"Math is Cool" Championships -- 2024-25
<mark>7th/8th grade</mark>
Mental Math Solutions

	Answer	Solution
1	13	What is the mean of the set of integers from eleven to fifteen inclusive? (11 + 12 + 13 + 14 + 15)/5 = 13
2	[$] 1750 [dollars]	Phoebe and Monica are going to evenly split the cost of a three-thousand-five-hundred dollar car. How many dollars will Phoebe pay for her share of the car? 3500/2 = 1750
3	7 [circles]	What is the maximum number of non-overlapping circles of radius one centimeter that can fit inside a two-centimeter-by-fifteen-centimeter rectangle? See figure:
4	2 [prime numbers]	How many prime numbers are between twenty-four and thirty-two? 29, 31
5	25	What is three-fourths of twelve plus four-thirds of twelve? $\frac{3}{4} \cdot 12 + \frac{4}{3} \cdot 12 = 9 + 16 = 25$
6	495 [minutes]	Hamta left home for school at seven thirty AM, and returned home on the same day at three forty-five PM. How many minutes was Hamta away from home? 7:30 am to 3:30 pm = 8 hours x 60 minutes/hour = 480 minutes, plus an extra 15 minutes = 495 minutes
7	[N =] 8	The first three terms of a geometric sequence are one hundred twelve, fifty-six, and twenty-eight. Let term number N be the first term in the sequence that is less than one. What is N? 112, 56, 28, 14, 7, 3.5, 1.75, 0.875, and 0.875 is the 8th term, so N = 8

| 8 | 120 [ways] | Min has four standard dice, where one is red, one is yellow, one is green, and one is blue. If she rolls them all together, in how many ways can three of the dice show the same number while the fourth die shows something different? |

The number of ways to get three 1s is $4 \cdot 5 = 20$, as shown in the table.

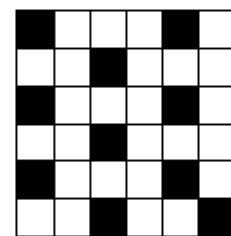| red	yellow	green	blue
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5
1	1	1	6
1	1	2	1
1	1	3	1
1	1	4	1
1	1	5	1
1	1	6	1
1	2	1	1
1	3	1	1
1	4	1	1
1	5	1	1
1	6	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1

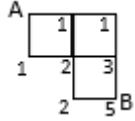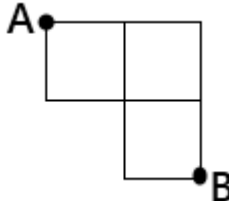There are also 20 ways to get three of each of the other five numbers, so the final answer is $6 \cdot 20 = 120$
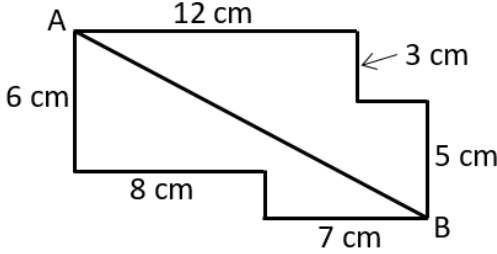
"Math is Cool" Championships -- 2024-25
<mark>7th/8th grade</mark>
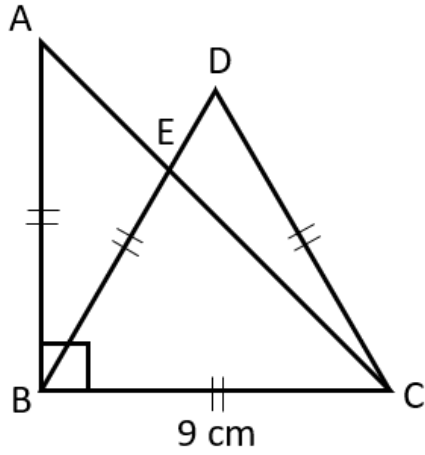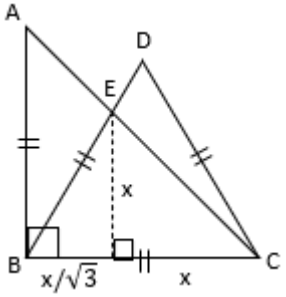## Individual Test Solutions

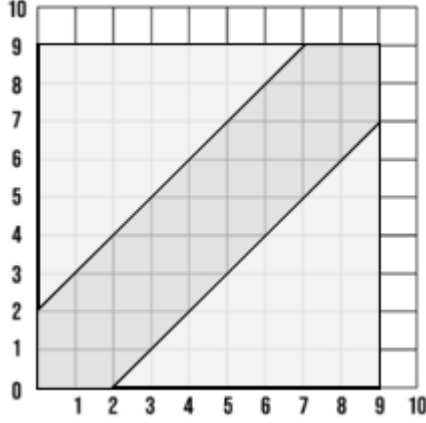|	Answer	Solution
1	17 [nickels]	How many nickels make 85 cents? 85/5 = 17
2	96	What is the largest 2-digit multiple of 12? 2-digit multiples of 12 are 12, 24, 36, 48, 60, 72, 84, and 96, and 96 is the largest.
3	52	The mean of three numbers is 30. If two of the numbers are 11 and 27, what is the third number? $30 \cdot 3 = 90$ and $90 - 11 - 27 = 52$
4	[A + B =] 4	Guillermo has one white hat, two blue hats, and three brown hats and he randomly selects one to wear on a given day. As a reduced common fraction, the probability that he takes a blue hat is A/B. What is A + B? P(blue) = 2/6 = 1/3, and 1 + 3 = 4
5	4132	Evaluate: 4444 – 333 + 22 – 1 4444 – 333 + 22 – 1 = 4132
6	208 [in^2]	How many square inches are in the area of a rectangle with a length of 13 inches and a width of 16 inches? $13 \cdot 16 = 208$
7	[A + B =] 18	In the grid of squares shown below, as a reduced common fraction, the ratio of black squares to white squares is A/B. What is A + B? There are a total of 36 squares, 10 are black and 26 are white, so 10/26 = 5/13 and 5 + 13 = 18.
8	5100 [meters]	How many meters are in 5.1 kilometers? $5.1 \cdot 1000 = 5100$
9	16 [marbles]	A dish has 4 green marbles, 7 clear marbles, and 8 orange marbles. If marbles are randomly taken from the dish, what is the minimum number needed to be taken that would guarantee at least one of each color? 8 + 7 + 1 = 16
10	[x =] 15	Solve the equation for x: 6x – 21 = 69 $6x - 21 = 69 \rightarrow 6x = 90 \rightarrow x = 15$
11	14 [min]	Lincoln can wash 12 plates in 3 minutes. In minutes, how long does it take him to wash 56 plates? 12 plates in 3 min is equivalent to 4 plates per minute and 56/4 = 14

