^2$. Say which ones are subspaces and which are not.





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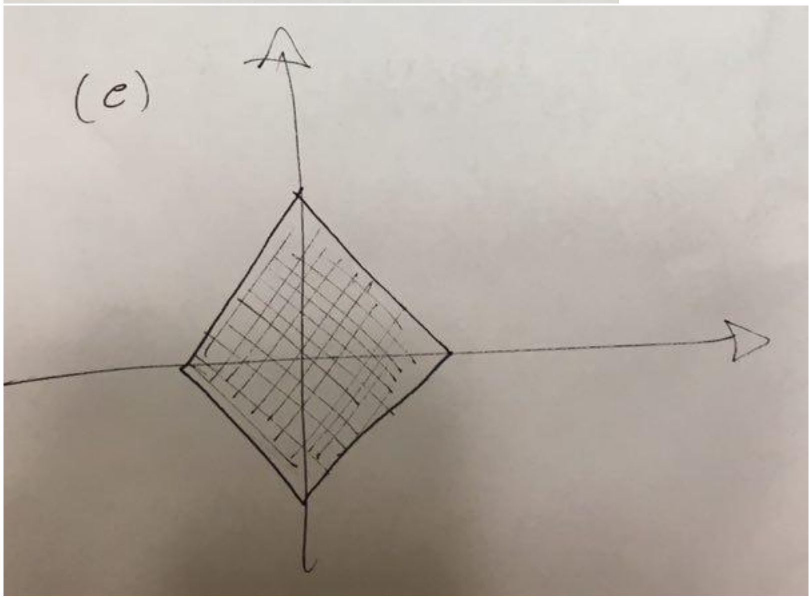
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