"Math is Cool" Masters -- 2024-25

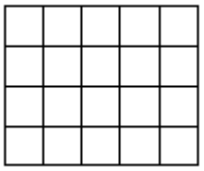
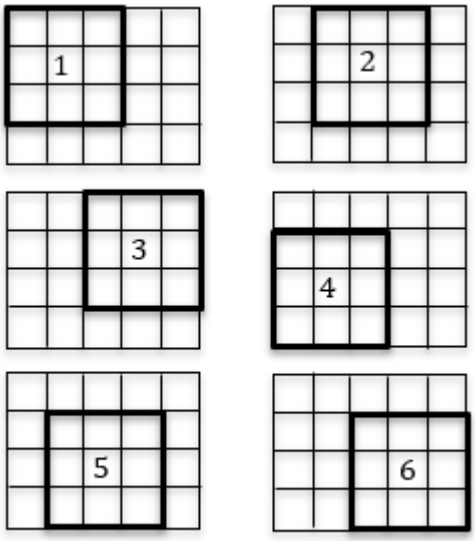
6th Grade

Mental Math Solutions

6th	Answer	Solution
1	9	What is one-half of ten plus one-third of twelve? $1/2 \cdot 10 + 1/3 \cdot 12 = 9$
2	54	What is the next term in the geometric sequence starting with two, six, and eighteen? 2, 6, 18, 54, pattern is x3
3	15 [quarters]	One-third of Ryan's quarters have a value of one dollar and twenty-five cents. How many quarters does Ryan have altogether? $\$1.25 \cdot 3 = \$3.75$ and $375/25 = 15$
4	16	What is the sum of the smallest two positive prime numbers that differ by six? $11 - 5 = 6$ and $11 + 5 = 16$
5	8 [= mean]	What is the mean of the integers four, six, eight, ten, and twelve? $(4 + 6 + 8 + 10 + 12)/5 = 8$
6	10 [cm]	A right triangle has integer side lengths and an area of twenty-four square centimeters. In centimeters, what is the length of the hypotenuse of the triangle? One Pythagorean Triple is (6, 8, 10) and the area would be $6 \cdot 8/2 = 24$, so the hypotenuse is 10.
7	[x =] 30	Solve the equation for X: three-point-five X minus forty-one equals sixty-four $3.5x - 41 = 64 \rightarrow 3.5x = 105 \rightarrow 7x = 210 \rightarrow x = 30$
8	13 [blue jellybeans]	The probability of randomly selecting one red jellybean from a jar containing red, orange, and blue jellybeans is one-sixth. The probability of randomly selecting one orange jellybean is two-fifths. What is the smallest number of blue jellybeans that could be in the jar? $P(\text{red}) = 1/6 = 5/30$, $P(\text{orange}) = 2/5 = 12/30$, and $30 - 5 - 12 = 13$

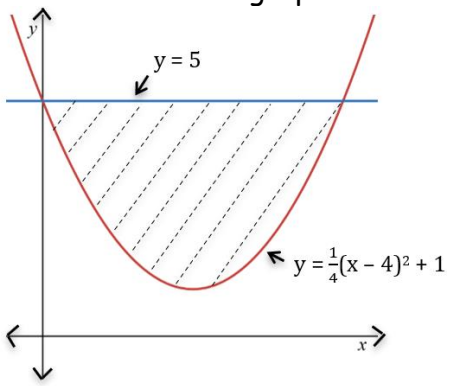
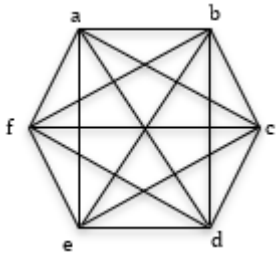
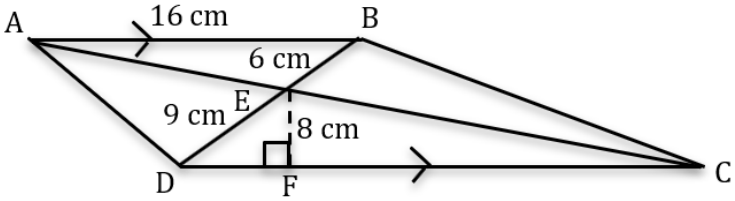
"Math is Cool" Masters -- 2024-25
6th Grade
Individual Test Solutions

6th	Answer	Solution
1	155	Evaluate: $5^3 + 5^2 + 5$ $5^3 + 5^2 + 5 = 125 + 25 + 5 = 155$
2	112	What is the sum of the prime numbers between 50 and 60? $53 + 59 = 112$
3	75 [minutes]	A car travels at an average rate of 32 miles per hour. How many minutes will it take for the car to travel 40 miles? $40 = 32 + 8$, so $60 + 0.25(60) = 75$
4	32 [inches]	In inches, what is the perimeter of a rectangle with a side length of 6 inches and an area of 60 in^2? $60/6 = 10$, so the perimeter is $2(6) + 2(10) = 32$
5	20 [years]	Sarina is 34 years old, and her only son is 7 years old. In how many years will she be twice her son's age? Sarina is $34 - 7 = 27$ years older than her son, so in 20 years, when her son is 27, she'll be 54 and she'll be twice his age.
6	60 [%]	What percent of 90 is 54? $54/90 = 6/10 = 60/100$
7	11	Evaluate the expression when $x = 8$: $x^2 - 6x - 5$ $8^2 - 6(8) - 5 = 64 - 48 - 5 = 11$
8	54	Starting with the 3rd term in the following sequence, each term is the product of the previous two terms. What is the fifth term? 1/2, 6, 3, . . . 1/2, 6, 3, 18, 54, . . . , the fifth term is 54
9	8 [= mean]	What is the mean of the following data set? {1, 12, 14, 5} $(1 + 12 + 14 + 5)/4 = 8$

10	6 [squares]	There are twenty 1 x 1 squares in the grid shown. How many 3 x 3 squares are in the grid? Note: the 3 x 3 squares may overlap each other.   There are six 3 x 3 squares.
11	25 [%]	The spinner shown here consists of 8 congruent sectors. As a percentage, what is the probability of the spinner landing on either 3 or 8 when it is spun one time? Two of the eight sectors contain a 3 or an 8 so the probability is 2/8 = 1/4, = 25%. 
12	352 [yards]	There are 5280 feet in a mile and 3 feet in a yard. How many yards are in 1/5 of a mile? 5280/3 = 1760 yards in a mile. 1760/5 = 352 yards = 1/5 of a mile.
13	25 [students]	In a middle school math class, 1/6 of the students prefer using pens and the rest prefer using pencils. If there are 30 students in the class, how many students prefer pencils? 1/6 = 5/30, so there are 30 – 5 = 25 students who prefer pencils.
14	1	Evaluate: $\frac{1}{2}+\frac{1}{3}+\frac{1}{10}+\frac{1}{15}$ 1/2 + 1/3 + 1/10 + 1/15 = (15 + 10 + 3 + 2)/30 = 30/30 = 1
15	[a + b =] 7	The prime factorization of 50 can be written in the form $a \cdot b^2$, where a and b are distinct prime numbers. What is $a + b$? $50 = 2 \cdot 5^2$, and 2 + 5 = 7
16	[ABC – DEF =] 200	In the expression ABC + DEF = 612, ABC and DEF represent two 3-digit integers consisting of single-digit integers A, B, C, D, E, and F, such that A = 2D, B = E, and C = F. What is ABC – DEF? Since A = 2D, A = 4 and D = 2, so the two numbers are 406 and 206, and 406 – 206 = 200
17	35	The 1st and 3rd terms of an arithmetic sequence are 4 and 8.6. What is the sum of the 2nd, 5th, and 6th terms? 8.6 – 4 = 4.6 and 4.6/2 = 2.3, so 2.3 is the amount that is added each time The first 6 terms are 4, 6.3, 8.6, 10.9, 13.2, 15.5, and 6.3 + 13.2 + 15.5 = 35

