Kuwa kabiri, 15. Nyakanga 2025

Ikibazo 1 Umurongo uri muri plane witwa *sunny* niba **utaringaniye** n' umwe muri iyi: x-axis, y-axis, cyangwa $x + y = 0$.

Reka $n \geqslant 3$ ibe integer itanzwe. Shaka integers zose k zitari nto kuri zeru kuburyo hari imirongo n itandukanye iri muri plane yujuje ibi byombi:

- Kuri integers ziruta zeru zose a na b zifite ko $a + b \leqslant n + 1$, point (a, b) iri nibura ku murongo umwe muri iyo mirongo; kandi
- imirongo k neza neza mu mirongo n iri sunny.

Ikibazo 2 Reka Ω na Γ bibe ibiziga bifite centres M and N, hakurikijwe uko bikurikirana, kuburyo radius ya Ω irutwa na radius ya Γ. Tuvuge ko ibiziga Ω na Γ bihurira kuri points ebyiri zitandukanye A na B. Umurongo MN uhura na Ω kuri C, ugahura na Γ kuri D, kuburyo points C, M, N na D ziri ku murongo zikurikiranye gutyo. Reka P ibe centre y'uruziga rwa triangle ACD. Umurongo AP uhura na Ω nanone kuri $E \neq A$. Umurongo AP uhura na Γ nanone kuri $F \neq A$. Reka H ibe orthocentre ya triangle PMN.

Erekana ko umurongo unyura muri H uringaniye na AP uri tangent k' uruziga rwa triangle BEF.

(The *orthocentre* of a triangle is the point of intersection of its altitudes.)

Ikibazo 3 Reka $\mathbb{N}$ yerekane set ya integers ziruta zeru. A function $f : \mathbb{N} \to \mathbb{N}$ iba iri *bonza* niba

$$f(a) \text{ igabanya } b^a - f(b)^{f(a)}$$

kuri positive integers zose a and b.

Shaka real constant c ntoya cyane kuburyo $f(n) \leqslant cn$ kuri bonza functions f zose na integers n zose ziruta zeru.

Kuwa gatatu, 16. Nyakanga 2025

Ikibazo 4 *Proper divisor* ya integer N iruta zeru ni umubare uruta zeru ugabanya N utari N ubwayo.

Urutonde rutarangira $a_1, a_2, \ldots$ rugizwe na integers ziruta zeru, buri yose ifite proper divisors nibura eshatu. Kuri buri $n \geqslant 1$, integer a_{n+1} ni igiteranyo cya proper divisors eshatu nini ziruta izindi za a_n.

Shaka a_1 zose zishoboka.

Ikibazo 5 Alice na Bazza bakina umukino witwa *inekoalaty*, umukino w' abantu babiri aho amategeko yawo agengwa n' umubare uruta zeru λ uba uzwi n' abakinnyi bombi. Ku nshuro ya n y' umukino ibikurikira biraba:

- Niba n iri odd, Alice ahitamo umubare utari muto kuri zeru x_n kuburyo

$$x_1 + x_2 + \cdots + x_n \leqslant \lambda n.$$

- Niba n iri even, Bazza ahitamo umubare utari muto kuri zeru x_n kuburyo

$$x_1^2 + x_2^2 + \cdots + x_n^2 \leqslant n.$$

Niba umukinnyi umwe adashobora guhitamo umubare ukwiriye x_n, umukino urarangira, undi mukinnyi agatsinda. Niba umukino ukomeza iteka, nta mukinnyi utsinda. Imibare yose bahisemo iba izwi n' abakinnyi bombi.

Shaka imibare yose λ kuburyo Alice afite uburyo bwo gutsinda ndetse n'imibare yose λ kuburyo Bazza afite uburyo bwo gutsinda.

Ikibazo 6 Fata 2025×2025 grid ya unit squares. Matilda ashaka gushyira rectangular tiles zimwe na zimwe kuri grid, birashoboka ko tiles zitangana, kuburyo buri ruhande rwa buri tile ruri ku murongo wa grid kandi buri unit square iriho tiles zitarenze imwe.

Shaka umubare wanyuma muto wa tiles Matilda akeneye gushyiraho kuburyo buri row na buri column ya grid ifite unit square imwe gusa itarimo tile iyo ari yo yose.