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Comp Phys Notes – Ch. 3

Reading Notes

<u>Intro</u>

- Oscillatory phenomena can be found in a lot of different areas of physics, and one common case is a pendulum.
- Pendulums exhibit a harmonic force.

<u>3.1</u>

- A simple pendulum can be connected to a mass on a rigid support, where the parallel forces all add up to zero.
- We then get the equation:
- F(theta) = -mgsin(theta)
- g = acceleration due to gravity
- this is negative because force is opposite of the displacement
- With Newton's second law representing F = ma, we can assume that the second derivative of theta (d^2*theta)/dt^2 = -(g/l)*theta, which is the equation of simple harmonic motion.
- We can also take a numerical approach to this problem; with the second-order DE that we made, it can be rewritten into two first0order DE's:
- d(omega)/dt = -(g/l)*theta
- d(theta)/dt = omega
- There is also a pseudo code that uses Euler's method to calculate values of theta and omega.
- The total energy equation can be defines as:
- E = 1/2(ml^2omega^2) + mgl(1 - cos(theta)
- KE = 1/2 mv^2

<u>3.2</u>

- We saw before that the equation of motion uses a frictionless pendulum.
- They incorporate a frictional force -q(d(theta)/dt), resulting in our equation of motion to be:
- (d^2*theta)/dt^2 = -(g/l)*theta - q d(theta)/dt
- The pendulum can be listed in three separate variants: underdamped, over damped, and critically damped.
- Resonance occurs when our amplitude (theta_0) can be large even with a small friction.
- With the equation of motion listed above in this section, this has a nonlinear pendulum without friction and a driving force. (according to Eq. 3.17)

- This means that there is no extra adding/removing energy to our system, and that the period cannot be independent of the amplitude.

3.3

- When we incorporate the sinusoidal driving force Fd sin(Omega_d t_), friction of the form -q(d(theta)/dt), and no expansion of sin(theta), we get the following equation of motion, which is for a **nonlinear damped driven pendulum**:
- d^2(theta)/dt^2 = -(g/l)*theta - q d(theta)/dt + Fd sin(Omega_d t_)
- In order to understand how theta acts as a function of time in this equation, we need to make a program to generate a numerical solution for this.
- By rewriting this into 2 differential equations:
- d(omega)/dt = -(g/l)*theta - q d(theta)/dt + Fd sin(Omega_d t_)
- d(theta)/dt = omega
- There are a lot of different theta vs. time graphs in Figure 3.6 that all show different behaviors for theta.
- The first one shows vertical leaps in theta when it is reset within the range of +/- pi.
- the second behavior of theta(t) for Fd = 1.2 has activity within a couple of oscillations for the decay. After this, the pendulum settles into a steady oscillation.
- The third graph shows behavior that corresponds to the angular velocity of the pendulum.
- When we have a larger value of Fd, we see that the changes in theta increase rapidly and irregularly with t. This can be indicated with a logarithmic scale to denote how fast the change in theta can be with time, and this also corresponds to a log relation which is similar to a value of Lyapunov*time:
- it implies that the change in theta is similar to e^Lyapunov*time.
- When there is a small driving force in a trajectory of omega, the phase space gets easier to understand due to how the pendulum can react. On Figure 3.8, we see two separate graphs that show the behavior of the absolute value fo theta = pi as well as how omega can behave as a function of theta for a pendulum.