Team Round

I2MC 2025

1. (20) In the image below, shade some of the white cells such that the sum of the shaded cells in each row and column matches the corresponding number to the left or above it. Write as your answer the product of all unshaded integers.

2. (25) Aryan and Aishwarya have some flower petals, which they are planning to incorporate into dessert items - pastries, cakes, and ice creams, each using 3, 4, and 5 petals respectively. They realize that if they ignore the ice cream and make an equal number of pastries and cakes, they will have 1 petal left over, while if they ignore the pastry and make an equal number of cakes and ice creams, they will have 7 petals left over. What is the smallest possible number of flower petals they could have?

3. (25) Let $ABCD$ be a rectangle with $AB = 6$ and $BC = 8$. Let M be the midpoint of AD and Q be the midpoint of MC. Given that P is the intersection of BD and MC, what is the ratio of the area of triangle MBQ to the area of triangle CBP?
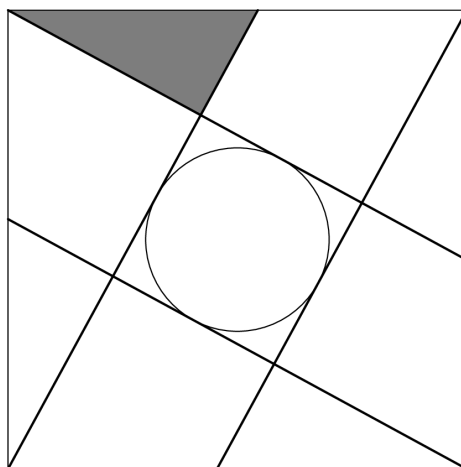
4. (30) Let $f(x) = x^2 + ax + b$ and $g(x) = x + c$ for real a, b, and c. Furthermore,

$$f(g(e)) = f(e) = g(e + 1) = 0.$$

for some real e. What is $a + b - c + e$?

5. (40) Christopher and Mingyue are playing a game. The game begins with an empty bag. Mingyue goes first, and each turn, a player adds either 3 or 4 marbles into the bag. At the end of the game, there are 26 marbles in the bag. There are x possible ways the game could have gone and Christopher moves last in y of them. What is $\frac{y}{x}$?

6. (45) Christopher has a triangle with area and perimeter both equal to 100. He draws a segment L parallel to one of its sides, splitting the triangle into two regions of equal area and equal perimeter. What is the length of segment L?

7. (50) Let x be an integer. Given that $x^{2025} - 1$ is divisible by $(x-1)^2$, what is the largest possible value of x?

8. (55) Aryan loves square numbers a bit too much. In an effort to increase Aryan's love for other numbers a bit more, Ian comes up with "quare" numbers, integers that can be written as a difference of two squares and are not squares themselves. How many "quare" numbers are there between 1 and 2025 inclusive?

9. (55) Four tangents are drawn from the vertices of a square to a circle concentric with the square, as shown. The square has area 100 and the circle has area π. What is the area of the shaded triangle?

10. (60) Each square on an 8×8 chessboard has some cake. Specifically, if the square is in column m and row n, place $\frac{m}{n}$ cakes there. A "hungry pawn" starts on row 1 and can either move forward one space or diagonally forward one space. Whenever it is on a square, it eats the cake there. It keeps moving until it can't anymore. A path is called "satisfying" if the hungry pawn devours a whole number amount of cake. How many satisfying paths are there?