



Kazakh (kaz), day 1

Dúisenbi, 9 shilde, 2018 j.

Esep 1. ABC súir buryshy úshburyshynyń syrttai syzylǵan sheńberi Γ bolsyn. $AD = AE$ bolatyn AB men AC kesindilerinde sáikesinshe D jáne E núkteleri belgilengen. BD men CE -niń ortaq perpendikuliarlary Γ sheńberiniń AB men AC kishi doğalardy sáikesinshe F jáne G núktelerinde qılysady. DE men FG parallel (nemese birdei bolatyn) ekenin dáledeńiz.

Esep 2. Kelesi shartty qanaǵattandyratyn barlyq bútin $n \geq 3$ sandaryn tabyńyz:

$$a_{n+1} = a_1, a_{n+2} = a_2, \text{ jáne } i = 1, 2, \dots, n \text{ úshin}$$

$$a_i a_{i+1} + 1 = a_{i+2}$$

bolatyn a_1, a_2, \dots, a_{n+2} naqty sandar tabylady.

Esep 3. *Anti-Pascal úshburyshy* ol kelesi ereje bobynsha sandardan qýralǵan teíbúirli úshburyshy ornalastyruy: tómengi qatardyń sandarynan basqa, árbir san dereu tómen turǵan eki sannyń aiymasynyń modýline teń. Mysaly, tórt qatardan turatyn jáne 1-den 10-ǵa dein barlyq bútin sandar kezdesetin osyndai anti-Pascal úshburyshy keltirilgen:

$$\begin{array}{ccccccc} & & & & 4 \\ & & & & 2 & 6 \\ & & & & 5 & 7 & 1 \\ & & & & 8 & 3 & 10 & 9 \end{array}$$

2018 qatardan turatyn jáne 1-den $1 + 2 + \dots + 2018$ sańga dein barlyq bútin sandar kezdesetindei anti-Pascal úshburyshy bar ma?

Seisenbi, 10 shilde, 2018

Esep 4. Eger x jáne y ekeýi naqty oí jane 20-dan kishi nemese teń bolsa, onda zhazyqtygynyń (x, y) núktesi *oryn* dep bolsyn.

Bastapqyda, 400 ornynan árqaisysy bos. Aruzhan men Berik osy oryndarǵa kezek-kezekpen tastar qoıady, jáne Aruzhan birinshi bastaídý. Aruzhan óziniń ár kezekte zhańa bir qyzyl tasty bos oryńga salady biraq qyzyl tastardyń kez kelgen eki ornynyń ara qashyqtygý $\sqrt{5}$ teń emes bolu qajet. Berik oziniń ár kezekte zhańa bir kók tasty bos oryńga salady. (Kók tasy bar orny basqa oryndardan ara qashyqtygý kez kelgen bolýy ruqsat.) Eger bir oiynshynyń tasty qoıýga mumkindigi zhoq bolsa, oïyn toqtайды.

Berik kók tastardy qalai qoisa da, Aruzhan K qyzyl tas saluǵa kepili bar bolatyndai, eí úlken K sanyn tabyńyz.

Esep 5. a_1, a_2, \dots naqty oí sandardan turatyn sheksiz tizbegi bolsyn. Barlyq $n \geq N$ úshin

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$$

sany bútin bolatyndai, bútin $N > 1$ sany bar. Barlyq $m \geq M$ úshin $a_m = a_{m+1}$ bolatyn M bar ekeni dáleldeneiz.

Esep 6. $AB \cdot CD = BC \cdot DA$ oryndalatyn $ABCD$ dóńes tórtburyshy berilgen.

$$\angle XAB = \angle XCD \quad \text{jáne} \quad \angle XBC = \angle XDA$$

bolatyn $ABCD$ ishinde X núktesi zhatyr. $\angle BXA + \angle DXC = 180^\circ$ ekenin dáleldeneiz.