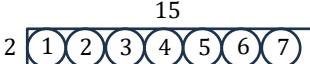


“Math is Cool” Championships -- 2024-25

6th Grade

Mental Math Solutions

	Answer	Solution
1	13	What is the mean of the set of integers from eleven to fifteen inclusive? $(11 + 12 + 13 + 14 + 15)/5 = 13$
2	[\$] 1750 [dollars]	Phoebe and Monica are going to evenly split the cost of a three-thousand-five-hundred dollar car. How many dollars will Phoebe pay for her share of the car? $3500/2 = 1750$
3	7 [circles]	What is the maximum number of non-overlapping circles of radius one centimeter that can fit inside a two-centimeter-by-fifteen-centimeter rectangle? See figure: 
4	2 [prime numbers]	How many prime numbers are between twenty-four and thirty-two? 29, 31
5	25	What is three-fourths of twelve plus four-thirds of twelve? $\frac{3}{4} \cdot 12 + \frac{4}{3} \cdot 12 = 9 + 16 = 25$
6	495 [minutes]	Hamta left home for school at seven thirty AM, and returned home on the same day at three forty-five PM. How many minutes was Hamta away from home? 7:30 am to 3:30 pm = 8 hours x 60 minutes/hour = 480 minutes, plus an extra 15 minutes = 495 minutes
7	[N =] 8	The first three terms of a geometric sequence are one hundred twelve, fifty-six, and twenty-eight. Let term number N be the first term in the sequence that is less than one. What is N? 112, 56, 28, 14, 7, 3.5, 1.75, 0.875, and 0.875 is the 8 th term, so N = 8

8

120 [ways]

Min has four standard dice, where one is red, one is yellow, one is green, and one is blue. If she rolls them all together, in how many ways can three of the dice show the same number while the fourth die shows something different?

The number of ways to get three 1s is $4 \cdot 5 = 20$, as shown in the table.

red	yellow	green	blue
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5
1	1	1	6
1	1	2	1
1	1	3	1
1	1	4	1
1	1	5	1
1	1	6	1
1	2	1	1
1	3	1	1
1	4	1	1
1	5	1	1
1	6	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1

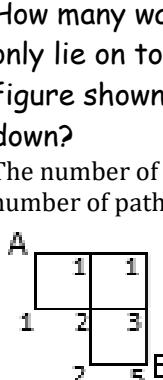
There are also 20 ways to get three of each of the other five numbers, so the final answer is $6 \cdot 20 = 120$

“Math is Cool” Championships -- 2024-25

6th Grade

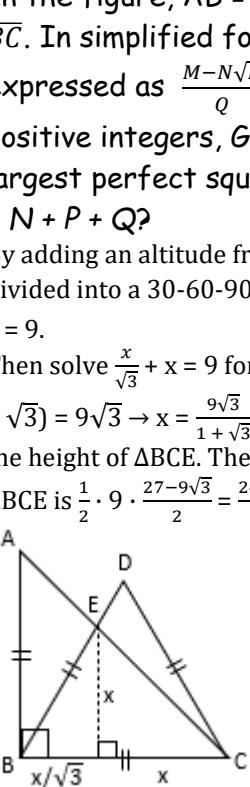
Individual Test Solutions

	Answer	Solution	
1	17 [nickels]	How many nickels make 85 cents? $85/5 = 17$	
2	90	What is the largest 2-digit multiple of 10? 2-digit multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80, and 90, and 90 is the largest.	
3	39	The mean of three numbers is 20. If two of the numbers are 8 and 13, what is the third number? $20 \cdot 3 = 60$ and $60 - 8 - 13 = 39$	
4	[A + B =] 4	Guillermo has one white hat, two blue hats, and three brown hats and he randomly selects one to wear on a given day. As a reduced common fraction, the probability that he takes a blue hat is A/B. What is A + B? $P(\text{blue}) = 2/6 = 1/3$, and $1 + 3 = 4$	
5	1010	Evaluate: $1111 - 111 + 11 - 1$ $1111 - 111 + 11 - 1 = 1010$	
6	84 [in ²]	How many square inches are in the area of a rectangle with a length of 7 inches and a width of 12 inches? $7 \cdot 12 = 84$	
7	[A + B =] 18	In the grid of squares shown below, as a reduced common fraction, the ratio of black squares to white squares is A/B. What is A + B? There are a total of 36 squares, 10 are black and 26 are white, so $10/26 = 5/13$ and $5 + 13 = 18$	
8	2300 [meters]	How many meters are in 2.3 kilometers? $2.3 \cdot 1000 = 2300$	
9	16 [marbles]	A dish has 4 green marbles, 7 clear marbles, and 8 orange marbles. If marbles are randomly taken from the dish, what is the minimum number needed to be taken that would guarantee at least one of each color? $8 + 7 + 1 = 16$	
10	[x =] 15	Solve the equation for x: $6x - 21 = 69$ $6x - 21 = 69 \rightarrow 6x = 90 \rightarrow x = 15$	
11	14 [min]	Lincoln can wash 12 plates in 3 minutes. In minutes, how long does it take him to wash 56 plates? 12 plates in 3 min is equivalent to 4 plates per minute and $56/4 = 14$	

12	40 [%]	What percent of 25 is 10? $10/25 = x/100 \rightarrow x = 40$
13	123	What is the 6 th term in the arithmetic sequence whose first three terms are 8, 31, and 54? 8, 31, 54, 77, 100, 123
14	20 [%]	A number is randomly selected from the integers 1 through 20. As a percent, what is the probability that the selected number is larger than 16? There are four numbers larger than 16, so $P(\text{larger than } 16) = 4/20 = 1/5 = 20\%$
15	5 [ways/pat hs]	How many ways are there to draw a path from A to B that must only lie on top of horizontal and/or vertical segments in the figure shown and in which your pencil can only move right or down? The number of paths from A to each vertex is shown in the figure, so the number of paths from A to B is 5. 
16	13	What is the largest prime factor of 104? $104 = 2^3 \cdot 13$ and 13 is the largest prime factor
17	[$A + B =$] 65	As a reduced common fraction, the sum of the following two numbers is A/B . What is $A + B$? $1\frac{2}{3} + 2\frac{3}{4}$ $1\frac{2}{3} + 2\frac{3}{4} = \frac{5}{3} + \frac{11}{4} = \frac{20}{12} + \frac{33}{12} = \frac{53}{12}$ and $53 + 12 = 65$
18	10	What is the median of the following set of numbers? 11, 6, 17, 9, 12, 4, 1, 25 The list in ascending order is 1, 4, 6, 9, 11, 12, 17, 25, and the median is $(9 + 11)/2 = 10$.
19	[$x =$] 7	What is the smallest integer solution to the inequality? $19 < 2x + 6$ $19 < 2x + 6 \rightarrow 13 < 2x \rightarrow 6.5 < x$, so the smallest integer value of x is 7
20	[$A =$] 25	The area of a circle with diameter 10 inches is $A\pi$ in ² . What is A? If d = 10, then r = 5, and A = 25π , so A = 25.
21	6 [ways]	How many ways are there to arrange the letters in the word CROW, if C must be the third letter? In the arrangement _ _ C _, there are $3! = 6$ ways to arrange the remaining 3 letters, so the answer is 6.
22	[$P + Q =$] 13	Let set A = {9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99}, set B = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95}, and set C = {15, 30, 45, 60, 75, 90}. As a reduced common fraction, the probability that a randomly chosen number from set A is also a member of both sets B and C is P/Q . What is $P + Q$? There are 11 members of set A and two of them, 45 and 90, are in both sets B and C, so the probability is $2/11$ and $2 + 11 = 13$.

