JAW Codebook

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1 Basic

1.1 vimrc

```
sy on
se sw=4 ts=4 sts=4 et nu sc hls
filet plugin indent on
no <F4> :vs input.txt<CR>
no <F5> :!./a.out<CR>
no <F6> :!./a.out < input.txt<CR>
no <F9> :g++ % -std=c++17 -O2 -g
    -fsanitize=undefined -Wall -Wextra -Wshadow
    -Wno-unused-result<CR>
no <expr> <silent> <Home> col('.') ==
    match(getline('.'),'\S') + 1 ? '0' : '^'
im <silent> <Home> <C-O><Home>
```

2 Data Structure

2.1 Disjoint Set

```
struct DisjointSet {
  // save() is like recursive
  // undo() is like return
  int n, fa[MXN], sz[MXN];
  vector<pair<int*,int>> h;
  vector<int> sp;
  void init(int tn) {
    n=tn;
    for (int i=0; i<n; i++) {
      fa[i]=i;
      sz[i]=1;
    }
    sp.clear(); h.clear();
  }
  void assign(int *k, int v) {
    h.PB({k, *k});
    *k=v;
  }
  void save() { sp.PB(SZ(h)); }
  void undo() {
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
    }
  }
  int f(int x) {
    while (fa[x]!=x) x=fa[x];
    return x;
  }
  void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x]<sz[y]) swap(x, y);
    assign(&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
  }
}djs;
```

2.2 Treap

```
const int MEM = 16000004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  char val;
  int size;
  Treap () : l(&nil), r(&nil), size(0) {}
  Treap (char _val) :
    l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem =
    Treap::mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
  t->size = size(t->l) + size(t->r) + 1;
}
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t;
  if (rand() % (size(a) + size(b)) < size(a)) {
    t = new (Treap::pmem++) Treap(*a);
    t->r = merge(a->r, b);
  } else {
    t = new (Treap::pmem++) Treap(*b);
    t->l = merge(a, b->l);
  }
  pull(t);
  return t;
}
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    a = new (Treap::pmem++) Treap(*t);
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
  } else {
    b = new (Treap::pmem++) Treap(*t);
    split(t->l, k, a, b->l);
    pull(b);
  }
}

int nv;
Treap *rt[50005];

void print(const Treap *t) {
  if (!size(t)) return;
  print(t->l);
  cout << t->val;
  print(t->r);
}

int main(int argc, char** argv) {
  IOS;
  rt[nv=0] = &Treap::nil;
  Treap::pmem = Treap::mem;
  int Q, cmd, p, c, v;
  string s;
  cin >> Q;
  while (Q--) {
    cin >> cmd;
    if (cmd == 1) {
      // insert string s after position p
      cin >> p >> s;
      Treap *tl, *tr;
      split(rt[nv], p, tl, tr);
      for (int i=0; i<SZ(s); i++)
        tl = merge(tl, new (Treap::pmem++)
            Treap(s[i]));
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 2) {
      // remove c characters starting at position
      Treap *tl, *tm, *tr;
      cin >> p >> c;
      split(rt[nv], p-1, tl, tm);
      split(tm, c, tm, tr);
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 3) {
      // print c characters starting at position p,
          in version v
      Treap *tl, *tm, *tr;
      cin >> v >> p >> c;
      split(rt[v], p-1, tl, tm);
      split(tm, c, tm, tr);
      print(tm);
      cout << "\n";
    }
  }
  return 0;
}
```

2.3 Heavy Light Decomposition

```
// only one segment tree / 0-base
// should call init after input N
// getPathSeg return the segment in order u->v
// fa[root] = root

typedef pair<int,int> pii;

int N,fa[MXN],belong[MXN],dep[MXN],sz[MXN],que[MXN];
int step,line[MXN],stPt[MXN],edPt[MXN];
vector<int> E[MXN], chain[MXN];

void init() {
  REP(i,N) {
    E[i].clear();
    chain[i].clear();
  }
}
void DFS(int u){
  vector<int> &c = chain[belong[u]];
  for (int i=c.size()-1; i>=0; i--){
    int v = c[i];
    stPt[v] = step;
    line[step++] = v;
  }
  for (int i=0; i<(int)c.size(); i++){
    u = c[i];
    for (auto v : E[u]){
      if (fa[u] == v || (i && v == c[i-1])) continue;
      DFS(v);
    }
    edPt[u] = step-1;
  }
}
void build_chain(int st){
  int fr,bk;
  fr=bk=0; que[bk++]=st; fa[st]=st; dep[st]=0;
  while (fr < bk){
    int u=que[fr++];
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      que[bk++] = v;
      dep[v] = dep[u]+1;
      fa[v] = u;
    }
  }
  for (int i=bk-1,u,pos; i>=0; i--){
    u = que[i]; sz[u] = 1; pos = -1;
    for (auto v : E[u]){
      if (v == fa[u]) continue;
      sz[u] += sz[v];
      if (pos==-1 || sz[v]>sz[pos]) pos=v;
    }
    if (pos == -1) belong[u] = u;
    else belong[u] = belong[pos];
    chain[belong[u]].PB(u);
  }
  step = 0;
  DFS(st);
}
int getLCA(int u, int v){
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]) u = fa[a];
    else v = fa[b];
  }
  return sz[u] >= sz[v] ? u : v;
}
vector<pii> getPathSeg(int u, int v){
  vector<pii> ret1,ret2;
  while (belong[u] != belong[v]){
    int a = chain[belong[u]].back();
    int b = chain[belong[v]].back();
    if (dep[a] > dep[b]){
      ret1.PB({stPt[a],stPt[u]});
      u = fa[a];
    } else {
      ret2.PB({stPt[b],stPt[v]});
      v = fa[b];
    }
  }
  if (dep[u] > dep[v]) swap(u,v);
  ret1.PB({stPt[u],stPt[v]});
  reverse(ret2.begin(), ret2.end());
  ret1.insert(ret1.end(),ret2.begin(),ret2.end());
  return ret1;
}
// Usage
void build(){
  build_chain(0); //change root
  init(0,step,0); //init segment tree
}
int get_answer(int u, int v){
  int ret = -2147483647;
  vector<pii> vec = getPathSeg(u,v);
  for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
  return ret;
}
```

2.4 Link-Cut Tree

```
const int MXN = 100005;
const int MEM = 100005;

struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  }
  Splay (int _val) : val(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  }
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  }
  int dir() {
    return f->ch[0] == this ? 0 : 1;
  }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
      if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
    }
  }
  void pull() {
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem =
    Splay::mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir())
        rotate(x->f),rotate(x);
    else rotate(x),rotate(x);
  }
}

Splay* access(Splay *x) {
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void evert(Splay *x) {
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y) {
//  evert(x);
  access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
//  evert(x);
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res;
}
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
  for (int i=1; i<=N; i++)
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
    char cmd[105];
    int u, v;
    scanf("%s", cmd);
    if (cmd[1] == 'i') {
      scanf("%d%d", &u, &v);
      link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
      scanf("%d", &v);
      cut(vt[1], vt[v]);
    } else {
      scanf("%d%d", &u, &v);
      int res=ask(vt[u], vt[v]);
      printf("%d\n", res);
    }
  }

  return 0;
}
```

3 Graph

3.1 BCC Edge

```
struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();
    djs.init(n);
  }
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
    m++;
  }
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
      if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
  }
}graph;
```