18	25,000 [cm^3]	A block of cheese in the shape of a triangular prism has a volume of 0.025 cubic meters. How many cubic centimeters are in the volume of the block of cheese? 1 cubic meter = 100 · 100 · 100 = 1,000,000 cubic centimeters, so 0.025 cubic meter = 25,000 cubic centimeters
19	6 [ways]	In how many distinct ways can the letters of the word BANANA be arranged such that the As are all next to each other and the Ns are all next to each other? Think of the AAA as one letter and the NN as one letter, then the answer is the same as the number of arrangements of 3 letters, or 3! = 6 - {AAABNN, AAANNB, BAAANN, BNNAAA, NNAAAB, NNBAAA}
20	19	Evaluate: $98 - (25/5)^2 + 2(4 - 7)^3$ $98 - (25/5)^2 + 2(4 - 7)^3 = 98 - 25 + 2(-27) = 98 - 25 - 54 = 98 - 79 = 19$
21	250 [attempts]	Currently, Downtown Freddy has made 28% of his 3-point attempts. If he makes 11 of his next 20 three-point attempts his percentage of attempts made will rise to 30%. How many total 3-point attempts does Downtown Freddy currently have? Let M = number of 3-pointers made and A = total 3-point attempts, then $M/A = 28/100 = 7/25$ and $(M + 11)/(A + 20) = 30/100 \rightarrow M = 7A/25 \rightarrow (7A/25 + 11)/(A + 20) = 30/100 \rightarrow 28A + 1100 = 30A + 600 \rightarrow 2A = 500 \rightarrow A = 250$
22	50 [minutes]	Biff can decorate two cakes in 25 minutes. If Eho helps, the two of them working together can decorate three cakes in 30 minutes. In minutes, how long would it take Eho to decorate a cake by himself? Biff can decorate 2/25 of a cake per minute and the two of them together can decorate 3/30 = 1/10 of a cake per minute. Eho can decorate 1/x of a cake per minute, where x = the number of minutes it takes Eho to decorate one cake. Then $2/25 + 1/x = 1/10 \rightarrow 4x + 50 = 5x \rightarrow x = 50$
23	[A =] 41	If $21_3 + 32_4 = A_5$, where A is a two-digit integer, what is A? Do not include the base in your answer. $2 \cdot 3^1 + 1 \cdot 3^0 = 7$ and $3 \cdot 4^1 + 2 \cdot 4^0 = 14$ and $7 + 14 = 21 = 4 \cdot 5^1 + 1 \cdot 5^0 = 41_5$, so A = 41
24	85	A set of 4 numbers has a mean of 15. When three positive integers are added to the set, its new mean is 21. What is the largest possible integer that could be among the three that were added to the set? Since $4 \cdot 15 = 60$ and $7 \cdot 21 = 147$, the sum of the 3 added integers is 147 – 60 = 87, and the largest possible value of any one of the 3 added integers is 85, if the other two are both 1s.
25	[A =] 3	The solutions to the equation $x + x + x = x^3$ are $x = 0$ and $x = \pm\sqrt{A}$. What is A? $x + x + x = x^3 \rightarrow 3x = x^3 \rightarrow 0 = x^3 - 3x \rightarrow 0 = x(x^2 - 3) \rightarrow 0 = x$ and $0 = x^2 - 3 \rightarrow 0 = x$ and $3 = x^2 \rightarrow 0 = x$ and $\pm\sqrt{3} = x$, so A = 3
26	12 [players]	Two-thirds of the players on Sanjay's baseball team bat right-handed. One-fourth of the players on Min's baseball team bat left-handed. Min's team has four-fifths the number of players of Sanjay's team. What is the number of players on Min's team? Note: there are no switch-hitters on either team, and both teams have fewer than 20 players. Let S = the number of players on Sanjay's team and M = the number of players on Min's team, then M = 0.8S and possible values for (M, S) include (4, 5), (8, 10), (12, 15), (16, 20), . . . The only one where both 2S/3 and M/4 are integers and S < 20 and M < 20 is (12, 15), so 12 is the number of players.

27	20 [palindromes]	How many three-digit palindromes are there in which the digits are all even? Note: zero is considered to be even. The ones and hundreds digits can be 2, 4, 6, or 8, and the tens digit can be 0, 2, 4, 6, or 8, so the answer is $4 \cdot 5 = 20$.
28	[a + b + c =] 10	When evaluating the given expression in scientific notation, the answer can be written in the form $a.b \cdot 10^c$, where $1 \leq a.b < 10$ and a, b, and c are integers. What is $a + b + c$? $39{,}000{,}000{,}000 \div 30{,}000 = a.b \cdot 10^c$ $39{,}000{,}000{,}000 = 39 \cdot 10^9$ and $30{,}000 = 3 \cdot 10^4$, and $(39 \cdot 10^9)/(3 \cdot 10^4) = 13 \cdot 10^5 = 1.3 \cdot 10^6$, and 1 + 3 + 6 = 10.
29	[A + B =] 14	Let $(\sqrt{A} + \sqrt{B})(\sqrt{A} - \sqrt{B}) = 4$, where A and B are single digit positive integers. What is the largest possible value of A + B? $(\sqrt{A} + \sqrt{B})(\sqrt{A} - \sqrt{B}) = A - B = 4$, so possible values of A and B include (5, 1), (6, 2), (7, 3), (8, 4), and (9, 5). The largest possible value of $A + B$ is $9 + 5 = 14$.
30	[A + B =] 73	Lorelei randomly draws 5 cards from a deck that consists of 13 hearts and 13 clubs. Three of the five cards are hearts and two are clubs. She then draws two more cards. As a simplified fraction, the probability that at least one of her next two draws will be hearts is A/B. What is A + B? After drawing the first 5 cards there are 10 hearts and 21 cards total in the deck, so P(at least 1 H in next two draws) = 1 – P(no hearts) = 1 – P(CC) = $1 - 11/21 \cdot 10/20 = 1 - 11/42 = 31/42$, and 31 + 42 = 73
31	240	What is the sum of the positive factors of 135? The factors of 135 are: 1, 3, 5, 9, 15, 27, 45, 135, and 1 + 3 + 5 + 9 + 15 + 27 + 45 + 135 = 240
32	2 [frogs to toads]	Ms. Badger gave the following homework assignment to her class of frogs and toads: "You must complete a swamp project by the end of the week. You may work alone or in frog-toad pairs." As it turned out, 1/3 of the frogs worked in frog-toad pairs, while 5/9 of the class worked alone. As an integer, what is the ratio of frogs to toads in the class? If there are F frogs in the class, then F/3 of them worked with F/3 of the toads in frog-toad pairs. If there are T toads in the class, then $(F + T - 2F/3)/(F + T) = 5/9 \rightarrow (9F + 9T - 18F/3) = 5F + 5T \rightarrow 27F + 27T - 18F = 15F + 15T \rightarrow 9F + 27T = 15F + 15T \rightarrow 12T = 6F$, so $F/T = 2/1 = 2$.
33	2990	An infinite sequence has 8 as its 1st term. In this sequence the 1st term is multiplied by 10 to get the 2nd term, the 2nd term is divided by 5 to get the 3rd term, 20 is added 3rd term to get the 4th term, and the 4th term is divided by 2 to get the 5th term. The four operations are repeated sequentially to generate the infinite sequence. What is the sum of the 77th, 78th, 79th, and 80th terms in the sequence? The sequence is 8, 80, 16, 36, 18, 180, 36, 56, 28, 280, 56, 76, . . ., and 8 + 80 + 16 + 36 = 140 18 + 180 + 36 + 56 = 290 28 + 280 + 56 + 76 = 440 and so on . . . So, the sum of each group of 4 terms increases by 150. The 77th, 78th, 79th, and 80th terms are the 20th group of four terms, so their sum will be 140 + 19(150) = 140 + 2850 = 2990