23	8	What is the sum of the distinct prime factors of the number 2025? $2025 = 3^4 \cdot 5^2$, and $3 + 5 = 8$
24	64	The Fibonacci Sequence is 1, 1, 2, 3, 5, 8, and so on. A different increasing sequence (Let's call it the New Sequence), has 10 as its first term. The second term in the New Sequence results from adding the first term in the Fibonacci Sequence to the first term of the New Sequence. The third term in the New Sequence results from adding the second term in the Fibonacci Sequence to the second term of the New Sequence. If this pattern continues, what is the ninth term of the New Sequence? The New Sequence is 10, 11, 12, 14, 17, 22, 30, 43, 64, and 64 is the 9 th term.
25	100 [seconds]	Leon takes 125 seconds to complete one lap on the track at school. Ayanna runs at a rate that is $\frac{5}{4}$ times as fast as Leon. How many seconds does it take Ayanna to complete one lap on the same track? If you run $\frac{5}{4}$ times as fast it takes you $\frac{4}{5}$ as long, so $(\frac{4}{5})(125) = 100$
26	17 [cm]	In centimeters, what is the length of \overline{AB} in the figure? Note: All external edges of the figure are either horizontal or vertical. The rectangle that encloses the figure is 8cm by 15 cm, and since $8^2 + 15^2 = 17^2$, then $AB = 17$ cm.
27	$[A + B =]$ 46	The following expression simplifies to the reduced common fraction A/B . What is $A + B$? $\frac{10^3 - 10}{5^5 + 5^2} = \frac{10(10^2 - 1)}{5^2(5^3 + 1)} = \frac{(10 \cdot 99)}{(25 \cdot 126)} = \frac{99}{(5 \cdot 126)} = \frac{99}{(5 \cdot 63)} = \frac{11}{(5 \cdot 7)} = \frac{11}{35}$ and $11 + 35 = 46$
28	60	What is the largest possible number in a data set with 5 distinct positive integers and a mean of 14? $5 \cdot 14 = 70$, so the data set could be 1, 2, 3, 4, and 60
29	$[P + Q =]$ 81	The lengths of two parallel sides of square ABCD are multiplied by 1.4 and the lengths of the other two parallel sides are multiplied by 1.6 to create rectangle EFGH. As a reduced common fraction, the ratio of the area of rectangle EFGH to the area of square ABCD is P/Q . What is $P + Q$? Let the side length of square ABCD be 10 and the area of square ABCD be 100. The sides of rectangle EFGH are 14 and 16 and the area of rectangle EFGH is 224. $224/100 = 56/25$ and $56 + 25 = 81$
30	22 [nickels]	Nayeli has 1.5 times as many dimes as nickels and the value of her dimes and nickels is \$4.40. How many nickels does she have? $D = 1.5N$ and $10D + 5N = 440 \rightarrow 15N + 5N = 440 \rightarrow 20N = 440 \rightarrow N = 22$

31	15 [minutes]	A bathtub takes 6 minutes to fill up when the drain is closed and 10 minutes to fill up when the drain is open. In minutes, how long does it take to empty a full tub? The tub fills at a rate of $1/6$ of the tub per minute when the drain is closed and $1/10$ of the tub per minute when the drain is open. The difference between these two is the rate of the tub emptying when the drain is open, which we'll call $1/x$, so $1/6 - 1/10 = 1/x$. In this equation, x represents the time in minutes that it takes to drain the tub. $1/6 - 1/10 = 1/x \rightarrow 10x - 6x = 60 \rightarrow 4x = 60 \rightarrow x = 15$
32	[b =] 15	The base-10 number $279 = 139_b$, where $b > 0$. What is the value of b ? $279 = b^2 + 3b + 9 \rightarrow b^2 + 3b - 270 = 0 \rightarrow (b + 18)(b - 15) = 0 \rightarrow b = -18$ or $b = 15$, so the positive base is 15.
33	105 [residents]	At a summer ping pong academy for teenagers $1/7$ of the residents are left-handed and $6/7$ of the residents are right-handed. Exactly $1/3$ of the left-handed residents and $3/10$ of the right-handed residents use a red paddle. What is the smallest possible number of residents at the academy? The ratio of left-handed to right-handed residents is $1/7 : 6/7 = 1 : 6$. Possible pairings of number of (left, right) include: (1, 6), (2, 12), (3, 18), (4, 24), (5, 30), (6, 36), (7, 42), (8, 48), (9, 54), (10, 60), (11, 66), (12, 72), (13, 78), (14, 84), (15, 90), and so on. The first pairing in which taking $1/3$ of the first number and $3/10$ of the second number both result in integers is (15, 90), so the minimum possible number of residents is $15 + 90 = 105$.
34	3	What is the mean of all integers that are 6 more than 55 times their reciprocal? Let x = any integer that is 6 more than 55 times its reciprocal. Then $x = 55/x + 6 \rightarrow x^2 = 55 + 6x \rightarrow x^2 - 6x - 55 = 0 \rightarrow (x - 11)(x + 5) = 0 \rightarrow x = 11$ and $x = -5$, and $(11 + -5)/2 = 3$
35	18	A data set consists of 5 positive integers, 4 of which are distinct from each other and one that is the same as one of the 4 distinct integers. The mean and the mode of the data set are both 9. What is the positive difference between the greatest possible range and the smallest possible range of a data set that meets these conditions? Two of the numbers must be 9, so the remaining three numbers must add up to 27. It is not possible for all three remaining numbers to be larger than 9 or for all of them to be smaller than 9. If two of the remaining numbers are smaller than 9, the largest and smallest possibilities for the range are {1, 2, 9, 9, 24} and {7, 8, 9, 9, 12}. If one of the remaining numbers is larger than 9, the largest and smallest possibilities for the range are {1, 9, 9, 10, 16} and {6, 9, 9, 10, 11}. The greatest possible range is $24 - 1 = 23$ and the smallest possible range is $12 - 7 = 5$ or $11 - 6 = 5$, and $23 - 5 = 18$.
36	[A + B =] 8	Josue and Felicity take turns rolling a fair 12-sided die with faces numbered 1 through 12. The first player to roll an odd prime number is the winner of the game. If Felicity goes first, as a reduced common fraction, the probability that she wins the game is A/B . What is $A + B$? $P(\text{Felicity wins}) = 1/3 + (2/3)^2 \cdot 1/3 + (2/3)^4 \cdot 1/3 + \dots = 1/3(1 + (2/3)^2 + (2/3)^4 + \dots) = S \rightarrow 3S = 1 + (2/3)^2 + (2/3)^4 + \dots \rightarrow (2/3)^2 \cdot 3S = (2/3)^2 + (2/3)^4 + (2/3)^6 + \dots \rightarrow 4S/3 = (2/3)^2 + (2/3)^4 + (2/3)^6 + \dots$ Subtract two of the equations: $3S = 1 + (2/3)^2 + (2/3)^4 + (2/3)^6 + \dots$ $- 4S/3 = (2/3)^2 + (2/3)^4 + (2/3)^6 + \dots$ $5S/3 = 1 \rightarrow S = 3/5$, and $3 + 5 = 8$

