2017 Fall - Math 355 - Homework 8

Due: Friday, October 27 *in class.*[1]
Unless specified otherwise, you must always show your work and justify your answers.

(1) Work out the following.
   (a) $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -2 & 3 \\ -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -2 & 3 \\ -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -1 & 0 \\ -1 & -3 & 1 \\ -1 & 0 & -1 \end{pmatrix}$

(2) Let
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
what is A^{-1}? Verify that $AA^{-1} = I = A^{-1}A$.

(3) Let $f\colon V \to W$ be a linear map. Does the number of zeros in $A = \mathrm{Rep}_{\mathbb{B},\mathbb{D}}$ depend on the bases $\mathbb{B}, \mathbb{D}$? Why, why not?

(4) Let $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the identity. Let $\mathbb{B}$ be the standard basis and let $\mathbb{D}$ be the ordered basis $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$.
   (a) Write down the matrix $\mathrm{Rep}_{\mathbb{B},\mathbb{D}}\, f$.
   (b) Using the previous point, write down the following vectors as a linear combination of elements of $\mathbb{D}$.
      (i) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$
      (ii) $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$
      (iii) $\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$

(5) Let $f\colon V \to W$ be a linear map. Suppose $\dim V = \dim W$.
   (a) Show that f is an isomorphism if and only if $\ker f = 0$.
   (b) Is this still true if $\dim V \neq \dim W$? Why/Why not?
   (c) Show that f is an isomorphism if and only if $\mathrm{Im} f = W$.
   (d) Is this still true if $\dim V \neq \dim W$? Why/Why not?

[1]This file was last updated at 09:09 on Sunday 22nd October, 2017.