

# “Math is Cool” Masters -- 2025-26

## High School

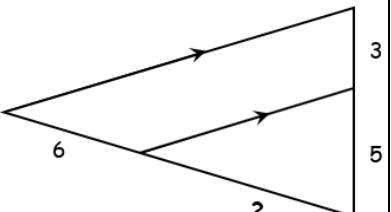
### Mental Math Solutions

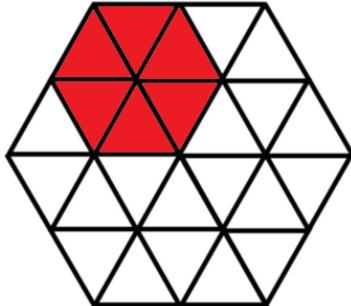
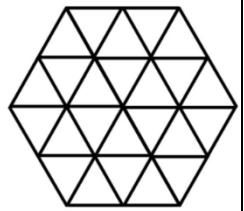
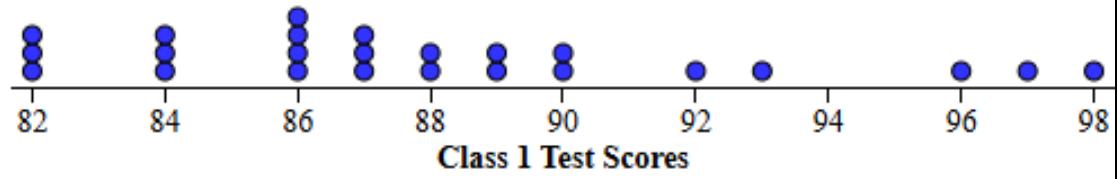
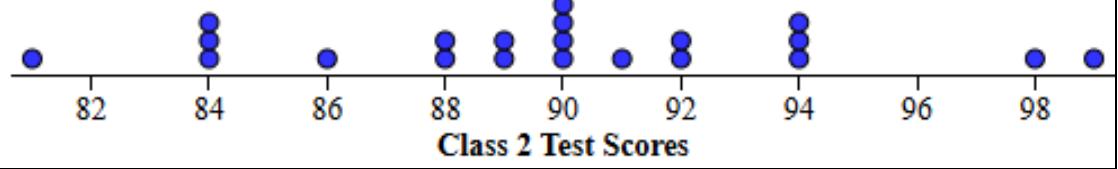
	<b>Answer</b>	<b>Solution</b>
<b>1</b>	170	What is twenty-five percent of six hundred and eighty? $680/4 = 170$
<b>2</b>	-24	What is the sum of the next three terms in the arithmetic sequence (pronounced air-ith-MET-ic) that begins as follows: Four, one, negative two, and so on. Subtract 3 each time: 4, 1, -2, -5, -8, -11 $-5 + (-8) + (-11) = -24$
<b>3</b>	240 [°]	In degrees, what is two times the measurement of an interior angle of a regular hexagon? Each interior angle of a regular hexagon = 120. $2(120) = 240$
<b>4</b>	12 [= median]	What is the median of the first eleven positive composite integers? We know that it will be the 6 <sup>th</sup> number: 4, 6, 8, 9, 10, <b>12</b> .
<b>5</b>	25	What is the smallest two-digit positive integer that is equal to its units digit squared? $5^2 = 25$
<b>6</b>	45 [= sum]	Dexter writes down seven consecutive positive integers. The sum of the smallest three is thirty-three. What is the sum of the largest three? 10, 11, 12, 13, 14, 15, 16 $14 + 15 + 16 = 45$
<b>7</b>	36 [%]	In the following sentence, if one letter is randomly selected, what is the probability as a percentage that it is a vowel? Loud wind shook the dark trees There are 25 total letters. Loud wind shook the dark trees 9 of them are vowels. $9/25 = 36/100 = 36\%$ .
<b>8</b>	14 [white marbles]	Caillou has fifty marbles, each of which is red, white or blue. The smallest number are red, and the largest number are blue. There are eleven times as many blue marbles as red ones. How many white marbles are there? R = 1 means B = 11 R = 2 means B = 22 R = 3 means B = 33, W = 50 – 36 = 14

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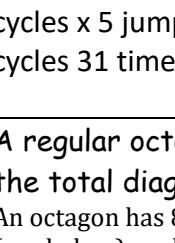
### Individual Test Solutions

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	3 [= x]	<p>Solve for x:</p> $-5\sqrt{x+1} + 12 = 2$ $-5\sqrt{x+1} = 2 - 12$ $\sqrt{x+1} = 2$ $x+1 = 4, x = 3$
<b>2</b>	30000	<p>As an integer, what is the following quotient?</p> $\frac{7.2 \times 10^2}{2.4 \times 10^{-2}} = \frac{7.2 \times 10^2}{2.4 \times 10^{-2}} = 3 \times 10^4 = 30000$
<b>3</b>	1800 [°]	<p>In degrees, what is the sum of all interior angles of a regular dodecagon (12 sides)?</p> <p>Sum of angles = <math>180(n - 2) = 180(12 - 2) = 1800</math>.</p>
<b>4</b>	300 [= sum]	<p>An arithmetic series begins as follows. What is the sum of the first 12 terms?</p> $3 + 7 + 11 + \dots$ $S_n = n/2(2a_1 + (n - 1)d)$ $= 12/2(2(3) + (12-1)(4)) = 300$
<b>5</b>	4 [%]	<p>The probability that any customer at Hot Mess Burgers will order a "Boring Mess" hamburger is 0.20. What is the probability as a percent that the next two customers will order a "Boring Mess"?</p> <p><math>P(\text{Boring Mess} \&amp; \text{ Boring Mess}) = (0.2)(0.2) = 0.04 = 4\%</math></p>
<b>6</b>	31245 [is the finishing order]	<p>Five students (1, 2, 3, 4 &amp; 5) were taking a math test. 1 finished before 2 but after 3. 4 finished before 5 but after 2. As a 5-digit integer, what was the finishing order from first to last?</p> <p>1 finished before 2 but after 3: 3 1 2          4 finished before 5 but after 2: 2 4 5          31245</p>
<b>7</b>	10 [factors]	<p>How many of the positive integer factors of 2025 are less than 100?</p> <p>Factors of 2025: 1, 3, 5, 9, 15, 25, 27, 45, 75, 81, 135, 225, 405</p>
<b>8</b>	10 [units]	<p>The triangle contains two parallel lines as shown.</p> <p>In units, what is the length of the missing segment labeled with the question mark?</p> $6/3 = ?/5$ $? = 30/3 = 10$ 

<b>9</b>	11 [= remainder]	<p>What is the remainder when the following division is performed?</p> $\begin{array}{r} x^2+3x+1 \\ \hline x-2 \end{array}$ <p>Can use the Remainder Theorem:  <math>F(2) = 2^2 + 3(2) + 1 = 11</math>  Or can perform the long division.</p>
<b>10</b>	705 [base 10]	<p>The number <math>2025_7</math> is equal to what number in base 10? You do not need to include the 'base 10' in your answer.</p> $2025_7 = (2 \times 7^3) + (0 \times 7^2) + (2 \times 7^1) + (5 \times 7^0) = 686 + 0 + 14 + 5 = 705$
<b>11</b>	13 [ounces]	<p>Bill and Echo collected <math>3\frac{1}{4}</math> pound of candy together on Halloween. They split the candy evenly into 4 piles. How many ounces does each pile weigh?</p> <p>1 pound = 16 ounces  <math>3.25 \times 16 = 52</math> ounces total  <math>52/4 = 13</math></p>
<b>12</b>	8 [regular hexagons]	<p>The figure shown here is made up of congruent equilateral triangles. How many regular hexagons of any size are in the figure?</p> <p>There is 1 big hexagon (the whole figure) and 7 little hexagons.</p>  
<b>13</b>	3 [= positive difference in medians]	<p>The following dotplots show test scores for 2 classes, where all test scores are integer values. What is the positive difference between the median test scores for the 2 classes?</p> <p>Class 1 median = 87  Class 2 median = 90  <math>90 - 87 = 3</math></p>  

<b>14</b>	324 [= 5 <sup>th</sup> term]	A geometric sequence begins as follows. What is the 5 <sup>th</sup> term in the sequence? 64, 96, ... $96/64 = 3/2$ , the common ratio. $96 * 3/2 = 144$ $144 * 3/2 = 216$ $216 * 3/2 = 324$
<b>15</b>	4 [= B]	When the following number is fully simplified, it can be written as $\frac{\sqrt{A}}{B}$ . What is B? $\frac{\sqrt{6}}{2\sqrt{8}}$ $\frac{\sqrt{6} \cdot \sqrt{8}}{2\sqrt{8} \cdot \sqrt{8}} = \frac{\sqrt{48}}{16} = \frac{4\sqrt{3}}{16} = \frac{\sqrt{3}}{4}$
<b>16</b>	7 [= n]	What is the smallest positive odd integer 'n' such that $n^3 + 2$ is not a prime number? $1^3 + 2 = 3$ , prime $3^3 + 2 = 29$ , prime $5^3 + 2 = 127$ , prime $7^3 + 2 = 345$ , not prime
<b>17</b>	95 [°]	The three angles of a triangle have the following measures in degrees: $(7x + 25)$ , $(5x + 18)$ , and $(3x - 13)$ . In degrees, what is the measure of the largest angle in the triangle? $(7x + 25) + (5x + 18) + (3x - 13) = 180$ $15x + 30 = 180$ $15x = 150$ $x = 10$ Largest angle is $7x + 25 = 7(10) + 25 = 95$ .
<b>18</b>	-3	What is the smallest integer that is a solution to the following inequality? $-3(2x + 1) < 21$ $-3(2x + 1) < 21$ $2x + 1 > -7$ $2x > -8$ $x > -4$
<b>19</b>	60 [%]	A game consists of flipping a fair coin and rolling a single 5-sided die (numbered 1 through 5). The player wins if the coin flip lands on heads or if the die shows a 5 (or both). What is the probability in percent of winning the game? There are 10 outcomes: H1 H2 H3 H4 H5 T1 T2 T3 T4 T5. Six of them are winning outcomes. $6/10 = 60\%$ .