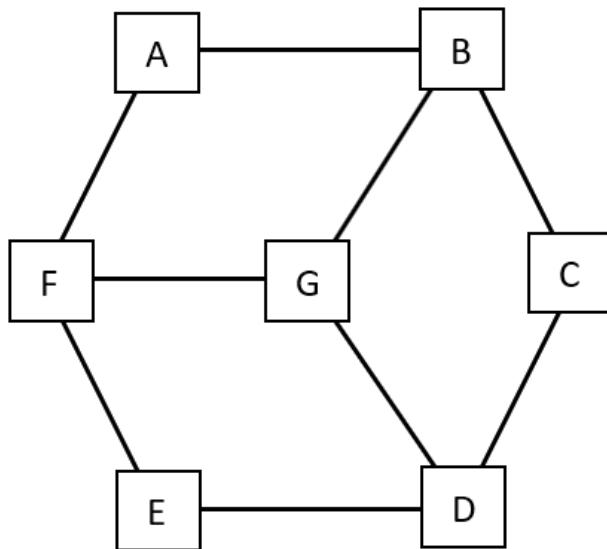
37	576 [integers]	<p>How many 5-digit positive integers can be created using the digits 2, 3, 4, 5, 6, 7, and 8, such that all 5 digits in a given integer are different and no two even digits may be next to each other?</p> <p>There are two cases.</p> <p>Case 1: there are 2 even digits in which case the number would have these patterns: EOEOO, OEOEO, OOEEO, EOOEO, OEOOE, or EOOOE. In all 6 patterns there are ${}_4P_2$ ways to arrange the even digits and $3!$ ways to arrange the odd digits, so the total for case 1 would be $6 \cdot {}_4P_2 \cdot 3! = 6 \cdot 12 \cdot 6 = 432$.</p> <p>Case 2: there are 3 even digits in which case the number would have this pattern: EOEOE. In this pattern there are ${}_4P_3$ ways to arrange the even digits and ${}_3P_2$ ways to arrange the odd digits, so the total for case 2 would be ${}_4P_3 \cdot {}_3P_2 = 24 \cdot 6 = 144$.</p> <p>$432 + 144 = 576$</p>
38	$[M + N + P + Q =] 331$	<p>In the figure, $AB = BC = CD = BD = 9$ cm and $\overline{AB} \perp \overline{BC}$. In simplified form, the area of $\triangle BCE$ can be expressed as $\frac{M-N\sqrt{P}}{Q}$ where M, N, P, and Q are positive integers, $\text{GCF}(M, N, Q) = 1$, and the largest perfect square factor of P is 1. What is $M + N + P + Q$?</p> <p>By adding an altitude from vertex E (see below), $\triangle BCE$ is divided into a 30-60-90 and a 45-45-90 triangle, so $BC = \frac{x}{\sqrt{3}} + x = 9$.</p> <p>Then solve $\frac{x}{\sqrt{3}} + x = 9$ for x, $\frac{x}{\sqrt{3}} + x = 9 \rightarrow x + x\sqrt{3} = 9\sqrt{3} \rightarrow x(1 + \sqrt{3}) = 9\sqrt{3} \rightarrow x = \frac{9\sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} \rightarrow x = \frac{9\sqrt{3} - 27}{-2} = \frac{27 - 9\sqrt{3}}{2}$, which is the height of $\triangle BCE$. The base of $\triangle BCE = 9$, so the area of $\triangle BCE$ is $\frac{1}{2} \cdot 9 \cdot \frac{27 - 9\sqrt{3}}{2} = \frac{243 - 81\sqrt{3}}{4}$, and $243 + 81 + 3 + 4 = 331$.</p> 

39	64 [%]	<p>Two real numbers from 1 to 6, inclusive, are chosen at random. As a percent, what is the probability that the positive difference between the two numbers is greater than 1?</p> <p>Let x = the first number selected and y = the second number selected. Then $x - y > 1$, which means $x - y > 1$ and $x - y < -1$. Put both inequalities in slope intercept form and you get $y < x - 1$ and $y > x + 1$. The graphs of these inequalities are the two lighter regions in the figure below and the area of the lighter regions as a fraction of the whole region is $16/25 = 64\%$.</p>
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40

10

Each of the letters A, B, C, D, E, F and G in the figure shown will be replaced with a different value from the set {2, 3, 4, 5, 6, 7, 8}. If the sums of the four numbers at the vertices of quadrilaterals ABGF, BCDG, and DEFG are each 18, what is the sum of all numbers that can replace G?



If each quadrilateral has a sum of 18, the total of the three quadrilaterals will be 54, and in than sum A, C, and E will each be used once, B, D, and F will each be used twice, and G will be used three times. So you can write the equation $a + c + e + 2(b + d + f) + 3g = 54$. The table shows the possibilities:

	Possible sums: $a + c + e$	Possible values: $2(b + d + f)$	
If $g = 2$, then $a + c + e + 2(b + d + f) = 48$	12 13 14 15 16 17 18 19 20 21	$2 \cdot 21 = 42$ $2 \cdot 20 = 40$ $2 \cdot 19 = 38$ $2 \cdot 18 = 36$ $2 \cdot 17 = 34$ $2 \cdot 16 = 32$ $2 \cdot 15 = 30$ $2 \cdot 14 = 28$ $2 \cdot 13 = 26$ $2 \cdot 12 = 24$	3 ways to make 48: $3 + 7 + 8 + 2(4 + 5 + 6)$ - doesn't work $4 + 6 + 8 + 2(3 + 5 + 7)$ - works $5 + 6 + 7 + 2(3 + 4 + 8)$ - doesn't work
If $g = 3$, then $a + c + e + 2(b + d + f) = 45$	11 12 13 14 15 16 17 18 19 20 21	$2 \cdot 21 = 42$ $2 \cdot 20 = 40$ $2 \cdot 19 = 38$ $2 \cdot 18 = 36$ $2 \cdot 17 = 34$ $2 \cdot 16 = 32$ $2 \cdot 15 = 30$ $2 \cdot 14 = 28$ $2 \cdot 13 = 26$ $2 \cdot 12 = 24$	2 ways to make 45: $4 + 7 + 8 + 2(2 + 5 + 6)$ - works $5 + 6 + 8 + 2(2 + 4 + 7)$ - doesn't work
If $g = 4$, then $a + c + e + 2(b + d + f) = 42$	10 11 12 13	$2 \cdot 21 = 42$ $2 \cdot 20 = 40$ $2 \cdot 19 = 38$ $2 \cdot 18 = 36$	1 way to make 42:

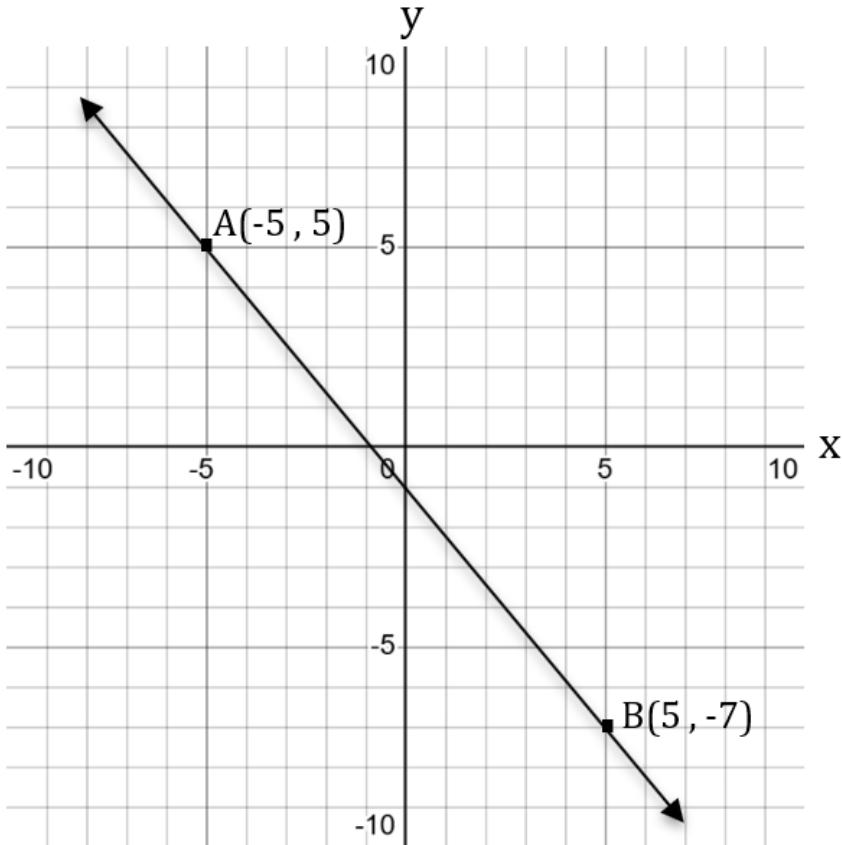
		+ f) =	14 15 16 17 18 19 20 21	2 · 17 = 34 2 · 16 = 32 2 · 15 = 30 2 · 14 = 28 2 · 13 = 26 2 · 12 = 24 2 · 11 = 22 2 · 10 = 20	5 + 7 + 8 + 2(2 + 3 + 6) – doesn't work	
		If g = 5, then a + c + e + 2(b + d + f) =	9 10 11 12 13 14 15 16 17 18 19 20 21	2 · 21 = 42 2 · 20 = 40 2 · 19 = 38 2 · 18 = 36 2 · 17 = 34 2 · 16 = 32 2 · 15 = 30 2 · 14 = 28 2 · 13 = 26 2 · 11 = 24 2 · 12 = 22 2 · 10 = 20 2 · 9 = 18	1 way to make 39: 6 + 7 = 8 + 2(2 + 3 + 4) – works	
		If g = 6, then a + c + e + 2(b + d + f) =	9 10 11 12 13 14 15 16 17 18 19 20	2 · 20 = 40 2 · 19 = 38 2 · 18 = 36 2 · 17 = 34 2 · 16 = 32 2 · 15 = 30 2 · 14 = 28 2 · 13 = 26 2 · 12 = 24 2 · 11 = 22 2 · 10 = 20 2 · 9 = 18	All of the possible sums here of a + c + e + 2(b + d + f) are greater than 36, so 6 cannot be g. For the same reason, 7 and 8 cannot be g either.	

