# Team Round

I2MC 2024

October 19th 2024

1. (20) August is building a snow fort and is attempting to claim the territory of a 12 meter by 12 meter square by walking around the perimeter, starting at one of the corners. However, 27 meters into his walk, he sees Daniel hiding behind a nearby tree with a large arsenal of snowballs, so he runs back to the first corner, taking a straight path. In meters, what is the length of the complete route that August took?

2. (25) Aryan has a $3 \times 3 \times 3$ cube. He dips it into paint so that all 6 faces of the cube are covered in paint. He then breaks the large cube into 27 cubes with side length 1. From these 27 cubes, he constructs three $2 \times 2 \times 2$ cubes and three $1 \times 1 \times 1$ cubes. What is the difference between the maximum and minimum possible area of paint on the external surfaces of the 6 cubes, in square units?

3. (25) An infinite geometric sequence has a common ratio of $\dfrac{1}{\sqrt[3]{7}}$, and the sum of its first three terms is 216. What is the sum of the entire infinite geometric series?

4. (30) A positive integer n leaves a remainder of 1 when divided by x and a remainder of x when divided by 6. What is the smallest possible value of n?

5. (40) Trapezoid $ABCD$ has right angles $\angle ABC$ and $\angle BCD$. Two equilateral triangles with side lengths 14 and 34 are inscribed in $ABCD$ as shown.
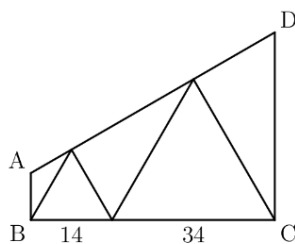
If the area of $ABCD$ is $k\sqrt{3}$, find $\lfloor \sqrt{k} \rfloor$.

6. (45) A sentient plastic knight is placed at the origin on the Cartesian plane. The knight may jump two units right and one unit up, or one unit right and two units up. How many ways can the knight reach $(23, 16)$?

7. (50) Benjamin writes down all 142 fractions with denominator 143 that are strictly between 0 and 1. He then simplifies all the fractions. What is the sum of all of the denominators of the 142 simplified fractions?

8. (55) There are 430 prime numbers less than 3000. How many numbers less than 3000 have a prime number of divisors?

9. (55) Let $ABCD$ be a parallelogram, and let M, N, and P denote the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{AD}$, respectively. Given that the region bounded by lines $\overline{AC}$, $\overline{BP}$, and $\overline{MN}$ is an equilateral triangle of side length 1, determine the area of $ABCD$.

10. (60) Let b and c be positive real numbers that satisfy $(b-1)^2 + (c-1)^2 = 7$ and $bc = 5$. Find the maximum possible value of b.