Kuwa gatandatu, 8. Nyakanga 2023

Ikibazo cya 1 Shaka ama composite integers $n > 1$ yose yujuje ibi bikurikira: Niba $d_1, d_2, \ldots, d_k$ ari ama positive divisors yose ya n, $1 = d_1 < d_2 < \cdots < d_k = n$, d_i izagabanya $d_{i+1} + d_{i+2}$ kuri buri $1 \leqslant i \leqslant k-2$.

Ikibazo cya 2 Reka ABC ibe triangle ifite buri interior angle irutwa na $90°$, Kandi $AB < AC$. Reka Ω ibe circumcircle ya ABC. Reka S ibe midpoint ya arc CB kuri Ω iriho akadomo A. Umurongo uri perpendicular kuri BC uturuka muri A uhura na BS kuri D ukanahura na Ω bwa kabiri kuri $E \neq A$. Umurongo uca muri D uri parallel na BC uhura n'umurongo BE kuri L. Reka circumcircle ya triangle BDL yitwe ω. Reka ω ihure na Ω bwakabiri kuri $P \neq B$.
Erekanako umurongo uri tangent na ω kuri P uhura n'umurongo BS kuri internal angle bisector ya $\angle BAC$.

Ikibazo cya 3 Kuri buri Integer $k \geqslant 2$, shaka ama infinite sequence yose y'ama positive integers $a_1, a_2, \ldots$ kuburyo hari polynomial P iteye itrya $P(x) = x^k + c_{k-1}x^{k-1} + \cdots + c_1 x + c_0$, mugihe $c_0, c_1, \ldots, c_{k-1}$ ari ama non-negative integers, hanyuma

$$P(a_n) = a_{n+1}a_{n+2}\cdots a_{n+k}$$

kuri buri integer $n \geqslant 1$.

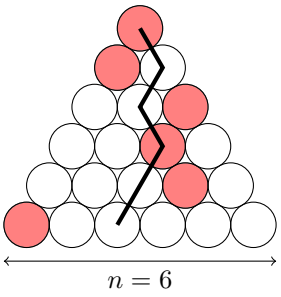
Ku cyumweru, 9. Nyakanga 2023

Ikibazo cya 4 reka $x_1, x_2, \ldots, x_{2023}$ zibe positive real numbers kuburyo nta n'umwe usa n'undi hagati yazo. Hanyuma

$$a_n = \sqrt{(x_1 + x_2 + \cdots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}\right)}$$

ikaba integer kuri buri $n = 1, 2, \ldots, 2023$. Erekanako $a_{2023} \geqslant 3034$.

Ikibazo cya 5 Reka n ibe positive integer. *Japanese triangle* igira utuziga $1+2+\cdots+n$ dutondetse dukoze equilateral triangular shape kuburyo kuri buri $i = 1, 2, \ldots, n$, buri i^{th} row izagira utuziga i , akaziga kamwe konyine gasa umutuku muri iyo row. Inzira yitwa *ninja path* muri Japanese triangle ni sequence y'utuziga n ubona uturutse kuri row yo hejuru, hanyuma ukagenda umanuka uva kukaziga ukajya kukaziga kamwe muri tubiri turi munsi y'ako uvuyeho, ukarangiriza urugendo kuri row yo hasi. Uru ni urugero rwa Japanese triangle ifite $n = 6$, harimo ninja path muri iyo triangle ifite utuziga dusa umutuku tubiri gusa.



in terms of n, shaka k nini ishoboka kuburyo muri buri Japanese triangle hari ninja path ifite nibura utuziga k dusa umutuku.

Ikibazo cya 6 Reka ABC ibe equilateral triangle. Reka A_1, B_1, C_1 tube utudomo turi mwimbere muri ABC kuburyo $BA_1 = A_1C$, $CB_1 = B_1A$, $AC_1 = C_1B$, kandi

$$\angle BA_1C + \angle CB_1A + \angle AC_1B = 480^\circ.$$

reka BC_1 na CB_1 zihurire kuri A_2, reka CA_1 na AC_1 zihurire kuri B_2, reka AB_1 na BA_1 zihurire kuri C_2.
Erekanako niba triangle $A_1B_1C_1$ nta mpande ebyiri ifte zingana, bivuzeko circumcircles eshatu za triangles AA_1A_2, BB_1B_2 na CC_1C_2 zose hari utudomo tubiri zicamo .

Language: Kinyarwanda

Igihe ntarengwa: amasaha 4 iminota 30.
Ikibazo cyose gifite amanota 7.