The values of g that have a solution are 2, 3, and 5, so the answer is $2 + 3 + 5 = 10$

“Math is Cool” Championships -- 2024-25

6th Grade

Multiple Choice Solutions

	Answer	Solution
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #3.		
		 <p>A coordinate plane with x and y axes ranging from -10 to 10. Grid lines are spaced at 1-unit intervals. A line is plotted passing through points A(-5, 5) and B(5, -7). The line has a negative slope and passes through the second, third, and fourth quadrants. The x-axis is labeled 'x' and the y-axis is labeled 'y'. Point A is labeled A(-5, 5) and point B is labeled B(5, -7).</p>
1	A	<p>The sum of the coordinates of point A is 0 and the sum of the coordinates of point B is -2. What is the sum of the coordinates of the point where \overleftrightarrow{AB} crosses the y-axis (aka: the y-intercept)?</p> <p>A) -1 B) -1/2 C) 0 D) $\frac{1}{2}$ E) 1</p> <p>The y-intercept is $(0, -1)$, and $0 + -1 = -1$</p>
2	C	<p>An ant walks straight up from point B until it can make a single 90° left turn and then walk straight to point A. If the length of one side of a grid square is one unit, how many units will the ant have traveled from A to B?</p> <p>A) 11 B) 20 C) 22 D) 24 E) 25</p> <p>$12 + 10 = 22$</p>

3	B	Point C is three units to the right of point A and point D is ten units straight up from point B. What is the distance between C and D? A) $2\sqrt{10}$ units B) $\sqrt{53}$ units C) $\sqrt{58}$ units D) $2\sqrt{17}$ units E) 9 units The distance between C(-2, 5) and D(5, 3) is $\sqrt{7^2 + 2^2} = \sqrt{53}$.
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USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #4 THROUGH #6.

When you divide two numbers, if the divisor does not go into the dividend evenly, there will be a remainder. For example, the divisor 8 goes into the dividend 50 six times, with a remainder of 2. In other words, $50 \div 8 = 6$ remainder 2. The following three problems have to do with remainders.

4	B	What is the remainder when 60 is divided by 7? A) 3 B) 4 C) 5 D) 6 E) 7 $60 \div 7 = 8 \text{ r}4$
5	D	When A is divided by 11, the remainder is 9. When A is divided by 13, the remainder is 3. What is the largest 2-digit value of A? A) 20 B) 29 C) 31 D) 42 E) 84 Let A = $11x + 9$ and let A = $13x + 3$, where x is an integer. Then $11x + 9 = 13x + 3 \rightarrow 2x = 6 \rightarrow x = 3$ and the smallest positive value of A is 42. The next value of A will be $42 + 143 = 185$, where $143 = \text{LCM}(11, 13)$, so 42 is also the largest 2-digit value of A.
6	E	What is the remainder when 2^{2025} is divided by 40? A) 4 B) 8 C) 16 D) 24 E) 32 $2^1 \div 40 = 0 \text{ r}2$ $2^2 \div 40 = 0 \text{ r}4$ $2^3 \div 40 = 0 \text{ r}8$ $2^4 \div 40 = 0 \text{ r}16$ $2^5 \div 40 = 0 \text{ r}32$ $2^6 \div 40 = 1 \text{ r}24$ $2^7 \div 40 = 3 \text{ r}8$ $2^8 \div 40 = 6 \text{ r}16$ $2^9 \div 40 = 12 \text{ r}32$ $2^{10} \div 40 = 25 \text{ r}24$ $2^{11} \div 40 = 51 \text{ r}8$ $2^{12} \div 40 = 102 \text{ r}16$ After the first four powers of 2, the pattern in the remainder is 32, 24, 8, 16, which repeats every 4 powers of 2. $2025 \div 4$ has a remainder of 1, so the first remainder in the pattern will be the remainder of $2^{2025} \div 40$, which is 32.

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #7 THROUGH #10.

The following four problems involve solving algebraic equations.

7	C	<p>Solve for x: $10x - 7 = 53$</p> <p>A) -6 B) -4.6 C) 6 D) 7 E) 9</p> $10x - 7 = 53 \rightarrow 10x = 60 \rightarrow x = 6$
8	D	<p>Solve the equation $C = \frac{5}{9}(F - 32)$ for F.</p> <p>A) $F = \frac{5}{9}C + 32$ B) $F = \frac{5}{9}(C - 32)$ C) $F = 1.8(C + 32)$ D) $F = \frac{9}{5}C + 32$ E) $F = 1.8C - 57.6$</p> $C = \frac{5}{9}(F - 32) \rightarrow \frac{9}{5}C = F - 32 \rightarrow F = \frac{9}{5}C + 32$
9	E	<p>If $x + 3/y = 1/7$ and $y + 3/x = 84$, what is the value of the product xy?</p> <p>A) -3 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) 3</p> <p>If $x + 3/y = 1/7$ and $y + 3/x = 84$, then $(x + 3/y)(y + 3/x) = (1/7)(84) \rightarrow xy + 3 + 9/xy = 12 \rightarrow xy + 9/xy = 6 \rightarrow (xy)^2 + 9 = 6xy \rightarrow (xy)^2 - 6xy + 9 = 0 \rightarrow (xy - 3)(xy - 3) = 0 \rightarrow xy = 3$</p>
10	B	<p>Given $a - b = 4$ and $a^2 + b^2 = 116$, what is the value of $a^3 - b^3$?</p> <p>A) 536 B) 664 C) 728 D) 856 E) Answer not given</p> <p>If $a - b = 4$, then $(a - b)^2 = 4^2 \rightarrow a^2 - 2ab + b^2 = 16 \rightarrow a^2 + b^2 - 2ab = 16 \rightarrow$ since $a^2 + b^2 = 116$, replace $a^2 + b^2$ with 116 to get $116 - 2ab = 16 \rightarrow -2ab = -100 \rightarrow ab = 50$.</p> <p>If $a - b = 4$, then $(a - b)^3 = 4^3 \rightarrow a^3 - 3a^2b + 3ab^2 - b^3 = 64 \rightarrow a^3 - b^3 - 3ab(a - b) = 64 \rightarrow$ Since $ab = 50$ and $a - b = 4$, replace ab with 50 and $a - b$ with 4 to get $a^3 - b^3 - 3(50)(4) = 64 \rightarrow a^3 - b^3 - 600 = 64 \rightarrow a^3 - b^3 = 664$</p>

