I2MC 2019 - Individual Finals

Interlake Math Club

December 15, 2019

Problems

1. Let $ABCD$ be a cyclic quadrilateral (in that there exists a circle passing through the 4 points A, B, C, D). Let $AB = 5, CD = 10, AC = 11, BD = 10$. Let segments AC and BD intersect at a point E, and let the midpoint of DC be M. If line EM hits line AB at X, find EX.

2. Let a sequence a_n be defined by $a_1 = 300$ and $a_n = 180 - \frac{1}{2}a_{n-1}$. Find the smallest positive integer n such that $|a_n - 120| < 0.2$.

3. Consider the regular nonagon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$. A *triangulation* of the nonagon is a partition into triangles with nonintersecting diagonals. Let $M(\triangle A_iA_jA_k)$ be the measure of the largest angle in $\triangle A_iA_jA_k$. Given an arbitrary triangulation, find the expected value, in degrees, of the minimum $M(A_iA_jA_k)$ across all triangles in the triangulation.