|		
12	40 [%]	What percent of 25 is 10? $10/25 = x/100 \rightarrow x = 40$
13	187	What is the 7th term in the arithmetic sequence whose first three terms are 13, 42, and 71? 13, 42, 71, 100, 129, 158, 187
14	20 [%]	A number is randomly selected from the integers 1 through 20. As a percent, what is the probability that the selected number is larger than 16? There four numbers larger than 16, so P(larger than 16) = 4/20 = 1/5 = 20%
15	5 [ways/pat hs]	How many ways are there to draw a path from A to B that must only lie on top of horizontal and/or vertical segments in the figure shown and in which your pencil can only move right or down? The number of paths from A to each vertex is shown in the figure, so the number of paths from A to B is 5.
16	13	What is the largest prime factor of 104? $104 = 2^3 \cdot 13$ and 13 is the largest prime factor
17	[A + B =] 186	As a reduced common fraction, the sum of the following two numbers is A/B. What is A + B? $1\frac{3}{5} + 2\frac{5}{7}$ $1\frac{3}{5} + 2\frac{5}{7} = \frac{8}{5} + \frac{19}{7} = \frac{56}{35} + \frac{95}{35} = \frac{151}{35}$ and 151 + 35 = 186
18	21	What is the median of the following set of numbers? 22, 15, 31, 18, 23, 17, 20, 32 The list in ascending order is 15, 17, 18, 20, 22, 23, 31, 32, and the median is (20 + 22)/2 = 21.
19	[x =] 22	What is the smallest integer solution to the inequality? 72 < 3x + 8 $72 < 3x + 8 \rightarrow 64 < 3x \rightarrow 21.33 \ldots < x$, so the smallest integer value of x is 22
20	[A =] 49	The area of a circle with diameter 14 inches is Aπ in². What is A? If d = 14, then r = 7, and A = 49π, so A = 49.
21	6 [ways]	How many ways are there to arrange the letters in the word CROW, if C must be the third letter? In the arrangement _ _ C _, there are 3! = 6 ways to arrange the remaining 3 letters, so the answer is 6.
22	[P + Q =] 13	Let set A = {9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99}, set B = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95}, and set C. = {15, 30, 45, 60, 75, 90}. As a reduced common fraction, the probability that a randomly chosen number from set A is also a member of both sets B and C is P/Q. What is P + Q? There are 11 members of set A and two of them, 45 and 90, are in both sets B and C, so the probability is 2/11 and 2 + 11 = 13

23	8	What is the sum of the distinct prime factors of the number 2025?
		$2025 = 3^4 \cdot 5^2$, and $3 + 5 = 8$
24	105	The Fibonacci Sequence is 1, 1, 2, 3, 5, 8, and so on. A different increasing sequence (Let's call it the New Sequence), has 17 as its first term. The second term in the New Sequence results from adding the first term in the Fibonacci Sequence to the first term of the New Sequence. The third term in the New Sequence results from adding the second term in the Fibonacci Sequence to the second term of the New Sequence. If this pattern continues, what is the tenth term of the New Sequence?
		The New Sequence is 17, 18, 19, 21, 24, 29, 37, 50, 71, 105, and 105 is the 10th term.
25	72 [seconds]	Leon takes 96 seconds to complete one lap on the track at school. Ayanna runs at a rate that is 4/3 times as fast as Leon. How many seconds does it take Ayanna to complete one lap on the same track?
		If you run 4/3 times as fast it takes you 3/4 as long, so (3/4)(96) = 72
26	17 [cm]	In centimeters, what is the length of $\overline{AB}$ in the figure below? Note: All external edges of the figure are either horizontal or vertical. The rectangle that encloses the figure is 8cm by 15 cm, and since $8^2 + 15^2 = 17^2$, then AB = 17 cm.
27	[A + B =] 46	The following expression simplifies to the reduced common fraction A/B. What is A + B? $$\frac{10^3 - 10}{5^5 + 5^2}$$ $\frac{10^3-10}{5^5+5^2}$ = [10(10² – 1)]/[5²(5³ + 1)] = (10·99)/(25·126) = (2·99)/(5·126) = 99/(5·63) = 11/(5·7) = 11/35 and 11 + 35 = 46
28	99	What is the largest possible number in a data set with 6 distinct positive integers and a mean of 19?
		6 · 19 = 114, so the data set could be 1, 2, 3, 4, 5, and 99
29	[P + Q =] 237	The lengths of two parallel sides of square ABCD are multiplied by 1.7 and the lengths of the other two parallel sides are multiplied by 2.2 to create rectangle EFGH. As a reduced common fraction, the ratio of the area of rectangle EFGH to the area of square ABCD is P/Q. What is P + Q?
		Let the side length of square ABCD be 10 and the area of square ABCD be 100. The sides of rectangle EFGH are 17 and 22 and the area of rectangle EFGH is 374. 374/100 = 187/50 and 187 + 50 = 237
30	32 [nickels]	Nayeli has 1.25 times as many dimes as nickels and the value of her dimes and nickels is $5.60. How many nickels does she have?
		D = 1.25N and 12.5D + 5N = 560 → 12.5N + 5N = 560 → 17.5N = 560 → N = 32

31	200 [seconds]	A bathtub takes 2 minutes to fill up when the drain is closed and 5 minutes to fill up when the drain is open. In seconds, how long does it take to empty a full tub?
		The tub fills at a rate of 1/2 of the tub per minute when the drain is closed and 1/5 of the tub per minute when the drain is open. The difference between these two is the rate of the tub emptying when the drain is open, which we'll call $1/x$, so $1/2 - 1/5 = 1/x$. In this equation, x represents the time in minutes that it takes to drain the tub. $1/2 - 1/5 = 1/x \rightarrow 5x - 2x = 10 \rightarrow 3x = 10 \rightarrow x = 10/3$ minutes = 200 seconds
32	[b =] 15	The base-10 number $279 = 139_b$, where $b > 0$. What is the value of b?
		$279 = b^2 + 3b + 9 \rightarrow b^2 + 3b - 270 = 0 \rightarrow (b + 18)(b - 15) = 0 \rightarrow b = -18$ or $b = 15$, so the positive base is 15.
33	128 [residents]	At a summer ping pong academy for teenagers 1/8 of the residents are left-handed and 7/8 of the residents are right-handed. Exactly 1/4 of the left-handed residents and 3/16 of the right-handed residents use a red paddle. What is the smallest possible number of residents at the academy?
		The ratio of left-handed to right-handed residents is $1/8 : 7/8 = 1 : 7$. Possible pairings of number of (left, right) include: (1, 7), (2, 14), (3, 21), (4, 28), (5, 35), (6, 42), (7, 49), (8, 56), (9, 63), (10, 70), (11, 77), (12, 84), (13, 91), 14, 98), (15, 105), (16, 112), and so on. The first pairing in which taking 1/4 of the first number and 3/16 of the second number both result in integers is (16, 112), so the minimum possible number of residents is 16 + 112 = 128.
34	2	What is the mean of all integers that are 4 more than 117 times their reciprocal?
		Let x = any integer that is 4 more than 117 times its reciprocal. Then $x = 117/x + 4 \rightarrow x^2 = 117 + 4x \rightarrow x^2 - 4x - 117 = 0 \rightarrow (x - 13)(x + 9) = 0 \rightarrow x = 13$ and $x = -9$, and $(13 + -9)/2 = 2$
35	29	A data set consists of 6 positive integers, 5 of which are distinct from each other and one that is the same as one of the 5 distinct integers. The mean and the mode of the data set are both 10. What is the positive difference between the greatest possible range and the smallest possible range of a data set that meets these conditions?
		Two of the numbers must be 10, so the remaining four numbers must add up to 40. It is not possible for all four remaining numbers to be larger than 10 or for all of them to be smaller than 10. If three of the remaining numbers are smaller than 10, the largest and smallest possibilities for the range are {1, 2, 3, 10, 10, 34} and {7, 8, 9, 10, 10, 16}. If three of the remaining numbers are larger than 10, the largest and smallest possibilities for the range are {1, 10, 10, 12, 13, 14} and {4, 10, 10, 11, 12, 13}. If two of the remaining numbers are less than 10 and two are greater than 10, the largest and smallest possibilities for the range are {1, 2, 10, 10, 11, 26} and {{8, 9, 10, 10, 11, 12}. The greatest possible range is $34 - 1 = 33$ and the smallest possible range is $12 - 8 = 4$, and $33 - 4 = 29$.
36	[A + B =] 8	Josue and Felicity take turns rolling a fair 12-sided die with faces numbered 1 through 12. The first player to roll an odd prime number is the winner of the game. If Felicity goes first, as a reduced common fraction, the probability that she wins the game is A/B. What is A + B?
		P(Felicity wins) = $1/3 + (2/3)^2 \cdot 1/3 + (2/3)^4 \cdot 1/3 + \ldots = 1/3(1 + (2/3)^2 + (2/3)^4 + \ldots) = S \rightarrow$ $3S = 1 + (2/3)^2 + (2/3)^4 + \ldots \rightarrow$ $(2/3)^2 \cdot 3S = (2/3)^2 + (2/3)^4 + (2/3)^6 + \ldots \rightarrow$ $4S/3 = (2/3)^2 + (2/3)^4 + (2/3)^6 + \ldots$ Subtract two of the equations: $3S = 1 + \cancel{(2/3)^2} + \cancel{(2/3)^4} + \cancel{(2/3)^6} + \ldots$ $- 4S/3 = \cancel{(2/3)^2} + \cancel{(2/3)^4} + \cancel{(2/3)^6} + \ldots$ $5S/3 = 1 \rightarrow S = 3/5$, and $3 + 5 = 8$