“Math is Cool” Championships -- 2024-25

6th Grade

Team Test Solutions

	Answer	Solution																
1	10	Evaluate: $\frac{(3-6)^3}{(-9)} - 17 + ((-5)^2 - 1)$ $\frac{(3-6)^3}{(-9)} - 17 + ((-5)^2 - 1) = \frac{-27}{-9} - 17 + (25 - 1) = 3 - 17 + 24 = 10$																
2	43 [students]	In the 8 th grade class of Park Middle School, there are 95 students. Two-fifths of these students have a dog. Forty-one of them have a cat and 27 of them have both a dog and a cat. How many of the 95 students don't have a cat or a dog? Based on the two-way table, 43 have neither. <table border="1"> <thead> <tr> <th></th> <th>Cat</th> <th>No Cat</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Dog</td> <td>27</td> <td>11</td> <td>38</td> </tr> <tr> <td>No Dog</td> <td>14</td> <td>43</td> <td>57</td> </tr> <tr> <td>Total</td> <td>41</td> <td>54</td> <td>95</td> </tr> </tbody> </table>		Cat	No Cat	Total	Dog	27	11	38	No Dog	14	43	57	Total	41	54	95
	Cat	No Cat	Total															
Dog	27	11	38															
No Dog	14	43	57															
Total	41	54	95															
3	4 [prime numbers]	How many positive prime numbers less than 50 have 7 as at least 1 of their digits? 7, 17, 27, 37 and 47 makes 4																
4	[A + B =] 3	As a reduced common fraction, the median of the following data set is A/B. What is A + B? {7/10, 1/2, 2/3, 2/5, 1/3} The list in ascending order is 1/3, 2/5, 1/2, 2/3, 7/10 So, 1/2 is the median, and 1 + 2 = 3.																
5	22 [coins]	Tanishi has \$4.37 in coins that consist only of quarters, dimes, nickels, and pennies. She has one nickel among her coins. What is the smallest total number of coins she could have? She could have \$4.25 in quarters, or 17 quarters, and 1 nickel and 7 pennies, or 25 coins total. Or she could have \$4.00 in quarters, or 16 quarters, 3 dimes, 1 nickel, and 2 pennies, for 22 coins total.																

6	<p>$[P + Q =]$ 111</p>	<p>In the figure, $AB = 10$ cm, $BC = 17$ cm, $CD = 25$ cm, and $BE = 8$ cm. The length of AD is $P\sqrt{Q}$ cm, where P and Q are integers, and Q does not have a factor that is a perfect square other than 1. What is $P + Q$?</p> <p>All 4 triangles are right triangles. Fill in the lengths of AE, CE, and DE using Pythagorean triples, and then the length of AD is $\sqrt{20^2 + 6^2} = \sqrt{436} = 2\sqrt{109}$, and $2 + 109 = 111$.</p>
7	15	<p>A particular number series increases alternately by adding 10 and then by multiplying by $1/2$. For example, if the first term in the series is 6, the next four terms would be 16, 8, 18, 9, and so on. What is the smallest possible integer in the series if the first term is 330?</p> <p>The series would be 330, 340, 170, 180, 90, 100, 50, 60, 30, 40, 20, 30, 15, 25, 12.5, ... and everything after 25 will not be an integer. The smallest integer in the list is 15.</p>
8	<p>$[N =]$ 1728</p>	<p>If N is a perfect cube and a factor of $11!$, what is the largest value of N?</p> $11! = 1 \cdot 2 \cdot 3 \cdot (2 \cdot 2) \cdot 5 \cdot (2 \cdot 3) \cdot 7 \cdot (2 \cdot 2 \cdot 2) \cdot (3 \cdot 3) \cdot (2 \cdot 5) \cdot 11$ $N = 2^6 \cdot 3^3 = 64 \cdot 27 = 1728$
9	19	<p>Let A and B be two distinct three-digit positive integers, such that $A + B > 1960$, $A > B$, and $A - B = D$. What is the median of all possible values of D?</p> <p>$999 + 962 = 1961 > 1960$, and $999 - 962 = 37$, which is the largest possible value of D.</p> <p>$981 + 980 = 1961 > 1960$, and $981 - 980 = 1$, which is the smallest possible value of D.</p> <p>Every value between 1 and 37 is also possible for D, and the median of $\{1, 2, 3, \dots, 35, 36, 37\}$ is $(1 + 37)/2 = 19$.</p>

10**[A + B =]****3**

In a hand of 8 cards there are 4 hearts and 4 clubs. The cards are randomly selected one at a time without replacement and laid on a table. As a reduced common fraction, the probability that the 6th card laid on the table is a heart is A/B. What is A + B?

There are three possible scenarios:

$$P(4C \text{ and } 1H \text{ followed by } 1H) = 5C_4 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} = 5 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} = 1/14$$

$$P(3C \text{ and } 2H \text{ followed by } 1H) = 5C_3 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = 10 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = 2/7$$

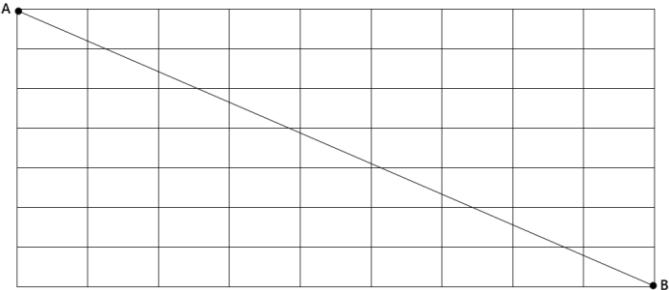
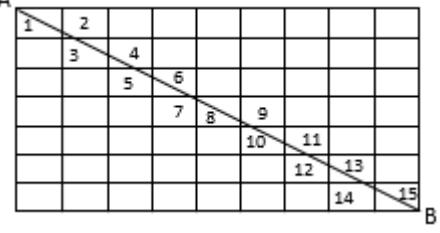
$$P(2C \text{ and } 3H \text{ followed by } 1H) = 5C_2 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 10 \cdot \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 1/7$$

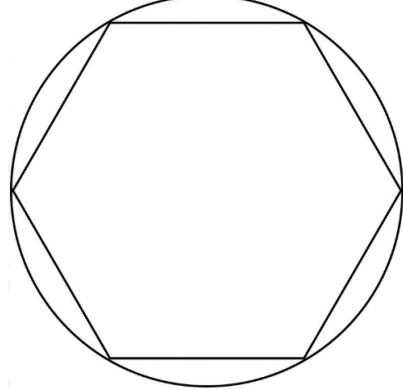
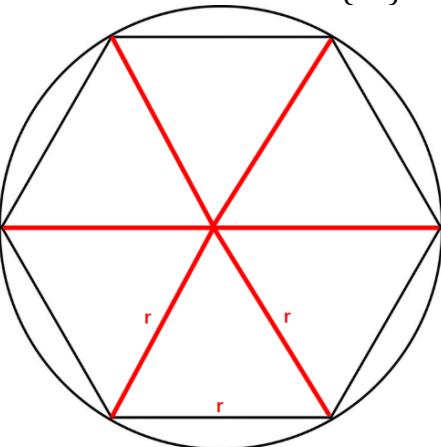
$$1/14 + 2/7 + 1/7 = 7/14 = 1/2, \text{ and } 1 + 2 = 3$$