3.2 BCC Vertex

```
struct BccVertex {
  int n,nBcc,step,root,dfn[MXN],low[MXN];
  vector<int> E[MXN], ap;
  vector<pii> bcc[MXN];
  int top;
  pii stk[MXN];
  void init(int _n) {
    n = _n;
    nBcc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    int son = 0;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        son++;
        stk[top++] = {u,v};
        DFS(v,u);
        if (low[v] >= dfn[u]) {
          if(v != root) ap.PB(v);
          do {
            assert(top > 0);
            bcc[nBcc].PB(stk[--top]);
          } while (stk[top] != pii(u,v));
          nBcc++;
        }
        low[u] = min(low[u], low[v]);
      } else {
        if (dfn[v] < dfn[u]) stk[top++] = pii(u,v);
        low[u] = min(low[u],dfn[v]);
      }
    }
    if (u == root && son > 1) ap.PB(u);
  }
  // return the edges of each bcc;
  vector<vector<pii>> solve() {
    vector<vector<pii>> res;
    for (int i=0; i<n; i++) {
      dfn[i] = low[i] = -1;
    }
    ap.clear();
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) {
        top = 0;
        root = i;
        DFS(i,i);
      }
    }
    REP(i,nBcc) res.PB(bcc[i]);
    return res;
  }
}graph;
```

3.3 Strongly Connected Components

```
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
    for (int i=0; i<n; i++){
      E[i].clear();
      rE[i].clear();
    }
  }
  void add_edge(int u, int v){
    E[u].PB(v);
    rE[v].PB(u);
  }
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u])
      if (!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1;
    bln[u] = nScc;
    for (auto v : rE[u])
      if (!vst[v]) rDFS(v);
  }
  void solve(){
    nScc = 0;
    vec.clear();
    for (int i=0; i<n; i++) vst[i] = 0;
    for (int i=0; i<n; i++)
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    for (int i=0; i<n; i++) vst[i] = 0;
    for (auto v : vec){
      if (!vst[v]){
        rDFS(v);
        nScc++;
      }
    }
  }
};
```

3.4 DMST_with_sol

```
const int INF = 1029384756;

struct edge_t{
  int u,v,w;
  set< pair<int,int> > add, sub;
  edge_t() : u(-1), v(-1), w(0) {}
  edge_t(int _u, int _v, int _w) {
    u = _u; v = _v; w = _w;
    add.insert({u, v});
  }
  edge_t& operator += (const edge_t& obj) {
    w += obj.w;
    FOR (it, obj.add) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    }
    FOR (it, obj.sub) {
      if (!add.count(*it)) sub.insert(*it);
      else add.erase(*it);
    }
    return *this;
  }
  edge_t& operator -= (const edge_t& obj) {
    w -= obj.w;
    FOR (it, obj.sub) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
    }
    for (auto it : obj.add) {
      if (!add.count(it)) sub.insert(it);
      else add.erase(it);
    }
    return *this;
  }
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MXN],contracted[MXN];
vector<int> E[MXN];

edge_t dmst(int rt){
  edge_t cost;
  for (int i=0; i<N; i++){
    contracted[i] = incyc[i] = 0;
    prv[i] = EDGE_INF;
  }
  cid = 0;
  int u,v;
  while (true){
    for (v=0; v<N; v++){
      if (v != rt && !contracted[v] && prv[v].w ==
          INF) break;
    }
    if (v >= N) break; // end
    for (int i=0; i<M; i++){
      if (eg[i].v == v && eg[i].w < prv[v].w)
        prv[v] = eg[i];
    }
    if (prv[v].w == INF) // not connected
      return EDGE_INF;
    cost += prv[v];
    for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
    if (u == -1) continue;
    incyc[v] = ++cid;
    for (u=prv[v].u; u!=v; u=prv[u].u){
      contracted[u] = 1;
      incyc[u] = cid;
    }
    for (int i=0; i<M; i++){
      if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
          cid){
        eg[i] -= prv[eg[i].v];
      }
    }
    for (int i=0; i<M; i++){
      if (incyc[eg[i].u] == cid) eg[i].u = v;
      if (incyc[eg[i].v] == cid) eg[i].v = v;
      if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
    }
    for (int i=0; i<N; i++){
      if (contracted[i]) continue;
      if (prv[i].u>=0 && incyc[prv[i].u] == cid)
        prv[i].u = v;
    }
    prv[v] = EDGE_INF;
  }
  return cost;
}

void solve(){
  edge_t cost = dmst(0);
  for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
  }
}
```

3.5 Dominator Tree

```
// idom[n] is the unique node that strictly
    dominates n but does
// not strictly dominate any other node that
    strictly dominates n.
// idom[n] = 0 if n is entry or the entry cannot
    reach n.
struct DominatorTree{
  static const int MAXN = 200010;
  int n,s;
  vector<int> g[MAXN],pred[MAXN];
  vector<int> cov[MAXN];
  int dfn[MAXN],nfd[MAXN],ts;
  int par[MAXN];
  int sdom[MAXN],idom[MAXN];
  int mom[MAXN],mn[MAXN];

  inline bool cmp(int u,int v) { return dfn[u] <
      dfn[v]; }

  int eval(int u) {
    if(mom[u] == u) return u;
    int res = eval(mom[u]);
    if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
      mn[u] = mn[mom[u]];
    return mom[u] = res;
  }

  void init(int _n, int _s) {
    n = _n;
    s = _s;
    REP1(i,1,n) {
      g[i].clear();
      pred[i].clear();
      idom[i] = 0;
    }
  }
  void add_edge(int u, int v) {
    g[u].push_back(v);
    pred[v].push_back(u);
  }
  void DFS(int u) {
    ts++;
    dfn[u] = ts;
    nfd[ts] = u;
    for(int v:g[u]) if(dfn[v] == 0) {
      par[v] = u;
      DFS(v);
    }
  }
  void build() {
    ts = 0;
    REP1(i,1,n) {
      dfn[i] = nfd[i] = 0;
      cov[i].clear();
      mom[i] = mn[i] = sdom[i] = i;
    }
    DFS(s);
    for (int i=ts; i>=2; i--) {
      int u = nfd[i];
      if(u == 0) continue ;
      for(int v:pred[u]) if(dfn[v]) {
        eval(v);
        if(cmp(sdom[mn[v]],sdom[u])) sdom[u] =
            sdom[mn[v]];
      }
      cov[sdom[u]].push_back(u);
      mom[u] = par[u];
      for(int w:cov[par[u]]) {
        eval(w);
        if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];
        else idom[w] = par[u];
      }
      cov[par[u]].clear();
    }
    REP1(i,2,ts) {
      int u = nfd[i];
      if(u == 0) continue ;
      if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
    }
  }
}dom;
```

3.6 Maximum Clique

```
class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }

    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a ; d++) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg ++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }

    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};
```

3.7 MinimumMeanCycle

```
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
    double avg=-inf;
    for(int k=0; k<n; k++) {
      if(d[n][i]<inf-eps)
          avg=max(avg,(d[n][i]-d[k][i])/(n-k));
      else avg=max(avg,inf);
    }
    if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  for(int i=0; i<n; i++) vst[i] = 0;
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  }
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
}
```