| 37 | 576 [integers] | How many 5-digit positive integers can be created using the digits 2, 3, 4, 5, 6, 7, and 8, such that all 5 digits in a given integer are different and no two even digits may be next to each other?

Case 1: there are 2 even digits in which case the number would have these patterns: EOEOO, OEOEO, OOEOE, EOOEO, OEOOE, or EOOOE. In all 6 patterns there are $_4P_2$ ways to arrange the even digits and 3! ways to arrange the odd digits, so the total for case 1 would be $6 \cdot _4P_2 \cdot 3! = 6 \cdot 12 \cdot 6 = 432$.
Case 2: there are 3 even digits in which case the number would have this pattern: EOEOE. In this pattern there are $_4P_3$ ways to arrange the even digits and $_3P_2$ ways to arrange the odd digits, so the total for case 2 would be $_4P_3 \cdot _3P_2 = 24 \cdot 6 = 144$.
$432 + 144 = 576$ |
| 38 | [M + N + P + Q =] 331 | In the figure, AB = BC = CD = BD = 9 cm and $\overline{AB} \perp \overline{BC}$. In simplified form, the area of ΔBCE can be expressed as $\frac{M-N\sqrt{P}}{Q}$, where M, N, P, and Q are positive integers, GCF(M, N, Q) = 1, and the largest perfect square factor of P is 1. What is M + N + P + Q?

By adding an altitude from vertex E (see below), ΔBCE is divided into a 30-60-90 and a 45-45-90 triangle, so BC = $\frac{x}{\sqrt{3}}$ + x = 9.
Then solve $\frac{x}{\sqrt{3}}$ + x = 9 for x, $\frac{x}{\sqrt{3}}$ + x = 9 → x + x$\sqrt{3}$ = 9$\sqrt{3}$ → x(1 + $\sqrt{3}$) = 9$\sqrt{3}$ → x = $\frac{9\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$ → x = $\frac{9\sqrt{3}-27}{-2}$ = $\frac{27-9\sqrt{3}}{2}$, which is the height of ΔBCE. The base of ΔBCE = 9, so the area of ΔBCE is $\frac{1}{2} \cdot 9 \cdot \frac{27-9\sqrt{3}}{2}$ = $\frac{243-81\sqrt{3}}{4}$, and 243 + 81 + 3 + 4 = 331.
 |

39	[A + B =] 130	Two real numbers from 0 to 9, inclusive, are chosen at random. As a reduced common fraction, the probability that the positive difference between the two numbers is greater than 2 is A/B. What is A + B?
		Let x = the first number selected and y = the second number selected. Then $|x - y| > 2$, which means x – y > 2 and x – y < -2. Put both inequalities in slope intercept form and you get y < x – 2 and y > x + 2. The graphs of these inequalities are the two lighter regions in the figure below and the area of the lighter regions as a fraction of the whole region is 49/81, and 49 + 81 = 130.

| 40 | 10 | Each of the letters A, B, C, D, E, F and G in the figure shown will be replaced with a different value from the set {2, 3, 4, 5, 6, 7, 8}. If the sums of the four numbers at the vertices of quadrilaterals ABGF, BCDG, and DEFG are each 18, what is the sum of all numbers that can replace G? |
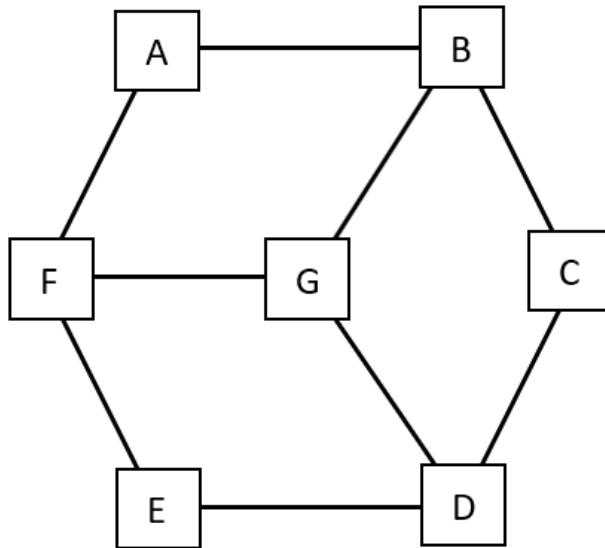
If each quadrilateral has a sum of 18, the total of the three quadrilaterals will be 54, and in than sum A, C, and E will each be used once, B, D, and F will each be used twice, and G will be used three times. So you can write the equation $a + c + e + 2(b + d + f) + 3g = 54$. The table shows the possibilities:

	Possible sums: a + c + e	Possible values: 2(b + d + f)	
If g = 2, then a + c + e + 2(b + d + f) = 48	12	2 · 21 = 42	3 ways to make 48:
	13	2 · 20 = 40	
	14	2 · 19 = 38	3 + 7 + 8 + 2(4 + 5 + 6) -doesn't work
	15	2 · 18 = 36	
	16	2 · 17 = 34	
	17	2 · 16 = 32	**4 + 6 + 8 + 2(3 + 5 + 7) -**
	18	2 · 15 = 30	**works**
	19	2 · 14 = 28	
	20	2 · 13 = 26	5 + 6 + 7 + 2(3 + 4 + 8) – doesn't work
	21	2 · 12 = 24	
If g = 3, then a + c + e + 2(b + d + f) = 45	11	2 · 21 = 42	2 ways to make 45:
	12	2 · 20 = 40	
	13	2 · 19 = 38	**4 + 7 + 8 + 2(2 + 5 + 6) –**
	14	2 · 18 = 36	**works**
	15	2 · 17 = 34	
	16	2 · 16 = 32	5 + 6 + 8 + 2(2 + 4 + 7) – doesn't work
	17	2 · 15 = 30	
	18	2 · 14 = 28	
	19	2 · 13 = 26	
	20	2 · 12 = 24	
	21	2 · 11 = 22	
If g = 4, then a + c + e + 2(b + d	10	2 · 21 = 42	1 way to make 42:
	11	2 · 20 = 40	
	12	2 · 19 = 38	
	13	2 · 18 = 36	