“Math is Cool” Championships -- 2024-25

6th Grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	528	Evaluate the following expression when $a = 1$ and $b = 4$: $5a^2b^2 + 7ab^3$ $5(1)^2(4)^2 + 7(1)(4)^3 = 5(1)(16) + 7(1)(64) = 80 + 448 = 528$
2	3785 [ml]	One gallon is equivalent to about 3.78541 liters. To the nearest integer number of milliliters, how many milliliters are in one gallon? $3.78541 \text{ L} = 3785.41 \text{ ml} = 3785 \text{ ml}$
3	[A + B =] 7	Horace takes 6 minutes to walk a quarter mile home from school. Horace's average rate of walking home from school is A.B miles per hour. What is A + B? $\frac{1}{4} \text{ mile in } 6 \text{ min} = 1 \text{ mile in } 24 \text{ min} = 2.5 \text{ miles in } 60 \text{ min} = 2.5 \text{ miles per hour, and } 2 + 5 = 7$
4	2	The first three terms of a geometric sequence are 1458, 486, and 162. What is the 7 th term in the sequence? $1458, 486, 162, 54, 18, 6, 2$
5	48 [rectangles]	The rectangular grid shown below consists of some number of congruent non-overlapping rectangles and point A is connected to point B with a segment. How many of the congruent non-overlapping rectangles does \overline{AB} not pass through?  <p>There are $7 \cdot 9 = 63$ total congruent non-overlapping rectangles and the segment passes through 15 of them. So, the answer is $63 - 15 = 48$.</p> 

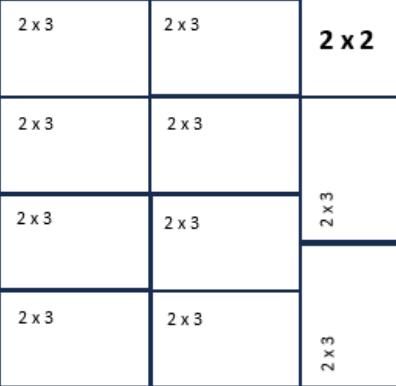
6	$[A + B =] 17$	<p>A pair of standard dice are rolled. As a reduced common fraction, the probability that the sum of the two numbers showing is greater than 6 and less than 10 is A/B. What is $A + B$? $P(7, 8, \text{ or } 9) = (6 + 5 + 4)/36 = 15/36 = 5/12$, and $5 + 12 = 17$</p>
7	$[A + B =] 105$	<p>As a reduced common fraction, the following expression simplifies to A/B. What is $A + B$?</p> $\frac{1}{2 + \frac{3}{4 + \frac{5}{6}}}$ $\frac{1}{2 + \frac{3}{2 + \frac{3}{(\frac{29}{6})}}} = \frac{1}{2 + \frac{3}{2 + \frac{18}{29}}} = \frac{1}{2 + \frac{18}{29}} = \frac{1}{\left(\frac{76}{29}\right)} = \frac{29}{76} \text{ and } 29 + 76 = 105$
8	$28 [= A]$	<p>A regular hexagon is inscribed in a circle, as shown here. Given that the area of the hexagon is $294\sqrt{3} \text{ cm}^2$, the circumference of the circle can be written as $A\pi \text{ cm}$. What is A?</p>  <p>The hexagon can be divided into 6 equilateral triangles, each of which has a side length = r, the radius of the circle. The area of a regular hexagon is:</p> $A = \frac{3\sqrt{3}}{2} r^2 = 294\sqrt{3}$ <p>Therefore, $r^2 = 196$, $r = 14$ $\text{Circumference} = 2\pi r = 2\pi(14) = 28\pi$</p> 

9	15 [days]	<p>Juana can build a large shed in 5 days less than it takes her brother Carlos. If they built it together it would take them 6 days. How many days would it take Carlos to build the shed by himself?</p> <p>$J = C - 5$, where J is the number of days it takes Juana and C is the number of days it takes Carlos to build the shed.</p> <p>Juana builds $1/J$ or $1/(C - 5)$ of the shed per day and Carlos builds $1/C$ of the shed per day, so together $1/(C - 5) + 1/C = 1/6 \rightarrow 6C + 6(C - 5) = C(C - 5) \rightarrow 6C + 6C - 30 = C^2 - 5C \rightarrow C^2 - 17C + 30 = 0 \rightarrow (C - 15)(C - 2) = 0 \rightarrow C = 15$ and $J = 10$ or $C = 2$ and $J = -3$. The second solution is not possible, so the answer is 15 days.</p>
10	3211000 _[7] [base 7]	<p>What is the only 7-digit base-7 number in which the first digit on the left tells how many 0s, the second digit from left tells how many 1s, the third digit tells how many 2s, the fourth digit tells how many 3s, the fifth digit tells how many 4s, the sixth digit tells how many 5s, and the seventh digit tells how many 6s are in the number? You do not need to include the "base 7" in your answer.</p> <p>The number must have exactly two 1s.</p> <p>To see why consider the cases where there are three 1s: In the case of 1311_ _ _ one of the remaining digits would need to be a 2, which would mean that there are also two 4s, two 5s, or two 6s and therefore no 0s. Or in the case of _ 3111 _ _ there needs to be a 2 and a 4 in the remaining digits, neither of which would work for the 1st digit and therefore no digit would be correct in the 1st digit's position. For these same reasons, any larger number of 1s would also be impossible.</p> <p>Also consider the cases in which there is one 1: In the case of _ 1 _ _ _ _ , whatever the number of 0s is would be indicated by a digit in the 1st digit's position. If the number of 0s is any other number other than 1, such as 2, then there would have to be another 1 in the number leading to a contradiction. In the case of 1 _ _ _ _ _ , then the one and only 0 would need to be the 2nd digit, in which case it would be impossible for none of the remaining digits to be 0.</p> <p>Also consider the cases in which there are no 1s: In the case of 202 _ _ _ , say any of the remaining digits was a 3, such as the 4th digit. Then there would have to be three 3s among the remaining digits meaning there would not only be three 3s, but also three 4s, three 5s, or three 6s, but this would be impossible.</p> <p>The two 1s would need to not be in the 1st or 2nd digit's positions, 2 would be in the 2nd digit's position and 1 would be in the 3rd digit's position, or _ 21 _ _ _ . If one of the remaining positions, 4th through 7th, is a 1, then there could be three 0s, so the digit in position 1 is a 3, and the number would be 3211000.</p>

“Math is Cool” Championships -- 2024-25

6th Grade

College Bowl Round #1 Solutions

	Answer	Solution
1	19 [even integers]	How many even integers are there between eleven and forty-nine? 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, and 48 make 19 even numbers
2	[y =] 8	If three X plus four Y equals sixty-five and X equals eleven, what does Y equal? $3x + 4y = 65, x = 11 \rightarrow 33 + 4y = 65 \rightarrow 4y = 32 \rightarrow y = 8$
3	15 [minutes]	Biff can prepare sixteen bags of tater tots in twenty minutes. Eho prepares tater tots at a rate that is twice as fast as Biff. How many minutes does it take Eho to prepare twenty-four bags of tater tots? Biff: 16 bags in 20 min Eho: 16 bags in 10 min or 8 bags in 5 min or 24 bags in 15 min
4	86 [%]	A certain bag of M-and-Ms contains ten green, twenty-two brown, six red, seven yellow, and five blue M-and-Ms. If one M-and-M is randomly chosen, what is the probability in percent that it is not yellow? 50 m&ms total 43 are not yellow $43/50 = 86/100 = 86\%$
5	8 [inches]	An eight-inch-by-eight-inch square is cut out from a piece of paper. It is then cut into as many two-inch-by-three-inch rectangles as possible, such that there is only one scrap leftover. In inches, what is the perimeter of the scrap? There are 10 congruent 2 x 3 rectangles in the figure below and the scrap is a 2 x 2 square with a perimeter of 8 inches. 

