Algebra and Number Theory Round

I2MC 2025

1. The Algebra and Number Theory Round is a 10-question test taken by 72 competitors. Once the test is completed, 15 graders simultaneously begin grading the questions, each at the same constant rate. If instead there were three fewer graders, how many more questions would each grader have to grade?
2. Aishwarya and Aryan are standing in a line. Aishwarya counts 43 people in front of her, and Aryan counts 14 people in between him and Aishwarya. Given that one of them is at the end of the line, find the sum of all possibilities for the number of people in line.
3. The least common multiple of three positive integers is 45. What is the least possible value of their sum?
4. Call a date of the form '$m/d/y$' *additive* if the sum of d and m is a factor of y. For example, June 19th, 2025 is additive since $6 + 19 = 25$ is a factor of 2025. Find the next year in which February 29th is an additive day.
5. In a non-constant arithmetic sequence of positive integers, the 1st, 5th, and 2025th terms form a geometric sequence. Find the common ratio of this geometric sequence.
6. Let the representation of a positive integer N in base 6 be A, and let its representation in base 7 be B. What is the minimum possible value of N such that if we treat A and B instead as integers written in base 10, their sum is divisible by $6 + 7$?
7. How many positive integers less than or equal to 210 are divisible by exactly one of 2, 3, 5, or 7?
8. Suppose that b and c are positive integers such that $x^2 + bx + c$ and $x^2 + bx + (c + 5)$ both have integer roots. What is the minimum possible value of $b + c$?
9. For each positive factor d of 60, Jeffrey calculates the sum of the positive factors of d. What is the sum of all of these sums?
10. Let a positive integer be *5-ish* if the sum of its digits is divisible by 5. Find the sum of all *5-ish* integers less than 1000.