+ f) = 42	14 15 16 17 18 19 20 21	$2 \cdot 17 = 34$ $2 \cdot 16 = 32$ $2 \cdot 15 = 30$ $2 \cdot 14 = 28$ $2 \cdot 13 = 26$ $2 \cdot 12 = 24$ $2 \cdot 11 = 22$ $2 \cdot 10 = 20$	5 + 7 + 8 + 2(2 + 3 + 6) – doesn't work	
If g = 5, then a + c + e + 2(b + d + f) = 39	9 10 11 12 13 14 15 16 17 18 19 20 21	$2 \cdot 21 = 42$ $2 \cdot 20 = 40$ $2 \cdot 19 = 38$ $2 \cdot 18 = 36$ $2 \cdot 17 = 34$ $2 \cdot 16 = 32$ $2 \cdot 15 = 30$ $2 \cdot 14 = 28$ $2 \cdot 13 = 26$ $2 \cdot 11 = 24$ $2 \cdot 12 = 22$ $2 \cdot 10 = 20$ $2 \cdot 9 = 18$	1 way to make 39: **6 + 7 = 8 + 2(2 + 3 + 4) – works**	
If g = 6, then a + c + e + 2(b + d + f) = 36	9 10 11 12 13 14 15 16 17 18 19 20	$2 \cdot 20 = 40$ $2 \cdot 19 = 38$ $2 \cdot 18 = 36$ $2 \cdot 17 = 34$ $2 \cdot 16 = 32$ $2 \cdot 15 = 30$ $2 \cdot 14 = 28$ $2 \cdot 13 = 26$ $2 \cdot 12 = 24$ $2 \cdot 11 = 22$ $2 \cdot 10 = 20$ $2 \cdot 9 = 18$	All of the possible sums here of a + c + e + 2(b + d + f) are greater than 36, so 6 cannot be g. For the same reason, 7 and 8 cannot be g either.	

The values of g that have a solution are 2, 3, and 5, so the answer is 2 + 3 + 5 = 10.

"Math is Cool" Championships -- 2024-25
<mark>7th/8th grade</mark>
Multiple Choice Solutions

	Answer	Solution
		USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #3.

1	A	The sum of the coordinates of point A is 0 and the sum of the coordinates of point B is -2. What is the sum of the coordinates of the point where $\overleftrightarrow{AB}$ crosses the y-axis (aka: the y-intercept)? A) -1 B) -1/2 C) 0 D) $\frac{1}{2}$ E) 1 The y-intercept is (0, -1), and 0 + -1 = -1
2	C	What is the slope of $\overleftrightarrow{AB}$? A) -13/10 B) -11/10 C) -6/5 D) 5/6 E) 10/11 (-7 − 5)/(5 − -5) = -12/10 = -6/5

3	B	Point C is three units to the right of point A and point D is ten units straight up from point B. What is the distance between C and D? A) $2\sqrt{10}$ units B) $\sqrt{53}$ units C) $\sqrt{58}$ units D) $2\sqrt{17}$ units E) 9 units The distance between C(-2, 5) and D(5, 3) is $\sqrt{7^2 + 2^2} = \sqrt{53}$.

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #4 THROUGH #6.

When you divide two numbers, if the divisor does not go into the dividend evenly, there will be a remainder. For example, the divisor 8 goes into the dividend 50 six times, with a remainder of 2. In other words, 50 ÷ 8 = 6 remainder 2. The following three problems have to do with remainders.

4	B	What is the remainder when 60 is divided by 7? A) 3 B) 4 C) 5 D) 6 E) 7 60/7 = 8r4
5	D	When A is divided by 12, the remainder is 11. When A is divided by 15, the remainder is 2. What is the largest 2-digit value of A? A) 23 B) 32 C) 35 D) 47 E) 94 Let A = 12x + 11 and let A = 15x + 2, where x is an integer. Then 12x + 11 = 15x + 2 → 3x = 9 → x = 3 and the smallest positive value of A is 47. The next value of A will be 47 + 60 = 107, where 60 = LCM(12, 15), so 47 is also the largest 2-digit value of A.
6	E	What is the remainder when 2^{2025} is divided by 40? A) 4 B) 8 C) 16 D) 24 E) 32 2^1/40 = 0r2 2^2/40 = 0r4 2^3/40 = 0r8 2^4/40 = 0r16 2^5/40 = 0r32 2^6/40 = 1r24 2^7/40 = 3r8 2^8/40 = 6r16 2^9/40 = 12r32 2^{10}/40 = 25r24 2^{11}/40 = 51r8 2^{12}/40 = 102r16 After the first four powers of 2, the pattern in the remainder is 32, 24, 8, 16, which repeats every 4 powers of 2. 2025/4 has a remainder of 1, so the first remainder in the pattern will be the remainder of 2^{2025}/40, which is 32.

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #7 THROUGH #10.

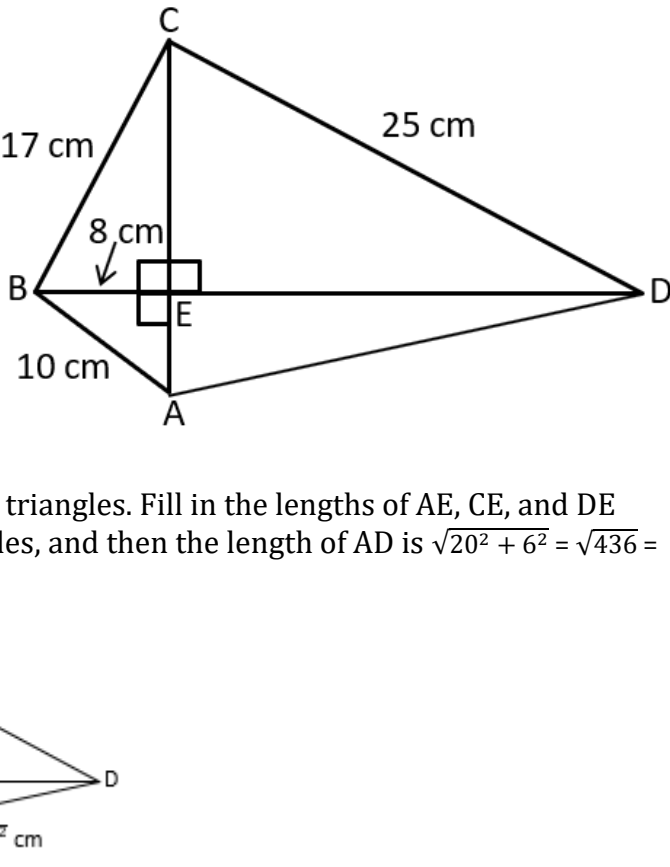The following four problems involve solving algebraic equations.