6	78	What is the largest two-digit integer that is the product of three distinct prime numbers? $2 \cdot 3 \cdot 5 = 30$, $2 \cdot 3 \cdot 7 = 42$, $2 \cdot 3 \cdot 11 = 55$, $2 \cdot 3 \cdot 13 = 78$, $2 \cdot 3 \cdot 17 = 102$, $2 \cdot 5 \cdot 7 = 70$, $2 \cdot 5 \cdot 11 = 110$, $2 \cdot 7 \cdot 11 = 154$, $3 \cdot 5 \cdot 7 = 105$, 78 is the largest 2-digit number in this list
7	$[A + B =] 20$	An infinite series begins with the terms one-third, two-fourths, three-fifths, and so on. The ninth term in the series is A over B. What is A plus B? $1/3, 2/4, 3/5, 4/6, 5/7, 6/8, 7/9, 8/10, 9/11$ $11 + 9 = 20$
8	55 [= mean]	What is the mean of the two-digit positive multiples of twenty-two? The multiples are: 22, 44, 66, 88. Since this is an arithmetic sequence, the mean is the same as the median, or $(44 + 66)/2 = 55$. Or, $22+44+66+88 = 220$, $220/4 = 55$
9	988	Let A be a single-digit positive integer, let B be a two-digit positive integer, and let C be a three-digit positive integer. What is the largest possible positive difference between C and the sum of A and B? If A = 1, B = 10, and C = 999, then $999 - 11 = 988$
10	18107	Let A equal one plus twenty plus three hundred plus four thousand plus fifty thousand. What is A divided by three? $(1 + 20 + 300 + 4000 + 50000)/3 = 54321/3 = 18107$

"Math is Cool" Championships -- 2024-25

6th Grade

College Bowl Round #2 Solutions

	Answer	Solution
1	27	What is the greatest positive difference between any two prime numbers less than thirty? $29 - 2 = 27$
2	360 [in ²]	How many square inches are in two and a half square feet? $2.5 \cdot 144 = 360$
3	[2A =] 28	As a reduced common fraction, one-third plus three-fifths equals A over B. What is two times A? $\frac{1}{3} + \frac{3}{5} = \frac{5}{15} + \frac{9}{15} = \frac{14}{15}$, $2(14) = 28$
4	4	The first three terms of a sequence are twelve, twenty, and eight, where starting with the third term, each successive term is the positive difference between the previous two terms. What is the one hundredth term in this sequence? $12, 20, 8, 12, 4, 8, 4, 4, 0, 4, 4, 0, 4, 4, 0, \dots, 4, 4, 0, 4$ After the first 6 terms the three number pattern 4, 4, 0, repeats indefinitely, so after repeating 31 times, the 99 th term is 0, and the 100 th term will be 4.
5	[B =] 8	The expression six X squared plus X minus forty can be factored into the quantity two X minus A times the quantity three X plus B. What is B? $6x^2 + x - 40 =$ $(2x - 5)(3x + 8)$
6	8 [integers]	How many positive integers less than two hundred and fifty are multiples of four and ten, but not six? Positive integer multiples of 4 and 10, but not 6: 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, there are 8 in the list that aren't crossed out.
7	24 [minutes]	Bert can decorate three cakes in an hour. Ernie decorates cakes at a rate that is one and a half times Bert's rate. How many minutes would it take the two of them working together to decorate three cakes? Bert: 3 cakes/hr Ernie: 4.5 cakes/hr Together 7.5 cakes/hr $3/7.5 = x/60 \rightarrow x = 24$

8	[A + B =] 170	As a reduced common fraction, the probability of drawing two cards one at a time with replacement, from a standard deck, that are both aces is A over B. What is A plus B? $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$, and $1 + 169 = 170$
9	500	Thirty-five is seven percent of what number? $\frac{7}{100} = \frac{35}{x} \rightarrow x = 500$
10	99 $^{\circ}$	The measures of three of the angles in a convex quadrilateral are sixty-one, ninety-two, and one hundred and eight. How many degrees are in the measure of the fourth angle? $360 - 61 - 92 - 108 = 99$

“Math is Cool” Championships -- 2024-25

6th Grade

College Bowl Round #3 Solutions

	Answer	Solution
1	[A + B =] 11	An ultimate frisbee team has twelve girls and nine boys. As a reduced common fraction, the ratio of girls to total players on the team is A over B. What is A plus B? $12/21 = 4/7$, and $4 + 7 = 11$
2	[A + B =] 101	As a reduced common fraction, the value of X in the equation five X over eight equals seven over nine is A over B. What is A plus B? $5x/8 = 7/9 \rightarrow 45x = 56 \rightarrow x = 56/45$, and $56 + 45 = 101$
3	530	What is thirty times fifty-three divided by three? $30 \cdot 53 \div 3 = 530$
4	17 [= 6 th term]	In a sequence the fourth term is the mean of the first three terms, and each successive term is the mean of the previous three terms. If the first three terms are negative fifteen, one, and thirty-two, what is the sixth term? -15, 1, 32, 6, 13, 17
5	48 [%]	The probability that the Mustangs will win any given game is sixty percent. As a percentage, what is the probability that they will win exactly one of their next two games? There are two ways they can win 1 of their next 2, WL, or LW, so the answer is $2 \cdot 6/10 \cdot 4/10 = 48/100 = 48\%$
6	24 [integers]	How many integers are greater than seven point nine and less than thirty-one point one? 8 through 31 makes 24 integers
7	50	A data set contains seven distinct positive integers, the median is thirteen, and the mean is fourteen. What is the largest possible number in the set? $7 \cdot 14 = 98$ $1 + 2 + 3 + 13 + 14 + 15 = 48$ $98 - 48 = 50$
8	26 [°]	In degrees, what is the measure of the complement of the supplement of a one-hundred and sixteen degree angle? $180 - 116 = 64$, and $90 - 64 = 26$
9	12 [years old]	Abe is six years older than Ben, who is currently ten years old. How old in years was Abe when he was twice as old as Ben? Abe = 16 and Ben = 10. Four years ago Abe = 12 and Ben = 6, so Abe was 12

10

75 [cabinets]

If three carpenters can build twenty-seven cabinets in six days, how many cabinets can five carpenters build in ten days?
3 carpenters : 27 cabinets : 6 days → 1 carpenter : 9 cabinets : 6 days → 1 carpenter : 15 cabinets : 10 days → 5 carpenters : 75 cabinets : 10 days

“Math is Cool” Championships -- 2024-25

6th Grade

College Bowl Round #4 Solutions

	Answer	Solution
1	6391	What is eighty-three times seventy-seven? $(80 + 3)(80 - 3) = 6400 - 9 = 6391$
2	364	What is the sum of the first six terms of the geometric sequence whose first three terms are one, three, and nine? $1 + 3 + 9 + 27 + 81 + 243 = 364$
3	12	Each of the smallest six positive prime numbers is doubled to make a set of six even integers. What is the median of the set of six even integers? The set is 4, 6, 10, 14, 22, 26, and the median is $(10 + 14)/2 = 12$
4	231 [minutes]	Jenna took sixty-three minutes to read fifty-seven pages of her current book which is two hundred and sixty-six pages long. At this rate, how many minutes will it take her to read the rest of the book? $266 - 57 = 209$ pages left. 57 pages in 63 minutes \rightarrow 19 pages in 21 minutes \rightarrow 190 pages in 210 minutes \rightarrow 209 pages in 231 minutes
5	27	What is the smallest two-digit composite number that has exactly two distinct composite numbers as factors? Prime numbers will have zero composite factors. This eliminates 11, 13, 17, 19, 23, 29, ... Numbers that are the product of two prime numbers will have one composite factor. This eliminates 10, 14, 15, 21, 22, 25, 26, 33, ... Checking what remains, 12, 16, 18, and 20 all have 3 composite factors. 24 has 4 composite factors. The factors of 27 are 1, 3, 9, and 27, and exactly two of the factors are composite, 9 and 27.
6	60 [%]	Let A be one hundred and fifty percent of B. Let C be two hundred and fifty percent of D. Let A equal C. What percent of B is D? $A = 1.5B$ and $C = 2.5D$, so $1.5B = 2.5D \rightarrow D = 1.5B/2.5 = 0.6B$, so D is 60 percent of B.