4 Flow

4.1 Push-relabel

```
#include <algorithm>
#include <list>
constexpr int maxn = 604;
int c[maxn][maxn], f[maxn][maxn], h[maxn], e[maxn],
    g[2 * maxn + 1];
int max_flow(int s, int t, int n) {
  for (int i = 0; i < n; i++)
    fill_n(f[i], n, 0);
  fill_n(h, n, 0);
  fill_n(e, n, 0);
  fill_n(g, 2 * n + 1, 0);

  for (int i = 0; i < n; i++) {
    f[s][i] = e[i] = c[s][i];
    f[i][s] = -c[s][i];
  }
  h[s] = n;
  e[s]++, e[t]++;
  g[0] = n - 1;
  g[n] = 1;

  list<int> fifo;
  for (int i = 0; i < n; i++)
    if (i != s && i != t && e[i])
      fifo.push_back(i);
  while (!fifo.empty()) {
    int u = fifo.front();
    fifo.pop_front();
    while (e[u]) {
      for (int v = 0; e[u] && v < n; v++) {
        if (h[u] == h[v] + 1 && f[u][v] < c[u][v]) {
          if (e[v] == 0)
            fifo.push_back(v);
          int x = min(e[u], c[u][v] - f[u][v]);
          e[u] -= x;
          e[v] += x;
          f[u][v] += x;
          f[v][u] -= x;
        }
      }
      if (e[u]) {
        if (--g[h[u]] == 0 && h[u] < n)
          for (int i = 0; i < n; i++)
            if (h[i] > h[u] && h[i] < n)
              h[i] = n + 1;
        h[u] = 2 * n;
        for (int v = 0; v < n; v++)
          if (f[u][v] < c[u][v])
            h[u] = min(h[u], h[v] + 1);
        g[h[u]]++;
      }
    }
  }
  return e[t] - 1;
}
```

4.2 Dinic

```
struct Dinic{
  static const int MXN = 10000;
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  }
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
        }
      }
    }
    return level[t] != -1;
  }
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
      }
    }
    if (!res) level[u] = -1;
    return res;
  }
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
  }
}flow;
```

4.3 Cost Flow

```
typedef pair<long long, long long> pll;
struct CostFlow {
  static const int MXN = 205;
  static const long long INF = 102938475610293847LL;
  struct Edge {
    int v, r;
    long long f, c;
  };
  int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
  long long dis[MXN], fl, cost;
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t) {
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
    fl = cost = 0;
  }
  void add_edge(int u, int v, long long f, long long
      c) {
    E[u].PB({v, SZ(E[v])  , f,  c});
    E[v].PB({u, SZ(E[u])-1, 0, -c});
  }
  pll flow() {
    while (true) {
      for (int i=0; i<n; i++) {
        dis[i] = INF;
        inq[i] = 0;
      }
      dis[s] = 0;
      queue<int> que;
      que.push(s);
      while (!que.empty()) {
        int u = que.front(); que.pop();
        inq[u] = 0;
        for (int i=0; i<SZ(E[u]); i++) {
          int v = E[u][i].v;
          long long w = E[u][i].c;
          if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
            prv[v] = u; prvL[v] = i;
            dis[v] = dis[u] + w;
            if (!inq[v]) {
              inq[v] = 1;
              que.push(v);
            }
          }
        }
      }
      if (dis[t] == INF) break;
      long long tf = INF;
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        tf = min(tf, E[u][l].f);
      }
      for (int v=t, u, l; v!=s; v=u) {
        u=prv[v]; l=prvL[v];
        E[u][l].f -= tf;
        E[v][E[u][l].r].f += tf;
      }
      cost += tf * dis[t];
      fl += tf;
    }
    return {fl, cost};
  }
}flow;
```

4.4 Kuhn Munkres

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect
    Match)
  static const int MXN = 650;
  static const int INF = 2147483647; // long long
  int n,match[MXN],vx[MXN],vy[MXN];
  int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
  // ^^^^ long long
  void init(int _n){
    n = _n;
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
  }
  void add_edge(int x, int y, int w){ // long long
    edge[x][y] = w;
  }
  bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){
      if (vy[y]) continue;
      if (lx[x]+ly[y] > edge[x][y]){
        slack[y] = min(slack[y],
            lx[x]+ly[y]-edge[x][y]);
      } else {
        vy[y] = 1;
        if (match[y] == -1 || DFS(match[y])){
          match[y] = x;
          return true;
        }
      }
    }
    return false;
  }
  int solve(){
    fill(match,match+n,-1);
    fill(lx,lx+n,-INF);
    fill(ly,ly+n,0);
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){
      fill(slack,slack+n,INF);
      while (true){
        fill(vx,vx+n,0);
        fill(vy,vy+n,0);
        if ( DFS(i) ) break;
        int d = INF; // long long
        for (int j=0; j<n; j++)
          if (!vy[j]) d = min(d, slack[j]);
        for (int j=0; j<n; j++){
          if (vx[j]) lx[j] -= d;
          if (vy[j]) ly[j] += d;
          else slack[j] -= d;
        }
      }
    }
    int res=0;
    for (int i=0; i<n; i++)
      res += edge[match[i]][i];
    return res;
  }
}graph;
```

4.5 SW-Mincut

```
struct SW{ // O(V^3) 0-base
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n;
    for (int i=0; i<n; i++) {
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
      del[i] = 0;
    }
  }
  void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
  }
  void search(int &s, int &t){
    for (int i=0; i<n; i++)
      vst[i] = wei[i] = 0;
    s = t = -1;
    while (true){
      int mx=-1, cur=0;
      for (int i=0; i<n; i++)
        if (!del[i] && !vst[i] && mx<wei[i])
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t;
      t = cur;
      for (int i=0; i<n; i++)
        if (!vst[i] && !del[i]) wei[i] +=
            edge[cur][i];
    }
  }
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  }
}graph;
```

4.6 Maximum Simple Graph Matching

```
struct GenMatch { // 1-base
  static const int MAXN = 514;
  int V;
  bool el[MAXN][MAXN];
  int pr[MAXN];
  bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue<int> qe;
  int st,ed;
  int nb;
  int bk[MAXN],djs[MAXN];
  int ans;
  void init(int _V) {
    V = _V;
    for(int i = 0; i <= V; i++) {
      for(int j = 0; j <= V; j++) el[i][j] = 0;
      pr[i] = bk[i] = djs[i] = 0;
      inq[i] = inp[i] = inb[i] = 0;
    }
    ans = 0;
  }
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  }
  int lca(int u,int v) {
    for(int i = 0; i <= V; i++) inp[i] = 0;
    while(1) {
      u = djs[u];
      inp[u] = true;
      if(u == st) break;
      u = bk[pr[u]];
    }
    while(1) {
      v = djs[v];
      if(inp[v]) return v;
      v = bk[pr[v]];
    }
    return v;
  }
  void upd(int u) {
    int v;
    while(djs[u] != nb) {
      v = pr[u];
      inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if(djs[u] != nb) bk[u] = v;
    }
  }
  void blo(int u,int v) {
    nb = lca(u,v);
    for (int i=0; i<=V; i++) inb[i] = 0;
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)
      if(inb[djs[tu]]) {
        djs[tu] = nb;
        if(!inq[tu]){
          qe.push(tu);
          inq[tu] = 1;
        }
      }
  }
  void flow() {
    for(int i = 1; i <= V; i++) {
      inq[i] = 0;
      bk[i] = 0;
      djs[i] = i;
    }

    while(qe.size()) qe.pop();
    qe.push(st);
    inq[st] = 1;
    ed = 0;
    while(qe.size()) {
      int u = qe.front(); qe.pop();
      for(int v = 1; v <= V; v++)
        if(el[u][v] && (djs[u] != djs[v]) && (pr[u]
            != v)) {
          if((v == st) || ((pr[v] > 0) && bk[pr[v]]
              > 0))
            blo(u,v);
          else if(bk[v] == 0) {
            bk[v] = u;
            if(pr[v] > 0) {
              if(!inq[pr[v]]) qe.push(pr[v]);
            } else {
              ed = v;
              return;
            }
          }
        }
    }
  }
  void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
      v = bk[u];
      w = pr[v];
      pr[v] = u;
      pr[u] = v;
      u = w;
    }
  }
  int solve() {
    for(int i = 0; i <= V; i++) pr[i] = 0;
    for(int u = 1; u <= V; u++)
      if(pr[u] == 0) {
        st = u;
        flow();
        if(ed > 0) {
          aug();
          ans ++;
        }
    }
    return ans;
  }
}G;

int main() {
  G.init(V);
  for(int i=0; i<E; i++) {
    int u, v;
    cin >> u >> v;
    G.add_edge(u, v);
  }
  cout << G.solve() << endl;
}
```

4.7 Minimum Weight Matching (Clique version)

