Algebra and Number Theory Round

I2MC 2024

October 19th 2024

1. Four years ago, Alice's age was three times Bob's age. One year ago, Alice's age was two times Bob's age. How old is Bob currently?

2. What is the greatest integer less than 100 that can be written as the product of two composite integers?

3. Daniel and August were making snowballs together. When they split apart, Daniel finds that he takes 50% more time to make the same number of snowballs, and August finds that he takes x% more time to make the same number of snowballs. What is x?

4. Find the value of $\dfrac{2025!}{2023! + 2024!}$.

5. The 2024 Interlake yearbook was labeled "Volume 57". The two numbers 2024 and 57 do not share any factors greater than 1. Assuming that the "Volume" number increases by 1 each year, when is the next year in which the year and volume number share a common divisor greater than 30?

6. Find the sum of all integers n such that the degree measure of an interior angle of a regular n-gon is odd.

7. The fraction $\dfrac{503}{12}$ is not an integer in base 10, but it is an integer for the Flaming Martians, who interpret both 503 and 12 in base k. What is k?

8. The number of letters in Edward's favorite word is divisible by 41. Also, adding 16 to this number results in a perfect square. What is the smallest possible number of letters in Edward's favorite word?

9. Call a number of the form $\frac{1}{n}$ where n is a positive integer *delectable* if it terminates when written in decimal form. What is the sum of all *delectable* numbers?

10. When the polynomial $(x-1)(x-2)(x-3)\ldots(x-19)(x-20)$ is expanded, what is the coefficient of x^{18}?