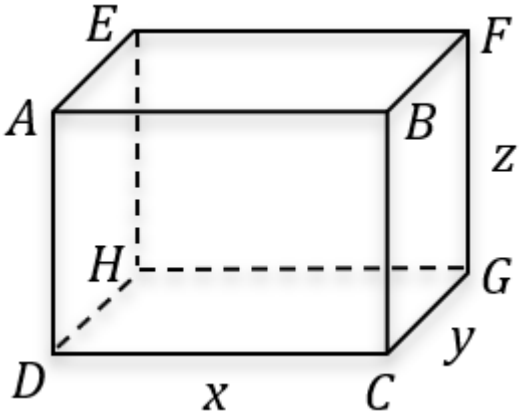
34	15 [grid points]	In figure below the graphs of the equations y = 5 and y = $\frac{1}{4}(x-4)^2 + 1$ are shown. How many grid points *(x, y)* exist inside the shaded region defined by the intersection of the two graphs such that *x* and *y* are integers, not including points that are on either graph.  Since (4, 1) is the vertex of the parabola, then (4, 2), (4, 3), (4, 4) are all inside the region. Substituting 3 and 5 for x results in y = 1.25, so (3, 2), (3, 3), (3, 4), and (5, 2), (5, 3), (5, 4) are all inside the region. Substituting 2 and 6 for x results in y = 2, so (2, 3), (2, 4), and (6, 3), (6, 4) are all inside the region. Substituting 1 and 7 for x results in y = 3.25, so (1, 4) and (7, 4) are both inside the region. There are a total of 3 + 6 + 4 + 2 = 15 points inside the region.
35	13	The vertices of a hexagon are labeled with the numbers 4 through 9 in random order. The diagonals of the hexagon are labeled with the sum of the two integers at their endpoints. What is the mean of the labels of the diagonals? If the vertices were labeled with a, b, c, d, e, and f, the sum of the labels of the diagonals would be (a + c) + (a + d) + (a + e) + (b + d) + (b + e) + (b + f) + (c + e) + (c + f) + (d + f) = 3a + 3b + 3c + 3d + 3e + 3f = 3(a + b + c + d + e + f). No matter how the numbers 4 through 9 replace the letters a through f, the sum of the labels of the diagonals will be 3(4 + 5 + 6 + 7 + 8 + 9) = 117, and the mean will be 117/9 = 13. 
36	160 [cm^2]	In the figure shown, $\overline{AB} \parallel \overline{CD}$, BE = 6 cm, DE = 9 cm, AB = 16 cm, and EF = 8 cm. How many square centimeters are in the area of ΔACD?  Since $\overline{AB} \parallel \overline{CD}$, then ΔABE ~ ΔCDE. Let h = the height of ΔABE, then h/8 = 6/9 → h = 16/3. The height of ΔACD = 8 + 16/3 = 40/3. Also, 16/CD = 6/9 → 6CD = 144 → CD = 24. The area of ΔACD = (1/2)(24)(40/3) = 160.

37	[A + B =] 131	An experiment consists of selecting a marble at random from a jar containing 7 blue and 7 red marbles, noticing its color, putting it back in the jar, shaking the jar, and selecting a 2nd marble at random. This experiment is repeated 4 times. As a simplified fraction, the probability that the result of 2 of the experiments is both marbles are blue, and the result of the other 2 experiments is both marbles are red, is A/B. What is A + B? P(BB) = P(RR) = 1/4 P(BB, BB, RR, RR) = $(1/4)^4 = 1/2^8$ But this doesn't have to happen in the order BB, BB, RR, RR. There are 4!/(2! · 2!) = 6 orders in which it can happen, so the final probability is $6 \cdot 1/2^8 = (3 \cdot 2)/2^8 = 3/2^7 = 3/128$, and 3 + 128 = 131.
38	270 [inches]	A rubber ball is dropped from a height of 54 inches. Each time it strikes the ground the ball bounces up 2/3 of the vertical distance of the previous fall. In inches, what is the total vertical distance that the ball travels? The vertical distances make the following geometric sequence: 54↓ + 36↑ + 36↓ + 24↑ + 24↓ + 16↑ + 16↓ + 32/3↑ + 32/3↓ . . . = 54 + 72 + 48 + 32 + 64/3 . . . = 54 + 72/(1 – 2/3) = 54 + 216 = 270
39	21 [ways]	How many ways are there to make a sum of 8 by adding two or more positive integers together? The order of the integers being added is not important. 2 integers: 1+7, 2+6, 3+5, 4+4 → 4 ways total 3 integers: 1+1+6, 1+2+5, 1+3+4, 2+2+4, 2+3+3 → 5 ways total 4 integers: 1+1+1+5, 1+1+2+4, 1+1+3+3, 1+2+2+3, 2+2+2+2 → 5 ways total 5 integers: 1+1+1+1+4, 1+1+1+2+3, 1+1+2+2+2 → 3 ways total 6 integers: 1+1+1+1+1+3, 1+1+1+1+2+2 → 2 ways total 7 integers: 1+1+1+1+1+1+2 → 1 way total 8 integers: 1+1+1+1+1+1+1+1 → 1 way total
40	200 [minutes]	Two candles of equal heights but different thicknesses are lit, and both burn at a constant linear rate. The first candle takes 4 hours to burn completely, and the second candle takes 5 hours to burn completely. If the two candles are lit at the same time, after how many minutes will the second candle be twice the length of the first candle? Let L = length of each candle Let t = time in hours Then 2(L – t · L/4) = L – t · L/5 → 2 – 2t/4 = 1 – t/5 → 1 = t/2 – t/5 → 1 = 3t/10 → t = 10/3 = 3.333 . . . hours = 200 minutes

"Math is Cool" Masters -- 2024-25

6th Grade

Multiple Choice Solutions

6th	Answer	Solution
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #3. Rectangular prism ABCDEFGH has dimensions x by y by z as shown.  		
1	B	Let x = 9 inches, y = 3 inches, and z = 5 inches. What is the volume of prism ABCDEFGH? A) 108 in^3 B) 135 in^3 C) 145 in^3 D) 162 in^3 E) 180 in^3 $9 \cdot 3 \cdot 5 = 135$
2	A	Let x = 15 m, y = 6 m, and z = 8 m. What is the area of rectangle DCFE? Hint: sketch the rectangle in the diagram. A) 150 m^2 B) 170 m^2 C) 225 m^2 D) $45\sqrt{29}$ m^2 E) 255 m^2 DC = 15 and CF = $\sqrt{6^2 + 8^2}$ = 10, so the area is $10 \cdot 15 = 150$.