```
struct Graph {
  // Minimum General Weighted Matching (Perfect
      Match) 0-base
  static const int MXN = 105;

  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;

  void init(int _n) {
    n = _n;
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
  }
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] +
            edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
        }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }

  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    }
    while (true){
      int found = 0;
      for (int i=0; i<n; i++)
        dis[i] = onstk[i] = 0;
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
          }
        }
      }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

4.8 (+1) SW-mincut $O(NM)$

```
// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
  public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly
        needed
      DisjointSet djs;
    vector<int> cut;
    //--8<------------------------------------------
      StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
        for(int i=0;i<n;i++)
          merged[i]=0;
        for(int i=0;i<n;i++)
          for(int j=0;j<n;j++)
            cost[i][j]=cost[j][i]=0;
      }
    void append(int v,int u,int c) {
      if(v==u) return;
      if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
      }
      cost[v][u]+=c;
      cost[u][v]+=c;
    }
    void merge(int v,int u) {
      merged[u]=1;
      for(int i=0;i<n;i++)
        append(v,i,cost[u][i]);
      // --8<-- include only if cut is explicitly
          needed
        djs.merge(v,u);
      //--8<------------------------------------------
    }
    void phase() {
      priority_queue<pii> pq;
      for(int v=0;v<n;v++) {
        if(merged[v]) continue;
        cs[v]=0;
        sel[v]=0;
        pq.push({0,v});
      }
      int v,s,pv;
      while(pq.size()) {
        if(cs[pq.top().S]>pq.top().F) {
          pq.pop();
          continue;
        }
        pv=v;
        v=pq.top().S;
        s=pq.top().F;
        pq.pop();
        sel[v]=1;
        for(int i=0;i<adj[v].size();i++) {
```

```
        int u=adj[v][i];
        if(merged[u]||sel[u]) continue;
        cs[u]+=cost[v][u];
        pq.push({cs[u],u});
      }
    }
    if(s<mc) {
      mc=s;
      // --8<-- include only if cut is explicitly
      needed ------
        cut.clear();
      for(int i=0;i<n;i++)
        if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
      //--8<----------------------------------------
    }
    merge(v,pv);
  }
  int mincut() {
    if(mc==inf) {
      for(int t=0;t<n-1;t++)
        phase();

    return mc;
  }
  // --8<-- include only if cut is explicitly
      needed
  ------
    vector<int> getcut() { // return one side of
        the cut
      mincut();
      return cut;
    }
  //--8<------------------------------------------
};
// }}}
```

5 Math

5.1 ax+by=gcd

```
typedef pair<int, int> pii;

pii gcd(int a, int b){
  if(b == 0) return make_pair(1, 0);
  else{
    int p = a / b;
    pii q = gcd(b, a % b);
    return make_pair(q.second, q.first - q.second *
        p);
  }
}
```

5.2 Fast Fourier Transform

```
// const int MAXN = 262144;
// (must be 2^k)

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);

cplx omega[MAXN+1];
void pre_fft()
{
  for(int i=0; i<=MAXN; i++)
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
void fft(int n, cplx a[], bool inv=false)
{
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {
      cplx w = omega[inv ? MAXN-(i*theta%MAXN) :
          i*theta%MAXN];
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    }
    theta = (theta * 2) % MAXN;
  }
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
  if (inv)
    for (i = 0; i < n; i++)
      a[i] /= n;
}
```

5.3 Fast Linear Recurrence

```
ll n,m,dp[N+N];
void pre_dp(){
  dp[0]= 1;
  ll bdr = min(m+m,n);
  for(ll i=1; i<=bdr; i++)
    for(ll j=i-1; j>=max(0ll,i-m); j--)
      dp[i]= add(dp[i],dp[j]);
}
vector<ll> Mul(const vector<ll>& v1,const
    vector<ll>& v2){
  int sz1 = (int)v1.size();
  int sz2 = (int)v2.size();
  assert(sz1 == m and sz2 == m);
  vector<ll> _v(m+m);
  for(int i=0; i<m+m; i++) _v[i]= 0;
  // expand
  for(int i=0; i<sz1; i++)
    for(int j=0; j<sz2; j++)
      _v[i+j+1]= add(_v[i+j+1],mul(v1[i],v2[j]));
  // shrink
  for(int i=0; i<m; i++)
    for(int j=1; j<=m; j++)
      _v[i + j]= add(_v[i + j],_v[i]);
  for(int i=0; i<m; i++)
    _v[i]= _v[i + m];
  _v.resize(m);
  return _v;
}
vector<ll> I,A;
ll solve(){
  pre_dp();
  if(n <= m+m)return dp[n];
  I.resize(m);
  A.resize(m);
  for(int i=0; i<m; i++) I[i]=A[i]=1;
  // dp[n]= /Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
  ll dlt = (n - m) / m;
  ll rdlt = dlt * m;
  while(dlt){
    if(dlt & 1ll) I = Mul(I,A);
    A = Mul(A,A);
    dlt >>= 1;
  }
  ll ans = 0;
  for(int i=0; i<m; i++)
    ans = add(ans,mul(I[i],dp[n-i-1-rdlt]));
  return ans;
}
```

5.4 (+1) ntt

```
int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
```

```
/*
p=a*2^n+1
n   2^n         p             a     root
5   32          97            3     5
6   64          193           3     5
7   128         257           2     3
8   256         257           1     3
9   512         7681          15    17
10  1024        12289         12    11
11  2048        12289         6     11
12  4096        12289         3     11
13  8192        40961         5     3
14  16384       65537         4     3
15  32768       65537         2     3
16  65536       65537         1     3
17  131072      786433        6     10
18  262144      786433        3     10 (605028353,
    2308, 3)
19  524288      5767169       11    3
20  1048576     7340033       7     3
21  2097152     23068673      11    3
22  4194304     104857601     25    3
23  8388608     167772161     20    3
24  16777216    167772161     10    3
25  33554432    167772161     5     3 (1107296257,
    33, 10)
26  67108864    469762049     7     3
27  134217728   2013265921    15    31
*/
int bigmod(long long a,int b){
  if(b==0)return 1;
  return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
}
int inv(int a,int b){
  if(a==1)return 1;
  return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
}
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
  std::vector<unsigned int> co;
  int n;//polynomial degree = n
  poly(int d){n=d;co.resize(n+1,0);}
  void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for(N=2;N<=NN;N<<=1,--r){
      for(st=0;st<NN;st+=N){
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=a+P-b; if(co[ss+i]>=P)co[ss+i]-=P;
        }
      }
    }
  }
  void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for(N=NN;N>1;N>>=1,++r){
      for(st=0;st<NN;st+=N){
        int i,ss=st+(N>>1);
        for(i=(N>>1)-1;i>=0;--i){
          a=co[st+i]; b=co[ss+i];
          co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
          co[ss+i]=((a+P-b)*ps[i<<r])%P;
        }
      }
    }
  }
  poly operator*(const poly& _b)const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
    a.trans1(N);b.trans1(N);
    for(i=0;i<N;++i)a.co[i]=((long
        long)a.co[i]*b.co[i])%P
      ;
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
    a.trans2(N);
    for(i=0;i<N;++i)a.co[i]=((long
        long)a.co[i]*Ni)%P;
    a.n=n+_b.n; return a;
  }
};
```

