1. (5 points) What is the area of a circle with diameter π?

2. (5 points) August is throwing snowballs at Daniel. He gathers snow from a rectangular area that is 4 feet wide and 5 feet long, and each snowball has a volume of 15 cubic inches. The snow accumulates at a rate of 2 inches per hour. If August can only throw whole snowballs, how many snowballs can he throw in the first minute of snowfall?

3. (5 points) Jeffrey draws a card from a standard 52-card deck and rolls a fair 6-sided die. What is the probability that the number on the card is smaller than the number on the die? (Face cards have a value of ten and aces have a value of 1.)

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4. (6 points) Let the answer to this question be x. What is $x^2 + 5x + 4$?

5. (6 points) Aryan decides that the name "I2MC" makes no sense. First, he replaces the "2" with any capital letter of the English alphabet. Then, he reorders the letters however he likes. How many four-character names could he end up with?

6. (6 points) Each letter in the 9-digit number $INTERLAKE$ stands for a digit between 0 and 9, with different letters standing for different digits. If $I \neq 0$, what is the smallest possible value of $INTERLAKE$ that is divisible by 3?

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7. (7 points) Consider right triangle ABC with a right angle at B. $AB = 6$ and $BC = 8$. Point D lies on $\overline{BC}$ so that $DC = 6$. A circle is drawn with diameter $\overline{DC}$ and it intersects $\overline{AC}$ at point E. What is the area of $\triangle CDE$?

8. (7 points) Let $S(n)$ denote the sum of the digits of a positive integer n. What is the minimum possible value of $S(77x)$, where x is a positive integer?

9. (? points) Submit any positive integer n. Your score on this question is

$$\max\left(0, \left\lfloor \frac{1}{8}n(15-n) \right\rfloor\right).$$

If your submission is not a positive integer, it will score 0 points.

10. (9 points) Let $A_1A_2\dots A_{2024}$ be a regular 2024-gon. What is the degree measure of $\angle A_{20}A_2A_4$?

11. (9 points) Simplify the expression $14\sqrt{34\sqrt{14\sqrt{34\sqrt{\cdots}}}}$. Your answer should be in the form $a\sqrt[n]{b}$, a, b, and n are integers, n is as small as possible, and a is as large as possible.

12. (9 points) What is the sum of all positive factors of 2025?

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13. (11 points) You want to guess a secret 4-digit number. You know that the second and fourth digits are equal, and that the digits sum to 12. Given that it is divisible by 7 and 5, find the secret number.

14. (11 points) Compute $\frac{1}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{3}{3^4}+\frac{5}{3^5}+\cdots$, where the numerators follow the Fibonacci sequence and the sequence continues infinitely.

15. (11 points) You think that 3-dimensional geometry is too easy. Unfortunately, the math club officers overheard this and decided to give you this problem as retribution: An ant is stranded at the origin of a 4-dimensional plane with axes w, x, y, and z. If the ant is currently at (w,x,y,z), it can reach either $(w+1,x,y,z)$, $(w,x+2,y,z)$, $(w,x,y+2,z)$, or $(w,x,y,z+4)$. How many ways can the ant reach $(4,4,4,4)$?

16. (12 points) In triangle $\triangle ABC$, $AB = 13$, $BC = 14$, and $CA = 15$. Let I be the incenter of $\triangle ABC$. Find the product of the lengths AI, BI, and CI.

17. (12 points) How many 10-letter strings consisting of only the letters A and B have the property that out of any three consecutive letters, exactly two of them are the same?

18. (12 points possible) August is running at a constant speed around a circle at a rate of 54 seconds per lap. Daniel stands outside the circle with a snowball in his hand. Daniel notices that August is directly east of him twice every lap: once when August is $10\sqrt{3}$ meters away, and another time 18 seconds later when August is just $6\sqrt{3}$ meters away.

    Submit a positive real number x. Daniel will choose the best time to throw the snowball x meters directly at August. If x is far enough, the snowball hits, and you get $\left\lfloor \frac{c}{x^3} \right\rfloor$ points, where c is some large positive real constant. If x is too close, the snowball falls harmlessly to the ground and you score 0 points.

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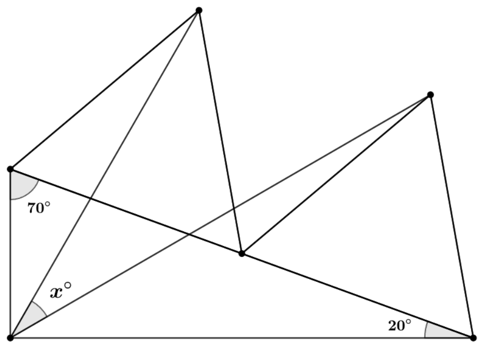
19. (14 points) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2 + 3n}$.

20. (14 points) A right triangle has angles 20° and 70°. Two congruent equilateral triangles sharing a vertex are constructed along its hypotenuse as shown below. If the marked angle measures x°, find x.



21. (14 points) The *median-mode-mean* of a set of positive integers is the mean of its median and its unique mode. Given the data set 8, 1, 4, b, 10, c, 8 has a *median-mode-mean* of 6 but does not contain the number 6, what is the mean of this data set?

22. (16 points) The number $\overline{aba}$ (where a is the hundreds digit, b is the tens digit, etc) in base 10 is equal to $\overline{7aa}$ in base b. Convert $\overline{7aa}$ in base 10 to base b.

23. (16 points) Find all real x satisfying

$$3\lfloor x\rfloor^2 + 2\{x\}^2 = \lceil x\rceil^2$$

$\{x\}$ denotes the fractional part of x.

24. (16 points) Let S be a set of 5 randomly selected (distinct) elements from the set $\{1, 2, 3, \ldots, 8\}$. What is the probability that the mean of S is greater than the median of S?

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25. (18 points) Let $ABCD$ be a convex quadrilateral with $AB = 4$, $BC = 5$, and $CD = 6$. Given that the quadrilateral formed by the midpoints of the sides of $ABCD$ is a square, determine the length of side $\overline{DA}$.

26. (18 points) One day during lunch, Jeffrey grabbed J fries from the school lunch line, planning to eat them all. However, Christopher swiftly snatched and gobbled groups of C fries from Jeffrey's tray until there were not enough fries to make another full group. This left poor Jeffrey with just 3 fries. If the least common multiple of C and J is 150, what is the value of $C + J$?

27. (18 points possible) Submit any positive integer n. You will get $\lfloor c \cdot 2^n \rfloor$ points, where c is a small positive real constant, if it is possible to do the following on an 8×8 chessboard:

    - Place 6 "blockers" on the chessboard on any squares you like, then
    - Place n rooks on the chessboard such that none of the rooks attack each other

    Two rooks attack each other if they are in the same row or same column, and there is not a blocker in between them.

    If it is not possible for your choice of n, you will score 0 points.