20	5 [= sum of solutions=]	<p>Find the sum of all real solutions to the following equation:</p> $x^2 - 4x = -6 + x$ $x^2 - 4x = -6 + x$ $x^2 - 5x + 6 = 0$ $(x - 3)(x - 2) = 0$ $X = 3 \text{ or } x = 2$ $3 + 2 = 5$
21	9 [nickels]	<p>Gibson told Leon that he had 9 coins worth 45 cents. Leon mentioned that there was more than one possibility, and asked how many pennies Gibson had. After Gibson answered, Leon said that he knew exactly which coins Gibson had. How many nickels did Gibson have?</p> <p>There are 3 possibilities:</p> <p>QNNNPPPPP DDDDPPPPP NNNNNNNNN</p> <p>Gibson must have answered that he had 0 pennies, otherwise Leon still would not have known the exact coins. Gibson has 9 nickels and 0 pennies.</p>
22	30375	<p>Evaluate:</p> $(3125)^{\frac{3}{5}} \times (729)^{\frac{5}{6}}$ $(3125)^{\frac{3}{5}} \times (729)^{\frac{5}{6}}$ $= 5^3 \times 3^5 = 125 \times 243 = 30375$
23	11 [angles]	<p>For angle measurements between <math>0^\circ</math> and <math>2000^\circ</math>, inclusive, how many angles have a tangent value equal to -1?</p> <p>Tangent is negative in the 2<sup>nd</sup> and 4<sup>th</sup> quadrants, at a reference angle of <math>45^\circ</math>.</p> <p>Therefore, between <math>0 - 360^\circ</math>, <math>\tan = -1</math> at <math>135^\circ</math> and <math>315^\circ</math>. <math>360 \times 5 = 1800</math>, so there are 5 full revolutions. The additional <math>200^\circ</math> to get to <math>2000^\circ</math> will take us past <math>135^\circ</math> but not to <math>315^\circ</math>. Therefore there are <math>5 \times 2 + 1 = 11</math> times that <math>\tan</math> will equal -1.</p>
24	144 [ways]	<p>How many ways are there to color the following diagram choosing from red, blue, green and yellow, such that each section is colored a solid color, and no two neighboring sections have the same color? A neighboring section is one that shares a boundary line or curve with another section.</p> <p>The rectangle cannot be the same color as any of the partial circles or the triangle, and you can choose its color 4 ways.</p> <p>Then, you can choose the outer circle's color 3 ways, the middle circle's color 2 ways, and the inner circle's color 2 ways, and the triangle's color 3 ways.</p> $4 \times 3 \times 2 \times 2 \times 3 = 144$

25	154	<p>Using a shift cipher, the word MATH is encoded as SGZN. As an integer, what does the following code represent?</p> <p><b>UTK NATJXKJ LOLZE-LUAX</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td><td>O</td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td><td>U</td><td>V</td><td>W</td><td>X</td><td>Y</td><td>Z</td></tr> <tr><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td><td>O</td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td><td>U</td><td>V</td><td>W</td><td>X</td><td>Y</td><td>Z</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr> <tr><td>ONE</td><td>H</td><td>U</td><td>N</td><td>D</td><td>R</td><td>E</td><td>D</td><td>F</td><td>I</td><td>F</td><td>T</td><td>Y</td><td>-</td><td>F</td><td>O</td><td>U</td><td>R</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>U</td><td>T</td><td>K</td><td>N</td><td>A</td><td>T</td><td>J</td><td>X</td><td>K</td><td>J</td><td>L</td><td>O</td><td>L</td><td>Z</td><td>E</td><td>-</td><td>L</td><td>U</td><td>A</td><td>X</td><td></td><td></td><td></td><td></td><td></td></tr> </table>	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	ONE	H	U	N	D	R	E	D	F	I	F	T	Y	-	F	O	U	R								U	T	K	N	A	T	J	X	K	J	L	O	L	Z	E	-	L	U	A	X					
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26	2 [= median]	<p>Nineteen consecutive integers have a sum of 38. What is the median of the nineteen integers?</p> <p>Let <math>m</math> = the median</p> <p>The 19 integers are: <math>m - 9, m - 8, \dots, m, m + 1, m + 2, \dots, m + 9</math></p> <p>The sum of all 19 integers = <math>19m = 38</math>, therefore <math>m = 38/19 = 2</math></p>																																																																																																						
27	6	<p>In Unluckyland, clocks are consecutively numbered 1 through 13 instead of 1 through 12. A jumping spider starts on number 1. It jumps one position to 2, then two positions to 4, three positions to 7, two positions to 9, and one position to 10. The spider continues jumping clockwise around the face following this 5-jump (1, 2, 3, 2, 1) pattern. What number does the spider land on after its 2025<sup>th</sup> jump?</p> <p>Draw the cycles of 5 jumps at a time (+1, +2, +3, +2, +1). It takes 13 of these “cycles” to end up back at the starting point (#1) at the exact end of a cycle. <math>13 \text{ cycles} \times 5 \text{ jumps/cycle} = 65 \text{ jumps}</math>. <math>2025 \div 65 = 31\text{r}10</math>. Therefore, will do the 13 cycles 31 times ending up on #1. Then do 10 more jumps, which takes us to #6.</p>																																																																																																						
28	20 [%]	<p>A regular octagon has multiple diagonals of varying lengths. What percent of the total diagonals have the longest possible length?</p> <p>An octagon has <math>8(5)/2 = 20</math> total diagonals and 4 of those 20 have the longest possible length (see below), and <math>4/20 = 1/5 = 20\%</math>.</p> 																																																																																																						

29	6 [%]	<p>The prevalence of obesity in 199 countries of the world in 2022 as a function of their prevalence in 1992 can be modeled with the following linear regression equation.</p> $\text{Predicted 2022 Obesity} = 1.2(\text{Actual 1992 Obesity}) + 10.3$ <p>The Obesity variables are given as a percentage of the country's adult population.</p> <p>The United States has the following actual data point: (20.0%, 40.3%). As an <b>integer</b> percent, what is the residual for this point?</p> <p style="text-align: center;">Prevalence of Obesity in 2022 vs. 1992</p> <table border="1"> <caption>Data points from the scatter plot</caption> <thead> <tr> <th>1992 (% obese)</th> <th>2022 (% obese)</th> </tr> </thead> <tbody> <tr> <td>20.0</td> <td>40.3</td> </tr> </tbody> </table> <p>Use the actual 1992 value to calculate the predicted 2022 value:</p> $\text{Predicted 2022 Obesity} = 1.2(20.0) + 10.3 = 34.3\%$ $\text{Residual} = \text{observed} - \text{predicted} = 40.3 - 34.3 = 6$	1992 (% obese)	2022 (% obese)	20.0	40.3
1992 (% obese)	2022 (% obese)					
20.0	40.3					
30	4 [= $\log_a \frac{x^2 z^2}{y^3}$ ]	<p>If <math>\log_a x = 4</math>, <math>\log_a y = 6</math>, and <math>\log_a z = 7</math>, what is the value of <math>\log_a \frac{x^2 z^2}{y^3}</math> ?</p> $\log_a \frac{x^2 z^2}{y^3} = 2 \log_a x + 2 \log_a z - 3 \log_a y = 2(4) + 2(7) - 3(6) = 8 + 14 - 18 = 4$				
31	25 [units]	<p>A right triangle has integral side lengths and an area of 84 square units. In units, what is the length of the hypotenuse?</p> <p>It must be a Pythagorean Triple, and the one that works is 7-24-25, because <math>\frac{1}{2}(7)(24) = 84</math>.</p>				

**32**

44

When a certain positive integer is divided by 24, the remainder of that division has a remainder of 2 when divided by 6. What is the sum of all possible values of this integer that are less than 25?

The values that satisfy all the given conditions are: 2, 8, 14, 20

The sum is 44.

N	r = mod(N, 24)	mod(r, 6)
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	0
7	7	1
8	8	2
9	9	3
10	10	4
11	11	5
12	12	0
13	13	1
14	14	2
15	15	3
16	16	4
17	17	5
18	18	0
19	19	1
20	20	2
21	21	3
22	22	4
23	23	5
24	0	0

**33**2  
[associate professors]

The mean age of a group consisting of assistant professors and full professors is 40 years. The mean age of the assistant professors is 35 years, and the mean age of the full professors is fifty years. How many associate professors are there for each full professor?

$$A = \# \text{ associate}, F = \# \text{ full}$$

$$\text{Sum of associates ages} = 35A$$

$$\text{Sum of full prof ages} = 50F$$

$$\frac{35A + 50F}{A + F} = 40$$

$$35A + 50F = 40A + 40F$$

$$10F = 5A$$

$$A/F = 10/5 = 2/1$$

Therefore, for every 1 full prof there are 2 associate profs.

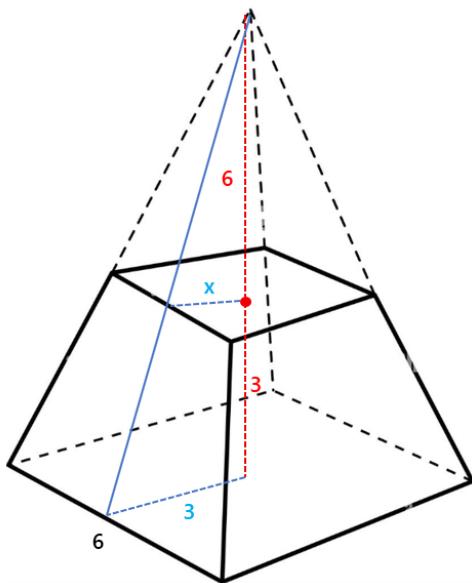
34	23 [= A + B]	<p>A standard 6-sided die is rolled three times. The probability that a two is rolled at least once, given that the third roll equals the sum of the first two rolls, can be written as a simplified fraction <math>A/B</math>. What is <math>A + B</math>?</p> <p>List the favorable outcomes, where the sum of the first 2 rolls equals the third roll:</p> <table border="1" data-bbox="456 295 840 939"> <thead> <tr> <th>1st</th><th>2nd</th><th>3rd</th></tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>2</td></tr> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>2</td><td>1</td><td>3</td></tr> <tr><td>1</td><td>3</td><td>4</td></tr> <tr><td>2</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>1</td><td>4</td></tr> <tr><td>1</td><td>4</td><td>5</td></tr> <tr><td>2</td><td>3</td><td>5</td></tr> <tr><td>3</td><td>2</td><td>5</td></tr> <tr><td>4</td><td>1</td><td>5</td></tr> <tr><td>1</td><td>5</td><td>6</td></tr> <tr><td>2</td><td>4</td><td>6</td></tr> <tr><td>3</td><td>3</td><td>6</td></tr> <tr><td>4</td><td>2</td><td>6</td></tr> <tr><td>5</td><td>1</td><td>6</td></tr> </tbody> </table> <p>There are 15 total outcomes, and 8 of them contain at least one 2. <math>P = 8/15</math>, <math>8 + 15 = 23</math>.</p>	1st	2nd	3rd	1	1	2	1	2	3	2	1	3	1	3	4	2	2	4	3	1	4	1	4	5	2	3	5	3	2	5	4	1	5	1	5	6	2	4	6	3	3	6	4	2	6	5	1	6
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35	9400 [triangles]	<p>Each side of a square is subdivided into 11 segments of equal length. How many different triangles can be created by selecting 3 of these points of subdivision as the vertices (excluding the corners of the square)? Count all triangles that have a unique set of three vertices.</p> <p>Either: (1) Each of the three vertices lies on a different side of the square, or (2) 2 vertices lie on one side of the square and the 3<sup>rd</sup> is on a different side.</p> <ul style="list-style-type: none"> <li>(1) Select 3 of the 4 sides of the square: <math>4C3 = 4</math>. Select 1 of the 10 points on each of the 3 sides: <math>10^3 = 1000</math>. Total = <math>4 \times 1000 = 4000</math></li> <li>(2) Select 1 side of the square (4 ways), and 2 division points on this side: <math>10C2 = 45</math> ways. Select a different side of the square (3 ways) and 1 division point on that side (10 ways). Total – <math>4 \times 45 \times 3 \times 10 = 5400</math> ways.</li> </ul> <p>Total = <math>4000 + 5400 = 9400</math></p>																																																

**36**76 [ $\text{m}^3$ ]

A pyramid has a square base 6 meters on a side and a height of 9 meters. In cubic meters, what is the volume of the portion of the pyramid that lies beneath a plane which is parallel to its base, and which is 3 meters above the base?