5.5 Mod

```
/// _fd(a,b)  floor(a/b).
/// _rd(a,m)  a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r
    }.

int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r)
{
    m=abs(m);
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r)
{
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}
```

5.6 (+1) Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383        6 :  pirmes <= 13
// n < 2^64                     7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2,  n  2 ]
    if
// you want to use magic.
long long power(long long x,long long p,long long
    mod){
  long long s=1,m=x;
  while(p) {
    if(p&1) s=mult(s,m,mod);
    p>>=1;
    m=mult(m,m,mod);
  }
  return s;
}
bool witness(long long a,long long n,long long u,int
    t){
  long long x=power(a,u,n);
  for(int i=0;i<t;i++) {
    long long nx=mult(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
}
```

```
bool miller_rabin(long long n,int s=100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;
  if(!(n&1)) return n==2;
  long long u=n-1;
  int t=0;
  // n-1 = u*2^t
  while(!(u&1)) {
    u>>=1;
    t++;
  }
  while(s--) {
    long long a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
  }
  return 1;
}
```

5.7 Pollard Rho

```
// does not work when n is prime
long long modit(long long x,long long mod) {
  if(x>=mod) x-=mod;
  //if(x<0) x+=mod;
  return x;
}
long long mult(long long x,long long y,long long
    mod) {
  long long s=0,m=x%mod;
  while(y) {
    if(y&1) s=modit(s+m,mod);
    y>>=1;
    m=modit(m+m,mod);
  }
  return s;
}
long long f(long long x,long long mod) {
  return modit(mult(x,x,mod)+1,mod);
}
long long pollard_rho(long long n) {
  if(!(n&1)) return 2;
  while (true) {
    long long y=2, x=rand()%(n-1)+1, res=1;
    for (int sz=2; res==1; sz*=2) {
      for (int i=0; i<sz && res<=1; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if (res!=0 && res!=n) return res;
  }
}
```

5.8 Algorithms about Primes

```
/*
 * 12721
 * 13331
 * 14341
 * 75577
 * 123457
 * 222557
 * 556679
 * 999983
 * 1097774749
 * 1076767633
 * 100102021
 * 999997771
 * 1001010013
 * 1000512343
 * 987654361
 * 999991231
 * 999888733
 * 98789101
 * 987777733
 * 999991921
 * 1010101333
 * 1010102101
 * 1000000000039
 * 1000000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
 * 18446744073709551557
 */

int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
  mu[1] = p_tbl[1] = 1;
  for (int i=2; i<MX; i++) {
    if (!p_tbl[i]) {
      p_tbl[i] = i;
      primes.PB(i);
      mu[i] = -1;
    }
    for (auto p : primes) {
      int x = i*p;
      if (x >= M) break;
      p_tbl[x] = p;
      mu[x] = -mu[i];
      if (i%p==0) {
        mu[x] = 0;
        break;
      }
    }
  }
}

vector<int> factor(int x) {
  vector<int> fac{1};
  while (x > 1) {
    int fn=SZ(fac), p=p_tbl[x], pos=0;
    while (x%p == 0) {
      x /= p;
      for (int i=0; i<fn; i++)
        fac.PB(fac[pos++]*p);
    }
  }
  return fac;
}
```

5.9 (+1) PolynomialGenerator

```
class PolynomialGenerator {
  /* for a nth-order polynomial f(x), *
   * given f(0), f(1), ..., f(n) *
   * express f(x) as sigma_i{c_i*C(x,i)} */
  public:
    int n;
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx
        should be
    // filled with f(0) to f(n)
      PolynomialGenerator(int _n,vector<long long>
          _fx):n(_n
          ),coef(_fx) {
        for(int i=0;i<n;i++)
          for(int j=n;j>i;j--)
            coef[j]-=coef[j-1];
      }
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
      long long m=1,ret=0;
      for(int i=0;i<=n;i++) {
        ret+=coef[i]*m;
        m=m*(x-i)/(i+1);
      }
      return ret;
    }
};
```

5.10 Pseudoinverse of Square matrix

```
Mat pinv(Mat m)
{
  Mat res = I;

  FZ(used);
  for(int i=0; i<W; i++)
  {
    int piv = -1;
    for(int j=0; j<W; j++)
    {
      if(used[j]) continue;
      if(abs(m.v[j][i]) > EPS)
      {
        piv = j;
        break;
      }
    }
    if(piv == -1)
      continue;
    used[i] = true;
    swap(m.v[piv], m.v[i]);
    swap(res.v[piv], res.v[i]);

    ld rat = m.v[i][i];
    for(int j=0; j<W; j++)
    {
      m.v[i][j] /= rat;
      res.v[i][j] /= rat;
    }

    for(int j=0; j<W; j++)
    {
      if(j == i) continue;
      rat = m.v[j][i];
      for(int k=0; k<W; k++)
      {
        m.v[j][k] -= rat * m.v[i][k];
        res.v[j][k] -= rat * res.v[i][k];
      }
    }
  }

  for(int i=0; i<W; i++)
  {
    if(used[i]) continue;
    for(int j=0; j<W; j++)
      res.v[i][j] = 0;
  }

  return res;
}
```

5.11 Simplex

```
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxm],
    d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxm], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j)
            d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s];
            ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i)
                if (i != r) {
                    for (int j = 0; j <= m; ++j)
                        if (j != s)
                            d[i][j] +=
                                d[r][j]*d[i][s];
                    d[i][s] *= d[r][s];
                }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps || (d[n +
                    1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i=0; i<n; ++i) if (d[i][s] < -eps) {
            if (r < 0 || (dd = d[r][m] / d[r][s] -
                d[i][m] / d[i][s]) < -eps || (dd <
                eps && ix[r + m] > ix[i + m])) r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not
        executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1)
        {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i-m][m];
        }
    }
    return ans;
}
```

6 Geometry

6.1 Point operators

```
#define x first
#define y second

#define cpdd const pdd
struct pdd : pair<double, double> {
    using pair<double, double>::pair;

    pdd operator + (cpdd &p) const {
        return {x+p.x, y+p.y};
    }

    pdd operator - () const {
        return {-x, -y};
    }

    pdd operator - (cpdd &p) const {
        return (*this) + (-p);
    }

    pdd operator * (double f) const {
        return {f*x, f*y};
    }

    double operator * (cpdd &p) const {
        return x*p.x + y*p.y;
    }
};

double abs(cpdd &p) { return hypot(p.x, p.y); }
double arg(cpdd &p) { return atan2(p.y, p.x); }
double cross(cpdd &p, cpdd &q) { return p.x*q.y -
    p.y*q.x; }
double cross(cpdd &p, cpdd &q, cpdd &o) { return
    cross(p-o, q-o); }
pdd operator * (double f, cpdd &p) { return p*f; }
    // !! Not f*p !!
```

6.2 Intersection of two circles

```
using ld = double;
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
    double r2) {
  ld d2 = (o1 - o2) * (o1 - o2);
  ld d = sqrt(d2);
  if (d < abs(r1-r2)) return {};
  if (d > r1+r2) return {};
  pdd u = 0.5*(o1+o2) +
      ((r2*r2-r1*r1)/(2*d2))*(o1-o2);
  double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d)
      * (-r1+r2+d));
  pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
  return {u+v, u-v};
}
```

6.3 Intersection of two lines