7	C	Solve for x: 20 + 15x = 260
		A) -16 B) -13.75 C) 16 D) 17 E) 56/3
		20 + 15x = 260 → 15x = 240 → x = 16

8	D	Solve the equation $C = \frac{5}{9}(F - 32)$ for F.
		A) $F = \frac{5}{9}C + 32$ B) $F = \frac{5}{9}(C - 32)$ C) F = 1.8(C + 32)
		D) $F = \frac{9}{5}C + 32$ E) F = 1.8C - 57.6
		$C = \frac{5}{9}(F - 32) \rightarrow \frac{9}{5}C = F - 32 \rightarrow F = \frac{9}{5}C + 32$

9	E	If x + 3/y = 1/7 and y + 3/x = 84, what is the value of the product xy?
		A) -3 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) 3
		If x + 3/y = 1/7 and y + 3/x = 84, then (x + 3/y)(y + 3/x) = (1/7)(84) → xy + 3 + 3 + 9/xy = 12 → xy + 9/xy = 6 → (xy)² + 9 = 6xy → (xy)² – 6xy + 9 = 0 → (xy – 3)(xy – 3) = 0 → xy = 3

10	B	Given a – b = 6 and a² + b² = 100, what is the value of a³ – b³?
		A) 360 B) 792 C) 1208 D) 1440 E) Answer not given
		If a – b = 6, then (a – b)² = 6² → a² – 2ab + b² = 36 → a² + b² – 2ab = 36 → since a² + b² = 100, replace a² + b² with 100 to get 100 – 2ab = 36 → -2ab = -64 → ab = 32.
		If a – b = 6, then (a – b)³ = 6³ → a³ – 3a²b + 3ab² – b³ = 216 → a³ – b³ – 3ab(a – b) = 216 → Since ab = 32 and a – b = 6, replace ab with 32 and a – b with 6 to get a³ – b³ – 3(32)(6) = 216 → a³ – b³ – 576 = 216 → a³ – b³ = 792

Team Test Solutions

	Answer	Solution
1	10	Evaluate: $\frac{(3-6)^3}{(-9)} - 17 + ((-5)^2 - 1)$ $\frac{(3-6)^3}{(-9)} - 17 + ((-5)^2 - 1) = \frac{-27}{-9} - 17 + (25 - 1) = 3 - 17 + 24 = 10$
2	50 [students]	In the 8th grade class of Park Middle School, there are 87 students. One-third of these students have a dog. Twenty-three of them have a cat and 15 of them have both a dog and a cat. How many of the 87 students don't have a cat or a dog? Based on the two-way table, 50 have neither. <table><tr><td></td><td>Cat</td><td>No Cat</td><td>Total</td></tr><tr><td>Dog</td><td>15</td><td>14</td><td>29</td></tr><tr><td>No Dog</td><td>8</td><td>50</td><td>58</td></tr><tr><td>Total</td><td>23</td><td>64</td><td>87</td></tr></table>
3	3 [prime numbers]	How many positive prime numbers less than 50 have 2 as at least 1 of their digits? 2, ~~21~~, 23, ~~25~~, ~~27~~, 29, ~~32~~, ~~42~~ makes 3
4	[A + B =] 8	As a common fraction, the median of the following data set is A/B. What is A + B? {4/7, 0.6, 5/8, 0.75, 5/12} 5/8 = 0.625 and 0.75 are both greater than 0.6. 5/12 is less than 1/2, so it is less than 0.6. 9/16 = 1/2 + 1/16 = 0.5 + 0.0625 = 0.5625, which is less than 0.6. So, 0.6 is the median, which equals 3/5 and 3 + 5 = 8.
5	22 [coins]	Tanishi has $4.37 in coins that consist only of quarters, dimes, nickels, and pennies. She has one nickel among her coins. What is the smallest total number of coins she could have? She could have $4.25 in quarters, or 17 quarters, and 1 nickel and 7 pennies, or 25 coins total. Or she could have $4.00 in quarters, or 16 quarters, 3 dimes, 1 nickel, and 2 pennies, for 22 coins total.

6	[P + Q =] 111	In the figure, AB = 10 cm, BC = 17 cm, CD = 25 cm, and BE = 8 cm. The length of AD is $P\sqrt{Q}$ cm, where P and Q are integers, and Q does not have a factor that is a perfect square other than 1. What is P + Q?

All 4 triangles are right triangles. Fill in the lengths of AE, CE, and DE using Pythagorean triples, and then the length of AD is $\sqrt{20^2 + 6^2} = \sqrt{436} = 2\sqrt{109}$, and 2 + 109 = 111.

7	5	A particular number series increases alternately by adding 14 and then by multiplying by 1/4. For example, if the first term in the series is 18, the next four terms would be 32, 8, 22, 22/4, and so on. What is the smallest possible integer in the series if the first term is 346?

The series would be 346, 360, 90, 104, 26, 40, 10, 24, 6, 20, 5, 19, 19/4, . . . and everything after 19 will not be an integer. The smallest integer in the list is 5.

8	[N =] 1728	If N is a perfect cube and a factor of 11!, what is the largest value of N?

$11! = 1 \cdot 2 \cdot 3 \cdot (2 \cdot 2) \cdot 5 \cdot (2 \cdot 3) \cdot 7 \cdot (2 \cdot 2 \cdot 2) \cdot (3 \cdot 3) \cdot (2 \cdot 5) \cdot 11$
$N = 2^6 \cdot 3^3 = 64 \cdot 27 = 1728$

9	[P + Q =] 31	Let A and B be two distinct three-digit positive integers, such that A + B > 1945, A > B, and A – B = D. The mean of all possible values of D is P.Q, where P is a two-digit integer and Q is a one-digit integer. What is P + Q?

999 + 947 = 1946 > 1945, and 999 – 947 = 52, which is the largest possible value of D.
974 + 973 = 1947 > 1945, and 974 – 973 = 1, which is the smallest possible value of D.
Every value between 1 and 52 is also possible for D, and the mean of {1, 2, 3, . . . , 50, 51, 52} is (1 + 52)/2 = 26.5 and 26 + 5 = 31.
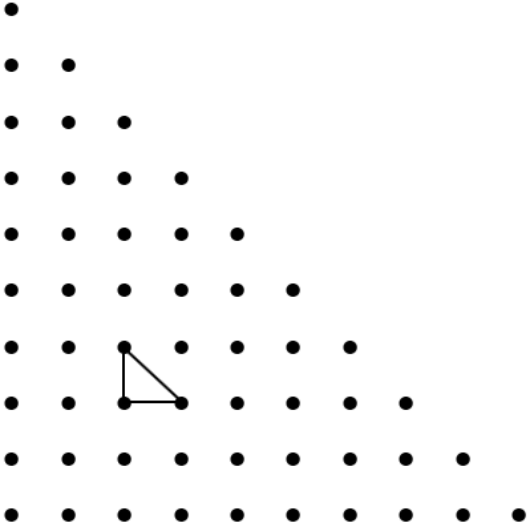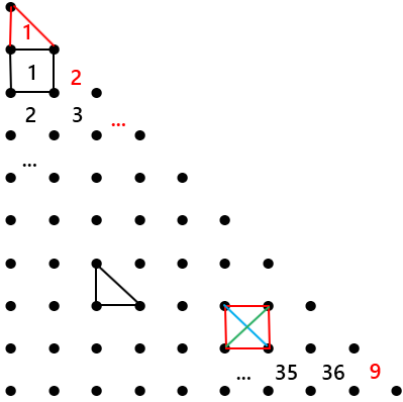
10	[A + B =] 4	In a hand of 6 cards there are 2 hearts, 2 diamonds, and 2 clubs. The cards are randomly selected one at a time without replacement and laid on a table. As a reduced common fraction, the probability that the 4th card laid on the table is a heart is A/B. What is A + B?