See figure (not to scale). We want the volume of the solid outlined frustum. The entire volume is:  $V = \frac{1}{3}b^2h = \frac{1}{3}6^2(9) = 108$ . To find the volume of the top pyramid, we need the base. Use similar triangles:  $6/x = 9/3$ ,  $x = 2$ , therefore the base = 4.  $V_{\text{top}} = \frac{1}{3}b^2h = \frac{1}{3}4^2(6) = 32$ .

Therefore,  $V_{\text{frustum}} = 108 - 32 = 76$ .

**37**

108 [= sum]

Find the sum of the x-intercepts of the graph of the following equation:

$$y = ||x - 54| - 32|$$

Can rewrite the equation:

$$y = |x - 54 - 32|, x \geq 54$$

$$y = |54 - x - 32|, x < 54$$

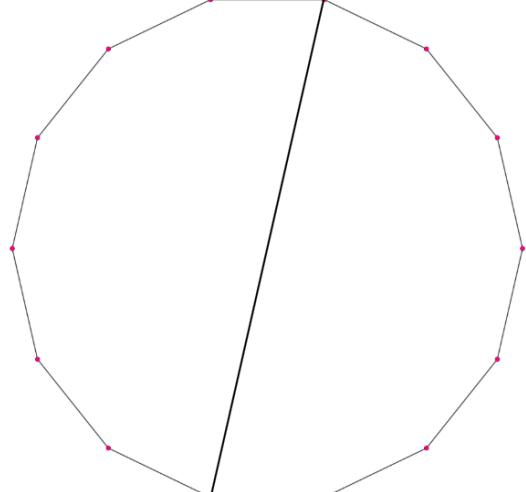
$$\text{If: } |x - 54 - 32| = 0$$

$$x - 86 = 0, x = 86$$

$$\text{If: } |54 - x - 32| = 0$$

$$22 - x = 0, x = 22$$

$$86 + 22 = 108$$

38	228 [= next term]	<p>Two arithmetic integer sequences have their corresponding terms multiplied together to form a third sequence, which starts as follows: 468, 462, 382, ... What is the next term of this third sequence?</p> <p><math>468 = 2 \cdot 234 = 2 \cdot 2 \cdot 117</math></p> <p><math>462 = 2 \cdot 231 = 2 \cdot 3 \cdot 77</math></p> <p><math>382 = 2 \cdot 191</math></p> <p>This works if the 2 sequences are as follows:</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>A (<math>d = -1</math>)</td> <td>B (<math>d = +37</math>)</td> </tr> <tr> <td>4</td> <td>117</td> </tr> <tr> <td>3</td> <td>154</td> </tr> <tr> <td>2</td> <td>191</td> </tr> <tr> <td>1</td> <td>228</td> </tr> </table> <p>Therefore the next term is <math>1 \cdot 228 = 228</math>.</p>	A ( $d = -1$ )	B ( $d = +37$ )	4	117	3	154	2	191	1	228
A ( $d = -1$ )	B ( $d = +37$ )											
4	117											
3	154											
2	191											
1	228											
39	84 [right triangles]	<p>How many different right triangles can be created by connecting three vertices of a regular tetradecagon (14 sides)? Count all right triangles that use three different vertices.</p> <p>The tetradecagon can be thought of as being inscribed in a circle, with all vertices lying on the circle. Using Thales' Theorem, any triangle formed using a diameter will necessarily be a right triangle. There are 7 "diameters" using the vertices of the polygon. For each of those, there are 12 other vertices that can be chosen to create a triangle. <math>12 \times 7 = 84</math>.</p> 										
40	288 [seconds]	<p>Biff and Eho each run at a constant speed around a track. They both start at the same time in the same location, and run in the same direction. Eho is running faster than Biff. Eho takes 3 minutes to cover one lap, and he overtakes Biff for the first time after 8 minutes. How many seconds does it take Biff to run one lap?</p> <p>After 8 minutes, Eho has run <math>\frac{8}{3}</math> laps and Biff has run <math>\frac{5}{3}</math> laps. Biff's rate is 8 min/<math>(\frac{5}{3})</math> lap = <math>24/5</math> min/lap = <math>4 \frac{4}{5}</math> min/lap = 4 min 48 sec/lap = 288 sec</p>										

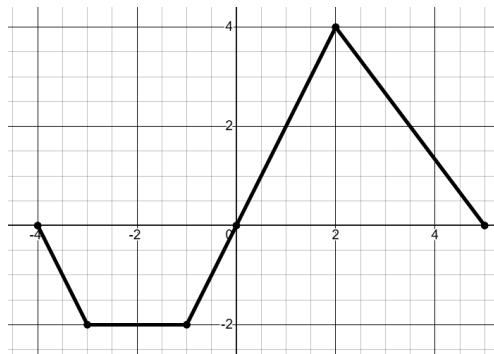
41	22 [unit squares]	<p>On the coordinate plane, consider the area that lies within the square boundary formed by the points <math>(10, 10)</math>, <math>(10, -10)</math>, <math>(-10, -10)</math>, and <math>(-10, 10)</math>. Within this area, there are a total of 400 unit squares, where each unit square has integer grid points <math>(x, y)</math>. When the following function is plotted, how many of these unit squares within the boundary does it pass through?</p> $f(x) = e^x - 1$ <p>First consider <math>f(x) = e^x</math>. The only integer grid point it passes through is <math>(0, 1)</math>. To the left, it will decrease and approach the <math>x</math>-axis, and pass through 10 squares. To the right, it will pass through <math>(1, e)</math> or about <math>(1, 2.7)</math>, and <math>(2, e^2)</math> or about <math>(2, 7.4)</math>. The next point is <math>(3, e^3)</math> or about <math>(3, 20.1)</math>. So from <math>(0, 1)</math>, the curve will increase continuously, crossing the vertical lines <math>x = 1</math> and <math>x = 2</math> before exiting the boundary area. From that we can see that it will pass through 11 squares in Quadrant 1.</p> <p>For <math>f(x) = e^x - 1</math>, the entire curve is shifted down 1 unit, so it will pass through <math>(0, 0)</math>. The left side will still pass through 10 squares. The right side now has an additional square, so 12 total on the right.</p> $10 + 12 = 22$
42	1	<p>Given the following matrices, perform the following operation: <math>3B - 2AC</math></p> <p>What is the sum of all elements in the matrix that results from the operation?</p> $A = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 4 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 0 \\ -1 & 6 \\ -3 & 7 \end{bmatrix}$ $3B = 3 \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 15 & -6 \\ 3 & 9 \end{bmatrix}$ $AC = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 4 & -6 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 6 \\ -3 & 7 \end{bmatrix}$ $= \begin{bmatrix} 3 \cdot 2 + 2 \cdot (-1) + 0 \cdot (-3) & 3 \cdot 0 + 2 \cdot 6 + 0 \cdot 7 \\ -1 \cdot 2 + 4 \cdot (-1) + (-6) \cdot (-3) & -1 \cdot 0 + 4 \cdot 6 + (-6) \cdot 7 \end{bmatrix}$ $AC = \begin{bmatrix} 4 & 12 \\ 12 & -18 \end{bmatrix} \quad \rightarrow 2AC = \begin{bmatrix} 8 & 24 \\ 24 & -36 \end{bmatrix}$ $3B - 2AC = \begin{bmatrix} 15 & -6 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 8 & 24 \\ 24 & -36 \end{bmatrix} = \begin{bmatrix} 7 & -30 \\ -21 & 45 \end{bmatrix}$ $7 - 21 - 30 + 45 = 1$

**43**

4 [= g(-2)]

The following graph shows the function  $f(x)$ . Define  $g(x)$  as follows:  
 $g(x) = -\frac{1}{2}f(x - 1) + 3$

Find the value of  $g(-2)$ .



$f(x - 1)$  means:  
**horizontal shift 1 unit to the right.**

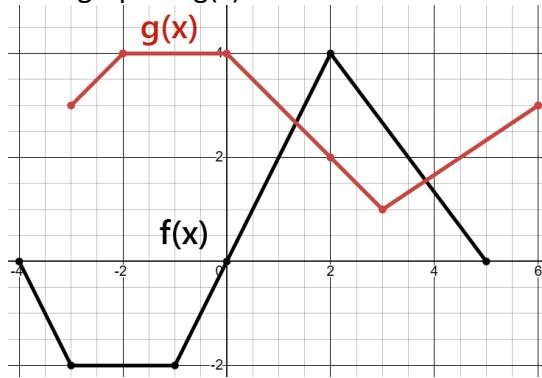
$-\frac{1}{2}f(x - 1)$  means:

**Compress vertically by a factor of  $\frac{1}{2}$ , then reflect across the x-axis.**

$-\frac{1}{2}f(x - 1) + 3$  means:

**Do a vertical shift 3 units up.**

Final graph of  $g(x)$  will look like this:

**44**

7 [=x]

Solve for  $x$ :  $e^{\ln 2x} - 1 = 13$

$$e^{\ln 2x} - 1 = 13$$

$$e^{\log_e 2x} = 14$$

$$2x = 14, x = 7$$

**45**

- 2
- [ $= (f^{-1})'(a)$ ]
- [ $= (f^{-1})'(3)$ ]

The following function  $f(x)$  has an inverse. For  $a = 3$ , find the value of  $(f^{-1})'(a)$ .

$$f(x) = \frac{x+6}{x-2}, x > 2$$

$$f(x) = \frac{x+6}{x-2}, x > 2$$

Find the inverse function by swapping  $x$  and  $y$ , then solving for  $y$ :

$$f^{-1}(x) = \frac{2x+6}{x-1}$$

Use the quotient rule to find the derivative of the inverse:

$$f'(x) = \frac{-8}{(x-1)^2}$$

Substitute  $a = 3$

$$f'(3) = \frac{-8}{(3-1)^2} = -2$$

**“Math is Cool” Masters -- 2025-26**  
**High School**  
**Multiple Choice Solutions**

<b>9/ 10<sup>th</sup></b>	<b>11/ 12<sup>th</sup></b>	<b>Answer</b>	<b>Solution</b>
<b>1</b>		<b>A</b>	<p>For what values of <math>q</math> will the following equation have exactly 2 real solutions?</p> $x\sqrt{14} + 7 = qx^2$ <p>A) <math>q &gt; -\frac{1}{2}</math>    B) <math>q &gt; 2</math>    C) <math>q &lt; 2</math>    D) <math>q &lt; -\frac{1}{2}</math>  E) None of the above.</p> $qx^2 - x\sqrt{14} - 7 = 0$ <p>All that matters is the value of the discriminant in the quadratic equation, it must be <math>&gt; 0</math> for the equation to have 2 real solutions:</p> $14 + 28q > 0$ $28q > -14$ $q > -\frac{1}{2}$
	<b>1</b>	<b>E</b>	<p>Which of the following intervals contains all of the real zeros of the following function?</p> $f(x) = 2x^3 - 3x^2 + x$ <p>A) <math>\left(\frac{1}{4}, 2\right)</math>    B) <math>\left[-1, \frac{1}{4}\right]</math>    C) <math>[1, 2]</math>    D) <math>(-2, 1)</math>  E) None of the above.</p> $2x^3 - 3x^2 + x$ $= x(2x^2 - 3x + 1)$ $= x(2x - 1)(x - 1)$ $x = 0, \frac{1}{2}, 1$ <p>None of the given intervals include all 3 values.</p>

