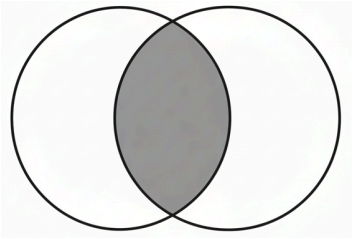
Combinatorics and Geometry Round

I2MC 2025

1. A Venn diagram is composed of two overlapping circles of radius 3 and covers a region of area 13π, as shown in the diagram below. Find the area of the shaded region.



2. Mingyue writes the integers from 1 to 100 in order with no spaces in between. How many times will "67" appear in this sequence?
3. Point D is drawn on $\overline{BC}$ of $\triangle ABC$, and point E is drawn on $\overline{AC}$. Given that $AE = ED = DC$ and $AD = DB = BA$, what is $\angle BDE$?
4. Ian has four books and a bookshelf. How many ways can he arrange at least one book onto the bookshelf? If there are multiple books on the bookshelf, their order matters.
5. Let ω be a circle with center O and let A and B be points on ω. Let ℓ be the tangent to ω at A. Line $\overline{BO}$ intersects ℓ at C. If minor arc AB on ω has measure $130°$, find $\angle ACB$.
6. Ryan, Aryan, Bryan, and Cryan are playing a 4-person game. On a player's turn, they roll a die. If the top face on the die is a 6, that player wins. Otherwise, they randomly give the die to another person. Given that Aryan plays first, what is the probability that either he or Cryan wins?
7. Let $ABCDEF$ be a regular hexagon. Suppose that the area of the circumcircle of $ABCDEF$ is x, and that the area of the incircle of triangle BDE is y. What is $\frac{x}{y}$?
8. Rivulet, a pedestrian, arrives at a traffic intersection. They would prefer to go forward or turn left, but in both directions there are red pedestrian signals. Each signal independently takes a uniformly random real number of seconds from 0 to 60 to turn white, and the moment one of the signals turns white, Rivulet will go in that direction. Rivulet also has the option to turn right, and will do so out of impatience after x seconds if neither signal has turned white. Find the value of x such that Rivulet has an equal chance of going in any of the three directions.

9. Aishwarya wants to place penguins in a 3×3 grid. As she wants no penguin to be lonely, she decides that every penguin must either be in the same row or same column as another penguin. Given that at most one penguin can be placed in each cell, how many ways can Aishwarya place penguins into the grid? She may place zero penguins.

10. A sphere is inscribed within a cube inscribed within a cone, such that one face of the cube lies on the base of the cone. A segment connecting the vertex of the cone to a vertex of the cube on the base of the cone is tangent to the sphere. Compute the ratio of the cone's height to the radius of its base.