There are five possible scenarios:

$$P(2D, 1C, \text{followed by } 1H) = {}_3C_2 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} = 3 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} = 1/15$$

$$P(1D, 2C, \text{followed by } 1H) = {}_3C_1 \cdot \frac{2}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} = 3 \cdot \frac{2}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} = 1/15$$

$$P(2D, 1H, \text{followed by } 1H) = {}_3C_2 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 3 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 1/30$$

$$P(2C, 1H, \text{followed by } 1H) = {}_3C_2 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 3 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 1/30$$

$$P(1C, 1D, 1H, \text{followed by } 1H) = 3! \cdot \frac{2}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 6 \cdot \frac{2}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = 2/15$$

1/15 + 1/15 + 1/30 + 1/30 + 2/15 = 10/30 = 1/3, and 1 + 3 = 4

"Math is Cool" Championships -- 2024-25
<mark>7th/8th grade</mark>

<u>Linda Moore Triple Jump Solutions</u>

	Answer	Solution
1	702	Evaluate the following expression when a = 2 and b = 3: $9a^2b^2 + 7ab^3$ $9(2)^2(3)^2 + 7(2)(3)^3 = 9(4)(9) + 7(2)(27) = 324 + 378 = 702$
2	3785 [ml]	One gallon is equivalent to about 3.78541 liters. To the nearest integer number of milliliters, how many milliliters are in one gallon? **3.78541 L = 3785.41 ml = 3785 ml**
3	[A + B =] 6	Horace takes 5 minutes to walk one-fifth of a mile home from school. Horace's average rate of walking home from school is A.B miles per hour. What is A + B? 1/5 mile in 5 min = 1 mile in 25 min = 2.4 miles in 60 min = 2.4 miles per hour, and 2 + 4 = 6
4	2	The first three terms of a geometric sequence are 1458, 486, and 162. What is the 7th term in the sequence? 1458, 486, 162, 54, 18, 6, 2
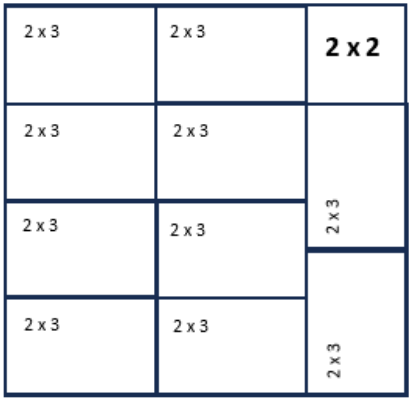
5	153 [triangles]	In the grid shown here, all dots are equally spaced both horizontally and vertically. One isosceles 45-45-90 triangle has been drawn. Including this triangle, how many triangles congruent to this one, in any orientation, can be drawn using the dots in the grid?
		Each unit square can be split into 4 congruent triangles, and there are 36 squares. 36*4 = 144. Additionally, there are 9 congruent triangles going down the diagonal. 144 + 9 = 153
6	[A + B =] 14	A pair of standard dice are rolled. As a reduced common fraction, the probability that the sum of the two numbers showing is greater than 4 and less than 9 is A/B. What is A + B? P(5, 6, 7, or 8) = (4 + 5 + 6 + 5)/36 = 20/36 = 5/9, and 5 + 9 = 14

| 7 | [A + B =]

189 | As a reduced common fraction, the following expression simplifies to A/B. What is A + B?

$$\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8}}}}$$

$\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\frac{7}{8}}}} = \cfrac{1}{2+\cfrac{3}{4+\frac{5}{\frac{55}{8}}}} = \cfrac{1}{2+\cfrac{3}{4+\frac{40}{55}}} = \cfrac{1}{2+\cfrac{3}{4+\frac{8}{11}}} = \cfrac{1}{2+\frac{3}{\frac{52}{11}}} = \cfrac{1}{2+\frac{33}{52}} = \cfrac{1}{\frac{137}{52}} = 52/137$, and

52 + 137 = 189 |
| 8 | 28 [= A] | A regular hexagon is inscribed in a circle, as shown here. Given that the area of the hexagon is $294\sqrt{3}\ cm^2$, the circumference of the circle can be written as $A\pi$ cm. What is A?

The hexagon can be divided into 6 equilateral triangles, each of which has a side length = r, the radius of the circle. The area of a regular hexagon is:

$$A = \frac{3\sqrt{3}}{2}r^2 = 294\sqrt{3}$$

Therefore, $r^2 = 196$, r = 14
Circumference = $2\pi r = 2\pi(14) = 28\pi$

 |

9	15 [days]	Juana can build a large shed in 5 days less than it takes her brother Carlos. If they built it together it would take them 6 days. How many days would it take Carlos to build the shed by himself? $J = C - 5$, where J is the number of days it takes Juana and C is the number of days it takes Carlos to build the shed. Juana builds $1/J$ or $1/(C - 5)$ of the shed per day and Carlos builds $1/C$ of the shed per day, so together $1/(C - 5) + 1/C = 1/6 \rightarrow 6C + 6(C - 5) = C(C - 5) \rightarrow 6C + 6C - 30 = C^2 - 5C \rightarrow C^2 - 17C + 30 = 0 \rightarrow (C - 15)(C - 2) = 0 \rightarrow C = 15$ and $J = 10$ or $C = 2$ and $J = -3$. The second solution is not possible, so the answer is 15 days.
10	$3211000_{[7]}$ [base 7]	What is the only 7-digit base-7 number in which the first digit on the left tells how many 0s, the second digit from left tells how many 1s, the third digit tells how many 2s, the fourth digit tells how many 3s, the fifth digit tells how many 4s, the sixth digit tells how many 5s, and the seventh digit tells how many 6s are in the number? You do not need to include the "base 7" in your answer. The number must have exactly two 1s. To see why consider the cases where there are three 1s: In the case of 1311_ _ _ one of the remaining digits would need to be a 2, which would mean that there are also two 4s, two 5s, or two 6s and therefore no 0s. Or in the case of _ 3111 _ _ there needs to be a 2 and a 4 in the remaining digits, neither of which would work for the 1st digit and therefore no digit would be correct in the 1st digit's position. For these same reasons, any larger number of 1s would also be impossible. Also consider the cases in which there is one 1: In the case of _ 1 _ _ _ _ _, whatever the number of 0s is would be indicated by a digit in the 1st digit's position. If the number of 0s is any other number other than 1, such as 2, then there would have to be another 1 in the number leading to a contradiction. In the case of 1 _ _ _ _ _ _, then the one and only 0 would need to be the 2nd digit, in which case it would be impossible for none of the remaining digits to be 0. Also consider the cases in which there are no 1s: In the case of 202 _ _ _ _, say any of the remaining digits was a 3, such as the 4th digit. Then there would have to be three 3s among the remaining digits meaning there would not only be three 3s, but also three 4s, three 5s, or three 6s, but this would be impossible. The two 1s would need to not be in the 1st or 2nd digit's positions, 2 would be in the 2nd digit's position and 1 would be in the 3rd digit's position, or _ 21 _ _ _ _. If one of the remaining positions, 4th through 7th, is a 1, then there could be three 0s, so the digit in position 1 is a 3, and the number would be 3211000.

College Bowl Round #1 Solutions

	Answer	**Solution**
1	19 [even integers]	How many even integers are there between eleven and forty-nine? 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, and 48 make 19 even numbers
2	[y =] 8	If three X plus four Y equals sixty-five and X equals eleven, what does Y equal? $3x + 4y = 65$, $x = 11 \rightarrow 33 + 4y = 65 \rightarrow 4y = 32 \rightarrow y = 8$
3	15 [minutes]	Biff can prepare sixteen bags of tater tots in twenty minutes. Eho prepares tater tots at a rate that is twice as fast as Biff. How many minutes does it take Eho to prepare twenty-four bags of tater tots? Biff: 16 bags in 20 min Eho: 16 bags in 10 min or 8 bags in 5 min or 24 bags in 15 min
4	86 [%]	A certain bag of M-and-Ms contains ten green, twenty-two brown, six red, seven yellow, and five blue M-and-Ms. If one M-and-M is randomly chosen, what is the probability in percent that it is not yellow? 50 m&ms total 43 are not yellow $43/50 = 86/100 = 86\%$
5	8 [inches]	An eight-inch-by-eight-inch square is cut out from a piece of paper. It is then cut into as many two-inch-by-three-inch rectangles as possible, such that there is only one scrap leftover. In inches, what is the perimeter of the scrap? There are 10 congruent 2 x 3 rectangles in the figure below and the scrap is a 2 x 2 square with a perimeter of 8 inches.