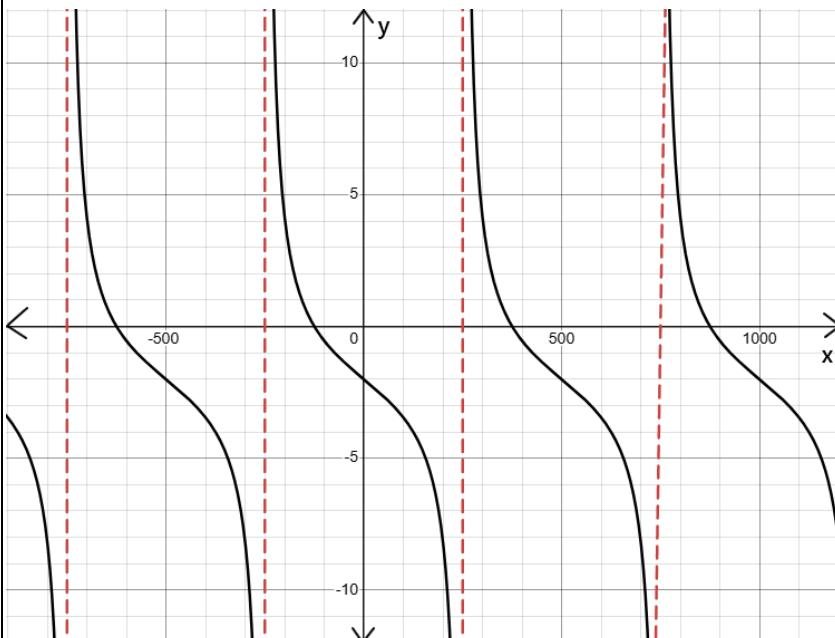
<b>2</b>	<b>C</b>	<p>If the International Space Station is orbiting the earth's equator at an altitude of 400 kilometers, how much farther (in kilometers) than the circumference of the equator does it fly when it circles the equator exactly once?</p> <p>A) 400 km    B) 800 km    C) <math>800\pi</math> km    D) <math>1200\pi</math> km  E) None of the above.</p> <p>Let <math>r</math> = radius of earth.  <math>C = 2\pi r</math></p> <p>Therefore <math>r + 400</math> is the radius of the ISS.  <math>C = 2\pi(r + 400) = 2\pi r + 800\pi</math>  <math>(2\pi r + 800\pi) - 2\pi r = 800\pi</math></p>
<b>2</b>	<b>D</b>	<p>All of the line segments shown here are vertical or horizontal, with their lengths in units indicated. What is the diagonal (shortest) distance from point X to point Y?</p> <p>A) <math>\sqrt{210}</math> units    B) <math>\sqrt{255}</math> units    C) <math>4\sqrt{15}</math> units    D) <math>5\sqrt{5}</math> units  E) None of the above.</p> <p>Place the diagram onto the coordinate axes, arbitrarily with point X at <math>(0, 0)</math>. Therefore point Y is at <math>(10, -5)</math>.</p> <p>Use the distance formula:</p> $d = \sqrt{(10 - 0)^2 + (-5 - 0)^2}$ $= \sqrt{125} = 5\sqrt{5}$

3	B	<p>For the sequence that begins as follows, determine either the common difference (if it is an arithmetic sequence) or the common ratio (if it is a geometric sequence).</p> $20\sqrt{5}, 40, 16\sqrt{5}, \dots$ <p>A) <math>2\sqrt{5}</math>      B) <math>\frac{2\sqrt{5}}{5}</math>      C) <math>\frac{\sqrt{5}}{5}</math>      D) <math>\sqrt{5}</math>      E) None of the above.</p> <p>By inspection, it looks like a geometric series, so find the common ratio by doing <math>a_2/a_1</math>:</p> $\frac{40}{20\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ <p>Verify by calculating <math>a_3/a_2</math>, which equals the same value of r.</p>
3	C	<p>Which of the following is a formula for the <math>n^{\text{th}}</math> term of the sequence that begins as follows? The first term corresponds to <math>n = 1</math>.</p> $48, 12, 3, \dots$ <p>A) <math>a_n = 192 \left(\frac{1}{4}\right)^{n-1}</math>      B) <math>a_n = 192 \left(\frac{1}{4}\right)^{1-n}</math>      C) <math>a_n = 48 \left(\frac{1}{4}\right)^{n-1}</math>      D) <math>a_n = 48 \left(\frac{1}{4}\right)^{1-n}</math>      E) None of the above.</p> <p>The only one that works is C, which divides by 4 every time. B and D are both increasing functions, and A has first term 192.</p>
4	E [5/3]	<p>Find the y-coordinate of the y-intercept of the given polynomial function.</p> $f(x) = \left(4x^2 - \frac{5}{2}\right) \left(2x + \frac{2}{3}\right) (x^2 - 1)$ <p>A) -1      B) 1      C) <math>-\frac{5}{3}</math>      D) <math>-\frac{20}{3}</math>      E) None of the above.</p> <p>Substitute <math>x = 0</math> into the equation:</p> $\begin{aligned} & \left(0 - \frac{5}{2}\right) \left(0 + \frac{2}{3}\right) (0 - 1) \\ &= \left(-\frac{5}{2}\right) \left(\frac{2}{3}\right) (-1) = \frac{5}{3} \end{aligned}$

4

B

Which of the following equations corresponds to the graph shown here?



- A)  $y = -2 \tan\left(\frac{\pi}{1000}x\right) - 2$     B)  $y = -2 \tan\left(\frac{\pi}{500}x\right) - 2$   
C)  $y = 2 \tan\left(\frac{\pi}{1000}x\right) - 2$     D)  $y = -2 \tan\left(\frac{\pi}{500}x\right) + 2$   
E) None of the above.

The graph has been reflected over the x-axis, the result of a negative multiplier, so it can't be C. We can specifically see that the multiplier is -2, because where the tangent term is 1 (when  $x = 125$ ), the y value is -4. The entire graph is shifted down 2 units, so it must be A or B. The period is calculated as  $\pi/b$  for  $\tan(bx)$ , and by inspection of the graph the period is 500. This matches graph B, where  $\pi/b = \pi/(\pi/500) = 500$ .

5	5	B	<p>Let <math>W</math> be the set of natural numbers <math>n</math> such that <math>n</math> is the sum of the squares of three consecutive positive integers. Which of the following statements must be true?</p> <p>A) All elements of <math>W</math> are even.      B) No element of <math>W</math> is divisible by 3.      C) No element of <math>W</math> is divisible by 5.      D) No element of <math>W</math> is divisible by 11.      E) All elements of <math>W</math> are odd.</p> <p>Try a few values to get an idea:</p> $1^2 + 2^2 + 3^2 = 14 \text{ (E is not true)}$ $2^2 + 3^2 + 4^2 = 29 \text{ (A is not true)}$ $3^2 + 4^2 + 5^2 = 50 \text{ (C is not true)}$ $4^2 + 5^2 + 6^2 = 77 \text{ (D is not true)}$ <p>All choices other than B have been eliminated.</p> <p>A way to verify is to write an expression for the sum:</p> $\text{Sum} = n^2 + (n+1)^2 + (n+2)^2 = 3n^2 + 6n + 5$ <p>Dividing this by 3 will give:</p> $n^2 + 2n + 5/3, \text{ so it is not divisible by 3.}$
6	6	D	<p>A hospital wing has 8 rooms on one side of the hallway and 5 rooms on the other side. A nurse randomly chooses one room to start in, then he randomly chooses a second and a third room to visit, for a total of 3 different rooms. What is the probability that the nurse crosses to the other side of the hallway at least once?</p> <p>A) <math>\frac{7}{11}</math>      B) <math>\frac{8}{15}</math>      C) <math>\frac{9}{14}</math>      D) <math>\frac{10}{13}</math>      E) Answer not given.</p> <p>There are a total of 13 rooms, so <math>13C3 = 286</math> ways to visit 3 rooms.</p> <p>Side A = 8 rooms and Side B = 5 rooms.</p> <p>There are <math>8C3 = 56</math> ways to visit 3 rooms on Side A only. There are <math>5C3 = 10</math> ways to visit 3 rooms on Side B only. The total ways to stay on one side of the hallway is <math>56 + 10 = 66</math>. Therefore, in <math>286 - 66 = 220</math> cases, the hallway will be crossed.</p> <p>P = <math>220/286 = 10/13</math>.</p>

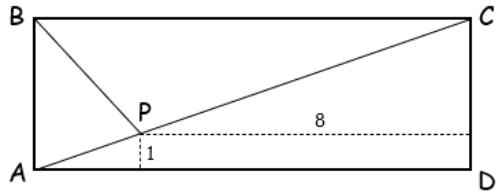
<b>7</b>	<b>C</b>	<p>Find the minimum distance from the point <math>(3, -1)</math> to the line <math>y = x</math> on the coordinate plane.</p> <p>A) <math>\sqrt{2}</math>      B) 2      C) <math>2\sqrt{2}</math>      D) <math>\frac{5}{4}</math>      E) None of the above.</p> <p>The shortest distance will be on the line perpendicular to <math>y = x</math>, which will have a slope = <math>-1</math>. Find equation of this line:</p> $y - (-1) = -(1x - 3)$ $y = -x + 2$ <p>The point of intersection of the 2 lines is <math>(1, 1)</math>. The distance from <math>(1, 1)</math> to <math>(3, -1)</math> is:</p> $d = \sqrt{(3 - 1)^2 + (-1 - 1)^2}$ $d = \sqrt{4 + 4} = 2\sqrt{2}$
<b>7</b>	<b>D</b>	<p>If <math>\cos \theta = \frac{5}{13}</math> and <math>\tan \theta &lt; 0</math>, what is the value of <math>\sin 2\theta</math>?</p> <p>A) <math>\frac{-5}{13}</math>    B) <math>\frac{12}{13}</math>    C) <math>\frac{-12}{13}</math>    D) <math>\frac{-120}{169}</math>    E) None of the above.</p> <p>Since <math>\cos</math> is + and <math>\tan</math> I -, the angle must be in Quadrant IV.</p> <p><math>\sin 2\theta = 2\sin\theta\cos\theta</math></p> $= 2 \left( \frac{-12}{13} \right) \left( \frac{5}{13} \right) = \frac{-120}{169}$

8

8

A

In rectangle ABCD, the angle bisector at B intersects the diagonal AC at point P. The perpendicular distance from P to CD is 8, and the perpendicular distance from P to AD is 1. What is the length of side AD?



- A)  $8 + 2\sqrt{2}$     B)  $11 - \sqrt{2}$     C)  $11 + \frac{\sqrt{2}}{2}$     D) 10

E) None of the above.

