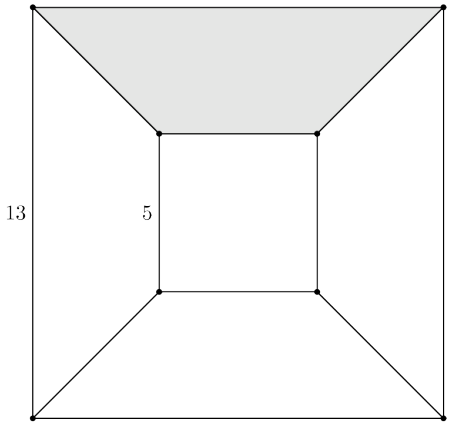
Combinatorics and Geometry Round

I2MC 2024

October 19th 2024

1. How many multiples of 77 are there between 7 and 777?

2. A square of side length 5 is placed perfectly at the center of a square with side length 13. What is the area of the shaded trapezoid?



3. How many of the first 100 positive integers have the property that the sum of their digits is a perfect square?

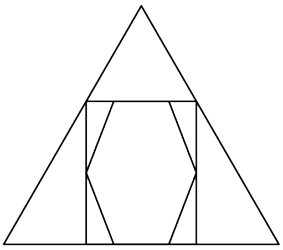
4. Right triangle $\triangle ABC$ has hypotenuse $\overline{AC}$. Define point D on $\overline{AB}$ such that $\overline{CD}$ bisects $\angle ACB$. Given that $AD = 13$ and $BD = 12$, find the area of $\triangle ABC$.

5. You have just entered the multiverse. There are 6 universes named A, B, C, D, E, and F. You can go from any universe to another thanks to a certain Sorcerer Supreme, except you cannot go from universe B to universe F. If you are currently at universe A and want to visit all of the other universes exactly once, in how many ways can you do so?

6. Mingyue randomly places a square of side length 2 inside a square of side length 4 with the same orientation (it's not rotated) such that the entire smaller square is contained within the bigger square. What is the probability that when Mingyue rotates the smaller square 45° around its center that it is still entirely contained within the bigger square?

7. Forty percent of Interlake High School students are in the IB program, and one out of every 6 non-IB students is sad. Given that 54% of all students are not sad, find the probability that a randomly selected sad student is in the IB program.

8. A regular hexagon is inscribed in a rectangle, which is inscribed in an equilateral triangle, as shown. What fraction of the triangle does the hexagon occupy?



9. Daniel is building a snow fort, which is divided into a 4×4 grid. Daniel wants to place 8 very large snowballs in the fort for defense purposes, and in order to optimally deter August, each row and column of the grid must contain 2 snowballs. How many ways can Daniel defend his fort?

10. Consider regular octagon $ABCDEFGH$ with side length 2. Let X be defined as the intersection of $\overline{AF}$ and $\overline{DH}$. Let Y be defined as the intersection of $\overline{CX}$ and $\overline{HG}$. What is XY?