3	*C*	Imagine an ant walks only along the edges of the prism and let x = 12 cm, y = 5 cm, and z = 6 cm. What is the shortest total distance the ant must travel to be able to walk the entire length of each edge at least once? A) 92 cm B) 104 cm C) 107 cm D) 108 cm E) 115 cm The minimum number of edges it takes to travel them all is 15 and it is possible for the ant to travel the longest edge 4 times, the second-longest edge 4 times, and the shortest edge 7 times. For example, this happens following the string of vertices: AEHDABCGHDCGFBFE. This covers all 12 edges in the following order: $\overline{AE}$, $\overline{EH}$, $\overline{HD}$, $\overline{DA}$, $\overline{AB}$, $\overline{BC}$, $\overline{CG}$, $\overline{GH}$, $\overline{HD}$, $\overline{DC}$, $\overline{CG}$, $\overline{GF}$, $\overline{FB}$, $\overline{BF}$, and $\overline{FE}$, with the three short edges $\overline{HD}$, $\overline{CG}$, and $\overline{FB}$ repeating twice each. So, the answer is 4(12) + 4(6) + 7(5) = 107.

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #4 THROUGH #6.

In a Magic Square the numbers in all rows, columns, and diagonals have the same sum, which is known as the magic constant.

In the 3-by-3 example shown here, the magic constant is 15. Also, it is important to note that all 9 numbers are different from each other.

In the 4-by-4 example shown here, the magic constant is 34 and similarly to the 3-by-3 example, all 16 numbers are different from each other.

4	3	8
9	5	1
2	7	6

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

4	*D*	In the 3-by-3 Magic Square shown here, what is the value of *A*? A) 1 B) 3 C) 4 D) 8 E) 14 Since 13 + 9 + 5 = 27, then A + 7 + 12 = 27 → A = 8

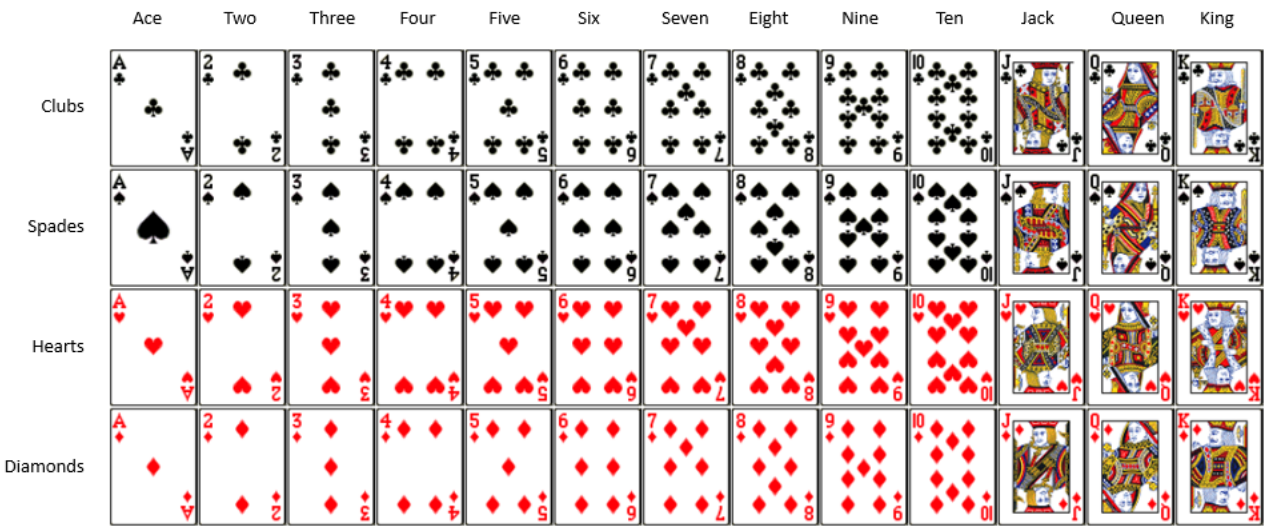
A	7	12
13	9	5
6	11	10

5	E	A 3-by-3 Magic Square consists of 9 consecutive odd integers whose mean is 9. What is the magic constant? A) 8 B) 9 C) 12 D) 18 E) 27 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81 and 81/3 = 27
6	D	In the partially filled in 4-by-4 Magic Square shown here, what is the value of the missing integer in the upper right-hand square? A) 7 B) 10 C) 11 D) 14 E) 19 The sum of 17 + 12 + 5 + 8 = 42, so you can fill in four of the missing squares with 3, 4, 6, and 9 as shown. Label the remaining four squares a, b, c, and d. We know that a + b = 24 and that a and b represent integers that are not already in the grid, so (a, b) could be (1, 23), (2, 22), (10, 14), (14, 10), (22, 2), or (23, 1). By similar reasoning, we also know that (c, d) is either (7, 11) or (11, 7). If c = 11, then a = 6, which is a repeated number. So, it must be that (c, d) = (7, 11), which results in (a, b) = (10, 14), so b = 14.

		15	?
17	12	5	8
		18	
16	13		

3	a	15	b
17	12	5	8
6	c	18	d
16	13	**4**	**9**

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #7 THROUGH #10.



There are 52 cards in a standard deck, shown here.
There are 4 suits: Clubs, Spades, Hearts and Diamonds.
"Face Cards" include Jacks, Queens, and Kings.
Black cards include all Clubs and Spades.
Red cards include all Hearts and Diamonds.
A **Euchre deck** contains 24 cards: the Nine, Ten, Jack, Queen, King and Ace of each suit.