The angle bisector theorem tells us that the perpendicular distance from point P (on the angle bisector) to sides AB and BC is equal, call it  $x$ . There are two similar triangles now: the one highlighted in yellow, and triangle ACD. Use this to write the following relationship:

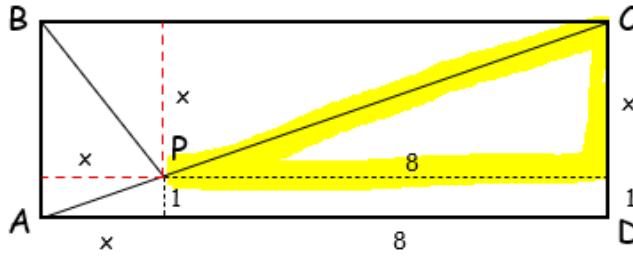
$$\frac{x}{8} = \frac{x+1}{x+8}$$

$$x^2 + 8x = 8x + 8$$

$$x^2 = 8$$

$$x = \sqrt{8} = 2\sqrt{2}$$

Therefore, the side length of AD is  $8+2\sqrt{2}$



9

9

D

Packard wrote a program for a computer science assignment. The program takes two real numbers, X and Y, as input. The program triples X, then squares the result, then subtracts nine times X. This value is called A. Second, the program squares Y, then increases that value by four times Y. This value is called B. The program outputs the sum of A and B.

What is the minimum possible output value for A + B?

- A) -4      B) 2      C)  $\frac{9}{4}$       D)  $-\frac{25}{4}$

E) None of the above.

To minimize the sum, we will minimize each value separately, then add them together.

$$A = (3X)^2 - 9X = 9X^2 - 9X$$

This is describing a parabola:

$y = 9x^2 - 9x$ , which will open upwards, so the minimum value will be the y-coordinate of the vertex.

$$x = -b/2a = 9/18 = \frac{1}{2}$$

$$y = 9(1/2)^2 - 9(1/2) = 9/4 - 9/2 = -9/4$$

Therefore, the minimum value of A is  $-9/4$ .

$$B = Y^2 + 4Y$$

Using the same method:

$$x = -b/2a = -4/2 = -2$$

$$y = (-2)^2 + 4(-2) = -4$$

$$\text{Minimum } A + B = -9/4 + (-4) = -25/4$$

**10****10****A**

The number  $\frac{1}{50^{2025}}$  is a terminating decimal. What is the last non-zero digit in the decimal representation of this number?

- A) 2   B) 4   C) 6   D) 8   E) None of the above.

The number can be rewritten:

$$\begin{aligned}\frac{1}{50^{2025}} &= \left(\frac{1}{50}\right)^{2025} \\ &= \left(\frac{1}{100} \times 2\right)^{2025} = 0.01^{2025} \times 2^{2025}\end{aligned}$$

So we want the last digit of that multiplication.

The last digit of  $0.01^{2025}$  will be 1, so the question is, what is the last digit of  $2^{2025}$ .

The last digit of powers of 2 has a cycle:

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

Then the cycle repeats, always ending in 2, 4, 8, 6.

$2025/4 = 506r1$ , so it will go through 506 full cycles of 4, and then the r1 puts us back at the beginning. Therefore the last digit will be 2.

# “Math is Cool” Masters -- 2025-26

## High School

### Team Test Solutions

9/ 10	11 / 12	Answer	Solution
1		119 [cents]	<p>You have an unlimited supply of the following U.S. coins: quarters, dimes, nickels and pennies. In cents, what is the largest amount of money you can have in these coins and not be able to make exactly one dollar in change?</p> <p>The maximum would be:</p> <p>QQQDDDDPPPP</p> <p>which equals 119 cents.</p>
1	0		<p>Consider the addition problem:</p> <p><math>8 + 88 + 888 + 8888 + \dots + 88888888888888888888</math> (20-digit number)</p> <p>What digit is in the hundreds place of the sum?</p> <p>The right-most column contains 20 8's. <math>20 \times 8 = 160</math>, so the 0 goes in the units place, and carry the 16.</p> <p>The next column contains 19 8's. <math>19 \times 8 + 16 = 168</math>. The 8 goes in the tens place, and carry the 16. The next column contains 18 8's. <math>18 \times 8 + 16 = 160</math>. The 0 goes in the hundreds place.</p>
2		77 [sq ft]	<p>The perimeter of a rectangle with integer side lengths is 36 feet, and its diagonal measures <math>\sqrt{170}</math> feet. What is the area of the rectangle in square feet?</p> $P = 2(x + y) = 36$ $x + y = 18$ $x^2 + y^2 = 170$ $7^2 + 11^2 = 49 + 121 = 170$ $\text{Area} = 7 \times 11 = 77$

	<b>2</b>	<b>126 [°]</b>	<p>Angle A of triangle ABC measures <math>60^\circ</math>, and angles B and C are in a ratio of 2 to 3, respectively. Point Q is the intersection of the angle bisectors of triangle ABC. Find the measure of angle AQB in degrees.</p> <p>Find angles B and C first. <math>2x + 3x = 120</math>, <math>x = 24</math>. Therefore angle B = <math>48^\circ</math>, and C = <math>72^\circ</math>. Bisecting the angles gives us a triangle with angles 30, 24 and Q. Therefore <math>Q = 180 - 30 - 24 = 126^\circ</math>.</p>
	<b>3</b>	<b>5 [= m]</b>	<p>The slope, <math>m</math>, of a line is positive. The points <math>(3, m)</math> and <math>(m, 15)</math> both lie on this line. What is the value of <math>m</math>?</p> $(15 - m)/(m - 3) = m$ $15 - m = m^2 - 3m$ $m^2 - 2m - 15 = 0$ $(m - 5)(m + 3) = 0$ $m = 5, \text{ or } m = -3$
	<b>3</b>	<b>16 [= x]</b>	<p>Solve the following equation for <math>x</math>:</p> $\log_2(\log_2 x) = 2$ $2^{\log_2(\log_2 x)} = 2^2$ $\log_2 x = 4$ $2^{\log_2 x} = 2^4$ $x = 16$
	<b>4</b>	<b>4 12 [distinct sums]</b>	<p>All non-empty subsets of the following set are selected. How many different sums do the elements of each of these subsets add to?</p> <p>{1, 3, 4, 6}</p> <p>Select 1: 1, 3, 4, 6</p> <p>Select 2: 1+3, 1+4, 1+6, 3+4, 3+6, 4+6</p> <p>Select 3: 1+3+4, 1+3+6, 1+4+6, 3+4+6</p> <p>Select 4: 1+3+4+6</p> <p>These result in 12 different sums: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14</p>

5	5	7 [=☺] [=smiley face]	<p>The digits one through nine are represented by nine different symbols in the equations below. Each symbol represents the same number throughout. If ♦ = 3, what is the value of ☺?</p> $\odot - \diamond = \circlearrowleft \quad \blacksquare + \blacktriangle = *$ $\blacktriangle \times \blacklozenge = \star \quad \circlearrowleft \div \diamond = \blacklozenge$ $\text{☺} = ?$ <p>Start by replacing ♦ with 3:</p> $\odot - 3 = \circlearrowleft \quad \blacksquare + \blacktriangle = *$ $\blacktriangle \times \blacklozenge = \star \quad \circlearrowleft \div 3 = \blacklozenge$ $\text{☺} = ?$ <p>Therefore, ☺ is either 6 or 9 (divisible by 3), but it must be 6 because of the top left equation. This also tells us that ☺ = 9, and ♦ = 2:</p> $9 - 3 = 6 \quad \blacksquare + \blacktriangle = *$ $\blacktriangle \times 2 = \star \quad 6 \div 3 = 2$ $\text{☺} = ?$ <p>The remaining digits are 1, 4, 5, 7, 8. △ must equal 4, and ★ = 8, so the only solution is:</p> $9 - 3 = 6 \quad 1 + 4 = 5$ $4 \times 2 = 8 \quad 6 \div 3 = 2$ $\text{☺} = ?$ <p>Therefore the ☺ is the last remaining digit which is 7.</p>
6	6	16 [=A + B]	<p>The United States Senate Committee on Indian Affairs has 11 total members, six of whom are Republicans, and five are Democrats (including Senator Maria Cantwell from WA). If four members are randomly chosen from the committee, the probability that two Republicans and two Democrats are selected can be written as a simplified fraction A/B. What is A + B?</p> <p>Total way to choose 4 out of 11 is <math>11C4 = 11!/(4!7!) = 330</math>.</p> <p>Number of ways to choose 2 from each group is:</p> $6C2 \times 5C2 = 15 \times 10 = 150$ $P = 150/330 = 15/33 = 5/11$ $5 + 11 = 16$

7	7	7 [= sum of x values]	If $f(x) = x^2 - x + 4$ , find the sum of all x values satisfying: $f(x - 3) = 34$ . $f(x - 3) = (x - 3)^2 - (x - 3) + 4$ $= x^2 - 6x + 9 - x + 3 + 4 = 34$ $\rightarrow x^2 - 7x - 18 = 0$ $(x - 9)(x + 2) = 0$ $x = 9, -2$ $9 - 2 = 7$																																	
8	8	121 [3-digit integers]	How many 3-digit positive integers have one digit that is equal to the mean of the other two? Case work, considering each integer 1 through 9: <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Mean</th> <th>Other 2 digits</th> <th># ways</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0, 2 or 1, 1</td> <td>5</td> </tr> <tr> <td>2</td> <td>0, 4 or 1, 3 or 2, 2</td> <td>11</td> </tr> <tr> <td>3</td> <td>0, 6 or 1, 5 or 2, 4 or 3, 3</td> <td>17</td> </tr> <tr> <td>4</td> <td>0, 8 or 1, 7 or 2, 6 or 3, 5 or 4, 4</td> <td>23</td> </tr> <tr> <td>5</td> <td>1, 9 or 2, 8 or 3, 7 or 4, 6 or 5, 5</td> <td>25</td> </tr> <tr> <td>6</td> <td>3, 9 or 4, 8 or 5, 7 or 6, 6</td> <td>19</td> </tr> <tr> <td>7</td> <td>5, 9 or 6, 8 or 7, 7</td> <td>13</td> </tr> <tr> <td>8</td> <td>7, 9 or 8, 8</td> <td>7</td> </tr> <tr> <td>9</td> <td>9, 9</td> <td>1</td> </tr> <tr> <td colspan="2" style="text-align: right;">Total =</td><td style="text-align: right;">121</td> </tr> </tbody> </table> If there are 3 different non-zero digits, there are $3! = 6$ different ways to make the number. If there are 3 different digits and one is a 0, there are 4 different ways. If the 3 digits are the same, there is 1 way.	Mean	Other 2 digits	# ways	1	0, 2 or 1, 1	5	2	0, 4 or 1, 3 or 2, 2	11	3	0, 6 or 1, 5 or 2, 4 or 3, 3	17	4	0, 8 or 1, 7 or 2, 6 or 3, 5 or 4, 4	23	5	1, 9 or 2, 8 or 3, 7 or 4, 6 or 5, 5	25	6	3, 9 or 4, 8 or 5, 7 or 6, 6	19	7	5, 9 or 6, 8 or 7, 7	13	8	7, 9 or 8, 8	7	9	9, 9	1	Total =		121
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9	9, 9	1																																		
Total =		121																																		

9

9 15 [= sum]

Distribute the digits 1 - 9 in the following 9 boxes, using each number exactly once, and placing one digit per box. Distribute the digits according to the following rules.

What is the sum of the digits in the boxes that have an asterisk in the upper corner?

1. The boxes containing the 1 and 2 and all boxes between them add up to 12.
2. The boxes containing the 2 and 3 and all boxes between them add up to 23.
3. The boxes containing the 3 and 4 and all boxes between them add up to 34.
4. The boxes containing the 4 and 5 and all boxes between them add up to 45.