6	78	What is the largest two-digit integer that is the product of three distinct prime numbers? $2 \cdot 3 \cdot 5 = 30$, $2 \cdot 3 \cdot 7 = 42$, $2 \cdot 3 \cdot 11 = 55$, $2 \cdot 3 \cdot 13 = 78$, $2 \cdot 3 \cdot 17 = 102$, $2 \cdot 5 \cdot 7 = 70$, $2 \cdot 5 \cdot 11 = 110$, $2 \cdot 7 \cdot 11 = 154$, $3 \cdot 5 \cdot 7 = 105$, 78 is the largest 2-digit number in this list
7	[A + B =] 20	An infinite series begins with the terms one-third, two-fourths, three-fifths, and so on. The ninth term in the series is A over B. What is A plus B? 1/3, 2/4, 3/5, 4/6, 5/7, 6/8, 7/9, 8/10, 9/11 11 + 9 = 20
8	55 [= mean]	What is the mean of the two-digit positive multiples of twenty-two? The multiples are: 22, 44, 66, 88. Since this is an arithmetic sequence, the mean is the same as the median, or (44 + 66)/2 = 55. Or, 22+44+66+88 = 220, 220/4 = 55
9	988	Let A be a single-digit positive integer, let B be a two-digit positive integer, and let C be a three-digit positive integer. What is the largest possible positive difference between C and the sum of A and B? If A = 1, B = 10, and C = 999, then 999 – 11 = 988
10	18107	Let A equal one plus twenty plus three hundred plus four thousand plus fifty thousand. What is A divided by three? (1 + 20 + 300 + 4000 + 50000)/3 = 54321/3 = 18107

College Bowl Round #2 Solutions

	Answer	Solution
1	27	What is the greatest positive difference between any two prime numbers less than thirty? 29 – 2 = 27
2	360 [in²]	How many square inches are in two and a half square feet? $2.5 \cdot 144 = 360$
3	[2A =] 28	As a reduced common fraction, one-third plus three-fifths equals A over B. What is two times A? 1/3 + 3/5 = 5/15 + 9/15 = 14/15, 2(14) = 28
4	4	The first three terms of a sequence are twelve, twenty, and eight, where starting with the third term, each successive term is the positive difference between the previous two terms. What is the one hundredth term in this sequence? 12, 20, 8, 12, 4, 8, 4, 4, 0, 4, 4, 0, 4, 4, 0, . . . , 4, 4, 0, 4 After the first 6 terms the three number pattern 4, 4, 0, repeats indefinitely, so after repeating 31 times, the 99th term is 0, and the 100th term will be 4.
5	[B =] 8	The expression six X squared plus X minus forty can be factored into the quantity two X minus A times the quantity three X plus B. What is B? $6x^2 + x - 40 =$ $(2x - 5)(3x + 8)$
6	8 [integers]	How many positive integers less than two hundred and fifty are multiples of four and ten, but not six? Positive integer multiples of 4 and 10, but not 6: 20, 40, ~~60~~, 80, 100, ~~120~~, 140, 160, ~~180~~, 200, 220, ~~240~~, there are 8 in the list that aren't crossed out.
7	24 [minutes]	Bert can decorate three cakes in an hour. Ernie decorates cakes at a rate that is one and a half times Bert's rate. How many minutes would it take the two of them working together to decorate three cakes? Bert: 3 cakes/hr Ernie: 4.5 cakes/hr Together 7.5 cakes/hr 3/7.5 = x/60 → x = 24

8	[A + B =] 170	As a reduced common fraction, the probability of drawing two cards one at a time with replacement, from a standard deck, that are both aces is A over B. What is A plus B? $4/52 \cdot 4/52 = 1/13 \cdot 1/13 = 1/169$, and $1 + 169 = 170$
9	500	Thirty-five is seven percent of what number? $7/100 = 35/x \rightarrow x = 500$
10	99 [o]	The measures of three of the angles in a convex quadrilateral are sixty-one, ninety-two, and one hundred and eight. How many degrees are in the measure of the fourth angle? $360 - 61 - 92 - 108 = 99$

College Bowl Round #3 Solutions

	Answer	Solution
1	[A + B =] 11	An ultimate frisbee team has twelve girls and nine boys. As a reduced common fraction, the ratio of girls to total players on the team is A over B. What is A plus B? 12/21 = 4/7, and 4 + 7 = 11
2	[A + B =] 101	As a reduced common fraction, the value of X in the equation five X over eight equals seven over nine is A over B. What is A plus B? 5x/8 = 7/9 → 45x = 56 → x = 56/45, and 56 + 45 = 101
3	530	What is thirty times fifty-three divided by three? 30 · 53 ÷ 3 = 530
4	17 [= 6th term]	In a sequence the fourth term is the mean of the first three terms, and each successive term is the mean of the previous three terms. If the first three terms are negative fifteen, one, and thirty-two, what is the sixth term? -15, 1, 32, 6, 13, 17
5	48 [%]	The probability that the Mustangs will win any given game is sixty percent. As a percentage, what is the probability that they will win exactly one of their next two games? There are two ways they can win 1 of their next 2, WL, or LW, so the answer is 2 · 6/10 · 4/10 = 48/100 = 48%
6	24 [integers]	How many integers are greater than seven point nine and less than thirty-one point one? 8 through 31 makes 24 integers
7	50	A data set contains seven distinct positive integers, the median is thirteen, and the mean is fourteen. What is the largest possible number in the set? 7 · 14 = 98 1 + 2 + 3 + 13 + 14 + 15 = 48 98 – 48 = 50
8	26 [°]	In degrees, what is the measure of the complement of the supplement of a one-hundred and sixteen degree angle? 180 – 116 = 64, and 90 – 64 = 26
9	12 [years old]	Abe is six years older than Ben, who is currently ten years old. How old in years was Abe when he was twice as old as Ben? Abe = 16 and Ben = 10. Four years ago Abe = 12 and Ben = 6, so Abe was 12

10	75 [cabinets]	If three carpenters can build twenty-seven cabinets in six days, how many cabinets can five carpenters build in ten days? 3 carpenters : 27 cabinets : 6 days → 1 carpenter : 9 cabinets : 6 days → 1 carpenter : 15 cabinets : 10 days → 5 carpenters : 75 cabinets : 10 days

College Bowl Round #4 Solutions

	Answer	Solution
1	6391	What is eighty-three times seventy-seven? (80 + 3)(80 – 3) = 6400 – 9 = 6391
2	364	What is the sum of the first six terms of the geometric sequence whose first three terms are one, three, and nine? 1 + 3 + 9+ 27 + 81 + 243 = 364
3	12	Each of the smallest six positive prime numbers is doubled to make a set of six even integers. What is the median of the set of six even integers? The set is 4, 6, 10, 14, 22, 26, and the median is (10 + 14)/2 = 12
4	231 [minutes]	Jenna took sixty-three minutes to read fifty-seven pages of her current book which is two hundred and sixty-six pages long. At this rate, how many minutes will it take her to read the rest of the book? 266 – 57 = 209 pages left. 57 pages in 63 minutes → 19 pages in 21 minutes → 190 pages in 210 minutes → 209 pages in 231 minutes
5	27	What is the smallest two-digit composite number that has exactly two distinct composite numbers as factors? Prime numbers will have zero composite factors. This eliminates 11, 13, 17, 19, 23, 29, . . . Numbers that are the product of two prime numbers will have one composite factor. This eliminates 10, 14, 15, 21, 22, 25, 26, 33, . . . Checking what remains, 12, 16, 18, and 20 all have 3 composite factors. 24 has 4 composite factors. The factors of 27 are 1, 3, 9, and 27, and exactly two of the factors are composite, 9 and 27.
6	60 [%]	Let A be one hundred and fifty percent of B. Let C be two hundred and fifty percent of D. Let A equal C. What percent of B is D? A = 1.5B and C = 2.5D, so 1.5B = 2.5D → D = 1.5B/2.5 = 0.6B, so D is 60 percent of B.

