Guts Round

I2MC 2025

Round 1

1. (5) Let $A_1A_2A_3...A_{12}$ be a regular 12-sided polygon. If $A_2A_4 = 8$, what is A_4A_8?
2. (5) There exists x such that $x + \frac{1}{x} = x^2 + \frac{1}{x^2}$. What is $x + \frac{1}{x}$?
3. (5) Aishwarya has the number

$$16! = 20922789888000$$

and replaces one of its digits with X. Aryan then receives this number and the information that it was divisible by 9. How many choices does Aishwarya have for which digit to replace such that Aryan can recover the original number?

Round 2

4. (6) Jai draws a card from a standard 52-card deck. What is the probability that the next card he draws has a higher rank? Suits do not matter in ranking cards.
5. (6) Unit cube $ABCDEFGH$ is perfectly balanced such that its space diagonal $\overline{AG}$ is perpendicular to a flat table. It is then filled to 50% volume with water. Find the height of the water level.
6. (6) Suppose $n = 2^a3^b$ for positive integers a and b. The *frime pactorization* of n is a^2b^3. What is the smallest n which is equal to its own frime pactorization?

Round 3

7. (7) A stellated octahedron is created by adding a regular tetrahedron to each face of an octahedron. How many edges does a stellated octahedron have?

8. (7) The positive integers 1 through 100 are written on a blackboard. Mingyue erases all 6s and replaces them with 8s. He then adds the numbers up. What does he get as the answer?

9. (14) For this problem, call a volunteer over for materials and proctoring.

   Aingyue the evil dictator has just taken power, forcing you and your secret resistance team to go underground. Split your team into two groups. Each group will be handed a triangle. You are not allowed to show the other group your triangle, and you are not allowed to exchange scratch paper. You know that:

   $$\triangle ABC \sim \triangle DEF$$

   If either group can figure out the area of $\triangle ABC$, you will create a secure communication tunnel unseen by Aingyue's spies, and you will have solved this problem.

Round 4

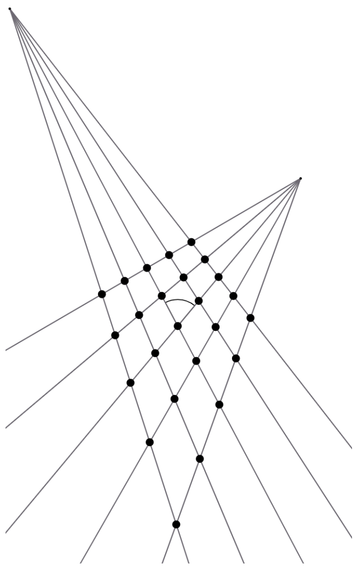
10. (9) A geometric series with first term a and ratio r has a sum of 3. A geometric series with first term a and ratio $-r$ has a sum of $\frac{3}{5}$. What is the sum of a geometric series with first term a and ratio r^2?

11. (9) One hundred one stairs lead up to a temple. The stairs are divided into three flights, with each flight having an odd number of stairs. How many ways can the stairs be divided such that some flight has at least 67 stairs?

12. (9) The integer $3^{18} - 2^{18}$ has five prime factors. Find the largest one.

Round 5

13. (11) The 5 rays originating from the point on the left are separated by $1°$ each, and the 5 rays originating from the point on the right are separated by $2°$ each. The marked angle is $45°$. Find the sum of the 25 acute angles at the 25 marked points.



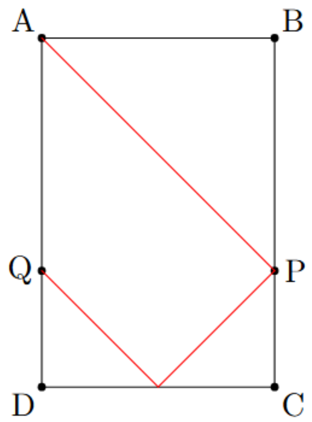
14. (11) A 6-sided die has each face labeled with a distinct positive factor of a number n with exactly 6 positive factors. When this die is rolled, the expected value is an integer k. What is the minimum possible value of k?