```
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool
    &res){
  double f1 = cross(p2, q1, p1);
  double f2 = -cross(p2, q2, p1);
  double f = (f1 + f2);

  if(fabs(f) < EPS) {
    res = false;
    return {};
    }

  res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
}
```

6.4 Half Plane Intersection

```
const double EPS = 1e-9;

pdd interPnt(Line l1, Line l2, bool &res){
    pdd p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
  double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
  double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
  return (f2 / f) * q1 + (f1 / f) * q2;
}

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    pdd p = interPnt(l1, l2, res);
    return cross(l0.S, p, l0.F) > EPS;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F)
     > 0
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
        pdd d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    }
    sort(ALL(ord), [&](int i, int j) {
        if (abs(ata[i] - ata[j]) < EPS) {
            return cross(lines[i].S, lines[j].S,
                lines[i].F) < 0;
        }
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; i++) {
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]])
            > EPS) {
            fin.PB(lines[ord[i]]);
        }
    }

    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {
        while(SZ(dq) >= 2 and
              not isin(fin[i], dq[SZ(dq)-2],
                  dq[SZ(dq)-1])) {
            dq.pop_back();
        }
        while(SZ(dq) >= 2 and
              not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        }
        dq.push_back(fin[i]);
    }

    while (SZ(dq) >= 3 and
           not isin(dq[0], dq[SZ(dq)-2],
               dq[SZ(dq)-1])) {
        dq.pop_back();
    }

    while (SZ(dq) >= 3 and
           not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
    }
    vector<Line> res(ALL(dq));
    return res;
}
```

6.5 2D Convex Hull

```
vector<pdd> convex_hull(vector<pdd> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<pdd> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){
    while (top >= 2 &&
        cross(stk[top-1],pt[i],stk[top-2]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-1],pt[i],
        stk[top-2]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  stk.resize(top-1);
  return stk;
}
```

6.6 3D Convex Hull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
  ld x,y,z;
  Point operator - (const Point &b) const {
    return (Point){x-b.x,y-b.y,z-b.z};
  }
  Point operator * (const ld &b) const {
    return (Point){x*b,y*b,z*b};
  }
  ld len() const { return sqrtl(x*x+y*y+z*z); }
  ld dot(const Point &a) const {
    return x*a.x+y*a.y+z*a.z;
  }
  Point operator * (const Point &b) const {
    return
        (Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y};
  }
};
Point ver(Point a, Point b, Point c) {
  return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
  int n = SZ(pt);
  REP(i,n) REP(j,n)
    flag[i][j] = 0;

  vector<Face> now;
  now.push_back((Face){0,1,2});
  now.push_back((Face){2,1,0});
  int ftop = 0;
  for (int i=3; i<n; i++){
    ftop++;
    vector<Face> next;
    REP(j, SZ(now)) {
      Face& f=now[j];
      ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt[f.b],
          pt[f.c]));
      if (d <= 0) next.push_back(f);
      int ff = 0;
      if (d > 0) ff=ftop;
      else if (d < 0) ff=-ftop;
      flag[f.a][f.b] = flag[f.b][f.c] =
          flag[f.c][f.a] = ff;
    }
    REP(j, SZ(now)) {
      Face& f=now[j];
      if (flag[f.a][f.b] > 0 and flag[f.a][f.b] !=
          flag[f.b][f.a])
        next.push_back((Face){f.a,f.b,i});
      if (flag[f.b][f.c] > 0 and flag[f.b][f.c] !=
          flag[f.c][f.b])
        next.push_back((Face){f.b,f.c,i});
      if (flag[f.c][f.a] > 0 and flag[f.c][f.a] !=
          flag[f.a][f.c])
        next.push_back((Face){f.c,f.a,i});
    }
    now=next;
  }
  return now;
}
```

6.7 Minimum Covering Circle

```
struct Mcc{
  // return pair of center and r^2
  static const int MAXN = 1000100;
  int n;
  pdd p[MAXN],cen;
  double r2;

  void init(int _n, pdd _p[]){
    n = _n;
    memcpy(p,_p,sizeof(pdd)*n);
  }
  double sqr(double a){ return a*a; }
  double abs2(pdd a){ return a*a; }
  pdd center(pdd p0, pdd p1, pdd p2) {
    pdd a = p1-p0;
    pdd b = p2-p0;
    double c1=abs2(a)*0.5;
    double c2=abs2(b)*0.5;
    double d = a % b;
    double x = p0.x + (c1 * b.y - c2 * a.y) / d;
    double y = p0.y + (a.x * c2 - b.x * c1) / d;
    return pdd(x,y);
  }

  pair<pdd,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){
      if (abs2(cen-p[i]) <= r2) continue;
      cen = p[i];
      r2 = 0;
      for (int j=0; j<i; j++){
        if (abs2(cen-p[j]) <= r2) continue;
        cen = 0.5 * (p[i]+p[j]);
        r2 = abs2(cen-p[j]);
        for (int k=0; k<j; k++){
          if (abs2(cen-p[k]) <= r2) continue;
          cen = center(p[i],p[j],p[k]);
          r2 = abs2(cen-p[k]);
        }
      }
    }
    return {cen,r2};
  }
}mcc;
```

6.8 KDTree (Nearest Point)

```
const int MXN = 100005;

struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root;

  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  }
  static bool cmpx(Node& a, Node& b){ return
      a.x<b.x; }
  static bool cmpy(Node& a, Node& b){ return
      a.y<b.y; }
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    }
    root = build_tree(0, n-1, 0);
  }
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f
        ? cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;

    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    }

    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    }

    return tree+M;
  }
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis ||
        y>r->y2+dis)
      return 0;
    return 1;
  }
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
      mID = r->id;
      md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
    } else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
    }
  }
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
  }
}tree;
```