7	C	How many cards in the given picture of a standard deck have letters on them? A) 8 B) 12 C) 16 D) 18 E) 20 4 Aces and 12 Face Cards makes 4 + 12 = 16
8	B	What is the ratio of face cards to non-face cards in a standard deck of cards? A) 3/13 B) 3/10 C) 4/13 D) 2/5 E) 15/37 12/40 =3/10
9	D	A card is randomly selected from a **Euchre deck** and removed from the deck. Then a second card is randomly selected. What is the probability that the second card is black? A) 11/46 B) 6/23 C) 11/23 D) 1/2 E) 12/23 P(R, then B) = 1/2 · 12/23 = 6/23 P(B, then B) = 1/2 · 11/23 = 11/46 6/23 + 11/46 = 23/46 = 1/2

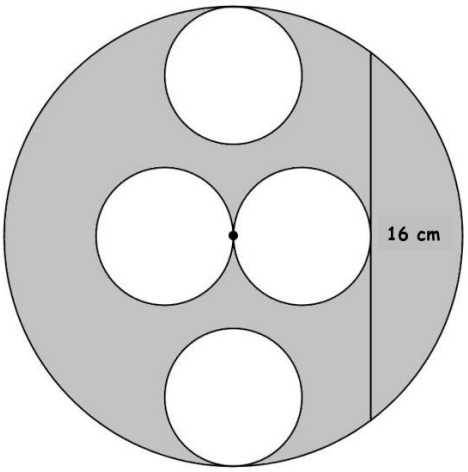
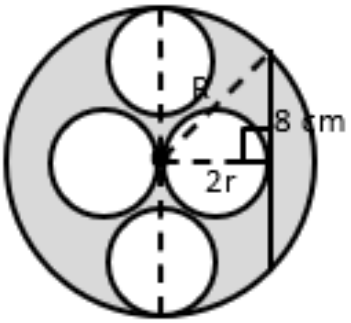
10	E	Bart and Lisa are playing a game using a **Euchre deck** in which they take turns drawing a card at random from the deck. Before each turn, the card that was drawn in the previous turn is returned to the deck which is then shuffled. If Bart draws a heart, he wins. If Lisa draws a face card, she wins. They keep taking turns until somebody wins. If Bart draws first, what is the probability that Lisa will be the winner of the game, no matter how long the game takes? A) 3/8 B) 33/64 C) 291/512 D) 4/5 E) Answer not given P(not H, then FC) = 3/4 · 1/2 = 3/8 that Lisa wins on her 1st draw. P(not H, then not FC, then not H, then FC) = 3/4 · 1/2 · 3/4 · 1/2 = 9/64 that Lisa wins on her 2nd draw. P(not H, then not FC, then not H, then not FC, then not H, then FC) = 3/4 · 1/2 · 3/4 · 1/2 · 3/4 · 1/2 = 27/512 that Lisa wins on her 3rd draw. P(Lisa wins) = 3/8 + 9/64 + 27/512 + . . . = $\frac{\frac{3}{8}}{1-\frac{3}{8}} = \frac{\frac{3}{8}}{\frac{5}{8}}$ = 3/5.

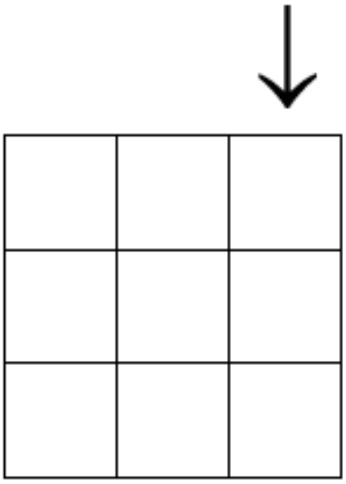
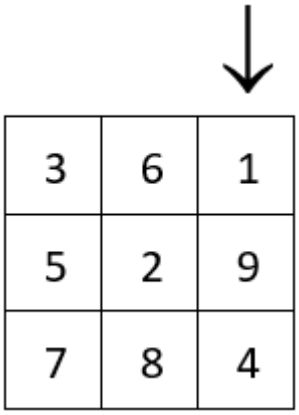
"Math is Cool" Masters -- 2024-25

6th Grade

Team Test Solutions

6th	Answer	Solution
1	13 [inches]	A regular hexagon has a perimeter of 78 inches. In inches, how long is one side of the hexagon? 78/6 = 13
2	[x =] 27	Solve for x: 17x – 61 = 20 + 14x 17x – 61 = 20 + 14x → 3x = 81 → x = 27
3	87 [inches]	How many inches are in 7.25 feet? 7.25 · 12 = 87
4	9	Evaluate the expression: $-\frac{170}{(7+3^3)} + 21 \cdot \frac{2}{3}$ $-\frac{170}{(7+3^3)} + 21 \cdot \frac{2}{3} = -\frac{170}{34} + 14$ = -5 + 14 = 9
5	20 [minutes]	It takes Drake 24 minutes to read his bedtime story. Kendrick can read the same story 1.2 times as fast as Drake. In minutes, how long does it take Kendrick to read the story? 1.2 = 6/5 and 6/5 times as fast means it takes 5/6 of the time, so 5/6 · 24 = 20.
6	[A + B =] 97	An infinite sequence progresses such that after the first two terms, each successive term is the product of all terms that have come before. If the first four terms are 5/6, 4/5, 2/3, and 4/9, the 5th term is the simplified fraction A/B. What is A + B? 5th term = 5/6 · 4/5 · 2/3 · 4/9 = 160/810 = 16/81 and 16 + 81 = 97
7	42 [= sum]	A data set includes the numbers 12, 18, 10, 16, and a number, N, that is a different positive integer from the other four. What is the sum of all values of N that make the mean and the median of the data set equal? Putting the four numbers in order 10, 12, 16, 18, shows that the median can be anywhere from 12 to 16 inclusive. If N = 4, the mean and the median are both 12. If N = 24, the mean and the median are both 16. One more possibility is the solution to the equation (10 + 12 + 16 + 18 + N)/5 = N → 56 + N = 5N → 4N = 56 → N = 56/4 = 14, so there are 3 values of N. 4+14+24 = 42

8	25	The sum of two numbers is 12 and the sum of their squares is 94. What is the product of the two numbers? Let $a + b = 12$ and $a^2 + b^2 = 94$. Then, $(a + b)^2 = a^2 + b^2 + 2ab = 144 \rightarrow 94 + 2ab = 144 \rightarrow 2ab = 50 \rightarrow ab = 25$
9	[A =] 64	In the figure shown here, there are 4 congruent smaller circles inside a larger circle. Two of the smaller circles are tangent to each other at the center of the larger circle and there is a chord of length 16 cm that is tangent to one of these circles. The other two smaller circles are internally tangent to the large circle, but do not intersect or overlap with the two circles that are tangent to each other at the center. The area of the shaded region in terms of π is Aπ cm^2. What is A?  Draw in a diameter of the large circle that is parallel to the chord. Then draw in a diameter of the small circle that is perpendicular to the chord at its midpoint. Also add a radius of the large circle from the center to one of the endpoints of the small chord. Then the Pythagorean equation would be $(2r)^2 + 8^2 = R^2 \rightarrow R^2 - 4r^2 = 64 \rightarrow \pi R^2 - 4\pi r^2 = 64\pi$, and since $\pi R^2 - 4\pi r^2$ represents the shaded area, then the shaded area is 64π, and $A = 64$. 