*		*		*		*		*

The 4 and 5 have to go on the 2 ends according to Rule 4, because the sum of all 9 digits is 45.

The 3 will go on the end with the 5, but not next to it. Between 3 and 4 adds up to 34:  $45 - 5 = 40$ . Subtract the 6 to get to 34, so put the 6 next to 5, then the 3.

Between 2 and 3 adds up to 23. Right now we have between 4 and 3 adding up to  $45 - 11 = 34$ . Subtract off the 4,  $34 - 4 = 30$ . Need to subtract 7 to get to 23, so put the 7 next to the 4, then the 2 next to the 7.

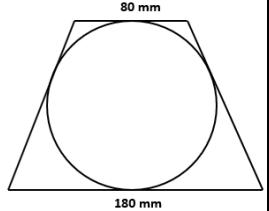
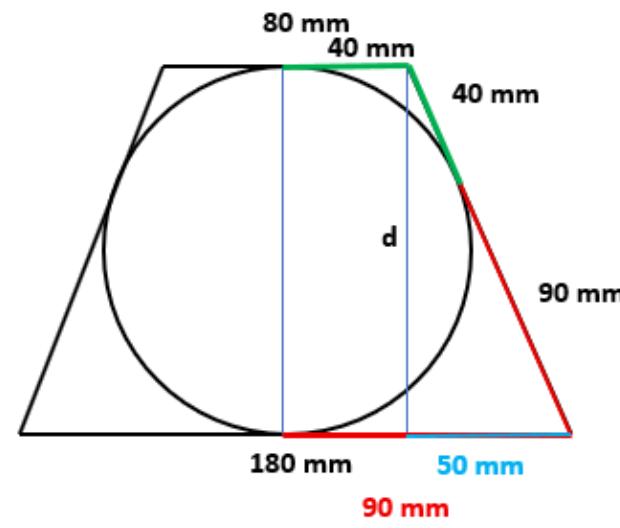
Between 1 and 2 adds up to 12, so it must be 2 9 1, then put 8 in the last box.

The sum is:  $4 + 2 + 1 + 3 + 5 = 15$ .

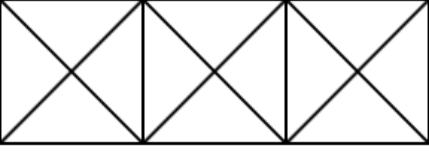
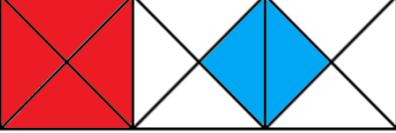
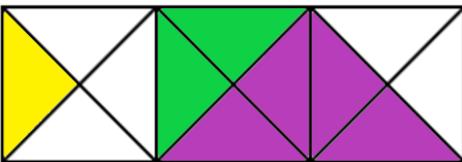
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*		*		*		*		*
4	7	2	9	1	8	3	6	5

10	10	2 [solutions]	<p>A 6-digit integer of the form <math>x2y308</math> is divisible by 33, where <math>x</math> and <math>y</math> represent single (not necessarily unique) digits. How many unique solutions are there for the number?</p> <p>If the number is divisible by 33, then it must be divisible by both 11 and 3. The divisibility rules for 11 states that the difference in the sum of the even and odd digits of the number must be either 0 or divisible by 11.</p> <p>Sum of odd: <math>x + y + 0 = x + y</math></p> <p>Sum of even: <math>2 + 3 + 8 = 13</math></p> <p>Therefore, <math>(x + y) - 13 = 0</math> or <math>-11</math> are the only possibilities.</p> <p>If <math>x + y = 13</math>, then the sum of all 6 digits = 26, which makes the number not divisible by 33.</p> <p>If <math>x + y = 2</math>, then the sum of all 6 digits = 15, which is divisible by 33.</p> <p>The possible ways to do this are:</p> <p><math>X = 1, y = 1</math>  <math>X = 2, y = 0</math></p> <p>Therefore there are 2 solutions.</p>
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**“Math is Cool” Masters -- 2025-26**  
**High School**  
Pressure Round Solutions

<b>9/ 10th</b>	<b>11/ 12th</b>	<b>Answer</b>	<b>Solution</b>
1	1	120 [mm]	<p>An isosceles trapezoid has bases measuring 80 mm and 180 mm. There is a circle inscribed in the trapezoid. In mm, what is the diameter of the circle?</p>  <p>See diagram. We know that the length of the tangent segments from the same external point will be equal. Given that, we can construct a right triangle with sides <math>d</math> (unknown), 50, and 130. Simplify the arithmetic by putting it into cm.  <math>d^2 + 5^2 = 13^2</math></p> $d = \sqrt{13^2 - 5^2} = 12 \text{ cm}$ <p>Therefore the diameter is 120 mm.</p> 
2		430 [jackrabbits]	<p>In a wildlife refuge near Burbank, WA, the jackrabbit population was 250 in the year 2020, and had grown to 340 in the year 2025. If the population is growing linearly, how many jackrabbits will there be in 2030?</p> <p><i>Rate of change</i> = <math>\frac{340-250}{2025-2020} = 18</math> rabbits/year.  <math>2025 \rightarrow 2030 = 5</math> years.  <math>340 + 5(18) = 430</math></p>

	2	10 [bags]	<p>Biff and Eho want to fill their raised garden bed with soil. The garden is 4 feet long, 5 feet 6 inches wide, and 9 inches deep. The soil can be purchased in full bags of 1.75 cubic feet of soil. How many total bags will they need to purchase?</p> <p><math>\text{Vol} = (4)(11/2)(3/4) = 33/2 \text{ ft}^3</math>  <math>1 \text{ bag} = 7/4 \text{ ft}^3</math>  <math>\text{Need: } (33/2)/(7/4) = 66/7 \text{ bags}</math>  <math>66/7 = 9.\text{something}, 9 \text{ bags is not enough, need 10.}</math></p>																																																																																																				
3	3	5 [= sum]	<p>Ten consecutive integers, each greater than 6 billion, are each raised to the 5<sup>th</sup> power. These powers are added together. What is the units digit of the sum? The 10 consecutive integers will have units digits 0, 1, 2, ..., 9, although not necessarily in that order.</p> <p>Consider what each of these digits raised to the 5 ends in (OR, just use the cyclic nature for each number raised to powers):</p> <p><math>0^5 = 0</math>  <math>1^5 = 1</math>  <math>2^5 = 32</math>  <math>3^5 = 243</math>  <math>4^5 = 2^{10} = 1024</math>  <math>5^5 = \text{ends in 5 (all powers do)}</math>  <math>6^5 = 7776</math>  <math>7^5 = 16807</math>  <math>8^5 = 32768</math>  <math>9^5 = 59049</math></p> <p>They end in the digits 1, 2, ..., 9. Therefore, the sum will end in the units digit of the sum of <math>1 + 2 + \dots + 9 = (9)(10)/2 = 45</math>, units digit = 5.</p>																																																																																																				
4		32 [= smallest integer]	<p>Counting up from 1, the sequence 8, 9, 10 is the first occurrence of three positive consecutive integers that are composite. What is the smallest integer in the second occurrence of five positive consecutive integers that are composite?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </tbody> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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	<b>4</b>	<b>48 [= smallest integer]</b>	<p>Counting up from 1, the sequence 8, 9, 10 is the first occurrence of three positive consecutive integers that are composite. What is the smallest integer in the third occurrence of five positive consecutive integers that are composite?</p> <table border="1"> <tbody> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </tbody> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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	<b>5</b>	<b>33 [total squares and triangles]</b>	<p>The figure shown here includes three unit squares and their diagonals. How many total squares and triangles of any size are included in the figure?</p>  <p>5 squares total, 3 red and 2 blue:</p>  <p>28 triangles total: 12 small, 12 medium, and 4 large.</p>  <p><math>28 + 5 = 33</math></p>																																																																																																				

# "Math is Cool" Masters -- 2025-26

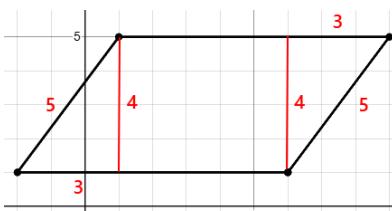
## High School

### College Bowl Round #1 Solutions

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	10 [= area of Q]	Rectangles Q and R are similar. Q has a length of five and R has a length of four. If the area of rectangle R is six point four, what is the area of rectangle Q? Scale factor = Q side/R side = 5/4 Area scale factor = linear scale factor squared = $(5/4)^2 = 25/16$ . Q area = R area x 25/16 = $6.4(25/16) = (0.4)(25) = 10$
<b>2</b>	121	What is the largest perfect square less than five hundred that does not contain any digits greater than three. $\begin{array}{ll} 1^2 = 1 & 11^2 = 121 \\ 2^2 = 4 & 12^2 = 144 \\ 3^2 = 9 & 13^2 = 169 \\ 4^2 = 16 & 14^2 = 196 \\ 5^2 = 25 & 15^2 = 225 \\ 6^2 = 36 & 16^2 = 256 \\ 7^2 = 49 & 17^2 = 289 \\ 8^2 = 64 & 18^2 = 324 \\ 9^2 = 81 & 19^2 = 361 \\ 10^2 = 100 & \end{array}$
<b>3</b>	20 [socks]	Ryan's sock drawer contains eighteen white socks and eighteen red socks. What is the smallest number of socks that can be randomly selected to guarantee a matching pair of each color? Worst case scenario: grab all socks (18) of one color first. Then need 2 more of the other color to have a pair of each color.
<b>4</b>	7 [years]	Two years ago Biff was three times as old as Eho. In three years, Biff will be twice as old as Eho. How many years old is Eho now? (1) $(B - 2) = 3(E - 2)$ $B - 3E = -4$ (2) $(B + 3) = 2(E + 3)$ $B - 2E = 3$ (1)-(2): $-E = -7$ , $E = 7$

5	21 [= A + B]	<p>A fair coin is tossed four times. The probability that there are more heads than tails can be written as the simplified fraction A over B. What is A + B?</p> <p>There are <math>2^4 = 16</math> total outcomes.</p> <p>We are looking for either 3 or 4 H (more than T).</p> <p>There is only 1 way to get 4 H.</p> <p>There are 4 ways to get 3 H: HHHT, HHTH, HTHH, THHH.</p> <p><math>P(3 \text{ or } 4) = 5/16</math></p>
6	10 [points]	<p>How many points in the coordinate plane with positive integer coordinates x and y satisfy the following inequality: x plus y is less than or equal to five</p>
7	12 [integers]	<p>How many five digit positive integers are there which only use the digits one, two and three, and in which consecutive digits always have a difference of one?</p> <p>If the number starts with 1:</p> <p>The second digit has to be 2, the third digit can be 1 or 3, the fourth digit must be 2, and the fifth digit can be 1 or 3. The number of ways to choose is: <math>1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 = 4</math>. The result will be the same if the number starts with 3. If the number starts with 2: The second digit can be 1 or 3, the third digit must be 2, the fourth digit can be 1 or 3, the 5<sup>th</sup> digit must be 2. The number of ways to choose is: <math>1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4</math>. The total number is <math>4+4+4 = 12</math>.</p>
8	2 [x intercepts]	<p>How many x-intercepts does the following equation have?</p> <p>Y equals two x to the fourth minus eight x cubed plus eight x squared</p> $\begin{aligned} 2x^4 - 8x^3 + 8x^2 \\ = 2x^2(x^2 - 4x + 4) \\ = 2x^2(x - 2)(x - 2) \end{aligned}$ <p>x-intercepts at <math>x = 0, x = 2</math></p>
9	3 [= d] [= common difference]	<p>The second term of an arithmetic (air-i-th-MET-ic) sequence is negative one, and the tenth term is twenty-three. What is the common difference for the sequence?</p> <p>To get from 2<sup>nd</sup> to 10<sup>th</sup> term, make 8 jumps. <math>23 - (-1) = 24</math>. <math>24/8 = 3</math>.</p>
10	130	<p>What is sixty divided by one-half plus ten?</p> $60/(1/2) + 10 = 120 + 10 = 130$