6.9 Triangulation

```
bool inCircle(pdd a, pdd b, pdd c, pdd d) {
    b = b - a;
    c = c - a;
    d = d - a;
    if (cross(b, c) < 0) swap(b, c);
    double m[3][3] = {
        {b.x, b.y, b*b},
        {c.x, c.y, c*c},
        {d.x, d.y, d*d}
    };

    double det = m[0][0] * (m[1][1]*m[2][2] -
        m[1][2]*m[2][1])
              + m[0][1] * (m[1][2]*m[2][0] -
                  m[1][0]*m[2][2])
              + m[0][2] * (m[1][0]*m[2][1] -
                  m[1][1]*m[2][0]);
    return det < 0;
}

bool intersect(pdd a, pdd b, pdd c, pdd d) {
    return cross(b, c, a) * cross(b, d, a) < 0 and
        cross(d, a, c) * cross(d, b, c) < 0;
}


const double EPS = 1e-12;
struct Triangulation {
    static const int MXN = 1e5+5;
    int N;
    vector<int> ord;
    vector<pdd> pts;
    set<int> E[MXN];
    vector<vector<int>> solve(vector<pdd> p) {
        N = SZ(p);
        ord.resize(N);
        for (int i=0; i<N; i++) {
            E[i].clear();
            ord[i] = i;
        }
        sort(ALL(ord), [&p](int i, int j) {
            return p[i] < p[j];
        });

        pts.resize(N);
        for (int i=0; i<N; i++) pts[i] = p[ord[i]];

        go(0, N);

        vector<vector<int>> res(N);
        for (int i=0; i<N; i++) {
            int o = ord[i];
            for (auto x: E[i]) {
                res[o].PB(ord[x]);
            }
        }
        return res;
    }

    void add_edge(int u, int v) {
        E[u].insert(v);
        E[v].insert(u);
    }

    void remove_edge(int u, int v) {
        E[u].erase(v);
        E[v].erase(u);
    }

    void go(int l, int r) {
        int n = r - l;

        if (n <= 3) {
            for (int i=l; i<r; i++)
                for (int j=i+1; j<r; j++)
                    add_edge(i, j);
            return;
        }
        int md = (l+r)/2;

        go(l, md);
        go(md, r);

        int il = l, ir = r-1;
        while (1) {
            int nx = -1;
            for (auto i: E[il]) {
                double cs = cross(pts[il], pts[i],
                    pts[ir]);
                if (cs > EPS ||
                    (abs(cs) < EPS and
                        abs(pts[i]-pts[ir]) <
                    abs(pts[il]-pts[ir]))) {
                nx = i;
                break;
            }
        }
        if (nx != -1) {
            il = nx;
            continue;
        }
        for (auto i: E[ir]) {
            double cs = cross(pts[ir], pts[i],
                pts[il]);
            if (cs < -EPS ||
                (abs(cs) < EPS and
                    abs(pts[i]-pts[il]) <
                    abs(pts[ir]-pts[il]))) {
                nx = i;
                break;
            }
        }

        if (nx != -1) {
            ir = nx;
        } else break;
    }

    add_edge(il, ir);

    while (1) {
        int nx = -1;
        bool is2 = false;

        for (int i: E[il]) {
            if (cross(pts[il], pts[i], pts[ir])
                < -EPS and
                (nx == -1 or inCircle(pts[il],
                    pts[ir], pts[nx], pts[i])))
                    nx = i;
        }

        for (int i: E[ir]) {
            if (cross(pts[ir], pts[i], pts[il])
                > EPS and
                (nx == -1 or inCircle(pts[il],
                    pts[ir], pts[nx], pts[i])))
                    nx = i, is2 = 1;
        }


        if (nx == -1) break;

        int a = il, b = ir;
        if (is2) swap(a, b);

        for (auto i: E[a]) {
            if (intersect(pts[a], pts[i],
                pts[b], pts[nx])) {
                remove_edge(a, i);
            }
        }
        if (is2) {
            add_edge(il, nx);
            ir = nx;
        } else {
            add_edge(ir, nx);
            il = nx;
        }
    }
  }
} tri;
```

7 Stringology

7.1 Suffix Array

```
const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX], sa[MAX], tsa[MAX],
    tp[MAX][2];

void suffix_array(char *ip){

  int len = strlen(ip);
  int alp = 256;

  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];

  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;

      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;

    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++)
        sa[ct[tp[tsa[j]][0]]++]=tsa[j];

    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }

  for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}
```

7.2 Suffix Array (SAIS TWT514)

```
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN],
        _p[MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0)
                : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans])
                ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool
        *t, int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmxz = -1, *nsa = sa + n, *ns =
            s + n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1])
            sa[x[s[sa[i]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] &&
            t[sa[i]-1]) sa[--x[s[sa[i]-1]]] =
            sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i;
            return; }
        for(int i = n - 2; i >= 0; i--) t[i] =
            (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1])
            sa[--x[s[i]]]=p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] &&
            !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmxz+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z,
            nn, nmxz + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
    }
}sa;

void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<l; i++) {
        hei[i] = sa.hei[i + 1];
        sa[i] = sa._sa[i + 1];
    }
}
```

7.3 Aho-Corasick Algorithm

```
struct ACautomata{
  struct Node{
    int cnt,dp;
    Node *go[26], *fail;
    Node (){
      cnt = 0;
      dp = -1;
      memset(go,0,sizeof(go));
      fail = 0;
    }
  };

  Node *root, pool[1048576];
  int nMem;

  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  }
  void init(){
    nMem = 0;
    root = new_Node();
  }
  void add(const string &str){
    insert(root,str,0);
  }
  void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
      cur->cnt++;
      return;
    }
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
      cur->go[c] = new_Node();
    }
    insert(cur->go[c],str,pos+1);
  }
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front();
      que.pop();
      for (int i=0; i<26; i++){
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          if (!ptr) fr->go[i]->fail = root;
          else fr->go[i]->fail = ptr->go[i];
          que.push(fr->go[i]);
        }
      }
    }
  }
};
```

7.4 KMP

```
#include<bits/stdc++.h>
using namespace std;

void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i ++) {
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos ++;
        fail[i] = pos;
    }
}

void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i ++) {
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];

        if (B[pos + 1] == A[i]) pos ++;

        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
        }
    }
}
```

7.5 Z value

```
void Zval(const char *s, int len, int *z) {
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
    }
}
```

7.6 Z value (palindrome ver.)

```
void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i],
            b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
               s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

7.7 palindromic tree

```
//bcw0x1bd2 {{{
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define F first
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;

#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
  freopen(name".in", "r", stdin); \
  freopen(name".out", "w", stdout);
#endif

#ifdef DARKHH
template<typename T>
void _dump( const char* s, T&& head ) {
    cerr<<s<<"="<<head<<endl; }

template<typename T, typename... Args>
void _dump( const char* s, T&& head, Args&&... tail
    ) {
  int c=0;
  while ( *s!=',' || c!=0 ) {
    if ( *s=='(' || *s=='[' || *s=='{' ) c++;
    if ( *s==')' || *s==']' || *s=='}' ) c--;
    cerr<<*s++;
  }
  cerr<<"="<<head<<",␣";
  _dump(s+1,tail...);
}

#define dump(...) do { \
  fprintf(stderr, "%s:%d␣-␣", __PRETTY_FUNCTION__,
      __LINE__); \
  _dump(#__VA_ARGS__, __VA_ARGS__); \
} while (0)

template<typename Iter>
ostream& _out( ostream &s, Iter b, Iter e ) {
  s<<"[";
  for ( auto it=b; it!=e; it++ ) s<<(it==b?"":",␣
      ")<<*it;
  s<<"]";
  return s;
}

template<typename A, typename B>
ostream& operator <<( ostream &s, const pair<A,B> &p
    ) { return s<<"("<<p.first<<","<<p.second<<")"; }
template<typename T>
ostream& operator <<( ostream &s, const vector<T> &c
    ) { return _out(s,ALL(c)); }
template<typename T, size_t N>
ostream& operator <<( ostream &s, const array<T,N>
    &c ) { return _out(s,ALL(c)); }
template<typename T>
ostream& operator <<( ostream &s, const set<T> &c )
    { return _out(s,ALL(c)); }
template<typename A, typename B>
ostream& operator <<( ostream &s, const map<A,B> &c
    ) { return _out(s,ALL(c)); }
#else
#define dump(...)
#endif
// }}}

struct palindromic_tree{
  struct node{
    int next[26],fail,len;
    int cnt,num,st,ed;
    node(int l=0):fail(0),len(l),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;
    }
  };
  vector<node> state;
  vector<char> s;
  int last,n;

  void init(){
    state.clear();
    s.clear();
    last=1;
    n=0;
    state.push_back(0);
    state.push_back(-1);
    state[0].fail=1;
    s.push_back(-1);
  }
  int get_fail(int x){
    while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
    return x;
  }
  void add(int c){
    s.push_back(c-='a');
    ++n;
    int cur=get_fail(last);
    if(!state[cur].next[c]){
      int now=state.size();
      state.push_back(state[cur].len+2);
      state[now].fail=state[get_fail(state[cur].fail)].next[c
      state[cur].next[c]=now;
      state[now].num=state[state[now].fail].num+1;
    }
    last=state[cur].next[c];
    ++state[last].cnt;
  }
  int size(){
    return state.size()-2;
  }
}pt;