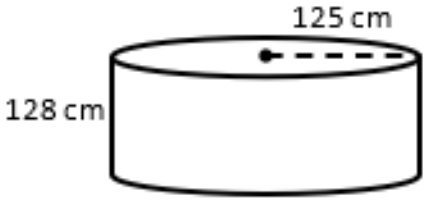
10	194	The digits from 1 to 9, each being used exactly once, can be arranged in the following grid such that each of the 3-digit numbers reading across the rows from left to right is a perfect square, and the 3-digit number reading down the diagonal from top left to bottom right is also a perfect square. What is the 3-digit number in the 3rd column, reading from top to bottom?  Start by writing down the perfect squares that are 3 digits. A number of them can be eliminated because they contain a 0 or they contain a repeated digit. Out of the remaining candidates, use logic and trial and error to discover the correct layout of the numbers. 

"Math is Cool" Masters -- 2024-25

6th Grade

Triple Jump Solutions

6th	Answer	Solution
1	9 [miles]	A scooter travels at an average rate of 15 miles per hour. In miles, how far will the scooter travel in thirty-six minutes? 15 miles in 1 hour = 15 miles in 60 min = 9 miles in 36 min
2	7 [units]	A circle is plotted on the rectangular coordinate system. The center of the circle is at the point (7, -10). The circle touches the y-axis at exactly one point. How many units is the radius of the circle? The circle is tangent to the y-axis at one point, therefore the radius must equal the straight-line distance from the center of the circle, which is 7 units.
3	7 [multiples]	How many positive multiples of 132 (including 132) are less than 1000? 132*7 = 924, which is the largest multiple less than 1000.
4	[A + B =] 11	Three fair coins are flipped. As a simplified fraction, the probability that exactly two of the coins show heads is A/B. What is A + B? There are 3 ways for there to be two heads: HHT, HTH, and THH, so P(exactly 2 heads) = $3 \cdot (1/2)^3 = 3/8$, and 3 + 8 = 11.
5	[R =] 22	A set of five distinct positive integers has a mean of 9, which is also the median. What is the largest possible range R, where R = the largest integer minus the smallest integer in the set. The data set with the largest range in which all the numbers are distinct positive integers would be: 1, 2, 9, 10, 23, so the range would be 23 – 1 = 22.
6	[a □ b =] 13	If a □ b = $\sqrt{a^2 + b^2}$, then what is $(\sqrt{71} \square \sqrt{73}) \square 5$? $(\sqrt{71} \square \sqrt{73}) \square 5 = \sqrt{\left(\sqrt{71}\right)^2 + \left(\sqrt{73}\right)^2} \square 5 = \sqrt{71 + 73} \square 5 = \sqrt{144} \square 5 = 12 \square 5 = \sqrt{144 + 25} = \sqrt{169} = 13$.

7	[A =] 2	A cylinder has a radius of 125 centimeters and a height of 128 centimeters. In terms of π, the number of cubic meters in the volume of the cylinder is Aπ m^3. What is A?  128 cm = 1.28 m = 32/25 m, and 125 cm = 1.25 m = 5/4 m, so the volume in cubic meters is $(32/25) \cdot (5/4)^2 \cdot \pi = (32/25) \cdot (25/16) \cdot \pi = 2\pi$, so A = 2.
8	[A + B =] 6	The base-5 number $4A1_5$, where A represents a digit from 0 to 4 inclusive, is equal to the base-6 number $2B4_6$, where B represents a digit from 0 to 5 inclusive. What is A + B? $4 \cdot 25 + 5A + 1 = 2 \cdot 36 + 6B + 4 \rightarrow 101 + 5A = 76 + 6B \rightarrow 6B - 5A = 25$. A can be 0, 1, 2, 3, or 4, and B can be 0, 1, 2, 3, 4, or 5, and with these possible values of A and B, the only way that 6B – 5A could equal 25 is if A = 1 and B = 5, and 5 + 1 = 6.
9	558 [minutes]	It is possible for an orchard's cherries to be harvested by 16 workers in 3 hours, with everyone working at the same constant rate. At this rate, if one worker starts the harvest at 8 AM, and one additional worker joins the harvest each hour on the hour, then how long in minutes will it take to complete the harvest? Note, the harvest may be completed with fewer than 16 workers. Each worker completes 1/48 of the harvest each hour. After t hours the amount of the harvest that will be complete will be $1/48 + 2/48 + 3/48 + \ldots + t/48 = \frac{1+2+3+\cdots+t}{48} = \frac{t(t+1)/2}{48} = \frac{t(t+1)}{96}$, and the largest value of t for which this expression is still less than 1 is t = 9, so after 9 hours, $\frac{9(9+1)}{96} = 90/96 = 15/16$ of the harvest will be complete. A 10^{th} worker will join after the first 9 hours. These 10 workers can complete 10/48 of the harvest in an hour, and solving the equation $10/48 \cdot t = 1/16 \rightarrow t = 3/10$, so it will take them 3/10 of an hour to complete the final 1/16 of the harvest, so altogether the harvest will be completed in 9.3 hours, and 9.3x60 = 558.

10	[A + B =] 281	A regular 6-sided (numbered 1 – 6), a regular 12-sided (numbered 1 – 12), and a regular 20-sided die (numbered 1 – 20) are rolled together. As a simplified fraction, the probability that the number showing on exactly 2 of the 3 dice is the same is A/B. What is A + B? There are $6 \cdot 12 \cdot 20 = 1440$ total possible sets of three numbers showing on the dice, so 1440 will be the denominator of the probability fraction. P(same on 6- and 12-sided dice, but not on all 3) = $6/6 \cdot 1/12 \cdot 19/20 = 114/1440$ P(same on 6- and 20-sided dice, but not on all 3) = $6/6 \cdot 1/20 \cdot 11/12 = 66/1440$ Case 1 (same number is 1 – 6): P(same on 12- and 20-sided dice, but on all 3) = $6/12 \cdot 1/20 \cdot 5/6 = 30/1440$ Case 2 (same number is 7 – 12): P(same on 12- and 20-sided dice, but on all 3) = $6/12 \cdot 1/20 \cdot 6/6 = 36/1440$ Final answer is $(114 + 66 + 30 + 36)/1440 = 246/1440 = 123/720 = 41/240$, and $41 + 240 = 281$.

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #1 Solutions

6th	Answer	Solution
1	76 [inches]	How many inches are in two and one-ninth of a yard? 1 yard = 36 inches, 2 yards = 72 inches, 1/9 yard = 4 inches, $2\frac{1}{9}$ yards = 72 + 4 = 76 inches
2	[A + B =] 27	As a simplified fraction, the probability of drawing a red ten from a standard deck is A over B. What is A plus B? P(red 10) = 2/52 = 1/26, and 1 + 26 = 27
3	[A =] 121	A circle has a circumference of twenty-two pi inches. In terms of pi, the area of the circle is 'A' pi square inches. What is A? Circumference = 22π → d = 22 and r = 11, so Area = $11^2\pi$ = 121π, and A = 121.
4	29	The product of three distinct positive prime numbers is seven hundred and fifteen. What is the sum of the three prime numbers? 715 = 5 · 11 · 13 and 5 + 11 + 13 = 29
5	10 [arrangements]	How many three-letter arrangements can be made with the letters in the word RATES, spelled R – A – T – E – S, such that the letters are in alphabetical order? A letter can only be used once. AER, AES, AET, ARS, ART, AST, ERS, ERT, EST, RST
6	[A + B =] 550	Out of eleven thousand species of birds there are sixty bird species that are flightless. As a simplified fraction, the ratio of birds that can fly to birds that can't is A over B. What is A plus B? 10940/60 = 1094/6 = 547/3 and 547 + 3 = 550
7	18 [minutes]	Brantley can build three sandcastles in two hours and fifteen minutes. Sawyer can build two sandcastles in an hour. In minutes, how long would it take Brantley and Sawyer to build one sandcastle working together? B: 3 c in 135 min = 1 c in 45 min Sawyer: 2 c in 60 min = 1 c in 30 min 1/30 + 1/45 = 1/x → 3x + 2x = 90 → 5x = 90 → x = 18