**“Math is Cool” Masters -- 2025-26**  
**High School**  
**College Bowl Round #2 Solutions**

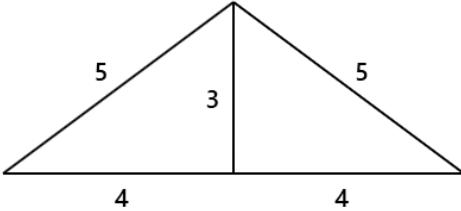
	<b>Answer</b>	<b>Solution</b>
<b>1</b>	18 [= median]	<p>Find the median of the following data set:</p> <p>Forty-eight, sixteen, eight, six, thirty-seven, twenty      6, 8, 16, 20, 37, 48</p> <p>The median is half-way between 16 and 20, which is 18.</p>
<b>2</b>	26 [units] [= perimeter]	<p>The following points form a parallelogram on the coordinate plane. In units, what is the perimeter of the parallelogram?</p> <p>Six comma one, nine comma five, one comma five, negative two comma 1</p> <p>A 3-4-5 triangle is formed on each end. The perimeter is: <math>8 + 8 + 5 + 5 = 26</math>.</p> 
<b>3</b>	11 [= largest]	<p>Three numbers, when added two at a time, give the sums of ten, fourteen and eighteen. What is the largest of the three numbers?</p> <p><math>A + b = 10</math>  <math>A + c = 14</math>  <math>B + c = 18</math></p> <p>Solve for: <math>a = 3, b = 7, c = 11</math></p>
<b>4</b>	10001 [base 2]	<p>What is the base ten number seventeen in base two?</p> <p><math>17 = 2^4 + 1 = 10001</math></p>
<b>5</b>	9	<p>A spinner is split into six equal sections, with five of them numbered two, four, six, seven and fifteen. What is the smallest positive integer greater than one that could be added to the sixth section that will make it more likely for a random spin to land on an odd number rather than a prime number?</p> <p>Currently have 2, 4, 6, 7, 15.</p> <p>Two are prime (2, 7).</p> <p>Two are odd (7, 15).</p> <p>Want to add the smallest odd number &gt; 1 that is not prime, which would be 9.</p>

<b>6</b>	10 [= x]	If $x$ factorial divided by six factorial equals seven factorial, what is the value of $x$ ? $\frac{x!}{6!} = 7!$ $x! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!$ By rearranging the factors, this becomes: $x! = 10 \cdot 9 \cdot 8 \cdot 7! = 10!$ $x = 10$
<b>7</b>	24 [eggs]	A hen and a half can lay an egg and a half in a day and a half. At this rate, how many eggs will six hens lay in six days? $\frac{3}{2} \text{ H: } \frac{3}{2} \text{ E: } \frac{3}{2} \text{ D} \rightarrow \frac{9}{4} \text{ H-D per } \frac{3}{2} \text{ E, or } \frac{3}{2} \text{ H-D per 1 E.}$ $36 \text{ H-D / } \frac{3}{2} \text{ H-D/E} = 36(\frac{2}{3}) = 24 \text{ E}$
<b>8</b>	48 [= product]	The squares of two positive integers sum to one hundred. What is the product of the two integers? $6^2 + 8^2 = 100$ $6 \times 8 = 48$
<b>9</b>	46 [students]	Yanett is ranked ninth from the top and thirty-eighth from the bottom in her class. How many students are in the class? 8 are above her, 37 are below her. $8 + 37 + 1 = 46$ .
<b>10</b>	9 [years]	There are eight kids in the Adler family, each born two years apart. If the oldest Adler child is twenty-three, how many years old is the youngest Adler child? 23, 21, 19, 17, 15, 13, 11, 9

# "Math is Cool" Masters -- 2025-26

## High School

### College Bowl Round #3 Solutions

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	8 [m&ms]	<p>Three friends eat a share size package of m&amp;ms which contains one hundred twenty candies. Felipe eats half of the candies, Oksana eats one-third and Buffy eats one-tenth. How many m&amp;ms are left?</p> <p>F: <math>\frac{1}{2}(120) = 60</math>      O: <math>1/3(120) = 40</math>      B: <math>1/10(120) = 12</math>  <math>120 - 60 - 40 - 12 = 8</math></p>
<b>2</b>	12 [sq ft]	<p>What is the area in square feet of a triangle with side lengths of five feet, five feet and eight feet?</p> <p>If a perpendicular is dropped from the vertex of this isosceles triangle, it splits it into two 3-4-5 triangles. Area = <math>\frac{1}{2} bh = \frac{1}{2}(8)(3) = 12</math>.</p> 
<b>3</b>	50 [%]	<p>An eight-sided die, numbered one through eight, is rolled. What is the probability as a percent that the number showing is a prime number?</p> <p>2, 3, 5, 7 are prime.  <math>4/8 = 50\%</math></p>
<b>4</b>	6 [sides]	<p>A sequence of shapes starts as follows, and then repeats itself indefinitely. How many sides does the two thousand twenty-fifth shape have?</p> <p>Triangle, pentagon, hexagon, square, octagon, trapezoid, and so on.      Triangle, pentagon, hexagon, square, octagon, trapezoid, ...      3, 5, 6, 4, 8, 4, repeating in a cycle of 6 shapes.  <math>2025/6 = 337 \text{ r } 3</math>      Starting from the beginning of the cycle, the 3<sup>rd</sup> element is a hexagon which has 6 sides.</p>

<b>5</b>	150 [minutes]	Ben takes two hours to wash five hundred dishes, and Jerry takes three hours to wash four hundred fifty dishes. How many minutes will it take them working together to wash one thousand dishes?  B: $500 \text{ d} / 2 \text{ h} = 250 \text{ d/h}$ J: $450 \text{ d} / 3 \text{ h} = 150 \text{ d/h}$ Together: $400 \text{ d/h}$ $1000 \text{ d} / 400 \text{ d/h} = 2.5 \text{ h}$ $2.5 \times 60 \text{ minutes/hour} = 150 \text{ minutes}$
<b>6</b>	0	Which digit, from zero to nine, appears least frequently when the integers from one to one thousand inclusive are written out? Without actually counting them, one can reasonably conclude that 0 will occur the least frequently as it cannot ever be a leading digit.
<b>7</b>	60 [%]	A biased coin has a probability of flipping two heads in a row of zero point one six. On a single flip, what is the probability as a percentage of flipping a tail? $P(H\&H) = P(H)xP(H) = 0.16$ Therefore, $P(H) = 0.4$ . $P(T) = 1 - P(H) = 0.6 = 60\%$ .
<b>8</b>	35	If $f$ of $x$ equals three $x$ plus 2, find the value of: $f$ of $f$ of three $f(x) = 3x + 2$ , Find $f(f(3))$ : $f(3) = 3(3) + 2 = 11$ $f(f(3)) = f(11) = 3(11) + 2 = 35$
<b>9</b>	540 [°]	What is the sum in degrees of the interior angles of a regular pentagon? $\text{Sum} = 180(n - 2) = 180(3) = 540$
<b>10</b>	-1 [= x]	Solve for $x$ in the following equation: Eight raised to the four $x$ equals sixteen raised to the quantity two $x$ minus 1 $8^{4x} = 16^{2x-1}$ $2^{3(4x)} = 2^{4(2x-1)}$ $12x = 8x - 4$ $4x = -4, x = -1$

# “Math is Cool” Masters -- 2025-26

## High School

### College Bowl Round #4 Solutions

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	500 [miles]	<p>Two space objects are traveling directly towards each other, one at nine thousand miles per hour and one at twenty-one thousand miles per hour. How many miles apart are they one minute before they collide?</p> <p>The closing speed is <math>9000 + 21000 = 30000</math> miles/hour.</p> $D = RT = 30000 \text{ mi/h} * 1 \text{ minute} * 1 \text{ hr}/60 \text{ minutes} = 500 \text{ miles}$
<b>2</b>	25 [%]	<p>An integer from one to twenty inclusive is randomly selected. What is the probability as a percent that it is a perfect square or a perfect cube?</p> <p>1, 4, 8, 9, 16</p> <p>5 favorable outcomes</p> $5/20 = \frac{1}{4} = 25\%$
<b>3</b>	32 [sq inches]	<p>A rectangle has a perimeter of twenty-four inches. The length is twice the width. What is the area of the rectangle in square inches?</p> <p><math>L = 2W</math></p> <p><math>6W = 24</math>, <math>W = 4</math>, <math>L = 8</math></p> <p><math>A = 4 \times 8 = 32</math></p>
<b>4</b>	19 [digits]	<p>How many digits are in the result when two raised to the twenty-one is multiplied by five raised to the seventeen?</p> $(2^{21})(5^{17})$ <p>Start with: <math>(2^{17})(5^{17}) = 10^{17}</math></p> <p>We have <math>2^4 = 16</math> leftover</p> <p><math>16 \times 10^{17} = 1.6 \times 10^{18}</math> which will have 19 digits when written out.</p>
<b>5</b>	125 [%]	<p>The area of circle P is sixty-four percent of the area of circle Q. The radius of circle Q is what percentage of the radius of circle P?</p> <p><math>\text{Area } P = \pi r_P^2</math></p> <p><math>\text{Area } Q = \pi r_Q^2</math></p> <p><math>\pi r_P^2 = 0.64 \pi r_Q^2</math></p> <p><math>r_Q^2 / r_P^2 = 1/0.64 = 100/64</math></p> <p><math>r_Q / r_P = 10/8 = 1.25 = 125\%</math></p>

<b>6</b>	7 [different lengths]	<p>Composite deck planks are available in six, eight or ten foot lengths. Placed end to end, how many different lengths can be achieved using exactly three planks?</p> <p>The distinct values that can be achieved are:</p> $\begin{aligned} 6 \ 6 \ 6 &= 18 \\ 6 \ 6 \ 8 &= 20 \\ 6 \ 6 \ 10 &= 22 \\ 6 \ 8 \ 10 &= 24 \\ 6 \ 10 \ 10 &= 26 \\ 8 \ 10 \ 10 &= 28 \\ 10 \ 10 \ 10 &= 30 \end{aligned}$
<b>7</b>	30 [= fifth term]	<p>The first term in a sequence is thirty-four. Each subsequent term is generated by adding eighteen to the product of the digits in the previous term. What is the fifth term?</p> $\begin{aligned} 34 \\ 3*4 + 18 &= 12 + 18 = 30 \\ 3*0 + 18 &= 18 \\ 1*8 + 18 &= 26 \\ 2*6 + 18 &= 30 \end{aligned}$
<b>8</b>	1260 [= LCM]	<p>What is the least common multiple of twenty, thirty-six and forty-two?</p> $\begin{aligned} 20 &= 2^2 \cdot 5 \\ 36 &= 2^2 \cdot 3^2 \\ 42 &= 2 \cdot 3 \cdot 7 \\ \text{LCM} &= 4 \cdot 5 \cdot 7 \cdot 9 = 1260 \end{aligned}$
<b>9</b>	85 [\$]	<p>Brent bought a calculator and a backpack for one hundred and five dollars. The backpack cost sixty-five dollars more than the calculator. How many dollars did the backpack cost?</p> $\begin{aligned} B + (B - 65) &= 105 \\ 2B &= 170, B = 85 \end{aligned}$
<b>10</b>	4 [candles]	<p>Mary lights a candle every ten minutes. Each candle burns for exactly forty minutes. How many candles are burning exactly fifty-five minutes after Mary lit the first candle?</p> <p>The ones that she lit at time = 0 and 10 minutes have burned out. The ones lit at time = 20, 30, 40 and 50 are still burning.</p>