15. (11) Let $f(x)$ be a cubic such that $f(1) = 2$,$f(2) = 0$, $f(3) = 2$, and $f(4) = 5$. What is the value of $f(8)$?

Round 6

16. (12) Jai has a 4×4 grid of dots. What is the maximum number of dots Jai can select such that no three of them are collinear (lie on a straight line)?

17. (12) Let $ABCD$ be a rectangle with $AB = 20$ and $BC = 25$. A laser is shot from vertex A and first intersects a side of $ABCD$ at point P on segment BC such that $BP = 16$. The laser reflects off of the sides of the rectangle. Let Q be the point where the laser first intersects side AD. Find AQ.



18. (24) Your team is pretending to be a personal circus for Aingyue the evil dictator to infiltrate his residence. Assign each of the four members of your team a different integer from 1 to 4. Aingyue is assigned the number 0.

    Since Aingyue's rule is in its *prime*, your performance will consists of your team members and Aingyue clapping in a certain order with regards to the primes. There will be 15 claps. On the nth clap, the person assigned the number equal to the remainder when the nth prime is divided by 5 must clap, and nobody else. Your performance ends after the 15th prime, 47.

    Once your performance begins, you may not communicate with each other, nor may you consult any scratch paper until your performance ends. When you are ready to perform, ask your grader. You have unlimited attempts to satisfy Aingyue with your performance. Good luck!

Round 7

19. (14) There exists exactly one four digit perfect square of the form $\overline{AABB}$. Find $A \cdot B$.

20. (14) The mean, median, and unique mode of 100 positive integers are increasing consecutive integers, in that order. Determine the least possible value of the mean.

21. (14) A *Latin square* puzzle consists of a grid of size $n \times n$. The solver must place the digits 1 through n inclusive into the grid, such that there is one digit in each cell and no two copies of the same digit appear in the same row or column.

    How many ways can a Latin square puzzle of size 4×4 be solved?

Round 8

22. (16) Points O and A are in the plane with $OA = 1$. Six equally spaced lines $\ell_1, ..., \ell_6$ pass through O, with A on ℓ_1. An immortal bug begins at A. On each step, the bug walks in a straight line from its current position to one of the nearest points in the plane that is on a different line. The bug is condemned to perform infinitely many steps to repent for its sins. Find the total length of the bug's path.

23. (16) Find the number of ordered positive integer triples $a, b, c \leq 100$ such that $a^2 + b^2 + c^2$ is a multiple of 20.

24. (16) An unordered set of positive integers $\{a, b, c\}$ is considered to be a *friend group* if every pair of numbers has a common factor greater than 1. Given that a, b, and c are all unique positive integers at most 15, how many friend groups are there?

Round 9

25. (18) Triangle ABC has $AB = 13$, $BC = 14$, and $CA = 15$. The altitude $\overline{AD}$ is drawn, intersecting the incircle at X and Y. What is $AX - DY$?

26. (18) Some number of horses are galloping around a circular racetrack with circumference 1600 meters. They all begin at the same point on the track and run counterclockwise with constant integer speeds (in meters per second). Suppose the first time that a horse passes another horse, one of those two horses has run 6000 meters in 400 seconds. Let $t > 0$ be the time in seconds between the first time and the second time a horse passes another horse. Find the sum of all possible values of t.

27. (36) Your team has gone through trials and tribulations, and you are now fighting the evil dictator Aingyue himself in a battle of wits. Here is how the game goes:

    Players take turns writing either a '1' or a '-1' onto a shared piece of paper. Writing a '1' takes one stroke, while writing a '-1' takes two. The game ends once 11 strokes are drawn onto the paper (you may not draw more). If the product of all written numbers is positive, you will best the evil dictator, and save the world. If the product of all written numbers is negative, then Aingyue will unfortunately win, and continue to oppress his subjects.

    Aingyue plays with perfect strategy and has given you the mercy of choosing to go first or second. How can you win? Try it out against him in real life! Ask your grader to call Aingyue over, for up to 2 attempts.