8	[A · B =] 14	If A minus B equals five and A plus B equals nine, then what is A times B? If A and B are positive integers, then (A, B) could be (8, 1), (7, 2), (6, 3), or (5, 4), and 7 + 2 = 9, 7 – 2 = 5, and 7 · 2 = 14.
9	59 [= mean]	What is the mean of the six numbers: twenty, thirty, forty, eighty, ninety, and ninety-four? (20 + 30 + 40 + 80 + 90 + 94)/6 = 59
10	[$x^2 \cdot y$ =] 539	Let X equal thirty-five over five and Y equal forty-four over four. What is X squared times Y? x = 35/5 = 7 and y = 44/4 = 11, and $7^2 \cdot 11 = 490 + 49 = 539$

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #2 Solutions

6th	Answer	Solution
1	15 [miles]	A cyclist rides one mile in eight minutes. At this rate, how far would the cyclist ride in two hours? 1 mi in 8 min = 15 mi in 120 min = 15 mi in 2 hr
2	27	What is the product of two and six to the third power divided by four to the second power? $2 \cdot 6^3/4^2 = 432/16 = 27$
3	206 [= 6th term]	The first two terms of an arithmetic sequence are eleven and fifty. What is the sixth term in the sequence? 11, 50, 89, 128, 167, 206 or 11 + 5(39) = 206
4	20 [multiples]	How many multiples of three are between fifty-five and one hundred and fifteen? $3 \cdot 19 = 57$ and $3 \cdot 38 = 114$, so there are 38 – 18 = 20 multiples of 3 between 55 and 115.
5	539 [= median]	What is the median of a data set consisting of all distinct three-digit multiples of one hundred and fifty-four? The set is: 154, 308, 462, 616, 770, 924, and the median is (462 + 616)/2 = 539
6	6 [chickens]	In a field there are only pigs and chickens. Altogether the animals in the field have thirty-four eyes and fifty-six legs. How many chickens are in the field? $4P + 2C = 56$ $2P + 2C = 34$ $\rightarrow$ $2P = 22 \rightarrow P = 11$ and $C = 6$
7	6 [ways]	As simplified fractions, the ratio A to B equals three over five and the ratio B to C equals one over three. In how many ways is this possible if A, B, and C are positive integers less than one hundred? A/B = 3/5 and B/C = 1/3 3/5 and 5/15, 6/10 and 10/30, 9/15 and 15/45, 12/20 and 20/60, 15/25 and 25/75, 18/30 and 30/90, makes 6 possible ways.
8	880 [yards]	How many yards are in half a mile? 5280/2 = 2640 and 2640/3 = 880
9	17 [feet]	A rectangle has an area of one hundred and twenty square feet. The length of the rectangle is fifteen feet. In feet, what is the length of the diagonal of the rectangle? $120/15 = 8$ and $8^2 + 15^2 = d^2 \rightarrow d = 17$

10	[A + B =] 28	Three standard dice are rolled. As a simplified fraction, the probability that each die shows either a three or a six is A over B. What is A plus B? P(3 or 6, 3 or 6, 3 or 6) = $1/3 \cdot 1/3 \cdot 1/3 = 1/27$ and 1 + 27 = 28

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #3 Solutions

6th	Answer	Solution
1	12,700 [cm]	How many centimeters are in one hundred and twenty-seven meters? $127 \cdot 100 = 12700$
2	16	The first two terms of a geometric sequence are eighty-one and fifty-four. What is the fifth term? 81, 54, 36, 24, 16
3	[A + B =] 20	A and B are positive integers. If A times B equals thirty-six and B divided by A equals nine, what is A plus B? If A and B are integers, then (A, B) could be (1, 36), (2, 18), (3, 12), (4, 9), or (6, 6), and 18/2 = 9, so 18 + 2 = 20
4	25 [grid points]	A ten-by-ten square is plotted on a coordinate plane, such that two of its sides are parallel with the y-axis and one of its vertices is the point five comma five. What is the number of grid points inside the square in which both coordinates are even? Assume the square has vertices (-5, 5), (5, 5), (5, -5), and (-5, -5). Then there would be 5 possible even x-values, -4, -2, 0, 2, and 4, and 5 possible even y-values, -4, -2, 0, 2, and 4. So, there would be $5 \cdot 5 = 25$ grid points in which both x and y are even. This would be true for the other three possible locations of the square in which one of its vertices is the point (5, 5).
5	[N =] 106	The number, N, has a prime factorization of A times B, where A and B are two distinct positive prime numbers. What is the smallest possible three-digit value of N? $100 = 2^2 \cdot 5^2$, 101 is prime, $102 = 2 \cdot 3 \cdot 17$, 103 is prime, $104 = 2^3 \cdot 13$, $105 = 3 \cdot 5 \cdot 7$, $106 = 2 \cdot 53$, so the answer is 106.
6	[A + B =] 242	A certain coin has a zero-point-six chance of landing heads when flipped and a zero-point-four chance of landing tails. When the coin is flipped three times, as a simplified fraction, the probability that at least one of the three flips results in heads is A over B. What is A plus B? P(at least one heads) = 1 – P(TTT) = $1 - (2/5)^3 = 117/125$, and 117 + 125 = 242.

7	3 [triangles]	How many triangles with integer side lengths are possible with a perimeter of twelve centimeters? (5, 3, 4), (5, 5, 2), and (4, 4, 4) are the only possibilities. Other combinations have the sum of the two shorter sides equal to or less than the length of the third side. For example, with (6, 5, 1), 5 + 1 = 6 and this means a triangle is impossible with these side lengths.
8	98 [%]	Three test scores are eighty-five percent, ninety-two percent, and ninety-three percent. What score in percent is needed on the next test to raise the average to ninety-two percent? $(85 + 92 + 93 + x)/4 = 92 \rightarrow 270 + x = 368 \rightarrow x = 98$
9	[x =] 30	Solve for X: zero-point-four X plus five equals seventeen $0.4x + 5 = 17 \rightarrow 4x + 50 = 170 \rightarrow 4x = 120 \rightarrow x = 30$
10	54	What is the sum of two squared plus three squared plus four squared plus five squared? $2^2 + 3^2 + 4^2 + 5^2 = 4 + 9 + 16 + 25 = 54$