# "Math is Cool" Masters -- 2025-26

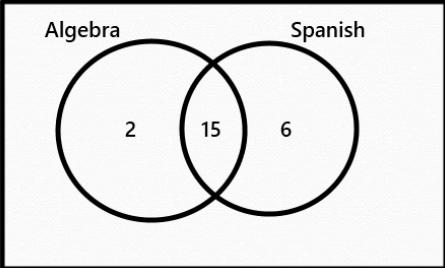
## High School

### College Bowl Round #5 Solutions

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	4091	What is one thousand plus forty plus one thousand plus thirty plus one thousand plus twenty plus one thousand plus one? $1000 + 40 + 1000 + 30 + 1000 + 20 + 1000 + 1 = 4091$
<b>2</b>	56 [cm]	Cookie added the lengths of three sides of a rectangle and got forty-four centimeters. Ron also added the lengths of three sides of the same rectangle and got forty centimeters. In centimeters, what is the perimeter of the rectangle? $2x + y = 40$ $x + 2y = 44$ Adding: $3x + 3y = 84$ $x + y = 28$ $2(x + y) = 2(28) = 56$
<b>3</b>	35 [= degree]	What is the degree of the following polynomial: In parentheses (pause) $x$ to the fifth plus $x$ squared end parentheses (pause) raised to the seventh $(x^5 + x^2)^7$ When multiplied out, the highest degree will be $(x^5)^7 = x^{35}$
<b>4</b>	3 [= 3 <sup>rd</sup> term]	The first term in a geometric sequence is three-fourths, and the fourth term is six. What is the third term? Make 3 jumps from $\frac{3}{4}$ to 6, or multiply by 'r' 3 times. $\frac{3}{4}(r^3) = 6$ $r^3 = 6/(3/4) = 6(4/3) = 8$ $r = 2$ Therefore the term before 6 is $6/2 = 3$ .
<b>5</b>	36 [integers]	How many different integers between one and one hundred contain the digits two or eight or both? For each, 2 or 8, there are 19 numbers containing the digit, for example: 2, 12, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 42, 52, 62, 72, 82, 92. $19 + 19 = 38$ But 28 and 82 have been counted twice each, so subtract off 2. $38 - 2 = 36$ .
<b>6</b>	194 [= product]	If the sum of two prime numbers is ninety-nine, what is the product of the two prime numbers? If $p + q = 99$ (odd), then either $p$ or $q$ must be even, so it must be 2, the only even prime number. Therefore the two numbers are 2 and 97, and $2 \times 97 = 194$ .

<b>7</b>	75 [%]	Two whole numbers are chosen at random. As a percent, what is the probability that their product is even? The probability of choosing even or odd is $\frac{1}{2}$ . The possible outcomes are: (even, even), (even, odd), (odd, even), or (odd, odd). Only (odd, odd) will give an odd product. $P(\text{even}) = \frac{3}{4} = 75\%$ .
<b>8</b>	582 [is the number]	A three digit whole number has three distinct digits. The tens digit is four times the units digit, and the hundreds digit is three less than the tens digit. What is the number? HTU $T = 4U$ , so $U = 1$ , $T = 4$ or $U = 2$ , $T = 8$ $H = T - 3$ , = 1 or 5 Number is 141 or 582, but only 582 has 3 distinct digits.
<b>9</b>	250 [\$]	One thousand seven hundred fifty dollars will be split between two people in the ratio of three to four. How many more dollars does one person get than the other? Ratio is 3:4, so 7 parts total. $1750/7 = 250$ , so each “part” equals \$250. Therefore, one person gets \$250 more.
<b>10</b>	120 [seconds]	A three hundred foot train going three hundred feet per minute travels through a tunnel that is three hundred feet long. How many seconds will it take the entire train to travel through the tunnel? The total distance from the front of the train entering the tunnel to the end of the train exiting is 600 feet. $600 \text{ ft}/300 \text{ ft/min} = 2 \text{ min} = 120 \text{ sec}$ .

**“Math is Cool” Masters -- 2025-26**  
**High School**  
**College Bowl Round #6 Solutions**

	<b>Answer</b>	<b>Solution</b>
<b>1</b>	100 [gumballs]	Gumballs cost twelve cents per dozen. How many gumballs can you get for one dollar? $12 \text{ cents}/12 \text{ gumballs} = 1 \text{ cent per gumball}$ . Times 100 cents = 100 gumballs.
<b>2</b>	12000 {\$}	Zach put half of his money into the stock market, half of that amount into bonds, and one-sixth of his money into his savings account. He kept one thousand dollars in cash. How many dollars does Zach have in total? $1/2M + 1/4M + 1/6M + 1000 = M$ $6M + 3M + 2M + 12000 = 12M$ $12000 = M$
<b>3</b>	20 [cm]	The two legs of a right triangle measure twelve centimeters and sixteen centimeters. In centimeters, what is the measure of the hypotenuse? $12^2 + 16^2 = 144 + 256 = 400$ Square root of 400 = 20.
<b>4</b>	92 [%]	In Ana's English class of twenty-five students, seventeen of them take Algebra and twenty-one of them take Spanish. Fifteen students take both Algebra and Spanish. If one student is chosen randomly, what is the probability in percent that they take Algebra or Spanish? Out of the 25, 23 are accounted for in Algebra or Spanish. $23/25 = 92/100 = 92\%$  Total = 25 

<b>5</b>	2 [= 8 times area]	The product of the circumference and the diameter of a circle is one. What is eight times the area of the circle? $CD = 1$ $(2\pi r)(2r) = 1$ $r^2 = 1/(4\pi)$ $A = \pi r^2 = \pi/(4\pi) = \frac{1}{4}$ $8A = 8(1/4) = 2$
<b>6</b>	-32 [= sum]	A sequence of numbers begins with one, negative one, negative one. From the third term on, each subsequent term is equal to the product of the two preceding terms. What is the sum of the first one hundred terms? The sequence will begin: 1, -1, -1, 1, -1, -1, ..., with the pattern repeating every 3 terms. The sum of each "cycle" of 1, -1, -1 equals -1. $100/3 = 33\text{r}1$ . $33 \times (-1) = -33$ . But the last term will be +1, so the sum is -32.
<b>7</b>	217 [integers]	How many positive three digit integers can be written using no zeros and at least one five? Three 5s: 555, 1 way Two 5s: 55X, 5X5, X55, 8 ways each = 24 One 5: 5XX, X5X, XX5, 8x8 = 64 ways each = 192 $192 + 24 + 1 = 217$
<b>8</b>	113 [= mean=]	What is the mean of the following data set: Sixty-six, One hundred and seven, eighty-four, One hundred and eighty, One hundred and twenty-eight $66 + 107 + 84 + 180 + 128 = 565$ $565/5 = 113$
<b>9</b>	172,800 [seconds]	How many seconds are in two days? 2 days = 48 hours 48 hours x 3600 seconds/hour = 172,800
<b>10</b>	8	The digits one, three, four and eight are each used once to form the smallest possible odd four-digit integer. What digit is in the tens place? 1, 3, 4, 8 1 must go first, so 3 is last, therefore: 1483

# "Math is Cool" Masters -- 2025-26

## High School

### College Bowl Extra Questions Solutions

	<b>Answer</b>	<b>Solution</b>						
<b>1</b>	41 [runners]	<p>In the finishing order of a race, there are twice as many runners behind Charles as there are before David, and one and a half times as many behind David as before Charles. Charles finished in twenty-first place. How many runners finished the race?</p> <p>1<sup>st</sup> place ----- last</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>20</td> <td>Charles (21<sup>st</sup>)</td> <td>2x</td> </tr> <tr> <td>X</td> <td>David</td> <td>30</td> </tr> </table> $21 + 2x = 31 + x$ $X = 10$ $\text{Total} = 10 + 1 + 30 = 41$	20	Charles (21 <sup>st</sup> )	2x	X	David	30
20	Charles (21 <sup>st</sup> )	2x						
X	David	30						
<b>2</b>	135 [= the sum]	<p>If the mean of five numbers is twenty-seven, what is the sum of the five numbers?</p> $27 \times 5 = 135$						
<b>3</b>	6 [= x]	<p>What is the smallest positive integer value of <math>x</math> for which fifty-four times <math>x</math> is a perfect square?</p> $54 = 2 \times 3 \times 3^2$ <p>Therefore, need additional factors of 2 and 3 to get a perfect square.</p>						
<b>4</b>	363 [ $\text{cm}^2$ ]	<p>Using a value of three to approximate pi, what is the surface area in square centimeters of a sphere with a diameter of eleven centimeters?</p> $\text{SA} = 4\pi r^2 = 4(3)(11/2)^2 = 3 \times 121 = 363$						
<b>5</b>	-12 [= cube root]	<p>What is the cube root of negative one thousand seven hundred twenty-eight?</p> $(-12)(-12)(-12) = -1728$						
<b>6</b>	85 [cents]	<p>A raffle has two hundred free tickets. One ticket will win a one-hundred seventy dollar prize. The remaining tickets will win nothing. If you have one ticket, what is your expected winnings in cents?</p> <p>The probability of winning is <math>1/200</math>. Since there is no cost or penalty to not winning, the expected value is <math>(1/200)(170) = 0.85 = 85</math> cents.</p>						
<b>7</b>	211 [base 10]	<p>The hexadecimal number D three is equal to what base ten number?</p> <p>D3 means <math>13 \times 16 + 3 \times 1 = 211</math></p>						

<b>8</b>	8 [%]	Biff buys a sandwich that costs twelve dollars and twenty-five cents, and pays thirteen dollars and twenty-three cents total with the tax. As a percentage, what was the tax rate? $\$13.23 - \$12.25 = 0.98$ $0.98/12.25 = 0.08 = 8\%$
<b>9</b>	2209 [sq cm]	What is the area in square centimeters of a square with a perimeter of one hundred eighty-eight centimeters? $188/4 = 47$ cm side length $47^2 = 2209$ sq cm area
<b>10</b>	-8 [= f inverse of 3]	The function $f$ of $x$ equals the quantity two $x$ minus five divided by the quantity $x$ plus one. What is $f$ inverse of three? $f(x) = (2x-5)/(x+1) = 3$ Solve for $x = -8$ If $f(-8) = 3$ , then $f^{-1}(3) = -8$