

1. (5 points) Evaluate  $I^2MC$  if  $I = 16$ ,  $M = 25$ , and  $C = 15$ .
2. (5 points) What is the sum of the mean, median, and mode of 1, 5, 2, 7, 2, 9, 4, and 2?
3. (5 points) Let  $\triangle ABC$  be a right triangle with  $AB = 20$ ,  $AC = 29$ ,  $\angle B = 90^\circ$ . Compute the length of the altitude from  $B$  to  $AC$ . Express your answer as a common fraction.
4. (7 points) Let the sequence  $(F_n)$  be defined by  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_{i+2} = F_{i+1} + F_i$ . Let the sequence  $(T_n)$  be defined by  $T_0 = 0$  and  $T_i = T_{i-1} + i$ . Find the sum of the smallest three nonnegative integers  $k$  such that  $F_k = T_k$ .
5. (7 points) Find the sum of the reciprocals of  $A$  and  $B$  if their product is 7 and their sum is 28.
6. (7 points) Mr. Kessler is trying to wrap his presents for Christmas. There is a  $\frac{2}{3}$  chance that he will wrap them correctly. What is the probability that he wraps exactly 4 of his 5 presents correctly?
7. (5 points, Estimation) Find the number of primes  $p$  between 1 and  $20^{19}$ , inclusive, such that the remainder when  $p$  is divided by 4 is 1. Express your answer as a power of 10, rounded down. You will get full points for the correct answer, 3 points for being 1 off, 1 point for being 2 off.
8. (7 points, Reading) Martin bribes his best friend Bob into giving him a bag of 38 identical chewy chewy chewy candies. Bob decides to surprise Martin by adding two identical, highly venomous, Super Super Super Spicy Surprising Supreme Cing Cobras to the bag of chewy chewy chewy candies. If Martin randomly chooses 1, 2, or 3 objects from the bag (with each of these three options being equally likely), what is the probability he receives at least one Super Super Super Spicy Surprising Supreme Cing Cobra?
9. (9 points, Meta) What will be the number of correct answers across all teams on question 10, taken modulo 9 (that is, remainder the sum of the scores when divided by 9)?
10. (9 points) A Ferris wheel has a radius of 120 meters and rotates at a rate of 12 degrees per second. At the beginning, a seat is at the lowest point, 2 meters off the ground. How high above the ground, in meters, is the seat after 185 seconds?
11. (9 points) Let  $x$  and  $y$  be positive numbers such that  $\frac{2xy}{y^2+1} = 1$ . What is the value of  $y$  when  $x$  is minimized?
12. (9 points) A tree breaks one fourth of the distance up the trunk from the ground, and its top lands 60 meters from its base, forming a triangle. How tall was the tree originally, in meters? Express your answer in simplest radical form.
13. (12 points) If  $3^{x+1} = 5$ , what is  $9^{2x}$ ? Express your answer as a common fraction.
14. (12 points) Let the solutions to  $x^3 - 4x^2 - x + 4 = 0$  be  $r$ ,  $s$ , and  $t$ . Compute  $r^3 + s^3 + t^3$ .
15. (12 points) Define  $a@b = \sqrt{a^2 + b^2}$ . Evaluate  $(6@(7@3))$  in simplest radical form.

16. (9 points, Estimation) Estimate  $1^1 \cdot 2^2 \cdot 3^3 \cdots 24^{24}$ . Express your answer as a power of 10, rounded down. You will get full points for the correct answer, 6 points for being 1 off, 3 points for being 2 off.
17. (12 points, Meta) Compute the number on the second hand of the stopwatch for this round (an integer) when this question is graded. We will give you the points if you are within 2 seconds of the answer.
18. (15 points, Reading) Frankie the friendly farmer recently decided to live on a friend farm amongst his farm friends, the Frases and the Fars. A Fra has 3 eyes and 4 legs, and a Far has 1 eye and 7 legs. There are 199 eyes and 412 legs on Frankie's friend farm. Frankie randomly picks a farm friend on his friend farm to find more friends for his friend farm. Find the probability that Frankie picks a Fra. Express your answer as a common fraction.
19. (15 points) The sum of two four-digit numbers,  $\overline{a2bd}$  and  $\overline{cb7d}$  is another four-digit number,  $\overline{660c}$ . Find the sum of all possible four-digit numbers  $\overline{abcd}$ .
20. (15 points) How many positive two-digit numbers  $\overline{ab}$  exist such that  $\frac{a}{2} \geq b$ ?
21. (15 points) Harry the horse is taking a nap on a field of grass at the point  $(3, 0)$  on the coordinate system. Before it gets dark, Harry wants to return to his barn at point  $(4, 5)$ . However, Harry wants to take a sip of water from the river that flows through the entire  $y$ -axis. What is the length of the shortest path that Harry must take to get home?
22. (18 points) Let  $S = \{1, 2, 3, \dots, 24, 25\}$ . What is the number of elements in the largest subset of  $S$  such that no two elements in the subset differ by a square of an integer?
23. (18 points) A rectangular box has dimensions of 12 inches, 16 inches, and  $x$  inches. Three faces of the box meet at a corner of the box. The center points of those three faces are the vertices of a triangle with an area of 30 square inches. Find  $x$ .
24. (18 points) Find the last two nonzero digits of  $93!$ .
25. (15 points, Estimation) How many ways can David pick 26 kids from the 91 competitors at I2MC to hand a slice of free pizza? Express your answer as a power of 10. You will get full points for the correct answer, 10 points for being 1 off, 5 points for being 2 off.
26. (18 points, Meta) Answer with a positive real number. The second-most popular answer to this question is the correct answer.
27. (20 points, Reading) The ExtremelyCelibate™ Pope Francis wants to hire 26 priests to drive his PriestPriestPrius. He can assign them each ranks of Uncelibate, mildly celibate, "celibate", and ExtremelyCelibate™. An Uncelibate priest can have between 5-7 partners. A mildly celibate priest may have 2-4 partners. A "celibate" priest may have 1 or 0 partners. An ExtremelyCelibate™ priest has exactly 0 partners. There are 8 Uncelibate priests, 7 mildly celibate priests, 6 "celibate" priests, and 5 ExtremelyCelibate™ priests. If Pope Francis invites all (only) the 26 priests and their partners to celebrate his PriestPriestPrius, what is the minimum number of guests that show up (assuming all invitees show up). Include Pope Francis in this count.