COMMON MATH SYMBOLS

The basic building block of modern math is the notion of *set*, which is a collection of *elements*. It's best to describe what a set is by example. For instance,

$$\mathrm{X} := \{\text{avocados on sale at the HEB on Bissonnet}\} \text{ or } \mathrm{Y} := \{\text{prime numbers}\}$$

The elements of X are all the avocados on sale at the HEB on Bissonnet, while the elements of Y are all prime numbers.

The symbol ":=" means *equal by definition*. So, one wouldn't write $9 := 3^2$ because that's not how you define the number 9.

Here are some frequently encountered sets.

$$\mathbb{N} := \{0, 1, 2, 3, \ldots\}, \quad \mathbb{Z} := \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}, \quad \mathbb{Q} := \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}$$

The last one reads as "the set of all symbols $\frac{a}{b}$ *such that* a, b are integers and b is different from zero". The *such that* is sometimes denoted as a colon, so

$$\left\{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \right\} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}.$$

The symbol $\in$ means "*belongs to*". For example, $3 \in \mathbb{Z}$ while $\frac{1}{2} \notin \mathbb{Z}$ (does not belong to).

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The symbol $\forall$ means "*for all*". The symbols $\subset$ and $\subseteq$ are interchangeable and mean "*is contained in*". For example,

$$\mathbb{N} \subset \mathbb{Z}$$

which reads "the set $\mathbb{N}$ is contained in the set $\mathbb{Z}$". Explicitly, what does it mean? It means that

$$\forall k \in \mathbb{N}, k \in \mathbb{Z}$$

which reads "for all elements k in the set $\mathbb{N}$, k belongs to the set $\mathbb{Z}$". More informally, this means that if you give me *any* natural number k then k is also an integer.

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The symbol $\exists$ means "*exists*". For example,

$$\mathrm{C} := \{n \in \mathbb{N} \mid \exists k \in \mathbb{N}, n = k^2\}$$

reads as "the set of all natural numbers n such that there exists a natural number k such that $k^2 = n$". In other words, C is the set of all squares.

Sometimes we can be more succinct, writing

$$\mathrm{D} = \{m^2 \mid m \in \mathbb{N}\}$$

which means D is the set of all the squares of natural numbers $m \in \mathbb{N}$.

Obviously, C = D. But how do we *rigorously* show that?

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To show that two sets X, Y *are the same, one must show that both* $X \subset Y$ *and* $Y \subset X$ *are true.*

Let's give it a shot.

Proposition 0.1. The sets C, D defined above are equal.

Proof. We must show both $C \subset D$ and $D \subset C$. Let us start with $C \subset D$. By definition, $n \in C$ if and only if there is $k \in \mathbb{N}$ such that $n = k^2$. Setting $m = k$, we see that $n = k^2 = m^2 \in D$. Hence, $C \subset D$.

Let us show that $D \subset C$. By definition, D consists of elements m^2 for $m \in \mathbb{N}$. Let us set $n := m^2$. We need to show that $n \in C$. But this is obvious! Set $m = k$, then $n = k^2$ so that $m^2 = n \in C$. Hence $D \subset C$. □

Of course, the proof above was incredibly easy. However, it's not always the case!

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A fundamental operation between sets is the *union*. If A, B are sets, we define

$$A \cup B := \{x \mid x \in A \text{ or } x \in B\}$$

so it is the set of elements x which belong to A or B (or both). For example,

$$A := \{x \in \mathbb{N} \mid x \text{ even}, x \geq 1\} = \{2, 4, 6, 8, 10, \ldots\}$$

$$B := \{x \in \mathbb{N} \mid x \text{ odd}, x \geq 8\} = \{9, 11, 13, 15, 17, \ldots\}$$

and

$$A \cup B := \{x \in \mathbb{N} \mid x \text{ even}, x \geq 1; \text{ or } x \text{ odd}, x \geq 8\} = \{2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, \cdots\}$$

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The second fundamental operation is that of *intersections*. If X, Y are two sets, we define

$$X \cap Y := \{x \mid x \in X \text{ and } x \in Y\}$$

so $X \cap Y$ consists of elements x which belong to *both* X and Y. For example,

$$X := \{x \in \mathbb{Z} \mid x \geq -3\}$$

$$Y := \{x \in \mathbb{Z} \mid \text{ x is odd}\}$$

$$X \cap Y = \{x \in \mathbb{Z} \mid \text{x is odd, and } x \geq -3\} = \{-3, -1, 1, 3, 5, 7, 9, \ldots\}$$

Sometimes, intersections can be a bit silly. Take the sets A, B from the example above. What is $A \cap B$? Well, it consists of all the natural numbers k, such that k is even, bigger than 1, odd and bigger than 8. But there aren't any such numbers! To express this, we write

$$A \cap B = \varnothing$$

where $\varnothing$ is called the "*empty set*", which is the only set with no elements.

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If $A \subset B$, we can form another set called the *complement* of A in B.

$$B \setminus A = \{b \in B | b \notin A\}$$

For example, if $B := \{2, 6, 10, \pi, -82\}$ and $A := \{\pi, -82, 2\}$ then $B \setminus A = \{6, 10\}$.