Kuwa kabiri, 16. Nyakanga 2024

Ikibazo 1 Shaka real numbers zose α kuburyo buri positive integer n, integer ingana na

$$\lfloor \alpha \rfloor + \lfloor 2\alpha \rfloor + \cdots + \lfloor n\alpha \rfloor$$

izaba ari multiple ya n.

(Wibuke ko $\lfloor z \rfloor$ bingana na positive integer nini ingana cyangwa irutwa na z. Urugero, $\lfloor -\pi \rfloor = -4$; $\lfloor 2 \rfloor = \lfloor 2.9 \rfloor = 2$.)

Ikibazo 2 Shaka pairs zose (a, b) za positive integers kuburyo hariho positive integer g aho

$$\gcd(a^n + b,\, b^n + a) = g$$

kuri buri positive integer n nini cyane ishoboka.

(Wibuke ko $\gcd(x, y)$ bivuga the greatest common divisor ya integers x na y.

Ikibazo 3 Reka $a_1, a_2, a_3, \ldots$ ibe sequense iri infinite ya positive integers, na N ibe positive integer. Reka nanone tuvuge ko kuri buri $n > N$, a_n ibe ingana n'inshuro a_{n-1} yagaragaye muri lisite $a_1, a_2, \ldots, a_{n-1}$. Erekana ko byibura imwe muri izi sequense $a_1, a_3, a_5, \ldots$ na $a_2, a_4, a_6, \ldots$ izarangira yisubiramo (periodic).

(Sequense iri infinite $b_1, b_2, b_3, \ldots$ ni sequense (izarangira yisubiramo) kuburyo hariho positive integers p and M kuburyo $b_{m+p} = b_m$ kuri buri $m \geqslant M$.)

Kuwa gatatu, 17. Nyakanga 2024

Ikibazo 4 Reka ABC ibe mpandeshatu aho $AB < AC < BC$. Reka incentre ya mpandeshatu ABC ibe I, na incirle ya mpandeshatu ABC ibe ω. Reka akadomo X, gatandukanye na C, kabe ku murongo BC kuburyo umurongo inyura muri X uri parallel na AC ube tangent kuri ω. Muburyo bumwe, reka akadomo Y, gatandukanye na C, kuburyo umurongo inyura muri Y uri paralle na AB ube tangent kuri ω. Reka nanone umurongo AI uhure na circumcircle ya mpandeshatu ABC ku kadomo $P \neq A$ (gatandukanye na A). Reka nanone akadomo K kabe midpoint ya AC na L ibe midpoint ya AB.

Erekana ko $\angle KIL + \angle YPX = 180°$.

Ikibazo 5 Turbo akina umukino kuri board ifite rows 2024 na columns 2023. Hari monsters 2022 zihishe mu tuzu twa board. Turbo ntabwo azi aho monsters zihishe, ariko azi neza ko muri buri row harimo monster imwe gusa, uretse muri row ya mbere na row ya nyuma, kandi ko no muri buri column harimo monster imwe gusa. Turbo agerageza inshuro zitandukanye kuva kuri row ya mbere ajya kuri row ya nyuma. Kuri buri nshuro, Turbo ashobora guhera mu kazu ashaka muri row ya mbere, akagenda ajya mu kazu gahuje uruhande n'ako arimo. (Yemerewe gusubira mu kazu yari yasuye mbere.) Iyo ageze mu kazu karimo monster, biba byanze, maze agasubizwa kuri row ya mbere gutangira. Monsters ntabwo ziva mu tuzu ntazo, kandi Turbo yibuka neza niba akazu yanyuzemo karimo monster. Iyo ageze mu kazu ako ari ko kose kari kuri row ya nyuma, Turbo aba atsinze, umukino ukarangira. Shaka inshuro n nkeya zishoboka Turbo azagerageza kuburyo Turbo azaba yamaze kubona strategy imwemerera kugera kuri row ya nyuma ku nshuro ya n, atitaye ku tuzu monsters zirimo.

Ikibazo 6 Reka $\mathbb{Q}$ ibe set ya rational numbers. Function $f : \mathbb{Q} \to \mathbb{Q}$ yitwa "aquaesulian" iyo yujuje ibi bikurikira: kuri buri $x, y \in \mathbb{Q}$,

$$f(x + f(y)) = f(x) + y \qquad \text{cyangwa} \qquad f(f(x) + y) = x + f(y).$$

Erekana ko hariho integer c kuburyo kuri buri function yose ya aquaesulian, hari rational numbers zitarenze c zifite form ya $f(r) + f(-r)$ kuri rational number r, kandi ushake umubare muto cyane ushobora wangana na c.