7	$[A + B =] 7$	<p>A cone and a cylinder have the same volume. The cylinder has a radius that is half the radius of the cone. As a reduced common fraction, the ratio of the height of the cylinder to the height of the cone is A over B. What is A plus B?</p> <p>Let r = the radius of the cone, h = the height of the cone, and H = the height of the cylinder, then $\pi r^2 h / 3 = \pi(r/2)^2 H \rightarrow r^2 h / 3 = r^2 H / 4 \rightarrow h / 3 = H / 4 \rightarrow 4h = 3H \rightarrow H/h = 4/3$, and $4 + 3 = 7$</p>
8	48 [inches]	<p>A fathom is equivalent to six feet. The average depth of Iceland's Blue Lagoon is two-thirds of a fathom. What is the average depth of Blue Lagoon in inches?</p> $\frac{2}{3} \cdot 6 = 4 \text{ and } 4 \cdot 12 = 48$
9	$[x =] 11$	<p>Solve the following equation for X: five X over two equals one hundred sixty-five over six</p> $\frac{5x}{2} = \frac{165}{6} \rightarrow 15x/6 = 165/6 \rightarrow 15x = 165 \rightarrow x = 11$
10	$[A + B =] 9$	<p>As a reduced common fraction, the ratio of the number of positive single-digit even integers to the number of positive single-digit odd integers is A over B. What is A plus B?</p> <p>Even: 2, 4, 6, 8 Odd: 1, 3, 5, 7, 9 The ratio is $4/5$, and $4 + 5 = 9$</p>

"Math is Cool" Championships -- 2024-25

6th Grade

College Bowl Round #5 Solutions

	Answer	Solution
1	30 [%]	Shira has six dimes, seven pennies, two nickels, and five quarters in her pocket. If she randomly selects one coin from her pocket, what is the probability in percent that it is a dime? $P(\text{dime}) = 6/(6 + 7 + 2 + 5) = 6/20 = 30/100 = 30\%$
2	26 [centimeters]	In centimeters, what is the diameter of a circle whose area is one hundred and sixty-nine pi square centimeters? $A = 169\pi \rightarrow r^2 = 169 \rightarrow r = 13 \rightarrow d = 26$
3	76	Serena's scores on her last three rounds of golf were eighty-two, eighty-five, and seventy-seven. What score does she need on her next round to lower her average to exactly eighty? $82 + 85 + 77 + x = 4 \cdot 80 \rightarrow 244 + x = 320 \rightarrow x = 76$
4	[D - E =] 16	The absolute value of A minus B equals eight and the absolute value of between B minus C equals nine. Let D equal the greatest possible absolute value of A minus C, and let E equal the smallest possible absolute value of A minus C. What is D - E? If A > B > C, then $ A - C = 17$, for example if A = 50, B = 42, and C = 33, then $ 50 - 33 = 17 = D$. If A < B and C < B, then $ A - C = 1$, for example if A = 42, B = 50, and C = 41, then $ 42 - 41 = 1 = E$. And $17 - 1 = 16$.
5	91	What is the smallest possible sum of two positive integers whose greatest common factor is thirteen and whose least common multiple is one hundred and thirty? $26 = 2 \cdot 13$ and $65 = 5 \cdot 13$, so $\text{GCF}(26, 65) = 13$, and $\text{LCM}(26, 65) = 130$, and $26 + 65 = 91$
6	500 [seconds]	Angela and Megan begin at the same place and time to run around a circular four-hundred-meter track. Angela's average rate is two hundred and forty meters per minute and Megan's average rate is twenty percent faster than Angela's. After how many seconds will Megan pass Angela for the first time? Angela runs 240 meters per minute and Megan runs $1.2 \cdot 240 = 288$ meters per minute, so she extends her lead by 48 meters per minute. $400/48 = 8\frac{1}{3}$ minutes = 500 seconds.

7	22 [times]	Every page of a one-hundred and twenty-one-page book is numbered. How many times does the digit six appear in the page numbering? 6, 16, 26, 36, 46, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 86, 96, 106, 116, 6 appears 22 times
8	6	What is three-eighths divided by one sixteenth? $3/8 \div 1/16 = 3/8 \cdot 16 = 6$
9	35 [orange or purple skittles]	One-fourth of the Skittles in a bag of sixty are green, one-sixth are yellow, three-tenths are purple, and the rest are orange. How many Skittles are orange or purple? Green: 15, yellow: 10, purple: 18, orange: 17 $18+17 = 35$
10	[x =] 18	Solve the following equation for X: seven X minus twenty-three equals negative two X plus one hundred thirty-nine $7x - 23 = -2x + 139 \rightarrow 9x = 162 \rightarrow x = 18$

“Math is Cool” Championships -- 2024-25

6th Grade

College Bowl Round #6 Solutions

	Answer	Solution
1	2800 [mm]	How many millimeters are in two and four-fifths of a meter? $2.8 \cdot 1000 = 2800$
2	[A =] 32	If A equals four B, and three A minus five B equals fifty-six, what is A? $a = 4b$ and $3a - 5b = 56 \rightarrow 12b - 5b = 56 \rightarrow 7b = 56 \rightarrow b = 8$ and $a = 4b = 32$
3	25 [boards]	Rae can paint ten boards in sixteen minutes. At this rate, how many boards can she paint in forty minutes? 10 boards in 16 minutes \rightarrow 2.5 boards in 4 minutes \rightarrow 25 boards in 40 minutes
4	40 [%]	A jar contains twenty marbles that are all either green or blue. After one-fourth of the green marbles are removed there are then seventeen marbles left in the jar. As a percentage, what is the probability of randomly selecting a blue marble from the original contents of the jar? $20 - 17 = 3$, so there are $3 \cdot 4 = 12$ green marbles in the jar originally, and $20 - 12 = 8$ blue marbles originally, so $P(\text{blue}) = 8/20 = 40/100 = 40\%$
5	4 [integers]	How many positive two-digit integers less than forty are there, such that each digit is a different positive factor of the integer? 11, 12 , 13, 14, 15 , 16, 17, 18, 19, 20, 21, 22, 23, 24 , 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 , 37, 38, 39
6	126	Let A, B, C, and D each be replaced randomly by a different member of the set six, nine, twelve, and fifteen. What is the largest possible value of A times B minus C times D? $12 \cdot 15 - 6 \cdot 9 = 180 - 54 = 126$
7	[B =] 28	As a reduced common fraction, the ratio A over B is three over seven, where A does not equal three and B does not equal seven. When four is added to A, the new ratio is four over seven as a reduced common fraction. What is B? $(3x + 4)/7x = 4x/7x \rightarrow 3x + 4 = 4x \rightarrow x = 4$, so $3x = 12 = A$ and $7x = 28 = B$
8	95	What is the positive difference between the largest two-digit prime number and the smallest one-digit prime number? $97 - 2 = 95$

9	37	What is the tenth term in the arithmetic sequence whose first term is ten and whose fourth term is nineteen? 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, or $10 + 3 \cdot 9 = 37$
10	54	What is the median of the following set of numbers: nineteen, one hundred and nine, nine, twenty-nine, seventy-nine, and eighty-nine? 9, 19, 29, 79, 89, 109, and the median is $(29 + 79)/2 = 54$