int main() {
  string s;
  cin >> s;
  pt.init();
  for (int i=0; i<SZ(s); i++) {
    int prvsz = pt.size();
    pt.add(s[i]);
    if (prvsz != pt.size()) {
      int r = i;
      int l = r - pt.state[pt.last].len + 1;
      cout << "Find␣pal␣@␣[" << l << "␣" << r << "]␣
          :␣" << s.substr(l,r-l+1) << endl;
    }
  }

  return 0;
}
```

7.8 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
```

7.9 Suffix Automaton

```
// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
  struct State{
    int par, go[26], val;
    State () : par(0), val(0){ FZ(go); }
    State (int _val) : par(0), val(_val){ FZ(go); }
  };
  vector<State> vec;
  int root, tail;

  void init(int arr[], int len){
    vec.resize(2);
    vec[0] = vec[1] = State(0);
    root = tail = 1;
    for (int i=0; i<len; i++)
      extend(arr[i]);
  }
  void extend(int w){
    int p = tail, np = vec.size();
    vec.PB(State(vec[p].val+1));
    for ( ; p && vec[p].go[w]==0; p=vec[p].par)
      vec[p].go[w] = np;
    if (p == 0){
      vec[np].par = root;
    } else {
      if (vec[vec[p].go[w]].val == vec[p].val+1){
        vec[np].par = vec[p].go[w];
      } else {
        int q = vec[p].go[w], r = vec.size();
        vec.PB(vec[q]);
        vec[r].val = vec[p].val+1;
        vec[q].par = vec[np].par = r;
        for ( ; p && vec[p].go[w] == q; p=vec[p].par)
          vec[p].go[w] = r;
      }
    }
    tail = np;
  }
};
```

8 Problems

8.1 Mo-Algorithm on Tree

```
#include<bits/stdc++.h>
using namespace std;
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))

const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;

struct BIT {
  int bit[MX];
  int lb(int x) { return x & -x; }
  void add(int p, int v) {
    p++;
    for (int i=p; i<MX; i+=lb(i)) bit[i] += v;
  }
  int qry() {
    int v = 0;
    for (int i=1<<LOG; i>0; i>>=1) {
      if ((v|i) < MX and bit[v|i]==i) v |= i;
    }
    return v;
  }
}bit;

struct Query {
  int l,r,qid;
}qry[MX];
struct Edge {
  int v,x;
};

int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector<Edge> E[MX];
vector<Edge> seq;

void DFS(int u, int f) {
  timestamp[u] = SZ(seq);
  for (auto it:E[u]) {
    if (it.v == f) continue;
    seq.push_back(it);
    DFS(it.v,u);
    seq.push_back(it);
  }
}
void poke(int id) {
  int v = seq[id].v;
  int x = seq[id].x;
  in[v] ^= 1;
  cnt[x] += in[v] ? 1 : -1;
  if (in[v] and cnt[x] == 1) bit.add(x, 1);
  if (!in[v] and cnt[x] == 0) bit.add(x, -1);
}
int main() {
  IOS;
  cin >> N >> Q;
  for (int i=0; i<N-1; i++) {
    int u,v,x;
    cin >> u >> v >> x;
    x = min(x,N);
    E[u].push_back({v,x});
    E[v].push_back({u,x});
  }
  DFS(1,1);
  for (int i=1; i<=Q; i++) {
    int u,v;
    cin >> u >> v;
    int l = timestamp[u], r = timestamp[v];
    if (l > r) swap(l,r);
    r--;
    qry[i] = {l,r,i};
  }
  sort(qry+1,qry+1+Q, [](Query a, Query b) {
      return make_pair(a.l/SQ,a.r) <
          make_pair(b.l/SQ,b.r);
      });

  int curL = 1, curR = 0;
  for (int i=1; i<=Q; i++) {
    int ql=qry[i].l,qr=qry[i].r;
    while (curL > ql) poke(--curL);
    while (curR < qr) poke(++curR);
    while (curL < ql) poke(curL++);
    while (curR > qr) poke(curR--);
    ans[qry[i].qid] = bit.qry();
  }

  for (int i=1; i<=Q; i++) cout << ans[i] << "\n";

  return 0;
}
```

8.2 Manhattan MST

```
#include<bits/stdc++.h>
#define REP(i,n) for(int i=0;i<n;i++)
using namespace std;
typedef long long LL;
const int N=200100;
int n,m;
struct PT {int x,y,z,w,id;}p[N];
inline int dis(const PT &a,const PT &b){return
    abs(a.x-b.x)+abs(a.y-b.y);}
inline bool cpx(const PT &a,const PT &b){return
    a.x!=b.x? a.x>b.x:a.y>b.y;}
inline bool cpz(const PT &a,const PT &b){return
    a.z<b.z;}
struct E{int a,b,c;}e[8*N];
bool operator<(const E&a,const E&b){return a.c<b.c;}
struct Node{
  int L,R,key;
}node[4*N];
int s[N];
int F(int x){return s[x]==x?x:s[x]=F(s[x]);}
void U(int a,int b){s[F(b)]=F(a);}
void init(int id,int L,int R) {
  node[id]=(Node){L,R,-1};
  if(L==R)return;
  init(id*2,L,(L+R)/2);
  init(id*2+1,(L+R)/2+1,R);
}
void ins(int id,int x) {
  if(node[id].key==-1 ||
      p[node[id].key].w>p[x].w)node[id].key=x;
  if(node[id].L==node[id].R)return;
  if(p[x].z<=(node[id].L+node[id].R)/2)ins(id*2,x);
  else ins(id*2+1,x);
}
int Q(int id,int L,int R){
  if(R<node[id].L || L>node[id].R)return -1;
  if(L<=node[id].L && node[id].R<=R)return
      node[id].key;
  int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
  if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;
  else return b;
}
void calc() {
  REP(i,n) {
    p[i].z=p[i].y-p[i].x;
    p[i].w=p[i].x+p[i].y;
  }
  sort(p,p+n,cpz);
  int cnt=0,j,k;
  for(int i=0;i<n;i=j){
    for(j=i+1;p[j].z==p[i].z && j<n;j++);
    for(k=i,cnt++;k<j;k++)p[k].z=cnt;
  }
  init(1,1,cnt);
  sort(p,p+n,cpx);
  REP(i,n) {
    j=Q(1,p[i].z,cnt);
    if(j!=-1)e[m++]=(E){p[i].id,p[j].id,dis(p[i],p[j])};
    ins(1,i);
  }
}
LL MST() {
  LL r=0;
  sort(e,e+m);
  REP(i,m) {
    if(F(e[i].a)==F(e[i].b))continue;
    U(e[i].a,e[i].b);
    r+=e[i].c;
  }
  return r;
}
int main(){
  int ts;
  scanf("%d", &ts);
  while (ts--) {
    m = 0;
    scanf("%d",&n);
    REP(i,n) {
      scanf("%d%d",&p[i].x,&p[i].y);
      p[i].id=s[i]=i;
    }
    calc();
    REP(i,n)p[i].y= -p[i].y;
    calc();
    REP(i,n)swap(p[i].x,p[i].y);
    calc();
    REP(i,n)p[i].x=-p[i].x;
    calc();
    printf("%lld\n",MST()*2);
  }
  return 0;
}
```

9 Miscellany

9.1 Made in Abyss