7	[A + B =] 7	A cone and a cylinder have the same volume. The cylinder has a radius that is half the radius of the cone. As a reduced common fraction, the ratio of the height of the cylinder to the height of the cone is A over B. What is A plus B? Let r = the radius of the cone, h = the height of the cone, and H = the height of the cylinder, then $\pi r^2 h/3 = \pi(r/2)^2 H \rightarrow r^2 h/3 = r^2 H/4 \rightarrow h/3 = H/4 \rightarrow 4h = 3H \rightarrow H/h = 4/3$, and 4 + 3 = 7
8	48 [inches]	A fathom is equivalent to six feet. The average depth of Iceland's Blue Lagoon is two-thirds of a fathom. What is the average depth of Blue Lagoon in inches? $2/3 \cdot 6 = 4$ and $4 \cdot 12 = 48$
9	[x =] 11	Solve the following equation for X: five X over two equals one hundred sixty-five over six $5x/2 = 165/6 \rightarrow 15x/6 = 165/6 \rightarrow 15x = 165 \rightarrow x = 11$
10	[A + B =] 9	As a reduced common fraction, the ratio of the number of positive single-digit even integers to the number of positive single-digit odd integers is A over B. What is A plus B? Even: 2, 4, 6, 8 Odd: 1, 3, 5, 7, 9 The ratio is 4/5, and 4 + 5 = 9

College Bowl Round #5 Solutions

	Answer	Solution
1	30 [%]	Shira has six dimes, seven pennies, two nickels, and five quarters in her pocket. If she randomly selects one coin from her pocket, what is the probability in percent that it is a dime? P(dime) = 6/(6 + 7 + 2 + 5) = 6/20 = 30/100 = 30%
2	26 [centimeters]	In centimeters, what is the diameter of a circle whose area is one hundred and sixty-nine pi square centimeters? A = $169\pi \to r^2$ =169 → r = 13 → d = 26
3	76	Serena's scores on her last three rounds of golf were eighty-two, eighty-five, and seventy-seven. What score does she need on her next round to lower her average to exactly eighty? 82 + 85 + 77 + x = 4 · 80 → 244 + x = 320 → x = 76
4	[D – E =] 16	The absolute value of A minus B equals eight and the absolute value of between B minus C equals nine. Let D equal the greatest possible absolute value of A minus C, and let E equal the smallest possible absolute value of A minus C. What is D – E? If A > B > C, then $\|A - C\|$ = 17, for example if A = 50, B = 42, and C = 33, then $\|50 - 33\|$ = 17 = D. If A < B and C < B, then $\|A - C\|$= 1, for example if A = 42, B = 50, and C = 41, then $\|42 - 41\|$= 1 = E. And 17 – 1 = 16.
5	91	What is the smallest possible sum of two positive integers whose greatest common factor is thirteen and whose least common multiple is one hundred and thirty? 26 = 2 · 13 and 65 = 5 · 13, so GCF(26, 65) = 13, and LCM(26, 65) = 130, and 26 + 65 = 91
6	500 [seconds]	Angela and Megan begin at the same place and time to run around a circular four-hundred-meter track. Angela's average rate is two hundred and forty meters per minute and Megan's average rate is twenty percent faster than Angela's. After how many seconds will Megan pass Angela for the first time? Angela runs 240 meters per minute and Megan runs 1.2 · 240 = 288 meters per minute, so she extends her lead by 48 meters per minute. 400/48 = $8\frac{1}{3}$ minutes = 500 seconds.

7	22 [times]	Every page of a one-hundred and twenty-one-page book is numbered. How many times does the digit six appear in the page numbering? 6, 16, 26, 36, 46, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 86, 96, 106, 116, 6 appears 22 times
8	6	What is three-eighths divided by one sixteenth? $3/8 \div 1/16 = 3/8 \cdot 16 = 6$
9	35 [orange or purple skittles]	One-fourth of the Skittles in a bag of sixty are green, one-sixth are yellow, three-tenths are purple, and the rest are orange. How many Skittles are orange or purple? Green: 15, yellow: 10, purple: 18, orange: 17 18+17 = 35
10	[x =] 18	Solve the following equation for X: seven X minus twenty-three equals negative two X plus one hundred thirty-nine $7x - 23 = -2x + 139 \rightarrow 9x = 162 \rightarrow x = 18$

College Bowl Round #6 Solutions

	Answer	Solution
1	2800 [mm]	How many millimeters are in two and four-fifths of a meter? 2.8 · 1000 = 2800
2	[A =] 32	If A equals four B, and three A minus five B equals fifty-six, what is A? a = 4b and 3a – 5b = 56 → 12b – 5b = 56 → 7b = 56 → b = 8 and a = 4b = 32
3	25 [boards]	Rae can paint ten boards in sixteen minutes. At this rate, how many boards can she paint in forty minutes? 10 boards in 16 minutes → 2.5 boards in 4 minutes → 25 boards in 40 minutes
4	40 [%]	A jar contains twenty marbles that are all either green or blue. After one-fourth of the green marbles are removed there are then seventeen marbles left in the jar. As a percentage, what is the probability of randomly selecting a blue marble from the original contents of the jar? 20 – 17 = 3, so there are 3 · 4 = 12 green marbles in the jar originally, and 20 – 12 = 8 blue marbles originally, so P(blue) = 8/20 = 40/100 = 40%
5	4 [integers]	How many positive two-digit integers less than forty are there, such that each digit is a different positive factor of the integer? ~~11~~, 12, ~~13, 14,~~ 15, ~~16, 17, 18, 19, 20, 21, 22, 23,~~ 24, ~~25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35,~~ 36, ~~37, 38, 39~~
6	126	Let A, B, C, and D each be replaced randomly by a different member of the set six, nine, twelve, and fifteen. What is the largest possible value of A times B minus C times D? 12 · 15 – 6 · 9 = 180 – 54 = 126
7	[B =] 28	As a reduced common fraction, the ratio A over B is three over seven, where A does not equal three and B does not equal seven. When four is added to A, the new ratio is four over seven as a reduced common fraction. What is B? (3x + 4)/7x = 4x/7x → 3x + 4 = 4x → x = 4, so 3x = 12 = A and 7x = 28 = B
8	95	What is the positive difference between the largest two-digit prime number and the smallest one-digit prime number? 97 – 2 = 95

9	37	What is the tenth term in the arithmetic sequence whose first term is ten and whose fourth term is nineteen? 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, or $10 + 3 \cdot 9 = 37$
10	54	What is the median of the following set of numbers: nineteen, one hundred and nine, nine, twenty-nine, seventy-nine, and eighty-nine? 9, 19, 29, 79, 89, 109, and the median is $(29 + 79)/2 = 54$