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #4 Solutions

6th	Answer	Solution
1	66 [integers]	How many two-digit integers are larger than thirty-three? There are 90 two-digit integers, and there are 33 – 9 = 24 of them from 10 to 33, and 90 – 24 = 66.
2	457	What is one thousand minus five hundred minus forty minus three? 1000 – 500 – 40 – 3 = 457
3	84 [cm^2]	The height of a triangle is twelve centimeters, and the base is fourteen centimeters. What is the number of square centimeters in the area of the triangle? $14 \cdot 12/2 = 84$
4	48	In a sequence, successive terms are generated by alternating between multiplying the previous term by ten and then dividing the resulting term by five. The first three terms of such a sequence are three, thirty, and six. What is the ninth term in this sequence? 3, 30, 6, 60, 12, 120, 24, 240, 48
5	[x =] 8	Solve the equation for X: two X over three plus four-fifths equals five X over six minus eight-fifteenths $2x/3 + 4/5 = 5x/6 - 8/15 \rightarrow 20x + 24 = 25x - 16 \rightarrow 40 = 5x \rightarrow x = 8$
6	43 [ice cream cones]	Ernesto can scoop ice cream for twenty cones in half an hour. Maria scoops ice cream at one-point-five times the rate of Ernesto. Bianca scoops at one-point-two times the rate of Maria. Working together, how many cones can the three of them scoop in fifteen minutes? Ernesto: 20 in 30 min Maria: 30 in 30 min Bianca: 36 in 30 min Total: 86 in 30 min = 43 in 15 min

7	50 [%]	Rectangle A has its length decreased by twenty percent and its width increased by one hundred and fifty percent to create Rectangle B. The area of Rectangle A is what percent of the area of Rectangle B? Let Rectangle A be an x-by-y rectangle, then Rectangle B is a 0.8x-by-2.5y rectangle. The ratio of the areas is xy/2xy = 1/2, so the area of Rectangle A is 50 percent of the area of Rectangle B.
8	35	What is the sum of the cubes of the two smallest positive prime numbers? $2^3 + 3^3 = 8 + 27 = 35$
9	[*A* + B + *C* =] 13	*A*, B, and *C* represent three distinct positive integers, and *A* is a two-digit integer. If *A* is greater than B, which is greater than *C*, then what is the smallest possible value of *A* plus B plus *C*? 10 + 2 + 1 = 13
10	390	What is the median of the set of numbers: one hundred, two hundred, three hundred fifty, four hundred thirty, five hundred, and six hundred? For the set 100, 200, 350, 430, 500, 600, the median is (350 + 430)/2 = 390

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #5 Solutions

6th	Answer	Solution
1	17	What is two squared plus eighteen minus the square root of twenty-five? $2^2 - \sqrt{25} + 18 = 17$
2	[x =] 7	Solve for X: thirteen X minus sixteen equals seventy-five $13x - 16 = 75 \rightarrow 13x = 91 \rightarrow x = 7$
3	12 [blue marbles]	A jar has twenty-one marbles that are either red or blue. The probability of randomly selecting a red marble from the jar is three-sevenths. How many blue marbles are in the jar? P(red) = 3/7 = 9/21, and 21 – 9 = 12
4	25	A set of five distinct positive integers has a mean of twenty-three. What is the smallest possible value of the largest integer in the set? The data set with the smallest possible largest number is the set in which the integers are consecutive: 21, 22, 23, 24, 25 and 25 is the smallest value of the largest integer.
5	52	In the set of positive integers less than thirty, what is the sum of the first and last in the longest string of consecutive composite numbers? The longest string of consecutive composite numbers is 24, 25, 26, 27, 28, and 24 + 28 = 52
6	36 [integers]	How many two-digit integers exist, such that the ones digit is larger than the tens digit? 12 through 19 – 8 23 through 29 – 7 34 through 39 – 6 45 through 49 – 5 56 through 59 – 4 67 through 69 – 3 78 through 79 – 2 89 – 1 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36
7	432 [in^2]	How many square inches are in one-third of a square yard? $1/3 \cdot 36 \cdot 36 = 12 \cdot 36 = 432$

8	40 [%]	What percent of eighty-five is thirty-four? $34/85 = 2/5 = 40\%$
9	33	The Fibonacci sequence begins with the terms one, one, two, three, and continues indefinitely. What is the sum of the first seven terms? $1 + 1 + 2 + 3 + 5 + 8 + 13 = 33$
10	7 [miles]	A car drives at an average rate of thirty-five miles per hour. How many miles does the car travel in twelve minutes? 35 miles per hour = 35 miles in 60 minutes = 7 miles in 12 minutes

"Math is Cool" Masters -- 2024-25

6th Grade

College Bowl Round #6 Solutions

6th	Answer	Solution
1	2300 [meters]	How many meters are in two point three kilometers? $2.3 \cdot 1000 = 2300$
2	64 [inches]	A square has an area of two hundred and fifty-six square inches. In inches, what is the perimeter of the square? $\sqrt{256} = 16$ and $4 \cdot 16 = 64$
3	[A + B =] 19	An infinite series begins with the terms two-fifths, five-eighths, eight-elevenths, and so on. The product of the first five terms as a simplified fraction is A over B. What is A plus B? $2/5 \cdot 5/8 \cdot 8/11 \cdot 11/14 \cdot 14/17 = 2/17$, and $2 + 17 = 19$
4	[A + B =] 5	One-eighteenth of the living beings in the mythical land of Dormor are goblins, one ninth are elves, one twelfth are trolls, and the rest are humans. As a simplified fraction, the ratio of goblins to trolls is A over B. What is A plus B? $(1/18)/(1/12) = 12/18 = 2/3$ $2 + 3 = 5$
5	10 [days]	It takes ten workers five days to build twenty birdhouses. How many days would it take three workers to build twelve birdhouses? $10W{:}5D{:}20B \rightarrow 1W{:}50D{:}20B \rightarrow 1W{:}30D{:}12B \rightarrow 3W{:}10D{:}12B$, so 10 days.
6	77 [°] or [degrees]	The formula for converting Fahrenheit "F" to Celsius "C" is: C equals five-ninths times the quantity F minus thirty-two. How many degrees Fahrenheit is twenty-five degrees Celsius? $25 = \frac{5}{9}(F - 32) \rightarrow 45 = F - 32 \rightarrow F = 77$

7	[A + B =] 89	A card is drawn at random from a standard deck and not replaced. If it is red, all remaining red cards are removed from the deck. If it is black, no cards are removed. Then a second card is drawn. As a simplified fraction, the probability that the second card is black is A over B. What is A plus B? P(R, then B) = $1/2 \cdot 1 = 1/2$ P(B, then B) = $1/2 \cdot 25/51 = 25/102$ P(2^{nd} card is black) = $1/2 + 25/102 = 76/102 = 38/51$, and $38 + 51 = 89$
8	4 [values]	Two integers are represented by A and B. Let A be thirty-one units away from zero on a number line and let B be fourteen units away from A. How many different values could B have? (A, B) = (31, 45), (31, 17), (-31, -17), or (-31, -45), B could be 45, 17, -17, or -45, so 4 values.
9	26	Four numbers have a mean of eighteen. Three of the numbers are one, sixteen, and twenty-nine. What is the fourth number? $4 \cdot 18 = 72$ and $72 - 1 - 16 - 29 = 26$
10	28	What is forty-nine times sixteen divided by twenty-eight? $49 \cdot 16/28 = 49 \cdot 4/7 = 28$