I2MC 2019 - Team

Interlake Math Club

December 15, 2019

Problems

1. How many unit squares must be attached to a 12 by 12 square to get a 20 by 20 square?

2. The product $33 \cdot 34 \cdot 57 \cdot 65$ is divisible by the square of a unique prime number p. Find p.

3. Consider a square with side length $\sqrt{5}$. This square is inscribed perfectly inside another larger square. What is the maximum area of this larger square?

4. Donald makes a plan to eat more fruit. On January 1, he will eat 1 watermelon. On January 2, he will eat 2 watermelons. Each day, he eats 1 more watermelon than the day before. On which month will he have reached his goal of eating 10000 watermelons in total?

5. A number n has 12 divisors and $n + 1$ has 9 divisors. Find the smallest possible value of n.

6. Ms. EEEEE the PE teacher has 77 students. She is providing written feedback on all of their report cards–either "NICE HUSTLE" or "NICE JOB", but never both. To the nearest whole percent, 27% of her students will receive the prestigious "NICE HUSTLE". How many times will Ms. EEEEE write the letter "E" while giving feedback?

7. On a hot summer day, Newton sits under a tree. Unfortunately, the tree decides to harass, intimidate, and bully Newton by dropping apples on his head. After the 1st apple hits Newton's head, Newton has a 1/100 chance of submitting a bullying form against the tree, with the 2nd apple, he has a 1/99 chance, with the 3rd apple, he has a 1/98 chance, and so on. What is the expected number of apples that will hit Newton's head before he submits a bullying form?

8. Sean is at point $(1, 0)$ on the cartesian plane. He would like to run to a point on the curve $x^2 + 2y^2 = 2$, then from that point, run to the point $(-1, 0)$. What is the minimal distance he can run? Express in simplest radical form.

9. Find the sum of all integers n such that $n^2 + n + 1$ is a divisor of $n^{2020} + 20$.

10. Let $ABCDEFGH$ be a regular octagon with side length 8. Let BF meet DH and EH at I and J, respectively and let BG meet DH and EH at K and L respectively. Find the area of quadrilateral $IJLK$.